MAH MCA CET Mathematics & Statistics

Sample Paper – 5

Duration: 27 Minutes **Maximum Marks: 60**

Instructions

- This paper contains **30** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics & Statistics portion of **MAH MCA CET** entrance.
- Each correct answer carries **+2 marks**. There is **no negative marking** for incorrect or unattempted answers.
- Only **one** option is correct. Choose carefully.
- Syllabus level: **Class 12 level — Algebra, Calculus, Coordinate Geometry, Matrices, Statistics & Probability, Trigonometry.**
- Use of mobile phones, calculators, or electronic gadgets is strictly prohibited.

Q1. An arithmetic progression has first term $a = 5$ and common difference $d = 3$. Find the 20th term a_{20}:

(A) 56

(B) 62

(C) 65

(D) 59

Q2. Evaluate $\log_3(81) - \log_3(9)$:

(A) 1

(B) 3

(C) 2

(D) 4

Q3. Find the coefficient of x^2 in the expansion of $(x+2)^4$:

(A) 24

(B) 16

(C) 8

(D) 32

Q4. For the quadratic equation $x^2 - 7x + 10 = 0$, find the sum of the squares of the roots:

(A) 49

(B) 10

(C) 21

(D) 29

Q5. How many 4-digit numbers can be formed using the digits $\{1, 2, 3, 4, 5\}$ with no repetition?

(A) 60

(B) 120

(C) 24

(D) 240

Q6. Find the value of $\binom{10}{4}$:

(A) 840

(B) 252

(C) 210

(D) 120

Q7. Using the product rule, find $\frac{d}{dx}[x \ln x]$:

(A) $1 + \ln x$

(B) $\ln x$

(C) $\dfrac{1}{x}$

(D) $x + \ln x$

Q8. Find $\dfrac{d}{dx}\left[e^{\sin x}\right]$:

(A) $e^{\sin x}$

(B) $\cos x \cdot e^{x}$

(C) $e^{\cos x}$

(D) $\cos x \cdot e^{\sin x}$

Q9. Evaluate $\lim\limits_{x\to\infty}\left(1+\dfrac{2}{x}\right)^{x}$:

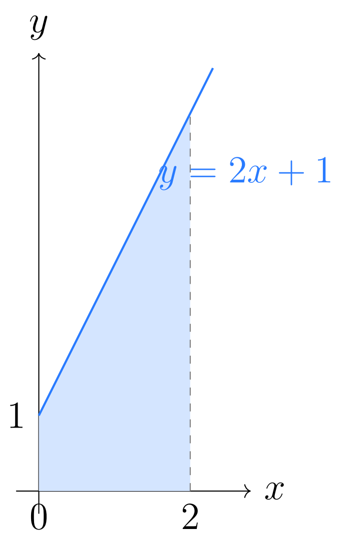
(A) 2

(B) e^2

(C) e

(D) ∞

Q10. The shaded region below represents the area under $y = 2x+1$ from $x = 0$ to $x = 2$.



Find the value of $\displaystyle\int_0^2 (2x+1)\,dx$:

(A) 4

(B) 8

(C) 6

(D) 5

Q11. Under exponential growth $P = P_0 e^{kt}$, a population doubles in 10 years. The growth rate k is:

(A) $\dfrac{\ln 2}{10}$

(B) $\ln 2$

(C) $\dfrac{10}{\ln 2}$

(D) $\dfrac{2}{10}$

Q12. Find the local maximum value of $f(x) = 2x^3 - 9x^2 + 12x - 4$:

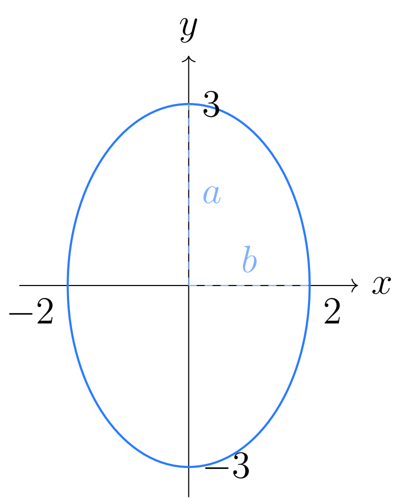
(A) 4

(B) 2

(C) -2

(D) 1

Q13. The ellipse $\dfrac{x^2}{4} + \dfrac{y^2}{9} = 1$ is shown below.



Identify the length of the semi-major axis of the ellipse:

(A) 2

(B) 3

(C) $\sqrt{13}$

(D) 6

Q14. Find the slope of the line joining the origin $O(0,0)$ to the midpoint M of the segment joining $A(2,6)$ and $B(4,10)$:

(A) 2

(B) 3

(C) $\frac{8}{3}$

(D) $\frac{4}{3}$

Q15. Find the equation of the tangent to the circle $x^2 + y^2 = 25$ at the point $(3,\ 4)$:

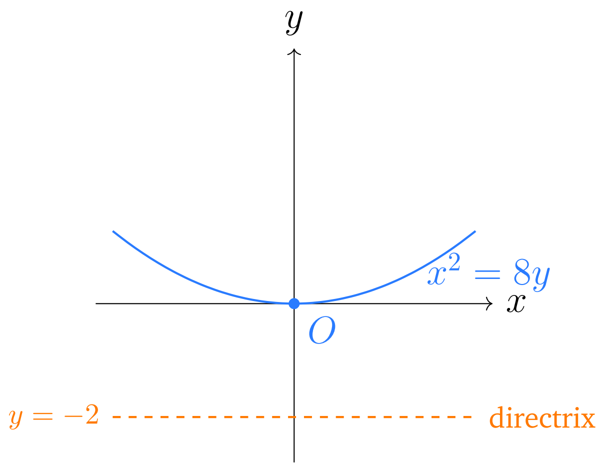
(A) $3x + 4y = 25$

(B) $4x + 3y = 25$

(C) $3x + 4y = 0$

(D) $3x - 4y = 25$

Q16. The parabola $x^2 = 8y$ is shown below with its directrix.



Find the directrix of the parabola $x^2 = 8y$:

(A) $y = 2$

(B) $x = -2$

(C) $x = 2$

(D) $y = -2$

Q17. Find the locus of a point P equidistant from $A(2, 0)$ and $B(-2, 0)$:

(A) $y = 0$

(B) $x = 0$

(C) $x + y = 0$

(D) $x^2 + y^2 = 4$

Q18. Find the minor M_{12} of the matrix shown below (the $(1, 2)$ entry is highlighted):

$$\begin{vmatrix} 1 & \mathbf{2} & 3 \\ 0 & 4 & 5 \\ 1 & 0 & 6 \end{vmatrix}$$

(A) 5

(B) -4

(C) -5

(D) 3

Q19. If A is a symmetric matrix, which of the following is always true?

(A) $A^T = A$

(B) $A^T = -A$

(C) $A^2 = I$

(D) $\det(A) = 0$

Q20. Find the inverse of the matrix $M = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}$:

(A) $\begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}$

(B) $\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}$

(C) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(D) $\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}$

Q21. Solve the system of equations: $3x + 2y = 7$ and $x + y = 3$:

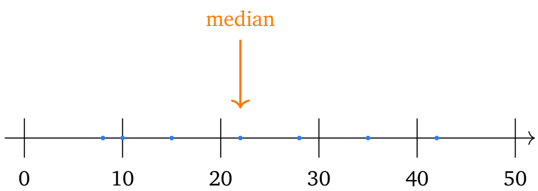
(A) $x = 2,\ y = 1$

(B) $x = 1,\ y = 2$

(C) $x = 3,\ y = 0$

(D) $x = 0,\ y = 3$

Q22. The data set $\{15, 22, 35, 8, 42, 10, 28\}$ is plotted on the number line below. Find its median.



(A) 15

(B) 28

(C) 22

(D) 35

Q23. Given $P(A \cap B) = 0.30$ and $P(B) = 0.50$, find $P(A \mid B)$:

(A) 0.60

(B) 0.50

(C) 0.15

(D) 0.80

Q24. For a Poisson random variable with parameter $\lambda = 2$, find $P(X = 0)$:

(A) 0.3679

(B) 0.2707

(C) 0.50

(D) e^{-2}

Q25. If the regression coefficient of y on x is $b_{yx} = 0.8$ and the regression coefficient of x on y is $b_{xy} = 0.2$, find the correlation coefficient r:

(A) 0.2

(B) 0.4

(C) 0.8

(D) 0.16

Q26. Find the standard deviation of the data set $\{10, 20, 30, 40, 50\}$:

(A) 10

(B) 100

(C) $10\sqrt{2}$

(D) 20

Q27. Using the identity $\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$, evaluate $\tan 75$:

(A) $2 + \sqrt{3}$

(B) $2 - \sqrt{3}$

(C) $\sqrt{3}$

(D) $\dfrac{1}{\sqrt{3}}$

Q28. Using the identity $\cos 30 = 1 - 2\sin^2 15$, find the exact value of $\sin^2 15$:

(A) $\dfrac{2+\sqrt{3}}{4}$

(B) $\dfrac{\sqrt{3}}{4}$

(C) $\dfrac{1}{4}$

(D) $\dfrac{2-\sqrt{3}}{4}$

Q29. The range (principal value branch) of $\cos^{-1}(x)$ is:

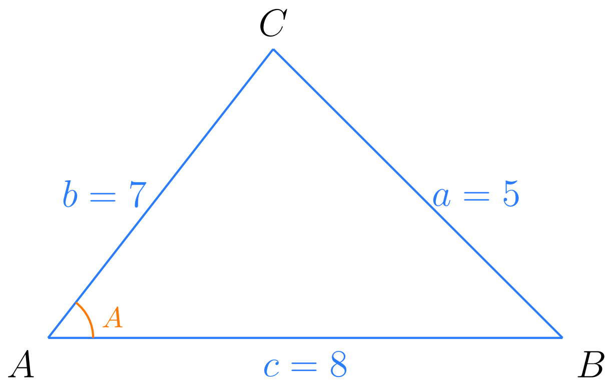
(A) $\left[-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right]$

(B) $[0, \pi]$

(C) $(-\pi, \pi)$

(D) $[0, 2\pi]$

Q30. In the triangle below, sides $a = 5$, $b = 7$, $c = 8$ are marked. Using the cosine rule $\cos A = \dfrac{b^2+c^2-a^2}{2bc}$, find $\cos A$ (the angle opposite side $a = 5$).



(A) $\dfrac{5}{7}$

(B) $\dfrac{7}{8}$

(C) $\dfrac{11}{14}$

(D) $\dfrac{1}{2}$

Detailed Solutions

Q1.

Solution

Concept — Arithmetic Progression (General Term): The nth term of an AP with first term a and common difference d is $a_n = a + (n-1)d$.

Step 1 — Identify parameters: Here $a = 5$, $d = 3$, $n = 20$.

Step 2 — Apply formula:

$$a_{20} = 5 + (20-1) \times 3 = 5 + 19 \times 3 = 5 + 57 = 62.$$

Why other options are wrong:

- Option A (56): Corresponds to $n = 18$: $5 + 17 \times 3 = 56$; uses the wrong term index.
- Option C (65): This is the 21st term: $5 + 20 \times 3 = 65$; off by one in n.
- Option D (59): This is the 19th term: $5 + 18 \times 3 = 59$; another off-by-one error.

Final Answer: $a_{20} = 62 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 1

Q2.

Solution

Concept — Logarithm Difference Law: $\log_b m - \log_b n = \log_b\left(\frac{m}{n}\right)$, and $\log_b b^k = k$.

Step 1 — Evaluate each term: $\log_3 81 = \log_3 3^4 = 4$ and $\log_3 9 = \log_3 3^2 = 2$.

Step 2 — Subtract:

$$\log_3 81 - \log_3 9 = 4 - 2 = 2.$$

Why other options are wrong:

- Option A (1): Incorrectly evaluates $\log_3 81 = 3$ (confusing with $\log_{81} 3$).
- Option B (3): Misidentifies $\log_3 9 = 1$ instead of 2; gives $4 - 1 = 3$.
- Option D (4): Reports only $\log_3 81 = 4$ without subtracting $\log_3 9$.

Final Answer: $\log_3 81 - \log_3 9 = 2 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 2

Q3.

Solution

Concept — Binomial Theorem (General Term): The $(r+1)$th term in the expansion of $(x+a)^n$ is $T_{r+1} = \binom{n}{r} x^{n-r} a^r$.

Step 1 — Identify the x^2 term: For $(x+2)^4$, $n = 4$. For $x^{n-r} = x^2$, we need $n - r = 2$, so $r = 2$.

Step 2 — Compute the coefficient:

$$T_3 = \binom{4}{2} x^2 \cdot 2^2 = 6 \cdot 4 \cdot x^2 = 24x^2.$$

Why other options are wrong:

- Option B (16): Uses $2^4 = 16$ as if all four factors of 2 appear; ignores $\binom{4}{2}$.
- Option C (8): Computes $\binom{4}{2} \cdot 2^1 = 6 \cdot 2 = 12$ or uses $r = 3$ giving $\binom{4}{3} \cdot 2 = 8$.
- Option D (32): Uses $\binom{4}{2} \cdot 2^3 = 6 \cdot 8 = 48$ (wrong r) or $2^5 = 32$ (index error).

Final Answer: Coefficient of $x^2 = 24 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 3

Q4.

Solution

Concept — Sum of Squares of Roots via Vieta's Formulae: For $x^2 + px + q = 0$, $\alpha + \beta = -p$ and $\alpha\beta = q$. Then $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$.

Step 1 — Identify sum and product: For $x^2 - 7x + 10 = 0$: $\alpha + \beta = 7$ and $\alpha\beta = 10$. (Roots are 2 and 5.)

Step 2 — Compute $\alpha^2 + \beta^2$:

$$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta = 49 - 20 = 29.$$

Why other options are wrong:

- Option A (49): Gives only $(\alpha+\beta)^2 = 7^2 = 49$; forgets to subtract $2\alpha\beta = 20$.
- Option B (10): Gives the product $\alpha\beta = 10$, not the sum of squares.

- Option C (21): Uses $3\alpha\beta$ instead of $2\alpha\beta$: $49 - 28 = 21$.

Final Answer: $\alpha^2 + \beta^2 = 29 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 4

Q5.

Solution

Concept — Permutations (without repetition): The number of ways to fill r positions from n distinct digits (no repetition) is $P(n, r) = \dfrac{n!}{(n-r)!}$.

Step 1 — Identify parameters: $n = 5$ (available digits), $r = 4$ (digit positions).

Step 2 — Calculate:

$$P(5, 4) = \frac{5!}{1!} = 5 \times 4 \times 3 \times 2 = 120.$$

Why other options are wrong:

- Option A (60): This is $P(5, 4)/2$; halves the answer without reason.
- Option C (24): This is $4! = P(4, 4)$; uses only 4 digits from 4, ignoring the 5th.
- Option D (240): This is $5 \times 4 \times 4 \times 3$; incorrectly allows repetition in one slot.

Final Answer: $P(5, 4) = 120 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 5

Q6.

Solution

Concept — Combinations Formula: $\dbinom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$.

Step 1 — Apply with $n = 10$, $r = 4$:

$$\binom{10}{4} = \frac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = \frac{5040}{24} = 210.$$

Why other options are wrong:

- Option A (840): This is $P(10, 4) = 10\times9\times8\times7 = 5040$; actually $840 = 5040/6$, divides by $3!$ instead of $4!$.

- Option B (252): This is $\binom{10}{5}$, not $\binom{10}{4}$.
- Option D (120): This is $\binom{10}{3} = \frac{10 \times 9 \times 8}{6} = 120$.

Final Answer: $\binom{10}{4} = 210 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 6

Q7.

Solution

Concept — Product Rule: If $u(x)$ and $v(x)$ are differentiable, then $\frac{d}{dx}[uv] = u'v + uv'$.

Step 1 — Differentiate each factor: Let $u = x \Rightarrow u' = 1$; let $v = \ln x \Rightarrow v' = \frac{1}{x}$.

Step 2 — Apply the rule:

$$\frac{d}{dx}[x \ln x] = 1 \cdot \ln x + x \cdot \frac{1}{x} = \ln x + 1.$$

Why other options are wrong:

- Option B ($\ln x$): Forgets the second term $x \cdot (1/x) = 1$; treats x as a constant.
- Option C ($1/x$): Only differentiates $\ln x$; ignores that x also varies.
- Option D ($x + \ln x$): Incorrectly writes $v' = 1$ (derivative of x) instead of $1/x$.

Final Answer: $\frac{d}{dx}[x \ln x] = 1 + \ln x \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 7

Q8.

Solution

Concept — Chain Rule: If $y = e^{f(x)}$, then $\frac{dy}{dx} = f'(x) \cdot e^{f(x)}$.

Step 1 — Identify the inner function: $f(x) = \sin x$, so $f'(x) = \cos x$.

Step 2 — Apply the chain rule:

$$\frac{d}{dx}\left[e^{\sin x}\right] = \cos x \cdot e^{\sin x}.$$

Why other options are wrong:

- Option A ($e^{\sin x}$): Forgets to multiply by the inner derivative $\cos x$.
- Option B ($\cos x \cdot e^x$): Replaces $e^{\sin x}$ with e^x; the exponent is $\sin x$, not x.
- Option C ($e^{\cos x}$): Differentiates $\sin x \to \cos x$ but then substitutes it as the new exponent rather than as a multiplier.

Final Answer: $\dfrac{d}{dx}\left[e^{\sin x}\right] = \cos x \cdot e^{\sin x} \Rightarrow \boxed{\text{D}}$

$\boxed{\textbf{Answer: (D)}}$ **Go Back to Q 8**

Q9.

Solution

Concept — Standard Limit: $\lim_{x\to\infty}\left(1+\dfrac{1}{x}\right)^x = e$. More generally, $\lim_{x\to\infty}\left(1+\dfrac{k}{x}\right)^x = e^k$.

Step 1 — Identify k: Comparing $\left(1+\dfrac{2}{x}\right)^x$ with the standard form, $k = 2$.

Step 2 — Apply the result:

$$\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^x = e^2.$$

Why other options are wrong:

- Option A (2): Confuses the constant $k = 2$ with the limit value e^2.
- Option C (e): Would arise from $k = 1$, i.e. $(1+1/x)^x \to e$; here $k = 2$.
- Option D (∞): Incorrect; the limit converges to the finite value $e^2 \approx 7.39$.

Final Answer: $\lim_{x\to\infty}\left(1+\dfrac{2}{x}\right)^x = e^2 \Rightarrow \boxed{\text{B}}$

$\boxed{\textbf{Answer: (B)}}$ **Go Back to Q 9**

Q10.

Solution

Concept — Definite Integral (Fundamental Theorem of Calculus): $\int_a^b f(x)\,dx = \big[F(x)\big]_a^b = F(b) - F(a)$.

Step 1 — Find the antiderivative:

$$\int (2x+1)\,dx = x^2 + x + C.$$

Step 2 — Evaluate at the bounds:

$$\int_0^2 (2x+1)\,dx = \big[x^2 + x\big]_0^2 = (4+2) - (0+0) = 6.$$

Why other options are wrong:

- Option A (4): Integrates only $2x$ and evaluates $[x^2]_0^2 = 4$; omits the $+1$ term.
- Option B (8): Uses $f(2) \times 2 = 5 \times 2 = 10$ (wrong method) or $2(2)^2 = 8$ (wrong antiderivative).
- Option D (5): Simply evaluates $f(2) = 2(2) + 1 = 5$, the function value, not the integral.

Final Answer: $\int_0^2 (2x+1)\,dx = 6 \Rightarrow \boxed{\text{C}}$

Answer: (C) **Go Back to Q 10**

Q11.

Solution

Concept — Exponential Growth Model: $P(t) = P_0 e^{kt}$. The population doubles when $P(t) = 2P_0$.

Step 1 — Set up the doubling condition at $t = 10$:

$$2P_0 = P_0\,e^{10k} \implies e^{10k} = 2.$$

Step 2 — Solve for k:

$$10k = \ln 2 \implies k = \frac{\ln 2}{10}.$$

Why other options are wrong:

- Option B ($\ln 2$): This corresponds to a doubling time of 1 year, not 10.
- Option C ($10/\ln 2$): This is the reciprocal; it equals the doubling time divided by $(\ln 2)^2$, not k.
- Option D ($2/10$): Confuses a linear ratio with the exponential growth rate.

Final Answer: $k = \dfrac{\ln 2}{10} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 11

Q12.

Solution

Concept — Maxima and Minima (First/Second Derivative Test): Stationary points satisfy $f'(x) = 0$; $f''(x) < 0$ indicates a local maximum.

Step 1 — Differentiate and find stationary points:

$$f'(x) = 6x^2 - 18x + 12 = 6(x-1)(x-2) = 0 \implies x = 1 \text{ or } x = 2.$$

Step 2 — Classify with $f''(x) = 12x - 18$: $f''(1) = -6 < 0$ (local max); $f''(2) = 6 > 0$ (local min).

Step 3 — Find the local maximum value:

$$f(1) = 2(1)^3 - 9(1)^2 + 12(1) - 4 = 2 - 9 + 12 - 4 = 1.$$

Why other options are wrong:

- Option A (4): This is $f(-1) = -2 - 9 - 12 - 4$; not relevant, or an arithmetic error.
- Option B (2): Partial evaluation; misses the -9 term.
- Option C (-2): This is $f(2) = 16 - 36 + 24 - 4 = 0$ (local min value is 0, not -2).

Final Answer: Local maximum value $= 1 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 12

Q13.

Solution

Concept — Standard Ellipse: For $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ with $a > b > 0$, the semi-major axis has length a (along the y-axis when $a > b$).

Step 1 — Read off a^2 and b^2: Comparing $\frac{x^2}{4} + \frac{y^2}{9} = 1$: $b^2 = 4 \Rightarrow b = 2$ and $a^2 = 9 \Rightarrow a = 3$.

Step 2 — Identify semi-major axis: Since $a = 3 > b = 2$, the major axis lies along the y-axis with semi-major length $\mathbf{a = 3}$.

Why other options are wrong:

- Option A (2): This is the semi-*minor* axis $b = 2$ along the x-axis.
- Option C ($\sqrt{13}$): Not derived from this ellipse; $c = \sqrt{9-4} = \sqrt{5}$, not $\sqrt{13}$.
- Option D (6): This is the full major axis length $2a = 6$, not the semi-major axis.

Final Answer: Semi-major axis $= 3 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 13

Q14.

Solution

Concept — Midpoint then Slope: First find the midpoint M of AB, then compute the slope of the line from the origin $O(0,0)$ to M.

Step 1 — Find midpoint M of $A(2,6)$ and $B(4,10)$:

$$M = \left(\frac{2+4}{2},\ \frac{6+10}{2}\right) = (3,\ 8).$$

Step 2 — Slope of OM:

$$\text{slope} = \frac{8-0}{3-0} = \frac{8}{3}.$$

Why other options are wrong:

- Option A (2): This is the slope of AB itself: $\frac{10-6}{4-2} = 2$, not the slope of OM.
- Option B (3): Uses only the x-coordinate of M; not a slope.
- Option D (4/3): Inverts the slope: run/rise instead of rise/run.

Final Answer: Slope of $OM = \frac{8}{3} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 14

Q15.

Solution

Concept — Tangent to a Circle at a Given Point: For the circle $x^2 + y^2 = r^2$, the tangent at point (x_1, y_1) is $xx_1 + yy_1 = r^2$ (using $T = 0$).

Step 1 — Verify the point lies on the circle: $3^2 + 4^2 = 9 + 16 = 25 = r^2$ ✓.

Step 2 — Write the tangent:

$$x(3) + y(4) = 25 \implies 3x + 4y = 25.$$

Why other options are wrong:

- Option B $(4x + 3y = 25)$: Swaps x_1 and y_1; gives the tangent at $(4, 3)$, not $(3, 4)$.
- Option C $(3x + 4y = 0)$: Sets the right-hand side to 0 instead of $r^2 = 25$.
- Option D $(3x - 4y = 25)$: Uses a minus sign; this is the tangent at $(3, -4)$.

Final Answer: Tangent: $3x + 4y = 25 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 15

Q16.

Solution

Concept — Parabola $x^2 = 4ay$ (upward-opening): The directrix is the horizontal line $y = -a$.

Step 1 — Compare with standard form:

$$x^2 = 8y \implies 4a = 8 \implies a = 2.$$

Step 2 — State the directrix: Directrix $= y = -a = -2$.

Why other options are wrong:

- Option A $(y = 2)$: This is the ordinate of the focus $y = a = 2$, not the directrix.

- Option B ($x = -2$): The directrix of the right-opening parabola $y^2 = 8x$ is $x = -2$, not $x^2 = 8y$.
- Option C ($x = 2$): A vertical line; the directrix of an upward-opening parabola is always horizontal.

Final Answer: Directrix: $y = -2 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 16

Q17.

Solution

Concept — Locus as Perpendicular Bisector: The set of all points equidistant from two fixed points is the perpendicular bisector of the segment joining them.

Step 1 — Set $|PA|^2 = |PB|^2$ for $P(x, y)$:

$$(x-2)^2 + y^2 = (x+2)^2 + y^2.$$

Step 2 — Expand and simplify:

$$x^2 - 4x + 4 = x^2 + 4x + 4 \implies -8x = 0 \implies x = 0.$$

Why other options are wrong:

- Option A ($y = 0$): This is the x-axis on which A and B lie; not equidistant from them.
- Option C ($x + y = 0$): Check: point $(-1, 1)$; $|PA|^2 = 9 + 1 = 10$, $|PB|^2 = 1 + 1 = 2$ — not equidistant.
- Option D ($x^2 + y^2 = 4$): A circle; equidistance from two distinct points gives a line, not a circle.

Final Answer: Locus: $x = 0 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 17

Q18.

Solution

Concept — Minor of a Matrix Entry: The minor M_{ij} is the determinant of the submatrix formed by deleting row i and column j.

Step 1 — Delete row 1 and column 2 from $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 1 & 0 & 6 \end{pmatrix}$:

$$\text{Remaining submatrix} = \begin{pmatrix} 0 & 5 \\ 1 & 6 \end{pmatrix}.$$

Step 2 — Compute the 2×2 determinant:

$$M_{12} = \begin{vmatrix} 0 & 5 \\ 1 & 6 \end{vmatrix} = 0 \cdot 6 - 5 \cdot 1 = 0 - 5 = -5.$$

Why other options are wrong:

- Option A (5): Takes the absolute value $|-5| = 5$; minors retain their sign.
- Option B (-4): Deletes the wrong row or uses the wrong entries in the submatrix.
- Option D (3): Confuses M_{12} with the $(1, 3)$ entry or a different minor.

Final Answer: $M_{12} = -5 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 18

Q19.

Solution

Concept — Symmetric Matrix: A square matrix A is symmetric if and only if it equals its own transpose: $A^T = A$ (equivalently, $A_{ij} = A_{ji}$ for all i, j).

Step 1 — Evaluate each option:

- $A^T = A$: the defining condition for symmetry — **always true**.
- $A^T = -A$: defines a *skew-symmetric* matrix — not symmetric.
- $A^2 = I$: defines an involutory matrix; symmetric matrices need not satisfy this (e.g. $\begin{pmatrix} 2 & 0 \\ 0 & 3 \end{pmatrix}$).
- $\det(A) = 0$: symmetric matrices can be non-singular (e.g. the identity I).

Why other options are wrong:

- Option B: $A^T = -A$ is the definition of *anti*-symmetry (skew-symmetry).
- Option C: Symmetry places no restriction on whether $A^2 = I$ holds.
- Option D: $\det(I) = 1 \neq 0$ yet I is symmetric; the determinant can be anything.

Final Answer: $A^T = A \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 19

Q20.

Solution

Concept — Inverse of a 2×2 Matrix: For $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $M^{-1} = \frac{1}{\det M}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.

Step 1 — Compute the determinant:

$$\det(M) = 2 \cdot 1 - 1 \cdot 1 = 1.$$

Step 2 — Apply the inverse formula:

$$M^{-1} = \frac{1}{1}\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix}.$$

Verification: $M \cdot M^{-1} = \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \checkmark.$

Why other options are wrong:

- Option A: Off-diagonal sign is wrong; $\begin{pmatrix} 1 & 1 \\ -1 & 2 \end{pmatrix}$ fails the check.
- Option B: Keeps original diagonal $\{2, 1\}$ instead of swapping to $\{1, 2\}$.
- Option C (I): The identity is the inverse only when $M = I$.

Final Answer: $M^{-1} = \begin{pmatrix} 1 & -1 \\ -1 & 2 \end{pmatrix} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 20

Q21.

Solution

Concept — Substitution Method for Linear Systems: Express one variable from one equation and substitute into the other.

Step 1 — From the second equation: $x + y = 3 \Rightarrow x = 3 - y$.

Step 2 — Substitute into the first equation:

$$3(3 - y) + 2y = 7 \implies 9 - 3y + 2y = 7 \implies 9 - y = 7 \implies y = 2.$$

Step 3 — Back-substitute: $x = 3 - 2 = 1$.

Why other options are wrong:

- Option A $(x = 2, y = 1)$: Swaps the values; check: $3(2) + 2(1) = 8 \neq 7$.
- Option C $(x = 3, y = 0)$: Check: $3(3) + 2(0) = 9 \neq 7$.
- Option D $(x = 0, y = 3)$: Check: $3(0) + 2(3) = 6 \neq 7$.

Final Answer: $x = 1,\ y = 2 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 21

Q22.

Solution

Concept — Median of a Data Set: Arrange values in ascending order; the median is the middle value when n is odd.

Step 1 — Sort the data:

$$8, \quad 10, \quad 15, \quad \mathbf{22}, \quad 28, \quad 35, \quad 42.$$

Step 2 — Locate the middle value: $n = 7$ (odd); the median is the $\frac{7+1}{2} = 4$th value $= \mathbf{22}$.

Why other options are wrong:

- Option A (15): This is the 3rd value in sorted order, not the 4th (median).
- Option B (28): This is the 5th value; one position after the median.
- Option D (35): This is the 6th value; two positions after the median.

Final Answer: Median $= 22 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 22

Q23.

Solution

Concept — Conditional Probability: $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$, provided $P(B) \neq 0$.

Step 1 — Substitute the given values:

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.30}{0.50} = 0.60.$$

Why other options are wrong:

- Option B (0.50): This is $P(B)$ alone; the formula requires dividing $P(A \cap B)$ by $P(B)$.
- Option C (0.15): Computes $P(A \cap B) \times P(B) = 0.30 \times 0.50$; multiplies instead of divides.
- Option D (0.80): No valid arithmetic from the given values produces 0.80.

Final Answer: $P(A \mid B) = 0.60 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 23

Q24.

Solution

Concept — Poisson Distribution: For $X \sim \text{Poisson}(\lambda)$, $P(X = k) = \dfrac{e^{-\lambda}\lambda^k}{k!}$.

Step 1 — Apply formula with $\lambda = 2$, $k = 0$:

$$P(X = 0) = \frac{e^{-2} \cdot 2^0}{0!} = \frac{e^{-2} \cdot 1}{1} = e^{-2}.$$

Step 2 — Note: $e^{-2} \approx 0.1353$, which is an exact closed-form answer.

Why other options are wrong:

- Option A (0.3679): This is $e^{-1} \approx 0.368$, corresponding to $\lambda = 1$, not $\lambda = 2$.
- Option B (0.2707): This is $P(X = 1) = e^{-2} \cdot 2 \approx 0.271$ for $\lambda = 2$; wrong k.
- Option C (0.50): Not derivable from the Poisson formula; a probability value, not $P(X = 0)$ here.

Final Answer: $P(X=0)=e^{-2} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 24

Q25.

Solution

Concept — Correlation Coefficient from Regression Coefficients: $r^2 = b_{yx} \cdot b_{xy}$, and r has the same sign as the regression coefficients (positive here).

Step 1 — Compute r^2:

$$r^2 = b_{yx} \cdot b_{xy} = 0.8 \times 0.2 = 0.16.$$

Step 2 — Find r:

$$r = \sqrt{0.16} = 0.4.$$

Why other options are wrong:

- Option A (0.2): This is b_{xy}, one regression coefficient, not r.
- Option C (0.8): This is b_{yx}, the other regression coefficient, not r.
- Option D (0.16): This is r^2; taking the square root gives $r = 0.4$, not 0.16.

Final Answer: $r = 0.4 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 25

Q26.

Solution

Concept — Standard Deviation: $\sigma = \sqrt{\dfrac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n}}$.

Step 1 — Find the mean:

$$\bar{x} = \frac{10+20+30+40+50}{5} = \frac{150}{5} = 30.$$

Step 2 — Compute squared deviations:

$$(10-30)^2+(20-30)^2+(30-30)^2+(40-30)^2+(50-30)^2 = 400+100+0+100+400 = 1000.$$

Step 3 — Variance and standard deviation:

$$\text{Variance} = \frac{1000}{5} = 200. \qquad \sigma = \sqrt{200} = 10\sqrt{2}.$$

Why other options are wrong:

- Option A (10): $10^2 = 100 \neq 200$; underestimates the variance.
- Option B (100): This is $\sqrt{10000}$; confuses $\sqrt{\text{variance}}$ with variance itself (times 5).
- Option D (20): $20^2 = 400 \neq 200$; overestimates.

Final Answer: $\sigma = 10\sqrt{2} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 26

Q27.

Solution

Concept — Tangent Addition Formula: $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B}$.

Step 1 — Write $75 = 45 + 30$ and use known values:

$$\tan 75 = \frac{\tan 45 + \tan 30}{1 - \tan 45 \cdot \tan 30} = \frac{1 + \frac{1}{\sqrt{3}}}{1 - \frac{1}{\sqrt{3}}}.$$

Step 2 — Rationalise by multiplying numerator and denominator by $\sqrt{3}$:

$$\tan 75 = \frac{\sqrt{3}+1}{\sqrt{3}-1} = \frac{(\sqrt{3}+1)^2}{(\sqrt{3})^2 - 1^2} = \frac{3 + 2\sqrt{3} + 1}{2} = \frac{4 + 2\sqrt{3}}{2} = 2 + \sqrt{3}.$$

Why other options are wrong:

- Option B $(2 - \sqrt{3})$: This is $\tan 15 = \cot 75$, the reciprocal of $\tan 75$.
- Option C $(\sqrt{3})$: This is $\tan 60$.
- Option D $(1/\sqrt{3})$: This is $\tan 30$.

Final Answer: $\tan 75 = 2 + \sqrt{3} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 27

Q28.

Solution

Concept — Half-Angle / Double-Angle Identity: $\cos\theta = 1 - 2\sin^2\left(\frac{\theta}{2}\right)$. Setting $\theta = 30$ gives $\cos 30 = 1 - 2\sin^2 15$.

Step 1 — Substitute $\cos 30 = \frac{\sqrt{3}}{2}$:

$$\frac{\sqrt{3}}{2} = 1 - 2\sin^2 15.$$

Step 2 — Solve for $\sin^2 15$:

$$2\sin^2 15 = 1 - \frac{\sqrt{3}}{2} = \frac{2-\sqrt{3}}{2} \implies \sin^2 15 = \frac{2-\sqrt{3}}{4}.$$

Why other options are wrong:

- Option A ($(2+\sqrt{3})/4$): This is $\cos^2 15$ via the identity $\cos 30 = 2\cos^2 15 - 1$.
- Option B ($\sqrt{3}/4$): Drops the constant 1; computes $(\sqrt{3}/2)/2$ only.
- Option C ($1/4$): Incorrectly uses $\cos 30 = 1/2$ (which is actually $\cos 60$).

Final Answer: $\sin^2 15 = \frac{2-\sqrt{3}}{4} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 28

Q29.

Solution

Concept — Range of Inverse Trigonometric Functions: The principal value branch of $\cos^{-1}(x)$ is chosen so the function is one-to-one. Its range is $[0,\ \pi]$.

Step 1 — Recall the definition: $y = \cos^{-1}(x)$ means $\cos y = x$ with y restricted to $[0, \pi]$. For example: $\cos^{-1}(1) = 0$, $\cos^{-1}(0) = \pi/2$, $\cos^{-1}(-1) = \pi$.

Why other options are wrong:

- Option A ($[-\pi/2,\ \pi/2]$): This is the range of $\sin^{-1}(x)$, not $\cos^{-1}(x)$.
- Option C ($(-\pi,\ \pi)$): Open interval and wrong bounds; $\cos^{-1}$ endpoints are included and the range is $[0, \pi]$.
- Option D ($[0,\ 2\pi]$): Too wide; $\cos$ is not one-to-one on $[0,\ 2\pi]$.

Final Answer: Range of $\cos^{-1}(x)$ is $[0,\ \pi] \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 29

Q30.

Solution

Concept — Cosine Rule: In a triangle with sides a, b, c opposite angles A, B, C:

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}.$$

Step 1 — Substitute $a = 5$, $b = 7$, $c = 8$:

$$\cos A = \frac{7^2 + 8^2 - 5^2}{2 \cdot 7 \cdot 8} = \frac{49 + 64 - 25}{112} = \frac{88}{112}.$$

Step 2 — Simplify the fraction:

$$\frac{88}{112} = \frac{11}{14}.$$

Why other options are wrong:

- Option A (5/7): Uses $a/(b) = 5/7$; this is just a ratio of sides, not $\cos A$.
- Option B (7/8): Uses $b/c = 7/8$; again a ratio of two sides, not the cosine rule result.
- Option D (1/2): Would imply $A = 60$; check: $49+64-25 = 88 \neq 56 = 112\times\frac{1}{2}$.

Final Answer: $\cos A = \dfrac{11}{14} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 30

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	C	3	A	4	D	5	B
6	C	7	A	8	D	9	B	10	C
11	A	12	D	13	B	14	C	15	A
16	D	17	B	18	C	19	A	20	D
21	B	22	C	23	A	24	D	25	B
26	C	27	A	28	D	29	B	30	C