GUJCET Mathematics

Sample Paper – 3

Duration: 60 Minutes **Maximum Marks: 40**

Instructions

- This paper contains **40** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics portion of **GUJCET** entrance.
- Each correct answer carries **+1 mark**. There is a **negative marking of** -0.25 **mark** for each incorrect answer. Unattempted questions carry no penalty.
- Only **one** option is correct. Choose carefully.
- Syllabus level: **Class 12 NCERT (80%) + Class 11 NCERT/GSEB (20%) — Calculus, Algebra, Vectors, 3D Geometry, Probability, Coordinate Geometry, and more.**
- Use of mobile phones, calculators, log tables, or any electronic gadgets is strictly prohibited. Rough work may be done in the space provided on the answer sheet.

Q1. Let $f : \mathbb{R} \to \mathbb{R}$ be defined by $f(x) = x^2 + 3$. Which of the following statements is **correct**?

(A) f is both one-one and onto

(B) f is one-one but not onto

(C) f is onto but not one-one

(D) f is neither one-one nor onto

Q2. Which of the following functions $f : \mathbb{R} \to \mathbb{R}$ is **one-one (injective)**?

(A) $f(x) = 3x - 7$

(B) $f(x) = |x| + 2$

(C) $f(x) = x^2 - 4$

(D) $f(x) = \sin x$

Q3. For $x > 0$, the value of $\tan^{-1} x + \tan^{-1}\left(\frac{1}{x}\right)$ equals:

(A) 0

(B) π

(C) $\frac{\pi}{2}$

(D) $\frac{\pi}{4}$

Q4. The principal value of $\sin^{-1}\left(\sin\frac{5\pi}{6}\right)$ is:

(A) $\frac{5\pi}{6}$

(B) $\frac{\pi}{6}$

(C) $-\frac{\pi}{6}$

(D) $\frac{\pi}{3}$

Q5. If A is a 3×3 matrix and I is the 3×3 identity matrix, then which of the following is **always** true?

(A) $AI = 2A$

(B) $IA = A^2$

(C) $AI \neq IA$

(D) $AI = IA = A$

Q6. Starting from $A = \begin{pmatrix} 4 & 8 \\ 2 & 6 \end{pmatrix}$, the row operation $R_1 \leftarrow R_1 - 2R_2$ is applied. The resulting matrix is:

(A) $\begin{pmatrix} 0 & -4 \\ 2 & 6 \end{pmatrix}$

(B) $\begin{pmatrix} 0 & 4 \\ 2 & 6 \end{pmatrix}$

(C) $\begin{pmatrix} 2 & 2 \\ 2 & 6 \end{pmatrix}$

(D) $\begin{pmatrix} 4 & 8 \\ 0 & -4 \end{pmatrix}$

Q7. Using Cramer's rule, solve for x in the system $3x + y = 7$ and $x - 2y = 1$:

(A) $x = 1$

(B) $x = \frac{15}{7}$

(C) $x = 2$

(D) $x = 3$

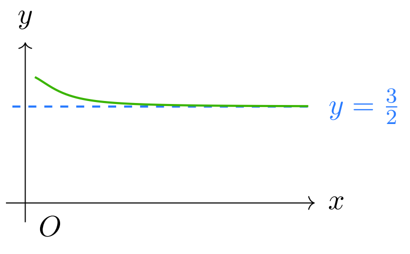
Q8. The matrix $A = \begin{pmatrix} 2k & 4 \\ 3 & 6 \end{pmatrix}$ is **singular**. The value of k is:

(A) $k = 4$

(B) $k = 2$

(C) $k = 1$

(D) $k = 3$



Q9.

The graph above shows a rational function. Evaluate $\lim_{x \to \infty} \frac{3x^2 + 1}{2x^2 + x - 4}$:

(A) 0

(B) 1

(C) ∞

(D) $\frac{3}{2}$

Q10. For the function $f(x) = \dfrac{|x-3|}{x-3}$, the value of $\lim_{x \to 3^-} f(x)$ is:

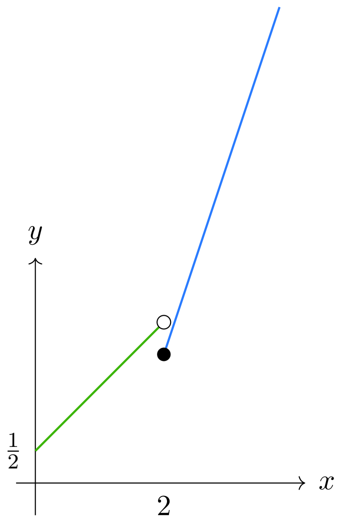
(A) -1

(B) 1

(C) 0

(D) The limit does not exist

Q11.



Consider the piecewise function $f(x) = \begin{cases} x + \frac{1}{2}, & x < 2 \\ 3x - 4, & x \geq 2 \end{cases}$. Then f is:

(A) Continuous at $x = 2$

(B) Discontinuous at $x = 2$

(C) Differentiable at $x = 2$

(D) Continuous everywhere on $\mathbb{R}$

Q12. If $y = e^{\sin^2 x}$, then $\dfrac{dy}{dx}$ equals:

(A) $e^{\sin^2 x} \cdot \cos x$

(B) $e^{\sin^2 x} \cdot 2\sin x$

(C) $e^{\sin^2 x} \cdot \sin 2x$

(D) $e^{\sin^2 x} \cdot 2\cos x$

Q13. If $y = 5^x$, then $\dfrac{dy}{dx}$ is:

(A) $x \cdot 5^{x-1}$

(B) 5^x

(C) $5^x \cdot \log_5 e$

(D) $5^x \ln 5$

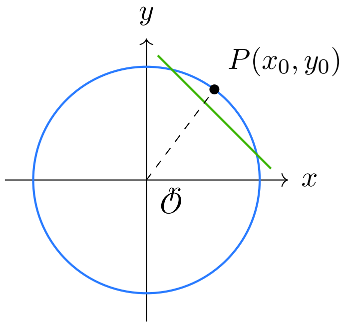
Q14. If $f(x) = 4x^4 - 3x^3 + 2x^2 - x + 6$, then $f'''(x)$ equals:

(A) $96x - 18$

(B) $16x^3 - 9x^2 + 4x$

(C) $48x^2 - 18x + 4$

(D) $96x^2 - 18x$



Q15.

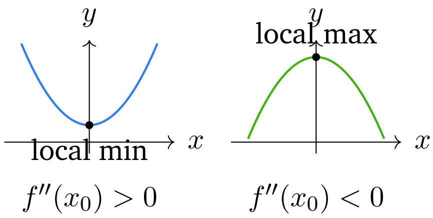
For the circle $x^2 + y^2 = r^2$, differentiating implicitly gives $\dfrac{dy}{dx}$ equal to:

(A) $\dfrac{y}{x}$

(B) $-\dfrac{x}{y}$

(C) $\dfrac{x}{y}$

(D) $-\dfrac{y}{x}$



Q16.

Using the second derivative test, a function $f(x)$ has a **local minimum** at $x = x_0$ if:

(A) $f'(x_0) > 0$ and $f''(x_0) > 0$

(B) $f'(x_0) = 0$ and $f''(x_0) < 0$

(C) $f'(x_0) = 0$ and $f''(x_0) > 0$

(D) $f'(x_0) < 0$ and $f''(x_0) = 0$

Q17. Using differentials, an approximate value of $\sqrt{36.4}$ is:

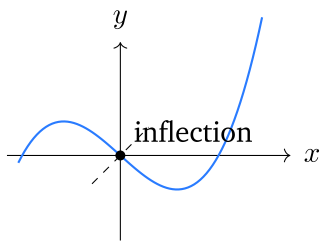
(A) 6.030

(B) 6.050

(C) 6.040

(D) 6.033

Q18.



For $f(x) = x^3 - 3x$, the **point of inflection** (where $f''(x) = 0$ and f'' changes sign) occurs at:

(A) $x = 0$

(B) $x = 1$

(C) $x = -1$

(D) $x = 3$

Q19. Evaluate $\int \tan x \, dx$:

(A) $\sec^2 x + C$

(B) $\ln|\sec x| + C$

(C) $\ln|\csc x| + C$

(D) $\ln|\sin x| + C$

Q20. Evaluate $\int x^2 e^x \, dx$:

(A) $x^2e^x - 2xe^x + C$

(B) $x^2e^x + 2xe^x - 2e^x + C$

(C) $e^x(x^2 - 2x + 2) + C$

(D) $e^x(x^2 + 2x + 2) + C$

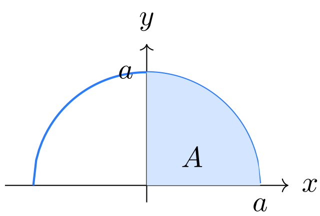
Q21. Evaluate $\int \sin^3 x \, dx$:

(A) $\cos x - \dfrac{\cos^3 x}{3} + C$

(B) $-\sin^3 x \cos x + C$

(C) $\dfrac{3\sin x - \sin 3x}{4} + C$

(D) $-\cos x + \dfrac{\cos^3 x}{3} + C$



Q22.

The formula $\int \sqrt{a^2 - x^2} \, dx$ equals:

(A) $\dfrac{x}{2}\sqrt{a^2 - x^2} + \dfrac{a^2}{2}\sin^{-1}\dfrac{x}{a} + C$

(B) $\dfrac{x}{2}\sqrt{a^2 - x^2} - \dfrac{a^2}{2}\sin^{-1}\dfrac{x}{a} + C$

(C) $\sqrt{a^2 - x^2} + \sin^{-1}\dfrac{x}{a} + C$

(D) $\dfrac{x^2}{2}\sqrt{a^2 - x^2} + C$

Q23. The value of $\int_0^{\pi/2} \ln(\tan x) \, dx$ is:

(A) 1

(B) 0

(C) $\dfrac{\pi}{2}$

(D) $-\dfrac{\pi}{2}$

Q24. Evaluate $\displaystyle\int_{-1}^{2} |2x-1|\,dx$:

(A) $\dfrac{7}{2}$

(B) 3

(C) $\dfrac{9}{2}$

(D) $\dfrac{13}{4}$

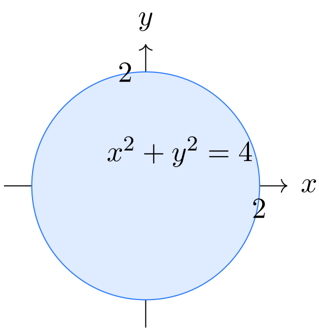
Q25. The definite integral $\displaystyle\int_{0}^{1} x^2\,dx$ gives the area under $y = x^2$ from $x = 0$ to $x = 1$. Its value is:

(A) $\dfrac{1}{2}$

(B) 1

(C) $\dfrac{3}{4}$

(D) $\dfrac{1}{3}$



Q26.

The area enclosed by the circle $x^2 + y^2 = 4$, computed using integration as $4\displaystyle\int_{0}^{2} \sqrt{4-x^2}\,dx$, equals:

(A) 4π

(B) 2π

(C) π

(D) 8π

Q27. The integrating factor of the linear differential equation $\frac{dy}{dx} + \frac{2}{x}y = x^3$ is:

(A) $e^{1/x}$

(B) x^2

(C) e^{2x}

(D) x^{-2}

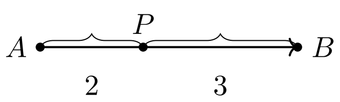
Q28. The differential equation formed by eliminating the arbitrary constant c from $y = ce^{3x}$ is:

(A) $\frac{dy}{dx} = 3y^2$

(B) $\frac{dy}{dx} = \frac{y}{3}$

(C) $\frac{dy}{dx} = 3y$

(D) $\frac{d^2y}{dx^2} = 9y$

Q29. 

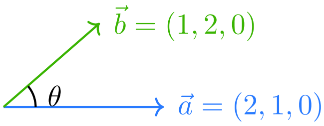
If $\vec{a}$ and $\vec{b}$ are position vectors of points A and B, the position vector of point P that divides AB in the ratio $2 : 3$ internally is:

(A) $\frac{2\vec{a} + 3\vec{b}}{5}$

(B) $\frac{2\vec{a} + 3\vec{b}}{2}$

(C) $\frac{\vec{a} + \vec{b}}{2}$

(D) $\frac{3\vec{a} + 2\vec{b}}{5}$

Q30. 

If $\vec{a} = 2\hat{i} + \hat{j}$ and $\vec{b} = \hat{i} + 2\hat{j}$, then $\cos\theta$ where θ is the angle between $\vec{a}$ and $\vec{b}$, equals:

(A) $\frac{4}{5}$

(B) $\frac{3}{5}$

(C) $\frac{2}{5}$

(D) $\frac{1}{5}$

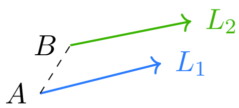
Q31. If $\vec{v} = 3\hat{i} - 4\hat{j} + 0\hat{k}$, then the unit vector $\hat{v}$ in the direction of $\vec{v}$ is:

(A) $\frac{3\hat{i} - 4\hat{j}}{7}$

(B) $\frac{3\hat{i} - 4\hat{j}}{5}$

(C) $3\hat{i} - 4\hat{j}$

(D) $\frac{3\hat{i} + 4\hat{j}}{5}$

Q32. 

Two lines with direction vectors $\vec{b_1}$ and $\vec{b_2}$ passing through A and B are **coplanar** if and only if:

(A) $(\overrightarrow{AB} \times \vec{b_1}) \cdot \vec{b_2} = 1$

(B) $\vec{b_1} \cdot \vec{b_2} = 0$

(C) $(\overrightarrow{AB} \times \vec{b_1}) \cdot \vec{b_2} = 0$

(D) $|\overrightarrow{AB}| = |\vec{b_1}|$

Q33. The foot of the perpendicular from the point $P(1, 0, 0)$ to the line $\frac{x-1}{2} = \frac{y}{3} = \frac{z}{1}$ (i.e. the parameter value t at foot) satisfies:

(A) $t = \frac{2}{14}$

(B) $t = \frac{3}{14}$

(C) $t = \frac{1}{7}$

(D) $t = 0$

Q34. The sine of the angle ϕ between the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{2}$ and the plane $2x - y + \sqrt{5}z = 4$ is:

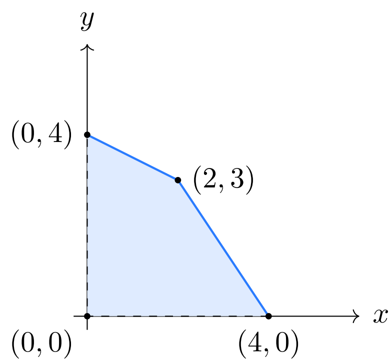
(A) $\frac{\sqrt{2}}{3}$

(B) $\frac{1}{\sqrt{2}}$

(C) $\frac{2}{3}$

(D) $\frac{\sqrt{2}}{6}$

Q35.



Maximise $Z = 5x + 4y$ subject to $3x + 2y \leq 12$, $x + 2y \leq 8$, $x, y \geq 0$. The maximum value of Z is:

(A) 20

(B) 22

(C) 18

(D) 24

Q36. If $P(A) = 0.6$, $P(B) = 0.5$, and $P(A \cup B) = 0.8$, then $P(A \cap B)$ equals:

(A) 0.2

(B) 0.4

(C) 0.3

(D) 0.5

Q37. In a binomial distribution with $n = 12$ and $p = \frac{1}{3}$, the variance is:

(A) 4

(B) 3

(C) $\frac{4}{3}$

(D) $\frac{8}{3}$

Q38. A random variable X takes values $1, 2, 3$ with probabilities $\frac{1}{2}, \frac{1}{3}, \frac{1}{6}$ respectively. Then $E(X)$ equals:

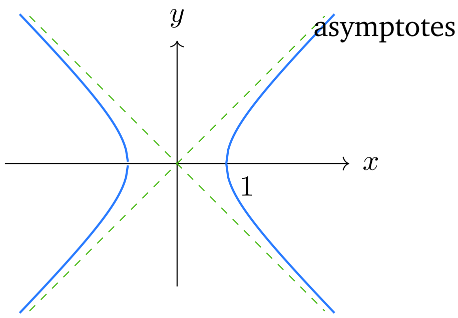
(A) $\frac{5}{3}$

(B) 2

(C) $\frac{7}{6}$

(D) $\frac{3}{2}$

Q39.



For the hyperbola $x^2 - y^2 = 1$ (i.e. $a = 1, b = 1$), the eccentricity e is:

(A) 1

(B) $\sqrt{2}$

(C) 2

(D) $\frac{\sqrt{3}}{2}$

Q40. If $\binom{n}{3} = 220$, then the value of n is:

(A) 10

(B) 11

(C) 12

(D) 13

Detailed Solutions

Q1.

Solution

Concept: A function $f : A \to B$ is *onto* (surjective) if every element of B has a pre-image in A; it is *one-one* (injective) if distinct inputs give distinct outputs.

Step 1 — Check one-one. $f(x) = x^2 + 3$. Take $x_1 = 1$ and $x_2 = -1$: $f(1) = 4 = f(-1)$. Since two distinct inputs produce the same output, f is **not one-one**.

Step 2 — Check onto. The range of $f(x) = x^2 + 3$ is $[3, \infty)$ because $x^2 \geq 0$ for all real x. The codomain is $\mathbb{R}$, which includes numbers like $1, 2, 2.9$ that are never achieved. Therefore f is **not onto**.

Why other options are wrong. (A) f cannot be both one-one and onto since neither property holds. (B) One-one fails as shown. (C) Onto fails since range $\subsetneq \mathbb{R}$.

Answer: (D) Go Back to Q 1

Q2.

Solution

Concept: A function is one-one if $f(x_1) = f(x_2) \Rightarrow x_1 = x_2$.

Step 1. Test $f(x) = 3x - 7$. Suppose $f(x_1) = f(x_2)$: $3x_1 - 7 = 3x_2 - 7 \Rightarrow 3x_1 = 3x_2 \Rightarrow x_1 = x_2$. **Injective.** ✓

Why other options are wrong. (B) $f(x) = |x| + 2$: $f(1) = f(-1) = 3$, not one-one. (C) $f(x) = x^2 - 4$: $f(2) = f(-2) = 0$, not one-one. (D) $f(x) = \sin x$: $\sin(0) = \sin(\pi) = 0$, not one-one on $\mathbb{R}$.

Answer: (A) Go Back to Q 2

Q3.

Solution

Concept: Standard inverse trig identity for $x > 0$:

$$\tan^{-1} x + \tan^{-1}\left(\tfrac{1}{x}\right) = \tfrac{\pi}{2}.$$

Step 1. Let $\alpha = \tan^{-1} x$, so $\tan \alpha = x$ and $\tan(\frac{\pi}{2} - \alpha) = \cot \alpha = \frac{1}{x}$. Thus $\tan^{-1}(\frac{1}{x}) = \frac{\pi}{2} - \alpha$.

Step 2. Adding: $\tan^{-1} x + \tan^{-1}(\frac{1}{x}) = \alpha + (\frac{\pi}{2} - \alpha) = \dfrac{\pi}{2}$.

Why other options are wrong. (A) 0: would require $\tan^{-1}(1/x) = -\tan^{-1}x$, which holds for $x < 0$, not $x > 0$. (B) π: that is the result for $x < 0$. (D) $\pi/4$: is just $\tan^{-1}(1)$, not the sum.

Answer: (C) Go Back to Q 3

Q4.

Solution

Concept: The principal value range of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. We use $\sin^{-1}(\sin\theta) = \pi - \theta$ when $\theta \in \left(\frac{\pi}{2}, \pi\right)$.

Step 1. $\frac{5\pi}{6} \in \left(\frac{\pi}{2}, \pi\right)$, so $\sin^{-1}\left(\sin\frac{5\pi}{6}\right) = \pi - \frac{5\pi}{6} = \frac{\pi}{6}$.

Verification: $\sin\left(\frac{5\pi}{6}\right) = \sin\left(\pi - \frac{\pi}{6}\right) = \sin\frac{\pi}{6} = \frac{1}{2}$, and $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$. ✓

Why other options are wrong. (A) $\frac{5\pi}{6}$ lies outside $[-\frac{\pi}{2}, \frac{\pi}{2}]$, so it is not a principal value. (C) $-\frac{\pi}{6}$ would give $\sin(-\frac{\pi}{6}) = -\frac{1}{2} \neq \sin\frac{5\pi}{6}$. (D) $\frac{\pi}{3}$ gives $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \neq \frac{1}{2}$.

Answer: (B) Go Back to Q 4

Q5.

Solution

Concept: The identity matrix I is the multiplicative identity for matrices: for any $n \times n$ matrix A, $AI = IA = A$.

Step 1. By definition of the identity matrix, $AI = A$ and $IA = A$ for all $n \times n$ matrices. This is analogous to $a \cdot 1 = 1 \cdot a = a$ for real numbers.

Why other options are wrong. (A) $AI = 2A$ is false; multiplying by I does not scale. (B) $IA = A^2$ is false in general. (C) $AI \neq IA$ is false; both equal A.

Answer: (D) Go Back to Q 5

Q6.

Solution

Concept: Elementary row operation $R_1 \leftarrow R_1 - 2R_2$ replaces every entry of row 1 with (entry of R_1) $-$ 2×(corresponding entry of R_2).

Step 1. $A = \begin{pmatrix} 4 & 8 \\ 2 & 6 \end{pmatrix}$. Row 2 is $(2, 6)$.

Step 2. New R_1: $(4 - 2 \cdot 2,\ 8 - 2 \cdot 6) = (4 - 4,\ 8 - 12) = (0, -4)$.

Result: $\begin{pmatrix} 0 & -4 \\ 2 & 6 \end{pmatrix}$.

Why other options are wrong. (B) $(0, 4)$: sign error in $8 - 12 = -4$, not $+4$. (C) $(2, 2)$: applies $R_1 \leftarrow R_1 - R_2$ (coefficient 1, not 2). (D) Applies the operation to R_2 instead of R_1.

Answer: (A) Go Back to Q 6

Q7.

Solution

Concept: Cramer's Rule: for $ax + by = e$, $cx + dy = f$, we have $x = \frac{D_x}{D}$, where $D = ad - bc$ and $D_x = ed - bf$.

Step 1. System: $3x + y = 7$, $x - 2y = 1$.

$$D = \begin{vmatrix} 3 & 1 \\ 1 & -2 \end{vmatrix} = 3(-2) - 1(1) = -7.$$

Step 2. Replace column 1 with constants:

$$D_x = \begin{vmatrix} 7 & 1 \\ 1 & -2 \end{vmatrix} = 7(-2) - 1(1) = -15.$$

Step 3. $x = \frac{D_x}{D} = \frac{-15}{-7} = \frac{15}{7}$.

Why other options are wrong. (A) $x = 1$: check $3(1) + y = 7 \Rightarrow y = 4$, but $1 - 2(4) = -7 \neq 1$. (C) $x = 2$: $3(2) + y = 7 \Rightarrow y = 1$; $2 - 2(1) = 0 \neq 1$. Fails. (D) $x = 3$: $9 + y = 7 \Rightarrow y = -2$; $3 - 2(-2) = 7 \neq 1$. Fails.

Answer: (B) Go Back to Q 7

Q8.

Solution

Concept: A matrix is singular if and only if its determinant equals zero.

Step 1. $A = \begin{pmatrix} 2k & 4 \\ 3 & 6 \end{pmatrix}$. $\det(A) = 2k \cdot 6 - 4 \cdot 3 = 12k - 12$.

Step 2. Set $\det(A) = 0$: $12k - 12 = 0 \Rightarrow k = 1$.

Verification: $\det = 12(1) - 12 = 0$. ✓

Why other options are wrong. (A) $k = 4$: $\det = 48 - 12 = 36 \neq 0$. (B) $k = 2$:

det $= 24 - 12 = 12 \neq 0$. (D) $k = 3$: det $= 36 - 12 = 24 \neq 0$.

Answer: (C) Go Back to Q 8

Q9.

Solution

Concept: For a rational function, divide numerator and denominator by the highest power of x in the denominator to evaluate the limit at infinity.

Step 1. $\lim_{x\to\infty} \dfrac{3x^2+1}{2x^2+x-4}$. Divide top and bottom by x^2:

$$\frac{3+\frac{1}{x^2}}{2+\frac{1}{x}-\frac{4}{x^2}}.$$

Step 2. As $x \to \infty$, every term $\frac{1}{x^k} \to 0$:

$$\frac{3+0}{2+0-0} = \frac{3}{2}.$$

This is the horizontal asymptote shown in the graph.

Why other options are wrong. (A) 0: occurs when degree of numerator $<$ degree of denominator. (B) 1: would require equal leading coefficients, but here they are 3 and 2. (C) ∞: would need degree of numerator $>$ degree of denominator.

Answer: (D) Go Back to Q 9

Q10.

Solution

Concept: $f(x) = \dfrac{|x-3|}{x-3}$. For $x < 3$, $|x-3| = -(x-3)$; for $x > 3$, $|x-3| = x-3$.

Step 1. Left-hand limit ($x \to 3^-$, so $x < 3$):

$$f(x) = \frac{-(x-3)}{x-3} = -1.$$

Therefore $\lim_{x\to 3^-} f(x) = -1$.

Step 2. Right-hand limit ($x \to 3^+$, so $x > 3$):

$$f(x) = \frac{x-3}{x-3} = 1. \Rightarrow \lim_{x\to 3^+} f(x) = 1.$$

Note: Left $\neq$ Right, so the two-sided limit does not exist, but the question asks only for the left-hand limit.

Why other options are wrong. (B) 1: that is the right-hand limit. (C) 0: f is never 0. (D) The two-sided limit does not exist, but the left-hand limit -1 does exist.

Answer: (A) Go Back to Q 10

Q11.

Solution

Concept: A function is continuous at $x = a$ if $\lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = f(a)$.

Step 1 — Left-hand limit. $\lim_{x \to 2^-} f(x) = \lim_{x \to 2^-} (x + \frac{1}{2}) = 2 + \frac{1}{2} = \frac{5}{2}$.

Step 2 — Value at $x = 2$. $f(2) = 3(2) - 4 = 6 - 4 = 2$.

Step 3 — Compare. $\dfrac{5}{2} \neq 2$, so f is **discontinuous** at $x = 2$ (jump discontinuity visible in the graph).

Why other options are wrong. (A) Continuous at $x = 2$: false, as the limits do not agree with the value. (C) Differentiable at $x = 2$: differentiability requires continuity, which fails here. (D) Continuous everywhere: fails at $x = 2$.

Answer: (B) Go Back to Q 11

Q12.

Solution

Concept: Chain rule: $\dfrac{d}{dx}[e^{g(x)}] = e^{g(x)} \cdot g'(x)$.

Step 1. Here $g(x) = \sin^2 x$. Differentiate g: $g'(x) = 2 \sin x \cdot \cos x = \sin 2x$ (using chain rule on $\sin^2 x$).

Step 2. Apply the outer derivative:

$$\frac{dy}{dx} = e^{\sin^2 x} \cdot \sin 2x.$$

Why other options are wrong. (A) $e^{\sin^2 x} \cos x$: derivative of $\sin^2 x$ is $2 \sin x \cos x$, not just $\cos x$. (B) $e^{\sin^2 x} \cdot 2 \sin x$: misses the $\cos x$ factor from the inner chain. (D) $e^{\sin^2 x} \cdot 2 \cos x$: misses the $\sin x$ factor.

Answer: (C) Go Back to Q 12

Q13.

Solution

Concept: For any constant base $a > 0$, $a \neq 1$:

$$\frac{d}{dx}(a^x) = a^x \ln a.$$

Step 1. With $y = 5^x$ and $a = 5$:

$$\frac{dy}{dx} = 5^x \ln 5.$$

Why other options are wrong. (A) $x \cdot 5^{x-1}$: this is the power rule, applicable only when the exponent is a constant, not when the variable is in the exponent. (B) 5^x: that would require $\ln 5 = 1$, i.e., $5 = e$, which is false. (C) $5^x \log_5 e$: $\log_5 e = \frac{1}{\ln 5} \neq \ln 5$ in general.

Answer: (D) Go Back to Q 13

Q14.

Solution

Concept: Differentiate term by term using the power rule $\frac{d}{dx}(x^n) = nx^{n-1}$, applied three times.

Step 1 — First derivative. $f'(x) = 16x^3 - 9x^2 + 4x - 1$.

Step 2 — Second derivative. $f''(x) = 48x^2 - 18x + 4$.

Step 3 — Third derivative. $f'''(x) = 96x - 18$.

Why other options are wrong. (B) $16x^3 - 9x^2 + 4x$: that is $f'(x)$ (missing the constant -1, actually the full first derivative). (C) $48x^2 - 18x + 4$: that is $f''(x)$, one differentiation short. (D) $96x^2 - 18x$: wrong powers; the third derivative of x^2 is 0, not x.

Answer: (A) Go Back to Q 14

Q15.

Solution

Concept: Implicit differentiation: differentiate both sides of $x^2 + y^2 = r^2$ with respect to x, treating y as a function of x.

Step 1. $\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(r^2)$.

$$2x + 2y\frac{dy}{dx} = 0.$$

Step 2. Solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx} = -\frac{x}{y}.$$

Geometric meaning: The slope of the tangent at (x_0, y_0) on the circle is $-x_0/y_0$, perpendicular to the radius whose slope is y_0/x_0.

Why other options are wrong. (A) y/x: that is the slope of the radius, not the tangent. (C) x/y: sign is wrong. (D) $-y/x$: confuses which variable is in the numerator.

Answer: (B) Go Back to Q 15

Q16.

Solution

Concept: Second Derivative Test: if $f'(x_0) = 0$, then

- $f''(x_0) > 0 \Rightarrow$ local minimum (concave up);
- $f''(x_0) < 0 \Rightarrow$ local maximum (concave down).

Step 1. For a local minimum, the function must have a stationary point: $f'(x_0) = 0$.

Step 2. The bowl-shaped (concave up) condition is $f''(x_0) > 0$.

Both conditions together confirm a local minimum.

Why other options are wrong. (A) $f'(x_0) > 0$: this means f is increasing at x_0, not a turning point. (B) $f'(x_0) = 0$ and $f''(x_0) < 0$: this is the condition for local *maximum*. (D) $f'(x_0) < 0$ and $f''(x_0) = 0$: the function is decreasing and the test is inconclusive.

Answer: (C) Go Back to Q 16

Q17.

Solution

Concept: Differentials: $\Delta y \approx f'(x)\,\Delta x$. For $f(x) = \sqrt{x}$, $f'(x) = \dfrac{1}{2\sqrt{x}}$.

Step 1. Choose $x = 36$ (perfect square near 36.4) and $\Delta x = 0.4$. $f(x) = \sqrt{36} = 6$.

Step 2. $f'(36) = \dfrac{1}{2\sqrt{36}} = \dfrac{1}{12}$.

Step 3. $\Delta y \approx \dfrac{1}{12} \times 0.4 = \dfrac{0.4}{12} = \dfrac{1}{30} \approx 0.0\overline{3}$.

Step 4. $\sqrt{36.4} \approx 6 + \dfrac{1}{30} = 6.0\overline{3} \approx 6.033$.

Why other options are wrong. (A) 6.030: uses $\Delta x = 0.36$ instead of 0.4. (B) 6.050: corresponds to $\Delta x = 0.6$. (C) 6.040: uses $f' = 1/10$, incorrect for $x = 36$.

$\boxed{\text{Answer: (D)}}$ Go Back to Q 17

Q18.

Solution

Concept: A *point of inflection* occurs where $f''(x) = 0$ **and** f'' changes sign (concavity changes).

Step 1. $f(x) = x^3 - 3x$. $f'(x) = 3x^2 - 3$, $\quad f''(x) = 6x$.

Step 2. Set $f''(x) = 0$: $6x = 0 \Rightarrow x = 0$.

Step 3. Check sign change: $f''(-1) = -6 < 0$ (concave down); $f''(1) = 6 > 0$ (concave up). Sign changes at $x = 0$. ✓

Why other options are wrong. (B) $x = 1$: $f''(1) = 6 \neq 0$; this is a critical point of f', not an inflection of f. (C) $x = -1$: $f''(-1) = -6 \neq 0$. (D) $x = 3$: not even a critical point of f''.

$\boxed{\text{Answer: (A)}}$ Go Back to Q 18

Q19.

Solution

Concept: $\displaystyle\int \tan x\,dx = \int \frac{\sin x}{\cos x}\,dx.$

Step 1. Let $u = \cos x$, $du = -\sin x\,dx$:

$$\int \frac{\sin x}{\cos x}\,dx = \int \frac{-du}{u} = -\ln|u| + C = -\ln|\cos x| + C.$$

Step 2. Since $-\ln|\cos x| = \ln\left(\frac{1}{|\cos x|}\right) = \ln|\sec x|$:

$$\int \tan x\,dx = \ln|\sec x| + C.$$

Why other options are wrong. (A) $\sec^2 x + C$: that is $\int \sec^2 x\,dx$, the derivative of $\tan x$. (C) $\ln|\csc x| + C$: that equals $\int \cot x\,dx$, not $\int \tan x\,dx$. (D) $\ln|\sin x| + C$: that equals $\int \cot x\,dx$.

Answer: (B) Go Back to Q 19

Q20.

Solution

Concept: Integration by Parts (IBP): $\int u\,dv = uv - \int v\,du$. Apply twice.

Step 1 — First IBP. Let $u = x^2$, $dv = e^x dx \Rightarrow du = 2x\,dx$, $v = e^x$.

$$\int x^2 e^x dx = x^2 e^x - 2\int xe^x dx.$$

Step 2 — Second IBP. For $\int xe^x dx$: let $u = x$, $dv = e^x dx \Rightarrow du = dx$, $v = e^x$.

$$\int xe^x dx = xe^x - e^x + C = e^x(x-1) + C.$$

Step 3 — Combine.

$$\int x^2 e^x dx = x^2 e^x - 2e^x(x-1) + C = e^x(x^2 - 2x + 2) + C.$$

Why other options are wrong. (A) $x^2e^x - 2xe^x + C$: missing the $+2e^x$ term from the second IBP. (B) $e^x(x^2+2x-2)+C$: sign errors in the IBP. (D) $e^x(x^2+2x+2)+C$: all signs in the bracket are wrong.

Answer: (C) Go Back to Q 20

Q21.

Solution

Concept: Use the identity $\sin^2 x = 1 - \cos^2 x$ to reduce the power.

Step 1.

$$\int \sin^3 x\,dx = \int \sin x \cdot \sin^2 x\,dx = \int \sin x(1 - \cos^2 x)\,dx.$$

Step 2. Let $u = \cos x$, $du = -\sin x\,dx$:

$$\int (1-u^2)(-du) = -u + \frac{u^3}{3} + C = -\cos x + \frac{\cos^3 x}{3} + C.$$

Why other options are wrong. (A) $\cos x - \frac{\cos^3 x}{3} + C$: wrong signs throughout. (B) $-\sin^3 x \cos x + C$: this would be the antiderivative of something different. (C) $\dfrac{3\sin x - \sin 3x}{4} + C$: this uses the triple angle formula and also gives the same result, but the expression doesn't simplify directly to any of the above; the given option (C) is an incorrect version of the triple-angle approach.

Answer: (D) Go Back to Q 21

Q22.

Solution

Concept: Standard formula (derived via trig substitution $x = a\sin\theta$):

$$\int \sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C.$$

Derivation sketch. Let $x = a\sin\theta$, $dx = a\cos\theta\,d\theta$, $\sqrt{a^2 - x^2} = a\cos\theta$:

$$\int a\cos\theta \cdot a\cos\theta\,d\theta = a^2 \int \cos^2\theta\,d\theta = \frac{a^2}{2}\int (1 + \cos 2\theta)\,d\theta = \frac{a^2}{2}\left(\theta + \frac{\sin 2\theta}{2}\right) + C.$$

Back-substituting gives the stated formula.

Why other options are wrong. (B) Has a minus sign before $\sin^{-1}$, which is wrong. (C) Missing the $x/2$ multiplier on the radical. (D) Incorrect structure; $\sin^{-1}$ term is essential.

Answer: (A) Go Back to Q 22

Q23.

Solution

Concept: Use the property $\displaystyle\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$.

Step 1. Let $I = \displaystyle\int_0^{\pi/2} \ln(\tan x)\,dx$. Apply the property with $a = \pi/2$:

$$I = \int_0^{\pi/2} \ln\left(\tan\left(\frac{\pi}{2} - x\right)\right) dx = \int_0^{\pi/2} \ln(\cot x)\,dx = \int_0^{\pi/2} \ln\left(\frac{1}{\tan x}\right) dx = -I.$$

Step 2. $I = -I \Rightarrow 2I = 0 \Rightarrow I = 0$.

Why other options are wrong. (A) 1: there is no reason for the integral to equal a non-zero constant. (C) $\pi/2$: this is the total length of the integration interval, not the value. (D) $-\pi/2$: would require $I < 0$ and a specific form, neither of which is justified here.

Answer: (B) Go Back to Q 23

Q24.

Solution

Concept: For $\int |g(x)|\,dx$, split the interval at the zero of $g(x)$.

Step 1. $2x - 1 = 0 \Rightarrow x = \frac{1}{2}$. For $x < \frac{1}{2}$: $|2x-1| = 1 - 2x$. For $x > \frac{1}{2}$: $|2x-1| = 2x - 1$.

Step 2.

$$\int_{-1}^{1/2} (1-2x)\,dx = \left[x - x^2\right]_{-1}^{1/2} = \left(\tfrac{1}{2} - \tfrac{1}{4}\right) - (-1-1) = \tfrac{1}{4} + 2 = \tfrac{9}{4}.$$

Step 3.

$$\int_{1/2}^{2} (2x-1)\,dx = \left[x^2 - x\right]_{1/2}^{2} = (4-2) - \left(\tfrac{1}{4} - \tfrac{1}{2}\right) = 2 + \tfrac{1}{4} = \tfrac{9}{4}.$$

Step 4. Total: $\dfrac{9}{4} + \dfrac{9}{4} = \dfrac{9}{2}$.

Why other options are wrong. (A) $7/2$: incorrect split or arithmetic. (B) 3: misses the negative region contribution. (D) $13/4$: error in the evaluation of either piece.

Answer: (C) Go Back to Q 24

Q25.

Solution

Concept: Apply the power rule $\displaystyle\int_a^b x^n\,dx = \left[\frac{x^{n+1}}{n+1}\right]_a^b$.

Step 1.

$$\int_0^1 x^2\,dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1^3}{3} - \frac{0^3}{3} = \frac{1}{3}.$$

Geometric interpretation: This equals the area of the region bounded by $y = x^2$,

$x = 0$, $x = 1$, and $y = 0$.

Why other options are wrong. (A) $\frac{1}{2}$: that equals $\int_0^1 x\,dx$. (B) 1: that equals $\int_0^1 1\,dx$. (C) $\frac{3}{4}$: no standard integral of this simple form gives $3/4$ on $[0, 1]$.

Answer: (D) Go Back to Q 25

Q26.

Solution

Concept: Area of circle $x^2 + y^2 = r^2$ is πr^2. Here $r = 2$.

Step 1. $y = \sqrt{4 - x^2}$ (upper semicircle). By symmetry:

$$\text{Area} = 4\int_0^2 \sqrt{4 - x^2}\,dx.$$

Step 2. Use the formula with $a = 2$:

$$4\int_0^2 \sqrt{4 - x^2}\,dx = 4 \cdot \left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2.$$

At $x = 2$: $\frac{2}{2} \cdot 0 + 2 \cdot \frac{\pi}{2} = \pi$. At $x = 0$: 0. Area $= 4 \cdot \pi = 4\pi$.

Alternatively: Area $= \pi r^2 = \pi(2)^2 = 4\pi$. ✓

Why other options are wrong. (B) 2π: area of a semicircle of radius 2. (C) π: area of the unit circle. (D) 8π: area of a circle with $r^2 = 8$, i.e., $r = 2\sqrt{2}$.

Answer: (A) Go Back to Q 26

Q27.

Solution

Concept: For a linear ODE $\frac{dy}{dx} + P(x)y = Q(x)$, the integrating factor (I.F.) is $e^{\int P(x)\,dx}$.

Step 1. The equation $\frac{dy}{dx} + \frac{2}{x}y = x^3$ has $P(x) = \frac{2}{x}$.

Step 2.

$$\text{I.F.} = e^{\int \frac{2}{x}\,dx} = e^{2\ln|x|} = e^{\ln x^2} = x^2.$$

Why other options are wrong. (A) $e^{1/x}$: would come from $P(x) = -1/x^2$. (C) e^{2x}: would come from $P(x) = 2$ (constant). (D) x^{-2}: would come from $P(x) = -2/x$.

Answer: (B) Go Back to Q 27

Q28.

Solution

Concept: To eliminate one arbitrary constant, differentiate once and substitute.

Step 1. $y = ce^{3x}$. Differentiate: $\frac{dy}{dx} = 3ce^{3x}$.

Step 2. From the original: $c = ye^{-3x}$. Substitute:

$$\frac{dy}{dx} = 3 \cdot ye^{-3x} \cdot e^{3x} = 3y.$$

The differential equation is $\frac{dy}{dx} = 3y$.

Why other options are wrong. (A) $dy/dx = 3y^2$: would require a Bernoulli equation structure, not a simple exponential. (B) $dy/dx = y/3$: gives $y = Ce^{x/3}$, a different exponential. (D) $d^2y/dx^2 = 9y$: this requires eliminating *two* constants; here we have only one.

Answer: (C) Go Back to Q 28

Q29.

Solution

Concept: Section formula (internal division): if P divides $\overrightarrow{AB}$ in ratio $m : n$, then

$$\vec{OP} = \frac{n\,\vec{a} + m\,\vec{b}}{m+n}.$$

Step 1. Here $m = 2$, $n = 3$:

$$\vec{OP} = \frac{3\,\vec{a} + 2\,\vec{b}}{2+3} = \frac{3\vec{a} + 2\vec{b}}{5}.$$

Why other options are wrong. (A) $\frac{2\vec{a} + 3\vec{b}}{5}$: this gives the point dividing in ratio $3 : 2$, not $2 : 3$. (B) $\frac{2\vec{a} + 3\vec{b}}{2}$: wrong denominator. (C) $\frac{\vec{a} + \vec{b}}{2}$: this is the midpoint formula (ratio $1 : 1$).

Answer: (D) Go Back to Q 29

Q30.

Solution

Concept: $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$.

Step 1. $\vec{a} = 2\hat{\imath} + \hat{\jmath}$, $\vec{b} = \hat{\imath} + 2\hat{\jmath}$. $\vec{a}\cdot\vec{b} = 2(1) + 1(2) = 4$.

Step 2. $|\vec{a}| = \sqrt{4+1} = \sqrt{5}$, $|\vec{b}| = \sqrt{1+4} = \sqrt{5}$.

Step 3. $\cos\theta = \dfrac{4}{\sqrt{5}\cdot\sqrt{5}} = \dfrac{4}{5}$.

Why other options are wrong. (B) $\frac{3}{5}$: would require $\vec{a}\cdot\vec{b} = 3$. (C) $\frac{2}{5}$: would require $\vec{a}\cdot\vec{b} = 2$. (D) $\frac{1}{5}$: would require $\vec{a}\cdot\vec{b} = 1$.

Answer: (A) **Go Back to Q 30**

Q31.

Solution

Concept: Unit vector $\hat{v} = \dfrac{\vec{v}}{|\vec{v}|}$.

Step 1. $\vec{v} = 3\hat{\imath} - 4\hat{\jmath}$. $|\vec{v}| = \sqrt{3^2 + (-4)^2} = \sqrt{9+16} = \sqrt{25} = 5$.

Step 2. $\hat{v} = \dfrac{3\hat{\imath} - 4\hat{\jmath}}{5}$.

Why other options are wrong. (A) $\dfrac{3\hat{\imath} - 4\hat{\jmath}}{7}$: $|\vec{v}| = 7$ only if $3^2 + 4^2 = 49$, which is false. (C) $3\hat{\imath} - 4\hat{\jmath}$: this is $\vec{v}$ itself, not the unit vector. (D) $\dfrac{3\hat{\imath} + 4\hat{\jmath}}{5}$: wrong sign on $\hat{\jmath}$.

Answer: (B) **Go Back to Q 31**

Q32.

Solution

Concept: Two lines through A (direction $\vec{b_1}$) and B (direction $\vec{b_2}$) are coplanar if and only if the scalar triple product $(\overrightarrow{AB} \times \vec{b_1}) \cdot \vec{b_2} = 0$.

Explanation: Coplanarity means the vector $\overrightarrow{AB}$ connecting one line to the other lies in the plane determined by $\vec{b_1}$ and $\vec{b_2}$. The scalar triple product equals zero iff $\overrightarrow{AB}$, $\vec{b_1}$, $\vec{b_2}$ are coplanar vectors.

Why other options are wrong. (A) $(\overrightarrow{AB} \times \vec{b_1}) \cdot \vec{b_2} = 1$: the value must be zero, not one. (B) $\vec{b_1} \cdot \vec{b_2} = 0$: this is the condition for perpendicular directions, not coplanarity. (D) $|\overrightarrow{AB}| = |\vec{b_1}|$: equal magnitudes have no bearing on coplanarity.

$\boxed{\text{Answer: (C)}}$ Go Back to Q 32

Q33.

Solution

Concept: To find the foot of perpendicular from P to a line, parametrize the line, form the vector from P to the general point, and set its dot product with the direction vector to zero.

Step 1. Line: $\dfrac{x-1}{2} = \dfrac{y}{3} = \dfrac{z}{1} = t$. General point on line: $Q = (1+2t,\, 3t,\, t)$. Direction vector: $\vec{d} = (2,3,1)$.

Step 2. $\overrightarrow{PQ} = Q - P = (1+2t-1,\, 3t-0,\, t-0) = (2t, 3t, t)$.

Step 3. Set $\overrightarrow{PQ}\cdot\vec{d} = 0$: $(2t)(2) + (3t)(3) + (t)(1) = 4t + 9t + t = 14t = 0 \Rightarrow t = 0$.

Verification: At $t = 0$, $Q = (1,0,0) = P$, meaning P already lies on the line, so the foot is P itself.

Why other options are wrong. (A) $t = 2/14 = 1/7$: incorrect, leads to $\overrightarrow{PQ}\cdot\vec{d} = 14/7 = 2 \neq 0$. (B) $t = 3/14$: gives $14(3/14) = 3 \neq 0$. (C) $t = 1/7$: same as (A).

$\boxed{\text{Answer: (D)}}$ Go Back to Q 33

Q34.

Solution

Concept: Angle ϕ between a line with direction $\vec{d}$ and a plane with normal $\vec{n}$: $\sin\phi = \dfrac{|\vec{d}\cdot\vec{n}|}{|\vec{d}||\vec{n}|}$.

Step 1. Line direction: $\vec{d} = (1,2,2)$; plane $2x - y + \sqrt{5}z = 4$, normal $\vec{n} = (2,-1,\sqrt{5})$.

Step 2. $\vec{d}\cdot\vec{n} = 1(2) + 2(-1) + 2(\sqrt{5}) = 2 - 2 + 2\sqrt{5} = 2\sqrt{5}$.

Step 3. $|\vec{d}| = \sqrt{1+4+4} = 3$, $|\vec{n}| = \sqrt{4+1+5} = \sqrt{10}$.

Step 4. $\sin\phi = \dfrac{2\sqrt{5}}{3\sqrt{10}} = \dfrac{2\sqrt{5}}{3\sqrt{10}} = \dfrac{2}{3\sqrt{2}} = \dfrac{2}{3\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{2\sqrt{2}}{6} = \dfrac{\sqrt{2}}{3}$.

Why other options are wrong. (B) $1/\sqrt{2}$: requires $|\vec{d}\cdot\vec{n}|/(|\vec{d}||\vec{n}|) = 1/\sqrt{2}$. (C) $2/3$: incorrect cancellation. (D) $\sqrt{2}/6$: missing a factor of 2.

$\boxed{\text{Answer: (A)}}$ Go Back to Q 34

Q35.

Solution

Concept: In Linear Programming, the maximum of a linear objective over a convex feasible polygon is attained at a corner (vertex) point.

Step 1 — Find vertices. Constraints: $3x + 2y \leq 12$, $x + 2y \leq 8$, $x, y \geq 0$.

Vertices: $O = (0, 0)$, $A = (4, 0)$ (from $3x = 12, y = 0$), $B = (2, 3)$ (solve $3x+2y = 12$ and $x + 2y = 8$: subtract to get $2x = 4, x = 2, y = 3$), $C = (0, 4)$ (from $x = 0$, $2y = 8$, $y = 4$).

Step 2 — Evaluate $Z = 5x + 4y$.

- $O = (0, 0)$: $Z = 0$
- $A = (4, 0)$: $Z = 20$
- $B = (2, 3)$: $Z = 10 + 12 = 22$
- $C = (0, 4)$: $Z = 16$

Maximum $Z = 22$ at $(2, 3)$.

Why other options are wrong. (A) 20: value at $(4, 0)$, not the global maximum. (C) 18: no corner gives this value. (D) 24: exceeds the maximum achievable under the constraints.

Answer: (B) Go Back to Q 35

Q36.

Solution

Concept: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Step 1. $0.8 = 0.6 + 0.5 - P(A \cap B)$.

Step 2. $P(A \cap B) = 0.6 + 0.5 - 0.8 = 0.3$.

Why other options are wrong. (A) 0.2: uses $P(A) - P(B) + P(A \cup B) = 0.6 - 0.5 + 0.8$, incorrect rearrangement. (B) 0.4: off by 0.1. (D) 0.5: equals $P(B)$, unrelated to this formula.

Answer: (C) Go Back to Q 36

Q37.

Solution

Concept: For a Binomial distribution $B(n, p)$, variance $= np(1-p) = npq$.

Step 1. $n = 12$, $p = \frac{1}{3}$, $q = 1 - \frac{1}{3} = \frac{2}{3}$.

Step 2. $\text{Var}(X) = npq = 12 \times \frac{1}{3} \times \frac{2}{3} = 4 \times \frac{2}{3} = \frac{8}{3}$.

Why other options are wrong. (A) 4: that is $np = 12 \times \frac{1}{3}$, the mean, not the variance. (B) 3: incorrect; $npq = 8/3 \neq 3$. (C) $\frac{4}{3}$: off by a factor of 2; possibly from $npq/2$.

$\boxed{\text{Answer: (D)}}$ Go Back to Q 37

Q38.

Solution

Concept: $E(X) = \sum_i x_i\, P(X = x_i)$.

Step 1. $E(X) = 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{3} + 3 \cdot \frac{1}{6}$.

Step 2. $= \frac{1}{2} + \frac{2}{3} + \frac{3}{6} = \frac{3}{6} + \frac{4}{6} + \frac{3}{6} = \frac{10}{6} = \frac{5}{3}$.

Verification of probabilities: $\frac{1}{2} + \frac{1}{3} + \frac{1}{6} = \frac{3+2+1}{6} = 1$. ✓

Why other options are wrong. (B) 2: equals the arithmetic mean of $\{1, 2, 3\}$, ignoring the unequal weights. (C) $7/6$: incorrectly computed using $1 \cdot \frac{1}{6} + 2 \cdot \frac{1}{3} + 3 \cdot \frac{1}{2}$ (probabilities reversed). (D) $3/2$: off by $\frac{1}{6}$.

$\boxed{\text{Answer: (A)}}$ Go Back to Q 38

Q39.

Solution

Concept: For a hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the eccentricity is $e = \sqrt{1 + \frac{b^2}{a^2}}$.

Step 1. The equation $x^2 - y^2 = 1$ can be written $\frac{x^2}{1} - \frac{y^2}{1} = 1$, so $a = 1$, $b = 1$.

Step 2. $e = \sqrt{1 + \frac{1}{1}} = \sqrt{2}$.

Asymptotes: $y = \pm\frac{b}{a}x = \pm x$ (as shown in the diagram).

Why other options are wrong. (A) $e = 1$: that would be a parabola. (C) $e = 2$: requires $b^2/a^2 = 3$, i.e. $b^2 = 3a^2$. (D) $\sqrt{3}/2 < 1$: eccentricity of a hyperbola must be > 1.

$\boxed{\text{Answer: (B)}}$ Go Back to Q 39

Q40.

Solution

Concept: $\binom{n}{3} = \dfrac{n(n-1)(n-2)}{6}$.

Step 1. Set $\dfrac{n(n-1)(n-2)}{6} = 220$, so $n(n-1)(n-2) = 1320$.

Step 2. Try $n = 12$: $12 \times 11 \times 10 = 1320$. ✓

Why other options are wrong. (A) $n = 10$: $10 \times 9 \times 8 = 720 \neq 1320$. (B) $n = 11$: $11 \times 10 \times 9 = 990 \neq 1320$. (D) $n = 13$: $13 \times 12 \times 11 = 1716 \neq 1320$.

$\boxed{\text{Answer: (C)}}$ Go Back to Q 40

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	D	2	A	3	C	4	B	5	D
6	A	7	B	8	C	9	D	10	A
11	B	12	C	13	D	14	A	15	B
16	C	17	D	18	A	19	B	20	C
21	D	22	A	23	B	24	C	25	D
26	A	27	B	28	C	29	D	30	A
31	B	32	C	33	D	34	A	35	B
36	C	37	D	38	A	39	B	40	C