# UP Board Class 12 Mathematics 2026 (Set 324 DD) Question Paper with Solutions

## General Instructions

**(i)** This booklet contains 9 questions, each provided with a complete, step-by-step solution.

**(ii)** It comprises 5 single-correct multiple-choice questions.

**(iii)** Attempt each question on your own before reviewing the given solution.

**1(a).** If the function $f : N \to N$ is defined as $f(x) = x^2$, then $f$ is:

(A) One-one and onto

(B) One-one but not onto

(C) Neither one-one nor onto

(D) Many-one and onto

**Correct Answer:** (B) One-one but not onto

**Solution:**

**Step 1:** Check one-one:

For $x_1, x_2 \in N$, if $f(x_1) = f(x_2)$ then $x_1^2 = x_2^2$. Since $x_1, x_2$ are natural numbers (positive), this forces $x_1 = x_2$. So $f$ is one-one.

**Step 2:** Check onto:

For $f$ to be onto, every $y \in N$ must have a pre-image $x \in N$ with $x^2 = y$, i.e. $x = \sqrt{y}$. Take $y = 2$: $\sqrt{2}$ is not a natural number, so 2 has no pre-image in $N$.

**Step 3:** Why the other options are wrong:

Option A and D claim $f$ is onto, which fails as shown. Option C claims $f$ is not one-one, which is false since we proved injectivity above.

**Final Answer:**

$f$ is one-one but not onto.

$$\boxed{\text{One-one but not onto}}$$

**Quick Tip:** Squaring is injective on naturals but its range misses non-perfect-squares like 2.

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**1(b).** The relation $R = \{(a, b) : a \leq b^2\}$ is defined in the set of real numbers. Then $R$ is:

(A) Reflexive and symmetric but not transitive

(B) Reflexive and transitive but not symmetric

(C) Symmetric and transitive but not reflexive

(D) Not reflexive, not symmetric and not transitive

**Correct Answer:** (D) Not reflexive, not symmetric and not transitive

**Solution:**

**Step 1:** Check reflexivity:

Reflexive needs $a \leq a^2$ for every real $a$, i.e. $a^2 - a \geq 0$, i.e. $a(a - 1) \geq 0$, true only for $a \leq 0$ or $a \geq 1$. Take $a = 0.5$: $0.5 \leq 0.25$ is false. So $R$ is not reflexive.

**Step 2:** Check symmetry:

Take $a = 0,\ b = 10$: $0 \leq 10^2 = 100$ is true, so $(0, 10) \in R$. But is $(10, 0)$

$\in R$? That needs $10 \le 0^2 = 0$, false. So $R$ is not symmetric.

**Step 3:** Check transitivity:

Take $a = 10,\ b = 4,\ c = 2$. $a \le b^2$: $10 \le 16$ true. $b \le c^2$: $4 \le 4$ true. But $a \le c^2$: $10 \le 4$ is false. So $R$ is not transitive.

**Final Answer:**

$R$ fails reflexivity, symmetry and transitivity all three.

$$\boxed{\text{Not reflexive, not symmetric, not transitive}}$$

**Quick Tip:** Test a=0.5 for reflexivity, (0,10) vs (10,0) for symmetry, and (10,4,2) for transitivity — all fail.

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**1(c).** If vectors $5\hat{i} - \lambda\hat{j} + 2\hat{k}$ and $2\hat{i} + 3\hat{j} + 4\hat{k}$ are perpendicular to each other, then the value of $\lambda$ is:

(A) 3

(B) 4

(C) 6

(D) 0

**Correct Answer:** (C) 6

**Solution:**

**Step 1:** Use the perpendicularity condition:

Two vectors are perpendicular exactly when their dot product is 0.

**Step 2:** Compute the dot product:

$$(5)(2) + (-\lambda)(3) + (2)(4) = 10 - 3\lambda + 8 = 18 - 3\lambda$$

**Step 3:** Solve for $\lambda$:

$$18 - 3\lambda = 0 \implies \lambda = 6$$

**Final Answer:**

$$\boxed{\lambda = 6}$$

**Quick Tip:** Set the dot product of the two vectors to zero and solve for $\lambda$.

**1(d).** If $x - y = \pi$, then the value of $\frac{dy}{dx}$ is:

(A) 0

(B) $\pi$

(C) 1

(D) $x$

**Correct Answer:** (C) 1

**Solution:**

**Step 1:** Differentiate both sides w.r.t. $x$:

$x - y = \pi$ is an equation between $x$ and $y$ with $\pi$ a constant.

Differentiating: $1 - \frac{dy}{dx} = 0.$

**Step 2:** Solve for $dy/dx$:

$$\frac{dy}{dx} = 1$$

**Final Answer:**

$$\boxed{\frac{dy}{dx} = 1}$$

**Quick Tip:** Differentiate x - y = constant directly; the derivative of a constant is 0.

**1(e).** Direction cosines of the $x$-axis are:

(A) (1, 0, 0)

(B) (0, 1, 0)

(C) (0, 0, 1)

(D) (0, 0, 0)

**Correct Answer:** (A) (1, 0, 0)

**Solution:**

**Step 1:** Recall the definition:

Direction cosines are the cosines of the angles the axis/line makes with the $x$, $y$ and $z$ axes respectively.

**Step 2:** Apply it to the $x$-axis itself:

The $x$-axis makes angle $0°$ with itself, and $90°$ with both the $y$-axis and the $z$-axis. So its direction cosines are $(\cos 0°, \cos 90°, \cos 90°)$ $= (1, 0, 0)$.

**Final Answer:**

$$\boxed{(1, 0, 0)}$$

**Quick Tip:** The x-axis makes 0 degrees with itself and 90 degrees with the y- and z-axes.

**2(a).** Find the principal value of $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$.

**Correct Answer:** —

**Solution:**

**Step 1:** Set up the equation:

Let $\theta = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$, so $\sin\theta = \frac{1}{\sqrt{2}}$, with $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ (the principal range).

**Step 2:** Identify the angle:

$\sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$, and $\frac{\pi}{4}$ lies in the principal range.

**Final Answer:**

$$\boxed{\theta = \frac{\pi}{4}}$$

**Quick Tip:** Recall sin(pi/4) = 1/sqrt2, and pi/4 lies in the principal range of sin^-1.

**2(b).** Find the value of $\int_2^3 \frac{1}{x}\,dx$.

**Correct Answer:** —

**Solution:**

**Step 1:** Find the antiderivative:
$\int \frac{1}{x}\,dx = \ln|x| + C.$

**Step 2:** Apply the limits:

$$\int_2^3 \frac{1}{x}\,dx = \ln 3 - \ln 2 = \ln\frac{3}{2}$$

**Final Answer:**

$$\boxed{\ln\frac{3}{2}}$$

**Quick Tip:** The antiderivative of 1/x is ln|x|; evaluate at the limits and subtract.

**2(c).** Find the angle between vectors $\hat{i} - 2\hat{j} + 3\hat{k}$ and $3\hat{i} - 2\hat{j} + \hat{k}$.

**Correct Answer:** —

**Solution:**

**Step 1:** Compute the dot product:

$$(1)(3) + (-2)(-2) + (3)(1) = 3 + 4 + 3 = 10$$

**Step 2:** Compute the magnitudes:

$$|\vec{a}| = \sqrt{1^2 + (-2)^2 + 3^2} = \sqrt{14}, \quad |\vec{b}| = \sqrt{3^2 + (-2)^2 + 1^2} = \sqrt{14}$$

**Step 3:** Use $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|}$:

$$\cos\theta = \frac{10}{\sqrt{14}\cdot\sqrt{14}} = \frac{10}{14} = \frac{5}{7}$$

**Final Answer:**

$$\boxed{\theta = \cos^{-1}\left(\frac{5}{7}\right)}$$

**Quick Tip:** Use cos(theta) = (a.b)/(|a||b|) with the dot product and magnitudes of the two vectors.

**2(d).** Direction ratios of a line are 2, $-1$ and $-2$. Find the direction cosines of the line.

**Correct Answer:** —

**Solution:**

**Step 1:** Find the magnitude of the direction ratio vector:

$$\sqrt{2^2 + (-1)^2 + (-2)^2} = \sqrt{4 + 1 + 4} = \sqrt{9} = 3$$

**Step 2:** Divide each ratio by the magnitude:

$$\left(\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}\right)$$

**Final Answer:**

$$\boxed{\left(\frac{2}{3}, -\frac{1}{3}, -\frac{2}{3}\right)}$$

**Quick Tip:** Divide each direction ratio by the square root of the sum of their squares.

**2(e).** If $P(A) = \frac{7}{13}$, $P(B) = \frac{9}{13}$ and $P(A \cap B) = \frac{4}{13}$, then find $P\left(\frac{A}{B}\right)$.

**Correct Answer:** —

**Solution:**

**Step 1:** Recall the conditional probability formula:

$$P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$$

**Step 2:** Substitute the given values:

$$P\left(\frac{A}{B}\right) = \frac{4/13}{9/13} = \frac{4}{9}$$

**Final Answer:**

$$\boxed{\frac{4}{9}}$$

**Quick Tip:** Use P(A|B) = P(A intersection B) / P(B); the 13-denominators cancel.

**3(a).** If functions $f : R \to R$ and $g : R \to R$ are defined respectively as $f(x) = \cos x$ and $g(x) = 3x^2$, then find $gof$ and $fog$.

**Correct Answer:** —

**Solution:**

**Step 1:** Compute $gof(x) = g(f(x))$:

$f(x) = \cos x$, so $g(f(x)) = g(\cos x) = 3(\cos x)^2 = 3\cos^2 x$.

**Step 2:** Compute $fog(x) = f(g(x))$:

$g(x) = 3x^2$, so $f(g(x)) = f(3x^2) = \cos(3x^2)$.

**Final Answer:**

$$\boxed{gof(x) = 3\cos^2 x, \qquad fog(x) = \cos(3x^2)}$$

**Quick Tip:** Substitute f into g for gof, and g into f for fog, in that order.

**3(b).** Find the value of the determinant $\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix}$.

**Correct Answer:** —

**Solution:**

**Step 1:** Compare the third column to the first two:

In every row, the third entry is the sum of the first two: $x + a$, $y + b$, $z + c$. So column 3 = column 1 + column 2.

**Step 2:** Apply the column operation $C_3 \to C_3 - C_1 - C_2$:

This is a legal determinant operation (adding a multiple of other columns doesn't change the value) and turns column 3 entirely to zeros.

**Step 3:** A determinant with a zero column is zero:

Expanding along the (now all-zero) third column gives 0.

**Final Answer:**

$$\boxed{0}$$

**Quick Tip:** Column 3 is exactly column 1 + column 2 in every row, so the columns are linearly dependent.

**3(c).** If $f(x) = \begin{cases} x+2, & x \neq 0 \\ 1, & x = 0 \end{cases}$, then prove that the function is not continuous at $x = 0$.

**Correct Answer:** —

**Solution:**

**Step 1:** Find the limit as $x \to 0$:

For $x \neq 0$, $f(x) = x + 2$, so $\lim_{x\to 0} f(x) = 0 + 2 = 2$.

**Step 2:** Find the function's actual value at $x = 0$:

By definition, $f(0) = 1$.

**Step 3:** Compare:

Continuity at $x = 0$ requires $\lim_{x\to 0} f(x) = f(0)$. Here $2 \neq 1$.

**Final Answer:**

Since the limit and the function value disagree, $f$ is not continuous at $x = 0$.

$$\boxed{\lim_{x\to 0} f(x) = 2 \neq f(0) = 1}$$

**Quick Tip:** Compute the limit of x+2 as x approaches 0 and compare it to the defined value f(0)=1.

**3(d).** Find the vector and Cartesian equations of a line which passes through the point $(1, 2, 3)$ and is parallel to the vector $2\hat{i} + 3\hat{j} + 2\hat{k}$.

**Correct Answer:** —

**Solution:**

**Step 1:** Write the vector equation:

A line through point $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ parallel to $\vec{b} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ has vector equation $\vec{r} = \vec{a} + \lambda\vec{b}$.

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 2\hat{k})$$

**Step 2:** Convert to Cartesian form:

With $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, matching components gives $x = 1 + 2\lambda,\ y = 2 + 3\lambda,\ z = 3 + 2\lambda$, so

$$\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{2}$$

**Final Answer:**

$$\boxed{\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 2\hat{k}), \qquad \frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{2}}$$

**Quick Tip:** Use r = a + lambda b for the vector form; equate components for the Cartesian symmetric form.

**4(a).** Find the general solution of the differential equation $\dfrac{dy}{dx} = \dfrac{1+y^2}{1+x^2}$.

**Correct Answer:** —

**Solution:**

**Step 1:** Separate the variables:

$$\frac{dy}{1+y^2} = \frac{dx}{1+x^2}$$

**Step 2:** Integrate both sides:

$$\int \frac{dy}{1+y^2} = \int \frac{dx}{1+x^2}$$

$$\tan^{-1} y = \tan^{-1} x + C$$

**Final Answer:**

$$\boxed{\tan^{-1} y - \tan^{-1} x = C}$$

**Quick Tip:** This is variable-separable; both sides integrate to an arctangent.

**4(b).** If $y = 5\cos x - 3\sin x$, then prove that $\dfrac{d^2y}{dx^2} + y = 0$.

**Correct Answer:** —

**Solution:**

**Step 1:** Differentiate once:

$$\frac{dy}{dx} = -5\sin x - 3\cos x$$

**Step 2:** Differentiate again:

$$\frac{d^2y}{dx^2} = -5\cos x + 3\sin x = -(5\cos x - 3\sin x) = -y$$

**Step 3:** Rearrange:

$$\frac{d^2y}{dx^2} + y = -y + y = 0$$

**Final Answer:**

$$\boxed{\frac{d^2y}{dx^2} + y = 0}$$

**Quick Tip:** Differentiate twice and notice the second derivative is exactly -y.

**4(c).** Find the projection of vector $\hat{\imath} + 3\hat{\jmath} + 7\hat{k}$ on vector $7\hat{\imath} - \hat{\jmath} + 8\hat{k}$.

**Correct Answer:** —

**Solution:**

**Step 1:** Recall the projection formula:

Projection of $\vec{a}$ on $\vec{b}$ is $\dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$.

**Step 2:** Compute the dot product:

$$(1)(7) + (3)(-1) + (7)(8) = 7 - 3 + 56 = 60$$

**Step 3:** Compute $|\vec{b}|$:

$$|\vec{b}| = \sqrt{7^2 + (-1)^2 + 8^2} = \sqrt{49 + 1 + 64} = \sqrt{114}$$

**Final Answer:**

$$\boxed{\frac{60}{\sqrt{114}}}$$

**Quick Tip:** Projection of a on b equals (a.b)/|b|; compute the dot product and the magnitude of b.

**4(d).** If $P(A) = \dfrac{1}{3}$, $P(B) = \dfrac{1}{2}$ and $P(A \cup B) = \dfrac{2}{3}$, then show that $A$ and $B$ are independent events.

**Correct Answer:** –

**Solution:**

**Step 1:** Find $P(A \cap B)$ using the addition rule:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$\frac{2}{3} = \frac{1}{3} + \frac{1}{2} - P(A \cap B) \implies P(A \cap B) = \frac{1}{3} + \frac{1}{2} - \frac{2}{3} = \frac{2+3-4}{6} = \frac{1}{6}$$

**Step 2:** Compute $P(A) \cdot P(B)$:

$$P(A) \cdot P(B) = \frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$$

**Step 3:** Compare:

$P(A \cap B) = \frac{1}{6} = P(A) \cdot P(B)$, which is exactly the independence condition.

**Final Answer:**

Since $P(A \cap B) = P(A)P(B)$, $A$ and $B$ are independent.

$$\boxed{P(A \cap B) = P(A)P(B) = \frac{1}{6}}$$

**Quick Tip:** Find P(A intersect B) from the addition rule, then check if it equals P(A) times P(B).

**5(a).** Prove that $\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$.

**Correct Answer:** —

**Solution:**

**Step 1:** Combine the first pair using $\tan^{-1} x + \tan^{-1} y = \tan^{-1} \dfrac{x+y}{1-xy}$:

$$\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{5}\cdot\frac{1}{7}} = \tan^{-1}\frac{12/35}{34/35} = \tan^{-1}\frac{12}{34} = \tan^{-1}\frac{6}{17}$$

**Step 2:** Combine the second pair the same way:

$$\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8} = \tan^{-1}\frac{\frac{1}{3}+\frac{1}{8}}{1-\frac{1}{3}\cdot\frac{1}{8}} = \tan^{-1}\frac{11/24}{23/24} = \tan^{-1}\frac{11}{23}$$

**Step 3:** Combine the two results:

$$\tan^{-1}\frac{6}{17} + \tan^{-1}\frac{11}{23} = \tan^{-1}\frac{\frac{6}{17}+\frac{11}{23}}{1-\frac{6}{17}\cdot\frac{11}{23}}$$

Numerator: $\dfrac{6\times 23 + 11\times 17}{17\times 23} = \dfrac{138+187}{391} = \dfrac{325}{391}$.
Denominator: $1 - \dfrac{66}{391} = \dfrac{325}{391}$.
So the fraction is $\dfrac{325/391}{325/391} = 1$.

**Step 4:** Conclude:

$$\tan^{-1}(1) = \frac{\pi}{4}$$

**Final Answer:**

$$\boxed{\tan^{-1}\frac{1}{5}+\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{8}=\frac{\pi}{4}}$$

**Quick Tip:** Pair the terms and repeatedly apply the tan inverse addition formula; the final ratio collapses to 1.

**5(b).** If $\cos y = x\cos(a+y)$ and $\cos a \neq \pm 1$, then prove that $\dfrac{dy}{dx} = \dfrac{\cos^2(a+y)}{\sin a}$.

**Correct Answer:** —

**Solution:**

**Step 1:** Differentiate both sides implicitly w.r.t. $x$:

$$-\sin y\cdot\frac{dy}{dx}=\cos(a+y)+x\cdot(-\sin(a+y))\frac{dy}{dx}$$

**Step 2:** Collect the $dy/dx$ terms:

$$-\sin y\cdot y'+x\sin(a+y)\cdot y'=\cos(a+y)$$

$$y'\left[x\sin(a+y)-\sin y\right]=\cos(a+y)$$

**Step 3:** Substitute $x=\dfrac{\cos y}{\cos(a+y)}$ from the original equation:

$$x\sin(a+y)-\sin y=\frac{\cos y\sin(a+y)}{\cos(a+y)}-\sin y=\frac{\cos y\sin(a+y)-\sin y\cos(a+y)}{\cos(a+y)}$$

The numerator is $\sin\left[(a+y)-y\right]=\sin a$ (sine subtraction formula), so this simplifies to $\dfrac{\sin a}{\cos(a+y)}$.

**Step 4:** Solve for $y'$:

$$y'\cdot\frac{\sin a}{\cos(a+y)}=\cos(a+y)\implies y'=\frac{\cos^2(a+y)}{\sin a}$$

**Final Answer:**

$$\boxed{\frac{dy}{dx}=\frac{\cos^2(a+y)}{\sin a}}$$

**Quick Tip:** Differentiate implicitly, then substitute x back from the original relation and use sin(A-B).

**5(c).** Solve the differential equation: $(1+x^2)\,dy+2xy\,dx=\cot x\,dx$.

**Correct Answer:** —

**Solution:**

**Step 1:** Write in standard linear form $\dfrac{dy}{dx}+Py=Q$:

Divide throughout by $dx$ and by $(1+x^2)$:

$$\frac{dy}{dx}+\frac{2x}{1+x^2}y=\frac{\cot x}{1+x^2}$$

Here $P = \dfrac{2x}{1+x^2}$, $Q = \dfrac{\cot x}{1+x^2}$.

**Step 2:** Find the integrating factor:

$$\text{IF} = e^{\int P\,dx} = e^{\int \frac{2x}{1+x^2}dx} = e^{\ln(1+x^2)} = 1 + x^2$$

**Step 3:** Multiply through by the IF and integrate:

$$\frac{d}{dx}\left[y(1+x^2)\right] = \cot x$$

$$y(1+x^2) = \int \cot x\,dx = \ln|\sin x| + C$$

**Final Answer:**

$$\boxed{y(1+x^2) = \ln|\sin x| + C}$$

**Quick Tip:** Convert to linear form dy/dx + Py = Q, find IF = 1+x^2, and integrate cot x.

**5(d).** Maximize $Z = 10x + 3y$ by the graphical method under the following constraints: $x \geq 0;\ y \geq 0;\ 5x + 3y \leq 15;\ 2x + 5y \leq 10$.

**Correct Answer:** —

**Solution:**

**Step 1:** Find the feasible-region corner points:

The boundary lines are $5x + 3y = 15$ and $2x + 5y = 10$, together with the axes. Intercepts: $5x + 3y = 15$ meets axes at $(3, 0)$ and $(0, 5)$; $2x + 5y = 10$ meets axes at $(5, 0)$ and $(0, 2)$.

**Step 2:** Check which intercepts are feasible (satisfy both constraints):

$(3, 0)$: check $2(3) + 5(0) = 6 \leq 10$ ✓ — feasible.

$(0, 2)$: check $5(0) + 3(2) = 6 \leq 15$ ✓ — feasible.

$(0, 5)$: check $2(0) + 5(5) = 25 \leq 10$? No — infeasible.

$(5, 0)$: check $5(5) + 3(0) = 25 \leq 15$? No — infeasible.

**Step 3:** Find the intersection of the two lines:

From $5x + 3y = 15$: $x = 3 - 0.6y$. Substitute into $2x + 5y = 10$: $2(3 - 0.6y) + 5y = 10 \Rightarrow 6 - 1.2y + 5y = 10 \Rightarrow 3.8y = 4 \Rightarrow y = \dfrac{20}{19}, x = \dfrac{45}{19}$.

**Step 4:** Evaluate $Z$ at every feasible corner: $(0, 0)$, $(3, 0)$, $(0, 2)$, $\left(\frac{45}{19}, \frac{20}{19}\right)$:

$Z(0, 0) = 0$. $Z(3, 0) = 30$. $Z(0, 2) = 6$. $Z\left(\frac{45}{19}, \frac{20}{19}\right) = \dfrac{450 + 60}{19} = \dfrac{510}{19} \approx 26.8$.

**Final Answer:**

The maximum occurs at $(3, 0)$.

$$\boxed{Z_{max} = 30 \text{ at } (x, y) = (3, 0)}$$

**Quick Tip:** Plot the two constraint lines, find all feasible corner points, and evaluate Z at each.

**5(e).** Find the shortest distance between the lines $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and $\vec{r} = (2\hat{i} - \hat{j} - \hat{k}) + \mu(2\hat{i} + \hat{j} + 2\hat{k})$.

**Correct Answer:** —

**Solution:**

**Step 1:** Identify $\vec{a}_1, \vec{b}_1, \vec{a}_2, \vec{b}_2$:

$\vec{a}_1 = (1, 2, 1), \vec{b}_1 = (1, -1, 1), \vec{a}_2 = (2, -1, -1), \vec{b}_2 = (2, 1, 2).$

**Step 2:** Compute $\vec{a}_2 - \vec{a}_1$:

$$\vec{a}_2 - \vec{a}_1 = (1, -3, -2)$$

**Step 3:** Compute $\vec{b}_1 \times \vec{b}_2$:

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = \hat{i}(-2-1) - \hat{j}(2-2) + \hat{k}(1+2) = (-3, 0, 3)$$

**Step 4:** Compute the shortest distance formula:

$$d = \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)|}{|\vec{b}_1 \times \vec{b}_2|}$$

Numerator: $(1)(-3) + (-3)(0) + (-2)(3) = -3 + 0 - 6 = -9$, so $|-9| = 9$.

Denominator: $|(-3, 0, 3)| = \sqrt{9 + 0 + 9} = \sqrt{18} = 3\sqrt{2}$.

**Final Answer:**

$$d = \frac{9}{3\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$

$$\boxed{d = \frac{3\sqrt{2}}{2}}$$

**Quick Tip:** Use d = |(a2-a1).(b1 x b2)| / |b1 x b2| for two skew lines in vector form.

**6(a).** Find the value of the determinant $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix}$.

**Correct Answer:** —

**Solution:**

**Step 1:** Factor each column:

Column 1 is $a, a^2, a^3 = a(1, a, a^2)$; similarly column 2 factors out $b$, column 3 factors out $c$. Pulling these common factors out of each column:

$$\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ a^3 & b^3 & c^3 \end{vmatrix} = abc \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix}$$

**Step 2:** Recognize the remaining determinant as a Vandermonde determinant:

$$\begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = (b-a)(c-a)(c-b)$$

**Step 3:** Rewrite the sign in the conventional $(a-b)(b-c)(c-a)$ form:

$(b-a)(c-a)(c-b) = (a-b)(b-c)(c-a)$ (each pair of sign flips cancels — two flips leave the product unchanged).

**Final Answer:**

$$\boxed{abc(a-b)(b-c)(c-a)}$$

**Quick Tip:** Factor a, b, c out of the columns to expose a Vandermonde determinant, then apply its known factorization.

**6(b).** Evaluate $\int \frac{x^2+1}{x^2-5x+6}\,dx.$

**Correct Answer:** —

**Solution:**

**Step 1:** Since numerator and denominator have equal degree, do polynomial division:

$$\frac{x^2+1}{x^2-5x+6} = 1 + \frac{5x-5}{x^2-5x+6}$$

**Step 2:** Factor the denominator and set up partial fractions:
$x^2 - 5x + 6 = (x-2)(x-3)$. Write $\dfrac{5x-5}{(x-2)(x-3)} = \dfrac{A}{x-2} + \dfrac{B}{x-3}$.

**Step 3:** Solve for A and B:
$5x - 5 = A(x-3) + B(x-2)$. At $x = 2$: $5 = A(-1) \Rightarrow A = -5$. At $x = 3$: $10 = B(1) \Rightarrow B = 10$.

**Step 4:** Integrate term by term:

$$\int \left[1 - \frac{5}{x-2} + \frac{10}{x-3}\right] dx = x - 5\ln|x-2| + 10\ln|x-3| + C$$

**Final Answer:**

$$\boxed{x - 5\ln|x-2| + 10\ln|x-3| + C}$$

**Quick Tip:** Do polynomial long division first (equal degree top and bottom), then partial fractions on the remainder.

**6(c).** Prove that vectors $2\hat{i} - \hat{j} + \hat{k}$, $\hat{i} - 3\hat{j} - 5\hat{k}$ and $3\hat{i} - 4\hat{j} - 4\hat{k}$ are vertices of a right-angled triangle.

**Correct Answer:** —

**Solution:**

**Step 1:** Label the points and find the side vectors:
Let $A = (2, -1, 1)$, $B = (1, -3, -5)$, $C = (3, -4, -4)$.
$\vec{AB} = B - A = (-1, -2, -6)$, $\vec{BC} = C - B = (2, -1, 1)$, $\vec{CA} = A - C$

$= (-1, 3, 5)$.

**Step 2:** Compute the squared lengths of the three sides:

$|AB|^2 = 1 + 4 + 36 = 41$, $|BC|^2 = 4 + 1 + 1 = 6$, $|CA|^2 = 1 + 9 + 25 = 35$.

**Step 3:** Check the Pythagorean relation:

$|BC|^2 + |CA|^2 = 6 + 35 = 41 = |AB|^2$. This matches, so the triangle is right-angled with the right angle at the vertex common to sides $BC$ and $CA$, which is $C$.

**Final Answer:**

Since $BC^2 + CA^2 = AB^2$, the triangle is right-angled at $C$.

$$\boxed{BC^2 + CA^2 = AB^2 = 41}$$

**Quick Tip:** Compute the three side-vectors, their squared lengths, and check which pair satisfies the Pythagorean relation.

**6(d).** If $y = Ae^{mx} + Be^{nx}$, then show that $\dfrac{d^2y}{dx^2} - (m+n)\dfrac{dy}{dx} + mny = 0$.

**Correct Answer:** —

**Solution:**

**Step 1:** Find the first derivative:

$$y' = Am\,e^{mx} + Bn\,e^{nx}$$

**Step 2:** Find the second derivative:

$$y'' = Am^2e^{mx} + Bn^2e^{nx}$$

**Step 3:** Substitute into $y'' - (m+n)y' + mny$ and group by $e^{mx}$ and $e^{nx}$:

Coefficient of $Ae^{mx}$: $m^2 - (m+n)m + mn = m^2 - m^2 - mn + mn = 0$.

Coefficient of $Be^{nx}$: $n^2 - (m+n)n + mn = n^2 - mn - n^2 + mn = 0$.

**Step 4:** Both coefficients vanish, so the whole expression is zero:

$$y'' - (m+n)y' + mny = 0 \cdot Ae^{mx} + 0 \cdot Be^{nx} = 0$$

**Final Answer:**

$$\boxed{\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0}$$

**Quick Tip:** Differentiate twice, substitute into the given expression, and show the coefficients of e^mx and e^nx each vanish.

**6(e).** If a die is thrown three times, then find the probability of getting at least one odd number.

**Correct Answer:** —

**Solution:**

**Step 1:** Find the probability of NOT getting an odd number on a single throw:

A die has 3 odd faces (1,3,5) and 3 even faces (2,4,6), so $P(\text{even}) = \frac{3}{6} = \frac{1}{2}$.

**Step 2:** Find the probability of getting NO odd number in all three throws:

The three throws are independent, so $P(\text{no odd in 3 throws}) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$.

**Step 3:** Use the complement rule:

$$P(\text{at least one odd}) = 1 - P(\text{no odd at all}) = 1 - \frac{1}{8} = \frac{7}{8}$$

**Final Answer:**

$$\boxed{\frac{7}{8}}$$

**Quick Tip:** Use the complement: 1 - P(all three throws are even).

**7.** If $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$, then find $A^{-1}$.

**Correct Answer:** —

**Solution:**

**Step 1:** Find $|A|$ by expanding along the first row:

$$|A| = 1(4 \cdot 6 - 5 \cdot 5) - 2(2 \cdot 6 - 5 \cdot 3) + 3(2 \cdot 5 - 4 \cdot 3)$$

$$= 1(24 - 25) - 2(12 - 15) + 3(10 - 12) = -1 + 6 - 6 = -1$$

**Step 2:** Find all nine cofactors:

$C_{11} = 24 - 25 = -1,\ C_{12} = -(12 - 15) = 3,\ C_{13} = 10 - 12 = -2$

$C_{21} = -(12 - 15) = 3,\ C_{22} = 6 - 9 = -3,\ C_{23} = -(5 - 6) = 1$

$C_{31} = 10 - 12 = -2,\ C_{32} = -(5 - 6) = 1,\ C_{33} = 4 - 4 = 0$

**Step 3:** Build the adjugate (transpose of the cofactor matrix):

$$\text{adj}(A) = \begin{bmatrix} -1 & 3 & -2 \\ 3 & -3 & 1 \\ -2 & 1 & 0 \end{bmatrix}$$

**Step 4:** Divide by $|A| = -1$:

$$A^{-1} = \frac{1}{-1}\begin{bmatrix} -1 & 3 & -2 \\ 3 & -3 & 1 \\ -2 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{bmatrix}$$

**Final Answer:**

$$\boxed{A^{-1} = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 3 & -1 \\ 2 & -1 & 0 \end{bmatrix}}$$

**Quick Tip:** Compute |A| via cofactor expansion, build the adjugate from all nine cofactors, then divide by |A|.

***OR***

Solve the following system of equations by using the Matrix Method:

$$x - y + 2z = 7$$

$$3x + 4y - 5z = -5$$

$$2x - y + 3z = 12$$

**Correct Answer:** —

**Solution:**

**Step 1:** Write as $AX = B$ and find $|A|$:

$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}, B = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}.$

$$|A| = 1(12 - 5) - (-1)(9 + 10) + 2(-3 - 8) = 7 + 19 - 22 = 4$$

**Step 2:** Compute all nine cofactors:

$C_{11} = 12 - 5 = 7,\ C_{12} = -(9 + 10) = -19,\ C_{13} = -3 - 8 = -11$

$C_{21} = -(-3 + 2) = 1,\ C_{22} = 3 - 4 = -1,\ C_{23} = -(-1 + 2) = -1$

$C_{31} = 5 - 8 = -3,\ C_{32} = -(-5 - 6) = 11,\ C_{33} = 4 + 3 = 7$

**Step 3:** Build the adjugate and divide by $|A| = 4$ to get $A^{-1}$:

$$\mathrm{adj}(A) = \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}, \qquad A^{-1} = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$

**Step 4:** Compute $X = A^{-1}B$:

$\mathrm{adj}(A) \cdot B$: row1 $= 7(7) + 1(-5) - 3(12) = 49 - 5 - 36 = 8$; row2 $= -19(7) - 1(-5) + 11(12) = -133 + 5 + 132 = 4$; row3 $= -11(7) - 1(-5) + 7(12) = -77 + 5 + 84 = 12$.

$$X = \frac{1}{4}\begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

**Final Answer:**

$$\boxed{x = 2,\ y = 1,\ z = 3}$$

**Quick Tip:** Find A^-1 via adjugate/determinant, then compute X = A^-1 B.

**8.** Prove that the height of the cylinder of maximum volume inscribed in a sphere of radius $R$ is $\frac{2R}{\sqrt{3}}$. Also find the maximum volume of the cylinder.

**Correct Answer:** —

**Solution:**

**Step 1:** Set up the constraint between cylinder half-height and radius:

Let the cylinder have height $H = 2h$ and base radius $r$. Since the cylinder is inscribed in a sphere of radius $R$, the center of the sphere, the center of the cylinder's axis and a top-rim point form a right triangle: $r^2 + h^2 = R^2$.

**Step 2:** Write the volume as a function of $h$ alone:

$$V = \pi r^2 H = \pi(R^2 - h^2)(2h) = 2\pi(R^2 h - h^3)$$

**Step 3:** Differentiate and set to zero:

$$\frac{dV}{dh} = 2\pi(R^2 - 3h^2) = 0 \implies h^2 = \frac{R^2}{3} \implies h = \frac{R}{\sqrt{3}}$$

Check it's a maximum: $\dfrac{d^2V}{dh^2} = 2\pi(-6h) < 0$ for $h > 0$, confirming a maximum.

**Step 4:** Find the height and the maximum volume:

$$H = 2h = \frac{2R}{\sqrt{3}} \quad \text{(as required)}$$

$$r^2 = R^2 - h^2 = R^2 - \frac{R^2}{3} = \frac{2R^2}{3}$$

$$V_{max} = \pi r^2 H = \pi \cdot \frac{2R^2}{3} \cdot \frac{2R}{\sqrt{3}} = \frac{4\pi R^3}{3\sqrt{3}} = \frac{4\sqrt{3}\,\pi R^3}{9}$$

**Final Answer:**

$$\boxed{H = \frac{2R}{\sqrt{3}}, \qquad V_{max} = \frac{4\pi R^3}{3\sqrt{3}}}$$

**Quick Tip:** Express V as a function of the half-height h using r^2+h^2=R^2, then maximize via dV/dh=0.

***OR***

Solve the differential equation $(x-y)\frac{dy}{dx} = x + 2y$.

**Correct Answer:** —

**Solution:**

**Step 1:** Recognize this as a homogeneous equation and substitute $y = vx$:

$\frac{dy}{dx} = v + x\frac{dv}{dx}$. The equation $(x-y)\frac{dy}{dx} = x + 2y$ becomes $x(1 - v)\left(v + x\frac{dv}{dx}\right) = x(1+2v)$, i.e. $(1-v)\left(v + x\frac{dv}{dx}\right) = 1 + 2v$.

**Step 2:** Expand and isolate the $x\,dv/dx$ term:

$$v - v^2 + x(1-v)\frac{dv}{dx} = 1 + 2v$$

$$x(1-v)\frac{dv}{dx} = 1 + 2v - v + v^2 = v^2 + v + 1$$

**Step 3:** Separate variables:

$$\frac{1-v}{v^2+v+1}\,dv = \frac{dx}{x}$$

**Step 4:** Integrate the left side by splitting $1-v = -\frac{1}{2}(2v+1)+\frac{3}{2}$:

$$\int \frac{1-v}{v^2+v+1}dv = -\frac{1}{2}\ln(v^2+v+1)+\sqrt{3}\tan^{-1}\left(\frac{2v+1}{\sqrt{3}}\right)$$

(using $\int \frac{2v+1}{v^2+v+1}dv = \ln(v^2+v+1)$ and completing the square $v^2 + v + 1 = (v+\frac{1}{2})^2+\frac{3}{4}$).

**Step 5:** Equate to $\ln|x|+C$ and substitute back $v = y/x$:

$$-\frac{1}{2}\ln\left(\frac{x^2+xy+y^2}{x^2}\right)+\sqrt{3}\tan^{-1}\left(\frac{x+2y}{x\sqrt{3}}\right) = \ln|x|+C$$

The $\ln x^2$ term combines with $\ln|x|$ on the right and simplifies to:

**Final Answer:**

$$\boxed{\sqrt{3}\tan^{-1}\left(\frac{x+2y}{\sqrt{3}\,x}\right)-\frac{1}{2}\ln(x^2+xy+y^2) = C}$$

**Quick Tip:** Substitute y=vx (homogeneous equation), separate variables, and complete the square in the denominator.

**9.** Find the value of: $\displaystyle\int_0^{\pi/2} \log\sin x\,dx$.

**Correct Answer:** —

**Solution:**

**Step 1:** Let $I = \int_0^{\pi/2} \log \sin x \, dx$ and use the property $\int_0^a f(x)dx = \int_0^a f(a-x)dx$:

$$I = \int_0^{\pi/2} \log \sin\left(\frac{\pi}{2} - x\right) dx = \int_0^{\pi/2} \log \cos x \, dx$$

**Step 2:** Add the two expressions for $I$:

$$2I = \int_0^{\pi/2} \left(\log \sin x + \log \cos x\right)dx = \int_0^{\pi/2} \log(\sin x \cos x)\, dx = \int_0^{\pi/2} \log\left(\frac{\sin 2x}{2}\right) dx$$

$$2I = \int_0^{\pi/2} \log \sin 2x \, dx - \int_0^{\pi/2} \log 2 \, dx$$

**Step 3:** Evaluate $\int_0^{\pi/2} \log \sin 2x \, dx$ via the substitution $t = 2x$:

$$\int_0^{\pi/2} \log \sin 2x \, dx = \frac{1}{2} \int_0^{\pi} \log \sin t \, dt = \frac{1}{2} \cdot 2 \int_0^{\pi/2} \log \sin t \, dt = I$$

(using symmetry of $\sin t$ about $t = \pi/2$ on $[0, \pi]$).

**Step 4:** Substitute back and solve for $I$:

$$2I = I - \frac{\pi}{2}\log 2 \implies I = -\frac{\pi}{2}\log 2$$

**Final Answer:**

$$\boxed{I = -\frac{\pi}{2}\ln 2}$$

**Quick Tip:** Use the King's rule (property P4) to relate I to log cos x, add, use sin2x, and self-reference the same integral.

***OR***

Solve: $\int \left[\sqrt{\cot x} + \sqrt{\tan x}\right] dx.$

**Correct Answer:** –

**Solution:**

**Step 1:** Combine the two terms over a common form:

$$\sqrt{\cot x} + \sqrt{\tan x} = \frac{\cos x}{\sqrt{\sin x \cos x}} + \frac{\sin x}{\sqrt{\sin x \cos x}} = \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}}$$

**Step 2:** Rewrite $\sin x \cos x$ using the double angle identity:

$\sin x \cos x = \frac{\sin 2x}{2}$, so the integral becomes $\int \frac{\sqrt{2}(\sin x + \cos x)}{\sqrt{\sin 2x}} dx.$

**Step 3:** Substitute $t = \sin x - \cos x$, so $dt = (\cos x + \sin x)\, dx$:

Also $t^2 = (\sin x - \cos x)^2 = 1 - 2\sin x \cos x = 1 - \sin 2x$, so $\sin 2x = 1 - t^2$.

The integral becomes $\int \frac{\sqrt{2}\, dt}{\sqrt{1 - t^2}}.$

**Step 4:** Integrate using the standard arcsine form:

$$\sqrt{2}\int \frac{dt}{\sqrt{1-t^2}} = \sqrt{2}\sin^{-1}(t) + C$$

**Final Answer:**

$$\boxed{\sqrt{2}\ \sin^{-1}(\sin x - \cos x) + C}$$

**Quick Tip:** Combine both terms over sqrt(sin x cos x), substitute t = sin x - cos x, and integrate to an arcsine.