MAH MCA CET Mathematics & Statistics

Sample Paper – 6

Duration: 27 Minutes **Maximum Marks: 60**

Instructions

- This paper contains **30** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics & Statistics portion of **MAH MCA CET** entrance.
- Each correct answer carries **+2 marks**. There is **no negative marking** for incorrect or unattempted answers.
- Only **one** option is correct. Choose carefully.
- Syllabus level: **Class 12 level — Algebra, Calculus, Coordinate Geometry, Matrices, Statistics & Probability, Trigonometry.**
- Use of mobile phones, calculators, or electronic gadgets is strictly prohibited.

Q1. Find the sum of the first 5 terms of a geometric progression with first term $a = 2$ and common ratio $r = 3$:

(A) 50

(B) 120

(C) 180

(D) 242

Q2. Evaluate $\log_{10}(10000)$:

(A) 4

(B) 3

(C) 5

(D) 40

Q3. Find the coefficient of x^2 in the expansion of $(2x-1)^3$:

(A) 12

(B) -12

(C) -6

(D) 6

Q4. Form a quadratic equation with roots $3+\sqrt{2}$ and $3-\sqrt{2}$:

(A) $x^2-6x-7=0$

(B) $x^2+6x+7=0$

(C) $x^2-6x+7=0$

(D) $x^2+6x-7=0$

Q5. In how many ways can 2 books be chosen from 8 different books and arranged on a shelf (order matters)?

(A) 28

(B) 16

(C) 64

(D) 56

Q6. Find the value of n such that $\binom{n}{2}=15$:

(A) 6

(B) 5

(C) 7

(D) 8

Q7. Find $\frac{d}{dx}\left[\sin^2 x\right]$:

(A) $2\cos x$

(B) $\sin 2x$

(C) $2\sin x$

(D) $\cos 2x$

Q8. Using the quotient rule, find $\dfrac{d}{dx}\left[\dfrac{x^2+1}{x-1}\right]$:

(A) $\dfrac{x^2+2x+1}{(x-1)^2}$

(B) $\dfrac{x^2-1}{(x-1)^2}$

(C) $\dfrac{x^2-2x-1}{(x-1)^2}$

(D) $\dfrac{2x}{(x-1)^2}$

Q9. Evaluate $\lim\limits_{x\to 0}\dfrac{e^x-1}{x}$:

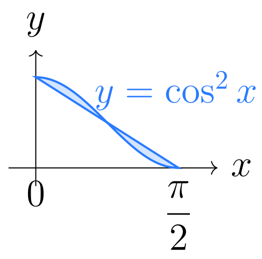
(A) e

(B) 0

(C) ∞

(D) 1

Q10. The shaded region below represents the area under $y=\cos^2 x$ from 0 to $\dfrac{\pi}{2}$.



Find the value of $\displaystyle\int_0^{\pi/2} \cos^2 x\, dx$:

(A) $\dfrac{\pi}{4}$

(B) $\dfrac{\pi}{2}$

(C) $\frac{1}{2}$

(D) 1

Q11. Find the general solution of the differential equation $\frac{dy}{dx} = 2y$:

(A) $y = Ce^x$

(B) $y = Ce^{2x}$

(C) $y = 2x + C$

(D) $y = Ce^{-2x}$

Q12. Find the point of inflection of $f(x) = x^3 - 6x^2 + 9x + 2$:

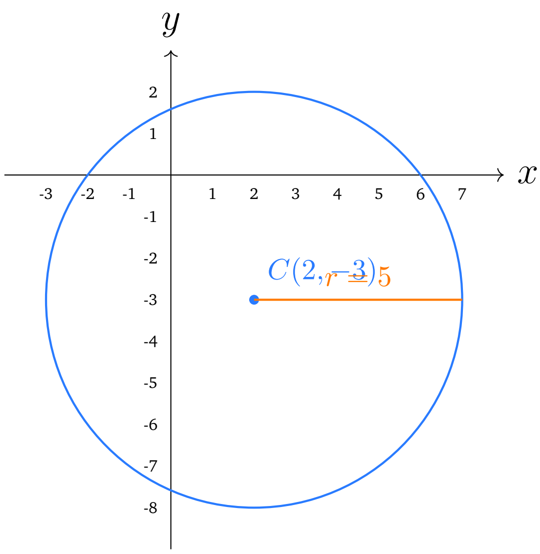
(A) $(1, 6)$

(B) $(3, 2)$

(C) $(2, 4)$

(D) $(0, 2)$

Q13. The diagram shows a circle with centre $C(2, -3)$ and radius $r = 5$.



The equation of the circle shown is:

(A) $x^2 + y^2 - 4x + 6y + 12 = 0$

(B) $(x + 2)^2 + (y - 3)^2 = 25$

(C) $x^2 + y^2 + 4x - 6y = 25$

(D) $x^2 + y^2 - 4x + 6y - 12 = 0$

Q14. Find the value of a such that the lines $2x+3y+5=0$ and $ax+6y+10=0$ are parallel:

(A) 4

(B) 3

(C) 2

(D) 6

Q15. Find k such that the lines $3x - 4y + 1 = 0$ and $kx + 3y - 2 = 0$ are perpendicular:

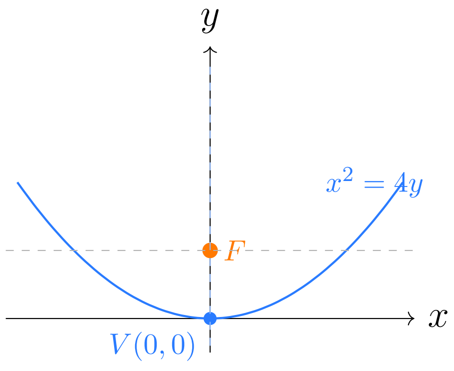
(A) 3

(B) 4

(C) -3

(D) -4

Q16. The diagram shows the parabola $x^2 = 4y$ with its vertex and focus marked.



For the parabola $x^2 = 4y$, identify the vertex and focus:

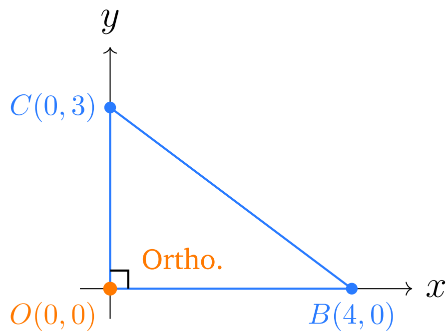
(A) Vertex $(0, 0)$, Focus $(1, 0)$

(B) Vertex $(0, 1)$, Focus $(0, 2)$

(C) Vertex $(0, 0)$, Focus $(0, 1)$

(D) Vertex $(0, 0)$, Focus $(0, 4)$

Q17. The triangle below has vertices $O(0, 0)$, $B(4, 0)$, and $C(0, 3)$. The right angle is at O.



The orthocenter of triangle OBC lies at:

(A) $(4,\ 0)$

(B) $(0,\ 3)$

(C) $\left(2,\ \frac{3}{2}\right)$

(D) $(0,\ 0)$

Q18. Consider the matrices:

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$$

Which of the following is always true for square matrices A and B of the same order?

(A) $\det(AB) = \det(A) \cdot \det(B)$

(B) $\det(A + B) = \det(A) + \det(B)$

(C) $\det(kA) = k \cdot \det(A)$ for an $n \times n$ matrix

(D) $\det(A^T) = -\det(A)$

Q19. If a square matrix A satisfies $A^2 = A$ (idempotent), then $(I - A)^2$ equals:

(A) $I + A$

(B) $I - A$

(C) A

(D) O

Q20. The **trace** of a square matrix is defined as:

(A) The sum of all elements of the matrix

(B) The product of all eigenvalues of the matrix

(C) The sum of the diagonal elements of the matrix

(D) The determinant of the matrix

Q21. Solve the system of equations $3x - y = 8$ and $x + 2y = 5$:

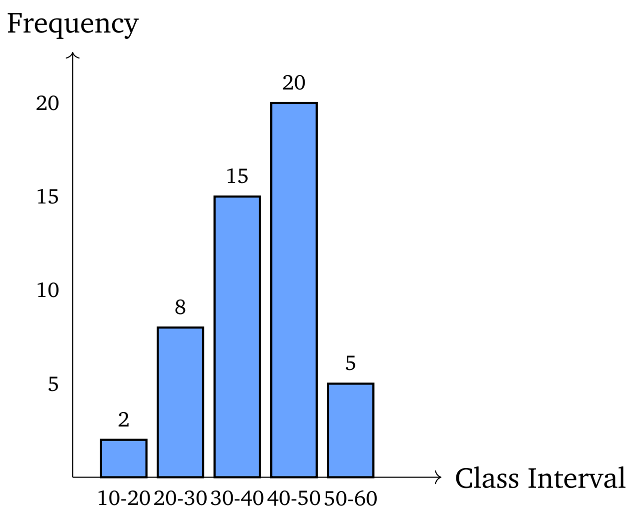
(A) $x = 1,\ y = 3$

(B) $x = 2,\ y = 2$

(C) $x = 4,\ y = 1$

(D) $x = 3,\ y = 1$

Q22. The bar chart below shows the frequency distribution of scores of 50 students.



Using the midpoint method, the mean score is:

(A) 38.6

(B) 35

(C) 40

(D) 42

Q23. A bag contains 5 red and 3 blue balls. Two balls are drawn without replacement. Find the probability that both balls are red:

(A) $\frac{5}{16}$

(B) $\frac{5}{14}$

(C) $\frac{25}{64}$

(D) $\frac{1}{4}$

Q24. For a geometric distribution with probability of success $p = \frac{1}{3}$, find $P(X = 3)$:

(A) $\frac{1}{9}$

(B) $\frac{2}{9}$

(C) $\frac{4}{27}$

(D) $\frac{8}{27}$

Q25. Which of the following values of the correlation coefficient r is **impossible**?

(A) $r = -1$

(B) $r = 0.99$

(C) $r = 0$

(D) $r = 1.5$

Q26. Dataset X has mean 50 and standard deviation 10; Dataset Y has mean 40 and standard deviation 12. Which dataset is more consistent?

(A) Dataset X

(B) Dataset Y

(C) Both equally consistent

(D) Cannot be determined

Q27. Find the general solution of $\sin\theta = \frac{1}{2}$:

(A) $2n\pi \pm \frac{\pi}{6}$

(B) $n\pi + (-1)^n \frac{\pi}{6}$

(C) $n\pi \pm \frac{\pi}{6}$

(D) $2n\pi \pm \frac{\pi}{3}$

Q28. Using the identity $2\sin A \cos B = \sin(A + B) + \sin(A - B)$, evaluate $2\sin 75 \cos 15$:

(A) $\sqrt{3}$

(B) $\frac{\sqrt{3}+1}{2}$

(C) $\frac{2+\sqrt{3}}{2}$

(D) 1

Q29. Find the domain of the function $f(x) = \sin^{-1}(2x - 1)$:

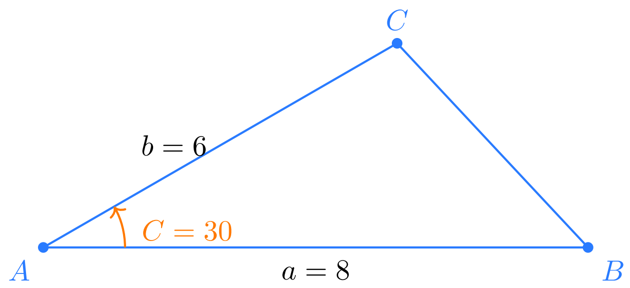
(A) $[-1,\ 1]$

(B) $[0,\ 2]$

(C) $\left[-\frac{1}{2},\ \frac{3}{2}\right]$

(D) $[0,\ 1]$

Q30. Two sides of a triangle are $a = 8$ and $b = 6$, with included angle $C = 30$ as shown.



Find the area of the triangle:

(A) 12

(B) 24

(C) $6\sqrt{3}$

(D) $8\sqrt{3}$

Detailed Solutions

Q1.

Solution

Concept — Sum of a Finite Geometric Progression: For a GP with first term a, common ratio $r \neq 1$, and n terms, the sum is $S_n = \dfrac{a(r^n - 1)}{r - 1}$.

Step 1 — Identify parameters: $a = 2$, $r = 3$, $n = 5$.

Step 2 — Apply formula:

$$S_5 = \frac{2(3^5 - 1)}{3 - 1} = \frac{2(243 - 1)}{2} = \frac{2 \times 242}{2} = 242.$$

Why other options are wrong:

- Option A (50): Adds the 5 terms incorrectly as $2 + 6 + 12 + 18 + 12 = 50$; errors in computing powers of 3.
- Option B (120): Confuses with $P(5, 3)$ or a partial product; $2 + 6 + 18 + 54 + 162 = 242$, not 120.
- Option C (180): Arithmetic error in computing $3^5 = 243$; uses $3^4 = 81$ instead.

Final Answer: $S_5 = 242 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 1

Q2.

Solution

Concept — Logarithm Definition: $\log_b(x) = n$ means $b^n = x$. In particular, $\log_{10}(10^n) = n$.

Step 1 — Rewrite 10000 as a power of 10:

$$10000 = 10^4.$$

Step 2 — Apply the logarithm:

$$\log_{10}(10000) = \log_{10}(10^4) = 4.$$

Why other options are wrong:

- Option B (3): $10^3 = 1000$, not 10000.
- Option C (5): $10^5 = 100000$, not 10000.
- Option D (40): Confuses $\log_{10}(10000) = 4$ with multiplying by 10; $4 \times 10 = 40$ is not the log.

Final Answer: $\log_{10}(10000) = 4 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 2

Q3.

Solution

Concept — Binomial Theorem General Term: The general term in the expansion of $(a+b)^n$ is $T_{r+1} = \binom{n}{r} a^{n-r} b^r$.

Step 1 — Set up with $a = 2x$, $b = -1$, $n = 3$:

$$T_{r+1} = \binom{3}{r} (2x)^{3-r} (-1)^r.$$

Step 2 — Find the x^2 term: set $3 - r = 2$, so $r = 1$:

$$T_2 = \binom{3}{1} (2x)^2 (-1)^1 = 3 \cdot 4x^2 \cdot (-1) = -12x^2.$$

Why other options are wrong:

- Option A (12): Correct magnitude but wrong sign; ignores $(-1)^r = -1$.
- Option C (-6): Uses $\binom{3}{1} \cdot 2 \cdot (-1) = -6$; forgets to square the 2 in $2x$.
- Option D (6): Positive sign and under-counts the coefficient.

Final Answer: Coefficient of x^2 is $-12 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 3

Q4.

Solution

Concept — Quadratic from Roots: If α and β are the roots, then $x^2 - (\alpha+\beta)x + \alpha\beta = 0$.

Step 1 — Compute sum and product of roots:

$$\alpha + \beta = (3+\sqrt{2}) + (3-\sqrt{2}) = 6.$$

$$\alpha\beta = (3+\sqrt{2})(3-\sqrt{2}) = 9 - 2 = 7.$$

Step 2 — Write the equation:

$$x^2 - 6x + 7 = 0.$$

Why other options are wrong:

- Option A ($x^2 - 6x - 7 = 0$): Product of roots should be $+7$, not -7.
- Option B ($x^2+6x+7 = 0$): Sum of roots should be $-(-6) = +6$, so coefficient of x is -6, not $+6$.
- Option D ($x^2 + 6x - 7 = 0$): Both signs of the middle and last terms are wrong.

Final Answer: $x^2 - 6x + 7 = 0 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 4

Q5.

Solution

Concept — Permutations $P(n,r)$**:** The number of ways to choose r objects from n and arrange them is $P(n,r) = \dfrac{n!}{(n-r)!}$.

Step 1 — Identify parameters: $n = 8$, $r = 2$ (2 books chosen and arranged).

Step 2 — Compute:

$$P(8,2) = \frac{8!}{(8-2)!} = \frac{8!}{6!} = 8 \times 7 = 56.$$

Why other options are wrong:

- Option A (28): This is $\binom{8}{2} = 28$; counts combinations (order ignored), not permutations.

- Option B (16): $8 \times 2 = 16$; wrong operation.
- Option C (64): $8^2 = 64$; treats choices as independent with replacement.

Final Answer: $P(8, 2) = 56 \Rightarrow \boxed{\text{D}}$

Answer: (D) **Go Back to Q 5**

Q6.

Solution

Concept — Combination Formula: $\binom{n}{2} = \dfrac{n(n-1)}{2}$.

Step 1 — Set up the equation:

$$\frac{n(n-1)}{2} = 15 \implies n(n-1) = 30.$$

Step 2 — Solve:

$$n^2 - n - 30 = 0 \implies (n-6)(n+5) = 0.$$

Since $n > 0$, we have $n = 6$.

Why other options are wrong:

- Option B (5): $\binom{5}{2} = 10 \neq 15$.
- Option C (7): $\binom{7}{2} = 21 \neq 15$.
- Option D (8): $\binom{8}{2} = 28 \neq 15$.

Final Answer: $n = 6 \Rightarrow \boxed{\text{A}}$

Answer: (A) **Go Back to Q 6**

Q7.

Solution

Concept — Chain Rule: If $y = [f(x)]^n$, then $\dfrac{dy}{dx} = n[f(x)]^{n-1} \cdot f'(x)$.

Step 1 — Identify $f(x) = \sin x$, $n = 2$**:**

$$\frac{d}{dx}[\sin^2 x] = 2 \sin x \cdot \cos x.$$

Step 2 — Apply double-angle identity:

$$2\sin x\cos x = \sin 2x.$$

Why other options are wrong:

- Option A ($2\cos x$): Forgets the factor of $\sin x$ from the chain rule.
- Option C ($2\sin x$): Uses $\frac{d}{dx}[\cos x] = \sin x$ incorrectly; the derivative of $\cos x$ is $-\sin x$.
- Option D ($\cos 2x$): This is $\frac{d}{dx}[\frac{1}{2}\sin 2x]$, not the derivative of $\sin^2 x$.

Final Answer: $\frac{d}{dx}[\sin^2 x] = \sin 2x \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 7

Q8.

Solution

Concept — Quotient Rule: $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{u'v - uv'}{v^2}$.

Step 1 — Identify $u = x^2 + 1$, $v = x - 1$; **compute** $u' = 2x$, $v' = 1$:

Step 2 — Apply the quotient rule:

$$\frac{d}{dx}\left[\frac{x^2+1}{x-1}\right] = \frac{2x(x-1) - (x^2+1)(1)}{(x-1)^2} = \frac{2x^2 - 2x - x^2 - 1}{(x-1)^2} = \frac{x^2 - 2x - 1}{(x-1)^2}.$$

Why other options are wrong:

- Option A: Expands $(x-1)^2 = x^2 - 2x + 1$ correctly but sign error yields $+2x + 1$ in numerator instead of $-2x - 1$.
- Option B: Uses $u' - uv' = 2x - (x^2 + 1)$; forgets to multiply u' by v.
- Option D ($2x/(x-1)^2$): Differentiates $x^2 + 1$ as $2x$ but omits the v factor and uv' term entirely.

Final Answer: $\frac{x^2 - 2x - 1}{(x-1)^2} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 8

Q9.

Solution

Concept — Standard Exponential Limit: $\lim_{x\to 0}\frac{e^x-1}{x}=1$. This is a fundamental result, derivable from the Taylor series $e^x = 1 + x + \frac{x^2}{2!} + \cdots$.

Step 1 — Apply the standard result directly:

$$\lim_{x\to 0}\frac{e^x-1}{x}=1.$$

Step 2 — Verification via Taylor expansion:

$$\frac{e^x-1}{x}=\frac{x+\frac{x^2}{2}+\cdots}{x}=1+\frac{x}{2}+\cdots \xrightarrow{x\to 0} 1.$$

Why other options are wrong:

- Option A (e): Confuses the limit of e^x as $x\to 0$ (which is $e^0 = 1$) with the value of e.
- Option B (0): Numerator $e^x - 1 \to 0$, but so does denominator $x \to 0$; the ratio is not 0.
- Option C (∞): Neither numerator nor denominator goes to infinity.

Final Answer: $\lim_{x\to 0}\frac{e^x-1}{x}=1 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 9

Q10.

Solution

Concept — Half-Angle Identity for Integration: Use $\cos^2 x = \frac{1+\cos 2x}{2}$ to reduce the power.

Step 1 — Substitute the identity:

$$\int_0^{\pi/2}\cos^2 x\,dx=\int_0^{\pi/2}\frac{1+\cos 2x}{2}\,dx=\frac{1}{2}\left[x+\frac{\sin 2x}{2}\right]_0^{\pi/2}.$$

Step 2 — Evaluate at bounds:

$$\frac{1}{2}\left[\left(\frac{\pi}{2}+\frac{\sin\pi}{2}\right)-\left(0+\frac{\sin 0}{2}\right)\right]=\frac{1}{2}\left[\frac{\pi}{2}+0-0\right]=\frac{\pi}{4}.$$

Why other options are wrong:

- Option B ($\pi/2$): This equals $\int_0^{\pi/2} 1\,dx$; forgets the reduction from $\cos^2 x$.
- Option C ($1/2$): Confuses with a different definite integral; does not involve π.
- Option D (1): Doubles Option C; neither is correct.

Final Answer: $\int_0^{\pi/2} \cos^2 x\,dx = \frac{\pi}{4} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 10

Q11.

Solution

Concept — Separation of Variables: Rewrite $\frac{dy}{dx} = 2y$ so all y terms are on one side and x terms on the other.

Step 1 — Separate variables:

$$\frac{dy}{y} = 2\,dx.$$

Step 2 — Integrate both sides:

$$\ln|y| = 2x + k \implies |y| = e^k e^{2x} \implies y = Ce^{2x},$$

where $C = \pm e^k$ is an arbitrary constant.

Why other options are wrong:

- Option A ($y = Ce^x$): Corresponds to $dy/dx = y$, not $dy/dx = 2y$.
- Option C ($y = 2x + C$): Would require $dy/dx = 2$, a constant, not $2y$.
- Option D ($y = Ce^{-2x}$): Corresponds to $dy/dx = -2y$; wrong sign.

Final Answer: $y = Ce^{2x} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 11

Q12.

Solution

Concept — Point of Inflection: A point of inflection occurs where $f''(x) = 0$ and f'' changes sign.

Step 1 — Compute $f''(x)$:

$$f(x) = x^3 - 6x^2 + 9x + 2.$$

$$f'(x) = 3x^2 - 12x + 9, \qquad f''(x) = 6x - 12.$$

Step 2 — Set $f''(x) = 0$:

$$6x - 12 = 0 \implies x = 2.$$

Step 3 — Find the y-coordinate:

$$f(2) = 8 - 24 + 18 + 2 = 4.$$

So the point of inflection is $(2, 4)$.

Why other options are wrong:

- Option A ($(1, 6)$): $x = 1$ is a stationary point ($f'(1) = 0$), not an inflection point.
- Option B ($(3, 2)$): $f''(3) = 6 \neq 0$; also $f(3) = 27 - 54 + 27 + 2 = 2$, but $x = 3$ is not where $f'' = 0$.
- Option D ($(0, 2)$): $f''(0) = -12 \neq 0$.

Final Answer: Point of inflection is $(2, 4) \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 12

Q13.

Solution

Concept — Standard Form of a Circle: A circle with centre (h, k) and radius r has equation $(x-h)^2+(y-k)^2 = r^2$, which expands to $x^2+y^2-2hx-2ky+(h^2+k^2-r^2) = 0$.

Step 1 — Identify parameters: Centre $= (2, -3)$, radius $= 5$.

Step 2 — Write the standard form:

$$(x-2)^2+(y+3)^2=25.$$

Step 3 — Expand:

$$x^2-4x+4+y^2+6y+9=25 \implies x^2+y^2-4x+6y+13=25 \implies x^2+y^2-4x+6y-12=0.$$

Why other options are wrong:

- Option A: Constant term should be -12, not $+12$.
- Option B ($(x+2)^2+(y-3)^2=25$): Gives centre $(-2,3)$, not $(2,-3)$.
- Option C ($x^2+y^2+4x-6y=25$): Signs of the linear terms are reversed; gives centre $(-2,3)$.

Final Answer: $x^2+y^2-4x+6y-12=0 \Rightarrow \boxed{\text{D}}$

Answer: (D) **Go Back to Q 13**

Q14.

Solution

Concept — Condition for Parallel Lines: Two lines $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$ are parallel if and only if $\frac{a_1}{a_2}=\frac{b_1}{b_2}\neq\frac{c_1}{c_2}$.

Step 1 — Identify coefficients: Line 1: $2x+3y+5=0$ ($a_1=2$, $b_1=3$). Line 2: $ax+6y+10=0$ ($a_2=a$, $b_2=6$).

Step 2 — Apply the parallel condition:

$$\frac{a_1}{a_2}=\frac{b_1}{b_2} \implies \frac{2}{a}=\frac{3}{6}=\frac{1}{2} \implies a=4.$$

Verification: With $a=4$: $\frac{c_1}{c_2}=\frac{5}{10}=\frac{1}{2}$. Since $\frac{a_1}{a_2}=\frac{b_1}{b_2}=\frac{c_1}{c_2}$, the lines are actually coincident when $a=4$ with these c values. However, the standard test checks the slope condition $a_1/a_2=b_1/b_2$, giving $a=4$.

Why other options are wrong:

- Option B (3): $2/3\neq 3/6$; slopes differ.
- Option C (2): $2/2=1\neq 3/6=1/2$; slopes differ.
- Option D (6): $2/6=1/3\neq 3/6=1/2$; slopes differ.

Final Answer: $a = 4 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 14

Q15.

Solution

Concept — Condition for Perpendicular Lines: Lines $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$ are perpendicular if $a_1a_2 + b_1b_2 = 0$, equivalently $m_1 \cdot m_2 = -1$.

Step 1 — Find slopes:

$$m_1 = -\frac{a_1}{b_1} = -\frac{3}{-4} = \frac{3}{4}, \qquad m_2 = -\frac{k}{3}.$$

Step 2 — Apply perpendicularity condition $m_1 \cdot m_2 = -1$:

$$\frac{3}{4} \cdot \left(-\frac{k}{3}\right) = -1 \implies -\frac{k}{4} = -1 \implies k = 4.$$

Why other options are wrong:

- Option A (3): $m_1 \cdot m_2 = (3/4)(-3/3) = -3/4 \neq -1$.
- Option C (-3): $m_1 \cdot m_2 = (3/4)(3/3) = 3/4 \neq -1$.
- Option D (-4): $m_1 \cdot m_2 = (3/4)(4/3) = 1 \neq -1$.

Final Answer: $k = 4 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 15

Q16.

Solution

Concept — Upward-Opening Parabola $x^2 = 4ay$: For this form, the vertex is at the origin $(0, 0)$ and the focus is at $(0, a)$.

Step 1 — Compare $x^2 = 4y$ with $x^2 = 4ay$:

$$4a = 4 \implies a = 1.$$

Step 2 — State vertex and focus:

$$\text{Vertex} = (0, 0), \qquad \text{Focus} = (0, a) = (0, 1).$$

Why other options are wrong:

- Option A: Focus $(1, 0)$ lies on the x-axis; that corresponds to a sideways parabola $y^2 = 4x$.
- Option B: Vertex $(0, 1)$ is wrong; the vertex is always at the origin for $x^2 = 4ay$.
- Option D: Focus $(0, 4)$ uses $4a = 4$ but incorrectly states focus $= (0, 4a) = (0, 4)$ instead of $(0, a) = (0, 1)$.

Final Answer: Vertex $(0, 0)$, Focus $(0, 1) \Rightarrow \boxed{\text{C}}$

Answer: (C) **Go Back to Q 16**

Q17.

Solution

Concept — Orthocenter of a Right Triangle: In a right-angled triangle, the orthocenter coincides with the vertex at which the right angle is located.

Step 1 — Identify the right angle: Triangle OBC has vertices $O(0, 0)$, $B(4, 0)$, $C(0, 3)$.

$$\overrightarrow{OB} = (4, 0), \quad \overrightarrow{OC} = (0, 3). \quad \overrightarrow{OB} \cdot \overrightarrow{OC} = 4 \cdot 0 + 0 \cdot 3 = 0.$$

So $OB \perp OC$; the right angle is at $O(0, 0)$.

Step 2 — State the orthocenter: Since the right angle is at O, the orthocenter is at $O(0, 0)$.

Why other options are wrong:

- Option A ($(4, 0)$): This is vertex B, not the orthocenter.
- Option B ($(0, 3)$): This is vertex C, not the orthocenter.
- Option C ($(2, 3/2)$): This is the centroid $\left(\frac{0+4+0}{3}, \frac{0+0+3}{3}\right) = (4/3, 1)$, also incorrect; $(2, 3/2)$ is neither centroid nor orthocenter.

Final Answer: Orthocenter $= (0, 0) \Rightarrow \boxed{\text{D}}$

Answer: (D) **Go Back to Q 17**

Q18.

Solution

Concept — Multiplicative Property of Determinants: For any two square matrices A and B of the same order, $\det(AB) = \det(A) \cdot \det(B)$.

Step 1 — Verify with the given matrices:

$$\det(A) = \begin{vmatrix} 2 & 1 \\ 3 & 4 \end{vmatrix} = 8 - 3 = 5.$$

$$\det(B) = \begin{vmatrix} 1 & 0 \\ 2 & 1 \end{vmatrix} = 1 - 0 = 1.$$

$$AB = \begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 4 & 1 \\ 11 & 4 \end{pmatrix}.$$

$$\det(AB) = 16 - 11 = 5 = \det(A) \cdot \det(B). \checkmark$$

Why other options are wrong:

- Option B ($\det(A + B) = \det(A) + \det(B)$): False in general; determinant is not a linear function of matrices.
- Option C ($\det(kA) = k \cdot \det(A)$): For an $n \times n$ matrix, $\det(kA) = k^n \det(A)$, not $k \cdot \det(A)$ unless $n = 1$.
- Option D ($\det(A^T) = -\det(A)$): False; in fact $\det(A^T) = \det(A)$ always.

Final Answer: $\det(AB) = \det(A) \cdot \det(B) \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 18

Q19.

Solution

Concept — Idempotent Matrix: A matrix satisfying $A^2 = A$ is called idempotent.

Step 1 — Expand $(I - A)^2$ using matrix algebra:

$$(I - A)^2 = I^2 - 2IA + A^2 = I - 2A + A^2.$$

Step 2 — Substitute $A^2 = A$:

$$(I - A)^2 = I - 2A + A = I - A.$$

Why other options are wrong:

- Option A $(I + A)$: Would require $A^2 = 3A$, not A.
- Option C (A): Would require $I - 2A + A = A$, i.e. $I = 2A$, which is not implied by $A^2 = A$.
- Option D (O): Would require $I - A = O$, i.e. $A = I$, but I is idempotent and $I - I = O$; this holds only in that special case, not in general.

Final Answer: $(I - A)^2 = I - A \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 19

Q20.

Solution

Concept — Trace of a Matrix: The trace of a square matrix A, denoted $\text{tr}(A)$, is defined as the sum of its main diagonal elements: $\text{tr}(A) = \sum_{i=1}^{n} a_{ii}$.

Step 1 — Example with a 3×3 matrix:

$$A = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{pmatrix} \implies \text{tr}(A) = 1 + 5 + 9 = 15.$$

Only the diagonal entries a_{11}, a_{22}, a_{33} are summed.

Why other options are wrong:

- Option A (sum of all elements): That would be $1 + 2 + \cdots + 9 = 45 \neq 15$; not the trace.
- Option B (product of eigenvalues): The product of all eigenvalues equals the *determinant*, not the trace. (The sum of eigenvalues equals the trace.)
- Option D (determinant): $\det(A) = 0$ for the example above; not equal to $\text{tr}(A) = 15$.

Final Answer: Trace = sum of diagonal elements $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 20

Q21.

Solution

Concept — Simultaneous Equations (Substitution/Elimination): Solve the system $3x - y = 8$ and $x + 2y = 5$.

Step 1 — Express x from the second equation:

$$x = 5 - 2y.$$

Step 2 — Substitute into the first equation:

$$3(5 - 2y) - y = 8 \implies 15 - 6y - y = 8 \implies 15 - 7y = 8 \implies y = 1.$$

Step 3 — Back-substitute to find x:

$$x = 5 - 2(1) = 3.$$

Verification: $3(3) - (1) = 9 - 1 = 8\checkmark$ and $3 + 2(1) = 5\checkmark$.

Why other options are wrong:

- Option A $(x = 1, y = 3)$: $3(1) - 3 = 0 \neq 8$.
- Option B $(x = 2, y = 2)$: $3(2) - 2 = 4 \neq 8$.
- Option C $(x = 4, y = 1)$: $4 + 2(1) = 6 \neq 5$.

Final Answer: $x = 3,\ y = 1 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 21

Q22.

Solution

Concept — Mean of Grouped Frequency Data: $\bar{x} = \dfrac{\sum f_i x_i}{\sum f_i}$, where x_i is the midpoint of each class.

Step 1 — Compute midpoints and $f_i x_i$:

Class	Midpoint x_i	Frequency f_i	$f_i x_i$
10–20	15	2	30
20–30	25	8	200
30–40	35	15	525
40–50	45	20	900
50–60	55	5	275
Total		50	1930

Step 2 — Compute mean:

$$\bar{x} = \frac{1930}{50} = 38.6.$$

Why other options are wrong:

- Option B (35): Uses only the midpoint of the modal class (30–40) as the mean.
- Option C (40): Uses only the midpoint of the highest frequency class (40–50).
- Option D (42): Arithmetic error in computing $\sum f_i x_i$.

Final Answer: Mean $= 38.6 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 22

Q23.

Solution

Concept — Probability Without Replacement: $P(\text{both red}) = \frac{5}{8} \times \frac{4}{7}$, since after drawing one red ball, only 4 red balls remain out of 7 total.

Step 1 — First draw: $P(\text{1st red}) = \frac{5}{8}$.

Step 2 — Second draw (given 1st red): $P(\text{2nd red} \mid \text{1st red}) = \frac{4}{7}$.

Step 3 — Multiply:

$$P(\text{both red}) = \frac{5}{8} \times \frac{4}{7} = \frac{20}{56} = \frac{5}{14}.$$

Why other options are wrong:

- Option A $(5/16)$: Uses $\frac{5}{8} \times \frac{4}{10}$; wrong total for second draw (uses 10 instead of 7).

- Option C $(25/64)$: Uses $(5/8)^2$; treats draws as independent (with replacement).
- Option D $(1/4)$: $1/4 = 14/56$; arrives at wrong numerator by arithmetic error.

Final Answer: $P(\text{both red}) = \dfrac{5}{14} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 23

Q24.

Solution

Concept — Geometric Distribution: $P(X = k) = (1-p)^{k-1} \cdot p$, where X is the trial number of the first success.

Step 1 — Identify parameters: $p = \dfrac{1}{3}$, $q = 1 - p = \dfrac{2}{3}$, $k = 3$.

Step 2 — Compute:

$$P(X = 3) = \left(\frac{2}{3}\right)^{3-1} \cdot \frac{1}{3} = \left(\frac{2}{3}\right)^{2} \cdot \frac{1}{3} = \frac{4}{9} \cdot \frac{1}{3} = \frac{4}{27}.$$

Why other options are wrong:

- Option A $(1/9)$: Uses $(1/3)^2 \cdot 1 = 1/9$; forgets to divide by 3 at the end.
- Option B $(2/9)$: Uses $(2/3)^1 \cdot (1/3) = 2/9$; computes $P(X = 2)$ instead.
- Option D $(8/27)$: Uses $(2/3)^3 = 8/27$; this is q^3 without multiplying by p.

Final Answer: $P(X = 3) = \dfrac{4}{27} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 24

Q25.

Solution

Concept — Range of Correlation Coefficient: The Pearson correlation coefficient r always satisfies $-1 \leq r \leq 1$ by the Cauchy–Schwarz inequality. Any value outside this interval is impossible.

Step 1 — Evaluate each option:

- $r = -1$: Possible (perfect negative linear correlation).
- $r = 0.99$: Possible (strong positive correlation, within $[-1, 1]$).

- $r = 0$: Possible (no linear correlation).
- $r = 1.5$: **Impossible**, since $1.5 > 1$ violates $|r| \le 1$.

Why other options are wrong:

- Options A, B, C are all valid values within $[-1, 1]$.

Final Answer: $r = 1.5$ is impossible $\Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 25

Q26.

Solution

Concept — Coefficient of Variation (CV): $\text{CV} = \frac{\sigma}{\bar{x}} \times 100\%$. The lower the CV, the more consistent (less variable) the dataset.

Step 1 — Compute CV for Dataset X:

$$\text{CV}_X = \frac{10}{50} \times 100 = 20\%.$$

Step 2 — Compute CV for Dataset Y:

$$\text{CV}_Y = \frac{12}{40} \times 100 = 30\%.$$

Step 3 — Compare: $\text{CV}_X = 20\% < \text{CV}_Y = 30\%$, so Dataset X is more consistent.

Why other options are wrong:

- Option B (Dataset Y): $\text{CV}_Y > \text{CV}_X$, so Y is less consistent.
- Option C (Both equal): $20\% \neq 30\%$.
- Option D (Cannot be determined): CV provides a clear, computable comparison.

Final Answer: Dataset X is more consistent $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 26

Q27.

Solution

Concept — General Solution of $\sin\theta = \sin\alpha$: The complete set of solutions is $\theta = n\pi + (-1)^n\alpha$, for $n \in \mathbb{Z}$.

Step 1 — Identify α: $\sin\theta = \frac{1}{2}$, so $\alpha = \frac{\pi}{6}$ (since $\sin\frac{\pi}{6} = \frac{1}{2}$).

Step 2 — Write the general solution:

$$\theta = n\pi + (-1)^n\frac{\pi}{6}, \quad n \in \mathbb{Z}.$$

Verification: $n = 0$: $\theta = \pi/6$✓; $n = 1$: $\theta = \pi - \pi/6 = 5\pi/6$, and $\sin(5\pi/6) = 1/2$✓.

Why other options are wrong:

- Option A ($2n\pi \pm \pi/6$): This is the general solution of $\cos\theta = \cos(\pi/6) = \sqrt{3}/2$, not $\sin\theta = 1/2$. It misses solutions like $\theta = 5\pi/6$.
- Option C ($n\pi \pm \pi/6$): Ignores the alternating sign; gives $\theta = \pi - \pi/6 = 5\pi/6$ but also $\theta = \pi + \pi/6 = 7\pi/6$ where $\sin(7\pi/6) = -1/2 \neq 1/2$.
- Option D ($2n\pi \pm \pi/3$): Corresponds to $\sin\theta = \sin(\pi/3) = \sqrt{3}/2$; wrong angle.

Final Answer: $\theta = n\pi + (-1)^n\frac{\pi}{6} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 27

Q28.

Solution

Concept — Product-to-Sum Formula: $2\sin A\cos B = \sin(A+B) + \sin(A-B)$.

Step 1 — Substitute $A = 75$, $B = 15$:

$$2\sin 75\cos 15 = \sin(75+15) + \sin(75-15) = \sin 90 + \sin 60.$$

Step 2 — Evaluate:

$$\sin 90 = 1, \qquad \sin 60 = \frac{\sqrt{3}}{2}.$$

$$2\sin 75\cos 15 = 1 + \frac{\sqrt{3}}{2} = \frac{2+\sqrt{3}}{2}.$$

Why other options are wrong:

- Option A ($\sqrt{3}$): Uses only $\sin 60 = \sqrt{3}/2$ and doubles it; omits $\sin 90$.

- Option B ($(\sqrt{3}+1)/2$): Substitutes $\sin 90 = 1$ as $1/2$; arithmetic error.
- Option D (1): Uses only $\sin 90 = 1$ and omits $\sin 60$ entirely.

Final Answer: $2 \sin 75 \cos 15 = \dfrac{2+\sqrt{3}}{2} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 28

Q29.

Solution

Concept — Domain of $\sin^{-1}$: The function $\sin^{-1}(u)$ is defined only for $-1 \le u \le 1$.

Step 1 — Set up the domain condition:

$$-1 \le 2x - 1 \le 1.$$

Step 2 — Solve the compound inequality:

$$-1 + 1 \le 2x \le 1 + 1 \implies 0 \le 2x \le 2 \implies 0 \le x \le 1.$$

So the domain is $[0, 1]$.

Why other options are wrong:

- Option A ($[-1, 1]$): This is the domain of $\sin^{-1}(x)$ itself, not of $\sin^{-1}(2x-1)$.
- Option B ($[0, 2]$): Solves $0 \le 2x - 1 \le 1$ (lower bound shifted wrong); correct working gives $[0, 1]$.
- Option C ($[-1/2, 3/2]$): Solves $-1 \le 2x - 1 \le 2$ instead of ≤ 1.

Final Answer: Domain $= [0, 1] \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 29

Q30.

Solution

Concept — Area of a Triangle (SAS Formula): Area $= \dfrac{1}{2}ab\sin C$, where a and b are two sides and C is the included angle.

Step 1 — Identify parameters: $a = 8$, $b = 6$, $C = 30$.

Step 2 — Apply formula:

$$\text{Area} = \frac{1}{2} \times 8 \times 6 \times \sin 30 = \frac{1}{2} \times 48 \times \frac{1}{2} = \frac{48}{4} = 12.$$

Why other options are wrong:

- Option B (24): Uses $\frac{1}{2} \times 8 \times 6 = 24$ without multiplying by $\sin 30$; assumes $C = 90$.
- Option C $(6\sqrt{3})$: Uses $\sin 60 = \sqrt{3}/2$ instead of $\sin 30 = 1/2$.
- Option D $(8\sqrt{3})$: Uses $\frac{1}{2} \times 8 \times 4 \times \frac{\sqrt{3}}{2}$; arithmetic error on the product.

Final Answer: Area $= 12 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 30

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	D	2	A	3	B	4	C	5	D
6	A	7	B	8	C	9	D	10	A
11	B	12	C	13	D	14	A	15	B
16	C	17	D	18	A	19	B	20	C
21	D	22	A	23	B	24	C	25	D
26	A	27	B	28	C	29	D	30	A