MAH MCA CET Mathematics & Statistics

Sample Paper – 7

Duration: 27 Minutes **Maximum Marks: 60**

Instructions

- This paper contains **30** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics & Statistics portion of **MAH MCA CET** entrance.
- Each correct answer carries **+2 marks**. There is **no negative marking** for incorrect or unattempted answers.
- Only **one** option is correct. Choose carefully.
- Syllabus level: **Class 12 level — Algebra, Calculus, Coordinate Geometry, Matrices, Statistics & Probability, Trigonometry.**
- Use of mobile phones, calculators, or electronic gadgets is strictly prohibited.

Q1. The first two terms of a Harmonic Progression (HP) are $\frac{1}{2}$ and $\frac{1}{3}$. Find the 4th term of the HP:

(A) $\frac{1}{4}$

(B) $\frac{1}{3}$

(C) $\frac{1}{5}$

(D) $\frac{1}{6}$

Q2. Using the change of base formula and natural logarithms, evaluate $\log_5 25$:

(A) $\ln 25$

(B) $\frac{\ln 5}{2}$

(C) $\frac{1}{2}$

(D) 2

Q3. Find the sum of all binomial coefficients in the expansion of $(1 + x)^6$:

(A) 64

(B) 32

(C) 128

(D) 63

Q4. Given $a + b = 10$ with $a, b > 0$, find the maximum value of the product ab:

(A) 50

(B) 25

(C) 20

(D) 100

Q5. Find the number of derangements of 3 distinct objects (arrangements where no object appears in its original position):

(A) 6

(B) 3

(C) 2

(D) 1

Q6. From 5 men and 4 women, how many committees of 3 persons can be formed containing exactly 2 men and 1 woman?

(A) 30

(B) 20

(C) 60

(D) 40

Q7. Given the parametric equations $x = t^2$ and $y = t^3$, find $\dfrac{dy}{dx}$:

(A) $\dfrac{3t}{2}$
(B) $\dfrac{2t}{3}$
(C) $\dfrac{3}{2}$
(D) $2t$

Q8. Which of the following functions satisfies **all** conditions of Rolle's Theorem on $[-1, 1]$?

(A) $f(x) = |x|$ (not differentiable at 0)

(B) $f(x) = 1 - x^2$

(C) $f(x) = \tan x$ (discontinuous on $[-1, 1]$)

(D) $f(x) = \dfrac{1}{x}$ (discontinuous at 0)

Q9. Using L'Hôpital's Rule, evaluate $\lim_{x\to 0} \dfrac{1 - \cos x}{x^2}$:

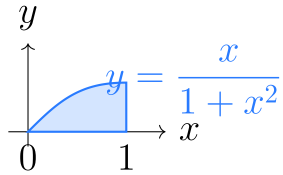
(A) 0

(B) 1

(C) $\dfrac{1}{2}$

(D) ∞

Q10. The shaded region below represents the area under $y = \dfrac{x}{1+x^2}$ from 0 to 1.



Evaluate $\displaystyle\int_0^1 \frac{x}{1+x^2}\,dx$:

(A) $\ln 2$

(B) $\dfrac{1}{2}$

(C) $\ln(\sqrt{2}+1)$

(D) $\dfrac{\ln 2}{2}$

Q11. Which of the following is a **homogeneous** differential equation?

(A) $\dfrac{dy}{dx} = \dfrac{x+y}{x-y}$

(B) $\dfrac{dy}{dx} = x + y$

(C) $\dfrac{dy}{dx} = xy$

(D) $\dfrac{dy}{dx} = \dfrac{x}{y^2}$

Q12. On which interval is $f(x) = x^2 - 4x + 5$ strictly increasing?

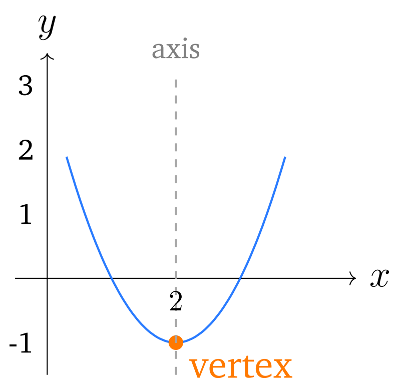
(A) $[0,\ 2]$

(B) $[2,\ 4]$

(C) $(-\infty,\ 2]$

(D) $[1,\ 3]$

Q13. Find the vertex and axis of symmetry of the parabola $y = x^2 - 4x + 3$.



(A) Vertex $(2,\ 1)$, axis of symmetry $x = 2$

(B) Vertex $(-2,\ -1)$, axis of symmetry $x = -2$

(C) Vertex $(2,\ -1)$, axis of symmetry $x = 2$

(D) Vertex $(4,\ 0)$, axis of symmetry $x = 4$

Q14. Three lines are concurrent if and only if:

(A) They all have the same slope

(B) The sum of their x-intercepts is zero

(C) The product of their slopes equals 1

(D) The determinant of the 3×3 matrix formed by their coefficients is zero

Q15. Find the power of the point $(6,\ 4)$ with respect to the circle $x^2 + y^2 - 4x - 6y + 4 = 0$:

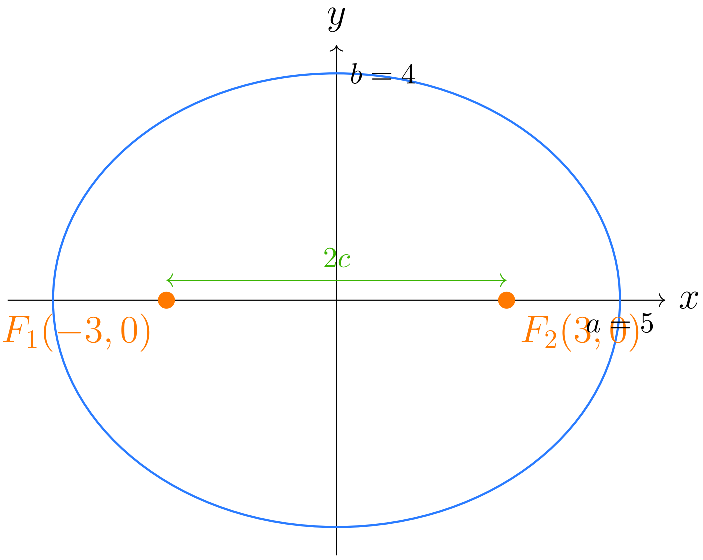
(A) 8

(B) 4

(C) 0

(D) -4

Q16. For the ellipse $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$ shown below, find the distance between the two foci.



(A) 3

(B) 6

(C) 10

(D) 4

Q17. The orthocenter of a right-angled triangle is located at:

(A) The centroid

(B) The midpoint of the hypotenuse

(C) The vertex of the right angle

(D) The circumcenter

Q18. Which of the following correctly states the transpose of a matrix product $(AB)^T$?

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}, \quad B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$$

(A) $A^T B^T$

(B) AB^T

(C) $A^T B$

(D) $B^T A^T$

Q19. For any square matrix A, the matrix $(A - A^T)$ is always:

(A) Skew-symmetric

(B) Symmetric

(C) The identity matrix

(D) The zero matrix

Q20. The system $AX = \mathbf{0}$ (where A is a square matrix) always has:

(A) No solution

(B) At least the trivial solution $X = \mathbf{0}$

(C) Exactly one non-trivial solution

(D) Infinitely many non-trivial solutions always

Q21. Find the inverse of the matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix}$:

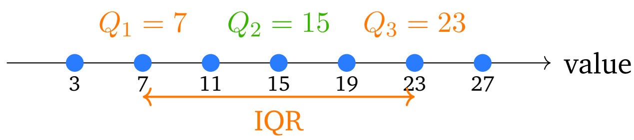
(A) $\begin{pmatrix} 7 & 2 \\ -3 & -1 \end{pmatrix}$

(B) $\begin{pmatrix} 7 & -2 \\ 3 & 1 \end{pmatrix}$

(C) $\begin{pmatrix} 7 & -2 \\ -3 & 1 \end{pmatrix}$

(D) $\begin{pmatrix} -7 & 2 \\ 3 & -1 \end{pmatrix}$

Q22. Find the Interquartile Range (IQR) of the data set $\{3, 7, 11, 15, 19, 23, 27\}$.



(A) 24

(B) 8

(C) 20

(D) 16

Q23. Two independent events A and B have $P(A) = 0.5$ and $P(B) = 0.4$. Find $P(A \cap B)$:

(A) 0.20

(B) 0.90

(C) 0.10

(D) 0.45

Q24. For the standard normal distribution $N(0, 1)$, the approximate probability $P(-1 < Z < 1)$ is:

(A) 0.5000

(B) 0.6827

(C) 0.9545

(D) 0.9973

Q25. Both lines of regression pass through the point:

(A) The origin $(0,\, 0)$

(B) $(\sigma_x,\, \sigma_y)$

(C) $(\bar{x},\, \bar{y})$ — the mean values of X and Y

(D) (the median of X, the median of Y)

Q26. If $\mathrm{Var}(X) = 4$, find $\mathrm{Var}(3X + 2)$:

(A) 14

(B) 38

(C) 12

(D) 36

Q27. If $\tan\theta = 2$, find $\sin 2\theta$:

(A) $\frac{4}{5}$

(B) $\frac{3}{5}$

(C) $-\frac{3}{5}$

(D) $\frac{5}{4}$

Q28. Using the identity $\cos 2A = 1 - 2\sin^2 A$, evaluate $\cos 60$:

(A) $\frac{\sqrt{3}}{2}$

(B) $\frac{1}{2}$

(C) 0

(D) 1

Q29. The range of the function $\tan^{-1}(x)$ is:

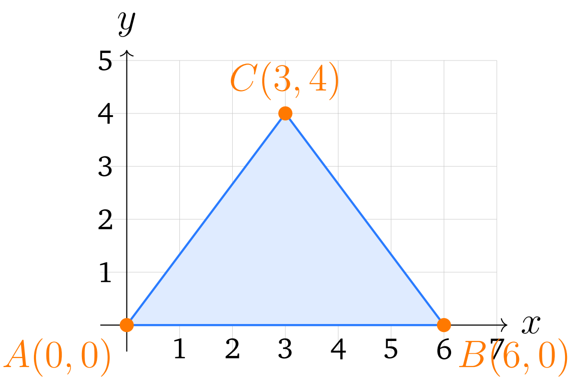
(A) $[0, \pi]$

(B) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

(C) $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (open interval)

(D) $(-\pi, \pi)$

Q30. Find the area of the triangle with vertices $A(0,0)$, $B(6,0)$, and $C(3,4)$ using the coordinate area formula.



(A) 6

(B) 9

(C) 18

(D) 12

Detailed Solutions

Q1.

Solution

Concept — Harmonic Progression: In a Harmonic Progression (HP), the reciprocals of the terms form an Arithmetic Progression (AP). If HP terms are $H_1, H_2, \ldots$, then $\frac{1}{H_1}, \frac{1}{H_2}, \ldots$ is an AP.

Step 1 — Find the AP of reciprocals: $H_1 = \frac{1}{2}$ and $H_2 = \frac{1}{3}$, so the AP is $2, 3, \ldots$ with common difference $d = 3 - 2 = 1$.

Step 2 — Find the 4th term of the AP:

$$a_4 = 2 + (4 - 1) \times 1 = 2 + 3 = 5.$$

Therefore the 4th HP term is $H_4 = \dfrac{1}{5}$.

Why other options are wrong:

- Option A $(1/4)$: Corresponds to the 3rd AP term $a_3 = 4$; off by one position.
- Option B $(1/3)$: This is H_2, the second term, not the fourth.
- Option D $(1/6)$: Would require $a_4 = 6$, but $d = 1$ gives $a_4 = 5$.

Final Answer: $H_4 = \dfrac{1}{5} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 1

Q2.

Solution

Concept — Change of Base Formula: For any valid base b and argument x, $\log_b x = \dfrac{\ln x}{\ln b}$.

Step 1 — Apply the formula:

$$\log_5 25 = \frac{\ln 25}{\ln 5}.$$

Step 2 — Simplify using $25 = 5^2$:

$$\frac{\ln 25}{\ln 5} = \frac{\ln 5^2}{\ln 5} = \frac{2 \ln 5}{\ln 5} = 2.$$

Why other options are wrong:

- Option A ($\ln 25$): Confuses $\log_5 25$ with $\ln 25$; the base-e logarithm is not the answer.
- Option B ($\ln 5/2$): Inverts the fraction; result of $\dfrac{\ln 5}{\ln 25} = \dfrac{1}{2}$.
- Option C ($1/2$): Result of $\log_{25} 5$, not $\log_5 25$.

Final Answer: $\log_5 25 = 2 \Rightarrow \boxed{\textbf{D}}$

Answer: (D) Go Back to Q 2

Q3.

Solution

Concept — Sum of Binomial Coefficients: Setting $x = 1$ in $(1+x)^n$ gives the sum of all coefficients: $\sum_{k=0}^{n} \binom{n}{k} = 2^n$.

Step 1 — Apply with $n = 6$:

$$\sum_{k=0}^{6} \binom{6}{k} = 2^6 = 64.$$

Step 2 — Verify the first few terms: $\binom{6}{0}+\binom{6}{1}+\cdots+\binom{6}{6} = 1+6+15+20+15+6+1 = 64.$ ✓

Why other options are wrong:

- Option B (32): This is 2^5, the sum for $(1+x)^5$.
- Option C (128): This is 2^7, the sum for $(1+x)^7$.
- Option D (63): Off by one; $2^6 - 1 = 63$ excludes the constant term $\binom{6}{0} = 1$.

Final Answer: Sum $= 64 \Rightarrow \boxed{\textbf{A}}$

Answer: (A) Go Back to Q 3

Q4.

Solution

Concept — AM–GM Inequality: For positive reals a and b, $\frac{a+b}{2} \geq \sqrt{ab}$, with equality when $a = b$.

Step 1 — Apply AM–GM: Given $a + b = 10$,

$$\frac{a+b}{2} \geq \sqrt{ab} \implies 5 \geq \sqrt{ab} \implies ab \leq 25.$$

Step 2 — Verify equality condition: Equality holds when $a = b = 5$, giving $ab = 25$.

Why other options are wrong:

- Option A (50): Confuses the sum $a + b = 10$ with the product; $2 \times 25 = 50$ is not the maximum product.
- Option C (20): A valid product (e.g., $a = 4, b = 6$) but not the maximum.
- Option D (100): Exceeds the AM–GM bound; impossible with $a + b = 10$.

Final Answer: Maximum $ab = 25 \Rightarrow \boxed{\textbf{B}}$

Answer: (B) Go Back to Q 4

Q5.

Solution

Concept — Derangements: A derangement D_n is a permutation of n objects where no object occupies its original position. The formula is:

$$D_n = n! \sum_{k=0}^{n} \frac{(-1)^k}{k!}.$$

Step 1 — Apply for $n = 3$**:**

$$D_3 = 3!\left(1 - 1 + \frac{1}{2!} - \frac{1}{3!}\right) = 6\left(\frac{1}{2} - \frac{1}{6}\right) = 6 \times \frac{1}{3} = 2.$$

Step 2 — List them explicitly: For objects $\{1, 2, 3\}$ with original positions $1 \to 1, 2 \to 2, 3 \to 3$, the derangements are $(2, 3, 1)$ and $(3, 1, 2)$. Exactly 2.

Why other options are wrong:

- Option A (6): This is $3! = 6$, the total number of permutations, not just derangements.
- Option B (3): Incorrect; listing all permutations confirms only 2 are derangements.
- Option D (1): There are two derangements, not one.

Final Answer: $D_3 = 2 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 5

Q6.

Solution

Concept — Combinations (Multiplication Principle): Select 2 men from 5 AND 1 woman from 4, independently.

Step 1 — Count ways to choose 2 men from 5:

$$\binom{5}{2} = \frac{5!}{2!\,3!} = \frac{5 \times 4}{2} = 10.$$

Step 2 — Count ways to choose 1 woman from 4:

$$\binom{4}{1} = 4.$$

Step 3 — Multiply: Total $= 10 \times 4 = 40$.

Why other options are wrong:

- Option A (30): Computes $\binom{5}{2} \times \binom{4}{0} +$ something; wrong woman count.
- Option B (20): Uses $\binom{5}{2} \times 2$ instead of $\times 4$.
- Option C (60): Uses $\binom{5}{3} \times \binom{4}{1} = 10 \times 4$ — wrong men selection.

Final Answer: $10 \times 4 = 40 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 6

Q7.

Solution

Concept — Parametric Differentiation: If $x = f(t)$ and $y = g(t)$, then $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$.

Step 1 — Differentiate with respect to t:

$$\frac{dy}{dt} = 3t^2, \qquad \frac{dx}{dt} = 2t.$$

Step 2 — Divide:

$$\frac{dy}{dx} = \frac{3t^2}{2t} = \frac{3t}{2}.$$

Why other options are wrong:

- Option B $(2t/3)$: Inverts the ratio dx/dy instead of dy/dx.
- Option C $(3/2)$: Drops the t factor; valid only at $t = 1$.
- Option D $(2t)$: This is just dx/dt, not the required ratio.

Final Answer: $\frac{dy}{dx} = \frac{3t}{2} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 7

Q8.

Solution

Concept — Rolle's Theorem Conditions: A function f satisfies Rolle's Theorem on $[a, b]$ if (i) f is continuous on $[a, b]$, (ii) f is differentiable on (a, b), and (iii) $f(a) = f(b)$.

Step 1 — Check $f(x) = 1 - x^2$ on $[-1, 1]$:

- Continuous everywhere: ✓
- Differentiable everywhere, $f'(x) = -2x$: ✓
- $f(-1) = 1 - 1 = 0 = f(1)$: ✓

All three conditions are satisfied.

Why other options are wrong:

- Option A $|x|$: Not differentiable at $x = 0$; condition (ii) fails.

- Option C $\tan x$: Discontinuous at $x = \pm\pi/2$; since $\pi/2 < 1$ is false but $\tan$ has a singularity inside any interval containing odd multiples of $\pi/2$; also $f(-1) \neq f(1)$ generically.
- Option D $1/x$: Discontinuous at $x = 0 \in [-1, 1]$; condition (i) fails.

Final Answer: $f(x) = 1 - x^2 \Rightarrow \boxed{\textbf{B}}$

Answer: (B) Go Back to Q 8

Q9.

Solution

Concept — L'Hôpital's Rule: If $\lim_{x\to c} \frac{f(x)}{g(x)}$ is of the form $\frac{0}{0}$ or $\frac{\infty}{\infty}$, then $\lim_{x\to c} \frac{f(x)}{g(x)} = \lim_{x\to c} \frac{f'(x)}{g'(x)}$ (provided the latter limit exists).

Step 1 — Verify indeterminate form: As $x \to 0$: numerator $1 - \cos 0 = 0$, denominator $0^2 = 0$. Form is $0/0$.

Step 2 — Apply L'Hôpital once:

$$\lim_{x\to 0} \frac{1 - \cos x}{x^2} = \lim_{x\to 0} \frac{\sin x}{2x}.$$

Step 3 — Apply again (still $0/0$) or use standard limit:

$$\lim_{x\to 0} \frac{\sin x}{2x} = \frac{1}{2} \lim_{x\to 0} \frac{\sin x}{x} = \frac{1}{2} \times 1 = \frac{1}{2}.$$

Why other options are wrong:

- Option A (0): Incorrectly evaluates the indeterminate form as 0.
- Option B (1): Forgets the factor of 2 in the denominator after differentiating x^2.
- Option D (∞): The limit converges to a finite value.

Final Answer: $\frac{1}{2} \Rightarrow \boxed{\textbf{C}}$

Answer: (C) Go Back to Q 9

Q10.

Solution

Concept — Substitution in Definite Integrals: For integrals of the form $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + C$.

Step 1 — Identify the substitution: Let $u = 1 + x^2$, so $du = 2x\,dx$, i.e., $x\,dx = \frac{du}{2}$. When $x = 0$, $u = 1$; when $x = 1$, $u = 2$.

Step 2 — Transform and evaluate:

$$\int_0^1 \frac{x}{1+x^2}\,dx = \frac{1}{2}\int_1^2 \frac{du}{u} = \frac{1}{2}\Big[\ln u\Big]_1^2 = \frac{1}{2}(\ln 2 - \ln 1) = \frac{\ln 2}{2}.$$

Why other options are wrong:

- Option A ($\ln 2$): Forgets the factor of $\frac{1}{2}$ from the substitution $du = 2x\,dx$.
- Option B ($1/2$): Confuses the coefficient $\frac{1}{2}$ with the full answer; $\ln 2 \neq 1$.
- Option C ($\ln(\sqrt{2}+1)$): Arises from a different integrand; not relevant here.

Final Answer: $\frac{\ln 2}{2} \Rightarrow \boxed{\textbf{D}}$

Answer: (D) Go Back to Q 10

Q11.

Solution

Concept — Homogeneous Differential Equation: A differential equation $\frac{dy}{dx} = F(x,y)$ is homogeneous if $F(tx,ty) = F(x,y)$ for all $t \neq 0$ (i.e., F is a homogeneous function of degree 0).

Step 1 — Test Option A, $F(x,y) = \frac{x+y}{x-y}$:

$$F(tx,ty) = \frac{tx+ty}{tx-ty} = \frac{t(x+y)}{t(x-y)} = \frac{x+y}{x-y} = F(x,y). \quad \checkmark$$

Why other options are wrong:

- Option B ($dy/dx = x+y$): $F(tx,ty) = tx+ty = t(x+y) \neq F(x,y)$; degree 1, not 0.
- Option C ($dy/dx = xy$): $F(tx,ty) = t^2xy \neq F(x,y)$; degree 2.
- Option D ($dy/dx = x/y^2$): $F(tx,ty) = tx/(ty)^2 = x/(ty^2) \neq F(x,y)$; degree -1.

Final Answer: $\frac{dy}{dx} = \frac{x+y}{x-y}$ **is homogeneous** $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 11

Q12.

Solution

Concept — Monotonically Increasing Functions: $f(x)$ is strictly increasing on an interval where $f'(x) > 0$.

Step 1 — Differentiate:

$$f'(x) = 2x - 4.$$

Step 2 — Find where $f'(x) > 0$:

$$2x - 4 > 0 \implies x > 2.$$

On $[2, 4]$: $f'(2) = 0$ (non-strict at endpoint) and $f'(x) > 0$ for $x \in (2, 4]$, so f is strictly increasing on $[2, 4]$.

Why other options are wrong:

- Option A ($[0, 2]$): Here $f'(x) = 2x - 4 \leq 0$; f is decreasing, not increasing.
- Option C ($(-\infty, 2]$): $f'(x) \leq 0$ throughout; f is non-increasing.
- Option D ($[1, 3]$): $f'(1) = -2 < 0$; f is decreasing near $x = 1$.

Final Answer: f **is strictly increasing on** $[2, 4] \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 12

Q13.

Solution

Concept — Vertex of a Parabola (Completing the Square): Write $y = a(x - h)^2 + k$; vertex is (h, k) and axis of symmetry is $x = h$.

Step 1 — Complete the square:

$$y = x^2 - 4x + 3 = (x^2 - 4x + 4) - 4 + 3 = (x - 2)^2 - 1.$$

Step 2 — Read off vertex and axis: Vertex $= (2, -1)$, axis of symmetry $x = 2$.

Why other options are wrong:

- Option A: Vertex $(2, 1)$ has the wrong y-coordinate; $k = -1$, not $+1$.
- Option B: Vertex $(-2, -1)$ takes the wrong sign for h; $h = +2$.
- Option D: Vertex $(4, 0)$ uses the un-completed constant term $c = 3$ and ignores the shift.

Final Answer: Vertex $(2, -1)$, **axis** $x = 2 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 13

Q14.

Solution

Concept — Condition for Concurrence of Three Lines: Three lines $a_i x + b_i y + c_i = 0$ ($i = 1, 2, 3$) are concurrent if and only if the determinant of the coefficient matrix vanishes:

$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = 0.$$

Step 1 — Geometric interpretation: Concurrent lines all pass through a single common point. The determinant condition ensures the system has a unique common solution.

Why other options are wrong:

- Option A (same slope): Parallel lines (same slope, different intercept) never meet; concurrent requires meeting at one point.
- Option B (sum of x-intercepts is zero): No standard theorem links concurrence to this property.
- Option C (product of slopes $= 1$): This is related to perpendicularity conditions, not concurrence.

Final Answer: Determinant of coefficient matrix $= 0 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 14

Q15.

Solution

Concept — Power of a Point: For a point (x_0, y_0) and circle $x^2+y^2+Dx+Ey+F=0$, the power equals $x_0^2+y_0^2+Dx_0+Ey_0+F$.

Step 1 — Identify coefficients: Circle: $x^2+y^2-4x-6y+4=0$, so $D=-4$, $E=-6$, $F=4$.

Step 2 — Substitute $(x_0, y_0) = (6, 4)$**:**

$$\text{Power} = 36+16+(-4)(6)+(-6)(4)+4 = 52-24-24+4 = 8.$$

Why other options are wrong:

- Option B (4): Arithmetic error; $52-48+4$ is computed correctly as 8.
- Option C (0): A power of 0 means the point lies on the circle; $(6, 4)$ does not.
- Option D (-4): Sign error in applying D and E.

Final Answer: Power $= 8 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 15

Q16.

Solution

Concept — Foci of an Ellipse: For $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ with $a>b>0$, the foci are at $(\pm c, 0)$ where $c^2=a^2-b^2$. The distance between foci is $2c$.

Step 1 — Identify a^2 and b^2: $a^2=25$, $b^2=16$, so $a=5$, $b=4$.

Step 2 — Compute c:

$$c^2=a^2-b^2=25-16=9 \implies c=3.$$

Step 3 — Distance between foci: $2c=6$.

Why other options are wrong:

- Option A (3): This is c, the distance from centre to one focus, not the full distance $2c$.
- Option C (10): This is $2a=10$, the length of the major axis, not the focal distance.

- Option D (4): This is b, the semi-minor axis.

Final Answer: Distance between foci $= 6 \Rightarrow \boxed{\textbf{B}}$

Answer: (B) Go Back to Q 16

Q17.

Solution

Concept — Orthocenter of a Triangle: The orthocenter is the intersection of the altitudes. For a right-angled triangle, the altitudes from the two acute vertices meet at the vertex containing the right angle.

Step 1 — Geometric argument: In a right triangle with the right angle at vertex C, the two legs CA and CB are themselves altitudes (each leg is perpendicular to the other leg). Their intersection is C itself.

Why other options are wrong:

- Option A (centroid): The centroid divides medians in ratio $2 : 1$; it is not generally the orthocenter.
- Option B (midpoint of hypotenuse): This is the circumcenter of a right triangle, not the orthocenter.
- Option D (circumcenter): The circumcenter of a right triangle is the midpoint of the hypotenuse, not the orthocenter.

Final Answer: Orthocenter is at the vertex of the right angle $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 17

Q18.

Solution

Concept — Transpose of a Product: For matrices A and B (with compatible dimensions), $(AB)^T = B^T A^T$. The order of multiplication reverses upon transposition.

Step 1 — Proof sketch: $[(AB)^T]_{ij} = [AB]_{ji} = \sum_k A_{jk}B_{ki} = \sum_k (B^T)_{ik}(A^T)_{kj} = [B^T A^T]_{ij}$.

Step 2 — Verify with given matrices $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 5 & 6 \\ 7 & 8 \end{pmatrix}$:

$$AB = \begin{pmatrix} 19 & 22 \\ 43 & 50 \end{pmatrix}, \quad (AB)^T = \begin{pmatrix} 19 & 43 \\ 22 & 50 \end{pmatrix}.$$

$$B^T A^T = \begin{pmatrix} 5 & 7 \\ 6 & 8 \end{pmatrix} \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 19 & 43 \\ 22 & 50 \end{pmatrix}. \quad \checkmark$$

Why other options are wrong:

- Option A ($A^T B^T$): Reversal not applied; the correct formula requires swapping A and B.
- Option B (AB^T): Transposes only B; inconsistent with the product rule.
- Option C ($A^T B$): Transposes only A; also incorrect.

Final Answer: $(AB)^T = B^T A^T \Rightarrow \boxed{\mathbf{D}}$

Answer: (D) Go Back to Q 18

Q19.

Solution

Concept — Skew-Symmetric Matrix: A matrix S is skew-symmetric if $S^T = -S$. We show $(A - A^T)$ always has this property.

Step 1 — Compute the transpose of $(A - A^T)$**:**

$$(A - A^T)^T = A^T - (A^T)^T = A^T - A = -(A - A^T).$$

Step 2 — Conclude: Since $(A - A^T)^T = -(A - A^T)$, the matrix $A - A^T$ is always skew-symmetric.

Why other options are wrong:

- Option B (symmetric): A symmetric matrix satisfies $S^T = +S$; here we get $-S$, so it is skew-symmetric.
- Option C (identity matrix): True only for a specific A; not a general result.
- Option D (zero matrix): $(A - A^T) = 0$ only when A is already symmetric; not always.

Final Answer: $(A - A^T)$ **is skew-symmetric** $\Rightarrow \boxed{\mathbf{A}}$

Answer: (A) Go Back to Q 19

Q20.

Solution

Concept — Homogeneous Linear System: The system $AX = \mathbf{0}$ is called homogeneous. It always has the *trivial solution* $X = \mathbf{0}$ (zero vector), regardless of the matrix A.

Step 1 — Verify: Substituting $X = \mathbf{0}$: $A \cdot \mathbf{0} = \mathbf{0}$. ✓This works for any matrix A.

Step 2 — When does a non-trivial solution exist? Non-trivial solutions exist if and only if $\det(A) = 0$. This is not always the case.

Why other options are wrong:

- Option A (no solution): Homogeneous systems always have at least the trivial solution; they are always consistent.
- Option C (exactly one non-trivial): Non-trivial solutions come in infinite families (spans); "exactly one" is never correct.
- Option D (infinitely many non-trivial always): Only true when $\det(A) = 0$; not always the case.

Final Answer: Always has at least the trivial solution $X = 0 \Rightarrow$ $\boxed{\textbf{B}}$

Answer: (B) Go Back to Q 20

Q21.

Solution

Concept — Inverse of a 2×2 Matrix: For $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $A^{-1} = \dfrac{1}{\det A} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.

Step 1 — Compute determinant:

$$\det(A) = 1 \times 7 - 2 \times 3 = 7 - 6 = 1.$$

Step 2 — Apply the formula:

$$A^{-1} = \frac{1}{1} \begin{pmatrix} 7 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} 7 & -2 \\ -3 & 1 \end{pmatrix}.$$

Step 3 — Verify: $AA^{-1} = I$:

$$\begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix} \begin{pmatrix} 7 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} 7-6 & -2+2 \\ 21-21 & -6+7 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \quad \checkmark$$

Why other options are wrong:

- Option A: Off-diagonal entries not negated; $b = -2$ should appear, not $+2$.
- Option B: Lower-left entry should be -3, not $+3$.
- Option D: Signs of both a and d are wrong; 7 becomes -7 and 1 becomes -1.

Final Answer: $A^{-1} = \begin{pmatrix} 7 & -2 \\ -3 & 1 \end{pmatrix} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 21

Q22.

Solution

Concept — Interquartile Range (IQR): $\text{IQR} = Q_3 - Q_1$, where Q_1 is the median of the lower half and Q_3 is the median of the upper half.

Step 1 — Sort and identify $n = 7$**:** Sorted data: $3, 7, 11, 15, 19, 23, 27$.

Step 2 — Find quartiles:

- Median (Q_2): 4th value $= 15$.
- Lower half: $\{3, 7, 11\}$; Q_1 = median $= 7$.
- Upper half: $\{19, 23, 27\}$; Q_3 = median $= 23$.

Step 3 — Compute IQR:

$$\text{IQR} = Q_3 - Q_1 = 23 - 7 = 16.$$

Why other options are wrong:

- Option A (24): This is $Q_3 + Q_1 = 23 + 7$, a sum rather than a difference.
- Option B (8): This is $Q_3 - Q_2 = 23 - 15$, not the IQR.
- Option C (20): Computes $27 - 7 = 20$, the full range, not IQR.

Final Answer: $\text{IQR} = 16 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 22

Q23.

Solution

Concept — Independent Events: If A and B are independent, then $P(A \cap B) = P(A) \cdot P(B)$.

Step 1 — Apply the multiplication rule:

$$P(A \cap B) = P(A) \cdot P(B) = 0.5 \times 0.4 = 0.20.$$

Why other options are wrong:

- Option B (0.90): This equals $P(A) + P(B) = 0.5 + 0.4$; the formula for $P(A \cup B)$ without subtraction.
- Option C (0.10): Arithmetic error; $0.5 \times 0.4 = 0.20$, not 0.10.
- Option D (0.45): Average of $P(A)$ and $P(B)$; not a probability formula.

Final Answer: $P(A \cap B) = 0.20 \Rightarrow \boxed{\textbf{A}}$

$\boxed{\textbf{Answer: (A)}}$ Go Back to Q 23

Q24.

Solution

Concept — Empirical Rule for Normal Distribution: For $Z \sim N(0, 1)$, the probability within k standard deviations of the mean follows the empirical rule:

- $P(-1 < Z < 1) \approx 0.6827$ (approximately 68%)
- $P(-2 < Z < 2) \approx 0.9545$ (approximately 95%)
- $P(-3 < Z < 3) \approx 0.9973$ (approximately 99.7%)

Step 1 — Identify the correct value: For $k = 1$, $P(-1 < Z < 1) \approx 0.6827$.

Why other options are wrong:

- Option A (0.5000): $P(Z > 0) = 0.5$ by symmetry; this is a one-sided probability.
- Option C (0.9545): This is $P(-2 < Z < 2)$, i.e., within 2 standard deviations.
- Option D (0.9973): This is $P(-3 < Z < 3)$, i.e., within 3 standard deviations.

Final Answer: $P(-1 < Z < 1) \approx 0.6827 \Rightarrow \boxed{\textbf{B}}$

$\boxed{\textbf{Answer: (B)}}$ Go Back to Q 24

Q25.

Solution

Concept — Lines of Regression: The two lines of regression (of Y on X, and of X on Y) both pass through the point $(\bar{x}, \bar{y})$, the bivariate mean.

Step 1 — Regression of Y on X: $y - \bar{y} = b_{yx}(x - \bar{x})$. At $x = \bar{x}$, $y = \bar{y}$. Point $(\bar{x}, \bar{y})$ lies on this line.

Step 2 — Regression of X on Y: $x - \bar{x} = b_{xy}(y - \bar{y})$. At $y = \bar{y}$, $x = \bar{x}$. Point $(\bar{x}, \bar{y})$ lies on this line too.

Why other options are wrong:

- Option A (origin): Regression lines pass through the origin only if $\bar{x} = \bar{y} = 0$; not in general.
- Option B ((σ_x, σ_y)): Standard deviations are not related to the regression intersection point.
- Option D (median, median): Regression theory uses means, not medians.

Final Answer: Both regression lines pass through $(\bar{x}, \bar{y}) \Rightarrow \boxed{\textbf{C}}$

Answer: (C) Go Back to Q 25

Q26.

Solution

Concept — Variance under Linear Transformation: $\operatorname{Var}(aX + b) = a^2 \operatorname{Var}(X)$. Adding a constant b does not change the variance.

Step 1 — Identify a and b: Here $a = 3$, $b = 2$, $\operatorname{Var}(X) = 4$.

Step 2 — Apply the formula:

$$\operatorname{Var}(3X + 2) = 3^2 \times \operatorname{Var}(X) = 9 \times 4 = 36.$$

Why other options are wrong:

- Option A (14): Uses $a^2 + b = 9 + 2 \neq$ a valid formula.
- Option B (38): Adds $b^2 = 4$ to the variance; constants do not contribute to variance.
- Option C (12): Uses $a \times \operatorname{Var}(X) = 3 \times 4$ instead of a^2.

Final Answer: $\operatorname{Var}(3X + 2) = 36 \Rightarrow \boxed{\textbf{D}}$

Answer: (D) Go Back to Q 26

Q27.

Solution

Concept — Double Angle Formula for Sine: $\sin 2\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$.

Step 1 — Substitute $\tan\theta = 2$**:**

$$\sin 2\theta = \frac{2\times 2}{1+2^2} = \frac{4}{1+4} = \frac{4}{5}.$$

Alternate verification using a right triangle: With $\tan\theta = 2$, take opposite $= 2$, adjacent $= 1$, hypotenuse $= \sqrt{5}$. Then $\sin\theta = \dfrac{2}{\sqrt{5}}$, $\cos\theta = \dfrac{1}{\sqrt{5}}$, so $\sin 2\theta = 2\sin\theta\cos\theta = 2\cdot\dfrac{2}{\sqrt{5}}\cdot\dfrac{1}{\sqrt{5}} = \dfrac{4}{5}$. ✓

Why other options are wrong:

- Option B $(3/5)$: Would follow from $\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta} = \dfrac{1-4}{5} = -\dfrac{3}{5}$; sign error and different trig function.
- Option C $(-3/5)$: This is $\cos 2\theta$, not $\sin 2\theta$.
- Option D $(5/4)$: Inverts the correct answer $4/5$.

Final Answer: $\sin 2\theta = \dfrac{4}{5} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 27

Q28.

Solution

Concept — Cosine Double Angle Identity: $\cos 2A = 1 - 2\sin^2 A$.

Step 1 — Set $2A = 60$**, so** $A = 30$**:**

$$\cos 60 = 1 - 2\sin^2 30.$$

Step 2 — Substitute $\sin 30 = \dfrac{1}{2}$**:**

$$\cos 60 = 1 - 2\left(\frac{1}{2}\right)^2 = 1 - 2\times\frac{1}{4} = 1 - \frac{1}{2} = \frac{1}{2}.$$

Why other options are wrong:

- Option A ($\sqrt{3}/2$): This is $\cos 30$, not $\cos 60$.
- Option C (0): This is $\cos 90$.
- Option D (1): This is $\cos 0$.

Final Answer: $\cos 60 = \frac{1}{2} \Rightarrow \boxed{\textbf{B}}$

Answer: (B) Go Back to Q 28

Q29.

Solution

Concept — Range of $\tan^{-1}(x)$: The inverse tangent function has domain $\mathbb{R}$ (all real numbers) and range the open interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. The endpoints are excluded because $\tan x \to \pm\infty$ as $x \to \pm\frac{\pi}{2}$; the inverse never reaches these boundary values.

Step 1 — Recall definition: $y = \tan^{-1}(x)$ means $\tan y = x$ with $y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ (open interval).

Why other options are wrong:

- Option A ($[0, \pi]$): This is the range of $\cos^{-1}(x)$.
- Option B ($[-\pi/2, \pi/2]$ closed): The endpoints $\pm\pi/2$ are not in the range since $\tan(\pm\pi/2)$ is undefined.
- Option D ($(-\pi, \pi)$): Too wide; this contains values like $\pm 3\pi/4$ where $\tan$ is not one-to-one.

Final Answer: Range of $\tan^{-1}(x)$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \Rightarrow \boxed{\textbf{C}}$

Answer: (C) Go Back to Q 29

Q30.

Solution

Concept — Area of a Triangle (Coordinate Formula):

$$\text{Area} = \frac{1}{2}\left|x_A(y_B - y_C) + x_B(y_C - y_A) + x_C(y_A - y_B)\right|.$$

Step 1 — Substitute $A(0,0)$, $B(6,0)$, $C(3,4)$:

$$\text{Area} = \frac{1}{2}\left|0(0-4) + 6(4-0) + 3(0-0)\right|.$$

Step 2 — Simplify:

$$\text{Area} = \frac{1}{2}\left|0 + 24 + 0\right| = \frac{1}{2} \times 24 = 12.$$

Alternate check (base $\times$ height): Base $AB = 6$ (along x-axis). Height $= y$-coordinate of $C = 4$. Area $= \frac{1}{2} \times 6 \times 4 = 12$. ✓

Why other options are wrong:

- Option A (6): Uses $\frac{1}{2} \times 6 \times 2$; incorrect height of 2 instead of 4.
- Option B (9): Arithmetic error; $\frac{1}{2} \times 6 \times 3 = 9$ uses height 3.
- Option C (18): Forgets the $\frac{1}{2}$ factor; $6 \times 4 \times \frac{1}{2} = 12$, not 18.

Final Answer: Area $= 12 \Rightarrow \boxed{\textbf{D}}$

Answer: (D) Go Back to Q 30

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	D	3	A	4	B	5	C
6	D	7	A	8	B	9	C	10	D
11	A	12	B	13	C	14	D	15	A
16	B	17	C	18	D	19	A	20	B
21	C	22	D	23	A	24	B	25	C
26	D	27	A	28	B	29	C	30	D