

MHT CET 2026 PCM Question Paper April 11 Shift 2 with Solutions



General Instructions

- (i) This booklet contains 150 questions, each provided with a complete, step-by-step solution.
- (ii) It comprises 150 single-correct multiple-choice questions.
- (iii) Attempt each question on your own before reviewing the given solution.

Physics

Questions 1 to 50 | 1 mark each | 50 marks

1. A quantity P is related as $X^{-2}Y^{-3/2}Z^{2/5}$ where X, Y, Z are independent parameters which have fractional errors of 0.1, 0.2 and 0.5 respectively in measurement. The maximum fractional error in P is

- (A) 0.7
- (B) 0.1
- (C) 0.8
- (D) 0.6

Correct Answer: (A) 0.7

Solution:

Step 1: Understanding the Concept:

When a quantity is $P = X^a Y^b Z^c$, the maximum fractional error is $\frac{\Delta P}{P} = |a| \frac{\Delta X}{X} + |b| \frac{\Delta Y}{Y} + |c| \frac{\Delta Z}{Z}$. Errors always add in the worst case, even for negative powers.

Step 2: Read off the powers:

Here $a = -2$, $b = -\frac{3}{2}$, $c = \frac{2}{5}$. The fractional errors are 0.1, 0.2, 0.5.

Step 3: Add the contributions:

$$\frac{\Delta P}{P} = 2(0.1) + \frac{3}{2}(0.2) + \frac{2}{5}(0.5) = 0.2 + 0.3 + 0.2 = 0.7$$

Step 4: Why the other options are wrong.

0.1 is just one of the inputs. 0.8 and 0.6 come from taking the powers with wrong magnitudes or from subtracting the terms for negative powers. Subtraction is never allowed for maximum error.

Final Answer:

The maximum fractional error in P is 0.7.

(A) 0.7

Quick Tip: Maximum fractional error is the sum of (power times fractional error) for each quantity.

2. The vectors $(\vec{A} + \vec{B})$ and $(\vec{A} - \vec{B})$ are perpendicular to each other. This is possible under the condition

- (A) $|\vec{A}| = |\vec{B}|$
 (B) $\vec{A} \cdot \vec{B} = 0$
 (C) $\vec{A} \times \vec{B} = 0$
 (D) $\vec{A} \cdot \vec{B} = 1$

Correct Answer: (A) $|\vec{A}| = |\vec{B}|$

Solution:

Step 1: Understanding the Concept:

Two vectors are perpendicular when their dot product is zero.

Step 2: Expand:

$$(\vec{A} + \vec{B}) \cdot (\vec{A} - \vec{B}) = \vec{A} \cdot \vec{A} - \vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{A} - \vec{B} \cdot \vec{B} = |\vec{A}|^2 - |\vec{B}|^2.$$

Step 3: Set to zero:

$$|\vec{A}|^2 - |\vec{B}|^2 = 0, \text{ so } |\vec{A}| = |\vec{B}|. \text{ Option A.}$$

Step 4: Why the other options are wrong.

$\vec{A} \cdot \vec{B} = 0$ means A is perpendicular to B, which is a different condition. $\vec{A} \times \vec{B} = 0$ means parallel vectors. $\vec{A} \cdot \vec{B} = 1$ has no link to the condition.

Final Answer:

A and B must have equal magnitudes.

$(A) |\vec{A}| = |\vec{B}|$

Quick Tip: Set the dot product $(A + B) \cdot (A - B)$ equal to zero.

3. The angular speed of the minute hand of a clock in degrees per second is

- (A) 0.6
- (B) 0.1
- (C) 0.12
- (D) 1

Correct Answer: (B) 0.1

Solution:

Step 1: Understanding the Concept:

Angular speed is the angle turned per unit time: $\omega = \frac{\theta}{t}$.

Step 2: Use one full turn:

The minute hand makes one complete revolution (360°) in 60 minutes, which is $60 \times 60 = 3600$ s.

Step 3: Calculate:

$$\omega = \frac{360^\circ}{3600 \text{ s}} = 0.1^\circ/\text{s}$$

Step 4: Why the other options are wrong.

The second hand turns $6^\circ/\text{s}$, so that is not the answer. A value of 0.6 would come from dividing 360 by 600 instead of 3600. 0.12 and 1 do not match a 3600 s period.

Final Answer:

The angular speed is 0.1 degree per second.

(B) 0.1

Quick Tip: The minute hand covers 360 degrees in 3600 seconds.

4. In non uniform circular motion, the ratio of radial acceleration to tangential acceleration is (V is the velocity, r is the radius and α is the angular acceleration)

- (A) $\frac{\alpha r}{V}$
- (B) $\frac{V^2}{r^2 \alpha}$
- (C) $\frac{r^2 \alpha}{V^2}$
- (D) $\frac{r \alpha^2}{V^2}$

Correct Answer: (B) $\frac{V^2}{r^2 \alpha}$

Solution:

Step 1: Understanding the Concept:

In non-uniform circular motion the speed changes. The total acceleration has two parts: radial (centripetal) and tangential.

Step 2: Write the two components:

Radial acceleration $a_r = \frac{V^2}{r}$. Tangential acceleration $a_t = \alpha r$, where α is the angular acceleration.

Step 3: Take the ratio:

$$\frac{a_r}{a_t} = \frac{V^2/r}{\alpha r} = \frac{V^2}{r^2 \alpha}$$

Option B.

Step 4: Why the other options are wrong.

Option A, $\frac{\alpha r}{V}$, has the wrong dimension (it is not dimensionless).

Option C is the inverse of the right ratio. Option D has the wrong powers of α and r .

Final Answer:

The ratio is $V^2 / (r^2 \alpha)$.

$$(B) \frac{V^2}{r^2 \alpha}$$

Quick Tip: Radial acceleration is V^2/r and tangential acceleration is α times r .

5. A mass M moving with velocity V along X-axis collides and sticks to another mass $2M$ which is moving along Y-axis with velocity $3V$. The velocity of the combination, after the collision is

- (A) $V\hat{i} + 2V\hat{j}$
- (B) $\frac{V}{3}\hat{i} + 2V\hat{j}$
- (C) $V\hat{i} + 3V\hat{j}$
- (D) $2V\hat{i} + 4V\hat{j}$

Correct Answer: (B) $\frac{V}{3}\hat{i} + 2V\hat{j}$

Solution:

Step 1: Understanding the Concept:

The masses stick together, so the collision is perfectly inelastic. Linear momentum is conserved in each direction.

Step 2: Momentum before the collision:

Along X: $M \cdot V = MV$. Along Y: $2M \cdot 3V = 6MV$.

Step 3: Momentum after the collision:

The combined mass is $3M$ with velocity (v_x, v_y) . $3Mv_x = MV$ gives $v_x = \frac{V}{3}$. $3Mv_y = 6MV$ gives $v_y = 2V$.

Step 4: Result:

$\vec{v} = \frac{V}{3}\hat{i} + 2V\hat{j}$. Option B.

Step 5: Why the other options are wrong.

Options A, C and D divide by 3 only partially or not at all, ignoring that the combined mass is $3M$.

Final Answer:

The velocity is $(V/3)\hat{i} + 2V\hat{j}$.

$(B) \frac{V}{3}\hat{i} + 2V\hat{j}$

Quick Tip: Conserve momentum separately along X and Y; the total mass is $3M$.

6. The moment of inertia of a ring about an axis passing through its centre and perpendicular to its plane is I . It is rotating with angular velocity ω . Another identical ring is gently placed on it so that their centres coincide. If both rings are rotating about the same axis then loss in kinetic energy is

- (A) $I\omega^2$
- (B) $\frac{I\omega^2}{2}$
- (C) $\frac{I\omega^2}{4}$
- (D) $\frac{I\omega^2}{8}$

Correct Answer: (C) $\frac{I\omega^2}{4}$

Solution:

Step 1: Understanding the Concept:

No external torque acts, so angular momentum is conserved. The second ring starts at rest and then moves with the first, so energy is lost as friction heat.

Step 2: Find the final angular velocity:

$$I\omega = (I + I)\omega', \text{ so } \omega' = \frac{\omega}{2}.$$

Step 3: Find the kinetic energies:

$$\text{Initial: } K_i = \frac{1}{2}I\omega^2. \text{ Final: } K_f = \frac{1}{2}(2I)\left(\frac{\omega}{2}\right)^2 = \frac{1}{4}I\omega^2.$$

Step 4: Find the loss:

$$K_i - K_f = \frac{1}{2}I\omega^2 - \frac{1}{4}I\omega^2 = \frac{1}{4}I\omega^2. \text{ Option C.}$$

Step 5: Why the other options are wrong.

$I\omega^2$ and $\frac{I\omega^2}{2}$ are bigger than or equal to the initial energy, which cannot be lost. $\frac{I\omega^2}{8}$ comes from halving the correct loss.

Final Answer:

The loss in kinetic energy is $I\omega^2 / 4$.

$$(C) \frac{I\omega^2}{4}$$

Quick Tip: Angular momentum is conserved, but kinetic energy is not.

7. Let M and L be the mass and length of thin uniform rod respectively. In first case, axis of rotation is passing through centre and perpendicular to length of rod. In second case axis of rotation is passing through one end and perpendicular to length of rod. The ratio of radius of gyration in first case to second case is

- (A) $\frac{1}{4}$
- (B) $\frac{1}{2}$
- (C) $\frac{1}{8}$
- (D) $\frac{1}{6}$

Correct Answer: (B) $\frac{1}{2}$

Solution:**Step 1: Understanding the Concept:**

The radius of gyration k is defined by $I = Mk^2$, so $k = \sqrt{I/M}$.

Step 2: Moment of inertia in each case:

Centre axis: $I_1 = \frac{ML^2}{12}$. End axis: $I_2 = \frac{ML^2}{3}$.

Step 3: Find the radii of gyration:

$$k_1 = \frac{L}{\sqrt{12}} \text{ and } k_2 = \frac{L}{\sqrt{3}}.$$

Step 4: Take the ratio:

$$\frac{k_1}{k_2} = \sqrt{\frac{3}{12}} = \sqrt{\frac{1}{4}} = \frac{1}{2}$$

Option B.

Step 5: Why the other options are wrong.

$\frac{1}{4}$ is the ratio of the moments of inertia, I_1/I_2 , not the radii. $\frac{1}{8}$ and $\frac{1}{6}$ do not follow from the formulas.

Final Answer:

The ratio of radii of gyration is $1/2$.

$$\boxed{\text{(B) } \frac{1}{2}}$$

Quick Tip: Use $I = ML^2/12$ about the centre and $ML^2/3$ about the end, with $I = M k^2$.

8. The density of a planet is three times that of earth and radius 2.5 times that of earth. If g_p and g_e represents acceleration due to gravity on planet and earth respectively then ratio g_p to g_e is

- (A) 7.5
- (B) 3.5
- (C) 6.5
- (D) 9.5

Correct Answer: (A) 7.5

Solution:

Step 1: Understanding the Concept:

For a spherical planet, $g = \frac{GM}{R^2}$ with $M = \frac{4}{3}\pi R^3 \rho$. So $g = \frac{4}{3}\pi G \rho R$.

Step 2: Compare planet and earth:

$$\frac{g_p}{g_e} = \frac{\rho_p}{\rho_e} \times \frac{R_p}{R_e} = 3 \times 2.5 = 7.5.$$

Step 3: Why the other options are wrong.

3.5, 6.5 and 9.5 are not the product of 3 and 2.5. Using $g \propto M/R^2$ with an incorrect cube would also give a different value.

Final Answer:

The ratio g_p / g_e is 7.5.

(A) 7.5

Quick Tip: g is proportional to density times radius.

9. A liquid drop having surface energy E is sprayed into 512 droplets of the same size. Then the final surface energy is

- (A) $4E$
- (B) $8E$
- (C) $2E$
- (D) E

Correct Answer: (B) $8E$

Solution:

Step 1: Understanding the Concept:

Surface energy equals surface tension times surface area. When the drop splits, the volume stays the same but the area changes.

Step 2: Find the small drop radius:

$$512 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3, \text{ so } r^3 = \frac{R^3}{512} \text{ and } r = \frac{R}{8}.$$

Step 3: Compare areas:

Final area = $512 \times 4\pi r^2 = 512 \times 4\pi \frac{R^2}{64} = 8 \times 4\pi R^2$. So the area is 8 times the initial area.

Step 4: Find the energy:

Final surface energy = $8E$. Option B.

Step 5: Why the other options are wrong.

$4E$, $2E$ and E are less than the correct factor. Energy rises because many small drops have more total surface than one big drop.

Final Answer:

The final surface energy is $8E$.

(B) $8E$

Quick Tip: Volume is conserved, so $r = R/8$; surface area (and energy) increases by a factor of 8.



10. In a capillary tube of radius ' R ', a straight thin metal wire of radius ' r ' is inserted symmetrically and one end of the combination is dipped vertically in water such that lower end of the capillary and thin wire are at same level. If ' T ' is the surface tension of water and ' ρ ' is the density of water then the rise of water in the capillary is

- (A) $\frac{T}{(R-r)\rho g}$
- (B) $\frac{2T}{(R-r)\rho g}$
- (C) $\frac{4T}{(R+r)\rho g}$
- (D) $\frac{T}{(R+r)\rho g}$

Correct Answer: (B) $\frac{2T}{(R-r)\rho g}$

Solution:

Step 1: Understanding the Concept:

The upward pull of surface tension acts along the wetted perimeter and balances the weight of the liquid column.

Step 2: Upward force:

Water touches the capillary wall (perimeter $2\pi R$) and the wire (perimeter $2\pi r$). Assuming zero contact angle, the total upward force is $T \cdot 2\pi(R + r)$.

Step 3: Weight of the column:

The area of cross-section of the water column is $\pi(R^2 - r^2)$. Weight
 $= \pi(R^2 - r^2)h\rho g$.

Step 4: Equate and solve:

$T \cdot 2\pi(R + r) = \pi(R^2 - r^2)h\rho g$. Cancel $\pi(R + r)$:

$$h = \frac{2T}{(R - r)\rho g}$$

Option B.

Step 5: Why the other options are wrong.

Option A drops the factor of 2. Options C and D have $R + r$ in the denominator, which comes from using the area $\pi(R + r)^2$ or other wrong geometry.

Final Answer:

The rise is $2T / ((R - r) \rho g)$.

(B) $\frac{2T}{(R - r)\rho g}$

Quick Tip: The wetted perimeter is $2\pi(R + r)$, and the water column cross-section is $\pi(R^2 - r^2)$.

11. Due to surface tension, the excess pressure inside a smaller drop is 9 units. If 27 smaller drops combine then excess pressure inside the bigger drop is

- (A) 3 units
- (B) 9 units
- (C) 18 units
- (D) 6 units

Correct Answer: (A) 3 units

Solution:

Step 1: Understanding the Concept:

The excess pressure inside a drop is $P = \frac{2T}{r}$, so it is inversely proportional to the radius.

Step 2: Find the radius of the big drop:

$$27 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3, \text{ so } R = 3r.$$

Step 3: Find the new pressure:

$$P_{big} = \frac{2T}{R} = \frac{2T}{3r} = \frac{1}{3}P_{small} = \frac{9}{3} = 3 \text{ units. Option A.}$$

Step 4: Why the other options are wrong.

9 units ignores the change of radius. 18 units and 6 units come from multiplying instead of dividing, or from using 3 as the factor on the wrong side.

Final Answer:

The excess pressure is 3 units.

(A) 3 units

Quick Tip: Volume conservation gives $R = 3r$; excess pressure is inversely proportional to radius.

12. A body cools in 7 minutes from 60°C to 40°C . What time (in minutes) does it take to cool from 40°C to 28°C , if its surrounding temperature is 10°C ? (Newton's law of cooling holds good)

- (A) 3.5
- (B) 10
- (C) 11
- (D) 7

Correct Answer: (D) 7

Solution:

Step 1: Understanding the Concept:

Newton's law of cooling in the average form is $\frac{T_1 - T_2}{t} = k \left(\frac{T_1 + T_2}{2} - T_0 \right)$.

Step 2: First interval:

From 60°C to 40°C in 7 min with $T_0 = 10^{\circ}\text{C}$: $\frac{20}{7} = k(50 - 10) = 40k$, so $k = \frac{1}{14}$ per min.

Step 3: Second interval:

From 40°C to 28°C : $\frac{12}{t} = k(34 - 10) = 24k = \frac{24}{14}$. So $t = \frac{12 \times 14}{24} = 7$ min.

Step 4: Why the other options are wrong.

3.5 and 10 and 11 come from assuming a constant rate of fall or from a wrong average temperature.

Final Answer:

The time is 7 minutes.

(D) 7

Quick Tip: Use the average-temperature form of Newton's law for each step.

13. A rigid diatomic gas undergoes adiabatic change. Its pressure P and temperature T are related as $P \propto T^x$ where x is

- (A) 1.5
- (B) 2.5
- (C) 3.5
- (D) 4.5

Correct Answer: (C) 3.5

Solution:

Step 1: Understanding the Concept:

For an adiabatic process of an ideal gas, $PV^\gamma = \text{constant}$ and $PV = nRT$. Eliminating V gives $P^{1-\gamma}T^\gamma = \text{constant}$.

Step 2: Find x :

$$P \propto T^{\gamma/(\gamma-1)}, \text{ so } x = \frac{\gamma}{\gamma-1}.$$

Step 3: Find gamma for a rigid diatomic gas:

$C_v = \frac{5}{2}R$, $C_p = \frac{7}{2}R$, so $\gamma = \frac{7}{5}$. Then $x = \frac{7/5}{2/5} = \frac{7}{2} = 3.5$. Option C.

Step 4: Why the other options are wrong.

For a monatomic gas with $\gamma = \frac{5}{3}$, the exponent is $\frac{5/3}{2/3} = 2.5$, which is option B. Values 1.5 and 4.5 do not match any ideal gas.

Final Answer:

$x = 3.5$.

(C) 3.5

Quick Tip: Use $T^\gamma P^{(1-\gamma)} = \text{constant}$ with $\gamma = 7/5$ for a rigid diatomic gas.



14. If ΔQ is the amount of heat supplied to 'n' moles of a rigid diatomic gas at constant pressure, ΔU is the change in internal energy and ΔW is the work done then $\Delta W : \Delta U : \Delta Q$ is

- (A) 2 : 5 : 7
- (B) 1 : 4 : 7
- (C) 2 : 3 : 4
- (D) 3 : 5 : 9

Correct Answer: (A) 2 : 5 : 7

Solution:

Step 1: Understanding the Concept:

For a rigid diatomic gas, $C_v = \frac{5}{2}R$ and $C_p = \frac{7}{2}R$. At constant pressure the first law reads $\Delta Q = \Delta U + \Delta W$.

Step 2: Write each quantity per unit n delta T:

$$\Delta Q = nC_p\Delta T = \frac{7}{2}nR\Delta T. \Delta U = nC_v\Delta T = \frac{5}{2}nR\Delta T. \Delta W = P\Delta V = nR\Delta T.$$

Step 3: Take the ratio:

$$\Delta W : \Delta U : \Delta Q = 1 : \frac{5}{2} : \frac{7}{2} = 2 : 5 : 7. \text{ Option A.}$$

Step 4: Why the other options are wrong.

The sum of the first two terms must equal the third. Only $2 + 5 = 7$ holds. For example $1 + 4 \neq 7$ in option B.

Final Answer:

The ratio is $2 : 5 : 7$.

(A) $2 : 5 : 7$

Quick Tip: At constant pressure, $Q = n C_p dT$, $dU = n C_v dT$ and $W = n R dT$.

15. An insulated container contains a monoatomic gas of molar mass 'm'. The container is moving with velocity 'V'. If it is stopped suddenly, the change in temperature of the gas is (R-gas constant)

- (A) $\frac{mv^2}{R}$
 (B) $\frac{mv^2}{2R}$
 (C) $\frac{mv^2}{3R}$
 (D) $\frac{mv^2}{5R}$

Correct Answer: (C) $\frac{mv^2}{3R}$

Solution:

Step 1: Understanding the Concept:

The container is insulated, so no heat leaves. When it stops suddenly, the bulk kinetic energy of the gas turns into random thermal energy of molecules.

Step 2: Write the energy balance:

Let the mass of the gas be M and its molar mass be m , so the number of moles is $n = \frac{M}{m}$. Kinetic energy lost = $\frac{1}{2}MV^2$.

Step 3: Internal energy gained:

For a monatomic gas, $\Delta U = n \cdot \frac{3}{2}R \Delta T = \frac{M}{m} \cdot \frac{3}{2}R \Delta T$.

Step 4: Solve:

$\frac{1}{2}MV^2 = \frac{3MR}{2m} \Delta T$, so $\Delta T = \frac{mV^2}{3R}$. Option C.

Step 5: Why the other options are wrong.

Options A and B ignore the $\frac{3}{2}R$ per mole for a monatomic gas (they use $C_v = R$ or $\frac{R}{2}$ instead). Option D uses $\frac{5}{2}R$, the value for a diatomic gas.

Final Answer:

The temperature rise is $\frac{m V^2}{3R}$.

$$(C) \frac{mV^2}{3R}$$

Quick Tip: Bulk kinetic energy becomes internal energy of the gas.

16. Carnot engine operating between temperatures T_1 and T_2 has efficiency 0.2. When T_2 is lowered by 45 K, its efficiency becomes 0.5. Temperatures T_1 and T_2 are respectively

- (A) 150 K, 120 K
- (B) 120 K, 150 K
- (C) 60 K, 80 K
- (D) 80 K, 60 K

Correct Answer: (A) 150 K, 120 K

Solution:**Step 1: Understanding the Concept:**

The efficiency of a Carnot engine is $\eta = 1 - \frac{T_2}{T_1}$.

Step 2: First condition:

$$0.2 = 1 - \frac{T_2}{T_1}, \text{ so } T_2 = 0.8T_1.$$

Step 3: Second condition:

The sink temperature is lowered to $T_2 - 45$: $0.5 = 1 - \frac{T_2 - 45}{T_1}$, so T_2

$$-45 = 0.5T_1.$$

Step 4: Solve:

Substitute $T_2 = 0.8T_1$: $0.8T_1 - 45 = 0.5T_1$, so $0.3T_1 = 45$ and $T_1 = 150$ K. Then $T_2 = 0.8 \times 150 = 120$ K. Option A.

Step 5: Why the other options are wrong.

B swaps source and sink (the source must be the hotter one). C and D give temperatures that do not satisfy $\eta = 0.2$ either.

Final Answer:

$T_1 = 150$ K and $T_2 = 120$ K.

(A) 150 K, 120 K

Quick Tip: Write efficiency $= 1 - T_2/T_1$ for both cases and solve.

17. For a gas, $\frac{R}{C_v} = 0.4$ where R is the universal gas constant and C_v is the molar specific heat at constant volume. The gas is made up of molecules which are

- (A) monoatomic
- (B) polyatomic
- (C) non rigid diatomic
- (D) rigid diatomic

Correct Answer: (D) rigid diatomic

Solution:

Step 1: Understanding the Concept:

The molar specific heat at constant volume is $C_v = \frac{f}{2}R$, where f is the number of degrees of freedom.

Step 2: Use the given ratio:

$\frac{R}{C_v} = 0.4$, so $C_v = \frac{R}{0.4} = 2.5R = \frac{5}{2}R$. That means $f = 5$.

Step 3: Match to the gas type:

A monatomic gas has $f = 3$. A rigid diatomic gas has $f = 5$ (3 translational + 2 rotational). A non-rigid diatomic gas has $f = 7$. So the gas is rigid diatomic. Option D.

Step 4: Why the other options are wrong.

Monatomic gas gives $C_v = 1.5R$, so $R/C_v = 0.67$. Non-rigid diatomic gives $C_v = 3.5R$, so $R/C_v = 0.286$.

Final Answer:

The gas is made of rigid diatomic molecules.

(D) rigid diatomic

Quick Tip: $R/C_v = 0.4$ gives $C_v = 5R/2$, which is a rigid diatomic gas.

18. An oscillating pendulum suspended from the roof of a lift which is at rest has time period T_1 . When lift moves up with acceleration 'a' its time period is T_2 . When lift moves down with acceleration 'a' its time period is T_3 . The relation between T_1 , T_2 and T_3 is

$$(A) \quad T_1 = \frac{2T_2T_3}{\sqrt{T_2^2 + T_3^2}}$$

$$(B) \quad T_1 = \frac{\sqrt{2}T_2T_3}{\sqrt{T_2^2 + T_3^2}}$$

$$(C) \quad T_1 = \frac{2T_2^2T_3^2}{\sqrt{T_2 + T_3}}$$

$$(D) \quad T_1 = \frac{\sqrt{2}T_2^2T_3^2}{\sqrt{T_2 + T_3}}$$

Correct Answer: (B) $T_1 = \frac{\sqrt{2}T_2T_3}{\sqrt{T_2^2 + T_3^2}}$

Solution:

Step 1: Understanding the Concept:

The time period of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g_{eff}}}$. In a lift, the effective gravity changes.

Step 2: Write the three periods:

At rest: $T_1 = 2\pi\sqrt{\frac{L}{g}}$. Lift going up with acceleration a : T_2
 $= 2\pi\sqrt{\frac{L}{g+a}}$. Lift going down: $T_3 = 2\pi\sqrt{\frac{L}{g-a}}$.

Step 3: Square and invert:

$$\frac{1}{T_2^2} = \frac{g+a}{4\pi^2L} \text{ and } \frac{1}{T_3^2} = \frac{g-a}{4\pi^2L}. \text{ Add: } \frac{1}{T_2^2} + \frac{1}{T_3^2} = \frac{2g}{4\pi^2L} = \frac{2}{T_1^2}.$$

Step 4: Solve for T1:

$$T_1^2 = \frac{2T_2^2T_3^2}{T_2^2 + T_3^2}, \text{ so } T_1 = \frac{\sqrt{2}T_2T_3}{\sqrt{T_2^2 + T_3^2}}. \text{ Option B.}$$

Step 5: Why the other options are wrong.

Option A has 2 instead of $\sqrt{2}$. Options C and D have the wrong powers

of T_2 and T_3 and do not match the derived form.

Final Answer:

$$T_1 = \sqrt{2 T_2 T_3} / \sqrt{T_2^2 + T_3^2}.$$

$$(B) T_1 = \frac{\sqrt{2} T_2 T_3}{\sqrt{T_2^2 + T_3^2}}$$

Quick Tip: Effective g is $g + a$ going up and $g - a$ going down; add $1/T^2$ values.

19. A mass 0.4 kg performs S.H.M. with a frequency $\frac{16}{\pi}$ Hz. At a certain displacement it has kinetic energy 2 J and potential energy 1.2 J. The amplitude of oscillation is

- (A) 0.125 m
- (B) 0.1 m
- (C) 0.05 m
- (D) 0.15 m

Correct Answer: (A) 0.125 m

Solution:

Step 1: Understanding the Concept:

In SHM the total energy is constant: $E = K + U = \frac{1}{2} m \omega^2 A^2$.

Step 2: Find the total energy and omega:

$$E = 2 + 1.2 = 3.2 \text{ J. } \omega = 2\pi f = 2\pi \times \frac{16}{\pi} = 32 \text{ rad/s.}$$

Step 3: Solve for the amplitude:

$$3.2 = \frac{1}{2}(0.4)(32)^2 A^2 = 0.2 \times 1024 A^2 = 204.8 A^2. \text{ So } A^2 = \frac{3.2}{204.8} \\ = 0.015625 \text{ and } A = 0.125 \text{ m. Option A.}$$

Step 4: Why the other options are wrong.

0.1, 0.05 and 0.15 m would give total energies 2.05 J, 0.51 J and 4.6 J, none equal to 3.2 J.

Final Answer:

The amplitude is 0.125 m.

(A) 0.125 m

Quick Tip: Total energy = KE + PE = $(1/2) m \omega^2 A^2$ with $\omega = 2\pi f$.

20. A mass attached to a spring performs S.H.M. whose displacement is $x = 3 \times 10^{-3} \cos 2\pi t$ m. The time taken to obtain maximum speed for the first time is (in s)

- (A) 1
- (B) $\frac{1}{2}$
- (C) $\frac{1}{4}$
- (D) $\frac{1}{8}$

Correct Answer: (C) $\frac{1}{4}$

Solution:

Step 1: Understanding the Concept:

In SHM the speed is maximum when the mass passes through the mean position ($x = 0$).

Step 2: Set $x = 0$:

$3 \times 10^{-3} \cos 2\pi t = 0$, so $\cos 2\pi t = 0$. The smallest positive solution is $2\pi t = \frac{\pi}{2}$, so $t = \frac{1}{4}$ s.

Step 3: Check with the period:

$\omega = 2\pi$, so $T = 1$ s. The body starts at the extreme position and reaches the mean position after $\frac{T}{4} = \frac{1}{4}$ s. Option C.

Step 4: Why the other options are wrong.

1 s is a full period, back at the extreme. $1/2$ s is the opposite extreme. $1/8$ s is not a special point of the motion.

Final Answer:

The time is $1/4$ s.

$(C) \frac{1}{4} \text{ s}$

Quick Tip: Maximum speed occurs at the mean position, where $x = 0$.

21. The closed and open organ pipe have same length and when they are vibrating simultaneously in first overtone produce 3 beats. The length of open pipe is made $(\frac{1}{3})^{rd}$ and closed pipe is made 3 times the original, the number of beats produced will be {neglect end correction}

- (A) 8
- (B) 10
- (C) 17
- (D) 33

Correct Answer: (D) 33

Solution:

Step 1: Understanding the Concept:

A closed pipe has a node at the closed end and an antinode at the open end. An open pipe has antinodes at both ends. Beats are the difference in frequency of two notes sounding together. Let v be the speed of sound and L the original length.

Step 2: Key Formula or Approach:

1. First overtone of a closed pipe is the third harmonic: $f_c = \frac{3v}{4L}$.
2. First overtone of an open pipe is the second harmonic: $f_o = \frac{2v}{2L} = \frac{v}{L}$.

Step 3: Use the first condition.

Both pipes have length L . The open pipe note is higher, since $\frac{v}{L} > \frac{3v}{4L}$. The beats are

$$f_o - f_c = \frac{v}{L} - \frac{3v}{4L} = \frac{v}{4L} = 3$$

So $\frac{v}{4L} = 3$, which gives $\frac{v}{L} = 12$ Hz.

Step 4: Change the lengths.

The open pipe is now $\frac{L}{3}$ long, so its first overtone is

$$f'_o = \frac{v}{L/3} = \frac{3v}{L} = 36 \text{ Hz}$$

The closed pipe is now $3L$ long, so its first overtone is

$$f'_c = \frac{3v}{4(3L)} = \frac{v}{4L} = 3 \text{ Hz}$$

Step 5: Find the new beats.

$$\text{Beats} = f'_o - f'_c = 36 - 3 = 33.$$

Final Answer:

The number of beats becomes 33, which is option (D).

33

Quick Tip: Write the first overtone of the closed pipe as $3v/4L$ and of the open pipe as v/L , use the 3 beats to find v/L , then rescale each frequency for its new length.

22. Two uniform wires of same material are vibrating under the same tension. If the first overtone of the first wire is equal to the second overtone of the second wire and radius of first wire is twice the radius of second wire then the ratio of length of first to second wire is

- (A) $\frac{1}{4}$
 (B) $\frac{1}{3}$
 (C) $\frac{2}{3}$
 (D) $\frac{1}{2}$

Correct Answer: (B) $\frac{1}{3}$

Solution:

Step 1: Understanding the Concept:

For a string fixed at both ends, $f_n = \frac{nv}{2L}$ with $v = \sqrt{T/\mu}$. For the same material, $\mu \propto r^2$, so $v \propto \frac{1}{r}$ at the same tension.

Step 2: Name the overtones:

First overtone is the second harmonic ($n = 2$): $f = \frac{v}{L}$. Second overtone is the third harmonic ($n = 3$): $f = \frac{3v}{2L}$.

Step 3: Equate the frequencies:

$$\frac{v_1}{L_1} = \frac{3v_2}{2L_2}, \text{ so } \frac{L_1}{L_2} = \frac{2}{3} \cdot \frac{v_1}{v_2}.$$

Step 4: Insert the radius ratio:

$$r_1 = 2r_2 \text{ gives } \frac{v_1}{v_2} = \frac{r_2}{r_1} = \frac{1}{2}. \text{ So } \frac{L_1}{L_2} = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}. \text{ Option B.}$$

Step 5: Why the other options are wrong.

$1/4$, $2/3$ and $1/2$ come from using $v \propto \frac{1}{r^2}$, ignoring the speed difference, or using the wrong harmonic numbers.

Final Answer:

The ratio of lengths is $1/3$.

$$(B) \frac{1}{3}$$

Quick Tip: Wires with same tension and material have v proportional to $1/r$; equate the overtone frequencies.



23. The equation of a progressive wave is $Y = 3\sin[\pi(\frac{t}{3} - \frac{x}{5}) + \frac{\pi}{4}]$ where X and Y are in metre and time in second. Which of the following is correct?

- (A) Velocity = 1.5 m/s
- (B) Amplitude = 30 cm
- (C) Wavelength = 10 m
- (D) Frequency = 0.2 Hz

Correct Answer: (C) Wavelength = 10 m

Solution:

Step 1: Understanding the Concept:

Compare the given wave with the standard form $Y = A \sin(\omega t - kx + \phi)$.

Step 2: Expand the equation:

$Y = 3 \sin\left(\frac{\pi}{3}t - \frac{\pi}{5}x + \frac{\pi}{4}\right)$. So $A = 3$ m, $\omega = \frac{\pi}{3}$ rad/s and $k = \frac{\pi}{5}$ rad/m.

Step 3: Find each quantity:

Wavelength $\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi/5} = 10$ m. Frequency $f = \frac{\omega}{2\pi} = \frac{1}{6}$ Hz.

$$\text{Speed } v = \frac{\omega}{k} = \frac{\pi/3}{\pi/5} = \frac{5}{3} \text{ m/s.}$$

Step 4: Check the options:

(A) speed 1.5 m/s is wrong, it is 1.67 m/s. (B) amplitude 30 cm is wrong, it is 3 m = 300 cm. (C) wavelength 10 m is correct. (D) frequency 0.2 Hz is wrong, it is 0.167 Hz. Option C.

Final Answer:

The wavelength is 10 m.

(C) Wavelength = 10 m

Quick Tip: Compare with $Y = A \sin(\omega t - kx + \phi)$ and find ω , k , wavelength and speed.



24. A source of sound is moving towards a stationary observer with velocity ' V_s ' and then moves away with velocity ' V_s '. Assume that medium through which the sound waves travel is at rest. If ' V ' is the velocity of sound and ' n ' is the frequency emitted by the source then the difference between the apparent frequencies heard by the observer is

- (A) $\frac{2nVV_s}{V^2 - V_s^2}$
- (B) $\frac{nVV_s}{V^2 - V_s^2}$
- (C) $\frac{nVV_s}{V_s^2 + V^2}$
- (D) $\frac{2nVV_s}{V_s^2 + V^2}$

Correct Answer: (A) $\frac{2nVV_s}{V^2 - V_s^2}$

Solution:

Step 1: Understanding the Concept:

When a source moves toward a stationary observer, the observed frequency rises. When it moves away, the frequency falls.

Step 2: Write the two frequencies:

Approaching: $n_1 = \frac{nV}{V - V_s}$. Receding: $n_2 = \frac{nV}{V + V_s}$.

Step 3: Subtract:

$$n_1 - n_2 = nV \left[\frac{1}{V - V_s} - \frac{1}{V + V_s} \right] = nV \cdot \frac{2V_s}{V^2 - V_s^2}$$

So the difference is $\frac{2nVV_s}{V^2 - V_s^2}$. Option A.

Step 4: Why the other options are wrong.

Option B misses the factor 2. Options C and D have $V^2 + V_s^2$ in the denominator, but the product $(V - V_s)(V + V_s)$ equals $V^2 - V_s^2$.

Final Answer:

The difference is $2nVV_s / (V^2 - V_s^2)$.

(A) $\frac{2nVV_s}{V^2 - V_s^2}$

Quick Tip: Use $f = nV/(V - V_s)$ for approach and $f = nV/(V + V_s)$ for recession.

25. If the radius of the spherical Gaussian surface is increased then the electric flux due to a point charge enclosed by the surface

- (A) increases
- (B) remains unchanged
- (C) is zero
- (D) decreases

Correct Answer: (B) remains unchanged

Solution:

Step 1: Understanding the Concept:

Gauss's law states that the net electric flux through a closed surface is

$$\Phi = \frac{q_{\text{enclosed}}}{\epsilon_0}.$$

Step 2: Apply:

The point charge stays inside when the sphere is enlarged, so q_{enclosed} does not change. The flux remains $\frac{q}{\epsilon_0}$.

Step 3: Why the other options are wrong.

The field falls as $1/r^2$, but the area grows as r^2 , so their product is constant. Flux does not rise, fall, or become zero.

Final Answer:

The flux remains unchanged.

(B) remains unchanged

Quick Tip: By Gauss law the flux depends only on enclosed charge.

26. The earth is assumed to be a charged conducting sphere having volume 'V' and surface area 'A'. The capacitance of the earth in free space is (ϵ_0 = permittivity of free space)

- (A) $2\pi\epsilon_0 V/A$
- (B) $4\pi\epsilon_0 V/A$
- (C) $8\pi\epsilon_0 V/A$
- (D) $12\pi\epsilon_0 V/A$

Correct Answer: (D) $12\pi\epsilon_0 V/A$

Solution:

Step 1: Understanding the Concept:

A charged conducting sphere of radius R has capacitance $C = 4\pi\epsilon_0 R$.

Step 2: Express R using V and A:

$$V = \frac{4}{3}\pi R^3 \text{ and } A = 4\pi R^2. \text{ So } \frac{V}{A} = \frac{R}{3}, \text{ giving } R = \frac{3V}{A}.$$

Step 3: Substitute:

$$C = 4\pi\epsilon_0 \cdot \frac{3V}{A} = \frac{12\pi\epsilon_0 V}{A}. \text{ Option D.}$$

Step 4: Why the other options are wrong.

Options A, B and C come from using $R = \frac{V}{2A}$, $R = \frac{V}{A}$ and $R = \frac{2V}{A}$ instead of $R = \frac{3V}{A}$. Note that $V/A = R/3$, not R .

Final Answer:

The capacitance is $12\pi\epsilon_0 V/A$.

$(D) \frac{12\pi\epsilon_0 V}{A}$

Quick Tip: Use $C = 4\pi\epsilon_0 R$ and express R through V and A .

27. Initially, ' n ' identical capacitors are joined in parallel, are charged to potential ' V '. Now they are separated and joined in series. Then

- (A) potential difference and total energy of the combination remain the same
- (B) potential difference remains the same and energy increases ' n ' times
- (C) potential difference becomes ' nV ' and energy remains the same
- (D) potential difference is ' nV ' and energy increases ' n ' times.

Correct Answer: (C) potential difference becomes ' nV ' and energy remains the same

Solution:

Step 1: Understanding the Concept:

Each capacitor of capacitance C is charged to V in parallel, so each carries charge $q = CV$ and energy $\frac{1}{2}CV^2$.

Step 2: Energy before:

$$\text{Total energy} = n \cdot \frac{1}{2}CV^2 = \frac{1}{2}nCV^2.$$

Step 3: Reconnect in series:

Each capacitor still holds charge $q = CV$ and the voltages add (with plates connected positive to negative): $V_{\text{total}} = nV$. The series capacitance is $\frac{C}{n}$.

Step 4: Energy after:

$$U = \frac{1}{2} \cdot \frac{C}{n} \cdot (nV)^2 = \frac{1}{2}nCV^2. \text{ The energy is unchanged. Option C.}$$

Step 5: Why the other options are wrong.

A and B keep the potential at V , which ignores that the voltages add.

D claims the energy rises n times, but the energy is just the sum of the n energies already stored.

Final Answer:

The potential becomes nV and the energy stays the same.

(C) potential difference becomes nV and energy remains the same

Quick Tip: Charge each capacitor in parallel, then reconnect in series: voltages add, energy stays the same.

28. Three parallel plate air capacitors are connected in parallel. Each capacitor has plate area $A/3$ and separation between the plates is d , $2d$ and $3d$ respectively. The equivalent capacity of the combination is (ϵ_0 -permittivity of free space)

- (A) $\frac{9\epsilon_0 A}{17d}$
- (B) $\frac{11\epsilon_0 A}{17d}$
- (C) $\frac{11\epsilon_0 A}{18d}$
- (D) $\frac{9\epsilon_0 A}{14d}$

Correct Answer: (C) $\frac{11\epsilon_0 A}{18d}$

Solution:

Step 1: Understanding the Concept:

For an air capacitor, $C = \frac{\epsilon_0 A_{plate}}{d}$. In a parallel connection, the equivalent capacitance is the sum of the capacitances.

Step 2: Find each capacitance:

Plate area is $\frac{A}{3}$. $C_1 = \frac{\epsilon_0 A}{3d}$, $C_2 = \frac{\epsilon_0 A}{6d}$, $C_3 = \frac{\epsilon_0 A}{9d}$.

Step 3: Add them:

$$C_{eq} = \frac{\epsilon_0 A}{d} \left(\frac{1}{3} + \frac{1}{6} + \frac{1}{9} \right) = \frac{\epsilon_0 A}{d} \cdot \frac{6 + 3 + 2}{18} = \frac{11\epsilon_0 A}{18d}. \text{ Option C.}$$

Step 4: Why the other options are wrong.

Options A, B and D have denominators 17 or 14, which would come from adding the separations in the denominator instead of adding the capacitances.

Final Answer:

The equivalent capacitance is $11 \epsilon_0 A / (18 d)$.

(C) $\frac{11\epsilon_0 A}{18d}$

Quick Tip: Capacitance is $\epsilon_0 A / d$ for each; in parallel they add.



29. In potentiometer experiment, null point is obtained at a particular point for a cell on potentiometer wire 'x' cm long. If length of potentiometer wire is increased by few cm without changing the cell, the balancing length will (Driving source is not changed)

- (A) decrease
- (B) increase
- (C) not change
- (D) become zero

Correct Answer: (B) increase

Solution:

Step 1: Understanding the Concept:

In a potentiometer, the potential gradient is $k = \frac{V}{L}$, where V is the driving voltage across the whole wire of length L . The null point is at $l = \frac{E}{k}$.

Step 2: Effect of lengthening the wire:

With the same driving source, the same voltage is spread over a longer wire (and the longer wire has more resistance, so the current drops). The potential gradient k becomes smaller.

Step 3: Effect on balancing length:

Since the cell emf E is the same, $l = \frac{E}{k}$ increases. Option B.

Step 4: Why the other options are wrong.

Decreasing or no change would need the gradient to rise or stay the same. A balancing length of zero is not possible for a real cell.

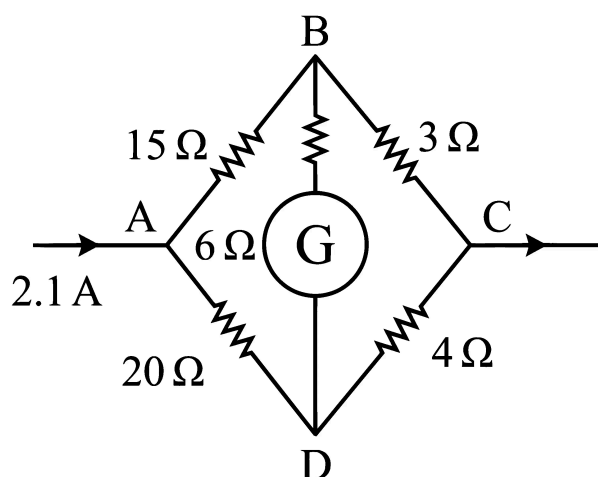
Final Answer:

The balancing length increases.

(B) increase

Quick Tip: The potential gradient falls when the wire becomes longer, so the balancing length increases.

30. In the following network, the current flowing through 15Ω resistance is



- (A) 2.1 A
- (B) 0.9 A
- (C) 1.2 A
- (D) 1.5 A

Correct Answer: (C) 1.2 A

Solution:

Step 1: Understanding the Figure:

The figure is a diamond-shaped bridge with corners A, B, C and D. A current of 2.1 A enters at A and leaves at C. Arm AB has 15 ohm, arm BC has 3 ohm, arm AD has 20 ohm and arm DC has 4 ohm. The middle branch BD carries a resistor and a galvanometer G.

Step 2: Key Formula or Approach:

A Wheatstone bridge is balanced when $\frac{R_{AB}}{R_{BC}} = \frac{R_{AD}}{R_{DC}}$. Then points B and D are at the same potential and no current flows through branch BD. The circuit reduces to two parallel paths.

Step 3: Check balance:

$$\frac{15}{3} = 5, \quad \frac{20}{4} = 5$$

The ratios are equal, so the bridge is balanced and BD can be ignored.

Step 4: Branch resistances:

Upper path A-B-C: $R_1 = 15 + 3 = 18 \Omega$. Lower path A-D-C: $R_2 = 20 + 4 = 24 \Omega$.

Step 5: Current divider:

Current splits in the inverse ratio of resistance. The 15 ohm resistor lies in the upper path:

$$I_{15} = 2.1 \times \frac{R_2}{R_1 + R_2} = 2.1 \times \frac{24}{42} = 1.2 \text{ A}$$

Step 6: Why the other options are wrong:

The value 0.9 A is the current in the lower path ($2.1 \times 18/42$). The value 2.1 A is the total current. The value 1.5 A does not follow from any split of the circuit.

Final Answer:

The current through the 15 ohm resistor is 1.2 A, option (C).

1.2 A

Quick Tip: Check if the bridge is balanced ($15/3 = 20/4$). If yes, split 2.1 A between the 18 ohm and 24 ohm paths.



31. A circular coil of N turns and diameter D carries current I . It is unwound and rewound to make another coil of diameter $2D$, current I remaining the same. The ratio of magnetic moments of the original coil to the new coil is

- (A) 2 : 1
- (B) 4 : 1
- (C) 1 : 2
- (D) 1 : 4

Correct Answer: (C) 1 : 2

Solution:

Step 1: Understanding the Concept:

The magnetic moment of a coil is $M = NIA$, where A is the area of each turn. The total wire length stays the same when the coil is rewound.

Step 2: Find the new number of turns:

Original wire length = $N \times \pi D$. New coil has diameter $2D$, so $N' \times \pi(2D) = N\pi D$, giving $N' = \frac{N}{2}$.

Step 3: Compare magnetic moments:

$$M_1 = NI\pi\frac{D^2}{4}. M_2 = \frac{N}{2}I\pi\frac{(2D)^2}{4} = \frac{N}{2}I\pi D^2.$$

$$\frac{M_1}{M_2} = \frac{N\pi D^2/4}{N\pi D^2/2} = \frac{1}{2}$$

So the ratio is 1 : 2. Option C.

Step 4: Why the other options are wrong.

2:1 and 4:1 would come from ignoring the change in turns. 1:4 would arise by multiplying area by 4 and forgetting turns also change.

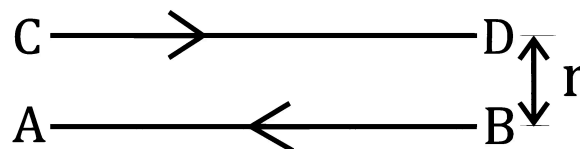
Final Answer:

The ratio of magnetic moments is 1 : 2.

(C) 1 : 2

Quick Tip: Wire length is fixed, so doubling the diameter halves the number of turns.

32. A wire AB is carrying steady current I_1 and is kept on the table. Another wire CD carrying current I_2 is held directly above as shown in figure. When the wire CD is left free and it remains suspended at its position, its mass per unit length is (g =acceleration due to gravity, μ_0 = permeability of free space)



- (A) $\frac{\mu_0 I_1 I_2}{2\pi r g}$
- (B) $\frac{\mu_0 I_1 I_2}{4\pi r g}$
- (C) $\frac{\mu_0 I_1 I_2}{\pi r g}$
- (D) $\frac{\mu_0 I_1 I_2}{\pi r^2 g}$

Correct Answer: (A) $\frac{\mu_0 I_1 I_2}{2\pi r g}$

Solution:

Step 1: Understanding the Figure:

Wire AB lies on the table and carries current to the left. Wire CD is held above it, parallel, at distance r , and carries current to the right. The currents are opposite, so the wires repel each other.

Step 2: Key Formula or Approach:

The magnetic force per unit length between two long parallel wires is

$$\frac{F}{L} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

For opposite currents it is repulsive.

Step 3: Direction of the force:

Wire CD is above AB, so the repulsion pushes CD upward. Gravity pulls it downward. For CD to stay suspended these two forces must be equal.

Step 4: Balance the forces:

Let λ be the mass per unit length of CD. Its weight per unit length is λg .

$$\lambda g = \frac{\mu_0 I_1 I_2}{2\pi r}$$

$$\lambda = \frac{\mu_0 I_1 I_2}{2\pi r g}$$

Step 5: Why the other options are wrong:

Options (B) and (C) carry factors of 4π and π instead of 2π , which do not match the force formula for parallel wires. Option (D) has r^2 , but the force between parallel wires falls as $1/r$, not $1/r^2$.

Final Answer:

The mass per unit length is $\frac{\mu_0 I_1 I_2}{2\pi r g}$, option (A).

$$\frac{\mu_0 I_1 I_2}{2\pi r g}$$

Quick Tip: Opposite currents repel, so the upward magnetic force per unit length equals the weight per unit length, λg .



33. Two parallel wires of equal lengths are separated by a distance of 3 m from each other. The currents flowing through first and second wire is 3 A and 4.5 A respectively in opposite directions. The resultant magnetic field at mid point of both the wires is (μ_0 =permeability of free space)

- (A) $\frac{\mu_0}{2\pi}$
- (B) $\frac{5\mu_0}{2\pi}$
- (C) $\frac{7\mu_0}{2\pi}$
- (D) $\frac{9\mu_0}{2\pi}$

Correct Answer: (B) $\frac{5\mu_0}{2\pi}$

Solution:

Step 1: Understanding the Concept:

A long straight wire gives $B = \frac{\mu_0 I}{2\pi r}$ at distance r . For two parallel wires with opposite currents, the fields between the wires point in the

same direction.

Step 2: Find the distances and fields:

The midpoint is 1.5 m from each wire. $B_1 = \frac{\mu_0(3)}{2\pi(1.5)} = \frac{2\mu_0}{2\pi} \cdot B_2$
 $= \frac{\mu_0(4.5)}{2\pi(1.5)} = \frac{3\mu_0}{2\pi}.$

Step 3: Add them:

$$B = B_1 + B_2 = \frac{2\mu_0}{2\pi} + \frac{3\mu_0}{2\pi} = \frac{5\mu_0}{2\pi}. \text{ Option B.}$$

Step 4: Why the other options are wrong.

$\frac{\mu_0}{2\pi}$ is the difference of the two fields, which holds for currents in the same direction. 7 and 9 come from using the wrong distance.

Final Answer:

The magnetic field is $5\mu_0 / (2\pi)$.

$(B) \frac{5\mu_0}{2\pi}$

Quick Tip: At the midpoint, fields of opposite currents point the same way and add.



34. A coil having 'n' turns and resistance 'R' is connected with a galvanometer of resistance '2R'. This combination is moved in time 't' from flux ϕ_1 to ϕ_2 Wb. The induced current in the circuit is

- (A) $\frac{n(\phi_1 - \phi_2)}{Rt}$
 (B) $\frac{n(\phi_2 - \phi_1)}{Rt}$
 (C) $\frac{n(\phi_2 - \phi_1)}{3Rt}$
 (D) $\frac{n(\phi_2 - \phi_1)}{2Rt}$

Correct Answer: (C) $\frac{n(\phi_2 - \phi_1)}{3Rt}$

Solution:

Step 1: Understanding the Concept:

By Faraday's law the induced emf of an n -turn coil is $\varepsilon = \frac{n \Delta \phi}{t}$.

Step 2: Find the emf:

$$\varepsilon = \frac{n(\phi_2 - \phi_1)}{t} \text{ in magnitude.}$$

Step 3: Find the total resistance:

The coil has resistance R and the galvanometer has $2R$, in series, so the total is $3R$.

Step 4: Find the current:

$$I = \frac{\varepsilon}{3R} = \frac{n(\phi_2 - \phi_1)}{3Rt}. \text{ Option C.}$$

Step 5: Why the other options are wrong.

Options A, B and D use a total resistance of R or $2R$ and so ignore one of the parts of the circuit.

Final Answer:

The induced current is $n(\phi_2 - \phi_1)/(3Rt)$.

$$(C) \frac{n(\phi_2 - \phi_1)}{3Rt}$$

Quick Tip: $\text{emf} = n (\text{change in flux})/t$ and the total resistance is $R + 2R$.



35. A light bulb connected in series with a capacitor and an a.c. source is glowing with certain brightness. On reducing the value of capacitance and frequency respectively, the brightness of the bulb

- (A) is reduced, is reduced
- (B) is more, is more
- (C) is more, is reduced
- (D) is reduced, is more

Correct Answer: (A) is reduced, is reduced

Solution:

Step 1: Understanding the Concept:

In a series circuit with a bulb (resistance R) and a capacitor, the

current is $I = \frac{V}{\sqrt{R^2 + X_C^2}}$ with $X_C = \frac{1}{2\pi fC}$.

Step 2: Reduce the capacitance:

A smaller C gives a larger X_C . So the impedance rises and the current falls. The bulb gets dimmer.

Step 3: Reduce the frequency:

A smaller f also gives a larger X_C . So again the current falls and the bulb gets dimmer.

Step 4: Conclusion:

The brightness is reduced in both cases. Option A.

Final Answer:

The brightness is reduced both times.

(A) is reduced, is reduced

Quick Tip: Capacitive reactance is $1/(2\pi f C)$, so reducing C or f increases it and lowers the current.

36. In an a.c. circuit containing L , C and R in series, the ratio of apparent power to the true power is
(Z = impedance of the circuit and R = resistance)

- (A) RZ
- (B) $\cos\phi$
- (C) $\cot\phi$
- (D) $\frac{Z}{R}$

Correct Answer: (D) $\frac{Z}{R}$

Solution:**Step 1: Understanding the Concept:**

Apparent power is $P_{app} = V_{rms}I_{rms}$. True power is $P_{true} = V_{rms}I_{rms} \cos \phi$.

Step 2: Take the ratio:

$$\frac{P_{app}}{P_{true}} = \frac{1}{\cos \phi}. \text{ For a series LCR circuit } \cos \phi = \frac{R}{Z}.$$

Step 3: Result:

$$\frac{P_{app}}{P_{true}} = \frac{Z}{R}. \text{ Option D.}$$

Step 4: Why the other options are wrong.

RZ is not even dimensionless. $\cos \phi$ is the inverse of the ratio. $\cot \phi$ equals X/R , which is a different quantity.

Final Answer:

The ratio is Z / R .

$$\boxed{(D) \frac{Z}{R}}$$

Quick Tip: True power = $V I \cos \phi$, apparent power = $V I$, and $\cos \phi = R/Z$.

37. In series LCR circuit, resistance is 18Ω and impedance 33Ω . An r.m.s. Voltage of 220 V is applied across the circuit. The true power consumed in a c. circuit is

- (A) 400 W
- (B) 600 W
- (C) 200 W
- (D) 800 W

Correct Answer: (D) 800 W

Solution:

Step 1: Understanding the Concept:

The true (average) power is $P = V_{rms} I_{rms} \cos \phi$, with $I_{rms} = \frac{V}{Z}$ and $\cos \phi = \frac{R}{Z}$.

Step 2: Find the current and power factor:

$$I = \frac{220}{33} = 6.67 \text{ A. } \cos \phi = \frac{18}{33} = 0.545.$$

Step 3: Compute the power:

$$P = 220 \times 6.67 \times 0.545 = 800 \text{ W. Equivalently } P = I^2 R = \left(\frac{220}{33} \right)^2 \times 18 = 44.44 \times 18 = 800 \text{ W. Option D.}$$

Step 4: Why the other options are wrong.

200, 400 and 600 W are not given by any of the above formulas with these values.

Final Answer:

The true power is 800 W.

(D) 800 W

Quick Tip: True power = $V^2 R / Z^2$.

38. An ideal transformer converts 220V a.c. to 3.3 KV a.c. to transmit a power of 4.4 KW. If primary coil has 600 turns then alternating current in the secondary coil is

- (A) $\frac{4}{3}$ A
- (B) $\frac{3}{4}$ A
- (C) $\frac{1}{3}$ A
- (D) $\frac{2}{3}$ A

Correct Answer: (A) $\frac{4}{3}$ A

Solution:

Step 1: Understanding the Concept:

An ideal transformer has no loss, so input power equals output power. The power delivered by the secondary is 4.4 kW at 3.3 kV.

Step 2: Find the secondary current:

$$I_s = \frac{P}{V_s} = \frac{4400}{3300} = \frac{4}{3} \text{ A. Option A.}$$

Step 3: Check with the turns ratio:

$$\begin{aligned} \text{The primary current is } I_p &= \frac{4400}{220} = 20 \text{ A. The turns ratio is } \frac{N_s}{N_p} \\ &= \frac{3300}{220} = 15, \text{ so } I_s = \frac{I_p}{15} = \frac{4}{3} \text{ A. This agrees.} \end{aligned}$$

Step 4: Why the other options are wrong.

$\frac{3}{4}$, $\frac{1}{3}$ and $\frac{2}{3}$ A would give powers of 2.5 kW, 1.1 kW and 2.2 kW, none equal to 4.4 kW.

Final Answer:

The secondary current is $\frac{4}{3}$ A.

(A) $\frac{4}{3}$ A

Quick Tip: For an ideal transformer, output power equals the transmitted power, so $I = P/V$ on the secondary.



39. Alternating current of peak value $(\frac{2}{\pi})$ A flows through the primary coil of a transformer. The coefficient of mutual inductance between primary and secondary coils is 1 H. The peak em.f. induced in secondary coil is (Frequency of a. c. is 50 Hz)

- (A) 25 V
- (B) 50 V
- (C) 100 V
- (D) 200 V

Correct Answer: (D) 200 V

Solution:

Step 1: Understanding the Concept:

The emf induced in the secondary is $\varepsilon_s = -M \frac{dI}{dt}$. For $I = I_0 \sin \omega t$, the peak emf is $M\omega I_0$.

Step 2: Find omega:

$$\omega = 2\pi f = 2\pi \times 50 = 100\pi \text{ rad/s.}$$

Step 3: Compute the peak emf:

$$\varepsilon_0 = M\omega I_0 = 1 \times 100\pi \times \frac{2}{\pi} = 200 \text{ V. Option D.}$$

Step 4: Why the other options are wrong.

25, 50 and 100 V come from missing the factor 2 in $\omega = 2\pi f$ or from using f instead of ω .

Final Answer:

The peak emf is 200 V.

(D) 200 V

Quick Tip: Peak emf = M times omega times peak current.

40. A thin prism in air produces the angle of minimum deviation δ . If the prism is immersed in water, the angle of minimum deviation for the same ray is (refractive index of water is $4/3$ and that of glass prism is $3/2$)

- (A) δ
- (B) $\frac{\delta}{2}$
- (C) $\frac{\delta}{4}$
- (D) 2δ

Correct Answer: (C) $\frac{\delta}{4}$

Solution:**Step 1: Understanding the Concept:**

For a thin prism of angle A , the angle of minimum deviation is $\delta = (\mu - 1)A$. In a liquid, μ is replaced by the relative index μ_g/μ_w .

Step 2: In air:

$$\delta = \left(\frac{3}{2} - 1\right)A = \frac{A}{2}.$$

Step 3: In water:

$$\delta' = \left(\frac{3/2}{4/3} - 1 \right) A = \left(\frac{9}{8} - 1 \right) A = \frac{A}{8}.$$

Step 4: Compare:

$$\frac{\delta'}{\delta} = \frac{A/8}{A/2} = \frac{1}{4}, \text{ so } \delta' = \frac{\delta}{4}. \text{ Option C.}$$

Step 5: Why the other options are wrong.

δ would apply with no medium change, and $\frac{\delta}{2}$ and 2δ come from wrong relative indices (for example using $\mu_w - 1$ as the factor).

Final Answer:

The deviation in water is $\delta / 4$.

$$\boxed{(C) \frac{\delta}{4}}$$

Quick Tip: For a thin prism $\delta = (\mu - 1) A$; in a liquid use the relative refractive index.



41. In biprism experiment, the 4th dark band is formed opposite to one of the slits. The wavelength of light used is λ (D=distance between source and screen, d= the distance between the slits)

- (A) $\frac{d^2}{9D}$
- (B) $\frac{d^2}{11D}$
- (C) $\frac{d^2}{14D}$
- (D) $\frac{d^2}{7D}$

Correct Answer: (D) $\frac{d^2}{7D}$

Solution:

Step 1: Understanding the Concept:

In a biprism setup the two virtual sources act as slits separated by d .

Dark bands are at $x_n = \frac{(2n - 1)\lambda D}{2d}$.

Step 2: Position of a slit:

A point opposite one of the slits is at distance $\frac{d}{2}$ from the central line.

Step 3: Use $n = 4$:

$$\frac{(2 \cdot 4 - 1)\lambda D}{2d} = \frac{d}{2}, \text{ so } \frac{7\lambda D}{2d} = \frac{d}{2}.$$

Step 4: Solve:

$$\lambda = \frac{d^2}{7D}. \text{ Option D.}$$

Step 5: Why the other options are wrong.

$\frac{d^2}{9D}$ would be right for the 5th dark band, and $\frac{d^2}{11D}$ for the 6th.

Option C, $\frac{d^2}{14D}$, comes from using $2n$ instead of $2n - 1$.

Final Answer:

The wavelength is $d^2 / (7D)$.

$(D) \frac{d^2}{7D}$

Quick Tip: Dark fringes are at $x = (2n - 1)\lambda D / (2d)$; a slit is at $x = d/2$.

42. In Young's double slit experiment, in an interference pattern, a minimum is observed exactly in front of one slit. The distance between the two coherent sources is 'd' and 'D' is the distance between source and screen. The possible wavelengths used are inversely proportional to

- (A) $D, 2D, 3D, \dots$
- (B) $D, 3D, 5D, \dots$
- (C) $\frac{1}{D}, \frac{2}{D}, \frac{3}{D}, \dots$
- (D) $\frac{1}{D^2}, \frac{2}{D^2}, \frac{3}{D^2}, \dots$

Correct Answer: (B) $D, 3D, 5D, \dots$

Solution:

Step 1: Understanding the Concept:

A minimum occurs where the path difference is an odd multiple of half a wavelength: $\Delta = (2n - 1)\frac{\lambda}{2}$.

Step 2: Path difference at a point opposite one slit:

The point is at $x = \frac{d}{2}$, so $\Delta = \frac{xd}{D} = \frac{d^2}{2D}$.

Step 3: Solve for lambda:

$\frac{d^2}{2D} = (2n - 1)\frac{\lambda}{2}$, so $\lambda = \frac{d^2}{(2n - 1)D}$ with $n = 1, 2, 3, \dots$

Step 4: Read off the dependence:

$\lambda \propto \frac{1}{(2n - 1)D}$, so λ is inversely proportional to $D, 3D, 5D, \dots$ Option B.

Step 5: Why the other options are wrong.

Even multiples $2D, 4D$ would mean maxima, not minima. Options C and D give the wrong dependence on D .

Final Answer:

Wavelength is inversely proportional to D , $3D$, $5D$, and so on.

$$(B) D, 3D, 5D, \dots$$

Quick Tip: Set path difference $d^2/(2D)$ equal to $(2n - 1) \lambda / 2$.

43. Converging lens of telescope 5 cm in diameter has focal length 25 cm. In the focal plane of the lens the distance between the centres of the Fraunhofer diffraction pattern is (wavelength of light used = $5000 \text{ \AA}$, Rayleigh's criterion is satisfied)

- (A) $25000 \text{ \AA}$
- (B) $28000 \text{ \AA}$
- (C) $30500 \text{ \AA}$
- (D) $32000 \text{ \AA}$

Correct Answer: (C) $30500 \text{ \AA}$

Solution:**Step 1: Understanding the Concept:**

Two point sources are just resolved when the angular separation equals $\theta = \frac{1.22\lambda}{D}$, where D is the lens diameter.

Step 2: Compute the angle:

$$\begin{aligned} \lambda &= 5000 \text{ \AA} = 5 \times 10^{-7} \text{ m}, D = 5 \text{ cm} = 0.05 \text{ m}. \theta = \frac{1.22 \times 5 \times 10^{-7}}{0.05} \\ &= 1.22 \times 10^{-5} \text{ rad}. \end{aligned}$$

Step 3: Find the linear separation at the focal plane:

$$y = f\theta = 0.25 \times 1.22 \times 10^{-5} = 3.05 \times 10^{-6} \text{ m} = 30500 \text{ \AA}. \text{ Option C.}$$

Step 4: Why the other options are wrong.

Without the 1.22 factor the answer would be 25000 Å (option A), which is the wrong criterion for a circular aperture.

Final Answer:

The separation is 30500 angstrom.

(C) 30500 Å

Quick Tip: Rayleigh criterion: angular limit $1.22 \lambda / D$; linear separation = f times angle.



44. Photoelectrons are emitted from two similar metal plates when wavelengths λ_1 and λ_2 are incident on them ($\lambda_1 = 1.5\lambda_2$). If maximum kinetic energy of emitted photoelectrons is E_1 and E_2 respectively then

- (A) $E_1 < 2E_2/3$
- (B) $E_1 = 2E_2/3$
- (C) $E_1 = E_2/3$
- (D) $E_1 = 2E_2$

Correct Answer: (A) $E_1 < 2E_2/3$

Solution:

Step 1: Understanding the Concept:

Einstein's equation: $E_k = \frac{hc}{\lambda} - \phi$, where $\phi > 0$ is the work function.

Both plates are similar, so ϕ is the same.

Step 2: Write for each wavelength:

$$E_1 = \frac{hc}{\lambda_1} - \phi = \frac{hc}{1.5\lambda_2} - \phi = \frac{2}{3} \cdot \frac{hc}{\lambda_2} - \phi. E_2 = \frac{hc}{\lambda_2} - \phi.$$

Step 3: Compare E_1 with $2E_2/3$:

$$\frac{2}{3}E_2 = \frac{2}{3} \cdot \frac{hc}{\lambda_2} - \frac{2}{3}\phi. \text{ So } E_1 - \frac{2}{3}E_2 = -\phi + \frac{2}{3}\phi = -\frac{\phi}{3} < 0.$$

Step 4: Conclusion:

$$E_1 < \frac{2E_2}{3}. \text{ Option A.}$$

Step 5: Why the other options are wrong.

B, C and D would need $\phi = 0$, which is not possible for a metal with a real threshold. They would hold only if the kinetic energy were proportional to the photon energy.

Final Answer:

E_1 is less than $2 E_2 / 3$.

$$\boxed{(A) E_1 < \frac{2E_2}{3}}$$

Quick Tip: Write Einstein equation for each wavelength and compare with the work function.



45. According to de-Broglie hypothesis, the ratio of the wave-length of a photon and that of an electron having same energy 'E' is (m = mass of electron, c = velocity of light)

- (A) $\frac{1}{c} \sqrt{\frac{E}{2m}}$
 (B) $c \sqrt{\frac{E}{2m}}$
 (C) $c \sqrt{\frac{2m}{E}}$
 (D) $\sqrt{\frac{2mc}{E}}$

Correct Answer: (C) $c \sqrt{\frac{2m}{E}}$

Solution:

Step 1: Understanding the Concept:

For a photon of energy E : $\lambda_p = \frac{hc}{E}$. For an electron with kinetic energy E : $p = \sqrt{2mE}$, so $\lambda_e = \frac{h}{\sqrt{2mE}}$.

Step 2: Take the ratio:

$$\frac{\lambda_p}{\lambda_e} = \frac{hc/E}{h/\sqrt{2mE}} = \frac{c\sqrt{2mE}}{E} = c \sqrt{\frac{2m}{E}}$$

Option C.

Step 3: Why the other options are wrong.

Options A and B have $\frac{E}{2m}$ inside the root, which gives the inverse ratio. Option D has c inside the root, which would give the wrong dimensions.

Final Answer:

The ratio is $c \sqrt{2m / E}$.

$$(C) \ c\sqrt{\frac{2m}{E}}$$

Quick Tip: Photon: $\lambda = hc/E$. Electron: $\lambda = h / \sqrt{2mE}$.

46. Bohr model is applied to a particle of mass 'm' and charge 'q' moving in a plane under the influence of a transverse magnetic field B. The energy of the charged particle in the n^{th} level will be (h=Planck's constant)

- (A) $\frac{2nhqB}{\pi m}$
 (B) $\frac{nhqB}{\pi m}$
 (C) $\frac{nhqB}{2\pi m}$
 (D) $\frac{nhqB}{4\pi m}$

Correct Answer: (D) $\frac{nhqB}{4\pi m}$

Solution:

Step 1: Understanding the Concept:

The particle moves in a circle in the plane perpendicular to the field. The magnetic force supplies the centripetal force: $qvB = \frac{mv^2}{r}$, so $mv = qBr$.

Step 2: Apply the Bohr quantisation:

Angular momentum: $mvr = \frac{nh}{2\pi}$. Using $mv = qBr$, we get $qBr^2 = \frac{nh}{2\pi}$, so $r^2 = \frac{nh}{2\pi qB}$.

Step 3: Find the energy:

All the energy is kinetic: $E = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{qBr}{m}\right)^2 = \frac{q^2B^2r^2}{2m}$.

$$E = \frac{q^2B^2}{2m} \cdot \frac{nh}{2\pi qB} = \frac{nhqB}{4\pi m}$$

Option D.

Step 4: Why the other options are wrong.

Options A, B and C are larger by factors of 8, 4 and 2. They come from dropping the factor $\frac{1}{2}$ in kinetic energy or the factor 2π in the quantisation rule.

Final Answer:

The energy is $nhqB / (4\pi m)$.

(D) $\frac{nhqB}{4\pi m}$

Quick Tip: Quantise angular momentum $mvr = nh / 2\pi$ and use $qvB = m v^2 / r$.

47. In Bohr's theory of hydrogen atom, 'r' is the radius of the orbit, 'V' is the speed of electron and E is the total energy of the electron. Out of the following which physical quantity is inversely proportional to principal quantum number?

- (A) $\frac{1}{Vr}$
- (B) Vr
- (C) V^2r
- (D) Vr^2

Correct Answer: (A) $\frac{1}{Vr}$

Solution:

Step 1: Recall the Bohr model results

In the Bohr model of hydrogen, the radius of the n th orbit grows as the square of the principal quantum number n .

The speed of the electron falls as n grows.

$$r_n \propto n^2, \quad V_n \propto \frac{1}{n}$$

Step 2: Test option (A)

Multiply the two proportionalities: $Vr \propto \frac{1}{n} \times n^2 = n$.

So Vr is directly proportional to n , and its reciprocal $\frac{1}{Vr}$ is proportional to $\frac{1}{n}$.

That is exactly "inversely proportional to n ", so (A) is correct.

Step 3: Why the other options are wrong

(B) $Vr \propto n$, so it is directly proportional to n , not inversely.

(C) $V^2r \propto \frac{1}{n^2} \times n^2 = n^0$, a constant that does not depend on n .

(D) $Vr^2 \propto \frac{1}{n} \times n^4 = n^3$, which grows with n .

Final Answer:

The reciprocal $\frac{1}{Vr}$ is proportional to $\frac{1}{n}$, which is option (A).

$$(A) \frac{1}{Vr}$$

Quick Tip: Use $r \propto n^2$ and $V \propto 1/n$, then combine them for each option.

48. In common emitter mode of transistor, the d.c. current gain is 20, the emitter current is 7 mA. The collector current is

- (A) $\frac{14}{3}$ mA
- (B) $\frac{20}{3}$ mA
- (C) $\frac{7}{3}$ mA
- (D) $\frac{8}{3}$ mA

Correct Answer: (B) $\frac{20}{3}$ mA

Solution:

Step 1: Write the current relations

In a transistor the emitter current is the sum of base and collector currents, $I_E = I_B + I_C$.

The d.c. current gain in common emitter mode is $\beta = \frac{I_C}{I_B}$.

Step 2: Express I_C through I_E

From $\beta = I_C/I_B$ we get $I_B = I_C/\beta$.

Then $I_E = \frac{I_C}{\beta} + I_C = I_C \frac{1+\beta}{\beta}$.

$$I_C = \frac{\beta}{1 + \beta} I_E$$

Step 3: Put in numbers

With $\beta = 20$ and $I_E = 7 \text{ mA}$:

$$I_C = \frac{20}{21} \times 7 = \frac{20}{3} \text{ mA}$$

Step 4: Check the other options

$\frac{14}{3} \text{ mA}$ would need $\beta = 2$ and $\frac{7}{3} \text{ mA}$ would need $\beta = 0.5$. These ignore that I_C is 20 times I_B . Only $\frac{20}{3} \text{ mA}$ fits.

Final Answer:

The collector current is $\frac{20}{3} \text{ mA}$, option (B).

$(B) \frac{20}{3} \text{ mA}$

Quick Tip: Use $I_C = \frac{\beta}{1+\beta} I_E$ because $I_E = I_B + I_C$.

49. In the case of NAND gate, if A and B are inputs and Y is the output then

- (A) $Y = \overline{A + B}$
- (B) $Y = A \cdot B$
- (C) $Y = \overline{A \cdot B}$
- (D) $Y = A + B$

Correct Answer: (C) $Y = \overline{A \cdot B}$

Solution:

Step 1: Understanding the Concept:

A NAND gate is a logic gate whose name means NOT-AND. It is an AND gate followed by a NOT gate.

Step 2: Key Formula or Approach:

The AND gate has output $A \cdot B$. Inverting it gives the NAND output:

$$Y = \overline{A \cdot B}$$

Step 3: Truth table:

For inputs (0,0), (0,1), (1,0), (1,1), the AND output is 0, 0, 0, 1.

Inverting gives the NAND output 1, 1, 1, 0. The output is low only when both inputs are high.

Step 4: Check the options:

Option (A) $\overline{A + B}$ is the NOR gate. Option (B) $A \cdot B$ is the AND gate.

Option (D) $A + B$ is the OR gate. Only option (C) is $\overline{A \cdot B}$, the NAND gate.

Final Answer:

The NAND gate output is $Y = \overline{A \cdot B}$, option (C).

$$Y = \overline{A \cdot B}$$

Quick Tip: NAND means NOT-AND, so put a bar over A times B.

50. In energy band diagram of insulators, a band gap and the conduction band is respectively

- (A) very high, empty
- (B) very low, empty
- (C) very high, completely filled
- (D) very low, partially filled

Correct Answer: (A) very high, empty

Solution:

Step 1: Recall the band picture

A solid has a valence band (filled with bound electrons) and a conduction band above it. The forbidden gap sits between them.

Step 2: Apply it to insulators

In an insulator the gap is very large, around 6 eV or more. Thermal energy at room temperature cannot lift electrons across it. So the conduction band stays empty and no current flows.

Step 3: Other options

(B) and (D) say the gap is very low, which describes semiconductors or conductors. (C) says the conduction band is completely filled, which would make a conductor. Only (A) fits.

Final Answer:

Insulators have a very high band gap and an empty conduction band, option (A).

(A) very high, empty

Quick Tip: An insulator has a very wide forbidden gap and nothing in the conduction band.

Chemistry

Questions 51 to 100 | 1 mark each | 50 marks

51. Calculate the volume of 99 g of CO_2 at STP ?

- (A) 22.4 L
- (B) 50.4 L
- (C) 67.2 L
- (D) 89.6 L

Correct Answer: (B) 50.4 L

Solution:

Step 1: Understanding the Concept:

One mole of any ideal gas occupies 22.4 L at STP. So we first convert the given mass into moles, then convert moles into volume.

Step 2: Key Formula or Approach:

$$n = \frac{\text{mass}}{\text{molar mass}} \text{ and } V_{\text{STP}} = n \times 22.4 \text{ L.}$$

Step 3: Find the molar mass of CO_2 :

$$M(\text{CO}_2) = 12 + 2(16) = 44 \text{ g/mol.}$$

Step 4: Find the number of moles:

$$n = \frac{99}{44} = 2.25 \text{ mol.}$$

Step 5: Find the volume:

$$V = 2.25 \times 22.4 = 50.4 \text{ L.}$$

Step 6: Why the other options are wrong.

22.4 L is the volume of just 1 mole (44 g). 67.2 L is the volume of 3 moles (132 g). 89.6 L is the volume of 4 moles (176 g). None of these match 99 g.

Final Answer:

The volume of 99 g of CO_2 at STP is 50.4 L.

(B) 50.4 L

Quick Tip: Find the moles of CO_2 from the mass, then multiply by 22.4 L, the molar volume at STP.



52. What are the total number of electrons present in s and p orbitals respectively in the argon?

- (A) 12 and 6
- (B) 8 and 12
- (C) 6 and 12
- (D) 18 and 6

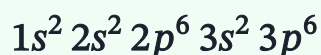
Correct Answer: (C) 6 and 12

Solution:

Step 1: Understanding the Question:

Argon has atomic number 18, so a neutral atom has 18 electrons. We must count all electrons sitting in s subshells and all electrons sitting in p subshells.

Step 2: Write the electronic configuration:



Step 3: Count the s electrons:

The s subshells are 1s, 2s and 3s. Each holds 2 electrons, so the total is $2 + 2 + 2 = 6$.

Step 4: Count the p electrons:

The p subshells are 2p and 3p. They hold $6 + 6 = 12$ electrons.

Step 5: Check the total:

$6 + 12 = 18$, which equals the number of electrons in argon. Option A (12 and 6) swaps the two counts. B and D do not add up to 18 in a meaningful way and miss the filled 2p and 3p subshells.

Final Answer:

Argon has 6 electrons in s orbitals and 12 in p orbitals.

(C) 6 and 12

Quick Tip: Write the configuration of argon (18 electrons) and count electrons in all s subshells and all p subshells.

53. Which of the following compounds is most acidic in nature?

- (A) H_2O
- (B) H_2S
- (C) H_2Se
- (D) H_2Te

Correct Answer: (D) H_2Te

Solution:

Step 1: Understanding the Concept:

The acid strength of a hydride H_2E depends on how easily it gives up H^+ . A weaker H-E bond means an easier release of H^+ and a stronger acid.

Step 2: Trend in bond strength:

Going down group 16 (O, S, Se, Te), the size of the atom E increases. The overlap between the H 1s orbital and the orbital of E gets poorer, so the H-E bond becomes longer and weaker.

Step 3: Compare the hydrides:

The H-E bond enthalpy decreases in the order $\text{H}_2\text{O} > \text{H}_2\text{S} > \text{H}_2\text{Se} > \text{H}_2\text{Te}$. So acidity increases in the order $\text{H}_2\text{O} < \text{H}_2\text{S} < \text{H}_2\text{Se} < \text{H}_2\text{Te}$.

Step 4: Why the other options are wrong.

Water has the strongest O-H bond and is the weakest acid. H_2S and H_2Se are more acidic than water but less than H_2Te , which has the longest and weakest bond.

Final Answer:

The most acidic hydride is H_2Te .

(D) H_2Te

Quick Tip: Acidity of group 16 hydrides rises down the group because the H-E bond gets weaker as E gets larger.

54. Identify the different types of bonds present in quaternary ammonium salts.

- (A) Covalent
- (B) Ionic
- (C) Covalent and co-ordinate
- (D) covalent, co-ordinate and ionic

Correct Answer: (D) covalent, co-ordinate and ionic

Solution:

Step 1: Understanding the Concept:

A quaternary ammonium salt has the formula $\text{R}_4\text{N}^+\text{X}^-$. We need to list every kind of bond inside this salt.

Step 2: Bonds inside the cation:

The N atom has 5 valence electrons. Three are used for three covalent N-R bonds, and one lone pair remains. This lone pair forms the fourth N-R bond as a coordinate (dative) bond, which gives N^+ . The R-groups themselves are joined internally by covalent bonds.

Step 3: Bond between ions:

The positive R_4N^+ cation and the negative X^- anion attract each other by an ionic bond.

Step 4: Compare the options.

Covalent alone (A) and ionic alone (B) miss part of the structure.

"Covalent and coordinate" (C) leaves out the ionic bond to X^- . Only D lists all three.

Final Answer:

Quaternary ammonium salts contain covalent, co-ordinate and ionic bonds.

(D) covalent, co-ordinate and ionic

Quick Tip: In $R_4N^+ X^-$, one N-R bond is formed from the lone pair (coordinate) and the ion pair is held by ionic bonding.

55. A syringe has a volume of 10.0cm^3 at a pressure of 1 atm. If the end is sealed and the plunger is pushed down (constant temperature), what will be the final volume when the pressure becomes 3.5 atm?

- (A) 35.0cm^3
- (B) 2.86cm^3
- (C) 0.286cm^3
- (D) 3.50cm^3

Correct Answer: (B) 2.86cm^3

Solution:

Step 1: Understanding the Concept:

The syringe is sealed and the temperature is constant, so the amount

of gas and temperature do not change. This is the situation described by Boyle's law.

Step 2: Key Formula:

$$P_1V_1 = P_2V_2$$

Step 3: Substitute the values:

$P_1 = 1 \text{ atm}$, $V_1 = 10.0 \text{ cm}^3$, $P_2 = 3.5 \text{ atm}$.

$$V_2 = \frac{P_1V_1}{P_2} = \frac{1 \times 10.0}{3.5} = 2.857 \text{ cm}^3 \approx 2.86 \text{ cm}^3$$

Step 4: Why the other options are wrong.

35.0 cm^3 multiplies the pressure by the volume instead of dividing, but pressure up must make volume go down. 0.286 cm^3 is off by a factor of 10. 3.50 cm^3 just reuses the pressure value and ignores the 10.0 cm^3 .

Final Answer:

The final volume is 2.86 cm^3 .

(B) 2.86 cm^3

Quick Tip: Use Boyle's law, $P_1V_1 = P_2V_2$, because temperature is constant.

56. The equilibrium constant for a reaction is 100. What will be the value of standard Gibbs energy change at 298 K ? ($R = 8.314 \text{ J K}^{-1}\text{mol}^{-1}$)

- (A) -11.411 KJ/mol
- (B) -5.744 KJ/mol
- (C) -570.584 KJ/mol
- (D) -57.058 KJ/mol

Correct Answer: (A) -11.411 KJ/mol

Solution:

Step 1: Understanding the Concept:

The standard Gibbs energy change of a reaction is linked to its equilibrium constant. A large K (more than 1) means products are favoured, so ΔG° must be negative.

Step 2: Key Formula:

$$\Delta G^\circ = -RT \ln K = -2.303 RT \log K$$

Step 3: Substitute the values:

$R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$, $T = 298 \text{ K}$, $K = 100$, $\ln 100 = 4.605$.

$$\Delta G^\circ = -(8.314)(298)(4.605) = -11409 \text{ J/mol}$$

$$\Delta G^\circ \approx -11.41 \text{ kJ/mol}$$

Step 4: Why the other options are wrong.

-5.744 kJ/mol comes from a slip in the logarithm step. -57.058 and -570.584 kJ/mol are off by factors of 5 and 50, from unit or arithmetic slips.

Final Answer:

The standard Gibbs energy change is -11.411 kJ/mol.

$$(A) -11.411 \text{ kJ/mol}$$

Quick Tip: Use $\Delta G^{\circ} = -RT \ln K$ and convert the answer from J to kJ.

57. The bond dissociation enthalpy of H_2 , Cl_2 and HCl are 434, 242 and 431 kJ mol⁻¹ respectively. Calculate the enthalpy of formation of HCl .

- (A) -93 kJ mol^{-1}
- (B) 245 kJ mol^{-1}
- (C) 93 kJ mol^{-1}
- (D) -245 kJ mol^{-1}

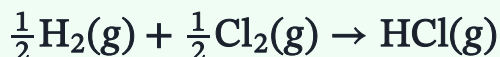
Correct Answer: (A) -93 kJ mol^{-1}

Solution:

Step 1: Understanding the Concept:

The enthalpy of formation of HCl is the enthalpy change for forming one mole of HCl(g) from its elements in their standard states. Bonds are broken (energy absorbed) and new bonds are formed (energy released).

Step 2: Write the reaction:



Step 3: Apply bond enthalpies:

$$\Delta_f H = \sum(\text{bonds broken}) - \sum(\text{bonds formed})$$

$$\Delta_f H = \frac{1}{2}(434) + \frac{1}{2}(242) - 431 = 217 + 121 - 431$$

$$\Delta_f H = 338 - 431 = -93 \text{ kJ/mol}$$

Step 4: Why the other options are wrong.

+93 kJ/mol has the wrong sign. The values 245 and -245 come from using full moles of H_2 and Cl_2 , but the equation for one mole of HCl needs only half a mole of each.

Final Answer:

The enthalpy of formation of HCl is -93 kJ per mole.

$$\boxed{\text{(A)} - 93 \text{ kJ mol}^{-1}}$$

Quick Tip: Break 1/2 mole H_2 and 1/2 mole Cl_2 , then form 1 mole of H-Cl , and subtract energy released from energy absorbed.

58. 2.8 g of KOH is dissolved in a 500 mL solution at 298 K. What is the p^H of solution ? (molar mass of KOH = 56 g/mol)

- (A) 1
- (B) 8
- (C) 11
- (D) 13

Correct Answer: (D) 13

Solution:

Step 1: Understanding the Concept:

KOH is a strong base and dissociates completely, so $[\text{OH}^-]$ equals the molarity of KOH. At 298 K, $\text{pH} + \text{pOH} = 14$.

Step 2: Find the moles and molarity:

$$n = \frac{2.8}{56} = 0.05 \text{ mol. Volume is } 500 \text{ mL} = 0.5 \text{ L.}$$

$$M = \frac{0.05}{0.5} = 0.1 \text{ M}$$

Step 3: Find pOH and pH:

$[\text{OH}^-] = 0.1 = 10^{-1} \text{ M}$, so $\text{pOH} = 1$.

$$\text{pH} = 14 - 1 = 13$$

Step 4: Why the other options are wrong.

A pH of 1 is what we get if we forget to convert pOH to pH. A pH of 8

or 11 would need much lower hydroxide concentration (10^{-6} M or 10^{-3} M).

Final Answer:

The pH of the solution is 13.

(D) 13

Quick Tip: KOH is a strong base: find $[\text{OH}^-]$ from moles and volume, then pOH and $\text{pH} = 14 - \text{pOH}$.



59. What is the solubility of BaSO_4 in g/dm^3 if its solubility product is 1.0×10^{-10} at 25°C . [Molar mass of $\text{BaSO}_4 = 233 \text{ g/mol}$]

- (A) $2.33 \times 10^{-3} \text{ g/dm}^3$
- (B) $4.66 \times 10^{-3} \text{ g/dm}^3$
- (C) 3.48 g/dm^3
- (D) $1.16 \times 10^{-3} \text{ g/dm}^3$

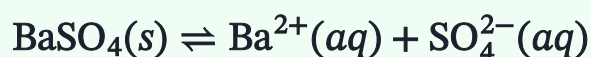
Correct Answer: (A) $2.33 \times 10^{-3} \text{ g/dm}^3$

Solution:

Step 1: Understanding the Concept:

BaSO_4 is a sparingly soluble salt. Its solubility product K_{sp} is the product of the ion concentrations at saturation.

Step 2: Write the dissociation:



If the solubility is s mol/dm³, then $[\text{Ba}^{2+}] = [\text{SO}_4^{2-}] = s$.

Step 3: Find the solubility in mol/dm³:

$$K_{sp} = s \times s = s^2 = 1.0 \times 10^{-10}$$

$$s = 1.0 \times 10^{-5} \text{ mol/dm}^3$$

Step 4: Convert to g/dm³:

$$s = 1.0 \times 10^{-5} \times 233 = 2.33 \times 10^{-3} \text{ g/dm}^3$$

Step 5: Why the other options are wrong.

4.66e-3 comes from taking $s = 2 \times 10^{-5}$, as if there were two ions of each. 1.16e-3 comes from halving the right value. 3.48 g/dm³ does not follow from any correct step.

Final Answer:

The solubility of BaSO₄ is 2.33×10^{-3} g/dm³.

$(A) 2.33 \times 10^{-3} \text{ g/dm}^3$

Quick Tip: For a 1:1 salt, $K_{sp} = s$ squared. Then multiply s (mol/dm³) by the molar mass.

60. What is the normal pH of human blood?

- (A) 7.0
- (B) 6.9
- (C) 7.4
- (D) 8.1

Correct Answer: (C) 7.4

Solution:

Step 1: Understanding the Concept:

Human blood must keep a nearly constant pH, because enzymes and cells work only in a narrow range. A buffer system in blood (mainly carbonic acid and bicarbonate) holds the pH steady.

Step 2: Recall the fact:

The normal pH of human blood is in the range 7.35 to 7.45, so its standard value is taken as 7.4. It is slightly basic.

Step 3: Why the other options are wrong.

A pH of 7.0 is neutral water. A pH of 6.9 is slightly acidic, which indicates a dangerous condition called acidosis if the blood falls to that level. A pH of 8.1 is too basic (alkalosis) and is not normal.

Final Answer:

The normal pH of human blood is 7.4.

(C) 7.4

Quick Tip: Blood is slightly basic and kept at about 7.35 to 7.45 by the



61. Find the oxidation number of Cl in ClO_4^{-1}

- (A) -9
- (B) $+7$
- (C) -7
- (D) $+9$

Correct Answer: (B) $+7$

Solution:

Step 1: Understanding the Concept:

In an ion, the sum of the oxidation numbers of all atoms equals the charge on the ion. Oxygen is -2 in most compounds.

Step 2: Set up the equation:

Let the oxidation number of Cl be x . The ion ClO_4^- has a charge of -1 .

$$x + 4(-2) = -1$$

Step 3: Solve:

$$x - 8 = -1, \text{ so } x = +7.$$

Step 4: Why the other options are wrong.

-9 , -7 and $+9$ are impossible or wrong: chlorine has only 7 valence electrons, so its maximum oxidation state is $+7$, and -7 would need it to gain 7 electrons.

Final Answer:

The oxidation number of chlorine in perchlorate is +7.

(B) + 7

Quick Tip: Let x be the oxidation number of Cl. Then $x + 4(-2) = -1$.

62. Which alkaline earth metal carbonate decomposes most easily?

- (A) MgCO_3
- (B) CaCO_3
- (C) SrCO_3
- (D) BaCO_3

Correct Answer: (A) MgCO_3

Solution:**Step 1: Understanding the Concept:**

Group 2 carbonates decompose on heating: $\text{MCO}_3 \rightarrow \text{MO} + \text{CO}_2$. The ease of decomposition depends on how strongly the metal ion polarises the carbonate ion.

Step 2: Apply the trend:

A small, highly charged cation polarises the large CO_3^{2-} anion strongly and weakens a C-O bond, so the carbonate breaks down at a lower temperature. Mg^{2+} is the smallest of the four ions here.

Step 3: Rank the carbonates:

Thermal stability: $\text{MgCO}_3 < \text{CaCO}_3 < \text{SrCO}_3 < \text{BaCO}_3$. So MgCO_3

decomposes most easily.

Step 4: Why the other options are wrong.

CaCO_3 , SrCO_3 and BaCO_3 have larger cations, polarise carbonate less, and need higher temperatures to decompose.

Final Answer:

Magnesium carbonate decomposes most easily.

(A) MgCO_3

Quick Tip: Thermal stability of group 2 carbonates increases down the group, so MgCO_3 decomposes first.



63. What is the difference in terms of molar masses of second and fourth members of homologous series?

- (A) 12
- (B) 24
- (C) 28
- (D) 36

Correct Answer: (C) 28

Solution:

Step 1: Understanding the Concept:

A homologous series is a family of compounds in which each member differs from the next by one $-\text{CH}_2-$ unit.

Step 2: Find the mass of one CH₂ unit:

$$12 + 2(1) = 14 \text{ g/mol.}$$

Step 3: Count the steps:

From the second member to the fourth member, there are two steps (second to third, third to fourth). So two $\text{—CH}_2\text{—}$ groups are added.

Step 4: Calculate the difference:

$$2 \times 14 = 28 \text{ g/mol.}$$

Step 5: Why the other options are wrong.

12 is the mass of one C atom only. 24 is two C atoms and ignores hydrogen. 36 would mean about three CH₂ units counted with a wrong mass.

Final Answer:

The difference in molar mass is 28.

(C) 28

Quick Tip: Consecutive homologues differ by CH₂ (14 g/mol), so the 2nd and 4th differ by two CH₂ units.

64. Cyclohexene on oxidation by KMnO_4 in dilute H_2SO_4 produces

- (A) benzene-1,4-dicarboxylic acid
- (B) pent-2-enoic acid
- (C) hexane-1,6-dioic acid
- (D) benzene-1,2-dicarboxylic acid

Correct Answer: (C) hexane-1,6-dioic acid

Solution:

Step 1: Understanding the Concept:

Hot acidified KMnO_4 is a strong oxidising agent that breaks the carbon-carbon double bond and converts each carbon of the $\text{C}=\text{C}$ into a carboxylic acid group (or a ketone if it is fully substituted).

Step 2: Apply to cyclohexene:

Cyclohexene has a double bond between two ring carbons. When it breaks, the ring opens. Each of the two former alkene carbons becomes a $-\text{COOH}$ group.

Step 3: Name the product:

The six carbons are now in a straight chain with $-\text{COOH}$ at both ends: $\text{HOOC}-(\text{CH}_2)_4-\text{COOH}$. This is hexane-1,6-dioic acid (adipic acid).

Step 4: Why the other options are wrong.

Benzene dicarboxylic acids (A and D) contain a benzene ring, which cannot form from cyclohexene by oxidation. Pent-2-enoic acid (B) has only 5 carbons and still has a double bond, but all 6 carbons must be retained.

Final Answer:

Cyclohexene gives hexane-1,6-dioic acid on oxidation.

(C) hexane-1,6-dioic acid

Quick Tip: Acidified KMnO_4 cleaves the $\text{C}=\text{C}$ bond; in a ring this opens the ring to a dicarboxylic acid.

65. Which of the following types of reactions is used to prepare Alkyl benzene?

- (A) Swartz reaction
- (B) Sandmeyer reaction
- (C) Finkelstein reaction
- (D) Wurtz Fittig reaction

Correct Answer: (D) Wurtz Fittig reaction

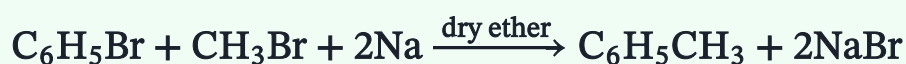
Solution:

Step 1: Understanding the Concept:

Alkyl benzenes (benzene with an alkyl side chain) are made by joining an aryl group to an alkyl group.

Step 2: Recall the reaction:

In the Wurtz-Fittig reaction, an aryl halide and an alkyl halide react with sodium metal in dry ether:



Step 3: Why the other options are wrong.

The Swarts reaction makes alkyl fluorides. The Sandmeyer reaction converts a diazonium salt into an aryl halide or nitrile using Cu(I) salts. The Finkelstein reaction makes alkyl iodides from alkyl chlorides or bromides using NaI in acetone.

Final Answer:

Alkyl benzenes are prepared by the Wurtz-Fittig reaction.

(D) Wurtz Fittig reaction

Quick Tip: Wurtz-Fittig reaction couples an aryl halide with an alkyl halide using sodium in dry ether.

66. 200 mL of ethylene gas and 150 mL of HCl gas were allowed to react at 1 bar. Pressure to form gaseous ethyl chloride. What is the work done during the reaction?

- (A) 15 J
- (B) 30 J
- (C) 150 J
- (D) 300 J

Correct Answer: (A) 15 J

Solution:

Step 1: Understanding the Concept:

Work done on a gas by a constant external pressure is $w = -P_{ext}\Delta V$.

We must find the net change in volume of the gas mixture during the reaction.

Step 2: Write the reaction and find the limiting reagent:



The ratio is 1:1 by volume. We have 200 mL of ethylene and 150 mL of HCl, so HCl is limiting and 150 mL of each reacts, forming 150 mL of product.

Step 3: Find the volume change:

Initial volume of the reacting gases: $150 + 150 = 300$ mL. Final volume of product: 150 mL. Excess ethylene is unchanged.

$$\Delta V = 150 - 300 = -150 \text{ mL} = -150 \times 10^{-6} \text{ m}^3$$

Step 4: Calculate the work:

$$P = 1 \text{ bar} = 10^5 \text{ Pa.}$$

$$w = -P\Delta V = -(10^5)(-150 \times 10^{-6}) = +15 \text{ J}$$

Step 5: Why the other options are wrong.

30 J and 300 J come from using a wrong volume change such as 300 mL or 3000 mL. 150 J forgets to convert mL to m^3 correctly. The key is that only 150 mL reacts.

Final Answer:

The work done on the system is 15 J.

(A) 15 J

Quick Tip: Find the volume consumed: the net volume change is minus 150 mL, so $w = -P \Delta V$ is positive.

67. Which of the following compounds does not decolourise Br_2 water?

- (A) Ethene
- (B) Ethyne
- (C) Benzene
- (D) Cyclohexene

Correct Answer: (C) Benzene

Solution:

Step 1: Understanding the Concept:

Bromine water is decolourised when bromine adds across a carbon-carbon double or triple bond. This is the standard test for unsaturation.

Step 2: Check each compound:

Ethene has a C=C and adds Br₂ readily. Ethyne has a triple bond and adds Br₂. Cyclohexene has a C=C in the ring and adds Br₂.

Step 3: Check benzene:

Benzene has delocalised pi electrons that make it unusually stable (aromatic stabilisation). It does not undergo addition with bromine water under ordinary conditions, so the orange-brown colour remains.

Step 4: Conclusion:

Benzene does not decolourise bromine water.

Final Answer:

Benzene does not decolourise bromine water.

(C) Benzene

Quick Tip: Compounds with C=C or C-triple-C add Br₂, but benzene has delocalised pi electrons and resists addition.



68. In a fcc arrangement of A and B atoms in which 'A' atoms are at the corners of the unit cell and 'B' atoms are at the face centers. One of the 'A' atoms is missing from one corner in unit cell. What is the simplest formula of a compound ?

- (A) A₇B₈
- (B) A₇B₃
- (C) AB₃
- (D) A₇B₂₄

Correct Answer: (D) A₇B₂₄

Solution:

Step 1: Understanding the Concept:

In a unit cell, an atom at a corner is shared by 8 cells (contributes 1/8) and an atom at a face centre is shared by 2 cells (contributes 1/2).

Step 2: Count B atoms:

A cube has 6 faces. B atoms at the face centres contribute $6 \times \frac{1}{2} = 3$.

Step 3: Count A atoms:

A cube has 8 corners, but one A is missing, so 7 corners are occupied. A atoms contribute $7 \times \frac{1}{8} = \frac{7}{8}$.

Step 4: Write the ratio:

A : B = $\frac{7}{8}$: 3 = 7 : 24. So the simplest formula is A₇B₂₄.

Step 5: Why the other options are wrong.

AB_3 is the formula with all 8 corners filled. A_7B_8 and A_7B_3 do not match the ratio of 7/8 to 3.

Final Answer:

The simplest formula is A_7B_{24} .

(D) A_7B_{24}

Quick Tip: Corners give 1/8 each, faces give 1/2 each; subtract the missing corner and write A_7B_{24} .

69. A compound forms bcc unit cell with edge length 400 pm. Determine the molar mass of the compound if the density of compound is 3.5 g cm^{-3} [$N_A = 6.022 \times 10^{23}$]

- (A) 65.2 g mol^{-1}
- (B) 67.4 g mol^{-1}
- (C) 69.8 g mol^{-1}
- (D) 71.6 g mol^{-1}

Correct Answer: (B) 67.4 g mol^{-1}

Solution:

Step 1: Understanding the Concept:

The density of a crystal is the mass of atoms in the unit cell divided by the volume of the cell.

Step 2: Key Formula:

$$\rho = \frac{Z \cdot M}{a^3 \cdot N_A}$$

For bcc, $Z = 2$.

Step 3: Convert units:

$a = 400 \text{ pm} = 400 \times 10^{-10} \text{ cm} = 4 \times 10^{-8} \text{ cm}$, so $a^3 = 64 \times 10^{-24} \text{ cm}^3$
 $= 6.4 \times 10^{-23} \text{ cm}^3$.

Step 4: Solve for M:

$$M = \frac{\rho a^3 N_A}{Z} = \frac{3.5 \times 6.4 \times 10^{-23} \times 6.022 \times 10^{23}}{2}$$

$$M = \frac{3.5 \times 38.54}{2} = \frac{134.9}{2} = 67.4 \text{ g/mol}$$

Step 5: Why the other options are wrong.

Using $Z = 4$ (fcc) would give 33.7 g/mol times $4 = 134.9 \text{ g/mol}$, and the wrong Z changes the answer by a whole-number factor. The other choices are close values that do not follow from the calculation.

Final Answer:

The molar mass of the compound is 67.4 g/mol .

(B) 67.4 g mol^{-1}

Quick Tip: Use $\text{density} = \frac{Z \times M}{a^3 \times N_A}$ with $Z = 2$ for bcc, and convert pm to cm.



70. What is the number of neighbouring spheres for any particle in the fcc crystal lattice ?

- (A) 6
- (B) 8
- (C) 10
- (D) 12

Correct Answer: (D) 12

Solution:

Step 1: Understanding the Concept:

The number of neighbouring spheres touching a given sphere is called its coordination number.

Step 2: Count in fcc:

In a face-centred cubic (cubic close packed) structure, a sphere touches 6 spheres in its own close-packed layer, 3 in the layer above and 3 in the layer below. So $6 + 3 + 3 = 12$.

Step 3: Why the other options are wrong.

6 is the value for simple cubic. 8 is the value for body-centred cubic. 10 does not occur in any of these standard packings.

Final Answer:

Each sphere in an fcc lattice has 12 neighbours.

Quick Tip: Each sphere in an fcc (ccp) lattice touches 12 others.

71. Identify a solution which contains solid as solute and liquid as solvent.

- (A) Hydrogen in Palladium
- (B) Sea water
- (C) Gels
- (D) Amalgum of mercury with metals

Correct Answer: (B) Sea water

Solution:

Step 1: Understanding the Concept:

A solution is named by the physical state of the solute and the solvent.
We need a solid solute dissolved in a liquid solvent.

Step 2: Check each option:

Hydrogen in palladium: gas solute in solid solvent. Sea water: solid salts (mainly NaCl) dissolved in liquid water, which is solid in liquid.
Gels: a liquid dispersed in a solid, a colloid, not a true solution.
Amalgam of mercury with metals: liquid (mercury) solute in a solid metal solvent.

Step 3: Conclusion:

Only sea water has a solid solute in a liquid solvent.

Final Answer:

Sea water is a solid-in-liquid solution.

(B) Sea water

Quick Tip: Sea water has solid salts dissolved in liquid water.

72. A solution of 50g of solute 'X' is dissolved in 150 g of 'Y' solvent boils at 357.27 K. What is the molar mass of solute?

(Given K_b and boiling point pure solvent is 2.77 K kg/mol and 350.06 K respectively.)

- (A) 132 g/mol
- (B) 128 g/mol
- (C) 150 g/mol
- (D) 15 g/mol

Correct Answer: (B) 128 g/mol

Solution:**Step 1: Understanding the Concept:**

Adding a non-volatile solute raises the boiling point of the solvent. The rise is proportional to the molality of the solution.

Step 2: Key Formula:

$$\Delta T_b = K_b \cdot m = K_b \cdot \frac{w_2 \times 1000}{M_2 \times w_1}$$

Step 3: Find the elevation:

$$\Delta T_b = 357.27 - 350.06 = 7.21 \text{ K.}$$

Step 4: Solve for the molar mass:

$$M_2 = \frac{K_b \times w_2 \times 1000}{\Delta T_b \times w_1} = \frac{2.77 \times 50 \times 1000}{7.21 \times 150}$$

$$M_2 = \frac{138500}{1081.5} = 128.06 \approx 128 \text{ g/mol}$$

Step 5: Why the other options are wrong.

132 and 150 are close guesses without the correct arithmetic. 15 g/mol is off by a factor of ten. Only 128 g/mol follows from the formula.

Final Answer:

The molar mass of the solute is 128 g/mol.

(B) 128 g/mol

Quick Tip: Use $\Delta T_b = K_b \times m$, then find moles of solute and the molar mass.

73. The partial pressure of a gas at 25°C is 0.18 atm, calculate the concentration of the gas dissolved at the same temperature, If K_H is $0.15 \text{ mol dm}^{-3} \text{ atm}^{-1}$

- (A) 0.027 M
- (B) 1.8 M
- (C) 4.5 M
- (D) 0.45 M

Correct Answer: (A) 0.027 M

Solution:

Step 1: Understanding the Concept:

Henry's law says that the amount of a gas dissolved in a liquid is directly proportional to its partial pressure above the liquid.

Step 2: Key Formula:

$$C = K_H \times P$$

Step 3: Substitute:

$K_H = 0.15 \text{ mol dm}^{-3} \text{ atm}^{-1}$ and $P = 0.18 \text{ atm}$.

$$C = 0.15 \times 0.18 = 0.027 \text{ M}$$

Step 4: Why the other options are wrong.

1.8 M and 4.5 M are far too large. Dividing instead of multiplying gives $0.18/0.15 = 1.2$, also wrong. 0.45 M comes from a decimal slip. The correct product is 0.027.

Final Answer:

The dissolved gas concentration is 0.027 M.

(A) 0.027 M

Quick Tip: Henry law: concentration = $K_H \times$ partial pressure.

74. The standard potential of Cu^{+2}/Cu is 0.34 V at 298 K. Calculate its potential at the same temperature when the Cu^{+2} ion concentration is 0.1 M.

(A) 0.64 V

(B) 0.34 V

(C) 0.31 V

(D) 0.37 V

Correct Answer: (C) 0.31 V

Solution:

Step 1: Understanding the Concept:

The electrode potential changes with the concentration of the ion. The Nernst equation gives this dependence.

Step 2: Key Formula:

For $\text{Cu}^{2+} + 2e^- \rightarrow \text{Cu}$, at 298 K:

$$E = E^\circ - \frac{0.0591}{n} \log \frac{1}{[\text{Cu}^{2+}]}$$

Step 3: Substitute:

$E^\circ = 0.34 \text{ V}$, $n = 2$, $[\text{Cu}^{2+}] = 0.1 \text{ M}$.

$$E = 0.34 - \frac{0.0591}{2} \log \frac{1}{0.1} = 0.34 - 0.02955 \times 1$$

$$E = 0.3105 \text{ V} \approx 0.31 \text{ V}$$

Step 4: Why the other options are wrong.

0.64 V adds 0.30 V, which is an error of sign and size. 0.34 V ignores the dilution. 0.37 V has the wrong sign for the correction: lower ion concentration lowers the reduction potential.

Final Answer:

The electrode potential is about 0.31 V.

(C) 0.31 V

Quick Tip: Use the Nernst equation with $n = 2$ and $\log(0.1) = -1$.

75. Which from following options is true when an electrolyte solution is diluted ?

- (A) both Λ and k increases
- (B) both Λ and k decreases
- (C) Λ increases and k decreases
- (D) Λ decreases and k increases

Correct Answer: (C) Λ increases and k decreases

Solution:

Step 1: Understanding the Concept:

Conductivity k is the conductance of a unit volume of solution. Molar conductivity Λ_m is k divided by the molarity: $\Lambda_m = \frac{k \times 1000}{M}$.

Step 2: Effect of dilution on k :

When the solution is diluted, the number of ions per unit volume falls. So conductivity k decreases.

Step 3: Effect of dilution on Λ :

Concentration falls faster than k does, and ions also get more freedom to move apart (and weak electrolytes dissociate more). So the ratio k/M rises, and Λ_m increases.

Step 4: Why the other options are wrong.

Both increasing (A) is impossible since fewer ions per volume lowers k . Both decreasing (B) and Λ decreasing with k increasing (D) contradict the trend of molar conductivity with dilution.

Final Answer:

On dilution, molar conductivity increases and conductivity decreases.

(C) Λ increases and k decreases

Quick Tip: On dilution, molar conductivity rises but conductivity falls.

76. Match List I with List II.

List I (Conversion)

List II (Number of Faraday required)

- | | |
|---|-----------|
| A. 1 mole of H_2O to O_2 | I. $3F$ |
| B. 1 mol of MnO_4^- to Mn^{2+} | II. $2F$ |
| C. 1.5 mol of Ca from molten CaCl_2 | III. $1F$ |
| D. 1 mol of FeO to Fe_2O_3 | IV. $5F$ |

Choose the correct answer from the options given below :

- (A) A-II, B-IV, C-I, D-III
(B) A-III, B-IV, C-I, D-II
(C) A-II, B-III, C-I, D-IV
(D) A-III, B-IV, C-II, D-I

Correct Answer: (A) A-II, B-IV, C-I, D-III

Solution:

Step 1: Understanding the Concept:

One Faraday (1 F) is the charge of one mole of electrons. So the Faradays needed equals the moles of electrons transferred.

Step 2: Match A:

$\text{H}_2\text{O} \rightarrow \frac{1}{2}\text{O}_2 + 2\text{H}^+ + 2e^-$. One mole of water gives 2 electrons, so it needs 2F. A matches II.

Step 3: Match B:

$\text{MnO}_4^- + 8\text{H}^+ + 5e^- \rightarrow \text{Mn}^{2+} + 4\text{H}_2\text{O}$. Manganese goes from +7 to +2, so 5 electrons per mole: 5F. B matches IV.

Step 4: Match C:

$\text{Ca}^{2+} + 2e^- \rightarrow \text{Ca}$. One mole needs 2F, so 1.5 mol needs $1.5 \times 2 = 3\text{F}$. C matches I.

Step 5: Match D:

FeO to Fe_2O_3 : $\text{Fe}^{2+} \rightarrow \text{Fe}^{3+} + e^-$, so 1 electron per mole of FeO : 1F. D matches III.

Step 6: Choose the option:

A-II, B-IV, C-I, D-III is option A. The other options mismatch at least one pair, for example B has A-III which would mean 1F for water oxidation.

Final Answer:

The correct matching is A-II, B-IV, C-I, D-III.

(A) A-II, B-IV, C-I, D-III

Quick Tip: Find the electrons transferred per mole in each half reaction; one Faraday is one mole of electrons.

77. The half life of first order reaction is 850 s. The initial concentration of the reactant is 0.06 mol dm^{-3} . What concentration would remain after 1200 s ?

- (A) $0.023 \text{ mol dm}^{-3}$
(B) $0.035 \text{ mol dm}^{-3}$
(C) 5.25 mol dm^{-3}
(D) 6.25 mol dm^{-3}

Correct Answer: (A) $0.023 \text{ mol dm}^{-3}$

Solution:

Step 1: Understanding the Concept:

For a first-order reaction, $k = \frac{0.693}{t_{1/2}}$ and $[A] = [A]_0 e^{-kt}$.

Step 2: Find the rate constant:

$$k = \frac{0.693}{850} = 8.15 \times 10^{-4} \text{ s}^{-1}.$$

Step 3: Use the integrated equation:

$$\log \frac{[A]_0}{[A]} = \frac{kt}{2.303} = \frac{8.15 \times 10^{-4} \times 1200}{2.303} = 0.4247$$

$$\frac{[A]_0}{[A]} = 10^{0.4247} = 2.66$$

Step 4: Find the concentration:

$$[A] = \frac{0.06}{2.66} = 0.0226 \approx 0.023 \text{ mol dm}^{-3}.$$

Step 5: Why the other options are wrong.

$0.035 \text{ mol dm}^{-3}$ would need less than one half-life to pass. 5.25 and 6.25 are larger than the initial 0.06 , which cannot be since the

concentration only falls.

Final Answer:

The remaining concentration is about 0.023 mol per dm³.

$$(A) \ 0.023 \text{ mol dm}^{-3}$$

Quick Tip: Find k from $k = 0.693 / t_{\text{half}}$, then use the integrated first order equation.

78. Half life period of a first order reaction is 1386 seconds. The rate constant of the reaction is

(A) $5.5 \times 10^{-2} \times s^{-1}$

(B) $0.5 \times 10^{-3} \times s^{-1}$

(C) $5 \times 10^{-2} \times s^{-1}$

(D) $5 \times 10^{-3} \times s^{-1}$

Correct Answer: (B) $0.5 \times 10^{-3} \times s^{-1}$

Solution:

Step 1: Understanding the Concept:

For a first-order reaction the half-life does not depend on the initial concentration. It is linked to the rate constant by a fixed relation.

Step 2: Key Formula:

$$k = \frac{0.693}{t_{1/2}}$$

Step 3: Substitute:

$$k = \frac{0.693}{1386} = 5 \times 10^{-4} \text{ s}^{-1} = 0.5 \times 10^{-3} \text{ s}^{-1}$$

This follows since $1386 = 2 \times 693$, so $\frac{693 \times 10^{-3}}{1386} = 0.5 \times 10^{-3}$.

Step 4: Why the other options are wrong.

5.5×10^{-2} , 5×10^{-2} and $5 \times 10^{-3} \text{ s}^{-1}$ are 110, 100 and 10 times too large, which would give half-lives of about 12.6 s, 13.9 s and 139 s.

Final Answer:

The rate constant is $0.5 \times 10^{-3} \text{ s}^{-1}$.

$(B) 0.5 \times 10^{-3} \text{ s}^{-1}$

Quick Tip: For a first order reaction, $k = 0.693 / t_{\text{half}}$.

79. What is the order of the following reaction ?



- (A) 0
- (B) 1
- (C) 2
- (D) 3

Correct Answer: (B) 1

Solution:

Step 1: Understanding the Concept:

The order of a reaction is the sum of the powers to which the concentration terms are raised in the experimentally found rate law. It does not come from the stoichiometric coefficients.

Step 2: Read the rate law:

The given rate law is $\text{rate} = k[\text{H}_2\text{O}_2]^1$. The power of $[\text{H}_2\text{O}_2]$ is 1.

Step 3: Find the order:

The order is 1, so the reaction is first order.

Step 4: Why the other options are wrong.

The coefficient 2 in the balanced equation does not set the order, so option C is a trap. Zero order would mean $\text{rate} = k$ with no concentration term. Order 3 has no basis here.

Final Answer:

The reaction is first order.

(B) 1

Quick Tip: The order is the sum of the powers of concentration terms in the rate law.

80. Which of the following nanomaterial has all the three dimensions $< 100 \text{ nm}$?

- (A) Nanotubes
- (B) Nanowires
- (C) Nanoshells
- (D) Thin films

Correct Answer: (C) Nanoshells

Solution:

Step 1: Understanding the Concept:

Nanomaterials are grouped by how many of their dimensions are larger than 100 nm. A 0D material has all three dimensions below 100 nm.

Step 2: Check each option:

Nanotubes are long in one direction (length in micrometres) so only two dimensions are nano. Nanowires are also long in one direction. Thin films are nano in thickness only, one dimension. Nanoshells are small spherical particles with a core and shell, and all three dimensions are in the nano range.

Step 3: Conclusion:

Nanoshells are the material with all three dimensions below 100 nm.

Final Answer:

Only nanoshells have all three dimensions below 100 nm.

(C) Nanoshells

Quick Tip: Zero-dimensional nanomaterials like nanoshells and quantum dots have all three dimensions at the nanoscale.

81. Identify the gas adsorbed by solid to the largest extent at their respective critical temperature.

- (A) $\text{N}_{2(g)}$
- (B) $\text{O}_{2(g)}$
- (C) $\text{H}_{2(g)}$
- (D) $\text{SO}_{2(g)}$

Correct Answer: (D) $\text{SO}_{2(g)}$

Solution:

Step 1: Understanding the Concept:

The extent of physical adsorption depends on how easily a gas can be liquefied. Easily liquefiable gases have stronger van der Waals forces and are adsorbed more.

Step 2: Link to critical temperature:

A higher critical temperature means the gas liquefies more easily. So the gas with the highest critical temperature is adsorbed most.

Step 3: Compare the gases:

Approximate critical temperatures: H_2 33 K, N_2 126 K, O_2 155 K, SO_2 431 K. SO_2 is the highest.

Step 4: Why the other options are wrong.

N_2 , O_2 and H_2 are permanent gases with weak intermolecular forces and low critical temperatures, so they are adsorbed far less.

Final Answer:

Sulphur dioxide is adsorbed the most.



Quick Tip: Gases with higher critical temperature are more easily liquefied, so they are adsorbed more.

82. Match the Xenon compounds in Column-I with its structure in Column-II.

Column – I

Column – II

- | | |
|---------------------|----------------------------|
| (a) XeF_4 | (i) pyramidal |
| (b) XeF_6 | (ii) square planar |
| (c) XeOF_4 | (iii) distorted octahedral |
| (d) XeO_3 | (iv) square pyramidal |

- (A) (a) - (ii), (b) - (iii), (c) - (i), (d) - (iv)
(B) (a) - (iii), (b) - (iv), (c) - (i), (d) - (ii)
(C) (a) - (i), (b) - (ii), (c) - (iii), (d) - (iv)
(D) (a) - (ii), (b) - (iii), (c) - (iv), (d) - (i)

Correct Answer: (D) (a) - (ii), (b) - (iii), (c) - (iv), (d) - (i)

Solution:

Step 1: Understanding the Concept:

The shapes of xenon compounds come from VSEPR theory using bond pairs plus lone pairs around Xe.

Step 2: Match (a) XeF_4 :

Xe has 4 bond pairs and 2 lone pairs (octahedral electron geometry).

The lone pairs sit opposite each other, so the shape is square planar:
(a) - (ii).

Step 3: Match (b) XeF_6 :

Xe has 6 bond pairs and 1 lone pair. The lone pair distorts the octahedron, so the shape is distorted octahedral: (b) - (iii).

Step 4: Match (c) XeOF_4 :

Xe has 5 bond regions (4 Xe-F and 1 Xe=O) and 1 lone pair. The shape is square pyramidal: (c) - (iv).

Step 5: Match (d) XeO_3 :

Xe has 3 bond regions and 1 lone pair, so the shape is trigonal pyramidal: (d) - (i).

Step 6: Conclusion:

The set (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i) is option D. Options A, B and C each mismatch at least one pair.

Final Answer:

The correct matching is (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i).

(D) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)

Quick Tip: Use VSEPR: XeF_4 square planar, XeF_6 distorted octahedral, XeOF_4 square pyramidal, XeO_3 pyramidal.

83. Which of the following lanthanoids is radioactive?

- (A) Tm
- (B) Dy
- (C) Eu
- (D) Pm

Correct Answer: (D) Pm

Solution:

Step 1: Understanding the Concept:

Among the 15 lanthanoids (La to Lu), all are stable or nearly stable except one element which has no stable isotope.

Step 2: Recall the fact:

Promethium (Pm, atomic number 61) is radioactive. It does not occur in significant amounts in nature.

Step 3: Why the other options are wrong.

Thulium (Tm), dysprosium (Dy) and europium (Eu) all have stable isotopes and are not radioactive.

Final Answer:

Promethium is the radioactive lanthanoid.

(D) Pm

Quick Tip: Promethium (Pm) is the only radioactive lanthanoid.

84. Which transition series includes elements Co and Mo respectively?

- (A) 4d and 5d
- (B) 5d and 6d
- (C) 3d and 4d
- (D) 3d and 6d

Correct Answer: (C) 3d and 4d

Solution:

Step 1: Understanding the Concept:

The transition series is named after the d subshell that is being filled: 3d (Sc to Zn), 4d (Y to Cd), 5d (La, Hf to Hg) and 6d (Ac, Rf onwards).

Step 2: Locate Co:

Cobalt has atomic number 27 and the configuration $[\text{Ar}]3d^74s^2$, so it belongs to the 3d series.

Step 3: Locate Mo:

Molybdenum has atomic number 42 and the configuration $[\text{Kr}]4d^55s^1$, so it belongs to the 4d series.

Step 4: Conclusion:

Co is in 3d and Mo is in 4d, which is option C. The other options place Co in 4d or 5d or Mo in 5d or 6d, which is wrong.

Final Answer:

Co is a 3d element and Mo is a 4d element.

(C) 3d and 4d

Quick Tip: Co ($Z = 27$) is in the 3d series and Mo ($Z = 42$) is in the 4d series.

85. Which of the following is an ambident ligand ?

- (A) Ethylene diamine
- (B) Cl^-
- (C) EDTA
- (D) SCN

Correct Answer: (D) SCN

Solution:

Step 1: Understanding the Concept:

An ambident ligand has two different atoms that can each donate a lone pair, but it uses only one at a time. Example: SCN^- can bind through S (thiocyanato) or N (isothiocyanato).

Step 2: Check the options:

Ethylenediamine has two identical N donors, so it is bidentate, not ambident. Cl^- has one donor atom, so it is monodentate. EDTA has six donor atoms (4 O and 2 N), so it is hexadentate (and the donors are used together). SCN has S and N as different donor atoms.

Step 3: Conclusion:

SCN is the ambident ligand, option D.

Final Answer:

SCN is the ambident ligand.

(D) SCN

Quick Tip: An ambident ligand has two different donor atoms but binds through only one at a time, like SCN⁻.

86. Which of the following sets of ligands does NOT contain ambident ligand in it?

- (A) Oxalate ion, NO₂⁻, NCS⁻
- (B) C₂O₄²⁻, Ethylene diamine, H₂O
- (C) NO₂⁻, C₂O₄²⁻, EDTA⁴⁻
- (D) EDTA⁴⁻, NCS⁻, Oxalate ion

Correct Answer: (B) C₂O₄²⁻, Ethylene diamine, H₂O

Solution:

Step 1: Understanding the Concept:

An ambident ligand has two different possible donor atoms. Common examples are NO₂⁻ (N or O) and SCN⁻/NCS⁻ (S or N).

Step 2: Check option A:

It contains NO₂⁻ and NCS⁻, which are ambident. So A has ambident ligands.

Step 3: Check option B:

Oxalate is a bidentate ligand with two equivalent O donors. Ethylenediamine has two equivalent N donors. Water has one O donor. None is ambident, so B has no ambident ligand.

Step 4: Check options C and D:

C contains NO_2^- , which is ambident. D contains NCS^- , which is ambident.

Step 5: Conclusion:

Only option B has no ambident ligand.

Final Answer:

Only set B has no ambident ligand.

(B) $\text{C}_2\text{O}_4^{2-}$, ethylene diamine, H_2O

Quick Tip: Ambident ligands include NO_2^- and NCS^- ; oxalate, en, water and EDTA are not ambident.

87. What type of color is observed when a compound absorbs red coloured light?

- (A) Red
- (B) Blue
- (C) Green
- (D) Yellow

Correct Answer: (C) Green

Solution:

Step 1: Understanding the Concept:

A coloured compound shows the colour that is left over after it takes some light out of white light. The colour we see is the complementary

colour of the light that is absorbed.

Step 2: Key Formula or Approach:

Complementary colours sit opposite each other on the colour wheel. The pairs are red and green, blue and orange, yellow and violet.

Step 3: Detailed Explanation:

The compound absorbs red light, so red is removed from the white light that falls on it.

The light that passes through or reflects back is white minus red, which looks green.

So the compound looks green to the eye.

Red is wrong because that is the colour absorbed, not the colour seen. Blue and yellow are not opposite red on the wheel, so they are not the complementary colour.

Final Answer:

The observed colour is green, option (C).

(C) Green

Quick Tip: The colour seen is the complementary colour of the light absorbed, and the complement of red is green.

88. Which of the following compounds is formed when chloroform is exposed to air and light?

- (A) Phosgene.
- (B) Phosphine.
- (C) Carbon dioxide
- (D) Carbon tetrachloride

Correct Answer: (A) Phosgene.

Solution:

Step 1: Understanding the Concept:

Chloroform (CHCl_3) is slowly oxidised by oxygen of the air in the presence of sunlight.

Step 2: Write the reaction:



The product COCl_2 is carbonyl chloride, known as phosgene, which is highly poisonous.

Step 3: Practical note:

Because of this reaction, chloroform is stored in dark brown bottles filled to the brim, and a little ethanol is added to turn any phosgene into harmless ethyl carbonate.

Step 4: Why the other options are wrong.

Phosphine (PH_3) has no source in this reaction. CO_2 is not formed directly. CCl_4 is made from chloroform with chlorine, not oxygen.

Final Answer:

Chloroform is converted to phosgene in air and light.

(A) Phosgene

Quick Tip: Chloroform is slowly oxidised by air and light to poisonous phosgene.

89. Identify the major product of dehydrohalogenation of $\text{CH}_3\text{-CH}_2\text{-CH(Br)-CH}_3$.

- (A) $\text{CH}_3\text{-CH}_2\text{-CH=CH}_2$
- (B) $\text{H}_3\text{C-CH=CH-CH}_3$
- (C) $\text{CH}_3\text{-(CH}_2)_2\text{CH}_3$
- (D) $\text{CH}_3\text{-(CH}_2)_2\text{CHO}$

Correct Answer: (B) $\text{H}_3\text{C-CH=CH-CH}_3$

Solution:

Step 1: Understanding the Concept:

Dehydrohalogenation removes H and Br from adjacent carbons to give an alkene. When two alkenes are possible, the more substituted (more stable) alkene is the major product. This is Saytzeff's rule.

Step 2: Find the possible products:

2-Bromobutane, $\text{CH}_3\text{CH}_2\text{CH(Br)CH}_3$, can lose H from C-1 to give but-1-ene ($\text{CH}_3\text{CH}_2\text{CH=CH}_2$), or from C-3 to give but-2-ene ($\text{CH}_3\text{CH=CHCH}_3$).

Step 3: Pick the major product:

But-2-ene has two alkyl groups on the double bond (disubstituted), while but-1-ene has only one (monosubstituted). So but-2-ene is more

stable and is the major product.

Step 4: Why the other options are wrong.

Option A is the minor product. Option C, butane, has no double bond and would need reduction. Option D, butanal, has an aldehyde group and would need an oxidation, not an elimination.

Final Answer:

But-2-ene is the major product.



Quick Tip: By Saytzeff's rule the more substituted alkene, but-2-ene, is the major product.



90. An organic compound with the molecular formula $\text{C}_6\text{H}_6\text{O}$ dissolves in NaOH , gives a characteristic colour with neutral FeCl_3 and on treatment with bromine water gives a tri bromo derivative. Which is that compound among the following?

- (A) Alcohol
- (B) Ether
- (C) Ketone
- (D) Phenol

Correct Answer: (D) Phenol

Solution:

Step 1: Understanding the Concept:

The three tests given are characteristic of phenol, $\text{C}_6\text{H}_5\text{OH}$.

Step 2: Test 1, solubility in NaOH:

Phenol is weakly acidic and reacts with NaOH to form soluble sodium phenoxide. Alcohols, ethers and ketones do not dissolve in NaOH in this way.

Step 3: Test 2, neutral FeCl_3 :

Phenol gives a violet colour with neutral ferric chloride.

Step 4: Test 3, bromine water:

The -OH group activates the ring, so bromine water gives a white precipitate of 2,4,6-tribromophenol.

Step 5: Conclusion:

The compound is phenol, option D. An alcohol, ether or ketone with formula $\text{C}_6\text{H}_6\text{O}$ cannot have all three properties.

Final Answer:

The compound is phenol.

(D) Phenol

Quick Tip: Phenol dissolves in NaOH, gives violet colour with FeCl_3 and gives 2,4,6-tribromophenol with bromine water.

91. What type of mechanism is followed by dehydration of alcohol to form ether?

- (A) Nucleophilic substitution unimolecular reaction
- (B) Nucleophilic substitution bimolecular reaction
- (C) Electrophilic substitution reaction
- (D) Electrophilic addition reaction

Correct Answer: (B) Nucleophilic substitution bimolecular reaction

Solution:

Step 1: Understanding the Concept:

At about 413 K with excess alcohol and an acid catalyst, primary alcohols form ethers by bimolecular dehydration.

Step 2: Mechanism:

Step 1: the alcohol is protonated to ROH_2^+ . Step 2: a second alcohol molecule uses its lone pair on O to attack the carbon of the protonated alcohol, pushing out water. Step 3: loss of a proton gives R-O-R .

Step 3: Name the mechanism:

Step 2 involves a nucleophile attacking in the same step in which the leaving group leaves, which is nucleophilic substitution bimolecular ($\text{S}_{\text{N}}2$).

Step 4: Why the other options are wrong.

$\text{S}_{\text{N}}1$ would need a stable carbocation, which a primary alcohol does not give. Electrophilic substitution and addition are reactions of aromatic rings and alkenes.

Final Answer:

Dehydration to ether follows the $\text{S}_{\text{N}}2$ mechanism.

(B) Nucleophilic substitution bimolecular reaction

Quick Tip: Dehydration of primary alcohols to ethers goes by $\text{S}_{\text{N}}2$: one alcohol attacks a protonated alcohol.

92. The number of primary isomeric alcohols of the molecular formula $\text{C}_4\text{H}_{10}\text{O}$ is

- (A) 1
- (B) 2
- (C) 3
- (D) 4

Correct Answer: (B) 2

Solution:

Step 1: Understanding the Concept:

A primary alcohol has the $-\text{OH}$ group on a carbon that is attached to only one other carbon, written as $-\text{CH}_2\text{OH}$.

Step 2: List all alcohol isomers of $\text{C}_4\text{H}_{10}\text{O}$:

There are four: butan-1-ol, butan-2-ol, 2-methylpropan-1-ol and 2-methylpropan-2-ol.

Step 3: Classify them:

Butan-1-ol, $\text{CH}_3\text{CH}_2\text{CH}_2\text{CH}_2\text{OH}$: primary. 2-Methylpropan-1-ol, $(\text{CH}_3)_2\text{CHCH}_2\text{OH}$: primary. Butan-2-ol: secondary. 2-Methylpropan-2-ol: tertiary.

Step 4: Count:

Two primary alcohols. The answer is option B.

Final Answer:

There are 2 primary isomeric alcohols.

(B) 2

Quick Tip: Butan-1-ol and 2-methylpropan-1-ol are the two primary alcohols of $C_4H_{10}O$.

93. In Gatterman reaction, which of the following catalyst is used?

- (A) CO, HCl and anhyd. $AlCl_3$
- (B) Copper powder and HBr
- (C) $Cu + Aq. NaNO_2$
- (D) $KOH + Br_2$

Correct Answer: (B) Copper powder and HBr

Solution:**Step 1: Understanding the Concept:**

The Gattermann reaction converts a benzenediazonium salt to an aryl halide using copper powder and the halogen acid (HCl or HBr).

Step 2: Write the reaction:

Step 3: Match with the options:

Option B, copper powder and HBr, is the Gattermann catalyst system. Option A (CO, HCl, anhydrous AlCl_3) is the Gattermann-Koch reaction. Option C is not a reagent set for aryl halides. Option D (KOH + Br_2) is the Hofmann bromamide reagent.

Step 4: Conclusion:

Copper powder and HBr, option B.

Final Answer:

The Gattermann reaction uses copper powder and HBr.

(B) Copper powder and HBr

Quick Tip: Gattermann reaction uses copper powder with HX, while Sandmeyer uses Cu(I) salts.



94. When ethanal and propanal undergo aldol condensation with dilute NaOH, the pair of products obtained by cross condensation reaction is

- (A) but-2-enal and 2-methylpent-2-enal
- (B) but-2-enal and pent-2-enal
- (C) 2-methylbut-2-enal and pent-2-enal
- (D) 2-methylbut-2-enal and 2-methylpent-2-enal

Correct Answer: (C) 2-methylbut-2-enal and pent-2-enal

Solution:

Step 1: Understanding the Concept:

In a cross aldol condensation, each aldehyde with alpha hydrogens can act as the enolate (nucleophile) and attack the other aldehyde. Four products form: two self-condensation and two cross products. We need the two cross products.

Step 2: Cross product 1:

Enolate of propanal attacks ethanal: $\text{CH}_3\text{CH}_2\text{CHO}$ gives $\text{CH}_3\text{CH}^-\text{CHO}$, which adds to CH_3CHO to give $\text{CH}_3\text{CH}(\text{OH})\text{CH}(\text{CH}_3)\text{CHO}$. Dehydration gives $\text{CH}_3\text{CH}=\text{C}(\text{CH}_3)\text{CHO}$, which is 2-methylbut-2-enal.

Step 3: Cross product 2:

Enolate of ethanal attacks propanal: $\text{CH}_3\text{CH}_2\text{CH}(\text{OH})\text{CH}_2\text{CHO}$. Dehydration gives $\text{CH}_3\text{CH}_2\text{CH}=\text{CHCHO}$, which is pent-2-enal.

Step 4: Compare with options:

But-2-enal (A and B) is the self-condensation product of ethanal. 2-Methylpent-2-enal (A and D) is the self-condensation product of propanal. So only C lists the two cross products.

Final Answer:

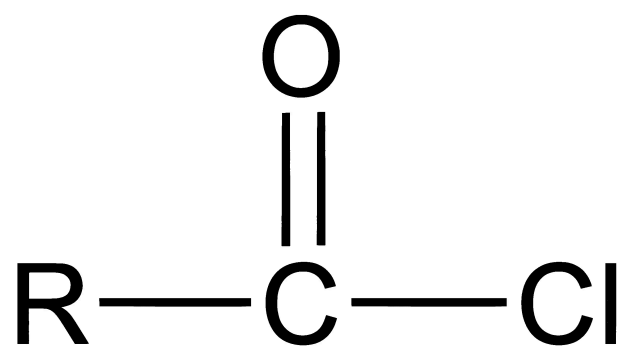
The cross products are 2-methylbut-2-enal and pent-2-enal.

(C) 2-methylbut-2-enal and pent-2-enal

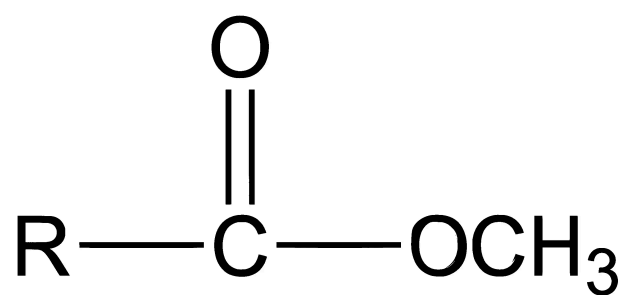
Quick Tip: Cross aldol of ethanal and propanal gives 2-methylbut-2-enal and pent-2-enal.

95. Which of the following species is used in Friedel - Craft reaction?

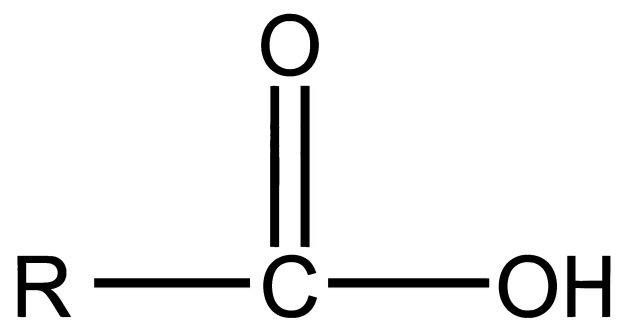
(A)



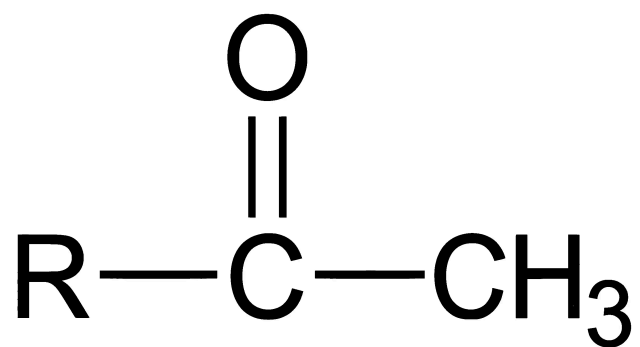
(B)



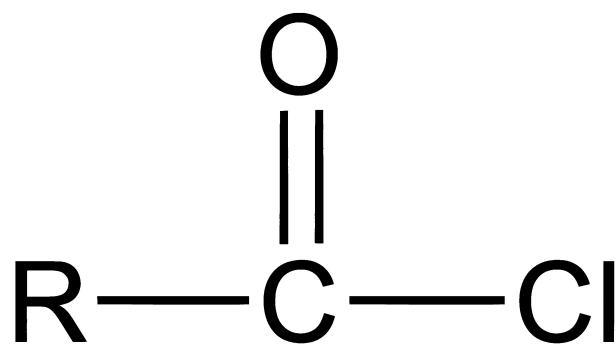
(C)



(D)



Correct Answer: (A)



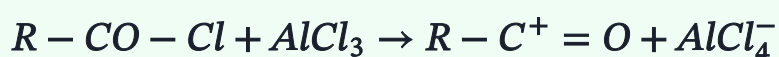
Solution:

Step 1: Understanding the Concept:

Friedel-Crafts reactions attach a carbon group to a benzene ring using a Lewis acid catalyst such as anhydrous AlCl_3 . The four options are carbonyl compounds, each with the group R on the left of the $\text{C}=\text{O}$ carbon and a different group on the right.

Step 2: Key Formula or Approach:

In Friedel-Crafts acylation, the attacking species is the acylium ion $\text{R}-\text{C}^+=\text{O}$. It forms when AlCl_3 pulls the chlorine off an acid chloride:



Step 3: Identify the compound in each option:

Option (A) is $\text{R}-\text{CO}-\text{Cl}$, an acyl chloride (acid chloride). Option (B) is $\text{R}-\text{CO}-\text{OCH}_3$, an ester. Option (C) is $\text{R}-\text{CO}-\text{OH}$, a carboxylic acid. Option (D) is $\text{R}-\text{CO}-\text{CH}_3$, a ketone.

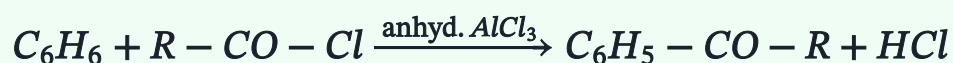
Step 4: Why the other options are wrong:

Chlorine is a good leaving group, so only the acid chloride gives the acylium ion easily. The ester has $-\text{OCH}_3$ and the acid has $-\text{OH}$, which

bind to $AlCl_3$ and do not leave readily. The ketone has a $-CH_3$ group, which cannot leave at all. So these three are not used as acylating agents.

Step 5: Product:

The acylium ion attacks the benzene ring and gives an aromatic ketone:



Final Answer:

The species used in the Friedel-Crafts reaction is the acid chloride $R - CO - Cl$, which is option (A).



Quick Tip: Friedel-Crafts acylation needs an acyl chloride (or anhydride) with anhydrous $AlCl_3$. Pick the option that has Cl on the carbonyl carbon.

96. What type of saccharide the maltose is?

- (A) Polysaccharide
- (B) Disaccharide
- (C) Trisaccharide
- (D) Monosaccharide

Correct Answer: (B) Disaccharide

Solution:

Step 1: Understanding the Concept:

Carbohydrates are classified by how many monosaccharide units they give on hydrolysis: monosaccharides (1), disaccharides (2), trisaccharides (3) and polysaccharides (many).

Step 2: Apply to maltose:

Maltose gives two molecules of glucose on hydrolysis. The two glucose units are joined by an alpha-1,4-glycosidic bond.

Step 3: Conclusion:

Maltose is a disaccharide, option B. Sucrose and lactose are other common disaccharides.

Final Answer:

Maltose is a disaccharide.

(B) Disaccharide

Quick Tip: Maltose is made of two glucose units, so it is a disaccharide.

97. Buna - S is an example of ,

- (A) elastomer
- (B) fibre
- (C) thermoplastic polymer
- (D) thermosetting polymer

Correct Answer: (A) elastomer

Solution:

Step 1: Understanding the Concept:

Polymers are classified by their intermolecular forces: elastomers (weak forces, rubber-like), fibres (strong forces), thermoplastics and thermosetting polymers.

Step 2: Apply to Buna-S:

Buna-S is a copolymer of 1,3-butadiene and styrene made with sodium (Na) as catalyst. It is a synthetic rubber with weak forces and high elasticity.

Step 3: Conclusion:

Buna-S is an elastomer, option A.

Final Answer:

Buna-S is an elastomer.

(A) Elastomer

Quick Tip: Buna-S is a synthetic rubber, so it is an elastomer.

98. Which of the following is the monomer of Teflon polymer?

- (A) $\text{FCH} = \text{CH}_2$
- (B) $\text{FCH} = \text{CHF}$
- (C) $\text{CF}_3 = \text{CF}_3$
- (D) $\text{CF}_2 = \text{CF}_2$

Correct Answer: (D) $\text{CF}_2 = \text{CF}_2$

Solution:

Step 1: Understanding the Concept:

Teflon is the common name of polytetrafluoroethene (PTFE).

Step 2: Write the polymerisation:

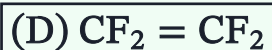


Step 3: Conclusion:

The monomer is tetrafluoroethene, $\text{CF}_2 = \text{CF}_2$, option D. Option C is not a valid structure, since a carbon with three fluorines cannot form a double bond to another such carbon.

Final Answer:

The monomer of Teflon is tetrafluoroethene.



Quick Tip: Teflon is polytetrafluoroethene, made from tetrafluoroethene $\text{CF}_2=\text{CF}_2$.

99. Which of the following statement about enzyme is correct ?

- (A) They alter the equilibrium constant of reactions
- (B) They increase the activation energy of reactions
- (C) They are specific in their action
- (D) They are consumed in the reaction

Correct Answer: (C) They are specific in their action

Solution:

Step 1: Understanding the Concept:

Enzymes are protein catalysts of biological reactions. A catalyst speeds up a reaction by giving it a path of lower activation energy.

Step 2: Check option A:

A catalyst does not change the equilibrium constant. So A is false.

Step 3: Check option B:

Enzymes lower, not raise, the activation energy. So B is false.

Step 4: Check option C:

Each enzyme acts on one substrate or one type of reaction, like a lock and key. So C is true.

Step 5: Check option D:

Enzymes are not consumed. They are regenerated at the end of the reaction. So D is false.

Final Answer:

Enzymes are specific in their action.

(C) They are specific in their action

Quick Tip: Enzymes are specific biocatalysts: they speed up reactions without being consumed.

100. What is the result when soap is used in hard water?

- (A) Micelles
- (B) Scum
- (C) Emulsion
- (D) Gel

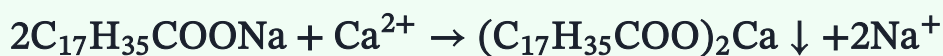
Correct Answer: (B) Scum

Solution:

Step 1: Understanding the Concept:

Soap is the sodium or potassium salt of a long-chain fatty acid. Hard water contains Ca^{2+} and Mg^{2+} ions.

Step 2: Write the reaction:



The calcium and magnesium salts of fatty acids are insoluble and appear as a greasy white scum.

Step 3: Why the other options are wrong.

Micelles form in soft water where soap stays soluble. Emulsion and gel are not the result of soap meeting hard water.

Final Answer:

Scum forms when soap is used in hard water.

(B) Scum

Quick Tip: Hard water contains Ca^{2+} and Mg^{2+} ions that form insoluble scum with soap.

Mathematics

Questions 101 to 150 | 2 marks each | 100 marks

101. If α and β are the roots of the equation $x^2 + x + 1 = 0$ then $\alpha^{2026} + \beta^{2026} =$

- (A) -1
- (B) 0
- (C) 1
- (D) 2

Correct Answer: (A) -1

Solution:

Step 1: Understanding the Concept:

The roots of $x^2 + x + 1 = 0$ are ω and ω^2 , the non-real cube roots of unity. They satisfy $\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$.

Step 2: Reduce the power:

Since $\omega^3 = 1$, only the remainder of 2026 divided by 3 matters. $2026 = 3(675) + 1$, so $\alpha^{2026} = \omega^1$ and $\beta^{2026} = (\omega^2)^{2026} = \omega^{4052}$.

$4052 = 3(1350) + 2$, so $\beta^{2026} = \omega^2$.

Step 3: Add them:

$$\alpha^{2026} + \beta^{2026} = \omega + \omega^2 = -1$$

This uses $1 + \omega + \omega^2 = 0$.

Step 4: Why the other options are wrong.

The value 2 or 0 would come from taking both powers as 1 or from wrongly cancelling the terms. The value 1 would need a remainder of 0 (power multiple of 3), but 2026 is not.

Final Answer:

The sum is -1.

$$\boxed{(A) - 1}$$

Quick Tip: The roots are the complex cube roots of unity, so powers repeat with period 3.

102. The number of ways that 8 beads of different colours can be strung to form a necklace is

- (A) 5040
- (B) 2520
- (C) 2840
- (D) 5080

Correct Answer: (B) 2520

Solution:

Step 1: Understanding the Concept:

Arranging n distinct objects in a circle gives $(n - 1)!$ arrangements. For a necklace, flipping it over gives the same pattern, so each arrangement is counted twice.

Step 2: Apply the formula:

$$\frac{(8-1)!}{2} = \frac{7!}{2} = \frac{5040}{2} = 2520$$

Step 3: Why the other options are wrong.

5040 is the count of circular arrangements without turning the necklace over. 2840 and 5080 do not match either formula.

Final Answer:

There are 2520 ways.

(B) 2520

Quick Tip: For a necklace of n distinct beads the count is $(n-1)!/2$.

103. In $\triangle ABC$, if $\angle A = 90^\circ$, then $\sin(B - C) =$

- (A) $\frac{a^2 - c^2}{a^2 + c^2}$
- (B) $\frac{c^2 - a^2}{c^2 + a^2}$
- (C) $\frac{b^2 - c^2}{b^2 + c^2}$
- (D) $\frac{c^2 - b^2}{c^2 + b^2}$

Correct Answer: (C) $\frac{b^2 - c^2}{b^2 + c^2}$

Solution:

Step 1: Understanding the Concept:

When $\angle A = 90^\circ$, side a is the hypotenuse and $a^2 = b^2 + c^2$.

Step 2: Write the ratios:

$$\sin B = \frac{b}{a}, \cos B = \frac{c}{a}, \sin C = \frac{c}{a}, \cos C = \frac{b}{a}.$$

Step 3: Expand sin(B-C):

$$\sin(B - C) = \sin B \cos C - \cos B \sin C = \frac{b}{a} \cdot \frac{b}{a} - \frac{c}{a} \cdot \frac{c}{a} = \frac{b^2 - c^2}{a^2}$$

Step 4: Replace a squared:

Since $a^2 = b^2 + c^2$, $\sin(B - C) = \frac{b^2 - c^2}{b^2 + c^2}$. This is option C.

Step 5: Why the other options are wrong.

Options A and B have a^2 and c^2 in the numerator, and D has the wrong sign (it equals $\sin(C - B)$).

Final Answer:

$\sin(B-C)$ equals $(b^2 - c^2)/(b^2 + c^2)$.

(C) $\frac{b^2 - c^2}{b^2 + c^2}$

Quick Tip: Use $\sin B = b/a$, $\cos B = c/a$, $\sin C = c/a$, $\cos C = b/a$ and $a^2 = b^2 + c^2$.

104. The value of $\cos^2 10^\circ - \cos 10^\circ \cos 50^\circ + \cos^2 50^\circ$ is equal to....

- (A) $\frac{1}{4}$
(B) $\frac{1}{2}$
(C) $\frac{3}{4}$
(D) 1

Correct Answer: (C) $\frac{3}{4}$

Solution:

Step 1: Understanding the Concept:

We use $\cos^2 \theta = \frac{1 + \cos 2\theta}{2}$ and $\cos A \cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$.

Step 2: Rewrite the squares:

$$\cos^2 10^\circ + \cos^2 50^\circ = 1 + \frac{1}{2}(\cos 20^\circ + \cos 100^\circ)$$

$\cos 20^\circ + \cos 100^\circ = 2 \cos 60^\circ \cos 40^\circ = \cos 40^\circ$, so the sum is $1 + \frac{1}{2}\cos 40^\circ$.

Step 3: Rewrite the product:

$$\cos 10^\circ \cos 50^\circ = \frac{1}{2}(\cos 60^\circ + \cos 40^\circ) = \frac{1}{4} + \frac{1}{2}\cos 40^\circ$$

Step 4: Subtract:

$$1 + \frac{1}{2}\cos 40^\circ - \frac{1}{4} - \frac{1}{2}\cos 40^\circ = \frac{3}{4}$$

Step 5: Why the other options are wrong.

$1/4$, $1/2$ and 1 would follow from dropping the "1" or the $1/4$ term. The cosine of 40 degrees cancels, which leaves exactly $3/4$.

Final Answer:

The value is $3/4$.

$$(C) \frac{3}{4}$$

Quick Tip: Convert squares and the product into sums of cosines using product-to-sum identities.

105. The general solution of $\cos 4\theta = \cos 3\theta$ is $\theta = \dots$

- (A) $\frac{2n\pi}{7}, n \in \mathbb{Z}$
- (B) $\frac{2n\pi}{5}, n \in \mathbb{Z}$
- (C) $n\pi, n \in \mathbb{Z}$
- (D) $\frac{4n\pi}{3}, n \in \mathbb{Z}$

Correct Answer: (A) $\frac{2n\pi}{7}, n \in \mathbb{Z}$

Solution:

Step 1: Understanding the Concept:

$\cos A = \cos B$ means $A = 2n\pi \pm B, n \in \mathbb{Z}$.

Step 2: Apply with $A = 4\theta$, $B = 3\theta$:

Case 1: $4\theta = 2n\pi + 3\theta$, so $\theta = 2n\pi$. Case 2: $4\theta = 2n\pi - 3\theta$, so $7\theta = 2n\pi$ and $\theta = \frac{2n\pi}{7}$.

Step 3: Combine:

Case 1 values ($\theta = 2n\pi$) are already contained in case 2 when n is a multiple of 7. So the full solution is $\theta = \frac{2n\pi}{7}, n \in \mathbb{Z}$.

Step 4: Why the other options are wrong.

$2n\pi/5$, $n\pi$ and $4n\pi/3$ do not satisfy the equation. For example $\theta = \pi$ gives $\cos 4\pi = 1$ but $\cos 3\pi = -1$.

Final Answer:

The general solution is $2n\pi / 7$.

$$(A) \frac{2n\pi}{7}, n \in \mathbb{Z}$$

Quick Tip: Use $\cos A = \cos B$ gives $A = 2n\pi$ plus or minus B .

106. The line $2x + y = 4$ intersects X and Y axes at points A and B respectively. Let $C(p, q)$ be the point such that the lines AC and BC are perpendicular. The distance of point C from the midpoint of segment AB is...

- (A) $\sqrt{2}$
- (B) $\sqrt{5}$
- (C) $2\sqrt{2}$
- (D) $2\sqrt{5}$

Correct Answer: (B) $\sqrt{5}$

Solution:

Step 1: Find A and B:

Put $y = 0$ in $2x + y = 4$: $x = 2$, so $A(2, 0)$. Put $x = 0$: $y = 4$, so $B(0, 4)$.

Step 2: Use the angle in a semicircle:

If $AC \perp BC$, then $\angle ACB = 90^\circ$, so C lies on the circle with AB as diameter.

Step 3: Find the distance:

The midpoint of AB is $M(1, 2)$, the centre of the circle. The distance CM equals the radius $= \frac{AB}{2}$.

$$AB = \sqrt{2^2 + 4^2} = \sqrt{20} = 2\sqrt{5}, \quad CM = \sqrt{5}$$

Step 4: Why the other options are wrong.

$\sqrt{2}$ and $2\sqrt{2}$ do not match half of AB . $2\sqrt{5}$ is the full length AB , not the radius.

Final Answer:

The distance is square root of 5.

(B) $\sqrt{5}$

Quick Tip: AC and BC perpendicular means C lies on the circle with AB as diameter.

107. If the distance between the lines represented by $(x - 2y)^2 + k(x - 2y) = 0$ is 3 units and $k > 0$ then the equations of the lines are

- (A) $x - 2y = 0$ and $x - 2y + \sqrt{3} = 0$.
 (B) $x - 2y = 0$ and $x - 2y + \sqrt{5} = 0$.
 (C) $x - 2y = 0$ and $x - 2y + 3\sqrt{3} = 0$.
 (D) $x - 2y = 0$ and $x - 2y + 3\sqrt{5} = 0$.

Correct Answer: (D) $x - 2y = 0$ and $x - 2y + 3\sqrt{5} = 0$.

Solution:

Step 1: Factor the equation:

$(x - 2y)^2 + k(x - 2y) = (x - 2y)(x - 2y + k) = 0$. The two lines are $x - 2y = 0$ and $x - 2y + k = 0$.

Step 2: Find the distance between them:

These lines are parallel. The distance is $\frac{|k - 0|}{\sqrt{1^2 + (-2)^2}} = \frac{k}{\sqrt{5}}$ for $k > 0$.

Step 3: Solve for k:

$\frac{k}{\sqrt{5}} = 3$, so $k = 3\sqrt{5}$. The lines are $x - 2y = 0$ and $x - 2y + 3\sqrt{5} = 0$.

Option D.

Step 4: Why the other options are wrong.

A, B, C have $k = \sqrt{3}, \sqrt{5}, 3\sqrt{3}$, which give distances $\sqrt{3}/\sqrt{5}, 1$ and $3\sqrt{3}/\sqrt{5}$, none equal to 3.

Final Answer:

The lines are $x - 2y = 0$ and $x - 2y + 3\sqrt{5} = 0$.

(D) $x - 2y = 0$ and $x - 2y + 3\sqrt{5} = 0$

Quick Tip: Factor the equation into $x-2y=0$ and $x-2y+k=0$, then use the parallel line distance formula.



108. If the slope of one of the two lines given by $\frac{x^2}{a} + \frac{2xy}{h} + \frac{y^2}{b} = 0$ is twice that of the other, then $h^2 : ab = \dots$

- (A) 1 : 2
- (B) 2 : 1
- (C) 8 : 9
- (D) 9 : 8

Correct Answer: (C) 8 : 9

Solution:

Step 1: Understanding the Concept:

Both lines pass through the origin, so each has the form $y = mx$.

Putting $y = mx$ in the pair equation gives a quadratic in m whose two roots are the slopes.

Step 2: Form the quadratic in slopes:

$$\frac{x^2}{a} + \frac{2mx^2}{h} + \frac{m^2x^2}{b} = 0 \Rightarrow \frac{m^2}{b} + \frac{2m}{h} + \frac{1}{a} = 0$$

Step 3: Use the given condition:

Let the slopes be m and $2m$.

Product of roots: $2m^2 = \frac{1/a}{1/b} = \frac{b}{a}$, so $m^2 = \frac{b}{2a}$.

Sum of roots: $3m = -\frac{2/h}{1/b} = -\frac{2b}{h}$.

Step 4: Eliminate m:

Square the sum relation: $9m^2 = \frac{4b^2}{h^2}$. Replace m^2 by $\frac{b}{2a}$:

$$\frac{9b}{2a} = \frac{4b^2}{h^2} \Rightarrow h^2 = \frac{8ab}{9}$$

So $h^2 : ab = 8 : 9$.

Step 5: Why the other options are wrong:

The ratios 1 : 2 and 2 : 1 only repeat the slope ratio and ignore the coefficients. 9 : 8 is the inverted ratio, which comes from writing $\frac{ab}{h^2}$ instead of $\frac{h^2}{ab}$.

Final Answer:

The required ratio is 8 : 9, which is option (C).

8 : 9

Quick Tip: Put $y = mx$ to get a quadratic in m , then use sum and product of roots with slopes m and $2m$.

109. If the equation $3x^2 + (3 - p)xy + qy^2 - 2px = 8pq$ represents a circle, then the area (in sq. units) of this circle is

- (A) 5π
- (B) 9π
- (C) 25π
- (D) 81π

Correct Answer: (C) 25π

Solution:

Step 1: Conditions for a circle:

The coefficient of xy must be 0 and the coefficients of x^2 and y^2 must be equal.

Step 2: Find p and q :

$3 - p = 0$ gives $p = 3$. Then $q = 3$.

Step 3: Rewrite the equation:

$3x^2 + 3y^2 - 6x = 8(3)(3) = 72$. Divide by 3: $x^2 + y^2 - 2x = 24$.

Complete the square: $(x - 1)^2 + y^2 = 25$.

Step 4: Find the area:

Radius = 5, area = $\pi r^2 = 25\pi$. Option C.

Step 5: Why the other options are wrong.

5π , 9π , 81π would need radii $\sqrt{5}$, 3 and 9, which come from sign or arithmetic slips.

Final Answer:

The area is 25 pi.

(C) 25π

Quick Tip: For a circle, the xy term vanishes and the coefficients of x^2 and y^2 are equal.

110. The ellipse $4x^2 + 9y^2 = 1$ intersects positive Y-axis at point A. If S and S' are its foci, then the area (in sq. units) of

Missing open brace for superscript is _____

- (A) $\frac{\sqrt{5}}{12}$
- (B) $\frac{\sqrt{5}}{18}$
- (C) $2\sqrt{5}$
- (D) $3\sqrt{5}$

Correct Answer: (B) $\frac{\sqrt{5}}{18}$

Solution:

Step 1: Write in standard form:

$4x^2 + 9y^2 = 1$ becomes $\frac{x^2}{1/4} + \frac{y^2}{1/9} = 1$. So $a^2 = \frac{1}{4}$ and $b^2 = \frac{1}{9}$. Since $a > b$, the foci lie on the x-axis.

Step 2: Find the foci and A:

$c^2 = a^2 - b^2 = \frac{1}{4} - \frac{1}{9} = \frac{5}{36}$, so $c = \frac{\sqrt{5}}{6}$. Foci: $S(\frac{\sqrt{5}}{6}, 0)$ and $S'(-\frac{\sqrt{5}}{6}, 0)$. On the positive Y-axis, $x = 0$, so $y = \frac{1}{3}$, and $A(0, \frac{1}{3})$.

Step 3: Find the area:

Base $SS' = 2c = \frac{\sqrt{5}}{3}$, height = $\frac{1}{3}$.

$$\text{Area} = \frac{1}{2} \cdot \frac{\sqrt{5}}{3} \cdot \frac{1}{3} = \frac{\sqrt{5}}{18}$$

Step 4: Why the other options are wrong.

$\sqrt{5}/12$ uses the wrong height, and $2\sqrt{5}$, $3\sqrt{5}$ result from reading $a^2 = 4$, $b^2 = 9$ wrongly.

Final Answer:

The area is $\sqrt{5} / 18$.

$$(B) \frac{\sqrt{5}}{18}$$

Quick Tip: Find a and b from the standard form, then $c = \sqrt{a^2 - b^2}$ and area = c times b.

111. $\lim_{x \rightarrow 1} \left[\frac{x-2}{x^2-x} - \frac{1}{x^3-3x^2+2x} \right] =$

- (A) 3
- (B) 4
- (C) 2
- (D) 1

Correct Answer: (C) 2

Solution:

Step 1: Factor the denominators:

$x^2 - x = x(x - 1)$ and $x^3 - 3x^2 + 2x = x(x^2 - 3x + 2) = x(x - 1)(x - 2)$. At $x = 1$ both fractions are of the form $\frac{\text{number}}{0}$, so we must combine them before putting $x = 1$.

Step 2: Take the common denominator:

The common denominator is $x(x - 1)(x - 2)$. The first fraction needs a factor $(x - 2)$ on top and bottom:

$$\frac{(x-2)^2 - 1}{x(x-1)(x-2)}$$

Step 3: Simplify the numerator:

$(x-2)^2 - 1 = x^2 - 4x + 3 = (x-1)(x-3)$. So the expression is

$$\frac{(x-1)(x-3)}{x(x-1)(x-2)} = \frac{x-3}{x(x-2)} \quad (x \neq 1)$$

Step 4: Take the limit:

Now the factor $(x-1)$ is gone, so we can put $x = 1$:

$$\frac{1-3}{1 \cdot (1-2)} = \frac{-2}{-1} = 2$$

Options (A), (B) and (D) do not come out of this working. They can only arise from a sign slip, such as reading $\frac{-2}{-1}$ as -2 , or from dropping the $(x-2)$ factor in the numerator.

Final Answer:

The limit is 2, which is option (C).

2

Quick Tip: Factor both denominators, combine the fractions and cancel the factor $(x-1)$.

112. If $A = \{1, 2, 3, 4, 5, 6\}$ then which of the following is not true?

- (A) $\exists x \in A$ such that $x + 3 = 9$.
- (B) $\exists x \in A$ such that $x + 2 < 9$.
- (C) $\forall x \in A, x + 6 \geq 10$.
- (D) $\exists x \in A$ such that $x + 6 < 10$.

Correct Answer: (C) $\forall x \in A, x + 6 \geq 10$.

Solution:

Step 1: Understanding the Concept:

$\exists$ needs at least one element that works. $\forall$ needs every element to work. A is $\{1, 2, 3, 4, 5, 6\}$.

Step 2: Check (A):

$x + 3 = 9$ gives $x = 6$, which is in A . TRUE.

Step 3: Check (B):

$x + 2 < 9$ is true for $x = 1$. TRUE.

Step 4: Check (C):

$x + 6 \geq 10$ needs $x \geq 4$. Take $x = 1$: $7 \geq 10$ is false. So the "for all" statement is FALSE.

Step 5: Check (D):

$x + 6 < 10$ is true for $x = 1$ (7 is less than 10). TRUE.

Step 6: Conclusion:

The statement that is not true is (C).

Final Answer:

Statement C is not true.

$$(C) \forall x \in A, x + 6 \geq 10$$

Quick Tip: Test each statement against the set; a "for all" statement fails if one element breaks it.

113. Consider the following statements

r : If $p \rightarrow q$ is false then $p \vee q$ is false.

s : If $p \leftrightarrow q$ is false then $p \vee q$ is false.

The truth values of $r \rightarrow s$ and $s \rightarrow r$ are respectively _____

- (A) T, T
- (B) T, F
- (C) F, T
- (D) F, F

Correct Answer: (A) T, T

Solution:**Step 1: Understanding the Concept:**

r and s are themselves implications. We must first decide if each is true or false for all values of p, q .

Step 2: Check r :

$p \rightarrow q$ is false only when $p = T, q = F$. Then $p \vee q = T$, not false. So the implication "false $\Rightarrow$ false" fails. Statement r is FALSE.

Step 3: Check s:

$p \leftrightarrow q$ is false when p and q differ, for example $p = T, q = F$. Then $p \vee q = T$, not false. Statement s is FALSE.

Step 4: Find the truth values of the compound statements:

$r \rightarrow s: F \rightarrow F$ is T. $s \rightarrow r: F \rightarrow F$ is T. So the answers are T, T, option A.

Final Answer:

Both implications are true.

(A) T, T

Quick Tip: Find the truth values of r and s by testing the antecedent cases.

114. In triangle ABC, with usual notations, if the angles are in Arithmetic Progression and $3c^2 = 2b^2$, then the measure of angle A is

- (A) $\frac{5\pi}{12}$
- (B) $\frac{7\pi}{12}$
- (C) $\frac{5\pi}{6}$
- (D) $\frac{7\pi}{6}$

Correct Answer: (A) $\frac{5\pi}{12}$

Solution:**Step 1: Use the AP condition:**

If A, B, C are in AP then $2B = A + C$. With $A + B + C = 180^\circ$ we get $3B = 180^\circ$, so $B = 60^\circ$.

Step 2: Use the sine rule:

$$\frac{c}{b} = \frac{\sin C}{\sin B}. \text{ From } 3c^2 = 2b^2, \frac{c^2}{b^2} = \frac{2}{3}, \text{ so } \frac{\sin^2 C}{\sin^2 60^\circ} = \frac{2}{3}.$$

Step 3: Find C:

$\sin^2 C = \frac{2}{3} \cdot \frac{3}{4} = \frac{1}{2}$, so $\sin C = \frac{1}{\sqrt{2}}$ and $C = 45^\circ$ or 135° . 135° is impossible since $B + C$ would exceed 180° . So $C = 45^\circ$.

Step 4: Find A:

$$A = 180^\circ - 60^\circ - 45^\circ = 75^\circ = \frac{5\pi}{12}. \text{ Option A.}$$

Final Answer:

Angle A is $5\pi / 12$.

$$\boxed{(A) \frac{5\pi}{12}}$$

Quick Tip: The middle angle of an AP of angles is 60 degrees; then use the sine rule.

115. If $A = \begin{bmatrix} 0 & 0 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$ then which of the following is true ?

- (A) $A^2 = A^{-1}$
- (B) $A = -(\text{adj}A)$
- (C) $A^{-1} = A$
- (D) $A^{-1} = -A$

Correct Answer: (C) $A^{-1} = A$

Solution:

Step 1: Understanding the Question:

We are given a 3×3 matrix A with -1 on the anti-diagonal and 0 elsewhere. We must find which relation among A^2 , A^{-1} , $\text{adj}A$ and A is true.

Step 2: Key Formula or Approach:

If $A^2 = I$ then $A \cdot A = I$, so A is its own inverse. Also $\text{adj}A = |A|A^{-1}$.

Step 3: Compute A^2 :

Row 1 of A is $(0, 0, -1)$. Multiplying it with the columns of A gives $(-1)(-1) = 1$ in column 1 and 0 in the others. Row 2 $(0, -1, 0)$ gives 1 only in column 2. Row 3 $(-1, 0, 0)$ gives 1 only in column 3. So

$$A^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

Step 4: Conclude about the inverse:

Since $A \cdot A = I$, the inverse of A is A itself, so $A^{-1} = A$. This is option (C).

Step 5: Check the other options:

Option (A): $A^2 = I$, but $A^{-1} = A \neq I$, so $A^2 \neq A^{-1}$. Option (D): $A^{-1} = -A$ would mean $A = -A$, which is false because $A \neq 0$. Option (B): $|A| = (-1) \cdot (-1)^3 = 1$, since the anti-diagonal arrangement is an odd permutation. So $\text{adj}A = |A|A^{-1} = A$, not $-A$, and (B) is false.

Final Answer:

Only $A^{-1} = A$ is true.

$$A^{-1} = A$$

Quick Tip: Square the matrix. If A times A gives the identity, A is its own inverse.

116. If $\begin{bmatrix} 2 & 1 & -1 \\ -1 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ -2 \end{bmatrix}$ then the distance of point $P(a, b, c)$ from the plane $2x + y + 2z = 10$ is

- (A) 2 units.
- (B) 3 units.
- (C) 4 units.
- (D) 5 units.

Correct Answer: (B) 3 units.

Solution:**Step 1: Understanding the Question:**

The matrix equation gives three linear equations in a, b, c . We first find the point $P(a, b, c)$, then find its distance from the plane $2x + y + 2z = 10$.

Step 2: Write the equations:

Multiplying out the matrix gives

$$2a + b - c = 4 \quad (1)$$

$$-a + 2b + c = 0 \quad (2)$$

$$a - b + 2c = -2 \quad (3)$$

Step 3: Solve the system:

Add (1) and (2) to remove c : $a + 3b = 4$. Add (2) and (3): $b + 3c = -2$. From $a = 4 - 3b$ and $c = (-2 - b)/3$, put both into (2): $-(4 - 3b) + 2b + (-2 - b)/3 = 0$. Multiply by 3 to get $14b = 14$, so $b = 1$, $a = 1$, $c = -1$. Check in (1): $2 + 1 + 1 = 4$. Check in (3): $1 - 1 - 2 = -2$. So $P = (1, 1, -1)$.

Step 4: Distance formula:

The distance of (x_1, y_1, z_1) from $Ax + By + Cz + D = 0$ is

$$d = \frac{|Ax_1 + By_1 + Cz_1 + D|}{\sqrt{A^2 + B^2 + C^2}}$$

The plane is $2x + y + 2z - 10 = 0$.

Step 5: Substitute:

$$d = \frac{|2(1) + 1(1) + 2(-1) - 10|}{\sqrt{4 + 1 + 4}} = \frac{|-9|}{3} = 3$$

Step 6: Why the others are wrong:

The numerator 9 divided by 3 gives exactly 3. The values 2, 4 and 5 would come from slips, such as forgetting the -10 or using $c = 1$ instead of -1 .

Final Answer:

The point is $P(1, 1, -1)$ and its distance from the plane is 3 units, option (B).

3 units

Quick Tip: First solve the three equations to get $P = (1, 1, -1)$, then use the point-to-plane distance formula.

117. If $\cot(\cos^{-1}x) = \sec(\tan^{-1}\frac{a}{\sqrt{b^2-a^2}})$, then the value of x is

- (A) $\frac{b}{\sqrt{2b^2+a^2}}$
(B) $\frac{\sqrt{2b^2-a^2}}{b}$
(C) $\frac{\sqrt{2b^2+a^2}}{b}$
(D) $\frac{b}{\sqrt{2b^2-a^2}}$

Correct Answer: (D) $\frac{b}{\sqrt{2b^2-a^2}}$

Solution:**Step 1: Simplify the left side:**

Let $\theta = \cos^{-1}x$, so $\cos \theta = x$, $\sin \theta = \sqrt{1-x^2}$. Then $\cot \theta = \frac{x}{\sqrt{1-x^2}}$.

Step 2: Simplify the right side:

Let $\phi = \tan^{-1}\frac{a}{\sqrt{b^2-a^2}}$. Then $\sec \phi = \sqrt{1 + \frac{a^2}{b^2-a^2}} = \frac{b}{\sqrt{b^2-a^2}}$.

Step 3: Equate and solve:

$$\frac{x}{\sqrt{1-x^2}} = \frac{b}{\sqrt{b^2-a^2}}$$

Square both sides: $x^2(b^2 - a^2) = b^2(1 - x^2)$, so $x^2(2b^2 - a^2) = b^2$.

$$x = \frac{b}{\sqrt{2b^2 - a^2}}$$

Step 4: Why the other options are wrong.

Option B and C give values with $\sqrt{2b^2 - a^2}$ in the numerator, which makes $x > 1$ for typical values. Option A uses a plus sign inside the root, which comes from a sign error when squaring.

Final Answer:

x equals $b / \sqrt{2b^2 - a^2}$.

(D) $\frac{b}{\sqrt{2b^2 - a^2}}$

Quick Tip: Express both sides algebraically using right triangles, then square and solve for x.

118. If $g(x) = x^2 + x - 2$ and $\frac{1}{2}(g \circ f)(x) = 2x^2 - 5x + 2$ then $f(x)$ is equal to

- (A) $2x + 3$
- (B) $2x - 3$
- (C) $2x^2 + 3x + 2$
- (D) $2x^2 - 3x - 2$

Correct Answer: (B) $2x - 3$

Solution:

Step 1: Write the composite:

$g(f(x)) = [f(x)]^2 + f(x) - 2$. The condition says $\frac{1}{2}g(f(x)) = 2x^2 - 5x + 2$, so $g(f(x)) = 4x^2 - 10x + 4$.

Step 2: Form an equation in f :

$f^2 + f - 2 = 4x^2 - 10x + 4$, so $f^2 + f = 4x^2 - 10x + 6$.

Step 3: Test the options:

Try $f = 2x - 3$: $f^2 = 4x^2 - 12x + 9$, so $f^2 + f = 4x^2 - 12x + 9 + 2x - 3 = 4x^2 - 10x + 6$. This matches.

Step 4: Why the other options are wrong.

$f = 2x + 3$ gives $4x^2 + 14x + 12$. The quadratic options for f give a degree 4 expression, which cannot equal a degree 2 polynomial.

Final Answer:

$f(x) = 2x - 3$.

(B) $2x - 3$

Quick Tip: Write $g(f(x)) = f^2 + f - 2$ and compare with the given polynomial.

119. Let the function f be defined by $f(x) = \frac{x-|x|}{x}$, for $x \neq 0$ and $f(0) = 2$ then f is

- (A) continuous nowhere
- (B) continuous for all x except at $x = 0$
- (C) continuous everywhere
- (D) continuous for all x except at $x = 1$

Correct Answer: (B) continuous for all x except at $x = 0$

Solution:

Step 1: Rewrite the function:

For $x > 0$: $|x| = x$, so $f(x) = \frac{x-x}{x} = 0$. For $x < 0$: $|x| = -x$, so $f(x) = \frac{2x}{x} = 2$.

Step 2: Check x not equal to 0:

On each side of 0 the function is a constant (0 or 2), so it is continuous at every $x \neq 0$.

Step 3: Check $x = 0$:

Left limit = 2, right limit = 0. These are not equal, so the limit does not exist at 0, and f is discontinuous at 0 even though $f(0) = 2$.

Step 4: Conclusion:

The function is continuous for all x except $x = 0$. Option B.

Final Answer:

f is continuous everywhere except at 0.

(B) continuous for all x except at $x = 0$

Quick Tip: Check the formula for x greater than 0 and less than 0, then compare limits at 0 with $f(0)$.

120. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ is...

- (A) $\frac{1}{2}$
- (B) $\frac{1}{4}$
- (C) 1
- (D) 2

Correct Answer: (B) $\frac{1}{4}$

Solution:

Step 1: Substitute $x = \tan \theta$:

Let $x = \tan \theta$. Then $\sqrt{1+x^2} = \sec \theta$.

Step 2: Simplify u :

$$u = \tan^{-1} \frac{\sec \theta - 1}{\tan \theta} = \tan^{-1} \frac{1 - \cos \theta}{\sin \theta} = \tan^{-1} \left(\tan \frac{\theta}{2} \right) = \frac{\theta}{2}. \text{ So } u = \frac{1}{2} \tan^{-1} x.$$

Step 3: Simplify v :

$$v = \sin^{-1} \frac{2x}{1+x^2} = \sin^{-1}(\sin 2\theta) = 2\theta = 2 \tan^{-1} x.$$

Step 4: Differentiate:

$$\frac{du}{dx} = \frac{1}{2} \cdot \frac{1}{1+x^2} \text{ and } \frac{dv}{dx} = \frac{2}{1+x^2}.$$

$$\frac{du}{dv} = \frac{1/2}{2} = \frac{1}{4}$$

Step 5: Why the other options are wrong.

1/2 or 2 come from forgetting one of the factors of 2, and 1 would mean u and v change equally.

Final Answer:

The derivative is 1/4.

$$\boxed{(B) \frac{1}{4}}$$

Quick Tip: Substitute $x = \tan \theta$ to simplify both functions to multiples of θ .

121. If the derivative of $\tan^{-1}(a + bx)$ with respect to x at $x = 0$ is 1, then $a^6 - b^3 =$

- (A) $3a^2b - 1$
- (B) $-3ab^2 + 1$
- (C) $-1 - 3a^2b$
- (D) $1 + 3ab^2$

Correct Answer: (C) $-1 - 3a^2b$

Solution:

Step 1: Differentiate:

$$\frac{d}{dx} \tan^{-1}(a + bx) = \frac{b}{1 + (a + bx)^2}.$$

Step 2: Apply the condition at $x = 0$:

$$\frac{b}{1 + a^2} = 1, \text{ so } b = 1 + a^2.$$

Step 3: Compute $a^6 - b^3$:

$$b^3 = (1 + a^2)^3 = 1 + 3a^2 + 3a^4 + a^6. \text{ So } a^6 - b^3 = -1 - 3a^2 - 3a^4.$$

Step 4: Match with the options:

Option C: $-1 - 3a^2b = -1 - 3a^2(1 + a^2) = -1 - 3a^2 - 3a^4$. This matches.

Step 5: Why the other options are wrong.

Options A, B and D carry the wrong sign or wrong power pattern, for example D, $1 + 3ab^2$, is positive and cannot be the negative expression found.

Final Answer:

$$a^6 - b^3 = -1 - 3a^2b.$$

$(C) \quad -1 - 3a^2b$

Quick Tip: Differentiate $\tan^{-1}(a + bx)$, set $x = 0$, and expand $(1 + a^2)^3$.

122. If $x^3 + y^3 = 6$ and $\frac{d^2y}{dx^2} \cdot \frac{d^2x}{dy^2} = \frac{m}{(xy)^n}$, where $m, n \in R$ then $\frac{m}{n} =$

- (A) 36
- (B) 9
- (C) 6
- (D) 4

Correct Answer: (A) 36

Solution:

Step 1: First derivative:

From $x^3 + y^3 = 6$: $3x^2 + 3y^2y' = 0$, so $y' = -\frac{x^2}{y^2}$.

Step 2: Second derivative d^2y/dx^2 :

$$y'' = -\frac{2xy^2 - x^2 \cdot 2yy'}{y^4} = -\frac{2xy^2 + 2x^4/y}{y^4} = -\frac{2x(y^3 + x^3)}{y^5} = -\frac{12x}{y^5},$$

using $x^3 + y^3 = 6$.

Step 3: Second derivative d^2x/dy^2 :

By symmetry of the equation in x and y : $\frac{d^2x}{dy^2} = -\frac{12y}{x^5}$.

Step 4: Multiply:

$$\frac{d^2y}{dx^2} \cdot \frac{d^2x}{dy^2} = \frac{144xy}{x^5y^5} = \frac{144}{(xy)^4}$$

So $m = 144$ and $n = 4$, giving $\frac{m}{n} = 36$. Option A.

Final Answer:

$m/n = 36$.

Quick Tip: Find y double prime and x double prime separately, then multiply.

123. If $x = \sin(t)$, $y = \cos(pt)$, then $(1 - x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} =$

- (A) 0
- (B) $-p^2y$
- (C) p^2y
- (D) p

Correct Answer: (B) $-p^2y$

Solution:

Step 1: First derivative:

$$\frac{dx}{dt} = \cos t, \quad \frac{dy}{dt} = -p \sin(pt). \quad \text{So } \frac{dy}{dx} = \frac{-p \sin(pt)}{\cos t}.$$

Step 2: Rearrange:

$$\sqrt{1 - x^2} \frac{dy}{dx} = -p \sin(pt), \quad \text{since } \cos t = \sqrt{1 - x^2}.$$

Step 3: Differentiate again with respect to x :

$$\sqrt{1 - x^2} y'' - \frac{x}{\sqrt{1 - x^2}} y' = -p^2 \cos(pt) \cdot \frac{dt}{dx} = \frac{-p^2 y}{\sqrt{1 - x^2}} \quad \text{after using } dt/dx = 1/\sqrt{1 - x^2}.$$

Step 4: Multiply through by $\sqrt{1 - x^2}$:

$$(1 - x^2)y'' - xy' = -p^2y$$

Option B.

Final Answer:

The expression equals $-p^2 y$.

$$(B) \quad -p^2 y$$

Quick Tip: Express dy/dx and d^2y/dx^2 using t , then substitute into the given expression.

124. If the function $f(x) = ax^3 + bx^2 + 11x - 6$, defined on $[1, 3]$, satisfies all the conditions of Rolle's theorem for $c = 2 + \frac{1}{\sqrt{3}}$, then

- (A) $a = -1, b = \frac{1}{2}$
- (B) $a = 1, b = -6$
- (C) $a = -1, b = 6$
- (D) $a = -2, b = 1$

Correct Answer: (B) $a = 1, b = -6$

Solution:

Step 1: Condition 1, $f(1) = f(3)$:

$f(1) = a + b + 11 - 6 = a + b + 5$. $f(3) = 27a + 9b + 33 - 6 = 27a + 9b + 27$. Setting them equal gives $26a + 8b + 22 = 0$, so $13a + 4b = -11$.

Step 2: Condition 2, f' prime of c is 0:

$f'(x) = 3ax^2 + 2bx + 11$. $f'(c) = 0$ at $c = 2 + \frac{1}{\sqrt{3}}$, where $c^2 = 4 + \frac{4}{\sqrt{3}} + \frac{1}{3}$.

Step 3: Test the options:

Option B: $a = 1$, $b = -6$. Check $13 - 24 = -11$ which is true. Then

$f'(x) = 3x^2 - 12x + 11$ with roots $x = \frac{12 \pm \sqrt{144 - 132}}{6} = 2 \pm \frac{1}{\sqrt{3}}$. So $c = 2 + \frac{1}{\sqrt{3}}$ works and lies in $(1, 3)$.

Step 4: Why the other options are wrong.

Option A satisfies $13a + 4b = -11$, but $f'(x) = -3x^2 + x + 11$ has roots $\frac{1 \pm \sqrt{133}}{6}$, not $2 \pm \frac{1}{\sqrt{3}}$. Option C gives $13a + 4b = 11$ and option D gives -22 , so both fail $f(1) = f(3)$.

Final Answer:

$a = 1$ and $b = -6$.

$$\boxed{(B) \ a = 1, \ b = -6}$$

Quick Tip: Use $f(1) = f(3)$ and $f'(c) = 0$ for $c = 2 + 1/\sqrt{3}$.

125. The tangent to the curve $y^2 - xy + 9 = 0$ is vertical when

- (A) $y = 0$
- (B) $y = \pm\sqrt{3}$
- (C) $y = \frac{1}{2}$
- (D) $y = \pm 3$

Correct Answer: (D) $y = \pm 3$

Solution:

Step 1: Differentiate implicitly:

$$2yy' - y - xy' = 0, \text{ so } y' = \frac{y}{2y - x}.$$

Step 2: Vertical tangent condition:

The slope is infinite when the denominator is zero: $2y - x = 0$, so $x = 2y$.

Step 3: Use the curve equation:

Substitute $x = 2y$: $y^2 - 2y^2 + 9 = 0$, so $y^2 = 9$ and $y = \pm 3$. Option D.

Step 4: Why the other options are wrong.

At $y = 0$ the curve equation gives $9 = 0$, impossible. The values $\pm\sqrt{3}$ and $\frac{1}{2}$ do not satisfy $y^2 = 9$.

Final Answer:

The tangent is vertical when $y = \text{plus or minus } 3$.

(D) $y = \pm 3$

Quick Tip: A vertical tangent means dy/dx is infinite, so the denominator of dy/dx is zero.

126. The value of k such that the function $f(x) = \sin x - \cos x - kx + b$ is strictly decreasing for all real x is

- (A) $k < 1$
- (B) $k > 1$
- (C) $k < \sqrt{2}$
- (D) $k > \sqrt{2}$

Correct Answer: (D) $k > \sqrt{2}$

Solution:

Step 1: Differentiate:

$$f'(x) = \cos x + \sin x - k.$$

Step 2: Condition for decreasing:

We need $f'(x) < 0$ for every real x , so $\cos x + \sin x < k$ for all x .

Step 3: Find the maximum of $\cos x + \sin x$:

$\cos x + \sin x = \sqrt{2} \sin\left(x + \frac{\pi}{4}\right)$, whose maximum value is $\sqrt{2}$. So we need $k > \sqrt{2}$. Option D.

Step 4: Why the other options are wrong.

$k < 1$, $k > 1$ and $k < \sqrt{2}$ do not guarantee $f' < 0$ everywhere. For example $k = 1.2$ fails at $x = \pi/4$, where $f' = \sqrt{2} - 1.2 > 0$.

Final Answer:

k must be greater than $\sqrt{2}$.

$(D) k > \sqrt{2}$

Quick Tip: f decreasing for all x needs f' negative for all x ; the maximum of $\cos x + \sin x$ is $\sqrt{2}$.

127. $\int \frac{\sin x}{\sin 4x} dx =$

- (A) $\frac{1}{4\sqrt{2}} \log \frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x} + \frac{1}{4} \log |\sec x + \tan x| + c$
- (B) $\frac{1}{4\sqrt{2}} \log \frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x} - \frac{1}{4} \log |\sec x + \tan x| + c$
- (C) $\frac{1}{4\sqrt{2}} \log \frac{1-\sqrt{2}\sin x}{1+\sqrt{2}\sin x} + \frac{1}{4} \log |\sec x + \tan x| + c$
- (D) $\frac{1}{4\sqrt{2}} \log \frac{1-\sqrt{2}\sin x}{1+\sqrt{2}\sin x} - \frac{1}{4} \log |\sec x + \tan x| + c$

Correct Answer: (B) $\frac{1}{4\sqrt{2}} \log \frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x} - \frac{1}{4} \log |\sec x + \tan x| + c$

Solution:

Step 1: Simplify the integrand:

Use $\sin 4x = 2 \sin 2x \cos 2x = 4 \sin x \cos x \cos 2x$. Then

$$\frac{\sin x}{\sin 4x} = \frac{1}{4 \cos x \cos 2x}$$

Step 2: Prepare the substitution:

Multiply the top and bottom by $\cos x$ and write $\cos 2x = 1 - 2 \sin^2 x$:

$$I = \frac{1}{4} \int \frac{\cos x \, dx}{\cos^2 x (1 - 2 \sin^2 x)} = \frac{1}{4} \int \frac{\cos x \, dx}{(1 - \sin^2 x)(1 - 2 \sin^2 x)}$$

Step 3: Substitute:

Let $t = \sin x$, so $dt = \cos x \, dx$:

$$I = \frac{1}{4} \int \frac{dt}{(1 - t^2)(1 - 2t^2)}$$

Step 4: Partial fractions:

Write $\frac{1}{(1-t^2)(1-2t^2)} = \frac{A}{1-t^2} + \frac{B}{1-2t^2}$. Then $A(1-2t^2) + B(1-t^2) = 1$, which gives $A + B = 1$ and $2A + B = 0$. So $A = -1$ and $B = 2$.

$$I = \frac{1}{4} \left[- \int \frac{dt}{1-t^2} + 2 \int \frac{dt}{1-2t^2} \right]$$

Step 5: Integrate:

Use $\int \frac{dt}{1-t^2} = \frac{1}{2} \log \frac{1+t}{1-t}$ and $\int \frac{dt}{1-2t^2} = \frac{1}{2\sqrt{2}} \log \frac{1+\sqrt{2}t}{1-\sqrt{2}t}$. Then

$$I = \frac{1}{4\sqrt{2}} \log \frac{1+\sqrt{2}t}{1-\sqrt{2}t} - \frac{1}{8} \log \frac{1+t}{1-t} + c$$

Step 6: Convert back:

With $t = \sin x$, $\frac{1}{2} \log \frac{1+\sin x}{1-\sin x} = \log |\sec x + \tan x|$. So the second term is $-\frac{1}{4} \log |\sec x + \tan x|$.

Step 7: Match the options:

The result has $+\frac{1}{4\sqrt{2}} \log \frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x}$ and $-\frac{1}{4} \log |\sec x + \tan x|$. Options (C) and (D) have the ratio inverted, which flips the sign of the first term. Option (A) has the wrong sign on the second term. Only (B) matches.

Final Answer:

The integral equals the expression in option (B).

$\frac{1}{4\sqrt{2}} \log \frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x} - \frac{1}{4} \log \sec x + \tan x + c$

Quick Tip: Write $\sin 4x = 4 \sin x \cos x \cos 2x$, put $t = \sin x$ and split into partial fractions.

128. $\int \sqrt{4^x(4^x + 4)} dx = \dots$

(A) $\frac{1}{\log 2} \left[\frac{2^x}{2} \sqrt{4^x + 4} + 2 \log |2^x + \sqrt{4^x + 4}| \right] + c$

(B) $\frac{2^x}{2} \sqrt{4^x + 4} + 2 \log |2^x + \sqrt{4^x + 4}| + c$

(C) $\frac{1}{\log 2} \left[\frac{2^x}{2} \sqrt{4^x + 4} + \log |2^x + \sqrt{4^x + 4}| \right] + c$

(D) $\left[\frac{2^x}{\log 4} \sqrt{4^x + 4} + \log |2^x + \sqrt{4^x + 4}| \right] + c$

Correct Answer: (A) $\frac{1}{\log 2} \left[\frac{2^x}{2} \sqrt{4^x + 4} + 2 \log |2^x + \sqrt{4^x + 4}| \right] + c$

Solution:

Step 1: Simplify:

$$\sqrt{4^x(4^x + 4)} = 2^x \sqrt{4^x + 4}.$$

Step 2: Substitute:

Let $u = 2^x$. Then $du = 2^x \ln 2 dx$, so $2^x dx = \frac{du}{\ln 2}$. Also $4^x = u^2$.

$$I = \frac{1}{\ln 2} \int \sqrt{u^2 + 4} du$$

Step 3: Use the standard result:

$$\int \sqrt{u^2 + a^2} du = \frac{u}{2} \sqrt{u^2 + a^2} + \frac{a^2}{2} \ln |u + \sqrt{u^2 + a^2}|. \text{ With } a = 2:$$
$$\frac{u}{2} \sqrt{u^2 + 4} + 2 \ln |u + \sqrt{u^2 + 4}|.$$

Step 4: Write the final form:

$$I = \frac{1}{\log 2} \left[\frac{2^x}{2} \sqrt{4^x + 4} + 2 \log |2^x + \sqrt{4^x + 4}| \right] + c$$

Option A.

Step 5: Why the other options are wrong.

Option B omits the $1/\ln 2$ factor. Options C and D carry $\ln |\cdot|$ with coefficient 1 instead of $a^2/2 = 2$, and D also has a wrong coefficient for the first term.

Final Answer:

The integral is given by option A.

$$(A) \frac{1}{\log 2} \left[\frac{2^x}{2} \sqrt{4^x + 4} + 2 \log |2^x + \sqrt{4^x + 4}| \right] + c$$

Quick Tip: Write $\sqrt{4^x}$ as 2^x , substitute $u = 2^x$, and use the standard integral of $\sqrt{u^2 + a^2}$.

129. The value of $\int \frac{1}{x^2-5x+8} dx$ is equal to...

- (A) $\frac{2}{\sqrt{7}} \tan^{-1}\left(\frac{2x+5}{\sqrt{7}}\right) + c$
- (B) $-\frac{2}{\sqrt{7}} \tan^{-1}\left(\frac{2x-5}{\sqrt{7}}\right) + c$
- (C) $\frac{2}{\sqrt{7}} \tan^{-1}\left(\frac{2x-5}{\sqrt{7}}\right) + c$
- (D) $-\frac{2}{\sqrt{7}} \tan^{-1}\left(\frac{2x+5}{\sqrt{7}}\right) + c$

Correct Answer: (C) $\frac{2}{\sqrt{7}} \tan^{-1}\left(\frac{2x-5}{\sqrt{7}}\right) + c$

Solution:

Step 1: Complete the square:

$$x^2 - 5x + 8 = \left(x - \frac{5}{2}\right)^2 + 8 - \frac{25}{4} = \left(x - \frac{5}{2}\right)^2 + \frac{7}{4}.$$

Step 2: Use the standard form:

$$\int \frac{dx}{u^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + c \text{ with } u = x - \frac{5}{2} \text{ and } a = \frac{\sqrt{7}}{2}.$$

Step 3: Substitute:

$$I = \frac{2}{\sqrt{7}} \tan^{-1} \left(\frac{x - 5/2}{\sqrt{7}/2} \right) + c = \frac{2}{\sqrt{7}} \tan^{-1} \left(\frac{2x - 5}{\sqrt{7}} \right) + c$$

Step 4: Why the other options are wrong.

Options A and D have $2x + 5$, which comes from a sign error in completing the square. Options B and D have a negative sign in front, but the integrand is positive, so the antiderivative must increase.

Final Answer:

The integral is $(2/\sqrt{7}) \tan^{-1}((2x - 5)/\sqrt{7}) + c$.

$$(C) \frac{2}{\sqrt{7}} \tan^{-1} \left(\frac{2x - 5}{\sqrt{7}} \right) + c$$

Quick Tip: Complete the square in the denominator and use the standard arctangent integral.

130. $\int \frac{2x}{4-3x-x^2} dx = \dots$

- (A) $\frac{8}{5} \log|4 + x| - \frac{2}{5} \log|1 - x| + c$
 (B) $2 \log|4 + x| + \frac{2}{5} \log|1 - x| + c$
 (C) $-\log|4 + x| + \frac{2}{5} \log|1 - x| + c$
 (D) $-\frac{8}{5} \log|4 + x| - \frac{2}{5} \log|1 - x| + c$

Correct Answer: (D) $-\frac{8}{5} \log|4 + x| - \frac{2}{5} \log|1 - x| + c$

Solution:

Step 1: Factor the denominator:

$$4 - 3x - x^2 = -(x^2 + 3x - 4) = -(x + 4)(x - 1) = (1 - x)(4 + x).$$

Step 2: Partial fractions:

$$\frac{2x}{(1 - x)(4 + x)} = \frac{A}{1 - x} + \frac{B}{4 + x}, \text{ so } 2x = A(4 + x) + B(1 - x). \text{ Put } x = 1: 2 = 5A, \text{ so } A = \frac{2}{5}. \text{ Put } x = -4: -8 = 5B, \text{ so } B = -\frac{8}{5}.$$

Step 3: Integrate:

$$\int \frac{A}{1 - x} dx = -A \ln |1 - x| \text{ and } \int \frac{B}{4 + x} dx = B \ln |4 + x|.$$

$$I = -\frac{2}{5} \ln |1 - x| - \frac{8}{5} \ln |4 + x| + c$$

Step 4: Why the other options are wrong.

Options A, B and C have wrong coefficients or a wrong sign on the $\ln |1 - x|$ term, which comes from forgetting the minus sign when integrating $\frac{1}{1 - x}$.

Final Answer:

The integral is $-(8/5) \log|4 + x| - (2/5) \log|1 - x| + c$.

$$(D) -\frac{8}{5}\log|4+x| - \frac{2}{5}\log|1-x| + c$$

Quick Tip: Factor the denominator as $(1-x)(4+x)$ and use partial fractions.

131. If $\int_2^e \left[\frac{1}{\log x} - \frac{1}{(\log x)^2} \right] dx = a + \frac{b}{\log 2}$, then values of a and b are

- (A) $a = e, b = 2$
- (B) $a = -e, b = 2$
- (C) $a = e, b = -2$
- (D) $a = -e, b = -2$

Correct Answer: (C) $a = e, b = -2$

Solution:

Step 1: Spot the antiderivative:

Consider $\frac{d}{dx} \left(\frac{x}{\ln x} \right) = \frac{\ln x \cdot 1 - x \cdot \frac{1}{x}}{(\ln x)^2} = \frac{\ln x - 1}{(\ln x)^2} = \frac{1}{\ln x} - \frac{1}{(\ln x)^2}$.

So the integrand is exactly this derivative.

Step 2: Apply the limits:

$$\left[\frac{x}{\ln x} \right]_2^e = \frac{e}{\ln e} - \frac{2}{\ln 2} = e - \frac{2}{\ln 2}$$

Step 3: Compare with $a + b/\log 2$:

So $a = e$ and $b = -2$. Option C.

Step 4: Why the other options are wrong.

Options A and B give $b = +2$, and options B and D give $a = -e$. The upper limit gives a positive e and the lower limit gives $-\frac{2}{\ln 2}$.

Final Answer:

$a = e$ and $b = -2$.

$$\boxed{(C) \ a = e, \ b = -2}$$

Quick Tip: The integrand is the derivative of $x / \ln x$.

132. If $\int_0^k \frac{1}{1+4^x} dx = \log_2\left(\frac{4}{3}\right)$, then $k =$ _____

- (A) $\frac{2}{\sqrt{3}}$
- (B) $\frac{3}{2}$
- (C) $\frac{\sqrt{3}}{2}$
- (D) $\frac{2}{3}$

Correct Answer: (B) $\frac{3}{2}$

Solution:

Step 1: Rewrite the integrand:

$$\frac{1}{1+4^x} = 1 - \frac{4^x}{1+4^x}.$$

Step 2: Integrate:

$$\int_0^k \frac{4^x}{1+4^x} dx = \frac{1}{\ln 4} [\ln(1+4^x)]_0^k = \frac{1}{\ln 4} \ln \frac{1+4^k}{2}. \text{ So } \int_0^k \frac{dx}{1+4^x} = k - \log_4 \frac{1+4^k}{2}.$$

Step 3: Set it equal to log base 2 of 4/3:

$$\log_2 \frac{4}{3} = \log_4 \frac{16}{9}. \text{ So } k - \log_4 \frac{1 + 4^k}{2} = \log_4 \frac{16}{9}, \text{ which gives}$$
$$\log_4 \frac{4^k \cdot 2}{1 + 4^k} = \log_4 \frac{16}{9}.$$

Step 4: Solve:

$$\frac{2 \cdot 4^k}{1 + 4^k} = \frac{16}{9}. \text{ Cross-multiply: } 18 \cdot 4^k = 16 + 16 \cdot 4^k, \text{ so } 2 \cdot 4^k = 16, 4^k$$
$$= 8, \text{ and } k = \frac{3}{2}. \text{ Option B.}$$

Step 5: Why the other options are wrong.

For $k = \frac{2}{\sqrt{3}}, \frac{\sqrt{3}}{2}, \frac{2}{3}, 4^k$ is not 8, so the equation fails.

Final Answer:

$$k = 3/2.$$

$$\boxed{\text{(B)} \frac{3}{2}}$$

Quick Tip: Write $1/(1+4^x) = 1 - 4^x/(1+4^x)$ and integrate.

133. The area bounded by the curve $y = x \sin(\frac{x}{2})$ and the X-axis between the lines $x = 0$ and $x = 4\pi$ (in square units), is....

- (A) 4π
- (B) 8π
- (C) 12π
- (D) 16π

Correct Answer: (D) 16π

Solution:

Step 1: Understanding the Question:

The area between the curve and the X-axis must count regions above and below the axis as positive. So we integrate $|x \sin(x/2)|$ from 0 to 4π .

Step 2: Find where the sign changes:

For $0 \leq x \leq 2\pi$, $x/2$ lies in $[0, \pi]$, so $\sin(x/2) \geq 0$. For $2\pi \leq x \leq 4\pi$, $x/2$ lies in $[\pi, 2\pi]$, so $\sin(x/2) \leq 0$. Hence

$$A = \int_0^{2\pi} x \sin \frac{x}{2} dx - \int_{2\pi}^{4\pi} x \sin \frac{x}{2} dx$$

Step 3: Substitute:

Put $t = x/2$, so $x = 2t$ and $dx = 2 dt$. The limits become 0 to 2π , and $x \sin(x/2) dx = 4t \sin t dt$. So

$$A = 4 \int_0^{2\pi} t |\sin t| dt$$

Step 4: Integration by parts:

$\int t \sin t dt = -t \cos t + \sin t$. On $[0, \pi]$: $[-t \cos t + \sin t]_0^\pi = \pi$. On $[\pi, 2\pi]$ we need $-\int t \sin t dt = [t \cos t - \sin t]_\pi^{2\pi} = 2\pi - (-\pi) = 3\pi$.

Step 5: Add up:

$$A = 4(\pi + 3\pi) = 16\pi$$

Step 6: Why the other options are wrong:

If the sign change is ignored, the integral is $4[\pi - 3\pi] = -8\pi$, whose size 8π is option (B). That counts the lower loop as negative area, which is wrong for an area. Options (A) and (C) do not come from either calculation.

Final Answer:

The area is 16π square units, option (D).

$$16\pi$$

Quick Tip: Split the integral where $\sin(x/2)$ changes sign at $x = 2\pi$ and take the positive area of each loop.

134. If the area enclosed between the curves $y^2 = 4kx$ and $y = kx, k > 0$ is $\frac{2k}{3}$ sq. units then $k =$

- (A) 1
- (B) 1.5
- (C) 2
- (D) 3

Correct Answer: (C) 2

Solution:

Step 1: Find the intersection:

$$k^2x^2 = 4kx \text{ gives } x = 0 \text{ or } x = \frac{4}{k}.$$

Step 2: Set up the area:

Between these limits the parabola $y = 2\sqrt{kx}$ lies above the line $y = kx$.

$$A = \int_0^{4/k} (2\sqrt{kx} - kx) dx = \left[\frac{4}{3} \sqrt{k} x^{3/2} - \frac{kx^2}{2} \right]_0^{4/k}$$

Step 3: Evaluate:

$$\frac{4}{3} \sqrt{k} \cdot \frac{8}{k^{3/2}} - \frac{k}{2} \cdot \frac{16}{k^2} = \frac{32}{3k} - \frac{8}{k} = \frac{8}{3k}.$$

Step 4: Solve for k:

$$\frac{8}{3k} = \frac{2k}{3}, \text{ so } k^2 = 4 \text{ and } k = 2 \text{ (since } k > 0\text{)}. \text{ Option C.}$$

Final Answer:

$$k = 2.$$

(C) 2

Quick Tip: Find the intersection points and integrate the difference of the curves.

135. The solution of the differential equation $2x \frac{dy}{dx} - y = 3$ represents a family of

- (A) straight lines
- (B) circles
- (C) parabola
- (D) ellipse

Correct Answer: (C) parabola

Solution:

Step 1: Separate variables:

$$2x \frac{dy}{dx} = y + 3, \text{ so } \frac{dy}{y + 3} = \frac{dx}{2x}.$$

Step 2: Integrate:

$$\ln |y + 3| = \frac{1}{2} \ln |x| + \ln C, \text{ so } \ln |y + 3| = \ln(C\sqrt{|x|}).$$

Step 3: Simplify:

Squaring both sides: $(y + 3)^2 = C'x$. This is of the form $Y^2 = 4aX$, a parabola.

Step 4: Why the other options are wrong.

A straight line would be of degree 1 in x and y . A circle or ellipse needs squares of both x and y , but here only y is squared.

Final Answer:

The solution is a family of parabolas.

(C) parabola

Quick Tip: Separate the variables, integrate, and identify the curve.

136. The solution of the differential equation $x \frac{dy}{dx} + y = x^3 y^6$, is...

(A) $y^5 = \frac{5}{2}x^{-3} + cx^5$

(B) $y^5 = \frac{5}{2}x^3 + cx^5$

(C) $\frac{1}{y^5} = \frac{5}{2}x^{-3} + cx^5$

(D) $\frac{1}{y^5} = \frac{5}{2}x^3 + cx^5$

Correct Answer: (D) $\frac{1}{y^5} = \frac{5}{2}x^3 + cx^5$

Solution:

Step 1: Rewrite in standard form:

Divide by x : $\frac{dy}{dx} + \frac{y}{x} = x^2 y^6$. This is a Bernoulli equation with $n = 6$.

Step 2: Substitute:

Divide by y^6 : $y^{-6}y' + \frac{1}{x}y^{-5} = x^2$. Let $v = y^{-5}$, so $v' = -5y^{-6}y'$. Then $-\frac{v'}{5} + \frac{v}{x} = x^2$, that is $v' - \frac{5}{x}v = -5x^2$.

Step 3: Solve the linear equation:

Integrating factor: $e^{\int -5/x dx} = x^{-5}$. Then $\frac{d}{dx}(vx^{-5}) = -5x^{-3}$, so $vx^{-5} = \frac{5}{2}x^{-2} + c$.

Step 4: Return to y :

$v = \frac{5}{2}x^3 + cx^5$, so $\frac{1}{y^5} = \frac{5}{2}x^3 + cx^5$. Option D.

Step 5: Why the other options are wrong.

Options A and B give y^5 instead of $1/y^5$ because they forgot that $v = y^{-5}$. Option C has x^{-3} from a sign slip in the integrating factor.

Final Answer:

The solution is $1/y^5 = (5/2)x^3 + cx^5$.

$$(D) \frac{1}{y^5} = \frac{5}{2}x^3 + cx^5$$

Quick Tip: This is a Bernoulli equation; substitute $v = 1/y^5$.

137. If $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = x\hat{i} + (x - 2)\hat{j} - \hat{k}$ are three vectors in which $\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$, then $x = \dots$

- (A) 0
- (B) 1
- (C) -4
- (D) -2

Correct Answer: (D) -2

Solution:

Step 1: Coplanarity condition:

$\vec{c}$ lies in the plane of $\vec{a}$ and $\vec{b}$ when $[\vec{a} \ \vec{b} \ \vec{c}] = 0$.

Step 2: Form the determinant:

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & x-2 & -1 \end{vmatrix} = 0$$

Step 3: Expand:

$$1[(-1)(-1) - 2(x - 2)] - 1[(1)(-1) - 2x] + 1[(x - 2) - (-1)x]$$

$$= (1 - 2x + 4) - (-1 - 2x) + (2x - 2) = 5 - 2x + 1 + 2x + 2x - 2 = 4 + 2x.$$

Step 4: Solve:

$4 + 2x = 0$ gives $x = -2$. Option D.

Final Answer:

$x = -2$.

(D) - 2

Quick Tip: Three vectors are coplanar when their scalar triple product is zero.

138. If $|\vec{a}| = 4$, $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 8$ then $[\vec{a}\vec{a} + \vec{b}\vec{a} \times \vec{b}] =$

- (A) 96
- (B) 80
- (C) 64
- (D) 120

Correct Answer: (B) 80

Solution:

Step 1: Expand the triple product:

$$[\vec{a}, \vec{a} + \vec{b}, \vec{a} \times \vec{b}] = \vec{a} \cdot [(\vec{a} + \vec{b}) \times (\vec{a} \times \vec{b})].$$

Step 2: Split the cross product:

$(\vec{a} + \vec{b}) \times (\vec{a} \times \vec{b}) = \vec{a} \times (\vec{a} \times \vec{b}) + \vec{b} \times (\vec{a} \times \vec{b})$. The first term is perpendicular to $\vec{a}$, so its dot with $\vec{a}$ is 0.

Step 3: Use the triple product identity:

$$\vec{b} \times (\vec{a} \times \vec{b}) = \vec{a}(\vec{b} \cdot \vec{b}) - \vec{b}(\vec{b} \cdot \vec{a}). \text{ So } \vec{a} \cdot [\dots] = |\vec{a}|^2 |\vec{b}|^2 - (\vec{a} \cdot \vec{b})^2.$$

Step 4: Substitute:

$$= 16 \cdot 9 - 8^2 = 144 - 64 = 80. \text{ Option B.}$$

Final Answer:

The value is 80.

(B) 80

Quick Tip: Expand the triple product and use the vector triple product identity.

139. Let $\vec{a}, \vec{b}, \vec{c}$ be the unit vectors such that $\vec{a}$ is perpendicular to $\vec{b}$ and the angle between $\vec{b}$ and $\vec{c}$ is 120° . If $\vec{a} + \vec{c}$ is perpendicular to $\vec{b} + \vec{c}$ then

- (A) $(\vec{a} + \vec{c}) \cdot (\vec{b} - \vec{c}) = 1$
- (B) $(\vec{a} - \vec{c}) \cdot (\vec{b} - \vec{c}) = -2$
- (C) $(\vec{a} - \vec{c}) \cdot (\vec{b} + \vec{c}) = 1$
- (D) $(\vec{a} - \vec{c}) \cdot (\vec{b} - \vec{c}) = 2$

Correct Answer: (D) $(\vec{a} - \vec{c}) \cdot (\vec{b} - \vec{c}) = 2$

Solution:

Step 1: List the dot products:

All three vectors are unit vectors. $\vec{a} \cdot \vec{b} = 0$ and $\vec{b} \cdot \vec{c} = \cos 120^\circ = -\frac{1}{2}$.

Step 2: Use the perpendicular condition:

$(\vec{a} + \vec{c}) \cdot (\vec{b} + \vec{c}) = 0$ gives $\vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} + \vec{c} \cdot \vec{b} + |\vec{c}|^2 = 0$. So $0 + \vec{a} \cdot \vec{c} - \frac{1}{2} + 1 = 0$, and $\vec{a} \cdot \vec{c} = -\frac{1}{2}$.

Step 3: Test the options:

$(\vec{a} - \vec{c}) \cdot (\vec{b} - \vec{c}) = \vec{a} \cdot \vec{b} - \vec{a} \cdot \vec{c} - \vec{c} \cdot \vec{b} + |\vec{c}|^2 = 0 + \frac{1}{2} + \frac{1}{2} + 1 = 2$. So D is true.

Step 4: Why the other options are wrong.

(A): $(\vec{a} + \vec{c}) \cdot (\vec{b} - \vec{c}) = 0 + (-\frac{1}{2}) + (-\frac{1}{2}) - 1 = -2$, not 1. (B): the value is 2, not -2. (C): $(\vec{a} - \vec{c}) \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c} - \vec{c} \cdot \vec{b} - 1 = 0 - \frac{1}{2} + \frac{1}{2} - 1 = -1$, not 1.

Final Answer:

The value $(\vec{a} - \vec{c}) \cdot (\vec{b} - \vec{c})$ equals 2.

$$\boxed{(D) (\vec{a} - \vec{c}) \cdot (\vec{b} - \vec{c}) = 2}$$

Quick Tip: Use $\vec{a} \perp \vec{b}$, $\vec{b} \cdot \vec{c} = -1/2$ and $(\vec{a} + \vec{c}) \perp (\vec{b} + \vec{c})$.

140. Let x_0 be the point of local maxima of $f(x) = \vec{a} \cdot (\vec{b} \times \vec{c})$ where $\vec{a} = x\hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = -2\hat{i} + x\hat{j} - \hat{k}$ and $\vec{c} = 7\hat{i} - 2\hat{j} + x\hat{k}$ then the value of $\vec{a} \cdot \vec{c}$ at $x = x_0$ is

- (A) 26
- (B) 0
- (C) -15
- (D) -26

Correct Answer: (D) -26

Solution:

Step 1: Write $f(x)$ as a determinant:

$$f(x) = \vec{a} \cdot (\vec{b} \times \vec{c}) = \begin{vmatrix} x & -2 & 3 \\ -2 & x & -1 \\ 7 & -2 & x \end{vmatrix}.$$

Step 2: Expand:

$$f(x) = x(x^2 - 2) + 2(-2x + 7) + 3(4 - 7x) = x^3 - 2x - 4x + 14 + 12 - 21x = x^3 - 27x + 26.$$

Step 3: Find the local maximum:

$f'(x) = 3x^2 - 27 = 0$, so $x = \pm 3$. $f''(x) = 6x$ is negative at $x = -3$, so $x_0 = -3$ is the point of local maximum.

Step 4: Compute $\vec{a} \cdot \vec{c}$ at $x = -3$:

$$\vec{a} = -3\hat{i} - 2\hat{j} + 3\hat{k}, \vec{c} = 7\hat{i} - 2\hat{j} - 3\hat{k}.$$

$$\vec{a} \cdot \vec{c} = -21 + 4 - 9 = -26$$

Option D.

Step 5: Why the other options are wrong.

Using $x = 3$ (a local minimum) gives $\vec{a} = (3, -2, 3)$, $\vec{c} = (7, -2, 3)$ and $\vec{a} \cdot \vec{c} = 21 + 4 + 9 = 34$, which is not an option. The values 26, 0 and -15

come from sign or arithmetic slips.

Final Answer:

The value of a . c is -26.

(D) - 26

Quick Tip: Expand the determinant to get a cubic in x, find its local maximum, then compute a dot c.

141. The parallelopiped is determined by vectors $\vec{a} = -2\hat{i} + 5\hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} + 3\hat{j} - 2\hat{k}$, $\vec{c} = -3\hat{i} + \hat{j} + 4\hat{k}$. The altitude of parallelopiped on the parallelogram base determined by vectors $\vec{b}$ and $\vec{c}$ is

- (A) $\frac{2\sqrt{3}}{5}$
- (B) $\frac{2}{5}$
- (C) 12
- (D) $\frac{5\sqrt{3}}{2}$

Correct Answer: (A) $\frac{2\sqrt{3}}{5}$

Solution:

Step 1: Key idea:

Volume = $|\vec{a} \cdot (\vec{b} \times \vec{c})|$, base area = $|\vec{b} \times \vec{c}|$, and altitude $h = \frac{\text{volume}}{\text{base area}}$.

Step 2: Find b x c:

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -2 \\ -3 & 1 & 4 \end{vmatrix} = \hat{i}(12 + 2) - \hat{j}(4 - 6) + \hat{k}(1 + 9) = 14\hat{i} + 2\hat{j} + 10\hat{k}.$$

Step 3: Volume and base area:

$$\vec{a} \cdot (\vec{b} \times \vec{c}) = -28 + 10 + 30 = 12. |\vec{b} \times \vec{c}| = \sqrt{196 + 4 + 100} = \sqrt{300} = 10\sqrt{3}.$$

Step 4: Altitude:

$$h = \frac{12}{10\sqrt{3}} = \frac{6}{5\sqrt{3}} = \frac{2\sqrt{3}}{5}$$

Option A.

Step 5: Why the other options are wrong.

12 is just the volume. $2/5$ and $5\sqrt{3}/2$ come from errors in the area of the base.

Final Answer:

The altitude is $2\sqrt{3}/5$.

$(A) \frac{2\sqrt{3}}{5}$

Quick Tip: Altitude = volume divided by base area, using the triple product and the cross product.

142. The vector equation of the plane passing through the point $A(-2, 7, 5)$ and parallel to the vectors $4\hat{i} - \hat{j} + 3\hat{k}$ and $\hat{i} + \hat{j} + \hat{k}$ is ...

(A) $\vec{r} \cdot (-2\hat{i} + 7\hat{j} + 5\hat{k}) = 26$

(B) $\vec{r} \cdot (4\hat{i} - \hat{j} + 3\hat{k}) = 27$

(C) $\vec{r} \cdot (4\hat{i} - \hat{j} + 5\hat{k}) = 26$

(D) $\vec{r} \cdot (-4\hat{i} - \hat{j} + 5\hat{k}) = 26$

Correct Answer: (D) $\vec{r} \cdot (-4\hat{i} - \hat{j} + 5\hat{k}) = 26$

Solution:

Step 1: Find the normal:

$$\begin{aligned}\vec{n} &= (4\hat{i} - \hat{j} + 3\hat{k}) \times (\hat{i} + \hat{j} + \hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 3 \\ 1 & 1 & 1 \end{vmatrix} \\ &= \hat{i}(-1 - 3) - \hat{j}(4 - 3) + \hat{k}(4 + 1) = -4\hat{i} - \hat{j} + 5\hat{k}.\end{aligned}$$

Step 2: Compute $\vec{a} \cdot \vec{n}$:

Position vector of A is $\vec{a} = -2\hat{i} + 7\hat{j} + 5\hat{k}$. $\vec{a} \cdot \vec{n} = (-2)(-4) + 7(-1) + 5(5) = 8 - 7 + 25 = 26$.

Step 3: Write the equation:

$\vec{r} \cdot (-4\hat{i} - \hat{j} + 5\hat{k}) = 26$. Option D.

Step 4: Why the other options are wrong.

Option A uses the position vector of A as the normal. Option B uses one of the parallel vectors as the normal. Option C has the right form but the wrong x-component of the normal.

Final Answer:

The plane is $\vec{r} \cdot (-4\hat{i} - \hat{j} + 5\hat{k}) = 26$.

(D) $\vec{r} \cdot (-4\hat{i} - \hat{j} + 5\hat{k}) = 26$

Quick Tip: The normal is the cross product of the two vectors; then use $\mathbf{r} \cdot \mathbf{n} = a \cdot \mathbf{n}$.

143. The point of intersection of the two lines $\frac{x-3}{3} = \frac{y-3}{-1}, z-1=0$ and $\frac{x-6}{2} = \frac{z-1}{3}, y-2=0$ is....

- (A) (0, 0, 0)
- (B) (1, 2, 6)
- (C) (3, -1, 0)
- (D) (6, 2, 1)

Correct Answer: (D) (6, 2, 1)

Solution:

Step 1: Use the given plane conditions:

Line 1 has $z - 1 = 0$, so every point on it has $z = 1$. Line 2 has $y - 2 = 0$, so every point on it has $y = 2$.

Step 2: Find x:

For line 2, $\frac{x-6}{2} = \frac{z-1}{3}$. With $z = 1$ this gives $\frac{x-6}{2} = 0$, so $x = 6$.

Step 3: Check the point (6, 2, 1) on line 1:

$\frac{x-3}{3} = \frac{6-3}{3} = 1$ and $\frac{y-3}{-1} = \frac{2-3}{-1} = 1$. The two ratios agree, and $z = 1$. So (6, 2, 1) is on both lines.

Step 4: Why the other options are wrong.

(0, 0, 0), (1, 2, 6) and (3, -1, 0) do not have $z = 1$ together with $y = 2$, so they cannot lie on both lines.

Final Answer:

The lines meet at (6, 2, 1).

(D) (6, 2, 1)

Quick Tip: Use the fixed plane coordinates $z = 1$ and $y = 2$ to fix the point.

144. If the centroid of a tetrahedron $OABC$ is $(1, 2, -1)$, where O is the origin, $A(a, 2, 3)$, $B(1, b, 2)$, $C(2, 1, c)$ are the other vertices, then the distance of the point $P(a, b, c)$ from the origin is...

- (A) 42 units
- (B) $\sqrt{107}$ units
- (C) 25 units
- (D) 15 units

Correct Answer: (B) $\sqrt{107}$ units

Solution:**Step 1: Centroid formula:**

For a tetrahedron with vertices $O(0, 0, 0)$, $A(a, 2, 3)$, $B(1, b, 2)$, $C(2, 1, c)$, the centroid is $\left(\frac{a+1+2}{4}, \frac{2+b+1}{4}, \frac{3+2+c}{4}\right)$.

Step 2: Equate with (1, 2, -1):

$$\frac{a+3}{4} = 1 \text{ gives } a = 1. \quad \frac{b+3}{4} = 2 \text{ gives } b = 5. \quad \frac{c+5}{4} = -1 \text{ gives } c = -9.$$

Step 3: Distance of $P(1, 5, -9)$ from the origin:

$$OP = \sqrt{1 + 25 + 81} = \sqrt{107}. \text{ Option B.}$$

Step 4: Why the other options are wrong.

42, 25 and 15 are not square roots of 107. They arise from adding squares without taking the square root or from wrong values of c .

Final Answer:

The distance is $\sqrt{107}$ units.

(B) $\sqrt{107}$ units

Quick Tip: The centroid is the mean of the four vertices; solve for a, b, c .



145. If the lines $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{4}$ and $\frac{x-3}{1} = \frac{y-c}{2} = \frac{z}{1}$ intersect, then the radius of circle $x^2 + y^2 - 4x + 10y + c = 0$ is

- (A) $\frac{9}{\sqrt{2}}$
- (B) $\frac{9}{2}$
- (C) $\frac{7}{\sqrt{2}}$
- (D) $\frac{7}{2}$

Correct Answer: (C) $\frac{7}{\sqrt{2}}$

Solution:

Step 1: Parametrise the lines:

Line 1: $(1 + 2s, -1 + 3s, 1 + 4s)$. Line 2: $(3 + t, c + 2t, t)$.

Step 2: Equate x and z :

$1 + 2s = 3 + t$ and $1 + 4s = t$. Substitute $t = 1 + 4s$: $1 + 2s = 4 + 4s$, so $s = -\frac{3}{2}$ and $t = 1 - 6 = -5$.

Step 3: Equate y to find c:

$$-1 + 3s = -1 - \frac{9}{2} = -\frac{11}{2} \text{ and } c + 2t = c - 10. \text{ So } c - 10 = -\frac{11}{2}, \text{ giving } c = \frac{9}{2}.$$

Step 4: Find the radius:

$$\text{For } x^2 + y^2 - 4x + 10y + c = 0: g = -2, f = 5. r = \sqrt{g^2 + f^2 - c} \\ = \sqrt{4 + 25 - \frac{9}{2}} = \sqrt{\frac{49}{2}} = \frac{7}{\sqrt{2}}. \text{ Option C.}$$

Step 5: Why the other options are wrong.

$\frac{9}{\sqrt{2}}$, $\frac{9}{2}$ and $\frac{7}{2}$ would need different values of c . They come from a wrong sign of c (using $-\frac{9}{2}$) or from forgetting the square root in r .

Final Answer:

The radius is $7 / \text{sqrt } 2$.

$$\boxed{(C) \frac{7}{\sqrt{2}}}$$

Quick Tip: Make the two lines meet to find c , then find the radius from c .

146. The corner points of the feasible region determined by a system of linear constraints are $(0, 3)$, $(1, 1)$ and $(3, 0)$. If the objective function is $z = px + qy$, where $p, q > 0$, then the condition on p and q such that the minimum of z occurs at both $(3, 0)$ and $(1, 1)$ is _____

- (A) $p = 3q$
- (B) $3p = q$
- (C) $p = \frac{q}{2}$
- (D) $p = 2q$

Correct Answer: (C) $p = \frac{q}{2}$

Solution:

Step 1: Understanding the Concept:

When the minimum of $z = px + qy$ occurs at two corner points, z has the same value at both (and at every point on the segment joining them).

Step 2: Equate z values:

At $(3, 0)$: $z = 3p$. At $(1, 1)$: $z = p + q$. Set them equal: $3p = p + q$, so $q = 2p$, or $p = \frac{q}{2}$.

Step 3: Check the third corner:

At $(0, 3)$: $z = 3q = 6p$, which is larger than $3p$ for $p > 0$. So the minimum is indeed $3p$, attained at both corners.

Step 4: Why the other options are wrong.

$p = 3q$, $3p = q$ and $p = 2q$ give different values of z at the two corners, so the minimum is at one of them only (or at $(0, 3)$).

Final Answer:

$$p = q/2.$$

$(C) \ p = \frac{q}{2}$

Quick Tip: If the minimum is at two corners, the objective function has equal value there.



147. If $X \sim B(n, p)$, then the value of $\frac{P(X=k)}{P(X=k-1)}$ is

- (A) $\left(\frac{n-k}{k}\right)\frac{p}{q}$
- (B) $\left(\frac{n-k+1}{k}\right)\frac{p}{q}$
- (C) $\left(\frac{n-k}{k-1}\right)\frac{p}{q}$
- (D) $\left(\frac{n-k+1}{k-1}\right)\frac{p}{q}$

Correct Answer: (B) $\left(\frac{n-k+1}{k}\right)\frac{p}{q}$

Solution:

Step 1: Write both probabilities:

$$P(X = k) = \binom{n}{k} p^k q^{n-k} \text{ and } P(X = k - 1) = \binom{n}{k-1} p^{k-1} q^{n-k+1}.$$

Step 2: Divide:

$$\frac{P(X = k)}{P(X = k - 1)} = \frac{\binom{n}{k}}{\binom{n}{k-1}} \cdot \frac{p}{q}.$$

Step 3: Use the ratio of binomial coefficients:

$$\frac{\binom{n}{k}}{\binom{n}{k-1}} = \frac{(k-1)!(n-k+1)!}{k!(n-k)!} = \frac{n-k+1}{k}.$$

Step 4: Result:

$$\frac{P(X = k)}{P(X = k - 1)} = \frac{n - k + 1}{k} \cdot \frac{p}{q}$$

Option B.

Final Answer:

The ratio is $((n - k + 1)/k)(p/q)$.

$$(B) \left(\frac{n - k + 1}{k} \right) \frac{p}{q}$$

Quick Tip: Write both binomial probabilities and divide; the powers of p and q partly cancel.

148. A player tosses two fair coins, he wins Rs. 5 if two heads appear, Rs. 3 if one head appears and Rs. 2 if no head appears, then the variance of the winning amount is

- (A) 1.1875
- (B) 1.8175
- (C) 1.1785
- (D) 1.8157

Correct Answer: (A) 1.1875

Solution:

Step 1: Probability of each outcome:

Two fair coins: $P(2 \text{ heads}) = \frac{1}{4}$, $P(1 \text{ head}) = \frac{1}{2}$, $P(\text{no head}) = \frac{1}{4}$.

Step 2: Winning amount X:

$X = 5$ with probability $\frac{1}{4}$, $X = 3$ with probability $\frac{1}{2}$, $X = 2$ with

probability $\frac{1}{4}$.

Step 3: Mean and mean of squares:

$$E(X) = \frac{5}{4} + \frac{3}{2} + \frac{2}{4} = 3.25. E(X^2) = \frac{25}{4} + \frac{9}{2} + \frac{4}{4} = 11.75.$$

Step 4: Variance:

$$\text{Var}(X) = 11.75 - (3.25)^2 = 11.75 - 10.5625 = 1.1875. \text{ Option A.}$$

Step 5: Why the other options are wrong.

1.8175, 1.1785 and 1.8157 are digit-shuffled values that come from small arithmetic slips.

Final Answer:

The variance is 1.1875.

(A) 1.1875

Quick Tip: Find the distribution of the winnings, then use $\text{Var} = E(X^2) - (E X)^2$.

149. The probability mass function of a random variable X is given by $P(X = x) = \frac{{}^5C_x}{2^5}$, for $x = 0, 1, 2, 3, 4, 5$ and $= 0$, otherwise. Then which of the following is correct?

- (A) $p(x \leq 2) = p(x \geq 3)$
- (B) $p(x \leq 2) > p(x \geq 3)$
- (C) $p(x \leq 2) < p(x \geq 3)$
- (D) $p(x \leq 2) = 2p(x \geq 3)$

Correct Answer: (A) $p(x \leq 2) = p(x \geq 3)$

Solution:

Step 1: Identify the distribution:

$P(X = x) = \frac{\binom{5}{x}}{2^5}$ is the binomial pmf with $n = 5$, $p = \frac{1}{2}$.

Step 2: Compute the two probabilities:

$$P(X \leq 2) = \frac{1 + 5 + 10}{32} = \frac{16}{32}, P(X \geq 3) = \frac{10 + 5 + 1}{32} = \frac{16}{32}.$$

Step 3: Compare:

Both equal $\frac{1}{2}$. So $P(X \leq 2) = P(X \geq 3)$. Option A.

Step 4: Why the other options are wrong.

B and C claim one side is larger, and D claims one side is double. All three contradict the equal values $\frac{1}{2}$ and $\frac{1}{2}$.

Final Answer:

$$p(x \leq 2) = p(x \geq 3).$$

$(A) p(x \leq 2) = p(x \geq 3)$

Quick Tip: Here X is binomial with $n = 5$ and $p = 1/2$, so the pmf is symmetric.

150. If events A and B are such that $P(A) = \frac{1}{4}$, $P(A \cap B) = \frac{1}{10}$ and $P(A/B) = 2P(B/A)$ then $P(B) = \dots\dots\dots$

- (A) $\frac{1}{2}$
- (B) $\frac{1}{5}$
- (C) $\frac{1}{8}$
- (D) $\frac{2}{5}$

Correct Answer: (C) $\frac{1}{8}$

Solution:

Step 1: Write the conditional probabilities:

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \text{ and } P(B/A) = \frac{P(A \cap B)}{P(A)}.$$

Step 2: Use the given relation:

$$\frac{P(A \cap B)}{P(B)} = 2 \cdot \frac{P(A \cap B)}{P(A)}. \text{ Since } P(A \cap B) = \frac{1}{10} \neq 0, \text{ cancel it: } \frac{1}{P(B)} = \frac{2}{P(A)}, \text{ so } P(A) = 2P(B).$$

Step 3: Substitute P(A):

$$P(B) = \frac{P(A)}{2} = \frac{1}{8}. \text{ Option C.}$$

Step 4: Why the other options are wrong.

The relation forces $P(B) = \frac{P(A)}{2} = \frac{1}{8}$, so $\frac{1}{2}$, $\frac{1}{5}$ and $\frac{2}{5}$ cannot hold. As a check, $P(A \cap B) = \frac{1}{10} \leq \frac{1}{8}$, which is consistent.

Final Answer:

$$P(B) = 1/8.$$

(C) $\frac{1}{8}$

Quick Tip: Write both conditional probabilities with $P(A \text{ and } B)$ and compare.