GAT B Mathematics

Sample Paper – 6

Duration: 23 Minutes **Maximum Marks: 15**

Instructions

- This paper contains **15 Multiple Choice Questions (MCQs)**, each with a single correct answer.
- Marking scheme: **+1** for each correct answer, -0.5 for each incorrect answer, **0** for unattempted questions.
- Syllabus: **Undergraduate Mathematics** — Calculus, Linear Algebra, Probability & Statistics, Ordinary Differential Equations, Coordinate Geometry, and Discrete Mathematics.
- **No electronic devices**, mobile phones, or programmable calculators are permitted during the examination.

Part A — Questions (Q1 – Q15)

Q1. The improper integral $\int_1^\infty \frac{1}{x^p}\,dx$ converges if and only if

(Concept check)

(A) $p < 1$

(B) $p > 1$

(C) $p = 1$

(D) $p \leq 1$

Q2. The slope of the tangent to the curve $y = x^3 - 3x$ at the point where $x = 2$ is

(Application)

(A) 9

(B) 6

(C) 12

(D) 3

Q3. Evaluate $\displaystyle\int x^2 e^x \, dx$.

(Application)

(A) $e^x(x^2 + 2x + 2) + C$

(B) $e^x(x^2 - 2x - 2) + C$

(C) $e^x(x^2 + 2x - 2) + C$

(D) $e^x(x^2 - 2x + 2) + C$

Q4. If A is an orthogonal matrix (i.e. $A^\top A = I$), which of the following must be true?

(Concept check)

(A) $\det(A) = 0$

(B) $\det(A) = 2$

(C) $\det(A) = \pm 1$

(D) $\det(A) = \frac{1}{2}$

Q5. A 2×2 matrix A has characteristic polynomial $p(\lambda) = \lambda^2 - 5\lambda + 6$. By the Cayley–Hamilton theorem, which equation does A necessarily satisfy?

(Application)

(A) $A^2 - 5A - 6I = O$

(B) $A^2 - 5A + 6I = O$

(C) $A^2 + 5A + 6I = O$

(D) $A^2 + 5A - 6I = O$

Q6. A random variable X follows a geometric distribution with success probability $p = \frac{1}{4}$ (counting the number of trials until the first success). The expected value $E[X]$ is

(Concept check)

(A) 4

(B) $\frac{1}{4}$

(C) $\frac{3}{4}$

(D) 16

Q7. For a bivariate data set of $n = 4$ pairs, the following summary statistics are given:

$$\sum x = 10, \quad \sum y = 10, \quad \sum xy = 29, \quad \sum x^2 = 30, \quad \sum y^2 = 30.$$

The Pearson correlation coefficient r is

(Application)

(A) 0.6

(B) 0.75

(C) 0.8

(D) 0.9

Q8. The general solution of the Cauchy–Euler equation $x^2y'' + xy' - y = 0 \ \ (x > 0)$ is

(Application)

(A) $y = c_1 x^2 + c_2 x^{-2}$

(B) $y = c_1 e^{x} + c_2 e^{-x}$

(C) $y = c_1 + c_2 x^{-2}$

(D) $y = c_1 x + c_2 x^{-1}$

Q9. Compute the Wronskian $W(y_1, y_2)$ for $y_1 = e^x$ and $y_2 = xe^x$, and state whether these functions are linearly independent.

(Application)

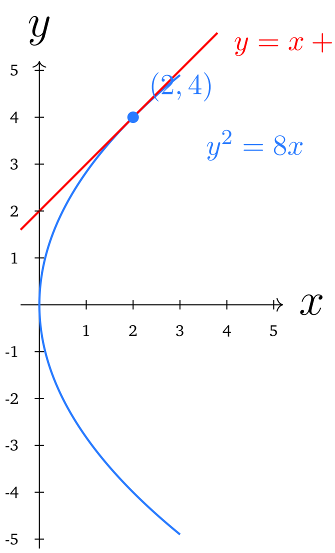
(A) $W = 0$; the functions are linearly dependent.

(B) $W = e^{2x} \neq 0$; the functions are linearly independent.

(C) $W = 2e^{2x}$; the functions are linearly independent.

(D) $W = e^{x}$; the functions are linearly independent.

Q10. Find the equation of the tangent to the parabola $y^2 = 8x$ at the point $(2, 4)$.



(Application)

(A) $x - y - 2 = 0$

(B) $x + y - 2 = 0$

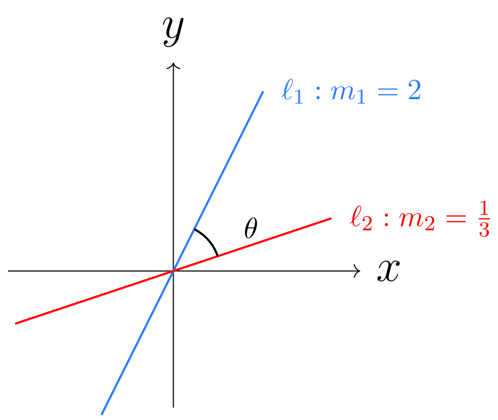
(C) $x - y + 2 = 0$

(D) $x + y + 2 = 0$

Q11. Two straight lines ℓ_1 and ℓ_2 have slopes $m_1 = 2$ and $m_2 = \frac{1}{3}$ respectively. Using the formula

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|,$$

find the acute angle θ between them.



(Application)

(A) $45°$

(B) $30°$

(C) $60°$

(D) $90°$

Q12. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^2$. Which of the following statements is correct?

(Concept check)

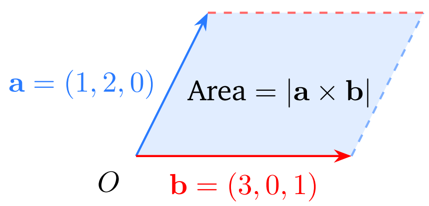
(A) f is injective (one-to-one) but not surjective (onto).

(B) f is surjective but not injective.

(C) f is both injective and surjective (bijective).

(D) f is neither injective nor surjective.

Q13. Vectors $\mathbf{a} = (1, 2, 0)$ and $\mathbf{b} = (3, 0, 1)$ form two adjacent sides of a parallelogram. Find the area of the parallelogram.



(Application)

(A) $\sqrt{45}$

(B) $\sqrt{41}$

(C) $\sqrt{40}$

(D) $\sqrt{38}$

Q14. In how many ways can 10 identical balls be distributed into 3 distinct boxes, with no restriction on the number of balls per box?

(Application)

(A) $\binom{10}{2} = 45$

(B) $\binom{11}{2} = 55$

(C) $\binom{12}{2} = 66$

(D) $\binom{13}{2} = 78$

Q15. The principal argument of the complex number $z = 1 + i\sqrt{3}$, taken in the interval $[0, 2\pi)$, is

(Application)

(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{6}$

(C) $\frac{2\pi}{3}$

(D) $\frac{\pi}{4}$

Part B — Detailed Solutions (Q1 – Q15)

Q1.

Solution

Concept — The p-integral test: Chapter: Integral Calculus For $p \neq 1$ and $p > 0$, direct integration gives

$$\int_1^b x^{-p}\,dx = \left[\frac{x^{1-p}}{1-p}\right]_1^b = \frac{b^{1-p}-1}{1-p}.$$

Step 1 — Analyse as $b \to \infty$.

- $p > 1$: then $1-p < 0$, so $b^{1-p} \to 0$. The limit is $\dfrac{1}{p-1}$ (finite). The integral **converges**.
- $p < 1$: then $1-p > 0$, so $b^{1-p} \to \infty$. The integral **diverges**.
- $p = 1$: $\displaystyle\int_1^b \frac{dx}{x} = \ln b \to \infty$. The integral **diverges**.

Step 2 — Conclusion. Convergence occurs *if and only if* $p > 1$.

Why other options are wrong:

- (A) $p < 1$: $b^{1-p} \to \infty$; the integral diverges.
- (C) $p = 1$: $\int_1^\infty x^{-1}dx = \ln b \to \infty$; diverges.
- (D) $p \leq 1$: union of the two divergent cases; incorrect.

Answer: (B) Go Back to Q 1

Q2.

Solution

Concept — Slope of tangent as a derivative: Chapter: Differential Calculus The slope of the curve $y = f(x)$ at a given point equals $f'(x)$ evaluated there.

Step 1 — Differentiate.

$$y = x^3 - 3x \implies \frac{dy}{dx} = 3x^2 - 3.$$

Step 2 — Evaluate at $x = 2$.

$$\left.\frac{dy}{dx}\right|_{x=2} = 3(4) - 3 = 12 - 3 = 9.$$

The tangent line is $y - 2 = 9(x - 2)$, i.e. $y = 9x - 16$, meeting the curve at $(2, 2)$.

Why other options are wrong:

- (B) 6: uses $3x - 3$ instead of $3x^2 - 3$; missing the square.
- (C) 12: evaluates $3x^2$ at $x = 2$ but forgets to subtract 3.
- (D) 3: the constant term in the derivative, not the full value.

$\boxed{\textbf{Answer: (A)}}$ Go Back to Q 2

Q3.

Solution

Concept — Integration by parts (IBP) applied twice: Chapter: Integral Calculus

$$\int u\,dv = uv - \int v\,du.$$

Step 1 — First IBP. Set $u = x^2$, $dv = e^x dx$, so $du = 2x\,dx$, $v = e^x$:

$$\int x^2 e^x\,dx = x^2 e^x - \int 2x\,e^x\,dx.$$

Step 2 — Second IBP. Set $u = 2x$, $dv = e^x dx$, so $du = 2\,dx$, $v = e^x$:

$$\int 2xe^x\,dx = 2xe^x - \int 2e^x\,dx = 2xe^x - 2e^x.$$

Step 3 — Combine.

$$\int x^2 e^x\,dx = x^2e^x - \left(2xe^x - 2e^x\right) + C = e^x(x^2 - 2x + 2) + C.$$

Verification by differentiation: $\dfrac{d}{dx}\left[e^x(x^2-2x+2)\right] = e^x(x^2-2x+2)+e^x(2x-2) = e^x x^2$. ✓

Why other options are wrong:

- (A) $e^x(x^2 + 2x + 2)$: sign error; the $2x$ term should be negative.

- (B) $e^x(x^2 - 2x - 2)$: sign error on the constant $+2$ term.
- (C) $e^x(x^2 + 2x - 2)$: both middle and constant terms have wrong signs.

Answer: (D) Go Back to Q 3

Q4.

Solution

Concept — Determinant of an orthogonal matrix: Chapter: Linear Algebra A is orthogonal $\iff A^\top A = I$.

Step 1 — Take determinants of both sides.

$$\det(A^\top A) = \det(I) = 1.$$

Step 2 — Use multiplicativity: $\det(A^\top) = \det(A)$**.**

$$\det(A^\top) \cdot \det(A) = [\det(A)]^2 = 1 \implies \det(A) = \pm 1.$$

Both values $+1$ (proper rotation) and -1 (improper rotation / reflection) are realised by explicit orthogonal matrices.

Why other options are wrong:

- (A) 0: a matrix with $\det = 0$ is singular and has no inverse, so $A^\top A = I$ would be impossible.
- (B) 2: $2^2 = 4 \neq 1$; impossible.
- (D) $\frac{1}{2}$: $(\frac{1}{2})^2 = \frac{1}{4} \neq 1$; impossible.

Answer: (C) Go Back to Q 4

Q5.

Solution

Concept — Cayley–Hamilton theorem: Chapter: Linear Algebra *Every square matrix satisfies its own characteristic equation.* If $p(\lambda)$ is the characteristic polynomial of A, then $p(A) = O$.

Step 1 — State the given polynomial. $p(\lambda) = \lambda^2 - 5\lambda + 6$.

Step 2 — Apply the theorem: replace λ by A.

$$p(A) = A^2 - 5A + 6I = O.$$

Step 3 — Concrete verification. Take $A = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}$ (eigenvalues $\lambda = 3, 2$; characteristic polynomial $(3-\lambda)(2-\lambda) = \lambda^2 - 5\lambda + 6$):

$$A^2 = \begin{pmatrix} 9 & 5 \\ 0 & 4 \end{pmatrix}, \; 5A = \begin{pmatrix} 15 & 5 \\ 0 & 10 \end{pmatrix}, \; 6I = \begin{pmatrix} 6 & 0 \\ 0 & 6 \end{pmatrix}.$$

$$A^2 - 5A + 6I = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = O. \checkmark$$

Why other options are wrong:

- (A) $A^2 - 5A - 6I = O$: wrong sign on $6I$; corresponds to $p(\lambda) = \lambda^2 - 5\lambda - 6$.
- (C) $A^2 + 5A + 6I = O$: both signs flipped; characteristic polynomial would be $\lambda^2 + 5\lambda + 6$.
- (D) $A^2 + 5A - 6I = O$: wrong sign on the linear term; not the given polynomial.

Answer: (B) Go Back to Q 5

Q6.

Solution

Concept — Expected value of the geometric distribution: Chapter: Probability & Statistics If $X \sim \mathrm{Geom}(p)$ counts the number of trials until the *first success*, then

$$E[X] = \frac{1}{p}.$$

Step 1 — Apply the formula with $p = \frac{1}{4}$.

$$E[X] = \frac{1}{1/4} = 4.$$

Intuition: Performing Bernoulli trials each with probability $\frac{1}{4}$ of success, you expect to wait *four* trials on average before the first success.

Why other options are wrong:

- (B) $\frac{1}{4}$: this is the success probability p itself, not $E[X]$.
- (C) $\frac{3}{4}$: this equals $1-p$, the probability of failure on one trial.
- (D) 16: this equals $(1/p)^2 = \left(E[X]\right)^2$; a confusion with $E[X]^2$ rather than $E[X]$.

Answer: (A) Go Back to Q 6

Q7.

Solution

Concept — Pearson correlation coefficient from summary statistics: Chapter: Probability & Statistics

$$r = \frac{n\sum xy - \sum x \sum y}{\sqrt{\left(n\sum x^2 - (\sum x)^2\right)\left(n\sum y^2 - (\sum y)^2\right)}}.$$

Step 1 — Compute the numerator.

$$n\sum xy - \sum x \sum y = 4(29) - 10 \cdot 10 = 116 - 100 = 16.$$

Step 2 — Compute each bracket in the denominator.

$$n\sum x^2 - (\sum x)^2 = 4(30) - 100 = 20.$$

$$n\sum y^2 - (\sum y)^2 = 4(30) - 100 = 20.$$

Step 3 — Evaluate r.

$$r = \frac{16}{\sqrt{20 \times 20}} = \frac{16}{20} = 0.8.$$

Why other options are wrong:

- (A) 0.6: requires numerator 12; $12/20 = 0.6$; wrong numerator.
- (B) 0.75: requires numerator 15; $15/20 = 0.75$; wrong numerator.
- (D) 0.9: requires numerator 18; $18/20 = 0.9$; wrong numerator.

Answer: (C) Go Back to Q 7

Q8.

Solution

Concept — Cauchy–Euler (equidimensional) ODE: Chapter: Differential Equations For $x^2y'' + \alpha xy' + \beta y = 0$, try $y = x^m$ to obtain the *indicial equation*.

Step 1 — Compute derivatives. $y = x^m,\ \ y' = mx^{m-1},\ \ y'' = m(m-1)x^{m-2}$.

Step 2 — Substitute into the ODE.

$$x^2 \cdot m(m-1)x^{m-2} + x \cdot mx^{m-1} - x^m = 0$$

$$x^m\big[m(m-1) + m - 1\big] = 0 \implies m^2 - 1 = 0.$$

Step 3 — Solve the indicial equation. $m = \pm 1$, giving two distinct real roots.

Step 4 — Write the general solution.

$$y = c_1x^1 + c_2x^{-1} = c_1x + \frac{c_2}{x}.$$

Why other options are wrong:

- (A) $c_1x^2 + c_2x^{-2}$: would follow from $m^2 - 4 = 0$; wrong ODE coefficients.
- (B) $c_1e^x + c_2e^{-x}$: applies to the constant-coefficient ODE $y'' - y = 0$, not Cauchy–Euler.
- (C) $c_1 + c_2x^{-2}$: incorrect roots $m = 0$ and $m = -2$.

Answer: (D) Go Back to Q 8

Q9.

Solution

Concept — Wronskian test for linear independence: Chapter: Differential Equations

$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix} = y_1y_2' - y_2y_1'.$$

If $W \neq 0$ on an interval, the functions are linearly independent on that interval.

Step 1 — Compute derivatives. $y_1 = e^x,\ \ y_1' = e^x;\quad y_2 = xe^x,\ \ y_2' = e^x + xe^x$.

Step 2 — Evaluate the Wronskian.

$$W = e^x(e^x + xe^x) - xe^x \cdot e^x = e^{2x} + xe^{2x} - xe^{2x} = e^{2x}.$$

Step 3 — Conclusion. $W = e^{2x} > 0$ for all $x \in \mathbb{R}$, so y_1 and y_2 are **linearly independent.**

Why other options are wrong:

- (A) $W = 0$: the xe^{2x} terms cancel; $W = e^{2x} \neq 0$.
- (C) $W = 2e^{2x}$: arithmetic error; the cross terms cancel exactly, leaving only e^{2x}.
- (D) $W = e^{x}$: incorrect exponent; the product of two e^{x} factors gives e^{2x}.

Answer: (B) Go Back to Q 9

Q10.

Solution

Concept — Tangent to a parabola via implicit differentiation: Chapter: Coordinate Geometry

Step 1 — Differentiate $y^2 = 8x$ implicitly with respect to x.

$$2y\frac{dy}{dx} = 8 \implies \frac{dy}{dx} = \frac{4}{y}.$$

Step 2 — Slope at the point $(2, 4)$.

$$m = \left.\frac{dy}{dx}\right|_{(2,4)} = \frac{4}{4} = 1.$$

Step 3 — Equation of the tangent. Using point-slope form $y - y_0 = m(x - x_0)$:

$$y - 4 = 1 \cdot (x - 2) \implies y = x + 2 \implies \boxed{x - y + 2 = 0}.$$

The TikZ figure in the question shows the parabola $y^2 = 8x$ (blue) and the tangent $y = x + 2$ (red) intersecting at the point $(2, 4)$.

Why other options are wrong:

- (A) $x - y - 2 = 0 \Rightarrow y = x - 2$: passes through $(2, 0)$, not $(2, 4)$.
- (B) $x + y - 2 = 0 \Rightarrow y = 2 - x$: slope -1; incorrect sign of derivative.
- (D) $x + y + 2 = 0 \Rightarrow y = -x - 2$: slope -1 and wrong intercept.

Answer: (C) Go Back to Q 10

Q11.

Solution

Concept — Acute angle between two lines: Chapter: Coordinate Geometry

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|, \quad \text{where } m_1 m_2 \neq -1.$$

Step 1 — Substitute $m_1 = 2$ and $m_2 = \frac{1}{3}$.

$$m_1 - m_2 = 2 - \frac{1}{3} = \frac{5}{3}, \qquad 1 + m_1 m_2 = 1 + \frac{2}{3} = \frac{5}{3}.$$

Step 2 — Compute $\tan\theta$.

$$\tan\theta = \left|\frac{5/3}{5/3}\right| = 1 \implies \theta = \arctan(1) = 45^\circ.$$

The TikZ figure in the question shows lines ℓ_1 (slope 2, making $\approx 63.4^\circ$ with the x-axis) and ℓ_2 (slope $\frac{1}{3}$, making $\approx 18.4^\circ$) with the 45° angle arc drawn between them.

Why other options are wrong:

- (B) 30°: $\tan 30^\circ = 1/\sqrt{3} \approx 0.577 \neq 1$.
- (C) 60°: $\tan 60^\circ = \sqrt{3} \approx 1.732 \neq 1$.
- (D) 90°: requires $1 + m_1 m_2 = 0$, i.e. $m_1 m_2 = -1$; but $2 \cdot \frac{1}{3} = \frac{2}{3} \neq -1$.

Answer: (A) Go Back to Q 11

Q12.

Solution

Concept — Injectivity and surjectivity of $f : \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$: Chapter: Functions

Step 1 — Test injectivity. $f(-2) = (-2)^2 = 4$ and $f(2) = 2^2 = 4$, yet $-2 \neq 2$. Since two distinct inputs produce the same output, f is **not injective**.

Step 2 — Test surjectivity. The range of f is $[0, \infty)$, since $x^2 \geq 0$ for all $x \in \mathbb{R}$. In particular, $y = -1$ has no preimage. Since range$\subsetneq \mathbb{R}$, f is **not surjective**.

Conclusion: f is **neither injective nor surjective**.

Why other options are wrong:

- (A): requires injectivity, which fails by Step 1.
- (B): requires surjectivity, which fails by Step 2.
- (C): requires both properties, both of which fail.

Answer: (D) Go Back to Q 12

Q13.

Solution

Concept — Area of parallelogram via cross product: Chapter: Vectors Area $= |\mathbf{a} \times \mathbf{b}|$.

Step 1 — Compute a × b.

$$\mathbf{a} \times \mathbf{b} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 2 & 0 \\ 3 & 0 & 1 \end{vmatrix} = \mathbf{i}(2 \cdot 1 - 0 \cdot 0) - \mathbf{j}(1 \cdot 1 - 0 \cdot 3) + \mathbf{k}(1 \cdot 0 - 2 \cdot 3).$$

$$= \mathbf{i}(2) - \mathbf{j}(1) + \mathbf{k}(-6) = (2,\ -1,\ -6).$$

Step 2 — Compute the magnitude.

$$|\mathbf{a} \times \mathbf{b}| = \sqrt{2^2 + (-1)^2 + (-6)^2} = \sqrt{4 + 1 + 36} = \sqrt{41}.$$

The TikZ figure in the question shows the parallelogram with the two vector sides labelled.

Why other options are wrong:

- (A) $\sqrt{45}$: uses $|\mathbf{j}| = 1$ but squares it as $+1 \to +9$; arithmetic slip.
- (C) $\sqrt{40}$: omits the $(-1)^2 = 1$ contribution from the $\mathbf{j}$ component.
- (D) $\sqrt{38}$: drops 3 from the sum of squares; arithmetic error.

Answer: (B) Go Back to Q 13

Q14.

Solution

Concept — Stars and bars (identical objects, distinct boxes): Chapter: Combinatorics The number of ways to distribute n identical objects into k distinct boxes (each box may be empty) is

$$\binom{n+k-1}{k-1}.$$

Step 1 — Identify n and k. $n = 10$ balls, $k = 3$ boxes.

Step 2 — Apply the formula.

$$\binom{10+3-1}{3-1} = \binom{12}{2} = \frac{12 \times 11}{2} = 66.$$

Combinatorial picture: Arrange 10 stars and 2 dividers in a row (12 positions total); choose 2 positions for the dividers: $\binom{12}{2} = 66$.

Why other options are wrong:

- (A) $\binom{10}{2} = 45$: uses $n = 10$ in place of $n + k - 1 = 12$; omits the $k - 1$ offset.
- (B) $\binom{11}{2} = 55$: adds only $k - 2 = 1$ to n instead of $k - 1 = 2$.
- (D) $\binom{13}{2} = 78$: adds $k = 3$ instead of $k - 1 = 2$; off by one.

Answer: (C) Go Back to Q 14

Q15.

Solution

Concept — Principal argument of a complex number: Chapter: Complex Numbers For $z = x + iy$ with $x > 0$, the principal argument in $[0, 2\pi)$ is

$$\arg(z) = \arctan\left(\frac{y}{x}\right).$$

Step 1 — Identify components. $z = 1 + i\sqrt{3}$: real part $x = 1 > 0$, imaginary part $y = \sqrt{3} > 0$. $\Rightarrow$ *first quadrant*.

Step 2 — Compute the argument.

$$\arg(z) = \arctan\left(\frac{\sqrt{3}}{1}\right) = \arctan(\sqrt{3}) = \frac{\pi}{3}.$$

Step 3 — Verify via polar form. $|z| = \sqrt{1+3} = 2$, so $z = 2\left(\cos\dfrac{\pi}{3} + i\sin\dfrac{\pi}{3}\right)$. $\cos\dfrac{\pi}{3} = \frac{1}{2}$, $\sin\dfrac{\pi}{3} = \frac{\sqrt{3}}{2}$; $2 \cdot \frac{1}{2} = 1$ and $2 \cdot \frac{\sqrt{3}}{2} = \sqrt{3}$. ✓

Why other options are wrong:

- (B) $\pi/6$: $\tan(\pi/6) = 1/\sqrt{3}$; this is the argument of $z = \sqrt{3}+i$, not $1+i\sqrt{3}$.
- (C) $2\pi/3$: second-quadrant angle; but $\mathrm{Re}(z) = 1 > 0$ places z in the first quadrant.
- (D) $\pi/4$: $\tan(\pi/4) = 1$; requires equal real and imaginary parts, i.e. $z = 1+i$.

Answer: (A) Go Back to Q 15

Answer Key — GAT B Mathematics Sample Paper 6

Q – Ans		Q – Ans		Q – Ans		Q – Ans		Q – Ans	
1	B	2	A	3	D	4	C	5	B
6	A	7	C	8	D	9	B	10	C
11	A	12	D	13	B	14	C	15	A