GAT B Mathematics

Sample Paper – 7

Duration: 23 Minutes **Maximum Marks: 15**

Instructions

1. This paper contains **15 Multiple Choice Questions (MCQs)**, each with a single correct answer.
2. Marking scheme: **+1** for every correct answer, **-0.5** for every incorrect answer, and **0** for unattempted questions.
3. The paper covers undergraduate-level **Mathematics**.
4. No electronic devices (calculators, mobile phones, etc.) are permitted during the examination.
5. All questions carry equal marks.

Section A — Mathematics (Q1–Q15)

Q1. Using L'Hôpital's rule, evaluate

$$\lim_{x\to 0} \frac{x - \sin x}{x^3}.$$

(Application)

(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) 0

(D) $\frac{1}{6}$

Q2. The function $f(x) = x^3 - 6x^2 + 9x + 1$ has

(Application)

(A) a local minimum at $x = 1$ with value $f(1) = 5$

(B) a local maximum at $x = 3$ with value $f(3) = 1$

(C) a local minimum at $x = 3$ with value $f(3) = 1$

(D) no local extrema, since f is a cubic polynomial

Q3. If $F(x) = \int_0^x \sin(t^2)\,dt$, then $F'(x)$ equals

(Concept check)

(A) $\sin(x)$

(B) $\sin(x^2)$

(C) $\cos(x^2)$

(D) $2x\cos(x^2)$

Q4. The eigenvalues of the diagonal matrix

$$A = \begin{pmatrix} 3 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 5 \end{pmatrix}$$

are

(Concept check)

(A) $3,\ -1,\ 5$

(B) $9,\ 1,\ 25$

(C) $-3,\ 1,\ -5$

(D) $1,\ 1,\ 1$

Q5. Let A be a 5×7 real matrix with $\mathrm{rank}(A) = 4$. By the Rank–Nullity theorem, $\mathrm{nullity}(A)$ equals

(Application)

(A) 1

(B) 2

(C) 4

(D) 3

Q6. If $X \sim \mathrm{Exp}(\lambda)$ with $s, t > 0$, which equation correctly states the *memoryless property* of the exponential distribution?

(Concept check)

(A) $P(X > s+t \mid X > s) = P(X > s)$

(B) $P(X > s+t \mid X > s) = P(X > s+t)$

(C) $P(X > s+t \mid X > s) = P(X > t)$

(D) $P(X > s+t \mid X > s) = e^{-\lambda s} P(X > t)$

Q7. Let X be any random variable with mean μ and variance $\sigma^2 < \infty$. Chebyshev's inequality states $P(|X-\mu| \geq k\sigma) \leq \dfrac{1}{k^2}$. For $k = 2$, the upper bound on $P(|X - \mu| \geq 2\sigma)$ is

(Concept check)

(A) $\dfrac{1}{2}$

(B) $\dfrac{1}{4}$

(C) $\dfrac{1}{8}$

(D) $\dfrac{1}{16}$

Q8. The most appropriate analytical method to find a particular solution of

$$y'' - y = \frac{e^x}{x}$$

is

(Concept check)

(A) Variation of parameters

(B) Method of undetermined coefficients

(C) Separation of variables

(D) Exact equation method

Q9. The radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{n^2}{3^n} x^n$ is

(Application)

(A) $\frac{1}{3}$

(B) 9

(C) 1

(D) 3

Q10. For the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$, the eccentricity is

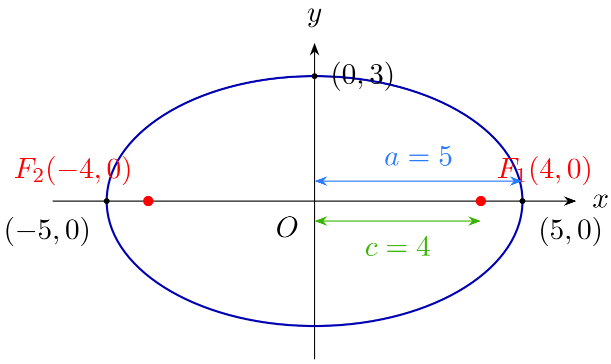
(Application)

(A) $\frac{3}{5}$

(B) $\frac{4}{5}$

(C) $\frac{5}{3}$

(D) $\dfrac{2}{5}$



Q11. From the external point $P(5,6)$, the equation of the chord of contact drawn to the circle $x^2+y^2=25$ is

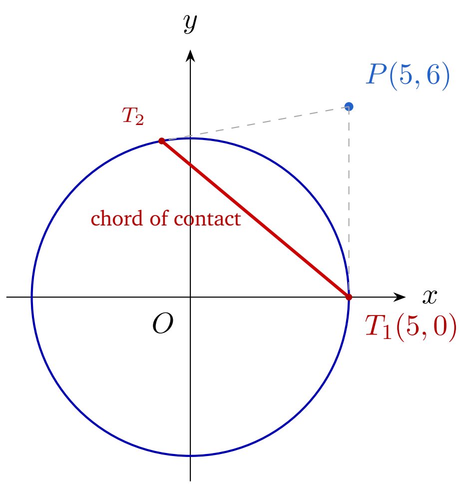
(Application)

(A) $5x+6y=61$

(B) $6x+5y=25$

(C) $5x+6y=25$

(D) $5x-6y=25$



Q12. The value of $\lfloor -3.7 \rfloor$ is

(Trap drill)

(A) -3

(B) 3

(C) -3.7

(D) -4

Q13. Let $\mathbf{a} = (1, 2, 2)$ and $\mathbf{b} = (3, 4, 0)$. The vector projection of b onto a is

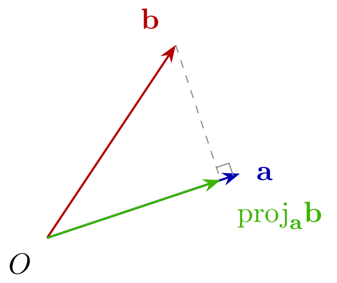
(Application)

(A) $\frac{11}{9}(1, 2, 2)$

(B) $\frac{11}{9}(3, 4, 0)$

(C) $\frac{11}{3}(1, 2, 2)$

(D) $\frac{11}{25}(1, 2, 2)$



Q14. Pascal's identity states $\binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r}$. With $n = 5$ and $r = 3$, the sum $\binom{5}{3} + \binom{5}{2}$ equals

(Application)

(A) $\binom{6}{4}$

(B) $\binom{6}{3}$

(C) $\binom{10}{1}$

(D) $\binom{5}{5} + \binom{5}{5}$

Q15. Using De Moivre's theorem, $(1+i)^8$ equals

(Application)

(A) $16i$

(B) -16

(C) 16

(D) $-16i$

Detailed Solutions — GAT B Mathematics Sample Paper 7

Q1.

Solution

Concept — L'Hôpital's Rule: Chapter: Limits/Continuity When a limit has the indeterminate form $\frac{0}{0}$ or $\frac{\infty}{\infty}$, differentiate the numerator and denominator separately and retry the limit. Repeat as needed.

Step 1 — Identify the indeterminate form:

$$\lim_{x\to 0}\frac{x-\sin x}{x^3}=\frac{0-0}{0}=\frac{0}{0}. \quad \checkmark$$

Step 2 — First application of L'Hôpital:

$$\lim_{x\to 0}\frac{(x-\sin x)'}{(x^3)'}=\lim_{x\to 0}\frac{1-\cos x}{3x^2}=\frac{0}{0} \quad \text{(still indeterminate)}.$$

Step 3 — Second application:

$$\lim_{x\to 0}\frac{(1-\cos x)'}{(3x^2)'}=\lim_{x\to 0}\frac{\sin x}{6x}=\frac{0}{0} \quad \text{(still indeterminate)}.$$

Step 4 — Third application:

$$\lim_{x\to 0}\frac{(\sin x)'}{(6x)'}=\lim_{x\to 0}\frac{\cos x}{6}=\frac{\cos 0}{6}=\frac{1}{6}.$$

(Alternatively, recall the Maclaurin series $\sin x = x-\frac{x^3}{6}+\cdots$, so $x-\sin x\sim\frac{x^3}{6}$, confirming the limit is $\frac{1}{6}$.)

Why other options are wrong:

- (A) $\frac{1}{3}$: Treating the whole limit as $\frac{1}{3x^2}$ and substituting $x=0$ prematurely gives an error; the form is still $0/0$ after one step.
- (B) $\frac{1}{2}$: Arises from a sign error, e.g. writing $(-\sin x)'=\cos x$ as $-\cos x$.
- (C) 0: The limit is non-zero; $x-\sin x$ vanishes to the same order as x^3.
- (D) $\frac{1}{6}$: **Correct.**

Answer: (D) Go Back to Q 1

Q2.

Solution

Concept — Second Derivative Test: Chapter: Differential Calculus For a twice-differentiable function with $f'(c) = 0$:

- $f''(c) > 0 \Rightarrow$ local minimum at $x = c$,
- $f''(c) < 0 \Rightarrow$ local maximum at $x = c$.

Step 1 — Find critical points of $f(x) = x^3 - 6x^2 + 9x + 1$:

$$f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x-1)(x-3) = 0 \Rightarrow x = 1 \text{ or } x = 3.$$

Step 2 — Compute the second derivative:

$$f''(x) = 6x - 12.$$

Step 3 — Classify each critical point:

$f''(1) = 6-12 = -6 < 0 \Rightarrow x = 1$ is a **local maximum**, $f(1) = 1-6+9+1 = 5.$

$f''(3) = 18-12 = 6 > 0 \Rightarrow x = 3$ is a **local minimum**, $f(3) = 27-54+27+1 = 1.$

Why other options are wrong:

- (A): $x = 1$ is a local *maximum* (value 5), not a minimum.
- (B): $x = 3$ is a local *minimum* (value 1), not a maximum.
- (C): **Correct.** Local minimum at $x = 3$ with $f(3) = 1$.
- (D): The discriminant of $f'(x) = 3x^2 - 12x + 9$ is $144 - 108 = 36 > 0$, giving two distinct real roots and hence two extrema.

Answer: (C) Go Back to Q 2

Q3.

Solution

Concept — Fundamental Theorem of Calculus, Part I (FTC I): Chapter: Integral Calculus

$$\text{If } F(x) = \int_a^x f(t)\,dt, \quad \text{then} \quad F'(x) = f(x).$$

Applying FTC I directly:

$$F(x) = \int_0^x \sin(t^2)\,dt \implies F'(x) = \sin\big(x^2\big).$$

The integrand $\sin(t^2)$ is simply evaluated at $t = x$; no chain rule is needed because the upper limit is x (not a function of x).

Why other options are wrong:

- (A) $\sin(x)$: This would be the answer for $\int_0^x \sin t\,dt$; the integrand here is $\sin(t^2)$.
- (B) $\sin(x^2)$: **Correct.** Direct application of FTC I.
- (C) $\cos(x^2)$: This is the derivative of $\sin(x^2)$ with respect to x^2, unrelated to FTC I here.
- (D) $2x\cos(x^2)$: Confuses $F'(x) = \sin(x^2)$ with its own derivative $\frac{d}{dx}[\sin(x^2)] = 2x\cos(x^2)$.

Answer: (B) Go Back to Q 3

Q4.

Solution

Concept — Eigenvalues of a Diagonal Matrix: Chapter: Linear Algebra The eigenvalues of a diagonal (or more generally, triangular) matrix are exactly its diagonal entries.

Verification via the characteristic polynomial:

$$\det(A - \lambda I) = (3 - \lambda)(-1 - \lambda)(5 - \lambda) = 0 \Rightarrow \lambda \in \{3, -1, 5\}.$$

The corresponding eigenvectors are the standard basis vectors $\mathbf{e}_1 = (1, 0, 0)^T$, $\mathbf{e}_2 = (0, 1, 0)^T$, $\mathbf{e}_3 = (0, 0, 1)^T$.

Why other options are wrong:

- (A) $\{3, -1, 5\}$: **Correct.**
- (B) $\{9, 1, 25\}$: These are the *squares* of the diagonal entries, not eigenvalues.
- (C) $\{-3, 1, -5\}$: The negatives of the diagonal entries.
- (D) $\{1, 1, 1\}$: Eigenvalues of the identity matrix I.

Answer: (A) Go Back to Q 4

Q5.

Solution

Concept — Rank–Nullity Theorem: Chapter: Linear Algebra For any $m \times n$ matrix A over a field,

$$\text{rank}(A) + \text{nullity}(A) = n \quad \text{(total number of columns)}.$$

Applying the theorem to a 5×7 **matrix with** $\text{rank}(A) = 4$**:**

$$\text{nullity}(A) = n - \text{rank}(A) = 7 - 4 = 3.$$

Geometrically, the null space (kernel) of A is a 3-dimensional subspace of $\mathbb{R}^7$.

Why other options are wrong:

- (A) 1: $7 - 4 \neq 1$.
- (B) 2: $7 - 4 \neq 2$.
- (C) 4: This is the *rank* itself, not the nullity.
- (D) 3: **Correct.**

Answer: (D) Go Back to Q 5

Q6.

Solution

Concept — Memoryless Property of the Exponential Distribution: Chapter: Probability & Statistics A non-negative continuous random variable X is memoryless if and only if $X \sim \text{Exp}(\lambda)$, satisfying

$$P(X > s+t \mid X > s) = P(X > t) \quad \text{for all } s, t > 0.$$

Derivation using the CDF $F(x) = 1 - e^{-\lambda x}$:

$$P(X > s+t \mid X > s) = \frac{P(X > s+t)}{P(X > s)} = \frac{e^{-\lambda(s+t)}}{e^{-\lambda s}} = e^{-\lambda t} = P(X > t).$$

The distribution "forgets" all elapsed time; the residual lifetime has the same exponential distribution regardless of how long it has already run.

Why other options are wrong:

- (A) $P(X > s)$: The remaining time cannot depend only on the already elapsed s; this would violate the definition.
- (B) $P(X > s+t)$: This is the *unconditional* probability, not the conditional one.
- (C) $P(X > t)$: **Correct.**
- (D) $e^{-\lambda s}P(X > t)$: Has an erroneous extra factor of $e^{-\lambda s}$ that cancels in the correct derivation.

$\boxed{\textbf{Answer: (C)}}$ Go Back to Q 6

Q7.

Solution

Concept — Chebyshev's Inequality: Chapter: Probability & Statistics For any random variable X with mean μ and finite variance σ^2, and for any real $k > 0$:

$$P(|X - \mu| \geq k\sigma) \leq \frac{1}{k^2}.$$

This bound holds for *any* distribution with finite variance; no normality assumption is required.

Substituting $k = 2$:

$$P(|X - \mu| \geq 2\sigma) \leq \frac{1}{2^2} = \frac{1}{4}.$$

At most 25% of the probability mass lies more than two standard deviations from the mean.

Why other options are wrong:

- (A) $\frac{1}{2}$: Corresponds to $k = \sqrt{2}$, not $k = 2$.
- (B) $\frac{1}{4}$: **Correct.**
- (C) $\frac{1}{8}$: No standard Chebyshev bound gives this value for integer k.
- (D) $\frac{1}{16}$: Would require $k = 4$.

Answer: (B) Go Back to Q 7

Q8.

Solution

Concept — Variation of Parameters vs. Undetermined Coefficients: Chapter: Differential Equations The method of *undetermined coefficients* applies only when the forcing function $g(x)$ is of the form $x^n e^{\alpha x}$, $\sin(\beta x)$, $\cos(\beta x)$, or products thereof. The term $\dfrac{e^x}{x}$ contains the non-polynomial factor $\frac{1}{x}$ and is *not* in this class.

Variation of parameters for $y'' - y = \dfrac{e^x}{x}$: The complementary solution is $y_c = C_1 e^x + C_2 e^{-x}$. Set $y_p = u_1(x)e^x + u_2(x)e^{-x}$ and determine u_1', u_2' from the Wronskian system:

$$u_1' e^x + u_2' e^{-x} = 0, \qquad u_1' e^x - u_2' e^{-x} = \frac{e^x}{x}.$$

Solving: $u_1' = \frac{1}{2x}$ and $u_2' = -\frac{e^{2x}}{2x}$, giving well-defined (if non-elementary) integrals.

Why other options are wrong:

- (A) Variation of parameters: **Correct.**
- (B) Undetermined coefficients: Inapplicable; e^x/x does not belong to the standard family of forcing functions.
- (C) Separation of variables: A technique for *first-order* ODEs of the form $\frac{dy}{dx} = f(x)g(y)$.
- (D) Exact equation method: Applies to first-order ODEs $M(x, y)\,dx + N(x, y)\,dy = 0$.

Answer: (A) Go Back to Q 8

Q9.

Solution

Concept — Radius of Convergence via the Ratio Test: Chapter: Integral Calculus For a power series $\sum_{n=0}^{\infty} a_n x^n$,

$$R = \lim_{n\to\infty} \left| \frac{a_n}{a_{n+1}} \right| \quad \text{(when the limit exists)}.$$

For $a_n = \frac{n^2}{3^n}$**:**

$$\frac{a_n}{a_{n+1}} = \frac{n^2/3^n}{(n+1)^2/3^{n+1}} = \frac{n^2 \cdot 3^{n+1}}{(n+1)^2 \cdot 3^n} = 3 \cdot \left(\frac{n}{n+1}\right)^2.$$

$$R = \lim_{n\to\infty} 3 \cdot \left(\frac{n}{n+1}\right)^2 = 3 \cdot 1^2 = 3.$$

The series converges absolutely for $|x| < 3$ and diverges for $|x| > 3$.

Why other options are wrong:

- (A) $\frac{1}{3}$: The reciprocal; arises if $a_n = 3^n/n^2$.
- (B) 9: Equals 3^2; no standard formula gives this.
- (C) 1: Ignores the exponential factor 3^n entirely.
- (D) 3: **Correct.**

Answer: (D) Go Back to Q 9

Q10.

Solution

Concept — Eccentricity of an Ellipse: Chapter: Coordinate Geometry For $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $a > b > 0$,

$$c = \sqrt{a^2 - b^2}, \qquad e = \frac{c}{a} \in (0, 1).$$

For $\dfrac{x^2}{25} + \dfrac{y^2}{9} = 1$**:**

$$a^2 = 25 \Rightarrow a = 5, \qquad b^2 = 9 \Rightarrow b = 3.$$

$$c = \sqrt{25 - 9} = \sqrt{16} = 4.$$

$$e = \frac{c}{a} = \frac{4}{5} = 0.8.$$

The foci lie at $F_1(4, 0)$ and $F_2(-4, 0)$, as shown in the diagram accompanying the question.

Why other options are wrong:

- (A) $\frac{3}{5}$: This is b/a, not the eccentricity.
- (B) $\frac{4}{5}$: **Correct.**
- (C) $\frac{5}{3}$: This is $a/b > 1$; the eccentricity of an ellipse is always strictly less than 1.
- (D) $\frac{2}{5}$: No standard ellipse formula yields this value.

Answer: (B) Go Back to Q 10

Q11.

Solution

Concept — Chord of Contact: Chapter: Coordinate Geometry From an external point (h, k) to the circle $x^2 + y^2 = r^2$, the equation of the chord of contact (the line joining the two points of tangency) is

$$hx + ky = r^2.$$

For the point $P(5, 6)$ **and circle** $x^2 + y^2 = 25$ **(**$r^2 = 25$**):**

Verify that P is external: $5^2 + 6^2 = 25 + 36 = 61 > 25$. ✓

Applying the formula with $h = 5$, $k = 6$:

$$5x + 6y = 25.$$

Check with tangent point $T_1(5, 0)$: $5(5) + 6(0) = 25$. ✓

Why other options are wrong:

- (A) $5x + 6y = 61$: Uses $h^2 + k^2 = 61$ (squared distance from origin to P) in place of $r^2 = 25$.

- (B) $6x + 5y = 25$: Coefficients of x and y are swapped.
- (C) $5x + 6y = 25$: **Correct.**
- (D) $5x - 6y = 25$: Incorrect sign on the y-term ($-k$ instead of $+k$).

Answer: (C) Go Back to Q 11

Q12.

Solution

Concept — Floor (Greatest Integer) Function: Chapter: Functions $\lfloor x \rfloor$ is the *greatest integer less than or equal to* x.

Locating -3.7 on the real line:

$$\cdots < -5 < -4 \; < \underbrace{-3.7}_{\text{here}} < \; -3 < -2 < \cdots$$

The two surrounding integers are -4 and -3. Since we require the greatest integer *not exceeding* -3.7:

$$-4 \leq -3.7 < -3 \implies \lfloor -3.7 \rfloor = -4.$$

Key contrast with truncation: For a negative non-integer, the floor moves *away from zero* (i.e. more negative), while truncation moves *toward zero*: $\lfloor -3.7 \rfloor = -4$ but $\text{trunc}(-3.7) = -3$.

Why other options are wrong:

- (A) -3: This equals $\lceil -3.7 \rceil$ (the ceiling function).
- (B) 3: Positive; entirely wrong sign.
- (C) -3.7: The floor function always returns an integer.
- (D) -4: **Correct.**

Answer: (D) Go Back to Q 12

Q13.

Solution

Concept — Vector Projection: Chapter: Vectors The vector projection of $\mathbf{b}$ onto $\mathbf{a}$ is

$$\mathrm{proj}_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a}\cdot\mathbf{b}}{|\mathbf{a}|^2}\,\mathbf{a}.$$

The result is a vector in the *direction of* $\mathbf{a}$, scaled by the signed length of the shadow of $\mathbf{b}$ on $\mathbf{a}$.

For $\mathbf{a} = (1, 2, 2)$ and $\mathbf{b} = (3, 4, 0)$:

$$\mathbf{a}\cdot\mathbf{b} = 1\cdot 3 + 2\cdot 4 + 2\cdot 0 = 3 + 8 + 0 = 11.$$

$$|\mathbf{a}|^2 = 1^2 + 2^2 + 2^2 = 1 + 4 + 4 = 9.$$

$$\mathrm{proj}_{\mathbf{a}}\mathbf{b} = \frac{11}{9}(1, 2, 2).$$

The diagram with the question illustrates the projection as the "shadow" of $\mathbf{b}$ dropped perpendicularly onto the line spanned by $\mathbf{a}$.

Why other options are wrong:

- (A) $\frac{11}{9}(1, 2, 2)$: **Correct.**
- (B) $\frac{11}{9}(3, 4, 0)$: Uses $\mathbf{b}$ (not $\mathbf{a}$) as the projection direction.
- (C) $\frac{11}{3}(1, 2, 2)$: Divides by $|\mathbf{a}| = 3$ instead of $|\mathbf{a}|^2 = 9$.
- (D) $\frac{11}{25}(1, 2, 2)$: Uses $|\mathbf{b}|^2 = 9 + 16 + 0 = 25$ in the denominator instead of $|\mathbf{a}|^2 = 9$.

Answer: (A) Go Back to Q 13

Q14.

Solution

Concept — Pascal's Identity: Chapter: Combinatorics

$$\binom{n}{r} + \binom{n}{r-1} = \binom{n+1}{r}.$$

This identity underlies the construction of Pascal's triangle and the binomial theorem.

Applying the identity with $n = 5$, $r = 3$:

$$\binom{5}{3} + \binom{5}{2} = \binom{6}{3}.$$

Numerical verification:

$$\binom{5}{3} = \frac{5!}{3!\,2!} = 10, \qquad \binom{5}{2} = \frac{5!}{2!\,3!} = 10, \qquad \binom{6}{3} = \frac{6!}{3!\,3!} = 20.$$

$$10 + 10 = 20 = \binom{6}{3}. \quad \checkmark$$

Why other options are wrong:

- (A) $\binom{6}{4} = 15 \neq 20$: Off by 5.
- (B) $\binom{6}{3} = 20$: **Correct.**
- (C) $\binom{10}{1} = 10 \neq 20$.
- (D) $\binom{5}{5} + \binom{5}{5} = 1 + 1 = 2 \neq 20$.

Answer: (B) Go Back to Q 14

Q15.

Solution

Concept — De Moivre's Theorem: Chapter: Complex Numbers

$$(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta).$$

Step 1 — Write $1 + i$ in polar form:

$$|1 + i| = \sqrt{1^2 + 1^2} = \sqrt{2}, \qquad \arg(1 + i) = \frac{\pi}{4}\ (= 45^\circ).$$

$$1 + i = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right).$$

Step 2 — Raise to the 8th power using De Moivre:

$$(1 + i)^8 = \left(\sqrt{2}\right)^8\left(\cos\frac{8\pi}{4} + i\sin\frac{8\pi}{4}\right) = 16\big(\cos 2\pi + i\sin 2\pi\big).$$

Step 3 — Evaluate:

$$\cos 2\pi = 1, \quad \sin 2\pi = 0, \quad \therefore\ (1+i)^8 = 16(1+0i) = 16.$$

Why other options are wrong:

- (A) $16i$: An imaginary result would require $\sin(n\theta) \neq 0$; here $\sin 2\pi = 0$.
- (B) -16: Would require $\cos(n\theta) = -1$ (i.e. $n\theta = \pi$); but $8 \cdot \frac{\pi}{4} = 2\pi$, giving $\cos 2\pi = +1$.
- (C) 16: **Correct.**
- (D) $-16i$: Both the sign and the imaginary unit are wrong; $\sin 2\pi = 0$ eliminates any imaginary part.

Answer: (C) Go Back to Q 15

Answer Key — GAT B Mathematics Sample Paper 7

Q – Ans		Q – Ans		Q – Ans		Q – Ans		Q – Ans	
1	D	2	C	3	B	4	A	5	D
6	C	7	B	8	A	9	D	10	B
11	C	12	D	13	A	14	B	15	C