GAT B Mathematics

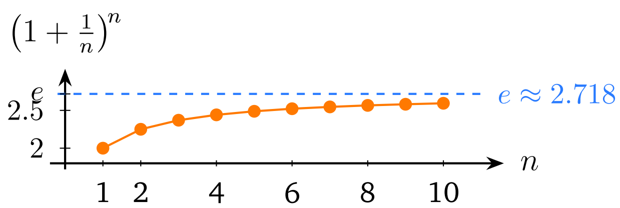
Sample Paper – 9

Duration: 23 Minutes **Maximum Marks: 15**

Instructions

- This paper contains **15** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics portion of **GAT B** entrance.
- Each correct answer carries **+1 mark**. There is a deduction of **0.5 mark** for each wrong answer. Unattempted questions carry zero marks.
- Only **one** option is correct. Choose carefully.
- Syllabus: Calculus, Linear Algebra, Probability & Statistics, Differential Equations, Coordinate Geometry, Sets & Functions, Vectors, Combinatorics (undergraduate level).
- Use of mobile phones, calculators, or electronic gadgets is strictly prohibited.

Q1. Evaluate $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$.



(Concept check)

(A) 1

(B) 2

(C) π

(D) e

Q2. Let $f(x) = x^2 - 3x$ on $[1, 4]$. By the **Mean Value Theorem**, find $c \in (1, 4)$ such that

$$f'(c) = \frac{f(4) - f(1)}{4 - 1}.$$

(Application)

(A) 1
(B) $\frac{5}{2}$
(C) 3
(D) $\frac{7}{2}$

Q3. Evaluate $\int \sin^2(x)\,dx$.

(Application)

(A) $\frac{x}{2} - \frac{\sin 2x}{4} + C$
(B) $-\cos(x)\sin(x) + C$
(C) $\frac{x}{2} + \frac{\sin 2x}{4} + C$
(D) $x - \sin 2x + C$

Q4. The matrix A shown below has $\text{rank}(A) = 2$. By the **Rank-Nullity Theorem**, what is the **nullity** of A?

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \quad \text{rank}(A) = 2, \quad n = 3$$

(Application)

(A) 0

(B) 2

(C) 1

(D) 3

Q5. The matrix $A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$ is diagonal with distinct diagonal entries. How many **linearly independent eigenvectors** does A possess?

(Concept check)

(A) 0

(B) 1

(C) It is not diagonalizable.

(D) 2

Q6. A bag contains 4 red and 6 blue balls. Three balls are drawn **without replacement**. What is the probability of drawing **exactly 2 red** balls?

(Application)

(A) $\dfrac{1}{5}$

(B) $\dfrac{3}{10}$

(C) $\dfrac{2}{5}$

(D) $\dfrac{1}{2}$

Q7. Find the **mode** of the data set $\{3,\ 7,\ 3,\ 5,\ 3,\ 7,\ 8,\ 7,\ 3\}$.

(Concept check)

(A) 3

(B) 5

(C) 7

(D) 8

Q8. The ODE $\dfrac{dy}{dx} = -\dfrac{x}{y}$ with initial condition $y(0) = 3$ describes which curve?

(Application)

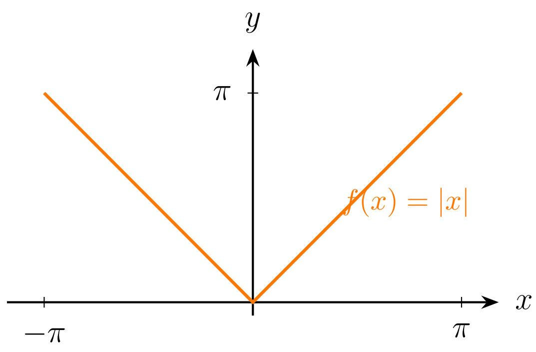
(A) Parabola

(B) Straight line

(C) Circle of radius 3

(D) Ellipse

Q9. For $f(x) = |x|$ on $[-\pi,\ \pi]$, the Fourier coefficient $a_0 = \dfrac{1}{\pi}\displaystyle\int_{-\pi}^{\pi} |x|\,dx$ equals:



(Application)

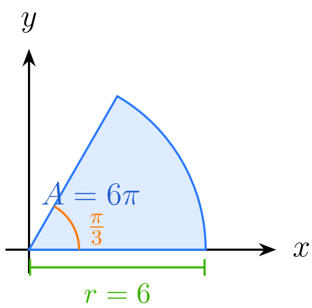
(A) π

(B) $\frac{\pi}{2}$

(C) 2π

(D) 0

Q10. Find the area of a circular sector with radius $r = 6$ and central angle $\theta = \frac{\pi}{3}$ radians.



(Application)

(A) 3π

(B) 4π

(C) 12π

(D) 6π

Q11. Which conic section does the equation $9x^2 + 4y^2 = 36$ represent?

(Concept check)

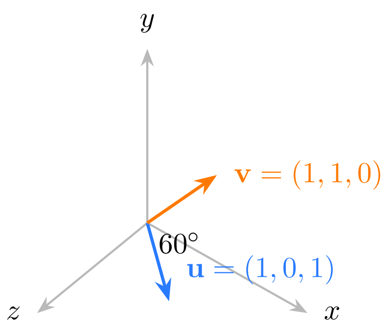
(A) Circle
(B) Ellipse
(C) Hyperbola
(D) Parabola

Q12. How many **bijections** (one-to-one onto functions) exist from the set $\{1, 2, 3\}$ to the set $\{a, b, c\}$?

(Concept check)

(A) 3
(B) 9
(C) 6
(D) 27

Q13. Find the angle between the vectors $\mathbf{u} = (1, 0, 1)$ and $\mathbf{v} = (1, 1, 0)$.



(Application)

(A) 30°

(B) 60°

(C) 90°

(D) 45°

Q14. In a class of 60 students, 35 study Mathematics, 25 study Physics, and 15 study **both**. How many students study **neither** subject?

(Application)

(A) 15

(B) 20

(C) 25

(D) 10

Q15. Compute $(1+i)(1-i)$.

(Concept check)

(A) 0

(B) $1+i^2$

(C) i

(D) 2

Detailed Solutions

Solution 1.

Solution

Concept: The limit $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ is the defining expression for Euler's number e.
Chapter: Limits/Continuity

Step 1 — Recognise the standard limit. This is one of the fundamental limits of analysis:

$$\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = e \approx 2.71828\ldots$$

Step 2 — Verify convergence numerically.

$$n=1:\ 2,\quad n=2:\ 2.25,\quad n=10:\approx 2.594,\quad n=100:\approx 2.705,\quad n\to\infty:\ \boxed{e}.$$

The sequence is strictly increasing and bounded above; it converges to e.

Why other options are wrong:

- (A) 1: the base $1+1/n\to 1$, but the exponent $n\to\infty$ creates the indeterminate form 1^∞, which resolves to e, not 1.
- (B) 2: the value only at $n=1$; the limit is strictly larger than 2.
- (C) $\pi\approx 3.14$: π is the circle constant; there is no connection to this limit.

Answer: (D) Go Back to Q 1

Solution 2.

Solution

Concept: Mean Value Theorem (MVT) — if f is continuous on $[a,b]$ and differentiable on (a,b), then there exists $c\in(a,b)$ with $f'(c)=\dfrac{f(b)-f(a)}{b-a}$. **Chapter:** Differential Calculus

Step 1 — Evaluate f at the endpoints.

$$f(x)=x^2-3x,\qquad f(4)=16-12=4,\qquad f(1)=1-3=-2.$$

Step 2 — Compute the average rate of change.

$$\frac{f(4)-f(1)}{4-1}=\frac{4-(-2)}{3}=\frac{6}{3}=2.$$

Step 3 — Solve $f'(c) = 2$.

$$f'(x) = 2x - 3 \;\Rightarrow\; 2c - 3 = 2 \;\Rightarrow\; c = \frac{5}{2} = \boxed{2.5}.$$

Check: $c = 5/2 \in (1, 4)$, confirming the MVT condition.

Why other options are wrong:

- (A) $c = 1$: an endpoint; the theorem requires c in the *open* interval $(1, 4)$.
- (C) $c = 3$: gives $f'(3) = 2(3) - 3 = 3 \neq 2$.
- (D) $c = 7/2$: gives $f'(7/2) = 2(7/2) - 3 = 4 \neq 2$.

Answer: (B) Go Back to Q 2

Solution 3.

Solution

Concept: Half-angle identity $\sin^2 x = \dfrac{1 - \cos 2x}{2}$. **Chapter:** Integral Calculus

Step 1 — Apply the identity.

$$\int \sin^2 x \, dx = \int \frac{1 - \cos 2x}{2} \, dx.$$

Step 2 — Integrate term by term.

$$= \frac{1}{2} \int 1 \, dx \;-\; \frac{1}{2} \int \cos 2x \, dx = \frac{x}{2} - \frac{\sin 2x}{4} + C.$$

Hence $\displaystyle\int \sin^2 x \, dx = \boxed{\frac{x}{2} - \frac{\sin 2x}{4} + C}$.

Why other options are wrong:

- (B) $-\cos x \sin x + C$: via the product-to-sum identity this equals $-\frac{1}{2}\sin 2x + C$, missing the linear term $x/2$.
- (C) $x/2 + \sin(2x)/4 + C$: wrong sign on the trigonometric part; integrating $-\cos 2x$ gives $-\sin 2x/2$, not $+\sin 2x/2$.
- (D) $x - \sin 2x + C$: each coefficient is twice the correct value.

Answer: (A) Go Back to Q 3

Solution 4.

Solution

Concept: Rank-Nullity Theorem: for an $m \times n$ matrix A, **Chapter:** Linear Algebra

$$\operatorname{rank}(A) + \operatorname{nullity}(A) = n.$$

Step 1 — Identify n (number of columns). A is 3×3, so $n = 3$.

Step 2 — Apply the theorem.

$$\operatorname{nullity}(A) = n - \operatorname{rank}(A) = 3 - 2 = \boxed{1}.$$

Verification: Row 3 of A equals $2 \times$ Row 2 − Row 1, confirming exactly one linearly dependent row and hence exactly one null-space vector (up to scaling).

Why other options are wrong:

- (A) 0: nullity $= 0$ only when the matrix has full column rank ($\operatorname{rank} = 3$), which is not the case.
- (B) 2: confuses nullity with rank.
- (D) 3: the total number of columns, not the nullity.

Answer: (C) Go Back to Q 4

Solution 5.

Solution

Concept: A diagonal matrix with *distinct* diagonal entries is automatically diagonalizable and has a full set of standard basis eigenvectors. **Chapter:** Linear Algebra

Step 1 — Read off eigenvalues.

$$A = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix} \implies \lambda_1 = 1, \quad \lambda_2 = 2.$$

Step 2 — Find eigenvectors.

$$\lambda_1 = 1: \quad (A - I)\mathbf{x} = \mathbf{0} \Rightarrow \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \mathbf{x} = \mathbf{0} \Rightarrow \mathbf{e}_1 = (1, 0)^\top.$$

$$\lambda_2 = 2: \quad (A - 2I)\mathbf{x} = \mathbf{0} \;\Rightarrow\; \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix}\mathbf{x} = \mathbf{0} \;\Rightarrow\; \mathbf{e}_2 = (0, 1)^{\top}.$$

Step 3 — Count. $\mathbf{e}_1$ and $\mathbf{e}_2$ are linearly independent, giving $\boxed{2}$ independent eigenvectors.

Why other options are wrong:

- (A) 0: every non-zero matrix with an eigenvalue has at least one eigenvector.
- (B) 1: a defective (non-diagonalizable) matrix of order 2 with a repeated eigenvalue may yield only 1; that is not the case here.
- (C) A diagonal matrix is always diagonalizable; this option is false.

Answer: (D) Go Back to Q 5

Solution 6.

Solution

Concept: Hypergeometric distribution: $P(\text{exactly } k \text{ red}) = \dfrac{\binom{K}{k}\binom{N-K}{n-k}}{\binom{N}{n}}$, where $N = 10$, $K = 4$, $n = 3$, $k = 2$. **Chapter:** Probability & Statistics

Step 1 — Favourable outcomes.

$$\binom{4}{2} \cdot \binom{6}{1} = 6 \times 6 = 36.$$

Step 2 — Total outcomes.

$$\binom{10}{3} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120.$$

Step 3 — Probability.

$$P = \frac{36}{120} = \frac{3}{10} = \boxed{0.3}.$$

Why other options are wrong:

- (A) $1/5$: uses $\binom{6}{1} = 5$ instead of the correct 6, or mis-evaluates $\binom{4}{2}$.
- (C) $2/5$: over-counts; arises from $72/120$ with an extra factor.
- (D) $1/2$: a rough over-estimate with no correct combinatorial basis.

Answer: (B) Go Back to Q 6

Solution 7.

Solution

Concept: The **mode** is the value appearing most frequently in a data set. **Chapter:** Probability & Statistics

Step 1 — Tally frequencies.

Value	Frequency
3	4
5	1
7	3
8	1

Step 2 — Identify the maximum frequency. The value 3 appears 4 times, more than any other value. Therefore

$$\text{Mode} = \boxed{3}.$$

Why other options are wrong:

- (B) 5: appears only once; the least frequent value alongside 8.
- (C) 7: appears 3 times, the second most frequent, but not the mode.
- (D) 8: appears only once.

Answer: (A) Go Back to Q 7

Solution 8.

Solution

Concept: Separate the ODE and integrate both sides. **Chapter:** Differential Equations

Step 1 — Separate variables.

$$\frac{dy}{dx} = -\frac{x}{y} \Rightarrow y\,dy = -x\,dx.$$

Step 2 — Integrate.

$$\int y\,dy = \int -x\,dx \;\Rightarrow\; \frac{y^2}{2} = -\frac{x^2}{2} + C \;\Rightarrow\; x^2 + y^2 = 2C.$$

Step 3 — Apply initial condition $y(0) = 3$.

$$0^2 + 3^2 = 2C \;\Rightarrow\; 2C = 9 \;\Rightarrow\; x^2 + y^2 = 9.$$

This is a **circle of radius** 3 centred at the origin: $\boxed{x^2 + y^2 = 9}$.

Why other options are wrong:

- (A) Parabola: arises from ODEs of the form $dy/dx = ax$, not $-x/y$.
- (B) Straight line: the ODE is non-linear; $y\,dy = -x\,dx$ cannot produce a linear curve.
- (D) Ellipse: requires unequal coefficients for x^2 and y^2; here both are 1.

Answer: (C) Go Back to Q 8

Solution 9.

Solution

Concept: Fourier coefficient $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$. **Chapter:** Integral Calculus

Step 1 — Exploit the even symmetry of $|x|$. Since $|x|$ is an even function:

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} |x|\,dx = \frac{2}{\pi}\int_0^{\pi} x\,dx.$$

Step 2 — Evaluate the integral.

$$\int_0^{\pi} x\,dx = \left[\frac{x^2}{2}\right]_0^{\pi} = \frac{\pi^2}{2}.$$

Step 3 — Compute a_0.

$$a_0 = \frac{2}{\pi}\cdot\frac{\pi^2}{2} = \boxed{\pi}.$$

Why other options are wrong:

- (B) $\pi/2$: arises if one does not use the symmetry doubling and evaluates $\frac{1}{\pi}\cdot\frac{\pi^2}{2}$.

- (C) 2π: off by a factor of 2; e.g. using $\frac{2}{\pi} \cdot \pi^2$ without halving.
- (D) 0: correct for an *odd* function; $|x|$ is even, so $a_0 \neq 0$.

Answer: (A) Go Back to Q 9

Solution 10.

Solution

Concept: Area of a circular sector with radius r and central angle θ (in radians):
Chapter: Coordinate Geometry

$$A = \frac{1}{2}r^2\theta.$$

Step 1 — Substitute $r = 6$ and $\theta = \pi/3$.

$$A = \frac{1}{2} \cdot 6^2 \cdot \frac{\pi}{3} = \frac{1}{2} \cdot 36 \cdot \frac{\pi}{3} = \frac{36\pi}{6} = \boxed{6\pi}.$$

Why other options are wrong:

- (A) 3π: uses $\frac{1}{2} \cdot r \cdot \theta = \frac{1}{2} \cdot 6 \cdot \frac{\pi}{3} = \pi$; or another combination missing r^2.
- (B) 4π: no standard formula for this sector gives 4π; likely a dimensional error.
- (C) 12π: omits the $\frac{1}{2}$ factor, computing $r^2\theta = 36 \cdot \frac{\pi}{3} = 12\pi$ instead.

Answer: (D) Go Back to Q 10

Solution 11.

Solution

Concept: Standard form of an ellipse: $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ with $a \neq b$. **Chapter:** Coordinate Geometry

Step 1 — Divide both sides by 36.

$$9x^2 + 4y^2 = 36 \Rightarrow \frac{x^2}{4} + \frac{y^2}{9} = 1.$$

Step 2 — Identify the conic. Here $a^2 = 4 \Rightarrow a = 2$ (semi-axis along x) and $b^2 = 9 \Rightarrow b = 3$ (semi-axis along y). Since $a \neq b$, this is an **ellipse** (not a circle).

Why other options are wrong:

- (A) Circle: requires equal denominators ($a = b$), i.e. coefficients 9 and 4 must match; they do not.
- (C) Hyperbola: requires one positive and one negative term: $x^2/a^2 - y^2/b^2 = 1$; both terms here are positive.
- (D) Parabola: contains only *one* squared variable; this equation has two.

Answer: (B) Go Back to Q 11

Solution 12.

Solution

Concept: The number of bijections from an n-element set to another n-element set equals $n!$. **Chapter:** Combinatorics

Step 1 — Apply the formula. Each bijection from $\{1, 2, 3\}$ to $\{a, b, c\}$ is a permutation of $\{a, b, c\}$:

$$\text{Number of bijections} = 3! = 3 \times 2 \times 1 = \boxed{6}.$$

Step 2 — List all bijections (verification).

- $1 \to a,\ 2 \to b,\ 3 \to c \quad 1 \to a,\ 2 \to c,\ 3 \to b \quad 1 \to b,\ 2 \to a,\ 3 \to c$
- $1 \to b,\ 2 \to c,\ 3 \to a \quad 1 \to c,\ 2 \to a,\ 3 \to b \quad 1 \to c,\ 2 \to b,\ 3 \to a$

Indeed exactly 6 distinct bijections.

Why other options are wrong:

- (A) 3: the cardinality of each set, not the count of bijections.
- (B) $9 = 3^2$: counts ordered pairs of elements, not functions.
- (D) $27 = 3^3$: the total number of *all* functions (including non-bijective ones) from $\{1, 2, 3\}$ to $\{a, b, c\}$.

Answer: (C) Go Back to Q 12

Solution 13.

Solution

Concept: Angle between vectors via $\cos\theta = \dfrac{\mathbf{u}\cdot\mathbf{v}}{|\mathbf{u}||\mathbf{v}|}$. **Chapter:** Vectors

Step 1 — Compute the dot product.

$$\mathbf{u}\cdot\mathbf{v} = (1)(1) + (0)(1) + (1)(0) = 1 + 0 + 0 = 1.$$

Step 2 — Compute the magnitudes.

$$|\mathbf{u}| = \sqrt{1^2 + 0^2 + 1^2} = \sqrt{2}, \qquad |\mathbf{v}| = \sqrt{1^2 + 1^2 + 0^2} = \sqrt{2}.$$

Step 3 — Find θ.

$$\cos\theta = \frac{1}{\sqrt{2}\cdot\sqrt{2}} = \frac{1}{2} \Rightarrow \theta = \cos^{-1}\left(\tfrac{1}{2}\right) = \boxed{60^\circ}.$$

Why other options are wrong:

- (A) 30°: corresponds to $\cos\theta = \sqrt{3}/2 \approx 0.866$; our value is $1/2$.
- (C) 90°: would require $\mathbf{u}\cdot\mathbf{v} = 0$; here it equals $1 \neq 0$.
- (D) 45°: corresponds to $\cos\theta = 1/\sqrt{2} \approx 0.707$; our value is 0.5.

Answer: (B) Go Back to Q 13

Solution 14.

Solution

Concept: Inclusion-Exclusion Principle: **Chapter:** Set Theory $|M \cup P| = |M| + |P| - |M \cap P|$.

Step 1 — Compute $|M \cup P|$.

$$|M \cup P| = 35 + 25 - 15 = 45.$$

Step 2 — Students studying neither.

$$\text{Neither} = \text{Total} - |M \cup P| = 60 - 45 = \boxed{15}.$$

Why other options are wrong:

- (B) 20: comes from $60 - 40 = 20$; 40 is an arithmetic error in applying inclusion-exclusion.
- (C) 25: the size of the Physics set; irrelevant by itself as an answer to "neither".
- (D) 10: arises from subtracting an extra 5 or from a double-counting error.

Answer: (A) Go Back to Q 14

Solution 15.

Solution

Concept: Product of a complex number and its conjugate: $(a+bi)(a-bi) = a^2+b^2$.
Chapter: Complex Numbers

Step 1 — Expand using FOIL.

$$(1+i)(1-i) = 1\cdot 1 - 1\cdot i + i\cdot 1 - i\cdot i = 1 - i + i - i^2.$$

Step 2 — Substitute $i^2 = -1$.

$$= 1 - (-1) = 1 + 1 = \boxed{2}.$$

Alternatively: Using the conjugate-product formula directly, $(1)^2+(1)^2 = 1+1 = 2$.

Why other options are wrong:

- (A) 0: would require $\mathrm{Re}(z_1\bar{z}_2) = 0$; here the product is purely real and positive.
- (B) $1 + i^2 = 1 + (-1) = 0$: this is just a partially written expression, not the full product.
- (C) i: the result is real; the imaginary parts cancel in $(1+i)(1-i)$.

Answer: (D) Go Back to Q 15

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	D	2	B	3	A	4	C	5	D
6	B	7	A	8	C	9	A	10	D
11	B	12	C	13	B	14	A	15	D