MHT CET 2026 PCM Question Paper April 13 Shift 2 with Solutions

General Instructions

(i) This booklet contains 150 questions, each provided with a complete, step-by-step solution.

(ii) It comprises 150 single-correct multiple-choice questions.

(iii) Attempt each question on your own before reviewing the given solution.

Physics

Questions 1 to 50 | 1 mark each | 50 marks

1. The length of a cylinder is measured with a meter rod having least count 0.1 cm. Its diameter is measured with vernier calipers having least count 0.01 cm. Length of cylinder is 8.0 cm and radius is 4.0 cm. Find the percentage error in the calculated value of the volume.

(A) 1.25%

(B) 1.75%

(C) 1.5%

(D) 2.75%

Correct Answer: (C) 1.5%

Solution:

Step 1: Understanding the Question:

The volume of a cylinder is $V = \pi r^2 l$. The maximum fractional error adds up the fractional errors, with the power multiplying the radius

error.

Step 2: Key Formula:

$$\frac{\Delta V}{V} = 2\frac{\Delta r}{r} + \frac{\Delta l}{l}$$

Step 3: Errors in l and r:

Length: $\frac{\Delta l}{l} = \frac{0.1}{8.0} = 0.0125 = 1.25\%$.

Diameter: the least count is 0.01 cm, so $\Delta d = 0.01$. The radius is half the diameter, so $\Delta r = 0.005$ cm, and $\frac{\Delta r}{r} = \frac{0.005}{4.0} = 0.125\%$.

Step 4: Combine:

$\frac{\Delta V}{V} \times 100 = 2(0.125) + 1.25 = 0.25 + 1.25 = 1.5\%$.

Final Answer:

The percentage error in the volume is 1.5%, option (C).

$$\boxed{1.5\%}$$

Quick Tip: For $V = \pi r^2 l$, $\frac{\Delta V}{V} = 2\frac{\Delta r}{r} + \frac{\Delta l}{l}$.

2. Calculate the values of a and b if vectors $a\hat{i} + b\hat{j} = \hat{n}$ and $(\hat{i} + \hat{j})$ are perpendicular to each other

(A) $\frac{1}{2}, \frac{1}{2}$

(B) $0, 0$

(C) $\sqrt{2}, -\frac{1}{\sqrt{2}}$

(D) $(\frac{1}{\sqrt{2}}), (-\frac{1}{\sqrt{2}})$

Correct Answer: (D) $(\frac{1}{\sqrt{2}}), (-\frac{1}{\sqrt{2}})$

Solution:

Step 1: Understanding the Question:

$\hat{n} = a\hat{i} + b\hat{j}$ is a unit vector (the hat shows it) perpendicular to $\hat{i} + \hat{j}$.

Step 2: Two conditions:

Perpendicular: $(a\hat{i} + b\hat{j}) \cdot (\hat{i} + \hat{j}) = a + b = 0$, so $b = -a$.

Unit magnitude: $a^2 + b^2 = 1$, so $2a^2 = 1$ and $a = \pm\frac{1}{\sqrt{2}}$.

Step 3: Pick the option:

$a = \frac{1}{\sqrt{2}}, b = -\frac{1}{\sqrt{2}}$, which is option D.

Option A: $a + b = 1 \neq 0$. Option B: the zero vector is not a unit vector.

Option C: $a + b \neq 0$, since $\sqrt{2} - \frac{1}{\sqrt{2}} \neq 0$.

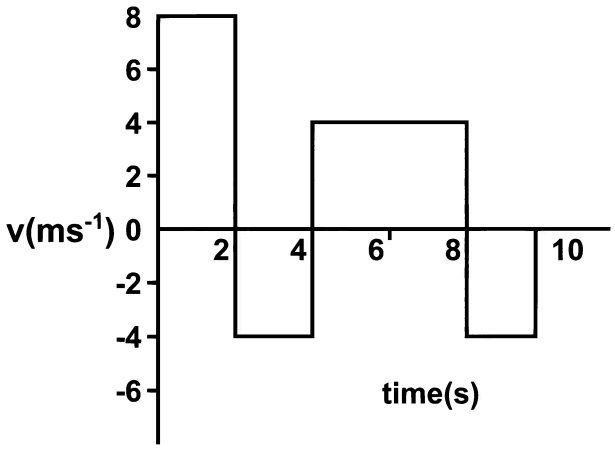
Final Answer:

The values are $a = \frac{1}{\sqrt{2}}$ and $b = -\frac{1}{\sqrt{2}}$, option (D).

$$\boxed{a = \frac{1}{\sqrt{2}}, \ b = -\frac{1}{\sqrt{2}}}$$

Quick Tip: Perpendicular means zero dot product; unit vector means unit magnitude.

3. The velocity time graph of a body moving in a straight line is shown in a figure. The ratio of displacement to distance travelled by the body in time 0 to 10 s is



(A) 1 : 1

(B) 1 : 4

(C) 1 : 2

(D) 1 : 3

Correct Answer: (D) 1 : 3

Solution:

Step 1: Read the graph

The v-t graph is made of rectangles. From 0 to 2 s, $v = +8$ m/s. From 2 to 4 s, $v = -4$ m/s. From 4 to 8 s, $v = +4$ m/s. From 8 to 10 s, $v = -4$ m/s.

Step 2: Rule

The signed area under a v-t graph gives displacement. The sum of the absolute areas gives distance travelled.

Step 3: Areas

$A_1 = 8 \times 2 = 16$ m. $A_2 = -4 \times 2 = -8$ m. $A_3 = 4 \times 4 = 16$ m. $A_4 = -4 \times 2 = -8$ m.

Step 4: Displacement and distance

Displacement $= 16 - 8 + 16 - 8 = 16$ m. Distance $= 16 + 8 + 16 + 8 = 48$ m.

Step 5: Ratio

$$\frac{\text{displacement}}{\text{distance}} = \frac{16}{48} = \frac{1}{3}$$

So the ratio is 1:3. A ratio of 1:1 would mean no reversal of direction, which is not the case, and 1:2 or 1:4 do not match the areas.

Final Answer:

The ratio is 1:3, option (D).

$$\boxed{1:3}$$

Quick Tip: Displacement is the signed area under the v-t graph and distance is the sum of absolute areas.

4. Spherical insulating ball and a spherical metallic ball of same size and mass are dropped from the same height. Choose the correct statement out of the following {Assume negligible air friction}

(A) Time taken by them to reach the earth's surface will be independent of the properties of their materials.

(B) Insulating ball will reach the earth's surface earlier than the metal ball.

(C) Both will reach the earth's surface simultaneously.

(D) Metal ball will reach the earth's surface earlier than the insulating ball.

Correct Answer: (C) Both will reach the earth's surface simultaneously.

Solution:

Step 1: Understanding the Concept:
With negligible air friction, the only force on each ball is gravity. A falling body then moves under free fall, and free fall does not depend on what the body is made of.

Step 2: Key Formula or Approach:
Newton's second law gives $ma = mg$, so the acceleration is $a = g$. The mass cancels. For a drop from rest through height h:

$$h = \frac{1}{2}gt^2 \quad \Rightarrow \quad t = \sqrt{\frac{2h}{g}}$$

Step 3: Detailed Explanation:
Both balls have the same size and the same mass, and both are dropped from the same height h with zero initial speed.
The time $t = \sqrt{2h/g}$ contains only h and g. It has no term for the material, conductivity or charge of the ball.
So the metallic ball and the insulating ball take exactly the same time

to reach the ground.
Option (B) says the insulating ball lands first and option (D) says the metal ball lands first. Both are wrong, because the equation gives one common time for the two balls.

Final Answer:
Both balls reach the ground at the same instant, so the correct option is (C).

(C) Both will reach the earth's surface simultaneously.

Quick Tip: Without air friction both balls fall with the same acceleration g, so the fall time $\sqrt{2h/g}$ is identical for both.

5. 100 balls each of mass m moving with speed v simultaneously strike a wall normally and reflected back with same speed, in time t s. The total magnitude of force exerted by the balls on the wall is

(A) $100mv/t$

(B) $200mv/t$

(C) $200mvt$

(D) $mv/100t$

Correct Answer: (B) $200mv/t$

Solution:

Step 1: Understanding the Concept:
Force equals the rate of change of momentum, $F = \frac{\Delta p}{t}$.

Step 2: Momentum change of one ball:

The ball hits normally with velocity $+v$ and comes back with $-v$. $\Delta p = mv - (-mv) = 2mv$.

Step 3: All 100 balls:

They strike together, so the total change is $100 \times 2mv = 200mv$.

$$F = \frac{200mv}{t}$$

Step 4: Other options:

$100mv/t$ would mean the balls stop, not reflect. $200mvt$ has the wrong dependence on time. $mv/100t$ is far too small.

Final Answer:

The total force on the wall is $\frac{200mv}{t}$, option (B).

$$\boxed{\frac{200mv}{t}}$$

Quick Tip: Each ball changes momentum by $2mv$; force is total change in momentum over time.

6. A solid sphere of mass 1 kg rolls without slipping on a plane surface. Its kinetic energy is 7×10^{-3} J. The speed of the center of mass of the sphere in cm s^{-1} is

(A) 1

(B) 10

(C) 100

(D) 1000

Correct Answer: (B) 10

Solution:

Step 1: Understanding the Concept:

A rolling sphere has translational and rotational kinetic energy. For rolling without slipping, $v = \omega R$, and for a solid sphere $I = \frac{2}{5}mR^2$.

Step 2: Total energy:

$$K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}\cdot\frac{2}{5}mR^2\cdot\frac{v^2}{R^2} = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$$

Step 3: Solve:

$\frac{7}{10}(1)v^2 = 7\times10^{-3}$, so $v^2 = 10^{-2}$, and $v = 0.1$ m/s $= 10$ cm/s.

Final Answer:

The speed is 10 cm/s, option (B).

$$\boxed{10\ \text{cm s}^{-1}}$$

Quick Tip: Total kinetic energy of a rolling sphere is $\frac{7}{10}mv^2$.

7. Moment of inertia of a disc of mass M and radius 'R' about any of its diameter is $MR^2/4$. The moment of inertia of this disc about an axis normal to the disc and passing through a point on its edge will be, $(x/2)MR^2$. The value of x is

(A) 1

(B) 3

(C) 5

(D) 7

Correct Answer: (B) 3

Solution:

Step 1: Understanding the Concept:

The perpendicular axis theorem gives the moment of inertia about the normal axis through the centre. The parallel axis theorem then shifts it to the edge.

Step 2: Step by step:

About a diameter: $I_d = \frac{MR^2}{4}$. By the perpendicular axis theorem, $I_z = I_d + I_d = \frac{MR^2}{2}$.

Parallel axis theorem to a point on the edge (distance R): $I = I_z + MR^2 = \frac{MR^2}{2} + MR^2 = \frac{3}{2}MR^2$.

Step 3: Find x:

Comparing with $\frac{x}{2}MR^2$: $x = 3$.

Final Answer:

The value of x is 3, option (B).

3

Quick Tip: Use the perpendicular axis theorem, then the parallel axis theorem.

8. A thin uniform rod of length 2 m cross-sectional area 'A' and density 'd' is rotated about an axis passing through the centre and perpendicular to its length with angular velocity ω. If the value of ω in terms of its rotational kinetic energy E is $(\alpha E/Ad)^{1/2}$, then the value of α is

(A) 2

(B) 3

(C) 4

(D) 5

Correct Answer: (B) 3

Solution:

Step 1: Understanding the Concept:

Rotational kinetic energy is $E = \frac{1}{2}I\omega^2$. For a thin uniform rod of mass M and length L about its centre, $I = \frac{ML^2}{12}$.

Step 2: Find the mass and I:

$L = 2$ m, so $M = \text{volume} \times \text{density} = A \cdot 2 \cdot d = 2Ad$.

$$I = \frac{2Ad \times 4}{12} = \frac{2Ad}{3}$$

Step 3: Solve for omega:

$E = \frac{1}{2} \cdot \frac{2Ad}{3}\omega^2 = \frac{Ad\,\omega^2}{3}$, so $\omega^2 = \frac{3E}{Ad}$, and $\omega = \left(\frac{3E}{Ad}\right)^{1/2}$.

Comparing with $\left(\frac{\alpha E}{Ad}\right)^{1/2}$: $\alpha = 3$.

Final Answer:

The value of α is 3, option (B).

$$\boxed{3}$$

Quick Tip: Moment of inertia of a rod about its centre is $\frac{ML^2}{12}$, with $M = AdL$.

9. If earth has a mass nine times and radius twice to the planet P. Then $\frac{v_e}{3}\sqrt{x}$ ms^{-1} will be the minimum velocity required by a rocket to pull out of gravitational force of P, where v_e is escape velocity on earth. The value of x is

(A) 2

(B) 3

(C) 18

(D) 1

Correct Answer: (A) 2

Solution:

Step 1: Understanding the Concept:

Escape speed is $v = \sqrt{\frac{2GM}{R}}$, so $v \propto \sqrt{\frac{M}{R}}$.

Step 2: Compare:

Earth has $M_e = 9M_P$ and $R_e = 2R_P$, so $M_P = \frac{M_e}{9}$ and $R_P = \frac{R_e}{2}$.

$$\frac{v_P}{v_e} = \sqrt{\frac{M_P/R_P}{M_e/R_e}} = \sqrt{\frac{1/9}{1/2}} = \sqrt{\frac{2}{9}} = \frac{\sqrt{2}}{3}$$

Step 3: Find x:

So $v_P = \frac{v_e}{3}\sqrt{2}$, and comparing with $\frac{v_e}{3}\sqrt{x}$ gives $x = 2$.

Final Answer:

The value of x is 2, option (A).

$$\boxed{2}$$

Quick Tip: $v_{esc} = \sqrt{2GM/R}$, so it scales as $\sqrt{M/R}$.

10. Determine the ratio of surface energy of 1 large drop to 1 small drop, if 1000 small drops combined to form 1 large drop.

(A) 100 : 1

(B) 10 : 1

(C) 1000 : 1

(D) 1 : 1

Correct Answer: (A) 100 : 1

Solution:

Step 1: Understanding the Concept:

Surface energy is surface tension times area, so it is proportional to r^2.

The volume stays the same when drops merge.

Step 2: Find the large drop radius:

$1000 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$, so $R^3 = 1000r^3$ and $R = 10r$.

Step 3: Ratio:

$$\frac{E_{\text{large}}}{E_{\text{small}}} = \frac{4\pi R^2 T}{4\pi r^2 T} = \frac{R^2}{r^2} = 100$$

So the ratio is $100 : 1$. Note this compares one drop with one drop. The total surface energy of the 1000 small drops is 1000 times that of one drop, which is 10 times that of the large drop, but that is a different ratio.

Final Answer:

The ratio is $100 : 1$, option (A).

$$\boxed{100 : 1}$$

Quick Tip: Volume conservation gives $R = 10r$; surface energy goes as radius squared.

11. A spherical drop of liquid splits into 1000 identical spherical drops. If u_i is the surface energy of the original drop and u_f is the total surface energy of the resulting drops, the (ignoring evaporation). $u_f/u_i = (10/x)$. Then value of x is

(A) 1

(B) 3

(C) 7

(D) 9

Correct Answer: (A) 1

Solution:

Step 1: Understanding the Concept:

Splitting conserves volume. If the original radius is R and each small drop has radius r, then $1000r^3 = R^3$, so $r = \frac{R}{10}$.

Step 2: Surface energies:

$u_i = 4\pi R^2 T$.

$u_f = 1000 \times 4\pi r^2 T = 1000 \times 4\pi \frac{R^2}{100} T = 10 \times 4\pi R^2 T$.

Step 3: Ratio:

$\frac{u_f}{u_i} = 10 = \frac{10}{x}$, so $x = 1$.

Final Answer:

The value of x is 1, option (A).

$\boxed{1}$

Quick Tip: Radius of each small drop is $R/10$; total area grows by a factor 10.

12. The surface of water in a water tank of cross-section area 750 cm^2 on the top of a house is h m above the tap level. The speed of water coming out through the tap of cross-section area 500 mm^2 is 30 cm/s. At that instant, dh/dt is $y \times 10^{-3}$ m/s. The value of y will be

(A) 2

(B) 3

(C) 4

(D) 5

Correct Answer: (A) 2

Solution:

Step 1: Understanding the Concept:

The volume leaving through the tap per second equals the volume lost from the tank per second: $A_1 \left|\frac{dh}{dt}\right| = A_2 v$.

Step 2: Convert units:

Tank area $A_1 = 750$ cm^2. Tap area $A_2 = 500$ $mm^2 = 5$ cm^2. Speed $v = 30$ cm/s.

Step 3: Calculate:

$$\left|\frac{dh}{dt}\right| = \frac{5 \times 30}{750} = 0.2 \text{ cm/s} = 2 \times 10^{-3} \text{ m/s}$$

So $y = 2$.

Final Answer:

The value of y is 2, option (A).

2

Quick Tip: Use the equation of continuity: $A_1\, dh/dt = A_2 v$.

13. What will be the ratio of rate of radiation of metal sphere at two different temperatures $T_1 = 527°C$ and $T_2 = 127°C$

(A) 4 : 1

(B) 16 : 1

(C) 8 : 1

(D) 64 : 1

Correct Answer: (B) 16 : 1

Solution:

Step 1: Key Formula:

By the Stefan-Boltzmann law, the radiated power is $P = \sigma A T^4$. For the same sphere, $P \propto T^4$.

Step 2: Convert to Kelvin:

$T_1 = 527 + 273 = 800$ K and $T_2 = 127 + 273 = 400$ K.

Step 3: Ratio:

$$\frac{P_1}{P_2} = \left(\frac{800}{400}\right)^4 = 2^4 = 16$$

The ratio is 16 : 1. A ratio of 4 : 1 would come from using T^2, and 8 : 1 from using T^3.

Final Answer:

The ratio of radiation rates is 16 : 1, option (B).

$$\boxed{16 : 1}$$

Quick Tip: Rate of radiation is proportional to T^4 (Kelvin).

14. The coefficient of thermal conductivity of a metal rod depends on its

(A) area of cross section

(B) mass

(C) material of the rod

(D) length used

Correct Answer: (C) material of the rod

Solution:

Step 1: Understanding the Concept:

The coefficient of thermal conductivity K is defined by $Q = \frac{KA\Delta T\, t}{l}$. It is the heat flow per unit area per unit temperature gradient.

Step 2: Reason:

The area A and length l already appear separately in the formula, so K does not depend on them. Mass also does not enter. K depends only on the material of the rod, for example copper has a high K and wood a low one.

Final Answer:

Thermal conductivity depends on the material of the rod, option (C).

$$\boxed{\text{Material of the rod}}$$

Quick Tip: Thermal conductivity is a property of the material only.

15. A sample of gas at temperature T is adiabatically expanded to double its volume. The work done by the gas in the process is (given, $\gamma = \frac{3}{2}$):

(A) $W = TR[\sqrt{2} - 2]$

(B) $W = \frac{T}{R}[\sqrt{2} - 2]$

(C) $W = \frac{R}{T}[2 - \sqrt{2}]$

(D) $W = RT[2 - \sqrt{2}]$

Correct Answer: (D) $W = RT[2 - \sqrt{2}]$

Solution:

Step 1: Understanding the Question:

The gas is taken as one mole, since the options contain R and T without n. In an adiabatic expansion $TV^{\gamma-1} = \text{constant}$.

Step 2: Final temperature:

$T_2 = T\left(\frac{V_1}{V_2}\right)^{\gamma-1} = T\left(\frac{1}{2}\right)^{1/2} = \frac{T}{\sqrt{2}}$.

Step 3: Work done:

$$W = \frac{R(T_1 - T_2)}{\gamma - 1} = \frac{R\left(T - \frac{T}{\sqrt{2}}\right)}{1/2} = 2RT\left(1 - \frac{1}{\sqrt{2}}\right) = RT(2 - \sqrt{2})$$

Step 4: Other options:

Options A and B have $\sqrt{2}-2$, which is negative. Work done by an expanding gas is positive, so these are wrong. Option C has R/T, which has the wrong units.

Final Answer:

The work done is $RT(2-\sqrt{2})$, option (D).

$$\boxed{RT(2-\sqrt{2})}$$

Quick Tip: Adiabatic: $TV^{\gamma-1}$ is constant and $W = \frac{nR(T_1-T_2)}{\gamma-1}$ for one mole.

16. Energy of a gas/litre is 600 joule, then what will be its pressure?

(A) 4×10^5 N/m^2

(B) 10^5 N/m^2

(C) 6×10^5 N/m^2

(D) 3×10^5 N/m^2

Correct Answer: (A) 4×10^5 N/m^2

Solution:

Step 1: Understanding the Concept:

For an ideal gas, the total translational kinetic energy is $E = \frac{3}{2}PV$. So the energy per unit volume is $\frac{E}{V} = \frac{3}{2}P$.

Step 2: Calculate:

Energy per litre: $600\,\text{J}$ in $1\,\text{L} = 10^{-3}\,\text{m}^3$, so $\frac{E}{V} = 6 \times 10^5\,\text{J/m}^3$.

$$P = \frac{2}{3} \times \frac{E}{V} = \frac{2}{3} \times 6 \times 10^5 = 4 \times 10^5\,\text{N/m}^2$$

Step 3: Other options:

6×10^5 is the energy density, not the pressure. 3×10^5 would come from using $P = \frac{E}{2V}$.

Final Answer:

The pressure is $4 \times 10^5\,\text{N/m}^2$, option (A).

$$\boxed{4 \times 10^5\,\text{N/m}^2}$$

Quick Tip: Translational kinetic energy per unit volume equals $\frac{3}{2}P$.

17. According to law of equipartition of energy the molar specific heat of a non rigid diatomic gas at constant volume is

(A) $(9/2)R$

(B) $(5/2)R$

(C) $(3/2)R$

(D) $(7/2)R$

Correct Answer: (D) $(7/2)R$

Solution:

Step 1: Understanding the Concept:

By the law of equipartition, each quadratic energy term contributes $\frac{1}{2}RT$ per mole.

Step 2: Count the terms:

A non-rigid diatomic gas has 3 translational and 2 rotational degrees of freedom. It also vibrates, which adds 2 more terms (kinetic and potential), so the energy has $3 + 2 + 2 = 7$ terms.

$$U = \frac{7}{2}RT, \quad C_V = \frac{dU}{dT} = \frac{7}{2}R$$

Step 3: Other options:

$\frac{5}{2}R$ is for a rigid diatomic molecule. $\frac{3}{2}R$ is for a monatomic gas.

Final Answer:

$C_V = \frac{7}{2}R$, option (D).

$$\boxed{\frac{7}{2}R}$$

Quick Tip: A non-rigid diatomic molecule has 7 degrees of freedom, so $C_V = \frac{7}{2}R$.

18. Let γ_1 be the ratio of molar specific heat at constant pressure and molar specific heat at constant volume of a mono-atomic gas and γ_2 be the similar ratio of diatomic gas. Considering the diatomic gas molecule as a rigid rotator, the ratio, γ_1/γ_2 is

(A) 27/35

(B) 35/27

(C) 25/21

(D) 21/25

Correct Answer: (C) 25/21

Solution:

Step 1: Key Formula:

$\gamma = \frac{C_p}{C_v} = 1 + \frac{2}{f}$, where f is the number of degrees of freedom.

Step 2: Values:

Monatomic gas: $f = 3$, so $\gamma_1 = \frac{5}{3}$.

Rigid diatomic gas: $f = 5$, so $\gamma_2 = 1 + \frac{2}{5} = \frac{7}{5}$.

Step 3: Ratio:

$$\frac{\gamma_1}{\gamma_2} = \frac{5/3}{7/5} = \frac{25}{21}$$

Final Answer:

The ratio is $\frac{25}{21}$, option (C).

$$\boxed{\frac{25}{21}}$$

Quick Tip: $\gamma = 1 + \frac{2}{f}$; monatomic $f = 3$, rigid diatomic $f = 5$.

19. A block is fastened to a horizontal spring. The block is pulled to a distance $x = 10$ cm from its equilibrium position (at $x = 0$) on a frictionless surface from rest. The kinetic energy of the block at $x = 5$ cm is 0.25 J. The spring constant of the spring is nearly (in Nm^{-1})

(A) 63

(B) 65

(C) 67

(D) 50

Correct Answer: (C) 67

Solution:

Step 1: Understanding the Concept:

The block is released from rest at $x = 10$ cm, so the amplitude is $A = 10$ cm. At displacement x, the total energy is $\frac{1}{2}kA^2$, and the potential energy is $\frac{1}{2}kx^2$. So the kinetic energy is $\frac{1}{2}k(A^2 - x^2)$.

Step 2: Calculate:

$A = 0.10$ m and $x = 0.05$ m. $A^2 - x^2 = 0.01 - 0.0025 = 0.0075$ m^2.

$$0.25 = \frac{1}{2}k(0.0075) \Rightarrow k = \frac{0.5}{0.0075} = 66.7 \text{ N/m} \approx 67 \text{ N/m}$$

Final Answer:

The spring constant is about 67 N/m, option (C).

$$\boxed{67 \text{ N/m}}$$

Quick Tip: In SHM, $KE = \frac{1}{2}k(A^2 - x^2)$.

20. The amplitude of a particle executing SHM is 3 cm. The displacement at which its kinetic energy will be 25% more than the potential energy is (in cm)

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (B) 2

Solution:

Step 1: Understanding the Concept:

In SHM with amplitude A, at displacement x: $PE = \frac{1}{2}kx^2$ and $KE = \frac{1}{2}k(A^2 - x^2)$.

Step 2: Set up:

KE is 25% more than PE, so $KE = 1.25\,PE = \frac{5}{4}PE$.

$$A^2 - x^2 = \frac{5}{4}x^2 \Rightarrow A^2 = \frac{9}{4}x^2$$

Step 3: Solve:

$x^2 = \frac{4}{9}A^2$, so $x = \frac{2}{3}A = \frac{2}{3} \times 3 = 2$ cm.

Check: PE $\propto 4$, KE $\propto 9 - 4 = 5$, and $5 = 1.25 \times 4$.

Final Answer:

The displacement is 2 cm, option (B).

2 cm

Quick Tip: Use $KE = \frac{1}{2}k(A^2 - x^2)$ and $PE = \frac{1}{2}kx^2$, with $KE = 1.25\,PE$.

21. The velocity of a particle executing SHM varies with displacement (x) as $4v^2 = 50 - x^2$.
The time period of oscillations is $x/7$ s.
The value of x is (Take $\pi = 22/7$)

(A) 82

(B) 84

(C) 88

(D) 90

Correct Answer: (C) 88

Solution:

Step 1: Understanding the Concept:
In SHM, $v^2 = \omega^2(A^2 - x^2)$.

Step 2: Compare:
The given relation is $v^2 = \frac{1}{4}(50 - x^2)$. So $\omega^2 = \frac{1}{4}$, giving $\omega = \frac{1}{2}$ rad/s, and $A^2 = 50$.

Step 3: Time period:

$$T = \frac{2\pi}{\omega} = 4\pi = 4 \times \frac{22}{7} = \frac{88}{7}\ \text{s}$$

The problem says $T = \frac{x}{7}$, so $x = 88$.

Final Answer:
The value of x is 88, option (C).

$$\boxed{88}$$

Quick Tip: Compare with $v^2 = \omega^2(A^2 - x^2)$ to find ω.

22. Two waves are represented by the equations, $y_1 = asin(\omega t + kx + 0.57)$ m and $y_2 = acos(\omega t + kx)$ m, where x is in metre and t is in second. What is the phase difference between them? ($\pi = 3.14$)

(A) 0.57 radian

(B) 1.57 radian

(C) 1.25 radian

(D) 1.0 radian

Correct Answer: (D) 1.0 radian

Solution:

Step 1: Understanding the Concept:
The phase difference is found when both waves are written in the same form, such as $\sin(\omega t + kx + \phi)$.

Step 2: Convert:
$y_2 = a\cos(\omega t + kx) = a\sin\left(\omega t + kx + \frac{\pi}{2}\right)$.
The phase of y_1 is 0.57, and the phase of y_2 is $\frac{\pi}{2} = 1.57$ (with π

$= 3.14$).

Step 3: Difference:

$$\Delta\phi = 1.57 - 0.57 = 1.0 \text{ rad}$$

Option A forgets the shift between sine and cosine. Option B uses $\frac{\pi}{2}$ alone.

Final Answer:
The phase difference is 1.0 radian, option (D).

$$\boxed{1.0 \text{ rad}}$$

Quick Tip: Convert the cosine to a sine with an extra $\pi/2$ phase.

23. Sound waves travel at 350 m/s through a warm air and at 3500 m/s through brass. Which of the following is correct about the wavelength of a 700 Hz accoustic wave as it enters brass from warm air?

(A) It increase by a factor 20

(B) It decrease by a factor 10

(C) It increase by a factor 10

(D) It decrease by a factor 20

Correct Answer: (C) It increase by a factor 10

Solution:

Step 1: Understanding the Concept:

When a wave goes from one medium to another, its frequency stays the same. The speed and the wavelength change, with $\lambda = \frac{v}{f}$.

Step 2: Calculate:

In air: $\lambda_1 = \frac{350}{700} = 0.5$ m. In brass: $\lambda_2 = \frac{3500}{700} = 5$ m.

$\frac{\lambda_2}{\lambda_1} = 10$, so the wavelength increases by a factor of 10.

Final Answer:

The wavelength increases by a factor of 10, option (C).

Increases by a factor of 10

Quick Tip: Frequency stays fixed while the speed changes, so $\lambda = v/f$ scales with speed.

24. The distance between two consecutive points with phase difference of 60° in a wave of frequency n is x meter. The velocity with which wave is traveling is

(A) nx

(B) $2nx$

(C) $3nx$

(D) $6nx$

Correct Answer: (D) $6nx$

Solution:

Step 1: Understanding the Concept:

A path difference of one wavelength λ corresponds to a phase difference of $360°$.

Step 2: Calculate:

A phase difference of $60°$ means a distance of $\frac{60}{360}\lambda = \frac{\lambda}{6}$. So $x = \frac{\lambda}{6}$ and $\lambda = 6x$.

$$v = n\lambda = 6nx$$

Final Answer:

The speed of the wave is $6nx$, option (D).

$$\boxed{6nx}$$

Quick Tip: A phase difference of $60°$ corresponds to a path difference of $\lambda/6$.

25. A traveling wave is described by the equation $y(x, t) = [0.05sin(8x - 4t)]$ m

The velocity of the wave is [All the quantities are in SI unit]

(A) $4\ ms^{-1}$

(B) $2\ ms^{-1}$

(C) $0.5\ ms^{-1}$

(D) $8\ ms^{-1}$

Correct Answer: (C) $0.5\ ms^{-1}$

Solution:

Step 1: Understanding the Concept:

A travelling wave is $y = A\sin(kx - \omega t)$, and its speed is $v = \frac{\omega}{k}$.

Step 2: Compare:

Here $k = 8$ rad/m and $\omega = 4$ rad/s.

$$v = \frac{4}{8} = 0.5 \text{ m/s}$$

Step 3: Other options:

4 m/s is just ω, 8 m/s is just k, and 2 m/s would come from k/ω inverted incorrectly.

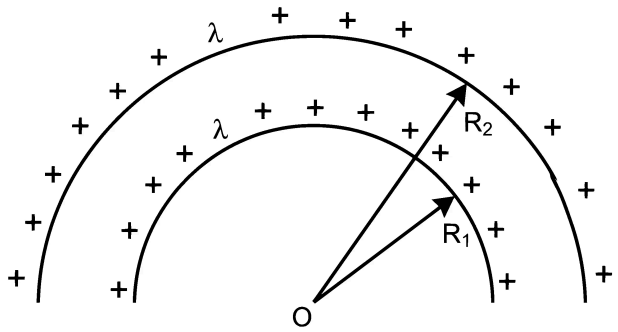
Final Answer:

The wave speed is 0.5 m/s, option (C).

$$\boxed{0.5 \text{ m/s}}$$

Quick Tip: Compare with $\sin(kx - \omega t)$; speed is ω/k.

26. The electric potential at the center of two concentric half rings of radii R_1 and R_2, having same linear charge density λ is (ε_0 = permittivity of free space)



(A) $2\lambda/\varepsilon_0$

(B) $\lambda/2\varepsilon_0$

(C) $\lambda/4\varepsilon_0$

(D) λ/ε_0

Correct Answer: (B) $\lambda/2\varepsilon_0$

Solution:

Step 1: Understand the figure

Two concentric half rings with centre O have radii R_1 and R_2. Both carry the same positive linear charge density λ.

Step 2: Potential of a half ring at its centre

Every bit of charge on a half ring is at the same distance R from the centre. So $V = \frac{1}{4\pi\epsilon_0}\frac{Q}{R}$. The length is πR, so $Q = \lambda\pi R$.

Step 3: Calculate

$$V = \frac{1}{4\pi\epsilon_0}\frac{\lambda\pi R}{R} = \frac{\lambda}{4\epsilon_0}$$

The radius cancels, so each half ring gives the same value.

Step 4: Add the two

Potential is a scalar, so we add directly:

$$V_{net} = \frac{\lambda}{4\epsilon_0} + \frac{\lambda}{4\epsilon_0} = \frac{\lambda}{2\epsilon_0}$$

Step 5: Other options

$\frac{2\lambda}{\epsilon_0}$ and $\frac{\lambda}{\epsilon_0}$ are too large, and $\frac{\lambda}{4\epsilon_0}$ counts only one ring.

Final Answer:

Net potential is $\frac{\lambda}{2\epsilon_0}$, option (B).

$$\boxed{\frac{\lambda}{2\epsilon_0}}$$

Quick Tip: Each half ring is at the same distance from the centre, so its potential is kQ/R with Q = lambda times pi R. Add the two.

27. If two charges q_1 and q_2 are separated with distance 'd' and placed in a medium of dielectric constant K. What will be the equivalent distance between charges in air for the same electrostatic force?

(A) $d[k]^{1/2}$

(B) $k[d]^{1/2}$

(C) $1.5d[k]^{1/2}$

(D) $2d[k]^{1/2}$

Correct Answer: (A) $d[k]^{1/2}$

Solution:

Step 1: Understanding the Concept:

A dielectric of constant K reduces the force between two charges by a factor of K.

Step 2: Equate:

In the medium: $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{Kd^2}$. In air at distance d': $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{d'^2}$.
Equal forces mean $d'^2 = Kd^2$, so $d' = d\sqrt{K} = d[K]^{1/2}$.

Final Answer:

The equivalent distance is $d\sqrt{K}$, option (A).

$$\boxed{d\,k^{1/2}}$$

Quick Tip: Force in a medium is $F = \frac{q_1q_2}{4\pi\varepsilon_0 Kd^2}$.

28. Two isolated metallic spheres of radii R and $2R$ are charged such that both have the same charge density σ. The spheres are then connected by a thin conducting wire. If the new charge density of the larger sphere is σ_1. The ratio σ_1 to σ is

(A) 9 : 4

(B) 4 : 3

(C) 5 : 3

(D) 5 : 6

Correct Answer: (D) 5 : 6

Solution:

Step 1: Understanding the Question:

The spheres have radii R and $2R$ and the same surface charge density σ. Charges: $q_1 = 4\pi R^2\sigma$ and $q_2 = 4\pi(2R)^2\sigma = 16\pi R^2\sigma$. Total charge $= 20\pi R^2\sigma$.

Step 2: After connection:

Connected spheres reach the same potential, $\frac{kq_1'}{R} = \frac{kq_2'}{2R}$, so $q_2' = 2q_1'$. Total is conserved: $3q_1' = 20\pi R^2\sigma$, so $q_2' = \frac{2}{3} \times 20\pi R^2\sigma = \frac{40\pi R^2\sigma}{3}$.

Step 3: New density:

$$\sigma_1 = \frac{q_2'}{4\pi(2R)^2} = \frac{40\pi R^2\sigma/3}{16\pi R^2} = \frac{5\sigma}{6}$$

So $\sigma_1 : \sigma = 5 : 6$.

Final Answer:

The ratio $\sigma_1 : \sigma$ is 5 : 6, option (D).

$$\boxed{5 : 6}$$

Quick Tip: After connection, potentials are equal, so charges share in proportion to radii.

29. A parallel plate capacitor with air between the plate has a capacitance of 15 pF. The separation between the plates becomes twice and the space between them is filled with a medium of dielectric constant 3.5. Then the capacitance becomes $x/4$ pF.
The value of x is

(A) 105

(B) 109

(C) 111

(D) 115

Correct Answer: (A) 105

Solution:

Step 1: Understanding the Concept:
The capacitance of a parallel plate capacitor is $C = \frac{K\varepsilon_0 A}{d}$. Initially (air, $K = 1$): $C_0 = \frac{\varepsilon_0 A}{d} = 15$ pF.

Step 2: New capacitance:
New separation is $2d$ and $K = 3.5$.

$$C = \frac{3.5\,\varepsilon_0 A}{2d} = \frac{3.5}{2} C_0 = 1.75 \times 15 = 26.25 \text{ pF}$$

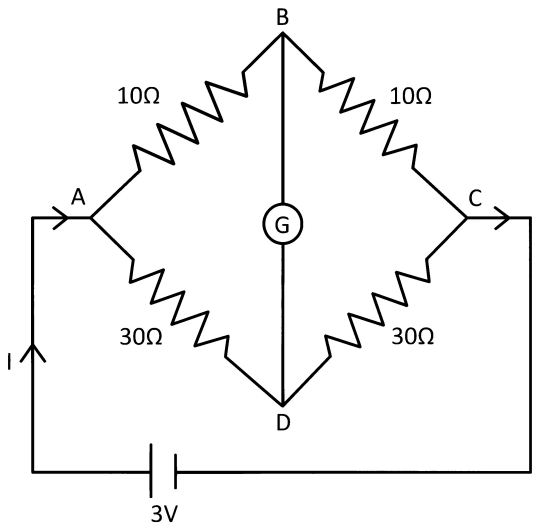
Step 3: Find x:
$26.25 = \frac{x}{4}$, so $x = 105$.

Final Answer:
The value of x is 105, option (A).

105

Quick Tip: $C = \frac{K\varepsilon_0 A}{d}$, so doubling d halves it and K multiplies it by 3.5.

30. Resistances are joint as shown in the figure. In the balanced condition, the current I drawn from the battery is



(A) 0.1 A

(B) 0.15 A

(C) 0.2 A

(D) 0.25 A

Correct Answer: (C) 0.2 A

Solution:

Step 1: Understand the figure

A Wheatstone bridge ABCD with a galvanometer between B and D. Arms: AB = 10 Ω, BC = 10 Ω, AD = 30 Ω, DC = 30 Ω. The battery is 3 V.

Step 2: Check the balance

$\frac{R_{AB}}{R_{BC}} = \frac{10}{10} = 1$ and $\frac{R_{AD}}{R_{DC}} = \frac{30}{30} = 1$. The ratios are equal, so the bridge is balanced and no current flows in the galvanometer branch BD.

Step 3: Equivalent resistance

Upper path ABC: $10 + 10 = 20\ \Omega$. Lower path ADC: $30 + 30 = 60\ \Omega$. They are in parallel:

$$\frac{1}{R_{eq}} = \frac{1}{20} + \frac{1}{60} = \frac{4}{60} \Rightarrow R_{eq} = 15\ \Omega$$

Step 4: Current

$$I = \frac{V}{R_{eq}} = \frac{3}{15} = 0.2\ \text{A}$$

Step 5: Other options

0.1 A would need 30 ohm, 0.15 A would need 20 ohm (only the upper branch, 3/20), and 0.25 A would need 12 ohm. None matches 15 ohm.

Final Answer:

The battery current is 0.2 A, option (C).

$$\boxed{0.2\ \text{A}}$$

Quick Tip: The bridge is balanced, so ignore the middle arm. Combine 20 ohm and 60 ohm in parallel and use Ohm's law.

31. In an experiment to find emf of a cell using potentiometer, the length of null point for a cell of emf 1.5 V is found to be 60 cm. If this cell is replaced by another cell of emf E, the length of null point increases by 40 cm. The value of E is $x/10$ V.
The value of x is

(A) 20

(B) 25

(C) 28

(D) 30

Correct Answer: (B) 25

Solution:

Step 1: Understanding the Concept:
In a potentiometer, the emf is proportional to the balancing length, since the wire has a uniform potential gradient.

Step 2: New length:
First null point: 60 cm for 1.5 V. The null point increases by 40 cm, so the second length is 100 cm.

Step 3: Calculate:

$$\frac{E}{1.5} = \frac{100}{60} \Rightarrow E = 2.5 \text{ V}$$

The problem writes $E = \frac{x}{10}$ V, so $x = 25$.

Final Answer:
The value of x is 25, option (B).

25

Quick Tip: Balancing length is proportional to emf: $E_1/l_1 = E_2/l_2$.

32. A charged particle q is accelerated by a potential difference of V enters a region of uniform magnetic field of induction B at right angles to the direction of the field. The charged particle completes semi circle of the radius r inside the magnetic field. The mass of the charged particle is (all quantities are in SI units)

(A) $q^2r^2B^2/2V$

(B) $qr^2B^2/2V$

(C) qBr/V

(D) $q^2r^2B/2V$

Correct Answer: (B) $qr^2B^2/2V$

Solution:

Step 1: Understanding the Concept:

The particle gains kinetic energy from the potential difference: $qV = \frac{1}{2}mv^2$. In the magnetic field it moves in a circle, with $\frac{mv^2}{r} = qvB$.

Step 2: Eliminate v:

From the circular motion, $v = \frac{qBr}{m}$.

Substitute: $qV = \frac{1}{2}m \cdot \frac{q^2B^2r^2}{m^2} = \frac{q^2B^2r^2}{2m}$.

Step 3: Solve for m:

$$m = \frac{q^2B^2r^2}{2qV} = \frac{qB^2r^2}{2V}$$

The other options have wrong powers of q or B. For example, option A has q^2, and option D has B to the first power, which would not fit the dimension of mass.

Final Answer:
The mass is $\frac{qr^2B^2}{2V}$, option (B).

$$\boxed{\frac{qr^2B^2}{2V}}$$

Quick Tip: Combine $qV = \frac{1}{2}mv^2$ with $r = \frac{mv}{qB}$ and eliminate v.

33. Two long straight wires are arranged parallel to each other and are kept 20 cm apart in vacuum. They carry currents of 2 A and 4 A respectively in the same direction. What will be the magnetic force on a length of 10 cm of either wire?
($\mu_0 = 4\pi \times 10^{-7}$ SI units)

(A) 10^{-7} N

(B) 2×10^{-7} N

(C) 4×10^{-7} N

(D) 8×10^{-7} N

Correct Answer: (D) 8×10^{-7} N

Solution:

Step 1: Key Formula:

Force between two long parallel wires: $F = \frac{\mu_0}{2\pi}\frac{I_1 I_2}{d} l$. It is attractive for currents in the same direction.

Step 2: Calculate:

$\frac{\mu_0}{2\pi} = 2 \times 10^{-7}$ SI units. $I_1 = 2$ A, $I_2 = 4$ A, $d = 0.2$ m, $l = 0.1$ m.

$$F = 2 \times 10^{-7} \times \frac{2 \times 4}{0.2} \times 0.1 = 2 \times 10^{-7} \times 4 = 8 \times 10^{-7} \text{ N}$$

Final Answer:

The force is 8×10^{-7} N, option (D).

$$\boxed{8 \times 10^{-7} \text{ N}}$$

Quick Tip: Force on length l is $F = \frac{\mu_0 I_1 I_2 l}{2\pi d}$.

34. A rod with a circular cross-section area 2 cm^2 and length 40 cm is wound uniformly with 400 turns of an insulated wire. If a current 0.4 A flows in the wire winding, the total magnetic flux produced inside the winding is $4\pi \times 10^{-6}$ Wb. The relative permeability of the rod is (Given permeability of vacuum $\mu_0 = 4\pi \times 10^{-7}$ N m A^{-2})

(A) 12.5

(B) 32/5

(C) 125

(D) 5/16

Correct Answer: (C) 125

Solution:

Step 1: Understanding the Concept:

Inside a solenoid with a core, $B = \mu_0 \mu_r n I$, where n is turns per metre. Flux $\Phi = BA$.

Step 2: Find B and n:

$A = 2\,\text{cm}^2 = 2 \times 10^{-4}\,\text{m}^2$. $B = \frac{\Phi}{A} = \frac{4\pi \times 10^{-6}}{2 \times 10^{-4}} = 2\pi \times 10^{-2}\,\text{T}$.

$n = \frac{400}{0.4} = 1000\,\text{turns/m}$.

Step 3: Solve:

$\mu_0 n I = 4\pi \times 10^{-7} \times 1000 \times 0.4 = 1.6\pi \times 10^{-4}\,\text{T}$.

$$\mu_r = \frac{2\pi \times 10^{-2}}{1.6\pi \times 10^{-4}} = 125$$

Final Answer:

The relative permeability of the rod is 125, option (C).

$$\boxed{125}$$

Quick Tip: $B = \mu_0 \mu_r n I$ and $B = \Phi/A$.

35. The magnetic moments associated with two closely wound circular coils a and b of radius r and $2r$ respectively are equal if (where N_a, I_a and N_b, I_b are number of turns and current in A and B respectively)

(A) $2N_aI_a = N_bI_b$

(B) $N_a = 2N_b$

(C) $N_aI_a = 4N_bI_b$

(D) $4N_aI_a = N_bI_b$

Correct Answer: (C) $N_aI_a = 4N_bI_b$

Solution:

Step 1: Key Formula:

The magnetic moment of a coil of N turns carrying current I with area A is $m = NIA$.

Step 2: Apply:

Coil a: $m_a = N_aI_a\pi r^2$. Coil b: $m_b = N_bI_b\pi(2r)^2 = 4N_bI_b\pi r^2$.

Equal moments: $N_aI_a = 4N_bI_b$.

Final Answer:

The condition is $N_aI_a = 4N_bI_b$, option (C).

$$\boxed{N_aI_a = 4N_bI_b}$$

Quick Tip: Magnetic moment is NIA with $A = \pi r^2$.

36. A conducting loop of radius $10/[\pi]^{1/2}$ cm is placed perpendicular to a uniform magnetic field 0.5 T. The magnetic field is decreased to zero in 0.5 s at a steady rate. The induced emf in the circular loop at 0.25 s is

(A) 1 mV

(B) 10 mV

(C) 100 mV

(D) 5 mV

Correct Answer: (B) 10 mV

Solution:

Step 1: Understanding the Concept:

By Faraday's law, $|\varepsilon| = \frac{d\Phi}{dt}$. The field falls at a steady rate, so the emf is the same at every instant, including 0.25 s.

Step 2: Area:

Radius $= \frac{10}{\sqrt{\pi}}$ cm $= \frac{0.1}{\sqrt{\pi}}$ m. Area $= \pi r^2 = \pi \times \frac{0.01}{\pi} = 0.01$ m^2.

Step 3: Emf:

$$\varepsilon = A\frac{\Delta B}{\Delta t} = 0.01 \times \frac{0.5}{0.5} = 0.01 \text{ V} = 10 \text{ mV}$$

Final Answer:

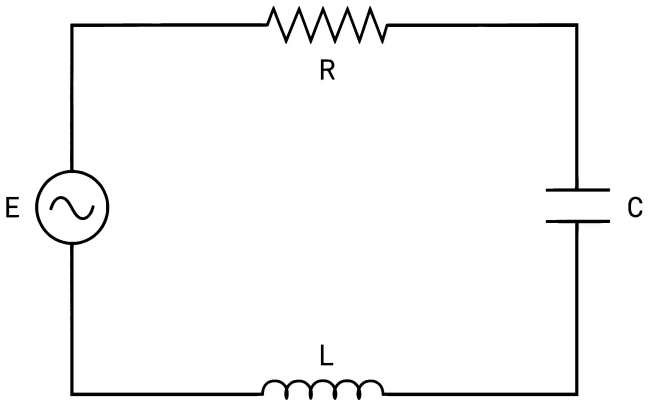
The induced emf is 10 mV, option (B).

$$\boxed{10 \text{ mV}}$$

Quick Tip: The emf is $A\frac{\Delta B}{\Delta t}$ and is constant for a steady rate.

37. In the circuit shown, the ratio of the quality factor and the band width is

(Given: band width = R/L)



(A) $\sqrt{\frac{1}{LC}}\frac{L^2}{R^2}$

(B) $\sqrt{\frac{1}{LC}}$

(C) $\sqrt{\frac{1}{LC}}\frac{L}{R}$

(D) $\sqrt{\frac{1}{LC}}\frac{R}{L}$

Correct Answer: (A) $\sqrt{\frac{1}{LC}}\frac{L^2}{R^2}$

Solution:

Step 1: Read the circuit:

The figure shows a source E, a resistor R, an inductor L and a capacitor C all joined in one loop. So this is a series LCR circuit driven by an AC source.

Step 2: Recall the formulas:

At resonance the angular frequency is $\omega_0 = \sqrt{\frac{1}{LC}}$. The quality factor of a series LCR circuit is $Q = \frac{\omega_0 L}{R}$. The question tells us to take the band width as $\Delta\omega = \frac{R}{L}$.

Step 3: Form the ratio:

We need Q divided by the band width.

$$\frac{Q}{\Delta\omega} = \frac{\omega_0 L/R}{R/L} = \omega_0 \cdot \frac{L}{R} \cdot \frac{L}{R} = \omega_0 \frac{L^2}{R^2}$$

Put $\omega_0 = \sqrt{1/(LC)}$:

$$\frac{Q}{\Delta\omega} = \sqrt{\frac{1}{LC}} \frac{L^2}{R^2}$$

Step 4: Check the other options:

Option (B) is just ω_0, which has lost the L^2/R^2 factor. Option (C) has only L/R, which is Q/ω_0, not $Q/\Delta\omega$. Option (D) has R/L, which is the band width itself. None of these matches the division we did.

Final Answer:

The ratio of quality factor to band width is $\omega_0 L^2/R^2$, which is option (A).

$$\boxed{\sqrt{\frac{1}{LC}} \frac{L^2}{R^2}}$$

Quick Tip: Use $Q = \omega_0 L/R$ with $\omega_0 = 1/\sqrt{LC}$, then divide by the given band width R/L.

38. An inductor of 0.5 mH, a capacitor of 20 μF and resistance 20 Ω are connected in series with a 220 V ac source. If the current is in phase with the emf, the amplitude of current of the circuit is $[x]^{1/2}$ A. The value of x is

(A) 61

(B) 121

(C) 242

(D) 442

Correct Answer: (C) 242

Solution:

Step 1: Understanding the Concept:

Current in phase with emf means $X_L = X_C$, which is resonance. The impedance is $Z = R = 20\ \Omega$.

Step 2: Peak voltage:

The 220 V source is a rms value, so the peak voltage is $V_0 = 220\sqrt{2}$ V.

Step 3: Current amplitude:

$$I_0 = \frac{V_0}{R} = \frac{220\sqrt{2}}{20} = 11\sqrt{2}\ \text{A} = \sqrt{242}\ \text{A}$$

So $x = 242$.

Final Answer:

The value of x is 242, option (C).

$$\boxed{242}$$

Quick Tip: In phase with the emf means resonance, so impedance equals R.

39. In a series LR circuit with $X_L = R$. Power factor is P_1. If a capacitor of capacitance C with $X_c = X_L$ is added to the circuit the power factor becomes P_2. The ratio of P_1 to P_2 will be :

(A) $1 : 3$

(B) $1 : \sqrt{2}$

(C) $1 : 1$

(D) $1 : 2$

Correct Answer: (B) $1 : \sqrt{2}$

Solution:

Step 1: Understanding the Concept:

The power factor is $\cos\phi = \frac{R}{Z}$.

Step 2: Series LR circuit:

$X_L = R$, so $Z = \sqrt{R^2 + X_L^2} = \sqrt{2}R$. $P_1 = \frac{R}{\sqrt{2}R} = \frac{1}{\sqrt{2}}$.

Step 3: After adding C:

$X_C = X_L$, so the net reactance is zero and $Z = R$. $P_2 = 1$.

Step 4: Ratio:

$\frac{P_1}{P_2} = \frac{1/\sqrt{2}}{1} = 1 : \sqrt{2}$.

Final Answer:

The ratio $P_1 : P_2$ is $1 : \sqrt{2}$, option (B).

$$\boxed{1 : \sqrt{2}}$$

Quick Tip: With $X_c = X_L$ the circuit is at resonance and its power factor is 1.

40. When a beam of white light is allowed to pass through convex lens parallel to principle axis, the different colors of light converge at different point on the principle axis after refraction. This is called

(A) scattering

(B) chromatic aberration

(C) spherical aberration

(D) polarisation

Correct Answer: (B) chromatic aberration

Solution:

Step 1: Understanding the Concept:

The refractive index of glass is different for each colour (higher for violet, lower for red). So a lens bends colours by different amounts and they come to focus at different points on the axis.

Step 2: Name of the effect:

This is chromatic aberration.

Scattering is the spreading of light by small particles. Spherical aberration is caused by the shape of the lens surface, where rays far from the axis focus at a different point, even for a single colour. Polarisation restricts the vibration of light to one plane.

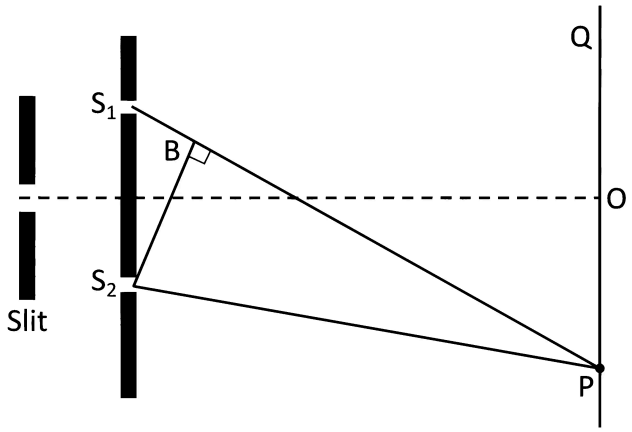
Final Answer:

This defect is chromatic aberration, option (B).

Chromatic aberration

Quick Tip: Different colours have different refractive indices, so they focus at different points.

41. In Young's double slit experiment, following figure shows that Q is the position of second bright fringe on the right side of point O. P is the eleventh bright fringe on the other side measured from point Q. If the wavelength of light used is 6000 Å, then what will be the value of S_1B?



(A) 3.142×10^{-7} m

(B) 3.138×10^{-7} m

(C) 6.6×10^{-6} m

(D) 5.4×10^{-6} m

Correct Answer: (D) 5.4×10^{-6} m

Solution:

Step 1: Understand the figure:

The figure shows the two slits S1 and S2, a screen, and a point P on

the screen. S2B is drawn perpendicular to the line S1P, so S1B is the path difference at P (the screen is far away, so S2P is almost equal to BP).

Step 2: Fix the order of the fringes:
Take the central bright fringe O as order 0. Q is the second bright fringe on the right, so it has order +2. P is the eleventh bright fringe counted from Q, on the other side of O. Moving 11 fringes to the left from Q lands at order $2 - 11 = -9$. So P is the 9th bright fringe on the left of O.

Step 3: Path difference at P:
For a bright fringe the path difference is a whole number of wavelengths. At order 9:

$$S_1B = 9\lambda$$

Step 4: Put in the numbers:
$\lambda = 6000$ Å $= 6 \times 10^{-7}$ m.

$$S_1B = 9 \times 6 \times 10^{-7} = 54 \times 10^{-7} = 5.4 \times 10^{-6} \text{ m}$$

Step 5: Why the other options are wrong:
The values 3.142×10^{-7} and 3.138×10^{-7} m are about half a wavelength, nowhere near 9 wavelengths. 6.6×10^{-6} m equals 11λ, which comes from counting 11 fringes from O and forgetting that the count starts at Q.

Final Answer:

The path difference at P is $9\lambda = 5.4 \times 10^{-6}$ m, option (D).

$$\boxed{5.4 \times 10^{-6} \text{ m}}$$

Quick Tip: Find the order of fringe P: it is 11 fringes from Q (order 2) on the other side, so order 9. Path difference is order times wavelength.

42. 'n' polarizing sheets are arranged such that each makes an angle 45° with the preceding sheet. An unpolarized light of intensity I is incident into this arrangement. The output intensity is found to be $I/64$. The value of n will be

(A) 3

(B) 6

(C) 5

(D) 4

Correct Answer: (B) 6

Solution:

Step 1: Understanding the Concept:

Unpolarised light passing through the first sheet loses half its intensity: $I_1 = \frac{I}{2}$. For every later sheet, Malus's law gives $I' = I\cos^2\theta$.

Step 2: Each later sheet:

With $\theta = 45°$, $\cos^2\theta = \frac{1}{2}$. So each of the other $(n-1)$ sheets halves the intensity.

Step 3: Solve:

$$I_{\text{out}} = \frac{I}{2}\left(\frac{1}{2}\right)^{n-1} = \frac{I}{2^n}$$

$\frac{I}{2^n} = \frac{I}{64}$, so $2^n = 64$ and $n = 6$.

Final Answer:

The number of sheets is 6, option (B).

$$\boxed{6}$$

Quick Tip: The first sheet halves the intensity; each next sheet multiplies by $\cos^2 45^\circ = \frac{1}{2}$.

43. In Young's double slit experiment, two slits S_1 and S_2 are 'd' distance apart and the separation from slits to screen is D. Now if two transparent slabs of equal thickness 0.1 mm but refractive index 1.51 and 1.55 are introduced in the path of beam ($\lambda = 4000$ Å) from S_1 and S_2 respectively. The central bright fringe spot will shift by ____ number of fringes.

(A) 5

(B) 10

(C) 15

(D) 20

Correct Answer: (B) 10

Solution:

Step 1: Understanding the Concept:

A slab of thickness t and index μ adds an extra optical path $(\mu - 1)t$. With slabs on both slits, the net extra path difference is $(\mu_2 - \mu_1)t$.

Step 2: Path difference:

$\Delta x = (1.55 - 1.51) \times 0.1 \text{ mm} = 0.04 \times 10^{-4} \text{ m} = 4 \times 10^{-6} \text{ m}$.

Step 3: Number of fringes:

$\lambda = 4000 \text{ Å} = 4 \times 10^{-7} \text{ m}$.

$$n = \frac{\Delta x}{\lambda} = \frac{4 \times 10^{-6}}{4 \times 10^{-7}} = 10$$

Final Answer:

The central fringe shifts by 10 fringes, option (B).

$$\boxed{10}$$

Quick Tip: Shift in fringes $= \frac{(\mu_2 - \mu_1)t}{\lambda}$.

44. The threshold frequency of metal is f_0. When the light of frequency $2f_0$ is incident on the metal plate, the maximum velocity of photoelectron is v_1. When the frequency of incident radiation is increased to $5f_0$ the maximum velocity of photoelectrons emitted is v_2. The ratio v_1 to v_2 is

(A) 1 : 2

(B) 1 : 8

(C) 1 : 16

(D) 1 : 4

Correct Answer: (A) 1 : 2

Solution:

Step 1: Understanding the Concept:

Einstein's equation: $\frac{1}{2}mv_{\max}^2 = h(f - f_0)$.

Step 2: Two cases:

At $f = 2f_0$: $\frac{1}{2}mv_1^2 = h(2f_0 - f_0) = hf_0$.

At $f = 5f_0$: $\frac{1}{2}mv_2^2 = h(5f_0 - f_0) = 4hf_0$.

Step 3: Ratio:

$$\frac{v_1^2}{v_2^2} = \frac{1}{4} \Rightarrow \frac{v_1}{v_2} = \frac{1}{2}$$

So $v_1 : v_2 = 1 : 2$. A ratio of $1 : 4$ would result from forgetting the square root.

Final Answer:

The ratio $v_1 : v_2$ is $1 : 2$, option (A).

$$\boxed{1 : 2}$$

Quick Tip: $\frac{1}{2}mv^2 = h(f - f_0)$, so $v \propto \sqrt{f - f_0}$.

45. An electron accelerated through a potential difference V_1 has a de-Broglie wavelength of λ. When the potential is changed to V_2, its de-Broglie wavelength increases to 2λ. The value of (V_1/V_2) is equal to

(A) 3 : 1

(B) 9 : 4

(C) 3 : 2

(D) 4 : 1

Correct Answer: (D) 4 : 1

Solution:

Step 1: Understanding the Concept:

An electron accelerated through V has $\lambda = \frac{h}{\sqrt{2meV}}$, so $\lambda \propto V^{-1/2}$.

Step 2: Compare:

$\frac{\lambda_2}{\lambda_1} = \sqrt{\frac{V_1}{V_2}} = 2$, so $\frac{V_1}{V_2} = 4$.

Step 3: Check:

Doubling the wavelength means quartering the voltage, which fits $V_1 : V_2 = 4 : 1$.

Final Answer:

The ratio $V_1 : V_2$ is 4 : 1, option (D).

$$\boxed{4 : 1}$$

Quick Tip: For an electron, $\lambda \propto \frac{1}{\sqrt{V}}$.

46. The wavelength of the radiation emitted is λ_0 when an electron jumps from the second excited state to the first excited state of hydrogen atom. If the electron jumps from the third excited state to the second orbit of the hydrogen atom, the wavelength of the radiation emitted will be $(20\lambda_0/x)$. The value of x is

(A) 17

(B) 21

(C) 27

(D) 29

Correct Answer: (C) 27

Solution:

Step 1: Understanding the Concept:

The ground state is $n = 1$, the first excited is $n = 2$, the second excited is $n = 3$ and the third excited is $n = 4$.

Step 2: First transition:

$n = 3 \to n = 2$: $\frac{1}{\lambda_0} = R\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R}{36}$.

Step 3: Second transition:

$n = 4 \to n = 2$: $\frac{1}{\lambda} = R\left(\frac{1}{4} - \frac{1}{16}\right) = \frac{3R}{16}$.

Step 4: Ratio:

$$\lambda = \frac{16}{3R}, \quad \lambda_0 = \frac{36}{5R}, \quad \frac{\lambda}{\lambda_0} = \frac{16}{3} \times \frac{5}{36} = \frac{20}{27}$$

So $\lambda = \frac{20\lambda_0}{27}$, and $x = 27$.

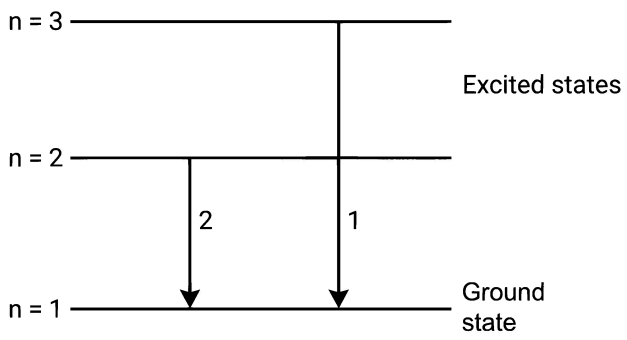
Final Answer:

The value of x is 27, option (C).

$$\boxed{27}$$

Quick Tip: Use $\frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$; second excited state is $n = 3$ and third excited is $n = 4$.

47. For hydrogen atom, λ_1 and λ_2 are the wavelengths corresponding to the transitions 1 and 2 respectively as shown in figure. The ratio of λ_1 and λ_2 is $x/32$. The value of x is



(A) 13

(B) 27

(C) 29

(D) 35

Correct Answer: (B) 27

Solution:

Step 1: Read the figure:

The figure is an energy level diagram of hydrogen. Arrow 1 goes from n = 3 down to n = 1. Arrow 2 goes from n = 2 down to n = 1.

Step 2: Use the Rydberg formula:

For a transition from n_i to n_f, $\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$.

Step 3: Transition 1 (3 to 1):

$$\frac{1}{\lambda_1} = R\left(1 - \frac{1}{9}\right) = \frac{8R}{9} \Rightarrow \lambda_1 = \frac{9}{8R}$$

Step 4: Transition 2 (2 to 1):

$$\frac{1}{\lambda_2} = R\left(1 - \frac{1}{4}\right) = \frac{3R}{4} \Rightarrow \lambda_2 = \frac{4}{3R}$$

Step 5: Take the ratio:

$$\frac{\lambda_1}{\lambda_2} = \frac{9}{8R} \times \frac{3R}{4} = \frac{27}{32}$$

Comparing with $x/32$ gives $x = 27$.

Step 6: Why the others are wrong:

If someone took the ratio the wrong way round, they would get 32/27, which does not fit the form $x/32$. Values 13, 29 and 35 do not come from any combination of 8/9 and 3/4.

Final Answer:

The ratio is 27/32, so $x = 27$, option (B).

$$\boxed{x = 27}$$

Quick Tip: Use the Rydberg formula for 3 to 1 and 2 to 1 transitions and divide the two wavelengths.

48. A pure silicon crystal at temperature 300 K has electron and hole concentration (n_i) 10^{16} per m^3 each and 10^{21} phosphorus atoms (n_e) are added per cubic metre. The new hole concentration in silicon crystal is

(A) 10^{19} per m^3

(B) 10^{21} per m^3

(C) 10^{5} per m^3

(D) 10^{11} per m^3

Correct Answer: (D) 10^{11} per m^3

Solution:

Step 1: Understanding the Concept:

In a doped semiconductor at a fixed temperature, $n_e n_h = n_i^2$.

Step 2: Calculate:

Phosphorus is a donor, so the electron concentration is about the donor density: $n_e \approx 10^{21}$ m^{-3}. Thermally generated electrons 10^{16} are negligible in comparison.

$$n_h = \frac{n_i^2}{n_e} = \frac{(10^{16})^2}{10^{21}} = 10^{11}\ m^{-3}$$

Final Answer:

The new hole concentration is 10^{11} per m^3, option (D).

$$\boxed{10^{11}\ \text{m}^{-3}}$$

Quick Tip: Use the mass action law $n_e n_h = n_i^2$.

49. If transistor having $\alpha = 0.9$, is used in CE configuration, then for a change of 0.4 mA in base current, what will be the change in collector current?

(A) 3.6 mA

(B) 4 mA

(C) 0.9 mA

(D) 36 mA

Correct Answer: (A) 3.6 mA

Solution:

Step 1: Understanding the Concept:

The current gain in CE configuration is $\beta = \frac{\Delta I_C}{\Delta I_B}$, and it is linked to α by $\beta = \frac{\alpha}{1-\alpha}$.

Step 2: Calculate:

$\beta = \frac{0.9}{0.1} = 9$.

$$\Delta I_C = \beta \Delta I_B = 9 \times 0.4 = 3.6\ \text{mA}$$

Final Answer:

The change in collector current is 3.6 mA, option (A).

$\boxed{3.6\ \text{mA}}$

Quick Tip: $\beta = \frac{\alpha}{1-\alpha}$ and $\Delta I_C = \beta \Delta I_B$.

50. Choose the correct statement about Zener diode:

(A) It works as a voltage regulator in reverse bias and behaves like simple pn junction diode in forward bias.

(B) It works as a voltage regulator in both forward and reverse bias.

(C) It works a voltage regulator only in forward bias.

(D) It works as a voltage regulator in forward bias and behaves like simple pn junction diode in reverse bias.

Correct Answer: (A) It works as a voltage regulator in reverse bias and behaves like simple pn junction diode in forward bias.

Solution:

Step 1: Understanding the Concept:
A Zener diode is a heavily doped p-n junction designed to work in reverse breakdown, where the voltage across it stays nearly constant over a wide range of current.

Step 2: Check each:
In reverse bias it acts as a voltage regulator. In forward bias it conducts like a normal p-n diode.
B and C say it regulates in forward bias, which is wrong, since a forward diode has a drop of only about 0.7 V and no sharp breakdown. D reverses the roles, so it is wrong too.

Final Answer:

A Zener diode regulates in reverse bias and acts as a normal diode in forward bias, option (A).

Option (A)

Quick Tip: A Zener diode regulates voltage in reverse breakdown and acts like an ordinary diode in forward bias.

Chemistry

Questions 51 to 100 | 1 mark each | 50 marks

51. 1 g of H_2 contains the same number of molecules as in

(A) 14 g of N_2

(B) 18 g of H_2O

(C) 16 g of CO

(D) 28 g of N_2

Correct Answer: (A) 14 g of N_2

Solution:

Step 1: Moles of H_2

$\frac{1}{2} = 0.5$ mol.

Step 2: Check each option

(A) 14 g N_2 is $\frac{14}{28} = 0.5$ mol. Matches.

(B) 18 g H_2O is 1 mol.

(C) 16 g CO is $\frac{16}{28} = 0.57$ mol.

(D) 28 g N_2 is 1 mol.

Step 3: Result

Only (A) has 0.5 mol.

Final Answer:

14 g of N_2 has the same number of molecules, option (A).

$\boxed{(A)}$

Quick Tip: Equal moles mean equal numbers of molecules.

52. If the energy of an electron in the first Bohr orbit of the H-atom is -2.18×10^{-18} J then the energy of the electron in the second orbit will be

(A) -2.18×10^{-18} J

(B) -4.36×10^{-18} J

(C) -0.545×10^{-18} J

(D) -0.273×10^{-18} J

Correct Answer: (C) -0.545×10^{-18} J

Solution:

Step 1: Formula

$E_n = -\frac{2.18 \times 10^{-18}}{n^2}$ J.

Step 2: Substitute

For $n = 2$: $E_2 = \frac{-2.18 \times 10^{-18}}{4} = -0.545 \times 10^{-18}$ J. Option (C).

Final Answer:

$E_2 = -0.545 \times 10^{-18}$ J, option (C).

$\boxed{\text{(C)}}$

Quick Tip: Use $E_n = \frac{E_1}{n^2}$.

53. In the compound $^{1}CH_2 =^{2} CH -^{3} CH_2 -^{4} CH_2 -^{5} C \equiv^{6} CH$, The $C_2 - C_3$ bond is of type

(A) $sp - sp^2$

(B) $sp^3 - sp^3$

(C) $sp - sp^3$

(D) $sp^2 - sp^3$

Correct Answer: (D) $sp^2 - sp^3$

Solution:

Step 1: Hybridisation of each carbon

C_2 is part of the $C = C$ double bond, so it is sp^2. C_3 is a CH_2 with only single bonds, so it is sp^3.

Step 2: Bond

The $C_2 - C_3$ bond is formed by sp^2 and sp^3 overlap. Option (D).

Final Answer:

The bond is $sp^2 - sp^3$, option (D).

$$\boxed{\text{(D)}}$$

Quick Tip: Double bonded carbon is sp^2 and saturated carbon is sp^3.

54. A gas occupies a volume of 0.35 dm^3 at 290 K and 2 atm. Pressure. Calculate the volume of gas at STP.

(A) $3.11\ dm^3$

(B) $0.66\ dm^3$

(C) $0.11\ dm^3$

(D) $3.5\ dm^3$

Correct Answer: (B) $0.66\ dm^3$

Solution:

Step 1: Combined law

$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$.

Step 2: Substitute

$\frac{2\times0.35}{290} = \frac{1\times V_2}{273}$.

$V_2 = \frac{0.7\times273}{290} = 0.659 \approx 0.66\ dm^3$. Option (B).

Final Answer:

The volume at STP is $0.66\ dm^3$, option (B).

$\boxed{(B)}$

Quick Tip: Use the combined gas law with STP of 273 K and 1 atm.

55. The enthalpy of combustion of cyclohexane, cyclohexene and H_2 are respectively $-3920, -3800$ and -241 KJ mol^{-1}. The enthalpy of hydrogenation of cyclohexene is

(A) -121 KJ mol^{-1}

(B) $+121$ KJ mol^{-1}

(C) -242 KJ mol^{-1}

(D) $+242$ KJ mol^{-1}

Correct Answer: (A) -121 KJ mol^{-1}

Solution:

Step 1: Reaction

Cyclohexene $+H_2 \rightarrow$ cyclohexane.

Step 2: Hess law

$\Delta H_{hyd} = \Delta H_c(\text{cyclohexene}) + \Delta H_c(H_2) - \Delta H_c(\text{cyclohexane})$.

$$= -3800 - 241 - (-3920) = -121 \text{ kJ mol}^{-1}$$

Step 3: Result

Option (A).

Final Answer:

$\Delta H_{hyd} = -121$ kJ mol^{-1}, option (A).

$$\boxed{\text{(A)}}$$

Quick Tip: Use Hess law: hydrogenation = combustion of reactants minus combustion of product.

56. A gas expands from a volume of 1 dm^3 to 1.25 dm^3 under a pressure of 1 bar. Find the change in internal energy if 150 J of heat is supplied to the system.

(A) 125 J

(B) 175 J

(C) −125 J

(D) −175 J

Correct Answer: (A) 125 J

Solution:

Step 1: Understanding the Concept:

The first law of thermodynamics links the change in internal energy to heat and work: $\Delta U = q + w$.

Here q is heat absorbed by the system and w is work done on the system. When a gas expands against a constant external pressure, the work done on the gas is negative: $w = -P\Delta V$.

Step 2: Find the Work:

$\Delta V = 1.25 - 1 = 0.25$ dm^3, and $P = 1$ bar.

$$w = -P\Delta V = -(1 \text{ bar})(0.25 \text{ dm}^3) = -0.25 \text{ bar dm}^3$$

Since 1 bar dm^3 = 100 J, we get $w = -25$ J. The sign is negative because the gas does work on the surroundings.

Step 3: Apply the First Law:

Heat is supplied to the system, so $q = +150$ J.

$$\Delta U = q + w = 150 + (-25) = 125 \text{ J}$$

Step 4: Why the other options are wrong:
175 J comes from adding 25 J instead of subtracting it, which treats the expansion work as work done on the gas. The negative values −125 J and −175 J would mean the internal energy falls, but the system gains 150 J of heat and loses only 25 J as work, so it must rise.

Final Answer:
The internal energy of the gas rises by 125 J, which is option (A).

$$\boxed{125\ \text{J}}$$

Quick Tip: Work done by the gas is $P\Delta V$; then use the first law $\Delta U = q + w$.

57. In which of the following processes the internal energy of system remain constant?

(A) Isobaric Process

(B) Isochoric Process

(C) Isothermal Process

(D) Adiabatic Process

Correct Answer: (C) Isothermal Process

Solution:

Step 1: Understanding the Concept:
For an ideal gas, internal energy U depends only on temperature. So U stays constant only when the temperature stays constant.

Step 2: Check each process:

Isobaric process: pressure is constant but the volume and temperature change, so U changes. This is wrong.

Isochoric process: volume is constant, so no work is done, but heat added raises the temperature and $\Delta U = q_V \neq 0$. This is wrong.

Isothermal process: temperature is constant, so $\Delta T = 0$ and $\Delta U = 0$. This is correct, and the heat absorbed equals the work done by the gas.

Adiabatic process: $q = 0$, but $\Delta U = w \neq 0$, so the temperature changes. This is wrong.

Final Answer:

Only the isothermal process keeps the internal energy constant, which is option (C).

Isothermal process

Quick Tip: Internal energy of an ideal gas depends only on temperature.

58. Match **List I** with **List II**

List I (Buffers)	**List II (Applications)**
(A) HCO_3^{-1} + H_2CO_3	(I) Maintain the pH of blood forms 7.36 to 7.42
(B) Phosphates	(II) In Medicine
(C) Sodium Citrate	(III) In Qualitative analysis of basic radicals
(D) NH_4Cl + NH_4OH	(IV) In fertilizers

Choose the **correct** answer from the options given below:

(A) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)

(B) (A) - (II), (B) - (IV), (C) - (I), (D) - (III)

(C) (A) - (I), (B) - (IV), (C) - (II), (D) - (III)

(D) (A) - (III), (B) - (IV), (C) - (I), (D) - (II)

Correct Answer: (C) (A) - (I), (B) - (IV), (C) - (II), (D) - (III)

Solution:

Step 1: Understanding the Concept:
A buffer resists a change in pH on adding small amounts of acid or base. Different buffers are chosen for different uses, so we match each by its known application.

Step 2: Match (A) and (D):
(A) HCO_3^-/H_2CO_3 is the main buffer in blood. It holds blood pH between 7.36 and 7.42. So (A) - (I).
(D) $NH_4Cl + NH_4OH$ gives a basic buffer of pH near 9. It is used in

qualitative analysis to precipitate group III hydroxides while keeping group IV and later radicals in solution. So (D) - (III).

Step 3: Match (B) and (C):
(B) Phosphate buffers supply nutrients and control soil pH in fertilizers. So (B) - (IV).
(C) Sodium citrate is a buffer used in medicine, for example in anticoagulants and oral preparations. So (C) - (II).

Step 4: Check the options:
The pairing (A)-I, (B)-IV, (C)-II, (D)-III is exactly option (C). Option (A) pairs the blood buffer with the right use but swaps the rest. Option (B) puts the blood buffer under 'In Medicine'. Option (D) gives the blood buffer to qualitative analysis.

Final Answer:
The correct matching is (A)-(I), (B)-(IV), (C)-(II), (D)-(III), which is option (C).

$$\boxed{(A)\text{-}(I),\ (B)\text{-}(IV),\ (C)\text{-}(II),\ (D)\text{-}(III)}$$

Quick Tip: Blood uses the bicarbonate buffer, and group analysis of basic radicals uses the ammonium buffer.

59. Find the pH of a solution formed by mixing equal volume of solution of 0.1 M sodium propionate and 0.1 molar propionic acid.
(Given The dissociation constant of propionic acid is 1.3×10^{-5})

(A) 2.45

(B) 4.89

(C) 5.98

(D) 6.89

Correct Answer: (B) 4.89

Solution:

Step 1: Understanding the Concept:

A mixture of a weak acid and its salt is an acidic buffer. Its pH follows the Henderson-Hasselbalch equation.

Step 2: Key Formula:

$$\text{pH} = \text{p}K_a + \log \frac{[\text{salt}]}{[\text{acid}]}$$

Step 3: Calculate:

Equal volumes of 0.1 M solutions are mixed, so both concentrations drop to 0.05 M, and the ratio is 1. Then $\log 1 = 0$ and pH = pK_a.

$$\text{p}K_a = -\log(1.3 \times 10^{-5}) = 5 - \log 1.3 = 5 - 0.114 = 4.886$$

So pH $\approx$ 4.89.

Step 4: Why the other options are wrong:

2.45 is far too acidic for a mixture that holds a large amount of the salt. 5.98 and 6.89 are too high, because with equal amounts of acid and salt the pH must sit exactly at pK_a.

Final Answer:

The buffer has pH equal to pK_a, about 4.89. This is option (B).

$$\boxed{4.89}$$

Quick Tip: Equal volumes of equal concentrations give salt/acid ratio of 1, so pH = pKa.

60. What is the concentration of hydrogen ion in the solution when the concentration of hydroxyl ion is 0.08 mol / dm^3 at 298 K ?

(A) 0.125 mol/L

(B) 0.125×10^{-12} mol/L

(C) 0.0125 mol/L

(D) 0.125×10^{-10} mol/L

Correct Answer: (B) 0.125×10^{-12} mol/L

Solution:

Step 1: Understanding the Concept:

In any aqueous solution at 298 K the ionic product of water is fixed: $K_w = [H^+][OH^-] = 1 \times 10^{-14}$.

Step 2: Calculate:

$$[H^+] = \frac{10^{-14}}{0.08} = 1.25 \times 10^{-13} \text{ mol/L}$$

This equals 0.125×10^{-12} mol/L.

Step 3: Why the other options are wrong:

0.125 mol/L and 0.0125 mol/L are far too big, since a basic solution has $[H^+] < 10^{-7}$. The value 0.125×10^{-10} comes from using 10^{-12} in place of 10^{-14}.

Final Answer:

$[H^+] = 0.125 \times 10^{-12}$ mol/L, which is option (B).

$$\boxed{0.125 \times 10^{-12} \text{ mol/L}}$$

Quick Tip: Use $K_w = [H^+][OH^-] = 10^{-14}$ at 298 K.

61. What is oxidation state of oxygen in OF_2 and in KO_2 respectively?

(A) +2 and +1

(B) +2 and −1/2

(C) +1 and −1/2

(D) −2 and −1

Correct Answer: (B) +2 and −1/2

Solution:

Step 1: Understanding the Concept:

Oxidation state is the charge an atom would have if all bonds were ionic. Oxygen is usually −2, but it changes when bonded to fluorine or in peroxides and superoxides.

Step 2: In OF_2:

Fluorine is the most electronegative element and is always -1. The molecule is neutral:

$$x + 2(-1) = 0 \Rightarrow x = +2$$

Step 3: In KO_2:

Potassium is $+1$, so the anion is O_2^-, a superoxide. Let the oxidation state of oxygen be y:

$$+1 + 2y = 0 \Rightarrow y = -\frac{1}{2}$$

Step 4: Conclusion:

The two values are $+2$ and $-1/2$. Option A ($+1$ for KO_2) is wrong because K, not O, carries $+1$. Option C gives $+1$ to OF_2 and ignores the two fluorines. Option D is the usual value of oxide and peroxide oxygen.

Final Answer:

Oxygen is $+2$ in OF_2 and $-1/2$ in KO_2, which is option (B).

$$\boxed{+2 \text{ and } -\frac{1}{2}}$$

Quick Tip: Fluorine is more electronegative than oxygen; KO2 is a superoxide containing O_2^-.

62. Which of the following ions are responsible for hardness of water?

(A) Ag^{+}, Au^{3+} ions

(B) Hg^{2+}, Pb^{2+} ions

(C) Ca^{2+} and Mg^{2+} ions.

(D) As^{3+}, CN^{-}

Correct Answer: (C) Ca^{2+} and Mg^{2+} ions.

Solution:

Step 1: Understanding the Concept:

Hard water does not form lather easily with soap. The cause is dissolved bicarbonates, chlorides and sulphates of calcium and magnesium.

Step 2: Explanation:

Soap is sodium stearate. The Ca^{2+} and Mg^{2+} ions react with it to form insoluble scum, so soap is wasted before lather forms.

$$2C_{17}H_{35}COONa + Ca^{2+} \rightarrow (C_{17}H_{35}COO)_2Ca \downarrow + 2Na^{+}$$

Step 3: Other options:

Ag^{+}, Au^{3+}, Hg^{2+}, Pb^{2+}, As^{3+} and CN^{-} are not present in natural water as hardness agents. Some are toxic pollutants, but they do not cause hardness.

Final Answer:

Hardness is due to Ca^{2+} and Mg^{2+} ions, which is option (C).

$$\boxed{Ca^{2+} \text{ and } Mg^{2+}}$$

Quick Tip: Hardness comes from dissolved calcium and magnesium salts.

63. Identify the elements of alkali metals and alkaline earth metals respectively having highest 1st ionization enthalpy.

(A) K and Ca

(B) Li and Be

(C) Rb and Ba

(D) K and Sr

Correct Answer: (B) Li and Be

Solution:

Step 1: Understanding the Concept:
Going down a group, atomic size grows and the outer electron is held less tightly. So first ionization enthalpy decreases down a group.

Step 2: Apply to the groups:
Alkali metals in order: Li, Na, K, Rb, Cs. Li is at the top, so it has the highest $\Delta_i H_1$ (about 520 kJ/mol).
Alkaline earth metals: Be, Mg, Ca, Sr, Ba. Be is at the top, so it has the highest $\Delta_i H_1$ (about 899 kJ/mol).

Step 3: Other options:
K, Rb, Ca, Sr and Ba are all lower in their groups. Their larger size gives lower ionization enthalpy, so options A, C and D are wrong.

Final Answer:
The pair is Li and Be, option (B).

Li and Be

Quick Tip: Ionization enthalpy falls down a group, so the top element of each group is highest.

64. What is the nature of Al_2O_3?

(A) Acidic

(B) Basic

(C) Amphoteric

(D) Neutral

Correct Answer: (C) Amphoteric

Solution:

Step 1: Understanding the Concept:

An amphoteric oxide reacts with both acids and bases to give salt and water.

Step 2: Evidence:

With acid: $Al_2O_3 + 6HCl \rightarrow 2AlCl_3 + 3H_2O$.

With base: $Al_2O_3 + 2NaOH + 3H_2O \rightarrow 2Na[Al(OH)_4]$.

Since it behaves as a base in the first reaction and as an acid in the second, it is amphoteric.

Step 3: Other options:

It is not purely acidic or basic because it reacts both ways, and it is not neutral because it does react with acids and bases (unlike CO or N_2O).

Final Answer:

Al_2O_3 is amphoteric, option (C).

Amphoteric

Quick Tip: Al2O3 reacts with both acids and bases.

65. Which of the following is correct decreasing order of acidic strength for the compounds listed below?

I: p-chlorophenol, II: p-Cresol, III: p-nitrophenol, IV: p-methoxyphenol.

(A) II > IV > I > III

(B) I > II > III > IV

(C) III > I > II > IV

(D) IV > III > I > II

Correct Answer: (C) III > I > II > IV

Solution:

Step 1: Understanding the Concept:

A phenol loses H^+ to give a phenoxide ion. Anything that stabilises the phenoxide makes the phenol more acidic.

Step 2: Effect of each group:

III: $-NO_2$ is strongly electron withdrawing ($-I$ and $-R$), so it stabilises the negative charge most. It is the most acidic.

I: $-Cl$ withdraws electrons by $-I$ effect (this outweighs its weak $+R$), so it is more acidic than phenol, but less than nitro.

II: $-CH_3$ donates electrons by $+I$ and hyperconjugation, which

destabilises the phenoxide, so it is less acidic than phenol.
IV: $-OCH_3$ donates electrons by a strong $+R$ effect, destabilising the phenoxide even more, so it is the least acidic.

Step 3: Order:

$$III > I > II > IV$$

This is option C. The other options put donor-substituted phenols above the acceptor-substituted ones, which is the wrong direction.

Final Answer:
The decreasing order of acidity is III > I > II > IV, option (C).

$$\boxed{III > I > II > IV}$$

Quick Tip: Electron withdrawing groups raise acidity of phenol and electron donating groups lower it.

66. Among the following, the most stable carbocation is

(A) CH_3^+

(B) $(CH_3)_2CH^+$

(C) $CH_3\text{-}CH_2^+$

(D) $(CH_3)_3C^+$

Correct Answer: (D) $(CH_3)_3C^+$

Solution:

Step 1: Understanding the Concept:
A carbocation is stabilised when the positive carbon is surrounded by alkyl groups. They donate electron density through the $+I$ effect and through hyperconjugation.

Step 2: Compare the ions:
CH_3^+ has no alkyl group, so it has no hyperconjugation at all and is least stable.
$CH_3CH_2^+$ (primary) has 3 alpha hydrogens.
$(CH_3)_2CH^+$ (secondary) has 6 alpha hydrogens.
$(CH_3)_3C^+$ (tertiary) has 9 alpha hydrogens, the most hyperconjugation and the strongest $+I$ effect.

Step 3: Order:

$$3^\circ > 2^\circ > 1^\circ > \text{methyl}$$

So the tert-butyl cation is most stable, option D.

Final Answer:
The tertiary carbocation $(CH_3)_3C^+$ is the most stable, option (D).

$$\boxed{(CH_3)_3C^+}$$

Quick Tip: More alkyl groups on the positive carbon give more hyperconjugation and inductive stabilisation.

67. What is the product of decarboxylation of anhydrous sodium benzoate?

(A) Benzene

(B) Cyclohexane

(C) Benzene hexachloride

(D) Chlorobenzene

Correct Answer: (A) Benzene

Solution:

Step 1: Understanding the Concept:

Decarboxylation is the loss of CO_2 from a carboxylic acid or its sodium salt. For sodium salts, it is done by heating with soda lime ($NaOH + CaO$).

Step 2: Reaction:

$$C_6H_5COONa + NaOH \xrightarrow{CaO,\ \Delta} C_6H_6 + Na_2CO_3$$

The $-COONa$ group is replaced by $-H$, so the product is benzene.

Step 3: Other options:

Cyclohexane needs hydrogenation. Benzene hexachloride needs addition of Cl_2 in light. Chlorobenzene needs a halogenation step. None happens in decarboxylation.

Final Answer:

The product is benzene, option (A).

$$\boxed{\text{Benzene}}$$

Quick Tip: Heating sodium benzoate with soda lime removes the COOH group as carbonate and gives benzene.

68. An ionic compound is formed by elements X and Y. Element Y forms a face-centered cubic (FCC) lattice, and element X occupies one-third of the tetrahedral voids. What is the formula of the compound?

(A) X_2Y_3

(B) XY

(C) XY_3

(D) X_2Y

Correct Answer: (A) X_2Y_3

Solution:

Step 1: Understanding the Concept:

In an FCC lattice with N atoms, there are N octahedral voids and $2N$ tetrahedral voids.

Step 2: Count atoms:

Y forms FCC, so $Y = 4$ atoms per unit cell. The number of tetrahedral voids is $2 \times 4 = 8$.

X occupies one-third of them: $X = \frac{1}{3} \times 8 = \frac{8}{3}$.

Step 3: Ratio:

$$X : Y = \frac{8}{3} : 4 = 8 : 12 = 2 : 3$$

The formula is X_2Y_3. Option B (XY) would need X in all 4 octahedral

voids. Option D (X_2Y) would need X in all 8 tetrahedral voids. Option C (XY_3) would need only $\frac{4}{3}$ X atoms per cell, which is one-sixth of the tetrahedral voids.

Final Answer:
The compound is X_2Y_3, option (A).

$$\boxed{X_2Y_3}$$

Quick Tip: FCC has 4 atoms and 8 tetrahedral voids per unit cell.

69. A metal crystallizes to form simple cubic unit cell. Calculate the void volume of unit cell. [$a = 3.36 \times 10^{-8}$ cm]

(A) 1.807×10^{-23} cm^3.

(B) 1.214×10^{-23} cm^3.

(C) 3.662×10^{-23} cm^3.

(D) 1.986×10^{-23} cm^3.

Correct Answer: (A) 1.807×10^{-23} cm^3.

Solution:

Step 1: Understanding the Concept:
In a simple cubic cell there is 1 atom per cell, and its radius is $r = a/2$. The packing efficiency is 52.4%, so 47.6% of the cell is empty.

Step 2: Cell volume:

$$a^3 = (3.36 \times 10^{-8})^3 = 37.93 \times 10^{-24} = 3.793 \times 10^{-23} \text{ cm}^3$$

Step 3: Void volume:

Volume of the one atom: $\frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\frac{a}{2}\right)^3 = 0.5236\,a^3 = 1.986 \times 10^{-23}$ cm^3.

$$V_{\text{void}} = 3.793 \times 10^{-23} - 1.986 \times 10^{-23} = 1.807 \times 10^{-23}\ \text{cm}^3$$

Option D (1.986×10^{-23}) is the volume of the atom, not the void. Options B and C do not match either quantity.

Final Answer:

The void volume is 1.807×10^{-23} cm^3, option (A).

$$\boxed{1.807 \times 10^{-23}\ \text{cm}^3}$$

Quick Tip: Void volume = cell volume a^3 minus atom volume (packing efficiency 52.4 percent).

70. Which from following ionic crystals exhibits Frenkel defect?

(A) ZnS

(B) NaCl

(C) KCl

(D) KBr

Correct Answer: (A) ZnS

Solution:

Step 1: Understanding the Concept:
In a Frenkel defect, a small cation leaves its lattice site and sits in an interstitial position. It needs a large difference in size between cation and anion, and low coordination number.

Step 2: Apply:
Zn^{2+} is small and S^{2-} is large, so ZnS shows Frenkel defect (and AgCl, AgBr also do).
NaCl, KCl and KBr have cations and anions of similar size, so they show Schottky defect instead.

Final Answer:
ZnS shows Frenkel defect, option (A).

$$\boxed{\text{ZnS}}$$

Quick Tip: Frenkel defect needs a small cation and a large size difference between ions, as in ZnS.

71. An aqueous solution has an osmotic pressure of 4.92 atm at 300 K. Calculate the molarity of the solution.
($R = 0.082$ L atm mol^{-1} K^{-1})

(A) 0.1 M

(B) 0.2 M

(C) 0.25 M

(D) 0.4 M

Correct Answer: (B) 0.2 M

Solution:

Step 1: Key Formula:

For a dilute solution, $\pi = CRT$, where C is the molarity.

Step 2: Calculate:

$$C = \frac{\pi}{RT} = \frac{4.92}{0.082 \times 300} = \frac{4.92}{24.6} = 0.2 \text{ M}$$

Step 3: Other options:

0.1 M would give $\pi = 2.46$ atm and 0.4 M would give 9.84 atm. 0.25 M gives 6.15 atm. None matches 4.92 atm.

Final Answer:

The molarity is 0.2 M, option (B).

$$\boxed{0.2 \text{ M}}$$

Quick Tip: Use $\pi = CRT$ and solve for C.

72. A solution is prepared by dissolving 68 g sucrose in 1 kg water. Calculate the vapour pressure of solution at 298 K. [vapour pressure of water at 298 K = 18 mm of Hg & mole fraction of solvent = 0.9964]

(A) 17.93 mm Hg

(B) 17.80 mm Hg

(C) 17.76 mm Hg

(D) 17.68 mm Hg

Correct Answer: (A) 17.93 mm Hg

Solution:

Step 1: Understanding the Concept:

For a non-volatile solute like sucrose, only the solvent contributes to vapour pressure. Raoult's law gives $p = x_1 p_1^0$.

Step 2: Calculate:

$$p = 0.9964 \times 18 = 17.935 \text{ mm Hg} \approx 17.93 \text{ mm Hg}$$

Step 3: Check:

The drop is only about 0.065 mm Hg, because 0.2 mol of sucrose sits in 55.5 mol of water. The other options show a larger drop than this.

Final Answer:

The vapour pressure is 17.93 mm Hg, option (A).

$$\boxed{17.93 \text{ mm Hg}}$$

Quick Tip: Raoult's law: $p = x_{\text{solvent}}\, p^0$.

73. Which of the following statements is NOT true about solubility?

(A) The solubility of gases in water usually decreases with increase of temperature.

(B) The solubility of gases in water usually increases with increase in pressure.

(C) The substances having similar inter-molecular forces are likely to be soluble in each other.

(D) Gases like NH_3 and CO_2 obey Henry's law

Correct Answer: (D) Gases like NH_3 and CO_2 obey Henry's law

Solution:

Step 1: Understanding the Concept:
We have to find the false statement, so we check each one.

Step 2: Check (A):
Dissolving a gas in water is exothermic, so by Le Chatelier's principle solubility drops as temperature rises. TRUE.

Step 3: Check (B):
By Henry's law, solubility of a gas is proportional to its partial pressure, so it rises with pressure. TRUE.

Step 4: Check (C):
This is 'like dissolves like'. Similar intermolecular forces allow mixing. TRUE.

Step 5: Check (D):
NH_3 and CO_2 react chemically with water (forming NH_4OH and H_2CO_3), so their solubility does not follow Henry's law. The statement is NOT true, so (D) is the answer.

Final Answer:

The statement that is NOT true is option (D).

$$\boxed{\text{(D)}}$$

Quick Tip: Gases that react with water, such as NH_3 and CO_2, do not obey Henry's law.

74. Which of the following reaction is possible at anode?

(A) $F_2 + 2e^- \rightarrow 2F^-$

(B) $2H^+ + \frac{1}{2}O_2 + 2e^- \rightarrow H_2O$

(C) $Fe^{2+} \rightarrow Fe^{3+} + 1e^-$

(D) $Cu^{2+} + 2e^- \rightarrow Cu(s)$

Correct Answer: (C) $Fe^{2+} \rightarrow Fe^{3+} + 1e^-$

Solution:

Step 1: Understanding the Concept:

The anode is the electrode where oxidation happens, meaning that electrons are lost.

Step 2: Check each:

A: $F_2 + 2e^- \rightarrow 2F^-$ gains electrons, so it is reduction (cathode).

B: O_2 with $2e^-$ gives water, again reduction.

C: $Fe^{2+} \rightarrow Fe^{3+} + e^-$ loses an electron, so it is oxidation. This occurs at the anode.

D: $Cu^{2+} + 2e^- \rightarrow Cu$ gains electrons, so it is reduction.

Final Answer:

Only $Fe^{2+} \rightarrow Fe^{3+} + e^-$ is an anode reaction, option (C).

$$\boxed{Fe^{2+} \rightarrow Fe^{3+} + e^-}$$

Quick Tip: Oxidation (loss of electrons) takes place at the anode.

75. Which of the following statements is correct regarding the cathode of electrochemical cell ?

(A) Oxidation occurs at the cathode

(B) Reduction occurs at cathode

(C) Usually denoted by a negative sign

(D) Is usually made up of non conducting material.

Correct Answer: (B) Reduction occurs at cathode

Solution:

Step 1: Understanding the Concept:

In every electrochemical cell, reduction occurs at the cathode and oxidation occurs at the anode.

Step 2: Check each:

A: Oxidation at cathode is wrong.

B: Reduction at cathode is correct.

C: In a galvanic cell the cathode is the positive electrode, and the negative sign belongs to the anode. This is wrong.

D: Electrodes are conductors such as metals or graphite, not non-

conductors. This is wrong.

Final Answer:
Reduction occurs at the cathode, option (B).

Reduction occurs at cathode

Quick Tip: Reduction takes place at the cathode.

76. Which of the following solutions will have the highest electrical conductivity?

(A) Distilled water

(B) Sugar solution

(C) Salt solution

(D) Alcohol solution

Correct Answer: (C) Salt solution

Solution:

Step 1: Understanding the Concept:
A solution conducts electricity by the movement of ions. More ions means higher conductivity.

Step 2: Compare:
Distilled water has very few ions (about 10^{-7} M), so it is a poor conductor.
Sugar and alcohol are non-electrolytes. They dissolve as molecules and give no ions.

Salt dissociates fully into Na^+ and Cl^-, so the salt solution conducts best.

Final Answer:
The salt solution has the highest conductivity, option (C).

Salt solution

Quick Tip: Conductivity needs free ions; salt gives ions, sugar and alcohol do not.

77. A first order reaction takes 16 minutes for it's 50% completion. What fraction that would react in 32 minutes from the beginning?

(A) 1/2

(B) 1/4

(C) 1/8

(D) 3/4

Correct Answer: (D) 3/4

Solution:

Step 1: Understanding the Concept:
For a first order reaction, the half life is constant. After each half life, half of what is left is used up.

Step 2: Calculate:
Given $t_{1/2} = 16$ min. In 32 min, number of half lives $= 32/16 = 2$.
Remaining fraction $= \left(\frac{1}{2}\right)^2 = \frac{1}{4}$.

Fraction reacted $= 1 - \frac{1}{4} = \frac{3}{4}$.

Step 3: Other options:

1/2 is the fraction reacted after one half life. 1/4 is the fraction remaining, not reacted. 1/8 is what remains after three half lives.

Final Answer:

The fraction reacted is 3/4, option (D).

$$\boxed{\frac{3}{4}}$$

Quick Tip: 32 minutes is two half-lives, so one quarter remains.

78. The half-life of a first-order reaction is 20 minutes. What is the rate constant (in min^{-1}) for this reaction?

(A) 0.0347

(B) 0.5

(C) 13.86

(D) 34.67

Correct Answer: (A) 0.0347

Solution:

Step 1: Key Formula:

For a first order reaction, $t_{1/2} = \frac{0.693}{k}$.

Step 2: Calculate:

$$k = \frac{0.693}{20\ \text{min}} = 0.03465\ \text{min}^{-1} \approx 0.0347\ \text{min}^{-1}$$

Step 3: Other options:

13.86 is 0.693×20, which multiplies instead of dividing. 34.67 is the same mistake with a factor of 1000 slip. 0.5 has no link to 0.693/20, and it would give a half life of only about 1.4 min.

Final Answer:

The rate constant is $0.0347\ \text{min}^{-1}$, option (A).

$$\boxed{0.0347\ \text{min}^{-1}}$$

Quick Tip: Use $k = 0.693/t_{1/2}$.

79. The rate constant of a reaction is $k = 3.28 \times 10^{-4}\ \text{s}^{-1}$. Find the order of the reaction.

(A) Zero order

(B) First order

(C) Second order

(D) Third order

Correct Answer: (B) First order

Solution:

Step 1: Understanding the Concept:

The unit of k for order n is $\text{mol}^{1-n}\ \text{L}^{n-1}\ \text{s}^{-1}$.

Step 2: Apply:
Zero order: mol L^{-1} s^{-1}. First order: s^{-1}. Second order: L mol^{-1} s^{-1}.
Third order: L^2 mol^{-2} s^{-1}.
The given $k = 3.28 \times 10^{-4}$ s^{-1} has only the unit of time, so the reaction is first order.

Final Answer:
The reaction is of first order, option (B).

First order

Quick Tip: A rate constant with units of s^-1 means first order.

80. Which of the following is a water-in-oil type of emulsion?

(A) Milk

(B) Vanishing Cream

(C) Paint

(D) Butter

Correct Answer: (D) Butter

Solution:

Step 1: Understanding the Concept:
In an oil-in-water (O/W) emulsion, oil droplets are dispersed in water. In a water-in-oil (W/O) emulsion, water droplets are dispersed in oil.

Step 2: Check each:

Milk: fat droplets in water, O/W. Vanishing cream: O/W (it feels non-greasy). Paint: a dispersion of pigment, not a W/O emulsion.

Butter: tiny water droplets dispersed in fat, so it is W/O.

Final Answer:

Butter is a water-in-oil emulsion, option (D).

Butter

Quick Tip: In a W/O emulsion, water droplets are spread in oil, like butter.

81. What is O-O bond length in ozone ?

(A) 178 pm

(B) 128 pm

(C) 256 pm

(D) 483 pm

Correct Answer: (B) 128 pm

Solution:

Step 1: Understanding the Concept:

Ozone is a bent molecule with two resonance forms. Each O-O bond has a character between a single and a double bond, so the two bonds are identical.

Step 2: Compare with reference lengths:

A typical O-O single bond (as in H_2O_2) is about 148 pm. A typical O=O

double bond (as in O_2) is about 121 pm.
Ozone lies between these two, at 128 pm, and the bond angle is about 117°.

Step 3: Other options:
178 pm, 256 pm and 483 pm are much longer than even a single O-O bond, so they cannot be bond lengths in ozone.

Final Answer:
The O-O bond length in ozone is 128 pm, option (B).

$$\boxed{128 \text{ pm}}$$

Quick Tip: Both O-O bonds in ozone are equal because of resonance, and measure 128 pm.

82. Which of the following ions exhibit same value of spin only magnetic moment?
(A). Ti^{3+}
(B). Cr^{3+}
(C). Mn^{2+}
(D). Fe^{3+}
(E). Sc^{3+}
Choose the most appropriate answer from the options given below.

(A) (B) and (D) only

(B) (A) and (E) only

(C) (C) and (D) only

(D) (A) and (D) only

Correct Answer: (C) (C) and (D) only

Solution:

Step 1: Key Formula:

Spin-only magnetic moment is $\mu = \sqrt{n(n+2)}$ BM, where n is the number of unpaired electrons. Equal n means equal μ.

Step 2: Find the unpaired electrons:

(A) Ti^{3+}: $3d^1$, so $n = 1$.

(B) Cr^{3+}: $3d^3$, so $n = 3$.

(C) Mn^{2+}: $3d^5$, so $n = 5$.

(D) Fe^{3+}: $3d^5$, so $n = 5$.

(E) Sc^{3+}: $3d^0$, so $n = 0$.

Step 3: Pick the pair:

Only (C) and (D) share the same $n = 5$, and so $\mu = \sqrt{35} = 5.92$ BM for both.

The other options fail: (B) and (D) have 3 and 5 unpaired electrons, (A) and (E) have 1 and 0, and (A) and (D) have 1 and 5.

Final Answer:

Mn(II) and Fe(III) have the same spin-only moment, option (C).

$$\boxed{\text{(C) and (D) only}}$$

Quick Tip: Count unpaired electrons: Mn2+ and Fe3+ are both d5 with 5 unpaired electrons.

83. Which of the following cations in their oxidation states forms colored compounds ?

(A) Sc^{3+}

(B) Ti^{4+}

(C) Cu^{+}

(D) V^{3+}

Correct Answer: (D) V^{3+}

Solution:

Step 1: Understanding the Concept:

Transition metal ions are coloured because an electron can jump between split d orbitals (a d-d transition) by absorbing visible light. This needs a partly filled d subshell.

Step 2: Check each ion:

Sc^{3+}: $3d^0$, no d electron to excite, colourless.

Ti^{4+}: $3d^0$, colourless.

Cu^{+}: $3d^{10}$, completely filled, no d-d transition, colourless.

V^{3+}: $3d^2$, partly filled, so it shows d-d transition and is coloured (green).

Final Answer:

V^{3+} forms coloured compounds, option (D).

$$\boxed{V^{3+}}$$

Quick Tip: Colour needs partly filled d orbitals, as in V3+ (d2).

84. Which of the following statements is correct with respect to $[Mn(CN)_6]^{3-}$?

(A) It is sp^3d^2 hybridised and tetrahedral.

(B) It is d^2sp^3 hybridised and octahedral.

(C) It is dsp^2 hybridised and square planar

(D) It is sp^3d^2 hybridised and octahedral.

Correct Answer: (B) It is d^2sp^3 hybridised and octahedral.

Solution:

Step 1: Understanding the Concept:
Hybridisation in a complex depends on the metal oxidation state and whether the ligand is strong or weak field. Strong field ligands pair up the d electrons and free inner d orbitals.

Step 2: Find the metal configuration:
Charge: $x + 6(-1) = -3$, so $x = +3$. Mn(III) is $3d^4$.

Step 3: Fill the orbitals:
CN^- is a strong field ligand, so the four d electrons pair up in two of the three t_{2g} orbitals. This leaves two $3d$ orbitals empty.
Two $3d$, one $4s$ and three $4p$ orbitals mix to give six d^2sp^3 hybrid orbitals, which accept six ligand pairs.

Step 4: Conclusion:
The complex is d^2sp^3 and octahedral (low spin, inner orbital). Options A and D say sp^3d^2, which is used with weak ligands such as F^-.
Option A also says tetrahedral, and option C says square planar, which fits d^8 ions.

Final Answer:

The complex is d^2sp^3 hybridised and octahedral, option (B).

$$\boxed{d^2sp^3 \text{ and octahedral}}$$

Quick Tip: CN- is a strong field ligand, so d electrons pair up and the complex is inner orbital, d2sp3.

85. What is the importance of Sandmeyer's reaction?

(A) To obtain R-I from R-Cl

(B) To obtain Ar-X by replacing diazo group of $\mathbf{AR\text{-}N_2^+\text{-}X^-}$ using **CuCl / HCl**

(C) To obtain R-F from R-Cl using AgF

(D) To obtain $\mathbf{R\text{-}CH_2\text{-}R}$ from R-CO-R using Zn-Hg / HCl

Correct Answer: (B) To obtain Ar-X by replacing diazo group of $\mathbf{AR\text{-}N_2^+\text{-}X^-}$ using **CuCl / HCl**

Solution:

Step 1: Understanding the Concept:

A benzene diazonium salt can be changed to an aryl halide in a Sandmeyer reaction. The diazonium group $-N_2^+$ is replaced by **−Cl** or **−Br** using **CuCl/HCl** or **CuBr/HBr**.

Step 2: Reaction:

$$\mathbf{Ar\text{-}N_2^+X^-} \xrightarrow{\mathbf{CuCl/HCl}} \mathbf{Ar\text{-}Cl + N_2}$$

Step 3: Other options:

A (R-Cl → R-I) is the Finkelstein reaction. C (using AgF) is the Swarts reaction. D (using Zn-Hg/HCl) is the Clemmensen reduction.

Final Answer:

Sandmeyer reaction gives Ar-X from the diazonium salt, option (B).

Ar-X from diazonium salt using CuCl/HCl

Quick Tip: Sandmeyer reaction replaces the diazonium group by Cl or Br using CuCl or CuBr.

86. Which of the following favors SN^1 reaction?

(A) Strong nucleophile

(B) Non - Polar solvent

(C) Bulky alkyl groups on the carbon atom attached to halogen atom

(D) Small alkyl groups on the carbon atom attached to halogen atom

Correct Answer: (C) Bulky alkyl groups on the carbon atom attached to halogen atom

Solution:

Step 1: Understanding the Concept:

The S_N1 reaction is a two step reaction. The slow step is the loss of the leaving group to give a carbocation. Anything that stabilises this carbocation favours S_N1.

Step 2: Check each option:

A: A strong nucleophile favours S_N2, since the nucleophile takes part in the rate step. In S_N1 it does not matter. So A is wrong.

B: S_N1 forms ions, which need a polar protic solvent to stabilise them. A non-polar solvent is wrong.

C: Bulky alkyl groups (as in tertiary halides) give a stable tertiary carbocation through the $+I$ effect and hyperconjugation. This favours S_N1. It is correct.

D: Small alkyl groups (methyl, primary) form unstable carbocations and are open to back-side attack, so they favour S_N2.

Final Answer:

Bulky alkyl groups on the carbon carrying the halogen favour S_N1, option (C).

Bulky alkyl groups

Quick Tip: SN1 goes through a carbocation, so bulky groups that stabilise it favour SN1.

87. How many moles of hydrogen are liberated when 6 moles of ethyl alcohol reacts with sodium metal?

(A) 2

(B) 3

(C) 4

(D) 6

Correct Answer: (B) 3

Solution:

Step 1: Understanding the Concept:

Alcohols react with sodium metal to give sodium alkoxide and hydrogen gas. The balanced equation tells the mole ratio.

Step 2: Balanced equation:

$$2C_2H_5OH + 2Na \rightarrow 2C_2H_5ONa + H_2$$

Step 3: Calculate:

2 mol of ethanol give 1 mol of H_2. So 6 mol of ethanol give $\frac{6}{2} = 3$ mol of H_2.

Step 4: Other options:

6 mol would be the answer if each ethanol released one H_2. Each molecule only has one acidic hydrogen (the O-H), and two such hydrogens are needed per H_2.

Final Answer:

Three moles of hydrogen are liberated, option (B).

$$\boxed{3 \text{ mol}}$$

Quick Tip: Two moles of ethanol give one mole of hydrogen with sodium.

88. Find the order of formation of alcohol from alkene from the following.

(A) $3^\circ > 1^\circ > 2^\circ$

(B) $3^\circ > 2^\circ > 1^\circ$

(C) $1^\circ > 2^\circ > 3^\circ$

(D) $1^\circ > 3^\circ > 2^\circ$

Correct Answer: (B) $3^\circ > 2^\circ > 1^\circ$

Solution:

Step 1: Understanding the Concept:
In acid-catalysed hydration, the alkene takes a proton to form a carbocation, and water then adds to it. The easier the carbocation forms, the faster that alcohol forms.

Step 2: Apply:
Carbocation stability is $3^\circ > 2^\circ > 1^\circ$, because alkyl groups stabilise the positive charge through the $+I$ effect and hyperconjugation.
By Markovnikov's rule, the proton adds so that the more stable carbocation forms. So tertiary alcohols form most readily and primary alcohols least readily.

Step 3: Order:

$$3^\circ > 2^\circ > 1^\circ$$

This is option B. The other options place primary alcohols too high, which would need the least stable carbocation to form fastest.

Final Answer:
The order of formation is $3^\circ > 2^\circ > 1^\circ$, option (B).

$$\boxed{3^\circ > 2^\circ > 1^\circ}$$

Quick Tip: Acid-catalysed hydration goes through carbocations, so 3 > 2 > 1.

89. Which of the following compounds dose NOT respond positively to the iodoform test ?

(A) Acetaldehyde

(B) Propan-2-ol

(C) Propanone

(D) Pentan-3-one

Correct Answer: (D) Pentan-3-one

Solution:

Step 1: Understanding the Concept:

The iodoform test is positive for compounds having a CH_3CO- group or a $CH_3CH(OH)$- group, which is oxidised to the former by I_2/NaOH.

Step 2: Check each:

A: Acetaldehyde, CH_3CHO, has CH_3CO-. Positive.

B: Propan-2-ol, $CH_3CH(OH)CH_3$, has $CH_3CH(OH)$-. Positive.

C: Propanone, CH_3COCH_3, has CH_3CO-. Positive.

D: Pentan-3-one, $CH_3CH_2COCH_2CH_3$, has a carbonyl flanked by two CH_2 groups and no CH_3CO- unit. Negative.

Final Answer:

Pentan-3-one does not respond to the iodoform test, option (D).

Pentan-3-one

Quick Tip: Iodoform test needs a CH_3CO- group or a $CH_3CH(OH)$- group.

90. Which of the following compounds can not form iodoform?

(A) Ethyl methyl ketone

(B) Isopropyl alcohol

(C) 3-Methyl - 2- butanone

(D) Isobutyl alcohol

Correct Answer: (D) Isobutyl alcohol

Solution:

Step 1: Understanding the Concept:

Iodoform forms from compounds containing CH_3CO- or $CH_3CH(OH)$-. Ethanol is the only primary alcohol that does.

Step 2: Check each:

A: Ethyl methyl ketone, $CH_3COCH_2CH_3$, has CH_3CO-. It gives iodoform.

B: Isopropyl alcohol, $CH_3CH(OH)CH_3$, has $CH_3CH(OH)$-. It gives iodoform.

C: 3-Methylbutan-2-one, $CH_3COCH(CH_3)_2$, has CH_3CO-. It gives iodoform.

D: Isobutyl alcohol, $(CH_3)_2CHCH_2OH$, is a primary alcohol. It has no $CH_3CH(OH)$- unit, so it cannot form iodoform.

Final Answer:

Isobutyl alcohol cannot form iodoform, option (D).

Isobutyl alcohol

Quick Tip: Look for CH3CO- or CH3CH(OH)-; isobutyl alcohol is a primary alcohol with neither.

91. Which is a weakest acid among the following ?

(A) $(CH_3)_2CHCOOH$

(B) CH_3COOH

(C) $ClCH_2COOH$

(D) Cl_3CCOOH

Correct Answer: (A) $(CH_3)_2CHCOOH$

Solution:

Step 1: Understanding the Concept:

A carboxylic acid is stronger if its carboxylate ion is more stable. Electron withdrawing groups (such as Cl) stabilise the anion. Electron donating groups (such as alkyl) destabilise it.

Step 2: Rank:

Cl_3CCOOH: three Cl atoms, strongest $-I$ effect, strongest acid (pK_a about 0.65).

$ClCH_2COOH$: one Cl, pK_a about 2.86.

CH_3COOH: pK_a about 4.76.

$(CH_3)_2CHCOOH$: two methyl groups push electrons towards the

carboxyl group ($+I$ effect), making the ion least stable. pK_a about 4.86.

Step 3: Order:

$$Cl_3CCOOH > ClCH_2COOH > CH_3COOH > (CH_3)_2CHCOOH$$

The weakest is option A.

Final Answer:

The weakest acid is $(CH_3)_2CHCOOH$, option (A).

$$\boxed{(CH_3)_2CHCOOH}$$

Quick Tip: Electron donating alkyl groups weaken the acid, and electron withdrawing Cl strengthens it.

92. Which of the following amines is not obtained by Gabriel phthalimide synthesis?

(A) Aniline

(B) Benzyl amine

(C) Methyl amine

(D) Isobutyl amine

Correct Answer: (A) Aniline

Solution:

Step 1: Understanding the Concept:

Gabriel phthalimide synthesis converts primary alkyl halides to

primary amines. Potassium phthalimide attacks the alkyl halide by an S_N2 route, and then hydrolysis frees the amine.

Step 2: Apply:
Aryl halides do not undergo nucleophilic substitution under these conditions, because the C-X bond has partial double bond character and the ring blocks back-side attack. So aniline cannot be made. Benzylamine (from benzyl chloride), methylamine (from methyl iodide) and isobutylamine (from isobutyl bromide) all come from primary alkyl halides, so they can be made.

Final Answer:
Aniline is not obtained by this method, option (A).

Aniline

Quick Tip: Gabriel synthesis needs an alkyl halide for SN2, so aryl amines cannot be made.

93. What type of amine is Iso propyl amine ?

(A) primary amine

(B) secondary amine

(C) tertiary amine

(D) quaternary ammonium salt

Correct Answer: (A) primary amine

Solution:

Step 1: Understanding the Concept:
Amines are classified by the number of alkyl or aryl groups attached to nitrogen: one is primary, two is secondary, three is tertiary.

Step 2: Apply:
Isopropylamine is $(CH_3)_2CH\text{-}NH_2$. Nitrogen carries one isopropyl group and two hydrogens.
So it is a primary amine. The classification does not depend on the carbon being secondary, only on the nitrogen. Options B, C and D need more groups on nitrogen.

Final Answer:
Isopropylamine is a primary amine, option (A).

Primary amine

Quick Tip: Count the carbon groups attached to nitrogen: isopropylamine has only one.

94. What is the correct sequence of the complementary strand for the given portion of a DNA molecule:- 5' - G A C G T A C -3' ?

(A) 3' - T T G C A T G - 5'

(B) 3' - C T G C A T C - 5'

(C) 3' - C T G C A T G - 5'

(D) 3' - C T G T A T G - 5'

Correct Answer: (C) 3' - C T G C A T G - 5'

Solution:

Step 1: Understanding the Concept:
The two strands of DNA are antiparallel and complementary. Adenine pairs with thymine (A-T), and guanine pairs with cytosine (G-C).

Step 2: Pair each base:
Given strand: 5′ G A C G T A C 3′.
G pairs with C, A with T, C with G, G with C, T with A, A with T, C with G.
The complement written along the same direction is: C T G C A T G, and it runs from 3′ to 5′.

Step 3: Result:
Complementary strand: 3′ C T G C A T G 5′. This is option C.
Option A uses T instead of C in the first position. Options B and D each have a base pair error (for example, ending in C or having T in place of C).

Final Answer:
The complementary strand is 3′-CTGCATG-5′, option (C).

$$\boxed{3'\text{-CTGCATG-}5'}$$

Quick Tip: Pair A with T and G with C, and write the new strand in the opposite direction.

95. Which of the following compounds forms starch on polymerization?

(A) α-D-glucose.

(B) β-D-glucose.

(C) α-L-glucose

(D) β-L-glucose

Correct Answer: (A) α-D-glucose.

Solution:

Step 1: Understanding the Concept:
Starch has two parts: amylose and amylopectin. Both are built from α-D-glucose units joined by glycosidic links.

Step 2: Details:
Amylose has α-1,4 links in a long unbranched chain. Amylopectin also has α-1,6 links at branch points.

Step 3: Other options:
β-D-glucose forms cellulose. L-glucose is not found in natural polysaccharides. So B, C and D are wrong.

Final Answer:
Starch is formed from α-D-glucose, option (A).

$\boxed{\alpha\text{-D-glucose}}$

Quick Tip: Starch is a polymer of alpha-D-glucose, while cellulose is a polymer of beta-D-glucose.

96. Name the catalyst used in the manufacture of HDPE polymer.

(A) Triethyl aluminum with titanium tetrachloride

(B) Triethyl aluminum with zinc chloride

(C) Triethyl aluminum with nickel chloride

(D) Triethyl aluminum with stannic chloride

Correct Answer: (A) Triethyl aluminum with titanium tetrachloride

Solution:

Step 1: Understanding the Concept:

High density polyethylene (HDPE) is a linear polymer with little branching. It is made at low pressure (6 to 7 atm) and 333 to 343 K.

Step 2: Catalyst:

The catalyst is the Ziegler-Natta catalyst, a mixture of triethylaluminium $Al(C_2H_5)_3$ and titanium tetrachloride $TiCl_4$. Low density polyethylene (LDPE) is instead made at high pressure with free radical initiators such as peroxides.

Step 3: Other options:

Zinc chloride, nickel chloride and stannic chloride are not used with triethylaluminium for this process.

Final Answer:

The catalyst is triethylaluminium with titanium tetrachloride, option (A).

$$\boxed{Al(C_2H_5)_3 + TiCl_4}$$

Quick Tip: HDPE is made with the Ziegler-Natta catalyst, triethylaluminium with titanium tetrachloride.

97. Identify the polymer obtained by addition polymerization method from following.

(A) Nylon 6

(B) Dacron

(C) Polythene

(D) Nylon 6,6

Correct Answer: (C) Polythene

Solution:

Step 1: Understanding the Concept:
Addition polymers form when unsaturated monomers add to each other with no loss of small molecules. Condensation polymers form with loss of a small molecule such as water.

Step 2: Check each:
Nylon 6: from caprolactam (ring opening, with water as the initiator), classed as a condensation polymer.
Dacron: from ethylene glycol and terephthalic acid, a polyester formed by condensation.
Polythene: from $CH_2 = CH_2$ by addition. Correct.
Nylon 6,6: from hexamethylenediamine and adipic acid, a condensation polymer.

Final Answer:
Polythene is the addition polymer, option (C).

Polythene

Quick Tip: Polythene is made from ethene by addition; nylons and Dacron are condensation polymers.

98. Which of the following is chain growth polymer?

(A) Nylon-6

(B) Teflon

(C) Nylon-66

(D) Terylene

Correct Answer: (B) Teflon

Solution:

Step 1: Understanding the Concept:
Chain growth polymers form by repeated addition of monomer to an active site (free radical, cation or anion). Step growth polymers form by repeated condensation between functional groups.

Step 2: Check each:
Nylon-6, Nylon-66 and Terylene are step growth (condensation) polymers, formed from amide or ester linkages.
Teflon is made from tetrafluoroethene, $CF_2 = CF_2$, by free radical addition. It is a chain growth polymer.

Final Answer:
Teflon is the chain growth polymer, option (B).

Teflon

Quick Tip: Chain growth polymers come from addition polymerisation, like Teflon from tetrafluoroethene.

99. Identify the use of Formalin from the followings.

(A) Preservation of biological and anatomical specimens

(B) Disinfectant

(C) Beverage

(D) Antiseptic

Correct Answer: (A) Preservation of biological and anatomical specimens

Solution:

Step 1: Understanding the Concept:
Formalin is a 40% aqueous solution of formaldehyde (methanal), HCHO.

Step 2: Use:
It hardens and preserves tissues by cross-linking proteins, so it is used to preserve biological and anatomical specimens. It is also used to make bakelite and urea-formaldehyde resins.

Step 3: Other options:
It is toxic, so it cannot be used as a beverage and is unsuitable as an antiseptic for living skin. Disinfectant is a minor use, but the standard answer to this question is preservation of specimens.

Final Answer:

Formalin is used for preservation of biological and anatomical specimens, option (A).

Preservation of biological and anatomical specimens

Quick Tip: Formalin is a 40 percent solution of methanal used to preserve specimens.

100. Identify the term used for drugs that reduce or relieve pain without causing loss of consciousness.

(A) Analgesic

(B) Antipyretic

(C) Antibiotic

(D) Antimicrobial

Correct Answer: (A) Analgesic

Solution:

Step 1: Understanding the Concept:
Drugs are classed by the effect they have. We need the class that relieves pain without causing unconsciousness, mental confusion or other disturbance of the nervous system.

Step 2: Check each:
Analgesic (such as aspirin and paracetamol) relieves pain without loss of consciousness. Correct.
Antipyretic lowers body temperature during fever.
Antibiotic kills or stops the growth of bacteria.

Antimicrobial acts against microbes such as bacteria, fungi and viruses in general.

Final Answer:

The term is analgesic, option (A).

Analgesic

Quick Tip: Analgesics reduce pain without affecting consciousness.

Mathematics

Questions 101 to 150 | 2 marks each | 100 marks

101. The quadratic polynomial $p(x)$ has roots 1 and α, while quadratic polynomial $q(x)$ has roots 1 and β. Let α and β be the roots of $r(x) = p(x) + q(x)$. Then $\lim_{x\to\infty}[\sqrt{p(x)} - \sqrt{q(x)}] =$

(A) 0

(B) -1

(C) 1

(D) $\frac{1}{2}$

Correct Answer: (A) 0

Solution:

Step 1: Understanding the Question:

We take p and q as monic quadratics, as the question intends. Then $p(x) = (x-1)(x-\alpha)$ and $q(x) = (x-1)(x-\beta)$. The roots of $r = p + q$ are required to be α and β.

Step 2: Form r(x):

$$r(x) = (x-1)\big[(x-\alpha) + (x-\beta)\big] = (x-1)(2x-\alpha-\beta)$$

So the roots of r are 1 and $\frac{\alpha+\beta}{2}$.

Step 3: Match the roots:

The set $\{\alpha, \beta\}$ must equal $\{1, \frac{\alpha+\beta}{2}\}$.

Case 1: $\alpha = 1$. Then $\beta = \frac{1+\beta}{2}$, which gives $\beta = 1$.

Case 2: $\alpha = \frac{\alpha+\beta}{2}$, so $\alpha = \beta$, and then $\beta = 1$ as the other root, so again α

$= \beta = 1$.
In both cases $\alpha = \beta = 1$.

Step 4: Evaluate the limit:
Now $p(x) = q(x) = (x-1)^2$, so $\sqrt{p(x)} - \sqrt{q(x)} = |x-1| - |x-1| = 0$ for every x.

$$\lim_{x \to \infty} \left[\sqrt{p(x)} - \sqrt{q(x)}\right] = 0$$

Step 5: Why the other options are wrong:
A nonzero limit such as ± 1 or $\frac{1}{2}$ would need the two quadratics to differ in their linear term, which would happen only if $\alpha \neq \beta$. The root condition rules this out.

Final Answer:
The two polynomials are identical, so the limit is 0, option (A).

$$\boxed{0}$$

Quick Tip: Show that the root condition forces $\alpha = \beta = 1$, so $p(x) = q(x)$.

102. If $x = \sqrt{-1 - \sqrt{-1 - \sqrt{-1 - \ldots \infty}}}$, where ω is a non-real complex cube root of unity, then the value of x is...

(A) 1

(B) -1

(C) $-\omega$

(D) ω^2

Correct Answer: (D) ω^2

Solution:

Step 1: Understanding the Question:

The radical repeats itself inside, so the whole expression equals x. That means $x = \sqrt{-1 - x}$.

Step 2: Form the equation:

Square both sides: $x^2 = -1 - x$, so

$$x^2 + x + 1 = 0$$

Step 3: Link to cube roots of unity:

The non-real cube roots of unity satisfy $1 + \omega + \omega^2 = 0$, and also $\omega^2 + \omega + 1 = 0$. So the roots of $x^2 + x + 1 = 0$ are $x = \omega$ and $x = \omega^2$.

Step 4: Match with the options:

Option A (1): $1 + 1 + 1 = 3 \neq 0$. Option B (-1): $1 - 1 + 1 = 1 \neq 0$. Option C ($-\omega$): $\omega^2 - \omega + 1 = -2\omega \neq 0$. Option D ($\omega^2$): $\omega^4 + \omega^2 + 1 = \omega + \omega^2 + 1 = 0$. It satisfies the equation.

Final Answer:

The value of x is ω^2, option (D).

$\boxed{\omega^2}$

Quick Tip: Put $x = \sqrt{-1-x}$, square it, and get $x^2 + x + 1 = 0$.

103. If $sin3\alpha = 4sin\alpha \cdot sin(x+\alpha) \cdot sin(x-\alpha)$ where $\alpha \neq n\pi, n \in Z$, then all possible values of x are given as

(A) $x = n\pi \pm \frac{\pi}{3}, n \in Z$

(B) $x = n\pi \pm \frac{\pi}{4}, n \in Z$

(C) $x = n\pi \pm \frac{\pi}{6}, n \in Z$

(D) $x = n\pi \pm \frac{\pi}{2}, n \in Z$

Correct Answer: (A) $x = n\pi \pm \frac{\pi}{3}, n \in Z$

Solution:

Step 1: Understanding the Question:

The equation has to hold for the given α, with $\sin\alpha \neq 0$. We can divide by $\sin\alpha$.

Step 2: Key identities:

$\sin(x+\alpha)\sin(x-\alpha) = \sin^2 x - \sin^2\alpha$.

$\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$.

Step 3: Solve:

Substitute: $3\sin\alpha - 4\sin^3\alpha = 4\sin\alpha(\sin^2 x - \sin^2\alpha)$.

Divide by $\sin\alpha$: $3 - 4\sin^2\alpha = 4\sin^2 x - 4\sin^2\alpha$.

The $\sin^2\alpha$ terms cancel, so $4\sin^2 x = 3$, i.e. $\sin^2 x = \frac{3}{4}$.

$$\sin x = \pm\frac{\sqrt{3}}{2} \Rightarrow x = n\pi \pm \frac{\pi}{3},\ n \in Z$$

Step 4: Other options:

$\frac{\pi}{4}$ gives $\sin^2 x = \frac{1}{2}$, $\frac{\pi}{6}$ gives $\frac{1}{4}$, and $\frac{\pi}{2}$ gives 1. None equals $\frac{3}{4}$.

Final Answer:

All values of x are $n\pi \pm \frac{\pi}{3}$, option (A).

$$\boxed{x = n\pi \pm \frac{\pi}{3},\ n \in Z}$$

Quick Tip: Use $\sin(x+\alpha)\sin(x-\alpha) = \sin^2 x - \sin^2 \alpha$ and $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$.

104. If $cos(\theta - \alpha) = a$ and $sin(\theta - \beta) = b$, then the value of $cos^2(\alpha - \beta) + 2absin(\alpha - \beta) + cos^2(\theta - \alpha)$ is...

(A) $a^2 - 2b^2$

(B) $a^2 + 2b^2$

(C) $2a^2 - b^2$

(D) $2a^2 + b^2$

Correct Answer: (D) $2a^2 + b^2$

Solution:

Step 1: Understanding the Question:

Let $u = \theta - \alpha$ and $v = \theta - \beta$. Then $\cos u = a$, $\sin v = b$ and $\alpha - \beta = v - u$.

Step 2: Expand:

$\cos(v-u) = \cos v \cos u + \sin v \sin u = a\cos v + b\sin u.$

$\sin(v-u) = \sin v \cos u - \cos v \sin u = ab - \cos v \sin u.$

Step 3: Combine the first two terms:

$$\cos^2(\alpha-\beta) = a^2\cos^2 v + 2ab\cos v \sin u + b^2\sin^2 u$$

$$2ab\sin(\alpha-\beta) = 2a^2b^2 - 2ab\cos v \sin u$$

Adding, the cross terms cancel: $a^2\cos^2 v + b^2\sin^2 u + 2a^2b^2$.

Step 4: Simplify:

Use $\cos^2 v = 1-b^2$ and $\sin^2 u = 1-a^2$: $a^2(1-b^2) + b^2(1-a^2) + 2a^2b^2 = a^2 + b^2$.

The last term is $\cos^2(\theta-\alpha) = a^2$. So the full expression is $a^2 + b^2 + a^2 = 2a^2 + b^2$.

Final Answer:

The value is $2a^2 + b^2$, option (D).

$$\boxed{2a^2 + b^2}$$

Quick Tip: Write $\alpha - \beta = (\theta - \beta) - (\theta - \alpha)$ and expand with the given a and b.

105. The general solution of $cos\theta - sin\theta = 1$ is

(A) $\theta = 2n\pi - \frac{\pi}{2}$ or $\theta = 2n\pi, n \in Z$

(B) $\theta = n\pi - \frac{\pi}{2}$ or $\theta = n\pi, n \in Z$

(C) $\theta = n\frac{\pi}{2} - \pi$ or $\theta = n\frac{\pi}{2}, n \in Z$

(D) $\theta = 2n\pi + \frac{\pi}{2}$ or $\theta = n\pi, n \in Z$

Correct Answer: (A) $\theta = 2n\pi - \frac{\pi}{2}$ or $\theta = 2n\pi, n \in Z$

Solution:

Step 1: Understanding the Question:

We rewrite the left side as a single cosine using the identity $a\cos\theta + b\sin\theta = R\cos(\theta - \phi)$, with $R = \sqrt{a^2 + b^2}$.

Step 2: Convert:

$$\cos\theta - \sin\theta = \sqrt{2}\left(\frac{1}{\sqrt{2}}\cos\theta - \frac{1}{\sqrt{2}}\sin\theta\right) = \sqrt{2}\cos\left(\theta + \frac{\pi}{4}\right)$$

Step 3: Solve:

$\sqrt{2}\cos(\theta + \frac{\pi}{4}) = 1$ gives $\cos(\theta + \frac{\pi}{4}) = \frac{1}{\sqrt{2}} = \cos\frac{\pi}{4}$.

So $\theta + \frac{\pi}{4} = 2n\pi \pm \frac{\pi}{4}$.

With the plus sign: $\theta = 2n\pi$. With the minus sign: $\theta = 2n\pi - \frac{\pi}{2}$.

Step 4: Other options:

Options B, C and D use $n\pi$ or $\frac{n\pi}{2}$ and so include angles such as π, where $\cos\pi - \sin\pi = -1 \neq 1$. They are wrong.

Final Answer:

The general solution is $\theta = 2n\pi$ or $2n\pi - \frac{\pi}{2}$, option (A).

$$\boxed{\theta = 2n\pi - \frac{\pi}{2} \text{ or } \theta = 2n\pi}$$

Quick Tip: Write $\cos\theta - \sin\theta = \sqrt{2}\cos(\theta + \pi/4)$.

106. The equation of the straight line passing through the point $(-5, 3)$ such that the portion of the line intercepted between the axes is divided by the point in the ratio $4 : 3$ (from the X-axis to the Y-axis) is...

(A) $9x + 20y + 105 = 0$

(B) $9x - 20y - 105 = 0$

(C) $9x - 20y + 105 = 0$

(D) $9x + 20y - 105 = 0$

Correct Answer: (C) $9x - 20y + 105 = 0$

Solution:

Step 1: Understanding the Question:

Let the line meet the X-axis at $A(a, 0)$ and the Y-axis at $B(0, b)$. The point $P(-5, 3)$ divides AB in the ratio $4 : 3$, measured from the X-axis side to the Y-axis side, so $AP : PB = 4 : 3$.

Step 2: Use the section formula:

$$P = \left(\frac{4 \cdot 0 + 3a}{7}, \frac{4b + 3 \cdot 0}{7}\right) = \left(\frac{3a}{7}, \frac{4b}{7}\right)$$

So $\frac{3a}{7} = -5$, which gives $a = -\frac{35}{3}$, and $\frac{4b}{7} = 3$, which gives $b = \frac{21}{4}$.

Step 3: Write the line:

Intercept form: $\frac{x}{a} + \frac{y}{b} = 1$, i.e. $-\frac{3x}{35} + \frac{4y}{21} = 1$.
Multiply by the LCM 105: $-9x + 20y = 105$, so $9x - 20y + 105 = 0$.

Step 4: Verify:
Check $P(-5, 3)$: $9(-5) - 20(3) + 105 = -45 - 60 + 105 = 0$. It lies on the line. Option A has $+20y$ and cannot pass through $(-5, 3)$ with the right intercepts, and options B and D have the wrong constant sign.

Final Answer:
The required line is $9x - 20y + 105 = 0$, option (C).

$$\boxed{9x - 20y + 105 = 0}$$

Quick Tip: Let the intercepts be $(a, 0)$ and $(0, b)$, then use the section formula to place $(-5, 3)$.

107. If the equation $kx^2 - 5xy + 6y^2 + x - 3y = 0$ represents a pair of straight lines, then the co-ordinates of their point of intersection are

(A) $(-3, -1)$

(B) $(-3, 1)$

(C) $(3, -1)$

(D) $(3, 1)$

Correct Answer: (A) $(-3, -1)$

Solution:

Step 1: Understanding the Question:
For $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ to be a pair of lines, abc

$+2fgh - af^2 - bg^2 - ch^2 = 0$.

Step 2: Find k:

Here $a = k, b = 6, h = -\frac{5}{2}, g = \frac{1}{2}, f = -\frac{3}{2}, c = 0$.

$$0 + 2\left(-\frac{3}{2}\right)\left(\frac{1}{2}\right)\left(-\frac{5}{2}\right) - k \cdot \frac{9}{4} - 6 \cdot \frac{1}{4} - 0 = 0$$

$\frac{15}{4} - \frac{9k}{4} - \frac{6}{4} = 0$, so $k = 1$.

Step 3: Factorise:

With $k = 1$: $x^2 - 5xy + 6y^2 + x - 3y = (x - 2y)(x - 3y) + (x - 3y) = (x - 3y)(x - 2y + 1)$.

The lines are $x - 3y = 0$ and $x - 2y + 1 = 0$.

Step 4: Intersect:

From the first, $x = 3y$. In the second: $3y - 2y + 1 = 0$, so $y = -1$ and $x = -3$. The point is $(-3, -1)$.

Final Answer:

The lines meet at $(-3, -1)$, option (A).

$$\boxed{(-3, -1)}$$

Quick Tip: Find k from the pair-of-lines condition, factorise, and solve the two lines.

108. The pair of straight lines represented by the equation $\sqrt{3}x^2 - 4xy + \sqrt{3}y^2 = 0$ are

(A) coincident.

(B) perpendicular.

(C) inclined at $30°$ to each other.

(D) inclined at $60°$ to each other.

Correct Answer: (C) inclined at $30°$ to each other.

Solution:

Step 1: Key Formula:

For $ax^2 + 2hxy + by^2 = 0$, the acute angle between the lines satisfies $\tan\theta = \dfrac{2\sqrt{h^2 - ab}}{|a + b|}$.

Step 2: Read off values:

$a = \sqrt{3}$, $b = \sqrt{3}$, $2h = -4$ so $h = -2$.

Step 3: Calculate:

$h^2 - ab = 4 - 3 = 1$. Then $\tan\theta = \frac{2 \cdot 1}{2\sqrt{3}} = \frac{1}{\sqrt{3}}$, so $\theta = 30°$.

The lines are real and distinct since $h^2 > ab$. They are not perpendicular since $a + b \neq 0$.

Final Answer:

The lines are inclined at $30°$, option (C).

$$\boxed{30°}$$

Quick Tip: Use $\tan\theta = \frac{2\sqrt{h^2 - ab}}{|a+b|}$.

109. The equation of the circle concentric with circle $x^2 + y^2 - 6x + 7 = 0$ and which touches the line $x + y + 3 = 0$ is ....

(A) $x^2 + y^2 - 6x + 9 = 0$

(B) $x^2 + y^2 - 6x - 9 = 0$

(C) $x^2 + y^2 - 6x + 3 = 0$

(D) $x^2 + y^2 - 6x - 3 = 0$

Correct Answer: (B) $x^2 + y^2 - 6x - 9 = 0$

Solution:

Step 1: Understanding the Question:

Concentric circles share a centre. The new circle touches the line, so its radius is the perpendicular distance from the centre to the line.

Step 2: Find the centre:

$x^2 + y^2 - 6x + 7 = 0$ has centre $(3, 0)$.

Step 3: Find the radius:

$$r = \frac{|3 + 0 + 3|}{\sqrt{1^2 + 1^2}} = \frac{6}{\sqrt{2}} = 3\sqrt{2}, \quad r^2 = 18$$

Step 4: Equation:

$(x - 3)^2 + y^2 = 18$, so $x^2 + y^2 - 6x + 9 - 18 = 0$, i.e. $x^2 + y^2 - 6x - 9 = 0$.

Option A has radius 0 (a point), and C and D have radius squared 6 and 12, which do not equal 18.

Final Answer:

The circle is $x^2 + y^2 - 6x - 9 = 0$, option (B).

$$\boxed{x^2 + y^2 - 6x - 9 = 0}$$

Quick Tip: Same centre $(3, 0)$; radius equals the distance from the centre to the tangent line.

110. The distance of the focus of the parabola $y^2 = 16x$ from its directrix is...

(A) 4

(B) 8

(C) 12

(D) 16

Correct Answer: (B) 8

Solution:

Step 1: Key Formula:

For $y^2 = 4ax$, the focus is $(a, 0)$ and the directrix is $x = -a$.

Step 2: Calculate:

$4a = 16$, so $a = 4$. The focus is $(4, 0)$ and the directrix is $x = -4$.

Distance $= 4 - (-4) = 8$.

Step 3: Other options:

4 is just a, and 16 is $4a$ (the length of the latus rectum). 12 has no meaning here.

Final Answer:

The distance is 8 units, option (B).

$$\boxed{8}$$

Quick Tip: For $y^2 = 4ax$ the distance from focus to directrix is $2a$.

111. The value of $\lim_{n\to\infty} \frac{(n+2)!+(n+1)!}{(n+2)!-(n+1)!}$ is _____

(A) -1

(B) 0

(C) 1

(D) 2

Correct Answer: (C) 1

Solution:

Step 1: Understanding the Question:

We write $(n+2)! = (n+2)(n+1)!$ so that every term has a common factor.

Step 2: Simplify:

Numerator: $(n+1)!\big[(n+2)+1\big] = (n+1)!\,(n+3)$.

Denominator: $(n+1)!\big[(n+2)-1\big] = (n+1)!\,(n+1)$.

The ratio is $\dfrac{n+3}{n+1}$.

Step 3: Take the limit:

$$\lim_{n\to\infty}\frac{n+3}{n+1}=\lim_{n\to\infty}\frac{1+3/n}{1+1/n}=1$$

Final Answer:

The limit is 1, option (C).

$$\boxed{1}$$

Quick Tip: Take $(n+1)!$ common from top and bottom.

112. If the truth value of the compound statement $[(p\vee q)\wedge(q\to r)\wedge(\sim r)]\to(p\wedge q)$ is false, then the truth values of $p\to q$ and $q\to p$ are respectively...

(A) (F,T)

(B) (T,F)

(C) (T,T)

(D) (F,F)

Correct Answer: (A) (F,T)

Solution:

Step 1: Understanding the Concept:

The statement has the form $A\to B$. It is false only when A is true and B is false. Here $A=(p\vee q)\wedge(q\to r)\wedge(\sim r)$ and $B=p\wedge q$.

Step 2: Make A true:

All three parts of A must be true. From $\sim r$ true we get $r=F$. Then q

$\rightarrow r$ is true with $r = F$ only when $q = F$. Then $p \vee q$ is true with $q = F$ only when $p = T$.

Step 3: Check B:
With $p = T$ and $q = F$ we get $p \wedge q = F$, as needed.

Step 4: Find the required values:
$p \rightarrow q = T \rightarrow F = F$. $q \rightarrow p = F \rightarrow T = T$. So the pair is (F, T).

Step 5: Why the other options are wrong:
Since $q = F$, the statement $q \rightarrow p$ is true, which rules out (T, F) and (F, F). Since $p = T$ and $q = F$, the statement $p \rightarrow q$ is false, which rules out (T, T).

Final Answer:
The truth values are (F, T), which is option (A).

Option A: (F, T)

Quick Tip: The whole statement is false only when the bracket is true and p and q is false; solve for r, then q, then p.

113. Which of the following statements is logically equivalent to $\sim (p \leftrightarrow q)$?

(A) $\sim p \rightarrow q$

(B) $\sim p \leftrightarrow \sim q$

(C) $\sim (q \rightarrow \sim p)$

(D) $p \leftrightarrow \sim q$

Correct Answer: (D) $p \leftrightarrow \sim q$

Solution:

Step 1: Meaning of the biconditional

$p \leftrightarrow q$ is true when p and q have the same truth value. So $\sim (p \leftrightarrow q)$ is true when they differ.

Step 2: Truth table

For (p,q) = (T,T): $p \leftrightarrow q$ = T, so $\sim$ gives F. For (T,F): F, so $\sim$ gives T. For (F,T): F, so $\sim$ gives T. For (F,F): T, so $\sim$ gives F. Thus $\sim (p \leftrightarrow q)$ has column F, T, T, F.

Step 3: Check option D

$p \leftrightarrow \sim q$: (T,T): $\sim q$=F, so F. (T,F): $\sim q$=T, so T. (F,T): $\sim q$=F, so T. (F,F): $\sim q$=T, so F. Column F, T, T, F, which matches.

Step 4: Check the others

(A) $\sim p \rightarrow q$ is $p \vee q$, column T, T, T, F. No. (B) $\sim p \leftrightarrow \sim q$ equals $p \leftrightarrow q$, column T, F, F, T. No. (C) $\sim (q \rightarrow \sim p)$ equals $q \wedge p$, column T, F, F, F. No.

Final Answer:

$\sim (p \leftrightarrow q) \equiv p \leftrightarrow \sim q$, option (D).

$$\boxed{p \leftrightarrow \sim q}$$

Quick Tip: Negating a biconditional gives true when p and q differ. Compare truth tables with the options.

114. If $A = \begin{bmatrix} sec\theta & -tan\theta \\ -tan\theta & sec\theta \end{bmatrix}$, $\theta \in (0, \frac{\pi}{2})$ such that $A + adjA = 4I$, then θ =

(A) $\frac{\pi}{12}$

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{4}$

Correct Answer: (C) $\frac{\pi}{3}$

Solution:

Step 1: Key Formula:

For $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $\text{adj}\, A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Step 2: Find adj A:

Here $a = d = \sec\theta$ and $b = c = -\tan\theta$. So $\text{adj}\, A = \begin{bmatrix} \sec\theta & \tan\theta \\ \tan\theta & \sec\theta \end{bmatrix}$.

Step 3: Add:

$A + \text{adj}\, A = \begin{bmatrix} 2\sec\theta & 0 \\ 0 & 2\sec\theta \end{bmatrix} = 2\sec\theta I$.

Set $2\sec\theta = 4$, so $\sec\theta = 2$ and $\cos\theta = \frac{1}{2}$. With $\theta \in (0, \frac{\pi}{2})$, $\theta = \frac{\pi}{3}$.

Final Answer:

The value is $\theta = \frac{\pi}{3}$, option (C).

$$\boxed{\frac{\pi}{3}}$$

Quick Tip: For a 2 by 2 matrix, adj$A$ swaps the diagonal and changes signs of the off-diagonal entries.

115. The element in 1st row and 2nd column of the inverse of the matrix $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$ is ...

(A) $-\frac{2}{5}$

(B) $-\frac{10}{3}$

(C) $-\frac{2}{25}$

(D) $\frac{1}{25}$

Correct Answer: (A) $-\frac{2}{5}$

Solution:

Step 1: Formula

$A^{-1} = \frac{1}{|A|}\,\text{adj}A$, and $(\text{adj}A)_{12} = C_{21}$, the cofactor of the element in row 2, column 1.

Step 2: Find the determinant

$$|A| = 1(0 \cdot 0 - (-5)(5)) - 3((-3)(0) - (-5)(2)) + (-2)((-3)(5) - 0 \cdot 2)$$

$$= 1(25) - 3(10) - 2(-15) = 25 - 30 + 30 = 25$$

Step 3: Find C21

Remove row 2 and column 1: the minor is $\begin{vmatrix} 3 & -2 \\ 5 & 0 \end{vmatrix} = 0 - (-10) = 10.$

So $C_{21} = (-1)^{2+1} \cdot 10 = -10.$

Step 4: Required element

$$(A^{-1})_{12} = \frac{C_{21}}{|A|} = \frac{-10}{25} = -\frac{2}{5}$$

Step 5: Check the options

Options (B) and (C) come from using a wrong determinant or cofactor. Option (D) has the wrong sign and size. Using C_{12} instead of C_{21} is a common slip, because the adjoint is the transpose of the cofactor matrix.

Final Answer:

The element is $-\frac{2}{5}$, option (A).

$$\boxed{-\frac{2}{5}}$$

Quick Tip: The (1,2) entry of the inverse is the cofactor C21 divided by the determinant. Find both.

116. If $y = tan^{-1}(\frac{1}{x^2+x+1}) + tan^{-1}(\frac{1}{x^2+3x+3}) + tan^{-1}(\frac{1}{x^2+5x+7}) + \ldots$ upto n terms, then Missing open brace for superscript

(A) $\frac{n^2}{n^2+1}$

(B) $-\frac{n^2}{n^2+1}$

(C) 0

(D) $\frac{1}{n^2+1}$

Correct Answer: (B) $-\frac{n^2}{n^2+1}$

Solution:

Step 1: Understanding the Question:

Each term has the form $\tan^{-1}\frac{1}{1+k(k+1)}$ with $k = x, x+1, x+2, \ldots$.

Check: $x^2 + x + 1 = 1 + x(x+1)$, $x^2 + 3x + 3 = 1 + (x+1)(x+2)$, $x^2 + 5x + 7 = 1 + (x+2)(x+3)$.

Step 2: Use the difference formula:

$\tan^{-1} A - \tan^{-1} B = \tan^{-1}\frac{A-B}{1+AB}$. With $A = x+1$ and $B = x$, we get $\tan^{-1}\frac{1}{1+x(x+1)} = \tan^{-1}(x+1) - \tan^{-1} x$.

So the terms are $[\tan^{-1}(x+1) - \tan^{-1} x] + [\tan^{-1}(x+2) - \tan^{-1}(x+1)] + \ldots$

Step 3: Telescope:

$$y = \tan^{-1}(x+n) - \tan^{-1} x$$

Step 4: Differentiate:

$$y' = \frac{1}{1+(x+n)^2} - \frac{1}{1+x^2}$$

At $x = 0$: $y'(0) = \frac{1}{1+n^2} - 1 = -\frac{n^2}{1+n^2}$.

Final Answer:

$y'(0) = -\dfrac{n^2}{n^2+1}$, option (B).

$$\boxed{-\frac{n^2}{n^2+1}}$$

Quick Tip: Write each term as a difference of two arctangents so the series telescopes.

117. Let $f(x) = (sin^{-1}x)^2 + (cos^{-1}x)^2$ be a real-valued function defined on its domain. Then the sum of the greatest and the least values of $f(x)$ is

(A) $\frac{\pi^2}{8}$

(B) $\frac{11\pi^2}{8}$

(C) $\frac{3\pi^2}{8}$

(D) $\frac{7\pi^2}{8}$

Correct Answer: (B) $\frac{11\pi^2}{8}$

Solution:

Step 1: Understanding the Question:

The domain is $x \in [-1, 1]$. Let $t = \sin^{-1} x$, so $t \in [-\frac{\pi}{2}, \frac{\pi}{2}]$ and $\cos^{-1} x = \frac{\pi}{2} - t$.

Step 2: Form the quadratic:

$$f = t^2 + \left(\frac{\pi}{2} - t\right)^2 = 2t^2 - \pi t + \frac{\pi^2}{4}$$

Step 3: Least value:

The parabola opens upwards, with vertex at $t = \frac{\pi}{4}$, which lies in the interval. $f_{\min} = 2 \cdot \frac{\pi^2}{16} - \frac{\pi^2}{4} + \frac{\pi^2}{4} = \frac{\pi^2}{8}$.

Step 4: Greatest value:

The maximum is at an end of the interval, whichever is farther from

$\frac{\pi}{4}$. That is $t = -\frac{\pi}{2}$.
$f(-\frac{\pi}{2}) = 2 \cdot \frac{\pi^2}{4} + \frac{\pi^2}{2} + \frac{\pi^2}{4} = \frac{5\pi^2}{4}$. At $t = \frac{\pi}{2}$, $f = \frac{\pi^2}{4}$, which is smaller.

Step 5: Sum:

$$\frac{\pi^2}{8} + \frac{5\pi^2}{4} = \frac{\pi^2 + 10\pi^2}{8} = \frac{11\pi^2}{8}$$

Final Answer:

The sum of the greatest and least values is $\frac{11\pi^2}{8}$, option (B).

$$\boxed{\frac{11\pi^2}{8}}$$

Quick Tip: Use $\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$ to get a quadratic in $t = \sin^{-1} x$.

118. The value of $\frac{\tan^{-1}(1)+\cos^{-1}(\frac{1}{2})+\sin^{-1}(\frac{1}{2})}{\tan^{-1}(\sqrt{3})-\sec^{-1}(-2)}$ is equal to

(A) $\frac{3}{4}$

(B) $\frac{-1}{3}$

(C) -4

(D) $\frac{-9}{4}$

Correct Answer: (D) $\frac{-9}{4}$

Solution:

Step 1: Numerator:

$\tan^{-1} 1 = \frac{\pi}{4}$, $\cos^{-1} \frac{1}{2} = \frac{\pi}{3}$, $\sin^{-1} \frac{1}{2} = \frac{\pi}{6}$.
Sum $= \frac{\pi}{4} + \frac{\pi}{3} + \frac{\pi}{6} = \frac{3\pi+4\pi+2\pi}{12} = \frac{9\pi}{12} = \frac{3\pi}{4}$.

Step 2: Denominator:

$\tan^{-1}\sqrt{3} = \frac{\pi}{3}$. Also $\sec^{-1}(-2) = \pi - \sec^{-1} 2 = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$, since $\sec^{-1}$ of a negative number lies in the second quadrant.

Denominator $= \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$.

Step 3: Divide:

$$\frac{3\pi/4}{-\pi/3} = -\frac{9}{4}$$

Final Answer:

The value is $-\frac{9}{4}$, option (D).

$$\boxed{-\frac{9}{4}}$$

Quick Tip: Evaluate each principal value in terms of pi, remembering $\sec^{-1}(-2) = \pi - \sec^{-1} 2$.

119. If $g(x)$ is the inverse function of $f(x)$, where $f(x) = \frac{5x+3}{4x-1}$, then $g(1) =$

(A) -4

(B) $\frac{-1}{4}$

(C) $\frac{8}{3}$

(D) $\frac{3}{8}$

Correct Answer: (A) -4

Solution:

Step 1: Understanding the Question:

If g is the inverse of f, then $g(1) = a$ means $f(a) = 1$. So we solve $f(x) = 1$.

Step 2: Solve:

$$\frac{5x+3}{4x-1} = 1 \Rightarrow 5x+3 = 4x-1 \Rightarrow x = -4$$

Check: $f(-4) = \frac{-20+3}{-16-1} = \frac{-17}{-17} = 1$.

Final Answer:

$g(1) = -4$, option (A).

$$\boxed{-4}$$

Quick Tip: $g(1)$ is the number x for which $f(x) = 1$.

120. If $f(x)$ is continuous at $x = 0$, where $f(x) = \frac{8^x - 2^x}{k^x - 1}$, for $x \neq 0$ and $f(0) = 2$, then the value of k is ...

(A) 0

(B) 4

(C) -2

(D) 2

Correct Answer: (D) 2

Solution:

Step 1: Understanding the Question:

For continuity at 0 we need $\lim_{x\to 0} f(x) = f(0) = 2$.

Step 2: Evaluate the limit:

$8^x - 2^x = 2^x(4^x - 1)$. So $f(x) = 2^x \cdot \dfrac{4^x - 1}{k^x - 1}$.

Divide top and bottom by x: $\dfrac{(4^x - 1)/x}{(k^x - 1)/x} \to \dfrac{\ln 4}{\ln k}$, and $2^x \to 1$.

$$\lim_{x\to 0} f(x) = \frac{\ln 4}{\ln k}$$

Step 3: Solve for k:

$\frac{\ln 4}{\ln k} = 2$, so $\ln k = \frac{\ln 4}{2} = \ln 2$, and $k = 2$.

$k = 0$ is undefined for the logarithm, $k = -2$ makes k^x undefined for general x, and $k = 4$ would give a limit of 1.

Final Answer:

The value of k is 2, option (D).

$$\boxed{2}$$

Quick Tip: Continuity needs the limit at 0 to equal 2; factor 2^x from the numerator and use $\lim \frac{a^x - 1}{x} = \ln a$.

121. $\frac{d}{dx}[sin^2\{cot^{-1}\sqrt{\frac{1-x}{1+x}}\}] = \ldots$

(A) -1

(B) $\frac{1}{2}$

(C) $\frac{-1}{2}$

(D) 1

Correct Answer: (B) $\frac{1}{2}$

Solution:

Step 1: Understanding the Question:

Simplify the inner expression first, then differentiate.

Step 2: Substitute:

Let $x = \cos 2\phi$. Then $\sqrt{\frac{1-x}{1+x}} = \sqrt{\frac{2\sin^2\phi}{2\cos^2\phi}} = \tan\phi$.

So $\cot^{-1}(\tan\phi) = \frac{\pi}{2} - \phi$, and $\sin^2\left(\frac{\pi}{2} - \phi\right) = \cos^2\phi = \frac{1+\cos 2\phi}{2} = \frac{1+x}{2}$.

Step 3: Differentiate:

$$\frac{d}{dx}\left(\frac{1+x}{2}\right) = \frac{1}{2}$$

Final Answer:

The derivative is $\frac{1}{2}$, option (B).

$$\boxed{\frac{1}{2}}$$

Quick Tip: Put $x = \cos 2\phi$; the expression simplifies to $\frac{1+x}{2}$.

122. If $f(x) = \sqrt{x^2+1}, g(x) = \frac{x+1}{x^2+1}, h(x) = 2x - 3$, then

Missing open brace for superscript

(A) 0

(B) $\frac{1}{\sqrt{x^2+1}}$

(C) $\frac{2}{\sqrt{5}}$

(D) $\frac{x}{\sqrt{x^2+1}}$

Correct Answer: (C) $\frac{2}{\sqrt{5}}$

Solution:

Step 1: Understanding the Question:

Work from the inside. We need $g'(x)$, then h' of it, then f' of that.

Step 2: Inner derivatives:

$h(x) = 2x - 3$, so $h'(x) = 2$ for every input. Hence $h'(g'(x)) = 2$, no matter what $g'(x)$ is.

(For reference, $g'(x) = \frac{1-2x-x^2}{(x^2+1)^2}$, but its value is not needed.)

Step 3: Outer derivative:

$f(x) = \sqrt{x^2+1}$, so $f'(x) = \frac{x}{\sqrt{x^2+1}}$.

$$f'(2) = \frac{2}{\sqrt{5}}$$

Step 4: Other options:

Options B and D still contain x, but the expression has no x left.

Option A would need $f'(0)$.

Final Answer:

The value is $\frac{2}{\sqrt{5}}$, option (C).

$$\boxed{\frac{2}{\sqrt{5}}}$$

Quick Tip: $h'(x)$ is the constant 2, so $f'(h'(g'(x))) = f'(2)$.

123. If $\sqrt{y+x} + \sqrt{y-x} = c$ such that $\frac{dy}{dx} = f(x) - \sqrt{[f(x)]^2 - 1}$, then $f(x)$ = _____

(A) $\frac{y}{x}$

(B) $-\frac{x}{y}$

(C) $-\frac{y}{x}$

(D) $\frac{x}{y}$

Correct Answer: (A) $\frac{y}{x}$

Solution:

Step 1: Differentiate:

From $\sqrt{y+x} + \sqrt{y-x} = c$: $\frac{y'+1}{2\sqrt{y+x}} + \frac{y'-1}{2\sqrt{y-x}} = 0.$

Cross multiply: $(y'+1)\sqrt{y-x} + (y'-1)\sqrt{y+x} = 0.$

Step 2: Solve for y':

$$y'\left(\sqrt{y-x} + \sqrt{y+x}\right) = \sqrt{y+x} - \sqrt{y-x}$$

$$y' = \frac{\sqrt{y+x} - \sqrt{y-x}}{\sqrt{y+x} + \sqrt{y-x}}$$

Step 3: Rationalise:

Multiply top and bottom by $\sqrt{y+x} - \sqrt{y-x}$. The denominator becomes $(y+x) - (y-x) = 2x$.

The numerator becomes $(y+x) + (y-x) - 2\sqrt{y^2 - x^2} = 2y - 2\sqrt{y^2 - x^2}$.

$$y' = \frac{y - \sqrt{y^2 - x^2}}{x} = \frac{y}{x} - \sqrt{\frac{y^2}{x^2} - 1}$$

Step 4: Compare:

We are told $\frac{dy}{dx} = f(x) - \sqrt{[f(x)]^2 - 1}$. Matching gives $f(x) = \frac{y}{x}$.

Final Answer:

$f(x) = \frac{y}{x}$, option (A).

$$\boxed{\frac{y}{x}}$$

Quick Tip: Differentiate implicitly and rationalise to get $\frac{dy}{dx} = \frac{y}{x} - \sqrt{\frac{y^2}{x^2} - 1}$.

124. If Rolle's theorem is applicable for the function $f(x) = log(\frac{x^2+a}{x})$ on $[3,4]$ with $c \in (3,4)$ such that Missing open brace for superscript, then the value of $f''(c)$ is

(A) $\frac{1}{48}$

(B) $\frac{1}{36}$

(C) $\frac{1}{24}$

(D) $\frac{1}{12}$

Correct Answer: (D) $\frac{1}{12}$

Solution:

Step 1: Understanding the Question:

Rolle's theorem needs $f(3) = f(4)$. Write $f(x) = \log\left(x + \frac{a}{x}\right)$.

Step 2: Find a:

$3 + \frac{a}{3} = 4 + \frac{a}{4}$, so $\frac{a}{3} - \frac{a}{4} = 1$, $\frac{a}{12} = 1$, $a = 12$.

Step 3: First derivative:

$$f'(x) = \frac{1 - a/x^2}{x + a/x} = \frac{x^2 - a}{x(x^2 + a)} = \frac{x^2 - 12}{x^3 + 12x}$$

$f'(c) = 0$ gives $c^2 = 12$, $c = 2\sqrt{3} \approx 3.46$, which lies in $(3, 4)$.

Step 4: Second derivative:

Differentiate $f' = \frac{N}{M}$, with $N = x^2 - 12$ and $M = x^3 + 12x$: $f'' = \frac{N'M - NM'}{M^2}$.

At $x = c$, $N = 0$, so $f''(c) = \frac{N'(c)}{M(c)} = \frac{2c}{c^3 + 12c} = \frac{2}{c^2 + 12} = \frac{2}{24} = \frac{1}{12}$.

Final Answer:

$f''(c) = \frac{1}{12}$, option (D).

$$\boxed{\frac{1}{12}}$$

Quick Tip: Use $f(3) = f(4)$ to get a, then $f'(c) = 0$ gives $c^2 = a$.

125. If the tangent to the curve $2y^3 = x^3 + ax^2$ at the point (a, a) cuts off intercepts α and β on the coordinate axes such that $\alpha^2 + \beta^2 = 61$, then the value of a is

(A) ± 61

(B) ± 36

(C) ± 30

(D) ± 25

Correct Answer: (C) ± 30

Solution:

Step 1: Check the point:

At (a, a): $2a^3 = a^3 + a^3$. The point lies on the curve.

Step 2: Slope:

Differentiate $2y^3 = x^3 + ax^2$: $6y^2 \frac{dy}{dx} = 3x^2 + 2ax$.

At (a, a): $6a^2 m = 3a^2 + 2a^2 = 5a^2$, so $m = \frac{5}{6}$.

Step 3: Tangent and intercepts:

Tangent: $y - a = \frac{5}{6}(x - a)$.

Put $y = 0$: $-a = \frac{5}{6}(x - a)$, so $x = a - \frac{6a}{5} = -\frac{a}{5}$. So $\alpha = -\frac{a}{5}$.

Put $x = 0$: $y = a - \frac{5a}{6} = \frac{a}{6}$. So $\beta = \frac{a}{6}$.

Step 4: Use the condition:

$$\alpha^2 + \beta^2 = \frac{a^2}{25} + \frac{a^2}{36} = \frac{61a^2}{900} = 61$$

So $a^2 = 900$, $a = \pm 30$.

Final Answer:

The value of a is ± 30, option (C).

$$\boxed{a = \pm 30}$$

Quick Tip: Find the tangent slope at (a, a) by implicit differentiation, then compute both intercepts.

126. Let $f(x)$ be the differentiable function for all x such that Missing open brace for superscript and $f(1) = 4$. The maximum value of $f(5)$ is...

(A) 16

(B) 20

(C) 24

(D) 25

Correct Answer: (C) 24

Solution:

Step 1: Understanding the Question:

The Mean Value Theorem says there is a $c \in (1, 5)$ with $f(5) - f(1) = f'(c)(5 - 1)$. The function is differentiable, so the theorem applies.

Step 2: Bound:

$f'(c) \leq 5$, so $f(5) - 4 = 4f'(c) \leq 20$, giving $f(5) \leq 24$.

The maximum value 24 is reached by $f(x) = 5x - 1$, where $f'(x) = 5$ and $f(1) = 4$.

Final Answer:

The maximum value of $f(5)$ is 24, option (C).

$$\boxed{24}$$

Quick Tip: Apply the Mean Value Theorem on $[1, 5]$.

127. If $\int x^2 \cdot e^x dx = e^x f(x) + c$, then the minimum value of $f(x)$ is ...

(A) 0

(B) -1

(C) 1

(D) $\frac{-1}{4}$

Correct Answer: (C) 1

Solution:

Step 1: Integrate by parts:

Take $u = x^2$, $dv = e^x dx$: $\int x^2 e^x dx = x^2 e^x - 2\int xe^x dx$.

Again: $\int xe^x dx = xe^x - e^x$.

$$\int x^2 e^x dx = x^2 e^x - 2xe^x + 2e^x + c = e^x(x^2 - 2x + 2) + c$$

So $f(x) = x^2 - 2x + 2$.

Step 2: Minimise:

$f(x) = (x-1)^2 + 1$, whose least value is 1 at $x = 1$.
Check with $f'(x) = 2x - 2 = 0$ at $x = 1$, and $f''(x) = 2 > 0$.

Final Answer:
The minimum value of $f(x)$ is 1, option (C).

$$\boxed{1}$$

Quick Tip: Integrate by parts twice to get $e^x(x^2 - 2x + 2)$.

128. $\int \frac{(x+1)}{x(1+xe^x)^2}\,dx =$

(A) $-log(\frac{xe^x}{1+xe^x}) + \frac{1}{(1+xe^x)} + c$

(B) $log(\frac{xe^x}{1+xe^x}) + \frac{1}{(1+xe^x)} + c$

(C) $log(\frac{1+xe^x}{xe^x}) + (1 + xe^x) + c$

(D) $-log(\frac{xe^x}{1+xe^x}) - \frac{1}{(1+xe^x)} + c$

Correct Answer: (B) $log(\frac{xe^x}{1+xe^x}) + \frac{1}{(1+xe^x)} + c$

Solution:

Step 1: Pick the substitution:
The bracket has $1 + xe^x$, so put $t = xe^x$. Then $dt = (e^x + xe^x)dx = e^x(x + 1)dx$. Dividing by $t = xe^x$ gives $\frac{dt}{t} = \frac{x+1}{x}\,dx$.

Step 2: Rewrite the integral:
The integrand is $\frac{1}{(1+xe^x)^2} \cdot \frac{x+1}{x}\,dx$. It becomes

$$\int \frac{dt}{t(1+t)^2}$$

Step 3: Use partial fractions:

Write $\frac{1}{t(1+t)^2} = \frac{A}{t} + \frac{B}{1+t} + \frac{C}{(1+t)^2}$. Putting $t = 0$ gives $A = 1$. Putting $t = -1$ gives $C = -1$. Matching the t^2 coefficient gives $A + B = 0$, so $B = -1$. Therefore

$$\frac{1}{t(1+t)^2} = \frac{1}{t} - \frac{1}{1+t} - \frac{1}{(1+t)^2}$$

Step 4: Integrate and substitute back:

$$\ln t - \ln(1+t) + \frac{1}{1+t} + c = \ln\left(\frac{xe^x}{1+xe^x}\right) + \frac{1}{1+xe^x} + c$$

Step 5: Why the other options are wrong:

The log term is $+\ln\frac{t}{1+t}$, so options (A) and (D) with a leading minus are wrong. The last term is $+\frac{1}{1+t}$, since $\int -\frac{dt}{(1+t)^2} = +\frac{1}{1+t}$, so option (D) fails again. Option (C) has $(1 + xe^x)$ added, which would come from a wrong integral of the squared term.

Final Answer:

The integral equals $\ln\left(\frac{xe^x}{1+xe^x}\right) + \frac{1}{1+xe^x} + c$, option (B).

$$\boxed{\log\left(\frac{xe^x}{1+xe^x}\right) + \frac{1}{1+xe^x} + c}$$

Quick Tip: Put t = x e^x; then (x+1)/x dx becomes dt/t and partial fractions finish it.

129. If $\int \frac{x+1}{x^2+1}dx = tan^{-1}x + g(x) + c$, where c is constant of integration, then the function $g(x)$ is monotonically increasing in the interval

(A) R^+

(B) R^-

(C) R

(D) $R - \{0\}$

Correct Answer: (A) R^+

Solution:

Step 1: Split the integral:

$\int \frac{x+1}{x^2+1}dx = \int \frac{x}{x^2+1}dx + \int \frac{1}{x^2+1}dx = \frac{1}{2}\ln(x^2+1) + \tan^{-1}x + c.$

So $g(x) = \frac{1}{2}\ln(x^2+1)$.

Step 2: Monotonicity:

$g'(x) = \frac{x}{x^2+1}$. The denominator is always positive, so the sign of g' is the sign of x.

$g'(x) > 0$ for $x > 0$, i.e. g is increasing on R^+. For $x < 0$, $g' < 0$, so g decreases on R^-.

Step 3: Other options:

R^-, R and $R - \{0\}$ all include negative values, where g is decreasing. So only R^+ works.

Final Answer:

The function g is increasing on R^+, option (A).

$$\boxed{R^+}$$

Quick Tip: Split the integrand; $g(x) = \frac{1}{2}\ln(x^2+1)$ and $g'(x) = \frac{x}{x^2+1}$.

130. $\int \sqrt{1+\sin x}\,dx$ is equal to

(A) $2\sqrt{2}\sin(\frac{x}{2} - \frac{\pi}{4}) + c$

(B) $2\sqrt{2}\sin(\frac{x}{2} + \frac{\pi}{4}) + c$

(C) $\sqrt{2}\sin(\frac{x}{2} - \frac{\pi}{4}) + c$

(D) $\sqrt{2}\sin(\frac{x}{2} + \frac{\pi}{4}) + c$

Correct Answer: (A) $2\sqrt{2}\sin(\frac{x}{2} - \frac{\pi}{4}) + c$

Solution:

Step 1: Turn 1 + sin x into a perfect square:

Write $1 = \sin^2\frac{x}{2} + \cos^2\frac{x}{2}$ and $\sin x = 2\sin\frac{x}{2}\cos\frac{x}{2}$. Adding them:

$$1 + \sin x = \left(\sin\frac{x}{2} + \cos\frac{x}{2}\right)^2$$

So $\sqrt{1+\sin x} = \sin\frac{x}{2} + \cos\frac{x}{2}$, taking the positive branch (the form used for this standard integral).

Step 2: Integrate term by term:

$\int \sin\frac{x}{2}\,dx = -2\cos\frac{x}{2}$ and $\int \cos\frac{x}{2}\,dx = 2\sin\frac{x}{2}$. So

$$\int \sqrt{1+\sin x}\,dx = 2\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right) + c$$

Step 3: Convert to a single sine:

Use $\sin A - \cos A = \sqrt{2}\sin\left(A - \frac{\pi}{4}\right)$, which follows from $\sin A \cos\frac{\pi}{4} - \cos A \sin\frac{\pi}{4} = \frac{\sin A - \cos A}{\sqrt{2}}$. With $A = \frac{x}{2}$:

$$2\left(\sin\frac{x}{2} - \cos\frac{x}{2}\right) = 2\sqrt{2}\sin\left(\frac{x}{2} - \frac{\pi}{4}\right)$$

Step 4: Why the other options are wrong:

Option (B) has $+\frac{\pi}{4}$, which equals $2(\sin\frac{x}{2} + \cos\frac{x}{2})$. Its derivative is $\cos\frac{x}{2} - \sin\frac{x}{2}$, not $\sin\frac{x}{2} + \cos\frac{x}{2}$. Options (C) and (D) carry only $\sqrt{2}$ as the coefficient. Differentiating $\sqrt{2}\sin\left(\frac{x}{2} - \frac{\pi}{4}\right)$ gives half of the required integrand, so they are too small by a factor of 2.

Final Answer:

The integral equals $2\sqrt{2}\sin\left(\frac{x}{2} - \frac{\pi}{4}\right) + c$, which is option (A).

$$\boxed{2\sqrt{2}\sin\left(\frac{x}{2} - \frac{\pi}{4}\right) + c}$$

Quick Tip: Write 1 + sin x as the square of (sin(x/2) + cos(x/2)), then integrate.

131. If $\int_0^a (x^2 - 4x + 1)dx = 6$, then the real value of a is

(A) 4

(B) 6

(C) 3

(D) -3

Correct Answer: (B) 6

Solution:

Step 1: Integrate:

$$\int_0^a (x^2 - 4x + 1)dx = \frac{a^3}{3} - 2a^2 + a$$

Step 2: Solve:

Set it equal to 6 and multiply by 3: $a^3 - 6a^2 + 3a - 18 = 0$.

Group: $a^2(a-6) + 3(a-6) = (a-6)(a^2+3) = 0$.

$a^2 + 3 = 0$ has no real root, so the only real value is $a = 6$.

Step 3: Check:

At $a = 6$: $\frac{216}{3} - 72 + 6 = 72 - 72 + 6 = 6$. It works.

Final Answer:

The real value is $a = 6$, option (B).

$$\boxed{6}$$

Quick Tip: Integrate, set equal to 6, and factor the cubic.

132. If $\int_{\pi/6}^{\pi/3} \frac{1}{1+sinx+cosx}dx = log2$, then the value of $\int_{\pi/6}^{\pi/3} \frac{cosx}{1+sinx+cosx}dx =$ .....

(A) $\frac{\pi}{12} - \frac{1}{2}log2$

(B) $\frac{\pi}{12} - \frac{1}{3}log2$

(C) $\frac{\pi}{6} - \frac{1}{2}log2$

(D) $\frac{\pi}{12} - \frac{1}{4}log2$

Correct Answer: (A) $\frac{\pi}{12} - \frac{1}{2}log2$

Solution:

Step 1: Understanding the Question:

Let $I_1 = \int \frac{1}{1+\sin x+\cos x}dx = \log 2$, $I_c = \int \frac{\cos x}{1+\sin x+\cos x}dx$ and $I_s = \int \frac{\sin x}{1+\sin x+\cos x}dx$, all from $\frac{\pi}{6}$ to $\frac{\pi}{3}$.

Step 2: Add:

$$I_c + I_s = \int \frac{\sin x + \cos x}{1 + \sin x + \cos x}dx = \int \left(1 - \frac{1}{1 + \sin x + \cos x}\right)dx$$

$$I_c + I_s = \left(\frac{\pi}{3} - \frac{\pi}{6}\right) - \log 2 = \frac{\pi}{6} - \log 2$$

Step 3: Use symmetry:

Replace x by $\frac{\pi}{2} - x$. The limits $\frac{\pi}{6}$ and $\frac{\pi}{3}$ swap with each other, and sin and cos swap. So $I_c = I_s$.

Hence $2I_c = \frac{\pi}{6} - \log 2$, so $I_c = \frac{\pi}{12} - \frac{1}{2}\log 2$.

Final Answer:

The integral equals $\frac{\pi}{12} - \frac{1}{2}\log 2$, option (A).

$$\boxed{\frac{\pi}{12} - \frac{1}{2}\log 2}$$

Quick Tip: The interval is symmetric under $x \to \frac{\pi}{2} - x$, so the cosine and sine integrals are equal.

133. The area of the region bounded by the curve $y = 2^{kx}$ and $x = 0, x = 2$, in first quadrant is $\frac{2}{log2}$, then the value of k is

(A) 1

(B) 2

(C) $-\frac{1}{2}$

(D) $\frac{1}{2}$

Correct Answer: (D) $\frac{1}{2}$

Solution:

Step 1: Set up the area:

$$A = \int_0^2 2^{kx} dx = \left[\frac{2^{kx}}{k \ln 2}\right]_0^2 = \frac{2^{2k} - 1}{k \ln 2}$$

Step 2: Use the given area:

$\frac{2^{2k}-1}{k \ln 2} = \frac{2}{\ln 2}$, so $2^{2k} - 1 = 2k$.

Step 3: Solve:

Test $k = \frac{1}{2}$: $2^1 - 1 = 1$, and $2k = 1$. It satisfies the equation.

Test the others: $k = 1$ gives $3 \neq 2$. $k = 2$ gives $15 \neq 4$. $k = -\frac{1}{2}$ gives $-\frac{1}{2}$

$\neq -1$.
So $k = \frac{1}{2}$.

Final Answer:
The value is $k = \frac{1}{2}$, option (D).

$$\boxed{\frac{1}{2}}$$

Quick Tip: Integrate 2^{kx} and test the options in $2^{2k} - 1 = 2k$.

134. The area of the region bounded by the curve $y = x^3$ and the lines $y = 8$ and $x = 0$ is ... square units.

(A) 8

(B) 12

(C) 16

(D) 10

Correct Answer: (B) 12

Solution:

Step 1: Understanding the Question:
The curve $y = x^3$ meets $y = 8$ at $x = 2$. The region lies between the Y-axis ($x = 0$), the curve and the line $y = 8$.

Step 2: Integrate along y:

$$A = \int_0^8 y^{1/3} dy = \frac{3}{4}\left[y^{4/3}\right]_0^8 = \frac{3}{4} \cdot 16 = 12$$

Step 3: Check:

The area is the rectangle $2 \times 8 = 16$ minus the area under the curve, $\int_0^2 x^3 dx = 4$. So $16 - 4 = 12$.

Final Answer:

The area is 12 square units, option (B).

$$\boxed{12}$$

Quick Tip: Integrate with respect to y: $x = y^{1/3}$ from 0 to 8.

135. The differential equation of $3y = \sqrt[3]{x + c}$ is

(A) $\frac{dy}{dx} = \frac{1}{9y^2}$

(B) $\frac{dy}{dx} = \frac{1}{27y^2}$

(C) $\frac{dy}{dx} = \frac{1}{81y^2}$

(D) $\frac{dy}{dx} = \frac{1}{36y^2}$

Correct Answer: (C) $\frac{dy}{dx} = \frac{1}{81y^2}$

Solution:

Step 1: Plan

The family has one arbitrary constant c, so we differentiate once and remove c to get a first order differential equation.

Step 2: Remove the cube root

Cube both sides: $(3y)^3 = x + c$, so $27y^3 = x + c$.

Step 3: Differentiate with respect to x

$$27 \cdot 3y^2 \frac{dy}{dx} = 1$$

$$81y^2 \frac{dy}{dx} = 1$$

Step 4: Write the result

$$\frac{dy}{dx} = \frac{1}{81y^2}$$

Step 5: Why the other options are wrong

Options (A), (B) and (D) have 9, 27 and 36 in place of 81. They come from forgetting to multiply 27 by the power 3, or from differentiating $3y$ instead of $27y^3$. Note $27 \cdot 3 = 81$.

Final Answer:

The differential equation is $\frac{dy}{dx} = \frac{1}{81y^2}$, option (C).

$$\boxed{\frac{dy}{dx} = \frac{1}{81y^2}}$$

Quick Tip: Cube both sides to get 27y^3 = x + c, then differentiate once so the constant disappears.

136. The general solution of the differential equation $(x + y)\frac{dy}{dx} = 1$ is

(A) $x + y + 1 = c$, where c is a constant of integration

(B) $x + y + 1 = ce^{y}$, where c is a constant of integration

(C) $x + y + 1 = ce^{-y}$, where c is a constant of integration

(D) $x + y - 1 = ce^{-y}$, where c is a constant of integration

Correct Answer: (B) $x + y + 1 = ce^{y}$, where c is a constant of integration

Solution:

Step 1: Understanding the Question:

The equation $(x + y)\frac{dy}{dx} = 1$ is not linear in y, but if we flip it, it becomes linear in x.

Step 2: Rewrite:

$\frac{dx}{dy} = x + y$, so $\frac{dx}{dy} - x = y$. This is linear with $P = -1$ and $Q = y$.

Step 3: Integrating factor:

I.F. $= e^{\int -1\, dy} = e^{-y}$.

$$xe^{-y} = \int ye^{-y}dy = -ye^{-y} - e^{-y} + c$$

Step 4: Simplify:

Multiply by e^{y}: $x = -y - 1 + ce^{y}$, so $x + y + 1 = ce^{y}$.

Option C and D have e^{-y}, which does not come from this integrating factor. Option A is not a family that satisfies the equation.

Final Answer:

The general solution is $x + y + 1 = ce^{y}$, option (B).

$$\boxed{x + y + 1 = ce^y}$$

Quick Tip: Treat x as a function of y; the equation becomes linear in x.

137. A body cools according to Newton's law of cooling from 100°C to 60°C in 20 minutes. The temperature of the surroundings being 20°C, then the total time required for the body to cool down to 30°C is

(A) 90 minutes

(B) 1 hour and 10 minutes

(C) 80 minutes

(D) 60 minutes

Correct Answer: (D) 60 minutes

Solution:

Step 1: Understanding the Question:

Newton's law of cooling: $\frac{dT}{dt} = -k(T - T_s)$, so $T - T_s = (T_0 - T_s)e^{-kt}$. The excess temperature falls exponentially.

Step 2: Find k:

At $t = 20$ min: $60 - 20 = (100 - 20)e^{-20k}$, so $40 = 80e^{-20k}$, giving $e^{-20k} = \frac{1}{2}$, and $20k = \ln 2$.

Step 3: Find the time for 30 degrees:

We need $30 - 20 = 80e^{-kt}$, so $e^{-kt} = \frac{1}{8}$ and $kt = \ln 8 = 3 \ln 2$.

Since $20k = \ln 2$, we get $t = 3 \times 20 = 60$ minutes.

Step 4: Check:

The excess temperature halves every 20 minutes: $80 \to 40 \to 20 \to 10$. The final excess is $30 - 20 = 10$, reached after three halvings, i.e. 60 minutes. Options of 80 and 90 minutes are too long, and 70 minutes does not match three halvings.

Final Answer:

The total time is 60 minutes, option (D).

$$\boxed{60 \text{ minutes}}$$

Quick Tip: Use $\ln \frac{T-T_s}{T_0-T_s} = -kt$ with surroundings at 20 degrees C.

138. If $\bar{a} = \hat{i} + \hat{j}$ and $\bar{b} = \hat{i} - \hat{k}$, then the point of intersection of the lines $\bar{r} \times \bar{a} = \bar{b} \times \bar{a}$ and $\bar{r} \times \bar{b} = \bar{a} \times \bar{b}$ is

(A) $(2, 1, -1)$

(B) $(2, -1, 1)$

(C) $(0, 1, 1)$

(D) $(0, -1, 1)$

Correct Answer: (A) $(2, 1, -1)$

Solution:

Step 1: Rewrite each line in a simple form:

The first equation is $\vec{r} \times \vec{a} = \vec{b} \times \vec{a}$, so $(\vec{r} - \vec{b}) \times \vec{a} = \vec{0}$. A zero cross product means $\vec{r} - \vec{b}$ is parallel to $\vec{a}$. So the first line is $\vec{r} = \vec{b} + \lambda\vec{a}$.

Step 2: Write the first line in components:

With $\vec{a} = (1, 1, 0)$ and $\vec{b} = (1, 0, -1)$:

$$\vec{r} = (1 + \lambda,\ \lambda,\ -1)$$

Step 3: Write the second line:

The second equation $\vec{r} \times \vec{b} = \vec{a} \times \vec{b}$ gives $(\vec{r} - \vec{a}) \times \vec{b} = \vec{0}$. So $\vec{r} = \vec{a} + \mu\vec{b}$ $= (1 + \mu,\ 1,\ -\mu)$.

Step 4: Find the point of intersection:

Equate the components of the two lines. From the y coordinates, λ $= 1$. From the z coordinates, $-1 = -\mu$, so $\mu = 1$. The x coordinates are $1 + \lambda = 2$ and $1 + \mu = 2$, which agree. So the point is $(2, 1, -1)$.

Step 5: Why the other options are wrong:

The first line always has $z = -1$, and the second always has $y = 1$. Options (B), (C) and (D) have $z = 1$, which breaks the first line, so none of them can be the common point.

Final Answer:

The lines meet at $(2, 1, -1)$, which is option (A).

$$\boxed{(2,\ 1,\ -1)}$$

Quick Tip: Each equation says r minus a fixed vector is parallel to another vector, so each is a line.

139. If $\bar{a}$ and $\bar{b}$ have the same magnitude and angle between them is $60°$ and their scalar product is $\frac{1}{2}$, then $|\bar{a}|$ is _____

(A) 2

(B) 3

(C) 7

(D) 1

Correct Answer: (D) 1

Solution:

Step 1: Key Formula:

$\vec{a} \cdot \vec{b} = |\vec{a}||\vec{b}| \cos\theta$.

Step 2: Calculate:

Since $|\vec{a}| = |\vec{b}|$, we get $\vec{a} \cdot \vec{b} = |\vec{a}|^2 \cos 60^\circ = \frac{1}{2}|\vec{a}|^2$.

This equals $\frac{1}{2}$, so $|\vec{a}|^2 = 1$ and $|\vec{a}| = 1$.

Step 3: Other options:

Values 2, 3 and 7 would make the dot product 2, 4.5 and 24.5, not $\frac{1}{2}$.

Final Answer:

The magnitude is 1, option (D).

$$\boxed{1}$$

Quick Tip: $\vec{a} \cdot \vec{b} = |a||b| \cos 60^\circ$ with $|a| = |b|$.

140. If $\bar{a} = \hat{i} + \hat{j} + \hat{k}, \bar{b} = \hat{i}, \bar{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ with $c_1 = 1, c_2 = 2$, then value of c_3 such that $\bar{a}, \bar{b}, \bar{c}$ are coplanar is _____

(A) 2

(B) -1

(C) 0

(D) -2

Correct Answer: (A) 2

Solution:

Step 1: Key Formula:

Vectors $\vec{a}, \vec{b}, \vec{c}$ are coplanar if $\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 0.$

Step 2: Set up:

$\vec{a} = (1, 1, 1), \vec{b} = (1, 0, 0), \vec{c} = (1, 2, c_3).$

Step 3: Expand:

Expand along the second row: $-1 \cdot \begin{vmatrix} 1 & 1 \\ 2 & c_3 \end{vmatrix} = -(c_3 - 2) = 2 - c_3.$

Setting the determinant to zero gives $c_3 = 2$.

Final Answer:

The value of c_3 is 2, option (A).

$$\boxed{2}$$

Quick Tip: Three vectors are coplanar when their scalar triple product (determinant) is zero.

141. If $\bar{a}, \bar{b}$ and $\bar{c}$ are non-coplanar unit vectors such that the angle between any two of them is $60°$, and the vector $\bar{d} = x\bar{a} + y\bar{b} + z\bar{c}$ is perpendicular to both $\bar{a}$ and $\bar{b}$, then the value of $\frac{(x+y)}{z}$ is

(A) $-\frac{1}{2}$

(B) $-\frac{2}{3}$

(C) -1

(D) 0

Correct Answer: (B) $-\frac{2}{3}$

Solution:

Step 1: Understanding the Question:

The vectors are unit vectors at $60°$, so $\vec{a} \cdot \vec{a} = 1$ and $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{c} = \vec{c} \cdot \vec{a} = \frac{1}{2}$.

Step 2: Use perpendicularity:

$\vec{d} \cdot \vec{a} = x + \frac{y}{2} + \frac{z}{2} = 0.$

$\vec{d} \cdot \vec{b} = \frac{x}{2} + y + \frac{z}{2} = 0.$

Step 3: Solve:

Subtract: $\frac{x}{2} - \frac{y}{2} = 0$, so $x = y$.

Put $y = x$ in the first equation: $x + \frac{x}{2} + \frac{z}{2} = 0$, so $3x + z = 0$ and $z = -3x$.

$$\frac{x+y}{z} = \frac{2x}{-3x} = -\frac{2}{3}$$

Final Answer:

The value is $-\frac{2}{3}$, option (B).

$$\boxed{-\frac{2}{3}}$$

Quick Tip: Dot $\vec{d}$ with $\vec{a}$ and $\vec{b}$; each dot product between different unit vectors is $\frac{1}{2}$.

142. If $\bar{a}, \bar{b}, \bar{c}$ are three non zero and non-coplanar vectors such that $\bar{a} \times (\bar{b} \times \bar{c}) = \frac{\bar{b}}{2}$, then the angle between $\bar{a}$ and $\bar{b}$ is ...

(A) $\frac{\pi}{6}$

(B) $\frac{\pi}{3}$

(C) $\frac{\pi}{2}$

(D) π

Correct Answer: (C) $\frac{\pi}{2}$

Solution:

Step 1: Key Formula:
$\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\,\vec{b} - (\vec{a} \cdot \vec{b})\,\vec{c}$.

Step 2: Apply:
Given equality: $(\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \frac{1}{2}\vec{b}$.
Rearrange: $\left(\vec{a} \cdot \vec{c} - \frac{1}{2}\right)\vec{b} - (\vec{a} \cdot \vec{b})\vec{c} = \vec{0}$.

Step 3: Use non-coplanarity:
$\vec{b}$ and $\vec{c}$ are not parallel (they are part of a non-coplanar set), so the coefficients of both must be zero.
So $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} \cdot \vec{c} = \frac{1}{2}$.
$\vec{a} \cdot \vec{b} = 0$ with non-zero vectors means the angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{2}$.

Final Answer:

The angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{2}$, option (C).

$$\boxed{\frac{\pi}{2}}$$

Quick Tip: Expand $\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}$ and use non-coplanarity.

143. The vector equation of the plane which is at a distance of 5 units from the origin and normal to the vector $2\hat{i} + \hat{j} - 2\hat{k}$ is

(A) $\vec{r} \cdot (2\hat{i} + \hat{j} - 2\hat{k}) = 12$

(B) $\vec{r} \cdot (2\hat{i} + \hat{j} - 2\hat{k}) = 15$

(C) $\vec{r} \cdot (2\hat{i} + \hat{j} - 2\hat{k}) = 9$

(D) $\vec{r} \cdot (2\hat{i} + \hat{j} - 2\hat{k}) = 18$

Correct Answer: (B) $\vec{r} \cdot (2\hat{i} + \hat{j} - 2\hat{k}) = 15$

Solution:

Step 1: Key Formula:

A plane at distance p from the origin with unit normal $\hat{n}$ is $\vec{r} \cdot \hat{n} = p$.

Step 2: Calculate:

$\vec{n} = 2\hat{i} + \hat{j} - 2\hat{k}$, $|\vec{n}| = \sqrt{4 + 1 + 4} = 3$. So $\hat{n} = \frac{\vec{n}}{3}$.

$\vec{r} \cdot \frac{\vec{n}}{3} = 5$, so $\vec{r} \cdot \vec{n} = 15$.

Step 3: Other options:

The values 12, 9 and 18 correspond to distances 4, 3 and 6 from the

origin, not 5.

Final Answer:

The plane is $\vec{r} \cdot (2\hat{i} + \hat{j} - 2\hat{k}) = 15$, option (B).

$$\boxed{\vec{r} \cdot (2\hat{i} + \hat{j} - 2\hat{k}) = 15}$$

Quick Tip: Divide the normal by its magnitude to get a unit normal, then $\vec{r} \cdot \hat{n} = 5$.

144. The equation of a line passing through a point $(4, -2, 3)$ and perpendicular to the XZ-plane is....

(A) $\bar{r} = (4\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} + \hat{k})$

(B) $\bar{r} = (4\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(\hat{i})$

(C) $\bar{r} = (4\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(\hat{j})$

(D) $\bar{r} = (4\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - \hat{k})$

Correct Answer: (C) $\bar{r} = (4\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda(\hat{j})$

Solution:

Step 1: Understanding the Question:

A line perpendicular to a plane runs along the normal to that plane.

Step 2: Find the direction:

The XZ-plane is the plane $y = 0$. Its normal is along the Y-axis, i.e. along $\hat{j}$.

Step 3: Write the line:

Through $(4, -2, 3)$ with direction $\hat{j}$: $\vec{r} = (4\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda\hat{j}$.
Options A and D have components along $\hat{i}$ and $\hat{k}$, which lie in the XZ-plane. Option B runs along the X-axis, which is parallel to the plane.

Final Answer:
The line is $\vec{r} = (4\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda\hat{j}$, option (C).

$$\boxed{\vec{r} = (4\hat{i} - 2\hat{j} + 3\hat{k}) + \lambda\hat{j}}$$

Quick Tip: The XZ-plane has the Y-axis as its normal, so the line runs along $\hat{j}$.

145. The equation of the plane passing through the intersection of planes $2x - y + z = 3$, $4x - 3y + 5z = -9$ and parallel to the line $\frac{x+1}{2} = \frac{y+3}{4} = \frac{z-3}{5}$ is...

(A) $11x - 3y - 2z - 54 = 0$

(B) $11x + 3y - 2z - 54 = 0$

(C) $11x - 3y + 2z - 54 = 0$

(D) $11x - 3y - 2z + 54 = 0$

Correct Answer: (A) $11x - 3y - 2z - 54 = 0$

Solution:

Step 1: Understanding the Question:
The required plane belongs to the family $(2x - y + z - 3) + \lambda(4x - 3y + 5z + 9) = 0$.

Step 2: Expand:

$$(2+4\lambda)x+(-1-3\lambda)y+(1+5\lambda)z+(-3+9\lambda)=0$$

Step 3: Use the parallel line:

The line has direction $(2,4,5)$. For a plane to be parallel to it, the normal must be perpendicular to $(2,4,5)$.

$2(2+4\lambda)+4(-1-3\lambda)+5(1+5\lambda)=0$, i.e. $4+8\lambda-4-12\lambda+5+25\lambda=0$, so $5+21\lambda=0$ and $\lambda=-\frac{5}{21}$.

Step 4: Substitute:

Coefficient of x: $2-\frac{20}{21}=\frac{22}{21}$. Of y: $-1+\frac{15}{21}=-\frac{6}{21}$. Of z: $1-\frac{25}{21}=-\frac{4}{21}$. Constant: $-3-\frac{45}{21}=-\frac{108}{21}$.

Multiply by 21: $22x-6y-4z-108=0$, i.e. $11x-3y-2z-54=0$.

Final Answer:

The plane is $11x-3y-2z-54=0$, option (A).

$$\boxed{11x-3y-2z-54=0}$$

Quick Tip: Use the family $P_1+\lambda P_2=0$ and make its normal perpendicular to the line direction.

146. If a line makes angles α,β,γ with the coordinate axes, then the sum of values of $sin^2\alpha+sin^2\beta+sin^2\gamma$ and $cos2\alpha+cos2\beta+cos2\gamma$ is ...

(A) 5

(B) 0

(C) 3

(D) 1

Correct Answer: (D) 1

Solution:

Step 1: Key Fact:

For a line making angles α, β, γ with the axes, $l^2 + m^2 + n^2 = 1$, i.e. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

Step 2: First sum:

$\sin^2\alpha + \sin^2\beta + \sin^2\gamma = 3 - 1 = 2$.

Step 3: Second sum:

$\cos 2\alpha + \cos 2\beta + \cos 2\gamma = 2(\cos^2\alpha + \cos^2\beta + \cos^2\gamma) - 3 = 2 - 3 = -1$.

Step 4: Add:

$2 + (-1) = 1$.

Final Answer:

The required sum is 1, option (D).

$$\boxed{1}$$

Quick Tip: Direction cosines satisfy $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

147. The difference between the maximum value and minimum value of the objective function $z = 3x + 5y$ of a linear programming problem subject to constraints $5x + 10y \leq 50$, $x + y \geq 1$, $y \leq 4$ and $x \geq 0, y \geq 0$ is 3λ. Then the value of λ is

(A) 3.

(B) 6.

(C) 9.

(D) 27.

Correct Answer: (C) 9.

Solution:

Step 1: Understanding the Concept:
In a linear programming problem, the maximum and minimum of the objective function occur at corner points of the feasible region.

Step 2: Draw the region:
The constraints are $x + 2y \leq 10$, $x + y \geq 1$, $y \leq 4$, $x, y \geq 0$. The boundaries meet at $(1, 0)$, $(10, 0)$, $(2, 4)$, $(0, 4)$ and $(0, 1)$. For example, $y = 4$ meets $x + 2y = 10$ at $x = 2$.

Step 3: Evaluate z = 3x + 5y:
At $(1, 0)$: 3. At $(10, 0)$: 30. At $(2, 4)$: 26. At $(0, 4)$: 20. At $(0, 1)$: 5.

Step 4: Find the difference:
Maximum is 30 and minimum is 3. The difference is 27. Given $3\lambda = 27$, we get $\lambda = 9$.

Step 5: Why the other options are wrong:
Option (D) 27 is the difference itself, not λ, so we still must divide by

3. Options (A) 3 and (B) 6 would need a difference of 9 or 18, which no pair of corner values gives.

Final Answer:

So $\lambda = 9$, which is option (C).

$$\boxed{9}$$

Quick Tip: Find the corner points of the feasible region and compare z at each one.

148. In a multiple-choice examination, there are 10 questions with one correct option out of 4 options for each question. A student gets 4 marks for each correct answer and 1 mark is deducted for each incorrect answer. The probability that a student, guessing randomly on every question, scores 30 marks in this exam is...

(A) $\frac{45}{4^{10}}$

(B) $\frac{135}{4^{10}}$

(C) $\frac{405}{4^{10}}$

(D) $\frac{810}{4^{10}}$

Correct Answer: (C) $\frac{405}{4^{10}}$

Solution:

Step 1: Understanding the Question:

Let k be the number of correct answers. The other $10 - k$ are wrong. The score is $4k - (10 - k) = 5k - 10$.

Step 2: Find k:

$5k - 10 = 30$, so $k = 8$.

Step 3: Binomial probability:

Each guess is right with probability $p = \frac{1}{4}$ and wrong with $q = \frac{3}{4}$.

$$P(X = 8) = \binom{10}{8}\left(\frac{1}{4}\right)^8\left(\frac{3}{4}\right)^2 = 45 \cdot \frac{9}{4^{10}} = \frac{405}{4^{10}}$$

Final Answer:

The probability is $\frac{405}{4^{10}}$, option (C).

$$\boxed{\frac{405}{4^{10}}}$$

Quick Tip: Find the number of correct answers from $4k - (10 - k) = 30$, then use the binomial formula.

149. It is known that a box of 8 batteries contains 3 defective pieces and a person randomly selects 2 batteries from this box. Then the probability distribution of the number of defective batteries is

(A)

$X = x$	0	1	2
$P(X = x)$	$\frac{10}{28}$	$\frac{15}{28}$	$\frac{3}{28}$

(B)

$X = x$	1	2	3
$P(X = x)$	$\frac{10}{28}$	$\frac{15}{28}$	$\frac{3}{28}$

(C)

$X = x$	0	1	2
$P(X = x)$	$\frac{15}{28}$	$\frac{10}{28}$	$\frac{3}{28}$

(D)

$X = x$	1	2	3
$P(X = x)$	$\frac{15}{28}$	$\frac{10}{28}$	$\frac{3}{28}$

Correct Answer: (A)

$X = x$	0	1	2
$P(X = x)$	$\frac{10}{28}$	$\frac{15}{28}$	$\frac{3}{28}$

Solution:

Step 1: Understanding the Question:

There are 8 batteries, 3 defective and 5 good. Two are drawn, so $X \in \{0, 1, 2\}$. Total ways: $\binom{8}{2} = 28$.

Step 2: Probabilities:

$P(X = 0) = \frac{\binom{5}{2}}{28} = \frac{10}{28}$.

$P(X=1) = \frac{\binom{3}{1}\binom{5}{1}}{28} = \frac{15}{28}$.
$P(X=2) = \frac{\binom{3}{2}}{28} = \frac{3}{28}$.
The total is $\frac{10+15+3}{28} = 1$.

Step 3: Match:
This is the table in option A. Options B and D start at $X = 1$, which cannot be (the number of defective ones can be 0). Option C swaps $\frac{10}{28}$ and $\frac{15}{28}$.

Final Answer:
The distribution is $P(0) = \frac{10}{28}$, $P(1) = \frac{15}{28}$, $P(2) = \frac{3}{28}$, option (A).

Option (A)

Quick Tip: X can be 0, 1 or 2, and the probabilities use combinations out of $\binom{8}{2} = 28$.

150. A certain disease has a prevalence of 1% in the population. A diagnostic test for the disease has a sensitivity of 98% and a specificity of 95%. If a person from this population tests positive, then the probability that they actually have the disease is...

(A) 98%

(B) 95%

(C) 16.5%

(D) 1%

Correct Answer: (C) 16.5%

Solution:

Step 1: Understanding the Question:

Let D be having the disease and $+$ a positive test. $P(D) = 0.01$, $P(+|D) = 0.98$ (sensitivity), and $P(+|D^c) = 1 - 0.95 = 0.05$ (specificity is 0.95).

Step 2: Bayes' theorem:

$$P(D|+) = \frac{P(+|D)P(D)}{P(+|D)P(D) + P(+|D^c)P(D^c)}$$

$$= \frac{0.98 \times 0.01}{0.98 \times 0.01 + 0.05 \times 0.99} = \frac{0.0098}{0.0098 + 0.0495} = \frac{0.0098}{0.0593} \approx 0.165$$

Step 3: Interpretation:

The answer is about 16.5%. It is small because the disease is rare, so false positives from the healthy majority outnumber true positives. Option A (98%) is the sensitivity, and B (95%) is the specificity. D (1%) is the prevalence.

Final Answer:

The probability that the person has the disease is about 16.5%, option (C).

$$\boxed{16.5\%}$$

Quick Tip: Apply Bayes' theorem with prevalence 1 percent, sensitivity 98 percent and false positive rate 5 percent.