GAT B Mathematics

Sample Paper – 8

Duration: 23 Minutes **Maximum Marks: 15**

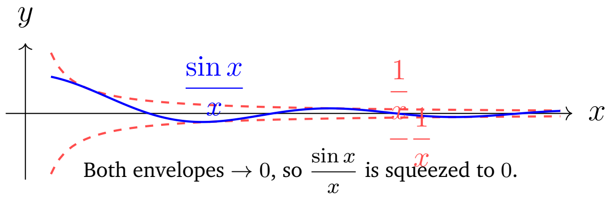
Instructions

- This paper contains **15 Multiple Choice Questions (MCQs)**, each with four options. Only **one** option is correct per question.
- **Marking Scheme:** $+1$ for each correct answer; -0.5 for each incorrect answer; 0 for unattempted questions.
- All questions are drawn from **undergraduate Mathematics** topics.
- Use of calculators, mobile phones, or any electronic devices is **strictly prohibited**.
- All questions carry **equal marks**.

Section A — Mathematics (Q 1 – Q 15)

Q1. Using the **Squeeze (Sandwich) Theorem** and the fact that $-1 \le \sin x \le 1$ for all $x \in \mathbb{R}$, evaluate

$$\lim_{x\to\infty} \frac{\sin x}{x}.$$



(Concept check)

(A) 0

(B) 1

(C) ∞

(D) Does not exist

Q2. A curve is defined parametrically by $x = t^2$ and $y = t^3$. What is $\dfrac{dy}{dx}$ at $t = 1$?

(Application)

(A) 1
(B) $\dfrac{2}{3}$
(C) $\dfrac{1}{2}$
(D) $\dfrac{3}{2}$

Q3. Evaluate the indefinite integral

$$\int \frac{dx}{x^2 + 4}.$$

(Application)

(A) $\arctan\left(\dfrac{x}{2}\right) + C$
(B) $\dfrac{1}{2}\arctan\left(\dfrac{x}{2}\right) + C$
(C) $\dfrac{1}{4}\arctan\left(\dfrac{x}{2}\right) + C$
(D) $2\arctan\left(\dfrac{x}{2}\right) + C$

Q4. A square matrix A of order n admits an LU factorisation *without row interchanges* if and only if:

(Concept check)

(A) All diagonal entries of A are nonzero
(B) A is symmetric and positive definite

(C) All leading principal minors of A are nonzero

(D) A is strictly diagonally dominant

Q5. Let $\mathcal{B} = \{(1, 2), (0, 1)\}$ be an ordered basis for $\mathbb{R}^2$. The coordinate vector of $\mathbf{v} = (3, 5)$ with respect to $\mathcal{B}$ is:

(Application)

(A) $(3, 5)$

(B) $(5, 3)$

(C) $(3, 1)$

(D) $(3, -1)$

Q6. Bag X contains 3 red and 2 white balls; Bag Y contains 2 red and 4 white balls. A fair coin is tossed: *heads* $\Rightarrow$ draw from Bag X; *tails* $\Rightarrow$ draw from Bag Y. What is the probability that the drawn ball is **red**?

(Application)

(A) $\frac{7}{15}$

(B) $\frac{3}{5}$

(C) $\frac{1}{3}$

(D) $\frac{2}{5}$

Q7. A dataset contains several very large values on the right (extreme outliers). Which single measure of **central tendency** is *least* affected by these extreme values?

(Concept check)

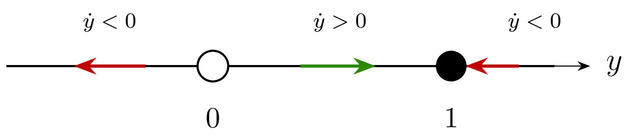
(A) Mean

(B) Median

(C) Variance

(D) Standard deviation

Q8. Consider the autonomous ODE $\dfrac{dy}{dt} = y(1-y)$. The phase line below shows the flow of solutions. The equilibrium points $y = 0$ and $y = 1$ are respectively:



(Application)

(A) stable and unstable (respectively)

(B) both stable

(C) unstable and stable (respectively)

(D) both unstable

Q9. According to **Picard's Existence–Uniqueness Theorem**, the IVP $\dfrac{dy}{dx} = f(x, y),\ y(x_0) = y_0$ has a *unique* local solution if:

(Concept check)

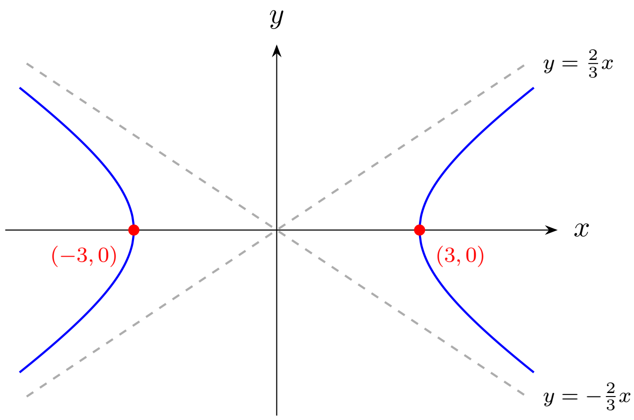
(A) $f(x, y)$ is bounded in a region containing (x_0, y_0)

(B) $f(x, y)$ is merely continuous in a region containing (x_0, y_0)

(C) $f(x,y)$ and $\dfrac{\partial^2 f}{\partial y^2}$ are both continuous in a region containing (x_0, y_0)

(D) $f(x,y)$ and $\dfrac{\partial f}{\partial y}$ are both continuous in a region containing (x_0, y_0)

Q10. The equations of the **asymptotes** of the hyperbola $\dfrac{x^2}{9} - \dfrac{y^2}{4} = 1$ are:



(Application)

(A) $y = \pm\dfrac{2}{3}x$

(B) $y = \pm\dfrac{3}{2}x$

(C) $y = \pm\dfrac{2}{9}x$

(D) $y = \pm\dfrac{1}{3}x$

Q11. The equation of the **normal** to the parabola $y^2 = 4ax$ at the point $P(at^2, 2at)$ is:

(Application)

(A) $y - tx = at^2 - 2at$

(B) $y + tx = at^3 + 2at$

(C) $y - tx = at^3 + 2at$

(D) $y + tx = at^3 - 2at$

Q12. Let R be the relation on $\mathbb{Z}$ defined by

$$a\,R\,b \iff 3 \mid (a-b).$$

Which statement correctly describes R?

(Concept check)

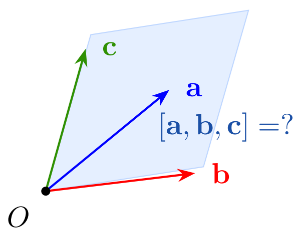
(A) R is reflexive and symmetric, but **not** transitive

(B) R is symmetric and transitive, but **not** reflexive

(C) R is an equivalence relation (reflexive, symmetric, and transitive)

(D) R is reflexive only

Q13. Let $\mathbf{a} = (1,2,3)$, $\mathbf{b} = (2,1,4)$, and $\mathbf{c} = (0,3,2)$. The scalar triple product $[\mathbf{a},\mathbf{b},\mathbf{c}] = \mathbf{a}\cdot(\mathbf{b}\times\mathbf{c})$ equals:



(Application)

(A) 0

(B) 6

(C) -6

(D) 12

Q14. Using the **Multinomial Theorem**, the coefficient of x^2y^2z in the expansion of $(x+y+z)^5$ is:

(Application)

(A) 10

(B) 20

(C) 60

(D) 30

Q15. Express $z = -\sqrt{3} + i$ in polar (modulus–argument) form $r\big(\cos\theta + i\sin\theta\big)$:

(Application)

(A) $2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

(B) $2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$

(C) $2\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

(D) $2\left(\cos\frac{7\pi}{6} + i\sin\frac{7\pi}{6}\right)$

Detailed Solutions — GAT B Mathematics Sample Paper 8

Q1.

Solution

Concept — Squeeze (Sandwich) Theorem: Chapter: Limits/Continuity If $g(x) \le f(x) \le h(x)$ for all sufficiently large x and $\lim_{x\to\infty} g(x) = \lim_{x\to\infty} h(x) = L$, then $\lim_{x\to\infty} f(x) = L$.

Step 1 — Establish the bounds: Since $-1 \le \sin x \le 1$ for all $x \in \mathbb{R}$, dividing by $x > 0$ gives

$$-\frac{1}{x} \le \frac{\sin x}{x} \le \frac{1}{x}.$$

Step 2 — Take the limit of both bounding functions:

$$\lim_{x\to\infty}\left(-\frac{1}{x}\right) = 0 \qquad \text{and} \qquad \lim_{x\to\infty}\frac{1}{x} = 0.$$

Step 3 — Conclude by the Squeeze Theorem:

$$\lim_{x\to\infty}\frac{\sin x}{x} = \boxed{0}.$$

Why other options are wrong:

- **(B)** 1**:** The famous limit $\lim_{x\to 0}(\sin x)/x = 1$ applies as $x \to 0$, *not* as $x \to \infty$.
- **(C)** ∞**:** Impossible; the function is bounded above by $1/x \to 0$.
- **(D) Does not exist:** The Squeeze Theorem proves the limit exists and equals 0.

Answer: (A) Go Back to Q 1

Q2.

Solution

Concept — Parametric differentiation: Chapter: Differential Calculus

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}, \qquad \frac{dx}{dt} \neq 0.$$

Step 1 — Differentiate each component with respect to t:

$$x = t^2 \;\Rightarrow\; \frac{dx}{dt} = 2t, \qquad y = t^3 \;\Rightarrow\; \frac{dy}{dt} = 3t^2.$$

Step 2 — Form the ratio:

$$\frac{dy}{dx} = \frac{3t^2}{2t} = \frac{3t}{2}.$$

Step 3 — Evaluate at $t = 1$:

$$\left.\frac{dy}{dx}\right|_{t=1} = \frac{3 \cdot 1}{2} = \boxed{\frac{3}{2}}.$$

Why other options are wrong:

- **(A)** 1: Results from incorrectly treating dy/dx as $dt/dt = 1$, ignoring the chain-rule formula.
- **(B)** $2/3$: This is dx/dy at $t = 1$ (the reciprocal); the ratio dy/dt and dx/dt are swapped.
- **(C)** $1/2$: Comes from $(dx/dt)|_{t=1} = 2(1)/2 = 1$ divided by the value 2; the dy/dt factor is neglected.

Answer: (D) Go Back to Q 2

Q3.

Solution

Concept — Standard arctangent integral: Chapter: Integral Calculus

$$\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C.$$

Step 1 — Identify a**:** $x^2 + 4 = x^2 + 2^2$, so $a = 2$.

Step 2 — Apply the formula directly:

$$\int \frac{dx}{x^2 + 4} = \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C.$$

Alternative via substitution: Let $u = x/2$, so $du = dx/2$:

$$\int \frac{dx}{x^2 + 4} = \int \frac{2\,du}{4(u^2 + 1)} = \frac{1}{2}\int \frac{du}{u^2 + 1} = \frac{1}{2}\arctan u + C = \frac{1}{2}\arctan\left(\frac{x}{2}\right) + C. \checkmark$$

Why other options are wrong:

- **(A)** $\arctan(x/2) + C$**:** Omits the factor $1/a = 1/2$ from the formula; the substitution's Jacobian is ignored.
- **(C)** $\frac{1}{4}\arctan(x/2) + C$**:** Uses $1/a^2 = 1/4$ instead of the correct $1/a = 1/2$.
- **(D)** $2\arctan(x/2) + C$**:** Multiplies by $a = 2$ instead of dividing.

Answer: (B) Go Back to Q 3

Q4.

Solution

Concept — LU Factorisation without row interchanges: Chapter: Linear Algebra Gaussian elimination on A avoids all row swaps if and only if every pivot is nonzero at the time it is used. The k-th pivot is nonzero precisely when the k-th *leading principal minor* $D_k = \det(A_{k\times k}) \neq 0$. Therefore:

$$A = LU \text{ without permutation} \iff D_1 \neq 0,\ D_2 \neq 0,\ \ldots,\ D_{n-1} \neq 0.$$

Why other options are wrong:

- **(A) Nonzero diagonal entries:** The diagonal of A does not equal the elimination pivots in general; off-diagonal entries modify the pivots during row reduction.
- **(B) Symmetric positive definite (SPD):** Sufficient (SPD $\Rightarrow$ all leading principal minors > 0), but not necessary—many non-symmetric matrices also have LU factorisation without row swaps.
- **(D) Strictly diagonally dominant:** Another sufficient condition (guarantees nonzero pivots), but far more restrictive than the necessary and sufficient condition (C).

Answer: (C) Go Back to Q 4

Q5.

Solution

Concept — Coordinate vector in a non-standard basis: Chapter: Linear Algebra
Find scalars a, b such that $a\,\mathbf{b}_1 + b\,\mathbf{b}_2 = \mathbf{v}$.

Step 1 — Write the system:

$$a\,(1,\ 2) + b\,(0,\ 1) = (3,\ 5) \implies \begin{cases} a = 3, \\ 2a + b = 5. \end{cases}$$

Step 2 — Solve: From the first equation $a = 3$. Substituting: $2(3) + b = 5 \ \Rightarrow \ b = -1$.

Step 3 — State the coordinate vector:

$$[\mathbf{v}]_{\mathcal{B}} = \boxed{(3,\ -1)}.$$

Check: $3(1, 2) + (-1)(0, 1) = (3, 6) + (0, -1) = (3, 5)$. ✓

Why other options are wrong:

- **(A)** $(3, 5)$**:** This is $\mathbf{v}$ expressed in the standard basis $\{(1, 0), (0, 1)\}$, not in $\mathcal{B}$.
- **(B)** $(5, 3)$**:** A simple reversal of components with no algebraic justification.
- **(C)** $(3, 1)$**:** Sign error; the correct second coordinate is $b = -1$, not $+1$.

Answer: (D) Go Back to Q 5

Q6.

Solution

Concept — Law of Total Probability: Chapter: Probability & Statistics

$$P(R) = P(R \mid X)\,P(X) + P(R \mid Y)\,P(Y).$$

Given data:

$$P(X) = P(Y) = \tfrac{1}{2}, \qquad P(R \mid X) = \tfrac{3}{5} \text{ (3 red out of 5)}, \qquad P(R \mid Y) = \tfrac{2}{6} = \tfrac{1}{3} \text{ (2 red out of 6)}.$$

Computation:

$$P(R) = \frac{3}{5} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{2} = \frac{3}{10} + \frac{1}{6} = \frac{9}{30} + \frac{5}{30} = \frac{14}{30} = \boxed{\frac{7}{15}}.$$

Why other options are wrong:

- **(B)** $3/5$**:** Probability of red given Bag X is chosen, ignoring Bag Y entirely.
- **(C)** $1/3$**:** Probability of red given Bag Y is chosen, ignoring Bag X entirely.
- **(D)** $2/5$**:** An arithmetic error when combining the two weighted conditional probabilities.

Answer: (A) Go Back to Q 6

Q7.

Solution

Concept — Robustness of measures of central tendency: Chapter: Probability & Statistics

The **mean** is computed from every data value, so a single extreme outlier can shift it substantially toward the tail. The **median** is the middle-ranked observation; because it depends only on the *ordering* of values—not their magnitudes—even extreme outliers leave it unchanged.

For a positively (right-)skewed dataset, the classical ordering is

$$\text{Mode} \; < \; \text{Median} \; < \; \text{Mean},$$

confirming that the mean is pulled furthest toward the outliers while the median remains a stable, representative centre.

Why other options are wrong:

- **(A) Mean:** Highly sensitive to outliers; it is inflated by large values, making it a poor summary when extreme observations are present.
- **(C) Variance:** A measure of *dispersion* (spread), not of central tendency.
- **(D) Standard deviation:** Also a measure of spread, not of central tendency, and it is itself severely inflated by outliers.

Answer: (B) Go Back to Q 7

Q8.

Solution

Concept — Phase-line stability: Chapter: Differential Equations For $\dot{y} = f(y)$, an equilibrium y^* is *stable* (sink) if $f'(y^*) < 0$ and *unstable* (source) if $f'(y^*) > 0$.

Find equilibria: $y(1-y) = 0 \;\Rightarrow\; y = 0$ or $y = 1$.

Linearise: $f(y) = y - y^2$, so $f'(y) = 1 - 2y$.

$$f'(0) = 1 - 0 = 1 > 0 \implies y = 0 \text{ is } \textbf{unstable} \text{ (source)}.$$

$$f'(1) = 1 - 2 = -1 < 0 \implies y = 1 \text{ is } \textbf{stable} \text{ (sink)}.$$

Sign analysis of $\dot{y} = y(1-y)$:

Region	$\operatorname{sgn}(\dot{y})$	Trajectory direction
$y < 0$	$(-)(+) < 0$	$\leftarrow$ (away from 0)
$0 < y < 1$	$(+)(+) > 0$	$\rightarrow$ (toward 1)
$y > 1$	$(+)(-) < 0$	$\leftarrow$ (toward 1)

$y = 0$ repels trajectories on both sides (**unstable**); $y = 1$ attracts trajectories from both sides (**stable**).

Why other options are wrong:

- **(A) stable/unstable:** The roles of $y = 0$ and $y = 1$ are interchanged; $f'(0) = 1 > 0$ confirms $y = 0$ is *unstable*.
- **(B) both stable:** $y = 0$ cannot be stable since trajectories diverge from it on both sides.
- **(D) both unstable:** $y = 1$ cannot be unstable since $f'(1) = -1 < 0$ and all nearby trajectories converge to it.

$\boxed{\textbf{Answer: (C)}}$ Go Back to Q 8

Q9.

Solution

Concept — Picard–Lindelöf Existence-Uniqueness Theorem: Chapter: Differential Equations The IVP $dy/dx = f(x, y)$, $y(x_0) = y_0$ has a *unique* local solution if f is continuous **and** satisfies a **Lipschitz condition in** y on a closed rectangle R containing (x_0, y_0). A sufficient and commonly stated form: $\partial f/\partial y$ exists and is continuous (hence bounded) on R.

Formal statement:

$$f \text{ and } \frac{\partial f}{\partial y} \text{ both continuous on a rectangle around } (x_0, y_0) \implies \text{unique local solution.}$$

Why other options are wrong:

- **(A) Bounded f alone:** Boundedness is neither necessary nor sufficient for uniqueness; the Lipschitz condition in y is the key requirement.
- **(B) Merely continuous f:** By *Peano's theorem* this guarantees only *existence*, not uniqueness. Counter-example: $dy/dx = y^{1/3}$, $y(0) = 0$ has infinitely many solutions (e.g. $y \equiv 0$ and $y = (2x/3)^{3/2}$).
- **(C) Continuous f and $\partial^2 f/\partial y^2$:** A stronger assumption than necessary; the theorem requires only the first partial $\partial f/\partial y$, not the second.

Answer: (D) Go Back to Q 9

Q10.

Solution

Concept — Asymptotes of a standard hyperbola: Chapter: Coordinate Geometry For $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, the asymptotes are $y = \pm\frac{b}{a}x$.

Identify a and b:

$$\frac{x^2}{9} - \frac{y^2}{4} = 1 \;\Rightarrow\; a^2 = 9,\; b^2 = 4 \;\Rightarrow\; a = 3,\; b = 2.$$

Write the asymptotes:

$$y = \pm\frac{b}{a}x = \pm\frac{2}{3}x.$$

These lines pass through the origin and serve as the "skeleton" that the hyperbola approaches (without crossing) as $|x| \to \infty$.

Why other options are wrong:

- **(B) $y = \pm\frac{3}{2}x$:** Uses the inverted ratio a/b instead of b/a.
- **(C) $y = \pm\frac{2}{9}x$:** Uses $b/a^2 = 2/9$ instead of $b/a = 2/3$.
- **(D) $y = \pm\frac{1}{3}x$:** Uses $1/a = 1/3$ and ignores $b = 2$ entirely.

Answer: (A) Go Back to Q 10

Q11.

Solution

Concept — Normal to the parabola $y^2 = 4ax$: Chapter: Coordinate Geometry

Step 1 — Find the slope of the tangent at $P(at^2, 2at)$: Differentiate $y^2 = 4ax$ implicitly:

$$2y\,\frac{dy}{dx} = 4a \;\Rightarrow\; \frac{dy}{dx} = \frac{2a}{y} = \frac{2a}{2at} = \frac{1}{t}.$$

Step 2 — Slope of the normal: $m_{\text{normal}} = -t$ (negative reciprocal of the tangent slope).

Step 3 — Equation of the normal through $P(at^2, 2at)$:

$$y - 2at = -t\,(x - at^2) \implies y = -tx + at^3 + 2at \implies \boxed{y + tx = at^3 + 2at.}$$

Why other options are wrong:

- **(A)** $y - tx = at^2 - 2at$**:** Uses the *tangent* slope $+1/t$ (not the normal slope $-t$) and has an incorrect constant.
- **(C)** $y - tx = at^3 + 2at$**:** Algebraic sign error on rearrangement; moving tx to the right gives $y + tx$, not $y - tx$.
- **(D)** $y + tx = at^3 - 2at$**:** Sign error on the $2at$ term; the correct expansion gives $+2at$, not $-2at$.

Answer: (B) Go Back to Q 11

Q12.

Solution

Concept — Equivalence Relation: reflexive + symmetric **Chapter:** Set Theory + transitive.

Reflexive: $a - a = 0 = 3 \cdot 0$, so $3 \mid (a - a)$ for all $a \in \mathbb{Z}$. ✓

Symmetric: Suppose $3 \mid (a - b)$, say $a - b = 3k$. Then $b - a = -3k = 3(-k)$, so $3 \mid (b - a)$. ✓

Transitive: Suppose $3 \mid (a - b)$ and $3 \mid (b - c)$, say $a - b = 3k$ and $b - c = 3m$.

$$a - c = (a - b) + (b - c) = 3k + 3m = 3(k + m), \text{ so } 3 \mid (a - c). \checkmark$$

All three properties hold; R is an **equivalence relation**. Its equivalence classes are the three residue classes modulo 3: $[0] = \{\ldots, -3, 0, 3, 6, \ldots\}$, $[1]$, $[2]$.

Why other options are wrong:

- **(A) Reflexive and symmetric only:** Transitivity was proved above, so this is an understatement.
- **(B) Symmetric and transitive only:** Reflexivity was proved above.
- **(D) Reflexive only:** Both symmetry and transitivity also hold.

Answer: (C) Go Back to Q 12

Q13.

Solution

Concept — Scalar triple product and coplanarity: Chapter: Vectors $[\mathbf{a}, \mathbf{b}, \mathbf{c}] = \mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = 0 \iff \mathbf{a}, \mathbf{b}, \mathbf{c}$ are coplanar.

Step 1 — Compute $\mathbf{b} \times \mathbf{c}$:

$$\mathbf{b} \times \mathbf{c} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1 & 4 \\ 0 & 3 & 2 \end{vmatrix} = \mathbf{i}(1 \cdot 2 - 4 \cdot 3) - \mathbf{j}(2 \cdot 2 - 4 \cdot 0) + \mathbf{k}(2 \cdot 3 - 1 \cdot 0) = (-10,\ -4,\ 6).$$

Step 2 — Dot product with $\mathbf{a}$:

$$\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (1)(-10) + (2)(-4) + (3)(6) = -10 - 8 + 18 = \boxed{0}.$$

Because $[\mathbf{a}, \mathbf{b}, \mathbf{c}] = 0$, the three vectors are **coplanar**—they all lie in a single plane through the origin.

Why other options are wrong:

- **(B)** 6**:** Arises from a sign error in the **j**-component of $\mathbf{b} \times \mathbf{c}$ (the cofactor expansion's minus sign on **j** is omitted).
- **(C)** -6**:** A global sign flip in the triple product, perhaps from swapping two vectors before computing.
- **(D)** 12**:** An arithmetic error in the final dot product (e.g. computing $+8$ instead of -8 for the second term).

Answer: (A) Go Back to Q 13

Q14.

Solution

Concept — Multinomial Theorem: Chapter: Combinatorics

$$(x+y+z)^n = \sum_{\substack{i+j+k=n \\ i,j,k\geq 0}} \frac{n!}{i!\,j!\,k!}\, x^i\, y^j\, z^k.$$

Identify the exponents for x^2y^2z: $i=2,\ j=2,\ k=1$, and $i+j+k=2+2+1=5=n$. ✓

Compute the multinomial coefficient:

$$\frac{5!}{2!\cdot 2!\cdot 1!} = \frac{120}{2\cdot 2\cdot 1} = \frac{120}{4} = \boxed{30}.$$

Why other options are wrong:

- **(A)** 10: Equals $\binom{5}{2} = 5!/(2!\,3!)$, the number of ways to choose 2 positions for x in 5 factors, without accounting for the split between y^2 and z.
- **(B)** 20: Equals $5!/(3!\,1!\,1!)$, the coefficient of x^3yz; the exponent distribution $(3,1,1)$ is used instead of $(2,2,1)$.
- **(C)** 60: May arise from a partial permutation $5\times 4\times 3 = 60$ without dividing by the correct denominator $2!\cdot 2!\cdot 1! = 4$.

Answer: (D) Go Back to Q 14

Q15.

Solution

Concept — Polar (modulus–argument) form: Chapter: Complex Numbers $z = r(\cos\theta + i\sin\theta)$, with $r=|z|\geq 0$ and $\theta = \arg(z)\in(-\pi,\pi]$.

Step 1 — Modulus:

$$r = |z| = \sqrt{(-\sqrt{3})^2 + 1^2} = \sqrt{3+1} = \sqrt{4} = 2.$$

Step 2 — Argument: $z = -\sqrt{3}+i$ has $\mathrm{Re}(z) < 0$ and $\mathrm{Im}(z) > 0$, so z lies in the **second quadrant**. The reference angle is

$$\alpha = \arctan\left(\frac{|\operatorname{Im}(z)|}{|\operatorname{Re}(z)|}\right) = \arctan\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}.$$

The actual argument in Quadrant II:

$$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}.$$

Step 3 — Polar form:

$$z = \boxed{2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right).}$$

Verify: $2\cos\frac{5\pi}{6} = 2\left(-\frac{\sqrt{3}}{2}\right) = -\sqrt{3}$; $2\sin\frac{5\pi}{6} = 2\cdot\frac{1}{2} = 1$. ✓

Why other options are wrong:

- **(A)** $\theta = \pi/6$**:** This is only the reference (acute) angle; using it directly places z in the first quadrant, giving $2(\frac{\sqrt{3}}{2} + \frac{1}{2}i) = \sqrt{3} + i \neq z$.
- **(C)** $\theta = 2\pi/3$**:** Off by $\pi/6$; gives $2(-\frac{1}{2} + i\frac{\sqrt{3}}{2}) = -1 + i\sqrt{3} \neq z$.
- **(D)** $\theta = 7\pi/6$**:** A third-quadrant angle where both components are negative, contradicting $\mathrm{Im}(z) = 1 > 0$.

$\boxed{\textbf{Answer: (B)}}$ Go Back to Q 15

Answer Key — GAT B Mathematics Sample Paper 8

Q – Ans		Q – Ans		Q – Ans		Q – Ans		Q – Ans	
1	A	2	D	3	B	4	C	5	D
6	A	7	B	8	C	9	D	10	A
11	B	12	C	13	A	14	D	15	B