MAH MCA CET Mathematics & Statistics

Sample Paper – 4

Duration: 27 Minutes **Maximum Marks: 60**

Instructions

- This paper contains **30** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics & Statistics portion of **MAH MCA CET** entrance.
- Each correct answer carries **+2 marks**. There is **no negative marking** for incorrect or unattempted answers.
- Only **one** option is correct. Choose carefully.
- Syllabus level: **Class 12 level — Algebra, Calculus, Coordinate Geometry, Matrices, Statistics & Probability, Trigonometry.**
- Use of mobile phones, calculators, or electronic gadgets is strictly prohibited.

Q1. If a, b, c are in **Harmonic Progression** (HP), which of the following relations correctly expresses b in terms of a and c?

(A) $2b = a + c$

(B) $b^2 = ac$

(C) $b = \dfrac{2ac}{a+c}$

(D) $\dfrac{1}{b} = \dfrac{1}{a} + \dfrac{1}{c}$

Q2. The value of $\log_{10}(0.001)$ is:

(A) -3

(B) -2

(C) 3

(D) 2

Q3. Using the Binomial Theorem, the coefficient of x^4 in the expansion of $(1+x)^7$ is:

(A) 21

(B) 35

(C) 42

(D) 28

Q4. For what value of k does the quadratic equation $kx^2+4x+1=0$ have equal roots?

(A) $k=1$

(B) $k=2$

(C) $k=3$

(D) $k=4$

Q5. A committee of 3 persons is to be formed from a group of 5 men and 3 women. In how many ways can this be done such that **at least one woman** is included?

(A) 36

(B) 42

(C) 46

(D) 52

Q6. Find the sum to infinity of the geometric progression $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\cdots$

(A) $\frac{3}{2}$

(B) $\frac{2}{3}$

(C) $\frac{4}{3}$

(D) $\frac{1}{2}$

Q7. Using the product rule, the derivative of $f(x) = x^2 \sin x$ with respect to x is:

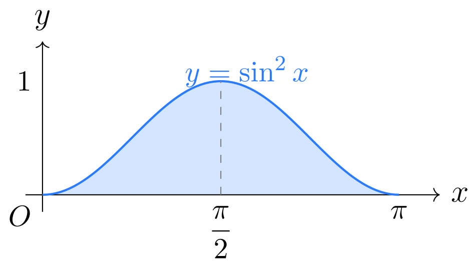
(A) $2x \cos x + x^2 \sin x$

(B) $2x \sin x + x^2 \cos x$

(C) $x^2 \cos x$

(D) $2x \cos x$

Q8. The graph of $y = \sin^2 x$ over $[0, \pi]$ is shown below. Evaluate $\int_0^{\pi} \sin^2 x \, dx$ using the identity $\cos 2x = 1 - 2\sin^2 x$.



(A) $\dfrac{\pi}{4}$

(B) $\dfrac{\pi}{2} + \dfrac{1}{4}$

(C) $\pi - \dfrac{1}{4}$

(D) $\dfrac{\pi}{2}$

Q9. Using the Squeeze Theorem (Sandwich Theorem), the value of $\lim_{x \to 0} x \sin\left(\frac{1}{x}\right)$ is:

(A) 1

(B) Does not exist

(C) 0

(D) ∞

Q10. Evaluate $\int \frac{2x+1}{(x+1)(x+2)} \, dx$ using partial fractions:

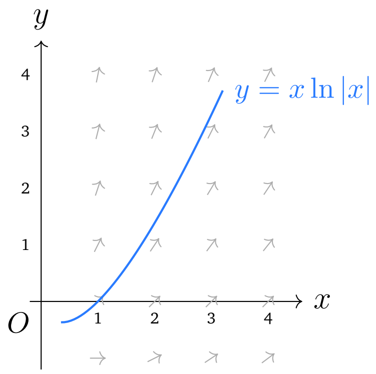
(A) $-\ln|x+1| + 3\ln|x+2| + C$

(B) $\ln|x+1| + 3\ln|x+2| + C$

(C) $-\ln|x+1| - 3\ln|x+2| + C$

(D) $3\ln|x+1| - \ln|x+2| + C$

Q11. The direction field below corresponds to the ODE $\dfrac{dy}{dx} = \dfrac{x+y}{x}$. Solving this homogeneous equation by the substitution $y = vx$, the general solution is:



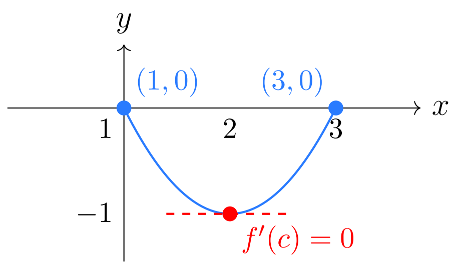
(A) $y = Cx$

(B) $y = x\big(\ln|x| + C\big)$

(C) $y = Ce^{x}$

(D) $y = x\ln|x|$

Q12. The graph of $f(x) = x^2 - 4x + 3$ on $[1, 3]$ is drawn below. By Rolle's Theorem, there exists $c \in (1, 3)$ such that $f'(c) = 0$. Find c.



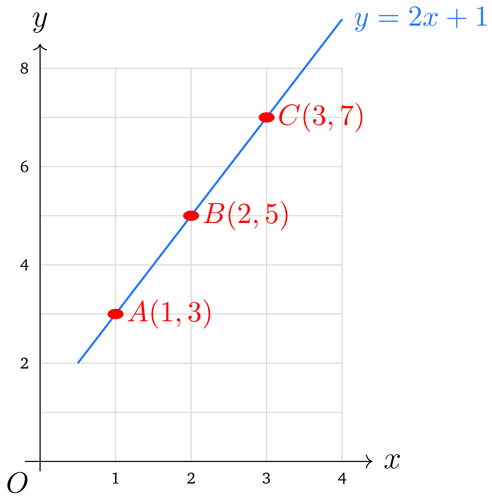
(A) $c = 1$

(B) $c = 3$

(C) $c = 1.5$

(D) $c = 2$

Q13. The three points $A(1, 3)$, $B(2, 5)$, $C(3, 7)$ are plotted on the grid below. Using the area-of-triangle formula, determine whether they are collinear.



(A) The points are collinear (area of $\triangle ABC = 0$)

(B) The area of $\triangle ABC$ is 4 sq. units

(C) The points form an isosceles triangle

(D) The slope of AB is 3

Q14. A straight line has x-intercept 3 and y-intercept 4. The equation of the line in standard form is:

(A) $3x + 4y = 12$

(B) $\dfrac{x}{4} + \dfrac{y}{3} = 1$

(C) $4x + 3y = 12$

(D) $x + y = 7$

Q15. The equation of the chord of contact drawn from the external point $P(2,\ 1)$ to the circle $x^2 + y^2 = 5$ is:

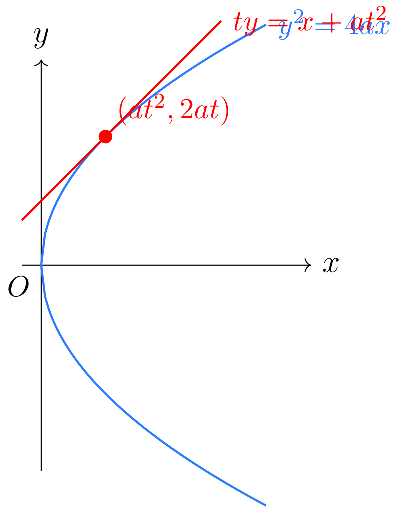
(A) $x + 2y = 5$

(B) $2x + y = 5$

(C) $2x - y = 5$

(D) $x - 2y = 5$

Q16. The diagram below shows the parabola $y^2 = 4ax$ (with $a = 1$ as example) and the tangent at point $T(at^2, 2at)$. The equation of the tangent to $y^2 = 4ax$ at the point $(at^2, 2at)$ is:



(A) $ty = x - at^2$

(B) $ty = 2x + at^2$

(C) $ty = x - 2at^2$

(D) $ty = x + at^2$

Q17. The centroid of the triangle with vertices $P(1, 2)$, $Q(3, 4)$, and $R(5, 6)$ is:

(A) $(3, 4)$

(B) $(2, 3)$

(C) $(4, 5)$

(D) $(9, 12)$

Q18. For the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, the Cayley–Hamilton theorem states that A satisfies its own characteristic equation. Which equation does A satisfy?

(A) $A^2 - 5A + 2I = \mathbf{0}$

(B) $A^2 + 5A - 2I = \mathbf{0}$

(C) $A^2 + 5A + 2I = \mathbf{0}$

(D) $A^2 - 5A - 2I = \mathbf{0}$

Q19. If A is a 3×3 matrix and k is any scalar, then $\det(kA)$ is equal to:

(A) $k\ \det(A)$

(B) $k^3\ \det(A)$

(C) $3k\ \det(A)$

(D) $k^2\ \det(A)$

Q20. The rank of the matrix $M = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$ is:

(A) 3

(B) 2

(C) 1

(D) 0

Q21. Which of the following statements is **always true** for every skew-symmetric matrix A?

(A) All diagonal elements of A are zero

(B) $\det(A) \neq 0$

(C) A must be of even order

(D) A^2 is skew-symmetric

Q22. A student scored marks $60, 70, 80, 90$ in four tests with corresponding weights $1, 2, 3, 4$. The weighted mean of the marks is:

(A) 75

(B) 78

(C) 80

(D) 82

Q23. Events A and B are mutually exclusive with $P(A) = 0.3$ and $P(B) = 0.5$. The value of $P(A \cup B)$ is:

(A) 0.15

(B) 0.8

(C) 0.85

(D) 0.65

Q24. A binomial distribution has parameters $n = 20$ and $p = 0.4$. The variance of this distribution is:

(A) 8

(B) 4

(C) 6

(D) 4.8

Q25. Karl Pearson's coefficient of correlation r always lies in the range $[-1, +1]$. Which value of r indicates **perfect positive correlation**?

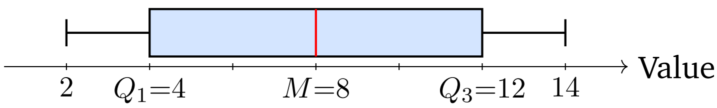
(A) $r = +1$

(B) $r = 0$

(C) $r = -1$

(D) $r = 0.5$

Q26. The box-plot below represents the data set $\{2, 4, 6, 8, 10, 12, 14\}$. Using the formula $QD = \dfrac{Q_3 - Q_1}{2}$, find the Quartile Deviation.



(A) 3

(B) 5

(C) 4

(D) 6

Q27. If $\sin\theta = \dfrac{5}{13}$ and θ is in the first quadrant, then $\cos\theta$ equals:

(A) $\dfrac{5}{13}$

(B) $\dfrac{12}{13}$

(C) $\dfrac{13}{12}$

(D) $\dfrac{13}{5}$

Q28. Using the identity $\sin 3A = 3\sin A - 4\sin^3 A$, the value of $\sin\left(3\times 30^\circ\right)$ is:

(A) 1

(B) $\dfrac{\sqrt{3}}{2}$

(C) $\dfrac{1}{2}$

(D) 0

Q29. Using the identity $\cos^{-1}(x) + \cos^{-1}(-x) = \pi$ for $x \in [-1, 1]$, the value of $\cos^{-1}\left(\dfrac{1}{2}\right) + \cos^{-1}\left(-\dfrac{1}{2}\right)$ is:

(A) $\dfrac{\pi}{2}$

(B) $\dfrac{\pi}{3}$

(C) $\dfrac{2\pi}{3}$

(D) π

Q30. In a triangle, $\angle A = 60^\circ$, $b = 5$, and $c = 7$. Using the cosine rule $a^2 = b^2 + c^2 - 2bc\cos A$, the side a equals:

(A) $\sqrt{29}$

(B) $\sqrt{34}$

(C) $\sqrt{39}$

(D) $\sqrt{44}$

Detailed Solutions

Q1.

Solution

Concept — Harmonic Progression: a, b, c are in HP if and only if their reciprocals $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in Arithmetic Progression (AP).

Step 1 — Condition for AP of reciprocals: If $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are in AP, then the middle term is the arithmetic mean:

$$\frac{1}{b} = \frac{1}{2}\left(\frac{1}{a} + \frac{1}{c}\right) \implies \frac{2}{b} = \frac{1}{a} + \frac{1}{c} = \frac{a+c}{ac}$$

Step 2 — Solve for b: Inverting both sides of $\frac{2}{b} = \frac{a+c}{ac}$ gives:

$$b = \frac{2ac}{a+c}$$

This is the *harmonic mean* of a and c.

Why other options are wrong:

- **Option A:** $2b = a + c \Rightarrow b = \frac{a+c}{2}$ is the condition for a, b, c in *AP*, not HP.
- **Option B:** $b^2 = ac$ is the condition for a, b, c in *GP* (geometric mean).
- **Option D:** $\frac{1}{b} = \frac{1}{a} + \frac{1}{c}$ would require $\frac{2}{b} = \frac{2}{a} + \frac{2}{c}$, which is not the AP condition.

Final Answer: $b = \dfrac{2ac}{a+c} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 1

Q2.

Solution

Concept — Common Logarithm: $\log_{10}(10^k) = k$ for any real k.

Step 1 — Express as a power of 10:

$$0.001 = \frac{1}{1000} = \frac{1}{10^3} = 10^{-3}$$

Step 2 — Apply the logarithm:

$$\log_{10}(0.001) = \log_{10}(10^{-3}) = -3$$

Why other options are wrong:

- **Option B:** -2 would correspond to $\log_{10}(0.01)$ since $0.01 = 10^{-2}$.
- **Option C:** 3 would correspond to $\log_{10}(1000) = 3$.
- **Option D:** 2 would correspond to $\log_{10}(100) = 2$.

Final Answer: $\log_{10}(0.001) = -3 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 2

Q3.

Solution

Concept — Binomial Theorem: The general term in the expansion of $(1+x)^n$ is $T_{r+1} = \binom{n}{r}x^r$.

Step 1 — Identify the required term: For the coefficient of x^4, set $r = 4$ with $n = 7$:

$$T_5 = \binom{7}{4} x^4$$

Step 2 — Compute the binomial coefficient:

$$\binom{7}{4} = \frac{7!}{4!\,3!} = \frac{7 \times 6 \times 5}{3 \times 2 \times 1} = \frac{210}{6} = 35$$

Why other options are wrong:

- **Option A:** $21 = \binom{7}{2}$, the coefficient of x^2, not x^4.
- **Option C:** 42 does not equal any binomial coefficient of $(1+x)^7$.
- **Option D:** $28 = \binom{8}{2}$; incorrect for this expansion.

Final Answer: Coefficient of x^4 is $35 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 3

Q4.

Solution

Concept — Equal Roots Condition: A quadratic $ax^2 + bx + c = 0$ has equal roots if and only if the discriminant $\Delta = b^2 - 4ac = 0$.

Step 1 — Set up the discriminant: For $kx^2 + 4x + 1 = 0$, we have $a = k$, $b = 4$, $c = 1$:

$$\Delta = 4^2 - 4 \cdot k \cdot 1 = 16 - 4k$$

Step 2 — Solve $\Delta = 0$:

$$16 - 4k = 0 \implies k = 4$$

Why other options are wrong:

- **Option A:** $k = 1 \Rightarrow \Delta = 12 \neq 0$; two distinct real roots.
- **Option B:** $k = 2 \Rightarrow \Delta = 8 \neq 0$; two distinct real roots.
- **Option C:** $k = 3 \Rightarrow \Delta = 4 \neq 0$; two distinct real roots.

Final Answer: $k = 4 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 4

Q5.

Solution

Concept — Complementary Counting: (At least 1 woman) = (Total selections) − (Selections with no women).

Step 1 — Total ways to choose 3 from 8 persons:

$$\binom{8}{3} = \frac{8 \times 7 \times 6}{3!} = 56$$

Step 2 — Ways with no woman (all 3 from 5 men):

$$\binom{5}{3} = 10$$

Therefore, selections with at least 1 woman $= 56 - 10 =$ **46**.

Verification by direct count: 1W+2M: $\binom{3}{1}\binom{5}{2} = 30$; 2W+1M: $\binom{3}{2}\binom{5}{1} = 15$; 3W: $\binom{3}{3} = 1$. Total $= 46$ ✓

Why other options are wrong:

- **Option A:** 36 omits the 2-women and 3-women cases.
- **Option B:** 42 arises from an arithmetic error in the complement.
- **Option D:** 52 overcounts by not excluding all-male groups correctly.

Final Answer: 46 ways $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 5

Q6.

Solution

Concept — Infinite GP Sum: For $|r| < 1$, the sum to infinity is $S_\infty = \dfrac{a}{1-r}$.

Step 1 — Identify a and r:

$$\text{First term } a = 1, \qquad \text{Common ratio } r = \frac{1/3}{1} = \frac{1}{3}$$

Step 2 — Apply the formula:

$$S_\infty = \frac{1}{1-\frac{1}{3}} = \frac{1}{\frac{2}{3}} = \frac{3}{2}$$

Why other options are wrong:

- **Option B:** $\frac{2}{3}$ is the common ratio, not the sum.
- **Option C:** $\frac{4}{3}$ would arise from incorrectly using $1 - r = 3/4$.
- **Option D:** $\frac{1}{2}$ corresponds to a GP with $r = \frac{1}{2}$, not $\frac{1}{3}$.

Final Answer: $S_\infty = \dfrac{3}{2} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 6

Q7.

Solution

Concept — Product Rule: If $f = u \cdot v$, then $f' = u'v + uv'$.

Step 1 — Identify u and v:

$$u = x^2 \implies u' = 2x; \qquad v = \sin x \implies v' = \cos x$$

Step 2 — Apply the product rule:

$$\frac{d}{dx}(x^2 \sin x) = u'v + uv' = 2x \sin x + x^2 \cos x$$

Why other options are wrong:

- **Option A:** $2x \cos x + x^2 \sin x$ confuses $v' = \cos x$ with v and vice versa.
- **Option C:** $x^2 \cos x$ omits the first term from the product rule.
- **Option D:** $2x \cos x$ treats only x^2 and forgets to keep $\sin x$ in the first term.

Final Answer: $2x \sin x + x^2 \cos x \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 7

Q8.

Solution

Concept — Half-Angle Identity: $\sin^2 x = \dfrac{1 - \cos 2x}{2}$ converts the integrand to an easily integrable form.

Step 1 — Substitute the identity:

$$\int \sin^2 x\, dx = \int \frac{1 - \cos 2x}{2}\, dx = \frac{1}{2} \int 1\, dx - \frac{1}{2} \int \cos 2x\, dx$$

Step 2 — Integrate term by term:

$$= \frac{x}{2} - \frac{1}{2} \cdot \frac{\sin 2x}{2} + C = \frac{x}{2} - \frac{\sin 2x}{4} + C$$

Evaluating from 0 to π: $\left[\frac{x}{2} - \frac{\sin 2x}{4}\right]_0^\pi = \frac{\pi}{2} - 0 = \dfrac{\pi}{2}$.

Why other options are wrong:

- **Option A:** $\frac{\pi}{4}$ omits the $x/2$ term entirely.
- **Option B:** $\frac{\pi}{2} + \frac{1}{4}$ has the wrong sign on $\sin 2x$.
- **Option C:** $\pi - \frac{1}{4}$ uses coefficient 1 instead of $\frac{1}{2}$ for the x term.

Final Answer: $\displaystyle\int_0^\pi \sin^2 x\, dx = \frac{\pi}{2} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 8

Q9.

Solution

Concept — Squeeze Theorem: If $g(x) \leq f(x) \leq h(x)$ near $x = a$ and $\lim_{x\to a} g(x) = \lim_{x\to a} h(x) = L$, then $\lim_{x\to a} f(x) = L$.

Step 1 — Bound the function: Since $|\sin(1/x)| \leq 1$ for all $x \neq 0$:

$$\left|x\sin(1/x)\right| = |x| \cdot |\sin(1/x)| \leq |x| \cdot 1 = |x|$$

Therefore $-|x| \leq x\sin(1/x) \leq |x|$.

Step 2 — Apply the Squeeze Theorem:

$$\lim_{x\to 0}(-|x|) = 0 \quad \text{and} \quad \lim_{x\to 0}|x| = 0$$

By the Squeeze Theorem: $\lim\limits_{x\to 0} x\sin(1/x) = 0$.

Why other options are wrong:

- **Option A:** 1 is $\lim_{x\to 0}\frac{\sin x}{x}$, a different limit.
- **Option B:** The limit *does exist* and equals 0; $\sin(1/x)$ oscillates but is bounded.
- **Option D:** The product $x \cdot [\text{bounded}]$ cannot tend to ∞ as $x \to 0$.

Final Answer: $\lim\limits_{x\to 0} x\sin(1/x) = 0 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 9

Q10.

Solution

Concept — Partial Fractions: Decompose $\dfrac{2x+1}{(x+1)(x+2)} = \dfrac{A}{x+1} + \dfrac{B}{x+2}$, then integrate logarithmically.

Step 1 — Find A and B:

$$2x + 1 = A(x+2) + B(x+1)$$

$x = -1$: $-1 = A(1) \Rightarrow A = -1$. $\quad x = -2$: $-3 = B(-1) \Rightarrow B = 3$.

Step 2 — Integrate:

$$\int \frac{2x+1}{(x+1)(x+2)}\,dx = \int \left(\frac{-1}{x+1} + \frac{3}{x+2}\right)dx = -\ln|x+1| + 3\ln|x+2| + C$$

Why other options are wrong:

- **Option B:** Uses $+\ln|x+1|$ instead of $-\ln|x+1|$ (wrong sign for A).
- **Option C:** Uses $-3\ln|x+2|$ (wrong sign for B).
- **Option D:** Swaps the roles of A and B (partial fraction coefficients reversed).

Final Answer: $-\ln|x+1| + 3\ln|x+2| + C \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 10

Q11.

Solution

Concept — Homogeneous ODE: If $\frac{dy}{dx} = f(y/x)$, substitute $y = vx$ so $\frac{dy}{dx} = v + x\frac{dv}{dx}$.

Step 1 — Substitute $y = vx$**:**

$$\frac{dy}{dx} = \frac{x+y}{x} = 1 + \frac{y}{x} = 1 + v$$

So $v + x\dfrac{dv}{dx} = 1 + v \implies x\dfrac{dv}{dx} = 1.$

Step 2 — Separate variables and integrate:

$$dv = \frac{dx}{x} \implies v = \ln|x| + C \implies \frac{y}{x} = \ln|x| + C$$

$$\therefore \quad y = x\big(\ln|x| + C\big)$$

Why other options are wrong:

- **Option A:** $y = Cx$ satisfies $dy/dx = C$, which is y/x, not $(x+y)/x$.
- **Option C:** $y = Ce^x$ is the solution of $dy/dx = y$, a different ODE.
- **Option D:** $y = x\ln|x|$ is only the particular solution ($C = 0$); the general solution needs $+C$.

Final Answer: $y = x\big(\ln|x| + C\big) \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 11

Q12.

Solution

Concept — Rolle's Theorem: If f is continuous on $[a,b]$, differentiable on (a,b), and $f(a)=f(b)$, then $\exists\, c\in(a,b)$ with $f'(c)=0$.

Step 1 — Verify conditions:

$$f(1)=1-4+3=0;\qquad f(3)=9-12+3=0$$

f is a polynomial, hence continuous and differentiable everywhere. Conditions are satisfied.

Step 2 — Find c:

$$f'(x)=2x-4;\qquad f'(c)=0 \implies 2c-4=0 \implies c=2$$

As shown in the graph, the tangent at $(2,-1)$ is horizontal.

Why other options are wrong:

- **Option A:** $c=1$ is an endpoint; Rolle's theorem requires $c\in(1,3)$, open interval.
- **Option B:** $c=3$ is also an endpoint.
- **Option C:** $f'(1.5)=2(1.5)-4=-1\neq 0$.

Final Answer: $c=2\Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 12

Q13.

Solution

Concept — Collinearity via Area: Three points P_1, P_2, P_3 are collinear if the area of the triangle they form is zero.

Step 1 — Apply the area formula:

$$\text{Area}=\tfrac{1}{2}\left|x_1(y_2-y_3)+x_2(y_3-y_1)+x_3(y_1-y_2)\right|$$

$$=\tfrac{1}{2}\left|1(5-7)+2(7-3)+3(3-5)\right|=\tfrac{1}{2}\left|-2+8+(-6)\right|=\tfrac{1}{2}\left|0\right|=0$$

Step 2 — Interpret the result: Area $=0\Rightarrow$ the three points are collinear (they lie on the line $y=2x+1$).

Why other options are wrong:

- **Option B:** Area $= 4$ is incorrect; the computed area is 0.
- **Option C:** An isosceles triangle cannot be formed if area is 0.
- **Option D:** Slope of $AB = \frac{5-3}{2-1} = 2$, not 3.

Final Answer: The points are collinear $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 13

Q14.

Solution

Concept — Intercept Form: A line with x-intercept a and y-intercept b has equation $\dfrac{x}{a} + \dfrac{y}{b} = 1$.

Step 1 — Write the intercept form:

$$\frac{x}{3} + \frac{y}{4} = 1$$

Step 2 — Convert to standard form: Multiply throughout by 12:

$$4x + 3y = 12$$

Why other options are wrong:

- **Option A:** $3x + 4y = 12$ swaps the roles of a and b (intercepts reversed).
- **Option B:** $\frac{x}{4} + \frac{y}{3} = 1$ implies x-intercept 4 and y-intercept 3, which are swapped.
- **Option D:** $x + y = 7$ gives intercepts $(7, 0)$ and $(0, 7)$, not $(3, 0)$ and $(0, 4)$.

Final Answer: $4x + 3y = 12 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 14

Q15.

Solution

Concept — Chord of Contact: For circle $x^2 + y^2 = r^2$, the chord of contact from external point (x_1, y_1) is $xx_1 + yy_1 = r^2$.

Step 1 — Identify the parameters: Circle: $x^2 + y^2 = 5$ ($r^2 = 5$). External point: $(x_1, y_1) = (2, 1)$.

Step 2 — Apply the formula:

$$x(2) + y(1) = 5 \implies 2x + y = 5$$

Why other options are wrong:

- **Option A:** $x + 2y = 5$ swaps the coefficients of x and y (uses $x_1 = 1, y_1 = 2$).
- **Option C:** $2x - y = 5$ has a wrong sign on y.
- **Option D:** $x - 2y = 5$ uses incorrect coefficient signs.

Final Answer: $2x + y = 5 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 15

Q16.

Solution

Concept — Tangent to Parabola: For $y^2 = 4ax$, the tangent at the parametric point $(at^2, 2at)$ is $ty = x + at^2$.

Step 1 — Derive by implicit differentiation: Differentiating $y^2 = 4ax$ gives $2y\,\frac{dy}{dx} = 4a$, so $\frac{dy}{dx} = \frac{2a}{y} = \frac{2a}{2at} = \frac{1}{t}$.

Step 2 — Write the tangent equation: At $(at^2, 2at)$ with slope $\frac{1}{t}$:

$$y - 2at = \frac{1}{t}(x - at^2) \implies ty - 2at^2 = x - at^2 \implies ty = x + at^2$$

Why other options are wrong:

- **Option A:** $ty = x - at^2$ has the wrong sign; gives $ty - 2at^2 = x + at^2$, inconsistent.
- **Option B:** $ty = 2x + at^2$ has coefficient 2 on x, which is incorrect.
- **Option C:** $ty = x - 2at^2$ has both the sign and coefficient of at^2 wrong.

Final Answer: $ty = x + at^2 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 16

Q17.

Solution

Concept — Centroid Formula: The centroid of a triangle with vertices (x_1, y_1), (x_2, y_2), (x_3, y_3) is $G = \left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$.

Step 1 — Sum the coordinates:

$$x_1 + x_2 + x_3 = 1 + 3 + 5 = 9; \qquad y_1 + y_2 + y_3 = 2 + 4 + 6 = 12$$

Step 2 — Divide by 3:

$$G = \left(\frac{9}{3}, \frac{12}{3}\right) = (3,\ 4)$$

Why other options are wrong:

- **Option B:** $(2, 3)$ would require coordinate sums of 6 and 9; not the case.
- **Option C:** $(4, 5)$ would require sums 12 and 15.
- **Option D:** $(9, 12)$ is the sum of coordinates, not divided by 3.

Final Answer: Centroid $= (3,\ 4) \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 17

Q18.

Solution

Concept — Cayley–Hamilton Theorem: Every square matrix satisfies its own characteristic equation.

Step 1 — Find the characteristic equation of A:

$$\det(A - \lambda I) = (1-\lambda)(4-\lambda) - 6 = \lambda^2 - 5\lambda - 2 = 0$$

(since $\mathrm{tr}(A) = 5$ and $\det(A) = 4 - 6 = -2$).

Step 2 — By Cayley–Hamilton, A satisfies this:

$$A^2 - 5A - 2I = \mathbf{0}$$

Verification: $A^2 = \begin{pmatrix} 7 & 10 \\ 15 & 22 \end{pmatrix}$, $5A = \begin{pmatrix} 5 & 10 \\ 15 & 20 \end{pmatrix}$, $2I = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$; difference is $\mathbf{0}$ ✓

Why other options are wrong:

- **Option A:** $A^2 - 5A + 2I = \mathbf{0}$ has wrong sign on the constant term (det is -2, not $+2$).
- **Option B:** $+5A$ contradicts the trace sign in the characteristic equation.
- **Option C:** $+5A + 2I$ has both signs wrong.

Final Answer: $A^2 - 5A - 2I = \mathbf{0} \Rightarrow \boxed{\text{D}}$

$\boxed{\textbf{Answer: (D)}}$ Go Back to Q 18

Q19.

Solution

Concept — Scalar Multiplication of Determinant: For an $n \times n$ matrix A and scalar k: $\det(kA) = k^n \det(A)$.

Step 1 — Reason: Multiplying every row of A by k multiplies the determinant by k each time. For a 3×3 matrix, all 3 rows are scaled, so:

$$\det(kA) = k \cdot k \cdot k \cdot \det(A) = k^3 \det(A)$$

Step 2 — Example check: Let $A = I_3$ so $\det(A) = 1$. Then $\det(kI_3) = k^3$. Indeed kI_3 is a diagonal matrix with all diagonal entries k, and $\det(kI_3) = k \cdot k \cdot k = k^3$ ✓

Why other options are wrong:

- **Option A:** $k \det(A)$ is the rule for scaling a single row, not the whole matrix.
- **Option C:** $3k \det(A)$ confuses scalar multiplication with addition of rows.
- **Option D:** $k^2 \det(A)$ would apply to a 2×2 matrix, not 3×3.

Final Answer: $\det(kA) = k^3 \det(A) \Rightarrow \boxed{\text{B}}$

$\boxed{\textbf{Answer: (B)}}$ Go Back to Q 19

Q20.

Solution

Concept — Rank by Row Reduction: The rank of a matrix equals the number of non-zero rows in its row echelon form.

Step 1 — Observe the row relationships:

$$R_2 = 2R_1 \quad \text{and} \quad R_3 = 3R_1$$

All three rows are proportional to the first row $(1, 2, 3)$.

Step 2 — Row reduce:

$$\xrightarrow{R_2-2R_1,\ R_3-3R_1} \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Only one non-zero row remains, so $\text{rank}(M) = 1$.

Why other options are wrong:

- **Option A:** Rank 3 would require 3 linearly independent rows; all are proportional here.
- **Option B:** Rank 2 would require 2 linearly independent rows; R_2 and R_3 are both multiples of R_1.
- **Option D:** Rank 0 would mean a zero matrix; the first row is non-zero.

Final Answer: $\text{rank}(M) = 1 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 20

Q21.

Solution

Concept — Skew-Symmetric Matrix: A is skew-symmetric if $A^T = -A$, which implies $a_{ij} = -a_{ji}$ for all i, j.

Step 1 — Diagonal elements: Setting $i = j$:

$$a_{ii} = -a_{ii} \implies 2a_{ii} = 0 \implies a_{ii} = 0$$

Every diagonal element must be zero.

Step 2 — Confirm with example: $\begin{pmatrix} 0 & -b \\ b & 0 \end{pmatrix}$ is skew-symmetric; diagonal entries are 0 ✓

Why other options are wrong:

- **Option B:** $\det(A)$ can be 0 (e.g., any odd-order skew-symmetric matrix has $\det = 0$).
- **Option C:** Skew-symmetric matrices of any order exist (both even and odd orders).
- **Option D:** $(A^2)^T = (A^T)^2 = (-A)^2 = A^2$, so A^2 is *symmetric*, not skew-symmetric.

Final Answer: All diagonal elements are zero $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 21

Q22.

Solution

Concept — Weighted Mean: $\bar{x}_w = \frac{\sum w_i x_i}{\sum w_i}$ where w_i are the weights.

Step 1 — Compute the weighted sum:

$$\sum w_i x_i = 1(60) + 2(70) + 3(80) + 4(90) = 60 + 140 + 240 + 360 = 800$$

Step 2 — Divide by total weight:

$$\sum w_i = 1 + 2 + 3 + 4 = 10; \qquad \bar{x}_w = \frac{800}{10} = 80$$

Why other options are wrong:

- **Option A:** 75 is the simple (unweighted) mean of 60 and 90; ignores middle values and weights.
- **Option B:** 78 results from an arithmetic error in the weighted sum.
- **Option D:** 82 overestimates by giving too much weight to high scores.

Final Answer: Weighted mean $= 80 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 22

Q23.

Solution

Concept — Mutually Exclusive Events: If $A \cap B = \varnothing$, then $P(A \cap B) = 0$ and $P(A \cup B) = P(A) + P(B)$.

Step 1 — Confirm mutual exclusivity: The problem states the events are mutually exclusive, so $P(A \cap B) = 0$.

Step 2 — Apply the addition law:

$$P(A \cup B) = P(A) + P(B) = 0.3 + 0.5 = 0.8$$

Why other options are wrong:

- **Option A:** $0.15 = P(A) \times P(B)$, which applies to *independent* events, not mutually exclusive.
- **Option C:** 0.85 uses $P(A \cup B) = P(A) + P(B) + 0.05$; unjustified.
- **Option D:** $0.65 = P(B) - P(A \cap B')$; an incorrect interpretation.

Final Answer: $P(A \cup B) = 0.8 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 23

Q24.

Solution

Concept — Binomial Distribution: For $X \sim B(n, p)$, Mean $= np$ and Variance $= npq$ where $q = 1 - p$.

Step 1 — Identify the parameters: $n = 20$, $p = 0.4$, $q = 1 - 0.4 = 0.6$.

Step 2 — Compute the variance:

$$\text{Variance} = npq = 20 \times 0.4 \times 0.6 = 4.8$$

Why other options are wrong:

- **Option A:** $8 = np$ is the *mean* of the distribution, not the variance.
- **Option B:** $4 = nq = 20 \times 0.2$; an incorrect formula.
- **Option C:** 6 does not correspond to any standard formula for this distribution.

Final Answer: Variance $= 4.8 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 24

Q25.

Solution

Concept — Pearson's Correlation Coefficient: r measures the linear relationship between two variables; $-1 \leq r \leq +1$.

Step 1 — Interpret $r = +1$: $r = +1$ means a perfect positive linear relationship: as one variable increases, the other increases proportionally with no scatter about the regression line.

Step 2 — Distinguish from other values:

- $r = 0$: no linear correlation.
- $r = -1$: perfect negative correlation.
- $0 < r < 1$: partial positive correlation.

Why other options are wrong:

- **Option B:** $r = 0$ indicates *no* linear relationship.
- **Option C:** $r = -1$ indicates perfect *negative* (inverse) correlation.
- **Option D:** $r = 0.5$ indicates moderate positive correlation, not perfect.

Final Answer: $r = +1 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 25

Q26.

Solution

Concept — Quartile Deviation: $QD = \dfrac{Q_3 - Q_1}{2}$, where Q_1 and Q_3 are the lower and upper quartiles.

Step 1 — Locate Q_1 and Q_3: Sorted data: $2, 4, 6, 8, 10, 12, 14$ ($n = 7$).

$$Q_1 = \text{median of lower half } \{2, 4, 6\} = 4$$

$$Q_3 = \text{median of upper half } \{10, 12, 14\} = 12$$

Step 2 — Compute QD:

$$QD = \frac{Q_3 - Q_1}{2} = \frac{12 - 4}{2} = \frac{8}{2} = 4$$

Why other options are wrong:

- **Option A:** 3 would require $Q_3 - Q_1 = 6$, but the interquartile range is 8.
- **Option B:** 5 would require $Q_3 - Q_1 = 10$; incorrect quartile positions.
- **Option D:** $6 = Q_3 - Q_1$ itself (the IQR), not divided by 2.

Final Answer: $QD = 4 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 26

Q27.

Solution

Concept — Pythagorean Identity: $\sin^2\theta + \cos^2\theta = 1$; in the first quadrant, all trig ratios are positive.

Step 1 — Apply the identity:

$$\cos^2\theta = 1 - \sin^2\theta = 1 - \left(\frac{5}{13}\right)^2 = 1 - \frac{25}{169} = \frac{144}{169}$$

Step 2 — Take the positive square root (first quadrant, $\cos\theta > 0$):

$$\cos\theta = \frac{12}{13}$$

Why other options are wrong:

- **Option A:** $\frac{5}{13}$ is $\sin\theta$, not $\cos\theta$.
- **Option C:** $\frac{13}{12}$ is $\sec\theta = 1/\cos\theta$.
- **Option D:** $\frac{13}{5}$ is $\csc\theta = 1/\sin\theta$.

Final Answer: $\cos\theta = \frac{12}{13} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 27

Q28.

Solution

Concept — Triple Angle Formula: $\sin 3A = 3\sin A - 4\sin^3 A$.

Step 1 — Substitute $A = 30°$:

$$\sin(3 \times 30°) = \sin 90° = 1$$

Step 2 — Verify using the formula: With $\sin 30° = \frac{1}{2}$:

$$3\sin 30° - 4\sin^3 30° = 3 \cdot \frac{1}{2} - 4 \cdot \frac{1}{8} = \frac{3}{2} - \frac{1}{2} = 1\checkmark$$

Why other options are wrong:

- **Option B:** $\frac{\sqrt{3}}{2} = \sin 60°$; this would be $\sin(3 \times 20°)$.
- **Option C:** $\frac{1}{2} = \sin 30°$; mistakenly returns A instead of $3A$.
- **Option D:** $0 = \sin 0°$ or $\sin 180°$; not applicable here.

Final Answer: $\sin 90° = 1 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 28

Q29.

Solution

Concept — Inverse Cosine Identity: For $x \in [-1, 1]$: $\cos^{-1}(x) + \cos^{-1}(-x) = \pi$.

Step 1 — Direct substitution with $x = \frac{1}{2}$:

$$\cos^{-1}\left(\tfrac{1}{2}\right) + \cos^{-1}\left(-\tfrac{1}{2}\right) = \pi$$

Step 2 — Verify by direct evaluation:

$$\cos^{-1}\left(\tfrac{1}{2}\right) = \frac{\pi}{3}; \qquad \cos^{-1}\left(-\tfrac{1}{2}\right) = \frac{2\pi}{3}$$

$$\frac{\pi}{3} + \frac{2\pi}{3} = \frac{3\pi}{3} = \pi\checkmark$$

Why other options are wrong:

- **Option A:** $\frac{\pi}{2}$ is $\cos^{-1}(0)$; the given identity is not applicable for that value.
- **Option B:** $\frac{\pi}{3}$ is only $\cos^{-1}(1/2)$; the second term is omitted.

- **Option C:** $\frac{2\pi}{3}$ is only $\cos^{-1}(-1/2)$; the first term is omitted.

Final Answer: $\cos^{-1}(1/2) + \cos^{-1}(-1/2) = \pi \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 29

Q30.

Solution

Concept — Law of Cosines: In any triangle, $a^2 = b^2 + c^2 - 2bc\cos A$.

Step 1 — Substitute the given values ($A = 60°$, $b = 5$, $c = 7$, $\cos 60° = \frac{1}{2}$):

$$a^2 = 5^2 + 7^2 - 2(5)(7)\cos 60° = 25 + 49 - 70 \cdot \frac{1}{2} = 74 - 35 = 39$$

Step 2 — Solve for a:

$$a = \sqrt{39}$$

Why other options are wrong:

- **Option A:** $\sqrt{29}$ arises from $74 - 45 = 29$, using $2bc\cos 60° = 45$ incorrectly.
- **Option B:** $\sqrt{34}$ comes from $74 - 40 = 34$; an incorrect value of the cosine term.
- **Option D:** $\sqrt{44}$ comes from $74 - 30 = 44$; uses $\cos 60° = \frac{3}{7}$ incorrectly.

Final Answer: $a = \sqrt{39} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 30

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	A	3	B	4	D	5	C
6	A	7	B	8	D	9	C	10	A
11	B	12	D	13	A	14	C	15	B
16	D	17	A	18	D	19	B	20	C
21	A	22	C	23	B	24	D	25	A
26	C	27	B	28	A	29	D	30	C