MAH MCA CET Mathematics & Statistics

Sample Paper – 10

Duration: 27 Minutes **Maximum Marks: 60**

Instructions

- This paper contains **30** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics & Statistics portion of **MAH MCA CET** entrance.
- Each correct answer carries **+2 marks**. There is **no negative marking** for incorrect or unattempted answers.
- Only **one** option is correct. Choose carefully.
- Syllabus level: **Class 12 level — Algebra, Calculus, Coordinate Geometry, Matrices, Statistics & Probability, Trigonometry.**
- Use of mobile phones, calculators, or electronic gadgets is strictly prohibited.

Q1. Find the sum of the series $1 + 3 + 5 + \cdots + (2n - 1)$ for $n = 12$:

(A) 132

(B) 144

(C) 156

(D) 128

Q2. Solve for x: $2^x = 64$.

(A) 6

(B) 8

(C) 32

(D) 4

Q3. Find the term independent of x (constant term) in the expansion of $\left(x+\dfrac{1}{x}\right)^6$:

(A) 15

(B) 6

(C) 20

(D) 35

Q4. Solve the inequality $x^2 - 5x + 6 > 0$:

(A) $2 < x < 3$

(B) $x > 3$ only

(C) $x < 2$ only

(D) $x < 2$ or $x > 3$

Q5. From 6 men and 4 women, a committee of 5 is formed with **at least 2 women**. The number of ways to form such a committee is:

(A) 162

(B) 186

(C) 246

(D) 126

Q6. Which of the following correctly states Pascal's rule relating binomial coefficients?

(A) $\binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r}$

(B) $\binom{n}{r} = \binom{n}{n-r-1}$

(C) $\binom{n}{r} + \binom{n}{r+1} = \binom{n-1}{r}$

(D) $\binom{n}{0} = n$

Q7. Using the chain rule, find $\frac{d}{dx}\left[\sqrt{x^2+1}\right]$:

(A) $\frac{1}{\sqrt{x^2+1}}$

(B) $\frac{-x}{\sqrt{x^2+1}}$

(C) $\frac{x}{\sqrt{x^2+1}}$

(D) $\sqrt{x^2+1}$

Q8. The n-th derivative of $\sin(ax+b)$ with respect to x is:

(A) $a\cos(ax+b)$

(B) $a^n\cos(ax+b)$

(C) $\sin(ax+b+n\pi)$

(D) $a^n\sin\left(ax+b+\frac{n\pi}{2}\right)$

Q9. Evaluate $\lim_{x\to 0}\frac{\tan 3x}{\sin 2x}$:

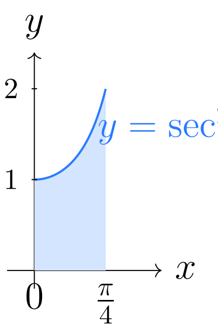
(A) 3

(B) $\frac{3}{2}$

(C) $\frac{2}{3}$

(D) 2

Q10. The shaded region below represents the area under $y=\sec^2 x$ from 0 to $\frac{\pi}{4}$.



Find the value of $\int_0^{\pi/4} \sec^2 x\, dx$:

(A) 1

(B) $\sqrt{2}-1$

(C) 0

(D) 2

Q11. What is the **degree** of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 = 5$?

(A) 1

(B) 2

(C) 3

(D) 6

Q12. Find the equation of the tangent to the curve $y = x^2$ at the point $(2, 4)$:

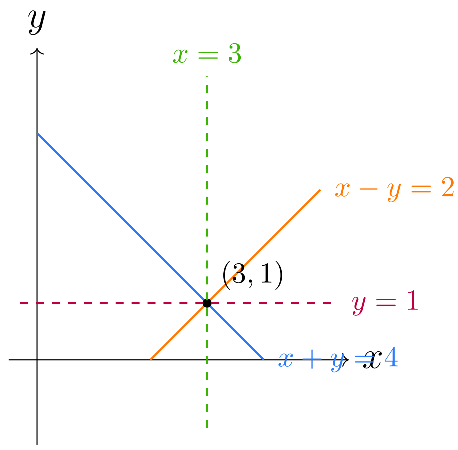
(A) $y = 2x$

(B) $y = 4x$

(C) $y = 2x - 4$

(D) $y = 4x - 4$

Q13. The diagram shows the lines $x + y = 4$ and $x - y = 2$ intersecting at the point $(3, 1)$.



Find the equations of the angle bisectors of the lines $x + y = 4$ and $x - y = 2$:

(A) $x = 1$ and $y = 3$

(B) $x = 3$ and $y = 1$

(C) $x + y = 4$

(D) $x - y = 2$

Q14. Find the value of k such that the points $A(k, 1)$, $B(4, 3)$ and $C(8, 5)$ are collinear:

(A) 0

(B) 1

(C) 2

(D) 4

Q15. The line $y = 2x + k$ is tangent to the parabola $y^2 = 8x$ when the value of k is:

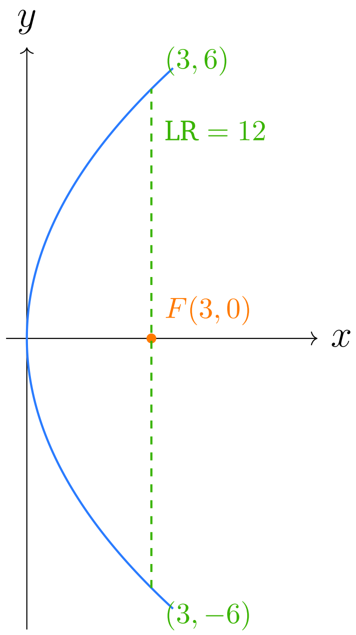
(A) -1

(B) 0

(C) 1

(D) 2

Q16. The parabola $y^2 = 12x$ is shown below with its focus F and latus rectum marked.



Find the length of the latus rectum of the parabola $y^2 = 12x$:

(A) 3

(B) 6

(C) 9

(D) 12

Q17. If $z = 3 + 4i$, then the modulus $|z|$ equals:

(A) 7

(B) 5

(C) 4

(D) 3

Q18. Find the value of k such that the matrix shown below is **singular** (i.e. determinant $= 0$):

$$\begin{bmatrix} k & 3 \\ 2 & 6 \end{bmatrix}, \quad \det = 0$$

(A) 1

(B) 2

(C) 3

(D) 6

Q19. Which matrix represents a **clockwise** rotation by $90°$?

(A) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

(B) $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

(C) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Q20. For the matrix $A = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$, the Cayley–Hamilton theorem states that A satisfies its own characteristic equation. The characteristic polynomial of A is $\lambda^2 - 5\lambda + 6 = 0$. What is $A^2 - 5A + 6I$?

(A) I

(B) A

(C) $5A$

(D) The zero matrix

Q21. The system $2x + y = 3$ and $4x + 2y = k$ is consistent if and only if k equals:

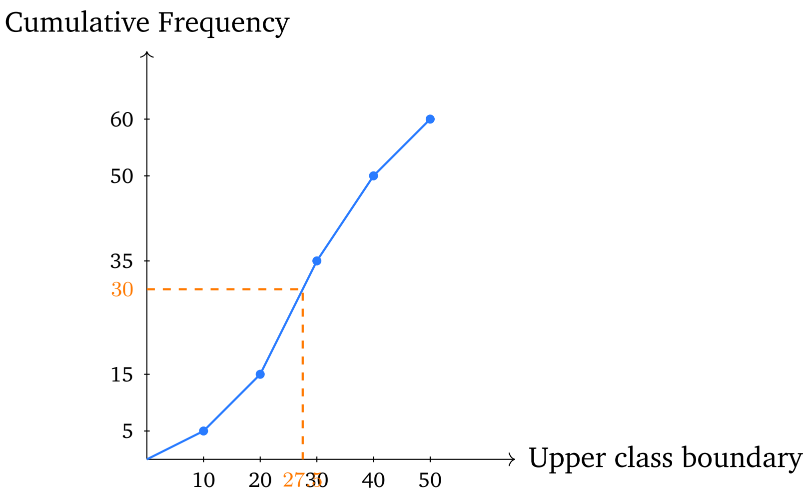
(A) 3

(B) 6

(C) 12

(D) Any value

Q22. The ogive (cumulative frequency curve) below is drawn from data with $N = 60$:



Using the ogive, find the **median** of the distribution:

(A) 27.5

(B) 25

(C) 30

(D) 28

Q23. If $P(A) = 0.4$ and $P(B) = 0.3$, and A and B are **mutually exclusive** events, find $P(A \cup B)$:

(A) 0.12

(B) 0.5

(C) 0.7

(D) 1.0

Q24. A discrete random variable X takes values $1, 2, 3, 4$ with probabilities $0.1,\ 0.3,\ 0.4,\ 0.2$ respectively. Find $E(X)$:

(A) 2.0

(B) 2.5

(C) 3.0

(D) 2.7

Q25. Given that $r^2 = 0.64$ and the regression coefficient $b_{yx} = 0.8$, find the regression coefficient b_{xy}:

(A) 0.64

(B) 0.8

(C) 1.0

(D) 0.512

Q26. In a **positively skewed** distribution, the relationship between mean, median, and mode is:

(A) Mean $>$ Median $>$ Mode

(B) Mode $>$ Median $>$ Mean

(C) Mean $=$ Median $=$ Mode

(D) Median $>$ Mean $>$ Mode

Q27. Using $\sin(A-B) = \sin A \cos B - \cos A \sin B$, find the exact value of $\sin 15$:

(A) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$

(B) $\dfrac{\sqrt{3}-1}{2}$

(C) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

(D) $\dfrac{\sqrt{3}}{2}$

Q28. Using the identity $\tan^2\theta + 1 = \sec^2\theta$, find $\sec 45$:

(A) 1

(B) 2

(C) $\dfrac{1}{\sqrt{2}}$

(D) $\sqrt{2}$

Q29. Find the principal value of $\sin^{-1}\left(\sin \dfrac{7\pi}{6}\right)$:

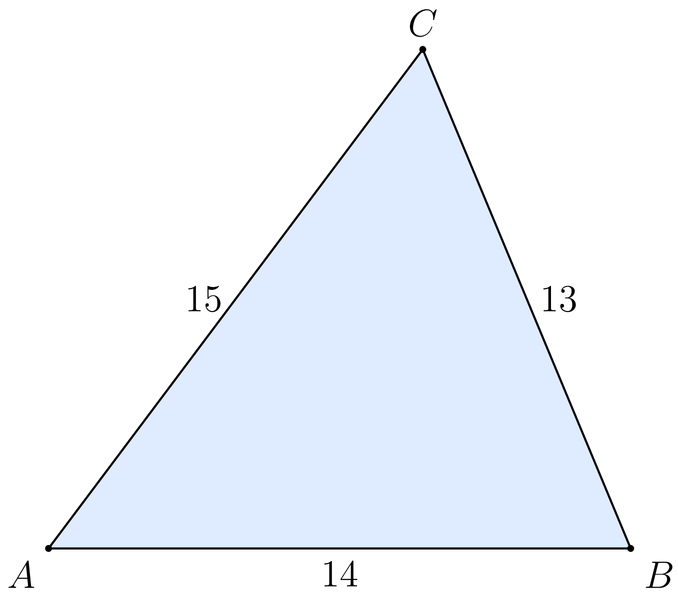
(A) $\frac{7\pi}{6}$

(B) $-\frac{\pi}{6}$

(C) $\frac{5\pi}{6}$

(D) $\frac{\pi}{6}$

Q30. The triangle below has sides of length 13, 14, and 15. Find its area using Heron's formula.



$$s = \frac{13+14+15}{2} = 21$$

Find the area of the triangle:

(A) 84

(B) 42

(C) 90

(D) 70

Detailed Solutions

Q1.

Solution

Concept — Sum of First n Odd Numbers (AP Sum Formula): The series $1+3+5+\cdots+(2n-1)$ is an arithmetic progression with first term $a=1$ and common difference $d=2$. A fundamental result states that the sum of the first n odd natural numbers equals n^2.

Step 1 — Identify n: The last term is $2n-1$. For $n=12$, the last term is $2(12)-1=23$. The series is $1+3+5+\cdots+23$.

Step 2 — Apply the formula:

$$S=n^2=12^2=144.$$

Verification using AP formula: $S_n=\frac{n}{2}[2a+(n-1)d]=\frac{12}{2}[2(1)+11(2)]=6\times24=144$. ✓

Why other options are wrong:

- Option A (132): Corresponds to $n=11$ since $11^2=121$; arithmetic slip.
- Option C (156): Result of $12\times13=156$; confuses with triangular number formula.
- Option D (128): Not a perfect square; no valid n gives this sum.

Final Answer: Sum $=144\Rightarrow\boxed{\text{B}}$

Answer: (B) Go Back to Q 1

Q2.

Solution

Concept — Exponential Equation: To solve $b^x=N$, express N as a power of b and compare exponents. If $b^x=b^k$, then $x=k$.

Step 1 — Express 64 as a power of 2:

$$64=2^6.$$

Step 2 — Equate exponents:

$$2^x = 2^6 \implies x = 6.$$

Why other options are wrong:

- Option B (8): $2^8 = 256 \neq 64$.
- Option C (32): 32 is 2^5; confuses the base with the exponent.
- Option D (4): $2^4 = 16 \neq 64$.

Final Answer: $x = 6 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 2

Q3.

Solution

Concept — Binomial Theorem (Term Independent of x): The general term in the expansion of $(x + 1/x)^6$ is

$$T_{r+1} = \binom{6}{r} x^{6-r} \cdot \left(\frac{1}{x}\right)^r = \binom{6}{r} x^{6-2r}.$$

For the term independent of x, set the exponent of x to zero.

Step 1 — Solve for r:

$$6 - 2r = 0 \implies r = 3.$$

Step 2 — Compute the constant term:

$$T_4 = \binom{6}{3} = \frac{6!}{3!\,3!} = \frac{6 \times 5 \times 4}{3 \times 2 \times 1} = 20.$$

Why other options are wrong:

- Option A (15): This is $\binom{6}{2}$ or $\binom{6}{4}$; wrong value of r.
- Option B (6): This is $\binom{6}{1}$; $r = 1$ gives x^4 term, not constant.
- Option D (35): This is $\binom{7}{3}$; uses wrong expansion.

Final Answer: Constant term $= 20 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 3

Q4.

Solution

Concept — Quadratic Inequality: To solve $f(x) > 0$ where $f(x) = (x - \alpha)(x - \beta)$ with $\alpha < \beta$, the product is positive when $x < \alpha$ or $x > \beta$.

Step 1 — Factorise:

$$x^2 - 5x + 6 = (x - 2)(x - 3).$$

Step 2 — Apply sign analysis: The product $(x - 2)(x - 3) > 0$ when both factors share the same sign:

- Both positive: $x > 3$.
- Both negative: $x < 2$.

Solution: $x < 2$ or $x > 3$.

Why other options are wrong:

- Option A ($2 < x < 3$): This is where $(x - 2)(x - 3) < 0$, i.e., the inequality is reversed.
- Option B ($x > 3$ only): Misses the branch $x < 2$.
- Option C ($x < 2$ only): Misses the branch $x > 3$.

Final Answer: $x < 2$ or $x > 3 \Rightarrow \boxed{\text{D}}$

$\boxed{\text{Answer: (D)}}$ **Go Back to Q 4**

Q5.

Solution

Concept — Combinatorics (Committee with Restrictions): "At least 2 women" means exactly 2, 3, or 4 women. Use $\binom{4}{w}\binom{6}{5-w}$ for each case and sum.

Step 1 — Case: exactly 2 women, 3 men:

$$\binom{4}{2}\binom{6}{3} = 6 \times 20 = 120.$$

Step 2 — Case: exactly 3 women, 2 men:

$$\binom{4}{3}\binom{6}{2} = 4 \times 15 = 60.$$

Step 3 — Case: exactly 4 women, 1 man:

$$\binom{4}{4}\binom{6}{1} = 1 \times 6 = 6.$$

Step 4 — Total:

$$120 + 60 + 6 = 186.$$

Why other options are wrong:

- Option A (162): Omits the case of 4 women; $120 + 42 = 162$ (wrong $\binom{4}{3}\binom{6}{2}$).
- Option C (246): Includes the case of 1 woman (which violates "at least 2").
- Option D (126): This is $\binom{9}{4}$; ignores the gender restriction entirely.

Final Answer: Number of committees $= 186 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 5

Q6.

Solution

Concept — Pascal's Rule: Pascal's identity states that for any integers $n \geq 1$ and $1 \leq r \leq n$:

$$\binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r}.$$

This follows directly from the definition $\binom{n}{r} = \dfrac{n!}{r!(n-r)!}$ by adding the two fractions.

Step 1 — Verify Option A: $\binom{n}{r}+\binom{n}{r-1} = \dfrac{n!}{r!(n-r)!}+\dfrac{n!}{(r-1)!(n-r+1)!}$. Factoring out $\dfrac{n!}{r!(n-r+1)!}$ gives $(n-r+1+r) = n+1$, yielding $\dfrac{(n+1)!}{r!(n-r+1)!} = \binom{n+1}{r}$. ✓

Why other options are wrong:

- Option B: The correct symmetry is $\binom{n}{r} = \binom{n}{n-r}$, not $\binom{n}{n-r-1}$.
- Option C: The right side should be $\binom{n+1}{r}$, not $\binom{n-1}{r}$; direction is reversed.
- Option D: $\binom{n}{0} = 1$ for all n, not n.

Final Answer: Pascal's rule: $\dbinom{n}{r} + \dbinom{n}{r-1} = \dbinom{n+1}{r} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 6

Q7.

Solution

Concept — Chain Rule: If $y = [f(x)]^n$, then $\dfrac{dy}{dx} = n[f(x)]^{n-1} \cdot f'(x)$.

Step 1 — Write as power function: $\sqrt{x^2+1} = (x^2+1)^{1/2}$.

Step 2 — Differentiate:

$$\frac{d}{dx}\left[(x^2+1)^{1/2}\right] = \frac{1}{2}(x^2+1)^{-1/2} \cdot 2x = \frac{x}{\sqrt{x^2+1}}.$$

Why other options are wrong:

- Option A ($1/\sqrt{x^2+1}$): Forgets the factor $2x$ from differentiating the inner function x^2+1.
- Option B ($-x/\sqrt{x^2+1}$): Sign error; $\dfrac{d}{dx}(x^2+1) = 2x > 0$ for $x > 0$.
- Option D ($\sqrt{x^2+1}$): This is the original function, not its derivative.

Final Answer: $\dfrac{d}{dx}\left[\sqrt{x^2+1}\right] = \dfrac{x}{\sqrt{x^2+1}} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 7

Q8.

Solution

Concept — Higher-Order Derivatives of Sine: Successive differentiation of $\sin(ax+b)$ introduces a factor of a each time and shifts the phase by $\pi/2$:

$$\frac{d^n}{dx^n}[\sin(ax+b)] = a^n \sin\left(ax+b+\frac{n\pi}{2}\right).$$

Step 1 — Verify with small n:

- $n=1$: $\frac{d}{dx}[\sin(ax+b)] = a\cos(ax+b) = a\sin(ax+b+\pi/2)$. ✓
- $n=2$: $= a^2(-\sin(ax+b)) = a^2\sin(ax+b+\pi)$. ✓

Why other options are wrong:

- Option A: Only the first derivative; misses the n-th power of a and phase shift.
- Option B ($a^n\cos(ax+b)$): Correct magnitude but wrong function; no phase shift applied.

- Option C ($\sin(ax + b + n\pi)$): Missing the factor a^n; phase shift is $n\pi$ instead of $n\pi/2$.

Final Answer: n-th derivative $= a^n \sin(ax + b + n\pi/2) \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 8

Q9.

Solution

Concept — Standard Trigonometric Limits: Use $\lim_{u\to 0} \frac{\tan u}{u} = 1$ and $\lim_{u\to 0} \frac{\sin u}{u} = 1$.

Step 1 — Rewrite by multiplying and dividing strategically:

$$\lim_{x\to 0} \frac{\tan 3x}{\sin 2x} = \lim_{x\to 0} \frac{\tan 3x}{3x} \cdot \frac{3x}{2x} \cdot \frac{2x}{\sin 2x}.$$

Step 2 — Apply standard limits:

$$= \left(\lim_{x\to 0} \frac{\tan 3x}{3x}\right) \cdot \frac{3}{2} \cdot \left(\lim_{x\to 0} \frac{2x}{\sin 2x}\right) = 1 \cdot \frac{3}{2} \cdot 1 = \frac{3}{2}.$$

Why other options are wrong:

- Option A (3): Only takes the numerator coefficient; ignores $\sin 2x \to 2x$ scaling.
- Option C ($2/3$): Inverts the ratio of coefficients.
- Option D (2): Uses only the denominator coefficient; ignores numerator scaling.

Final Answer: $\lim_{x\to 0} \frac{\tan 3x}{\sin 2x} = \frac{3}{2} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 9

Q10.

Solution

Concept — Definite Integral of $\sec^2 x$: Recall that $\dfrac{d}{dx}(\tan x) = \sec^2 x$, so $\displaystyle\int \sec^2 x\,dx = \tan x + C$.

Step 1 — Antiderivative:

$$\int_0^{\pi/4} \sec^2 x\,dx = \Big[\tan x\Big]_0^{\pi/4}.$$

Step 2 — Evaluate at bounds:

$$= \tan\left(\frac{\pi}{4}\right) - \tan(0) = 1 - 0 = 1.$$

Why other options are wrong:

- Option B ($\sqrt{2}-1$): Result of $\displaystyle\int_0^{\pi/4} (\sec x - 1)\,dx$; wrong integrand.
- Option C (0): $\sec^2 x > 0$ on $[0, \pi/4]$; the integral cannot be zero.
- Option D (2): $\tan(\pi/2)$ is undefined; upper limit confusion with $\pi/2$.

Final Answer: $\displaystyle\int_0^{\pi/4} \sec^2 x\,dx = 1 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 10

Q11.

Solution

Concept — Order and Degree of an ODE: The *order* is the order of the highest derivative present. The *degree* is the power of the highest-order derivative, provided the equation is polynomial in the derivatives (no transcendental functions of derivatives).

Step 1 — Identify the highest-order derivative: The highest derivative is $\dfrac{d^2y}{dx^2}$ (second-order).

Step 2 — Read its power:

$$\left(\frac{d^2y}{dx^2}\right)^{\mathbf{3}} + \left(\frac{dy}{dx}\right)^2 = 5.$$

The second-order derivative appears to the power **3**. Hence **degree** $= 3$.

Why other options are wrong:

- Option A (1): Confuses degree with order of the first-derivative term.
- Option B (2): This is the power of dy/dx, which is a lower-order term.
- Option D (6): Adds the powers $3 + 2 = 5$ or multiplies them; neither is correct.

Final Answer: Degree $= 3 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 11

Q12.

Solution

Concept — Equation of Tangent to a Curve: The tangent to $y = f(x)$ at (x_0, y_0) has slope $m = f'(x_0)$ and equation $y - y_0 = m(x - x_0)$.

Step 1 — Differentiate: $f(x) = x^2 \Rightarrow f'(x) = 2x$.

Step 2 — Find slope at $(2, 4)$**:** $m = 2(2) = 4$.

Step 3 — Write tangent equation:

$$y - 4 = 4(x - 2) \implies y = 4x - 8 + 4 = 4x - 4.$$

Why other options are wrong:

- Option A $(y = 2x)$: Slope $= 2$ is $f'(1)$, not $f'(2)$; also does not pass through $(2, 4)$.
- Option B $(y = 4x)$: Correct slope but wrong intercept; passes through origin, not $(2, 4)$.
- Option C $(y = 2x - 4)$: Wrong slope (uses $f'(1) = 2$) and wrong intercept.

Final Answer: Tangent $y = 4x - 4 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 12

Q13.

Solution

Concept — Angle Bisectors of Two Lines: For lines $L_1 \equiv a_1x + b_1y + c_1 = 0$ and $L_2 \equiv a_2x + b_2y + c_2 = 0$, the angle bisectors satisfy:

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm\frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}.$$

Step 1 — Write lines in standard form: $L_1 : x + y - 4 = 0$ and $L_2 : x - y - 2 = 0$. Both have $\sqrt{1^2 + 1^2} = \sqrt{2}$.

Step 2 — Case 1 (positive sign):

$$\frac{x + y - 4}{\sqrt{2}} = \frac{x - y - 2}{\sqrt{2}} \implies x + y - 4 = x - y - 2 \implies 2y = 2 \implies y = 1.$$

Step 3 — Case 2 (negative sign):

$$\frac{x + y - 4}{\sqrt{2}} = -\frac{x - y - 2}{\sqrt{2}} \implies x + y - 4 = -(x - y - 2) \implies 2x = 6 \implies x = 3.$$

Bisectors: $y = 1$ and $x = 3$.

Why other options are wrong:

- Option A ($x = 1$ and $y = 3$): Values are swapped.
- Option C ($x + y = 4$): This is one of the original lines, not its bisector.
- Option D ($x - y = 2$): Also an original line, not a bisector.

Final Answer: Angle bisectors $x = 3$ and $y = 1 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 13

Q14.

Solution

Concept — Collinearity Condition: Three points A, B, C are collinear if and only if the slope of AB equals the slope of BC (or equivalently, the area of triangle ABC is zero).

Step 1 — Compute slope AB:

$$\text{slope}_{AB} = \frac{3 - 1}{4 - k} = \frac{2}{4 - k}.$$

Step 2 — Compute slope BC:

$$\text{slope}_{BC} = \frac{5-3}{8-4} = \frac{2}{4} = \frac{1}{2}.$$

Step 3 — Set equal and solve:

$$\frac{2}{4-k} = \frac{1}{2} \implies 4 = 4 - k \implies k = 0.$$

Why other options are wrong:

- Option B ($k = 1$): slope$_{AB} = 2/3 \neq 1/2$.
- Option C ($k = 2$): slope$_{AB} = 2/2 = 1 \neq 1/2$.
- Option D ($k = 4$): $4 - k = 0$; slope AB is undefined (vertical line), not $1/2$.

Final Answer: $k = 0 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 14

Q15.

Solution

Concept — Condition for Tangency (Line and Parabola): Substitute the line into the parabola equation and set the discriminant of the resulting quadratic to zero.

Step 1 — Substitute $y = 2x + k$ into $y^2 = 8x$:

$$(2x+k)^2 = 8x \implies 4x^2 + 4kx + k^2 = 8x \implies 4x^2 + (4k-8)x + k^2 = 0.$$

Step 2 — Apply discriminant $= 0$ for tangency:

$$\Delta = (4k-8)^2 - 4(4)(k^2) = 0.$$

$$16k^2 - 64k + 64 - 16k^2 = 0 \implies -64k + 64 = 0 \implies k = 1.$$

Why other options are wrong:

- Option A ($k = -1$): $\Delta = (-4-8)^2 - 16 = 144 - 16 = 128 \neq 0$; two intersections.
- Option B ($k = 0$): $\Delta = (-8)^2 - 0 = 64 \neq 0$; line passes through origin, not tangent.

- Option D ($k = 2$): $\Delta = (0)^2 - 64 = -64 < 0$; no real intersection.

Final Answer: $k = 1 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 15

Q16.

Solution

Concept — Latus Rectum of a Parabola: For the standard parabola $y^2 = 4ax$, the focus is at $(a, 0)$ and the length of the latus rectum is $4a$.

Step 1 — Compare with standard form:

$$y^2 = 12x \implies 4a = 12 \implies a = 3.$$

Step 2 — Compute latus rectum length:

$$\text{Length of latus rectum} = 4a = 12.$$

Verification: The latus rectum passes through the focus $(3, 0)$ perpendicularly to the axis. Points on the parabola with $x = 3$: $y^2 = 36 \Rightarrow y = \pm 6$. Length $= 6 - (-6) = 12$. ✓

Why other options are wrong:

- Option A (3): This is the value of a, not $4a$.
- Option B (6): This is $2a = 6$, half the latus rectum.
- Option C (9): This is $3a = 9$; no standard formula gives $3a$.

Final Answer: Latus rectum $= 12 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 16

Q17.

Solution

Concept — Modulus of a Complex Number: For $z = a + bi$, the modulus is $|z| = \sqrt{a^2 + b^2}$.

Step 1 — Identify real and imaginary parts: $z = 3 + 4i$, so $a = 3$, $b = 4$.

Step 2 — Compute:

$$|z| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5.$$

Note: The pair $(3, 4, 5)$ is a Pythagorean triple; this result should be memorised.

Why other options are wrong:

- Option A (7): Adds $3 + 4 = 7$; modulus is not the sum of parts.
- Option C (4): Uses only the imaginary part.
- Option D (3): Uses only the real part.

Final Answer: $|z| = 5 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 17

Q18.

Solution

Concept — Singular Matrix: A matrix is singular when its determinant equals zero. For a 2×2 matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $\det = ad - bc$.

Step 1 — Compute the determinant:

$$\det \begin{pmatrix} k & 3 \\ 2 & 6 \end{pmatrix} = 6k - 3 \times 2 = 6k - 6.$$

Step 2 — Set determinant to zero:

$$6k - 6 = 0 \implies k = 1.$$

Why other options are wrong:

- Option B $(k = 2)$: $\det = 12 - 6 = 6 \neq 0$.

- Option C ($k = 3$): $\det = 18 - 6 = 12 \neq 0$.
- Option D ($k = 6$): $\det = 36 - 6 = 30 \neq 0$.

Final Answer: $k = 1 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 18

Q19.

Solution

Concept — Rotation Matrices: A counter-clockwise rotation by angle θ has matrix $\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}$. A **clockwise** rotation by θ is the transpose (or $-\theta$ substitution): $\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$.

Step 1 — Clockwise 90° (substitute $\theta = 90$):

$$\begin{pmatrix} \cos 90 & \sin 90 \\ -\sin 90 & \cos 90 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

Step 2 — Verify with a test vector: Under clockwise 90°, $(1, 0) \to (0, -1)$ and $(0, 1) \to (1, 0)$. Check: $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$. ✓

Why other options are wrong:

- Option A: $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ is counter-clockwise rotation by 90.
- Option B: $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ is rotation by 180.
- Option D: $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ is the identity (rotation by 0).

Final Answer: Clockwise 90 matrix $= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 19

Q20.

Solution

Concept — Cayley-Hamilton Theorem: Every square matrix satisfies its own characteristic equation. For A, if the characteristic polynomial is $p(\lambda)$, then $p(A) = \mathbf{0}$ (the zero matrix).

Step 1 — Verify the characteristic polynomial: For $A = \begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$, eigenvalues are $\lambda = 2$ and $\lambda = 3$. Characteristic polynomial: $(\lambda - 2)(\lambda - 3) = \lambda^2 - 5\lambda + 6$.

Step 2 — Compute $A^2 - 5A + 6I$:

$$A^2 = \begin{pmatrix} 4 & 0 \\ 0 & 9 \end{pmatrix}, \quad 5A = \begin{pmatrix} 10 & 0 \\ 0 & 15 \end{pmatrix}, \quad 6I = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}.$$

$$A^2 - 5A + 6I = \begin{pmatrix} 4 - 10 + 6 & 0 \\ 0 & 9 - 15 + 6 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

Why other options are wrong:

- Option A (I): Cayley-Hamilton gives $p(A) = \mathbf{0}$, not I.
- Option B (A): Would require $A^2 - 4A + 6I = A$; not the correct relation.
- Option C ($5A$): $A^2 + 6I = 10A \Rightarrow$ doesn't match with $p(A) = \mathbf{0}$.

Final Answer: $A^2 - 5A + 6I =$ the zero matrix $\Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 20

Q21.

Solution

Concept — Consistency of a Linear System: A system $a_1x + b_1y = c_1$ and $a_2x + b_2y = c_2$ is consistent (has at least one solution) when the lines are not parallel, or when they coincide. If $\frac{a_1}{a_2} = \frac{b_1}{b_2}$, the lines are parallel (or the same); for coincidence we also need $\frac{a_1}{a_2} = \frac{c_1}{c_2}$.

Step 1 — Check the ratio of coefficients:

$$\frac{2}{4} = \frac{1}{2} = \frac{y\text{-coeff ratio}}{} : \frac{1}{2}.$$

Both lines have the same ratio $1 : 2$, so they are parallel or coincident.

Step 2 — Condition for coincidence:

$$\frac{c_1}{c_2} = \frac{3}{k} = \frac{1}{2} \implies k = 6.$$

When $k = 6$ the two equations are identical (consistent). For any other k, they are parallel with no solution (inconsistent).

Why other options are wrong:

- Option A ($k = 3$): $3/3 = 1 \neq 1/2$; the system is inconsistent (parallel lines).
- Option C ($k = 12$): $3/12 = 1/4 \neq 1/2$; inconsistent.
- Option D (any value): Only $k = 6$ works; any other value makes the system inconsistent.

Final Answer: $k = 6 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 21

Q22.

Solution

Concept — Median from Cumulative Frequency (Ogive): The median corresponds to $N/2$ on the cumulative frequency axis. Locate $N/2$ on the ogive, then read off the corresponding class value. Use the interpolation formula:

$$\text{Median} = L + \frac{\frac{N}{2} - \text{CF}_{\text{prev}}}{f} \times h,$$

where L is the lower boundary of the median class, CF_{prev} is the cumulative frequency before the median class, f is the frequency of the median class, and h is the class width.

Step 1 — Frequency table from CF data:

Class	CF	Frequency f
0–10	5	5
10–20	15	10
20–30	35	20
30–40	50	15
40–50	60	10

Step 2 — Locate median class: $N/2 = 30$. CF crosses 30 in the class 20–30 (CF goes from 15 to 35).

Step 3 — Apply formula: $L = 20$, $\text{CF}_{\text{prev}} = 15$, $f = 20$, $h = 10$:

$$\text{Median} = 20 + \frac{30 - 15}{20} \times 10 = 20 + \frac{15}{20} \times 10 = 20 + 7.5 = 27.5.$$

Why other options are wrong:

- Option B (25): Uses $\text{CF}_{\text{prev}} = 20$ instead of 15; incorrect table reading.
- Option C (30): Reads the lower boundary of the next class.
- Option D (28): Arithmetic error in the interpolation step.

Final Answer: Median $= 27.5 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 22

Q23.

Solution

Concept — Addition Rule for Mutually Exclusive Events: If A and B are mutually exclusive, $P(A \cap B) = 0$, so $P(A \cup B) = P(A) + P(B)$.

Step 1 — Apply the rule directly:

$$P(A \cup B) = P(A) + P(B) = 0.4 + 0.3 = 0.7.$$

Why other options are wrong:

- Option A (0.12): This is $P(A) \times P(B) = 0.4 \times 0.3$; applies to independent events, and gives $P(A \cap B)$, not $P(A \cup B)$.
- Option B (0.5): Neither the sum nor the product; no valid formula gives this.
- Option D (1.0): Only true if A and B together cover the entire sample space (exhaustive), which is not stated.

Final Answer: $P(A \cup B) = 0.7 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 23

Q24.

Solution

Concept — Expected Value of a Discrete Random Variable: $E(X) = \sum_i x_i P(X = x_i)$.

Step 1 — Compute each term:

x_i	$P(X = x_i)$	$x_i P(X = x_i)$
1	0.1	0.1
2	0.3	0.6
3	0.4	1.2
4	0.2	0.8

Step 2 — Sum:

$$E(X) = 0.1 + 0.6 + 1.2 + 0.8 = 2.7.$$

Why other options are wrong:

- Option A (2.0): Uses equal probabilities $1/4$ each; $E(X) = (1+2+3+4)/4 = 2.5$, not 2.0.
- Option B (2.5): Result under uniform probabilities; ignores the actual probability weights.
- Option C (3.0): Ignores probabilities entirely and takes the simple average of $\{1, 2, 3, 4\}$ incorrectly.

Final Answer: $E(X) = 2.7 \Rightarrow \boxed{\text{D}}$

Answer: (D) **Go Back to Q 24**

Q25.

Solution

Concept — Regression Coefficients and Correlation: The two regression coefficients b_{yx} and b_{xy} satisfy:

$$r^2 = b_{yx} \cdot b_{xy},$$

where r is Pearson's correlation coefficient.

Step 1 — Substitute known values:

$$r^2 = b_{yx} \cdot b_{xy} \implies 0.64 = 0.8 \cdot b_{xy}.$$

Step 2 — Solve for b_{xy}:

$$b_{xy} = \frac{0.64}{0.8} = 0.8.$$

Why other options are wrong:

- Option A (0.64): Confuses r^2 with b_{xy}; $r^2 = b_{yx} \cdot b_{xy}$, not $b_{xy} = r^2$.
- Option C (1.0): Would require $b_{yx} = b_{xy} = 1$, meaning perfect correlation; not the case here.
- Option D (0.512): Uses $b_{xy} = r^2 \times b_{yx} = 0.64 \times 0.8$; wrong formula.

Final Answer: $b_{xy} = 0.8 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 25

Q26.

Solution

Concept — Skewness and Measures of Central Tendency: In a distribution:

- **Positively skewed** (right-skewed): The tail extends to the right. The mean is pulled towards the tail, so **Mean** $>$ **Median** $>$ **Mode**.
- Negatively skewed: Mode $>$ Median $>$ Mean.
- Symmetric: Mean $=$ Median $=$ Mode.

Step 1 — Identify the relationship for positive skew: A positively skewed distribution has most data concentrated on the left with a long right tail. High outliers pull the mean rightward beyond the median, which is beyond the mode.

Why other options are wrong:

- Option B: This is the order for a negatively skewed (left-skewed) distribution.
- Option C: This holds for a perfectly symmetric (normal) distribution.
- Option D: No standard distribution produces Median $>$ Mean $>$ Mode in positive skew.

Final Answer: Mean $>$ Median $>$ Mode $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 26

Q27.

Solution

Concept — Sine Subtraction Formula: $\sin(A-B) = \sin A \cos B - \cos A \sin B$.

Step 1 — Express $15 = 45 - 30$:

$$\sin 15 = \sin(45 - 30) = \sin 45 \cos 30 - \cos 45 \sin 30.$$

Step 2 — Substitute standard values:

$$\sin 15 = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} = \frac{\sqrt{6}-\sqrt{2}}{4}.$$

Why other options are wrong:

- Option A ($(\sqrt{6}+\sqrt{2})/4$): This is $\sin 75$ or $\cos 15$; the signs are added, not subtracted.
- Option B ($(\sqrt{3}-1)/2$): Missing the factor of $\sqrt{2}/2$ and has wrong denominator.
- Option D ($\sqrt{3}/2$): This is $\sin 60$, not $\sin 15$.

Final Answer: $\sin 15 = \dfrac{\sqrt{6}-\sqrt{2}}{4} \Rightarrow \boxed{\text{C}}$

Answer: (C) **Go Back to Q 27**

Q28.

Solution

Concept — Pythagorean Identity for Secant: The identity $\tan^2\theta + 1 = \sec^2\theta$ connects tangent and secant.

Step 1 — Substitute $\theta = 45$:

$$\tan^2 45 + 1 = \sec^2 45.$$

Step 2 — Use known value $\tan 45 = 1$:

$$1^2 + 1 = \sec^2 45 \implies \sec^2 45 = 2 \implies \sec 45 = \sqrt{2}.$$

Alternative check: $\sec 45 = \dfrac{1}{\cos 45} = \dfrac{1}{1/\sqrt{2}} = \sqrt{2}.$ ✓

Why other options are wrong:

- Option A (1): $\sec 45 = 1$ would require $\cos 45 = 1$, but $\cos 45 = 1/\sqrt{2}$.
- Option B (2): $\sec^2 45 = 2$, not $\sec 45$; fails to take the square root.
- Option C ($1/\sqrt{2}$): This is $\cos 45$, not $\sec 45$ (which is the reciprocal).

Final Answer: $\sec 45 = \sqrt{2} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 28

Q29.

Solution

Concept — Principal Value of Inverse Trigonometric Functions: The principal value branch of $\sin^{-1}$ has range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. So $\sin^{-1}(\sin\theta)$ is not necessarily θ; it equals the unique $\phi \in [-\pi/2, \pi/2]$ such that $\sin\phi = \sin\theta$.

Step 1 — Find $\sin(7\pi/6)$**:**

$$\frac{7\pi}{6} = \pi + \frac{\pi}{6} \implies \sin\left(\frac{7\pi}{6}\right) = -\sin\left(\frac{\pi}{6}\right) = -\frac{1}{2}.$$

Step 2 — Find the principal value: We need $\phi \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ such that $\sin\phi = -\frac{1}{2}$.

$$\phi = -\frac{\pi}{6}.$$

Why other options are wrong:

- Option A ($7\pi/6$): This lies outside the principal range $[-\pi/2, \pi/2]$.
- Option C ($5\pi/6$): $\sin(5\pi/6) = 1/2$, not $-1/2$.
- Option D ($\pi/6$): $\sin(\pi/6) = 1/2$, not $-1/2$; wrong sign.

Final Answer: $\sin^{-1}\left(\sin\frac{7\pi}{6}\right) = -\frac{\pi}{6} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 29

Q30.

Solution

Concept — Heron's Formula: For a triangle with sides a, b, c, the semi-perimeter is $s = \dfrac{a+b+c}{2}$ and the area is:

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)}.$$

Step 1 — Compute semi-perimeter:

$$s = \frac{13+14+15}{2} = \frac{42}{2} = 21.$$

Step 2 — Compute each factor:

$$s-a = 21-13 = 8, \quad s-b = 21-14 = 7, \quad s-c = 21-15 = 6.$$

Step 3 — Apply Heron's formula:

$$\text{Area} = \sqrt{21 \times 8 \times 7 \times 6} = \sqrt{7056} = 84.$$

Verification: $21 \times 8 = 168$, $7 \times 6 = 42$, $168 \times 42 = 7056$, $\sqrt{7056} = 84$. ✓

Why other options are wrong:

- Option B (42): Half of the correct answer; forgets that $\sqrt{7056} = 84$, not 42.
- Option C (90): Approximation error; $\sqrt{8100} = 90$ but $21 \times 8 \times 7 \times 6 = 7056 \neq 8100$.
- Option D (70): Uses incorrect semi-perimeter or factor computation.

Final Answer: Area $= 84 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 30

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	A	3	C	4	D	5	B
6	A	7	C	8	D	9	B	10	A
11	C	12	D	13	B	14	A	15	C
16	D	17	B	18	A	19	C	20	D
21	B	22	A	23	C	24	D	25	B
26	A	27	C	28	D	29	B	30	A