GAT B Mathematics

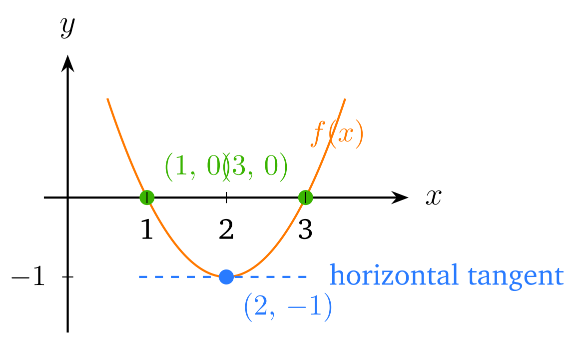
Sample Paper – 5

Duration: 23 Minutes **Maximum Marks: 15**

Instructions

- This paper contains **15** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics portion of **GAT B** entrance.
- Each correct answer carries **+1 mark**. There is a deduction of **0.5 mark** for each wrong answer. Unattempted questions carry zero marks.
- Only **one** option is correct. Choose carefully.
- Syllabus: Calculus, Linear Algebra, Probability & Statistics, Differential Equations, Coordinate Geometry, Sets & Functions, Vectors, Combinatorics (undergraduate level).
- Use of mobile phones, calculators, or electronic gadgets is strictly prohibited.

Q1. Let $f(x) = x^2 - 4x + 3$ on the closed interval $[1, 3]$. Rolle's theorem guarantees at least one $c \in (1, 3)$ such that $f'(c) = 0$. The TikZ figure below shows the parabola with a horizontal tangent. Find the value of c.



(Application)

(A) $c = 0$

(B) $c = 1$

(C) $c = 2$

(D) $c = 3$

Q2. Let $f(x, y) = x^2y + 3xy^2$. Compute the partial derivative $\dfrac{\partial f}{\partial x}$ (treat y as a constant).

(Application)

(A) $2x + 6y$

(B) $x^2 + 6xy$

(C) $2xy + 6xy$

(D) $2xy + 3y^2$

Q3. The curve $y = \sqrt{x}$ is rotated about the x-axis from $x = 0$ to $x = 4$. Using the disk method,

$$V = \pi \int_0^4 \left[\sqrt{x}\right]^2 dx.$$

Evaluate V.

(Application)

(A) 8π

(B) 4π

(C) 16π

(D) 2π

Q4. The *trace* of a square matrix is the sum of its main diagonal entries. Find $\mathrm{tr}(A)$ for the matrix A shown below (diagonal entries are highlighted).

$$\begin{bmatrix} \boxed{} & 1 & -2 \\ 0 & \boxed{} & 4 \\ 2 & -1 & \boxed{} \end{bmatrix} \qquad \text{diagonal: } 3,\ 5,\ 7$$

(Application)

(A) 12

(B) 15

(C) 9

(D) 18

Q5. Consider the vectors $\mathbf{v}_1 = (1, 0, 0)$, $\mathbf{v}_2 = (1, 1, 0)$, $\mathbf{v}_3 = (1, 1, 1)$ in $\mathbb{R}^3$. Which statement correctly describes their linear independence?

(Application)

(A) Linearly *dependent*; the determinant of the matrix $[\mathbf{v}_1\ \mathbf{v}_2\ \mathbf{v}_3]$ equals 0.

(B) Linearly *independent*; the determinant equals -1.

(C) Linearly *independent*; the determinant of the matrix $[\mathbf{v}_1\ \mathbf{v}_2\ \mathbf{v}_3]$ equals 1.

(D) Linearly *dependent*; $\mathbf{v}_3$ is a scalar multiple of $\mathbf{v}_1$.

Q6. A random experiment has probability of success $p = \frac{1}{4}$ on each independent trial. Let X be the number of trials until the first success, so $P(X = k) = \big(1 - p\big)^{k-1} p$ for $k = 1, 2, 3, \ldots$ Find the expected value $E[X]$.

(Application)

(A) 1

(B) 2

(C) 3

(D) 4

Q7. For the data set $\{2,\ 4,\ 6,\ 8\}$, find the variance using $\mathrm{Var}(X) = E[X^2] - \big(E[X]\big)^2$, where expectations are taken over the uniform distribution on the four values.

(Application)

(A) 5

(B) 4

(C) 6

(D) 25

Q8. Find a particular solution y_p of the second-order ODE

$$y'' + y = 2\cos x.$$

(Application)

(A) $y_p = \sin x$

(B) $y_p = x \sin x$

(C) $y_p = x \cos x$

(D) $y_p = x \sin x + x \cos x$

Q9. The Laplace transform of $f(t) = e^{at}$ (a a real constant) is defined by $\mathcal{L}\{e^{at}\}(s) = \int_0^{\infty} e^{-st}\, e^{at}\, dt$. Which expression is correct?

(Concept check)

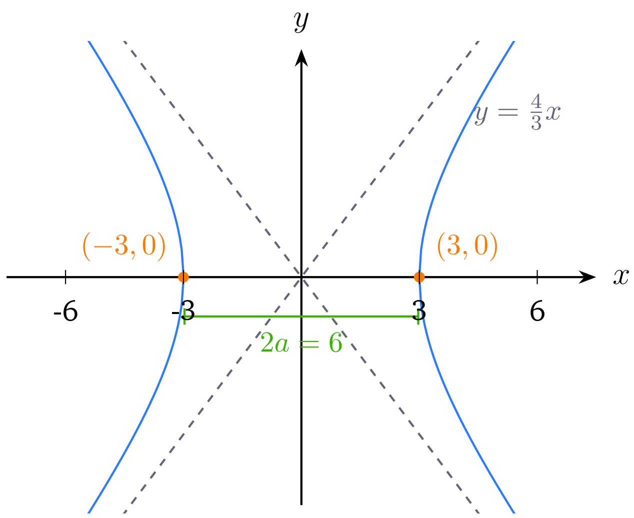
(A) $\dfrac{1}{s+a}, \quad s > -a$

(B) $\dfrac{a}{s-a}, \quad s > a$

(C) $\dfrac{1}{s-a}, \quad s > a$

(D) $\dfrac{1}{s}, \quad s > 0$

Q10. For the hyperbola $\dfrac{x^2}{9} - \dfrac{y^2}{16} = 1$, the transverse axis lies along the x-axis and has length $2a$. Identify the length of the transverse axis.



(Application)

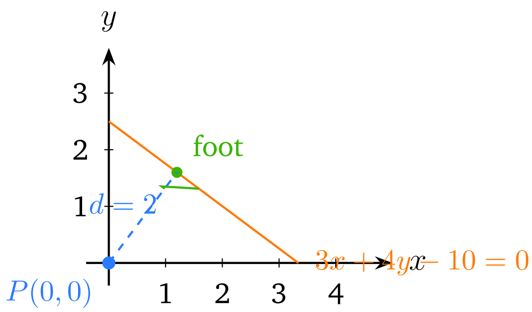
(A) 6

(B) 8

(C) 3

(D) 4

Q11. Find the perpendicular distance from the point $P(0, 0)$ to the line $3x + 4y - 10 = 0$.



(Application)

(A) 1

(B) $\frac{3}{5}$

(C) 5

(D) 2

Q12. Two sets A and B satisfy $|A| = 5$, $|B| = 7$, and $|A \cap B| = 2$. Recall that the symmetric difference is $A \triangle B = (A \cup B) \setminus (A \cap B)$. Find $|A \triangle B|$.

(Concept check)

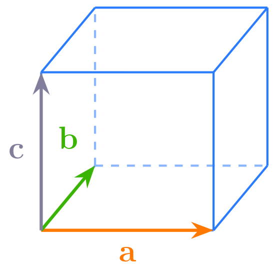
(A) 10

(B) 8

(C) 6

(D) 12

Q13. Let $\mathbf{a} = (2, 1, 0)$, $\mathbf{b} = (1, 3, 1)$, $\mathbf{c} = (0, 1, 2)$. The volume of the parallelepiped with edge vectors $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ equals $|\mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})|$. Compute this volume.



(Application)

(A) 4

(B) 6

(C) 8

(D) 12

Q14. A *derangement* of n elements is a permutation in which no element occupies its original position. The count of derangements of n elements is

$$D_n = n! \sum_{k=0}^{n} \frac{(-1)^k}{k!}.$$

Find D_3.

(Concept check)

(A) 1

(B) 3

(C) 4

(D) 2

Q15. An arithmetic series has first term $a = 2$ and common difference $d = 3$. Using the formula

$$S_n = \frac{n}{2}\big(2a + (n-1)d\big),$$

find the sum of the first 10 terms, S_{10}.

(Concept check)

(A) 155

(B) 150

(C) 145

(D) 160

Detailed Solutions

Solution 1.

Solution

Concept — Rolle's Theorem: Chapter: Differential Calculus If f is continuous on $[a, b]$, differentiable on (a, b), and $f(a) = f(b)$, then there exists $c \in (a, b)$ such that $f'(c) = 0$.

Step 1 — Verify the endpoint condition.

$$f(1) = 1 - 4 + 3 = 0, \qquad f(3) = 9 - 12 + 3 = 0.$$

Since $f(1) = f(3) = 0$ and f is a polynomial (continuous and differentiable everywhere), Rolle's theorem applies.

Step 2 — Solve $f'(c) = 0$.

$$f'(x) = 2x - 4 = 0 \Rightarrow x = 2.$$

Since $2 \in (1, 3)$, we have $\boxed{c = 2}$.

Why other options are wrong:

- (A) $c = 0$: outside the interval $(1, 3)$.
- (B) $c = 1$: an endpoint, not an interior point; Rolle's theorem requires $c \in (a, b)$.
- (D) $c = 3$: also an endpoint, not interior.

Answer: (C) Go Back to Q 1

Solution 2.

Solution

Concept — Partial Derivative: Chapter: Differential Calculus When computing $\partial f/\partial x$, treat every occurrence of y as a constant and differentiate with respect to x only.

Step 1 — Differentiate term by term.

$$f(x, y) = x^2y + 3xy^2.$$

$$\frac{\partial}{\partial x}(x^2y) = y \cdot \frac{d}{dx}(x^2) = y \cdot 2x = 2xy.$$

$$\frac{\partial}{\partial x}(3xy^2) = 3y^2 \cdot \frac{d}{dx}(x) = 3y^2.$$

Step 2 — Combine.

$$\frac{\partial f}{\partial x} = 2xy + 3y^2 = \boxed{2xy + 3y^2}.$$

Why other options are wrong:

- (A) $2x + 6y$: confuses partial with total derivative and ignores the y factors.
- (B) $x^2 + 6xy$: differentiates x^2y as x^2 (dropping y) and applies product rule incorrectly to $3xy^2$.
- (C) $2xy + 6xy$: treats $3y^2$ as $6xy$ by incorrectly differentiating with respect to y as well.

Answer: (D) Go Back to Q 2

Solution 3.

Solution

Concept — Disk Method: Chapter: Integral Calculus When the region bounded by $y = f(x)$, $x = a$, $x = b$, and the x-axis is rotated about the x-axis, the volume is $V = \pi \displaystyle\int_a^b [f(x)]^2\, dx$.

Step 1 — Simplify the integrand.

$$\left[\sqrt{x}\right]^2 = x.$$

Step 2 — Evaluate the integral.

$$V = \pi \int_0^4 x\, dx = \pi \left[\frac{x^2}{2}\right]_0^4 = \pi \cdot \frac{16}{2} = \boxed{8\pi}.$$

Why other options are wrong:

- (B) 4π: evaluates $\pi \int_0^4 \sqrt{x}\, dx$ instead of $\pi \int_0^4 x\, dx$ (forgets to square).
- (C) 16π: uses x^2 instead of x as the integrand (squares again without the square root cancellation).
- (D) 2π: divides by 4 instead of evaluating the antiderivative correctly.

Answer: (A) Go Back to Q 3

Solution 4.

Solution

Concept — Trace of a Matrix: Chapter: Linear Algebra For a square matrix A, $\mathrm{tr}(A) = \sum_i A_{ii}$ (sum of diagonal entries).

Step 1 — Identify the diagonal entries. From the highlighted matrix:

$$A_{11} = 3, \quad A_{22} = 5, \quad A_{33} = 7.$$

Step 2 — Sum.

$$\mathrm{tr}(A) = 3 + 5 + 7 = \boxed{15}.$$

Key property: The trace equals the sum of the eigenvalues (counted with multiplicity), and it is invariant under similarity transformations.

Why other options are wrong:

- (A) 12: omits the third diagonal entry ($12 = 3 + 5 + 4$ — uses off-diagonal $A_{23} = 4$).
- (C) 9: sums only two entries ($3 + 6$ or similar arithmetic error).
- (D) 18: likely adds $3 + 5 + 7 + 3$ or includes an off-diagonal entry twice.

Answer: (B) Go Back to Q 4

Solution 5.

Solution

Concept — Linear Independence Test: Chapter: Linear Algebra Vectors $\mathbf{v}_1, \ldots, \mathbf{v}_n$ are linearly independent if and only if the determinant of the matrix formed by these vectors (as columns) is non-zero.

Step 1 — Form the matrix $M = [\mathbf{v}_1\ \mathbf{v}_2\ \mathbf{v}_3]$.

$$M = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

This is an upper-triangular matrix.

Step 2 — Compute the determinant. For a triangular matrix, $\det =$ product of diagonal entries:

$$\det(M) = 1 \cdot 1 \cdot 1 = \boxed{1}.$$

Since $\det(M) \neq 0$, the vectors are **linearly independent**.

Why other options are wrong:

- (A) Det $= 0$, dependent: incorrect — the determinant is 1, not 0.
- (B) Det $= -1$: sign error; for this upper-triangular matrix the determinant is $+1$.
- (D) $\mathbf{v}_3$ scalar multiple of $\mathbf{v}_1$: $(1,1,1) \neq k(1,0,0)$ for any scalar k.

Answer: (C) Go Back to Q 5

Solution 6.

Solution

Concept — Geometric Distribution: Chapter: Probability & Statistics If $X \sim \text{Geom}(p)$, then $E[X] = \dfrac{1}{p}$.

Step 1 — Derive from first principles.

$$E[X] = \sum_{k=1}^{\infty} k\,(1-p)^{k-1}p = p\sum_{k=1}^{\infty} k\,q^{k-1} \quad (q = 1-p).$$

Using $\displaystyle\sum_{k=1}^{\infty} k\,q^{k-1} = \frac{1}{(1-q)^2} = \frac{1}{p^2}$:

$$E[X] = p \cdot \frac{1}{p^2} = \frac{1}{p}.$$

Step 2 — Substitute $p = \frac{1}{4}$.

$$E[X] = \frac{1}{1/4} = \boxed{4}.$$

Interpretation: On average, 4 trials are needed before the first success when each trial succeeds with probability $\frac{1}{4}$.

Why other options are wrong:

- (A) 1: would require $p = 1$ (certain success on first trial).
- (B) 2: corresponds to $p = \frac{1}{2}$.
- (C) 3: corresponds to $p = \frac{1}{3}$.

Answer: (D) Go Back to Q 6

Solution 7.

Solution

Concept — Variance Formula: Chapter: Probability & Statistics $\mathrm{Var}(X) = E[X^2] - (E[X])^2$, where expectations are computed over the uniform distribution on the given data set.

Step 1 — Compute $E[X]$.

$$E[X] = \frac{2+4+6+8}{4} = \frac{20}{4} = 5.$$

Step 2 — Compute $E[X^2]$.

$$E[X^2] = \frac{2^2+4^2+6^2+8^2}{4} = \frac{4+16+36+64}{4} = \frac{120}{4} = 30.$$

Step 3 — Compute the variance.

$$\mathrm{Var}(X) = E[X^2] - (E[X])^2 = 30 - 5^2 = 30 - 25 = \boxed{5}.$$

Why other options are wrong:

- (B) 4: off by one; likely uses $n-1$ denominator but miscalculates.
- (C) 6: arithmetic error in summing squares.
- (D) 25: gives $(E[X])^2$ alone, forgetting to subtract it from $E[X^2]$.

Answer: (A) Go Back to Q 7

Solution 8.

Solution

Concept — Method of Undetermined Coefficients (Resonance Case): Chapter: Differential Equations When the forcing term overlaps with the homogeneous solution, multiply the trial particular solution by x.

Step 1 — Homogeneous solution. Characteristic equation: $r^2 + 1 = 0 \Rightarrow r = \pm i$. Homogeneous solution: $y_h = C_1 \cos x + C_2 \sin x$.

Step 2 — Identify resonance. The right-hand side $2\cos x$ belongs to the same family as y_h, so we multiply by x:

$$y_p = x(A\cos x + B\sin x).$$

Step 3 — Compute y_p'' and substitute.

$$y_p' = (A\cos x + B\sin x) + x(-A\sin x + B\cos x),$$
$$y_p'' = 2(-A\sin x + B\cos x) + x(-A\cos x - B\sin x).$$

$$y_p''+y_p = 2(-A\sin x+B\cos x)+x(-A\cos x-B\sin x)+x(A\cos x+B\sin x) = -2A\sin x+2B\cos x.$$

Setting equal to $2\cos x$:

$$-2A = 0 \Rightarrow A = 0, \qquad 2B = 2 \Rightarrow B = 1.$$

Step 4 — Particular solution.

$$y_p = x\cdot(0\cdot\cos x + 1\cdot\sin x) = \boxed{x\sin x}.$$

Why other options are wrong:

- (A) $y_p = \sin x$: ignores resonance; substituting gives 0, not $2\cos x$.
- (C) $y_p = x\cos x$: would produce only $\sin x$ terms after differentiation, not $\cos x$.
- (D) $y_p = x\sin x + x\cos x$: two unknowns both survive but the $\cos x$ coefficient turns out to be zero.

Answer: (B) Go Back to Q 8

Solution 9.

Solution

Concept — Laplace Transform of Exponential: Chapter: Differential Equations
Compute $\mathcal{L}\{e^{at}\}(s)$ directly from the definition.

Step 1 — Evaluate the integral.

$$\mathcal{L}\{e^{at}\}(s) = \int_0^\infty e^{-st}\,e^{at}\,dt = \int_0^\infty e^{-(s-a)t}\,dt.$$

For convergence, we need $s > a$ (so the exponent is negative):

$$= \left[\frac{e^{-(s-a)t}}{-(s-a)}\right]_0^\infty = 0 - \frac{1}{-(s-a)} = \boxed{\frac{1}{s-a}}, \quad s > a.$$

Why other options are wrong:

- (A) $\frac{1}{s+a}$: sign error in the exponent — uses $+(s+a)t$ instead of $-(s-a)t$.
- (B) $\frac{a}{s-a}$: multiplies by a from a wrong normalisation step.
- (D) $\frac{1}{s}$: this is $\mathcal{L}\{1\}$, valid only for $a = 0$.

Answer: (C) Go Back to Q 9

Solution 10.

Solution

Concept — Hyperbola Standard Form: Chapter: Coordinate Geometry For $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the transverse axis lies along the x-axis with length $2a$.

Step 1 — Read off a^2.

$$\frac{x^2}{9} - \frac{y^2}{16} = 1 \;\Rightarrow\; a^2 = 9 \;\Rightarrow\; a = 3.$$

Step 2 — Compute the transverse axis length.

$$\text{Length} = 2a = 2 \times 3 = \boxed{6}.$$

The vertices are at $(\pm 3, 0)$, and the asymptotes are $y = \pm\frac{4}{3}x$ (since $b = 4$, $a = 3$).

Why other options are wrong:

- (B) 8: this is $2b = 2 \times 4$, the length of the *conjugate* axis, not the transverse axis.
- (C) 3: gives a, not $2a$.
- (D) 4: gives b, the semi-conjugate axis length.

Answer: (A) Go Back to Q 10

Solution 11.

Solution

Concept — Point-to-Line Distance Formula: Chapter: Coordinate Geometry The distance from point $(x_0,\, y_0)$ to the line $ax + by + c = 0$ is

$$d = \frac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}.$$

Step 1 — Identify parameters. Point: $(x_0, y_0) = (0, 0)$. Line: $3x + 4y - 10 = 0$, so $a = 3$, $b = 4$, $c = -10$.

Step 2 — Apply the formula.

$$d = \frac{|3(0) + 4(0) + (-10)|}{\sqrt{3^2 + 4^2}} = \frac{|-10|}{\sqrt{25}} = \frac{10}{5} = \boxed{2}.$$

Verification: The foot of the perpendicular is at $(x_0, y_0) - \dfrac{ax_0 + by_0 + c}{a^2 + b^2}(a, b) = (0, 0) - \dfrac{-10}{25}(3, 4) = \left(\frac{6}{5}, \frac{8}{5}\right)$, and $\sqrt{(6/5)^2 + (8/5)^2} = \sqrt{100/25} = 2$. ✓

Why other options are wrong:

- (A) 1: omits the denominator $\sqrt{a^2 + b^2} = 5$ (uses $10/10$ instead of $10/5$).
- (B) $\frac{3}{5}$: substitutes only $|a|/\sqrt{a^2 + b^2} = 3/5$, ignoring the numerator.
- (C) 5: gives $\sqrt{a^2 + b^2}$ alone, the denominator not the distance.

Answer: (D) Go Back to Q 11

Solution 12.

Solution

Concept — Symmetric Difference: Chapter: Set Theory $A\triangle B = (A\cup B)\setminus(A\cap B)$, so $|A\triangle B| = |A \cup B| - |A \cap B|$.

Step 1 — Find $|A \cup B|$ using inclusion-exclusion.

$$|A \cup B| = |A| + |B| - |A \cap B| = 5 + 7 - 2 = 10.$$

Step 2 — Subtract the intersection.

$$|A\triangle B| = |A \cup B| - |A \cap B| = 10 - 2 = \boxed{8}.$$

Equivalently: $|A\triangle B| = |A \setminus B| + |B \setminus A| = (5 - 2) + (7 - 2) = 3 + 5 = 8.$

Why other options are wrong:

- (A) 10: gives $|A \cup B|$, not $|A \triangle B|$ (forgets to subtract $|A \cap B|$).
- (C) 6: uses $|A| - |A \cap B| = 3$ alone (counts only $A \setminus B$, not $B \setminus A$).
- (D) 12: gives $|A| + |B| - 2 \cdot 0 = 12$, ignoring the intersection entirely.

Answer: (B) Go Back to Q 12

Solution 13.

Solution

Concept — Scalar Triple Product: Chapter: Vectors $V = |\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})|$ gives the volume of the parallelepiped with edge vectors **a**, **b**, **c**.

Step 1 — Compute $\mathbf{b} \times \mathbf{c}$.

$$\mathbf{b} \times \mathbf{c} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 3 & 1 \\ 0 & 1 & 2 \end{vmatrix} = \mathbf{i}(3 \cdot 2 - 1 \cdot 1) - \mathbf{j}(1 \cdot 2 - 1 \cdot 0) + \mathbf{k}(1 \cdot 1 - 3 \cdot 0) = (5,\ -2,\ 1).$$

Step 2 — Dot with $\mathbf{a}$.

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (2,\ 1,\ 0) \cdot (5,\ -2,\ 1) = 2(5) + 1(-2) + 0(1) = 10 - 2 + 0 = 8.$$

Step 3 — Volume.

$$V = |8| = \boxed{8} \text{ cubic units.}$$

Why other options are wrong:

- (A) 4: halves the result, which would give the volume of a tetrahedron, not a parallelepiped.
- (B) 6: arithmetic slip — uses $2(3) + 1(2)$ ignoring the correct cross product components.
- (D) 12: multiplies a components directly without computing the cross product first.

Answer: (C) Go Back to Q 13

Solution 14.

Solution

Concept — Derangement Formula: Chapter: Combinatorics

$$D_n = n!\sum_{k=0}^{n}\frac{(-1)^k}{k!} = n!\left(1-1+\frac{1}{2!}-\frac{1}{3!}+\cdots+\frac{(-1)^n}{n!}\right).$$

Step 1 — Substitute $n = 3$.

$$D_3 = 3!\left(1-1+\frac{1}{2!}-\frac{1}{3!}\right) = 6\left(1-1+\frac{1}{2}-\frac{1}{6}\right) = 6\cdot\frac{2}{6} = \boxed{2}.$$

Step 2 — Enumerate to verify. The $3! = 6$ permutations of $\{1,2,3\}$ are: $(1,2,3)$, $(1,3,2)$, $(2,1,3)$, $(2,3,1)$, $(3,1,2)$, $(3,2,1)$. Only $(2,3,1)$ and $(3,1,2)$ have no fixed point. Hence $D_3 = 2$. ✓

Why other options are wrong:

- (A) 1: the value of D_2 (only one derangement of $\{1,2\}$: $(2,1)$).
- (B) 3: counts permutations with at least one displacement, not all displacements.
- (C) 4: overcounts by including permutations that fix one element.

Answer: (D) Go Back to Q 14

Solution 15.

Solution

Concept — Sum of Arithmetic Series: Chapter: Combinatorics

$$S_n = \frac{n}{2}\big(2a+(n-1)d\big).$$

Step 1 — Substitute $n = 10$, $a = 2$, $d = 3$.

$$S_{10} = \frac{10}{2}\big(2(2)+(10-1)(3)\big) = 5\,(4+27) = 5\times 31 = \boxed{155}.$$

Verification: The sequence is $2, 5, 8, 11, 14, 17, 20, 23, 26, 29$. Sum $= 2+5+\cdots+29$. Using $S = \frac{n}{2}(\text{first}+\text{last}) = \frac{10}{2}(2+29) = 5\times 31 = 155$. ✓

Why other options are wrong:

- (B) 150: uses $n = 10$ with $a = 2$, $d = 3$ but computes $5(4+26)$ (uses $(n-2)d$

instead of $(n-1)d$).

- (C) 145: arithmetic slip — computes $5(4+25)$ with d one step short.
- (D) 160: uses $5(4+28) = 5 \times 32$, taking nd instead of $(n-1)d$.

Answer: (A) Go Back to Q 15

Answer Key — GAT B Mathematics Sample Paper 5

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	D	3	A	4	B	5	C
6	D	7	A	8	B	9	C	10	A
11	D	12	B	13	C	14	D	15	A