GAT B Mathematics
Sample Paper – 3

Exam	Questions	Duration	Max Marks
GAT B (DBT)	15 (Single-Correct MCQ)	23 Minutes	15
Marking Scheme: $+1$ correct | -0.5 wrong | 0 unattempted			

Instructions

- This paper contains **15 single-correct MCQ** questions from the Mathematics section of GAT B.
- Each correct answer carries $+1$ **mark**; each wrong answer carries -0.5 **marks**; unattempted questions carry 0 **marks**.
- Total duration: **23 minutes**. Manage your time carefully.
- Read each question carefully before selecting your answer.
- For more sample papers and resources, scan the QR code in the footer or visit https://collegedunia.com/exams/gat-b/sample-paper.

SECTION: MATHEMATICS (15 Questions)

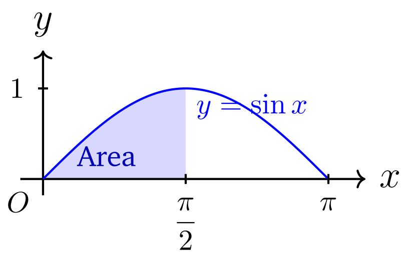
Q1. $\lim_{x \to \infty} \dfrac{3x^2 + 2x}{x^2 - 1}$ equals: *(Concept check)*

(A) 0
(B) 2
(C) ∞
(D) 3

Q2. Using logarithmic differentiation, $\dfrac{d}{dx}[x^x]$ equals: *(Application)*

(A) x^{x-1}
(B) $x \cdot x^{x-1}$
(C) $x^x(1 + \ln x)$
(D) $x^x \ln x$

Q3. The figure below shows the graph of $y = \sin x$ from $x = 0$ to $x = \pi$, with the shaded region indicating the area under the curve from 0 to $\pi/2$. Find $\int_0^{\pi/2} \sin x \, dx$:



(Application)

(A) 0
(B) 1
(C) 2
(D) $\pi/2$

Q4. The rank of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$ is: *(Concept check)*

(A) 1
(B) 2
(C) 3
(D) 0

Q5. If A is a 3×3 singular matrix, then $A\mathbf{x} = \mathbf{0}$ has: *(Concept check)*

(A) a unique solution
(B) no solution
(C) exactly two solutions
(D) infinitely many solutions

Q6. Given $P(A \mid B) = 0.8$, $P(B) = 0.3$, and $P(A) = 0.6$, find $P(B \mid A)$ using Bayes' theorem. *(Application)*

(A) 0.24
(B) 0.8
(C) 0.4
(D) 0.6

Q7. The standard deviation of the data set $\{2, 4, 6, 8, 10\}$ is: *(Application)*

(A) 4
(B) $2\sqrt{2}$
(C) $\sqrt{10}$
(D) 5

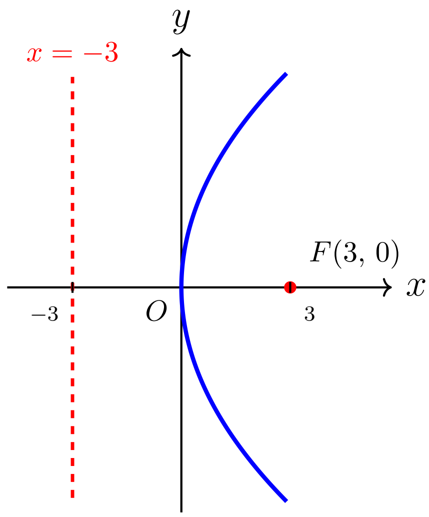
Q8. The general solution of $y'' - y = 0$ is: *(Concept check)*

(A) $Ae^{x} + Be^{-x}$
(B) $A\sin x + B\cos x$
(C) $Ae^{2x} + B$
(D) $A + Bx$

Q9. In the Taylor series $e^{x} = \sum_{n=0}^{\infty} \frac{x^n}{n!}$, the coefficient of x^3 is: *(Concept check)*

(A) 1
(B) $\frac{1}{3}$
(C) $\frac{1}{2}$
(D) $\frac{1}{6}$

Q10. The figure shows the parabola $y^2 = 12x$ with its focus F and directrix. The focus is located at:



(Concept check)

(A) $(0,\ 3)$
(B) $(0,\ -3)$
(C) $(3,\ 0)$
(D) $(-3,\ 0)$

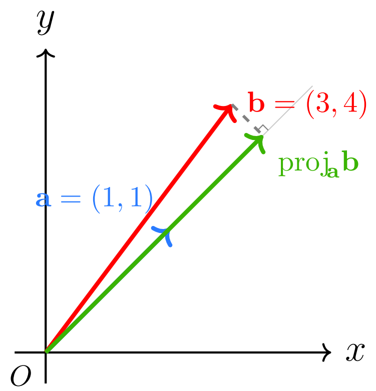
Q11. The centroid of the triangle with vertices $A(1, 2)$, $B(4, 5)$, $C(7, 8)$ is: *(Application)*

(A) $(3, 4)$
(B) $(4, 5)$
(C) $(5, 6)$
(D) $(6, 7)$

Q12. The number of elements in the power set $\mathcal{P}(A)$ when $|A| = 4$ is: *(Concept check)*

(A) 16
(B) 8
(C) 4
(D) 12

Q13. The diagram below shows vectors $\mathbf{a} = (1, 1)$ and $\mathbf{b} = (3, 4)$. The dashed line indicates the perpendicular from $\mathbf{b}$ to the line of $\mathbf{a}$, and the green arrow shows the vector projection. Given that the scalar projection $= \dfrac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|}$, its value is:



(Application)

(A) 3.5
(B) 7
(C) 5
(D) $\dfrac{7}{\sqrt{2}}$

Q14. The number of ways to seat 5 people around a circular table is: *(Application)*

(A) 120
(B) 24
(C) 60
(D) 12

Q15. The sum of the infinite geometric series $2 + \dfrac{2}{3} + \dfrac{2}{9} + \cdots$ is: *(Concept check)*

(A) 3
(B) 2
(C) 6
(D) 1

DETAILED SOLUTIONS

Q1.

Solution

Concept — Limits at Infinity: For rational functions as $x \to \infty$, divide numerator and denominator by the highest power of x in the denominator. Each remaining term of the form $1/x^k$ vanishes, leaving only the ratio of leading coefficients. **Chapter:** Limits/Continuity

Step 1 — Divide numerator and denominator by x^2:

$$\lim_{x\to\infty} \frac{3x^2 + 2x}{x^2 - 1} = \lim_{x\to\infty} \frac{3 + \frac{2}{x}}{1 - \frac{1}{x^2}}$$

Step 2 — Evaluate the limit: As $x \to \infty$, $\frac{2}{x} \to 0$ and $\frac{1}{x^2} \to 0$, so:

$$= \frac{3+0}{1-0} = 3$$

Why other options are wrong:

- Option A (0): Would require the numerator degree to be strictly less than the denominator degree.
- Option B (2): Arises from comparing the coefficient of x rather than x^2.
- Option C (∞): The degrees are equal, so the limit is finite; it equals the ratio of leading coefficients.
- Option D (3): Correct — ratio of leading coefficients $\frac{3}{1} = 3$.

Final Answer: $\lim_{x\to\infty} \frac{3x^2 + 2x}{x^2 - 1} = 3 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 1

Q2.

Solution

Concept — Logarithmic Differentiation: When a function has the variable in both the base and the exponent (e.g., x^x), the power rule alone is insufficient. We take $\ln$ of both sides and differentiate implicitly, using the product rule on the right side.
Chapter: Differential Calculus

Step 1 — Set $y = x^x$ and take the natural logarithm:

$$\ln y = x \ln x$$

Step 2 — Differentiate both sides with respect to x (product rule on right):

$$\frac{1}{y}\frac{dy}{dx} = \ln x + x \cdot \frac{1}{x} = \ln x + 1$$

Step 3 — Multiply both sides by $y = x^x$:

$$\frac{dy}{dx} = x^x (1 + \ln x)$$

Why other options are wrong:

- Option A (x^{x-1}): Power rule applied as if the exponent were a constant; invalid when the exponent depends on x.
- Option B ($x \cdot x^{x-1}$): Simplifies to x^x; the $\ln x$ correction from the product rule is missing.
- Option C ($x^x(1 + \ln x)$): Correct.
- Option D ($x^x \ln x$): The product rule on $x \ln x$ gives $1 + \ln x$, not just $\ln x$.

Final Answer: $\frac{d}{dx}[x^x] = x^x(1 + \ln x) \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 2

Q3.

Solution

Concept — Definite Integral (Fundamental Theorem of Calculus): $\int_a^b f(x)\,dx = \left[F(x)\right]_a^b = F(b) - F(a)$, where F is any antiderivative of f. The result equals the net signed area between the curve and the x-axis over $[a, b]$. **Chapter:** Integral Calculus

Step 1 — Antiderivative of $\sin x$:

$$\int \sin x\,dx = -\cos x + C$$

Step 2 — Evaluate between the limits:

$$\int_0^{\pi/2} \sin x\,dx = \Big[-\cos x\Big]_0^{\pi/2} = -\cos\tfrac{\pi}{2} - \big(-\cos 0\big) = -(0) + 1 = 1$$

The shaded area in the figure is exactly 1 square unit.

Why other options are wrong:

- Option A (0): $\int_0^{\pi} \sin x\,dx = 0$ due to symmetric cancellation; here the upper limit is only $\pi/2$.
- Option B (1): Correct.
- Option C (2): $\int_0^{\pi} \sin x\,dx = 2$, integrating over the full half-period $[0, \pi]$.
- Option D $(\pi/2)$: Would result from integrating $1\,dx$ from 0 to $\pi/2$, not $\sin x$.

Final Answer: $\displaystyle\int_0^{\pi/2} \sin x\,dx = 1 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 3

Q4.

Solution

Concept — Matrix Rank: The rank of a matrix is the maximum number of linearly independent rows (equivalently, columns). Row reduction (Gaussian elimination) reduces the matrix to row-echelon form; the number of non-zero rows equals the rank. **Chapter:** Linear Algebra

Step 1 — Observe row relationships in $M = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$**:**

$R_2 = 2\,R_1$ and $R_3 = 3\,R_1$, so all three rows are multiples of the first row.

Step 2 — Perform row reduction:

$$\xrightarrow{R_2 \leftarrow R_2 - 2R_1} \xrightarrow{R_3 \leftarrow R_3 - 3R_1} \begin{pmatrix} 1 & 2 & 3 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Only one non-zero row remains, so $\operatorname{rank}(M) = 1$.

Why other options are wrong:

- Option A (1): Correct — only one linearly independent row.
- Option B (2): Would require two independent rows; R_2 and R_3 are not independent of R_1.
- Option C (3): Requires $\det(M) \neq 0$; but $\det(M) = 0$ since rows are linearly dependent.
- Option D (0): Only the zero matrix has rank 0.

Final Answer: $\operatorname{rank}(M) = 1 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 4

Q5.

Solution

Concept — Null Space and Singular Matrices: A matrix A is singular if and only if $\det(A) = 0$, which implies $\operatorname{rank}(A) < n$. By the rank-nullity theorem, $\dim(\ker A) = n - \operatorname{rank}(A) \geq 1$, guaranteeing non-trivial solutions to $A\mathbf{x} = \mathbf{0}$. **Chapter:** Linear Algebra

Step 1 — Apply rank-nullity for a 3×3 singular A:

$$\dim(\ker A) = 3 - \operatorname{rank}(A) \geq 1$$

Step 2 — Conclude on solution set: Since $\dim(\ker A) \geq 1$, there exists a non-zero vector $\mathbf{x}_0$ with $A\mathbf{x}_0 = \mathbf{0}$. Every scalar multiple $\lambda\mathbf{x}_0$ ($\lambda \in \mathbb{R}$) is also a solution, giving infinitely many solutions.

Why other options are wrong:

- Option A (unique solution): Unique solution $\mathbf{x} = \mathbf{0}$ occurs only when A is invertible (non-singular).

- Option B (no solution): Homogeneous systems always admit the trivial solution $\mathbf{x} = \mathbf{0}$.
- Option C (exactly two solutions): Solution sets of linear systems are vector subspaces; a subspace contains either one element or infinitely many.
- Option D (infinitely many): Correct.

Final Answer: $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions $\Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 5

Q6.

Solution

Concept — Bayes' Theorem: Chapter: Probability & Statistics

$$P(B \mid A) = \frac{P(A \mid B)\, P(B)}{P(A)}$$

This formula reverses the direction of a conditional probability when the individual probabilities $P(A)$, $P(B)$ and $P(A \mid B)$ are known.

Step 1 — Compute $P(A \cap B)$:

$$P(A \cap B) = P(A \mid B) \cdot P(B) = 0.8 \times 0.3 = 0.24$$

Step 2 — Apply Bayes' theorem:

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} = \frac{0.24}{0.6} = 0.4$$

Why other options are wrong:

- Option A (0.24): This is $P(A \cap B)$, the joint probability, not the conditional.
- Option B (0.8): This is $P(A \mid B)$, the given conditional in the opposite direction.
- Option C (0.4): Correct.
- Option D (0.6): This is $P(A)$, one of the given inputs.

Final Answer: $P(B \mid A) = 0.4 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 6

Q7.

Solution

Concept — Population Standard Deviation: Chapter: Probability & Statistics

$$\sigma = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

First find the mean, compute each squared deviation from the mean, average them (variance), then take the square root.

Step 1 — Compute the mean:

$$\bar{x} = \frac{2+4+6+8+10}{5} = \frac{30}{5} = 6$$

Step 2 — Squared deviations:

$$(2-6)^2 = 16, \quad (4-6)^2 = 4, \quad (6-6)^2 = 0, \quad (8-6)^2 = 4, \quad (10-6)^2 = 16$$

Step 3 — Variance and standard deviation:

$$\sigma^2 = \frac{16+4+0+4+16}{5} = \frac{40}{5} = 8 \implies \sigma = \sqrt{8} = 2\sqrt{2} \approx 2.83$$

Why other options are wrong:

- Option A (4): $\sqrt{16}$; the largest individual deviation, not the SD.
- Option B ($2\sqrt{2}$): Correct.
- Option C ($\sqrt{10} \approx 3.16$): $\sqrt{10} \neq \sqrt{8} = 2\sqrt{2}$.
- Option D (5): This is the mean of the data, not the standard deviation.

Final Answer: $\sigma = 2\sqrt{2} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 7

Q8.

Solution

Concept — Linear 2nd-Order ODE with Constant Coefficients: For $ay'' + by' + cy = 0$, substitute $y = e^{rx}$ to obtain the characteristic equation $ar^2 + br + c = 0$. Two distinct real roots r_1, r_2 give the general solution $y = Ae^{r_1 x} + Be^{r_2 x}$. **Chapter:** Differential Equations

Step 1 — Write the characteristic equation for $y'' - y = 0$:

$$r^2 - 1 = 0 \implies (r-1)(r+1) = 0 \implies r = +1 \text{ or } r = -1$$

Step 2 — General solution (two distinct real roots $r_1 = 1$, $r_2 = -1$):

$$y = Ae^{x} + Be^{-x}$$

where A and B are arbitrary real constants.

Why other options are wrong:

- Option A ($Ae^x + Be^{-x}$): Correct.
- Option B ($A\sin x + B\cos x$): Solves $y'' + y = 0$ (complex roots $r = \pm i$).
- Option C ($Ae^{2x} + B$): Corresponds to roots $r = 2$ and $r = 0$, arising from $y'' - 2y' = 0$.
- Option D ($A + Bx$): Solves $y'' = 0$ (double root at $r = 0$).

Final Answer: $y = Ae^x + Be^{-x} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 8

Q9.

Solution

Concept — Maclaurin (Taylor) Series for e^x: The expansion $e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$ provides exact coefficients: the coefficient of x^n is $\frac{1}{n!}$. **Chapter:** Integral Calculus

Step 1 — Write out the series to the x^3 term explicitly:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots = 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \cdots$$

Step 2 — Identify the coefficient of x^3:

$$\text{Coefficient of } x^3 = \frac{1}{3!} = \frac{1}{6}$$

Why other options are wrong:

- Option A (1): Coefficient of x^0 (or x^1), not x^3.
- Option B $(\frac{1}{3})$: Confuses $\frac{1}{3!} = \frac{1}{6}$ with $\frac{1}{3}$.
- Option C $(\frac{1}{2})$: This is $\frac{1}{2!}$, the coefficient of x^2, not x^3.
- Option D $(\frac{1}{6})$: Correct — $\frac{1}{3!} = \frac{1}{6}$.

Final Answer: Coefficient of x^3 in e^x is $\frac{1}{6} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 9

Q10.

Solution

Concept — Standard Parabola $y^2 = 4ax$: This parabola opens to the right with vertex at the origin $O(0, 0)$, focus at $F(a, 0)$, and directrix $x = -a$. The parameter $a > 0$ is the focal distance. **Chapter:** Coordinate Geometry

Step 1 — Match $y^2 = 12x$ with the standard form $y^2 = 4ax$:

$$4a = 12 \implies a = 3$$

Step 2 — State the focus and directrix:

$$\text{Focus } F = (a, 0) = (3, 0), \qquad \text{Directrix: } x = -a = -3$$

Both are clearly marked in the diagram: the focus $F(3, 0)$ lies to the right of the vertex and the directrix $x = -3$ is to the left.

Why other options are wrong:

- Option A $((0, 3))$: Focus of the upward-opening parabola $x^2 = 12y$.
- Option B $((0, -3))$: Focus of the downward-opening parabola $x^2 = -12y$.
- Option C $((3, 0))$: Correct.
- Option D $((-3, 0))$: This is the point on the directrix at the axis; the directrix equation is $x = -3$, not a focus.

Final Answer: Focus $= (3,\ 0) \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 10

Q11.

Solution

Concept — Centroid of a Triangle: The centroid G is the intersection of the three medians and has coordinates equal to the arithmetic mean of the vertex coordinates: **Chapter:** Coordinate Geometry

$$G = \left(\frac{x_1 + x_2 + x_3}{3}, \ \frac{y_1 + y_2 + y_3}{3} \right)$$

Step 1 — Substitute $A(1,2)$**,** $B(4,5)$**,** $C(7,8)$**:**

$$G_x = \frac{1+4+7}{3} = \frac{12}{3} = 4, \qquad G_y = \frac{2+5+8}{3} = \frac{15}{3} = 5$$

Step 2 — State the centroid: $G = (4,\ 5)$.

Note: The vertices are collinear (each consecutive pair differs by $(3,3)$), so this is a degenerate triangle; the centroid formula still applies and gives $G = (4,5)$.

Why other options are wrong:

- Option A ($(3,4)$): Off by $(1,1)$; arises from averaging only two of the three vertices.
- Option B ($(4,5)$): Correct.
- Option C ($(5,6)$): Off by $(1,1)$; a common arithmetic slip.
- Option D ($(6,7)$): Average of $B(4,5)$ and $C(7,8)$ only; ignores vertex A.

Final Answer: Centroid $G = (4,\ 5) \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 11

Q12.

Solution

Concept — Power Set: For any set A with n elements, the power set $\mathcal{P}(A)$ contains all possible subsets of A — from the empty set $\emptyset$ to A itself. The number of elements is $|\mathcal{P}(A)| = 2^n$. **Chapter:** Set Theory

Step 1 — Apply the formula with $n = |A| = 4$:

$$|\mathcal{P}(A)| = 2^4 = 16$$

Step 2 — Verify by counting subsets of each size:

$$\binom{4}{0} + \binom{4}{1} + \binom{4}{2} + \binom{4}{3} + \binom{4}{4} = 1 + 4 + 6 + 4 + 1 = 16 \checkmark$$

Why other options are wrong:

- Option A (16): Correct — $2^4 = 16$.
- Option B (8): 2^3; power set size for a 3-element set.
- Option C (4): Equals n; confuses the size of A with the size of $\mathcal{P}(A)$.
- Option D (12): No standard formula yields 12 for $n = 4$.

Final Answer: $|\mathcal{P}(A)| = 16 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 12

Q13.

Solution

Concept — Scalar Projection: The scalar projection of $\mathbf{b}$ onto $\mathbf{a}$ measures the signed length of the component of $\mathbf{b}$ in the direction of $\mathbf{a}$: **Chapter:** Vectors

$$\text{scalar proj} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|}$$

It equals the length of the green vector shown in the diagram.

Step 1 — Compute the dot product $\mathbf{a} \cdot \mathbf{b}$:

$$\mathbf{a} \cdot \mathbf{b} = (1)(3) + (1)(4) = 3 + 4 = 7$$

Step 2 — Compute $|\mathbf{a}|$:

$$|\mathbf{a}| = \sqrt{1^2 + 1^2} = \sqrt{2}$$

Step 3 — Scalar projection:

$$\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|} = \frac{7}{\sqrt{2}}$$

Why other options are wrong:

- Option A (3.5): This is $\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|^2} = \frac{7}{2}$, the scalar coefficient in the vector projection formula, not the scalar projection itself.
- Option B (7): This is the raw dot product $\mathbf{a} \cdot \mathbf{b}$; has not been divided by $|\mathbf{a}|$.
- Option C (5): This is $|\mathbf{b}| = \sqrt{3^2 + 4^2} = 5$, the magnitude of $\mathbf{b}$.
- Option D $(\frac{7}{\sqrt{2}})$: Correct.

Final Answer: Scalar projection $= \frac{7}{\sqrt{2}} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 13

Q14.

Solution

Concept — Circular Permutations: In a circular arrangement, rotating all n elements by any amount yields the same arrangement. One element is fixed to remove this rotational equivalence, leaving $(n-1)!$ distinct arrangements. **Chapter:** Combinatorics

Step 1 — Apply the formula for $n = 5$ people:

$$\text{Number of circular arrangements} = (5-1)! = 4! = 4 \times 3 \times 2 \times 1 = 24$$

Step 2 — Justify: Fix person 1 at a designated seat. The remaining 4 people can be arranged in $4! = 24$ distinct ways in the other seats.

Why other options are wrong:

- Option A (120): $5! = 120$; counts linear (row) arrangements, over-counting rotations by a factor of 5.
- Option B (24): Correct — $(5-1)! = 4! = 24$.
- Option C (60): $5!/2 = 60$; used for necklace problems where clockwise and counterclockwise are indistinguishable.

- Option D (12): $4!/2 = 12$; applies additional reflective symmetry, not required for seating around a table.

Final Answer: $(5-1)! = 24$ ways $\Rightarrow \boxed{\text{B}}$

$\boxed{\textbf{Answer: (B)}}$ **Go Back to Q 14**

Q15.

Solution

Concept — Sum of Infinite Geometric Series: For $|r| < 1$, the series $a + ar + ar^2 + \cdots$ converges and its sum is $S_\infty = \dfrac{a}{1-r}$. **Chapter:** Combinatorics

Step 1 — Identify a (first term) and r (common ratio):

$$a = 2, \qquad r = \frac{2/3}{2} = \frac{1}{3} \quad \left(|r| = \tfrac{1}{3} < 1,\ \text{series converges}\right)$$

Step 2 — Apply the sum formula:

$$S_\infty = \frac{a}{1-r} = \frac{2}{1-\frac{1}{3}} = \frac{2}{\frac{2}{3}} = 2 \times \frac{3}{2} = 3$$

Why other options are wrong:

- Option A (3): Correct.
- Option B (2): This is just the first term $a = 2$; ignores all subsequent terms.
- Option C (6): Arises from misapplying the formula, perhaps using $1 - r = 1/3$ instead of $2/3$.
- Option D (1): Impossible since $S_\infty > a = 2 > 1$ for positive terms.

Final Answer: $S_\infty = 3 \Rightarrow \boxed{\text{A}}$

Answer: (A) **Go Back to Q 15**

ANSWER KEY

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	D	2	C	3	B	4	A	5	D
6	C	7	B	8	A	9	D	10	C
11	B	12	A	13	D	14	B	15	A