KEAM Mathematics

Sample Paper – 1

Duration: 150 Minutes **Maximum Marks: 480**

Instructions

- This paper contains **120 Multiple Choice Questions** (Single Correct Answer), modelled on **Paper II (Mathematics)** of the **KEAM** (Kerala Engineering Architecture Medical) entrance examination.
- Each question has **five options** (A, B, C, D, E); only **one** option is correct.
- Each correct answer carries $+4$ **marks**. Each incorrect answer carries -1 **mark**. Unattempted questions carry **0 marks**.
- If more than one option is marked, the response is treated as incorrect and attracts -1 mark.
- Personal calculators, log tables, mobile phones, and other electronic gadgets are **strictly prohibited**. Rough work must be done only in the space provided in the question booklet.
- Syllabus: **Class XI & XII Mathematics** (Kerala State Board / NCERT) — Algebra, Trigonometry, Coordinate Geometry, Vectors & 3D Geometry, Calculus, Statistics & Probability.

Q1. If $A = \{1, 2, 3, 4, 5\}$ and $B = \{4, 5, 6, 7\}$, the number of elements in $A \cup B$ is:

(A) 8

(B) 5

(C) 4

(D) 9

(E) 7

Q2. Let $A = \{1, 2, 3\}$ and $R = \{(1, 2), (2, 3), (3, 1)\}$ be a relation on A. The number of ordered pairs in $R \circ R$ is:

(A) 2

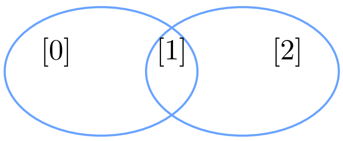
(B) 4

(C) 3

(D) 5

(E) 6

Q3. Which of the following is an equivalence relation on $\mathbb{Z}$?



Equivalence classes of $\equiv \pmod 3$

(A) aRb if $a > b$

(B) aRb if $a \neq b$

(C) aRb if $a \equiv b \pmod 3$

(D) aRb if a divides b

(E) aRb if $a + b = 0$

Q4. If $f(x) = \sqrt{x - 1}$ and $g(x) = \dfrac{1}{x - 2}$, the domain of $f \circ g$ is:

(A) $(1,\ 2]$

(B) $[2,\ 3)$

(C) $(1,\ 3]$

(D) $[1,\ 3)$

(E) $(2,\ 3]$

Q5. The range of the function $f(x) = |x| - 1$ is:

(A) $(-\infty,\ \infty)$

(B) $[-1, \infty)$

(C) $(-1, \infty)$

(D) $[0, \infty)$

(E) $(0, \infty)$

Q6. If $A = \{a, b, c\}$, the number of subsets of the power set $\mathcal{P}(A)$ is:

(A) 256

(B) 64

(C) 16

(D) 128

(E) 32

Q7. The polar form of $z = \sqrt{3} + i$ is:

(A) $2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

(B) $\sqrt{3}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

(C) $\sqrt{3}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

(D) $2\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

(E) $2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

Q8. If $z_1 = 3 + 4i$ and $z_2 = -4i$, then $|z_1 + z_2|$ equals:

(A) 5

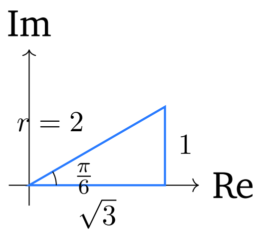
(B) $\sqrt{5}$

(C) 1

(D) 7

(E) 3

Q9. Using De Moivre's theorem, $\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^6$ equals:



(A) -1

(B) 0

(C) i

(D) 1

(E) $-i$

Q10. If $z = \dfrac{2+3i}{1-i}$, the real and imaginary parts of z are respectively:

(A) $-\dfrac{1}{2}$ and $\dfrac{5}{2}$

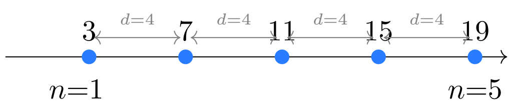
(B) $-\dfrac{1}{2}$ and $-\dfrac{5}{2}$

(C) 1 and 5

(D) 2 and 3

(E) $\dfrac{1}{2}$ and $\dfrac{5}{2}$

Q11. An arithmetic progression has first term $a = 3$ and common difference $d = 4$. The n-th term a_n is:



(A) $4n - 3$

(B) $4n + 1$

(C) $3n + 4$

(D) $4n - 1$

(E) $3n - 1$

Q12. The sum of the first 5 terms of a geometric progression with first term $a = 2$ and common ratio $r = 3$ is:

(A) 243

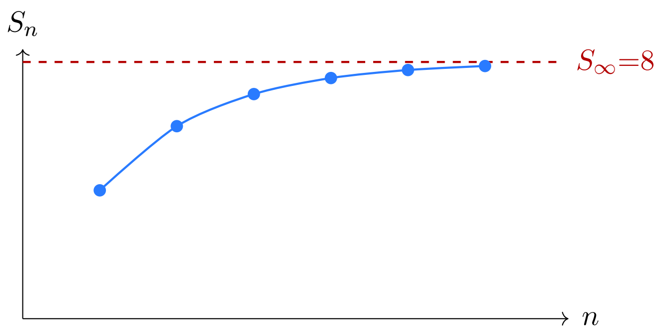
(B) 245

(C) 240

(D) 244

(E) 242

Q13. A geometric progression has first term $a = 4$ and common ratio $r = \frac{1}{2}$. Its sum to infinity is:



(A) 8

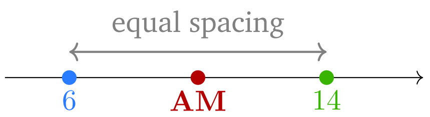
(B) 6

(C) 4

(D) 10

(E) 12

Q14. The arithmetic mean of the numbers 6 and 14 is:



(A) 8

(B) 10

(C) 12

(D) 9

(E) 11

Q15. In a geometric progression, the first term is $a = 6$ and the common ratio is $r = \frac{1}{3}$. The 4th term is:

(A) $\frac{2}{9}$

(B) $\frac{6}{9}$

(C) $\frac{1}{3}$

(D) $\frac{1}{9}$

(E) $\frac{2}{3}$

Q16. For the quadratic equation $2x^2 - 5x + 3 = 0$, the nature of roots is:

(A) Irrational and distinct

(B) Real and equal

(C) Complex (non-real)

(D) Real and distinct

(E) Purely imaginary

Q17. For the equation $x^2 - 7x + 12 = 0$, the sum and product of roots are respectively:

(A) Sum $= 7$, Product $= 12$

(B) Sum $= 12$, Product $= 7$

(C) Sum $= -7$, Product $= 12$

(D) Sum $= 7$, Product $= -12$

(E) Sum $= -7$, Product $= -12$

Q18. The quadratic equation whose roots are $\alpha = 2$ and $\beta = 5$ is:

(A) $x^2 + 7x + 10 = 0$

(B) $x^2 - 7x - 10 = 0$

(C) $x^2 - 5x + 10 = 0$

(D) $x^2 + 7x - 10 = 0$

(E) $x^2 - 7x + 10 = 0$

Q19. A quadratic equation with rational coefficients has one root $2 + \sqrt{3}$. The other root is:

(A) $2 + \sqrt{3}$

(B) $\sqrt{3} - 2$

(C) $2 - \sqrt{3}$

(D) $-2 + \sqrt{3}$

(E) $2\sqrt{3}$

Q20. The number of different arrangements of the letters of the word **MATHS** is:

(A) 24

(B) 60

(C) 120

(D) 100

(E) 90

Q21. Five people are to be arranged in a row. In how many ways can this be done if a particular person must be at one end of the row?

(A) 120

(B) 24

(C) 60

(D) 48

(E) 36

Q22. The value of $\binom{8}{3}$ is:

(A) 48

(B) 64

(C) 56

(D) 72

(E) 80

Q23. The number of ways of selecting a committee of 4 students from a group of 10 students is:

(A) 120

(B) 180

(C) 252

(D) 240

(E) 210

Q24. The coefficient of x^3 in the expansion of $(1 + x)^{10}$ is:

(A) 45

(B) 90

(C) 120

(D) 100

(E) 210

Q25. The coefficient of x^3 in the expansion of $(1 + 2x)^6$ is:

(A) 60

(B) 160

(C) 120

(D) 140

(E) 80

Q26. The middle term in the expansion of $\left(x + \dfrac{1}{x}\right)^8$ is:

(A) 28

(B) 56

(C) 70

(D) 8

(E) 1

Q27. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}$, the $(1,1)$ entry of AB is:

(A) 2

(B) 10

(C) 6

(D) 8

(E) 4

Q28. If A is a matrix of order 2×3, then the order of $A^\top$ is:

(A) 3×2

(B) 3×3

(C) 2×3

(D) 2×2

(E) 6×1

Q29. If A is of order 3×2 and B is of order 2×4, then AB has order:

(A) 2×2

(B) 3×2

(C) 2×4

(D) 3×4

(E) 4×3

Q30. For matrices $A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, which statement is correct?

(A) $AB \neq BA$ in general

(B) $AB = A^{\top}$

(C) $BA = B^{\top}$

(D) $AB = B$

(E) $AB = BA$ always holds

Q31. The value of $\begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{vmatrix}$ is:

(A) 0

(B) -1

(C) 1

(D) 2

(E) -2

Q32. The value of $\begin{vmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \\ 0 & 1 & 2 \end{vmatrix}$ is:

(A) 2

(B) -2

(C) 0

(D) 4

(E) -4

Q33. For the matrix $\begin{pmatrix} 3 & k & 2 \\ 1 & 4 & 1 \\ 2 & 3 & 1 \end{pmatrix}$ to be singular, k must equal:

(A) 7

(B) 5

(C) 3

(D) 9

(E) 11

Q34. The value of $\begin{vmatrix} 1 & 2 & 0 \\ 3 & 4 & 1 \\ 0 & 1 & 2 \end{vmatrix}$ is:

(A) -7

(B) -6

(C) -4

(D) -5

(E) -3

Q35. If $\sin A = \frac{3}{5}$ and $\sin B = \frac{5}{13}$, where A and B are acute angles, then $\sin(A + B)$ equals:

(A) $\frac{33}{65}$

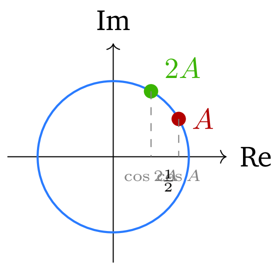
(B) $\frac{40}{65}$

(C) $\frac{56}{65}$

(D) $\frac{52}{65}$

(E) $\frac{48}{65}$

Q36. If $\sin A = \frac{1}{2}$, then $\cos 2A$ equals:



(A) $\frac{1}{2}$

(B) 0

(C) $\frac{\sqrt{3}}{2}$

(D) $-\frac{1}{2}$

(E) 1

Q37. If $\sin A = -\frac{4}{5}$ and A is in the third quadrant, then $\tan A$ equals:

(A) $-\frac{4}{3}$

(B) $-\frac{3}{4}$

(C) $\frac{3}{4}$

(D) $\frac{4}{3}$

(E) $\frac{5}{3}$

Q38. The period of the function $f(x) = \sin(3x)$ is:

(A) 2π

(B) π

(C) 3π

(D) $\frac{\pi}{3}$

(E) $\frac{2\pi}{3}$

Q39. The range of $f(x) = 2\sin x + 1$ is:

(A) $[-2, 2]$

(B) $[-1, 3]$

(C) $[0, 3]$

(D) $[-1, 1]$

(E) $[-2, 3]$

Q40. Which of the following statements about $f(x) = \cos x$ is correct?

(A) $\cos(0) = 0$

(B) $\cos(\pi) = 1$

(C) $\cos(0) = 1$

(D) The maximum value of $\cos x$ is 2

(E) $\cos\left(\frac{\pi}{2}\right) = 1$

Q41. The exact value of $\sin 60$ is:

(A) $\frac{1}{2}$

(B) $\frac{\sqrt{3}}{2}$

(C) 1

(D) $\frac{\sqrt{2}}{2}$

(E) 0

Q42. The principal value of $\sin^{-1}\left(\frac{1}{2}\right)$ is:

(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{2}$

(E) π

Q43. The value of $\cos^{-1}(-1)$ is:

(A) 0

(B) $\frac{\pi}{2}$

(C) π

(D) $\frac{3\pi}{2}$

(E) 2π

Q44. The value of $\tan^{-1}(1) + \tan^{-1}(\sqrt{3})$ is:

(A) $\dfrac{\pi}{4}$

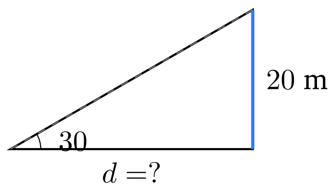
(B) $\dfrac{\pi}{3}$

(C) $\dfrac{7\pi}{12}$

(D) $\dfrac{\pi}{2}$

(E) $\dfrac{5\pi}{12}$

Q45. A vertical tower is $20\,\text{m}$ tall. The angle of elevation of the top of the tower from a point on the ground is 30. The distance of the point from the base of the tower is:



(A) $10\sqrt{3}$ m

(B) $20\sqrt{3}$ m

(C) 20 m

(D) $25\sqrt{3}$ m

(E) $15\sqrt{3}$ m

Q46. A flagpole casts a shadow of length $10\,\text{m}$ when the angle of elevation of the sun is 45. The height of the flagpole is:

(A) $5\sqrt{2}$ m

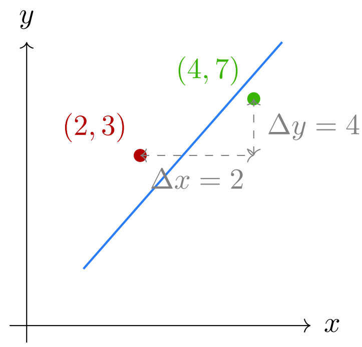
(B) 5 m

(C) $10\sqrt{2}$ m

(D) 10 m

(E) 20 m

Q47. The slope of the line passing through the points $(2, 3)$ and $(4, 7)$ is:



(A) 1

(B) 3

(C) 2

(D) 4

(E) $\frac{1}{2}$

Q48. The equation of the line with slope 3 passing through the point $(0, 2)$ is:

(A) $y = 3x + 2$

(B) $y = 2x + 3$

(C) $y = 3x - 2$

(D) $y = 3x$

(E) $y = 2x - 3$

Q49. The x-intercept of the line $3x + 4y = 12$ is:

(A) 3

(B) 6

(C) 2

(D) 8

(E) 4

Q50. The distance between the parallel lines $3x + 4y = 5$ and $3x + 4y = 20$ is:

(A) 3

(B) $\frac{5}{3}$

(C) 2

(D) $\frac{5}{2}$

(E) 1

Q51. The perpendicular distance from the point $(3,\ 4)$ to the line $3x - 4y + 5 = 0$ is:

(A) $\frac{1}{5}$

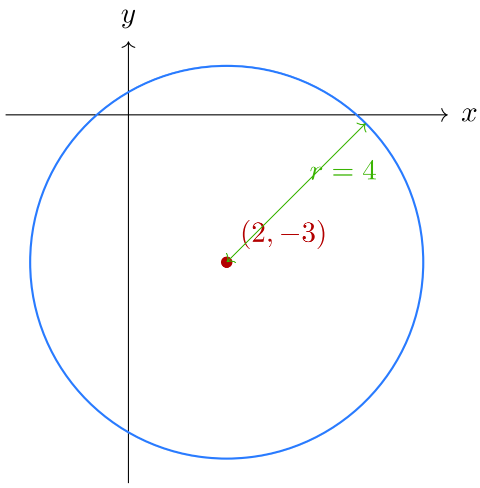
(B) $\frac{3}{5}$

(C) 1

(D) 2

(E) $\frac{2}{5}$

Q52. The equation of the circle with centre $(2,\ -3)$ and radius 4 is:



(A) $(x+2)^2 + (y-3)^2 = 16$

(B) $(x-2)^2 + (y+3)^2 = 4$

(C) $(x-2)^2 + (y-3)^2 = 16$

(D) $(x+2)^2 + (y+3)^2 = 16$

(E) $(x-2)^2+(y+3)^2=16$

Q53. The length of the tangent from the point $(5,\ 12)$ to the circle $x^2+y^2=144$ is:

(A) $\sqrt{17}$

(B) $3\sqrt{2}$

(C) 5

(D) $2\sqrt{5}$

(E) $\sqrt{10}$

Q54. The equation of the circle passing through the points $(0,0)$, $(4,0)$, and $(0,3)$ is:

(A) $x^2+y^2+4x+3y=0$

(B) $x^2+y^2-4x+3y=0$

(C) $x^2+y^2+4x-3y=0$

(D) $x^2+y^2-4x-3y=0$

(E) $x^2+y^2+3x+4y=0$

Q55. The radius of the circle $x^2+y^2-6x+4y-3=0$ is:

(A) 2

(B) 3

(C) 4

(D) $2\sqrt{3}$

(E) $\sqrt{3}$

Q56. The centre of the circle $2x^2+2y^2-4x+6y-10=0$ is:

(A) $(2,\ -3)$

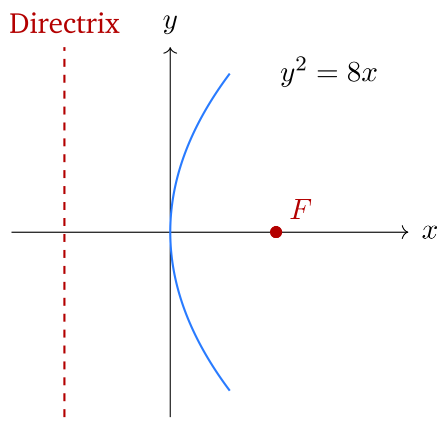
(B) $\left(-1,\ \frac{3}{2}\right)$

(C) $\left(1, -\frac{3}{2}\right)$

(D) $(-2, 3)$

(E) $\left(1, \frac{3}{2}\right)$

Q57. For the parabola $y^2 = 8x$, the focus and directrix are:



(A) Focus $(4, 0)$, Directrix $x = -4$

(B) Focus $(0, 4)$, Directrix $y = -4$

(C) Focus $(8, 0)$, Directrix $x = -8$

(D) Focus $(0, 2)$, Directrix $y = -2$

(E) Focus $(2, 0)$, Directrix $x = -2$

Q58. The vertex of the parabola $y = x^2 - 4x + 5$ is:

(A) $(2, 1)$

(B) $(2, -1)$

(C) $(0, 5)$

(D) $(-2, 1)$

(E) $(4, 5)$

Q59. For the ellipse $\frac{x^2}{25} + \frac{y^2}{9} = 1$, the semi-major axis a and semi-minor axis b are:

(A) $a = 9,\ b = 5$

(B) $a = 3,\ b = 5$

(C) $a = 5,\ b = 25$

(D) $a = 5,\ b = 3$

(E) $a = 25,\ b = 9$

Q60. The eccentricity of the ellipse $\dfrac{x^2}{16} + \dfrac{y^2}{7} = 1$ is:

(A) $\dfrac{3}{4}$

(B) $\dfrac{1}{2}$

(C) $\dfrac{\sqrt{7}}{4}$

(D) $\dfrac{7}{4}$

(E) $\dfrac{7}{16}$

Q61. The vertices of the hyperbola $\dfrac{x^2}{9} - \dfrac{y^2}{4} = 1$ are:

(A) $(0,\ \pm 3)$

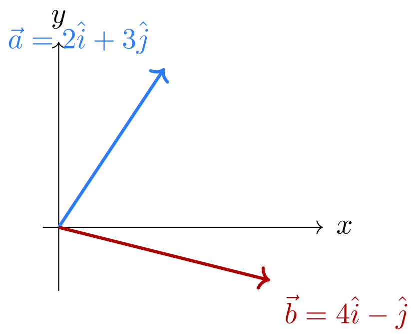
(B) $(\pm 3,\ 0)$

(C) $(\pm 2,\ 0)$

(D) $(0,\ \pm 2)$

(E) $(\pm 9,\ 0)$

Q62. If $\vec{a} = 2\hat{i} + 3\hat{j}$ and $\vec{b} = 4\hat{i} - \hat{j}$, then $\vec{a} \cdot \vec{b}$ equals:



(A) 3

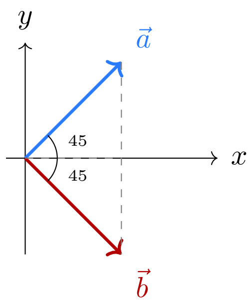
(B) 11

(C) 7

(D) 5

(E) 13

Q63. The angle between $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = \hat{i} - \hat{j}$ is:



(A) 0

(B) 45

(C) 120

(D) 90

(E) 135

Q64. The unit vector along $\vec{a} = 3\hat{i} + 4\hat{j}$ is:

(A) $\frac{1}{5}(3\hat{i} + 4\hat{j})$

(B) $\frac{1}{3}(3\hat{i} + 4\hat{j})$

(C) $\frac{1}{5}(4\hat{i} + 3\hat{j})$

(D) $\frac{1}{5}(3\hat{i} - 4\hat{j})$

(E) $\frac{1}{4}(3\hat{i} + 4\hat{j})$

Q65. If $\vec{a} = 2\hat{i} + k\hat{j}$ and $\vec{b} = 6\hat{i} + 3\hat{j}$ are collinear, the value of k is:

(A) 2

(B) 1

(C) 4

(D) 6

(E) 3

Q66. If $|\vec{a}| = 3$, $|\vec{b}| = 4$, and the angle between them is 30, then $|\vec{a} \times \vec{b}|$ equals:

(A) 6

(B) 12

(C) $6\sqrt{3}$

(D) $12\sqrt{3}$

(E) 3

Q67. If $\vec{a} = \hat{i}$, $\vec{b} = \hat{j}$, $\vec{c} = \hat{k}$, then $\vec{a} \cdot (\vec{b} \times \vec{c})$ equals:

(A) 0

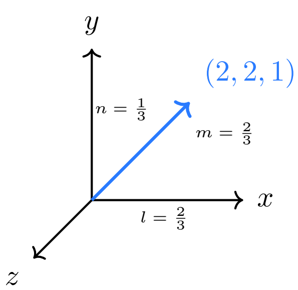
(B) 2

(C) -1

(D) 1

(E) 3

Q68. The direction cosines of the line with direction ratios $(2,\ 2,\ 1)$ are:



(A) $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$

(B) $\left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right)$

(C) $\left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$

(D) $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$

(E) $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$

Q69. Which of the following is a valid set of direction cosines?

(A) $\left(\frac{1}{3}, \frac{2}{3}, \frac{1}{3}\right)$

(B) $\left(\frac{1}{2}, \frac{1}{3}, \frac{1}{6}\right)$

(C) $(0, 1, 1)$

(D) $\left(\frac{2}{7}, \frac{3}{7}, \frac{6}{7}\right)$

(E) $\left(\frac{1}{2}, \frac{1}{2}, 1\right)$

Q70. The angle between two lines with direction ratios $(1, 0, 0)$ and $(0, 1, 0)$ is:

(A) 0

(B) 90

(C) 45

(D) 60

(E) 30

Q71. The distance between the points $(1, 2, 3)$ and $(4, 6, 3)$ is:

(A) 5

(B) 7

(C) $\sqrt{5}$

(D) $\sqrt{7}$

(E) 3

Q72. The midpoint of the line segment joining $(2, 4, 6)$ and $(8, 10, 12)$ is:

(A) $(3, 5, 7)$

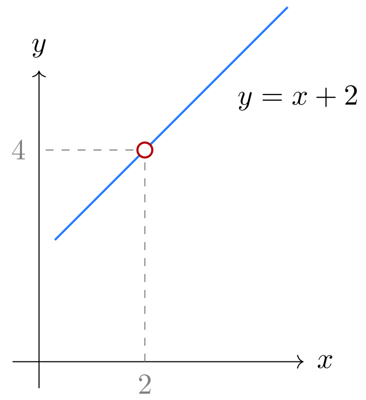
(B) $(5, 6, 9)$

(C) $(6, 7, 9)$

(D) $(4, 6, 8)$

(E) $(5, 7, 9)$

Q73. $\lim_{x \to 2} \dfrac{x^2 - 4}{x - 2}$ equals:



(A) 0

(B) 2

(C) 6

(D) 4

(E) 8

Q74. $\lim_{x \to 3} \dfrac{x^2 - 9}{x - 3}$ equals:

(A) 0

(B) 6

(C) 9

(D) 3

(E) ∞

Q75. Let $f(x) = \dfrac{x^2-1}{x-1}$ for $x \neq 1$ and $f(1) = k$. For f to be continuous at $x = 1$, k must equal:

(A) 2

(B) -1

(C) 1

(D) 3

(E) 0

Q76. $\lim_{x\to 1} \dfrac{x^3-1}{x-1}$ equals:

(A) 3

(B) 0

(C) 1

(D) 2

(E) ∞

Q77. For $f(x) = \dfrac{|x|}{x}$, $x \neq 0$, the value of $\lim_{x\to 0} f(x)$ is:

(A) 0

(B) ∞

(C) 1

(D) -1

(E) Does not exist

Q78. $\lim_{x\to 0} \dfrac{\sin x}{x}$ equals:

(A) 0

(B) ∞

(C) π

(D) -1

(E) 1

Q79. Using the product rule, $\frac{d}{dx}[x^2 \sin x]$ equals:

(A) $2x \cos x$

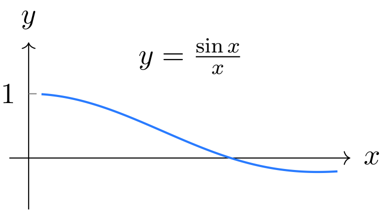
(B) $2x \sin x$

(C) $x^2 \cos x + \sin x$

(D) $2x \sin x + x^2 \cos x$

(E) $2x \sin x - x^2 \cos x$

Q80. Using the quotient rule, $\frac{d}{dx}\left[\frac{\sin x}{x}\right]$ equals:



(A) $\frac{\cos x}{x}$

(B) $\frac{\sin x - x \cos x}{x^2}$

(C) $\frac{x \cos x + \sin x}{x^2}$

(D) $\frac{x \cos x - \sin x}{x^2}$

(E) $-\frac{\cos x}{x}$

Q81. The derivative of $f(x) = x^5$ with respect to x is:

(A) $5x$

(B) $4x^5$

(C) x^5

(D) $5x^3$

(E) $5x^4$

Q82. The derivative of $\sin x$ with respect to x is:

(A) $-\cos x$

(B) $\cos x$

(C) $\sin x$

(D) $-\sin x$

(E) $\tan x$

Q83. The derivative of $\ln x$ with respect to x is:

(A) x

(B) $\frac{1}{x^2}$

(C) $-\frac{1}{x}$

(D) $\frac{\ln x}{x}$

(E) $\frac{1}{x}$

Q84. The derivative of $\sin(3x + 2)$ with respect to x is:

(A) $\cos(3x + 2)$

(B) $3\sin(3x + 2)$

(C) $-3\cos(3x + 2)$

(D) $3\cos(3x + 2)$

(E) $-\cos(3x + 2)$

Q85. The derivative of e^{2x} with respect to x is:

(A) e^{2x}

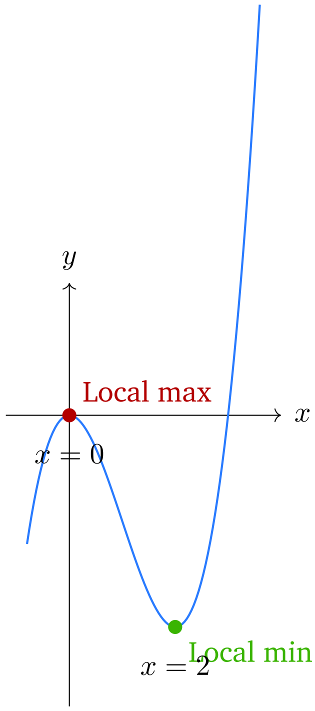
(B) e^x

(C) $2e^x$

(D) $2e^{2x}$

(E) $\dfrac{e^{2x}}{2}$

Q86. The critical points of $f(x) = x^3 - 3x^2$ are:



(A) $x = 0$ and $x = 3$

(B) $x = 1$ and $x = 3$

(C) $x = 0$ and $x = 2$

(D) $x = -1$ and $x = 2$

(E) $x = 1$ and $x = 2$

Q87. For $f(x) = x^3 - 6x^2 + 9x + 1$, the local maximum value of f is:

(A) 3

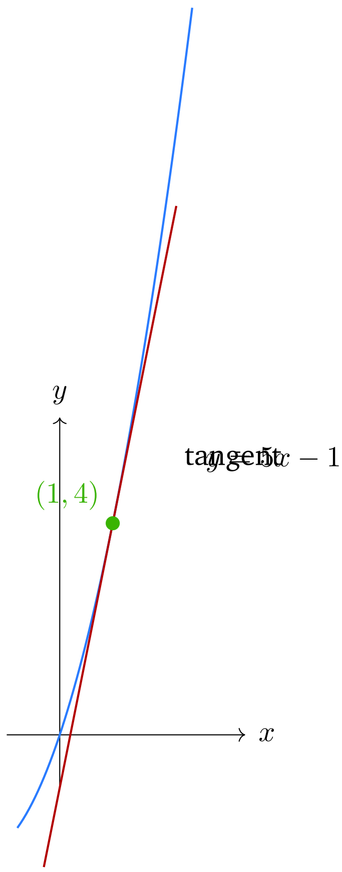
(B) 9

(C) 7

(D) 5

(E) 1

Q88. The equation of the tangent to the curve $y = x^2 + 3x$ at the point where $x = 1$ is:



(A) $y = 3x + 1$

(B) $y = 4x - 1$

(C) $y = x + 4$

(D) $y = 2x + 3$

(E) $y = 5x - 1$

Q89. The function $f(x) = x^2 - 4x + 3$ is increasing on the interval:

(A) $(-\infty,\ 2)$

(B) $(2,\ \infty)$

(C) $(0,\ 2)$

(D) $(-\infty,\ -2)$

(E) $(4,\ \infty)$

Q90. The slope of the normal to the curve $y = x^3$ at the point where $x = 1$ is:

(A) -3

(B) 3

(C) $\frac{1}{3}$

(D) $-\frac{1}{3}$

(E) 1

Q91. The area of a circle is $A = \pi r^2$. The rate of change of area with respect to radius r when $r = 5$ is:

(A) 25π

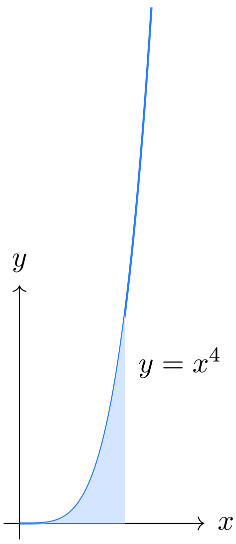
(B) 5π

(C) 10π

(D) 2π

(E) π

Q92. $\int x^4 \, dx$ equals:



(A) $\frac{x^3}{3} + C$

(B) $\frac{x^5}{5} + C$

(C) $4x^3 + C$

(D) $x^4 + C$

(E) $5x^5 + C$

Q93. $\int \sin x \, dx$ equals:

(A) $\cos x + C$

(B) $-\cos x + C$

(C) $-\sin x + C$

(D) $\sin x + C$

(E) $\cos x - \sin x + C$

Q94. $\int e^{3x} \, dx$ equals:

(A) $\dfrac{e^{3x}}{3} + C$

(B) $e^x + C$

(C) $e^{3x} + C$

(D) $3e^{3x} + C$

(E) $3e^x + C$

Q95. $\int \frac{1}{x} \, dx$ equals:

(A) $-\dfrac{1}{x^2} + C$

(B) $\ln|x| + C$

(C) $-\ln|x| + C$

(D) $\dfrac{1}{x} + C$

(E) $\dfrac{1}{x^2} + C$

Q96. $\int (2x+1)^3 \, dx$ equals:

(A) $\dfrac{(2x+1)^3}{3} + C$

(B) $\dfrac{(2x+1)^4}{4} + C$

(C) $\dfrac{(2x+1)^4}{2} + C$

(D) $\dfrac{(2x+1)^4}{8} + C$

(E) $2(2x+1)^4 + C$

Q97. Using the identity $\cos^2 x = \dfrac{1+\cos 2x}{2}$, $\displaystyle\int \cos^2 x\, dx$ equals:

(A) $\dfrac{\sin 2x}{2} + C$

(B) $\dfrac{x}{2} + \dfrac{\sin 2x}{4} + C$

(C) $x + \dfrac{\sin 2x}{2} + C$

(D) $\dfrac{x}{2} - \dfrac{\sin 2x}{4} + C$

(E) $\dfrac{\cos 2x}{4} + C$

Q98. $\displaystyle\int \cos x\, dx$ equals:

(A) $-\sin x + C$

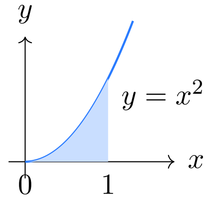
(B) $\cos x + C$

(C) $\sin x + C$

(D) $\tan x + C$

(E) $-\cos x + C$

Q99. The value of $\displaystyle\int_0^1 x^2\, dx$ is:



(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) $\frac{1}{4}$

(D) 1

(E) $\frac{2}{3}$

Q100. The value of $\int_0^{\pi} \sin x \, dx$ is:

(A) 0

(B) 2

(C) 1

(D) -2

(E) π

Q101. The value of $\int_0^{2} x \, dx$ is:

(A) 2

(B) 4

(C) 1

(D) 8

(E) 3

Q102. The area under the curve $y = x^2$ from $x = 0$ to $x = 3$ is:

(A) 6

(B) 27

(C) 12

(D) 18

(E) 9

Q103. The value of $\int_0^1 e^x \, dx$ is:

(A) e

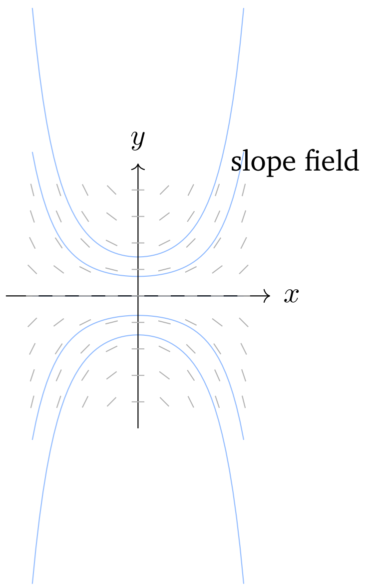
(B) $e - 1$

(C) 1

(D) $e + 1$

(E) $2e - 1$

Q104. The general solution of $\frac{dy}{dx} = xy$ is:



(A) $y = e^x + C$

(B) $y = Ce^x$

(C) $y = Ce^{x^2}$

(D) $y = Ce^{x/2}$

(E) $y = Ce^{x^2/2}$

Q105. The general solution of $\frac{dy}{dx} = x^2 + 1$ is:

(A) $y = \frac{x^3}{3} + x + C$

(B) $y = x^3 + x + C$

(C) $y = \dfrac{x^3}{3} + C$

(D) $y = \dfrac{x^2}{2} + x + C$

(E) $y = x^2 + C$

Q106. The order and degree of the differential equation $\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^3 + y = 0$ are:

(A) Order 3, Degree 1

(B) Order 1, Degree 2

(C) Order 2, Degree 1

(D) Order 2, Degree 3

(E) Order 1, Degree 3

Q107. The general solution of $\dfrac{dy}{dx} = \dfrac{y}{x}$ is:

(A) $y = x + C$

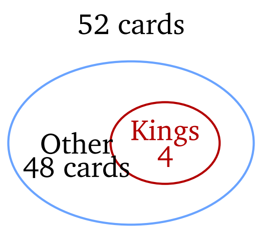
(B) $y = e^x + C$

(C) $y = kx^2$

(D) $y = \ln x + C$

(E) $y = kx$ (where k is an arbitrary constant)

Q108. A card is drawn at random from a well-shuffled standard deck of 52 cards. The probability that it is a king is:



(A) $\dfrac{1}{4}$

(B) $\dfrac{1}{26}$

(C) $\frac{1}{13}$

(D) $\frac{4}{13}$

(E) $\frac{1}{52}$

Q109. If $P(A) = 0.4$, $P(B) = 0.5$, and $P(A \cap B) = 0.2$, then $P(A \cup B)$ is:

(A) 0.5

(B) 0.7

(C) 0.9

(D) 0.6

(E) 0.8

Q110. If $P(A) = \frac{3}{7}$, then $P(A^c)$ (probability of complement of A) is:

(A) $\frac{4}{7}$

(B) $\frac{1}{7}$

(C) $\frac{1}{3}$

(D) $\frac{3}{4}$

(E) $\frac{3}{7}$

Q111. Two fair dice are rolled simultaneously. The probability that the sum of the numbers on the two dice is 7 is:

(A) $\frac{1}{9}$

(B) $\frac{1}{6}$

(C) $\frac{5}{36}$

(D) $\frac{7}{36}$

(E) $\frac{1}{12}$

Q112. If $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$, and A and B are independent events, then $P(A \cap B)$ is:

(A) $\frac{7}{12}$

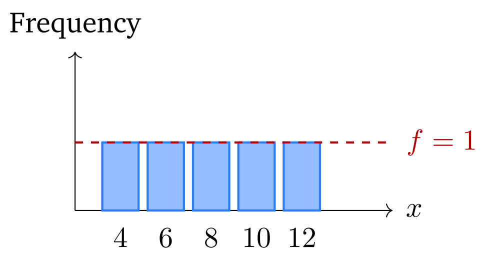
(B) $\frac{5}{12}$

(C) $\frac{1}{2}$

(D) $\frac{1}{6}$

(E) $\frac{1}{12}$

Q113. The arithmetic mean of the data $4,\ 6,\ 8,\ 10,\ 12$ is:



(A) 8

(B) 7

(C) 6

(D) 9

(E) 10

Q114. The variance of the data $2,\ 4,\ 6,\ 8,\ 10$ is:

(A) 4

(B) 8

(C) 2

(D) 6

(E) 10

Q115. The mean deviation about the mean of the data $3,\ 5,\ 7,\ 9,\ 11$ is:

(A) 0

(B) 1

(C) 1.5

(D) 2

(E) 2.4

Q116. If p is **True** and q is **True**, the truth value of $p \wedge q$ is:

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

(A) Depends on context

(B) False

(C) True

(D) Undefined

(E) Neither True nor False

Q117. The negation of $p \vee q$ (by De Morgan's law) is:

(A) $\neg p \vee \neg q$

(B) $\neg(p \wedge q)$

(C) $p \wedge q$

(D) $\neg p \wedge \neg q$

(E) $\neg p \vee q$

Q118. The feasible region determined by the constraints $x + y \leq 6$, $x \geq 0$, $y \geq 0$ is:

(A) Unbounded

(B) Empty

(C) A line segment

(D) A bounded triangle

(E) A rectangle

Q119. The vertices of the feasible region for the constraints $2x + y \leq 6$, $x + 2y \leq 6$, $x \geq 0$, $y \geq 0$ are:

(A) $(0, 0)$, $(6, 0)$, $(0, 6)$

(B) $(0, 0)$, $(3, 0)$, $(2, 2)$, $(0, 3)$

(C) $(0, 0)$, $(2, 0)$, $(0, 2)$

(D) $(0, 0)$, $(3, 0)$, $(0, 3)$

(E) $(0, 0)$, $(6, 0)$, $(2, 2)$, $(0, 6)$

Q120. Maximize $Z = 3x + 5y$ subject to $x + y \leq 4$, $x \geq 0$, $y \geq 0$. The maximum value of Z is:

(A) 12

(B) 20

(C) 16

(D) 18

(E) 15

Detailed Solutions

Solution

Q1.

Concept — Set Union and Cardinality: The union $A \cup B$ contains every element that belongs to at least one of the sets. By the inclusion-exclusion principle, $|A \cup B| = |A| + |B| - |A \cap B|$.

Step 1 — Identify the sets: $A = \{1, 2, 3, 4, 5\}$, so $|A| = 5$. $B = \{4, 5, 6, 7\}$, so $|B| = 4$.

Step 2 — Find the intersection: $A \cap B = \{4, 5\}$, so $|A \cap B| = 2$.

Step 3 — Apply inclusion-exclusion: $|A \cup B| = 5 + 4 - 2 = 7$.

Why other options are wrong:

- **Option B:** 5 ignores the elements of B not in A.
- **Option C:** 4 is just $|B|$, not the union.
- **Option D:** 9 = $|A| + |B|$ without subtracting the overlap.
- **Option A:** 8 overcounts by one.

Final Answer: $|A \cup B| = 7 \Rightarrow$ **Option (E)**

Answer: (E) ↑ Go Back to Q 1

Solution

Q2.

Concept — Composition of Relations: For relations R on a set, $R \circ R$ consists of all pairs (a, c) such that there exists an intermediate element b with $(a, b) \in R$ and $(b, c) \in R$.

Step 1 — List the relation: $R = \{(1, 2), (2, 3), (3, 1)\}$ on $A = \{1, 2, 3\}$.

Step 2 — Find $R \circ R$:

- $(1, 2) \in R$ and $(2, 3) \in R \Rightarrow (1, 3) \in R \circ R$
- $(2, 3) \in R$ and $(3, 1) \in R \Rightarrow (2, 1) \in R \circ R$
- $(3, 1) \in R$ and $(1, 2) \in R \Rightarrow (3, 2) \in R \circ R$

So $R \circ R = \{(1, 3), (2, 1), (3, 2)\}$, which has **3** ordered pairs.

Why other options are wrong:

- **Option A:** 2 — there are 3 valid compositions, not 2.
- **Option B:** 4 — only 3 pairs can be formed from 3 pairs in a cyclic relation.
- **Option D:** 5 and **Option E:** 6 — exceed what the 3-element relation can

generate.

Final Answer: $|R \circ R| = 3 \Rightarrow$ **Option (C)**

Answer: (C) ↑ Go Back to Q 2

Q3.

Solution

Concept — Equivalence Relations: A relation is an equivalence relation if it is simultaneously (i) reflexive, (ii) symmetric, and (iii) transitive.

Step 1 — Test Option C: aRb **iff** $a \equiv b \pmod 3$**:**

- *Reflexive:* $a \equiv a \pmod 3$ always. ✓
- *Symmetric:* If $a \equiv b$, then $b \equiv a$. ✓
- *Transitive:* If $a \equiv b$ and $b \equiv c$, then $a \equiv c$. ✓

All three properties hold, so this is an equivalence relation.

Why other options are wrong:

- **Option A:** $a > b$ is not reflexive ($a \not> a$).
- **Option B:** $a \neq b$ is not reflexive ($a = a$, so aRa fails).
- **Option D:** Divisibility is not symmetric ($2|4$ but $4 \nmid 2$).
- **Option E:** $a + b = 0$ is not transitive in general (if aRb and bRc, that gives $a = -b$ and $b = -c$, so $a = c$, but $a + c = 2a \neq 0$ unless $a = 0$).

Final Answer: Congruence mod 3 is an equivalence relation $\Rightarrow$ **Option (C)**

Answer: (C) ↑ Go Back to Q 3

Solution

Concept — Domain of Composite Functions: For $f \circ g$, we need x in the domain of g such that $g(x)$ lies in the domain of f.

Step 1 — Identify the functions: $g(x) = \dfrac{1}{x-2}$ requires $x \neq 2$. $f(x) = \sqrt{x-1}$ requires its argument ≥ 1.

Step 2 — Compose: $(f \circ g)(x) = f\left(\dfrac{1}{x-2}\right) = \sqrt{\dfrac{1}{x-2} - 1}$.

Step 3 — Set the radicand condition: $\dfrac{1}{x-2} - 1 \geq 0 \;\Rightarrow\; \dfrac{1-(x-2)}{x-2} \geq 0 \;\Rightarrow\; \dfrac{3-x}{x-2} \geq 0$.

Step 4 — Solve the inequality: The fraction is non-negative when both numerator and denominator have the same sign.

Q4.

- Both positive: $3 - x \geq 0$ and $x - 2 > 0$, i.e. $2 < x \leq 3$.
- Both negative: $3 - x < 0$ and $x - 2 < 0$, i.e. $x > 3$ and $x < 2$ — impossible.

Domain $= (2,\ 3]$.

Why other options are wrong:

- **Option A:** $(1, 2]$ includes $x < 2$ where $g(x) < 0$ and f is undefined.
- **Option B:** $[2, 3)$ includes $x = 2$ where g is undefined.
- **Option C:** $(1, 3]$ includes values between 1 and 2 that fail the inequality.
- **Option D:** $[1, 3)$ has similar issues and excludes $x = 3$ incorrectly.

Final Answer: Domain of $f \circ g$ is $(2,\ 3] \Rightarrow$ **Option (E)**

Answer: (E) ↑ Go Back to Q 4

Solution

Concept — Range of Absolute Value Functions: Since $|x| \geq 0$ for all real x, any expression of the form $|x| + k$ achieves a minimum of k.

Step 1 — Find the minimum: $|x| \geq 0$, so $|x| - 1 \geq -1$. The minimum -1 is achieved at $x = 0$.

Step 2 — Find the upper bound: As $|x| \to \infty$, $|x| - 1 \to \infty$. There is no upper bound.

Step 3 — State the range: Range $= [-1,\ \infty)$.

Why other options are wrong:

Q5.

- **Option A:** $(-\infty, \infty)$ would require f to take negative values less than -1, which it does not.
- **Option E:** $(0, \infty)$ excludes $[-1, 0)$ where f takes values (e.g. $f(0) = -1$).
- **Option C:** $(-1, \infty)$ excludes the minimum value -1 itself.
- **Option D:** $[0, \infty)$ excludes $[-1, 0)$.

Final Answer: Range of $f(x) = |x| - 1$ is $[-1,\ \infty) \Rightarrow$ **Option (B)**

Answer: (B) ↑ Go Back to Q 5

Q6.

Solution

Concept — Power Set and Its Cardinality: For a set A with n elements, $|\mathcal{P}(A)| = 2^n$. The number of subsets of $\mathcal{P}(A)$ is 2^{2^n}.

Step 1 — Find $|A|$: $A = \{a, b, c\}$, so $|A| = 3$.

Step 2 — Find $|\mathcal{P}(A)|$: $|\mathcal{P}(A)| = 2^3 = 8$.

Step 3 — Find the number of subsets of $\mathcal{P}(A)$: $\mathcal{P}(A)$ has 8 elements, so the number of its subsets $= 2^8 = 256$.

Why other options are wrong:

- **Option B:** $64 = 2^6$, which corresponds to subsets of $\mathcal{P}(A)$ only if $|A| = \log_2 6$, which is not an integer.
- **Option C:** $16 = 2^4$; this would be correct for a 2-element set.
- **Option D:** $128 = 2^7$; off by a factor of 2.
- **Option E:** $32 = 2^5$; this applies for a different interpretation.

Final Answer: Number of subsets of $\mathcal{P}(A)$ is $256 \Rightarrow$ **Option (A)**

Answer: (A) ↑ Go Back to Q 6

Q7.

Solution

Concept — Polar Form of a Complex Number: For $z = x + iy$, the polar form is $r(\cos\theta + i\sin\theta)$ where $r = |z| = \sqrt{x^2 + y^2}$ and $\theta = \arg(z) = \tan^{-1}(y/x)$.

Step 1 — Compute the modulus: $r = |\sqrt{3} + i| = \sqrt{(\sqrt{3})^2 + 1^2} = \sqrt{3+1} = 2$.

Step 2 — Compute the argument: $\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$.

Step 3 — Write the polar form: $z = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$.

Why other options are wrong:

- **Option A:** $\pi/4$ gives $\arg(1+i)$, not $\arg(\sqrt{3}+i)$.
- **Option C:** Modulus $\sqrt{3}$ is incorrect.
- **Option D:** $\pi/3$ would correspond to $z = 1 + i\sqrt{3}$ (roles of real and imaginary parts swapped).
- **Option B:** Both modulus and argument are wrong.

Final Answer: $\sqrt{3} + i = 2\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right) \Rightarrow$ **Option (E)**

Answer: (E) ↑ Go Back to Q 7

Q8.

Solution

Concept — Modulus of a Sum of Complex Numbers: First add the complex numbers, then find the modulus of the result.

Step 1 — Add the complex numbers: $z_1 + z_2 = (3+4i) + (-4i) = 3 + 0i = 3$.

Step 2 — Find the modulus: $|z_1 + z_2| = |3| = 3$.

Why other options are wrong:

- **Option A:** $5 = |z_1|$ alone; the imaginary parts cancel.
- **Option B:** $\sqrt{5}$ is incorrect numerically.
- **Option D:** 7 has no basis here.
- **Option C:** 1 has no basis here.

Final Answer: $|z_1 + z_2| = 3 \Rightarrow$ **Option (E)**

Answer: (E) ↑ Go Back to Q 8

Q9.

Solution

Concept — De Moivre's Theorem: $(\cos\theta + i\sin\theta)^n = \cos(n\theta) + i\sin(n\theta)$.

Step 1 — Apply the theorem: $\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)^6 = \cos\left(6\cdot\frac{\pi}{3}\right) + i\sin\left(6\cdot\frac{\pi}{3}\right) = \cos(2\pi) + i\sin(2\pi)$.

Step 2 — Evaluate: $\cos(2\pi) = 1$, $\sin(2\pi) = 0$. So the result is $1 + 0i = 1$.

Why other options are wrong:

- **Option A:** -1 would result from $n = 3$ (giving $\cos\pi + i\sin\pi$).
- **Option B:** 0 is not the real part at 2π.
- **Option C:** i would require $\cos\theta = 0$, $\sin\theta = 1$.
- **Option E:** $-i$ would require $\cos\theta = 0$, $\sin\theta = -1$.

Final Answer: The expression equals $1 \Rightarrow$ **Option (D)**

Answer: (D) ↑ Go Back to Q 9

Solution

Concept — Division of Complex Numbers: Multiply numerator and denominator by the conjugate of the denominator to obtain the standard form $a + bi$.

Step 1 — Multiply by conjugate: $\frac{2+3i}{1-i}\cdot\frac{1+i}{1+i} = \frac{(2+3i)(1+i)}{(1-i)(1+i)}$.

Step 2 — Expand: Denominator: $(1-i)(1+i) = 1 + 1 = 2$. Numerator: $(2+$

$3i)(1+i) = 2+2i+3i+3i^2 = 2+5i-3 = -1+5i$.

Step 3 — Simplify: $z = \dfrac{-1+5i}{2} = -\dfrac{1}{2} + \dfrac{5}{2}i$.

So $\mathrm{Re}(z) = -\dfrac{1}{2}$ and $\mathrm{Im}(z) = \dfrac{5}{2}$.

Why other options are wrong:

Q10.

- **Option E:** $1/2$ and $5/2$ — the real part has the wrong sign.
- **Option B:** $-1/2$ and $-5/2$ — the imaginary part has the wrong sign.
- **Option C:** 1 and 5 — fails to divide by 2.
- **Option D:** 2 and 3 — simply reads off the numerator coefficients.

Final Answer: Real part $= -\frac{1}{2}$, Imaginary part $= \frac{5}{2} \Rightarrow$ **Option (A)**

Answer: (A) ↑ Go Back to Q 10

Solution

Concept — n-th Term of an Arithmetic Progression: For an AP with first term a and common difference d, $a_n = a + (n-1)d$.

Step 1 — Substitute values: $a_n = 3 + (n-1)\cdot 4 = 3 + 4n - 4 = 4n - 1$.

Verification: $a_1 = 4(1) - 1 = 3$ ✓, $a_2 = 7$ ✓, $a_5 = 19$ ✓.

Why other options are wrong:

Q11.

- **Option B:** $4n+1$ gives $a_1 = 5 \neq 3$.
- **Option C:** $3n+4$ gives $a_1 = 7 \neq 3$.
- **Option A:** $4n-3$ gives $a_1 = 1 \neq 3$.
- **Option E:** $3n-1$ gives $a_1 = 2 \neq 3$.

Final Answer: $a_n = 4n - 1 \Rightarrow$ **Option (D)**

Answer: (D) ↑ Go Back to Q 11

Solution

Concept — Sum of n Terms of a GP: $S_n = \dfrac{a(r^n - 1)}{r-1}$ for $r \neq 1$.

Step 1 — Apply formula: $S_5 = \dfrac{2(3^5-1)}{3-1} = \dfrac{2(243-1)}{2} = \dfrac{2\times 242}{2} = 242$.

Verification by listing terms: $2+6+18+54+162 = 242$ ✓.

Why other options are wrong:

Q12.

- **Option A:** $243 = 3^5$, the fifth term, not the sum.
- **Option C:** 240 — short by 2.
- **Option D:** 244 — off by 2.
- **Option B:** 245 — off by 3.

Final Answer: $S_5 = 242 \Rightarrow$ **Option (E)**

Answer: (E) ↑ Go Back to Q 12

Solution

Concept — Sum to Infinity of a GP: When $|r| < 1$, the GP converges and $S_\infty = \dfrac{a}{1-r}$.

Step 1 — Check convergence: $|r| = \dfrac{1}{2} < 1$, so the series converges.

Step 2 — Apply formula: $S_\infty = \dfrac{4}{1-\frac{1}{2}} = \dfrac{4}{\frac{1}{2}} = 8$.

Why other options are wrong:

Q13.

- **Option C:** 4 is the first term, not the sum to infinity.
- **Option B:** 6 is $S_2 = 4 + 2$.
- **Option D:** 10 would require a different ratio.
- **Option E:** 12 would require $r = 2/3$.

Final Answer: $S_\infty = 8 \Rightarrow$ **Option (A)**

Answer: (A) ↑ Go Back to Q 13

Solution

Concept — Arithmetic Mean: The AM of two numbers p and q is $\dfrac{p+q}{2}$.

Step 1 — Compute: AM $= \dfrac{6+14}{2} = \dfrac{20}{2} = 10$.

Why other options are wrong:

Q14.

- **Option A:** 8 is $(14-6)/2$, the half-difference, not the mean.
- **Option D:** 9 is incorrect.
- **Option C:** 12 is incorrect.
- **Option E:** 11 is incorrect.

Final Answer: AM of 6 and 14 is $10 \Rightarrow$ **Option (B)**

Answer: (B) ↑ Go Back to Q 14

Solution

Concept — n-th Term of a GP: The n-th term of a GP is $t_n = ar^{n-1}$.

Step 1 — Find the 4th term: $t_4 = 6 \cdot \left(\frac{1}{3}\right)^{4-1} = 6 \cdot \frac{1}{27} = \frac{6}{27} = \frac{2}{9}$.

Why other options are wrong:

Q15.

- **Option E:** $\frac{2}{3} = 6 \cdot \frac{1}{3^2}$, which is t_3.
- **Option B:** $\frac{6}{9} = \frac{2}{3}$, same as t_3.
- **Option C:** $\frac{1}{3}$ corresponds to $a = 1$.
- **Option D:** $\frac{1}{9}$ corresponds to $a = 1, n = 3$.

Final Answer: The 4th term is $\frac{2}{9} \Rightarrow$ **Option (A)**

Answer: (A) ↑ Go Back to Q 15

Solution

Concept — Nature of Roots via Discriminant: For $ax^2 + bx + c = 0$, the discriminant $\Delta = b^2 - 4ac$. If $\Delta > 0$: real and distinct; $\Delta = 0$: real and equal; $\Delta < 0$: complex.

Step 1 — Identify coefficients: $2x^2 - 5x + 3 = 0$: $a = 2,\ b = -5,\ c = 3$.

Step 2 — Compute Δ: $\Delta = (-5)^2 - 4(2)(3) = 25 - 24 = 1 > 0$.

Since $\Delta > 0$, the roots are real and distinct.

Why other options are wrong:

Q16.

- **Option B:** Equal roots require $\Delta = 0$, but $\Delta = 1 \neq 0$.
- **Option C:** Complex roots require $\Delta < 0$, but $\Delta = 1 > 0$.
- **Option A:** Irrational roots require Δ to be a non-perfect-square positive integer; here $\Delta = 1$ is a perfect square, so roots are rational.
- **Option E:** Purely imaginary roots require $\Delta < 0$ and real part zero.

Final Answer: Roots are real and distinct ($\Delta = 1 > 0$) $\Rightarrow$ **Option (D)**

Answer: (D) ↑ Go Back to Q 16

Solution

Concept — Vieta's Formulae: For $x^2-(\alpha+\beta)x+\alpha\beta=0$: sum of roots $=\alpha+\beta=-b/a$, product $=\alpha\beta=c/a$.

Step 1 — Apply Vieta's formulae to $x^2-7x+12=0$: $a=1,\ b=-7,\ c=12$.

$$\text{Sum}=\frac{-(-7)}{1}=7,\qquad \text{Product}=\frac{12}{1}=12.$$

Why other options are wrong:

Q17.

- **Option B:** Swaps sum and product.
- **Option C:** Sum is -7 (wrong sign).
- **Option D:** Product is -12 (wrong sign).
- **Option E:** Both have wrong signs.

Final Answer: Sum $=7$, Product $=12\Rightarrow$ **Option (A)**

Answer: (A) ↑ Go Back to Q 17

Solution

Concept — Forming a Quadratic from Its Roots: If roots are α and β, then the quadratic is $x^2-(\alpha+\beta)x+\alpha\beta=0$.

Step 1 — Compute sum and product: $\alpha+\beta=2+5=7.\quad \alpha\beta=2\times5=10.$

Step 2 — Write the equation: $x^2-7x+10=0$.

Why other options are wrong:

Q18.

- **Option A:** $x^2+7x+10=0$ has sum $=-7$ (wrong sign on middle term).
- **Option B:** Negative product -10 contradicts $\alpha\beta=10$.
- **Option D:** Both sum and product signs are wrong.
- **Option C:** $x^2-5x+10=0$ uses β alone as the sum coefficient.

Final Answer: Quadratic is $x^2-7x+10=0\Rightarrow$ **Option (E)**

Answer: (E) ↑ Go Back to Q 18

Solution

Concept — Conjugate Irrational Roots: If a polynomial with rational coefficients has an irrational root $p+\sqrt{q}$, then $p-\sqrt{q}$ must also be a root (irrational conjugate theorem).

Step 1 — Identify the conjugate: Given root: $2+\sqrt{3}$. Conjugate root: $2-\sqrt{3}$.

Verification: Sum $= (2+\sqrt{3}) + (2-\sqrt{3}) = 4$ (rational). Product $= (2+\sqrt{3})(2-\sqrt{3}) = 4-3 = 1$ (rational). The equation $x^2 - 4x + 1 = 0$ has rational coefficients. ✓

Why other options are wrong:

Q19.

- **Option A:** $2+\sqrt{3}$ — a quadratic has at most 2 roots; both cannot be the same irrational.
- **Option B:** $\sqrt{3}-2 = -(2-\sqrt{3})$ — this is the negative of the conjugate.
- **Option D:** $-2+\sqrt{3}$ — differs in both parts from the correct conjugate.
- **Option E:** $2\sqrt{3}$ — not related to the conjugate pair.

Final Answer: The other root is $2-\sqrt{3} \Rightarrow$ **Option (C)**

Answer: (C) ↑ Go Back to Q 19

Solution

Concept — Permutations of Distinct Letters: The number of arrangements of n distinct objects is $n!$.

Step 1 — Count the letters: MATHS has 5 distinct letters: M, A, T, H, S.

Step 2 — Apply the formula: Number of arrangements $= 5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$.

Why other options are wrong:

Q20.

- **Option A:** 24 $= 4!$, valid for 4-letter arrangements.
- **Option B:** 60 $= 5!/2!$, which applies when one letter repeats.
- **Option E:** 90 and **Option D:** 100 have no combinatorial basis here.

Final Answer: Arrangements of MATHS $= 120 \Rightarrow$ **Option (C)**

Answer: (C) ↑ Go Back to Q 20

Solution

Concept — Permutations with Restrictions: Fix the constrained person first, then count arrangements of the remaining people.

Step 1 — Place the fixed person: The particular person can occupy either end of the row: 2 choices.

Step 2 — Arrange the remaining 4 people: The remaining 4 people can be arranged in $4! = 24$ ways.

Step 3 — Multiply: Total $= 2 \times 24 = 48$.

Why other options are wrong:

Q21.

- **Option B:** 24 counts only one end.
- **Option C:** 60 overcounts arrangements.
- **Option A:** $120 = 5!$ ignores the restriction.
- **Option E:** 36 has no combinatorial basis here.

Final Answer: Total arrangements $= 48 \Rightarrow$ **Option (D)**

Answer: (D) ↑ Go Back to Q 21

Solution

Concept — Combination Formula: $\binom{n}{r} = \dfrac{n!}{r!(n-r)!}$.

Step 1 — Compute $\binom{8}{3}$: $\binom{8}{3} = \dfrac{8!}{3! \cdot 5!} = \dfrac{8 \times 7 \times 6}{3 \times 2 \times 1} = \dfrac{336}{6} = 56$.

Why other options are wrong:

Q22.

- **Option A:** $48 = 8 \times 6$, partial product without denominator.
- **Option B:** 64 is $8^2/1$, no combinatorial meaning here.
- **Option D:** 72 and **Option E:** 80 are incorrect computations.

Final Answer: $\binom{8}{3} = 56 \Rightarrow$ **Option (C)**

Answer: (C) ↑ Go Back to Q 22

Solution

Concept — Selecting a Committee: Order does not matter for committee selection, so use combinations.

Step 1 — Apply $\binom{10}{4}$: $\binom{10}{4} = \dfrac{10!}{4! \cdot 6!} = \dfrac{10 \times 9 \times 8 \times 7}{4 \times 3 \times 2 \times 1} = \dfrac{5040}{24} = 210$.

Why other options are wrong:

Q23.

- **Option A:** $120 = \binom{10}{3}$, selecting 3 not 4.
- **Option B:** 180 is incorrect.
- **Option D:** 240 is incorrect.
- **Option C:** $252 = \binom{10}{5}$, selecting 5 not 4.

Final Answer: Number of committees $= 210 \Rightarrow$ **Option (E)**

Answer: (E) ↑ Go Back to Q 23

Solution

Concept — Binomial Coefficients: The coefficient of x^r in $(1+x)^n$ is $\binom{n}{r}$.

Step 1 — Apply the formula: Coefficient of x^3 in $(1+x)^{10} = \binom{10}{3} = \dfrac{10 \times 9 \times 8}{3 \times 2 \times 1} = \dfrac{720}{6} = 120$.

Why other options are wrong:

Q24.

- **Option A:** $45 = \binom{10}{2}$, the coefficient of x^2.
- **Option B:** $90 = 2\binom{10}{2}$, incorrect.
- **Option D:** 100 is incorrect.
- **Option E:** $210 = \binom{10}{4}$, the coefficient of x^4.

Final Answer: Coefficient of x^3 is $120 \Rightarrow$ **Option (C)**

Answer: (C) ↑ Go Back to Q 24

Solution

Concept — Binomial Theorem with a Coefficient: The general term in $(1+ax)^n$ is $T_{r+1} = \binom{n}{r}(ax)^r$. The coefficient of x^r is $\binom{n}{r}a^r$.

Step 1 — Find the coefficient of x^3 in $(1+2x)^6$: $\binom{6}{3}(2)^3 = 20 \times 8 = 160$.

Why other options are wrong:

Q25.

- **Option A:** $60 = \binom{6}{3} \times 3$, uses factor 3 instead of $2^3 = 8$.
- **Option E:** $80 = \binom{6}{3} \times 4$, uses $2^2 = 4$ instead of 2^3.
- **Option C:** $120 = \binom{6}{3} \times 6$, incorrect multiplier.
- **Option D:** 140 is incorrect.

Final Answer: Coefficient of x^3 is $160 \Rightarrow$ **Option (B)**

Answer: (B) ↑ Go Back to Q 25

Solution

Concept — Middle Term of a Binomial Expansion: For $(a+b)^n$ with even n, the middle term is $T_{n/2+1}$.

Step 1 — Identify the middle term: $n = 8$, so the middle term is T_5 (the 5th term).

Step 2 — Compute T_5: $T_5 = \binom{8}{4}x^{8-4}\left(\dfrac{1}{x}\right)^4 = \binom{8}{4} \cdot x^4 \cdot x^{-4} = \binom{8}{4} = \dfrac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} =$

$\frac{1680}{24} = 70.$

Why other options are wrong:

Q26.

- **Option B:** $56 = \binom{8}{3}$, corresponds to T_4.
- **Option A:** $28 = \binom{8}{2}$, corresponds to T_3.
- **Option D:** $8 = \binom{8}{1}$, corresponds to T_2.
- **Option E:** $1 = \binom{8}{0}$, corresponds to T_1.

Final Answer: Middle term (coefficient) $= 70 \Rightarrow$ **Option (C)**

Answer: (C) ↑ Go Back to Q 26

Solution

Concept — Matrix Multiplication: The (i, j) entry of AB is the dot product of the i-th row of A with the j-th column of B.

Step 1 — Compute the $(1, 1)$ **entry of** AB**:** Row 1 of $A = [1, 2]$. Column 1 of $B = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$. $(AB)_{11} = 1 \times 0 + 2 \times 2 = 0 + 4 = 4.$

Why other options are wrong:

Q27.

- **Option A:** 2 would result from $1 \times 0 + 2 \times 1 = 2$, mixing columns.
- **Option C:** 6, **Option D:** 8, **Option B:** 10 are incorrect row-column products.

Final Answer: $(AB)_{11} = 4 \Rightarrow$ **Option (E)**

Answer: (E) ↑ Go Back to Q 27

Solution

Concept — Transpose of a Matrix: If A has order $m \times n$, then $A^\top$ has order $n \times m$.

Step 1 — Apply the rule: A is 2×3, so $A^\top$ is 3×2.

Why other options are wrong:

Q28.

- **Option C:** 2×3 — that is A itself, not its transpose.
- **Option B:** 3×3 — square matrices arise only when $m = n$.
- **Option D:** 2×2 — incorrect dimensions.
- **Option E:** 6×1 — that would be a vectorization, not transpose.

Final Answer: $A^\top$ has order $3 \times 2 \Rightarrow$ **Option (A)**

Answer: (A) ↑ Go Back to Q 28

Solution

Concept — Order of a Matrix Product: If A is $m \times p$ and B is $p \times n$, then AB is $m \times n$.

Step 1 — Identify dimensions: A is 3×2 and B is 2×4. The inner dimensions match (both 2).

Step 2 — State the result order: AB is 3×4.

Why other options are wrong:

Q29.

- **Option A:** 2×2 — wrong on both dimensions.
- **Option B:** 3×2 — retains A's size.
- **Option C:** 2×4 — retains B's size.
- **Option E:** 4×3 — reverses the correct order.

Final Answer: AB has order $3 \times 4 \Rightarrow$ **Option (D)**

Answer: (D) ↑ Go Back to Q 29

Solution

Concept — Matrix Multiplication is Not Commutative: In general, $AB \neq BA$ for matrices, even when both products exist.

Step 1 — Compute AB: $AB = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$.

Step 2 — Compute BA: $BA = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$.

Since $AB = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \neq \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} = BA$, we confirm $AB \neq BA$.

Why other options are wrong:

Q30.

- **Option E:** False; we just showed $AB \neq BA$.
- **Option B:** $A^\top = A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \neq AB$.
- **Option C:** $B^\top = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \neq BA = 0$.
- **Option D:** $AB \neq B$.

Final Answer: $AB \neq BA$ in general $\Rightarrow$ **Option (A)**

Answer: (A) ↑ Go Back to Q 30

Solution

Concept — Cofactor Expansion: Expand along the first row: $\det(A) = a_{11}M_{11} - a_{12}M_{12} + a_{13}M_{13}$, where M_{ij} are the 2×2 minors.

Step 1 — Expand along row 1:

$$\begin{vmatrix} 1 & 2 & 3 \\ 0 & 1 & 4 \\ 5 & 6 & 0 \end{vmatrix} = 1\begin{vmatrix} 1 & 4 \\ 6 & 0 \end{vmatrix} - 2\begin{vmatrix} 0 & 4 \\ 5 & 0 \end{vmatrix} + 3\begin{vmatrix} 0 & 1 \\ 5 & 6 \end{vmatrix}.$$

Step 2 — Compute each 2×2 minor: $\begin{vmatrix} 1 & 4 \\ 6 & 0 \end{vmatrix} = 0 - 24 = -24$. $\begin{vmatrix} 0 & 4 \\ 5 & 0 \end{vmatrix} = 0 - 20 = -20$. $\begin{vmatrix} 0 & 1 \\ 5 & 6 \end{vmatrix} = 0 - 5 = -5$.

Step 3 — Combine: $= 1(-24) - 2(-20) + 3(-5) = -24 + 40 - 15 = 1$.

Why other options are wrong:

Q31.
- **Option B:** -1 — sign error in one minor.
- **Option A:** 0 — incorrect; the determinant is non-zero.
- **Option D:** 2 and **Option E:** -2 — arithmetic errors.

Final Answer: Determinant $= 1 \Rightarrow$ **Option (C)**

Answer: (C) ↑ Go Back to Q 31

Solution

Concept — Row Operations and Determinants: Expanding directly or noting linear dependence of rows: if one row is a linear combination of others, the determinant is zero.

Step 1 — Check for row dependency: Observe $R_1 = 2R_2$ fails (R1 = [2,4,6], 2R2 = [2,6,10]). Let us expand directly.

Step 2 — Expand along row 1:

$$\begin{vmatrix} 2 & 4 & 6 \\ 1 & 3 & 5 \\ 0 & 1 & 2 \end{vmatrix} = 2(3 \cdot 2 - 5 \cdot 1) - 4(1 \cdot 2 - 5 \cdot 0) + 6(1 \cdot 1 - 3 \cdot 0).$$

$$= 2(6 - 5) - 4(2 - 0) + 6(1 - 0) = 2(1) - 4(2) + 6(1) = 2 - 8 + 6 = 0.$$

Why other options are wrong:

Q32.
- **Option A:** 2 — sign or coefficient error.

- **Option B:** -2 — sign error.
- **Option D:** 4 and **Option E:** -4 — incorrect computations.

Final Answer: Determinant $= 0 \Rightarrow$ **Option (C)**

Answer: (C) ↑ Go Back to Q 32

Solution

Concept — Singular Matrix: A matrix is singular when its determinant equals zero.

Step 1 — Set up the determinant:

$$\begin{vmatrix} 3 & k & 2 \\ 1 & 4 & 1 \\ 2 & 3 & 1 \end{vmatrix} = 0.$$

Step 2 — Expand along row 1: $= 3(4 \cdot 1 - 1 \cdot 3) - k(1 \cdot 1 - 1 \cdot 2) + 2(1 \cdot 3 - 4 \cdot 2)$ $= 3(4-3) - k(1-2) + 2(3-8) = 3(1) - k(-1) + 2(-5) = 3 + k - 10 = k - 7.$

Step 3 — Solve: $k - 7 = 0 \Rightarrow k = 7.$

Why other options are wrong:

Q33.

- **Option C:** $k = 3$ gives determinant $= 3 - 7 = -4 \neq 0.$
- **Option B:** $k = 5$ gives determinant $= -2 \neq 0.$
- **Option D:** $k = 9$ gives determinant $= 2 \neq 0.$
- **Option E:** $k = 11$ gives determinant $= 4 \neq 0.$

Final Answer: $k = 7 \Rightarrow$ **Option (A)**

Answer: (A) ↑ Go Back to Q 33

Solution

Concept — Cofactor Expansion of a 3×3 Determinant.

Step 1 — Expand along row 1:

$$\begin{vmatrix} 1 & 2 & 0 \\ 3 & 4 & 1 \\ 0 & 1 & 2 \end{vmatrix} = 1 \begin{vmatrix} 4 & 1 \\ 1 & 2 \end{vmatrix} - 2 \begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix} + 0 \cdot (\ldots).$$

Step 2 — Compute the 2×2 minors: $\begin{vmatrix} 4 & 1 \\ 1 & 2 \end{vmatrix} = 8 - 1 = 7$. $\begin{vmatrix} 3 & 1 \\ 0 & 2 \end{vmatrix} = 6 - 0 = 6$.

Step 3 — Combine: $= 1(7) - 2(6) + 0 = 7 - 12 = -5$.

Why other options are wrong:

Q34.

- **Option A:** -7 — uses the first minor but forgets to subtract the second.
- **Option B:** -6 — arithmetic error.
- **Option C:** -4 — arithmetic error.
- **Option E:** -3 — arithmetic error.

Final Answer: Determinant $= -5 \Rightarrow$ **Option (D)**

Answer: (D) ↑ Go Back to Q 34

Solution

Concept — Addition Formula for Sine: $\sin(A + B) = \sin A \cos B + \cos A \sin B$.

Step 1 — Find missing values: $\sin A = \frac{3}{5}$, A acute $\Rightarrow \cos A = \frac{4}{5}$. $\sin B = \frac{5}{13}$, B acute $\Rightarrow \cos B = \frac{12}{13}$.

Step 2 — Apply the formula: $\sin(A + B) = \dfrac{3}{5} \cdot \dfrac{12}{13} + \dfrac{4}{5} \cdot \dfrac{5}{13} = \dfrac{36}{65} + \dfrac{20}{65} = \dfrac{56}{65}$.

Why other options are wrong:

Q35.

- **Option A:** $\frac{33}{65}$ — incorrect partial products.
- **Option B:** $\frac{40}{65}$ — only one term counted.
- **Option E:** $\frac{48}{65}$ — arithmetic error.
- **Option D:** $\frac{52}{65}$ — arithmetic error.

Final Answer: $\sin(A + B) = \dfrac{56}{65} \Rightarrow$ **Option (C)**

Answer: (C) ↑ Go Back to Q 35

Solution

Concept — Double Angle Formula: $\cos 2A = 1 - 2\sin^2 A$.

Step 1 — Substitute $\sin A = \frac{1}{2}$: $\cos 2A = 1 - 2\left(\dfrac{1}{2}\right)^2 = 1 - 2 \cdot \dfrac{1}{4} = 1 - \dfrac{1}{2} = \dfrac{1}{2}$.

Verification: $A = 30$, so $2A = 60$ and $\cos 60 = \frac{1}{2}$. ✓

Why other options are wrong:

Q36.

- **Option B:** 0 would require $\sin A = \frac{1}{\sqrt{2}}$.

- **Option C:** $\frac{\sqrt{3}}{2}$ would require $A = 15$, giving $\sin 15 \neq \frac{1}{2}$.
- **Option D:** $-\frac{1}{2}$ would require $\sin A = \frac{\sqrt{3}}{2}$.
- **Option E:** 1 would require $\sin A = 0$.

Final Answer: $\cos 2A = \dfrac{1}{2} \Rightarrow$ **Option (A)**

Answer: (A) ↑ Go Back to Q 36

Q37.

Solution

Concept — Signs of Trigonometric Functions by Quadrant: In quadrant III, both $\sin$ and $\cos$ are negative, so $\tan = \sin / \cos$ is positive.

Step 1 — Find $\cos A$**:** $\sin^2 A + \cos^2 A = 1 \Rightarrow \cos^2 A = 1 - \frac{16}{25} = \frac{9}{25} \Rightarrow \cos A = -\frac{3}{5}$ (negative in Q3).

Step 2 — Compute $\tan A$**:** $\tan A = \dfrac{\sin A}{\cos A} = \dfrac{-4/5}{-3/5} = \dfrac{4}{3}$.

Why other options are wrong:

- **Option A:** $-\frac{4}{3}$ — would be $\tan A$ in Q2 or Q4.
- **Option C:** $\frac{3}{4}$ — reciprocal of the correct answer.
- **Option B:** $-\frac{3}{4}$ — wrong sign and inverted.
- **Option E:** $\frac{5}{3}$ — involves the hypotenuse, not a valid ratio here.

Final Answer: $\tan A = \dfrac{4}{3} \Rightarrow$ **Option (D)**

Answer: (D) ↑ Go Back to Q 37

Q38.

Solution

Concept — Period of Trigonometric Functions: The period of $\sin(bx)$ is $\dfrac{2\pi}{|b|}$.

Step 1 — Apply the formula: For $f(x) = \sin(3x)$, $b = 3$. Period $= \dfrac{2\pi}{3}$.

Why other options are wrong:

- **Option A:** 2π is the period of $\sin x$ (when $b = 1$).
- **Option B:** π is the period of $\sin(2x)$.
- **Option D:** $\dfrac{\pi}{3}$ is too small by a factor of 2.
- **Option C:** 3π — inverts the relationship between b and period.

Final Answer: Period of $\sin(3x)$ is $\frac{2\pi}{3} \Rightarrow$ **Option (E)**

Answer: (E) ↑ Go Back to Q 38

Solution

Q39.

Concept — Range of Transformed Sine Functions: Since $-1 \leq \sin x \leq 1$, we have $-2 \leq 2\sin x \leq 2$, and thus $-1 \leq 2\sin x + 1 \leq 3$.

Step 1 — Find bounds: Minimum: $2(-1) + 1 = -1$. Maximum: $2(1) + 1 = 3$.

Step 2 — State range: Range $= [-1, 3]$.

Why other options are wrong:

- **Option A:** $[-2, 2]$ is the range of $2\sin x$ before adding 1.
- **Option D:** $[-1, 1]$ is the range of $\sin x$.
- **Option C:** $[0, 3]$ misses the minimum -1.
- **Option E:** $[-2, 3]$ uses an incorrect lower bound.

Final Answer: Range of $2\sin x + 1$ is $[-1, 3] \Rightarrow$ **Option (B)**

Answer: (B) ↑ Go Back to Q 39

Solution

Q40.

Concept — Key Values of $\cos x$: $\cos(0) = 1$, $\cos(\pi/2) = 0$, $\cos(\pi) = -1$.

Step 1 — Evaluate $\cos(0)$: $\cos(0) = 1$.

Why other options are wrong:

- **Option A:** $\cos(0) = 0$ is incorrect; it is $\sin(0)$ that equals 0.
- **Option B:** $\cos(\pi) = -1$, not 1.
- **Option E:** $\cos(\pi/2) = 0$, not 1.
- **Option D:** Maximum of $\cos x$ is 1, not 2.

Final Answer: $\cos(0) = 1 \Rightarrow$ **Option (C)**

Answer: (C) ↑ Go Back to Q 40

Solution

Concept — Standard Trigonometric Values: From the 30-60-90 triangle with sides 1, $\sqrt{3}$, 2: $\sin 60 = \frac{\sqrt{3}}{2}$.

Why other options are wrong:

Q41.

- **Option A:** $\frac{1}{2} = \sin 30$, not $\sin 60$.
- **Option C:** $1 = \sin 90$, not $\sin 60$.
- **Option D:** $\frac{\sqrt{2}}{2} = \sin 45$.
- **Option E:** $0 = \sin 0$.

Final Answer: $\sin 60 = \frac{\sqrt{3}}{2} \Rightarrow$ **Option (B)**

Answer: (B) ↑ Go Back to Q 41

Solution

Concept — Principal Value of Inverse Sine: $\sin^{-1}$ has principal range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
We need the angle θ in this range with $\sin\theta = \frac{1}{2}$.

Step 1 — Identify the angle: $\sin\frac{\pi}{6} = \frac{1}{2}$, and $\frac{\pi}{6} \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Why other options are wrong:

Q42.

- **Option A:** $\frac{\pi}{3}$ — $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} \neq \frac{1}{2}$.
- **Option C:** $\frac{\pi}{4}$ — $\sin\frac{\pi}{4} = \frac{\sqrt{2}}{2} \neq \frac{1}{2}$.
- **Option D:** $\frac{\pi}{2}$ — $\sin\frac{\pi}{2} = 1 \neq \frac{1}{2}$.
- **Option E:** π — outside the principal range.

Final Answer: $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6} \Rightarrow$ **Option (B)**

Answer: (B) ↑ Go Back to Q 42

Solution

Concept — Principal Value of Inverse Cosine: $\cos^{-1}$ has principal range $[0, \pi]$.
We need $\theta \in [0, \pi]$ with $\cos\theta = -1$.

Step 1 — Identify the angle: $\cos\pi = -1$, and $\pi \in [0, \pi]$.

Why other options are wrong:

Q43.

- **Option A:** $\cos(0) = 1 \neq -1$.
- **Option B:** $\cos(\pi/2) = 0 \neq -1$.

- **Option D:** $\frac{3\pi}{2}$ is outside the principal range $[0, \pi]$.
- **Option E:** 2π is outside the principal range.

Final Answer: $\cos^{-1}(-1) = \pi \Rightarrow$ **Option (C)**

Answer: (C) ↑ Go Back to Q 43

Solution

Concept — Standard Values of Inverse Tangent: $\tan^{-1}(1) = \frac{\pi}{4}$ and $\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$.

Step 1 — Add the values: $\tan^{-1}(1) + \tan^{-1}(\sqrt{3}) = \frac{\pi}{4} + \frac{\pi}{3} = \frac{3\pi}{12} + \frac{4\pi}{12} = \frac{7\pi}{12}$.

Why other options are wrong:

Q44.

- **Option A:** $\frac{\pi}{4}$ — only the first term.
- **Option B:** $\frac{\pi}{3}$ — only the second term.
- **Option D:** $\frac{\pi}{2}$ — would require both values to sum to 6/12.
- **Option E:** $\frac{5\pi}{12}$ — arithmetic error in common denominator.

Final Answer: Sum $= \frac{7\pi}{12} \Rightarrow$ **Option (C)**

Answer: (C) ↑ Go Back to Q 44

Solution

Concept — Angle of Elevation and Tangent Ratio: If the angle of elevation is α, height is h, and horizontal distance is d, then $\tan\alpha = h/d$.

Step 1 — Set up the equation: $\tan 30 = \frac{20}{d}$.

Step 2 — Solve for d: $d = \frac{20}{\tan 30} = \frac{20}{1/\sqrt{3}} = 20\sqrt{3}$ m.

Why other options are wrong:

Q45.

- **Option A:** $10\sqrt{3}$ corresponds to height 10 m or angle 60.
- **Option E:** $15\sqrt{3}$ corresponds to height 15 m.
- **Option C:** 20 m would require $\tan\alpha = 1$, i.e. $\alpha = 45$.
- **Option D:** $25\sqrt{3}$ corresponds to height 25 m.

Final Answer: Distance from base $= 20\sqrt{3}$ m $\Rightarrow$ **Option (B)**

Answer: (B) ↑ Go Back to Q 45

Solution

Concept — Shadow Length and Angle of Elevation: $\tan(\text{angle of elevation}) = \dfrac{\text{height}}{\text{shadow length}}$.

Step 1 — Set up: $\tan 45 = \dfrac{h}{10}$.

Step 2 — Solve: $h = 10 \times \tan 45 = 10 \times 1 = 10$ m.

Why other options are wrong:

Q46.

- **Option B:** 5 m — halves the shadow length unnecessarily.
- **Option C:** $10\sqrt{2}$ — applies $\sin 45$ relation incorrectly.
- **Option A:** $5\sqrt{2}$ — incorrect combination.
- **Option E:** 20 m — doubles the shadow length unnecessarily.

Final Answer: Height of flagpole $= 10$ m $\Rightarrow$ **Option (D)**

Answer: (D) ↑ Go Back to Q 46

Solution

Concept — Slope Formula: The slope of the line through (x_1, y_1) and (x_2, y_2) is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.

Step 1 — Substitute values: $m = \dfrac{7-3}{4-2} = \dfrac{4}{2} = 2$.

Why other options are wrong:

Q47.

- **Option A:** 1 — divides the difference of y by 4 instead of 2.
- **Option B:** 3 — adds rather than divides.
- **Option D:** 4 — uses only Δy without dividing by Δx.
- **Option E:** $\frac{1}{2}$ — inverts Δy and Δx.

Final Answer: Slope $= 2 \Rightarrow$ **Option (C)**

Answer: (C) ↑ Go Back to Q 47

Solution

Concept — Slope-Intercept Form: $y = mx + c$, where m is slope and c is y-intercept.

Step 1 — Identify m and c: Slope $m = 3$, passes through $(0, 2)$ so y-intercept $c = 2$.

Step 2 — Write the equation: $y = 3x + 2$.

Why other options are wrong:

Q48.

- **Option C:** $y = 3x - 2$ — wrong sign on the intercept.
- **Option B:** $y = 2x + 3$ — slope and intercept are swapped.
- **Option D:** $y = 3x$ — zero intercept, doesn't pass through $(0, 2)$.
- **Option E:** $y = 2x - 3$ — both slope and intercept are wrong.

Final Answer: Equation is $y = 3x + 2 \Rightarrow$ **Option (A)**

Answer: (A) ↑ Go Back to Q 48

Solution

Concept — x-intercept: The x-intercept is found by setting $y = 0$ in the equation.

Step 1 — Set $y = 0$: $3x + 4(0) = 12 \Rightarrow 3x = 12 \Rightarrow x = 4$.

Why other options are wrong:

Q49.

- **Option A:** 3 is the coefficient of x, not the intercept.
- **Option B:** 6 — arithmetic error.
- **Option C:** 2 — arithmetic error.
- **Option D:** 8 — arithmetic error.

Final Answer: x-intercept $= 4 \Rightarrow$ **Option (E)**

Answer: (E) ↑ Go Back to Q 49

Solution

Concept — Distance Between Parallel Lines: For lines $ax+by = c_1$ and $ax+by = c_2$: $d = \dfrac{|c_2 - c_1|}{\sqrt{a^2 + b^2}}$.

Step 1 — Apply formula: $d = \dfrac{|20 - 5|}{\sqrt{3^2 + 4^2}} = \dfrac{15}{\sqrt{9 + 16}} = \dfrac{15}{\sqrt{25}} = \dfrac{15}{5} = 3$.

Why other options are wrong:

Q50.

- **Option E:** 1 — divides $|c_2 - c_1|$ by 15 instead of 5.
- **Option B:** $\frac{5}{3}$ — uses wrong denominator.
- **Option C:** 2 — arithmetic error.
- **Option D:** $\frac{5}{2}$ — uses denominator 6 instead of 5.

Final Answer: Distance $= 3 \Rightarrow$ **Option (A)**

Answer: (A) ↑ Go Back to Q 50

Solution

Concept — Perpendicular Distance Formula: Distance from (x_0, y_0) to line $ax + by + c = 0$ is $d = \dfrac{|ax_0 + by_0 + c|}{\sqrt{a^2 + b^2}}$.

Step 1 — Substitute values: Line: $3x - 4y + 5 = 0$. Point: $(3, 4)$. $d = \dfrac{|3(3) - 4(4) + 5|}{\sqrt{9 + 16}} = \dfrac{|9 - 16 + 5|}{\sqrt{25}} = \dfrac{|-2|}{5} = \dfrac{2}{5}$.

Why other options are wrong:

Q51.

- **Option B:** $\frac{3}{5}$ — arithmetic error in numerator.
- **Option C:** 1 — uses wrong denominator 2.
- **Option D:** 2 — doesn't divide by $\sqrt{a^2 + b^2}$.
- **Option A:** $\frac{1}{5}$ — arithmetic error in numerator.

Final Answer: Perpendicular distance $= \dfrac{2}{5} \Rightarrow$ **Option (E)**

Answer: (E) ↑ Go Back to Q 51

Solution

Concept — Standard Equation of a Circle: Centre (h, k), radius r: $(x - h)^2 + (y - k)^2 = r^2$.

Step 1 — Substitute $h = 2$, $k = -3$, $r = 4$**:** $(x - 2)^2 + (y - (-3))^2 = 4^2 \Rightarrow (x - 2)^2 + (y + 3)^2 = 16$.

Why other options are wrong:

Q52.

- **Option A:** $(x + 2)^2 + (y - 3)^2 = 16$ — both signs of center are flipped.
- **Option C:** $(x - 2)^2 + (y - 3)^2 = 16$ — wrong sign for $k = -3$.
- **Option D:** $(x + 2)^2 + (y + 3)^2 = 16$ — wrong sign for $h = 2$.
- **Option B:** Uses $r^2 = 4$ instead of 16.

Final Answer: Equation is $(x - 2)^2 + (y + 3)^2 = 16 \Rightarrow$ **Option (E)**

Answer: (E) ↑ Go Back to Q 52

Solution

Concept — Length of Tangent from an External Point: For circle $x^2 + y^2 = r^2$ and external point (x_1, y_1): tangent length $= \sqrt{x_1^2 + y_1^2 - r^2}$.

Step 1 — Apply formula: $L = \sqrt{5^2 + 12^2 - 144} = \sqrt{25 + 144 - 144} = \sqrt{25} = 5$.

Why other options are wrong:

Q53.

- **Option A:** $\sqrt{17}$ — incorrect computation.
- **Option B:** $3\sqrt{2}$ — incorrect computation.
- **Option D:** $2\sqrt{5}$ — arithmetic error.
- **Option E:** $\sqrt{10}$ — arithmetic error.

Final Answer: Length of tangent $= 5 \Rightarrow$ **Option (C)**

Answer: (C) ↑ Go Back to Q 53

Solution

Concept — Circle Through Three Points: Use the general form $x^2 + y^2 + Dx + Ey + F = 0$ and substitute each point to find D, E, F.

Step 1 — Substitute $(0, 0)$: $0 + 0 + 0 + 0 + F = 0 \Rightarrow F = 0$.

Step 2 — Substitute $(4, 0)$: $16 + 0 + 4D + 0 + 0 = 0 \Rightarrow D = -4$.

Step 3 — Substitute $(0, 3)$: $0 + 9 + 0 + 3E + 0 = 0 \Rightarrow E = -3$.

Step 4 — Write the equation: $x^2 + y^2 - 4x - 3y = 0$.

Why other options are wrong:

Q54.

- **Option A:** $+4x$ and $+3y$ — both signs flipped.
- **Option B:** $-4x$ but $+3y$ — E has wrong sign.
- **Option C:** $+4x$ and $-3y$ — D has wrong sign.
- **Option E:** Swaps D and E.

Final Answer: Equation is $x^2 + y^2 - 4x - 3y = 0 \Rightarrow$ **Option (D)**

Answer: (D) ↑ Go Back to Q 54

Solution

Concept — Radius from the General Circle Equation: For $x^2+y^2+Dx+Ey+F = 0$, the radius is $r = \sqrt{\frac{D^2}{4} + \frac{E^2}{4} - F}$.

Step 1 — Identify D, E, F: $x^2 + y^2 - 6x + 4y - 3 = 0$: $D = -6$, $E = 4$, $F = -3$.

Step 2 — Compute r: $r = \sqrt{\frac{36}{4} + \frac{16}{4} + 3} = \sqrt{9+4+3} = \sqrt{16} = 4.$

Why other options are wrong:

Q55.

- **Option A:** $r = 2$ — uses only partial terms.
- **Option B:** $r = 3$ — arithmetic error.
- **Option E:** $\sqrt{3}$ — arithmetic error.
- **Option D:** $2\sqrt{3}$ — arithmetic error.

Final Answer: Radius $= 4 \Rightarrow$ **Option (C)**

Answer: (C) ↑ Go Back to Q 55

Solution

Concept — Centre from the General Circle Equation: For $x^2+y^2+Dx+Ey+F = 0$, the centre is $\left(-\frac{D}{2}, -\frac{E}{2}\right)$.

Step 1 — Simplify the given equation: $2x^2 + 2y^2 - 4x + 6y - 10 = 0 \;\Rightarrow\; x^2 + y^2 - 2x + 3y - 5 = 0$. So $D = -2$, $E = 3$.

Step 2 — Find centre: $\left(-\frac{-2}{2}, -\frac{3}{2}\right) = \left(1, -\frac{3}{2}\right)$.

Why other options are wrong:

Q56.

- **Option B:** $(-1, 3/2)$ — both signs flipped.
- **Option A:** $(2, -3)$ — fails to divide by 2 after simplifying.
- **Option D:** $(-2, 3)$ — fails to divide and flips signs.
- **Option E:** $(1, 3/2)$ — wrong sign on y-coordinate.

Final Answer: Centre $= \left(1, -\frac{3}{2}\right) \Rightarrow$ **Option (C)**

Answer: (C) ↑ Go Back to Q 56

Solution

Concept — Parabola $y^2 = 4ax$: The focus is at $(a, 0)$ and the directrix is $x = -a$, where $4a$ is the coefficient of x.

Step 1 — Identify a: $y^2 = 8x \Rightarrow 4a = 8 \Rightarrow a = 2$.

Step 2 — State focus and directrix: Focus $= (2, 0)$, Directrix: $x = -2$.

Why other options are wrong:

Q57.

- **Option A:** Uses $a = 4$ (value of $4a$, not a).
- **Option C:** Uses $a = 8$ (value of the whole RHS coefficient).
- **Option D, E:** Give vertical focus/directrix, which applies to $x^2 = 4ay$.

Final Answer: Focus $(2, 0)$, Directrix $x = -2 \Rightarrow$ **Option (E)**

Answer: (E) ↑ Go Back to Q 57

Solution

Concept — Vertex of a Parabola by Completing the Square: Write $y = a(x - h)^2 + k$; the vertex is (h, k).

Step 1 — Complete the square: $y = x^2 - 4x + 5 = (x^2 - 4x + 4) + 1 = (x - 2)^2 + 1$.

Step 2 — Read the vertex: $(h, k) = (2, 1)$.

Why other options are wrong:

Q58.

- **Option C:** $(0, 5)$ — evaluates $y(0)$, not the vertex.
- **Option B:** $(2, -1)$ — wrong sign on k.
- **Option D:** $(-2, 1)$ — wrong sign on h.
- **Option E:** $(4, 5)$ — no mathematical basis.

Final Answer: Vertex $= (2, 1) \Rightarrow$ **Option (A)**

Answer: (A) ↑ Go Back to Q 58

Solution

Concept — Standard Ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ ($a > b$): a is the semi-major axis (along x-axis when larger) and b is the semi-minor axis.

Step 1 — Read off a^2 and b^2: $\frac{x^2}{25} + \frac{y^2}{9} = 1 \Rightarrow a^2 = 25,\ b^2 = 9$.

Step 2 — Find a and b: $a = 5,\ b = 3$.

Why other options are wrong:

Q59.

- **Option A:** $a = 9, b = 5$ — swaps values and takes wrong square roots.
- **Option B:** $a = 3, b = 5$ — swaps a and b.
- **Option C:** $b = 25$ — uses the squared value.
- **Option E:** Uses squared values as the axes.

Final Answer: $a = 5,\ b = 3 \Rightarrow$ **Option (D)**

Answer: (D) ↑ Go Back to Q 59

Solution

Concept — Eccentricity of an Ellipse: $e = \frac{c}{a}$ where $c^2 = a^2 - b^2$.

Step 1 — Identify a^2 and b^2: $\frac{x^2}{16} + \frac{y^2}{7} = 1 \Rightarrow a^2 = 16,\ b^2 = 7$.

Step 2 — Compute c and e: $c^2 = 16 - 7 = 9 \Rightarrow c = 3$. $e = \frac{3}{4}$.

Why other options are wrong:

Q60.

- **Option E:** $\frac{7}{16}$ — uses b^2/a^2, not c/a.
- **Option B:** $\frac{1}{2}$ — corresponds to different a, b values.
- **Option C:** $\frac{\sqrt{7}}{4}$ — uses $\sqrt{b^2}/a$ instead of c/a.
- **Option D:** $\frac{7}{4}$ — uses b^2/c, no standard meaning.

Final Answer: $e = \frac{3}{4} \Rightarrow$ **Option (A)**

Answer: (A) ↑ Go Back to Q 60

Solution

Concept — Vertices of a Hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$: The vertices are at $(\pm a, 0)$.

Step 1 — Identify a^2: $\frac{x^2}{9} - \frac{y^2}{4} = 1 \Rightarrow a^2 = 9 \Rightarrow a = 3$.

Step 2 — State vertices: $(\pm 3,\ 0)$.

Why other options are wrong:

Q61.

- **Option A:** $(0, \pm 3)$ — vertices on y-axis apply to $y^2/a^2 - x^2/b^2 = 1$.
- **Option C:** $(\pm 2, 0)$ — uses $b = 2$ instead of $a = 3$.
- **Option D:** $(0, \pm 2)$ — both axis and value wrong.
- **Option E:** $(\pm 9, 0)$ — uses a^2 instead of a.

Final Answer: Vertices at $(\pm 3,\ 0) \Rightarrow$ **Option (B)**

Answer: (B) ↑ Go Back to Q 61

Solution

Concept — Dot (Scalar) Product: $\vec{a} \cdot \vec{b} = a_1 b_1 + a_2 b_2$ for 2D vectors.

Step 1 — Compute: $\vec{a} \cdot \vec{b} = (2)(4) + (3)(-1) = 8 - 3 = 5$.

Why other options are wrong:

Q62.

- **Option A:** 3 — uses only the $\hat{j}$-component product with wrong sign.
- **Option C:** 7 — arithmetic error.
- **Option B:** 11 — adds instead of subtracts.
- **Option E:** 13 — sums magnitudes squared.

Final Answer: $\vec{a} \cdot \vec{b} = 5 \Rightarrow$ **Option (D)**

Answer: (D) ↑ Go Back to Q 62

Solution

Concept — Angle Between Vectors: $\cos\theta = \dfrac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$. If $\vec{a} \cdot \vec{b} = 0$, the vectors are perpendicular ($\theta = 90$).

Step 1 — Compute dot product: $\vec{a} \cdot \vec{b} = (1)(1) + (1)(-1) = 1 - 1 = 0$.

Step 2 — Conclude: Since $\vec{a} \cdot \vec{b} = 0$, the angle between them is 90.

Why other options are wrong:

Q63.

- **Option A:** 0 would require $\vec{a}$ parallel to $\vec{b}$.
- **Option B:** 45 — the dot product would need to be $\frac{1}{\sqrt{2}}$ of $|\vec{a}||\vec{b}|$.
- **Option C:** 120 and **Option E:** 135 — require a negative dot product.

Final Answer: Angle between $\vec{a}$ and $\vec{b}$ is $90 \Rightarrow$ **Option (D)**

Answer: (D) ↑ Go Back to Q 63

Solution

Concept — Unit Vector: $\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$.

Step 1 — Compute $|\vec{a}|$: $|\vec{a}| = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$.

Step 2 — Divide by magnitude: $\hat{a} = \dfrac{1}{5}(3\hat{i} + 4\hat{j})$.

Why other options are wrong:

Q64.

- **Option D:** Wrong sign on $\hat{j}$ component.
- **Option B:** Uses $\frac{1}{3}$ instead of $\frac{1}{5}$.
- **Option C:** Swaps the $\hat{i}$ and $\hat{j}$ coefficients.
- **Option E:** Uses $\frac{1}{4}$ instead of $\frac{1}{5}$.

Final Answer: Unit vector $= \frac{1}{5}(3\hat{i} + 4\hat{j}) \Rightarrow$ **Option (A)**

Answer: (A) ↑ Go Back to Q 64

Solution

Concept — Collinearity of Vectors: Two vectors are collinear (parallel) iff their components are proportional: $\frac{a_1}{b_1} = \frac{a_2}{b_2}$.

Step 1 — Set up proportion: $\frac{2}{6} = \frac{k}{3} \Rightarrow k = \frac{2 \times 3}{6} = 1$.

Why other options are wrong:

Q65.

- **Option A:** $k = 2$ gives ratio $\frac{2}{3} \neq \frac{2}{6} = \frac{1}{3}$.
- **Option E:** $k = 3$ gives ratio $\frac{3}{3} = 1 \neq \frac{1}{3}$.
- **Option C:** $k = 4$ gives ratio $\frac{4}{3} \neq \frac{1}{3}$.
- **Option D:** $k = 6$ gives ratio $\frac{6}{3} = 2 \neq \frac{1}{3}$.

Final Answer: $k = 1 \Rightarrow$ **Option (B)**

Answer: (B) ↑ Go Back to Q 65

Solution

Concept — Magnitude of the Cross Product: $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\theta$.

Step 1 — Substitute values: $|\vec{a} \times \vec{b}| = 3 \times 4 \times \sin 30 = 12 \times \frac{1}{2} = 6$.

Why other options are wrong:

Q66.

- **Option B:** 12 — forgets to multiply by $\sin 30 = \frac{1}{2}$.
- **Option C:** $6\sqrt{3}$ — uses $\sin 60$ instead of $\sin 30$.
- **Option D:** $12\sqrt{3}$ — uses $\sin 60$ and forgets the half.
- **Option E:** 3 — only uses one magnitude.

Final Answer: $|\vec{a} \times \vec{b}| = 6 \Rightarrow$ **Option (A)**

Answer: (A) ↑ Go Back to Q 66

Solution

Concept — Scalar Triple Product: $\vec{a} \cdot (\vec{b} \times \vec{c}) = [\vec{a}\ \vec{b}\ \vec{c}]$, the volume of the parallelepiped. For standard basis vectors, $\hat{i} \cdot (\hat{j} \times \hat{k}) = \hat{i} \cdot \hat{i} = 1$.

Step 1 — Compute $\vec{b} \times \vec{c}$: $\hat{j} \times \hat{k} = \hat{i}$.

Step 2 — Compute the dot product: $\hat{i} \cdot \hat{i} = 1$.

Why other options are wrong:

Q67.

- **Option A:** 0 — would result if the vectors were coplanar.
- **Option C:** -1 — would result from $\hat{k} \times \hat{j}$.
- **Option B:** 2 and **Option E:** 3 — exceed the maximum scalar triple product for unit vectors.

Final Answer: $\hat{i} \cdot (\hat{j} \times \hat{k}) = 1 \Rightarrow$ **Option (D)**

Answer: (D) ↑ Go Back to Q 67

Solution

Concept — Direction Cosines from Direction Ratios: If the DRs are (a, b, c), the DCs are $\left(\frac{a}{r}, \frac{b}{r}, \frac{c}{r}\right)$ where $r = \sqrt{a^2 + b^2 + c^2}$.

Step 1 — Compute r: $r = \sqrt{2^2 + 2^2 + 1^2} = \sqrt{4 + 4 + 1} = \sqrt{9} = 3$.

Step 2 — Divide: DCs $= \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right)$.

Why other options are wrong:

Q68.

- **Option A:** $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$ — ignores the actual direction ratios and uses only r.
- **Option B:** $\left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right)$ — incorrectly uses 2 for all components.
- **Option D:** $\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$ — uses wrong $r = 2$.
- **Option E:** $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$ — swaps components 2 and 3.

Final Answer: DCs $= \left(\frac{2}{3}, \frac{2}{3}, \frac{1}{3}\right) \Rightarrow$ **Option (C)**

Answer: (C) ↑ Go Back to Q 68

Solution

Concept — Validity of Direction Cosines: A set (l, m, n) forms valid DCs iff $l^2 + m^2 + n^2 = 1$.

Step 1 — Check Option D: $\left(\frac{2}{7}, \frac{3}{7}, \frac{6}{7}\right)$: $\left(\frac{2}{7}\right)^2 + \left(\frac{3}{7}\right)^2 + \left(\frac{6}{7}\right)^2 = \frac{4 + 9 + 36}{49} = \frac{49}{49} = 1$. ✓

Why other options are wrong:

Q69.

- **Option A:** $\frac{1+4+1}{9} = \frac{6}{9} \neq 1$.
- **Option B:** $\frac{1/4+1/9+1/36}{1} = \frac{9+4+1}{36} = \frac{14}{36} \neq 1$.

- **Option C:** $0 + 1 + 1 = 2 \neq 1$.
- **Option E:** $\frac{1}{4} + \frac{1}{4} + 1 = \frac{3}{2} \neq 1$.

Final Answer: Valid DCs are $\left(\frac{2}{7}, \frac{3}{7}, \frac{6}{7}\right) \Rightarrow$ **Option (D)**

Answer: (D) ↑ Go Back to Q 69

Q70.

Solution

Concept — Angle Between Two Lines: $\cos\theta = |l_1 l_2 + m_1 m_2 + n_1 n_2|$ for direction cosines (or $\vec{a} \cdot \vec{b} / |\vec{a}||\vec{b}|$ for direction ratios).

Step 1 — Compute the dot product: $(1)(0) + (0)(1) + (0)(0) = 0$.

Step 2 — Find the angle: $\cos\theta = 0 \Rightarrow \theta = 90$.

Why other options are wrong:

- **Option A:** 0 requires parallel lines.
- **Option E:** 30, **Option C:** 45, **Option D:** 60 — require non-zero dot products.

Final Answer: Angle between the lines is $90 \Rightarrow$ **Option (B)**

Answer: (B) ↑ Go Back to Q 70

Q71.

Solution

Concept — Distance Formula in 3D: $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$.

Step 1 — Substitute: $d = \sqrt{(4-1)^2 + (6-2)^2 + (3-3)^2} = \sqrt{9 + 16 + 0} = \sqrt{25} = 5$.

Why other options are wrong:

- **Option B:** 7 — arithmetic error.
- **Option C:** $\sqrt{5}$ — omits squaring the differences before summing.
- **Option D:** $\sqrt{7}$ — arithmetic error.
- **Option E:** 3 — uses only the z-difference (which is 0 here).

Final Answer: Distance $= 5 \Rightarrow$ **Option (A)**

Answer: (A) ↑ Go Back to Q 71

Solution

Concept — Midpoint Formula in 3D: Midpoint $= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$.

Step 1 — Compute: $M = \left(\frac{2+8}{2}, \frac{4+10}{2}, \frac{6+12}{2}\right) = (5,\ 7,\ 9).$

Why other options are wrong:

Q72.

- **Option A:** $(3, 5, 7)$ — subtracts rather than averages.
- **Option C:** $(6, 7, 9)$ — first coordinate is wrong.
- **Option D:** $(4, 6, 8)$ — arithmetic error.
- **Option B:** $(5, 6, 9)$ — arithmetic error in y-coordinate.

Final Answer: Midpoint $= (5, 7, 9) \Rightarrow$ **Option (E)**

Answer: (E) ↑ Go Back to Q 72

Solution

Concept — Removing the Indeterminate Form: Factor the numerator using the difference of squares: $x^2 - 4 = (x-2)(x+2)$.

Step 1 — Simplify and evaluate: $\lim_{x\to 2} \frac{x^2-4}{x-2} = \lim_{x\to 2} \frac{(x-2)(x+2)}{x-2} = \lim_{x\to 2}(x+2) = 2+2 = 4.$

Why other options are wrong:

Q73.

- **Option A:** 0 — evaluating at $x = 2$ before factoring gives $0/0$.
- **Option B:** 2 — evaluates x alone at the limit.
- **Option C:** 6 — arithmetic error.
- **Option E:** 8 — arithmetic error.

Final Answer: Limit $= 4 \Rightarrow$ **Option (D)**

Answer: (D) ↑ Go Back to Q 73

Solution

Concept — Factoring to Resolve Indeterminate Forms.

Step 1 — Factor and simplify: $\lim_{x\to 3} \frac{x^2-9}{x-3} = \lim_{x\to 3} \frac{(x-3)(x+3)}{x-3} = \lim_{x\to 3}(x+3) = 3+3 = 6.$

Why other options are wrong:

Q74.

- **Option A:** 0 — direct substitution gives $0/0$, not 0.
- **Option D:** 3 — evaluates numerator at $x = 0$ not at limit.
- **Option C:** 9 — evaluates $(x+3)$ with $x = 9$.

- **Option E:** ∞ — valid only if the denominator $\to 0$ with numerator $\neq 0$.

Final Answer: Limit $= 6 \Rightarrow$ **Option (B)**

Answer: (B) ↑ Go Back to Q 74

Q75.

Solution

Concept — Continuity Condition: f is continuous at $x = c$ iff $\lim_{x\to c} f(x) = f(c)$.

Step 1 — Find the limit as $x \to 1$: $\lim_{x\to 1} \dfrac{x^2-1}{x-1} = \lim_{x\to 1} \dfrac{(x-1)(x+1)}{x-1} = \lim_{x\to 1}(x+1) = 2$.

Step 2 — Set k **equal to the limit:** $k = 2$.

Why other options are wrong:

- **Option E:** $k = 0$ — causes a jump discontinuity.
- **Option B:** $k = -1$ — causes a jump discontinuity.
- **Option C:** $k = 1$ — causes a jump discontinuity.
- **Option D:** $k = 3$ — causes a jump discontinuity.

Final Answer: $k = 2 \Rightarrow$ **Option (A)**

Answer: (A) ↑ Go Back to Q 75

Q76.

Solution

Concept — Factoring $x^3 - 1$: $x^3 - 1 = (x-1)(x^2+x+1)$.

Step 1 — Apply the factoring: $\lim_{x\to 1} \dfrac{x^3-1}{x-1} = \lim_{x\to 1}(x^2+x+1) = 1+1+1 = 3$.

Why other options are wrong:

- **Option B:** 0 — direct substitution gives $0/0$.
- **Option C:** 1 — evaluates only x at the limit.
- **Option D:** 2 — evaluates $x^2 + x$ without the constant.
- **Option E:** ∞ — incorrect; the form resolves to 3.

Final Answer: Limit $= 3 \Rightarrow$ **Option (A)**

Answer: (A) ↑ Go Back to Q 76

Q77.

Solution

Concept — One-Sided Limits for $|x|/x$: A limit exists iff both one-sided limits are equal.

Step 1 — Left-hand limit ($x \to 0^-$): For $x < 0$: $|x| = -x$, so $\frac{|x|}{x} = \frac{-x}{x} = -1$. LHL $= -1$.

Step 2 — Right-hand limit ($x \to 0^+$): For $x > 0$: $|x| = x$, so $\frac{|x|}{x} = \frac{x}{x} = 1$. RHL $= 1$.

Step 3 — Conclusion: LHL $\neq$ RHL, so $\lim_{x\to 0} f(x)$ does not exist.

Why other options are wrong:

- **Option A:** 0 — $f(x)$ never takes the value 0 here.
- **Option C:** 1 — only the right-hand limit.
- **Option D:** -1 — only the left-hand limit.
- **Option B:** ∞ — the function is bounded.

Final Answer: The limit does not exist $\Rightarrow$ **Option (E)**

Answer: (E) ↑ Go Back to Q 77

Q78.

Solution

Concept — Fundamental Trigonometric Limit: $\lim_{x\to 0} \frac{\sin x}{x} = 1$ is a standard result (proved using the squeeze theorem).

Why other options are wrong:

- **Option A:** 0 — incorrect; both $\sin x \to 0$ and $x \to 0$ but their ratio $\to 1$.
- **Option B:** ∞ — the function is bounded near 0.
- **Option D:** -1 — the function is positive near $x = 0^+$.
- **Option C:** π — no mathematical basis.

Final Answer: $\lim_{x\to 0} \frac{\sin x}{x} = 1 \Rightarrow$ **Option (E)**

Answer: (E) ↑ Go Back to Q 78

Solution

Concept — Product Rule: $\frac{d}{dx}[u \cdot v] = u'v + uv'$.

Step 1 — Identify u and v: $u = x^2 \Rightarrow u' = 2x$. $\quad v = \sin x \Rightarrow v' = \cos x$.

Step 2 — Apply product rule: $\frac{d}{dx}[x^2 \sin x] = 2x \sin x + x^2 \cos x$.

Why other options are wrong:

Q79.

- **Option A:** $2x \cos x$ — uses chain rule incorrectly; ignores the $\sin x$ term.
- **Option B:** $2x \sin x$ — misses the second term $x^2 \cos x$.
- **Option C:** $x^2 \cos x + \sin x$ — uses $u' = 1$ instead of $2x$.
- **Option E:** Wrong sign on second term.

Final Answer: $\frac{d}{dx}[x^2 \sin x] = 2x \sin x + x^2 \cos x \Rightarrow$ **Option (D)**

Answer: (D) ↑ Go Back to Q 79

Solution

Concept — Quotient Rule: $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{u'v - uv'}{v^2}$.

Step 1 — Identify u and v: $u = \sin x \Rightarrow u' = \cos x$. $\quad v = x \Rightarrow v' = 1$.

Step 2 — Apply quotient rule: $\frac{d}{dx}\left[\frac{\sin x}{x}\right] = \frac{x \cos x - \sin x \cdot 1}{x^2} = \frac{x \cos x - \sin x}{x^2}$.

Why other options are wrong:

Q80.

- **Option A:** $\frac{\cos x}{x}$ — ignores the second term in the numerator.
- **Option B:** $\frac{\sin x - x \cos x}{x^2}$ — numerator sign is flipped.
- **Option C:** $\frac{x \cos x + \sin x}{x^2}$ — uses addition instead of subtraction.
- **Option E:** $-\frac{\cos x}{x}$ — incomplete and wrong sign.

Final Answer: $\frac{d}{dx}\left[\frac{\sin x}{x}\right] = \frac{x \cos x - \sin x}{x^2} \Rightarrow$ **Option (D)**

Answer: (D) ↑ Go Back to Q 80

Solution

Concept — Power Rule: $\frac{d}{dx}[x^n] = nx^{n-1}$.

Step 1 — Apply the power rule: $\frac{d}{dx}[x^5] = 5x^{5-1} = 5x^4$.

Why other options are wrong:

Q81.

- **Option B:** $4x^5$ — decrements coefficient but not the exponent.
- **Option C:** x^5 — no differentiation applied.

- **Option D:** $5x^3$ — wrong exponent (decremented by 2).
- **Option A:** $5x$ — wrong exponent (decremented by 4).

Final Answer: $\frac{d}{dx}[x^5] = 5x^4 \Rightarrow$ **Option (E)**

Answer: (E) ↑ Go Back to Q 81

Q82.

Solution

Concept — Standard Derivative: $\frac{d}{dx}[\sin x] = \cos x$.

Why other options are wrong:

- **Option A:** $-\cos x = \frac{d}{dx}[-\sin x]$, not $\sin x$.
- **Option C:** $\sin x$ — function equals its own derivative only for e^x.
- **Option D:** $-\sin x = \frac{d}{dx}[\cos x]$.
- **Option E:** $\tan x$ — no derivative relation here.

Final Answer: $\frac{d}{dx}[\sin x] = \cos x \Rightarrow$ **Option (B)**

Answer: (B) ↑ Go Back to Q 82

Q83.

Solution

Concept — Derivative of Natural Logarithm: $\frac{d}{dx}[\ln x] = \frac{1}{x}$ for $x > 0$.

Why other options are wrong:

- **Option A:** x — that is the antiderivative of $x^0 = 1$, not the derivative of $\ln x$.
- **Option B:** $\frac{1}{x^2}$ — that is the derivative of $-\frac{1}{x}$.
- **Option D:** $\frac{\ln x}{x}$ — that is the derivative of $\frac{(\ln x)^2}{2}$.
- **Option C:** $-\frac{1}{x}$ — wrong sign.

Final Answer: $\frac{d}{dx}[\ln x] = \frac{1}{x} \Rightarrow$ **Option (E)**

Answer: (E) ↑ Go Back to Q 83

Q84.

Solution

Concept — Chain Rule: $\frac{d}{dx}[\sin(f(x))] = \cos(f(x)) \cdot f'(x)$.

Step 1 — Identify $f(x)$: $f(x) = 3x + 2$, $f'(x) = 3$.

Step 2 — Apply chain rule: $\frac{d}{dx}[\sin(3x + 2)] = \cos(3x + 2) \cdot 3 = 3\cos(3x + 2)$.

Why other options are wrong:

- **Option A:** $\cos(3x + 2)$ — misses the factor of 3 from the chain rule.
- **Option B:** $3\sin(3x + 2)$ — differentiates $\sin$ to $\sin$ instead of $\cos$.
- **Option C:** $-3\cos(3x + 2)$ — wrong sign.
- **Option E:** $-\cos(3x + 2)$ — wrong sign and missing factor 3.

Final Answer: $\frac{d}{dx}[\sin(3x + 2)] = 3\cos(3x + 2) \Rightarrow$ **Option (D)**

Answer: (D) ↑ Go Back to Q 84

Q85.

Solution

Concept — Derivative of Exponential (Chain Rule): $\frac{d}{dx}[e^{ax}] = ae^{ax}$.

Step 1 — Apply with $a = 2$: $\frac{d}{dx}[e^{2x}] = 2e^{2x}$.

Why other options are wrong:

- **Option A:** e^{2x} — misses the factor of 2 from the chain rule.
- **Option B:** e^{x} — reduces the exponent incorrectly.
- **Option C:** $2e^{x}$ — applies the factor but uses wrong exponent.
- **Option E:** $\frac{e^{2x}}{2}$ — divides instead of multiplying by 2.

Final Answer: $\frac{d}{dx}[e^{2x}] = 2e^{2x} \Rightarrow$ **Option (D)**

Answer: (D) ↑ Go Back to Q 85

Solution

Concept — Critical Points: Critical points occur where $f'(x) = 0$ or $f'(x)$ is undefined.

Step 1 — Differentiate: $f(x) = x^3 - 3x^2 \Rightarrow f'(x) = 3x^2 - 6x = 3x(x - 2)$.

Step 2 — Solve $f'(x) = 0$: $3x(x - 2) = 0 \Rightarrow x = 0$ or $x = 2$.

Verification: $f(0) = 0$ (local max), $f(2) = -4$ (local min). ✓

Why other options are wrong:

Q86.
- **Option B:** $x = 1, 3$ — gives $f'(1) = -3 \neq 0$, $f'(3) = 9 \neq 0$.
- **Option A:** $x = 0, 3$ — $f'(3) = 9 \neq 0$.
- **Option D:** $x = -1, 2$ — $f'(-1) = 9 \neq 0$.
- **Option E:** $x = 1, 2$ — $f'(1) = -3 \neq 0$.

Final Answer: Critical points at $x = 0$ and $x = 2 \Rightarrow$ **Option (C)**

Answer: (C) ↑ Go Back to Q 86

Solution

Concept — Second Derivative Test for Local Extrema.

Step 1 — Find critical points: $f(x) = x^3 - 6x^2 + 9x + 1$. $f'(x) = 3x^2 - 12x + 9 = 3(x-1)(x-3)$. Critical points: $x = 1$ and $x = 3$.

Step 2 — Second derivative test: $f''(x) = 6x - 12$. $f''(1) = 6 - 12 = -6 < 0 \Rightarrow$ local maximum at $x = 1$. $f''(3) = 18 - 12 = 6 > 0 \Rightarrow$ local minimum at $x = 3$.

Step 3 — Find the local maximum value: $f(1) = 1 - 6 + 9 + 1 = 5$.

Why other options are wrong:

Q87.
- **Option A:** 3 — does not correspond to f evaluated at any critical point.
- **Option C:** 7 — arithmetic error.
- **Option B:** 9 — evaluates $3x^2$ at $x = 3$ (the minimum location).
- **Option E:** 1 — this is the constant term, not the maximum value.

Final Answer: Local maximum value $= 5 \Rightarrow$ **Option (D)**

Answer: (D) ↑ Go Back to Q 87

Solution

Concept — Tangent Line to a Curve: The tangent at (x_0, y_0) has equation $y - y_0 = m(x - x_0)$ where $m = f'(x_0)$.

Step 1 — Find the point: At $x = 1$: $y = 1^2 + 3(1) = 4$. Point: $(1, 4)$.

Step 2 — Find the slope: $f'(x) = 2x + 3$. $f'(1) = 2 + 3 = 5$.

Step 3 — Write the tangent equation: $y - 4 = 5(x - 1) \Rightarrow y = 5x - 5 + 4 = 5x - 1$.

Why other options are wrong:

Q88.
- **Option A:** $y = 3x + 1$ — uses slope 3, not 5.

- **Option B:** $y = 4x - 1$ — uses slope 4.
- **Option D:** $y = 2x + 3$ — uses slope 2 and wrong intercept.
- **Option C:** $y = x + 4$ — uses slope 1.

Final Answer: Tangent equation is $y = 5x - 1 \Rightarrow$ **Option (E)**

Answer: (E) ↑ Go Back to Q 88

Q89.

Solution

Concept — Increasing/Decreasing by Sign of $f'(x)$: f is increasing where $f'(x) > 0$ and decreasing where $f'(x) < 0$.

Step 1 — Find $f'(x)$: $f(x) = x^2 - 4x + 3 \Rightarrow f'(x) = 2x - 4$.

Step 2 — Solve $f'(x) > 0$: $2x - 4 > 0 \Rightarrow x > 2$.

Step 3 — State the interval: f is increasing on $(2, \infty)$.

Why other options are wrong:

- **Option A:** $(-\infty, 2)$ — $f'(x) < 0$ there, so f is decreasing.
- **Option D:** $(-\infty, -2)$ — incorrect; the critical point is $x = 2$, not $x = -2$.
- **Option C:** $(0, 2)$ — $f'(x) < 0$ on this interval.
- **Option E:** $(4, \infty)$ — correct direction but starts too late; f increases from $x > 2$.

Final Answer: f is increasing on $(2, \infty) \Rightarrow$ **Option (B)**

Answer: (B) ↑ Go Back to Q 89

Q90.

Solution

Concept — Slope of Normal: If slope of tangent is m, then slope of normal $= -\dfrac{1}{m}$.

Step 1 — Find slope of tangent at $x = 1$: $y = x^3 \Rightarrow \dfrac{dy}{dx} = 3x^2$. At $x = 1$: slope $= 3$.

Step 2 — Find slope of normal: Slope of normal $= -\dfrac{1}{3}$.

Why other options are wrong:

- **Option A:** -3 — that is the negative of the tangent slope, not the reciprocal.
- **Option B:** 3 — slope of the tangent, not the normal.

- **Option C:** $\frac{1}{3}$ — positive reciprocal; normal has negative reciprocal.
- **Option E:** 1 — incorrect.

Final Answer: Slope of normal $= -\frac{1}{3} \Rightarrow$ **Option (D)**

Answer: (D) ↑ Go Back to Q 90

Q91.

Solution

Concept — Rate of Change: $\frac{dA}{dr}$ gives the rate of change of area with respect to radius.

Step 1 — Differentiate $A = \pi r^2$: $\frac{dA}{dr} = 2\pi r$.

Step 2 — Evaluate at $r = 5$: $\left.\frac{dA}{dr}\right|_{r=5} = 2\pi(5) = 10\pi$.

Why other options are wrong:

- **Option B:** 5π — uses πr instead of $2\pi r$.
- **Option A:** 25π — evaluates A itself at $r = 5$, not the derivative.
- **Option D:** 2π — evaluates the formula at $r = 1$.
- **Option E:** π — evaluates at $r = 1/2$.

Final Answer: Rate of change of area at $r = 5$ is $10\pi \Rightarrow$ **Option (C)**

Answer: (C) ↑ Go Back to Q 91

Q92.

Solution

Concept — Power Rule for Integration: $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ for $n \neq -1$.

Step 1 — Apply with $n = 4$: $\int x^4\,dx = \frac{x^5}{5} + C$.

Why other options are wrong:

- **Option A:** $\frac{x^3}{3}$ — decrements instead of increments the exponent.
- **Option C:** $4x^3$ — differentiates instead of integrating.
- **Option D:** x^4 — integrates to same power (off by one).
- **Option E:** $5x^5$ — multiplies instead of divides by the new exponent.

Final Answer: $\int x^4\,dx = \frac{x^5}{5} + C \Rightarrow$ **Option (B)**

Answer: (B) ↑ Go Back to Q 92

Solution

Concept — Standard Integral: $\int \sin x\, dx = -\cos x + C$.

Verification: $\frac{d}{dx}[-\cos x] = \sin x$. ✓

Why other options are wrong:

Q93.
- **Option A:** $\cos x$ — has the wrong sign.
- **Option C:** $-\sin x$ — that is $\frac{d}{dx}[\cos x]$, not an integral.
- **Option D:** $\sin x$ — unchanged function.
- **Option E:** $\cos x - \sin x$ — incorrect combination.

Final Answer: $\int \sin x\, dx = -\cos x + C \Rightarrow$ **Option (B)**

Answer: (B) ↑ Go Back to Q 93

Solution

Concept — Integration of Exponential Functions: $\int e^{ax}\, dx = \frac{e^{ax}}{a} + C$.

Step 1 — Apply with $a = 3$: $\int e^{3x}\, dx = \frac{e^{3x}}{3} + C$.

Why other options are wrong:

Q94.
- **Option D:** $3e^{3x}$ — differentiates rather than integrates.
- **Option B:** e^{x} — wrong exponent.
- **Option C:** e^{3x} — misses the $1/3$ factor.
- **Option E:** $3e^{x}$ — both exponent and factor are wrong.

Final Answer: $\int e^{3x}\, dx = \frac{e^{3x}}{3} + C \Rightarrow$ **Option (A)**

Answer: (A) ↑ Go Back to Q 94

Solution

Concept — Logarithmic Integration: $\int \frac{1}{x}\, dx = \ln|x| + C$.

Why other options are wrong:

Q95.
- **Option A:** $-\frac{1}{x^2}$ — result of differentiating $\frac{1}{x}$ (with wrong sign).

- **Option E:** $\frac{1}{x^2}$ — positive version of (A), both incorrect.
- **Option C:** $-\ln|x|$ — wrong sign.
- **Option D:** $\frac{1}{x}$ — unchanged function.

Final Answer: $\int \frac{1}{x}\,dx = \ln|x| + C \Rightarrow$ **Option (B)**

Answer: (B) ↑ Go Back to Q 95

Solution

Concept — Integration by Substitution: $\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C.$

Step 1 — Apply with $a = 2$, $b = 1$, $n = 3$: Let $u = 2x+1$, $du = 2\,dx$, so $dx = \frac{du}{2}$.

$$\int (2x+1)^3\,dx = \frac{1}{2}\int u^3\,du = \frac{1}{2}\cdot\frac{u^4}{4} + C = \frac{(2x+1)^4}{8} + C.$$

Why other options are wrong:

Q96.

- **Option B:** $\frac{(2x+1)^4}{4}$ — omits the factor $\frac{1}{2}$ from substitution.
- **Option C:** $\frac{(2x+1)^4}{2}$ — uses $a = 1$ in the denominator.
- **Option A:** $\frac{(2x+1)^3}{3}$ — does not increment the exponent.
- **Option E:** $2(2x+1)^4$ — multiplies rather than divides.

Final Answer: $\int (2x+1)^3\,dx = \frac{(2x+1)^4}{8} + C \Rightarrow$ **Option (D)**

Answer: (D) ↑ Go Back to Q 96

Solution

Concept — Half-Angle Identity: $\cos^2 x = \frac{1+\cos 2x}{2}$.

Step 1 — Substitute and integrate: $\int \cos^2 x\,dx = \int \frac{1+\cos 2x}{2}\,dx = \frac{1}{2}\int dx + \frac{1}{2}\int \cos 2x\,dx = \frac{x}{2} + \frac{1}{2}\cdot\frac{\sin 2x}{2} + C = \frac{x}{2} + \frac{\sin 2x}{4} + C.$

Why other options are wrong:

Q97.

- **Option A:** $\frac{\sin 2x}{2}$ — misses the $x/2$ term.
- **Option C:** $x + \frac{\sin 2x}{2}$ — both terms are doubled.

- **Option D:** Negative sign on $\sin 2x$ term is incorrect.
- **Option E:** $\frac{\cos 2x}{4}$ — integrating $\cos^2$ does not give $\cos 2x$.

Final Answer: $\int \cos^2 x\,dx = \frac{x}{2} + \frac{\sin 2x}{4} + C \Rightarrow$ **Option (B)**

Answer: (B) ↑ Go Back to Q 97

Q98.

Solution

Concept — Standard Integral: $\int \cos x\,dx = \sin x + C$.

Verification: $\frac{d}{dx}[\sin x] = \cos x$. ✓

Why other options are wrong:

- **Option A:** $-\sin x$ — that is $\frac{d}{dx}[\cos x]$, not an integral.
- **Option B:** $\cos x$ — unchanged function.
- **Option D:** $\tan x$ — unrelated to $\int \cos x\,dx$.
- **Option E:** $-\cos x = \int \sin x\,dx$, not $\int \cos x\,dx$.

Final Answer: $\int \cos x\,dx = \sin x + C \Rightarrow$ **Option (C)**

Answer: (C) ↑ Go Back to Q 98

Q99.

Solution

Concept — Fundamental Theorem of Calculus: $\int_a^b f(x)\,dx = [F(x)]_a^b = F(b) - F(a)$.

Step 1 — Integrate and evaluate: $\int_0^1 x^2\,dx = \left[\frac{x^3}{3}\right]_0^1 = \frac{1}{3} - 0 = \frac{1}{3}$.

Why other options are wrong:

- **Option D:** 1 — evaluates x^2 at the upper limit.
- **Option B:** $\frac{1}{2}$ — uses $n = 1$ (integral of x).
- **Option C:** $\frac{1}{4}$ — uses $n = 3$ in denominator.
- **Option E:** $\frac{2}{3}$ — arithmetic error.

Final Answer: $\int_0^1 x^2\,dx = \frac{1}{3} \Rightarrow$ **Option (A)**

Answer: (A) ↑ Go Back to Q 99

Solution

Concept — Definite Integral of $\sin x$**:**

Step 1 — Integrate: $\int_0^\pi \sin x\,dx = [-\cos x]_0^\pi = -\cos\pi - (-\cos 0) = -(-1) + 1 = 1 + 1 = 2.$

Why other options are wrong:

Q100.

- **Option A:** 0 — that would be $\int_0^{2\pi} \sin x\,dx$.
- **Option E:** π — no integration performed.
- **Option C:** 1 — partial computation (only one endpoint).
- **Option D:** -2 — sign error.

Final Answer: $\int_0^\pi \sin x\,dx = 2 \Rightarrow$ **Option (B)**

Answer: (B) ↑ Go Back to Q 100

Solution

Concept — Definite Integral of x**:**

Step 1 — Integrate and evaluate: $\int_0^2 x\,dx = \left[\frac{x^2}{2}\right]_0^2 = \frac{4}{2} - 0 = 2.$

Why other options are wrong:

Q101.

- **Option B:** 4 — evaluates x^2 at the upper limit without dividing by 2.
- **Option C:** 1 — evaluates at $x = \sqrt{2}$ or $x = 1$.
- **Option D:** 8 — evaluates $x^3/3$ with wrong formula.
- **Option E:** 3 — arithmetic error.

Final Answer: $\int_0^2 x\,dx = 2 \Rightarrow$ **Option (A)**

Answer: (A) ↑ Go Back to Q 101

Solution

Concept — Area Under $y = x^2$ from 0 to 3.

Step 1 — Integrate: $\int_0^3 x^2\,dx = \left[\frac{x^3}{3}\right]_0^3 = \frac{27}{3} - 0 = 9.$

Why other options are wrong:

Q102.

- **Option A:** 6 — arithmetic error.
- **Option C:** 12 — arithmetic error.
- **Option D:** 18 — uses $x^2/2$ instead of $x^3/3$.
- **Option B:** 27 — evaluates x^3 without dividing by 3.

Final Answer: Area $= 9 \Rightarrow$ **Option (E)**

Answer: (E) ↑ Go Back to Q 102

Solution

Concept — Definite Integral of e^x:

Step 1 — Integrate and evaluate: $\int_0^1 e^x\,dx = [e^x]_0^1 = e^1 - e^0 = e - 1.$

Why other options are wrong:

Q103.

- **Option A:** e — ignores the lower limit $e^0 = 1$.
- **Option C:** 1 — evaluates e^0 alone.
- **Option D:** $e + 1$ — adds instead of subtracts the lower-limit value.
- **Option E:** $2e - 1$ — incorrect.

Final Answer: $\int_0^1 e^x\,dx = e - 1 \Rightarrow$ **Option (B)**

Answer: (B) ↑ Go Back to Q 103

Solution

Concept — Separation of Variables: Write $\frac{dy}{y} = x\,dx$, then integrate both sides.

Step 1 — Separate variables: $\frac{dy}{y} = x\,dx.$

Step 2 — Integrate: $\ln|y| = \frac{x^2}{2} + C_1 \Rightarrow y = e^{C_1} \cdot e^{x^2/2} = Ce^{x^2/2}$ (where $C = e^{C_1}$).

Why other options are wrong:

Q104.

- **Option A:** $y = e^x + C$ — does not come from separating $dy/y = x\,dx$.

- **Option B:** $y = Ce^x$ — would be the solution of $dy/dx = y$.
- **Option C:** $y = Ce^{x^2}$ — exponent should be $x^2/2$, not x^2.
- **Option D:** $y = Ce^{x/2}$ — would be solution of $dy/dx = y/2$.

Final Answer: General solution is $y = Ce^{x^2/2} \Rightarrow$ **Option (E)**

Answer: (E) ↑ Go Back to Q 104

Q105.

Solution

Concept — Direct Integration: Integrate the right-hand side with respect to x.

Step 1 — Integrate: $y = \int (x^2 + 1)\, dx = \frac{x^3}{3} + x + C$.

Why other options are wrong:

- **Option E:** $x^2 + C$ — integrates $2x$ not $x^2 + 1$.
- **Option B:** $x^3 + x + C$ — uses x^3 instead of $x^3/3$.
- **Option C:** $\frac{x^3}{3} + C$ — misses the $+x$ term from integrating 1.
- **Option D:** $\frac{x^2}{2} + x + C$ — integrates x instead of x^2.

Final Answer: $y = \frac{x^3}{3} + x + C \Rightarrow$ **Option (A)**

Answer: (A) ↑ Go Back to Q 105

Q106.

Solution

Concept — Order and Degree of a DE:

- **Order**: the highest derivative present.
- **Degree**: the power to which the highest derivative is raised, once the equation is polynomial in all derivatives.

Step 1 — Identify the highest derivative: The highest derivative in $\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 + y = 0$ is $\frac{d^2y}{dx^2}$. Order $= 2$.

Step 2 — Identify the degree: The highest derivative $\frac{d^2y}{dx^2}$ appears to the power 1. Degree $= 1$.

Why other options are wrong:

- **Option B:** Order 1, Degree 2 — the highest derivative is 2nd order, not 1st.

- **Option A:** Order 3 — there is no 3rd derivative in the equation.
- **Option D:** Degree 3 — the power 3 applies to dy/dx, not to the highest derivative d^2y/dx^2.
- **Option E:** Order 1, Degree 3 — both are wrong.

Final Answer: Order 2, Degree 1 $\Rightarrow$ **Option (C)**

Answer: (C) ↑ Go Back to Q 106

Q107.

Solution

Concept — Separation of Variables for Homogeneous ODE.

Step 1 — Separate variables: $\dfrac{dy}{y} = \dfrac{dx}{x}$.

Step 2 — Integrate: $\ln|y| = \ln|x| + C_1 \;\Rightarrow\; \ln|y| - \ln|x| = C_1 \;\Rightarrow\; \ln\left|\dfrac{y}{x}\right| = C_1$.

Step 3 — Exponentiate: $\dfrac{y}{x} = k$ (where $k = \pm e^{C_1}$) $\;\Rightarrow\; y = kx$.

Why other options are wrong:

- **Option A:** $y = x + C$ — solution of $dy/dx = 1$.
- **Option C:** $y = kx^2$ — solution of $dy/dx = 2y/x$.
- **Option D:** $y = \ln x + C$ — solution of $dy/dx = 1/x$.
- **Option B:** $y = e^x + C$ — solution of $dy/dx = e^x$.

Final Answer: General solution is $y = kx \Rightarrow$ **Option (E)**

Answer: (E) ↑ Go Back to Q 107

Q108.

Solution

Concept — Classical Definition of Probability: $P(E) = \dfrac{\text{number of favourable outcomes}}{\text{total number of equally likely outcomes}}$.

Step 1 — Count outcomes: Total cards $= 52$. Kings in a deck $= 4$ (one per suit).

Step 2 — Compute probability: $P(\text{King}) = \dfrac{4}{52} = \dfrac{1}{13}$.

Why other options are wrong:

- **Option A:** $\dfrac{1}{4}$ — probability of any suit, not a king.
- **Option B:** $\dfrac{1}{26}$ — half of the correct answer.

- **Option D:** $\frac{4}{13}$ — corresponds to 16 kings, not 4.
- **Option E:** $\frac{1}{52}$ — probability of one specific card.

Final Answer: $P(\text{King}) = \frac{1}{13} \Rightarrow$ **Option (C)**

Answer: (C) ↑ Go Back to Q 108

Q109.

Solution

Concept — Addition Rule for Probabilities: $P(A\cup B) = P(A)+P(B)-P(A\cap B)$.

Step 1 — Apply the formula: $P(A \cup B) = 0.4 + 0.5 - 0.2 = 0.7$.

Why other options are wrong:

- **Option A:** 0.5 — uses only $P(A)$.
- **Option D:** 0.6 — uses $P(A) + P(B) - P(A) - P(B) \times P(A)$ incorrectly.
- **Option C:** 0.9 — forgets to subtract $P(A \cap B)$.
- **Option E:** 0.8 — arithmetic error.

Final Answer: $P(A \cup B) = 0.7 \Rightarrow$ **Option (B)**

Answer: (B) ↑ Go Back to Q 109

Q110.

Solution

Concept — Complement Rule: $P(A^c) = 1 - P(A)$.

Step 1 — Compute: $P(A^c) = 1 - \frac{3}{7} = \frac{7-3}{7} = \frac{4}{7}$.

Why other options are wrong:

- **Option E:** $\frac{3}{7} = P(A)$, not $P(A^c)$.
- **Option B:** $\frac{1}{7}$ — arithmetic error.
- **Option C:** $\frac{1}{3}$ — reciprocal of $P(A)$.
- **Option D:** $\frac{3}{4}$ — incorrect denominator.

Final Answer: $P(A^c) = \frac{4}{7} \Rightarrow$ **Option (A)**

Answer: (A) ↑ Go Back to Q 110

Q111.

Solution

Concept — Classical Probability for Dice: Total outcomes when rolling two dice $= 6 \times 6 = 36$.

Step 1 — Count favourable outcomes (sum $=$ 7): $(1,6), (2,5), (3,4), (4,3), (5,2), (6,1)$ — 6 pairs.

Step 2 — Compute probability: $P(\text{sum} = 7) = \frac{6}{36} = \frac{1}{6}$.

Why other options are wrong:

- **Option A:** $\frac{1}{9} = \frac{4}{36}$ — only 4 favourable outcomes (undercounting).
- **Option C:** $\frac{5}{36}$ — corresponds to sum $= 6$ or 8.
- **Option D:** $\frac{7}{36}$ — no sum has exactly 7 pairs.
- **Option E:** $\frac{1}{12} = \frac{3}{36}$ — corresponds to sum $= 5$ or 9.

Final Answer: $P(\text{sum} = 7) = \frac{1}{6} \Rightarrow$ **Option (B)**

Answer: (B) ↑ Go Back to Q 111

Q112.

Solution

Concept — Multiplication Rule for Independent Events: If A and B are independent, $P(A \cap B) = P(A) \cdot P(B)$.

Step 1 — Apply the rule: $P(A \cap B) = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$.

Why other options are wrong:

- **Option A:** $\frac{7}{12} = P(A) + P(B) - P(A \cap B) = P(A \cup B)$, not $P(A \cap B)$.
- **Option C:** $\frac{1}{2}$ — arithmetic error.
- **Option D:** $\frac{1}{6}$ — uses $P(A) + P(B)$ divided by a wrong factor.
- **Option B:** $\frac{5}{12}$ — arithmetic error.

Final Answer: $P(A \cap B) = \frac{1}{12} \Rightarrow$ **Option (E)**

Answer: (E) ↑ Go Back to Q 112

Q113.

Solution

Concept — Arithmetic Mean: $\bar{x} = \dfrac{\text{sum of all observations}}{\text{number of observations}}$.

Step 1 — Sum the data: $4 + 6 + 8 + 10 + 12 = 40$.

Step 2 — Divide by count: $\bar{x} = \dfrac{40}{5} = 8$.

Why other options are wrong:

- **Option C:** 6 — the second value, not the mean.
- **Option B:** 7 — arithmetic error.
- **Option D:** 9 — arithmetic error.
- **Option E:** 10 — the fourth value, not the mean.

Final Answer: Arithmetic mean $= 8 \Rightarrow$ **Option (A)**

Answer: (A) ↑ Go Back to Q 113

Q114.

Solution

Concept — Variance: $\sigma^2 = \dfrac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$.

Step 1 — Find the mean: $\bar{x} = \dfrac{2+4+6+8+10}{5} = \dfrac{30}{5} = 6$.

Step 2 — Compute squared deviations: $(2-6)^2 = 16$, $(4-6)^2 = 4$, $(6-6)^2 = 0$, $(8-6)^2 = 4$, $(10-6)^2 = 16$.

Step 3 — Compute variance: $\sigma^2 = \dfrac{16+4+0+4+16}{5} = \dfrac{40}{5} = 8$.

Why other options are wrong:

- **Option A:** 4 — half of the correct answer.
- **Option D:** 6 — equals the mean, not the variance.
- **Option C:** 2 — standard deviation of different data.
- **Option E:** 10 — equals the largest value, not the variance.

Final Answer: Variance $= 8 \Rightarrow$ **Option (B)**

Answer: (B) ↑ Go Back to Q 114

Solution

Concept — Mean Deviation About the Mean: $\text{MD} = \dfrac{1}{n}\sum_{i=1}^{n}|x_i - \bar{x}|$.

Step 1 — Find the mean: $\bar{x} = \frac{3+5+7+9+11}{5} = \frac{35}{5} = 7.$

Step 2 — Compute absolute deviations: $|3-7| = 4$, $|5-7| = 2$, $|7-7| = 0$, $|9-7| = 2$, $|11-7| = 4$.

Step 3 — Compute mean deviation: $\text{MD} = \frac{4+2+0+2+4}{5} = \frac{12}{5} = 2.4.$

Why other options are wrong:

Q115.

- **Option A:** 0 — sum of signed deviations, not absolute deviations.
- **Option B:** 1 — underestimates by a factor of more than 2.
- **Option C:** 1.5 — arithmetic error.
- **Option D:** 2 — uses sum $= 10$ instead of 12.

Final Answer: Mean deviation $= 2.4 \Rightarrow$ **Option (E)**

Answer: (E) ↑ Go Back to Q 115

Solution

Concept — Conjunction ($\wedge$): $p \wedge q$ is True only when *both* p and q are True.

Step 1 — Apply truth table: $p =$ True, $q =$ True. $p \wedge q =$ True $\wedge$ True $=$ **True**.

Why other options are wrong:

Q116.

- **Option B:** False — $p \wedge q$ is False only when at least one of p, q is False.
- **Option A:** "Depends on context" — truth tables are context-independent.
- **Option D:** "Undefined" — $p \wedge q$ is always well-defined.
- **Option E:** "Neither True nor False" — every proposition has a definite truth value.

Final Answer: $p \wedge q =$ True $\Rightarrow$ **Option (C)**

Answer: (C) ↑ Go Back to Q 116

Solution

Concept — De Morgan's Law: $\neg(p \vee q) \equiv \neg p \wedge \neg q$.

Step 1 — Apply De Morgan's law: The negation of a disjunction $(p \vee q)$ is the conjunction of negations: $\neg p \wedge \neg q$.

Why other options are wrong:

Q117.

- **Option A:** $\neg p \vee \neg q = \neg(p \wedge q)$ (the other De Morgan law — negation of conjunction).

- **Option C:** $p \wedge q$ — this is a different compound statement, not the negation.
- **Option B:** $\neg(p \wedge q) = \neg p \vee \neg q$ — correct for negating $p \wedge q$, not $p \vee q$.
- **Option E:** $\neg p \vee q$ — partial negation; only p is negated.

Final Answer: $\neg(p \vee q) = \neg p \wedge \neg q \Rightarrow$ **Option (D)**

Answer: (D) ↑ Go Back to Q 117

Q118.

Solution

Concept — Feasible Region: The feasible region is the set of all points satisfying all constraints simultaneously. Non-negativity constraints $x \geq 0$, $y \geq 0$ restrict us to the first quadrant.

Step 1 — Identify the constraints: $x + y \leq 6$, $x \geq 0$, $y \geq 0$.

Step 2 — Determine the shape: The line $x + y = 6$ intersects the axes at $(6, 0)$ and $(0, 6)$. With the non-negativity constraints, the feasible region is a triangle with vertices $(0, 0)$, $(6, 0)$, $(0, 6)$. This region is bounded.

Why other options are wrong:

- **Option A:** Unbounded — the non-negativity constraints and $x + y \leq 6$ together bound the region.
- **Option B:** Empty — there are clearly feasible points (e.g. $(0, 0)$).
- **Option C:** A line segment — the region is 2-dimensional, not 1-dimensional.
- **Option E:** A rectangle — a rectangle would require at least four bounding constraints.

Final Answer: Feasible region is a bounded triangle $\Rightarrow$ **Option (D)**

Answer: (D) ↑ Go Back to Q 118

Solution

Concept — Vertices of Feasible Region: Vertices occur at the origin, intercepts, and intersections of constraint boundaries.

Step 1 — Find the intercepts: $2x + y = 6$: x-intercept $(3, 0)$; y-intercept $(0, 6)$. $x + 2y = 6$: x-intercept $(6, 0)$; y-intercept $(0, 3)$.

Step 2 — Find the intersection of the two lines: $2x + y = 6$ and $x + 2y = 6$. Subtract: $x - y = 0 \Rightarrow x = y$. Substitute: $2x + x = 6 \Rightarrow x = 2$, $y = 2$. Intersection: $(2, 2)$.

Step 3 — Identify vertices of the feasible region: $(0, 0)$, $(3, 0)$, $(2, 2)$, $(0, 3)$.

Why other options are wrong:

Q119.

- **Option A:** Uses $(6,0),(0,6)$ — these lie outside the feasible region (violating the other constraint).
- **Option D:** $(0,0),(3,0),(0,3)$ — misses the intersection vertex $(2,2)$.
- **Option C:** $(0,0),(2,0),(0,2)$ — uses wrong intercepts.
- **Option E:** Uses $(6,0)$ and $(0,6)$ — infeasible points.

Final Answer: Vertices are $(0,0)$, $(3,0)$, $(2,2)$, $(0,3) \Rightarrow$ **Option (B)**

Answer: (B) ↑ Go Back to Q 119

Solution

Concept — Corner Point Theorem: The maximum of a linear objective function over a bounded feasible region occurs at one of the corner points (vertices).

Step 1 — Identify vertices: Constraints: $x+y \leq 4$, $x \geq 0$, $y \geq 0$. Vertices: $(0,0)$, $(4,0)$, $(0,4)$.

Step 2 — Evaluate $Z = 3x + 5y$ at each vertex:

Q120.

- $(0,0)$: $Z = 0$.
- $(4,0)$: $Z = 12$.
- $(0,4)$: $Z = 20$.

Step 3 — State the maximum: Maximum $Z = 20$ at $(0,4)$.

Why other options are wrong:

- **Option A:** 12 — value at $(4,0)$; not the maximum.
- **Option E:** 15 — no vertex gives this value.
- **Option C:** 16 — no vertex gives this value.
- **Option D:** 18 — no vertex gives this value.

Final Answer: Maximum value of $Z = 20$ at $(0,4) \Rightarrow$ **Option (B)**

Answer: (B) ↑ Go Back to Q 120

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	E	2	C	3	C	4	E	5	B
6	A	7	E	8	E	9	D	10	A
11	D	12	E	13	A	14	B	15	A
16	D	17	A	18	E	19	C	20	C
21	D	22	C	23	E	24	C	25	B
26	C	27	E	28	A	29	D	30	A
31	C	32	C	33	A	34	D	35	C
36	A	37	D	38	E	39	B	40	C
41	B	42	B	43	C	44	C	45	B
46	D	47	C	48	A	49	E	50	A
51	E	52	E	53	C	54	D	55	C
56	C	57	E	58	A	59	D	60	A
61	B	62	D	63	D	64	A	65	B
66	A	67	D	68	C	69	D	70	B
71	A	72	E	73	D	74	B	75	A
76	A	77	E	78	E	79	D	80	D
81	E	82	B	83	E	84	D	85	D
86	C	87	D	88	E	89	B	90	D
91	C	92	B	93	B	94	A	95	B
96	D	97	B	98	C	99	A	100	B
101	A	102	E	103	B	104	E	105	A
106	C	107	E	108	C	109	B	110	A
111	B	112	E	113	A	114	B	115	E
116	C	117	D	118	D	119	B	120	B