MHT CET 2026 PCM Question Paper April 11 Shift 1 with Solutions

General Instructions

(i) This booklet contains 150 questions, each provided with a complete, step-by-step solution.

(ii) It comprises 150 single-correct multiple-choice questions.

(iii) Attempt each question on your own before reviewing the given solution.

Physics

Questions 1 to 50 | 1 mark each | 50 marks

1. The period of oscillation of a simple pendulum is given by $T = 2\pi\sqrt{\frac{l}{g}}$ where l is about 80 cm and is known to have 0.1 cm accuracy. The period is about 1.5 s. The time of 50 oscillations is measured by a stop watch of least count 0.1 s. The percentage error in g is nearly

(A) 0.8

(B) 0.4

(C) 0.1

(D) 1

Correct Answer: (B) 0.4

Solution:

Step 1: Write the formula for g

From $T = 2\pi\sqrt{l/g}$ we get $g = \frac{4\pi^2 l}{T^2}$.

Errors add in a product or quotient, and a power counts that many

times: $\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T}$.

Step 2: Error in l

$\frac{\Delta l}{l} = \frac{0.1}{80} = 0.00125$, that is 0.125%.

Step 3: Error in T

The time for 50 oscillations is $50 \times 1.5 = 75$ s, measured with a least count of 0.1 s. So $\frac{\Delta T}{T} = \frac{0.1}{75} = 0.00133$, that is 0.133%. Timing many oscillations keeps this error small.

Step 4: Combine

$\frac{\Delta g}{g} \times 100 = 0.125 + 2 \times 0.133 = 0.39\%$, which is nearly 0.4%. Option (B).

Final Answer:

The percentage error in g is nearly 0.4, option (B).

$$\boxed{\text{(B) } 0.4}$$

Quick Tip: For $g = 4\pi^2 l/T^2$, $\frac{\Delta g}{g} = \frac{\Delta l}{l} + 2\frac{\Delta T}{T}$.

2. A car passes three points A, B and C at 3 m/s, 6 m/s and 9 m/s respectively in a straight line with uniform acceleration. If distance $AB = 60$ m then find the distance BC.

(A) 100 m

(B) 75 m

(C) 200 m

(D) 125 m

Correct Answer: (A) 100 m

Solution:

Step 1: Find the acceleration from AB

$v_B^2 - v_A^2 = 2a \cdot AB$ gives $36 - 9 = 2a \times 60$, so $a = \frac{27}{120} = 0.225$ m/s^2.

Step 2: Use it on BC

$v_C^2 - v_B^2 = 2a \cdot BC$ gives $81 - 36 = 2 \times 0.225 \times BC$, so $45 = 0.45\,BC$.

Step 3: Solve

$BC = 100$ m. Option (A).

Final Answer:

$BC = 100$ m, option (A).

$$\boxed{\text{(A) } 100 \text{ m}}$$

Quick Tip: Use $v^2 - u^2 = 2as$ on each stretch and divide.

3. A particle of mass m is moving in a circular path of constant radius r such that its centripetal acceleration a_c is varying with time t as, $a_c = k^2rt^2$. The power delivered to the particle by the forces acting on it is (K = constant)

(A) $m^2k^2r^2t^2$

(B) mk^2r^2t

(C) $2\pi mk^2r^2t$

(D) $\frac{mk^4r^2t^5}{3}$

Correct Answer: (B) mk^2r^2t

Solution:

Step 1: Find the speed

$a_c = \frac{v^2}{r} = k^2rt^2$, so $v^2 = k^2r^2t^2$ and $v = krt$.

Step 2: Tangential acceleration

$a_t = \frac{dv}{dt} = kr$. The tangential force is $F_t = mkr$. The centripetal force does no work.

Step 3: Power

$P = F_tv = mkr \times krt = mk^2r^2t$. Option (B).

Step 4: Why not the others

(D) would come from integrating the wrong expression, and (A) and (C) have wrong powers of m or extra factors of π.

Final Answer:

The power is mk^2r^2t, option (B).

$$\boxed{(B)}$$

Quick Tip: Find v from $a_c = v^2/r$, then use $P = F_tv$ with $F_t = m\,dv/dt$.

4. The position vector of a particle is $\vec{r} = a\cos\omega t\hat{i} + a\sin\omega t\hat{j}$. Calculate the angle between its position vector and the velocity vector.

$(\cos 0 = 1, \cos 90 = 0, \cos 60 = 0.5, \sin 30 = 0.5, \sin 0 = 0)$

(A) 30°

(B) 60°

(C) 90°

(D) 0°

Correct Answer: (C) 90°

Solution:

Step 1: Find velocity

$\vec{v} = \frac{d\vec{r}}{dt} = -a\omega \sin \omega t\, \hat{i} + a\omega \cos \omega t\, \hat{j}.$

Step 2: Dot product

$\vec{r} \cdot \vec{v} = -a^2\omega \sin \omega t \cos \omega t + a^2\omega \sin \omega t \cos \omega t = 0.$

Step 3: Angle

A zero dot product means the vectors are perpendicular, so the angle is 90°. Option (C).

Final Answer:

The angle is 90°, option (C).

$$\boxed{\text{(C) } 90^\circ}$$

Quick Tip: Check the dot product of $\vec{r}$ and $\vec{v}$.

5. Two massless springs of spring constants K_1 and K_2 are connected one after the other forming a single chain, suspended vertically and a certain mass is attached to the free end. If x_1 and x_2 are their respective extensions and 'F' is their stretching force, the total extension produced is

(A) $F(K_1 + K_2)$

(B) $F(1/K_1 - 1/K_2)$

(C) $F(K_1 - K_2)$

(D) $F(1/K_1 + 1/K_2)$

Correct Answer: (D) $F(1/K_1 + 1/K_2)$

Solution:

Step 1: Same force

In a series chain the same force F acts on each spring.

Step 2: Extensions

$x_1 = \frac{F}{K_1}$ and $x_2 = \frac{F}{K_2}$.

Step 3: Total

$x = x_1 + x_2 = F\left(\frac{1}{K_1} + \frac{1}{K_2}\right)$. Option (D).

Final Answer:

Total extension is $F(1/K_1 + 1/K_2)$, option (D).

$$\boxed{\text{(D)}}$$

Quick Tip: Springs in series carry the same force, and extensions add.

6. A wheel is rotating at 600 rpm about its axis of rotation. When the power is cut off, it comes to rest in half a minute. Its angular retardation expressed in rad/s^2 is (retardation is uniform)

(A) $\frac{2\pi}{3}$

(B) $\frac{\pi}{4}$

(C) π

(D) $\frac{\pi}{6}$

Correct Answer: (A) $\frac{2\pi}{3}$

Solution:

Step 1: Convert

600 rpm $= 10$ rev/s $= 20\pi$ rad/s.

Step 2: Use the equation

It stops in 30 s, so $0 = 20\pi - \alpha \times 30$.

Step 3: Solve

$\alpha = \frac{20\pi}{30} = \frac{2\pi}{3}$ rad/s^2. Option (A).

Final Answer:

The retardation is $\frac{2\pi}{3}$ rad/s^2, option (A).

$$\boxed{\text{(A)}}$$

Quick Tip: Convert rpm to rad/s, then use $\omega = \omega_0 - \alpha t$.

7. A flywheel rotating about a fixed axis has a kinetic energy of 500 joule, if its angular frequency is 5 Hz, then calculate the moment of inertia of the wheel about the axis of rotation. ($\pi^2 = 10$)

(A) $0.6\ \text{kg m}^2$

(B) $1\ \text{kg m}^2$

(C) $0.75\ \text{kg m}^2$

(D) $0.15\ \text{kg m}^2$

Correct Answer: (B) $1\ \text{kg m}^2$

Solution:

Step 1: Find ω

The frequency is 5 Hz, so $\omega = 2\pi \times 5 = 10\pi$ rad/s.

Step 2: Use the energy formula

$K = \frac{1}{2}I\omega^2$, so $500 = \frac{1}{2}I \times 100\pi^2 = 50\pi^2 I$.

Step 3: Solve

With $\pi^2 = 10$: $500 = 500I$, so $I = 1\ \text{kg m}^2$. Option (B).

Final Answer:

$I = 1\ \text{kg m}^2$, option (B).

$$\boxed{\text{(B) } 1\ \text{kg m}^2}$$

Quick Tip: Use $\omega = 2\pi f$ and $K = \frac{1}{2}I\omega^2$.

8. Given below are two statements:

Statement I: Acceleration due to earth's gravity decreases as you go 'up' or 'down' from earth's surface.

Statement II: Acceleration due to earth's gravity is same at a height 'h' and depths 'd' from earth's surface, if $h = d$.

In the light of above statements, choose the most appropriate answer from the options given below.

(A) Statement I is incorrect but statement II is correct

(B) Both statement I and statement II are incorrect

(C) Statement I is correct but statement II is incorrect

(D) Both Statement I and II are correct.

Correct Answer: (C) Statement I is correct but statement II is incorrect

Solution:

Step 1: Statement I

Above the surface, g falls as $\frac{1}{r^2}$. Below the surface, $g = g_0\frac{r}{R}$ falls linearly to zero at the centre. g is greatest at the surface. So statement I is correct.

Step 2: Statement II

For small h: $g_h = g\left(1 - \frac{2h}{R}\right)$. For small d: $g_d = g\left(1 - \frac{d}{R}\right)$.

Setting $h = d$ gives different values. They are equal only when $d = 2h$.

So statement II is incorrect.

Step 3: Result

Statement I is correct and II is incorrect. Option (C).

Final Answer:

Statement I is correct, II is incorrect, option (C).

(C)

Quick Tip: Compare $g_h \approx g(1 - \frac{2h}{R})$ and $g_d = g(1 - \frac{d}{R})$.

9. Two rain drops of same radius 'r' falling with same terminal velocity 'V' merge and form bigger drop of radius 'R'. The terminal velocity of big drop is

(A) $\frac{VR^2}{r^2}$

(B) $\frac{2VR}{r}$

(C) $\frac{VR}{r}$

(D) Vr^2/R^2

Correct Answer: (A) $\frac{VR^2}{r^2}$

Solution:

Step 1: Terminal velocity law

$v_T = \frac{2r^2(\rho-\sigma)g}{9\eta}$, so $v_T \propto r^2$.

Step 2: Volume conservation

Two drops merge: $\frac{4}{3}\pi R^3 = 2 \cdot \frac{4}{3}\pi r^3$, so $R^3 = 2r^3$.

Step 3: Result

$\frac{V'}{V} = \frac{R^2}{r^2}$, so $V' = \frac{VR^2}{r^2}$. Option (A).

Final Answer:

The terminal velocity is $\frac{VR^2}{r^2}$, option (A).

(A)

Quick Tip: Terminal velocity varies as the square of the radius, and volume is conserved on merging.

10. A spherical ball of radius 1 mm and density 10.5 g/cc is dropped in glycerine of coefficient of viscosity 9.8 poise and density 1.5 g/cc. Viscous force on the ball when it attains constant velocity is 3696×10^{-x} N. The value of x is
(Given, $g = 9.8\ \text{m/s}^2$ and $\pi = \frac{22}{7}$)

(A) 5

(B) 6

(C) 7

(D) 8

Correct Answer: (C) 7

Solution:

Step 1: Use equilibrium
At terminal speed: viscous force = weight − upthrust = $\frac{4}{3}\pi r^3(\rho - \sigma)g$.

Step 2: Convert units
$r = 1\ \text{mm} = 10^{-3}$ m. $\rho - \sigma = 10.5 - 1.5 = 9\ \text{g/cc} = 9000\ \text{kg/m}^3$.

Step 3: Calculate

$$F = \frac{4}{3} \times \frac{22}{7} \times 10^{-9} \times 9000 \times 9.8 = 3.696 \times 10^{-4}\ \text{N}$$

$3.696 \times 10^{-4} = 3696 \times 10^{-7}$, so $x = 7$. Option (C).

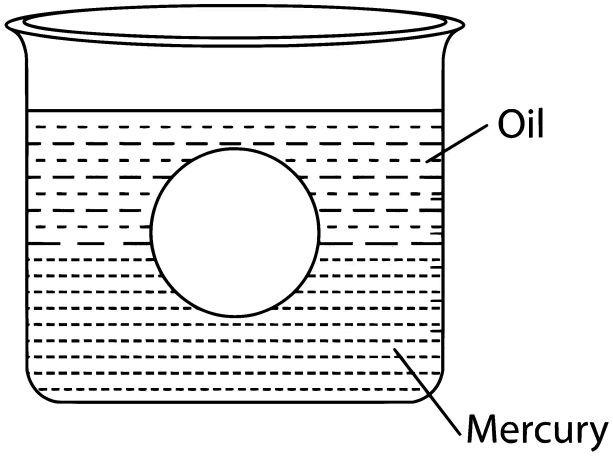
Final Answer:

$x = 7$, option (C).

$$\boxed{\text{(C) } 7}$$

Quick Tip: At constant velocity the viscous force equals the weight minus the buoyant force.

11. A vessel contains oil (density $= 0.8\ \text{gm/cm}^3$) over mercury (density $= 13.6\ \text{gm/cm}^3$). A homogeneous sphere floats with half of its volume immersed in mercury and the other half in oil. The density of the material of the sphere in gm/cm^3 is



(A) 12.8

(B) 7.2

(C) 6.4

(D) 3.3

Correct Answer: (B) 7.2

Solution:

Step 1: Understanding the Figure:

A beaker holds mercury at the bottom and oil above it. A homogeneous sphere floats at the boundary, with half its volume in mercury and half in oil.

Step 2: Key Formula or Approach:

A floating body is in equilibrium: its weight equals the total buoyant force. The buoyant force from each liquid is (volume immersed in that liquid) $\times$ (density of the liquid) $\times$ g.

Step 3: Set up:

Let the sphere's volume be V and density ρ.

$$V\rho g = \frac{V}{2}\rho_{Hg}\, g + \frac{V}{2}\rho_{oil}\, g$$

Step 4: Solve:

Cancel Vg from every term:

$$\rho = \frac{\rho_{Hg} + \rho_{oil}}{2} = \frac{13.6 + 0.8}{2} = \frac{14.4}{2} = 7.2\ \text{g/cm}^3$$

Step 5: Why the other options are wrong:

The value 12.8 is $13.6 - 0.8$, which subtracts instead of averaging. The value 6.4 is half of 12.8. The value 3.3 does not come from any correct step. Only the average of the two densities is right, because the sphere is split half and half.

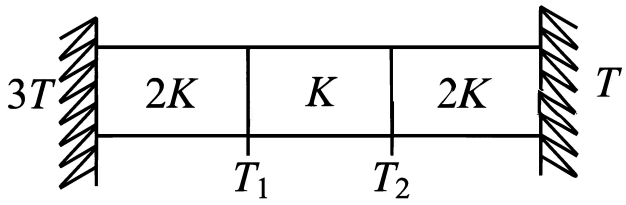
Final Answer:

The density of the sphere is 7.2 g/cm^3, option (B).

$$\boxed{7.2\ \text{g/cm}^3}$$

Quick Tip: Weight of the sphere equals the sum of upthrusts from the two liquids, each acting on half the volume. That gives the mean of the two densities.

12. Three identical heat conducting rods are connected in series as shown in the figure. The rods on the side have thermal conductivity $2K$ while that in the middle has thermal conductivity K. The left end of the combination is maintained at temperature $3T$ and the right end at T. The rods are thermally insulated from outside. In steady state, temperature at the left junction is T_1 and that at the right junctions is T_2. The ratio T_1/T_2 is



(A) $\frac{3}{2}$

(B) $\frac{4}{3}$

(C) $\frac{5}{3}$

(D) $\frac{4}{5}$

Correct Answer: (C) $\frac{5}{3}$

Solution:

Step 1: Understand the figure

Three identical rods are joined end to end. The two outer rods have conductivity $2K$ and the middle rod has K. The left end is held at $3T$ and the right end at T. Let each rod have length L and area A. T_1 is the temperature at the left junction and T_2 at the right junction.

Step 2: Use the steady state condition

The rods are insulated from the sides, so no heat leaks out. In steady state the same heat current flows through every rod. For one rod the heat current is $\frac{kA\,\Delta T}{L}$.

Step 3: Write the equal heat flow equation

$$\frac{2KA(3T - T_1)}{L} = \frac{KA(T_1 - T_2)}{L} = \frac{2KA(T_2 - T)}{L}$$

Cancel $\frac{KA}{L}$ from all three parts:

$$2(3T - T_1) = T_1 - T_2 = 2(T_2 - T)$$

Step 4: Solve for T1 and T2

From the first and third parts, $3T - T_1 = T_2 - T$, so $T_1 + T_2 = 4T$. From the middle and third parts, $T_1 - T_2 = 2T_2 - 2T$, so $T_1 = 3T_2 - 2T$. Put this in the first result: $3T_2 - 2T + T_2 = 4T$, so $T_2 = \frac{3T}{2}$. Then $T_1 = 4T - \frac{3T}{2} = \frac{5T}{2}$.

Step 5: Find the ratio and check the options

$$\frac{T_1}{T_2} = \frac{5T/2}{3T/2} = \frac{5}{3}$$

Option (A) 3/2, option (B) 4/3 and option (D) 4/5 do not satisfy the two equations. For example 4/5 would make T_1 smaller than T_2, but

heat flows from the hot left end, so T_1 must be larger than T_2.

Final Answer:

The ratio T_1/T_2 is $5/3$, which is option (C).

$$\boxed{\frac{5}{3}\ (C)}$$

Quick Tip: Heat current is the same in all three rods because there is no side loss. Write k A (temperature difference) / L for each rod and equate them.

13. A wire of length L and diameter d is used in a bulb. The temperature of wire is T and power radiated by the wire is P. Its emissivity is (σ = Stefan's constant) (Assume that emissivity of wire material is same at all wavelength)

(A) $\frac{P}{\sigma T^4 \pi d L}$

(B) $\frac{P}{\sigma T^2 \pi d L}$

(C) $\frac{P}{\sigma T^2 \pi d^2 L^2}$

(D) $\frac{P^2}{\sigma T^4 \pi d L}$

Correct Answer: (A) $\frac{P}{\sigma T^4 \pi d L}$

Solution:

Step 1: Write the law

$P = e\sigma A T^4$ where A is the radiating surface area.

Step 2: Surface area

The wire is a cylinder of diameter d and length L, so the curved area is

$A = \pi dL$.

Step 3: Solve for e

$e = \frac{P}{\sigma T^4 \pi dL}$. Option (A).

Final Answer:

The emissivity is $\frac{P}{\sigma T^4 \pi dL}$, option (A).

$$\boxed{\text{(A)}}$$

Quick Tip: Use the Stefan-Boltzmann law $P = e\sigma AT^4$ with the curved surface area πdL.

14. The energy spectrum of a black body exhibits a maximum of wavelength λ_0. The temperature of the black body is now changed such that the energy is maximum at wavelength $\frac{3\lambda_0}{2}$. The ratio of power radiated by the black body will be

(A) $\frac{16}{9}$

(B) $\frac{16}{81}$

(C) $\frac{64}{27}$

(D) $\frac{4}{3}$

Correct Answer: (B) $\frac{16}{81}$

Solution:

Step 1: Wien displacement law

$\lambda_m T = b$. When λ_m changes from λ_0 to $\frac{3\lambda_0}{2}$, $T' = T \cdot \frac{2}{3}$.

Step 2: Stefan law

Radiated power $\propto T^4$.

$$\frac{P'}{P} = \left(\frac{2}{3}\right)^4 = \frac{16}{81}$$

Step 3: Result

The ratio of the new power to the old is $\frac{16}{81}$. Option (B).

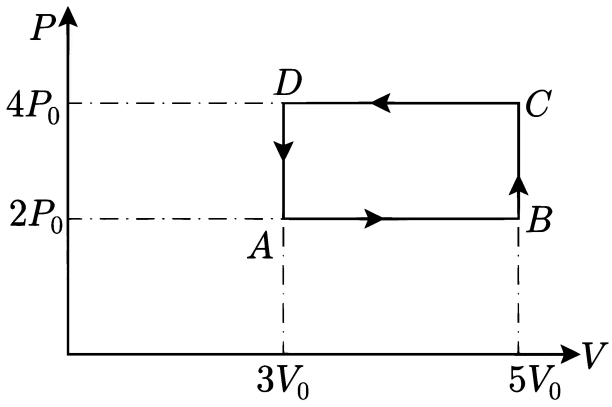
Final Answer:

The ratio is $\frac{16}{81}$, option (B).

$$\boxed{\text{(B) } \frac{16}{81}}$$

Quick Tip: Wien law $\lambda_m T =$ constant, and power $\propto T^4$.

15. An ideal gas undergoes cyclic process ABCDA as shown in given p-V diagram. What is the magnitude of amount of work done by the gas?



(A) $6P_0V_0$

(B) $4P_0V_0$

(C) $2P_0V_0$

(D) P_0V_0

Correct Answer: (B) $4P_0V_0$

Solution:

Step 1: Understand the figure

The path ABCDA on the p-V diagram is a closed rectangle. The lower side is at pressure $2P_0$ and the upper side at pressure $4P_0$. The left side is at volume $3V_0$ and the right side at $5V_0$. The arrows show the gas moves up the right side and back along the top, so the loop is traced as a cycle.

Step 2: Use the cyclic process rule

In a cycle the gas returns to its starting state, so the net change in internal energy is zero. The net work done by the gas equals the area enclosed by the loop on the p-V diagram. The sign depends on the direction of the loop, but the question asks only for the magnitude.

Step 3: Find the sides of the rectangle

Width (change in volume): $\Delta V = 5V_0 - 3V_0 = 2V_0$.

Height (change in pressure): $\Delta P = 4P_0 - 2P_0 = 2P_0$.

Step 4: Calculate the area

$$|W| = \Delta P \times \Delta V = (2P_0)(2V_0) = 4P_0V_0$$

Step 5: Why the other options are wrong

$6P_0V_0$ would come from a wrong width or height, such as $2P_0 \times 3V_0$. $2P_0V_0$ and P_0V_0 are too small because they leave out part of the rectangle. Only $4P_0V_0$ equals width times height.

Final Answer:

The enclosed area is $4P_0V_0$, so the magnitude of work is option (B).

$$\boxed{4P_0V_0}$$

Quick Tip: In a cycle the work equals the area enclosed on the p-V graph. Find the width and height of the rectangle and multiply.

16. Two samples A and B of gas initially at the same pressure and temperature. They are compressed from volume V to $\frac{V}{2}$ (A isothermally and B adiabatically). The relation between pressure of the gas A (P_A) and pressure of gas B (P_B) is

(A) $P_A < P_B$

(B) $P_A > P_B$

(C) $P_A = P_B$

(D) $P_A = 2P_B$

Correct Answer: (A) $P_A < P_B$

Solution:

Step 1: Isothermal (A)

$P_AV = P_A \cdot \frac{V}{2}$ gives $P_A = 2P$.

Step 2: Adiabatic (B)

$PV^\gamma = P_B\left(\frac{V}{2}\right)^\gamma$ gives $P_B = 2^\gamma P$.

Step 3: Compare

Since $\gamma > 1$, $2^\gamma > 2$, so $P_B > P_A$. Option (A).

Final Answer:

$P_A < P_B$, option (A).

$$\boxed{\text{(A)}}$$

Quick Tip: Adiabatic compression heats the gas, so its pressure rises more.

17. A vessel has 6 gram of hydrogen at pressure P and temperature 500 K. A small hole is made in it so that hydrogen leaks out. How much hydrogen leaks out if the final pressure is $\frac{P}{2}$ and the temperature falls to 300 K?

(A) 2 gram

(B) 3 gram

(C) 4 gram

(D) 1 gram

Correct Answer: (D) 1 gram

Solution:

Step 1: Use the gas equation

$PV = nRT$ with fixed V gives $n \propto \frac{P}{T}$.

Step 2: Ratio

$\frac{n_2}{n_1} = \frac{P/2}{300} \times \frac{500}{P} = \frac{5}{6}$.

Step 3: Mass left and lost

Mass left $= \frac{5}{6} \times 6 = 5$ g, so $6 - 5 = 1$ g leaks out. Option (D).

Final Answer:

One gram of hydrogen leaks out, option (D).

$$\boxed{\text{(D) 1 g}}$$

Quick Tip: The vessel volume is fixed, so $n \propto P/T$.

18. A particle executes simple harmonic motion between $x = -A$ and $x = +A$. If time taken by particle to go from $x = 0$ to $\frac{A}{2}$ is 2s, then time taken by particle in going from $x = \frac{A}{2}$ to A is

(A) 3 s

(B) 2 s

(C) 1.5 s

(D) 4 s

Correct Answer: (D) 4 s

Solution:

Step 1: Time to reach $\frac{A}{2}$

With $x = A \sin \omega t$, $x = \frac{A}{2}$ when $\omega t = \frac{\pi}{6}$, so $t = \frac{T}{12}$.

Step 2: Find T

$\frac{T}{12} = 2$ s gives $T = 24$ s.

Step 3: Time from $\frac{A}{2}$ to A

Time from 0 to A is $\frac{T}{4} = 6$ s. So the time from $\frac{A}{2}$ to A is $6 - 2 = 4$ s. Option (D).

Final Answer:

The time is 4 s, option (D).

$$\boxed{\text{(D) 4 s}}$$

Quick Tip: Use $x = A \sin \omega t$ to find the time to reach $A/2$.

19. There is a body having mass m and performing SHM with amplitude 'a'. There is a restoring force $F = -Kx$, where x is the displacement. The total energy of the body depends upon which of the following?

(A) K, a, x

(B) K, a, v

(C) K, x

(D) K, a

Correct Answer: (D) K, a

Solution:

Step 1: Write the total energy

At the extreme position the speed is zero and all energy is potential: $E = \frac{1}{2}Ka^2$.

Step 2: Result

It depends only on K and the amplitude a, not on x or v at some instant (they change, but E does not). Option (D).

Final Answer:

Total energy depends on K and a, option (D).

$$\boxed{\text{(D)}}$$

Quick Tip: Total energy is $\frac{1}{2}Ka^2$.

20. What is the effect of humidity on sound waves when humidity increases?

(A) Speed of sound waves is more.

(B) Speed of sound waves is less.

(C) Speed of sound waves remains same.

(D) Speed of sound waves becomes zero.

Correct Answer: (A) Speed of sound waves is more.

Solution:

Step 1: Reason

Water vapour (molar mass 18) replaces heavier nitrogen and oxygen (molar mass 28 and 32) in air. So humid air has lower density.

Speed of sound is $v = \sqrt{\frac{\gamma P}{\rho}}$, so lower density gives a higher speed.

Option (A).

Final Answer:

Speed of sound increases, option (A).

(A)

Quick Tip: Moist air is less dense, so sound travels faster.

21. A tuning fork of frequency n produces x beats per second when sounded with a vibrating sonometer string. What must have been the frequency of the string, when a slight increase in tension produces lesser beats per second than before?

(A) $n + x$

(B) $n - x$

(C) $(n + x)^2$

(D) $(n - x)^2$

Correct Answer: (B) $n - x$

Solution:

Step 1: Effect of tension

$f \propto \sqrt{T}$, so a slight increase in tension raises the string frequency.

Step 2: Two cases

If $f_s = n + x$, a rise moves it farther from n and the beats increase.

If $f_s = n - x$, a rise moves it closer to n and the beats decrease.

Step 3: Choose

The beats decreased, so $f_s = n - x$. Option (B).

Final Answer:

The string frequency was $n - x$, option (B).

(B)

Quick Tip: Tension up means string frequency up. Beats fall only if the string was below the fork frequency.

22. A string of length 0.5 m and mass 10^{-3} kg is tightly clamped at its ends. The tension in the string is 0.8 N. Identical wave pulses are produced at one end at equal intervals of time Δt. What is the minimum value of Δt which allows constructive interference between successive pulses?

(A) 0.40 s

(B) 0.20 s

(C) 0.10 s

(D) 0.05 s

Correct Answer: (D) 0.05 s

Solution:

Step 1: Wave speed

Linear density $\mu = \frac{10^{-3}}{0.5} = 2 \times 10^{-3}$ kg/m. $v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{0.8}{2\times10^{-3}}} = 20$ m/s.

Step 2: Round trip

A pulse reflects at a clamped end with a phase change of π. After reflecting at both ends it has two phase changes, so it returns in its original form after $\frac{2L}{v}$.

$\frac{2L}{v} = \frac{2\times0.5}{20} = 0.05$ s.

Step 3: Result

New pulses must be produced every **0.05** s so that they add constructively with the returning pulse. Option (D).

Final Answer:

The minimum interval is **0.05** s, option (D).

(D) 0.05 s

Quick Tip: Find the wave speed, then the round trip time between the clamped ends.

23. Which of the following statements is NOT TRUE?

a. Work done to move a charge on an equipotential surface is not zero.

b. Equipotential surfaces are the surfaces where the potential is constant.

c. Equipotential surfaces for a uniform electric field are parallel and equidistant from each other.

d. Electric field is always perpendicular to an equipotential surface.

(A) c

(B) a

(C) d

(D) b

Correct Answer: (B) a

Solution:

Step 1: Check each statement

(a) Work along an equipotential surface is $W = q\Delta V = 0$. The

statement says it is not zero, so (a) is false.
(b) True by definition.
(c) True: in a uniform field the surfaces are parallel planes with equal spacing for equal potential steps.
(d) True: the field is perpendicular to the surface.

Step 2: Result
The false statement is (a), which is option (B).

Final Answer:
Statement (a) is not true, option (B).

$\boxed{\text{(B)}}$

Quick Tip: No work is done moving a charge along an equipotential surface.

24. Two metal sphere of radius R and $3R$ have same surface charge density σ. If they are brought in contact and then separated, the surface charge density on smaller and bigger sphere becomes σ_1 and σ_2, respectively. The ratio $\frac{\sigma_1}{\sigma_2}$ is.

(A) $\frac{1}{9}$

(B) 9

(C) $\frac{1}{3}$

(D) 3

Correct Answer: (D) 3

Solution:

Step 1: Same potential

Spheres in contact share a potential: $\frac{kq_1}{R} = \frac{kq_2}{3R}$, so $q_2 = 3q_1$.

Step 2: Surface densities

$\sigma_1 = \frac{q_1}{4\pi R^2}$ and $\sigma_2 = \frac{q_2}{4\pi(3R)^2} = \frac{3q_1}{36\pi R^2}$.

$\frac{\sigma_1}{\sigma_2} = \frac{q_1/(4\pi R^2)}{3q_1/(36\pi R^2)} = \frac{36}{12} = 3$. Option (D).

Final Answer:

The ratio $\frac{\sigma_1}{\sigma_2} = 3$, option (D).

$$\boxed{\text{(D) } 3}$$

Quick Tip: After contact both spheres reach the same potential, so $\sigma \propto \frac{1}{R}$.

25. Three charges q, Q and $+4q$ are placed in a straight line of length d at points at distance 0, $\frac{d}{3}$, d respectively. In order to make the net force on q be zero, the value of Q should be

(A) $\frac{-q}{2}$

(B) $\frac{-3q}{2}$

(C) $\frac{-4q}{3}$

(D) $\frac{-4q}{9}$

Correct Answer: (D) $\frac{-4q}{9}$

Solution:

Step 1: Understand the setup

Three charges lie on a line of length d. Charge q is at 0, charge Q is at $\frac{d}{3}$ and charge $+4q$ is at d. We want the net force on q to be zero.

Step 2: Find the sign of Q

The charge $+4q$ is positive, so it repels q away from itself. The push on q is therefore directed away from $+4q$. To cancel it, Q must pull q the other way. Pulling means attraction, so Q must be negative.

Step 3: Write the distances

Distance of Q from q: $r_1 = \frac{d}{3}$. Distance of $+4q$ from q: $r_2 = d$.

Step 4: Balance the two forces

$$\frac{1}{4\pi\epsilon_0}\frac{q|Q|}{(d/3)^2} = \frac{1}{4\pi\epsilon_0}\frac{q(4q)}{d^2}$$

Cancel the common factor and q:

$$\frac{9|Q|}{d^2} = \frac{4q}{d^2} \Rightarrow |Q| = \frac{4q}{9}$$

With the negative sign, $Q = -\frac{4q}{9}$.

Step 5: Check the other options

$-\frac{q}{2}$, $-\frac{3q}{2}$ and $-\frac{4q}{3}$ have the right sign but the wrong size. For example $-\frac{4q}{3}$ would give a pull on q three times too strong, so q would not stay at rest.

Final Answer:

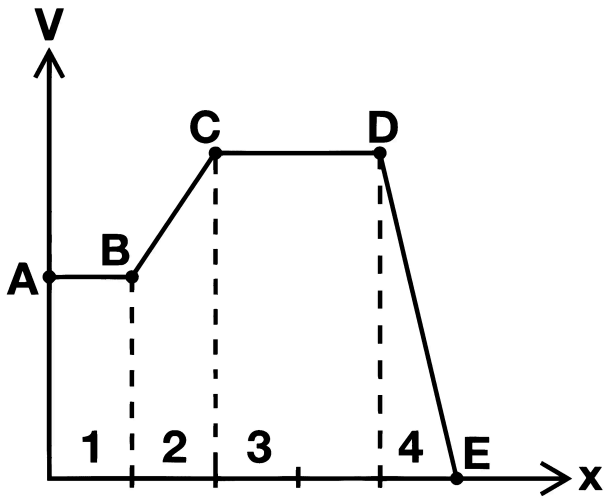
The charge needed is $-\frac{4q}{9}$, option (D).

$$\boxed{Q = -\frac{4q}{9}}$$

Quick Tip: The middle charge must attract q to cancel the push from +4q,

so it is negative. Equate Coulomb forces using distances d/3 and d.

26. Following figure indicates the electric potential V as a function through 4 regions on x-axis. Which of the following is correct for the electric field E in these regions?



(A) $E_1 < E_2 < E_3 < E_4$

(B) $E_2 = E_4$ and $E_1 < E_2$

(C) $E_1 = E_3$ and $E_2 < E_4$

(D) $E_1 > E_2 > E_3 > E_4$

Correct Answer: (C) $E_1 = E_3$ and $E_2 < E_4$

Solution:

Step 1: Understand the figure

The graph shows potential V against position x split into four regions. In region 1 the line is flat. In region 2 the potential rises steeply to point C. In region 3 it is flat again (C to D). In region 4 it falls steeply from D down to zero at E.

Step 2: Use the field-potential relation

$E = -\frac{dV}{dx}$. So the size of the field in a region is the size of the slope of the V-x graph there.

Step 3: Regions 1 and 3

Both are horizontal lines. The slope is zero, so $E_1 = 0$ and $E_3 = 0$. Hence $E_1 = E_3$.

Step 4: Regions 2 and 4

Region 2 rises by $V_C - V_B$ over a width Δx_2. Region 4 falls by the full V_C over a width Δx_4. From the figure region 4 is narrower and has the larger potential change, so its slope is steeper. This gives $E_2 < E_4$.

Step 5: Check the options

(A) says all four fields increase in order, but $E_1 = E_3 = 0$ cannot be strictly increasing. (B) says $E_2 = E_4$, which is false. (D) says the fields decrease in order, which is also false. Only (C) fits.

Final Answer:

$E_1 = E_3$ and $E_2 < E_4$, option (C).

$$\boxed{E_1 = E_3 \text{ and } E_2 < E_4}$$

Quick Tip: Field is minus the slope of the V-x graph. Flat parts give zero field, and the steeper part gives the larger field.

27. A parallel plate capacitor has plate area 40 cm^2 and plates separation 2 mm. The space between the plates is filled with a dielectric medium of a thickness 1 mm and dielectric constant 5. The capacitance of the system is

(A) $24\varepsilon_0$ F

(B) $\frac{3}{10}\varepsilon_0$ F

(C) $\frac{10}{3}\varepsilon_0$ F

(D) $10\varepsilon_0$ F

Correct Answer: (C) $\frac{10}{3}\varepsilon_0$ F

Solution:

Step 1: Write the formula

With a dielectric slab of thickness t and constant K in a gap d: $C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$.

Step 2: Substitute

$A = 40 \times 10^{-4}$ m^2, $d = 2 \times 10^{-3}$ m, $t = 10^{-3}$ m, $K = 5$.

Denominator: $2 \times 10^{-3} - 10^{-3} + \frac{10^{-3}}{5} = 1.2 \times 10^{-3}$ m.

Step 3: Calculate

$C = \frac{\varepsilon_0 \times 4 \times 10^{-3}}{1.2 \times 10^{-3}} = \frac{10}{3}\varepsilon_0$ F. Option (C).

Final Answer:

$C = \frac{10}{3}\varepsilon_0$ F, option (C).

$\boxed{\text{(C)}}$

Quick Tip: Use $C = \frac{\varepsilon_0 A}{d - t + t/K}$.

28. In a metre bridge experiment the balance point is obtained if the gaps are closed by 2 Ω and 3Ω. A shunt of X Ω is added to 3 Ω resistor to shift the balancing point by 22.5 cm. The value of X is

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (B) 2

Solution:

Step 1: Initial balance

$\frac{l}{100-l} = \frac{2}{3}$ gives $l = 40$ cm.

Step 2: New balance

A shunt lowers the 3 Ω resistance, so the balance point moves toward the other side by 22.5 cm: $l' = 62.5$ cm.

$\frac{62.5}{37.5} = \frac{2}{R'}$, so $R' = 1.2\,\Omega$.

Step 3: Shunt

$\frac{3X}{3+X} = 1.2$ gives $3X = 3.6 + 1.2X$, so $X = 2\,\Omega$. Option (B).

Final Answer:

$X = 2\,\Omega$, option (B).

$$\boxed{\text{(B) } 2\,\Omega}$$

Quick Tip: First find the initial balance length, then the shifted one.

29. A galvanometer has a current range of 10 mA and a voltage range of 0.75 V. To convert this galvanometer into an ammeter of range 10 A, what is the shunt resistance?

(A) $\frac{100}{999}\,\Omega$

(B) $\frac{50}{999}\,\Omega$

(C) $\frac{200}{999}\,\Omega$

(D) $\frac{75}{999}\,\Omega$

Correct Answer: (D) $\frac{75}{999}\,\Omega$

Solution:

Step 1: Galvanometer resistance

$G = \frac{0.75}{10\times10^{-3}} = 75\,\Omega.$

Step 2: Shunt

$S = \frac{I_g G}{I - I_g} = \frac{0.01\times75}{10-0.01} = \frac{0.75}{9.99} = \frac{75}{999}\,\Omega$. Option (D).

Final Answer:

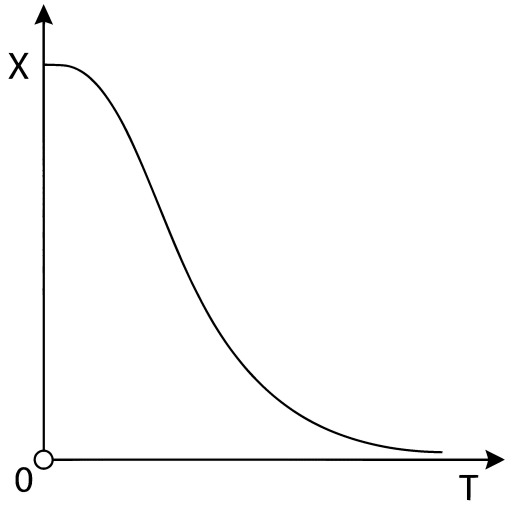
The shunt is $\frac{75}{999}\,\Omega$, option (D).

$$\boxed{\text{(D)}}$$

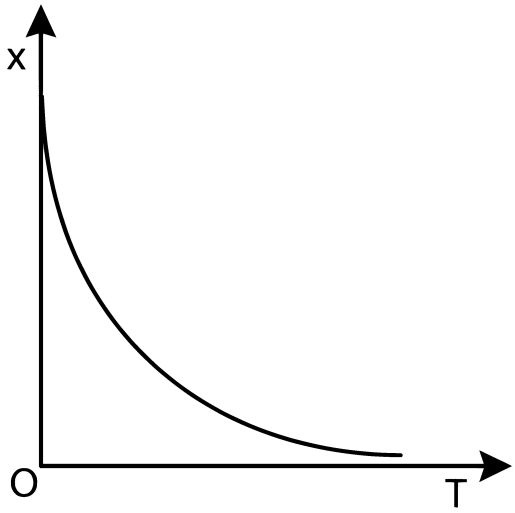
Quick Tip: Galvanometer resistance is $G = V/I_g$, and $S = \frac{I_g G}{I - I_g}$.

30. Which of the following figure best represents the variation of magnetic susceptibility (χ) with temperature for a diamagnetic substance?

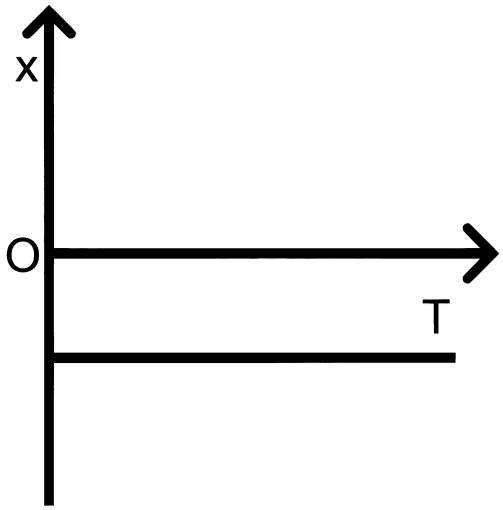
(A)



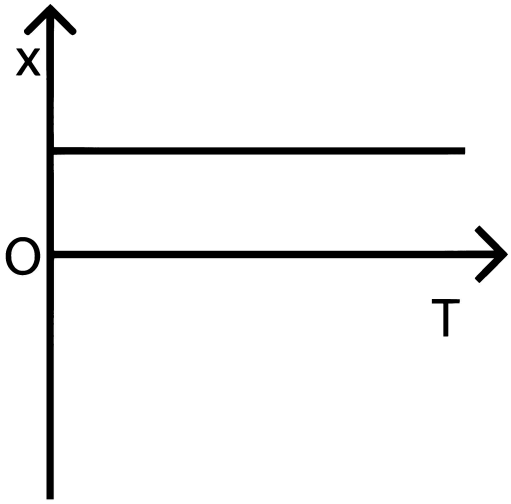
(B)



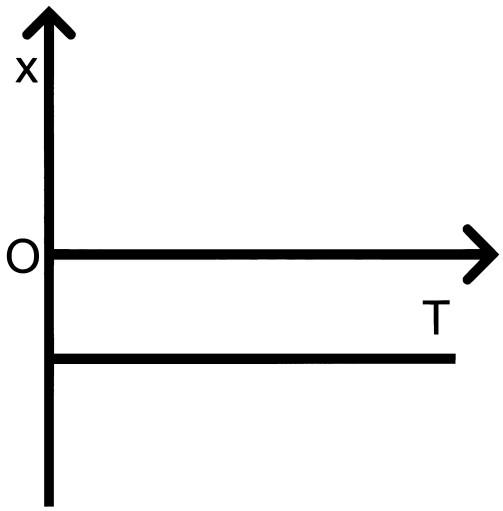
(C)



(D)



Correct Answer: (C)



Solution:

Step 1: Recall the property

A diamagnetic material has a small negative susceptibility, $\chi \approx -10^{-5}$. It does not depend on temperature, because diamagnetism comes from induced magnetic moments in every atom and not from the alignment of permanent dipoles.

Step 2: What the graph must look like

The graph of χ against T must be a horizontal straight line, and it must lie below the T-axis because χ is negative.

Step 3: Check each figure

Figure A is a curve that stays flat and then falls with temperature, like a Curie-Weiss law for a ferromagnet above its Curie point, so it is not diamagnetic. Figure B is a decreasing hyperbola $\chi \propto 1/T$, the Curie law for a paramagnet. Figure C is a horizontal line below the T-axis, which is what we need. Figure D is a horizontal line above the T-axis, which would mean positive χ, so it is wrong.

Final Answer:

Figure C shows a constant negative susceptibility, so the answer is option (C).

Option (C)

Quick Tip: Diamagnetic susceptibility is negative and almost independent of temperature, so look for a flat line below the axis.

31. A wire of length L is bent in the form of a circular coil and current 'i' is passed through it. This coil is kept in magnetic field. The torque acting on the coil will be maximum, when the number of turns is ____

(A) 1

(B) 2

(C) 4

(D) As large as possible

Correct Answer: (A) 1

Solution:

Step 1: Relation

The wire of length L gives N turns of radius r: $L = N \cdot 2\pi r$, so $r = \frac{L}{2\pi N}$.

Step 2: Torque

$\tau = NiAB = Ni\pi r^2 B = Ni\pi B \frac{L^2}{4\pi^2 N^2} = \frac{iBL^2}{4\pi N}$.

Step 3: Maximise

$\tau \propto \frac{1}{N}$, so it is largest for the smallest N, that is $N = 1$. Option (A).

Final Answer:

The torque is maximum for $N = 1$, option (A).

$$\boxed{\text{(A) 1}}$$

Quick Tip: Radius shrinks as turns increase, so area falls faster than N rises.

32. Two long parallel wires carrying currents 8A and 15A in opposite directions are placed at a distance of 7 cm from each other. A point P is at equidistant from both the wires such that the lines joining the point P to the wires are perpendicular to each other. The magnetic field at P is ____ $\times 10^{-6}$ T. (Given: $\sqrt{2} = 1.4$, $\mu_0 = 4\pi \times 10^{-7}$ SI unit)

(A) 62

(B) 65

(C) 68

(D) 70

Correct Answer: (C) 68

Solution:

Step 1: Find the distances

Lines from P to the wires are perpendicular and equal, so the wires and P form a right isosceles triangle with hypotenuse 7 cm. Each distance is $r = \frac{7}{\sqrt{2}} = \frac{7}{1.4} = 5$ cm.

Step 2: Fields from each wire

$B_1 = \frac{\mu_0 I_1}{2\pi r} = \frac{2\times10^{-7}\times8}{0.05} = 3.2 \times 10^{-5}$ T.

$B_2 = \frac{2\times10^{-7}\times15}{0.05} = 6 \times 10^{-5}$ T.

Step 3: Combine

Each field is perpendicular to its line from the wire. These lines are perpendicular, so $B_1 \perp B_2$.

$B = \sqrt{(3.2)^2 + (6)^2} \times 10^{-5} = \sqrt{46.24} \times 10^{-5} = 6.8 \times 10^{-5}$ T $= 68 \times 10^{-6}$ T. Option (C).

Final Answer:

The field is 68×10^{-6} T, option (C).

(C) 68

Quick Tip: The point is 5 cm from each wire, and the two fields are perpendicular.

33. A square loop of area $25\ \text{cm}^2$ has a resistance of $10\ \Omega$. The loop is placed in uniform magnetic field of magnitude 40 T. The plane of the loop is perpendicular to the magnetic field. The work done in pulling the loop out of the magnetic field slowly and uniformly in one second will be

(A) 2.5×10^{-3} J

(B) 1.0×10^{-3} J

(C) 1.0×10^{-4} J

(D) 5×10^{-3} J

Correct Answer: (B) 1.0×10^{-3} J

Solution:

Step 1: Find the emf

$A = 25 \times 10^{-4}$ m^2. Flux $= BA = 40 \times 25 \times 10^{-4} = 0.1$ Wb. Removing it in 1 s gives $\varepsilon = 0.1$ V.

Step 2: Power and work

$P = \frac{\varepsilon^2}{R} = \frac{0.01}{10} = 10^{-3}$ W. Work $= Pt = 10^{-3} \times 1 = 1.0 \times 10^{-3}$ J. Option (B).

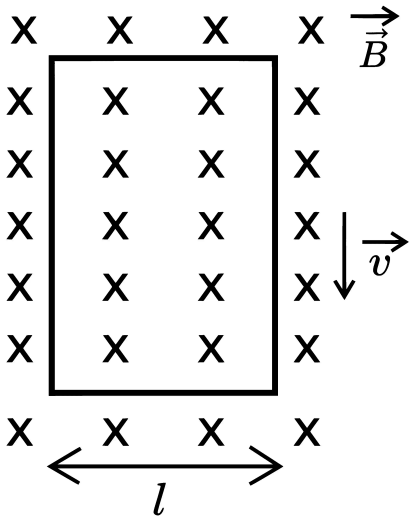
Final Answer:

The work done is 1.0×10^{-3} J, option (B).

(B)

Quick Tip: Find emf $= \frac{BA}{t}$, then work $= \frac{\varepsilon^2}{R}t$.

34. A long rectangular conducting loop of width 'l' mass 'm' and resistance 'R' is placed partly in a perpendicular magnetic field 'B'. With what velocity should it be pushed downwards so that it may continue to fall without any acceleration? (g = acceleration due to gravity)



(A) $\frac{B^2l^2R^2}{mg}$

(B) $\frac{mgR}{B^2l^2}$

(C) $\frac{mgl}{B^2R^2}$

(D) $\frac{mgR^2}{Bl}$

Correct Answer: (B) $\frac{mgR}{B^2l^2}$

Solution:

Step 1: Understand the figure

A rectangular loop of width l, mass m and resistance R falls with

velocity v downward. The magnetic field B points into the page and covers part of the loop, so a horizontal side of length l is inside the field.

Step 2: Find the induced emf

As the loop falls, the flux through it changes. The motional emf is $\varepsilon = Blv$.

Step 3: Find the current and the force

Current: $I = \frac{\varepsilon}{R} = \frac{Blv}{R}$. This current runs through the side of length l in the field, so the magnetic force is

$$F = IlB = \frac{B^2 l^2 v}{R}$$

By Lenz's law this force acts upward and opposes the fall.

Step 4: Apply the no acceleration condition

The loop falls at constant velocity when the net force is zero: $F = mg$.

$$\frac{B^2 l^2 v}{R} = mg \Rightarrow v = \frac{mgR}{B^2 l^2}$$

Step 5: Check the options

Options (A), (C) and (D) have R, B or l to the wrong power. A quick unit check helps: only $\frac{mgR}{B^2 l^2}$ comes from $F = \frac{B^2 l^2 v}{R}$ and so has units of speed.

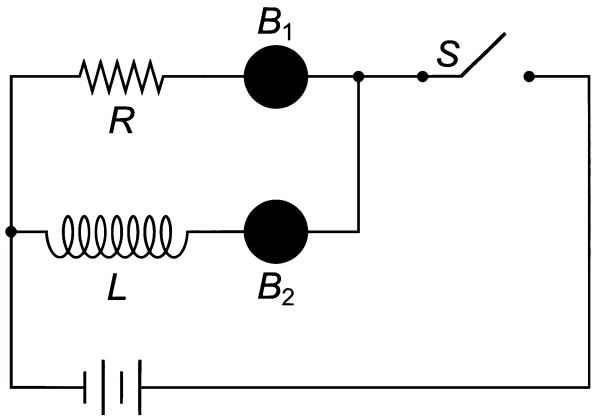
Final Answer:

The required speed is $\frac{mgR}{B^2 l^2}$, option (B).

$$\boxed{v = \frac{mgR}{B^2 l^2}}$$

Quick Tip: At constant speed the upward magnetic force on the side in the field equals the weight. Use emf = Blv and force = B I l.

35. The following figure represents two bulbs B_1 and B_2, resistor R and an inductor L. When the switch S is turned off, which of the following statement is true?



(A) B_1 becomes off promptly but B_2 with some delay

(B) B_2 becomes off promptly but B_1 with some delay

(C) Both B_1 and B_2 becomes off with same delay

(D) Both B_1 and B_2 becomes off promptly

Correct Answer: (C) Both B_1 and B_2 becomes off with same delay

Solution:

Step 1: Understand the circuit

The battery and switch S sit in one branch. Resistor R and bulb B_1 are in series in the top branch. Inductor L and bulb B_2 are in series in the

lower branch. The two branches share the same two junctions.

Step 2: While S is closed

The battery drives current through both branches. The inductor stores energy in its magnetic field.

Step 3: When S is opened

The battery is cut off. The inductor opposes the sudden fall of its current by producing an induced emf that keeps the current going in the same direction. The only closed path left is $L \rightarrow B_2 \rightarrow$ junction $\rightarrow B_1 \rightarrow R \rightarrow L$. In this loop all four parts are in series.

Step 4: Compare the bulbs

Because they are in series, the same current flows through B_1 and B_2. Both glow while the current decays and both go off together after the same delay.

Step 5: Why the other options fail

(A) and (B) say one bulb goes off at once, but both bulbs are inside the closed loop. (D) says both go off at once, but the inductor does not let the current drop to zero instantly.

Final Answer:

Both bulbs go off after the same delay, option (C).

Option (C)

Quick Tip: After the switch opens the inductor keeps the current flowing around a series loop containing both bulbs.

36. An inductor of reactance **100 Ω**, a capacitor of reactance **50 Ω**, and a resistor of resistance **50 Ω** are connected in series with an AC source of **10** V, **50** Hz. Average power dissipated by the circuit is _____

(A) 1 W

(B) 2 W

(C) 3 W

(D) 4 W

Correct Answer: (A) 1 W

Solution:

Step 1: Impedance

$X_L - X_C = 100 - 50 = 50\,\Omega$ and $R = 50\,\Omega$. $Z = \sqrt{50^2 + 50^2} = 50\sqrt{2}\,\Omega$.

Step 2: Current

Taking 10 V as the rms voltage, $I = \frac{10}{50\sqrt{2}} = \frac{1}{5\sqrt{2}}$ A.

Step 3: Power

$P = I^2R = \frac{1}{50} \times 50 = 1$ W. Option (A).

Final Answer:

The average power is 1 W, option (A).

$$\boxed{\text{(A) 1 W}}$$

Quick Tip: Average power is $I_{rms}^2 R$ with $Z = \sqrt{R^2 + (X_L - X_C)^2}$.

37. In an AC circuit $E = 50sin(500t)$, $I = 600sin(500t + \frac{\pi}{3})mA$. What is the power dissipated in the circuit? [$cos60° = 0.5$]

(A) 10 W

(B) 75 W

(C) 7.5 W

(D) 50 W

Correct Answer: (C) 7.5 W

Solution:

Step 1: Peak values

$E_0 = 50$ V and $I_0 = 600$ mA $= 0.6$ A. The phase difference is $\phi = \frac{\pi}{3} = 60°$.

Step 2: Average power

$P = \frac{E_0 I_0}{2}\cos\phi = \frac{50 \times 0.6}{2} \times 0.5 = 7.5$ W. Option (C).

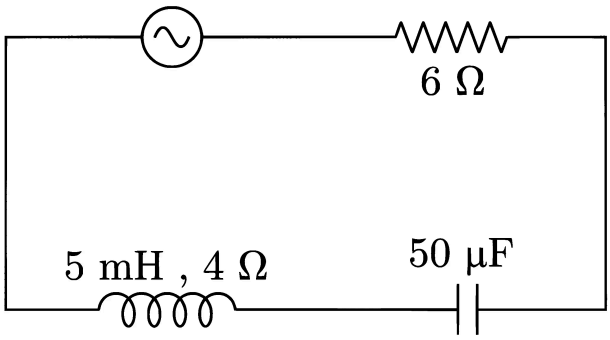
Final Answer:

The power dissipated is 7.5 W, option (C).

$$\boxed{\text{(C) 7.5 W}}$$

Quick Tip: Use $P = V_{rms} I_{rms} \cos\phi$ with phase $\frac{\pi}{3}$.

38. In the circuit shown below, the ac source has voltage $V = 30cos(\omega t)$ volt with $\omega = 2000$ rad/s. What will be the amplitude of the current?



(A) $\sqrt{5}$ A

(B) $\frac{3}{\sqrt{5}}$ A

(C) 3 A

(D) 3.3 A

Correct Answer: (C) 3 A

Solution:

Step 1: Read the circuit

The ac source is $V = 30\cos\omega t$ with $\omega = 2000$ rad/s. In series: a $6\ \Omega$ resistor, a coil of 5 mH with $4\ \Omega$ resistance, and a $50\ \mu$F capacitor.

Step 2: Total resistance

$R = 6 + 4 = 10\ \Omega$.

Step 3: Reactances

$$X_L = \omega L = 2000 \times 5 \times 10^{-3} = 10\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2000 \times 50 \times 10^{-6}} = 10\ \Omega$$

Since $X_L = X_C$, the circuit is at resonance.

Step 4: Impedance and current

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{10^2 + 0} = 10\ \Omega$$

$$I_0 = \frac{V_0}{Z} = \frac{30}{10} = 3\ \text{A}$$

Step 5: Check the options

$\sqrt{5}$ A and $\frac{3}{\sqrt{5}}$ A would need a nonzero net reactance. 3.3 A would need an impedance of about 9 ohm, which is smaller than the total resistance 10 ohm, and that is impossible. Only 3 A fits.

Final Answer:

The current amplitude is 3 A, option (C).

$$\boxed{3\ \text{A}}$$

Quick Tip: Find X_L and X_C. They turn out to be equal, so the impedance equals the total resistance of 10 ohm.

39. For large magnifying power of a telescope

(A) object must be large

(B) focal length of objective is large

(C) focal length of eyepiece is large

(D) focal length of objective and eyepiece must be large

Correct Answer: (B) focal length of objective is large

Solution:

Step 1: Formula

Magnifying power of an astronomical telescope in normal adjustment is $M = \frac{f_o}{f_e}$.

Step 2: Meaning

To make M large, f_o must be large and f_e small. Among the options, only "focal length of objective is large" fits. Option (B).

Final Answer:

A large objective focal length gives a large magnification, option (B).

$$\boxed{(B)}$$

Quick Tip: For a telescope $M = \frac{f_o}{f_e}$.

40. A ray of light is incident of the surface of a glass plate at an angle of incidence equal to Brewster's angle Φ. If μ denotes the refractive index of glass w.r.t. air, then what will be the angle between reflected and refracted rays?

(A) $90° - sin^{-1}(sin\Phi/\mu)$

(B) $90°$

(C) $sin^{-1}(\mu cos\Phi)$

(D) $90° + \Phi$

Correct Answer: (B) $90°$

Solution:

Step 1: Brewster law

At the polarising angle, $\tan\Phi = \mu$, and $\Phi + r = 90°$.

Step 2: Angle between rays

The reflected ray makes Φ with the normal and the refracted ray makes $r = 90° - \Phi$. The angle between them is $180° - \Phi - r = 90°$. Option (B).

Final Answer:

The angle is $90°$, option (B).

$$\boxed{\text{(B) } 90°}$$

Quick Tip: At Brewster angle the reflected and refracted rays are perpendicular.

41. Two polaroid A and B placed in such a way that the pass-axis of polaroid are perpendicular to each other. Now, another polaroid C is placed between A and B bisecting angle between them. If intensity of unpolarised light is I_0 then intensity of transmitted light after passing through polaroid B will be

(A) $\frac{I_0}{4}$

(B) $\frac{I_0}{2}$

(C) $\frac{I_0}{8}$

(D) zero

Correct Answer: (C) $\frac{I_0}{8}$

Solution:

Step 1: After A

Unpolarised light of intensity I_0 falls to $\frac{I_0}{2}$ after the first polaroid.

Step 2: After C

C bisects the angle, so its axis is at $45°$ to A. $I = \frac{I_0}{2}\cos^2 45° = \frac{I_0}{4}$.

Step 3: After B

B is at $45°$ to C. $I = \frac{I_0}{4}\cos^2 45° = \frac{I_0}{8}$. Option (C).

Final Answer:

The transmitted intensity is $\frac{I_0}{8}$, option (C).

$$\boxed{\text{(C) } \frac{I_0}{8}}$$

Quick Tip: Use Malus law at each polaroid. The first polaroid halves unpolarised light.

42. In a Young's double slit experiment, the intensities at two points, for the path difference $\frac{\lambda}{4}$ and $\frac{\lambda}{3}$ (λ being the wavelength of light used) are I_1 and I_2 respectively. If I_0 denotes the intensity produced by each one of the individual slits, then $\frac{I_1+I_2}{I_0} =$
$(cos45° = \frac{1}{\sqrt{2}}, cos60 = \frac{1}{2})$

(A) 2

(B) 3

(C) 4

(D) 5

Correct Answer: (B) 3

Solution:

Step 1: First point

$\Delta = \frac{\lambda}{4}$ gives $\phi = \frac{\pi}{2}$. $I_1 = 4I_0 \cos^2 45° = 4I_0 \times \frac{1}{2} = 2I_0$.

Step 2: Second point

$\Delta = \frac{\lambda}{3}$ gives $\phi = \frac{2\pi}{3}$. $I_2 = 4I_0 \cos^2 60° = 4I_0 \times \frac{1}{4} = I_0$.

Step 3: Sum

$\frac{I_1+I_2}{I_0} = 2 + 1 = 3$. Option (B).

Final Answer:

The ratio is 3, option (B).

$$\boxed{\text{(B) } 3}$$

Quick Tip: Use $I = 4I_0 \cos^2 \frac{\phi}{2}$ with $\phi = \frac{2\pi}{\lambda}\Delta$.

43. Two light waves amplitudes in the ratio 3 : 1 produce interference. The ratio of the maximum to minimum intensity is

(A) 9 : 4

(B) 16 : 9

(C) 3 : 2

(D) 4 : 1

Correct Answer: (D) 4 : 1

Solution:

Step 1: Write the formula

$\frac{I_{max}}{I_{min}} = \frac{(a_1+a_2)^2}{(a_1-a_2)^2}$.

Step 2: Substitute

With $a_1 : a_2 = 3 : 1$: $\frac{(3+1)^2}{(3-1)^2} = \frac{16}{4} = 4 : 1$. Option (D).

Final Answer:

The ratio is $4 : 1$, option (D).

$$\boxed{\text{(D) } 4 : 1}$$

Quick Tip: Intensity ratio is $\frac{(a_1+a_2)^2}{(a_1-a_2)^2}$.

44. Photoelectric emission is observed from a metallic surface for frequencies v_1 and v_2 of the incident light rays ($v_1 > v_2$). If the maximum values of kinetic energy of the photoelectrons emitted in the two cases are in the ratio of $k : 1$, then what is the threshold frequency of the metallic surface?

(A) $\frac{v_1-v_2}{k}$

(B) $\frac{v_1-v_2}{k-1}$

(C) $\frac{kv_1-v_2}{k-1}$

(D) $\frac{kv_2-v_1}{k-1}$

Correct Answer: (D) $\frac{kv_2-v_1}{k-1}$

Solution:

Step 1: Two equations

$K_1 = h(\nu_1 - \nu_0)$ and $K_2 = h(\nu_2 - \nu_0)$.

Step 2: Use the ratio

$\frac{K_1}{K_2} = k$ gives $\nu_1 - \nu_0 = k(\nu_2 - \nu_0)$.

Step 3: Solve

$\nu_1 - \nu_0 = k\nu_2 - k\nu_0$, so $(k-1)\nu_0 = k\nu_2 - \nu_1$ and $\nu_0 = \frac{k\nu_2 - \nu_1}{k-1}$. Option (D).

Final Answer:

The threshold frequency is $\frac{k\nu_2 - \nu_1}{k-1}$, option (D).

$\boxed{\text{(D)}}$

Quick Tip: Use Einstein equation $K = h(\nu - \nu_0)$ for both frequencies.

45. A photon and an electron have equal energy E. The ratio of λ(electron) to λ(photon) is proportional to

(A) $1 : \sqrt{E}$

(B) $\sqrt{E} : 1$

(C) $1 : E$

(D) $1 : E^2$

Correct Answer: (B) $\sqrt{E} : 1$

Solution:

Step 1: Wavelengths

$\lambda_e = \frac{h}{\sqrt{2mE}}$ and $\lambda_{ph} = \frac{hc}{E}$.

Step 2: Ratio

$\frac{\lambda_e}{\lambda_{ph}} = \frac{E}{c\sqrt{2mE}} = \frac{\sqrt{E}}{c\sqrt{2m}}$.

Step 3: Proportion

The ratio is proportional to $\sqrt{E}$, that is $\sqrt{E} : 1$. Option (B).

Final Answer:

The ratio is proportional to $\sqrt{E} : 1$, option (B).

$\boxed{\text{(B)}}$

Quick Tip: Electron: $\lambda = \frac{h}{\sqrt{2mE}}$. Photon: $\lambda = \frac{hc}{E}$.

46. The kinetic energy of the electron in an orbit of radius r in hydrogen atom is proportional to (e = electronic charge)

(A) $\frac{e^2}{2r^2}$

(B) $\frac{e^2}{r}$

(C) $\frac{e^2}{2r}$

(D) $\frac{e^2}{r^2}$

Correct Answer: (C) $\frac{e^2}{2r}$

Solution:

Step 1: Force balance

$\frac{mv^2}{r} = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$.

Step 2: Kinetic energy

$\frac{1}{2}mv^2 = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{2r}$.

So the kinetic energy is proportional to $\frac{e^2}{2r}$. Option (C).

Final Answer:

The kinetic energy is proportional to $\frac{e^2}{2r}$, option (C).

$\boxed{\text{(C)}}$

Quick Tip: Set the Coulomb force equal to the centripetal force.

47. The number of revolutions per second made by an electron in the first Bohr orbit of hydrogen atom is (h = Planck's constant, m is the mass of electron and r is the radius of the orbit.)

(A) $h/4\pi^2mr^2$

(B) $h/4\pi^2mr$

(C) $h/4\pi mr$

(D) $h/4\pi^2m^2r^2$

Correct Answer: (A) $h/4\pi^2mr^2$

Solution:

Step 1: Frequency

Revolutions per second $f = \frac{v}{2\pi r}$.

Step 2: Speed from quantisation

For the first orbit $mvr = \frac{h}{2\pi}$, so $v = \frac{h}{2\pi mr}$.

Step 3: Substitute

$f = \frac{h}{2\pi mr} \times \frac{1}{2\pi r} = \frac{h}{4\pi^2 mr^2}$. Option (A).

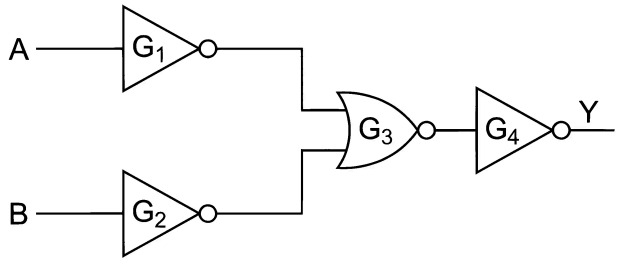
Final Answer:

The frequency is $\frac{h}{4\pi^2 mr^2}$, option (A).

$$\boxed{\text{(A)}}$$

Quick Tip: Frequency is $\frac{v}{2\pi r}$, and Bohr gives $mvr = \frac{h}{2\pi}$.

48. What does the following combination of gates produce?



(A) NAND gate

(B) NOR gate

(C) XOR gate

(D) AND gate

Correct Answer: (A) NAND gate

Solution:

Step 1: Read the circuit

Input A goes through gate G_1, a NOT gate. Input B goes through G_2, also a NOT gate. Their outputs $\bar{A}$ and $\bar{B}$ enter G_3, a NOR gate. The

output of G_3 goes to G_4, a NOT gate, giving Y.

Step 2: Write the expression

G_3 output: $\overline{\bar{A} + \bar{B}}$. G_4 inverts it:

$$Y = \overline{\overline{\bar{A} + \bar{B}}} = \bar{A} + \bar{B}$$

Step 3: Apply De Morgan's theorem

$\bar{A} + \bar{B} = \overline{A \cdot B}$, so $Y = \overline{AB}$.

Step 4: Truth table check

A=0,B=0 gives $Y = 1$. A=0,B=1 gives $Y = 1$. A=1,B=0 gives $Y = 1$. A=1,B=1 gives $Y = 0$. This is the NAND truth table.

Step 5: Why the others fail

NOR gives 1 only for 0,0. XOR gives 0 for 0,0. AND gives 1 only for 1,1. None matches the table above.

Final Answer:

The combination acts as a NAND gate, option (A).

$$\boxed{Y = \overline{A \cdot B}\,(\text{NAND})}$$

Quick Tip: Two NOT gates feed a NOR followed by a NOT. Write the Boolean expression and use De Morgan's theorem.

49. In common emitter transistor amplifier, the output resistance is 500 k Ω and the current gain $\beta = 50$. If power gain of amplifier is 5×10^6, what is the input resistance?

(A) 325Ω

(B) 150Ω

(C) 350Ω

(D) 250Ω

Correct Answer: (D) 250Ω

Solution:

Step 1: Formula

In a common emitter amplifier, power gain $= \beta^2 \times \frac{R_{out}}{R_{in}}$.

Step 2: Substitute

$5 \times 10^6 = 2500 \times \frac{5 \times 10^5}{R_{in}}$.

Step 3: Solve

$R_{in} = \frac{2500 \times 5 \times 10^5}{5 \times 10^6} = 250\,\Omega$. Option (D).

Final Answer:

The input resistance is 250 Ω, option (D).

$$\boxed{\text{(D) } 250\,\Omega}$$

Quick Tip: Power gain $= \beta^2 \frac{R_{out}}{R_{in}}$.

50. When the conductivity of a semiconductor is only due to breaking of covalent bonds, the semiconductor is called

(A) extrinsic.

(B) intrinsic.

(C) p-type.

(D) n-type.

Correct Answer: (B) intrinsic.

Solution:

Step 1: Define

In a pure semiconductor, conduction comes from electron-hole pairs made by breaking covalent bonds. This is an intrinsic semiconductor.

Step 2: Others

Extrinsic, p-type and n-type semiconductors are doped, so some carriers come from impurities. Option (B).

Final Answer:

It is an intrinsic semiconductor, option (B).

$$\boxed{(B)}$$

Quick Tip: A pure semiconductor with no doping is called intrinsic.

Chemistry

Questions 51 to 100 | 1 mark each | 50 marks

51. Calculate the mass of 100 molecules of oxygen in amu and in grams.

(A) 3200 u and 5.314×10^{-21} g

(B) 1600 u and 2.657×10^{-21} g

(C) 32 u and 1.328×10^{-21} g

(D) 8 u and 3.978×10^{-21} g

Correct Answer: (A) 3200 u and 5.314×10^{-21} g

Solution:

Step 1: Mass of one molecule

The molecular mass of O_2 is $2 \times 16 = 32$ u, so one molecule has a mass of 32 u.

Step 2: Mass of 100 molecules in u

$$100 \times 32 \text{ u} = 3200 \text{ u}$$

Step 3: Convert to grams

Use $1 \text{ u} = 1.66 \times 10^{-24}$ g.

$$3200 \times 1.66 \times 10^{-24} = 5.31 \times 10^{-21} \text{ g}$$

Step 4: Check options

Option (A) gives 3200 u and 5.314×10^{-21} g. Options (B) to (D) use

wrong masses (1600, 32, 8 u), so they fail.

Final Answer:

The mass is 3200 u or 5.314×10^{-21} g, option (A).

$$\boxed{\text{(A)}}$$

Quick Tip: One O_2 molecule weighs 32 u and $1\ \text{u} = 1.66 \times 10^{-24}$ g.

52. Match **List I** with **List II**

List I	**List II**
Element	**Number of Neutron**
(A). $^{24}_{12}Mg$	(I). 5
(B). $^{27}_{13}Al$	(II). 12
(C). $^{9}_{4}Be$	(III). 6
(D). $^{12}_{6}C$	(IV). 14

Choose the **correct** answer from the options given below:

(A) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)

(B) (A) - (II), (B) - (IV), (C) - (I), (D) - (III)

(C) (A) - (I), (B) - (II), (C) - (IV), (D) - (III)

(D) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Correct Answer: (B) (A) - (II), (B) - (IV), (C) - (I), (D) - (III)

Solution:

Step 1: State the rule

Number of neutrons $= A - Z$, where A is the mass number (top) and Z is the atomic number (bottom).

Step 2: Calculate each

(A) Mg: $24 - 12 = 12$, which is (II).

(B) Al: $27 - 13 = 14$, which is (IV).

(C) Be: $9 - 4 = 5$, which is (I).

(D) C: $12 - 6 = 6$, which is (III).

Step 3: Match with options

The pairing is A-II, B-IV, C-I, D-III, which is option (B). The other options pair at least one element with a wrong neutron count.

Final Answer:

The matching is A-II, B-IV, C-I, D-III, option (B).

$\boxed{(B)}$

Quick Tip: Neutrons = mass number minus atomic number.

53. Which of the following hydrides of Group 16 elements has highest reducing property?

(A) H_2O

(B) H_2S

(C) H_2Se

(D) H_2Te

Correct Answer: (D) H_2Te

Solution:

Step 1: Recall the trend

The hydrides of group 16 are H_2O, H_2S, H_2Se and H_2Te. A reducing agent donates hydrogen or electrons.

Step 2: Link bond strength to reduction

Down the group, the size of the central atom grows and the E-H bond becomes longer and weaker.

A weaker bond breaks more easily, so the hydride reduces more strongly.

Step 3: Conclude

The order of reducing power is $H_2O < H_2S < H_2Se < H_2Te$. So H_2Te is the strongest, option (D).

Final Answer:

H_2Te has the weakest bond and is the best reducing agent, option (D).

$$\boxed{\text{(D) } H_2Te}$$

Quick Tip: Going down the group, the H-E bond gets weaker, so the hydride gives up hydrogen more easily.

54. Identify the shape of IF_5 molecule from the following options.

(A) Trigonal bipyramidal

(B) Square planar

(C) Hexagonal

(D) Square pyramidal

Correct Answer: (D) Square pyramidal

Solution:

Step 1: Count electron pairs on iodine

Iodine has 7 valence electrons. Five are used in bonds with fluorine, leaving 2 electrons, which is one lone pair.

So there are 5 bond pairs and 1 lone pair, 6 pairs in total.

Step 2: Find the geometry

Six electron pairs give an octahedral arrangement (sp^3d^2 hybridisation).

The lone pair takes one octahedral position, leaving five atoms in a square pyramid shape.

Step 3: Other options

Trigonal bipyramidal needs 5 pairs in all. Square planar is the shape of XeF_4 (4 bonds, 2 lone pairs). Hexagonal does not occur for such molecules.

Final Answer:

IF_5 is square pyramidal, option (D).

(D) Square pyramidal

Quick Tip: Count bond pairs and lone pairs on iodine: 5 bond pairs and 1 lone pair.

55. According to the kinetic molecular theory of gases, the average kinetic energy of gas molecules:

(A) Increases with increase in pressure

(B) Decreases with increase in volume

(C) Is directly proportional to the absolute temperature

(D) Is the same for all gases at the same volume

Correct Answer: (C) Is directly proportional to the absolute temperature

Solution:

Step 1: Recall the result

For gas molecules the average kinetic energy is $\bar{E}_k = \frac{3}{2}k_B T$ per molecule.

It depends only on the absolute temperature.

Step 2: Check each option

(A) Pressure can rise with no change in T, so $\bar{E}_k$ does not depend on it.

(B) Volume changes do not alter $\bar{E}_k$ at fixed T.

(C) $\bar{E}_k \propto T$, which is true.

(D) Volume has no role. The kinetic energy is the same for all gases at the same temperature, not at the same volume.

Final Answer:

Option (C) is correct.

$$\boxed{(C)}$$

Quick Tip: In kinetic theory, the average kinetic energy depends only on temperature.

56. Calculate the enthalpy change for the following reaction, using given bond energy (kJ/mol)

(C-H = 414, H-O = 463, H-Cl = 431, C-Cl = 326 and C-O = 335)

$CH_3OH_{(g)} + HCl_{(g)} \rightarrow CH_3Cl_{(g)} + H_2O_{(g)}$

(A) $-23\ kJmol^{-1}$

(B) $-42\ kJmol^{-1}$

(C) $-59\ kJmol^{-1}$

(D) $-51\ kJmol^{-1}$

Correct Answer: (A) $-23\ kJmol^{-1}$

Solution:

Step 1: Write the rule

$\Delta H = \sum(\text{bond energies of reactants}) - \sum(\text{bond energies of products})$.

Step 2: Bonds broken

In CH_3OH we break one C-O bond (335) and one O-H bond (463). In HCl we break H-Cl (431).

$$335 + 463 + 431 = 1229\ kJ$$

Step 3: Bonds formed

In CH_3Cl we form one C-Cl bond (326). In H_2O we form two O-H bonds ($2 \times 463 = 926$).

$$326 + 926 = 1252 \text{ kJ}$$

Step 4: Compute

$$\Delta H = 1229 - 1252 = -23 \text{ kJ mol}^{-1}$$

The C-H bonds are unchanged and cancel.

Final Answer:

$\Delta H = -23$ kJ mol^{-1}, option (A).

$$\boxed{\text{(A) } -23 \text{ kJ mol}^{-1}}$$

Quick Tip: ΔH = bonds broken − bonds formed. Remember water has two O-H bonds.

57. Calculate the work done when 1 mole of an ideal gas is expanded reversibly and isothermally from initial pressure 10 bar to final pressure 1 bar at constant temperature 300 K. [$R = 8.314$ J K^{-1} mol^{-1}]

(A) -5.744 kJ

(B) -5.123 kJ

(C) -6.514 kJ

(D) -4.981 kJ

Correct Answer: (A) -5.744 kJ

Solution:

Step 1: Write the formula

For a reversible isothermal expansion, $w = -2.303\, nRT \log_{10} \frac{V_2}{V_1}$. At constant T, $\frac{V_2}{V_1} = \frac{P_1}{P_2}$.

Step 2: Substitute

$n = 1$, $R = 8.314$, $T = 300$ K, $\frac{P_1}{P_2} = 10$, so $\log 10 = 1$.

$$w = -2.303 \times 8.314 \times 300 \times 1 = -5744 \text{ J}$$

Step 3: Convert

$w = -5.744$ kJ. The negative sign means the gas does work on the surroundings. Option (A).

Final Answer:

Work done is -5.744 kJ, option (A).

$$\boxed{\text{(A) } -5.744 \text{ kJ}}$$

Quick Tip: Use $w = -2.303\, nRT \log \frac{P_1}{P_2}$ for reversible isothermal expansion.

58. Which of the following processes exhibits, $\Delta U = 0$?

(A) Adiabatic process

(B) Isothermal process

(C) Isobaric process

(D) Isochoric process

Correct Answer: (B) Isothermal process

Solution:

Step 1: Recall the concept

For an ideal gas, internal energy depends only on temperature, $\Delta U = nC_V\Delta T$.

Step 2: Check each process

(A) Adiabatic: $q = 0$ but $\Delta U = w \neq 0$.

(B) Isothermal: $\Delta T = 0$ so $\Delta U = 0$.

(C) Isobaric: T changes, so $\Delta U \neq 0$.

(D) Isochoric: $w = 0$ but $\Delta U = q \neq 0$.

Final Answer:

Isothermal process gives $\Delta U = 0$, option (B).

$$\boxed{\text{(B)}}$$

Quick Tip: For an ideal gas ΔU depends only on temperature.

59. Which of the following aqueous solution of salt have $pH < 7$ at 298 K?

(A) KNO_3

(B) Na_2CO_3

(C) CH_3COONH_4

(D) $CuCl_2$

Correct Answer: (D) $CuCl_2$

Solution:

Step 1: Classify each salt

(A) KNO_3: strong base + strong acid, neutral, pH $= 7$.

(B) Na_2CO_3: strong base + weak acid, basic, pH > 7.

(C) CH_3COONH_4: weak acid + weak base with nearly equal strengths, about neutral.

(D) $CuCl_2$: weak base $Cu(OH)_2$ + strong acid HCl, acidic.

Step 2: Reason for (D)

Cu^{2+} hydrolyses: $Cu^{2+} + 2H_2O \rightleftharpoons Cu(OH)_2 + 2H^+$. Extra H^+ makes pH < 7.

Final Answer:

$CuCl_2$ solution is acidic, option (D).

$$\boxed{\text{(D) } CuCl_2}$$

Quick Tip: A salt of a strong acid and a weak base gives an acidic solution.

60. What happens when solid Na_2CO_3 dissolved in water?

(A) an increase in $[OH^-]$

(B) no change in pH

(C) an increase in $[H^+]$ ions

(D) turns blue litmus red.

Correct Answer: (A) an increase in $[OH^-]$

Solution:

Step 1: Write the hydrolysis

Na_2CO_3 is a salt of a strong base (NaOH) and a weak acid (H_2CO_3). The carbonate ion reacts with water:

$$CO_3^{2-} + H_2O \rightleftharpoons HCO_3^- + OH^-$$

Step 2: Read off the result

$[OH^-]$ rises, so the solution is basic, pH > 7.

So (A) is right, (B) is wrong because pH changes, (C) is wrong because $[H^+]$ falls, and (D) is wrong because a basic solution turns red litmus blue, not blue litmus red.

Final Answer:

Dissolving Na_2CO_3 raises $[OH^-]$, option (A).

$$\boxed{(A)}$$

Quick Tip: Carbonate ion is the anion of a weak acid, so it hydrolyses and releases OH^-.

61. What happens when $CuSO_4$ is dissolved in water?
Identify correct statements from the following

(A) Anion of the salt react with water.

(B) Its solution will turn red litmus into blue.

(C) The pH of the solution will be less than 7.

(D) The $CuSO_4$ will not undergo hydrolysis reaction.

Correct Answer: (C) The pH of the solution will be less than 7.

Solution:

Step 1: Identify the salt

$CuSO_4$ comes from the weak base $Cu(OH)_2$ and the strong acid H_2SO_4.

Step 2: Hydrolysis

Only the cation hydrolyses: $Cu^{2+} + 2H_2O \rightleftharpoons Cu(OH)_2 + 2H^+$.
This releases H^+, so pH < 7.

Step 3: Check options

(A) is false, since the anion SO_4^{2-} does not react (it is the conjugate base of a strong acid).
(B) is false, an acidic solution turns blue litmus red.
(C) is true.
(D) is false, since the salt does hydrolyse.

Final Answer:

$CuSO_4$ solution has pH below 7, option (C).

$\boxed{(C)}$

Quick Tip: $CuSO_4$ is a salt of a weak base and a strong acid, so the solution is acidic.

62. Which of the following is obtained when dihydrogen reacts with the alkali metals at high temperature?

(A) Alkoxides

(B) Metal hydrides

(C) Peroxides

(D) Metal nitrites

Correct Answer: (B) Metal hydrides

Solution:

Step 1: Recall the reaction

At high temperature, dihydrogen combines with alkali metals to give ionic hydrides:

$$2M + H_2 \xrightarrow{\Delta} 2MH$$

Step 2: Check

Alkoxides need an alcohol. Peroxides form with oxygen. Nitrites need nitrogen and oxygen. So only metal hydrides fit, option (B).

Final Answer:

The product is a metal hydride, option (B).

$$\boxed{(B)}$$

Quick Tip: Hydrogen takes an electron from the alkali metal to form H^-.

63. What type of isomerism is exhibited by neopentane?

(A) position

(B) chain

(C) geometrical

(D) tautomerism

Correct Answer: (B) chain

Solution:

Step 1: Identify the compound

Neopentane is 2,2-dimethylpropane, C_5H_{12}.

Step 2: Find the isomer relation

C_5H_{12} has three structures: n-pentane, isopentane and neopentane. They differ only in the chain of carbon atoms, so this is chain isomerism.

Step 3: Others

Position isomerism needs a movable substituent or double bond. Geometrical isomerism needs a double bond or ring. Tautomerism needs a labile hydrogen with a carbonyl-like group. None applies.

Final Answer:

Neopentane shows chain isomerism, option (B).

$$\boxed{\text{(B) chain}}$$

Quick Tip: Neopentane is one of the three isomers of C_5H_{12}, which differ in carbon skeleton.

64. Which from following is obtained as major product when methoxybenzene is treated with nitrating mixture?

(A) 2-Nitroanisole

(B) 4-Nitroanisole

(C) 4-Nitrophenol

(D) 2-Nitrophenol

Correct Answer: (B) 4-Nitroanisole

Solution:

Step 1: Identify the directing group

The methoxy group donates electrons by resonance, so it activates the ring and directs to ortho and para.

Step 2: Nitration

With HNO_3/H_2SO_4 the electrophile is NO_2^+. It attacks the ortho and para positions.

Step 3: Major product

The para product is major because the ortho positions are hindered by the OCH_3 group. So the major product is 4-nitroanisole. Nitrophenols are wrong because the ether group is not cleaved under these conditions.

Final Answer:

The major product is 4-nitroanisole, option (B).

$$\boxed{(B)}$$

Quick Tip: $-OCH_3$ is an ortho/para director and para is favoured by sterics.

65. Which of the following compounds is carcinogenic and toxic?

(A) Alkane

(B) Alkyne

(C) Benzene

(D) Alkene

Correct Answer: (C) Benzene

Solution:

Step 1: Recall the facts

Benzene is a toxic liquid and a known carcinogen. Long exposure damages bone marrow and can cause leukaemia.

Step 2: Others

Simple alkanes, alkenes and alkynes are not classed as carcinogens in this sense. So option (C).

Final Answer:

Benzene is carcinogenic and toxic, option (C).

$$\boxed{(C)}$$

Quick Tip: Benzene is a known human carcinogen.

66. Pt has FCC structure with an edge length of unit cell 392 pm. What is the radius of Pt?

(A) 138.6 pm

(B) 175.6 pm

(C) 185.6 pm

(D) 205.6 pm

Correct Answer: (A) 138.6 pm

Solution:

Step 1: Use FCC geometry

In an FCC cell, atoms touch along the face diagonal. The face diagonal is $\sqrt{2}a$ and holds $4r$.

$$4r = \sqrt{2}\,a \Rightarrow r = \frac{a}{2\sqrt{2}}$$

Step 2: Substitute

$$r = \frac{392}{2 \times 1.414} = \frac{392}{2.828} = 138.6 \text{ pm}$$

Step 3: Check

The value 196 pm would be $a/2$, which applies to a simple cubic cell, not FCC. So option (A).

Final Answer:

$r = 138.6$ pm, option (A).

$$\boxed{\text{(A) } 138.6 \text{ pm}}$$

Quick Tip: In FCC the face diagonal is $4r$, so $r = \frac{a}{2\sqrt{2}}$.

67. In a cubic unit cell of an ionic compound, the eight corners are occupied by anions and cations at the center of the cube. Calculate the volume of the unit cell if density of the unit cell is $4\ \text{g cm}^{-3}$.
[Molar mass of compound $= 168.6\ \text{g mol}^{-1}$ and $N_A = 6.022 \times 10^{23}$]

(A) $5.0 \times 10^{-23}\ \text{cm}^3$

(B) $6.0 \times 10^{-23}\ \text{cm}^3$

(C) $7.0 \times 10^{-23}\ \text{cm}^3$

(D) $8.0 \times 10^{-23}\ \text{cm}^3$

Correct Answer: (C) $7.0 \times 10^{-23}\ \text{cm}^3$

Solution:

Step 1: Count formula units

Anions at 8 corners give $8 \times \frac{1}{8} = 1$ anion. One cation at the centre. So the cell holds $Z = 1$ formula unit (like CsCl).

Step 2: Use the density formula

$$\rho = \frac{ZM}{N_A V} \Rightarrow V = \frac{ZM}{\rho N_A}$$

Step 3: Substitute

$$V = \frac{1 \times 168.6}{4 \times 6.022 \times 10^{23}} = \frac{168.6}{2.409 \times 10^{24}} = 7.0 \times 10^{-23}\ \text{cm}^3$$

Step 4: Check

If one used $Z = 2$ the answer would double, which is not an option. So option (C).

Final Answer:

$V = 7.0 \times 10^{-23}\ \text{cm}^3$, option (C).

$$\boxed{\text{(C)}}$$

Quick Tip: Count Z first: corners give $8 \times \frac{1}{8} = 1$ anion, center gives 1 cation.

68. What is the total number of spheres that surrounds the tetrahedral hole in the close-packed three-dimensional structure?

(A) 2

(B) 4

(C) 6

(D) 8

Correct Answer: (B) 4

Solution:

Step 1: Define the hole

A tetrahedral hole is formed when one sphere sits in the depression of three spheres in a layer, with a fourth sphere on top.

Step 2: Count

Three spheres in the lower layer plus one above gives **4** spheres whose centres form a tetrahedron. Octahedral holes have **6** neighbours instead. Option (B).

Final Answer:

A tetrahedral hole has **4** surrounding spheres, option (B).

$$\boxed{\text{(B)}}$$

Quick Tip: A tetrahedral void sits among 4 touching spheres.

69. Which of the following is a green alternative for dry cleaning solvents?

(A) Benzene

(B) Carbon tetrachloride

(C) Perchloroethylene

(D) Supercritical CO_2

Correct Answer: (D) Supercritical CO_2

Solution:

Step 1: Recall green chemistry

Dry cleaning used tetrachloroethene (perchloroethylene) and CCl_4,

which are toxic and pollute groundwater. Benzene is carcinogenic.

Step 2: Alternative
Liquid or supercritical CO_2 with a detergent is non-toxic, recyclable and leaves no harmful waste. Option (D).

Final Answer:
Supercritical CO_2 is the green solvent, option (D).

$$\boxed{(D)}$$

Quick Tip: The green choice is non-toxic and recyclable: supercritical carbon dioxide.

70. Vapour pressure of CCl_4 at 25°C is 143 mm Hg. If 0.5 g of a non-volatile solute is dissolved in 100 cm^3 of CCl_4. Find the vapour pressure of the solution.
(Density of $CCl_4 = 1.58\ g/cm^3$ and molecular weight of solute is 65)

(A) 141.93 mm

(B) 194.39 mm

(C) 199.34 mm

(D) 143.99 mm

Correct Answer: (A) 141.93 mm

Solution:

Step 1: Find the moles
Mass of $CCl_4 = 100 \times 1.58 = 158$ g. Molar mass of $CCl_4 = 154$, so

$n_{\text{solvent}} = \frac{158}{154} = 1.026$ mol.
Moles of solute $= \frac{0.5}{65} = 0.00769$ mol.

Step 2: Mole fraction of solute

$$x_2 = \frac{0.00769}{0.00769 + 1.026} = 0.00744$$

Step 3: Lowering of vapour pressure

$$P^0 - P = 0.00744 \times 143 = 1.064 \text{ mm}$$

$$P = 143 - 1.064 = 141.93 \text{ mm}$$

Step 4: Check

Other options exceed 143 mm, which is impossible since a solute lowers the pressure. Option (A).

Final Answer:

The vapour pressure is 141.93 mm Hg, option (A).

$$\boxed{\text{(A)}}$$

Quick Tip: Use Raoult's law: $\frac{P^0 - P}{P^0} = x_{\text{solute}}$.

71. The solubility of N_2 gas in water at 25°C and 1 bar is 6.85×10^{-4} mol L^{-1}. Calculate the solubility of N_2 gas in water at the same temperature when the partial pressure of N_2 is 0.70 bar

(A) 5.138×10^{-4} mol L^{-1}

(B) 5.480×10^{-4} mol L^{-1}

(C) 4.795×10^{-4} mol L^{-1}

(D) 4.875×10^{-4} mol L^{-1}

Correct Answer: (C) 4.795×10^{-4} mol L^{-1}

Solution:

Step 1: State Henry's law

The solubility of a gas is directly proportional to its partial pressure: $S \propto p$.

Step 2: Compute

$$S_2 = 6.85 \times 10^{-4} \times \frac{0.70}{1} = 4.795 \times 10^{-4} \text{ mol } L^{-1}$$

Step 3: Result

Option (C).

Final Answer:

Solubility is 4.795×10^{-4} mol L^{-1}, option (C).

$$\boxed{\text{(C)}}$$

Quick Tip: Henry's law: solubility is directly proportional to partial pressure.

72. Identify from the following pair of solutions that exhibits the same osmotic pressure.
[molar mass of urea = 60 g mol^{-1} and molar mass of glucose = 180 g mol^{-1}]

(A) 6 g/L urea solution and 18 g/L glucose solution.

(B) 12 g/L urea solution and 18 g/L glucose solution.

(C) 18 g/L urea solution and 18 g/L glucose solution.

(D) 6 g/L urea solution and 6 g/L glucose solution.

Correct Answer: (A) 6 g/L urea solution and 18 g/L glucose solution.

Solution:

Step 1: Use $\pi = CRT$
Osmotic pressure depends on molarity at the same T. Urea and glucose do not ionise, so only molarity matters.

Step 2: Compute molarities
(A) Urea $\frac{6}{60} = 0.1$ M, glucose $\frac{18}{180} = 0.1$ M. Equal.
(B) 0.2 M vs 0.1 M. Not equal.
(C) 0.3 M vs 0.1 M. Not equal.
(D) 0.1 M vs 0.033 M. Not equal.

Final Answer:
Both solutions in (A) are 0.1 M, option (A).

(A)

Quick Tip: Equal molar concentrations of non-electrolytes give equal osmotic pressure.

73. At 298 K, the specific conductance (κ) of a 0.0020 M NaCl solution is 2.50×10^{-4} S cm^{-1}. Calculate the molar conductivity (Λ_m) of the solution in S cm^2 mol^{-1}.

(A) 125

(B) 62.5

(C) 12.5

(D) 250

Correct Answer: (A) 125

Solution:

Step 1: Write the formula

Molar conductivity links to specific conductance by $\Lambda_m = \frac{\kappa \times 1000}{M}$, where M is the molarity and the factor 1000 converts cm^3 to litres.

Step 2: Substitute

$$\Lambda_m = \frac{2.50 \times 10^{-4} \times 1000}{0.0020} = \frac{0.25}{0.0020} = 125 \text{ S cm}^2 \text{ mol}^{-1}$$

Step 3: Check the options

62.5 comes from using 0.004 M, 12.5 from dropping a factor of 10, and

250 from using 0.001 M. The correct value is 125, option (A).

Final Answer:

$\Lambda_m = 125$ S cm^2 mol^{-1}, option (A).

(A) 125

Quick Tip: Use $\Lambda_m = \frac{\kappa \times 1000}{M}$ with κ in S cm^{-1}.

74. Which of the following statements is correct regarding electrolysis of molten NaCl?

(A) Pale green Cl_2 gas released at anode

(B) Molten silvery white sodium is deposited at cathode

(C) Decomposition of NaCl into Na metal and Cl_2 gas

(D) Both options A and B

Correct Answer: (D) Both options A and B

Solution:

Step 1: Write the cell reactions

Molten NaCl contains Na^+ and Cl^- ions.

Cathode: $Na^+ + e^- \rightarrow Na$ (reduction).

Anode: $2Cl^- \rightarrow Cl_2 + 2e^-$ (oxidation).

Step 2: Check the statements

(A) Pale green Cl_2 gas appears at the anode: true.

(B) Silvery white molten sodium forms at the cathode: true.

(C) The overall change is NaCl into Na and Cl_2: also true, but option

(D) combines (A) and (B), the two statements the question builds on. So (D) is the answer.

Final Answer:
Both A and B are correct, option (D).

$$\boxed{(D)}$$

Quick Tip: Chlorine forms at the anode and sodium at the cathode.

75. Which of the following statements is true about the voltaic cell?

(A) It converts chemical energy into electrical energy.

(B) It converts electrical energy into chemical energy.

(C) The anode of voltaic cells is positive.

(D) The cathode of voltaic cell is negative.

Correct Answer: (A) It converts chemical energy into electrical energy.

Solution:

Step 1: Recall the idea
A voltaic (galvanic) cell uses a spontaneous redox reaction to produce an electric current. Chemical energy turns into electrical energy.

Step 2: Check the options
(A) is true.
(B) describes an electrolytic cell, so it is false.
(C) In a voltaic cell the anode is the negative electrode (oxidation releases electrons), so it is false.

(D) The cathode is positive in a voltaic cell, so it is false.

Final Answer:
A voltaic cell converts chemical energy to electrical energy, option (A).

$$\boxed{(A)}$$

Quick Tip: A voltaic cell makes electricity from a spontaneous chemical reaction.

76. Identify the function of salt bridge.

(A) As cathode

(B) As anode

(C) To connect two half cells.

(D) To measure electrode potential.

Correct Answer: (C) To connect two half cells.

Solution:

Step 1: Recall the role
The salt bridge is a U-tube holding an inert electrolyte such as KCl in agar. It joins the two half cells and lets ions move between them.

Step 2: Why that matters
It completes the electrical circuit and keeps each half cell electrically neutral as ions are made or used. It is not an electrode, so (A) and (B) fail. It cannot measure potential, so (D) fails. Option (C) is correct.

Final Answer:

The salt bridge connects the two half cells, option (C).

$$\boxed{\text{(C)}}$$

Quick Tip: The salt bridge closes the circuit and keeps both solutions neutral.

77. A first order reaction complete 60% in 20 minutes. How long will the reaction take to complete 84%?

(A) 54 min

(B) 68 min

(C) 40 min

(D) 76 min

Correct Answer: (C) 40 min

Solution:

Step 1: Find k

For first order, $k = \frac{2.303}{t}\log\frac{a}{a-x}$. After 60% completion, $a - x = 40$ when $a = 100$.

$$k = \frac{2.303}{20}\log\frac{100}{40} = \frac{2.303 \times 0.3979}{20} = 0.0458\ \text{min}^{-1}$$

Step 2: Time for 84%

Now $a - x = 16$.

$$t = \frac{2.303}{0.0458}\log\frac{100}{16} = \frac{2.303 \times 0.7959}{0.0458} = 40 \text{ min}$$

Step 3: Check

Option (C) is 40 min. Others come from assuming linear progress.

Final Answer:

The reaction takes 40 min, option (C).

$$\boxed{\text{(C) 40 min}}$$

Quick Tip: Find k from the 60% data, then use $t = \frac{2.303}{k}\log\frac{100}{100-x}$.

78. The rate constant for a first order reaction is 60 s^{-1}. How much time will it take to reduce the concentration of the reactant to $1/20^{th}$ of its initial value?

(A) 0.0529 s

(B) 0.0852 s

(C) 0.0499 s

(D) 0.0357 s

Correct Answer: (C) 0.0499 s

Solution:

Step 1: Write the integrated law

$$t = \frac{2.303}{k}\log\frac{[A]_0}{[A]}$$

Step 2: Substitute

The concentration falls to $\frac{1}{20}$, so the ratio is 20 and $\log 20 = 1.301$.

$$t = \frac{2.303 \times 1.301}{60} = \frac{2.996}{60} = 0.0499 \text{ s}$$

Step 3: Result

Option (C), 0.0499 s.

Final Answer:

The time is 0.0499 s, option (C).

$$\boxed{\text{(C)}}$$

Quick Tip: Use $t = \frac{2.303}{k} \log \frac{[A]_0}{[A]}$ with $[A]_0/[A] = 20$.

79. Which of the following statements is NOT true about the rate constant?

(A) It is proportionality constant in rate law that relates reaction rate to reactant concentrations.

(B) It is independent of concentration of reactant.

(C) It varies with temperature.

(D) Greater the value of rate constant slower is the reaction.

Correct Answer: (D) Greater the value of rate constant slower is the reaction.

Solution:

Step 1: Recall the meaning

In the rate law $r = k[A]^n$, k is the proportionality constant. It does not depend on concentration but changes with temperature.

Step 2: Check the options

(A), (B) and (C) are true.

(D) is false: for the same concentrations a larger k gives a larger rate, so the reaction is faster.

Final Answer:

Statement (D) is not true, option (D).

$$\boxed{\text{(D)}}$$

Quick Tip: A larger rate constant means a faster reaction, not a slower one.

80. Which of the following assertions about the extent of physisorption is correct?

(A) Increases with increase in temperature

(B) Decreases with increase in surface area

(C) Decreases with increase in the strength of Van der Waals forces

(D) Decreases with increase in temperature

Correct Answer: (D) Decreases with increase in temperature

Solution:

Step 1: Recall the facts

Physisorption is caused by weak Van der Waals forces and is exothermic.

Step 2: Check the options

(A) False. By Le Chatelier, raising temperature opposes an exothermic process.

(B) False. A larger surface area increases adsorption.

(C) False. Stronger Van der Waals forces increase adsorption.

(D) True.

Final Answer:

Physisorption decreases with temperature, option (D).

$\boxed{(D)}$

Quick Tip: Physisorption is exothermic, so it falls as temperature rises.

81. Which of the following is not characteristics of interhalogen compounds?

(A) Covalent nature

(B) Volatile but not explosive

(C) Form addition products with unsaturated hydrocarbons.

(D) Behave as strong reducing agent.

Correct Answer: (D) Behave as strong reducing agent.

Solution:

Step 1: Recall the properties

Interhalogen compounds are covalent, volatile solids or liquids. They are not explosive, though they are reactive. They add across C=C bonds of unsaturated hydrocarbons.

Step 2: Find the odd one out

They are strong oxidising agents because the halogen in the positive oxidation state readily takes electrons. So (D) is not a characteristic.

Final Answer:

They are strong oxidising agents, so option (D) is not a characteristic.

$$\boxed{\text{(D)}}$$

Quick Tip: Interhalogens are oxidising agents, not reducing agents.

82. Identify the set of paramagnetic ions among the following.

(A) V^{+2}, Co^{+2}, Ti^{+4}

(B) $Ni^{+2}, Cu^{+2}, Zn^{+2}$

(C) $Ti^{+2}, Cu^{+2}, Mn^{+3}$

(D) Sc^{+3}, Ti^{+3}, V^{+3}

Correct Answer: (C) $Ti^{+2}, Cu^{+2}, Mn^{+3}$

Solution:

Step 1: Write configurations

Ti^{2+}: $3d^2$ (2 unpaired). Cu^{2+}: $3d^9$ (1 unpaired). Mn^{3+}: $3d^4$ (4 unpaired).

Step 2: Check the other sets

(A) has Ti^{4+} ($3d^0$), which is diamagnetic.

(B) has Zn^{2+} ($3d^{10}$), which is diamagnetic.

(D) has Sc^{3+} ($3d^0$), which is diamagnetic.

Only (C) has all three paramagnetic.

Final Answer:

All ions in set (C) have unpaired electrons, option (C).

$\boxed{(C)}$

Quick Tip: Paramagnetic ions have at least one unpaired d electron.

83. Why zinc does not exhibit variable oxidation states?

(A) Completely filled $4s$ subshell

(B) Completely filled $3d$ subshell

(C) Incomplete $3d$ subshell

(D) Incomplete $4s$ subshell

Correct Answer: (B) Completely filled $3d$ subshell

Solution:

Step 1: Write the configuration

Zn is $[Ar]3d^{10}4s^2$.

Step 2: Reason

The $3d$ subshell is completely filled and very stable. Zinc loses only its

two $4s$ electrons to form Zn^{2+} and does not disturb the $3d^{10}$ set. So it shows only the $+2$ state.
The $4s$ subshell is also full, but the full $3d$ shell is the reason given.
Option (B).

Final Answer:
Zinc has a completely filled $3d$ subshell, option (B).

$$\boxed{\text{(B)}}$$

Quick Tip: Zinc has a stable $3d^{10}$ configuration.

84. Identify the hybridization in central metal of $[Fe(CO)_5]$ complex.

(A) dsp^3

(B) sp^3d^2

(C) d^2sp^3

(D) sp^3

Correct Answer: (A) dsp^3

Solution:

Step 1: Configuration of Fe
Fe has $3d^6 4s^2$. CO is a strong field ligand, so the electrons rearrange to $3d^8$, with the $4s^2$ electrons pushed into $3d$.

Step 2: Hybrid orbitals
Five CO ligands need five empty orbitals: one $3d$, one $4s$ and three $4p$.
This is dsp^3 hybridisation.

The shape is trigonal bipyramidal.

Step 3: Other options

sp^3d^2 and d^2sp^3 would give six ligands. sp^3 would give four. Option (A).

Final Answer:

Fe in $Fe(CO)_5$ is dsp^3 hybridised, option (A).

$$\boxed{\text{(A) } dsp^3}$$

Quick Tip: Fe in $Fe(CO)_5$ is $3d^8 4s^0$ after pairing electrons, giving dsp^3.

85. In a complex compound, central metal ion is

(A) Arrhenius acid

(B) Arrhenius base

(C) Lewis acid

(D) Lewis base

Correct Answer: (C) Lewis acid

Solution:

Step 1: Recall the bonding

Ligands donate lone pairs to the central metal ion through coordinate bonds.

Step 2: Name the role

The species that accepts an electron pair is a Lewis acid, so the central

metal ion is a Lewis acid. The ligands are Lewis bases. Option (C).

Final Answer:
The metal ion is a Lewis acid, option (C).

$$\boxed{(C)}$$

Quick Tip: The central metal ion accepts lone pairs from ligands.

86. Which one of the following factors proceeds more rapidly, unimolecular nucleophilic substitution reaction?

(A) Aprotic solvent

(B) Stronger nucleophile

(C) Weaker nucleophile

(D) Polar protic solvent

Correct Answer: (D) Polar protic solvent

Solution:

Step 1: Recall the SN1 mechanism
SN1 is a two-step reaction. The slow step is ionisation to a carbocation, so the rate depends only on the substrate.

Step 2: Which factor helps
A polar protic solvent solvates the carbocation and the leaving group, so it speeds up the slow step. Nucleophile strength does not change the rate. Aprotic solvents favour SN2. Option (D).

Final Answer:

A polar protic solvent speeds up SN1, option (D).

$$\boxed{\text{(D)}}$$

Quick Tip: SN1 goes through a carbocation, which polar protic solvents stabilise.

87. Wurtz reaction is NOT possible with

(A) $(CH_3)_3CCl$

(B) CH_3Br

(C) C_2H_5-Cl

(D) $(C_2H_5)_2$-CH-I

Correct Answer: (A) $(CH_3)_3CCl$

Solution:

Step 1: Recall the reaction

The Wurtz reaction couples two alkyl halides with sodium in dry ether: $2RX + 2Na \rightarrow R\text{-}R + 2NaX$.

Step 2: Which halide fails

Tertiary halides such as $(CH_3)_3CCl$ mainly undergo elimination to give alkenes, because the strongly basic alkyl sodium removes a β-hydrogen.

Primary halides (B, C) work well, and the secondary halide (D) can also couple. So (A) is correct.

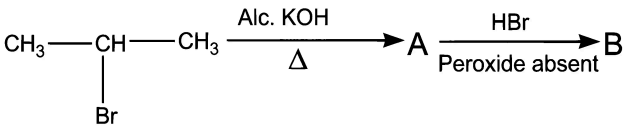
Final Answer:

Wurtz reaction fails for the tertiary halide, option (A).

$\boxed{\text{(A)}}$

Quick Tip: Tertiary halides eliminate instead of coupling.

88. The major product 'B' in the below mentioned reaction is-



(A) Bromoethane

(B) 1-Bromopropane

(C) 2-Bromopropane

(D) Carbontetrabromide

Correct Answer: (C) 2-Bromopropane

Solution:

Step 1: Understanding the Figure:

The reactant is 2-bromopropane, $CH_3 - CH(Br) - CH_3$. It is heated with alcoholic KOH to give compound A. Then A reacts with HBr in the absence of peroxide to give B.

Step 2: Step A: Elimination:

Alcoholic KOH removes HBr from an alkyl halide (dehydrohalogenation). The H comes from a carbon next to the one holding Br.

$$CH_3 - CH(Br) - CH_3 \xrightarrow{\text{Alc. KOH},\Delta} CH_3 - CH = CH_2 + KBr + H_2O$$

So A is propene.

Step 3: Step B: Addition of HBr:

Without peroxide, HBr adds by Markovnikov's rule. The hydrogen goes to the double-bond carbon that already has more hydrogens (the terminal CH_2). The bromine goes to the middle carbon, which gives the more stable secondary carbocation.

$$CH_3 - CH = CH_2 + HBr \rightarrow CH_3 - CH(Br) - CH_3$$

Step 4: Name of B:

B is $CH_3 - CH(Br) - CH_3$, which is 2-bromopropane.

Step 5: Why the other options are wrong:

1-Bromopropane would form only by anti-Markovnikov addition, which needs peroxide. Bromoethane has only two carbons and carbon tetrabromide has no chain, so neither can form from a three-carbon alkene.

Final Answer:

The major product B is 2-bromopropane, option (C).

2-Bromopropane

Quick Tip: Alcoholic KOH gives propene, and HBr without peroxide adds as per Markovnikov to give back 2-bromopropane.

89. Which from following is a suitable method for preparation of ether?

(A) Reacting sodium methoxide and tertiary butyl bromide

(B) Reacting sodium tertiarybutoxide and methyl bromide

(C) Reacting methyl alcohol and ethene

(D) Reacting methyl alcohol and methyl bromide

Correct Answer: (B) Reacting sodium tertiarybutoxide and methyl bromide

Solution:

Step 1: Recall the method

In the Williamson synthesis an alkoxide attacks an alkyl halide by an SN2 reaction. A tertiary halide would eliminate to alkene.

Step 2: Check the options

(A) Tertiary butyl bromide with methoxide gives isobutene by elimination, so it is unsuitable.

(B) Sodium tert-butoxide with methyl bromide gives methyl tert-butyl ether, which is suitable.

(C) Methyl alcohol and ethene do not form an ether this way.

(D) Methyl alcohol and methyl bromide do not react without a base.

Final Answer:

Sodium tert-butoxide and methyl bromide work, option (B).

$\boxed{(B)}$

Quick Tip: Williamson synthesis needs a primary alkyl halide and an alkoxide.

90. The molecule with a maximum boiling point is

(A) $CH_3 - CH(OH) - CH_2 - CH_2 - OH$

(B) $CH_3 - CH_2 - Br$

(C) $CH_3 - CH_2 - CH_2 - Br$

(D) $CH_3 - C(CH_3)(Cl) - OH$

Correct Answer: (A) $CH_3 - CH(OH) - CH_2 - CH_2 - OH$

Solution:

Step 1: Compare intermolecular forces

Compound (A) is butane-1,3-diol with two OH groups, so it forms strong hydrogen bonds in many directions.

(B) and (C) are alkyl bromides with only dipole and dispersion forces.

(D) has one OH group and a smaller molar mass.

Step 2: Decide

Two OH groups give the strongest hydrogen bonding, so (A) has the highest boiling point.

Final Answer:

The diol (A) has the highest boiling point.

$$\boxed{(A)}$$

Quick Tip: Hydrogen bonding raises the boiling point most, and a diol has two OH groups.

91. Which of the following reactions is an example of a disproportionation reaction?

(A) Wolf-Kishner reduction.

(B) Aldol condensation.

(C) Clemmenson reduction.

(D) Cannizzaro reaction.

Correct Answer: (D) Cannizzaro reaction.

Solution:

Step 1: Recall the definition

Disproportionation means one species is oxidised and reduced at the same time.

Step 2: Check the reactions

Wolff-Kishner and Clemmensen are reductions of carbonyl to CH_2.
Aldol condensation forms a C-C bond.
In the Cannizzaro reaction, two molecules of an aldehyde with no alpha hydrogen give an alcohol (reduction) and an acid salt (oxidation). So (D).

Final Answer:

Cannizzaro reaction is a disproportionation, option (D).

$\boxed{\text{(D)}}$

Quick Tip: In a disproportionation, the same species is both oxidised and reduced.

92. Acetaldehyde, when heated in the presence of sodium hydroxide and iodine forms

(A) Sodium formate

(B) Sodium acetate

(C) Ethanol

(D) Iodoethane

Correct Answer: (A) Sodium formate

Solution:

Step 1: Recognise the test

Compounds with a CH_3CO- group give the iodoform reaction with I_2 and NaOH.

Step 2: Write the reaction

$$CH_3CHO + 3I_2 + 4NaOH \rightarrow CHI_3 + HCOONa + 3NaI + 3H_2O$$

Step 3: Result

The salt formed is sodium formate. Option (A).

Final Answer:

The product salt is sodium formate, option (A).

$$\boxed{(A)}$$

Quick Tip: This is the iodoform reaction: CH_3CHO gives CHI_3 and sodium formate.

93. Identify the product obtained when Grignard reagent reacts with alkyl cyanide followed by acid hydrolysis.

(A) an aldehyde

(B) a ketone

(C) a primary alcohol

(D) a secondary alcohol

Correct Answer: (B) a ketone

Solution:

Step 1: Write the reaction

$$R'MgX + RC \equiv N \rightarrow RC(=NMgX)R' \xrightarrow{H_3O^+} RCOR'$$

Step 2: Result

The imine intermediate hydrolyses to a ketone. Option (B).

Final Answer:

A ketone is obtained, option (B).

$$\boxed{(B)}$$

Quick Tip: RMgX adds to the C-N triple bond, and hydrolysis gives a ketone.

94. Which of following amines exhibits carbylamine test?

(A) Tertiary amine

(B) Secondary amine

(C) Primary amine

(D) Both Secondary and Tertiary amine

Correct Answer: (C) Primary amine

Solution:

Step 1: Recall the test

In the carbylamine test, a primary amine is heated with chloroform and alcoholic KOH to give a foul-smelling isocyanide.

Step 2: Conclude

It needs two N-H bonds to lose, so secondary and tertiary amines do not respond. Option (C).

Final Answer:

Primary amines respond, option (C).

$\boxed{(C)}$

Quick Tip: Only primary amines form isocyanides with $CHCl_3$ and KOH.

95. Identify the product obtained when RCN is treated with sodium and alcohol.

(A) $RCONH_2$

(B) $RCOONH_4$

(C) RCH_2NH_2

(D) $R(CH_2)_3NH_2$

Correct Answer: (C) RCH_2NH_2

Solution:

Step 1: Recall the reaction

Sodium with alcohol is a reducing mixture (nascent hydrogen). It reduces $R\text{-}C \equiv N$ to RCH_2NH_2.

Step 2: Result

The amine has one more carbon than R. Option (C).

Final Answer:

The product is RCH_2NH_2, option (C).

$$\boxed{(C)}$$

Quick Tip: Na/C_2H_5OH reduces a nitrile to a primary amine.

96. The reaction of glucose with acetic Anhydride, confirms the

(A) straight chain structure of glucose

(B) presence of carbonyl group

(C) presence of keto group

(D) presence of five hydroxyl group

Correct Answer: (D) presence of five hydroxyl group

Solution:

Step 1: Recall the reaction

Glucose reacts with acetic anhydride to give glucose pentaacetate.

Step 2: Meaning

Five acetyl groups attach, one on each hydroxyl group, so glucose has five OH groups. Option (D).

Final Answer:

It confirms five hydroxyl groups, option (D).

(D)

Quick Tip: Five acetyl groups attach, so there are five OH groups.

97. The secondary structure of protein is determined by

(A) co-ordinate bond

(B) covalent bond

(C) ionic bond

(D) hydrogen bond

Correct Answer: (D) hydrogen bond

Solution:

Step 1: Recall the structures

The secondary structure is the alpha helix or beta pleated sheet.

Step 2: Reason

They are held in shape by hydrogen bonds between the C=O and N-H groups of the peptide backbone. Option (D).

Final Answer:

Hydrogen bonds decide the secondary structure, option (D).

$\boxed{(D)}$

Quick Tip: Alpha helix and beta sheet are held by hydrogen bonds.

98. Which of the following compounds are used to obtain terylene polymer?

(A) Ethane-1,2-diol and Benzene-1,3 dicarboxylic acid

(B) Propane-1, 2-diol and Benzene-1, 4 dicarboxylic acid

(C) Ethane-1,2-diol and Benzene-1, 4 dicarboxylic acid

(D) Ethane-1,2-diol and Benzene-1, 2 dicarboxylic acid

Correct Answer: (C) Ethane-1,2-diol and Benzene-1, 4 dicarboxylic acid

Solution:

Step 1: Recall the monomers

Terylene (Dacron, PET) is a polyester from ethane-1,2-diol and benzene-1,4-dicarboxylic acid (terephthalic acid).

Step 2: Check

Isophthalic acid (1,3) and phthalic acid (1,2) give other polymers, and

propane-1,2-diol is not used. Option (C).

Final Answer:
Ethane-1,2-diol and benzene-1,4-dicarboxylic acid, option (C).

(C)

Quick Tip: Terylene is made from ethylene glycol and terephthalic acid.

99. Which of the following polymers is also called as dacron?

(A) Teflon

(B) PVC

(C) Bakelite

(D) Terylene

Correct Answer: (D) Terylene

Solution:

Step 1: Recall the names
Terylene, Dacron and PET are the same polyester.

Step 2: Others
Teflon is PTFE, PVC is polyvinyl chloride and Bakelite is a phenol-formaldehyde resin. Option (D).

Final Answer:
Terylene is Dacron, option (D).

(D)

Quick Tip: Dacron is another name for terylene.

100. What type of drug the aspirin is?

(A) Analgesic

(B) Antimicrobial

(C) Anti-oxidant

(D) Antibiotic

Correct Answer: (A) Analgesic

Solution:

Step 1: Recall the use

Aspirin relieves pain and reduces fever and also thins blood. It belongs to the non-narcotic analgesics.

Step 2: Others

It does not kill microbes, so it is not an antimicrobial or antibiotic, and it is not an anti-oxidant. Option (A).

Final Answer:

Aspirin is an analgesic, option (A).

(A)

Quick Tip: Aspirin relieves pain.

Mathematics

Questions 101 to 150 | 2 marks each | 100 marks

101. If $A = \{1, 2, 3, 4, 5\}$, then which of the following statements is not true?

(A) $\exists x \in A$, such that $x + 4 \leq 9$

(B) $\exists x \in A$, such that $12 - x = 7$

(C) $\forall x \in A, x + 3 \geq 4$

(D) $\exists x \in A$, such that $x + 2 < 3$

Correct Answer: (D) $\exists x \in A$, such that $x + 2 < 3$

Solution:

Step 1: Understand the quantifiers

$\exists$ means "there is at least one x in A" and $\forall$ means "every x in A". We test each statement on $x = 1, 2, 3, 4, 5$.

Step 2: Check (A)

$x + 4 \leq 9$ holds for $x = 5$ (and all others). So (A) is true.

Step 3: Check (B)

$12 - x = 7$ gives $x = 5$, which is in A. So (B) is true.

Step 4: Check (C)

The smallest value of $x + 3$ is $1 + 3 = 4 \geq 4$, so it holds for all x. So (C) is true.

Step 5: Check (D)

$x + 2 < 3$ means $x < 1$. No element of A is below 1. So (D) is false, and it is the statement that is not true.

Final Answer:

Only statement (D) fails, option (D).

$$\boxed{\text{(D)}}$$

Quick Tip: Test each statement against the five elements of A.

102. If $-1+\sqrt{-3}=re^{i\theta}$, then the value of θ is

(A) $-\frac{2\pi}{3}$

(B) $\frac{\pi}{3}$

(C) $-\frac{\pi}{3}$

(D) $\frac{2\pi}{3}$

Correct Answer: (D) $\frac{2\pi}{3}$

Solution:

Step 1: Write the number

$\sqrt{-3}=i\sqrt{3}$, so the number is $z=-1+i\sqrt{3}$. Its real part is -1 and imaginary part is $\sqrt{3}$.

Step 2: Find modulus

$r=\sqrt{1+3}=2$.

Step 3: Find the argument

$\cos\theta=\frac{-1}{2}$ and $\sin\theta=\frac{\sqrt{3}}{2}$. Both conditions hold for the second quadrant angle $\theta=\frac{2\pi}{3}$.

The reference angle is $\tan^{-1}\sqrt{3}=\frac{\pi}{3}$, and in the second quadrant θ

$= \pi - \frac{\pi}{3} = \frac{2\pi}{3}$.

Step 4: Why not the others

$-\frac{2\pi}{3}$ is in the third quadrant, $\frac{\pi}{3}$ is in the first quadrant and $-\frac{\pi}{3}$ is in the fourth. None of those has a negative real part with a positive imaginary part, except $\frac{2\pi}{3}$.

Final Answer:

$\theta = \frac{2\pi}{3}$, option (D).

$$\boxed{\text{(D) } \frac{2\pi}{3}}$$

Quick Tip: Write $-1 + i\sqrt{3}$ in polar form and find its argument.

103. There are 5 boys and 3 girls seated in a row for a photograph. The number of ways in which a photograph can be taken if the girls occupy only odd places is...

(A) 5040

(B) 2880

(C) 720

(D) 40320

Correct Answer: (B) 2880

Solution:

Step 1: Count the odd places

In a row of 8 seats the odd places are 1, 3, 5, 7. So there are 4 odd places.

Step 2: Place the girls

The 3 girls choose 3 of these 4 places in order: ${}^4P_3 = 4 \times 3 \times 2 = 24$ ways.

Step 3: Place the boys

The 5 boys fill the remaining 5 seats in $5! = 120$ ways.

Step 4: Multiply

$$24 \times 120 = 2880$$

Final Answer:

The number of arrangements is 2880, option (B).

$$\boxed{\text{(B) } 2880}$$

Quick Tip: There are 4 odd positions among 8 seats. Place girls first, then boys.

104. The value of the expression $tan(x - 9\pi)$ is equal to:

(A) $-tanx$

(B) $-cotx$

(C) $tanx$

(D) $cotx$

Correct Answer: (C) $tanx$

Solution:

Step 1: Use periodicity

$\tan(\theta + n\pi) = \tan\theta$ for every integer n. The period of tangent is π.

Step 2: Apply

Here $n = -9$, so $\tan(x - 9\pi) = \tan x$. Option (C).

Option (A) and (B) would need a sign change or a swap to cotangent, which happens for shifts by $\frac{\pi}{2}$ not π.

Final Answer:

$\tan(x - 9\pi) = \tan x$, option (C).

$$\boxed{\text{(C) } \tan x}$$

Quick Tip: The tangent has period π, so subtracting 9π changes nothing.

105. The principal solutions of $\text{cosec}(\theta) = -2$ are...

(A) $\frac{\pi}{6}, \frac{5\pi}{6}$

(B) $\frac{5\pi}{6}, \frac{7\pi}{6}$

(C) $\frac{7\pi}{6}, \frac{11\pi}{6}$

(D) $\frac{\pi}{6}, \frac{11\pi}{6}$

Correct Answer: (C) $\frac{7\pi}{6}, \frac{11\pi}{6}$

Solution:

Step 1: Convert

$\text{cosec}\,\theta = -2 \Rightarrow \sin\theta = -\frac{1}{2}$.

Step 2: Find the quadrants

Sine is negative in the third and fourth quadrants. The reference angle is $\frac{\pi}{6}$.

Step 3: Principal solutions

Third quadrant: $\pi + \frac{\pi}{6} = \frac{7\pi}{6}$. Fourth quadrant: $2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$.
So the answer is $\frac{7\pi}{6}, \frac{11\pi}{6}$.

Final Answer:

The principal solutions are $\frac{7\pi}{6}, \frac{11\pi}{6}$, option (C).

$$\boxed{\text{(C)}}$$

Quick Tip: Convert to $\sin\theta = -\frac{1}{2}$ and find angles in $[0, 2\pi)$.

106. The equation of a line passing through the point of the intersection of the lines $x - y + 1 = 0$ and $2x + 3y - 8 = 0$ and having x intercept 3 is

(A) $x + y + 3 = 0$

(B) $x + y - 3 = 0$

(C) $x - y + 3 = 0$

(D) $-x + y - 2 = 0$

Correct Answer: (B) $x + y - 3 = 0$

Solution:

Step 1: Find the intersection

From $x - y + 1 = 0$, $x = y - 1$. Put into $2x + 3y - 8 = 0$: $2y - 2 + 3y = 8$, so $y = 2$ and $x = 1$. The point is $(1, 2)$.

Step 2: Use the intercept

x-intercept 3 means the line passes through $(3, 0)$.

Step 3: Equation

Slope $= \frac{0-2}{3-1} = -1$. Line: $y - 0 = -1(x - 3)$, so $x + y - 3 = 0$.

Step 4: Check

Option (B) is $x + y - 3 = 0$. Test: $(1, 2)$ gives $1 + 2 - 3 = 0$, and $(3, 0)$ gives 0. Both lie on it.

Final Answer:

The line is $x + y - 3 = 0$, option (B).

$$\boxed{\text{(B)}}$$

Quick Tip: Find the intersection point, then use the points $(1, 2)$ and $(3, 0)$.

107. If the equation $x^2 - ky^2 - 4x + 6y - 5 = 0$ represents a pair of straight lines, then their point of intersection is

(A) $(1, 2)$

(B) $(2, 3)$

(C) $(3, 4)$

(D) $(4, 5)$

Correct Answer: (B) $(2, 3)$

Solution:

Step 1: Understanding the Concept:

A general second degree equation represents a pair of lines when its determinant $\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix}$ is zero. The point of intersection satisfies $\frac{\partial S}{\partial x} = 0$ and $\frac{\partial S}{\partial y} = 0$.

Step 2: Find k:

Here $a = 1, b = -k, c = -5, h = 0, g = -2, f = 3$. The condition $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ gives

$$5k - 9 + 4k = 0 \Rightarrow k = 1$$

Step 3: Find the intersection:

$\frac{\partial S}{\partial x} = 2x - 4 = 0$ gives $x = 2$. $\frac{\partial S}{\partial y} = -2ky + 6 = 0$ gives $y = \frac{3}{k} = 3$.

Step 4: Check:

Put $k = 1$: $x^2 - y^2 - 4x + 6y - 5 = (x - 2)^2 - (y - 3)^2 = 0$. This is the pair $x - y + 1 = 0$ and $x + y - 5 = 0$, which meet at $(2, 3)$. The other options do not make both factors vanish.

Final Answer:

The lines meet at $(2, 3)$, option (B).

$$\boxed{(2, 3)}$$

Quick Tip: Find k from the pair-of-lines determinant condition, then solve the two partial derivatives equal to zero.

108. The equation of a circle which passes through the points $(2, 3)$ and $(4, 5)$ and whose center lies on the straight line $y - 4x + 3 = 0$ is

(A) $x^2 + y^2 - 4x - 10y + 25 = 0$

(B) $x^2 + y^2 - 4x - 10y - 25 = 0$

(C) $x^2 + y^2 - 4x + 10y - 25 = 0$

(D) $x^2 + y^2 + 4x - 10y + 25 = 0$

Correct Answer: (A) $x^2 + y^2 - 4x - 10y + 25 = 0$

Solution:

Step 1: Set up

Let the centre be (h, k). It lies on $y - 4x + 3 = 0$, so $k = 4h - 3$.

Step 2: Equal distances

The distance to $(2, 3)$ equals the distance to $(4, 5)$:

$$(h - 2)^2 + (k - 3)^2 = (h - 4)^2 + (k - 5)^2$$

$$-4h - 6k + 13 = -8h - 10k + 41 \Rightarrow h + k = 7$$

Step 3: Solve

Put $k = 4h - 3$: $5h - 3 = 7$, so $h = 2$ and $k = 5$. Centre $(2, 5)$.

Radius$^2 = (2 - 2)^2 + (3 - 5)^2 = 4$.

Step 4: Equation

$$(x - 2)^2 + (y - 5)^2 = 4 \Rightarrow x^2 + y^2 - 4x - 10y + 25 = 0$$

This is option (A).

Final Answer:

The circle is $x^2 + y^2 - 4x - 10y + 25 = 0$, option (A).

$$\boxed{\text{(A)}}$$

Quick Tip: The centre is equidistant from both points and lies on $y = 4x - 3$.

109. The value of $\lim_{x \to 3} \frac{[x]-3}{x-3}$, where $[\cdot]$ denotes the greatest integer function, is.....

(A) ∞

(B) 1

(C) 0

(D) does not exist

Correct Answer: (D) does not exist

Solution:

Step 1: Right-hand limit

For x slightly above 3, $[x] = 3$, so the fraction is $\frac{0}{x-3} = 0$. The limit is 0.

Step 2: Left-hand limit

For x slightly below 3, $[x] = 2$, so the fraction is $\frac{-1}{x-3}$. As $x \to 3^-$, $x - 3 \to 0^-$, and the fraction tends to $+\infty$.

Step 3: Compare

The two sided limits differ (0 and $+\infty$), so the limit does not exist. Option (D).

Final Answer:

The limit does not exist, option (D).

$$\boxed{\text{(D)}}$$

Quick Tip: Compare the left-hand and right-hand limits.

110. If the truth value of the compound statement $[(p \vee q) \wedge (q \rightarrow r) \wedge (\sim r)] \rightarrow (p \wedge q)$ is False then the truth values of $p \rightarrow q$ and $q \rightarrow p$ are respectively.....

(A) (T, T)

(B) (T, F)

(C) (F, T)

(D) (F, F)

Correct Answer: (C) (F, T)

Solution:

Step 1: Understanding the Concept:

The statement has the form $A \rightarrow B$. It is false only when A is true and B is false. Here $A = (p \vee q) \wedge (q \rightarrow r) \wedge (\sim r)$ and $B = p \wedge q$.

Step 2: Make A true:

All three parts of A must be true. From $\sim r$ true we get $r = F$. Then $q \to r$ is true with $r = F$ only when $q = F$. Then $p \vee q$ is true with $q = F$ only when $p = T$.

Step 3: Check B:
With $p = T$ and $q = F$ we get $p \wedge q = F$, as needed.

Step 4: Find the required values:
$p \to q = T \to F = F$. $q \to p = F \to T = T$. So the pair is (F, T).

Step 5: Why the other options are wrong:
Since $q = F$, the statement $q \to p$ is true, which rules out (T, F) and (F, F). Since $p = T$ and $q = F$, the statement $p \to q$ is false, which rules out (T, T).

Final Answer:
The truth values are (F, T), which is option (C).

Option C: (F, T)

Quick Tip: The whole statement is false only when the bracket is true and p and q is false; solve for r, then q, then p.

111. The statement pattern $(p \vee q) \to \sim r$ is logically equivalent to

(A) $(\sim p \vee \sim q) \vee \sim r$

(B) $(\sim p \wedge \sim q) \wedge \sim r$

(C) $(\sim p \wedge \sim q) \vee \sim r$

(D) $(\sim p \vee \sim q) \wedge \sim r$

Correct Answer: (C) $(\sim p \wedge \sim q) \vee \sim r$

Solution:

Step 1: Understanding the Question:

We need a statement pattern that is logically equivalent to $(p \vee q) \rightarrow \sim r$.

Step 2: Key Formula or Approach:

An implication can be rewritten as $a \rightarrow b \equiv \sim a \vee b$. De Morgan's law says $\sim (p \vee q) \equiv \sim p \wedge \sim q$.

Step 3: Apply the implication rule:

Take $a = p \vee q$ and $b = \sim r$:

$$(p \vee q) \rightarrow \sim r \equiv \sim (p \vee q) \vee \sim r$$

Step 4: Apply De Morgan's law:

$$\sim (p \vee q) \vee \sim r \equiv (\sim p \wedge \sim q) \vee \sim r$$

This is option (C).

Step 5: Check the other options:

Take $p = T, q = F, r = T$. The original gives $T \rightarrow F = F$, but option (A) gives $F \vee T \vee F = T$, so (A) differs. Now take $p = F, q = F, r = T$. The original gives $F \rightarrow F = T$. Option (B) gives $T \wedge T \wedge F = F$, and option (D) gives $(T) \wedge F = F$. So (B) and (D) also differ.

Final Answer:

The pattern is equivalent to $(\sim p \wedge \sim q) \vee \sim r$, option (C).

$$\boxed{(\sim p \wedge \sim q) \vee \sim r}$$

Quick Tip: Use a implies b = not a or b, then apply De Morgan's law to not (p or q).

112. In $\triangle ABC$, with usual notations, if $\cos A = \frac{1}{2}, a = 3, \angle B = \frac{\pi^c}{6}$, then the values of b and c are.....

(A) $b = \sqrt{3}, c = \sqrt{3}$

(B) $b = 2\sqrt{3}, c = \sqrt{3}$

(C) $b = \sqrt{3}, c = 2\sqrt{3}$

(D) $b = \sqrt{3}, c = \frac{1}{\sqrt{3}}$

Correct Answer: (C) $b = \sqrt{3}, c = 2\sqrt{3}$

Solution:

Step 1: Find the angles

$\cos A = \frac{1}{2} \Rightarrow A = 60°$. With $B = 30°$, $C = 180° - 90° = 90°$.

Step 2: Use the sine rule

$$\frac{a}{\sin A} = \frac{3}{\sqrt{3}/2} = 2\sqrt{3}$$

Step 3: Find b and c

$b = 2\sqrt{3} \sin 30° = \sqrt{3}$.

$c = 2\sqrt{3} \sin 90° = 2\sqrt{3}$.

This is option (C).

Final Answer:
$b = \sqrt{3}$ and $c = 2\sqrt{3}$, option (C).

$$\boxed{\text{(C)}}$$

Quick Tip: Find angle A, then C, then use the sine rule.

113. If $A = \frac{1}{2}\begin{bmatrix} -1 & -\sqrt{3} \\ \sqrt{3} & -1 \end{bmatrix}$ then $A^{-1} - A^2$ is not

(A) a null-matrix

(B) a unit matrix

(C) a diagonal matrix

(D) a scalar matrix

Correct Answer: (B) a unit matrix

Solution:

Step 1: Recognise A

$$A = \begin{bmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} \end{bmatrix} = \begin{bmatrix} \cos 120^\circ & -\sin 120^\circ \\ \sin 120^\circ & \cos 120^\circ \end{bmatrix}$$, a rotation by 120°.

Step 2: Find A^2 and A^{-1}

A^2 is a rotation by 240°. A^{-1} is a rotation by -120°, which is the same as 240°. So $A^{-1} = A^2$.

Step 3: Conclude

$A^{-1} - A^2 = O$, the null matrix. A null matrix is also diagonal and

scalar (with scalar 0), but it is not a unit matrix because its diagonal entries are not 1. So the answer is (B).

Final Answer:
The difference is the null matrix, so it is not a unit matrix, option (B).

$$\boxed{\text{(B)}}$$

Quick Tip: A is a rotation matrix by 120°.

114. If $A = \begin{bmatrix} 2i & i^3 \\ i^2 & 1 \end{bmatrix}$, then A^{-1} is equal to

(A) $\begin{bmatrix} -i & 1 \\ -i & 2 \end{bmatrix}$

(B) $\begin{bmatrix} i & 1 \\ i & -2 \end{bmatrix}$

(C) $\begin{bmatrix} -i & -1 \\ i & -2 \end{bmatrix}$

(D) $\begin{bmatrix} i & -1 \\ -i & 2 \end{bmatrix}$

Correct Answer: (A) $\begin{bmatrix} -i & 1 \\ -i & 2 \end{bmatrix}$

Solution:

Step 1: Simplify
$i^2 = -1$ and $i^3 = -i$, so $A = \begin{bmatrix} 2i & -i \\ -1 & 1 \end{bmatrix}$.

Step 2: Determinant
$\det A = 2i \cdot 1 - (-i)(-1) = 2i - i = i$.

Step 3: Adjoint

Swap the diagonal entries and change the signs of the others: $\text{adj}A = \begin{bmatrix} 1 & i \\ 1 & 2i \end{bmatrix}$.

Step 4: Inverse

Divide by i, using $\frac{1}{i} = -i$: $A^{-1} = -i\begin{bmatrix} 1 & i \\ 1 & 2i \end{bmatrix} = \begin{bmatrix} -i & 1 \\ -i & 2 \end{bmatrix}$. This is option (A).

Final Answer:

$A^{-1} = \begin{bmatrix} -i & 1 \\ -i & 2 \end{bmatrix}$, option (A).

$$\boxed{\text{(A)}}$$

Quick Tip: Simplify the powers of i first, then use $A^{-1} = \frac{1}{\det A}\text{adj}A$.

115. The minimum value of $(sin^{-1}x)^2 + (cos^{-1}x)^2$ is.....

(A) $\frac{\pi^2}{8}$

(B) $\frac{3\pi^2}{8}$

(C) $\frac{5\pi^2}{8}$

(D) $\frac{7\pi^2}{8}$

Correct Answer: (A) $\frac{\pi^2}{8}$

Solution:

Step 1: Set up

Let $t = \sin^{-1} x$. Then $\cos^{-1} x = \frac{\pi}{2} - t$.

Step 2: Form the expression

$$t^2 + \left(\frac{\pi}{2} - t\right)^2 = 2t^2 - \pi t + \frac{\pi^2}{4}$$

Step 3: Minimise

The minimum of $2t^2 - \pi t + \frac{\pi^2}{4}$ occurs at $t = \frac{\pi}{4}$, which lies in $[-\frac{\pi}{2}, \frac{\pi}{2}]$.

$$2 \cdot \frac{\pi^2}{16} - \frac{\pi^2}{4} + \frac{\pi^2}{4} = \frac{\pi^2}{8}$$

Final Answer:

The minimum value is $\frac{\pi^2}{8}$, option (A).

$$\boxed{\text{(A)}}$$

Quick Tip: Use $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ and complete the square.

116. If $tan^{-1}(1) + tan^{-1}(3) + tan^{-1}(5) + tan^{-1}(\frac{1}{4}) = \pi + tan^{-1}(\frac{\alpha}{2})$, then the value of α is...

(A) $\frac{46}{41}$

(B) $\frac{23}{41}$

(C) $\frac{42}{41}$

(D) $\frac{44}{41}$

Correct Answer: (A) $\frac{46}{41}$

Solution:

Step 1: Combine $\tan^{-1} 1 + \tan^{-1} 3$

Since $1 \times 3 > 1$, the sum is $\pi + \tan^{-1} \frac{1+3}{1-3} = \pi + \tan^{-1}(-2) = \pi - \tan^{-1} 2$.

Step 2: Combine $\tan^{-1} 5 + \tan^{-1} \frac{1}{4}$

$5 \times \frac{1}{4} = \frac{5}{4} > 1$, so the sum is $\pi + \tan^{-1} \frac{5+\frac{1}{4}}{1-\frac{5}{4}} = \pi + \tan^{-1}(-21) = \pi - \tan^{-1} 21$.

Step 3: Add them

Total $= 2\pi - (\tan^{-1} 2 + \tan^{-1} 21)$. Here $2 \times 21 > 1$, so $\tan^{-1} 2 + \tan^{-1} 21 = \pi + \tan^{-1} \frac{23}{1-42} = \pi - \tan^{-1} \frac{23}{41}$.

Step 4: Result

Total $= 2\pi - \pi + \tan^{-1} \frac{23}{41} = \pi + \tan^{-1} \frac{23}{41}$.

Compare with $\pi + \tan^{-1} \frac{\alpha}{2}$: $\frac{\alpha}{2} = \frac{23}{41}$, so $\alpha = \frac{46}{41}$.

Final Answer:

$\alpha = \frac{46}{41}$, option (A).

$$\boxed{\text{(A) } \frac{46}{41}}$$

Quick Tip: Combine the inverse tangents two at a time and watch the π corrections.

117. The domain of the function $f(x) = \sqrt{x-1} + \sqrt{3-x}$ is ...

(A) $[0, \infty)$

(B) R

(C) $[1, 3]$

(D) $[2, 2]$

Correct Answer: (C) $[1, 3]$

Solution:

Step 1: Conditions

$x - 1 \geq 0$ gives $x \geq 1$, and $3 - x \geq 0$ gives $x \leq 3$.

Step 2: Intersect

Both hold together only for $1 \leq x \leq 3$, so the domain is $[1, 3]$. Option (C).

Final Answer:

The domain is $[1, 3]$, option (C).

$$\boxed{\text{(C) } [1, 3]}$$

Quick Tip: Both square roots need non-negative inputs.

118. Let the function $f(x)$ be defined as:

$$f(x) = \begin{cases} [tan(\frac{\pi}{4} + x)]^{\frac{1}{x}}, & x \neq 0 \\ k, & x = 0 \end{cases}$$

If $f(x)$ is continuous at $x = 0$, then the value of k is...

(A) e

(B) e^2

(C) $\frac{1}{e^2}$

(D) $\frac{1}{e}$

Correct Answer: (B) e^2

Solution:

Step 1: Understanding the Question:

For f to be continuous at $x = 0$, we need $k = \lim_{x \to 0} f(x)$.

Step 2: Identify the form:

As $x \to 0$, $\tan(\pi/4 + x) \to \tan(\pi/4) = 1$ and the power $1/x \to \infty$. This is the 1^∞ form, so we use logarithms.

Step 3: Take the logarithm:

Let $L = \lim [\tan(\pi/4 + x)]^{1/x}$. Then

$$\ln L = \lim_{x \to 0} \frac{\ln \tan(\pi/4 + x)}{x}$$

This is of the form $0/0$, since $\ln \tan(\pi/4) = \ln 1 = 0$.

Step 4: Differentiate (L'Hopital's rule):

The derivative of $\ln \tan(\pi/4 + x)$ is $\frac{\sec^2(\pi/4+x)}{\tan(\pi/4+x)}$. At $x = 0$:

$$\frac{\sec^2(\pi/4)}{\tan(\pi/4)} = \frac{2}{1} = 2$$

Step 5: Finish:

So $\ln L = 2$, and $L = e^2$. Therefore $k = e^2$.

Step 6: Cross-check:

Near $x = 0$, $\tan(\pi/4 + x) \approx 1 + 2x$. Then $(1 + 2x)^{1/x} \to e^2$. The options e, $1/e$ and $1/e^2$ would need slopes 1, -1 and -2, which do not match.

Final Answer:

The value of k is e^2, option (B).

$$\boxed{k = e^2}$$

Quick Tip: Continuity needs k to equal the limit. The limit is of 1 to the power infinity form, so take the log and differentiate.

119. If $y = f\left(\frac{3+2x}{3-2x}\right)$, where $f(x) = tan(logx)$ and $\frac{dy}{dx} = \left(\frac{A}{B+Cx^2}\right) \cdot sec^2\left(log\left(\frac{3+2x}{3-2x}\right)\right)$, then the respective values of A, B and C are .....

(A) $3, 2, -4$

(B) $12, -9, 4$

(C) $6, 6, -2$

(D) $12, 9, -4$

Correct Answer: (D) $12, 9, -4$

Solution:

Step 1: Understanding the Concept:

The function is $y = \tan\left(\log \frac{3+2x}{3-2x}\right)$. We use the chain rule: $\frac{d}{dx}\tan u = \sec^2 u \cdot u'$.

Step 2: Differentiate the outer function:

$$\frac{dy}{dx} = \sec^2\left(\log\frac{3+2x}{3-2x}\right)\cdot\frac{d}{dx}\left[\log\frac{3+2x}{3-2x}\right]$$

Step 3: Simplify the inner derivative:

$\log\frac{3+2x}{3-2x} = \log(3+2x) - \log(3-2x)$. So

$$\frac{2}{3+2x} + \frac{2}{3-2x} = \frac{2(3-2x)+2(3+2x)}{9-4x^2} = \frac{12}{9-4x^2}$$

Step 4: Compare with the given form:

The form $\frac{A}{B+Cx^2}$ matches $\frac{12}{9-4x^2}$, so $A = 12, B = 9, C = -4$.

Step 5: Why the other options are wrong:

Option (B) $12, -9, 4$ gives $\frac{12}{-9+4x^2}$, which has the wrong sign. Options (A) and (C) have the wrong numerator. Only $(12, 9, -4)$ matches.

Final Answer:

The values are $A = 12, B = 9, C = -4$, option (D).

$$\boxed{D}$$

Quick Tip: Use the chain rule on tan(log u), then simplify the derivative of log((3+2x)/(3-2x)) with the log difference rule.

120. If $y = \sqrt{x^2+8x+3}$, then the value of $\frac{d^2y}{dx^2}$ at $x = 3$ is

(A) $\frac{-13}{216}$

(B) $\frac{13}{216}$

(C) $\frac{15}{216}$

(D) $\frac{-15}{216}$

Correct Answer: (A) $\frac{-13}{216}$

Solution:

Step 1: First derivative

$y^2 = x^2 + 8x + 3$, so $2yy' = 2x + 8$ and $y' = \frac{x+4}{y}$.

Step 2: Second derivative

Use the quotient rule: $y'' = \frac{y-(x+4)y'}{y^2} = \frac{y^2-(x+4)^2}{y^3}$.

Now $y^2 - (x+4)^2 = x^2 + 8x + 3 - x^2 - 8x - 16 = -13$, so $y'' = \frac{-13}{y^3}$.

Step 3: Evaluate at $x = 3$

$y^2 = 9 + 24 + 3 = 36$, so $y = 6$ and $y^3 = 216$.

$$y'' = \frac{-13}{216}$$

Final Answer:

$\frac{d^2y}{dx^2} = -\frac{13}{216}$, option (A).

$$\boxed{(A)\ -\frac{13}{216}}$$

Quick Tip: Use $y^2 = x^2 + 8x + 3$ and differentiate twice.

121. If $y = \sqrt{tanx + \sqrt{tanx + \sqrt{tanx + \cdots\cdots\cdots\infty}}}$, then $(\frac{dy}{dx})^2$ at $x = \frac{\pi}{4}$ is

(A) $\frac{5}{4}$

(B) $\frac{-5}{4}$

(C) $\frac{4}{5}$

(D) $\frac{-4}{5}$

Correct Answer: (C) $\frac{4}{5}$

Solution:

Step 1: Use the repeating structure

The expression under the first root is $\tan x + y$ itself, so $y^2 = \tan x + y$.

Step 2: Differentiate

$2yy' = \sec^2 x + y'$, so $y' = \frac{\sec^2 x}{2y-1}$.

Step 3: Find y at $x = \frac{\pi}{4}$

$\tan \frac{\pi}{4} = 1$, so $y^2 - y - 1 = 0$ and $y = \frac{1+\sqrt{5}}{2}$ (positive root).

Then $2y - 1 = \sqrt{5}$ and $\sec^2 \frac{\pi}{4} = 2$.

Step 4: Square

$y' = \frac{2}{\sqrt{5}}$, so $(y')^2 = \frac{4}{5}$.

Final Answer:

$(y')^2 = \frac{4}{5}$, option (C).

$$\boxed{(C)\ \frac{4}{5}}$$

Quick Tip: Since the root repeats, $y^2 = \tan x + y$.

122. If $y = \sqrt{x + \sqrt{x^2 + 1}}$, then the value of $\frac{dy}{dx}$ is

(A) $\frac{\sqrt{x^2+1}+x}{2y\sqrt{x^2+1}}$

(B) $\frac{\sqrt{x^2+1}+x}{y\sqrt{x^2+1}}$

(C) $\frac{\sqrt{x^2+1}-x}{2y\sqrt{x^2+1}}$

(D) $\frac{\sqrt{x^2+1}+x}{2y\sqrt{x^2-1}}$

Correct Answer: (A) $\frac{\sqrt{x^2+1}+x}{2y\sqrt{x^2+1}}$

Solution:

Step 1: Square both sides

$y^2 = x + \sqrt{x^2 + 1}$.

Step 2: Differentiate

$2yy' = 1 + \frac{x}{\sqrt{x^2+1}} = \frac{\sqrt{x^2+1}+x}{\sqrt{x^2+1}}$.

Step 3: Solve for y'

$$y' = \frac{\sqrt{x^2 + 1} + x}{2y\sqrt{x^2 + 1}}$$

This is option (A).

Final Answer:

$\frac{dy}{dx} = \frac{\sqrt{x^2+1}+x}{2y\sqrt{x^2+1}}$, option (A).

$\boxed{\text{(A)}}$

Quick Tip: Square the equation, differentiate, and keep y in the answer.

123. If $y = sin(e^{log 2x})$, then the value of $\frac{dy}{dx}$ at $x = \frac{\pi}{2}$ is

(A) 2

(B) -2

(C) 1

(D) -1

Correct Answer: (B) -2

Solution:

Step 1: Simplify

$e^{\log 2x} = 2x$, so $y = \sin 2x$.

Step 2: Differentiate

$\frac{dy}{dx} = 2\cos 2x$.

Step 3: Evaluate

At $x = \frac{\pi}{2}$: $2\cos \pi = -2$. Option (B).

Final Answer:

$\frac{dy}{dx} = -2$, option (B).

$$\boxed{\text{(B) } -2}$$

Quick Tip: Use $e^{\log a} = a$ to simplify before differentiating.

124. The difference between the local extreme values of the function $f(x) = 2x^3 - 15x^2 + 36x + 40$ is .....

(A) 4

(B) 3

(C) 2

(D) 1

Correct Answer: (D) 1

Solution:

Step 1: Critical points

$f'(x) = 6x^2 - 30x + 36 = 6(x-2)(x-3)$, so $x = 2, 3$.

Step 2: Classify

$f''(x) = 12x - 30$. At $x = 2$, $f'' = -6 < 0$ (local max). At $x = 3$, $f'' = 6 > 0$ (local min).

Step 3: Values

$f(2) = 16 - 60 + 72 + 40 = 68$ and $f(3) = 54 - 135 + 108 + 40 = 67$.

Difference $= 68 - 67 = 1$.

Final Answer:

The difference is 1, option (D).

$$\boxed{\text{(D) } 1}$$

Quick Tip: Find $f' = 0$, then compute f at both critical points.

125. A wire of length 64 m is to be bent to form a rectangle such that its area is maximum, then its area is.....

(A) 156 sq.m

(B) 324 sq.m

(C) 256 sq.m

(D) 320 sq.m

Correct Answer: (C) 256 sq.m

Solution:

Step 1: Set up

Perimeter $= 64$ m gives $l + b = 32$. Area $A = l(32 - l)$.

Step 2: Maximise

$\frac{dA}{dl} = 32 - 2l = 0$ gives $l = 16$, and $\frac{d^2A}{dl^2} = -2 < 0$, so it is a maximum.

Then $b = 16$ and $A = 16 \times 16 = 256$ sq. m.

Final Answer:

The maximum area is 256 sq. m, option (C).

$$\boxed{\text{(C) } 256}$$

Quick Tip: For a fixed perimeter the rectangle of maximum area is a square.

126. The approximate value of $\sqrt[3]{63}$ is

(A) 3.9791

(B) 3.8791

(C) 3.9581

(D) 4.0205

Correct Answer: (A) 3.9791

Solution:

Step 1: Understanding the Question:

We need an approximate value of $\sqrt[3]{63}$. The nearest perfect cube is $64 = 4^3$, so we use differentials.

Step 2: Key Formula or Approach:

For a small change Δx, $f(x+\Delta x) \approx f(x) + f'(x)\,\Delta x$. Take $f(x) = x^{1/3}$, so $f'(x) = \frac{1}{3x^{2/3}}$.

Step 3: Choose values:

Let $x = 64$ and $\Delta x = -1$. Then $f(64) = 4$ and $f'(64) = \frac{1}{3\times 16} = \frac{1}{48}$.

Step 4: Calculate:

$$\sqrt[3]{63} \approx 4 + \frac{1}{48}(-1) = 4 - 0.0208 = 3.9792$$

A slightly more careful value (adding the second-order term) is 3.97906, which rounds to 3.9791.

Step 5: Check the options:

Options (B) 3.8791 and (C) 3.9581 are too far below 4. Option (D) 4.0205 is above 4, but $\sqrt[3]{63}$ must be less than $\sqrt[3]{64} = 4$. So (A) is the closest.

Final Answer:

The approximate value is 3.9791, option (A).

$$\boxed{3.9791}$$

Quick Tip: Use differentials near 64: cube root of 64 is 4, derivative is 1/(3 x^(2/3)) = 1/48.

127. The equation of the tangent to the curve $y = 3x^3 - 3x^2 + x$ at $x = 1$ is

(A) $4x - y + 3 = 0$

(B) $4x + y - 3 = 0$

(C) $4x - y - 3 = 0$

(D) $4x + y + 3 = 0$

Correct Answer: (C) $4x - y - 3 = 0$

Solution:

Step 1: Understanding the Concept:

The tangent at a point has slope equal to $\frac{dy}{dx}$ at that point. We need the point and the slope.

Step 2: Find the point:

At $x = 1$: $y = 3 - 3 + 1 = 1$. The point is $(1, 1)$.

Step 3: Find the slope:

$\frac{dy}{dx} = 9x^2 - 6x + 1$. At $x = 1$ this is $9 - 6 + 1 = 4$.

Step 4: Write the equation:

Use the point-slope form $y - y_1 = m(x - x_1)$ with $m = 4$ and $(x_1, y_1) = (1, 1)$.

$y - 1 = 4(x - 1)$, so $y = 4x - 3$, that is $4x - y - 3 = 0$.

Step 5: Why the other options are wrong:

Option (A) $4x - y + 3 = 0$ does not pass through $(1, 1)$ because $4 - 1 + 3 = 6$. Options (B) and (D) have slope -4, so they are wrong. Only option (C) has slope 4 and passes through $(1, 1)$.

Final Answer:

The tangent is $4x - y - 3 = 0$, option (C).

$$\boxed{4x - y - 3 = 0}$$

Quick Tip: Find the point on the curve at x = 1 and the slope dy/dx there, then write y - y1 = m(x - x1).

128. $\int \frac{x+2}{x^2-7x+12} dx$ is equal to

(A) $-5log|x - 3| + 6log|x - 4| + c$

(B) $-5log|x - 3| - 6log|x - 4| + c$

(C) $5log|x - 3| - 6log|x - 4| + c$

(D) $5log|x - 3| + 6log|x - 4| + c$

Correct Answer: (A) $-5log|x - 3| + 6log|x - 4| + c$

Solution:

Step 1: Factor

$x^2 - 7x + 12 = (x - 3)(x - 4)$.

Step 2: Partial fractions

Write $\frac{x+2}{(x-3)(x-4)} = \frac{A}{x-3} + \frac{B}{x-4}$.

Put $x = 3$: $A = \frac{5}{-1} = -5$. Put $x = 4$: $B = \frac{6}{1} = 6$.

Step 3: Integrate

$$\int \left(\frac{-5}{x-3} + \frac{6}{x-4} \right) dx = -5 \log |x-3| + 6 \log |x-4| + c$$

Final Answer:

The integral is $-5 \log |x-3| + 6 \log |x-4| + c$, option (A).

$$\boxed{\text{(A)}}$$

Quick Tip: Factor the denominator as $(x-3)(x-4)$ and split into partial fractions.

129. If $\int \frac{dx}{\sqrt{2ax-x^2}} = f(g(x)) + c$, where c is constant of integration, then $f(x), g(x)$ are respectively equal to

(A) $sin^{-1}x, \frac{x-a}{a}$

(B) $sin^{-1}x, \frac{x+a}{a}$

(C) $tan^{-1}x, \frac{x-a}{a}$

(D) $tan^{-1}x, \frac{x+a}{a}$

Correct Answer: (A) $sin^{-1}x, \frac{x-a}{a}$

Solution:

Step 1: Complete the square

$2ax - x^2 = a^2 - (x-a)^2$.

Step 2: Use the standard form

$\int \frac{dx}{\sqrt{a^2-(x-a)^2}} = \sin^{-1}\frac{x-a}{a} + c$.

Step 3: Read f and g

So $f(x) = \sin^{-1} x$ and $g(x) = \frac{x-a}{a}$. Option (A).

Final Answer:

$f = \sin^{-1} x$ and $g = \frac{x-a}{a}$, option (A).

$$\boxed{\text{(A)}}$$

Quick Tip: Complete the square: $2ax - x^2 = a^2 - (x-a)^2$.

130. $\int \frac{x}{x+2} dx$ is equal to

(A) $x + 2log(x+2) + c$

(B) $x + log(x+2) + c$

(C) $x - log(x+2) + c$

(D) $x - 2log(x+2) + c$

Correct Answer: (D) $x - 2log(x+2) + c$

Solution:

Step 1: Understanding the Concept:

When the numerator has the same degree as the denominator, divide first. Here we rewrite $x = (x+2) - 2$.

Step 2: Split the fraction:

$$\frac{x}{x+2} = \frac{(x+2)-2}{x+2} = 1 - \frac{2}{x+2}$$

Step 3: Integrate:

The integral of 1 is x. The integral of $\frac{1}{x+2}$ is $\log|x+2|$, because the derivative of $x+2$ is 1. The constant 2 stays outside.

$$\int \left(1 - \frac{2}{x+2}\right) dx = x - 2\log|x+2| + c$$

Here c is the constant of integration.

Step 4: Check by differentiating:

$\frac{d}{dx}[x - 2\log(x+2)] = 1 - \frac{2}{x+2} = \frac{x}{x+2}$, which is the original integrand.

Step 5: Why the other options are wrong:

Options (A) and (B) have a plus sign, but the split gives a minus. Option (C) has the coefficient 1 instead of 2, which loses the factor 2 in the numerator.

Final Answer:

The integral is $x - 2\log(x+2) + c$, option (D).

$$\boxed{x - 2\log(x+2) + c}$$

Quick Tip: Write x as (x+2) - 2 so the fraction splits into 1 minus 2/(x+2).

131. If $[\cdot]$ denotes the greatest integer function, then $\int_1^4 x[x]dx$ is equal to

(A) 34

(B) -17

(C) -34

(D) 17

Correct Answer: (D) 17

Solution:

Step 1: Split

On $[1,2)$, $[x]=1$; on $[2,3)$, $[x]=2$; on $[3,4)$, $[x]=3$.

Step 2: Integrate each part

$$\int_1^2 x\,dx = \frac{4-1}{2} = \frac{3}{2}$$

$$\int_2^3 2x\,dx = 9-4 = 5$$

$$\int_3^4 3x\,dx = \frac{3(16-9)}{2} = \frac{21}{2}$$

Step 3: Add

$\frac{3}{2} + 5 + \frac{21}{2} = 17.$

Final Answer:

The value is 17, option (D).

$$\boxed{\text{(D) } 17}$$

Quick Tip: Split the interval at the integers where $[x]$ jumps.

132. If $\int_1^a (2x+1)dx = 5$, then the sum of the values of a is..

(A) 1

(B) -1

(C) 7

(D) -7

Correct Answer: (B) -1

Solution:

Step 1: Integrate

$\int_1^a (2x+1)dx = [x^2+x]_1^a = a^2+a-2.$

Step 2: Set equal to 5

$a^2+a-2=5$ gives $a^2+a-7=0.$

Step 3: Sum of roots

The sum of the roots is $-\frac{b}{a} = -1$. Option (B).

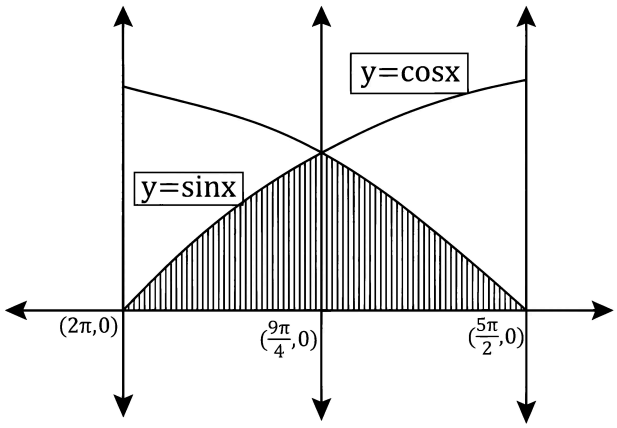
Final Answer:

The sum is -1, option (B).

(B) -1

Quick Tip: Evaluate the integral and use the sum of roots of the quadratic.

133. The area of the shaded region is ... sq.units.



(A) $2-\sqrt{2}$

(B) $2+\sqrt{2}$

(C) $\sqrt{2}$

(D) 2

Correct Answer: (A) $2-\sqrt{2}$

Solution:

Step 1: Understanding the Figure:

The graph shows $y = \sin x$ rising from the point $(2\pi, 0)$ and $y = \cos x$ falling to the point $(5\pi/2, 0)$. The two curves meet at $x = 9\pi/4$. The shaded region lies under both curves, above the X-axis.

Step 2: Key Formula or Approach:

Split the area at the intersection $x = 9\pi/4$. From 2π to $9\pi/4$ the top boundary is $\sin x$. From $9\pi/4$ to $5\pi/2$ the top boundary is $\cos x$.

$$A = \int_{2\pi}^{9\pi/4} \sin x\, dx + \int_{9\pi/4}^{5\pi/2} \cos x\, dx$$

Step 3: First part:

$$\int_{2\pi}^{9\pi/4} \sin x\, dx = [-\cos x]_{2\pi}^{9\pi/4} = -\cos\frac{9\pi}{4} + \cos 2\pi = 1 - \frac{1}{\sqrt{2}}$$

Step 4: Second part:

$$\int_{9\pi/4}^{5\pi/2} \cos x\, dx = [\sin x]_{9\pi/4}^{5\pi/2} = \sin\frac{5\pi}{2} - \sin\frac{9\pi}{4} = 1 - \frac{1}{\sqrt{2}}$$

Step 5: Add:

$$A = 2 - \frac{2}{\sqrt{2}} = 2 - \sqrt{2}$$

Step 6: Why the other options are wrong:
$2 + \sqrt{2}$ comes from adding $\frac{1}{\sqrt{2}}$ instead of subtracting it. $\sqrt{2}$ and 2 do not match either integral. The region is small, and $2 - \sqrt{2} \approx 0.586$ fits.

Final Answer:
The shaded area is $2 - \sqrt{2}$ square units, option (A).

$$\boxed{2-\sqrt{2}}$$

Quick Tip: Split the area at x = 9pi/4 and integrate sin x first, then cos x. The two parts are equal.

134. $\int_0^3 \sqrt{9-x^2}dx =$

(A) $\frac{3\pi}{4}$

(B) $\frac{9\pi}{4}$

(C) $\frac{7\pi}{4}$

(D) $\frac{5\pi}{4}$

Correct Answer: (B) $\frac{9\pi}{4}$

Solution:

Step 1: Understanding the Concept:

The expression $y = \sqrt{9-x^2}$ can be squared to give $x^2 + y^2 = 9$. This is a circle with centre at the origin and radius 3. The positive root means only the upper half, where $y \geq 0$.

Step 2: Key Formula or Approach:

A definite integral of a positive function equals the area under its curve. The area of a full circle is πr^2.

Step 3: Detailed Explanation:

The limits $x = 0$ to $x = 3$ select the right half of the upper semicircle. That is the quarter of the circle lying in the first quadrant, between the axes and the arc.

Area of the full circle: $\pi(3)^2 = 9\pi$.
Area of one quarter:

$$\frac{1}{4} \times 9\pi = \frac{9\pi}{4}$$

The other options are wrong because only $\frac{9\pi}{4}$ is exactly one quarter of 9π. Option (A), $\frac{3\pi}{4}$, is only $\frac{1}{12}$ of the circle. Option (C), $\frac{7\pi}{4}$, is $\frac{7}{36}$ of it, and option (D), $\frac{5\pi}{4}$, is $\frac{5}{36}$ of it. None of these fractions matches a quarter-circle region.

Final Answer:
The integral equals the area of a quarter circle of radius 3, which is option (B).

$$\boxed{\frac{9\pi}{4}}$$

Quick Tip: The curve is the upper half of x^2 + y^2 = 9; the limits cover a quarter of the circle.

135. The particular solution of the differential equation $xdy + 2ydx = 0$, when $x = 2$ and $y = 1$ is

(A) $x\sqrt{y} = 2$

(B) $x\sqrt{y} = -2$

(C) $\sqrt{xy} = 2$

(D) $x\sqrt{y} = 1$

Correct Answer: (A) $x\sqrt{y} = 2$

Solution:

Step 1: Separate

$x\,dy = -2y\,dx$ gives $\frac{dy}{y} = -2\frac{dx}{x}$.

Step 2: Integrate

$\log y = -2\log x + \log C$, so $yx^2 = C$.

Step 3: Use the condition

At $x = 2$, $y = 1$: $C = 4$. So $x^2y = 4$, and taking positive roots, $x\sqrt{y} = 2$. Option (A).

Final Answer:

The solution is $x\sqrt{y} = 2$, option (A).

$$\boxed{\text{(A)}}$$

Quick Tip: Separate the variables and integrate.

136. The differential equation representing the family of curves $xsinx + y^3 = 4ax$ is

(A) $\frac{dy}{dx} = \frac{y^3+x^2cosx}{3xy^2}$

(B) $\frac{dy}{dx} = \frac{y^3-x^2cosx}{3xy^2}$

(C) $\frac{dy}{dx} = \frac{y^3-x^2cosx}{3xy}$

(D) $\frac{dy}{dx} = \frac{y^3-x^2cosx}{3x^2y}$

Correct Answer: (B) $\frac{dy}{dx} = \frac{y^3-x^2cosx}{3xy^2}$

Solution:

Step 1: Isolate the constant

$\sin x + \frac{y^3}{x} = 4a.$

Step 2: Differentiate

$\cos x + \frac{3y^2y'x - y^3}{x^2} = 0.$

Step 3: Simplify

Multiply by x^2: $x^2 \cos x + 3xy^2y' - y^3 = 0.$

So $\frac{dy}{dx} = \frac{y^3 - x^2 \cos x}{3xy^2}$, option (B).

Final Answer:

$\frac{dy}{dx} = \frac{y^3 - x^2 \cos x}{3xy^2}$, option (B).

$\boxed{\text{(B)}}$

Quick Tip: Divide by x to isolate the constant $4a$, then differentiate once.

137. The order and degree of the differential equation $\sqrt{2 + (\frac{d^2y}{dx^2})^3} = (\frac{d^3y}{dx^3})^{5/2}$ respectively are

(A) 2, 3

(B) 3, 5

(C) 3, 2

(D) 5, 3

Correct Answer: (B) 3, 5

Solution:

Step 1: Clear the roots

Square both sides: $2+\left(\frac{d^2y}{dx^2}\right)^3=\left(\frac{d^3y}{dx^3}\right)^5$.

Step 2: Order

The highest derivative is $\frac{d^3y}{dx^3}$, so the order is 3.

Step 3: Degree

Its power is 5 and the equation is polynomial in derivatives, so the degree is 5. Option (B).

Final Answer:

Order 3, degree 5, option (B).

$$\boxed{\text{(B) } 3, 5}$$

Quick Tip: Remove the roots by squaring, then read the highest derivative.

138. The vector $\bar{a}+3\bar{b}$ is perpendicular to $7\bar{a}-5\bar{b}$ and the vector $\bar{a}-4\bar{b}$ is perpendicular to $7\bar{a}-2\bar{b}$. Then the angle between $\bar{a}$ and $\bar{b}$ is

(A) $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{3}$

Correct Answer: (D) $\frac{\pi}{3}$

Solution:

Step 1: Expand the first condition

$(\vec{a}+3\vec{b})\cdot(7\vec{a}-5\vec{b})=0$ gives $7|a|^2+16\,\vec{a}\cdot\vec{b}-15|b|^2=0$.

Step 2: Expand the second condition

$(\vec{a}-4\vec{b})\cdot(7\vec{a}-2\vec{b})=0$ gives $7|a|^2-30\,\vec{a}\cdot\vec{b}+8|b|^2=0$.

Step 3: Solve

Subtract: $46\,\vec{a}\cdot\vec{b}-23|b|^2=0$, so $\vec{a}\cdot\vec{b}=\frac{|b|^2}{2}$.

Substitute into the second equation: $7|a|^2-15|b|^2+8|b|^2=0$, so $|a|=|b|$.

Step 4: Angle

$\cos\theta=\frac{|b|^2/2}{|a||b|}=\frac{1}{2}$, so $\theta=\frac{\pi}{3}$.

Final Answer:

The angle is $\frac{\pi}{3}$, option (D).

$$\boxed{\text{(D) } \frac{\pi}{3}}$$

Quick Tip: Use dot product zero for each perpendicular pair.

139. If $7\hat{j}+10\hat{k}$, $-\hat{i}+6\hat{j}+6\hat{k}$ and $-4\hat{i}+9\hat{j}+6\hat{k}$ are the position vectors of the vertices A, B and C repectively of $\triangle ABC$. Then the position vector of the point where the bisector of the angle A meets side BC is

(A) $(2+3\sqrt{2})\hat{i}+(3+3\sqrt{3})\hat{j}+6\hat{k}$

(B) $(2-3\sqrt{2})\hat{i}+(3+3\sqrt{2})\hat{j}+6\hat{k}$

(C) $(2-3\sqrt{2})\hat{i}+(3-3\sqrt{2})\hat{j}+6\hat{k}$

(D) $(2-3\sqrt{2})\hat{i}+(3+3\sqrt{2})\hat{j}-6\hat{k}$

Correct Answer: (B) $(2-3\sqrt{2})\hat{i}+(3+3\sqrt{2})\hat{j}+6\hat{k}$

Solution:

Step 1: Find the side lengths

$A(0,7,10)$, $B(-1,6,6)$, $C(-4,9,6)$.

$AB=\sqrt{1+1+16}=3\sqrt{2}$ and $AC=\sqrt{16+4+16}=6$.

Step 2: Use the angle bisector theorem

$BD:DC=AB:AC=3\sqrt{2}:6=\sqrt{2}:2$.

Step 3: Section formula

$$D=\frac{2B+\sqrt{2}C}{2+\sqrt{2}}$$

Numerator: $(-2-4\sqrt{2},\ 12+9\sqrt{2},\ 12+6\sqrt{2})$. Multiply by $\frac{2-\sqrt{2}}{2}$ to rationalise:

$x=\frac{(-2-4\sqrt{2})(2-\sqrt{2})}{2}=2-3\sqrt{2}$, $y=\frac{(12+9\sqrt{2})(2-\sqrt{2})}{2}=3+3\sqrt{2}$, $z=\frac{(12+6\sqrt{2})(2-\sqrt{2})}{2}=6$.

Final Answer:

The point is $(2-3\sqrt{2})\hat{i}+(3+3\sqrt{2})\hat{j}+6\hat{k}$, option (B).

$$\boxed{\text{(B)}}$$

Quick Tip: The bisector divides BC in the ratio $AB : AC$.

140. If a, b, c are distinct non-negative numbers and the vectors $a\hat{i} + a\hat{j} + c\hat{k}, \hat{i} + \hat{k}, c\hat{i} + c\hat{j} + b\hat{k}$ lie in the same plane, then the value of c is...

(A) The arithmetic mean of a and b

(B) The geometric mean of a and b

(C) The harmonic mean of a and b

(D) 0

Correct Answer: (B) The geometric mean of a and b

Solution:

Step 1: Set up the determinant

$$\begin{vmatrix} a & a & c \\ 1 & 0 & 1 \\ c & c & b \end{vmatrix} = 0$$

Step 2: Expand

$a(0 \cdot b - c) - a(b - c) + c(c - 0) = -ac - ab + ac + c^2 = c^2 - ab.$

Step 3: Conclude

$c^2 = ab$, so $c = \sqrt{ab}$, the geometric mean of a and b. Option (B).

Final Answer:

c is the geometric mean of a and b, option (B).

(B)

Quick Tip: Three vectors are coplanar when their scalar triple product is zero.

141. Let $A(2, 3, 0)$, $B(0, 3, 2)$ and $C(4, 0, 3)$ be vertices of a triangle, then the area of the triangle is

(A) $\frac{\sqrt{171}}{2}$ sq. units

(B) $\frac{\sqrt{172}}{2}$ sq. units

(C) $\frac{\sqrt{173}}{2}$ sq. units

(D) $\frac{\sqrt{174}}{2}$ sq. units

Correct Answer: (B) $\frac{\sqrt{172}}{2}$ sq. units

Solution:

Step 1: Edge vectors

$\vec{AB} = (-2, 0, 2)$ and $\vec{AC} = (2, -3, 3)$.

Step 2: Cross product

$$\vec{AB} \times \vec{AC} = (0 \cdot 3 - 2 \cdot (-3))\hat{i} - ((-2)(3) - 2 \cdot 2)\hat{j} + ((-2)(-3) - 0)\hat{k} = 6\hat{i} + 10\hat{j} + 6\hat{k}$$

Step 3: Area

$|\ldots| = \sqrt{36 + 100 + 36} = \sqrt{172}$, so the area is $\frac{\sqrt{172}}{2}$. Option (B).

Final Answer:

The area is $\frac{\sqrt{172}}{2}$ square units, option (B).

$$\boxed{\text{(B)}}$$

Quick Tip: Area $= \frac{1}{2}|\vec{AB} \times \vec{AC}|$.

142. If D and E are the midpoints of the sides BA and BC of triangle ABC, then $\overline{AE} + \overline{DC} =$

(A) $\overline{AC}$

(B) $\frac{3}{2}\overline{BC}$

(C) $\frac{3}{2}\overline{AC}$

(D) $\frac{1}{2}\overline{AC}$

Correct Answer: (C) $\frac{3}{2}\overline{AC}$

Solution:

Step 1: Understanding the Question:

In triangle ABC, D is the midpoint of BA and E is the midpoint of BC. We need to express $\overline{AE} + \overline{DC}$ in terms of the sides.

Step 2: Key Formula or Approach:

Use position vectors $\vec{a}, \vec{b}, \vec{c}$ for A, B, C. A midpoint has the position vector half the sum of its endpoints. The vector from P to Q is $\vec{q} - \vec{p}$.

Step 3: Midpoints:

$$\vec{e} = \frac{\vec{b} + \vec{c}}{2}, \qquad \vec{d} = \frac{\vec{a} + \vec{b}}{2}$$

Step 4: Find each vector:

$$\overline{AE} = \vec{e} - \vec{a} = \frac{\vec{b} + \vec{c} - 2\vec{a}}{2}$$

$$\overline{DC} = \vec{c} - \vec{d} = \frac{2\vec{c} - \vec{a} - \vec{b}}{2}$$

Step 5: Add:

$$\overline{AE} + \overline{DC} = \frac{3\vec{c} - 3\vec{a}}{2} = \frac{3}{2}(\vec{c} - \vec{a}) = \frac{3}{2}\overline{AC}$$

The $\vec{b}$ terms cancel.

Step 6: Check the others:

Option (A) $\overline{AC}$ would need a factor 1, but we found 3/2. Option (B) is in terms of $\overline{BC}$, but no $\vec{b}$ remains in the sum. Option (D) $\frac{1}{2}\overline{AC}$ is also not the factor found.

Final Answer:

The sum equals $\frac{3}{2}\overline{AC}$, option (C).

$$\boxed{\frac{3}{2}\overline{AC}}$$

Quick Tip: Use position vectors with midpoints, then add AE and DC. The b terms cancel and 3/2 (c - a) remains.

143. If the points $(1, 1, \mu)$ and $(-3, 0, 1)$ are equidistant from the plane $\vec{r} \cdot (3\hat{i} + 4\hat{j} - 12\hat{k}) + 13 = 0$, then the value of μ are

(A) $1, \frac{-7}{3}$

(B) $1, \frac{7}{3}$

(C) $-1, \frac{7}{3}$

(D) $1, \frac{3}{7}$

Correct Answer: (B) $1, \frac{7}{3}$

Solution:

Step 1: Plane in Cartesian form

$3x + 4y - 12z + 13 = 0$ and $\sqrt{9 + 16 + 144} = 13$.

Step 2: Distance of $(-3, 0, 1)$

$\frac{|-9+0-12+13|}{13} = \frac{8}{13}$.

Step 3: Distance of $(1, 1, \mu)$

$\frac{|3+4-12\mu+13|}{13} = \frac{|20-12\mu|}{13}$.

Step 4: Equate

$|20 - 12\mu| = 8$ gives $12\mu = 12$ or $12\mu = 28$, so $\mu = 1$ or $\frac{7}{3}$. Option (B).

Final Answer:

$\mu = 1, \frac{7}{3}$, option (B).

$$\boxed{\text{(B)}}$$

Quick Tip: Set the two distances from the plane equal.

144. If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-2}{1} = \frac{y+m}{2} = \frac{z-2}{1}$ intersect each other, then the value of m is....

(A) 1

(B) 2

(C) -1

(D) 4

Correct Answer: (C) -1

Solution:

Step 1: Points and directions

Line 1 passes through $(1, -1, 1)$ with direction $(2, 3, 4)$. Line 2 passes through $(2, -m, 2)$ with direction $(1, 2, 1)$.

Join vector: $(1, 1 - m, 1)$.

Step 2: Coplanarity determinant

$$\begin{vmatrix} 1 & 1-m & 1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{vmatrix} = 0$$

Step 3: Expand

$1(3 - 8) - (1 - m)(2 - 4) + 1(4 - 3) = -5 + 2(1 - m) + 1 = -2 - 2m.$

Setting it to 0 gives $m = -1$. Option (C).

Final Answer:

$m = -1$, option (C).

$$\boxed{\text{(C) } -1}$$

Quick Tip: Lines intersect when the scalar triple product of the join vector and both directions is zero.

145. If the lines $L_1 : \frac{x-1}{-3} = \frac{y-2}{2k} = \frac{z-3}{2}, L_2 : \frac{x-1}{3k} = \frac{y-5}{1} = \frac{z-6}{-5}$ are perpendicular to each other, then the equation of a plane containing the line L_1 and parallel to the line L_2 for the value of k that satisfies this condition is ...

(A) $31x + 21y + 46z - 211 = 0$

(B) $602x - 1155y - 747z + 3949 = 0$

(C) $43x + 105y - 154z + 209 = 0$

(D) $2x - 3y + 5z - 11 = 0$

Correct Answer: (B) $602x - 1155y - 747z + 3949 = 0$

Solution:

Step 1: Find k

Direction ratios: $L_1 = (-3, 2k, 2)$, $L_2 = (3k, 1, -5)$. Perpendicular: $-9k + 2k - 10 = 0$, so $k = -\frac{10}{7}$.

Step 2: Directions

$L_1 : (-3, -\frac{20}{7}, 2) \propto (-21, -20, 14)$. $L_2 : (-\frac{30}{7}, 1, -5) \propto (-30, 7, -35)$.

Step 3: Normal

$$\vec{n} = (-21, -20, 14) \times (-30, 7, -35) = (602, -1155, -747)$$

Step 4: Plane

It passes through $(1, 2, 3)$, which lies on L_1. Constant: $602 - 2310 - 2241 = -3949$.

So $602x - 1155y - 747z + 3949 = 0$, option (B).

Final Answer:

The plane is $602x - 1155y - 747z + 3949 = 0$, option (B).

$$\boxed{\text{(B)}}$$

Quick Tip: Find k from the perpendicular condition, then take the cross product of the directions as the normal.

146. A plane meets the coordinate axes at points A, B and C, such that the centroid of a triangle ABC is $(2, -\frac{2}{3}, \frac{1}{2})$. The perpendicular distance from the origin to this plane is...

(A) $\frac{6}{\sqrt{26}}$

(B) $\frac{5}{\sqrt{26}}$

(C) $\frac{4}{\sqrt{26}}$

(D) $\frac{3}{\sqrt{26}}$

Correct Answer: (A) $\frac{6}{\sqrt{26}}$

Solution:

Step 1: Find the intercepts

If the plane meets the axes at $(a, 0, 0)$, $(0, b, 0)$, $(0, 0, c)$, the centroid is $(\frac{a}{3}, \frac{b}{3}, \frac{c}{3})$.

So $a = 6$, $b = -2$, $c = \frac{3}{2}$.

Step 2: Plane equation

$\frac{x}{6} - \frac{y}{2} + \frac{2z}{3} = 1$. Multiply by 6: $x - 3y + 4z = 6$.

Step 3: Distance

$$d = \frac{6}{\sqrt{1+9+16}} = \frac{6}{\sqrt{26}}$$

Option (A).

Final Answer:

The distance is $\frac{6}{\sqrt{26}}$, option (A).

$$\boxed{\text{(A)}}$$

Quick Tip: Centroid coordinates are one third of the intercepts.

147. The point at which the maximum value of $x + y$ subject to constraints $x + 2y \leq 70$ and $2x + y \leq 95, x \geq 0, y \geq 0$ is.

(A) $(47.5, 0)$

(B) $(0, 95)$

(C) $(0, 35)$

(D) $(40, 15)$

Correct Answer: (D) $(40, 15)$

Solution:

Step 1: Corner points

The constraints are $x + 2y \leq 70$ and $2x + y \leq 95$ with $x, y \geq 0$.

Corners: $(0, 0)$, $(47.5, 0)$, $(0, 35)$, and the intersection of $x + 2y = 70$ and $2x + y = 95$.

Step 2: Intersection

Solve: $x = 40$, $y = 15$.

Step 3: Compare values

$(0, 0)$: 0. $(47.5, 0)$: 47.5. $(0, 35)$: 35. $(40, 15)$: 55.

The maximum is 55 at $(40, 15)$. Option (D).

Final Answer:

The maximum occurs at $(40, 15)$, option (D).

$$\boxed{\text{(D) } (40, 15)}$$

Quick Tip: Evaluate $x + y$ at each corner of the feasible region.

148. Let $X \sim B(n, p)$. If $E(X) = 2$ and $\text{Var}(X) = 1$, then the probability of getting at most one success is .....

(A) 0.0625

(B) 0.2500

(C) 0.3125

(D) 0.6875

Correct Answer: (C) 0.3125

Solution:

Step 1: Find p and n

Mean $np = 2$ and variance $npq = 1$. Dividing, $q = \frac{1}{2}$, so $p = \frac{1}{2}$ and $n = 4$.

Step 2: Compute

$$P(X \leq 1) = \binom{4}{0}\left(\frac{1}{2}\right)^4 + \binom{4}{1}\left(\frac{1}{2}\right)^4 = \frac{1+4}{16} = \frac{5}{16} = 0.3125$$

Final Answer:

The probability is **0.3125**, option (C).

$$\boxed{\text{(C) } 0.3125}$$

Quick Tip: Use $np = 2$ and $npq = 1$ to find n and p.

149. If the following function is probability density function of r.v. X $f(x) = kx^2(1 - x)$, for $0 < x < 1 = 0$, otherwise, then the value of k is

(A) -12

(B) $\frac{1}{12}$

(C) $\frac{1}{6}$

(D) 12

Correct Answer: (D) 12

Solution:

Step 1: Set the total to 1

$\int_0^1 kx^2(1-x)\,dx = 1$.

Step 2: Integrate

$k\left(\frac{1}{3} - \frac{1}{4}\right) = \frac{k}{12} = 1$, so $k = 12$. Option (D).

Final Answer:

$k = 12$, option (D).

$$\boxed{\text{(D) } 12}$$

Quick Tip: A pdf must integrate to 1.

150. Two cards are drawn at random from a standard pack of 52 cards. Then the probability that the draw consists of exactly one face card and exactly one ace card is...

(A) $\frac{7}{221}$

(B) $\frac{5}{221}$

(C) $\frac{8}{221}$

(D) $\frac{9}{221}$

Correct Answer: (C) $\frac{8}{221}$

Solution:

Step 1: Count cases

There are 12 face cards (J, Q, K) and 4 aces. Face cards are taken as not including aces.

Favourable pairs: $12 \times 4 = 48$.

Step 2: Total

$\binom{52}{2} = 1326$.

Step 3: Probability

$\frac{48}{1326} = \frac{8}{221}$. Option (C).

Final Answer:

The probability is $\frac{8}{221}$, option (C).

$$\boxed{\text{(C)}}$$

Quick Tip: Count favourable pairs and divide by $\binom{52}{2}$.