MAH MCA CET Mathematics & Statistics

Sample Paper – 9

Duration: 27 Minutes **Maximum Marks: 60**

Instructions

- This paper contains **30** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics & Statistics portion of **MAH MCA CET** entrance.
- Each correct answer carries **+2 marks**. There is **no negative marking** for incorrect or unattempted answers.
- Only **one** option is correct. Choose carefully.
- Syllabus level: **Class 12 level — Algebra, Calculus, Coordinate Geometry, Matrices, Statistics & Probability, Trigonometry.**
- Use of mobile phones, calculators, or electronic gadgets is strictly prohibited.

Q1. In an arithmetic progression with first term $\frac{1}{2}$ and common difference $\frac{1}{2}$, which term equals $\frac{7}{2}$?

(A) 5th term

(B) 6th term

(C) 8th term

(D) 7th term

Q2. Simplify: $3^{\log_3 5}$

(A) 3

(B) $\log_3 5$

(C) 5

(D) 15

Q3. Find the constant term in the expansion of $\left(x - \frac{1}{x}\right)^6$.

(A) 20

(B) -20

(C) 15

(D) -15

Q4. Solve $x^2 - 4x + 1 = 0$ by completing the square.

(A) $2 \pm \sqrt{3}$

(B) $2 \pm \sqrt{2}$

(C) $4 \pm \sqrt{3}$

(D) $\pm\sqrt{3}$

Q5. A student must choose 3 courses from 5 compulsory and 3 optional courses, with **at least one** optional course. In how many ways can this be done?

(A) 30

(B) 36

(C) 40

(D) 46

Q6. For $n = 10$, the greatest binomial coefficient $\binom{10}{r}$ equals:

(A) 210

(B) 120

(C) 252

(D) 45

Q7. Find $\frac{d}{dx}\left[\tan^{-1} x\right]$:

(A) $\dfrac{1}{\sqrt{1-x^2}}$

(B) $\dfrac{1}{1+x^2}$

(C) $\tan x$

(D) $\sec^2 x$

Q8. According to the Mean Value Theorem, for a function f continuous on $[a,b]$ and differentiable on (a,b), there exists $c \in (a,b)$ such that:

(A) $f'(c) = \dfrac{f(b)-f(a)}{b-a}$

(B) $f(c) = f(a) + f(b)$

(C) $f'(c) = 0$

(D) $f'(c) = \dfrac{f(a)}{f(b)}$

Q9. Evaluate $\lim_{x\to 2} \dfrac{x^3-8}{x-2}$:

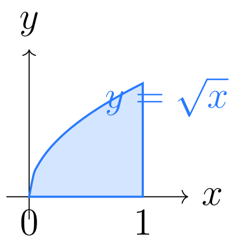
(A) 4

(B) 6

(C) 8

(D) 12

Q10. The shaded region below represents the area under $y = \sqrt{x}$ from 0 to 1.



Find the value of $\int_0^1 \sqrt{x}\, dx$:

(A) $\dfrac{1}{2}$

(B) $\dfrac{1}{3}$

(C) $\frac{2}{3}$

(D) $\frac{3}{2}$

Q11. For the ODE $\frac{dy}{dx} = \frac{y}{x} + \frac{y^2}{x^2}$, using the substitution $v = \frac{y}{x}$, the general solution is:

(A) $y = Cx$

(B) $y = \frac{-x}{\ln|x| + C}$

(C) $y = \frac{Ce^x}{x}$

(D) $y = x\ln(Cx)$

Q12. On which interval is $f(x) = x^2 - 4x + 5$ a **decreasing** function?

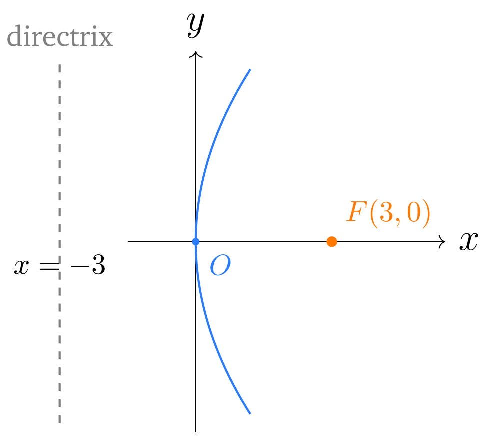
(A) $[0,\ 2]$

(B) $[2,\ 4]$

(C) $[1,\ 3]$

(D) $[2,\ \infty)$

Q13. The parabola $y^2 = 12x$ is shown below with its directrix and focus marked.



Find the equation of the directrix of the parabola $y^2 = 12x$:

(A) $x = 3$

(B) $y = 3$

(C) $y = -3$

(D) $x = -3$

Q14. The three lines $2x + 3y = 4$, $x - y = 1$, and $3x + ky = 7$ are concurrent. Find the value of k:

(A) $k = 3$

(B) $k = 5$

(C) $k = 7$

(D) $k = 9$

Q15. Determine whether the point $(2, 3)$ lies inside, on, or outside the circle $x^2 + y^2 = 9$:

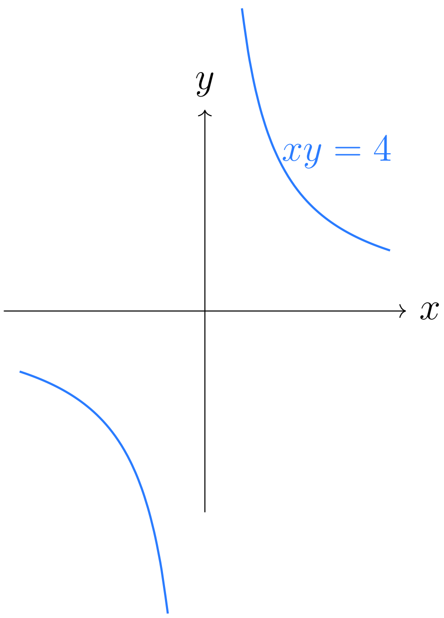
(A) Inside the circle

(B) Outside the circle

(C) On the circle

(D) Cannot be determined

Q16. Which of the following is an example of a **rectangular hyperbola**? The figure shows one candidate curve.



(A) $xy = 4$

(B) $\dfrac{x^2}{4} - \dfrac{y^2}{9} = 1$

(C) $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$

(D) $x^2 - 4y^2 = 1$

Q17. Find the reflection of the point $(3, 4)$ in the x-axis:

(A) $(-3,\ 4)$

(B) $(-3,\ -4)$

(C) $(4,\ 3)$

(D) $(3,\ -4)$

Q18. Find the cofactor A_{21} of the matrix shown below:

$$\begin{bmatrix} 1 & 2 & 3 \\ \mathbf{0} & 1 & 4 \\ 5 & 6 & 0 \end{bmatrix}$$

(The element $a_{21} = 0$ is in row 2.)

(A) -18

(B) -6

(C) 18

(D) 6

Q19. If A is an orthogonal matrix (i.e. $AA^T = I$), then which of the following is necessarily true?

(A) $\det(A) = 0$

(B) $\det(A) = \pm 1$

(C) $A = A^T$

(D) $A^2 = I$ always

Q20. Write the augmented matrix $[A \mid \mathbf{b}]$ for the system of equations $x + y = 2$ and $2x + 3y = 5$:

(A) $\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 2 & 3 & 5 \end{array}\right]$

(B) $\begin{bmatrix} 1 & 2 & 1 \\ 3 & 2 & 5 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 1 \\ 2 & 3 \end{bmatrix}$

(D) $\begin{bmatrix} 2 & 1 & 1 \\ 5 & 3 & 2 \end{bmatrix}$

Q21. The system of equations $2x + 4y = 6$ and $x + 2y = 3$ has:

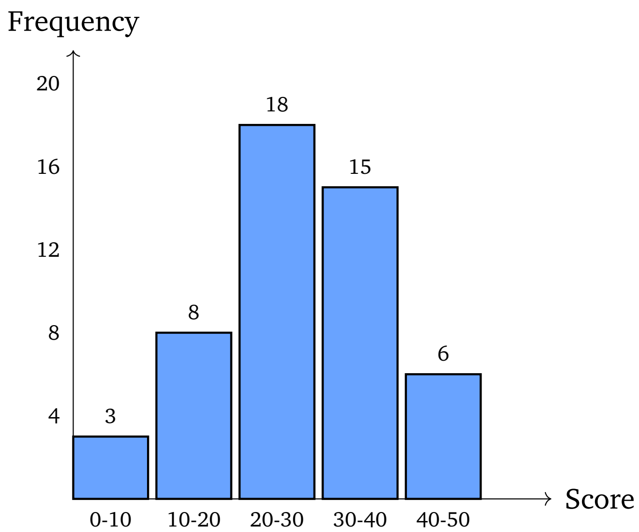
(A) A unique solution

(B) No solution

(C) Exactly two solutions

(D) Infinitely many solutions

Q22. The histogram below shows the scores of 50 students. Which class interval has the **highest** frequency?



(A) 10–20

(B) 30–40

(C) 20–30

(D) 40–50

Q23. Three unbiased coins are tossed simultaneously. Find $P(\text{at least one head})$:

(A) $\frac{3}{8}$

(B) $\frac{7}{8}$

(C) $\frac{1}{2}$

(D) $\frac{5}{8}$

Q24. For a binomial distribution with $n = 15$ and $p = 0.4$, find the **variance**:

(A) 3.6

(B) 6

(C) 0.24

(D) 9

Q25. Given regression coefficients $b_{yx} = 0.75$ and $b_{xy} = 0.48$, find the Karl Pearson's correlation coefficient r:

(A) 0.36

(B) 0.75

(C) 0.48

(D) 0.6

Q26. If the **variance** of a dataset is 25, what is the **standard deviation**?

(A) 25

(B) 10

(C) 5

(D) 625

Q27. Using $\cos(A-B) = \cos A\cos B+\sin A\sin B$, find the exact value of $\cos 15$:

(A) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

(B) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$

(C) $\dfrac{\sqrt{3}}{2}$

(D) $\dfrac{1}{2}$

Q28. Simplify $\sin 3x - \sin x$ using the sum-to-product identity:

(A) $2\cos(2x)\sin(x)$

(B) $2\sin(2x)\cos(x)$

(C) $2\cos(2x)\cos(x)$

(D) $\cos(2x)\sin(x)$

Q29. Find the principal value of $\cos^{-1}\left(-\dfrac{1}{2}\right)$:

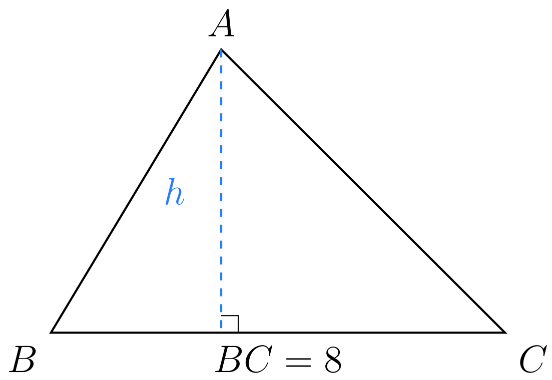
(A) $\dfrac{\pi}{3}$

(B) $\dfrac{5\pi}{6}$

(C) $\dfrac{\pi}{2}$

(D) $\dfrac{2\pi}{3}$

Q30. The area of a triangle is 20 sq. units and its base $BC = 8$ units. The diagram below shows the altitude h from A to BC.



Find the altitude h from A to BC:

(A) 2.5

(B) 4

(C) 5

(D) 8

Detailed Solutions

Q1.

Solution

Concept — n-th Term of an Arithmetic Progression: For an AP with first term a and common difference d, the n-th term is $T_n = a + (n-1)d$.

Step 1 — Identify parameters: $a = \frac{1}{2}$, $d = \frac{1}{2}$, $T_n = \frac{7}{2}$.

Step 2 — Set up and solve:

$$\frac{1}{2} + (n-1)\cdot\frac{1}{2} = \frac{7}{2} \implies \frac{1+(n-1)}{2} = \frac{7}{2} \implies n = 7.$$

Why other options are wrong:

- Option A (5th): $T_5 = \frac{1}{2} + \frac{4}{2} = \frac{5}{2} \neq \frac{7}{2}$.
- Option B (6th): $T_6 = \frac{1}{2} + \frac{5}{2} = 3 \neq \frac{7}{2}$.
- Option C (8th): $T_8 = \frac{1}{2} + \frac{7}{2} = 4 \neq \frac{7}{2}$.

Final Answer: $n = 7 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 1

Q2.

Solution

Concept — Exponential–Logarithm Identity: For any base $a > 0$, $a^{\log_a x} = x$.

Step 1 — Apply the identity directly:

$$3^{\log_3 5} = 5.$$

This follows because $\log_3 5$ is the exponent to which 3 must be raised to obtain 5, so raising 3 to that exponent recovers 5.

Why other options are wrong:

- Option A (3): Ignores the exponent; $3^1 = 3$ only if $\log_3 5 = 1$, which is false.
- Option B ($\log_3 5$): Confuses the exponent with the result.
- Option D (15): Incorrectly multiplies base by argument: $3 \times 5 = 15$.

Final Answer: $3^{\log_3 5} = 5 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 2

Q3.

Solution

Concept — General Term in Binomial Expansion: The general term in $\left(x - \frac{1}{x}\right)^6$ is

$$T_{r+1} = \binom{6}{r} x^{6-r} \cdot \left(-\frac{1}{x}\right)^r = \binom{6}{r} (-1)^r \, x^{6-2r}.$$

Step 1 — Find r for the constant term: Set $6 - 2r = 0 \Rightarrow r = 3$.

Step 2 — Compute the term:

$$T_4 = \binom{6}{3} (-1)^3 = 20 \times (-1) = -20.$$

Why other options are wrong:

- Option A (20): Correct magnitude but wrong sign; $(-1)^3 = -1$ is missed.
- Option C (15): Uses $\binom{6}{2} = 15$ instead of $\binom{6}{3} = 20$.
- Option D (-15): Correct sign but wrong coefficient.

Final Answer: Constant term $= -20 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 3

Q4.

Solution

Concept — Completing the Square: Rewrite $x^2 - 4x + 1 = 0$ by adding and subtracting $\left(\frac{4}{2}\right)^2 = 4$.

Step 1 — Complete the square:

$$(x-2)^2 - 4 + 1 = 0 \implies (x-2)^2 = 3.$$

Step 2 — Solve:

$$x - 2 = \pm\sqrt{3} \implies x = 2 \pm \sqrt{3}.$$

Why other options are wrong:

- Option B ($2 \pm \sqrt{2}$): Uses $\sqrt{2}$ instead of $\sqrt{3}$; $(x-2)^2 = 3$, not 2.
- Option C ($4 \pm \sqrt{3}$): Uses 4 in place of 2 as the centre.
- Option D ($\pm\sqrt{3}$): Forgets to add the 2 back after taking the square root.

Final Answer: $x = 2 \pm \sqrt{3} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 4

Q5.

Solution

Concept — Complementary Counting: Count all selections, then subtract those with *no* optional course.

Step 1 — Total ways to choose 3 from 8 courses:

$$\binom{8}{3} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = 56.$$

Step 2 — Ways with no optional (all 3 from compulsory):

$$\binom{5}{3} = 10.$$

Step 3 — At least one optional:

$$56 - 10 = 46.$$

Why other options are wrong:

- Option A (30): Undercounts by missing several mixed combinations.
- Option B (36): Arithmetic error; not a valid intermediate result.
- Option C (40): Subtracts 16 instead of 10 from 56.

Final Answer: 46 ways $\Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 5

Q6.

Solution

Concept — Greatest Binomial Coefficient: For even n, the greatest binomial coefficient is $\binom{n}{n/2}$.

Step 1 — Apply for $n = 10$:

$$\binom{10}{5} = \frac{10!}{5!\,5!} = \frac{10 \times 9 \times 8 \times 7 \times 6}{5 \times 4 \times 3 \times 2 \times 1} = \frac{30240}{120} = 252.$$

Why other options are wrong:

- Option A (210): This is $\binom{10}{4} = \binom{10}{6}$, not the middle term.
- Option B (120): This is $5! = \binom{10}{3}$; not a binomial coefficient of $(1+x)^{10}$.
- Option D (45): This is $\binom{10}{2}$, far from the middle.

Final Answer: Greatest coefficient $= 252 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 6

Q7.

Solution

Concept — Derivative of Inverse Trigonometric Functions: The standard result is $\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2}$.

Derivation sketch: Let $y = \tan^{-1} x$, so $\tan y = x$. Differentiating implicitly: $\sec^2 y \frac{dy}{dx} = 1$. Since $\sec^2 y = 1 + \tan^2 y = 1 + x^2$:

$$\frac{dy}{dx} = \frac{1}{1+x^2}.$$

Why other options are wrong:

- Option A $\left(\frac{1}{\sqrt{1-x^2}}\right)$: This is $\frac{d}{dx}[\sin^{-1} x]$, not $\tan^{-1} x$.
- Option C ($\tan x$): That is the function itself; it is not the derivative.
- Option D ($\sec^2 x$): That is $\frac{d}{dx}[\tan x]$; the inverse is different.

Final Answer: $\frac{d}{dx}[\tan^{-1} x] = \frac{1}{1+x^2} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 7

Q8.

Solution

Concept — Mean Value Theorem (MVT): If f is continuous on $[a, b]$ and differentiable on (a, b), then there exists at least one $c \in (a, b)$ such that the instantaneous rate of change equals the average rate of change over the interval.

The theorem states:

$$f'(c) = \frac{f(b) - f(a)}{b - a}.$$

Geometric meaning: The tangent to the curve at c is parallel to the secant through $(a, f(a))$ and $(b, f(b))$.

Why other options are wrong:

- Option B ($f(c) = f(a) + f(b)$): This has no basis in MVT; it resembles a linearity assumption.
- Option C ($f'(c) = 0$): This is Rolle's Theorem, a special case where $f(a) = f(b)$.
- Option D ($f'(c) = f(a)/f(b)$): Not related to any standard theorem.

Final Answer: $f'(c) = \dfrac{f(b) - f(a)}{b - a} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 8

Q9.

Solution

Concept — Factoring to Evaluate a Limit: Use the factorisation $x^3 - a^3 = (x - a)(x^2 + ax + a^2)$.

Step 1 — Factor the numerator:

$$x^3 - 8 = (x - 2)(x^2 + 2x + 4).$$

Step 2 — Cancel and evaluate:

$$\lim_{x \to 2} \frac{(x - 2)(x^2 + 2x + 4)}{x - 2} = \lim_{x \to 2}(x^2 + 2x + 4) = 4 + 4 + 4 = 12.$$

Why other options are wrong:

- Option A (4): Only substitutes $x^2 = 4$; forgets the full expression $x^2 + 2x + 4$.

- Option B (6): Computes $2 + 4 = 6$; partial evaluation of the expression.
- Option C (8): Uses $2 \times 4 = 8$; incorrect multiplication of partial terms.

Final Answer: $\lim_{x \to 2} \frac{x^3 - 8}{x - 2} = 12 \Rightarrow \boxed{\text{D}}$

$\boxed{\textbf{Answer: (D)}}$ Go Back to Q 9

Q10.

Solution

Concept — Power Rule for Integration: $\int x^n \, dx = \frac{x^{n+1}}{n+1} + C$ for $n \neq -1$.

Step 1 — Write $\sqrt{x} = x^{1/2}$ and integrate:

$$\int_0^1 x^{1/2} \, dx = \left[\frac{x^{3/2}}{3/2} \right]_0^1 = \left[\frac{2x^{3/2}}{3} \right]_0^1.$$

Step 2 — Evaluate at the bounds:

$$\frac{2(1)^{3/2}}{3} - \frac{2(0)^{3/2}}{3} = \frac{2}{3} - 0 = \frac{2}{3}.$$

Why other options are wrong:

- Option A ($\frac{1}{2}$): Would result from $\int_0^1 x^0 \, dx \cdot \frac{1}{2}$; incorrect exponent handling.
- Option B ($\frac{1}{3}$): Forgets the factor $\frac{2}{3}$; uses $\frac{x^{3/2}}{3}$ without the coefficient.
- Option D ($\frac{3}{2}$): Uses $\frac{3}{2}$ directly instead of its reciprocal as the divisor.

Final Answer: $\int_0^1 \sqrt{x} \, dx = \frac{2}{3} \Rightarrow \boxed{\text{C}}$

$\boxed{\textbf{Answer: (C)}}$ Go Back to Q 10

Q11.

Solution

Concept — Homogeneous ODE via Substitution: Let $v = y/x$, so $y = vx$ and $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$.

Step 1 — Substitute:

$$v + x\frac{dv}{dx} = v + v^2 \implies x\frac{dv}{dx} = v^2.$$

Step 2 — Separate and integrate:

$$\frac{dv}{v^2} = \frac{dx}{x} \implies \int v^{-2}\,dv = \int \frac{dx}{x} \implies -\frac{1}{v} = \ln|x| + C.$$

Step 3 — Back-substitute $v = y/x$:

$$-\frac{x}{y} = \ln|x| + C \implies y = \frac{-x}{\ln|x| + C}.$$

Why other options are wrong:

- Option A ($y = Cx$): Solves $dy/dx = y/x$ (missing y^2/x^2 term).
- Option C ($y = Ce^x/x$): Does not satisfy the original ODE.
- Option D ($y = x\ln(Cx)$): Arises from a different substitution error.

Final Answer: $y = \dfrac{-x}{\ln|x| + C} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 11

Q12.

Solution

Concept — Monotone Decreasing Function: f is decreasing on an interval where $f'(x) < 0$.

Step 1 — Differentiate:

$$f'(x) = 2x - 4.$$

Step 2 — Find where $f'(x) < 0$:

$$2x - 4 < 0 \implies x < 2.$$

So f is decreasing for $x < 2$. On the interval $[0, 2]$, $f'(x) \leq 0$ (equals 0 only at $x = 2$), so f is decreasing there.

Why other options are wrong:

- Option B ($[2, 4]$): $f'(3) = 2 > 0$, so f is *increasing* there.
- Option C ($[1, 3]$): $f'(2.5) = 1 > 0$, so part of this interval has f increasing.
- Option D ($[2, \infty)$): $f'(x) \geq 0$ for $x \geq 2$; f is increasing, not decreasing.

Final Answer: f is decreasing on $[0, 2] \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 12

Q13.

Solution

Concept — Directrix of a Parabola: For the standard parabola $y^2 = 4ax$, the directrix is $x = -a$.

Step 1 — Compare with standard form:

$$y^2 = 12x \implies 4a = 12 \implies a = 3.$$

Step 2 — Write the directrix:

$$x = -a = -3.$$

Why other options are wrong:

- Option A ($x = 3$): This is the *focus*, not the directrix.
- Option B ($y = 3$): Directrix of this parabola is vertical ($x =$const), not horizontal.
- Option C ($y = -3$): Similarly incorrect axis for a horizontally-opening parabola.

Final Answer: Directrix: $x = -3 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 13

Q14.

Solution

Concept — Condition for Concurrent Lines: Three lines are concurrent if their intersection point satisfies all three equations.

Step 1 — Solve the first two equations:

$$2x + 3y = 4 \quad \text{and} \quad x - y = 1.$$

From the second: $x = 1+y$. Substituting: $2(1+y)+3y = 4 \Rightarrow 2+5y = 4 \Rightarrow y = \frac{2}{5}$, $x = \frac{7}{5}$.

Step 2 — Substitute into the third line:

$$3 \cdot \frac{7}{5} + k \cdot \frac{2}{5} = 7 \implies \frac{21+2k}{5} = 7 \implies 21 + 2k = 35 \implies k = 7.$$

Why other options are wrong:

- Option A ($k = 3$): $3(7/5) + 3(2/5) = \frac{21+6}{5} = \frac{27}{5} \neq 7$.
- Option B ($k = 5$): $\frac{21+10}{5} = \frac{31}{5} \neq 7$.
- Option D ($k = 9$): $\frac{21+18}{5} = \frac{39}{5} \neq 7$.

Final Answer: $k = 7 \Rightarrow \boxed{\text{C}}$

Answer: (C) **Go Back to Q 14**

Q15.

Solution

Concept — Position of a Point Relative to a Circle: For the circle $x^2 + y^2 = r^2$, a point (h, k) is inside if $h^2 + k^2 < r^2$, on if equal, and outside if greater.

Step 1 — Substitute $(2, 3)$**:**

$$2^2 + 3^2 = 4 + 9 = 13.$$

Step 2 — Compare with $r^2 = 9$**:**

$$13 > 9.$$

So the point lies **outside** the circle.

Why other options are wrong:

- Option A (inside): Requires $h^2 + k^2 < 9$; here $13 > 9$.
- Option C (on): Requires $h^2 + k^2 = 9$; here $13 \neq 9$.
- Option D (cannot be determined): The calculation is straightforward and deterministic.

Final Answer: The point lies outside the circle $\Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 15

Q16.

Solution

Concept — Rectangular Hyperbola: A hyperbola is called *rectangular* when its asymptotes are perpendicular to each other. The standard form is $xy = c^2$, where the asymptotes are the coordinate axes ($x = 0$ and $y = 0$), which are perpendicular.

Step 1 — Check Option A ($xy = 4$): This is of the form $xy = c^2$ with $c^2 = 4$. The asymptotes are the coordinate axes, which meet at 90. Hence it is a rectangular hyperbola.

Why other options are wrong:

- Option B $\left(\frac{x^2}{4} - \frac{y^2}{9} = 1\right)$: Standard hyperbola; asymptotes have slopes $\pm\frac{3}{2}$, not perpendicular ($a^2 \neq b^2$).
- Option C $\left(\frac{x^2}{9} + \frac{y^2}{4} = 1\right)$: This is an *ellipse*, not a hyperbola.
- Option D ($x^2 - 4y^2 = 1$): Asymptotes have slopes $\pm\frac{1}{2}$; not perpendicular.

Final Answer: $xy = 4$ is a rectangular hyperbola $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 16

Q17.

Solution

Concept — Reflection in the x-axis: Reflecting a point (x, y) in the x-axis gives $(x, -y)$. The x-coordinate stays the same while the y-coordinate changes sign.

Step 1 — Apply the rule:

$$(3, 4) \xrightarrow{\text{reflect in } x\text{-axis}} (3, -4).$$

Why other options are wrong:

- Option A ($(-3, 4)$): Negates x instead of y; this would be reflection in the y-axis.
- Option B ($(-3, -4)$): Negates both coordinates; this is a 180 rotation about the origin.
- Option C ($(4, 3)$): Swaps coordinates; this is reflection in the line $y = x$.

Final Answer: Reflection of $(3, 4)$ in the x-axis is $(3, -4) \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 17

Q18.

Solution

Concept — Cofactor of a Matrix Entry: The cofactor $A_{ij} = (-1)^{i+j} M_{ij}$, where M_{ij} is the minor (determinant of the submatrix obtained by deleting row i and column j).

Step 1 — Find the minor M_{21} (delete row 2 and column 1):

$$M_{21} = \begin{vmatrix} 2 & 3 \\ 6 & 0 \end{vmatrix} = (2)(0) - (3)(6) = 0 - 18 = -18.$$

Step 2 — Apply the sign:

$$A_{21} = (-1)^{2+1} M_{21} = (-1)^3 \times (-18) = (-1)(-18) = 18.$$

Why other options are wrong:

- Option A (-18): This is the minor M_{21} without the sign factor.
- Option B (-6): Arithmetic error in computing the 2×2 determinant.
- Option D (6): Incorrect minor value with wrong sign applied.

Final Answer: $A_{21} = 18 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 18

Q19.

Solution

Concept — Orthogonal Matrix: A square matrix A is orthogonal if $AA^T = I$. Taking determinants of both sides: $\det(AA^T) = \det(I) = 1$. Since $\det(AA^T) = \det(A) \cdot \det(A^T) = [\det(A)]^2$, we get $[\det(A)]^2 = 1$, so $\det(A) = \pm 1$.

Why other options are wrong:

- Option A ($\det(A) = 0$): A singular matrix has no inverse; if $\det(A) = 0$ then $AA^T \neq I$.
- Option C ($A = A^T$): A symmetric orthogonal matrix satisfies $A^2 = I$, but not all orthogonal matrices are symmetric (e.g., rotation matrices).
- Option D ($A^2 = I$ always): True only if A is also symmetric. In general, $A^2 \neq I$ (e.g., a 90 rotation matrix satisfies $A^4 = I$ but $A^2 \neq I$).

Final Answer: $\det(A) = \pm 1 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 19

Q20.

Solution

Concept — Augmented Matrix: For a linear system $A\mathbf{x} = \mathbf{b}$, the augmented matrix $[A|\mathbf{b}]$ is formed by appending the column vector $\mathbf{b}$ to the coefficient matrix A, separated by a vertical bar.

Step 1 — Identify coefficients and constants:

$$x + y = 2: \text{ row } [1,\ 1 \mid 2]. \qquad 2x + 3y = 5: \text{ row } [2,\ 3 \mid 5].$$

Step 2 — Write the augmented matrix:

$$\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 2 & 3 & 5 \end{array}\right].$$

Why other options are wrong:

- Option B: Mixes up coefficients and has wrong column arrangement.
- Option C: Shows only the coefficient matrix A without the $\mathbf{b}$ column.
- Option D: Reverses the arrangement of rows and coefficients.

Final Answer: $\left[\begin{array}{cc|c} 1 & 1 & 2 \\ 2 & 3 & 5 \end{array}\right] \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 20

Q21.

Solution

Concept — Consistent and Dependent Systems: If two equations represent the same line, the system has infinitely many solutions.

Step 1 — Check whether the equations are multiples of each other:

$$2x + 4y = 6 \xrightarrow{\div 2} x + 2y = 3.$$

The second equation $x + 2y = 3$ is *identical* to the scaled first equation. Both equations represent the same line.

Step 2 — Conclusion: Every point on the line $x + 2y = 3$ is a solution; hence infinitely many solutions.

Why other options are wrong:

- Option A (unique): Unique solution requires independent equations (different lines intersecting once).
- Option B (no solution): No solution occurs for parallel but distinct lines; here the lines coincide.
- Option C (exactly two): A pair of linear equations can never have exactly two solutions.

Final Answer: Infinitely many solutions $\Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 21

Q22.

Solution

Concept — Reading a Histogram: The class interval with the tallest bar has the highest frequency.

Step 1 — Read the frequencies from the histogram:

Class	Frequency
0–10	3
10–20	8
20–30	18
30–40	15
40–50	6

Step 2 — Identify the maximum: The class 20–30 has frequency 18, which is the highest.

Why other options are wrong:

- Option A (10–20): Frequency $8 < 18$.
- Option B (30–40): Frequency $15 < 18$.
- Option D (40–50): Frequency $6 < 18$.

Final Answer: Class 20–30 has the highest frequency $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 22

Q23.

Solution

Concept — Complementary Events: $P(\text{at least one head}) = 1 - P(\text{no heads})$.

Step 1 — Find P(no heads):

$$P(\text{all tails}) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}.$$

Step 2 — Apply complement:

$$P(\text{at least one head}) = 1 - \frac{1}{8} = \frac{7}{8}.$$

Why other options are wrong:

- Option A ($\frac{3}{8}$): This is $P(\text{exactly one head}) = \binom{3}{1}(1/2)^3 = 3/8$.
- Option C ($\frac{1}{2}$): This is $P(\text{exactly one or two heads})$ only partially; incorrect total.
- Option D ($\frac{5}{8}$): No standard event gives $5/8$ for three fair coins.

Final Answer: $P(\text{at least one head}) = \dfrac{7}{8} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 23

Q24.

Solution

Concept — Variance of Binomial Distribution: For $X \sim B(n,p)$, $\text{Var}(X) = npq$, where $q = 1 - p$.

Step 1 — Identify parameters: $n = 15$, $p = 0.4$, $q = 1 - 0.4 = 0.6$.

Step 2 — Compute variance:

$$\text{Var}(X) = npq = 15 \times 0.4 \times 0.6 = 15 \times 0.24 = 3.6.$$

Why other options are wrong:

- Option B (6): This is the *mean* $np = 15 \times 0.4 = 6$, not the variance.
- Option C (0.24): This is $pq = 0.4 \times 0.6$; forgets to multiply by n.
- Option D (9): Uses $n \cdot p^2 = 15 \times 0.16 \approx 2.4$ or similar arithmetic error.

Final Answer: $\text{Var}(X) = 3.6 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 24

Q25.

Solution

Concept — Correlation from Regression Coefficients: $r = \sqrt{b_{yx} \cdot b_{xy}}$ (with the sign of r matching the sign of the regression coefficients).

Step 1 — Compute the product:

$$b_{yx} \cdot b_{xy} = 0.75 \times 0.48 = 0.36.$$

Step 2 — Take the square root:

$$r = \sqrt{0.36} = 0.6.$$

Since both regression coefficients are positive, $r = +0.6$.

Why other options are wrong:

- Option A (0.36): This is $b_{yx} \cdot b_{xy}$, the product before taking the square root.
- Option B (0.75): This is just b_{yx}, not the correlation coefficient.
- Option C (0.48): This is just b_{xy}, not the correlation coefficient.

Final Answer: $r = 0.6 \Rightarrow \boxed{\text{D}}$

Answer: (D) **Go Back to Q 25**

Q26.

Solution

Concept — Relationship Between Standard Deviation and Variance: $\text{SD} = \sqrt{\text{Variance}}$.

Step 1 — Apply the formula:

$$\text{SD} = \sqrt{25} = 5.$$

Why other options are wrong:

- Option A (25): This is the variance itself, not the standard deviation.
- Option B (10): $10^2 = 100 \neq 25$; incorrect square root.
- Option D (625): This is 25^2; squares the variance instead of taking its root.

Final Answer: $\text{SD} = 5 \Rightarrow \boxed{\text{C}}$

Answer: (C) **Go Back to Q 26**

Q27.

Solution

Concept — Cosine Subtraction Formula: $\cos(A - B) = \cos A \cos B + \sin A \sin B$.

Step 1 — Express $15 = 45 - 30$:

$$\cos 15 = \cos(45 - 30) = \cos 45 \cos 30 + \sin 45 \sin 30.$$

Step 2 — Substitute exact values:

$$\cos 15 = \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2} \cdot \frac{1}{2} = \frac{\sqrt{6}}{4} + \frac{\sqrt{2}}{4} = \frac{\sqrt{6} + \sqrt{2}}{4}.$$

Why other options are wrong:

- Option A $\left(\frac{\sqrt{6} - \sqrt{2}}{4}\right)$: This is $\cos 75$; obtained by *subtracting* rather than adding the two terms.
- Option C $\left(\frac{\sqrt{3}}{2}\right)$: This is $\cos 30$.
- Option D $\left(\frac{1}{2}\right)$: This is $\cos 60$.

Final Answer: $\cos 15 = \frac{\sqrt{6} + \sqrt{2}}{4} \Rightarrow \boxed{\text{B}}$

Answer: (B) **Go Back to Q 27**

Q28.

Solution

Concept — Sum-to-Product Identity: $\sin C - \sin D = 2\cos\left(\frac{C + D}{2}\right) \sin\left(\frac{C - D}{2}\right)$.

Step 1 — Set $C = 3x$, $D = x$:

$$\sin 3x - \sin x = 2\cos\left(\frac{3x + x}{2}\right) \sin\left(\frac{3x - x}{2}\right) = 2\cos(2x)\sin(x).$$

Why other options are wrong:

- Option B $(2\sin(2x)\cos(x))$: This is the expansion of $\sin(3x) + \sin(x)$ (sum, not difference).

- Option C ($2\cos(2x)\cos(x)$): This is $\cos(x) - \cos(3x)$ by the product formula.
- Option D ($\cos(2x)\sin(x)$): Missing the factor of 2.

Final Answer: $\sin 3x - \sin x = 2\cos(2x)\sin(x) \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 28

Q29.

Solution

Concept — Principal Value of $\cos^{-1}$: The range of $\cos^{-1}$ is $[0, \pi]$. We need the angle $\theta \in [0, \pi]$ such that $\cos\theta = -\frac{1}{2}$.

Step 1 — Recall standard value:

$$\cos\frac{\pi}{3} = \frac{1}{2} \implies \cos\left(\pi - \frac{\pi}{3}\right) = -\cos\frac{\pi}{3} = -\frac{1}{2}.$$

Step 2 — State the principal value:

$$\pi - \frac{\pi}{3} = \frac{2\pi}{3} \in [0, \pi].$$

So $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$.

Why other options are wrong:

- Option A ($\pi/3$): $\cos(\pi/3) = +1/2$, not $-1/2$.
- Option B ($5\pi/6$): $\cos(5\pi/6) = -\sqrt{3}/2 \neq -1/2$.
- Option C ($\pi/2$): $\cos(\pi/2) = 0 \neq -1/2$.

Final Answer: $\cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 29

Q30.

Solution

Concept — Area of a Triangle: Area $= \frac{1}{2} \times \text{base} \times \text{height}$.

Step 1 — Set up the equation:

$$20 = \frac{1}{2} \times 8 \times h \implies 20 = 4h.$$

Step 2 — Solve for h:

$$h = \frac{20}{4} = 5.$$

Why other options are wrong:

- Option A (2.5): Computes $20/8 = 2.5$; forgets the factor of $\frac{1}{2}$ in the area formula.
- Option B (4): Uses $20/5 = 4$; substitutes wrong values.
- Option D (8): This is the given base BC, not the altitude.

Final Answer: Altitude $h = 5$ units $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 30

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	D	2	C	3	B	4	A	5	D
6	C	7	B	8	A	9	D	10	C
11	B	12	A	13	D	14	C	15	B
16	A	17	D	18	C	19	B	20	A
21	D	22	C	23	B	24	A	25	D
26	C	27	B	28	A	29	D	30	C