

4.7 PROJECTION OF SOLIDS

After having the study of projections of points, lines and planes, we can now proceed to the projection of solids. Solids are kept in different positions and its projections are done in the following topics.

4.7.1 PROJECTION OF SOLIDS WHEN ITS AXIS PERPENDICULAR TO ONE REF. PLANE AND II TO THE OTHER

Case (i) Axis perpendicular to the H.P. & Parallel to the V.P.

Suppose a cone rests on H.P. with its base (Fig. 4.85) and axis perpendicular to H.P., the projections are done as shown in Fig. 4.86

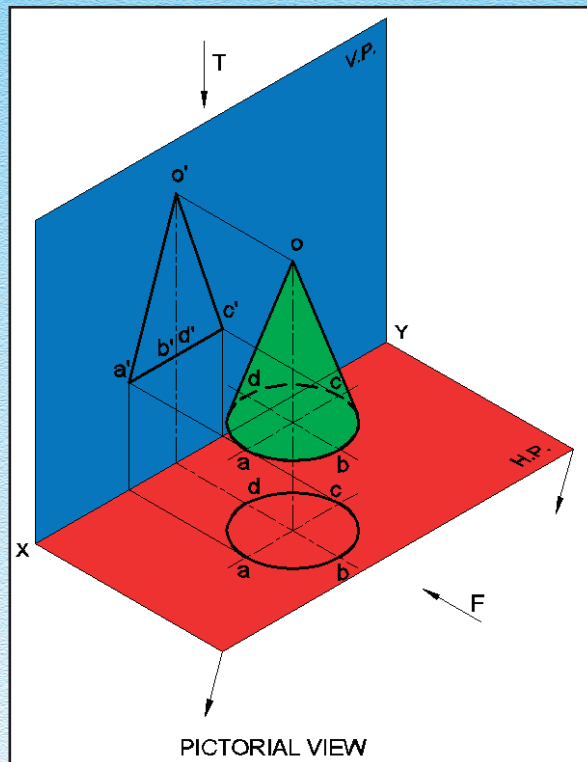


Fig. 4.85

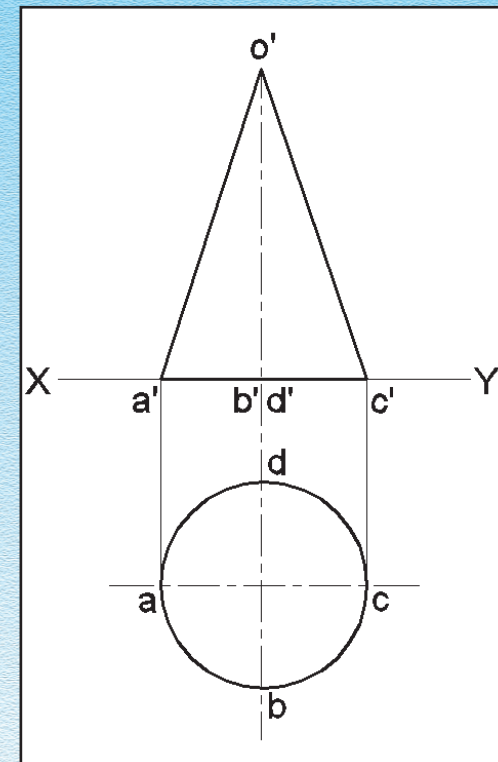


Fig. 4.86

Since the Top View shows more detail of the object, the Top View should be drawn first and then the Front View is to be projected from it.

DO YOU KNOW ?

In third angle projections, the solid can not rest easily but it will fall down as it has got no support. In order to place the solid in correct position, a third plane $\parallel$ to H.P. is considered and is known as Auxiliary Horizontal plane (A.H.P.) is also known as ground plane.

Example 4.15 : Project the frontview and topview of a hexagonal prism of 25 mm base edges and 50 mm height, having two of its vertical rectangular faces parallel to V.P. and its base resting on H.P.

Solution : refer fig. 4.87

Steps Involved :

Here the base of the solid rests on H.P., So its axis is $\perp$ to H.P.

- (i) Start with the Top View, which is a hexagon of side 25 mm
- (ii) Project the Front View from the Top View, which comprises three rectangles.

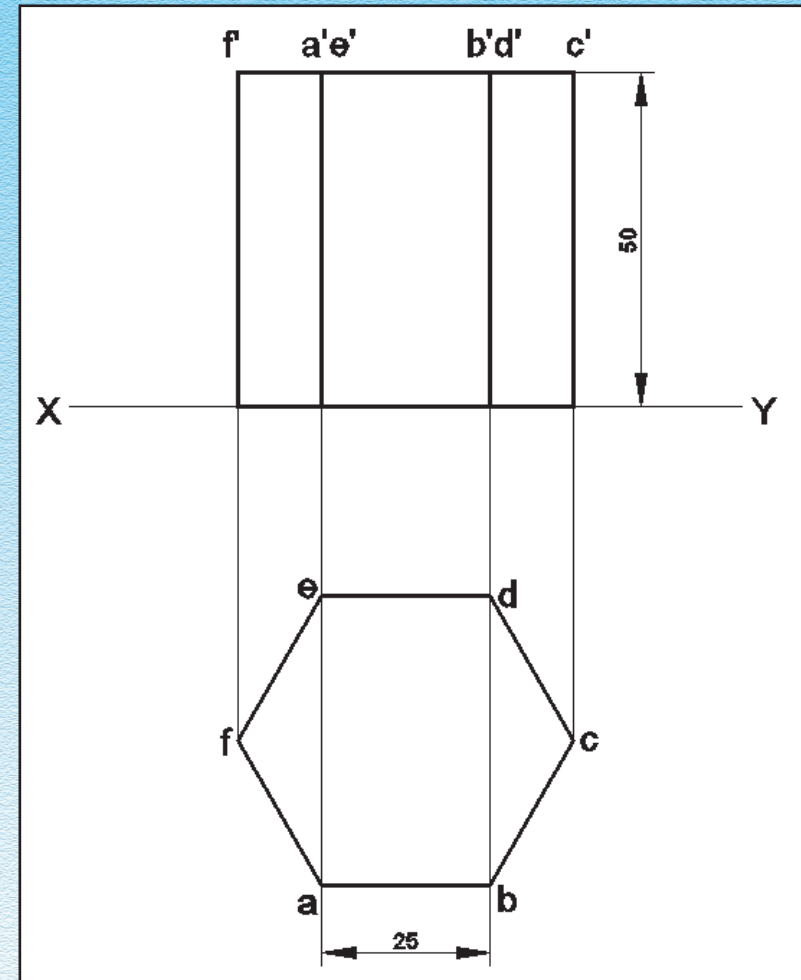


Fig. 4.87

Example 4.16 : A cube of 40 mm long edge rests on H.P. and its vertical faces are equally inclined to V.P. Draw the projections of the solid.

Solution : refer fig. 4.88

Steps (i) Start with the Top View

Construct a square on a line 'ad' 40 mm long and inclined at 45° to XY line.

(ii) Project the Top View above XY to get the Front View which comprises 2 rectangles.

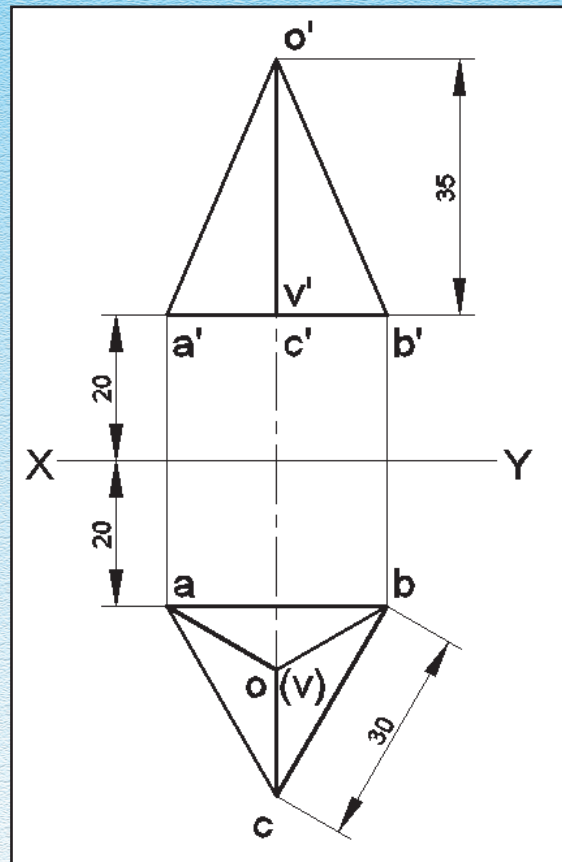


Fig. 4.89

Example 4.17 : A triangular pyramid with 30 mm edge at its base and 35 mm long axis resting on its base with an edge of the base near the VP, parallel to and 20 mm from the V.P. Draw the projections of the pyramid, if the base is 20 mm above the H.P.

Solution :

Given Data

- Triangular pyramid, 30×35 mm
- Base on ground, $\therefore$ axis perpendicular to HP and base $\parallel$ to the HP. We recall the fact that, the projections of vertical solids to be started with the Top View.

Steps

- (i) Draw an equilateral triangle.
- (ii) Name its corners and mark its centre O.
- (iii) Complete the Top View by drawing lines joining the Centre with the corners.
- (iv) Now, project the Front View as shown in the fig. 4.89

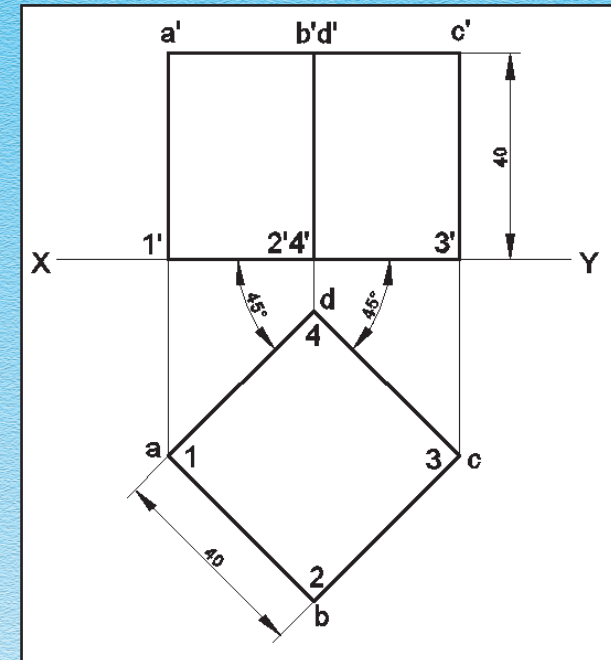


Fig. 4.88

Example 4.18 : Project the Front View and Top View of a sphere of 50 mm diameter, resting on the H.P.

Solution : see fig. 4.90

We should remember the fact that the projection of a sphere in any position, on any plane is always a circle whose diameter is equal to the diameter of the sphere.

∴ Draw a circle each for Front View and Top View which are the required projections.

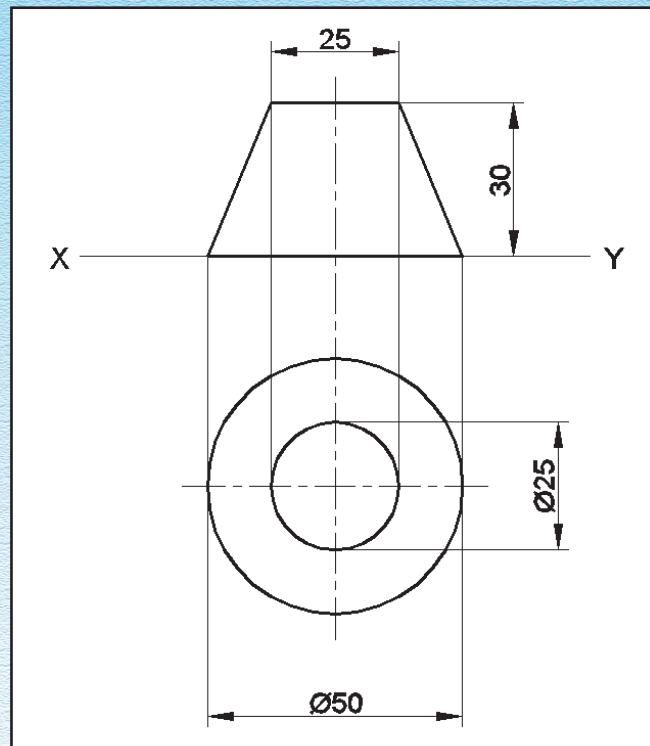


Fig. 4.91

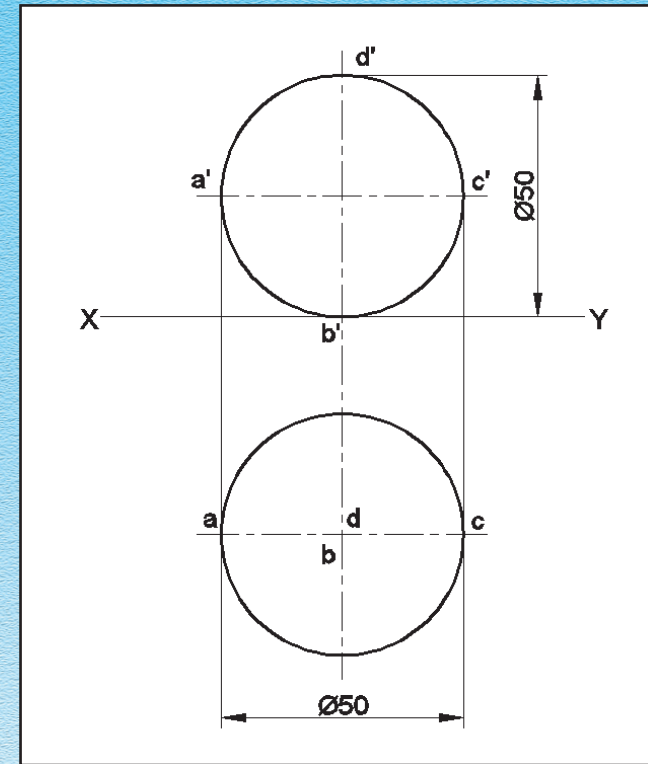


Fig. 4.90

Example 4.19 : The frustum of a cone with base dia = 50 mm, top face diameter = 25 mm and vertical axis = 30 mm is resting on its base on H.P. Project its Front View and Top View.

Solution : see fig. 4.91

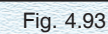
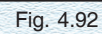
Given Data

- Frustum of a cone.
- Resting on H.P., so axis $\perp$ to H.P.
- (i) Start with the Top View Draw two concentric circles of ϕ 25 and ϕ 50.
- (ii) Project the Top View above to get the Front View, which is a trapezium.

ASSIGNMENT

1. Project the Front View and Top View of a square prism of 35 mm base edges and 50 mm vertical height, rests on H.P., with two of its vertical rectangular faces parallel to V.P.
2. A triangular prism of 40 mm base edges and 60 mm height, standing on its on H.P. with one of its vertical rectangular faces on the rear, || to V.P. Draw its projections.
3. Draw the projections of a cylinder, which rests on H.P. on its base, with 30 mm base dia and 40 mm long axis.
4. Project the Front View and Top View of a hemisphere which rests on H.P. with its circular face on Top. ($\phi = 60$ mm)
5. Project the Front View and Top View of the frustum of a hexagonal pyramid, of 25 mm base edges and 70 mm height, cut at mid-height, parallel to its base.

Suppose a cylinder is kept in I quadrant in such a way that, the axis is $\perp$ to V.P. (Fig. 4.92), its projections are done as shown in Fig. 4.93



Here Front View shows more details of the object, so the Front View should be drawn first and then the Top View is to be projected from it.

Example 4.20 : Draw the frontview and topview of a square pyramid of base edge 40 mm and axis 50 mm long, which is perpendicular to V.P., and the vertex is in front.

Solution refer Fig. 4.94

GIVEN DATA

- Square pyramid
 - axis $\perp$ to V.P.
 - Vertex in front
- (i) Since axis perpendicular to V.P., start with the Front View, which is a square with four triangular faces, in it.
 - (ii) Project down the Front View to get the Top View. Top View is a triangle showing one of the triangular face of the pyramid.

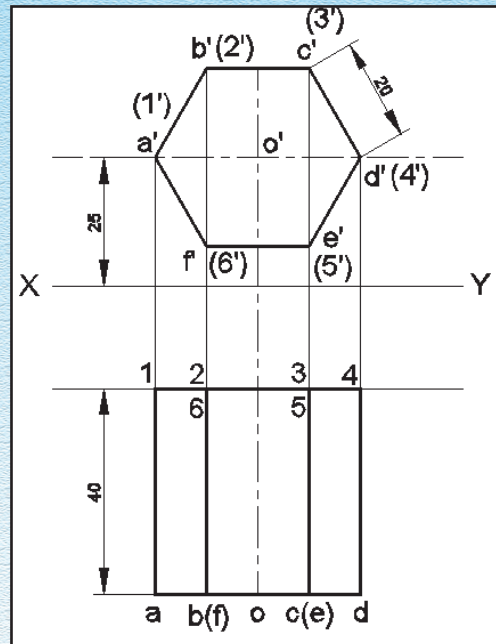


Fig. 4.95

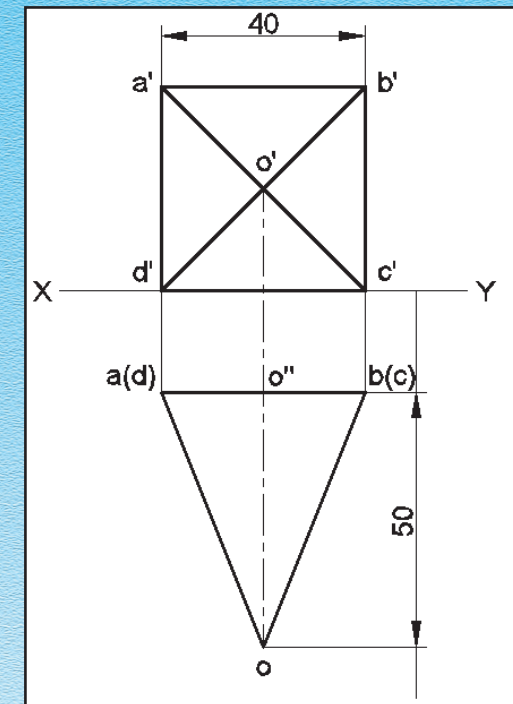


Fig. 4.94

Example 4.21 : A hexagonal prism, base 20 mm side and axis 40 mm long is lying on one of its rectangular faces. Its axis is perpendicular to V.P. and 25 mm above the ground. The nearer end is 20 mm in front of V.P. Draw its projections.

Solution refer Fig. 4.95

GIVEN DATA

- hexagonal prism
- axis $\perp$ to V.P.

Since the axis is perpendicular to V.P., Front View to be drawn first.

- (i) Front View is a hexagon
- (ii) Front View is projected down to get the Top View which comprises 3 rectangles.

Example 4.22 : The frustum of a cone of 40 mm base diameter and 20 mm cut face diameter, rests on H.P. with its 40 mm long axis parallel to H.P. and at right angles to V.P. the cut face is in front. Project its Front View and Top View.

Solution refer Fig. 4.96

GIVEN DATA

- Frustum of a cone
 - axis $\perp$ to V.P.
 - cut face is in front
- (i) Draw a XY line
 - (ii) Draw 2 concentric circles of diameter 20 and 40 above XY which is the required Front View
 - (iii) Project down the Top View which is in the shape of a trapezium.

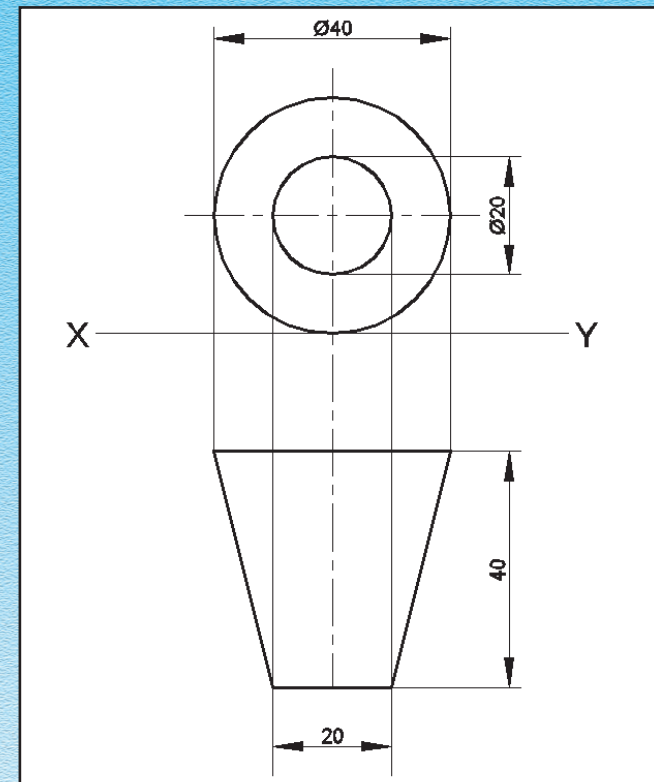


Fig. 4.96

ASSIGNMENT

- Q1. Project the Front View and topview of a hollow cylinder (Pipe) having outer diameter = 50 mm, inner diameter = 40 mm and length = 50 mm, resting on the H.P., with its axis $\perp$ to V.P.
- Q2. A triangular pyramid, of 50 mm base and 50 mm axis, is resting on its base corner on the H.P., so that the upper edge, of the base is horizontal. The base of the pyramid is on the rear and $\parallel$ to V.P. Draw its projections.
- Q3. Project the frontview and topview of a pentagonal prism of 30 mm base edges and 60 mm long edges which are $\perp$ to V.P., and its rectangular face on top is parallel to H.P.
- Q4. The frustum of a square pyramid of 40 mm base edges and 20 mm cut face (top) edges is resting on H.P. on a base edge with its 50 mm long axis horizontal and at right angles to V.P. the cut face is in front. Draw its projections.
- Q5. A hexagonal prism, of 25mm base and 60 mm axis, is resting on one of the its base edges on the H.P. and its axis is perpendicular to V.P. Project its Front View and Top View.
- Q6. Project the frontview and Top View of a cylinder, with base diameter = 50 mm and height = 70 mm, resting on H.P., with its axis perpendicular to V.P.
- Q7. Draw the Front View and topview of a cone of base diameter = 30 mm and axis = 65 mm, with its axis perpendicular to V.P., keeping the vertex in front.
- Q8. A square prism, base 40 mm side and axis 70 mm long is lying on one of its rectangular faces. Its axis is perpendicular to V.P. Draw its Front View and Top View.
- Q9. The frustum of a triangular pyramid of 50 mm base edge and 20 mm top edge, rests on H.P. With its base edge on it and the 60 mm long axis parallel to H.P. and at right angles to V.P. The cut face is in front. Project its frontview and topview.
- Q10. A right regular pentagonal pyramid of base edge = 25 mm and height = 60 mm, having its axis perpendicular to V.P., with its base parallel to V.P. Draw its projections.

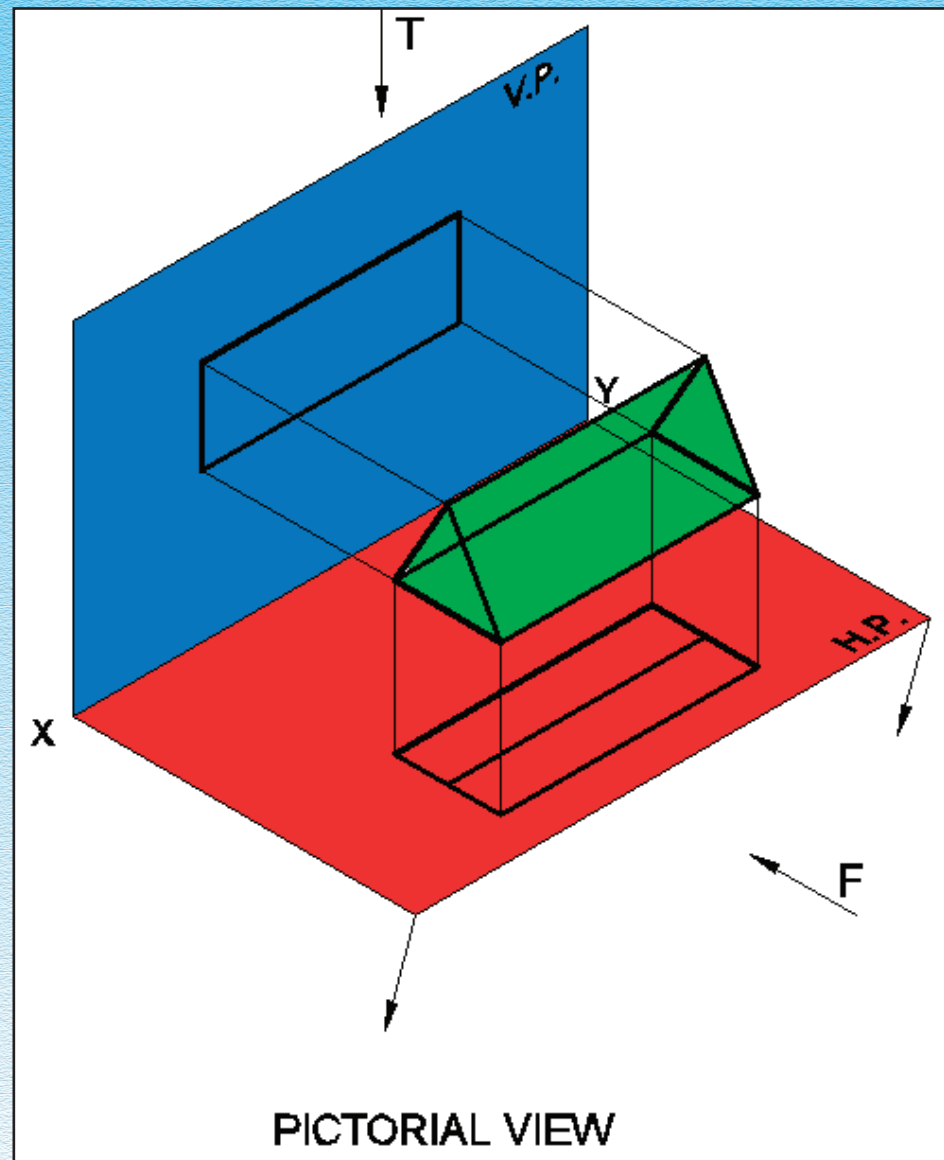
4.7.2 PROJECTION OF SOLIDS WHEN ITS AXIS IS PARALLEL TO BOTH THE REFERENCE PLANES

Fig. 4.97

Suppose, a triangular prism is placed in I quadrant, in such a way that the axis is parallel to both V.P. and H.P. (Fig. 4.90) its projections are done as shown in Fig. 4.98

In this position of the solid, the Front View does not reveal about the base of the solid, even Top View does not show it. We have to take the side view projected on A.V.P. So, side view must be drawn first, then Front View and Top View are projected from it.

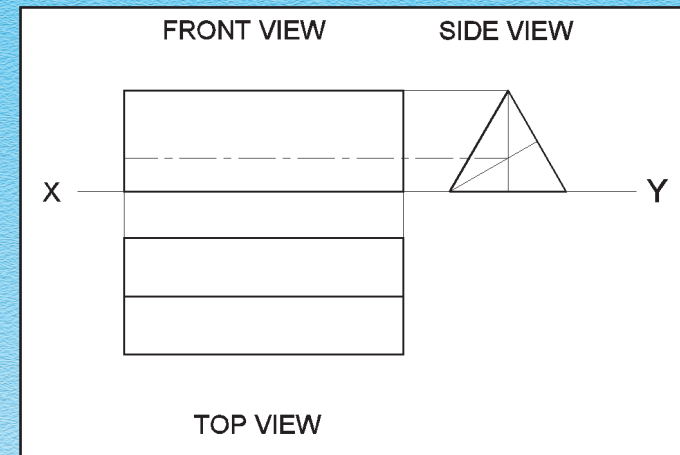


Fig. 4.98

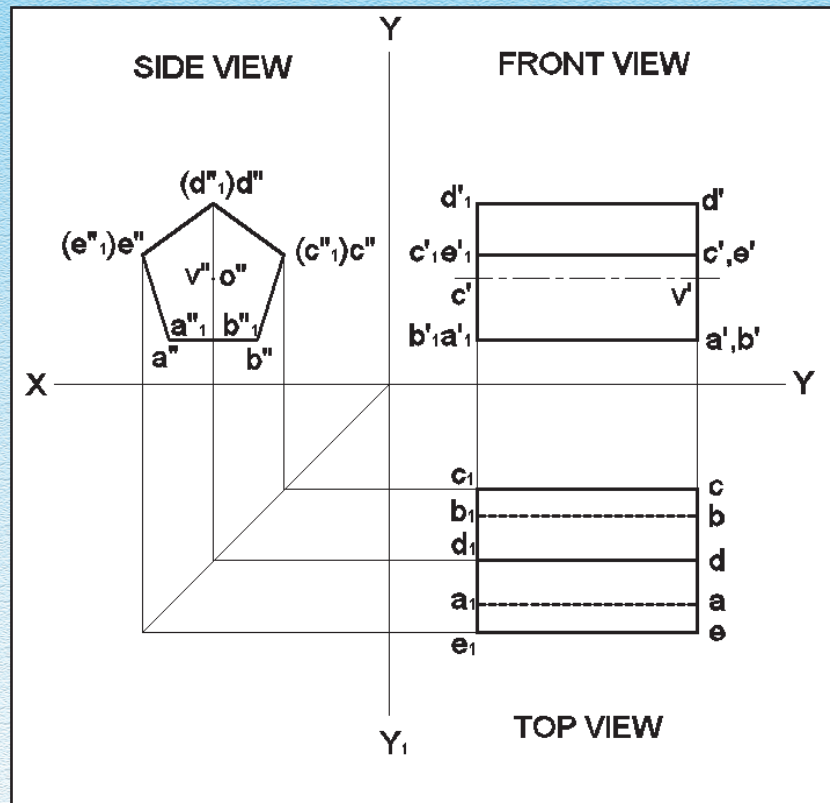


Fig. 4.99

Example 4.23 : A pentagonal prism having a 20 mm edge of its base and an axis of 50 mm length is resting on one of its rectangular faces with the axis perpendicular to the side plane. Draw the projections of the prism.

Solution refer Fig. 4.99

GIVEN DATA

- pentagonal prism
- axis perpendicular profile planed side plane
- rectangular face parallel to V.P.

Steps :

- (i) Draw a XY line
- (ii) Here, Axis perpendicular to P.P/side plane means the axis is parallel to both V.P. & H.P.

So start with the side view which is a triangle with true shape and size.

- (iii) Then, project the corresponding Front View and Top View, which are rectangles.

Example 4.24 : Project the Front View and Top View of a pentagonal pyramid of 30 mm base edges and 70 mm long horizontal axis, parallel to V.P., when it is resting on one corner of its base with one edge of its base on top, $\parallel$ to H.P.

Solution refer Fig. 4.100

GIVEN DATA

- pentagonal pyramid
- Axis parallel to both V.P. & H.P.
- Standing on its corner

Steps :

- (i) Draw a XY line
- (ii) Since axis parallel to VP & HP helping view side view is drawn first, which is a pentagon with the edge $\parallel$ to HP on the top side.
- (iii) Then, using the projectors, get the Front View and Top View.

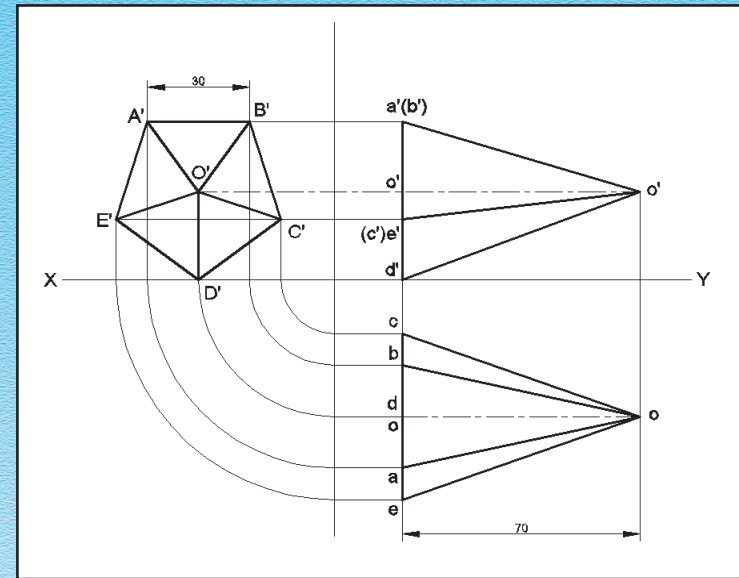


Fig. 4.100

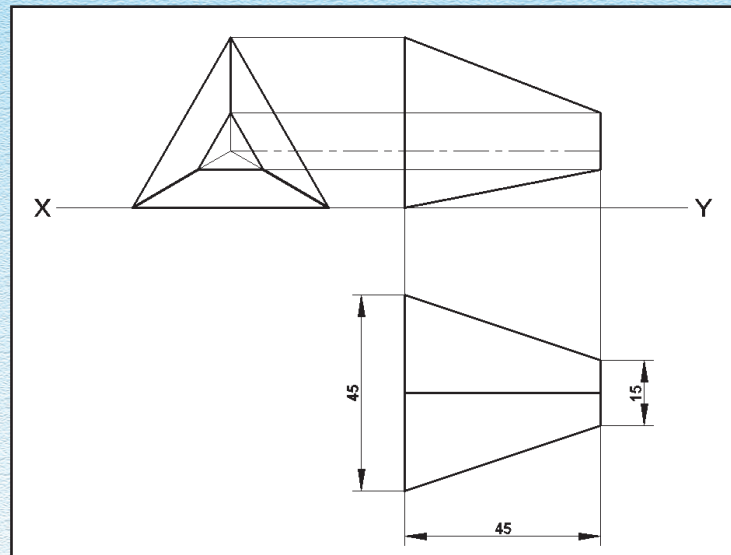


Fig. 4.101

Example 4.25 : The frustum of a triangular pyramid of 45 mm base edges and 15 mm cut face to edges, is standing on one of its base edges, which is at right angles to VP. The axis is $\parallel$ to both V.P. and H.P. Draw its projections.

Solution refer Fig. 4.101

GIVEN DATA

Helping view, side view must be drawn first. Then project it to get the Front View and Top View.

ASSIGNMENT

1. A hexagonal prism, base 25 mm side, axis 60 mm long, is lying on the ground on one of its faces with the axis parallel to both V.P. and H.P. Draw its projections.
2. A triangular pyramid, base 25 mm side axis 50 mm long, is resting on the ground on one of its edges of the base. Its axis is $\parallel$ to both the planes. Draw its projections.
3. A cylinder, base 40 mm diameter, axis 60 mm long, is lying on the ground on its generators with the axis $\parallel$ to both V.P. and H.P. Draw its projections.
4. The frustum of a hexagonal pyramid of 20 mm base edges and 10 mm cut face top edges is resting on H.P. on a base edge with its 50 mm long axis horizontal and parallel to V.P. The cut top face is in front. Draw its projections.

4.7.3 PROJECTION OF SOLIDS WHEN ITS AXIS PARALLEL TO REFERENCE PLANE & INCLINED TO THE OTHER CASE (I) WHEN THE AXIS INCLINED TO H.P. & PARALLEL TO V.P.

Fig. 4.102 shows pictorially a square pyramid with its axis $\perp$ to the H.P. and $\parallel$ to the V.P. in the first step and having its axis inclined at θ to the H.P.

This kind of two steps are required, when the axis of a solid inclined to any one of the reference principal plane.

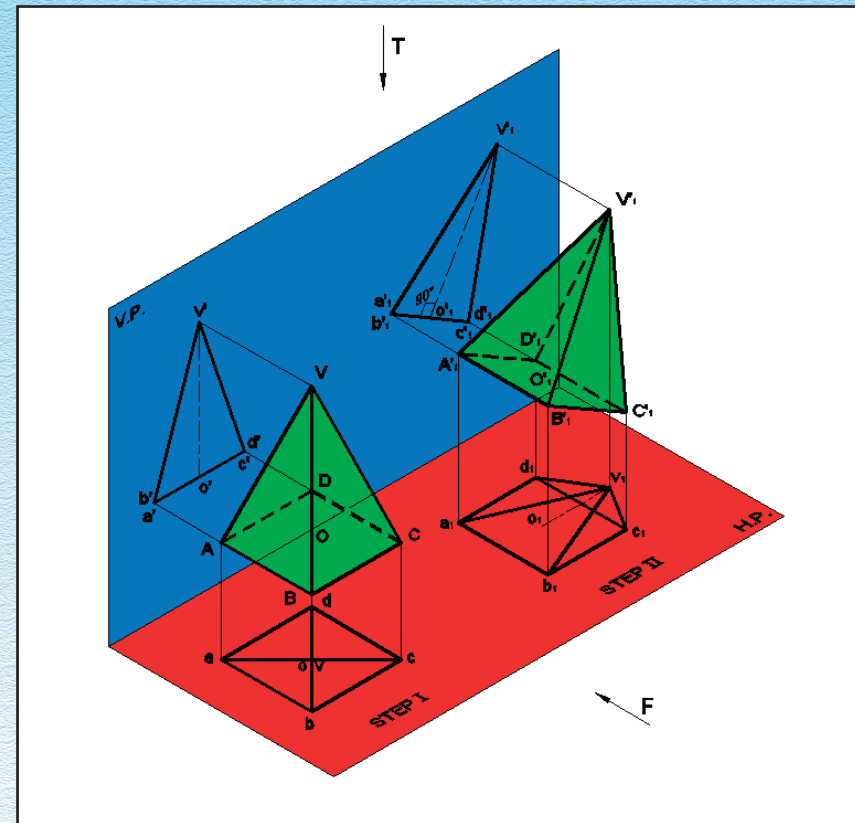


Fig. 4.102

Its projections are done as follows. Fig. 4.103

- (i) Initially, the solid is assumed to have its axis perpendicular to H.P. Then views are drawn for this simple position.
- (ii) Then in the second step, the solid is tilted to the given inclination with H.P. Then redraw to Front View in the previous step to the required inclination.
- (iii) Required Top View will be projected from the Front View.

Example 4.26 : A pentagonal prism having 20 mm edges at its base and axis of 70 mm length is resting on one of the edges of its base with its axis parallel to the V.P. and inclined at 30° to the H.P.

GIVEN DATA

- Pentagonal prism 20 mm $\times$ 70 mm
- axis $\parallel$ to VP and inclined to H.P. at 30°

Steps 1 : Assume, the axis is perpendicular to H.P. start drawing with the Top View, which is a pentagon i.e., true shape of the base Fig. 4.104

Steps 2 : Project the Front View above XY

Steps 3 : Then redraw this Front View with axis inclined at 30° to the XY line.

Steps 4 : Draw vertical projectors from each of the redrawn points in the Front View and horizontal lines from corresponding points in the topview drawn in step I.

Steps 5 : Note the corresponding points of intersection and join them to obtain projections of all the surface boundaries.

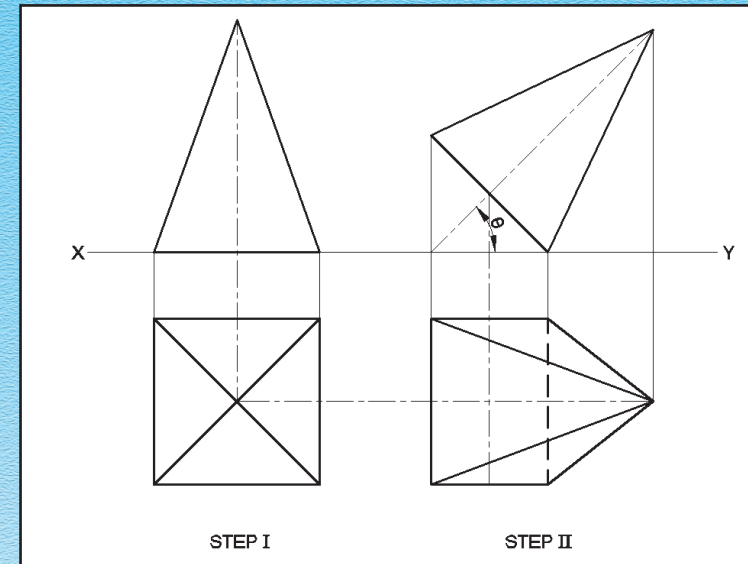


Fig. 4.103

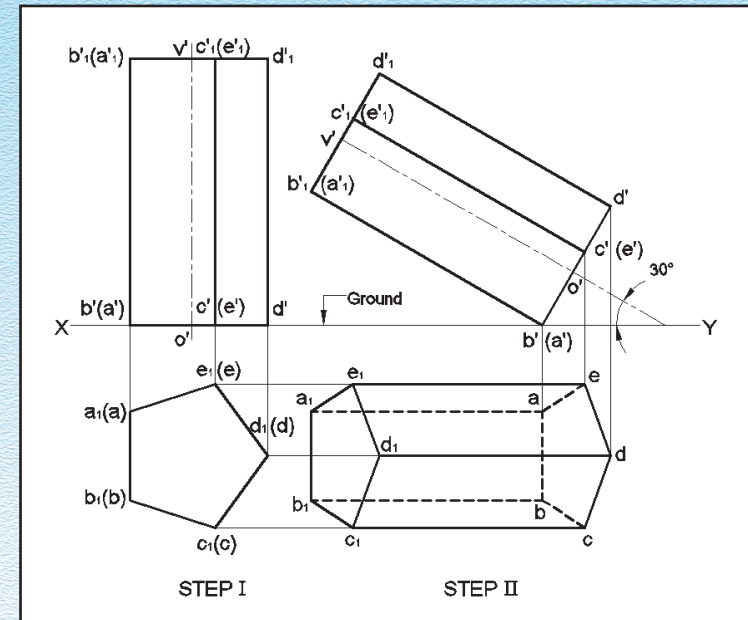


Fig. 4.104

Example 4.27 : Draw the projections of a hexagonal pyramid, base 20 mm side and axis 45 mm long, has an edge of its base on the ground. Its axis is inclined at 60° to the ground and parallel to the V.P.

Solution see Fig. 4.105

- (i) Assuming the axis to be $\perp$ to the ground, draw the Top View abcdef below XY.
- (ii) Project the Front View as shown in Fig. 4.105
- (iii) Now tilt the pyramid about the edge. On tilting, the axis will become inclined to the ground but will remain $\parallel$ to V.P. The axis makes 30° angle with XY.
- (iv) Now from this Front View project all the points downwards and draw horizontal lines from first Top View.
- (v) Reproduce the new Top View by joining the apex with the corners of the base and also draw lines for the edges of the base as shown in the fig. 4.105.

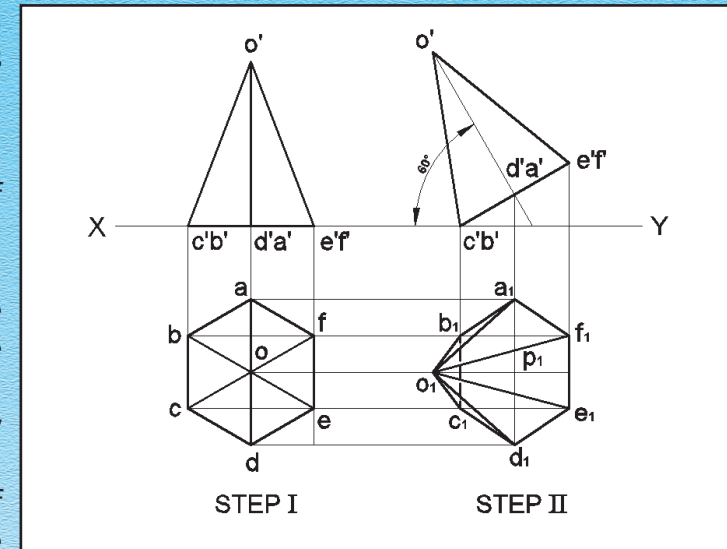


Fig. 4.105

Hidden portion of the pyramid is shown by dashed (dotted) lines.

Example 4.28 : The frustum of a cone of 45 mm base dia, 25 mm cut face diameter and 50 mm axis, rests on H.P. so that its axis is $\parallel$ to V.P. and inclined at 30° towards the right. Draw its projections, when the cut-face is one top.

Solution Fig. 4.106

- Steps I :**
- (i) The frustum is assumed to be in the simple position is axis $\perp$ to H.P. So Draw its Top View first which are concentric circle ϕ 25 and ϕ 45
 - (ii) Project the Top View above XY to get the Front View.
- Steps II :**
- (iii) Tilt the Front View to the required inclination is axis makes 30° with XY line.
 - (iv) Project down this Front View and project horizontally from the previous Top View, to draw the required Top View, matching the corresponding points.

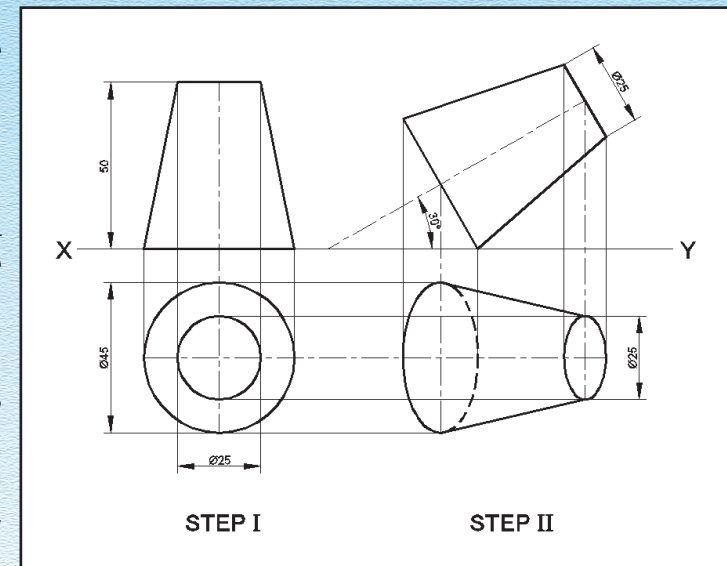


Fig. 4.106

Note : The titled circular ends are seen as ellipses in the topview, i.e. compressed (foreshortened) circle is ellipse.

ASSIGNMENT

1. A triangular prism with 25 mm edges at its base and the axis 60 mm long is resting on one of the edges of its base with axis $\parallel$ to V.P. and inclined at 30° to the H.P. Draw the projections of the prism.
2. A pentagonal pyramid of 25 mm edges of its base and axis 50 mm, has its axis perpendicular to the V.P. and 50 mm above the H.P. Draw the projections of the pyramid if one edge of its base is inclined at 30° to the H.P.
3. A frustum of square pyramid of 20 mm edges at the top, 40 mm edges at the bottom and 50 mm length of the axis has its side surface (face) inclined at 45° to the H.P. with axis $\parallel$ to V.P. Draw the projections of the frustum.
4. The frustum of a cone, which is 90 mm base diameter and 30 mm top diameter. Draw the projections of the cone frustum when its axis is parallel to the V.P. and inclined at (i) 30° to H.P. (ii) 60° to the H.P.

ACTIVITY

You will be studying about the development of surfaces in the later unit of this book. Using the knowledge of development of surfaces, develop different types of solids like prisms, pyramids, cones etc. and study their projections in different possible positions. Make sphere out of plasticine or clay. Cut the sphere into 2 halves to get himisphere.

Case (ii) When the axis inclined to V.P. & parallel to H.P.

The above given figure Fig. 4.107 shows pictorially a square pyramid with its axis perpendicular to V.P. and parallel to H.P. in the first step and having its axis inclined to V.P. at ϕ and parallel to H.P. in the second step.

Such problems are solved in two steps as shown in Fig. 4.108

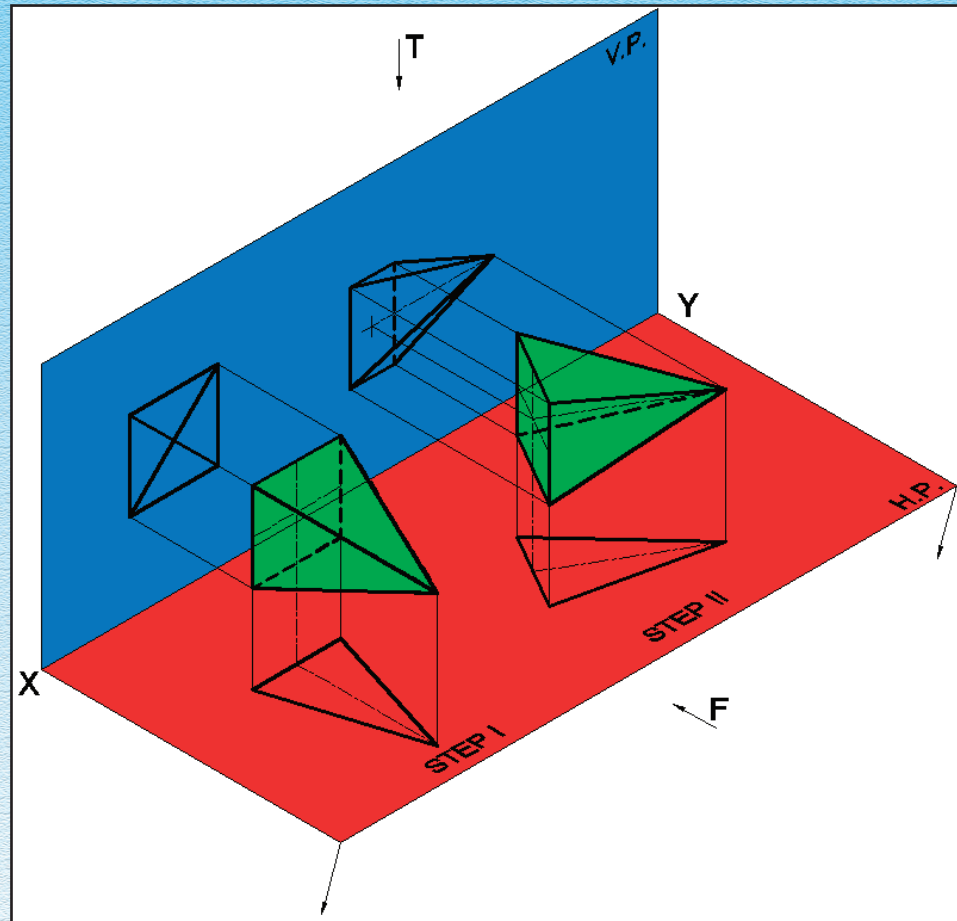


Fig. 4.107

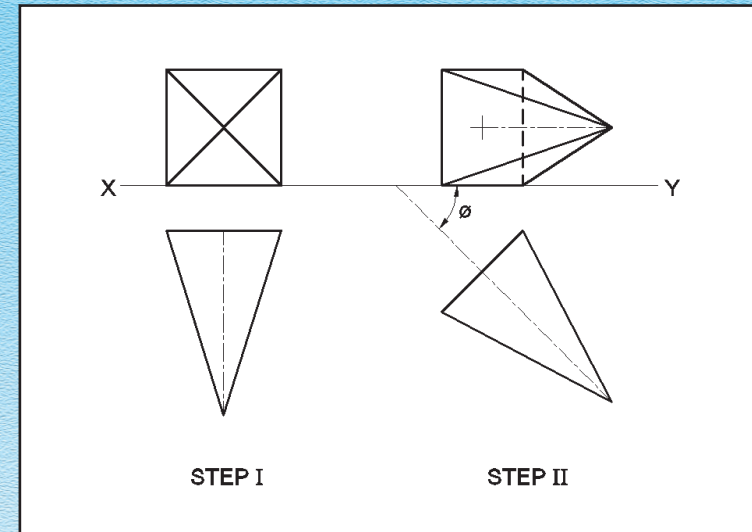


Fig. 4.108

PROCEDURE FOR DRAWING PROJECTIONS OF SOLIDS HAVING AXIS PARALLEL TO THE H.P. & INCLINED AT ϕ TO THE VP

- (i) When the axis is inclined to V.P., assume it to be perpendicular to V.P. and draw, the true shape of the base in Front View and project the Top View from it.
- (ii) Using proper conventional lines, redraw the Top View so that the axis is inclined at the given angle ϕ to XY line.
- (iii) Draw vertical projectors from various points of redrawn Top View and horizontal lines from the Front View in the I step. The corresponding points of intersection of these horizontal and vertical lines locate the positions of the concerned points in the Front View, in the II step.
- (iv) Complete the Front View of the solid by drawing all the surface boundaries using outlines or short dashed (dotted) lines, depending upon their visibility.

Example 4.29 : A hexagonal pyramid having 20 mm sides at its base and an axis 70 mm long has one of the corners of its base in the V.P. and its axis inclined at 45° to the V.P. and parallel to the H.P.

Solution see Fig. 4.109

- (i) First assume the axis to be perpendicular to V.P. and draw the true shape of the base, is hexagon in Front View locate the axis and join it to the corners of the hexagon.
- (ii) To view in this simple position is drawn by projecting from the Front View.
- (iii) Then redraw the Top View in the I step, so that the axis is inclined at 45° to XY line.
- (iv) Draw vertical projectors from this Top View and horizontal projectors from the Front View of Step I.
- (v) Complete the Front View of the solid by drawing all the surface boundaries.

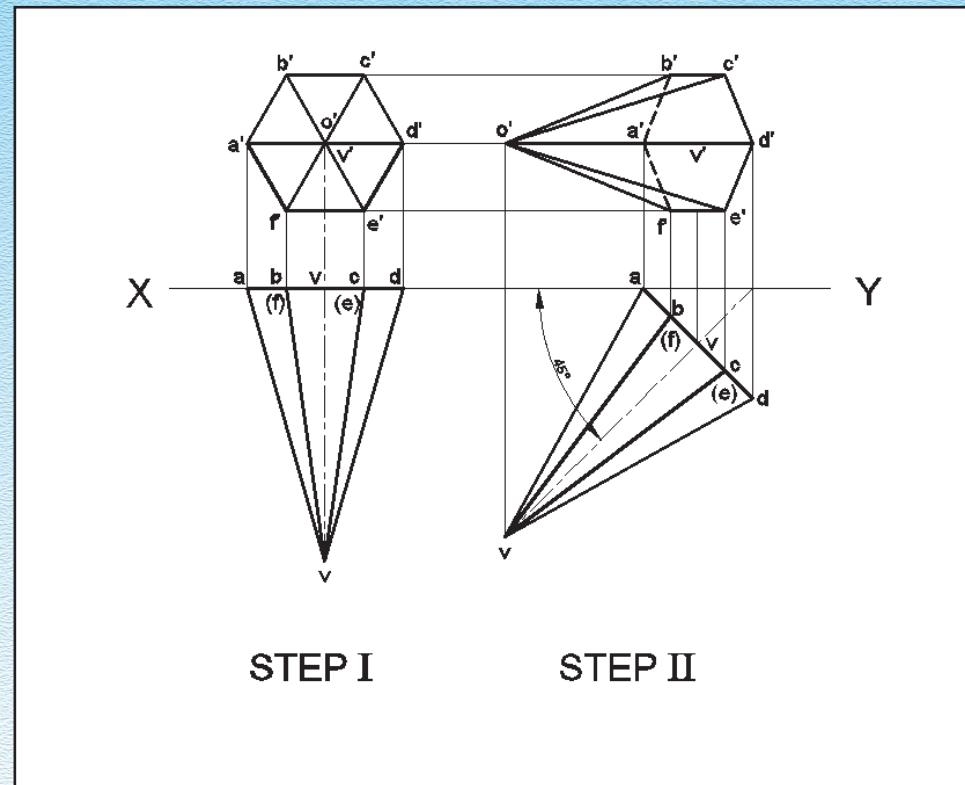


Fig. 4.109

Example 4.30 : Draw the projections of a cylinder 30 mm dia, and axis 50 mm long has its axis parallel to H.P and inclined at 45° to the V.P.

Solution see Fig. 4.110

- (i) Assume the axis perpendicular to the V.P. and draw the Front View which is a circle.
- (ii) Project the Top View, which is a rectangle
- (iii) Redraw the Top View, such that the axis is inclined at 45° to XY line
- (iv) Project this Top View and the previous Front View to get the required Front View.
- (v) Circular ends are shown by drawing ellipses.

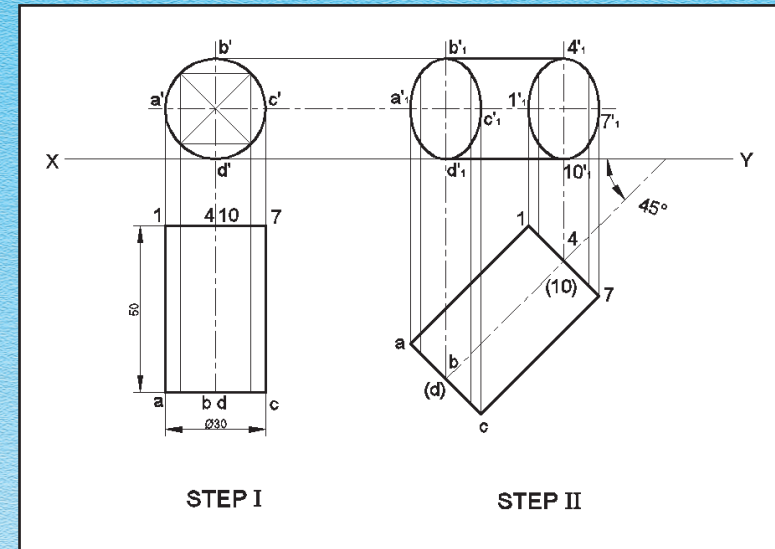


Fig. 4.110

Example 4.31 : The frustum of a square pyramid with the base edge of 40 mm and top edge of 20 mm with axis 50 mm, kept in such a way that its axis is parallel to H.P. and inclined to V.P. at 30° . The cut face is in front.

Solution see Fig. 4.111

- (i) In Step I Assume the axis perpendicular to V.P. and draw the Front View which shows two square ends.
- (ii) Draw the corresponding Top View by projection.
- (iii) In step (ii), redraw the previous Top View such that the axis is inclined at 30°
- (iv) Projectors should be drawn from this redrawn Top View and the previous Front View to get the required Front View.

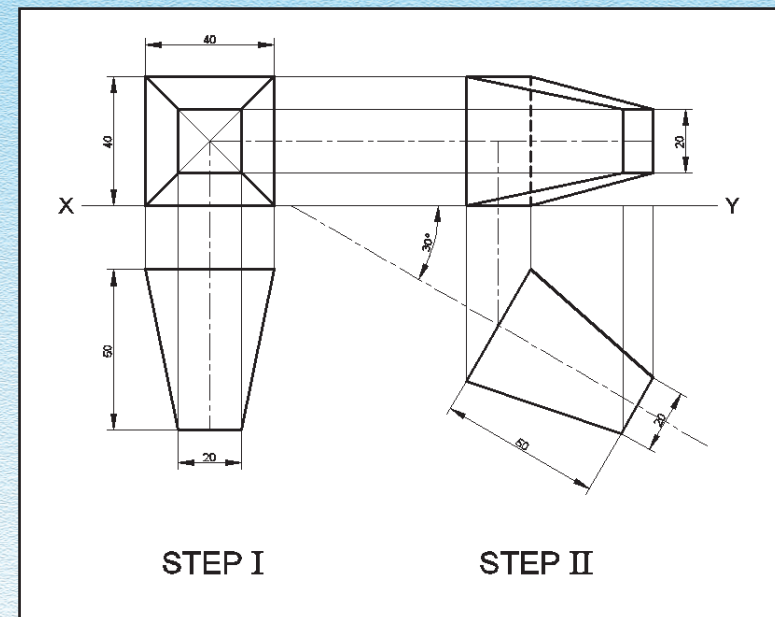


Fig. 4.111

WHAT WE HAVE LEARNT

We now list some of the important conclusions for the projections of solid in the form of a table given below :

S.No.	Position of Solid	Hints for Drawing the Proj	Number of Steps
1.	When the axis perpendicular to H.P.	Start with the Top View	one
2.	When the axis perpendicular to V.P.	Start with the Front View	one
3.	When the axis perpendicular to P.P. is axis parallel to both V.P. & H.P	Start with the side view	one
4.	Axis parallel to V.P. and inclined to H.P. at θ	(i) Assume the axis is $\perp$ to H.P. (ii) Redraw the Front View as the inclined view	Two
5.	Axis parallel to H.P. and inclined to V.P. at ϕ	(i) Assume the axis is $\perp$ to V.P. (ii) Redraw the Top View as the inclined view	Two

Table 4.5 : Steps involved in projections of solids

ASSIGNMENT

- Q1. Draw the projections of a pentagonal prism having 25 mm edge of its base and the axis 50 mm long when it is resting on its base with an edge of its base inclined at 30° to the V.P.
- Q2. A triangular pyramid of 50 mm edges of the base and axis 60 mm long has one of its corners of the base touching V.P. with axis parallel to H.P. and inclined at 45° to the V.P. Draw the projections of the pyramid.
- Q3. The frustum of a cone of 40 mm base diameter and 20 mm cutface diameter, rests on H.P., with its axis 50 mm long, parallel to H.P. and inclined to V.P. at 30° towards right. Project the topview and Front View.

- Q4. A square duct is in the form of a frustum of a square pyramid. The sides of top and bottom are 90 mm and 60 mm respectively, and the length is 110 mm. It is situated in such a way that its axis is parallel to H.P. and inclined at 60° to V.P. Draw the projections of the duct, assuming the thickness of the duct sheet to be negligible.
- Q5. Draw the projections of a square prism having 30 mm edge of its base and the axis 55 mm long when it is resting on its base with its axis inclined at 30° to V.P.
- Q6. Draw the projections of a hexagonal prism having 20 mm edge of its base and the axis 50 mm long when it is resting on its base, with its axis parallel to H.P. and inclined at 40° to the V.P.
- Q7. A triangular prism of 50 mm base edges of the base and axis 60 mm long, resting on its base and its axis inclined at 45° to the V.P. Draw the projections of the prism.
- Q8. A hexagonal pyramid of 25 mm edges of the base and 60 mm long axis, resting on its base, has its axis inclined to V.P. at 30° . Draw its projections.