GAT B Mathematics

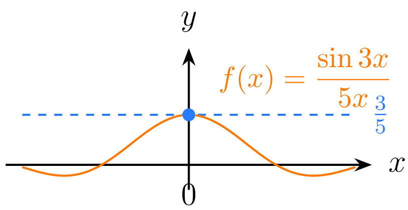
Sample Paper – 1

Duration: 23 Minutes **Maximum Marks: 15**

Instructions

- This paper contains **15** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics portion of **GAT B** entrance.
- Each correct answer carries **+1 mark**. There is a deduction of **0.5 mark** for each wrong answer. Unattempted questions carry zero marks.
- Only **one** option is correct. Choose carefully.
- Syllabus: Calculus, Linear Algebra, Probability & Statistics, Differential Equations, Coordinate Geometry, Sets & Functions, Vectors, Combinatorics (undergraduate level).
- Use of mobile phones, calculators, or electronic gadgets is strictly prohibited.

Q1. Evaluate $\lim_{x\to 0} \dfrac{\sin 3x}{5x}$.



(Concept check)

(A) $\dfrac{1}{5}$

(B) $\dfrac{3}{5}$

(C) $\dfrac{5}{3}$

(D) 3

Q2. Find $\dfrac{d}{dx}\left[\sin(x^2)\right]$.

(Application)

(A) $\cos(x^2)$

(B) $-2x\cos(x^2)$

(C) $2x\sin(x^2)$

(D) $2x\cos(x^2)$

Q3. Evaluate $\displaystyle\int x\,e^x\,dx$.

(Application)

(A) $(x-1)e^x + C$

(B) $x\,e^x + C$

(C) $(x+1)e^x + C$

(D) $e^x + C$

Q4. Find the value of the determinant of the matrix shown below.

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix}$$

(Trap drill)

(A) 54

(B) -54

(C) 0

(D) 27

Q5. Solve the system of equations using Cramer's rule:

$$2x + 3y = 7, \qquad x - y = 1.$$

(Application)

(A) $x = 1,\ y = 2$

(B) $x = 3,\ y = 0$

(C) $x = 0,\ y = \frac{7}{3}$

(D) $x = 2,\ y = 1$

Q6. Two events A and B are such that $P(A \cap B) = 0.2$ and $P(B) = 0.5$. Find the conditional probability $P(A \mid B)$.

(Concept check)

(A) 0.2

(B) 0.4

(C) 0.5

(D) 0.7

Q7. Find the arithmetic mean of the data set $\{2,\ 4,\ 6,\ 8,\ 10\}$.

(Concept check)

(A) 6

(B) 5

(C) 7

(D) 8

Q8. Solve the separable differential equation $\dfrac{dy}{dx} = \dfrac{y}{x}$ with the initial condition $y(1) = 2$.

(Application)

(A) $y = x^2$

(B) $y = \dfrac{2}{x}$

(C) $y = 2x$

(D) $y = e^x$

Q9. The logistic growth model is governed by the ODE

$$\frac{dP}{dt} = rP\left(1 - \frac{P}{K}\right),$$

where $r > 0$ and $K > 0$ are constants. What is the equilibrium population that the solution approaches as $t \to \infty$?

(Concept check)

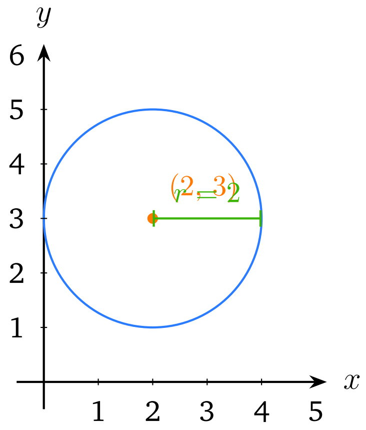
(A) K

(B) 0

(C) $\dfrac{r}{K}$

(D) r

Q10. The equation $x^2 + y^2 - 4x - 6y + 9 = 0$ represents a circle. By completing the square, identify the centre and radius.



(Application)

(A) Centre $(2, 3)$, $r = 4$

(B) Centre $(-2, -3)$, $r = 2$

(C) Centre $(2, 3)$, $r = 3$

(D) Centre $(2, 3)$, $r = 2$

Q11. Find the midpoint of the segment joining $A(1,\ 2)$ and $B(5,\ 6)$.

(Concept check)

(A) $(2, 3)$

(B) $(3, 4)$

(C) $(4, 5)$

(D) $(6, 8)$

Q12. If $|A| = m$, $|B| = n$, and $|A \cap B| = k$, which expression gives $|A \cup B|$?

(Concept check)

(A) $m + n$

(B) $mn - k$

(C) $m + n - k$

(D) $m + n + k$

Q13. Compute the dot product of the vectors $\mathbf{u} = (1, 2, 3)$ and $\mathbf{v} = (2, -1, 0)$.

(Application)

(A) 0

(B) 4

(C) -1

(D) 5

Q14. Evaluate $\binom{8}{3}$.

(Concept check)

(A) 24

(B) 56

(C) 112

(D) 336

Q15. Find the modulus $|3 + 4i|$.

(Application)

(A) 3

(B) 4

(C) 7

(D) 5

Detailed Solutions

Solution 1.

Solution

Concept: Standard limit $\lim_{x\to 0}\dfrac{\sin\theta}{\theta} = 1$ **Chapter:** Limits/Continuity with a scaling argument.

Step 1 — Rewrite the argument.

$$\frac{\sin 3x}{5x} = \frac{3}{5}\cdot\frac{\sin 3x}{3x}.$$

Step 2 — Apply the standard limit. As $x \to 0$, the substitution $u = 3x \to 0$ gives

$$\lim_{x\to 0}\frac{\sin 3x}{3x} = \lim_{u\to 0}\frac{\sin u}{u} = 1.$$

Therefore

$$\lim_{x\to 0}\frac{\sin 3x}{5x} = \frac{3}{5}\times 1 = \boxed{\frac{3}{5}}.$$

Why other options are wrong:

- (A) $\frac{1}{5}$: ignores the factor of 3 in the numerator.
- (C) $\frac{5}{3}$: inverts the correct ratio.
- (D) 3: ignores the denominator's factor of 5.

Answer: (B) Go Back to Q 1

Solution 2.

Solution

Concept: Chain rule — if $y = f(g(x))$ then $y' = f'(g(x))\cdot g'(x)$. **Chapter:** Differential Calculus

Step 1 — Identify the composite functions. Let $u = x^2$ (inner), $f(u) = \sin u$ (outer).

Step 2 — Differentiate.

$$\frac{d}{dx}\left[\sin(x^2)\right] = \cos(x^2)\cdot\frac{d}{dx}(x^2) = \cos(x^2)\cdot 2x = \boxed{2x\cos(x^2)}.$$

Why other options are wrong:

- (A) $\cos(x^2)$: forgets to multiply by the inner derivative $2x$.
- (B) $-2x\cos(x^2)$: wrong sign; $\frac{d}{dx}(\sin u) = +\cos u$.
- (C) $2x\sin(x^2)$: keeps $\sin$ instead of differentiating it to $\cos$.

Answer: (D) Go Back to Q 2

Solution 3.

Solution

Concept: Integration by parts — $\int u\,dv = uv - \int v\,du$. **Chapter:** Integral Calculus

Step 1 — Choose u and dv. Let $u = x$ (so $du = dx$) and $dv = e^x\,dx$ (so $v = e^x$).

Step 2 — Apply the formula.

$$\int x\,e^x\,dx = x\,e^x - \int e^x\,dx = x\,e^x - e^x + C = \boxed{(x-1)e^x + C}.$$

Why other options are wrong:

- (B) $xe^x + C$: omits the $-e^x$ term arising from the second integral.
- (C) $(x+1)e^x + C$: sign error — the second term should be $-e^x$, not $+e^x$.
- (D) $e^x + C$: ignores the x factor entirely.

Answer: (A) Go Back to Q 3

Solution 4.

Solution

Concept: Determinant of a 3×3 matrix by cofactor expansion. **Chapter:** Linear Algebra

Step 1 — Observe the rows. Row $3 =$ Row $1+$ Row 2 would not hold, but notice: each row forms an arithmetic progression with common difference 1, and Row 3 $= 2\times$ Row 2 $-$ Row 1. Hence the rows are *linearly dependent*.

Step 2 — Conclude immediately. Linearly dependent rows $\Rightarrow \det = 0$.

Verification by cofactor expansion along row 1:

$$\det = 1(5\cdot9-6\cdot8)-2(4\cdot9-6\cdot7)+3(4\cdot8-5\cdot7) = 1(-3)-2(-6)+3(-3) = -3+12-9 = \boxed{0}.$$

Why other options are wrong:

- (A) 54 and (B) -54: common computational errors when signs are mishandled.
- (D) 27: a product of diagonal entries, not the determinant.

Answer: (C) Go Back to Q 4

Solution 5.

Solution

Concept: Cramer's rule — $x = D_x/D$, $y = D_y/D$. **Chapter:** Linear Algebra

Step 1 — Main determinant.

$$D = \begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} = (2)(-1) - (3)(1) = -2 - 3 = -5.$$

Step 2 — Numerator determinants.

$$D_x = \begin{vmatrix} 7 & 3 \\ 1 & -1 \end{vmatrix} = (7)(-1)-(3)(1) = -7-3 = -10, \qquad D_y = \begin{vmatrix} 2 & 7 \\ 1 & 1 \end{vmatrix} = (2)(1)-(7)(1) = 2-7 = -5.$$

Step 3 — Solve.

$$x = \frac{D_x}{D} = \frac{-10}{-5} = 2, \qquad y = \frac{D_y}{D} = \frac{-5}{-5} = 1. \quad \Rightarrow \quad \boxed{x = 2,\ y = 1}.$$

Why other options are wrong:

- (A) $x = 1, y = 2$: values are swapped.
- (B) $x = 3, y = 0$: does not satisfy $x - y = 1$ ($3 - 0 = 3 \neq 1$).
- (C) $x = 0, y = 7/3$: does not satisfy $x - y = 1$ ($0 - 7/3 \neq 1$).

Answer: (D) Go Back to Q 5

Solution 6.

Solution

Concept: Definition of conditional probability: **Chapter:** Probability & Statistics $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$.

Step 1 — Substitute.

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.5} = \boxed{0.4}.$$

Why other options are wrong:

- (A) 0.2: that is $P(A \cap B)$, not the conditional probability.
- (C) 0.5: that is $P(B)$ itself.
- (D) 0.7: likely from adding $0.2 + 0.5$, which has no probabilistic meaning here.

Answer: (B) Go Back to Q 6

Solution 7.

Solution

Concept: Arithmetic mean $= \dfrac{\text{sum of observations}}{\text{number of observations}}$. **Chapter:** Probability & Statistics

Step 1 — Compute the sum.

$$2 + 4 + 6 + 8 + 10 = 30.$$

Step 2 — Divide by count.

$$\bar{x} = \frac{30}{5} = \boxed{6}.$$

Why other options are wrong:

- (B) 5: the median of the set, not the mean.
- (C) 7: no standard average formula yields 7 for this data.
- (D) 8: the fourth element, sometimes confused with the mean for evenly spaced data.

Answer: (A) Go Back to Q 7

Solution 8.

Solution

Concept: Separable ODE — separate variables and integrate both sides. **Chapter:** Differential Equations

Step 1 — Separate.

$$\frac{dy}{y} = \frac{dx}{x}.$$

Step 2 — Integrate.

$$\ln|y| = \ln|x| + C_1 \;\Rightarrow\; y = Ax \quad (A = e^{C_1}).$$

Step 3 — Apply initial condition $y(1) = 2$.

$$2 = A \cdot 1 \;\Rightarrow\; A = 2.$$

Hence $\boxed{y = 2x}$.

Why other options are wrong:

- (A) $y = x^2$: arises from $dy/dx = 2x/y$, not y/x.
- (B) $y = 2/x$: would give $y(1) = 2$ but does *not* satisfy $dy/dx = y/x$ since $d(2/x)/dx = -2/x^2 \neq (2/x)/x$.
- (D) $y = e^x$: comes from a different separable equation $dy/dx = y$.

Answer: (C) Go Back to Q 8

Solution 9.

Solution

Concept: Equilibrium of the logistic ODE — set $dP/dt = 0$. **Chapter:** Differential Equations

Step 1 — Find equilibria.

$$rP\left(1 - \frac{P}{K}\right) = 0 \;\Rightarrow\; P = 0 \text{ or } P = K.$$

Step 2 — Stability analysis. $P = 0$ is an *unstable* equilibrium (for $0 < P < K$, $dP/dt > 0$). $P = K$ is a *stable* (carrying-capacity) equilibrium, so

$$\lim_{t\to\infty} P(t) = \boxed{K}.$$

Why other options are wrong:

- (B) 0: the unstable trivial equilibrium; solutions starting above zero move *away* from 0.
- (C) r/K: not a meaningful quantity in this context.
- (D) r: the intrinsic growth rate, a parameter, not an equilibrium value.

Answer: (A) Go Back to Q 9

Solution 10.

Solution

Concept: Complete the square to convert general form to standard form **Chapter:** Coordinate Geometry $(x-h)^2+(y-k)^2=r^2$.

Step 1 — Group and complete the square.

$$\begin{aligned} x^2-4x+y^2-6y+9&=0\\ (x^2-4x+4)-4+(y^2-6y+9)-9+9&=0\\ (x-2)^2+(y-3)^2&=4. \end{aligned}$$

Step 2 — Read off centre and radius.

$$\text{Centre}=(2,3),\quad r=\sqrt{4}=\boxed{2}.$$

Why other options are wrong:

- (A) $r=4$: takes $r^2=4$ as r without taking the square root.
- (B) Centre $(-2,-3)$: sign error when completing the square.
- (C) $r=3$: confuses the y-coefficient 6 (half gives 3) with the radius.

Answer: (D) Go Back to Q 10

Solution 11.

Solution

Concept: Midpoint formula: $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$. **Chapter:** Coordinate Geometry

Step 1 — Substitute.

$$M = \left(\frac{1+5}{2}, \frac{2+6}{2}\right) = \left(\frac{6}{2}, \frac{8}{2}\right) = \boxed{(3,\ 4)}.$$

Why other options are wrong:

- (A) $(2, 3)$: uses only one coordinate of each point instead of averaging.
- (C) $(4, 5)$: off by 1 in both coordinates; likely an arithmetic slip.
- (D) $(6, 8)$: gives the *sum* $(x_1 + x_2, y_1 + y_2)$ without dividing by 2.

Answer: (B) Go Back to Q 11

Solution 12.

Solution

Concept: Inclusion-Exclusion Principle for two sets: **Chapter:** Set Theory $|A \cup B| = |A| + |B| - |A \cap B|$.

Step 1 — Apply the principle. Elements in $A \cap B$ are counted twice (once in $|A|$, once in $|B|$), so

$$|A \cup B| = m + n - k = \boxed{m + n - k}.$$

Why other options are wrong:

- (A) $m + n$: over-counts the intersection by k.
- (B) $mn - k$: mn is a product, not a set-cardinality sum; formula is incorrect.
- (D) $m + n + k$: adds the intersection instead of subtracting it.

Answer: (C) Go Back to Q 12

Solution 13.

Solution

Concept: Dot product $\mathbf{u} \cdot \mathbf{v} = u_1v_1 + u_2v_2 + u_3v_3$. **Chapter:** Vectors

Step 1 — Compute component-wise products.

$$(1)(2) + (2)(-1) + (3)(0) = 2 - 2 + 0 = \boxed{0}.$$

Geometric note: $\mathbf{u} \cdot \mathbf{v} = 0$ means the two vectors are *orthogonal* (perpendicular to each other).

Why other options are wrong:

- (B) 4: ignores the -1 sign on the second component of $\mathbf{v}$.
- (C) -1: partial sum $2 + (-1) = 1$ with a sign error.
- (D) 5: sums the absolute values of all components without respecting signs.

Answer: (A) Go Back to Q 13

Solution 14.

Solution

Concept: Combination formula $\binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$. **Chapter:** Combinatorics

Step 1 — Substitute $n = 8$, $r = 3$.

$$\binom{8}{3} = \frac{8!}{3! \cdot 5!} = \frac{8 \times 7 \times 6}{3 \times 2 \times 1} = \frac{336}{6} = \boxed{56}.$$

Why other options are wrong:

- (A) $24 = 4!$: value of $4!$, unrelated to $\binom{8}{3}$.
- (C) $112 = 2 \times \binom{8}{3}$: the combination doubled by a factor-of-2 error.
- (D) 336: the numerator $8 \times 7 \times 6$ before dividing by $3! = 6$; it is $P(8,3)$, the permutation, not the combination.

Answer: (B) Go Back to Q 14

Solution 15.

Solution

Concept: Modulus of a complex number $a + bi$: $|a + bi| = \sqrt{a^2 + b^2}$. **Chapter:** Complex Numbers

Step 1 — Substitute $a = 3$, $b = 4$.

$$|3 + 4i| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = \boxed{5}.$$

Note: This is a classic $(3, 4, 5)$ Pythagorean triple.

Why other options are wrong:

- (A) 3: only the real part, not the modulus.
- (B) 4: only the imaginary part coefficient, not the modulus.
- (C) 7: the *sum* $3 + 4$, not the Euclidean magnitude.

Answer: (D) Go Back to Q 15

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	D	3	A	4	C	5	D
6	B	7	A	8	C	9	A	10	D
11	B	12	C	13	A	14	B	15	D