GAT B Mathematics

Sample Paper – 10

Duration: 23 Minutes **Maximum Marks: 15**

Instructions

- This paper contains **15** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics portion of **GAT B** entrance.
- Each correct answer carries **+1 mark**. There is a deduction of **0.5 mark** for each wrong answer. Unattempted questions carry zero marks.
- Only **one** option is correct. Choose carefully.
- Syllabus: Calculus, Linear Algebra, Probability & Statistics, Differential Equations, Coordinate Geometry, Sets & Functions, Vectors, Combinatorics (undergraduate level).
- Use of mobile phones, calculators, or electronic gadgets is strictly prohibited.

Q1. Evaluate $\lim_{x \to 0} \frac{e^x - 1}{x}$.

(Concept check)

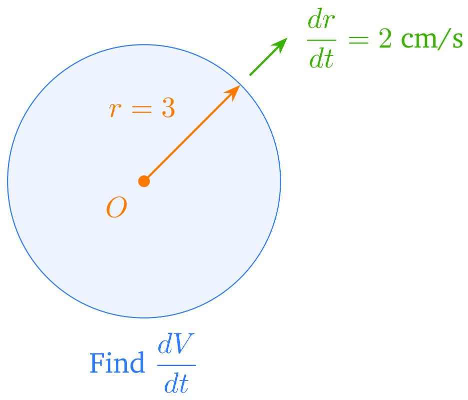
(A) 0

(B) 1

(C) e

(D) ∞

Q2. A spherical balloon is being inflated so that its radius increases at a rate of 2 cm/s. How fast is the volume increasing when $r = 3$ cm? (Recall: $V = \frac{4}{3}\pi r^3$.)



(Application)

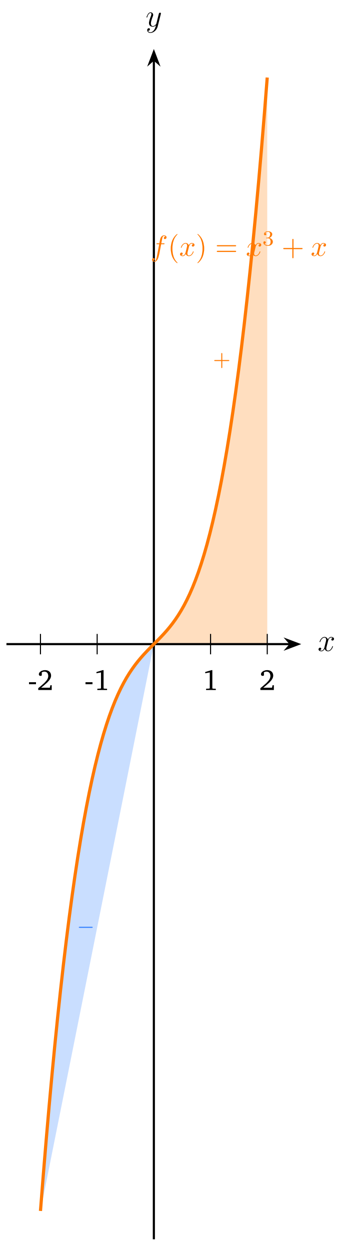
(A) 24π cm^3/s

(B) 36π cm^3/s

(C) 72π cm^3/s

(D) 108π cm^3/s

Q3. Evaluate $\int_{-2}^{2}(x^3 + x)\,dx$.



Areas cancel: f is odd

(Trap drill)

(A) 8

(B) -8

(C) 16

(D) 0

Q4. Which of the following 2×2 matrices is **positive definite**?

$$\overset{>0}{\left[\boxed{\begin{matrix} 2 & 0 \\ 0 & 3 \end{matrix}}\right]}\ {>0}$$

All eigenvalues positive $\Rightarrow$ positive definite

(Concept check)

(A) $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$

(B) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

(C) $\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$

(D) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Q5. Apply the Gram–Schmidt process to $\mathbf{v}_1 = (1, 1)$ and $\mathbf{v}_2 = (0, 1)$ in $\mathbb{R}^2$. What is the second orthonormal vector $\mathbf{e}_2$?

(Application)

(A) $(0,\ 1)$

(B) $\left(-\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right)$

(C) $\left(\frac{1}{\sqrt{2}},\ -\frac{1}{\sqrt{2}}\right)$

(D) $\left(\frac{1}{\sqrt{2}},\ \frac{1}{\sqrt{2}}\right)$

Q6. Events A and B are independent with $P(A) = 0.4$ and $P(B) = 0.5$. Find $P(A \cup B)$.

(Application)

(A) 0.2

(B) 0.9

(C) 0.7

(D) 0.5

Q7. A data set has mean $\mu = 50$ and standard deviation $\sigma = 10$. What is the z-score for the observation $x = 65$?

(Concept check)

(A) 0.5

(B) 1.0

(C) 2.0

(D) 1.5

Q8. The system of ODEs

$$\frac{dx}{dt} = 2x, \qquad \frac{dy}{dt} = 3y$$

has solutions $x(t) = x_0\, e^{2t}$ and $y(t) = y_0\, e^{3t}$. With initial conditions $x(0) = 1$ and $y(0) = 1$, find $x(1) + y(1)$.

(Application)

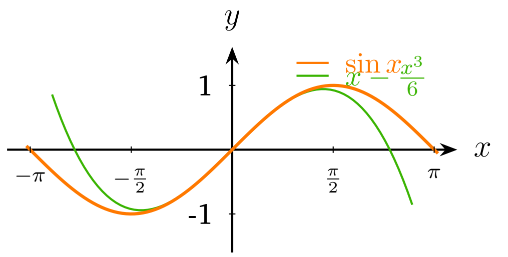
(A) $e^2 + e^3$

(B) e^5

(C) $2e^3$

(D) e^6

Q9. The coefficient of x^3 in the Maclaurin series of $\sin x$ is:



(Trap drill)

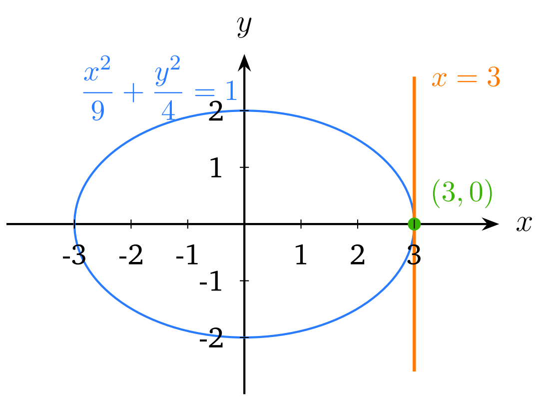
(A) $\dfrac{1}{6}$

(B) $\dfrac{1}{3}$

(C) $-\dfrac{1}{6}$

(D) $-\dfrac{1}{3}$

Q10. Find the equation of the tangent to the ellipse $\dfrac{x^2}{9} + \dfrac{y^2}{4} = 1$ at the point $(3,\, 0)$.



(Application)

(A) $y = 0$

(B) $x = 3$

(C) $x + y = 3$

(D) $3x + 2y = 6$

Q11. Find the acute angle between the lines $y = 0$ (the x-axis) and $y = x$.

(Concept check)

(A) 45

(B) 30

(C) 60

(D) 90

Q12. Which of the following sets is **uncountable**?

(Concept check)

(A) $\mathbb{Z}$ (integers)

(B) $\mathbb{R}$ (real numbers)

(C) $\mathbb{Q}$ (rational numbers)

(D) $\mathbb{N}$ (natural numbers)

Q13. Find the unit vector in the direction of $\mathbf{a} = (3, 4)$.

(Application)

(A) $(3, 4)$

(B) $\left(\frac{4}{5}, \frac{3}{5}\right)$

(C) $\left(\frac{3}{5}, \frac{4}{5}\right)$

(D) $\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

Q14. If 13 people each independently choose a month of the year, what is the minimum number of people that **must** share the same chosen month?

(Concept check)

(A) 1

(B) 3

(C) 4

(D) 2

Q15. Compute $\frac{1+2i}{1-i}$.

(Application)

(A) $-\frac{1}{2}+\frac{3}{2}i$

(B) $\frac{1}{2}+\frac{3}{2}i$

(C) $1+i$

(D) $\frac{3}{2}-\frac{1}{2}i$

Detailed Solutions

Solution 1.

Solution

Concept: The standard limit $\lim_{x\to 0}\frac{e^x-1}{x}=1$ **Chapter:** Limits/Continuity is the definition of the derivative of e^x evaluated at $x=0$.

Step 1 — Recall the derivative definition.

$$\frac{d}{dx}e^x\bigg|_{x=0}=\lim_{x\to 0}\frac{e^x-e^0}{x-0}=\lim_{x\to 0}\frac{e^x-1}{x}.$$

Step 2 — Evaluate. Since $\frac{d}{dx}e^x=e^x$ and $e^0=1$, we get

$$\lim_{x\to 0}\frac{e^x-1}{x}=e^0=\boxed{1}.$$

Why other options are wrong:

- (A) 0: the expression is not zero at the limit; $e^x-1\approx x$ for small x, so the ratio approaches 1.
- (C) e: confuses the limit with the value of e^x at $x=1$.
- (D) ∞: incorrect; the numerator and denominator both vanish at the same rate, giving a finite limit.

Answer: (B) Go Back to Q 1

Solution 2.

Solution

Concept: Related rates — differentiate a geometric formula with respect to time. **Chapter:** Differential Calculus

Step 1 — Differentiate the volume formula with respect to t.

$$V=\frac{4}{3}\pi r^3 \;\Rightarrow\; \frac{dV}{dt}=4\pi r^2\frac{dr}{dt}.$$

Step 2 — Substitute $r = 3$ and $\frac{dr}{dt} = 2$.

$$\frac{dV}{dt} = 4\pi(3)^2 \cdot 2 = 4\pi \cdot 9 \cdot 2 = \boxed{72\pi \text{ cm}^3/\text{s}}.$$

Why other options are wrong:

- (A) 24π: uses $r = 3$ but omits dr/dt, computing $4\pi \cdot 3 \cdot 2 = 24\pi$ instead.
- (B) 36π: uses $4\pi r \cdot dr/dt = 4\pi \cdot 3 \cdot 3 = 36\pi$, incorrectly applying a linear (not squared) radius.
- (D) 108π: uses $\pi r^3/dr/dt$ or a related error giving $4\pi \cdot 27/2 \approx 108\pi/2$; a factor-of-$3/2$ error on the exponent.

Answer: (C) Go Back to Q 2

Solution 3.

Solution

Concept: Symmetry of definite integrals — the integral of an *odd* function **Chapter:** Integral Calculus over a symmetric interval $[-a, a]$ is zero.

Step 1 — Verify that $f(x) = x^3 + x$ is odd.

$$f(-x) = (-x)^3 + (-x) = -x^3 - x = -f(x). \quad \checkmark$$

Step 2 — Apply the odd-function theorem.

$$\int_{-2}^{2} (x^3 + x)\,dx = 0 \quad \text{since } f \text{ is odd and the interval is symmetric.}$$

Direct verification:

$$\int_{-2}^{2} (x^3 + x)\,dx = \left[\frac{x^4}{4} + \frac{x^2}{2}\right]_{-2}^{2} = (4+2) - (4+2) = \boxed{0}.$$

Why other options are wrong:

- (A) 8: might come from integrating only the positive half $[0, 2]$ without accounting for cancellation.
- (B) -8: integrating only the negative half $[-2, 0]$ without the positive contribution.

- (C) 16: adding both halves' magnitudes instead of recognizing cancellation.

Answer: (D) Go Back to Q 3

Solution 4.

Solution

Concept: A symmetric matrix is positive definite if and only if all its **Chapter:** Linear Algebra eigenvalues are strictly positive. For a diagonal matrix, eigenvalues equal the diagonal entries.

Step 1 — Check each option.

- (A) $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$: eigenvalues $2, 3$ — both positive. **Positive definite.**
- (B) $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$: eigenvalue $-1 < 0$ — *indefinite*, not positive definite.
- (C) $\begin{bmatrix} -1 & 0 \\ 0 & -2 \end{bmatrix}$: eigenvalues $-1, -2$ — both negative. *Negative definite.*
- (D) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$: eigenvalues $+1, -1$ — one negative, so *indefinite*.

Step 2 — Conclusion.

$$\boxed{\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}}$$

is the unique positive definite matrix among the options.

Answer: (A) Go Back to Q 4

Solution 5.

Solution

Concept: Gram–Schmidt orthonormalization. **Chapter:** Linear Algebra Given vectors $\mathbf{v}_1, \mathbf{v}_2$, compute $\mathbf{e}_1 = \mathbf{v}_1/\|\mathbf{v}_1\|$, then subtract the projection onto $\mathbf{e}_1$ and normalize.

Step 1 — Compute $\mathbf{e}_1$.

$$\mathbf{e}_1 = \frac{\mathbf{v}_1}{\|\mathbf{v}_1\|} = \frac{(1,1)}{\sqrt{2}} = \left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right).$$

Step 2 — Project $\mathbf{v}_2 = (0, 1)$ onto $\mathbf{e}_1$.

$$\text{proj}_{\mathbf{e}_1}\mathbf{v}_2 = (\mathbf{v}_2 \cdot \mathbf{e}_1)\,\mathbf{e}_1 = \frac{1}{\sqrt{2}} \cdot \left(\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}\right) = \left(\tfrac{1}{2}, \tfrac{1}{2}\right).$$

Step 3 — Subtract to get the orthogonal component.

$$\mathbf{v}_2' = \mathbf{v}_2 - \text{proj}_{\mathbf{e}_1}\mathbf{v}_2 = (0, 1) - \left(\tfrac{1}{2}, \tfrac{1}{2}\right) = \left(-\tfrac{1}{2}, \tfrac{1}{2}\right).$$

Step 4 — Normalize.

$$\|\mathbf{v}_2'\| = \sqrt{\tfrac{1}{4} + \tfrac{1}{4}} = \frac{1}{\sqrt{2}}. \qquad \mathbf{e}_2 = \frac{\mathbf{v}_2'}{\|\mathbf{v}_2'\|} = \sqrt{2} \cdot \left(-\tfrac{1}{2}, \tfrac{1}{2}\right) = \boxed{\left(-\tfrac{1}{\sqrt{2}}, \tfrac{1}{\sqrt{2}}\right)}.$$

Why other options are wrong:

- (A) $(0, 1)$: this is $\mathbf{v}_2$ itself, not orthogonal to $\mathbf{e}_1$.
- (C) $(1/\sqrt{2}, -1/\sqrt{2})$: the sign of both components is flipped; this is $-\mathbf{e}_2$.
- (D) $(1/\sqrt{2}, 1/\sqrt{2})$: this is $\mathbf{e}_1$, not the second orthonormal vector.

Answer: (B) Go Back to Q 5

Solution 6.

Solution

Concept: For independent events, $P(A \cap B) = P(A) \cdot P(B)$, and **Chapter:** Probability & Statistics $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

Step 1 — Compute $P(A \cap B)$.

$$P(A \cap B) = P(A) \cdot P(B) = 0.4 \times 0.5 = 0.2.$$

Step 2 — Apply the union formula.

$$P(A \cup B) = 0.4 + 0.5 - 0.2 = \boxed{0.7}.$$

Why other options are wrong:

- (A) 0.2: this is only $P(A \cap B)$, not the union.
- (B) 0.9: adds $P(A) + P(B)$ without subtracting the intersection; over-counts elements in both.

- (D) 0.5: equals $P(B)$ alone, ignoring $P(A)$'s contribution.

Answer: (C) Go Back to Q 6

Solution 7.

Solution

Concept: z-score measures how many standard deviations an observation lies from the mean: $z = \dfrac{x-\mu}{\sigma}$. **Chapter:** Probability & Statistics

Step 1 — Substitute $x = 65$, $\mu = 50$, $\sigma = 10$.

$$z = \frac{65-50}{10} = \frac{15}{10} = \boxed{1.5}.$$

Why other options are wrong:

- (A) 0.5: uses $(65-50)/30$ or otherwise incorrectly scales the deviation.
- (B) 1.0: computes $(65-55)/10$, using the wrong mean.
- (C) 2.0: computes $(65-45)/10$, shifting the mean incorrectly.

Answer: (D) Go Back to Q 7

Solution 8.

Solution

Concept: The system $dx/dt = 2x$, $dy/dt = 3y$ has independent exponential solutions. **Chapter:** Differential Equations

Step 1 — Write general solutions.

$$x(t) = x_0\, e^{2t}, \qquad y(t) = y_0\, e^{3t}.$$

Step 2 — Apply initial conditions $x(0) = 1$, $y(0) = 1$.

$$x_0 = 1, \qquad y_0 = 1.$$

Step 3 — Evaluate at $t = 1$.

$$x(1) = e^2, \qquad y(1) = e^3. \qquad \Rightarrow \qquad x(1) + y(1) = \boxed{e^2 + e^3}.$$

Why other options are wrong:

- (B) e^5: applies the rule $e^a \cdot e^b = e^{a+b}$ (multiplication, not addition) or confuses the product with the sum.
- (C) $2e^3$: assumes both components decay at rate 3, ignoring the different exponent in $x(t)$.
- (D) e^6: takes the product $e^2 \cdot e^3 = e^5$ or incorrectly combines exponents by multiplication.

Answer: (A) Go Back to Q 8

Solution 9.

Solution

Concept: The Maclaurin series of $\sin x$ is obtained from the general formula **Chapter:** Integral Calculus $f^{(n)}(0)/n!$ for each coefficient.

Step 1 — Write out the Maclaurin series of $\sin x$.

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots = x - \frac{x^3}{6} + \frac{x^5}{120} - \cdots$$

Step 2 — Read off the coefficient of x^3.

$$\text{Coefficient of } x^3 = -\frac{1}{3!} = -\frac{1}{6} = \boxed{-\frac{1}{6}}.$$

Verification: $\frac{d^3}{dx^3} \sin x = -\cos x$, so the third derivative at 0 is -1, and the coefficient is $-1/3! = -1/6$.

Why other options are wrong:

- (A) $1/6$: correct magnitude but wrong sign; forgets the alternating minus sign.
- (B) $1/3$: uses $1/3$ instead of $1/3! = 1/6$; confuses the factorial.
- (D) $-1/3$: correct sign but uses 3 instead of $3! = 6$ in the denominator.

Answer: (C) Go Back to Q 9

Solution 10.

Solution

Concept: The tangent to the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ at **Chapter:** Coordinate Geometry the point (x_1, y_1) on the ellipse is given by $\dfrac{x\,x_1}{a^2}+\dfrac{y\,y_1}{b^2}=1$.

Step 1 — Identify parameters. $a^2=9$, $b^2=4$, $(x_1,y_1)=(3,\ 0)$.

Step 2 — Apply the tangent formula.

$$\frac{x\cdot 3}{9}+\frac{y\cdot 0}{4}=1 \;\Rightarrow\; \frac{x}{3}=1 \;\Rightarrow\; \boxed{x=3}.$$

The tangent is a vertical line through the right vertex of the ellipse.

Why other options are wrong:

- (A) $y=0$: the x-axis passes through $(3,0)$ but is the *major axis*, not the tangent; the tangent must be perpendicular to the radius at the vertex.
- (C) $x+y=3$: a line with slope -1 that passes through $(3,0)$, but it intersects the ellipse at two points — not a tangent.
- (D) $3x+2y=6$: simplifies to $x+2y/3=2$, which does not satisfy the tangent formula at $(3,0)$.

Answer: (B) Go Back to Q 10

Solution 11.

Solution

Concept: The acute angle θ between two lines with slopes m_1 and m_2 **Chapter:** Coordinate Geometry satisfies $\tan\theta=\left|\dfrac{m_2-m_1}{1+m_1m_2}\right|$.

Step 1 — Identify slopes. Line $y=0$ (the x-axis): $m_1=0$. Line $y=x$: $m_2=1$.

Step 2 — Compute $\tan\theta$**.**

$$\tan\theta=\left|\frac{1-0}{1+(0)(1)}\right|=\frac{1}{1}=1.$$

Step 3 — Find θ**.**

$$\theta=\arctan(1)=\boxed{45}.$$

Why other options are wrong:

- (B) 30: $\tan 30 = 1/\sqrt{3} \neq 1$.
- (C) 60: $\tan 60 = \sqrt{3} \neq 1$.
- (D) 90: would require the lines to be perpendicular ($m_1 m_2 = -1$), but $0 \cdot 1 = 0 \neq -1$.

Answer: (A) Go Back to Q 11

Solution 12.

Solution

Concept: Countability of standard number sets. **Chapter:** Set Theory A set is *countable* if it can be put in bijection with $\mathbb{N}$; otherwise it is *uncountable*.

Step 1 — Classify each option.

- $\mathbb{N}$: countably infinite by definition.
- $\mathbb{Z}$: countable — can be listed as $0, 1, -1, 2, -2, 3, -3, \ldots$
- $\mathbb{Q}$: countable — Cantor's diagonal enumeration of fractions shows a bijection with $\mathbb{N}$.
- $\mathbb{R}$: **uncountable** — Cantor's diagonal argument shows no enumeration of all reals is possible.

Step 2 — Conclude.

$$\boxed{\mathbb{R}} \text{ is the uncountable set.}$$

Why other options are wrong:

- (A) $\mathbb{Z}$: countable — integers can be listed in a sequence.
- (C) $\mathbb{Q}$: countable, despite being dense in $\mathbb{R}$.
- (D) $\mathbb{N}$: the prototypical countably infinite set.

Answer: (B) Go Back to Q 12

Solution 13.

Solution

Concept: The unit vector in the direction of a is **Chapter:** Vectors $\hat{\mathbf{a}} = \frac{\mathbf{a}}{|\mathbf{a}|}$.

Step 1 — Compute the magnitude.

$$|\mathbf{a}| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5.$$

Step 2 — Divide by the magnitude.

$$\hat{\mathbf{a}} = \frac{(3,4)}{5} = \boxed{\left(\frac{3}{5}, \frac{4}{5}\right)}.$$

Note: $|\hat{\mathbf{a}}| = \sqrt{(3/5)^2 + (4/5)^2} = \sqrt{9/25 + 16/25} = \sqrt{1} = 1.$ ✓

Why other options are wrong:

- (A) $(3,4)$: the original vector; magnitude $5 \neq 1$, so not a unit vector.
- (B) $(4/5, 3/5)$: components are swapped (x- and y-components reversed).
- (D) $(1/\sqrt{2}, 1/\sqrt{2})$: unit vector in the direction of $(1,1)$, not $(3,4)$.

Answer: (C) Go Back to Q 13

Solution 14.

Solution

Concept: Pigeonhole Principle — if n objects are placed into k containers, **Chapter:** Combinatorics at least one container must hold $\lceil n/k \rceil$ objects.

Step 1 — Identify pigeons and pigeonholes.

- Pigeons: $n = 13$ people.
- Pigeonholes: $k = 12$ months.

Step 2 — Apply the Pigeonhole Principle.

$$\left\lceil \frac{n}{k} \right\rceil = \left\lceil \frac{13}{12} \right\rceil = \lceil 1.08\overline{3} \rceil = \boxed{2}.$$

At least 2 people must share the same month.

Why other options are wrong:

- (A) 1: with only 1 person per month and 12 months, we can fit 12 people without sharing; but the 13th forces a repeat.
- (B) 3 and (C) 4: these are only guaranteed when the number of people is large enough to force 3 or 4 into a single month (e.g., 25 or 37 people for months).

Answer: (D) Go Back to Q 14

Solution 15.

Solution

Concept: To divide complex numbers, multiply numerator and denominator by the **Chapter:** Complex Numbers *conjugate* of the denominator.

Step 1 — Multiply by the conjugate of $1 - i$, which is $1 + i$.

$$\frac{1+2i}{1-i} = \frac{(1+2i)(1+i)}{(1-i)(1+i)}.$$

Step 2 — Expand the denominator.

$$(1-i)(1+i) = 1 - i^2 = 1 - (-1) = 2.$$

Step 3 — Expand the numerator.

$$(1+2i)(1+i) = 1 + i + 2i + 2i^2 = 1 + 3i + 2(-1) = -1 + 3i.$$

Step 4 — Divide.

$$\frac{-1+3i}{2} = \boxed{-\frac{1}{2} + \frac{3}{2}i}.$$

Why other options are wrong:

- (B) $1/2 + 3i/2$: sign error on the real part; computes $+1$ instead of -1 in the numerator.
- (C) $1 + i$: a common guess from simply dividing component-wise without multiplying by the conjugate.
- (D) $3/2 - i/2$: real and imaginary parts are swapped and the signs are incorrect.

Answer: (A) Go Back to Q 15

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	C	3	D	4	A	5	B
6	C	7	D	8	A	9	C	10	B
11	A	12	B	13	C	14	D	15	A