MAH MCA CET Mathematics & Statistics

Sample Paper – 8

Duration: 27 Minutes **Maximum Marks: 60**

Instructions

- This paper contains **30** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics & Statistics portion of **MAH MCA CET** entrance.
- Each correct answer carries **+2 marks**. There is **no negative marking** for incorrect or unattempted answers.
- Only **one** option is correct. Choose carefully.
- Syllabus level: **Class 12 level — Algebra, Calculus, Coordinate Geometry, Matrices, Statistics & Probability, Trigonometry.**
- Use of mobile phones, calculators, or electronic gadgets is strictly prohibited.

Q1. The 3rd term of a geometric progression with first term $a = 5$ and common ratio $r = 2$ is:

(A) 20

(B) 15

(C) 10

(D) 25

Q2. Evaluate: $\ln(e^3) + \ln(1)$

(A) 1

(B) 3

(C) e^3

(D) 4

Q3. Find the coefficient of the middle term in the expansion of $(1+x)^8$:

(A) 28

(B) 56

(C) 35

(D) 70

Q4. The condition for the quadratic equation $ax^2+bx+c=0$ to have real and distinct roots is:

(A) $b^2-4ac=0$

(B) $b^2-4ac<0$

(C) $b^2-4ac>0$

(D) $b^2+4ac>0$

Q5. Find the number of distinct arrangements of the letters in the word **LEVEL**:

(A) 30

(B) 60

(C) 120

(D) 20

Q6. From a group of 10 people, the number of ways to select **at least 8** people is:

(A) 45

(B) 56

(C) 111

(D) 100

Q7. Using the product rule, find $\frac{d}{dx}[x^2\cos x]$:

(A) $x^2\cos x$

(B) $-2x \sin x$

(C) $2x \cos x + x^2 \sin x$

(D) $2x \cos x - x^2 \sin x$

Q8. Find $\dfrac{d^2y}{dx^2}$ for $y = \sin x$:

(A) $\sin x$

(B) $\cos x$

(C) $-\sin x$

(D) $-\cos x$

Q9. Evaluate $\lim\limits_{x \to 1} \dfrac{x^2 - 1}{x - 1}$:

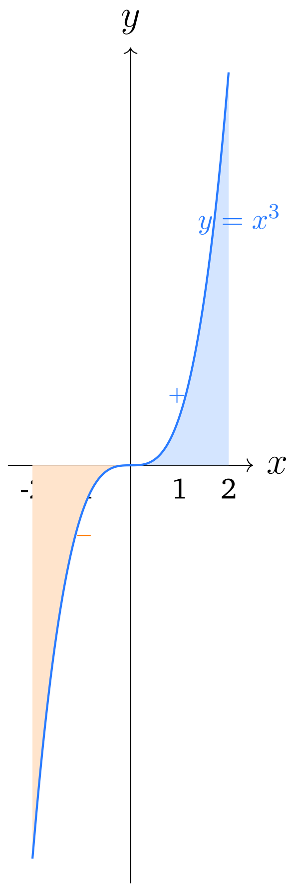
(A) 2

(B) 0

(C) 1

(D) ∞

Q10. The diagram below shows the curve $y = x^3$ on the interval $[-2, 2]$, with the positive region (blue) and negative region (orange) shaded to illustrate symmetry:



odd function: symmetry cancels

Evaluate $\int_{-2}^{2} x^3\,dx$:

(A) 4

(B) 0

(C) 8

(D) -4

Q11. The general solution of the differential equation $\frac{dy}{dx} = -\frac{y}{x}$ is:

(A) $y = Cx$

(B) $y = Ce^x$

(C) $y = Cx^2$

(D) $y = \frac{C}{x}$

Q12. The radius of a circle increases at the rate of 2 cm/s. The rate of increase of its area when the radius is $r = 5$ cm is:

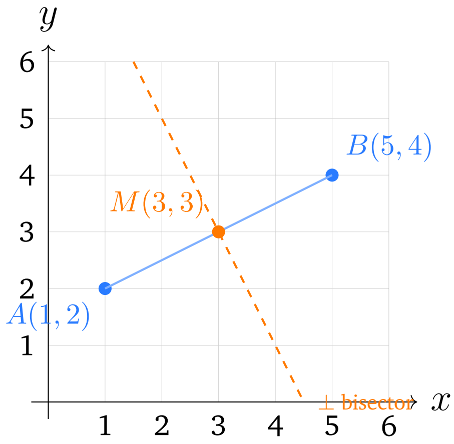
(A) 10π cm^2/s

(B) 4π cm^2/s

(C) 20π cm^2/s

(D) 25π cm^2/s

Q13. The diagram shows points $A(1,2)$ and $B(5,4)$ with their midpoint $M(3,3)$ marked. Find the equation of the perpendicular bisector of segment AB:



(A) $2x + y = 9$

(B) $x + 2y = 9$

(C) $2x - y = 3$

(D) $x - 2y + 1 = 0$

Q14. Find the equation of the line with x-intercept 3 and y-intercept 4:

(A) $3x + 4y = 12$

(B) $4x + 3y = 12$

(C) $\frac{x}{4} + \frac{y}{3} = 1$

(D) $x + y = 7$

Q15. The condition for the line $y = mx+c$ to be a tangent to the circle $x^2+y^2 = r^2$ is:

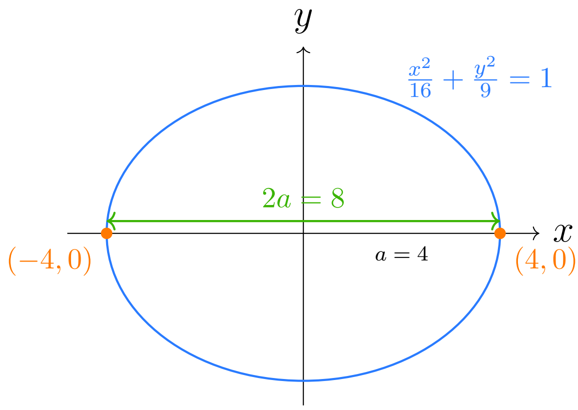
(A) $c = r$

(B) $c = \dfrac{r}{m}$

(C) $c^2 = r^2 m^2$

(D) $c^2 = r^2(1 + m^2)$

Q16. The ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{9} = 1$ is shown below. Find the length of its major axis:



(A) 4

(B) 6

(C) 8

(D) 10

Q17. The points $A(2, 3)$, $B(4, 6)$, and $C(6, 9)$ are:

(A) collinear

(B) vertices of an equilateral triangle

(C) vertices of a right triangle

(D) vertices of an isosceles triangle

Q18. Consider the matrix below where the first two rows are identical. What is its determinant?

$$\begin{vmatrix} 1 & 2 & 3 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{vmatrix}$$

(A) 1

(B) 0

(C) 2

(D) undefined

Q19. A matrix A (with $A \neq 0$) such that $A^2 = 0$ is called:

(A) idempotent

(B) involutory

(C) orthogonal

(D) nilpotent

Q20. Solve for X if $\begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix} X = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$:

(A) $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$

(B) $\begin{pmatrix} 7 \\ 3 \end{pmatrix}$

(C) $\begin{pmatrix} 7 \\ -3 \end{pmatrix}$

(D) $\begin{pmatrix} -7 \\ 3 \end{pmatrix}$

Q21. Solve the system of equations $x + y = 3$ and $x - y = 1$:

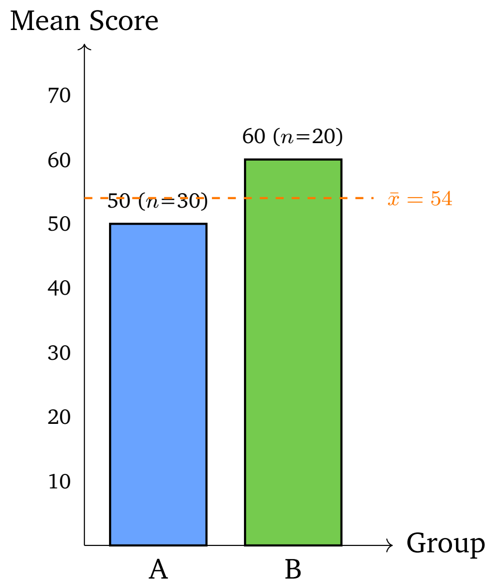
(A) $x = 2,\ y = 1$

(B) $x = 1,\ y = 2$

(C) $x = 3,\ y = 0$

(D) $x = 0,\ y = 3$

Q22. Group A has 30 students with mean score 50, and Group B has 20 students with mean score 60. The bar chart is shown below. Find the combined mean:



(A) 55

(B) 54

(C) 53

(D) 56

Q23. Using the law of total probability: $P(A) = 0.4$, $P(B) = 0.6$, $P(E|A) = 0.3$, $P(E|B) = 0.5$. Find $P(E)$:

(A) 0.38

(B) 0.45

(C) 0.35

(D) 0.42

Q24. For a Binomial distribution $B(n, p)$ with $n = 20$ and $p = 0.3$, the mean $E(X)$ is:

(A) 3

(B) 4

(C) 6

(D) 8

Q25. The formula for Spearman's rank correlation coefficient ρ is:

(A) $1 - \dfrac{6\sum d^2}{n(n^2-1)}$

(B) $\dfrac{\sum d^2}{n-1}$

(C) $\dfrac{6\sum d}{n(n^2-1)}$

(D) $\dfrac{\sum d}{n}$

Q26. Find the range of the data set: $\{15, 21, 18, 33, 27, 12, 45\}$:

(A) 27

(B) 33

(C) 30

(D) 36

Q27. Simplify $\sin(\pi - \theta)$:

(A) $-\sin\theta$

(B) $\cos\theta$

(C) $-\cos\theta$

(D) $\sin\theta$

Q28. Evaluate $\cos^2 30^\circ - \sin^2 30^\circ$ using the identity $\cos 2A = \cos^2 A - \sin^2 A$:

(A) $\dfrac{\sqrt{3}}{2}$

(B) 0

(C) $\dfrac{1}{2}$

(D) $-\dfrac{1}{2}$

Q29. Evaluate $\tan^{-1}(1) + \tan^{-1}(0)$:

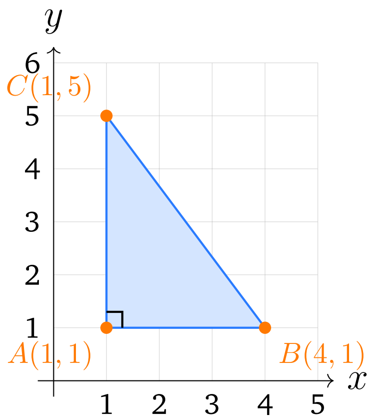
(A) $\dfrac{\pi}{4}$

(B) $\dfrac{\pi}{2}$

(C) π

(D) 0

Q30. Find the area of the triangle with vertices $A(1, 1)$, $B(4, 1)$, and $C(1, 5)$, shown below:



(A) 4

(B) 6

(C) 8

(D) 12

Detailed Solutions

Q1.

Solution

Concept — Geometric Progression (General Term): The n-th term of a GP with first term a and common ratio r is $a_n = a \cdot r^{n-1}$.

Step 1 — Identify parameters: $a = 5$, $r = 2$, $n = 3$.

Step 2 — Apply formula:

$$a_3 = 5 \cdot 2^{3-1} = 5 \cdot 2^2 = 5 \cdot 4 = 20.$$

Why other options are wrong:

- Option B (15): Applies the AP formula $a + (n-1)d$ with $d = 2$: $5 + 2 \cdot 2 = 9$; wrong formula for GP.
- Option C (10): Uses r^1 instead of r^2: $5 \times 2 = 10$.
- Option D (25): Arithmetic error; perhaps adds $a + r^2 = 5 + 20 = 25$.

Final Answer: $a_3 = 20 \Rightarrow \boxed{\text{A}}$

$\boxed{\text{Answer: (A)}}$ Go Back to Q 1

Q2.

Solution

Concept — Natural Logarithm Properties: $\ln(e^k) = k$ for any real k, and $\ln(1) = 0$ since $e^0 = 1$.

Step 1 — Evaluate each term:

$$\ln(e^3) = 3, \qquad \ln(1) = 0.$$

Step 2 — Add:

$$\ln(e^3) + \ln(1) = 3 + 0 = 3.$$

Why other options are wrong:

- Option A (1): Confuses $\ln(e^3)$ with $\ln(e) = 1$; ignores the exponent 3.
- Option C (e^3): Returns the argument of $\ln$, not its value.
- Option D (4): Mistakenly uses $\ln(1) = 1$ instead of 0, giving $3 + 1 = 4$.

Final Answer: $\ln(e^3) + \ln(1) = 3 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 2

Q3.

Solution

Concept — Binomial Theorem (Middle Term): For $(1 + x)^n$ with n even, the single middle term is $T_{n/2+1}$ with $r = n/2$. Its coefficient is $\binom{n}{n/2}$.

Step 1 — Identify middle term: $n = 8$ (even). Middle term is T_5, corresponding to $r = 4$.

Step 2 — Compute coefficient:

$$\binom{8}{4} = \frac{8!}{4!\,4!} = \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} = \frac{1680}{24} = 70.$$

Why other options are wrong:

- Option A (28): This is $\binom{8}{2}$, the coefficient of x^2.
- Option B (56): This is $\binom{8}{3}$, the coefficient of x^3.
- Option C (35): This is $\binom{7}{3}$; uses wrong n.

Final Answer: Coefficient $= 70 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 3

Q4.

Solution

Concept — Discriminant of a Quadratic: For $ax^2 + bx + c = 0$, the discriminant $\Delta = b^2 - 4ac$ determines the nature of roots.

Step 1 — Recall conditions:

- $\Delta > 0$: two real and distinct roots.
- $\Delta = 0$: two real and equal (repeated) roots.
- $\Delta < 0$: two complex (non-real) roots.

Step 2 — Identify correct condition: For real and distinct roots, we need $b^2 - 4ac > 0$.

Why other options are wrong:

- Option A ($b^2 - 4ac = 0$): Gives equal (repeated) roots, not distinct.
- Option B ($b^2 - 4ac < 0$): Gives complex (imaginary) roots.
- Option D ($b^2+4ac > 0$): Incorrect formula; the discriminant uses subtraction.

Final Answer: $b^2 - 4ac > 0 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 4

Q5.

Solution

Concept — Permutations with Repeated Letters: Number of distinct arrangements of n letters with repetitions $n_1, n_2, \ldots$ is $\frac{n!}{n_1!\, n_2! \cdots}$.

Step 1 — Count letters in LEVEL: L occurs 2 times, E occurs 2 times, V occurs 1 time. Total $n = 5$.

Step 2 — Apply formula:

$$\frac{5!}{2! \times 2!} = \frac{120}{2 \times 2} = \frac{120}{4} = 30.$$

Why other options are wrong:

- Option B (60): Divides by only one $2!$; accounts for only one repeated pair.
- Option C (120): This is $5!$; treats all letters as distinct.
- Option D (20): Uses an incorrect denominator $3! \times 1! = 6$.

Final Answer: Distinct arrangements $= 30 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 5

Q6.

Solution

Concept — Combinations (At Least k): "At least 8 from 10" means selecting exactly 8, 9, or 10 people. Sum the corresponding binomial coefficients.

Step 1 — Compute each term:

$$\binom{10}{8} = \frac{10!}{8!\,2!} = 45, \qquad \binom{10}{9} = 10, \qquad \binom{10}{10} = 1.$$

Step 2 — Sum:

$$45 + 10 + 1 = 56.$$

Why other options are wrong:

- Option A (45): Only counts $\binom{10}{8}$; omits selections of 9 and 10 people.
- Option C (111): Incorrectly sums more terms or makes arithmetic error.
- Option D (100): Not a valid combinatorial result for this problem.

Final Answer: Total ways $= 56 \Rightarrow \boxed{\textbf{B}}$

Answer: (B) Go Back to Q 6

Q7.

Solution

Concept — Product Rule: If $y = u(x) \cdot v(x)$, then $\dfrac{dy}{dx} = u'v + uv'$.

Step 1 — Identify u and v: $u = x^2 \Rightarrow u' = 2x; \quad v = \cos x \Rightarrow v' = -\sin x$.

Step 2 — Apply:

$$\frac{d}{dx}[x^2 \cos x] = 2x \cdot \cos x + x^2 \cdot (-\sin x) = 2x \cos x - x^2 \sin x.$$

Why other options are wrong:

- Option A ($x^2 \cos x$): Returns the original function; no differentiation applied.
- Option B ($-2x \sin x$): Takes derivative of v only; ignores the $u'v$ term.
- Option C ($2x \cos x + x^2 \sin x$): Sign error on v'; uses $+\sin x$ instead of $-\sin x$.

Final Answer: $\dfrac{d}{dx}[x^2 \cos x] = 2x \cos x - x^2 \sin x \Rightarrow \boxed{\textbf{D}}$

Answer: (D) Go Back to Q 7

Q8.

Solution

Concept — Second Derivative: Differentiate the function twice successively.

Step 1 — First derivative of $y = \sin x$:

$$\frac{dy}{dx} = \cos x.$$

Step 2 — Second derivative:

$$\frac{d^2y}{dx^2} = \frac{d}{dx}(\cos x) = -\sin x.$$

Why other options are wrong:

- Option A ($\sin x$): This is the original function y, not its second derivative.
- Option B ($\cos x$): This is the first derivative dy/dx, not the second.
- Option D ($-\cos x$): This is the third derivative $d^3y/dx^3 = \frac{d}{dx}(-\sin x) = -\cos x$.

Final Answer: $\frac{d^2y}{dx^2} = -\sin x \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 8

Q9.

Solution

Concept — Limit via Factorisation: Direct substitution gives $0/0$ (indeterminate). Factor and cancel the common factor.

Step 1 — Factor the numerator:

$$x^2 - 1 = (x-1)(x+1).$$

Step 2 — Cancel and evaluate:

$$\lim_{x\to 1} \frac{(x-1)(x+1)}{x-1} = \lim_{x\to 1}(x+1) = 1 + 1 = 2.$$

Why other options are wrong:

- Option B (0): Plugs in numerator $1^2 - 1 = 0$ alone without cancelling the denominator.

- Option C (1): Evaluates $x = 1$ in the denominator $x - 1$ directly, getting 0; or uses wrong simplified form.
- Option D (∞): Assumes denominator $\to 0$ while numerator $\not\to 0$, which is false here.

Final Answer: $\lim_{x \to 1} \frac{x^2 - 1}{x - 1} = 2 \Rightarrow \boxed{\mathbf{A}}$

Answer: (A) Go Back to Q 9

Q10.

Solution

Concept — Odd Function Property: If $f(x)$ is odd (i.e., $f(-x) = -f(x)$), then $\int_{-a}^{a} f(x)\,dx = 0$.

Step 1 — Verify $f(x) = x^3$ is odd:

$$f(-x) = (-x)^3 = -x^3 = -f(x). \quad \checkmark$$

Step 2 — Apply the property:

$$\int_{-2}^{2} x^3\,dx = 0.$$

Direct verification:

$$\left[\frac{x^4}{4}\right]_{-2}^{2} = \frac{16}{4} - \frac{16}{4} = 4 - 4 = 0. \quad \checkmark$$

Why other options are wrong:

- Option A (4): Computes $\int_0^2 x^3\,dx = 4$; takes only the positive half.
- Option C (8): Doubles $\int_0^2 x^3\,dx$ instead of recognising cancellation.
- Option D (-4): Computes only $\int_{-2}^{0} x^3\,dx = -4$; takes only the negative half.

Final Answer: $\int_{-2}^{2} x^3\,dx = 0 \Rightarrow \boxed{\mathbf{B}}$

Answer: (B) Go Back to Q 10

Q11.

Solution

Concept — Separation of Variables: Rearrange so y-terms are on one side and x-terms on the other, then integrate.

Step 1 — Separate:

$$\frac{dy}{dx} = -\frac{y}{x} \implies \frac{dy}{y} = -\frac{dx}{x}.$$

Step 2 — Integrate both sides:

$$\ln|y| = -\ln|x| + k \implies \ln|y| + \ln|x| = k \implies \ln|xy| = k \implies xy = e^k.$$

$$\therefore \quad y = \frac{C}{x}, \quad C = e^k.$$

Why other options are wrong:

- Option A ($y = Cx$): Solves $dy/dx = y/x$ (positive sign); sign is opposite.
- Option B ($y = Ce^x$): Solves $dy/dx = y$, a different ODE.
- Option C ($y = Cx^2$): Would give $dy/dx = 2Cx = 2y/x$; inconsistent.

Final Answer: $y = C/x \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 11

Q12.

Solution

Concept — Rate of Change (Chain Rule): $A = \pi r^2$. Differentiating implicitly with respect to time:

$$\frac{dA}{dt} = 2\pi r \cdot \frac{dr}{dt}.$$

Step 1 — Substitute given values: $r = 5$ cm, $\frac{dr}{dt} = 2$ cm/s.

Step 2 — Compute:

$$\frac{dA}{dt} = 2\pi(5)(2) = 20\pi \text{ cm}^2/\text{s}.$$

Why other options are wrong:

- Option A (10π): Uses $\frac{dA}{dt} = 2\pi r = 2\pi(5) = 10\pi$; forgets to multiply by $dr/dt = 2$.
- Option B (4π): Substitutes $r = 1$; wrong radius used.

- Option D (25π): Uses $\frac{dA}{dt} = \pi r^2 \cdot \frac{dr}{dt} = \pi(25)(2)/2$; incorrect differentiation of r^2.

Final Answer: $\frac{dA}{dt} = 20\pi$ **cm²/s** $\Rightarrow \boxed{\textbf{C}}$

Answer: (C) Go Back to Q 12

Q13.

Solution

Concept — Perpendicular Bisector: The perpendicular bisector of AB passes through the midpoint of AB and is perpendicular to AB.

Step 1 — Midpoint M:

$$M = \left(\frac{1+5}{2}, \frac{2+4}{2}\right) = (3,\ 3).$$

Step 2 — Slope of AB and slope of perpendicular bisector:

$$\text{slope of } AB = \frac{4-2}{5-1} = \frac{2}{4} = \frac{1}{2}.$$

$$\text{slope of perp. bisector} = -\frac{1}{1/2} = -2.$$

Step 3 — Equation through $M(3,3)$ with slope -2:

$$y - 3 = -2(x-3) \implies y - 3 = -2x + 6 \implies 2x + y = 9.$$

Why other options are wrong:

- Option B ($x + 2y = 9$): Has slope $-1/2$ (same as AB); not perpendicular.
- Option C ($2x - y = 3$): Has slope 2; not the negative reciprocal of slope of AB.
- Option D ($x - 2y + 1 = 0$): Check with $M(3,3)$: $3 - 6 + 1 = -2 \neq 0$; does not pass through midpoint.

Final Answer: $2x + y = 9 \Rightarrow \boxed{\textbf{A}}$

Answer: (A) Go Back to Q 13

Q14.

Solution

Concept — Intercept Form of a Line: The line with x-intercept a and y-intercept b is $\frac{x}{a} + \frac{y}{b} = 1$.

Step 1 — Apply with $a = 3$, $b = 4$:

$$\frac{x}{3} + \frac{y}{4} = 1.$$

Step 2 — Multiply through by lcm$(3, 4) = 12$:

$$4x + 3y = 12.$$

Why other options are wrong:

- Option A ($3x + 4y = 12$): Swaps intercepts; this line has x-intercept 4 and y-intercept 3.
- Option C ($x/4 + y/3 = 1$): Assigns $a = 4$ and $b = 3$; intercepts are reversed.
- Option D ($x + y = 7$): Simply sums the intercepts; not derived from the intercept form.

Final Answer: $4x + 3y = 12 \Rightarrow \boxed{\textbf{B}}$

Answer: (B) Go Back to Q 14

Q15.

Solution

Concept — Tangency Condition: The line $y = mx + c$ is tangent to $x^2 + y^2 = r^2$ iff the perpendicular distance from the centre $(0, 0)$ to the line equals r.

Step 1 — Line in standard form: $mx - y + c = 0$.

Step 2 — Perpendicular distance from $(0, 0)$:

$$d = \frac{|m(0) - 0 + c|}{\sqrt{m^2 + 1}} = \frac{|c|}{\sqrt{1 + m^2}}.$$

Step 3 — Set $d = r$ and square:

$$\frac{c^2}{1 + m^2} = r^2 \implies c^2 = r^2(1 + m^2).$$

Why other options are wrong:

- Option A ($c = r$): Valid only for horizontal tangent ($m = 0$); not general.
- Option B ($c = r/m$): Has no standard geometric derivation.
- Option C ($c^2 = r^2m^2$): Missing the $+1$ under the square root; omits the constant term.

Final Answer: $c^2 = r^2(1 + m^2) \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 15

Q16.

Solution

Concept — Standard Ellipse: For $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $a > b > 0$, the major axis is along the x-axis and has length $2a$.

Step 1 — Read a^2:

$$a^2 = 16 \implies a = 4.$$

Step 2 — Length of major axis:

$$2a = 2 \times 4 = 8.$$

Why other options are wrong:

- Option A (4): This is a, the semi-major axis length, not the full axis.
- Option B (6): This is $2b = 2 \times 3 = 6$, the length of the minor axis.
- Option D (10): Sums $a + b = 4 + 3 = 7$; not a standard axis-length formula.

Final Answer: Major axis length $= 8 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 16

Q17.

Solution

Concept — Collinearity Test: Three points are collinear if the slope between every pair of points is the same, or equivalently if the area of the triangle they form equals zero.

Step 1 — Compute slopes:

$$\text{slope } AB = \frac{6-3}{4-2} = \frac{3}{2}, \qquad \text{slope } BC = \frac{9-6}{6-4} = \frac{3}{2}.$$

Step 2 — Conclusion: Equal slopes $\Rightarrow A, B, C$ are collinear.

Area verification:

$$\text{Area} = \tfrac{1}{2}|2(6-9)+4(9-3)+6(3-6)| = \tfrac{1}{2}|-6+24-18| = \tfrac{1}{2}\cdot 0 = 0.$$

Why other options are wrong:

- Options B, C, D all require the three points to form a non-degenerate triangle (area > 0), which is not the case here.

Final Answer: The points are collinear $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 17

Q18.

Solution

Concept — Identical-Row Property: If any two rows (or columns) of a square matrix are identical, its determinant is 0.

Step 1 — Observe: Rows 1 and 2 are both $[1,\ 2,\ 3]$. By the property, $\det = 0$.

Step 2 — Explicit verification (cofactor expansion along Row 1):

$$\det = 1(2\cdot 6 - 3\cdot 5) - 2(1\cdot 6 - 3\cdot 4) + 3(1\cdot 5 - 2\cdot 4)$$

$$= 1(12-15) - 2(6-12) + 3(5-8) = -3 + 12 - 9 = 0. \quad \checkmark$$

Why other options are wrong:

- Option A (1): Determinant of the identity matrix; does not apply here.
- Option C (2): Arithmetic error; property guarantees 0.

- Option D (undefined): Determinant always exists for any square matrix; identical rows give 0, not undefined.

Final Answer: $\det = 0 \Rightarrow \boxed{\textbf{B}}$

Answer: (B) Go Back to Q 18

Q19.

Solution

Concept — Special Matrix Types:

- **Idempotent:** $A^2 = A$.
- **Involutory:** $A^2 = I$ (identity matrix).
- **Orthogonal:** $AA^T = I$.
- **Nilpotent:** $A^k = 0$ for some positive integer k (with $A \neq 0$).

Step 1 — Match condition: The given condition is $A^2 = 0$ with $A \neq 0$. This is the definition of a **nilpotent** matrix (with nilpotency index 2).

Why other options are wrong:

- Option A (idempotent): Requires $A^2 = A$; result must equal A, not zero.
- Option B (involutory): Requires $A^2 = I$; result must be the identity.
- Option C (orthogonal): Defined by $AA^T = I$; unrelated to the nilpotency condition.

Final Answer: The matrix is nilpotent $\Rightarrow \boxed{\textbf{D}}$

Answer: (D) Go Back to Q 19

Q20.

Solution

Concept — Matrix Equation $AX = b$**:** Pre-multiply both sides by A^{-1} to get $X = A^{-1}b$.

Step 1 — Compute $\det(A)$ **for** $A = \begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix}$**:**

$$\det(A) = 1 \cdot 7 - 2 \cdot 3 = 7 - 6 = 1.$$

Step 2 — Find A^{-1}:

$$A^{-1} = \frac{1}{1}\begin{pmatrix} 7 & -2 \\ -3 & 1 \end{pmatrix} = \begin{pmatrix} 7 & -2 \\ -3 & 1 \end{pmatrix}.$$

Step 3 — Compute $X = A^{-1}b$ with $b = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$:

$$X = \begin{pmatrix} 7 & -2 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 7(1) + (-2)(0) \\ (-3)(1) + (1)(0) \end{pmatrix} = \begin{pmatrix} 7 \\ -3 \end{pmatrix}.$$

Why other options are wrong:

- Option A ($\begin{pmatrix} 1 \\ -3 \end{pmatrix}$): Uses $X = b$ directly without applying A^{-1}.
- Option B ($\begin{pmatrix} 7 \\ 3 \end{pmatrix}$): Sign error on lower entry; $-3 \cdot 1 = -3$, not $+3$.
- Option D ($\begin{pmatrix} -7 \\ 3 \end{pmatrix}$): Negates entries of correct answer incorrectly.

Final Answer: $X = \begin{pmatrix} 7 \\ -3 \end{pmatrix} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 20

Q21.

Solution

Concept — Elimination Method: Add or subtract the equations to eliminate one variable.

Step 1 — Add both equations:

$$(x + y) + (x - y) = 3 + 1 \implies 2x = 4 \implies x = 2.$$

Step 2 — Substitute $x = 2$ into $x + y = 3$:

$$2 + y = 3 \implies y = 1.$$

Verification: $x - y = 2 - 1 = 1$ ✓.

Why other options are wrong:

- Option B ($x = 1, y = 2$): Swaps x and y; check: $1 - 2 = -1 \neq 1$.
- Option C ($x = 3, y = 0$): Check: $3 - 0 = 3 \neq 1$.
- Option D ($x = 0, y = 3$): Check: $0 - 3 = -3 \neq 1$.

Final Answer: $x = 2,\ y = 1 \Rightarrow \boxed{\textbf{A}}$

Answer: (A) Go Back to Q 21

Q22.

Solution

Concept — Combined (Weighted) Mean:

$$\bar{x}_{\text{combined}} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}.$$

Step 1 — Identify values: $n_1 = 30,\ \bar{x}_1 = 50; \quad n_2 = 20,\ \bar{x}_2 = 60.$

Step 2 — Compute:

$$\bar{x}_{\text{combined}} = \frac{30 \times 50 + 20 \times 60}{30 + 20} = \frac{1500 + 1200}{50} = \frac{2700}{50} = 54.$$

Why other options are wrong:

- Option A (55): Simple (unweighted) average $(50 + 60)/2 = 55$; ignores the different group sizes.
- Option C (53): Arithmetic error in the numerator.
- Option D (56): Overweights Group B; incorrect numerator calculation.

Final Answer: Combined mean $= 54 \Rightarrow \boxed{\textbf{B}}$

Answer: (B) Go Back to Q 22

Q23.

Solution

Concept — Law of Total Probability: If A and B partition the sample space (mutually exclusive and exhaustive), then:

$$P(E) = P(E|A) \cdot P(A) + P(E|B) \cdot P(B).$$

Step 1 — Verify partition: $P(A) + P(B) = 0.4 + 0.6 = 1$ ✓.

Step 2 — Compute:

$$P(E) = (0.3)(0.4) + (0.5)(0.6) = 0.12 + 0.30 = 0.42.$$

Why other options are wrong:

- Option A (0.38): Arithmetic error; perhaps uses $0.3 \times 0.4 + 0.4 \times 0.6 = 0.12 + 0.24 = 0.36$.
- Option B (0.45): Simple average of conditional probabilities $(0.3 + 0.5)/2$; ignores weights.
- Option C (0.35): Uses incorrect weighting or arithmetic error.

Final Answer: $P(E) = 0.42 \Rightarrow \boxed{\textbf{D}}$

Answer: (D) Go Back to Q 23

Q24.

Solution

Concept — Mean of Binomial Distribution: For $X \sim B(n, p)$, the mean is $E(X) = np$.

Step 1 — Identify parameters: $n = 20$, $p = 0.3$.

Step 2 — Compute:

$$E(X) = np = 20 \times 0.3 = 6.$$

Why other options are wrong:

- Option A (3): Uses $E(X) = np/2$; incorrect halving.
- Option B (4): Arithmetic error; perhaps $20 \times 0.2 = 4$.
- Option D (8): Uses $nq = 20 \times 0.7$ mistakenly as the mean, or arithmetic error.

Final Answer: $E(X) = 6 \Rightarrow \boxed{\textbf{C}}$

Answer: (C) Go Back to Q 24

Q25.

Solution

Concept — Spearman's Rank Correlation: Spearman's coefficient ρ measures the monotonic association between two ranked variables. The formula is:

$$\rho = 1 - \frac{6\sum d_i^2}{n(n^2-1)},$$

where d_i is the rank difference for the i-th observation and n is the sample size.

Step 1 — Key features of the formula:

- Factor of 6 in the numerator.
- $\sum d^2$ (sum of *squared* rank differences; squaring removes sign).
- Denominator $n(n^2-1)$.

Why other options are wrong:

- Option B: Missing the factor 6 and uses $n-1$ in denominator; not Spearman's formula.
- Option C: Uses $\sum d$ instead of $\sum d^2$; sum of differences (not squared) cancels out to zero always.
- Option D: Merely averages rank differences; no squaring or standard scaling.

Final Answer: $\rho = 1 - \frac{6\sum d^2}{n(n^2-1)} \Rightarrow \boxed{\textbf{A}}$

Answer: (A) Go Back to Q 25

Q26.

Solution

Concept — Range: Range $=$ Maximum value $-$ Minimum value.

Step 1 — Sort the data: $\{12, 15, 18, 21, 27, 33, 45\}$.

$$\max = 45, \qquad \min = 12.$$

Step 2 — Compute:

$$\text{Range} = 45 - 12 = 33.$$

Why other options are wrong:

- Option A (27): This is one of the data values, not the range.
- Option C (30): Arithmetic error; perhaps $45 - 15 = 30$.
- Option D (36): Arithmetic error; perhaps $45 - 9 = 36$ (wrong minimum).

Final Answer: Range $= 33 \Rightarrow \boxed{\textbf{B}}$

Answer: (B) Go Back to Q 26

Q27.

Solution

Concept — Supplementary Angle Identity: Using the sine addition formula with π and $-\theta$:

$$\sin(\pi - \theta) = \sin \pi \cos \theta - \cos \pi \sin \theta = 0 \cdot \cos \theta - (-1) \cdot \sin \theta = \sin \theta.$$

Step 1 — Apply directly:

$$\sin(\pi - \theta) = \sin \theta.$$

Why other options are wrong:

- Option A ($-\sin \theta$): This is $\sin(\pi + \theta)$, not $\sin(\pi - \theta)$.
- Option B ($\cos \theta$): This is $\sin\left(\frac{\pi}{2} - \theta\right)$, a complementary angle identity.
- Option C ($-\cos \theta$): This is $\cos(\pi - \theta)$, not $\sin(\pi - \theta)$.

Final Answer: $\sin(\pi - \theta) = \sin \theta \Rightarrow \boxed{\textbf{D}}$

Answer: (D) Go Back to Q 27

Q28.

Solution

Concept — Double Angle Identity: $\cos 2A = \cos^2 A - \sin^2 A$.

Step 1 — Set $A = 30$**:**

$$\cos^2 30^\circ - \sin^2 30^\circ = \cos(2 \times 30^\circ) = \cos 60^\circ = \frac{1}{2}.$$

Direct verification: $\cos 30 = \dfrac{\sqrt{3}}{2}$, $\sin 30 = \dfrac{1}{2}$.

$$\cos^2 30^\circ - \sin^2 30^\circ = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}. \quad \checkmark$$

Why other options are wrong:

- Option A ($\sqrt{3}/2$): This is $\cos 30$ or $\sin 60$; not $\cos 60$.
- Option B (0): Would require $A = 45$ where $\cos^2 A = \sin^2 A$.
- Option D ($-1/2$): This is $\cos 120$; sign error.

Final Answer: $\cos^2 30^\circ - \sin^2 30^\circ = \dfrac{1}{2} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 28

Q29.

Solution

Concept — Inverse Tangent Values:

$$\tan^{-1}(1) = \frac{\pi}{4} \quad (\text{since } \tan\tfrac{\pi}{4} = 1), \qquad \tan^{-1}(0) = 0 \quad (\text{since } \tan 0 = 0).$$

Step 1 — Add:

$$\tan^{-1}(1) + \tan^{-1}(0) = \frac{\pi}{4} + 0 = \frac{\pi}{4}.$$

Why other options are wrong:

- Option B ($\pi/2$): This equals $\tan^{-1}(1) + \tan^{-1}(1) = \pi/4 + \pi/4$; adds $\tan^{-1}(1)$ twice.
- Option C (π): The range of $\tan^{-1}$ is $(-\pi/2, \pi/2)$; sum cannot reach π.
- Option D (0): Ignores the $\tan^{-1}(1) = \pi/4$ contribution.

Final Answer: $\tan^{-1}(1) + \tan^{-1}(0) = \dfrac{\pi}{4} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 29

Q30.

Solution

Concept — Area of Triangle (Coordinate Formula):

$$\text{Area} = \tfrac{1}{2}\left|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)\right|.$$

Step 1 — Substitute $A(1,1)$, $B(4,1)$, $C(1,5)$:

$$\text{Area} = \tfrac{1}{2}|1(1-5) + 4(5-1) + 1(1-1)|$$

$$= \tfrac{1}{2}|1(-4) + 4(4) + 1(0)| = \tfrac{1}{2}|-4 + 16 + 0| = \tfrac{1}{2} \times 12 = 6.$$

Alternative (right-triangle method): The right angle is at $A(1,1)$.
Base $AB = 4 - 1 = 3$ (horizontal). Height $AC = 5 - 1 = 4$ (vertical).

$$\text{Area} = \tfrac{1}{2} \times 3 \times 4 = 6. \quad \checkmark$$

Why other options are wrong:

- Option A (4): Uses $\frac{1}{2} \times$ base without multiplying by height; incomplete formula.
- Option C (8): Uses base $= 4$ instead of 3; $\frac{1}{2} \times 4 \times 4 = 8$.
- Option D (12): Computes base $\times$ height $= 3 \times 4 = 12$ without the factor $\frac{1}{2}$.

Final Answer: Area $= 6 \Rightarrow \boxed{\textbf{B}}$

Answer: (B) Go Back to Q 30

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	D	4	C	5	A
6	B	7	D	8	C	9	A	10	B
11	D	12	C	13	A	14	B	15	D
16	C	17	A	18	B	19	D	20	C
21	A	22	B	23	D	24	C	25	A
26	B	27	D	28	C	29	A	30	B