GAT B Mathematics
Sample Paper – 2

Duration	Total Questions	Max Marks
23 Minutes	15	15

Correct Answer	$+1$
Wrong Answer	-0.5
Unattempted	0

Instructions

1. This paper contains **15 single-correct MCQ** questions.
2. Duration of the test is **23 minutes**.
3. Each correct response carries $+1$ mark; each incorrect response carries -0.5 mark.
4. Choose the single best option from (A), (B), (C), (D).
5. Use of calculators or electronic devices is not permitted.
6. All questions are compulsory.

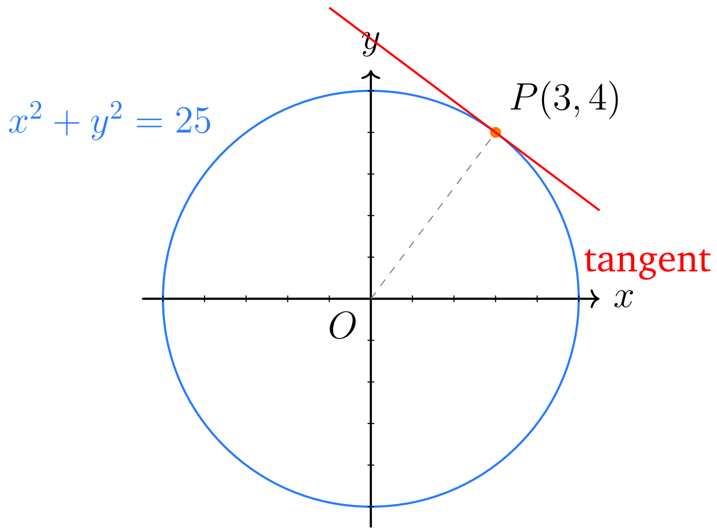
Section: Mathematics

Q1. Using the Squeeze theorem, evaluate $\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right)$.

(Concept check)

(A) 1
(B) ∞
(C) 0
(D) -1

Q2. A point $P(3,\ 4)$ lies on the circle $x^2 + y^2 = 25$. Using implicit differentiation, the slope of the tangent at P is:



(Application)

(A) $-\frac{3}{4}$
(B) $\frac{3}{4}$
(C) $-\frac{4}{3}$
(D) $\frac{4}{3}$

Q3. Evaluate $\int 2x\sqrt{x^2+1}\,dx$.

(Application)

(A) $\sqrt{x^2+1}+C$
(B) $x^2\sqrt{x^2+1}+C$
(C) $\frac{1}{2}(x^2+1)^{3/2}+C$
(D) $\frac{2}{3}(x^2+1)^{3/2}+C$

Q4. The inverse of $A=\begin{pmatrix}2 & 1\\ 5 & 3\end{pmatrix}$ is:

(Application)

(A) $\begin{pmatrix}3 & 1\\ 5 & 2\end{pmatrix}$
(B) $\begin{pmatrix}3 & -1\\ -5 & 2\end{pmatrix}$
(C) $\begin{pmatrix}2 & 5\\ 1 & 3\end{pmatrix}$
(D) $\begin{pmatrix}-3 & 1\\ 5 & -2\end{pmatrix}$

Q5. A homogeneous system $A\mathbf{x}=\mathbf{0}$ where $\det(A)\neq 0$ has:

(Concept check)

(A) Infinitely many solutions
(B) No solution
(C) Only the trivial solution $\mathbf{x}=\mathbf{0}$
(D) Exactly two solutions

Q6. If $X\sim B(10,\, 0.3)$, the mean of X is:

(Concept check)

(A) 3
(B) 7
(C) 2.1
(D) 0.3

Q7. For a Poisson random variable with parameter $\lambda = 4$, the probability $P(X = 0)$ equals:

(Concept check)

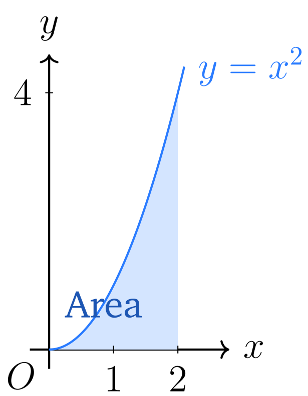
(A) $e^{-4}/4$
(B) $4e^{-4}$
(C) $1 - e^{-4}$
(D) e^{-4}

Q8. For the ODE $\dfrac{dy}{dx} + 2y = e^{x}$, the integrating factor is:

(Concept check)

(A) e^{x}
(B) e^{2x}
(C) $2e^{x}$
(D) e^{-2x}

Q9. The area under the parabola $y = x^2$ from $x = 0$ to $x = 2$ is $\displaystyle\int_0^2 x^2\,dx =$



(Application)

(A) 4
(B) 2
(C) $\dfrac{8}{3}$
(D) $\dfrac{4}{3}$

Q10. The point $(6,\ 8)$ lies on the circle $x^2 + y^2 = r^2$ centred at the origin. The radius r is:

(Concept check)

(A) 10
(B) 8
(C) 14
(D) 100

Q11. The area of the triangle with vertices $A(0,0)$, $B(4,0)$, $C(0,3)$ is:

(Application)

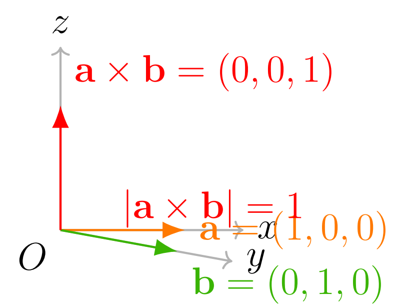
(A) 12
(B) 3
(C) 4
(D) 6

Q12. If $f(x) = 2x - 3$, then $f^{-1}(x)$ equals:

(Application)

(A) $\dfrac{x-3}{2}$
(B) $\dfrac{x+3}{2}$
(C) $2x + 3$
(D) $\dfrac{1}{2x-3}$

Q13. For vectors $\mathbf{a} = (1,0,0)$ and $\mathbf{b} = (0,1,0)$, the magnitude $|\mathbf{a} \times \mathbf{b}|$ is:



(Application)

(A) 0
(B) 2
(C) 1
(D) $\sqrt{2}$

Q14. The number of ways to arrange all 4 letters of the word MATH in a row is:

(Concept check)

(A) 24
(B) 12
(C) 4
(D) 16

Q15. The sum of the first 10 positive integers $1 + 2 + \cdots + 10$ equals:

(Concept check)

(A) 45
(B) 55
(C) 50
(D) 100

Answer Key

Quick Answer Reference (click an answer to jump to its solution):

Q	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans	C	A	D	B	C	A	D	B	C	A	D	B	C	A	B

Detailed Solutions

Sol. 1.

Solution

Concept — Squeeze (Sandwich) Theorem: Chapter: Limits/Continuity If $g(x) \leq f(x) \leq h(x)$ near $x = a$ and $\lim_{x\to a} g(x) = \lim_{x\to a} h(x) = L$, then $\lim_{x\to a} f(x) = L$.

Step 1 — Bound the sine factor. For all $x \neq 0$, we know $-1 \leq \sin(1/x) \leq 1$.

Step 2 — Multiply through by $x^2 \geq 0$.

$$-x^2 \;\leq\; x^2 \sin\!\left(\frac{1}{x}\right) \;\leq\; x^2.$$

Step 3 — Apply the Squeeze Theorem. Since $\lim_{x\to 0}(-x^2) = 0$ and $\lim_{x\to 0} x^2 = 0$, by the Squeeze Theorem,

$$\lim_{x\to 0} x^2 \sin\!\left(\frac{1}{x}\right) = 0.$$

Why other options are wrong:

- Option A (1): $\sin(1/x)$ oscillates; x^2 crushes it to 0, not 1.
- Option B (∞): $x^2 \to 0$ dominates $\sin(1/x)$, so the product cannot diverge.
- Option D (-1): The product is always bounded by $\pm x^2 \to 0$.

Final Answer: $\lim_{x\to 0} x^2 \sin(1/x) = 0 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 1

Sol. 2.

Solution

Concept — Implicit Differentiation: Chapter: Differential Calculus Differentiate both sides of the relation with respect to x, treating y as a function of x.

Step 1 — Differentiate $x^2 + y^2 = 25$ implicitly.

$$\frac{d}{dx}(x^2) + \frac{d}{dx}(y^2) = \frac{d}{dx}(25) \implies 2x + 2y\frac{dy}{dx} = 0.$$

Step 2 — Solve for dy/dx.

$$\frac{dy}{dx} = -\frac{x}{y}.$$

Step 3 — Substitute the point $P(3,4)$.

$$\left.\frac{dy}{dx}\right|_{(3,4)} = -\frac{3}{4}.$$

Why other options are wrong:

- Option B $(3/4)$: Misses the negative sign from the chain rule.
- Option C $(-4/3)$: Inverts x and y; this is the slope of the radius OP, not the tangent.
- Option D $(4/3)$: Both the sign and the fraction are wrong.

Final Answer: Slope of tangent at $P(3,4) = -3/4 \Rightarrow \boxed{\text{A}}$

Answer: (A) **Go Back to Q 2**

Sol. 3.

Solution

Concept — Integration by Substitution (u-substitution): Chapter: Integral Calculus Choose a substitution $u = g(x)$ so that $g'(x)$ appears in the integrand.

Step 1 — Choose the substitution. Let $u = x^2 + 1$, so $du = 2x\,dx$.

Step 2 — Rewrite the integral.

$$\int 2x\sqrt{x^2+1}\,dx = \int \sqrt{u}\,du = \int u^{1/2}\,du.$$

Step 3 — Integrate.

$$\int u^{1/2}\,du = \frac{u^{3/2}}{3/2} + C = \frac{2}{3}u^{3/2} + C.$$

Step 4 — Back-substitute $u = x^2 + 1$.

$$\int 2x\sqrt{x^2+1}\,dx = \frac{2}{3}(x^2+1)^{3/2} + C.$$

Why other options are wrong:

- Option A: Differentiating $\sqrt{x^2+1}$ gives $x/\sqrt{x^2+1}$, not $2x\sqrt{x^2+1}$.
- Option B: Would require product rule; differentiating it does not recover the integrand.
- Option C ($\frac{1}{2}(x^2+1)^{3/2}$): The correct coefficient is $\frac{2}{3}$, not $\frac{1}{2}$.

Final Answer: $\dfrac{2}{3}(x^2+1)^{3/2} + C \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 3

Sol. 4.

Solution

Concept — Inverse of a 2×2 Matrix: Chapter: Linear Algebra For $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, $A^{-1} = \dfrac{1}{\det(A)}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$.

Step 1 — Compute the determinant.

$$\det(A) = 2\cdot3 - 1\cdot5 = 6 - 5 = 1.$$

Step 2 — Apply the inverse formula. Since $\det(A) = 1$:

$$A^{-1} = \frac{1}{1}\begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix} = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix}.$$

Step 3 — Verify: $A \cdot A^{-1} = I$**.**

$$\begin{pmatrix} 2 & 1 \\ 5 & 3 \end{pmatrix}\begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix} = \begin{pmatrix} 6-5 & -2+2 \\ 15-15 & -5+6 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I. \checkmark$$

Why other options are wrong:

- Option A: Signs of off-diagonal elements are wrong.
- Option C: This is the transpose of A, not the inverse.
- Option D: Signs of diagonal elements are wrong.

Final Answer: $A^{-1} = \begin{pmatrix} 3 & -1 \\ -5 & 2 \end{pmatrix} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 4

Sol. 5.

Solution

Concept — Homogeneous Linear Systems and Determinants: Chapter: Linear Algebra A homogeneous system $A\mathbf{x} = \mathbf{0}$ always has the trivial solution $\mathbf{x} = \mathbf{0}$. Non-trivial solutions exist if and only if $\det(A) = 0$.

Step 1 — Recall the invertibility criterion. If $\det(A) \neq 0$, then A is invertible, and the unique solution is

$$\mathbf{x} = A^{-1}\mathbf{0} = \mathbf{0}.$$

Step 2 — Conclusion. The system has *only* the trivial solution $\mathbf{x} = \mathbf{0}$.

Why other options are wrong:

- Option A (infinitely many): This happens only when $\det(A) = 0$.
- Option B (no solution): Homogeneous systems always have at least the trivial solution.
- Option D (exactly two): A linear system can never have exactly two solutions.

Final Answer: Only the trivial solution $\mathbf{x} = \mathbf{0} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 5

Sol. 6.

Solution

Concept — Mean of Binomial Distribution: Chapter: Probability & Statistics If $X \sim B(n, p)$, the mean (expected value) is $\mu = np$.

Step 1 — Identify parameters. Here $n = 10$ and $p = 0.3$.

Step 2 — Compute the mean.

$$\mu = np = 10 \times 0.3 = 3.$$

Why other options are wrong:

- Option B (7): This is $n - np = nq$, i.e. 10×0.7, the mean of the complementary distribution.

- Option C (2.1): This is the variance $npq = 10 \times 0.3 \times 0.7$, not the mean.
- Option D (0.3): This is p alone, not np.

Final Answer: $\mu = np = 3 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 6

Sol. 7.

Solution

Concept — Poisson Probability Formula: Chapter: Probability & Statistics For $X \sim \text{Poisson}(\lambda)$,

$$P(X = k) = \frac{e^{-\lambda}\lambda^k}{k!}.$$

Step 1 — Substitute $k = 0$ and $\lambda = 4$.

$$P(X = 0) = \frac{e^{-4} \cdot 4^0}{0!} = \frac{e^{-4} \cdot 1}{1} = e^{-4}.$$

Why other options are wrong:

- Option A ($e^{-4}/4$): Incorrectly divides by $\lambda = 4$.
- Option B ($4e^{-4}$): This equals $P(X = 1) = e^{-4} \cdot 4^1/1! = 4e^{-4}$, not $P(X = 0)$.
- Option C ($1 - e^{-4}$): This is $P(X \geq 1)$, the complementary event.

Final Answer: $P(X = 0) = e^{-4} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 7

Sol. 8.

Solution

Concept — Integrating Factor for First-Order Linear ODE: Chapter: Differential Equations For $\frac{dy}{dx} + P(x)\,y = Q(x)$, the integrating factor is $\mu(x) = e^{\int P(x)\,dx}$.

Step 1 — Identify $P(x)$. The ODE is $\frac{dy}{dx} + 2y = e^x$, so $P(x) = 2$.

Step 2 — Compute the integrating factor.

$$\mu(x) = e^{\int 2\,dx} = e^{2x}.$$

Why other options are wrong:

- Option A (e^x): This would arise from $P(x) = 1$, not $P(x) = 2$.
- Option C ($2e^x$): The integrating factor is a pure exponential; a coefficient of 2 is extraneous.
- Option D (e^{-2x}): This arises from $P(x) = -2$; the sign must match the equation as written.

Final Answer: Integrating factor $= e^{2x} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 8

Sol. 9.

Solution

Concept — Definite Integral as Area: Chapter: Integral Calculus The area under $y = f(x)$ from a to b equals $\displaystyle\int_a^b f(x)\,dx$ (when $f \geq 0$).

Step 1 — Set up the integral.

$$\int_0^2 x^2\,dx.$$

Step 2 — Apply the power rule.

$$\int_0^2 x^2\,dx = \left[\frac{x^3}{3}\right]_0^2.$$

Step 3 — Evaluate.

$$\frac{2^3}{3} - \frac{0^3}{3} = \frac{8}{3} - 0 = \frac{8}{3}.$$

Why other options are wrong:

- Option A (4): This is the value of $y = x^2$ at $x = 2$, not the integral.
- Option B (2): Off by a factor; $\int_0^2 x\,dx = 2$, but the integrand is x^2.
- Option D (4/3): This equals $\int_0^1 x^2\,dx \cdot \frac{4}{3}$; wrong limits.

Final Answer: $\displaystyle\int_0^2 x^2\,dx = \frac{8}{3} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 9

Sol. 10.

Solution

Concept — Equation of a Circle Centred at the Origin: Chapter: Coordinate Geometry The circle $x^2 + y^2 = r^2$ has radius r. Any point (x_0, y_0) on it satisfies $x_0^2 + y_0^2 = r^2$.

Step 1 — Substitute the point $(6, 8)$.

$$r^2 = 6^2 + 8^2 = 36 + 64 = 100.$$

Step 2 — Solve for r.

$$r = \sqrt{100} = 10.$$

(Note: this is a 6-8-10 Pythagorean triple.)

Why other options are wrong:

- Option B (8): This is only one coordinate, not the distance to the origin.
- Option C (14): This is $6 + 8$; radii are not found by adding coordinates.
- Option D (100): This is r^2, not r.

Final Answer: $r = 10 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 10

Sol. 11.

Solution

Concept — Area of a Triangle with One Vertex at the Origin: Chapter: Coordinate Geometry For a right triangle with legs along the axes, Area $= \frac{1}{2} \times \text{base} \times \text{height}$.

Step 1 — Identify the base and height. $B(4, 0)$ is on the x-axis at distance 4 from O; $C(0, 3)$ is on the y-axis at height 3.

Step 2 — Compute the area.

$$\text{Area} = \tfrac{1}{2} \times 4 \times 3 = \frac{12}{2} = 6.$$

Alternative — Shoelace formula:

$$\text{Area} = \tfrac{1}{2}\left|x_A(y_B - y_C) + x_B(y_C - y_A) + x_C(y_A - y_B)\right| = \tfrac{1}{2}|0(0-3) + 4(3-0) + 0(0-0)| = \tfrac{1}{2} \cdot 12 = 6.$$

Why other options are wrong:

- Option A (12): This omits the factor of $\frac{1}{2}$.
- Option B (3): This is the height alone, not the area.
- Option C (4): This is the base alone, not the area.

Final Answer: Area $= 6 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 11

Sol. 12.

Solution

Concept — Finding an Inverse Function: Chapter: Functions To find f^{-1}, write $y = f(x)$, swap x and y, then solve for y.

Step 1 — Write the function. $y = 2x - 3$.

Step 2 — Swap x and y. $x = 2y - 3$.

Step 3 — Solve for y.

$$x + 3 = 2y \implies y = \frac{x+3}{2}.$$

Therefore $f^{-1}(x) = \dfrac{x+3}{2}$.

Verification: $f(f^{-1}(x)) = 2 \cdot \dfrac{x+3}{2} - 3 = x + 3 - 3 = x$. ✓

Why other options are wrong:

- Option A $(\frac{x-3}{2})$: Subtracts 3 instead of adding; this corresponds to $y = 2x + 3$.
- Option C $(2x + 3)$: This would be the inverse of $y = \frac{x-3}{2}$, not of $y = 2x - 3$.
- Option D $(\frac{1}{2x-3})$: This is the reciprocal, not the inverse function.

Final Answer: $f^{-1}(x) = \dfrac{x+3}{2} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 12

Sol. 13.

Solution

Concept — Cross Product of Perpendicular Unit Vectors: Chapter: Vectors For $\mathbf{a} \times \mathbf{b}$, $|\mathbf{a} \times \mathbf{b}| = |\mathbf{a}||\mathbf{b}| \sin\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$.

Step 1 — Compute the cross product directly.

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = (0 \cdot 0 - 0 \cdot 1)\,\mathbf{i} - (1 \cdot 0 - 0 \cdot 0)\,\mathbf{j} + (1 \cdot 1 - 0 \cdot 0)\,\mathbf{k} = \mathbf{k} = (0, 0, 1).$$

Step 2 — Find the magnitude.

$$|\mathbf{a} \times \mathbf{b}| = |(0, 0, 1)| = \sqrt{0^2 + 0^2 + 1^2} = 1.$$

Alternative — Formula approach. $\mathbf{a}$ and $\mathbf{b}$ are unit vectors ($|\mathbf{a}| = |\mathbf{b}| = 1$) perpendicular to each other ($\theta = 90^\circ$, $\sin 90^\circ = 1$), so $|\mathbf{a} \times \mathbf{b}| = 1 \cdot 1 \cdot 1 = 1$.

Why other options are wrong:

- Option A (0): The dot product $\mathbf{a} \cdot \mathbf{b} = 0$, not the cross product magnitude.
- Option B (2): There is no factor of 2; both vectors are unit vectors.
- Option D $(\sqrt{2})$: This would arise if $\theta = 45^\circ$; here $\theta = 90^\circ$.

Final Answer: $|\mathbf{a} \times \mathbf{b}| = 1 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 13

Sol. 14.

Solution

Concept — Permutations of Distinct Objects: Chapter: Combinatorics The number of ways to arrange n distinct objects in a row is $n!$.

Step 1 — Identify the number of distinct letters. The word MATH has 4 letters: M, A, T, H — all distinct.

Step 2 — Apply the formula.

$$4! = 4 \times 3 \times 2 \times 1 = 24.$$

Why other options are wrong:

- Option B (12): This is $4!/2! = 12$, as if two letters were identical.
- Option C (4): This counts only the number of letters, not their arrangements.
- Option D (16): This is 4^2; arrangements are computed with factorials, not powers.

Final Answer: Number of arrangements $= 4! = 24 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 14

Sol. 15.

Solution

Concept — Sum of an Arithmetic Series: Chapter: Combinatorics The sum of the first n positive integers is given by

$$S_n = \frac{n(n+1)}{2}.$$

Step 1 — Substitute $n = 10$.

$$S_{10} = \frac{10 \times 11}{2} = \frac{110}{2} = 55.$$

Verification — Gauss's pairing method: Pair the terms: $(1+10)+(2+9)+(3+8)+(4+7)+(5+6) = 11 \times 5 = 55$. ✓

Why other options are wrong:

- Option A (45): This is $S_9 = 9 \times 10/2$; the upper limit is 10, not 9.
- Option C (50): Off by 5; a common miscount from not using the correct formula.
- Option D (100): This is 10^2 or 10×10; the formula has a factor of $\frac{1}{2}$.

Final Answer: $1 + 2 + \cdots + 10 = 55 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 15