

MHT CET 2026 PCM Question Paper April 13 Shift 1 with Solutions



General Instructions

- (i) This booklet contains 150 questions, each provided with a complete, step-by-step solution.
- (ii) It comprises 150 single-correct multiple-choice questions.
- (iii) Attempt each question on your own before reviewing the given solution.

Physics

Questions 1 to 50 | 1 mark each | 50 marks

1. The period of oscillation of a simple pendulum is $T = 2\pi(L/g)^{1/2}$. Measured value of 'L' is 10 cm known to 1 mm accuracy and time for 100 oscillations is 50 second, using a watch of 1 second resolution. The percentage error in the measurement of 'g' is

- (A) 2%
- (B) 4%
- (C) 5%
- (D) 8%

Correct Answer: (C) 5%

Solution:

Step 1: Understanding the Concept:

Errors add up when quantities are combined. For a formula $g = 4\pi^2 L/T^2$, the fractional error in g is the fractional error in L plus

twice the fractional error in T , because T is squared.

Step 2: Key Formula or Approach:

$$T = 2\pi\sqrt{\frac{L}{g}} \Rightarrow g = \frac{4\pi^2 L}{T^2} \Rightarrow \frac{\Delta g}{g} = \frac{\Delta L}{L} + 2\frac{\Delta T}{T}$$

Step 3: Error in L .

$L = 10$ cm and $\Delta L = 1$ mm = 0.1 cm. So $\frac{\Delta L}{L} = \frac{0.1}{10} = 0.01 = 1\%$.

Step 4: Error in T .

100 oscillations take 50 s, with a watch resolution of 1 s. The error in the total time is 1 s. So $\frac{\Delta T}{T} = \frac{1}{50} = 0.02 = 2\%$ (the ratio is the same whether we use the total time or the period).

Step 5: Combine.

$$\frac{\Delta g}{g} \times 100 = 1\% + 2 \times 2\% = 5\%$$

Final Answer:

The percentage error in g is 5 %, option (C).

5%

Quick Tip: Write g in terms of L and T , then add the fractional error in L to twice the fractional error in T .

2. The vector sum of the two forces $\vec{A}$ and $\vec{B}$ is perpendicular to their vector difference. Hence forces $\vec{A}$ and $\vec{B}$ are

- (A) perpendicular to each other
- (B) unequal in magnitude
- (C) parallel to each other
- (D) equal in magnitude

Correct Answer: (D) equal in magnitude

Solution:

Step 1: Understanding the Concept:

Two vectors are perpendicular when their dot product is zero.

Step 2: Set up.

$$(\vec{A} + \vec{B}) \cdot (\vec{A} - \vec{B}) = 0$$

Step 3: Expand.

$$\vec{A} \cdot \vec{A} - \vec{A} \cdot \vec{B} + \vec{B} \cdot \vec{A} - \vec{B} \cdot \vec{B} = A^2 - B^2 = 0$$

The cross terms cancel because the dot product is commutative. So $A = B$.

Step 4: Check the options.

Only (D), equal in magnitude, follows. The vectors need not be perpendicular or parallel, and they cannot be unequal.

Final Answer:

The two forces are equal in magnitude, option (D).

$$|\vec{A}| = |\vec{B}|$$

Quick Tip: Perpendicular vectors have zero dot product, so set the dot product of the sum and difference to zero.



3. According to Boyle's law, the product PV remains constant. The SI unit of PV is same as that of

- (A) momentum
- (B) energy
- (C) force
- (D) impulse

Correct Answer: (B) energy

Solution:**Step 1: Understanding the Concept:**

The SI unit of a product is the product of units.

Step 2: Detailed Explanation:

Pressure has the unit N/m^2 and volume has the unit m^3 .

$$[PV] = \frac{\text{N}}{\text{m}^2} \times \text{m}^3 = \text{N m} = \text{J}$$

Newton metre is the joule, the unit of energy (and work).

Step 3: Check the options.

Momentum is kg m/s , force is N , and impulse is N s . None equals N m .

Final Answer:

PV has the unit of energy, option (B).

Energy

Quick Tip: Pressure times volume has the units of force per area times area times length, which is force times length.

4. A string of length ' L ' fixed at one end carries a mass ' m ' at the other end. The string makes $\frac{3}{\pi}$ r.p.s. around the vertical axis through fixed end. The tension in the string is

- (A) $72mL$
- (B) $36mL$
- (C) $18mL$
- (D) $9mL$

Correct Answer: (B) $36mL$

Solution:

Step 1: Understanding the Concept:

The mass moves in a circle of radius L . Taking the string as horizontal, the tension alone gives the needed centripetal force.

Step 2: Key Formula or Approach:

$$T = m\omega^2 L, \quad \omega = 2\pi f$$

Step 3: Detailed Explanation:

$$f = \frac{3}{\pi} \text{ rps, so } \omega = 2\pi \times \frac{3}{\pi} = 6 \text{ rad/s.}$$

$$T = m(6)^2 L = 36 mL$$

Step 4: Check the options.

72 mL would need $\omega^2 = 72$, 18 mL needs $\omega^2 = 18$ and 9 mL needs $\omega^2 = 9$. Only 36 mL fits $\omega = 6$.

Final Answer:

The tension is $36 mL$, option (B).

$$\boxed{36 mL}$$

Quick Tip: The tension provides the centripetal force, so use $T = m \omega^2 L$ with $\omega = 2\pi f$.

5. A stationary body explodes into two parts of masses M_1 and M_2 . They move in opposite directions with velocities V_1 and V_2 . The ratio of their kinetic energies is

- (A) $\frac{M_1}{M_2}$
- (B) $\frac{M_2}{M_1}$
- (C) $\frac{M_1^2}{M_2^2}$
- (D) $\frac{M_2^2}{M_1^2}$

Correct Answer: (B) $\frac{M_2}{M_1}$

Solution:

Step 1: Understanding the Concept:

A stationary body has zero momentum. After the explosion, total momentum is still zero, so the two parts have equal momentum with opposite directions.

Step 2: Key Formula or Approach:

$$M_1 V_1 = M_2 V_2 = p, \quad K = \frac{p^2}{2M}$$

Step 3: Detailed Explanation:

$$\frac{K_1}{K_2} = \frac{p^2/2M_1}{p^2/2M_2} = \frac{M_2}{M_1}$$

The lighter fragment gets more kinetic energy.

Step 4: Check the options.

(A) is the inverse of the correct ratio. (C) and (D) are squared and do not come from $K = p^2/2M$.

Final Answer:

The ratio $K_1 : K_2 = M_2 : M_1$, option (B).

$$\frac{M_2}{M_1}$$

Quick Tip: Momentum is conserved and starts at zero, so the fragments have equal and opposite momenta.

6. A body slides down a smooth inclined plane of inclination θ and reaches the bottom with velocity 'V'. If the same body is a ring which rolls down the same inclined plane then linear velocity at the bottom of the plane is

- (A) $\frac{V}{\sqrt{2}}$
- (B) $\frac{V}{2}$
- (C) V
- (D) $2V$

Correct Answer: (A) $\frac{V}{\sqrt{2}}$

Solution:**Step 1: Understanding the Concept:**

On a smooth plane the body slides, so all the lost potential energy becomes translational kinetic energy. A ring rolling without slipping shares the energy between translation and rotation.

Step 2: Sliding body.

$$mgh = \frac{1}{2}mV^2 \Rightarrow V^2 = 2gh$$

Step 3: Rolling ring.

For a ring $I = mR^2$ and $\omega = v/R$. The kinetic energy is

$$\frac{1}{2}mv^2 + \frac{1}{2}mR^2\left(\frac{v}{R}\right)^2 = mv^2$$

Energy conservation gives $mgh = mv^2$, so $v^2 = gh$.

Step 4: Compare.

$$v^2 = \frac{V^2}{2}, \text{ so } v = \frac{V}{\sqrt{2}}.$$

Step 5: Check the options.

A speed of V would mean no energy goes into rotation, and $V/2$ or $2V$ do not match the equal split.

Final Answer:

The ring reaches the bottom with speed $\frac{V}{\sqrt{2}}$, option (A).

$$\boxed{\frac{V}{\sqrt{2}}}$$

Quick Tip: Compare energy conservation for sliding with energy conservation for rolling, where rotation takes half the energy for a ring.

7. The relative angular speed of hour hand and minute hand of clock is

- (A) $\frac{2\pi}{60}$
(B) $\frac{11\pi}{360}$
(C) $\frac{11\pi}{21600}$
(D) $\frac{2\pi}{3600}$

Correct Answer: (C) $\frac{11\pi}{21600}$

Solution:

Step 1: Understanding the Concept:

Both hands rotate in the same direction. The relative angular speed is the difference of their angular speeds.

Step 2: Angular speeds.

The minute hand turns 2π rad in 60 min = 3600 s, so $\omega_m = \frac{2\pi}{3600}$ rad/s.

The hour hand turns 2π rad in 12 h = 43200 s, so $\omega_h = \frac{2\pi}{43200}$ rad/s.

Step 3: Subtract.

$$\omega_m - \omega_h = 2\pi \left(\frac{1}{3600} - \frac{1}{43200} \right) = 2\pi \cdot \frac{12 - 1}{43200} = \frac{22\pi}{43200} = \frac{11\pi}{21600} \text{ rad/s}$$

Step 4: Check the options.

Option (A) is $2\pi/60$, the minute hand speed in rad per minute. Option (D) is $2\pi/3600$, the minute hand speed alone. Option (B) has the wrong denominator.

Final Answer:

The relative angular speed is $\frac{11\pi}{21600}$ rad/s, option (C).

$$\frac{11\pi}{21600}$$

Quick Tip: Find the angular speed of each hand and subtract.

8. A disc of radius 0.4 m and mass 1 kg rotates about an axis passing through its centre and perpendicular to its plane. The angular acceleration is 10 rad/s^2 . The tangential force applied to the rim of the disc is

- (A) 1 N
- (B) 2 N
- (C) 3 N
- (D) 4 N

Correct Answer: (B) 2 N

Solution:

Step 1: Understanding the Concept:

A tangential force at the rim gives a torque $\tau = FR$, which produces angular acceleration α in a body of moment of inertia I .

Step 2: Key Formula or Approach:

$$FR = I\alpha, \quad I_{disc} = \frac{1}{2}MR^2$$

Step 3: Detailed Explanation:

$$F = \frac{I\alpha}{R} = \frac{\frac{1}{2}MR^2\alpha}{R} = \frac{1}{2}MR\alpha$$

$$F = \frac{1}{2} \times 1 \times 0.4 \times 10 = 2 \text{ N}$$

Step 4: Check the options.

1 N, 3 N and 4 N do not fit $\frac{1}{2}MR\alpha$. The common mistake is to use $MR\alpha = 4 \text{ N}$ without the factor half.

Final Answer:

The tangential force is 2 N, option (B).

2 N

Quick Tip: Torque equals moment of inertia times angular acceleration, and torque also equals force times radius.

9. A body is projected vertically from earth's surface of radius R with velocity equal to half the escape velocity. The maximum height reached by the body is

- (A) $\frac{R}{3}$
- (B) R
- (C) $\frac{R}{2}$
- (D) $\frac{R}{4}$

Correct Answer: (A) $\frac{R}{3}$

Solution:

Step 1: Understanding the Concept:

The escape speed is $v_e = \sqrt{2GM/R}$. A launch speed of $v = v_e/2$ means

$$v^2 = \frac{v_e^2}{4} = \frac{GM}{2R}.$$

Step 2: Energy conservation.

At the top the speed is zero. The loss of kinetic energy equals the gain of gravitational potential energy:

$$\frac{1}{2}mv^2 = GMm\left(\frac{1}{R} - \frac{1}{R+h}\right)$$

Step 3: Solve.

Put $v^2 = \frac{GM}{2R}$:

$$\frac{1}{4R} = \frac{1}{R} - \frac{1}{R+h} \Rightarrow \frac{1}{R+h} = \frac{3}{4R} \Rightarrow R+h = \frac{4R}{3}$$

$$h = \frac{R}{3}$$

Step 4: Check the options.

$h = R$ would need launch speed $\sqrt{GM/R}$, which is $v_e/\sqrt{2}$, and the other options do not come from the equation.

Final Answer:

The maximum height is $\frac{R}{3}$, option (A).

$$\frac{R}{3}$$

Quick Tip: Use energy conservation with the gravitational potential energy and the value of v squared from half the escape speed.



10. Two small drops of liquid of same radius coalesce to form a big drop. The ratio of the total surface energies after and before the change is

- (A) 2 : 1
- (B) $\sqrt{2} : 1$
- (C) $2^{-1/3} : 1$
- (D) 1 : 2

Correct Answer: (C) $2^{-1/3} : 1$

Solution:

Step 1: Understanding the Concept:

The surface energy of a drop is $E = T \times \text{area}$. When two drops merge, the total volume stays the same but the area changes.

Step 2: Find the radius of the big drop.

$$\frac{4}{3}\pi R^3 = 2 \times \frac{4}{3}\pi r^3 \Rightarrow R = 2^{1/3}r$$

Step 3: Compare the areas.

Before: $2 \times 4\pi r^2 = 8\pi r^2$. After: $4\pi R^2 = 4\pi 2^{2/3}r^2$.

$$\frac{E_{after}}{E_{before}} = \frac{4\pi 2^{2/3} r^2}{8\pi r^2} = \frac{2^{2/3}}{2} = 2^{-1/3}$$

Step 4: Check the options.

The ratio is less than 1 because area decreases on coalescing. So options (A) and (B), which are above 1, cannot be right. 1 : 2 would hold only if the radius did not change.

Final Answer:

The ratio is $2^{-1/3} : 1$, option (C).

$$2^{-1/3} : 1$$

Quick Tip: Volume is conserved when drops merge, which gives the radius of the big drop. Surface energy is surface tension times area.

11. Three liquids have same surface tension and have densities ρ_1, ρ_2 and ρ_3 ($\rho_1 > \rho_2 > \rho_3$). In three identical capillaries rise of liquid is same. The corresponding angles of contact θ_1, θ_2 and θ_3 are related as

- (A) $\theta_1 > \theta_2 > \theta_3$
- (B) $\theta_1 < \theta_2 < \theta_3$
- (C) $\theta_1 = \theta_2 = \theta_3$
- (D) $\theta_1 > \theta_2 < \theta_3$

Correct Answer: (B) $\theta_1 < \theta_2 < \theta_3$

Solution:

Step 1: Understanding the Concept:

Capillary rise is given by $h = \frac{2T \cos \theta}{r\rho g}$.

Step 2: Apply the condition.

The surface tension T , the capillary radius r and the rise h are the same for all three. So $\frac{\cos \theta}{\rho}$ must be the same, which means $\cos \theta \propto \rho$.

Step 3: Compare.

Given $\rho_1 > \rho_2 > \rho_3$, we get $\cos \theta_1 > \cos \theta_2 > \cos \theta_3$. On the range 0 to 90° , the cosine is smaller for a larger angle. So $\theta_1 < \theta_2 < \theta_3$.

Step 4: Check the options.

Option (A) is the reverse. Option (C) would need equal densities.

Option (D) is not a consistent ordering.

Final Answer:

The relation is $\theta_1 < \theta_2 < \theta_3$, option (B).

$$\theta_1 < \theta_2 < \theta_3$$

Quick Tip: Use the capillary rise formula $h = \frac{2T \cos(\theta)}{r\rho g}$ with h , T and r equal.

12. The work done in splitting a water drop of radius 'R' into 64 droplets is (T is the surface tension of water)

- (A) $8\pi R^2 T$
- (B) $12\pi R^2 T$
- (C) $4\pi R^2 T$
- (D) $16\pi R^2 T$

Correct Answer: (B) $12\pi R^2 T$

Solution:

Step 1: Understanding the Concept:

Splitting a drop creates new surface area. The work done equals surface tension times the increase in area.

Step 2: Find the droplet radius.

$$\frac{4}{3}\pi R^3 = 64 \cdot \frac{4}{3}\pi r^3 \Rightarrow r = \frac{R}{4}$$

Step 3: Find the increase in area.

Final area: $64 \times 4\pi \left(\frac{R}{4}\right)^2 = 64 \times 4\pi \times \frac{R^2}{16} = 16\pi R^2$. Initial area: $4\pi R^2$.

$$\Delta A = 16\pi R^2 - 4\pi R^2 = 12\pi R^2$$

Step 4: Work done.

$$W = T \Delta A = 12\pi R^2 T$$

Step 5: Check the options.

Option (A) is $8\pi R^2 T$, and (C) is only the initial area term. Option (D) is the final area term without subtracting the initial area.

Final Answer:

The work done is $12\pi R^2 T$, option (B).

$$12\pi R^2 T$$

Quick Tip: Volume is conserved, so the radius of each droplet is R divided by the cube root of 64. Work equals tension times the increase in area.



13. A celsius and a Fahrenheit thermometer are dipped in boiling water. The temperature of water is lowered until the Fahrenheit thermometer shows 140° . What is the fall in temperature on the Celsius thermometer?

- (A) 60°C
- (B) 40°C
- (C) 50°C
- (D) 80°C

Correct Answer: (B) 40°C

Solution:

Step 1: Understanding the Concept:

A Celsius degree is larger than a Fahrenheit degree. The scales are linked by $\frac{C}{5} = \frac{F - 32}{9}$, so a change of 9 F equals a change of 5 C.

Step 2: Find the fall in F.

Boiling water is at 212°F . It cools to 140°F , so the fall is $212 - 140 = 72$

F degrees.

Step 3: Convert.

$$\Delta C = \frac{5}{9} \times 72 = 40$$

Step 4: Check the options.

60, 50 and 80 degrees do not match. A common slip is to use $\frac{9}{5}$ in place of $\frac{5}{9}$.

Final Answer:

The fall in temperature on the Celsius thermometer is 40°C , option (B).

40°C

Quick Tip: Boiling water is 100 C or 212 F. Convert the fall in Fahrenheit degrees into Celsius degrees using the ratio 5 to 9.

14. Rate of radiation by a black body is 'R' at temperature 'T'. Another body has same area but emissivity is 0.2 and temperature $3T$. Its rate of radiation is

- (A) R
- (B) $2R$
- (C) $16.2R$
- (D) $0.2R$

Correct Answer: (C) $16.2R$

Solution:

Step 1: Understanding the Concept:

By the Stefan-Boltzmann law, the power radiated by a surface is $P = e\sigma AT^4$, where e is the emissivity. A perfect black body has $e = 1$.

Step 2: Black body.

$$R = \sigma AT^4.$$

Step 3: Second body.

Same area, $e = 0.2$ and temperature $3T$:

$$P = 0.2 \sigma A(3T)^4 = 0.2 \times 81 \sigma AT^4 = 16.2 R$$

Step 4: Check the options.

Option (B) uses 3^1 and (D) ignores the temperature. Only $16.2R$ fits 0.2×3^4 .

Final Answer:

The rate of radiation is $16.2R$, option (C).

$16.2R$

Quick Tip: Radiated power varies as emissivity times the fourth power of temperature, with the black body having emissivity 1.

15. A black rectangular surface of area A emits energy E per unit time at 27°C . If length and breadth is reduced to $(\frac{1}{4})^{\text{th}}$ of initial value and temperature is raised to 327°C , then energy emitted per unit time becomes

- (A) E
- (B) $2E$
- (C) $\frac{E}{2}$
- (D) $\frac{E}{4}$

Correct Answer: (A) E

Solution:

Step 1: Understanding the Concept:

A black body emits energy per unit time $E = \sigma AT^4$, with T in kelvin.

Step 2: Convert temperatures.

$T_1 = 27 + 273 = 300\text{ K}$ and $T_2 = 327 + 273 = 600\text{ K}$, so $T_2 = 2T_1$.

Step 3: Find the change in area.

Length and breadth are each reduced to $\frac{1}{4}$, so the area becomes $\frac{1}{16}$ of A .

Step 4: Calculate.

$$\frac{E_2}{E_1} = \frac{A_2}{A_1} \left(\frac{T_2}{T_1} \right)^4 = \frac{1}{16} \times 2^4 = 1$$

Step 5: Check the options.

The two changes cancel exactly, so the emission is unchanged.

Final Answer:

The new energy per second is E , option (A).

$$\boxed{E}$$

Quick Tip: Energy per second goes as area times the fourth power of absolute temperature. Work in kelvin.

16. The change in internal energy of the mass of gas, when the volume changes from V to $2V$ at constant pressure P is (γ is the ratio of specific heat at constant pressure to that of volume) ($\gamma = \frac{C_p}{C_v}$)

- (A) $\frac{PV}{\gamma-1}$
- (B) $\frac{PV}{\gamma+1}$
- (C) $\frac{(\gamma-1)}{PV}$
- (D) $\frac{(\gamma+1)}{PV}$

Correct Answer: (A) $\frac{PV}{\gamma-1}$

Solution:**Step 1: Understanding the Concept:**

The internal energy of an ideal gas changes by $\Delta U = nC_v\Delta T$, whatever the process.

Step 2: Find ΔT .

At constant pressure, $P\Delta V = nR\Delta T$. Here $\Delta V = 2V - V = V$, so $nR\Delta T = PV$.

Step 3: Write C_v .

From $C_p - C_v = R$ and $\gamma = C_p/C_v$, $C_v = \frac{R}{\gamma - 1}$.

Step 4: Calculate.

$$\Delta U = n \frac{R}{\gamma - 1} \Delta T = \frac{nR\Delta T}{\gamma - 1} = \frac{PV}{\gamma - 1}$$

Step 5: Check the options.

Option (B) has $\gamma + 1$ and options (C) and (D) invert the PV factor, which would give the wrong units for energy.

Final Answer:

$$\Delta U = \frac{PV}{\gamma - 1}, \text{ option (A).}$$

$$\boxed{\frac{PV}{\gamma - 1}}$$

Quick Tip: Use the ideal gas law for the temperature change, then $dU = n C_v dT$ with $C_v = R/(\gamma - 1)$.

17. An ideal gas expands adiabatically ($\gamma = 1.5$). To reduce the r.m.s. velocity of the molecules 3 times the gas has to be expanded

- (A) 9 times
- (B) 81 times
- (C) 3 times
- (D) 27 times

Correct Answer: (B) 81 times

Solution:

Step 1: Understanding the Concept:

The rms speed of gas molecules is $v_{rms} = \sqrt{3RT/M}$, so $v_{rms} \propto \sqrt{T}$. For an adiabatic change, $TV^{\gamma-1} = \text{constant}$.

Step 2: Find the temperature ratio.

If v_{rms} becomes $\frac{1}{3}$ of its value, T becomes $\frac{1}{9}$ of its value. So $\frac{T_1}{T_2} = 9$.

Step 3: Use the adiabatic relation.

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1} \Rightarrow \left(\frac{V_2}{V_1} \right)^{\gamma-1} = \frac{T_1}{T_2} = 9$$

With $\gamma - 1 = 0.5$:

$$\frac{V_2}{V_1} = 9^{1/0.5} = 9^2 = 81$$

Step 4: Check the options.

A factor of 9 is the temperature ratio, not the volume ratio. 3 times and 27 times do not follow from the power 2.

Final Answer:

The gas must expand 81 times, option (B).

81 times

Quick Tip: The rms speed varies as the square root of temperature, and an adiabatic process obeys $T V^{\gamma-1} = \text{constant}$.



18. A simple pendulum oscillates with an angular amplitude θ . If the maximum tension in the string is twice the minimum tension then θ is

- (A) $\cos^{-1}(0.75)$
- (B) $\cos^{-1}(0.5)$
- (C) $\sin^{-1}(0.5)$
- (D) $\sin^{-1}(0.75)$

Correct Answer: (A) $\cos^{-1}(0.75)$

Solution:

Step 1: Understanding the Concept:

The tension is smallest at the extreme position (speed zero) and largest at the mean position (speed maximum).

Step 2: Minimum tension.

At the extreme position, the speed is zero, so there is no centripetal force needed along the string. The tension balances the component of weight along the string:

$$T_{\min} = mg \cos \theta$$

Step 3: Maximum tension.

Energy conservation from the extreme to the bottom: $\frac{1}{2}mv^2 = mgL(1 - \cos \theta)$, so $v^2 = 2gL(1 - \cos \theta)$. At the bottom, $T - mg = \frac{mv^2}{L}$:

$$T_{max} = mg + 2mg(1 - \cos \theta) = mg(3 - 2 \cos \theta)$$

Step 4: Use the condition.

$$T_{max} = 2T_{min} \Rightarrow 3 - 2 \cos \theta = 2 \cos \theta \Rightarrow \cos \theta = \frac{3}{4}$$

Step 5: Check the options.

So $\theta = \cos^{-1}(0.75)$, option (A). The options with $\cos^{-1}(0.5)$, $\sin^{-1}(0.5)$ and $\sin^{-1}(0.75)$ do not satisfy the equation.

Final Answer:

The angular amplitude is $\cos^{-1}(0.75)$.

$$\theta = \cos^{-1}(0.75)$$

Quick Tip: Find tension at the lowest point using energy conservation and centripetal force, and at the extreme point where speed is zero.

19. A spring executes S.H.M. with mass 10 kg attached to it. The force constant of spring is 10 N/m. If at any instant its velocity is 40 cm/s, the displacement at that instant is
(Amplitude of S.H.M. is 0.5 m)

- (A) 0.1 m
- (B) 0.3 m
- (C) 0.5 m
- (D) 0.6 m

Correct Answer: (B) 0.3 m

Solution:

Step 1: Understanding the Concept:

In SHM the speed at displacement x is $v = \omega\sqrt{A^2 - x^2}$, where $\omega = \sqrt{k/m}$.

Step 2: Find ω .

$$\omega = \sqrt{\frac{10}{10}} = 1 \text{ rad/s.}$$

Step 3: Use the speed.

$v = 40 \text{ cm/s} = 0.4 \text{ m/s}$ and $A = 0.5 \text{ m}$.

$$0.4^2 = 1^2(0.5^2 - x^2) \Rightarrow 0.16 = 0.25 - x^2 \Rightarrow x^2 = 0.09 \Rightarrow x = 0.3 \text{ m}$$

Step 4: Check the options.

$x = 0.5 \text{ m}$ would be the extreme position where the speed is zero. $x = 0.6 \text{ m}$ is bigger than the amplitude, which is impossible. $x = 0.1 \text{ m}$ gives $v = \sqrt{0.24} \approx 0.49 \text{ m/s}$, not 0.4.

Final Answer:

The displacement is 0.3 m, option (B).

0.3 m

Quick Tip: Find omega from the mass and force constant, then use $v^2 = \omega^2 (A^2 - x^2)$.

20. Waves having a velocity 10m/s, strike a stationary boat near the sea shore. The distance between two consecutive crests of the waves is 40m. After how many second, the consecutive waves strike the boat?

- (A) 8 s
- (B) 6 s
- (C) 4 s
- (D) 2 s

Correct Answer: (C) 4 s

Solution:

Step 1: Understanding the Concept:

Consecutive crests reach a fixed point after a time equal to the period T of the wave.

Step 2: Key Formula or Approach:

$$v = \frac{\lambda}{T} \Rightarrow T = \frac{\lambda}{v}$$

Step 3: Calculate.

$$T = \frac{40}{10} = 4 \text{ s}$$

Step 4: Check the options.

8 s would need a wavelength of 80 m and 2 s would need 20 m.

Final Answer:

Consecutive waves strike the boat every 4 s, option (C).

4 s

Quick Tip: The time between consecutive crests is the period, which equals wavelength divided by wave speed.

21. A musical instrument P produces sound waves of frequency 'n' and amplitude 'A'. Another musical instrument Q produces sound waves of frequency $n/4$. The waves produced by P and Q have equal energies. The amplitude of waves produced by Q will be

- (A) A
- (B) 2A
- (C) 4A
- (D) 8A

Correct Answer: (C) 4A

Solution:

Step 1: Understanding the Concept:

The energy of a wave per unit volume depends on $\omega^2 A^2$, so $E \propto f^2 A^2$.

Step 2: Set the energies equal.

$$n^2 A^2 = \left(\frac{n}{4}\right)^2 A_Q^2$$

Step 3: Solve.

$$A_Q^2 = 16A^2 \Rightarrow A_Q = 4A$$

Step 4: Check the options.

A would be right only if the frequencies were equal. $2A$ and $8A$ do not satisfy the equation.

Final Answer:

The amplitude of Q is $4A$, option (C).

$$\boxed{4A}$$

Quick Tip: Wave energy is proportional to the square of the amplitude times the square of the frequency.

22. In a stationary wave, all the particles

- (A) except at node vibrate in S.H.M. of same period but of different amplitudes
- (B) vibrate in S.H.M. of same period and amplitude
- (C) except at nodes vibrate in S.H.M. of same period and same amplitude
- (D) vibrate in S.H.M. of different periods and different amplitudes

Correct Answer: (A) except at node vibrate in S.H.M. of same period but of different amplitudes

Solution:

Step 1: Understanding the Concept:

A stationary wave is formed by the superposition of two identical waves travelling in opposite directions.

Step 2: Detailed Explanation:

Every particle between nodes executes SHM with the same period (the period of the original wave). The amplitude changes from point to point: it is zero at the nodes and largest at the antinodes. So all particles except those at nodes vibrate in SHM with the same period but different amplitudes.

Step 3: Check the options.

(B) and (C) say the amplitude is the same, which is wrong. (D) says the periods are different, which is wrong. Only (A) is correct.

Final Answer:

Option (A) is correct.

(A)

Quick Tip: In a stationary wave all particles oscillate with the same period but with amplitude depending on position, and nodes do not move.



23. In fundamental mode, the time required for the sound wave to reach upto the closed end of a pipe filled with air is t second. The frequency of vibration of air column is

(λ = wavelength of the wave)

- (A) $\frac{1}{t}$
- (B) $\frac{0.25}{t}$
- (C) $\frac{\lambda}{4t}$
- (D) t

Correct Answer: (B) $\frac{0.25}{t}$

Solution:

Step 1: Understanding the Concept:

A closed pipe has a node at the closed end and an antinode at the open end. In the fundamental mode, the length of the pipe is $L = \frac{\lambda}{4}$.

Step 2: Travel time.

Sound travels the length L at speed v , so $t = \frac{L}{v} = \frac{\lambda}{4v}$.

Step 3: Frequency.

Since $f = \frac{v}{\lambda}$, we get $t = \frac{1}{4f}$, so

$$f = \frac{1}{4t} = \frac{0.25}{t}$$

Step 4: Check the options.

Option (C) $\frac{\lambda}{4t}$ is the speed, not the frequency. Options (A) and (D) do not match the fundamental mode relation.

Final Answer:

The frequency is $\frac{0.25}{t}$, option (B).

$$\boxed{\frac{0.25}{t}}$$

Quick Tip: In the fundamental mode a closed pipe has length λ over 4, so the travel time to the closed end is a quarter of the period.

24. A sonometer wire is vibrating in third overtone. There are

- (A) 5 nodes and 4 antinodes
- (B) 4 nodes and 5 antinodes
- (C) 4 nodes and 4 antinodes
- (D) 5 nodes and 5 antinodes

Correct Answer: (A) 5 nodes and 4 antinodes

Solution:

Step 1: Understanding the Concept:

A sonometer wire is fixed at both ends. Its fundamental has one loop (2 nodes, 1 antinode). The n th overtone is the $(n + 1)$ th harmonic.

Step 2: Detailed Explanation:

Third overtone = 4th harmonic, so the wire vibrates in 4 loops. A wire with 4 loops has 4 antinodes and $4 + 1 = 5$ nodes (including the two fixed ends).

Step 3: Check the options.

Only option (A) matches 5 nodes and 4 antinodes.

Final Answer:

There are 5 nodes and 4 antinodes, option (A).

5 nodes, 4 antinodes

Quick Tip: The third overtone is the fourth harmonic, which has four loops.



25. A charge 'Q' C is placed at the centre of a cube. The electric flux through two opposite faces of the cube is
(ϵ_0 = permittivity of free space)

- (A) $\frac{Q}{6\epsilon_0}$
- (B) $\frac{Q}{3\epsilon_0}$
- (C) $\frac{Q}{\epsilon_0}$
- (D) $\frac{Q}{2\epsilon_0}$

Correct Answer: (B) $\frac{Q}{3\epsilon_0}$

Solution:

Step 1: Understanding the Concept:

By Gauss law, the total flux through a closed surface is $\frac{Q}{\epsilon_0}$. A charge at the centre of a cube is symmetric with respect to all faces.

Step 2: Flux per face.

The cube has 6 identical faces, so each gets $\frac{Q}{6\epsilon_0}$.

Step 3: Two opposite faces.

$$2 \times \frac{Q}{6\epsilon_0} = \frac{Q}{3\epsilon_0}$$

Step 4: Check the options.

$\frac{Q}{6\epsilon_0}$ is one face, $\frac{Q}{\epsilon_0}$ is the whole cube and $\frac{Q}{2\epsilon_0}$ is three faces.

Final Answer:

The flux through two opposite faces is $\frac{Q}{3\epsilon_0}$, option (B).

$$\boxed{\frac{Q}{3\epsilon_0}}$$

Quick Tip: Gauss law gives the total flux Q over epsilon naught, and symmetry splits it equally over the six faces.

26. The capacitance of a capacitor becomes $\frac{7}{6}$ times the original value if dielectric slab of thickness $t = \frac{2d}{3}$ is introduced between the plates, where 'd' is the distance of separation between the plates. The dielectric constant of the slab is

- (A) $\frac{9}{11}$
- (B) $\frac{14}{11}$
- (C) $\frac{8}{11}$
- (D) $\frac{12}{11}$

Correct Answer: (B) $\frac{14}{11}$

Solution:

Step 1: Understanding the Concept:

A dielectric slab of thickness t and constant K in a gap d makes the effective gap $d - t + \frac{t}{K}$.

Step 2: Key Formula or Approach:

$$C = \frac{\epsilon_0 A}{d - t + t/K}, \quad C_0 = \frac{\epsilon_0 A}{d}$$

Step 3: Use the condition.

$$\frac{C}{C_0} = \frac{d}{d - t + t/K} = \frac{7}{6} \Rightarrow d - t + \frac{t}{K} = \frac{6d}{7}$$

Put $t = \frac{2d}{3}$: $d - \frac{2d}{3} + \frac{2d}{3K} = \frac{6d}{7}$, so

$$\frac{2}{3K} = \frac{6}{7} - \frac{1}{3} = \frac{18 - 7}{21} = \frac{11}{21}$$

Step 4: Solve for K.

$$K = \frac{2}{3} \times \frac{21}{11} = \frac{14}{11}$$

Step 5: Check.

Strictly a dielectric constant should be at least 1, and $14/11 \approx 1.27$ satisfies that. The other options $9/11$, $8/11$, $12/11$ do not satisfy the equation.

Final Answer:

The dielectric constant is $\frac{14}{11}$, option (B).

$$K = \frac{14}{11}$$

Quick Tip: With a partial dielectric slab, the capacitance is epsilon naught A divided by $(d - t + t/K)$. Set it equal to $7/6$ of the original.

27. Two identical parallel plate air capacitors are connected in series to a battery of e.m.f. 'V'. If one of the capacitor is inserted in liquid of dielectric constant 'K' then potential difference of the other capacitor will become

- (A) $\frac{KV}{K+1}$
- (B) $\frac{KV}{K-1}$
- (C) $\frac{K+1}{KV}$
- (D) $\frac{K-1}{KV}$

Correct Answer: (A) $\frac{KV}{K+1}$

Solution:

Step 1: Understanding the Concept:

Two capacitors in series connected to a battery carry the same charge. The voltage across each is $V = Q/C$, so the capacitor with smaller capacitance takes the larger voltage.

Step 2: Capacitances.

Each capacitor has capacitance C in air. After one is placed in a liquid of dielectric constant K , its capacitance becomes KC .

Step 3: Find the charge.

$$C_{eq} = \frac{C \cdot KC}{C + KC} = \frac{KC}{K + 1}, \quad Q = C_{eq}V = \frac{KCV}{K + 1}$$

Step 4: Voltage across the air capacitor.

$$V_{air} = \frac{Q}{C} = \frac{KV}{K + 1}$$

Step 5: Check.

The voltage across the liquid capacitor is $\frac{V}{K + 1}$. The two add to V , as they should.

Final Answer:

The potential difference of the other capacitor becomes $\frac{KV}{K + 1}$, option (A).

$$\frac{KV}{K+1}$$

Quick Tip: In a series combination the charge is the same on both, so the potential divides in inverse ratio of the capacitances.



28. A condenser of capacity C_1 is charged to potential V_1 and then disconnected. Uncharged capacitor of capacity C_2 is connected in parallel with C_1 . The resultant potential V_2 is

- (A) $\frac{C_1 V_1}{C_2}$
- (B) $\frac{C_1 V_1}{C_1 + C_2}$
- (C) $\frac{C_2 V_1}{C_1}$
- (D) $\frac{C_2 V_1}{C_1 + C_2}$

Correct Answer: (B) $\frac{C_1 V_1}{C_1 + C_2}$

Solution:

Step 1: Understanding the Concept:

When the charged capacitor, already disconnected from its battery, is connected in parallel with an uncharged one, charge flows until both have the same potential. The total charge does not change.

Step 2: Initial charge.

$$Q = C_1 V_1.$$

Step 3: Final state.

The capacitors in parallel have capacitance $C_1 + C_2$ and common potential V_2 , so

$$V_2 = \frac{Q}{C_1 + C_2} = \frac{C_1 V_1}{C_1 + C_2}$$

Step 4: Check the options.

Options (A), (C) and (D) divide by the wrong capacitance or use the wrong charge.

Final Answer:

The common potential is $\frac{C_1 V_1}{C_1 + C_2}$, option (B).

$$\boxed{\frac{C_1 V_1}{C_1 + C_2}}$$

Quick Tip: The total charge is conserved, and the combined capacitance in parallel is the sum.



29. A potentiometer wire is 4 m long and potential difference of 3V is maintained between the ends. The e.m.f. of the cell which balances against a length of 100 cm of the potentiometer wire is

- (A) 0.25 V
- (B) 0.50 V
- (C) 0.75 V
- (D) 1.0 V

Correct Answer: (C) 0.75 V

Solution:

Step 1: Understanding the Concept:

In a potentiometer, the potential drop is uniform along the wire. The emf of a cell is the potential drop over the balancing length.

Step 2: Potential gradient.

$$k = \frac{3 \text{ V}}{4 \text{ m}} = 0.75 \text{ V/m}$$

Step 3: EMF.

Balancing length $l = 100 \text{ cm} = 1 \text{ m}$.

$$E = kl = 0.75 \times 1 = 0.75 \text{ V}$$

Step 4: Check the options.

0.25 V would result from using $1/12$ of the wire, and 1.0 V from using $4/3 \text{ m}$. Only 0.75 V fits.

Final Answer:

The emf of the cell is 0.75 V, option (C).

0.75 V

Quick Tip: The potential gradient is the total voltage over the wire length, and the emf equals gradient times balancing length.

30. Two wires A and B of equal lengths are connected in left and right gap respectively of a metre bridge, null point is obtained at 40 cm from left end. Diameters of the wires A and B are in the ratio 3:1 respectively, the ratio of specific resistance of A to that of B is

- (A) 2 : 1
- (B) 3 : 1
- (C) 6 : 1
- (D) 12 : 1

Correct Answer: (C) 6 : 1

Solution:

Step 1: Understanding the Concept:

In a metre bridge, $\frac{R_A}{R_B} = \frac{l}{100 - l}$, where l is the null point from the left end where A is connected.

Step 2: Use the null point.

$$\frac{R_A}{R_B} = \frac{40}{60} = \frac{2}{3}$$

Step 3: Write resistances.

$R = \rho \frac{L}{A}$ with area $A = \frac{\pi d^2}{4}$. The wires have the same length, so

$$\frac{R_A}{R_B} = \frac{\rho_A}{\rho_B} \times \frac{A_B}{A_A} = \frac{\rho_A}{\rho_B} \times \frac{d_B^2}{d_A^2} = \frac{\rho_A}{\rho_B} \times \frac{1}{9}$$

Step 4: Solve.

$$\frac{\rho_A}{\rho_B} = 9 \times \frac{2}{3} = 6$$

Step 5: Check the options.

Using $d_A : d_B$ instead of squares gives 2, which is option (A), a common error.

Final Answer:

The ratio of specific resistance A to B is 6 : 1, option (C).

6 : 1

Quick Tip: Use the metre bridge balance condition, then write resistance in terms of resistivity, length and area.

31. An iron rod is placed parallel to magnetic field intensity 2000 A/m. The magnetic flux through rod is 6×10^{-4} wb and its cross sectional area is 3 cm^2 . The magnetic permeability of the rod in wb/A-m is

- (A) 10^{-1}
- (B) 10^{-2}
- (C) 10^{-3}
- (D) 10^{-4}

Correct Answer: (C) 10^{-3}

Solution:

Step 1: Understanding the Concept:

Flux density is $B = \frac{\Phi}{A}$ and the permeability of a material is $\mu = \frac{B}{H}$.

Step 2: Convert the units.

$$A = 3 \text{ cm}^2 = 3 \times 10^{-4} \text{ m}^2.$$

Step 3: Find B.

$$B = \frac{6 \times 10^{-4}}{3 \times 10^{-4}} = 2 \text{ Wb/m}^2$$

Step 4: Find μ .

$$\mu = \frac{B}{H} = \frac{2}{2000} = 10^{-3} \text{ Wb/A m}$$

Final Answer:

The permeability is 10^{-3} Wb/A-m , option (C).

10^{-3} Wb/A m

Quick Tip: Find the flux density B from flux over area, then use $\mu = B/H$.

32. The magnetic flux near the axis and inside the air core solenoid of length 60 cm carrying current I is $\frac{\pi}{2} \times 10^{-6} \text{ Wb}$. Its magnetic moment will be (cross-sectional area is very small as compared to length of solenoid, μ_0 = Permeability of free space = $4\pi \times 10^{-7} \text{ SI unit}$)

- (A) 0.75 Am^2
- (B) 0.25 Am^2
- (C) 0.50 Am^2
- (D) 1.0 Am^2

Correct Answer: (A) 0.75 Am^2

Solution:

Step 1: Understanding the Concept:

Inside a long solenoid, $B = \mu_0 n I$, where n is the number of turns per unit length. The magnetic moment is $m = N I A$, where $N = n L$.

Step 2: Link flux and moment.

$\Phi = B A = \mu_0 n I A$. Then

$$m = N I A = n L I A = \frac{L}{\mu_0} (\mu_0 n I A) = \frac{L \Phi}{\mu_0}$$

Step 3: Substitute.

$L = 0.6 \text{ m}$, $\Phi = \frac{\pi}{2} \times 10^{-6} \text{ Wb}$, $\mu_0 = 4\pi \times 10^{-7}$.

$$m = \frac{0.6 \times \frac{\pi}{2} \times 10^{-6}}{4\pi \times 10^{-7}} = \frac{0.3\pi \times 10^{-6}}{4\pi \times 10^{-7}} = \frac{3}{4} = 0.75 \text{ A m}^2$$

Step 4: Check the options.

Only option (A), 0.75 A m^2 , matches.

Final Answer:

The magnetic moment is 0.75 A m^2 .

$$0.75 \text{ A m}^2$$

Quick Tip: Use $B = \mu_0 n I$ and the magnetic moment $m = N I A$, then eliminate n , I and A using the flux.



33. If the number of turns in the coil of a moving coil galvanometer decreases, the resistance of the galvanometer,

- (A) decreases
- (B) increases
- (C) remains the same
- (D) may increase or decrease

Correct Answer: (A) decreases

Solution:

Step 1: Understanding the Concept:

The coil of a galvanometer is made of copper wire. The resistance of a wire is $R = \rho \frac{l}{A}$, so a longer wire has more resistance.

Step 2: Detailed Explanation:

Fewer turns means a shorter length of wire, which means a smaller resistance. So the resistance of the galvanometer decreases.

Step 3: Check the options.

It cannot stay the same because the length changes, and it does not increase.

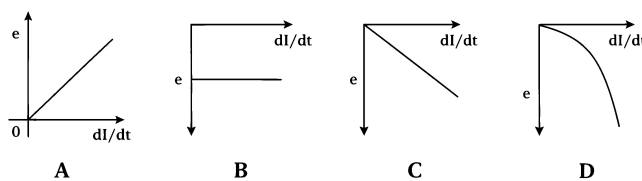
Final Answer:

The resistance decreases, option (A).

decreases

Quick Tip: The resistance of the coil is proportional to the length of its wire, which falls with the number of turns.

34. The current flowing through the inductor of self inductance 'L' is continuously increasing at constant rate. The variation of induced e.m.f. (e) versus $\frac{dI}{dt}$ is shown graphically by figure



(A) A

(B) B

(C) C

(D) D

Correct Answer: (C) C

Solution:**Step 1: Recall the law:**

For an inductor with self inductance L , the induced e.m.f. is $e = -L \frac{dI}{dt}$. The minus sign is Lenz's law: the induced e.m.f. opposes the change in current.

Step 2: Compare with a straight line:

Put $y = e$ and $x = dI/dt$. Then $e = (-L) \frac{dI}{dt}$ has the form $y = mx$ with slope $m = -L$. So the graph is a straight line through the origin, and the slope is negative because L is positive.

Step 3: Read the figures:

In the figure, graph A is a straight line through the origin with positive slope. Graph B is a horizontal line (e does not change with dI/dt). Graph C is a straight line through the origin that falls into the negative e region as dI/dt increases. Graph D is a curve.

Step 4: Match:

Only graph C is a straight line from the origin with negative e for positive dI/dt . Graph A has the wrong sign, graph B would mean e is independent of dI/dt , and graph D would mean e is not proportional to dI/dt .

Final Answer:

The e versus dI/dt graph is a straight line of slope $-L$, which is figure C.

C

Quick Tip: Write $e = -L dI/dt$. It is a straight line through the origin with negative slope.

35. Two coils P and Q have mutual inductance 'M' H. If the current in the coil P is $I = I_0 \sin \omega t$, then the maximum value of e.m.f. induced in coil Q is

- (A) $\frac{\omega}{MI_0}$
(B) $\frac{M\omega}{I_0}$
(C) $M\omega I_0$
(D) $\frac{MI_0}{\omega}$

Correct Answer: (C) $M\omega I_0$

Solution:

Step 1: Understanding the Concept:

The emf induced in coil Q is $e = -M \frac{dI}{dt}$, where I is the current in coil P.

Step 2: Differentiate.

$$\frac{dI}{dt} = I_0 \omega \cos \omega t \Rightarrow e = -MI_0 \omega \cos \omega t$$

Step 3: Find the maximum.

The largest value of $|\cos \omega t|$ is 1, so $e_{max} = M\omega I_0$.

Step 4: Check the options.

Options (A), (B) and (D) put ω or I_0 in the denominator, which gives the wrong dimensions.

Final Answer:

The maximum emf is $M\omega I_0$, option (C).

$$\boxed{M\omega I_0}$$

Quick Tip: The induced emf is M times the rate of change of current in the primary.



36. In series LCR circuit $C = 2 \mu\text{F}$, $L = 1 \text{ mH}$ and $R = 10 \Omega$. What is the ratio of energies stored in the capacitor and inductor when maximum current flows through circuit?

- (A) 0.1 : 1
- (B) 0.4 : 1
- (C) 0.2 : 1
- (D) 1 : 1

Correct Answer: (D) 1 : 1

Solution:

Step 1: Understanding the Concept:

In a series LCR circuit the current is largest when the circuit is at resonance. This is the condition where the inductive reactance equals the capacitive reactance.

Step 2: Key Formula or Approach:

At resonance $X_L = X_C$, that is $\omega L = \frac{1}{\omega C}$, which gives $\omega^2 = \frac{1}{LC}$.

The energy in the inductor is $U_L = \frac{1}{2}LI^2$. The energy in the capacitor is $U_C = \frac{1}{2}CV_C^2$, where $V_C = IX_C$.

Step 3: Detailed Explanation:

Put $V_C = IX_C = \frac{I}{\omega C}$ into the capacitor energy:

$$U_C = \frac{1}{2}C \frac{I^2}{\omega^2 C^2} = \frac{1}{2} \frac{I^2}{\omega^2 C}$$

Now use $\omega^2 = \frac{1}{LC}$, so $\frac{1}{\omega^2 C} = L$:

$$U_C = \frac{1}{2}LI^2 = U_L$$

So the two energies are equal and

$$U_C : U_L = 1 : 1$$

The values $C = 2 \mu\text{F}$, $L = 1 \text{ mH}$ and $R = 10 \Omega$ are not needed, because the result holds for any L , C and R . The resistance only fixes how large the current is, not the ratio.

Options (A), (B) and (C) give ratios different from 1, and none of them can come from the resonance condition.

Final Answer:

The ratio is 1:1, option (D).

$$\boxed{1 : 1}$$

Quick Tip: Maximum current occurs at resonance where $X_L = X_C$, and then the energy in the capacitor equals the energy in the inductor.

37. In series LCR resonant circuit, $R=800 \Omega$, $C = 2 \mu\text{F}$ and voltage across resistance is 200 V. The angular frequency is 250 rad/s. At resonance the voltage across the capacitance is

- (A) 250 V
- (B) 500 V
- (C) 1000 V
- (D) 750 V

Correct Answer: (B) 500 V

Solution:

Step 1: Understanding the Concept:

In a series LCR circuit at resonance, the impedance is just R . The current is set by the voltage across the resistor, and the capacitor voltage is IX_C .

Step 2: Find the current.

$$I = \frac{V_R}{R} = \frac{200}{800} = 0.25 \text{ A}$$

Step 3: Find the reactance.

$$X_C = \frac{1}{\omega C} = \frac{1}{250 \times 2 \times 10^{-6}} = \frac{1}{5 \times 10^{-4}} = 2000 \Omega$$

Step 4: Find the voltage.

$$V_C = IX_C = 0.25 \times 2000 = 500 \text{ V}$$

Step 5: Check the options.

250 V, 750 V and 1000 V do not match 0.25×2000 .

Final Answer:

The voltage across the capacitor is 500 V, option (B).

500 V

Quick Tip: At resonance the current is the resistor voltage divided by R, and the capacitor voltage is the current times its reactance.

38. An alternating e.m.f. is given by $e = e_0 \sin \omega t$. In what time the e.m.f. will have half its maximum value, if 'e' starts from zero? ($\sin 30^\circ = 0.5$)

- (A) $\frac{T}{4}$
- (B) $\frac{T}{8}$
- (C) $\frac{T}{12}$
- (D) $\frac{T}{16}$

Correct Answer: (C) $\frac{T}{12}$

Solution:

Step 1: Understanding the Concept:

The emf is $e = e_0 \sin \omega t$, and $\omega = \frac{2\pi}{T}$.

Step 2: Set up.

$$\frac{e_0}{2} = e_0 \sin \omega t \Rightarrow \sin \omega t = \frac{1}{2} \Rightarrow \omega t = 30^\circ = \frac{\pi}{6}$$

Step 3: Solve for t.

$$t = \frac{\pi}{6} \times \frac{T}{2\pi} = \frac{T}{12}$$

Step 4: Check the options.

$T/4$ is when the emf reaches the maximum value. $T/8$ and $T/16$ correspond to angles of 45° and 22.5° .

Final Answer:

The emf is half its maximum after $\frac{T}{12}$, option (C).

$$\boxed{\frac{T}{12}}$$

Quick Tip: Set $e_0 \sin(\omega t)$ equal to e_0 divided by 2 and use $\omega = 2\pi / T$.

39. A resistor of $100 \, \Omega$, inductor of self inductance $(\frac{4}{\pi^2})$ H and a capacitor of unknown capacity are connected in series to an a.c. source of 200 V and 50 Hz. When the current and voltage are in phase, the value of capacity is

- (A) $40\mu F$
- (B) $50\mu F$
- (C) $20\mu F$
- (D) $25\mu F$

Correct Answer: (D) $25\mu F$

Solution:

Step 1: Understanding the Concept:

In a series LCR circuit, current and voltage are in phase when the inductive reactance equals the capacitive reactance (resonance).

Step 2: Key Formula or Approach:

$$\omega^2 LC = 1 \Rightarrow C = \frac{1}{\omega^2 L}$$

Step 3: Calculate.

$$\omega = 2\pi f = 2\pi \times 50 = 100\pi \text{ rad/s, and } L = \frac{4}{\pi^2} \text{ H.}$$

$$\omega^2 L = 10^4 \pi^2 \times \frac{4}{\pi^2} = 4 \times 10^4$$

$$C = \frac{1}{4 \times 10^4} = 2.5 \times 10^{-5} \text{ F} = 25 \mu\text{F}$$

Step 4: Check the options.

Only $25 \mu\text{F}$ matches. The resistance and the supply voltage do not matter at resonance.

Final Answer:

The capacitance is $25 \mu\text{F}$, option (D).

$25 \mu\text{F}$

Quick Tip: Current and voltage are in phase at resonance, so $X_L = X_C$.

40. If a parallel beam of light is incident on spherical mirrors then after reflection they pass through focal point. If F is the focal length and R is the radius of curvature of mirror then

- (A) $R = F$
- (B) $R = 2F$
- (C) $R = F/2$
- (D) $R = F/4$

Correct Answer: (B) $R = 2F$

Solution:

Step 1: Understanding the Concept:

For a spherical mirror of small aperture, the focal point is at the middle of the line joining the pole and the centre of curvature.

Step 2: Detailed Explanation:

So $F = \frac{R}{2}$, which means $R = 2F$.

Step 3: Check the options.

$R = F$ would put the focus at the centre of curvature, and $R = F/2$ or $F/4$ would put it beyond the centre.

Final Answer:

$R = 2F$, option (B).

$$\boxed{R = 2F}$$

Quick Tip: For a spherical mirror the focus lies halfway between the pole and the centre of curvature.



41. Choose the correct statement from the following.

Brewster's angle for a transparent medium is

- (A) different for light of different colours
- (B) same for lights of different colours
- (C) independent of refractive index of the medium
- (D) different for lights of same colour

Correct Answer: (A) different for light of different colours

Solution:

Step 1: Understanding the Concept:

Brewster law says that reflected light is fully polarised when $\tan i_p = \mu$, where μ is the refractive index of the medium.

Step 2: Detailed Explanation:

The refractive index changes with wavelength, so μ is different for different colours. So $i_p = \tan^{-1} \mu$ is different for different colours.

Step 3: Check the options.

(B) says the same for all colours, which is wrong. (C) says it does not depend on refractive index, which is the opposite of the law. (D) says it differs for light of the same colour, which is wrong because the same colour has one wavelength and one index.

Final Answer:

Brewster's angle differs for different colours, option (A).

(A)

Quick Tip: Brewster angle is tan inverse of the refractive index, which depends on the colour of light.



42. In Young's double slit experiment, the angular width of a fringe is found to be 0.2° on a screen placed 1m away. The wavelength of light used is 600 nm. If the entire apparatus is immersed in water of refractive index $4/3$, the angular width of the fringe will be

- (A) 0.10°
- (B) 0.15°
- (C) 0.20°
- (D) 0.25°

Correct Answer: (B) 0.15°

Solution:**Step 1: Understanding the Concept:**

The angular fringe width is $\theta = \frac{\lambda}{d}$. When the apparatus goes into a medium of index μ , the wavelength becomes λ/μ , while d stays the same.

Step 2: Calculate.

$$\theta_{water} = \frac{\theta_{air}}{\mu} = \frac{0.2^\circ}{4/3} = 0.2^\circ \times \frac{3}{4} = 0.15^\circ$$

Step 3: Check the options.

0.10° would need $\mu = 2$. 0.20° means no change, which would be true only in air. 0.25° is bigger and would mean the fringes widened.

Final Answer:

The angular width becomes 0.15° , option (B).

0.15°

Quick Tip: The wavelength in water is the air wavelength divided by the refractive index, and angular width is proportional to wavelength.



43. According to corpuscular theory of light, Which is NOT the property of light?

- (A) The velocity of light does not change after reflection
- (B) Light travels in a straight line
- (C) The velocity of light in air is greater than in glass
- (D) The velocity of light changes after refraction

Correct Answer: (C) The velocity of light in air is greater than in glass

Solution:

Step 1: Understanding the Concept:

Newton corpuscular theory treated light as tiny particles. To explain refraction towards the normal, it assumed that light speeds up on entering a denser medium.

Step 2: Check statement (A).

The speed stays the same after reflection in a given medium. This agrees with the theory.

Step 3: Check statement (B).

Particles travel in straight lines, so light does too. This agrees with the theory.

Step 4: Check statement (C).

The theory predicts a greater speed in glass than in air. Statement (C) says the opposite, so it is NOT a property according to this theory. Experiments (Foucault) later showed that light is slower in glass, which supported the wave theory.

Step 5: Check statement (D).

The speed changes after refraction, because the medium changes. This agrees with the theory.

Final Answer:

Statement (C) is not a property under the corpuscular theory, option (C).

(C)

Quick Tip: The corpuscular theory says light is faster in a denser medium, so claiming it is faster in air is not its prediction.

44. Energy of the incident photons on the photosensitive metal surface is $3W$ and then $5W$ where ' W ' is the work function of that metal. The ratio of velocities of the emitted photoelectrons is

- (A) $1 : 1$
- (B) $1 : \sqrt{2}$
- (C) $1 : 2$
- (D) $1 : 4$

Correct Answer: (B) $1 : \sqrt{2}$

Solution:

Step 1: Understanding the Concept:

Einstein equation: $KE_{max} = h\nu - W$. The speed of the emitted electron is $v = \sqrt{2KE/m}$.

Step 2: Find the kinetic energies.

First case: $3W - W = 2W$. Second case: $5W - W = 4W$.

Step 3: Find the speed ratio.

$$\frac{v_1}{v_2} = \sqrt{\frac{2W}{4W}} = \frac{1}{\sqrt{2}}$$

Step 4: Check the options.

$1 : 2$ is the ratio of kinetic energies, and $1 : 4$ would be the case if kinetic energy were proportional to speed. $1 : 1$ is not possible with different energies.

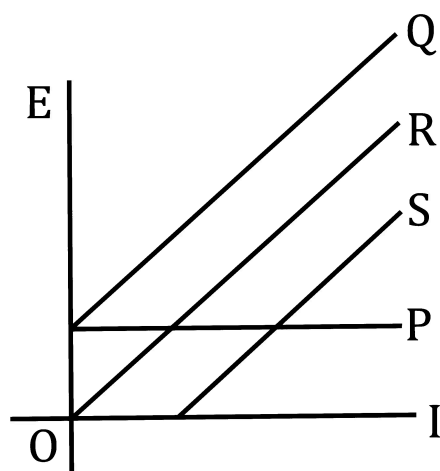
Final Answer:

The ratio of velocities is $1 : \sqrt{2}$, option (B).

$$1 : \sqrt{2}$$

Quick Tip: Use Einstein photoelectric equation to get the kinetic energies, then the speed ratio from the square root.

45. Which one of the four graphs showing lines P, Q, R and S between maximum kinetic energy (E) and intensity of incident radiation (I) is correct?



- (A) P
- (B) Q
- (C) R
- (D) S

Correct Answer: (A) P

Solution:

Step 1: Recall Einstein's equation:

The maximum kinetic energy of photoelectrons is $E_{max} = h\nu - \phi$. It depends on the frequency ν of the light and on the work function ϕ of the metal.

Step 2: Role of intensity:

Intensity only tells us how many photons fall on the metal per second. More photons release more electrons, so the photocurrent grows, but each electron still gets the same energy $h\nu - \phi$.

Step 3: Read the graphs:

In the figure, Q, R and S are inclined straight lines, which would mean the kinetic energy grows with intensity. P is a horizontal line parallel to the intensity axis.

Step 4: Pick the line:

At fixed frequency, E_{max} stays constant as I changes, so the graph is horizontal. Lines Q, R and S are wrong because they show E_{max} rising with I .

Final Answer:

The correct graph is the horizontal line P, option (A).

P

Quick Tip: Maximum kinetic energy depends on frequency, not on intensity, so the graph against intensity is horizontal.

46. The ratio of centripetal acceleration for an electron revolving in 3rd and 5th Bohr orbit of hydrogen atom is

- (A) 25 : 9
- (B) 125 : 27
- (C) 625 : 81
- (D) 5 : 3

Correct Answer: (C) 625 : 81

Solution:

Step 1: Understanding the Concept:

In Bohr model the speed of the electron is $v \propto \frac{1}{n}$ and the radius is $r \propto n^2$.

Step 2: Find the dependence.

$$a_c = \frac{v^2}{r} \propto \frac{1/n^2}{n^2} = \frac{1}{n^4}$$

Step 3: Take the ratio.

$$\frac{a_3}{a_5} = \frac{5^4}{3^4} = \frac{625}{81}$$

Step 4: Check the options.

25 : 9 is $(5/3)^2$, which comes from using $1/n^2$. 125 : 27 is $(5/3)^3$, which also does not follow. 5 : 3 is the ratio of the principal quantum numbers.

Final Answer:

The ratio is 625 : 81, option (C).

$$625 : 81$$

Quick Tip: Speed varies as $1/n$ and radius as n^2 , so centripetal acceleration varies as $1/n^3$ to the fourth.

47. An electron is revolving in a circular orbit of radius 'r' in hydrogen atom. Using Bohr's theory, the angular momentum of the electron is (M = magnetic dipole moment, m = mass of electron)

- (A) $\frac{mM}{e}$
- (B) $\frac{e}{mM}$
- (C) $\frac{2mM}{e}$
- (D) $\frac{e}{2mM}$

Correct Answer: (C) $\frac{2mM}{e}$

Solution:**Step 1: Understanding the Concept:**

A revolving electron acts like a current loop. Its magnetic dipole moment is $M = IA$.

Step 2: Find the moment.

Current $I = \frac{e}{T}$ with $T = \frac{2\pi r}{v}$, so $I = \frac{ev}{2\pi r}$. Area $A = \pi r^2$.

$$M = \frac{ev}{2\pi r} \times \pi r^2 = \frac{evr}{2}$$

Step 3: Link to angular momentum.

Angular momentum $L = mvr$, so $vr = \frac{L}{m}$.

$$M = \frac{e}{2} \cdot \frac{L}{m} \Rightarrow L = \frac{2mM}{e}$$

Step 4: Check the options.

Option (A) misses the factor 2. Options (B) and (D) have e in the numerator, which is dimensionally wrong.

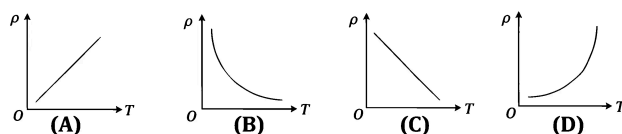
Final Answer:

The angular momentum is $\frac{2mM}{e}$, option (C).

$$\boxed{\frac{2mM}{e}}$$

Quick Tip: Express the magnetic moment through the orbital current, then relate it to angular momentum $m v r$.

48. Which of the following graph represents the temperature (T) dependence of resistivity (ρ) of a semiconductor?



- (A) A
- (B) B
- (C) C
- (D) D

Correct Answer: (B) B

Solution:

Step 1: Recall the behaviour:

In a metal, resistivity rises with temperature. In a semiconductor, the opposite happens: resistivity falls as temperature rises.

Step 2: Reason:

Heating gives more electrons enough energy to jump from the valence band to the conduction band. The number of free charge carriers grows fast, in an exponential way, so $\rho \propto e^{E_g/(2k_B T)}$ falls quickly at low T and flattens at high T.

Step 3: Read the graphs:

Graph A is a straight line rising with T (typical of a metal). Graph B is a curve falling steeply and then flattening, as expected. Graph C is a straight falling line. Graph D is a curve that rises.

Step 4: Pick the graph:

Graph B matches the exponential fall. Graph C is also falling, but a straight line would mean a constant rate of fall, which does not match an exponential law. A and D rise, so they are wrong for a semiconductor.

Final Answer:

The curve of resistivity falling non-linearly with temperature is graph B.

B

Quick Tip: In a semiconductor more carriers are freed as temperature rises, so resistivity falls in a curved, exponential way.

49. In common emitter mode of transistor, the d.c. current gain is 20, the emitter current is 7 mA. The collector current is

- (A) $\frac{20}{3}$ mA
- (B) $\frac{14}{5}$ mA
- (C) $\frac{1}{3}$ mA
- (D) $\frac{8}{13}$ mA

Correct Answer: (A) $\frac{20}{3}$ mA

Solution:

Step 1: Understanding the Concept:

In a transistor, $I_E = I_B + I_C$, and the dc current gain in common emitter mode is $\beta = \frac{I_C}{I_B}$.

Step 2: Detailed Explanation:

$$\beta = 20, \text{ so } I_B = \frac{I_C}{20}.$$

$$I_E = \frac{I_C}{20} + I_C = \frac{21}{20} I_C \Rightarrow I_C = \frac{20}{21} \times 7 = \frac{20}{3} \text{ mA}$$

Step 3: Check the options.

$\frac{20}{3} \approx 6.67$ mA is slightly less than the emitter current, as it should be.

The other options are far too small.

Final Answer:

The collector current is $\frac{20}{3}$ mA, option (A).

$$\boxed{\frac{20}{3} \text{ mA}}$$

Quick Tip: The emitter current is the sum of the base and collector currents, and the dc current gain beta is I_c over I_b .

50. In transistor amplifier, base emitter junction is forward biased and collector base junction is reversed biased. The current gain is

- (A) $\frac{\Delta I_C}{\Delta I_B}$
- (B) $\frac{\Delta I_B}{\Delta I_C}$
- (C) $\frac{\Delta I_E}{\Delta I_B}$
- (D) $\frac{\Delta I_B}{\Delta I_E}$

Correct Answer: (A) $\frac{\Delta I_C}{\Delta I_B}$

Solution:

Step 1: Understanding the Concept:

In the normal working of a transistor amplifier, the emitter-base junction is forward biased and the collector-base junction is reverse

biased.

Step 2: Detailed Explanation:

The current gain β is the ratio of change in collector current to change in base current: $\beta = \frac{\Delta I_C}{\Delta I_B}$.

Step 3: Check the options.

Option (B) is the inverse. Option (C) divides emitter current change by base current change, which equals $\beta + 1$. Option (D) is the inverse of that kind of ratio.

Final Answer:

The current gain is $\frac{\Delta I_C}{\Delta I_B}$, option (A).

$$\boxed{\frac{\Delta I_C}{\Delta I_B}}$$

Quick Tip: The common emitter current gain is the ratio of change in collector current to change in base current.

Chemistry

Questions 51 to 100 | 1 mark each | 50 marks

51. Which of the following contains maximum number of molecules?

- (A) 0.4 g H_2 gas
- (B) 3.2 g of O_2 gas
- (C) 16.8 dm^3 of H_2 gas at STP
- (D) 11.2 dm^3 of O_2 gas at STP

Correct Answer: (C) 16.8 dm^3 of H_2 gas at STP

Solution:

Step 1: Understanding the Concept:

The number of molecules is proportional to the number of moles, $N = nN_A$.

Step 2: Find moles:

A: $\frac{0.4}{2} = 0.2 \text{ mol of } \text{H}_2$.

B: $\frac{3.2}{32} = 0.1 \text{ mol of } \text{O}_2$.

C: $\frac{16.8}{22.4} = 0.75 \text{ mol of } \text{H}_2 \text{ at STP}$.

D: $\frac{11.2}{22.4} = 0.5 \text{ mol of } \text{O}_2 \text{ at STP}$.

Step 3: Compare:

$0.75 > 0.5 > 0.2 > 0.1$, so C has the most molecules.

Final Answer:

Sample C has the most molecules, option (C).

16.8 dm³ of H₂ at STP

Quick Tip: Convert each sample to moles; the largest number of moles has the most molecules.



52. What is the atomic number of an element having ns^1 electronic configuration and belonging to 3d transition series?

- (A) Only 24
- (B) Only 25
- (C) Only 29
- (D) 24 and 29

Correct Answer: (D) 24 and 29

Solution:

Step 1: Understanding the Concept:

Most 3d elements have $4s^2$, but two elements take one electron from $4s$ to give a half-filled or completely filled $3d$ subshell, which is more stable.

Step 2: The two cases:

Chromium ($Z = 24$): $3d^5 4s^1$, a half-filled d subshell.

Copper ($Z = 29$): $3d^{10} 4s^1$, a fully filled d subshell.

Step 3: Conclusion:

Mn ($Z = 25$) is $3d^5 4s^2$, so it does not have ns^1 . Therefore the atomic numbers are 24 and 29.

Final Answer:

The atomic numbers are 24 and 29, option (D).

24 and 29

Quick Tip: Chromium and copper have $4s^1$ because of the extra stability of half-filled and full d subshells.



53. The electronic configurations of elements are as-



Which of the following is formula of ionic compound that could be formed between these two elements A and B?

- (A) A_2B
- (B) AB_2
- (C) AB_5
- (D) A_5B_2

Correct Answer: (B) AB_2

Solution:**Step 1: Understanding the Concept:**

Identify the elements from the configurations, then balance the charges.

Step 2: Identify:

A: $1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$ has 20 electrons, so it is calcium. It loses 2 electrons to give A^{2+} .

B: $1s^2 2s^2 2p^6 3s^2 3p^5$ has 17 electrons, so it is chlorine. It gains 1 electron to give B^- .

Step 3: Balance charges:

One A^{2+} needs two B^- , so the formula is AB_2 (like $CaCl_2$).

The other formulae do not balance: A_2B gives +4 and -1, AB_5 gives +2 and -5, A_5B_2 gives +10 and -2.

Final Answer:

The formula of the ionic compound is AB_2 , option (B).



Quick Tip: Element A has two valence electrons (calcium) and B has seven (chlorine).

54. Which of the following pairs represents isobars?

- (A) ${}^3\text{He}_2$ and ${}^4\text{He}_2$
- (B) ${}^{24}\text{Mg}_{12}$ and ${}^{25}\text{Mg}_{12}$
- (C) ${}^{40}\text{K}_{19}$ and ${}^{40}\text{Ca}_{20}$
- (D) ${}^{40}\text{K}_{19}$ and ${}^{39}\text{K}_{19}$

Correct Answer: (C) ${}^{40}\text{K}_{19}$ and ${}^{40}\text{Ca}_{20}$

Solution:

Step 1: Understanding the Concept:

Isobars are atoms of different elements with the same mass number A . Isotopes have the same atomic number Z but different A .

Step 2: Check each pair:

A: ${}^3\text{He}_2$ and ${}^4\text{He}_2$: same Z , different A , so they are isotopes.

B: ${}^{24}\text{Mg}_{12}$ and ${}^{25}\text{Mg}_{12}$: same Z , different A , isotopes.

C: ${}^{40}\text{K}_{19}$ and ${}^{40}\text{Ca}_{20}$: same $A = 40$, different Z . These are isobars.

D: ${}^{40}\text{K}_{19}$ and ${}^{39}\text{K}_{19}$: isotopes.

Final Answer:

The isobars are ${}^{40}\text{K}_{19}$ and ${}^{40}\text{Ca}_{20}$, option (C).



Quick Tip: Isobars have the same mass number but different atomic numbers.

55. Identify the type of hybridization of carbon that is attached to the OH group, in benzylic alcohol?

- (A) sp
- (B) sp^2
- (C) sp^3
- (D) dsp^2

Correct Answer: (C) sp^3

Solution:

Step 1: Understanding the Concept:

Benzyl alcohol is $\text{C}_6\text{H}_5\text{-CH}_2\text{-OH}$. The carbon carrying OH is the CH_2 carbon, not a ring carbon.

Step 2: Count bonds:

This carbon forms four sigma bonds (with the ring carbon, two hydrogens and oxygen) and no pi bonds. Four sigma bonds mean sp^3 hybridisation.

Step 3: Other options:

The ring carbons are sp^2 , which might tempt one to choose B, but the question asks about the carbon attached to OH. sp is for triple bonds, and dsp^2 is for square planar metal complexes.

Final Answer:

The carbon is sp^3 hybridised, option (C).



Quick Tip: The carbon bonded to OH in benzyl alcohol is a CH_2 group with four single bonds.

56. Calculate the temperature of 2 moles of gas occupying a volume of 5.0 liters at 24.6 atmospheres?

($R = 0.0821 \text{ L atm K}^{-1} \text{ mol}^{-1}$)

- (A) 476°C
- (B) 480°C
- (C) 440°C
- (D) 450°C

Correct Answer: (A) 476°C

Solution:

Step 1: Key Formula:

The ideal gas equation is $PV = nRT$, so $T = \frac{PV}{nR}$.

Step 2: Calculate:

$$T = \frac{24.6 \times 5.0}{2 \times 0.0821} = \frac{123}{0.1642} = 749.1 \text{ K}$$

Convert to Celsius: $749.1 - 273 = 476.1^\circ\text{C}$, about 476°C .

Step 3: Other options:

Values 440 , 450 and 480°C do not match $T = 749 \text{ K}$, and 749 K itself is not listed, since the options are in Celsius.

Final Answer:

The temperature is about 476°C , option (A).

476°C

Quick Tip: Use $PV = nRT$ and convert the Kelvin result to Celsius.

57. For a certain reaction $\Delta H = -50 \text{ kJ}$ and $\Delta S = -100 \text{ J/K}$ find the temperature so that the reaction is spontaneous.

- (A) 650 K
- (B) 600 K
- (C) 550 K
- (D) 450 K

Correct Answer: (D) 450 K

Solution:

Step 1: Understanding the Concept:

A reaction is spontaneous when its Gibbs free energy change is negative. Here both the enthalpy change and the entropy change are negative, so temperature decides the sign of ΔG .

Step 2: Key Formula or Approach:

$$\Delta G = \Delta H - T\Delta S$$

The condition for spontaneity is $\Delta G < 0$. Keep the units of ΔH and ΔS consistent, so convert kJ to J.

Step 3: Detailed Explanation:

$\Delta H = -50 \text{ kJ} = -50000 \text{ J}$ and $\Delta S = -100 \text{ J/K}$.

$$\Delta G = -50000 - T(-100) = -50000 + 100 T$$

For $\Delta G < 0$ we need $100 T < 50000$, so $T < 500 \text{ K}$.

The reaction is spontaneous only below 500 K. At higher temperature the negative entropy term $-T\Delta S$ becomes larger than the favourable ΔH .

Step 4: Check the options.

650 K, 600 K and 550 K are all above 500 K, so ΔG is positive there and the reaction is not spontaneous. Only 450 K is below 500 K.

Final Answer:

The reaction is spontaneous only for $T < 500 \text{ K}$, so the required temperature is 450 K, option (D).

450 K

Quick Tip: A reaction is spontaneous when the Gibbs energy change is negative, so write the Gibbs equation and solve for the temperature.



58. Calculate the enthalpy change of the reaction,



if bond energies (kJ mol^{-1}):

$\text{H}-\text{H} = 436$, $\text{Cl}-\text{Cl} = 242$, $\text{H}-\text{Cl} = 431$

(A) -184 kJ/mol

(B) -246 kJ/mol

(C) -242 kJ/mol

(D) -431 kJ/mol

Correct Answer: (A) -184 kJ/mol

Solution:

Step 1: Understanding the Concept:

Breaking bonds takes energy and forming bonds releases it. So the reaction enthalpy is found by comparing the two totals.

Step 2: Key Formula or Approach:

$$\Delta H = \sum (\text{bond energies of bonds broken}) - \sum (\text{bond energies of bonds formed})$$

Step 3: Detailed Explanation:

Bonds broken: one H-H (436) and one Cl-Cl (242). Total = $436 + 242 = 678$ kJ/mol.

Bonds formed: two H-Cl bonds, since 2 moles of HCl form. Total = $2 \times 431 = 862$ kJ/mol.

$$\Delta H = 678 - 862 = -184 \text{ kJ/mol}$$

The negative sign shows the reaction releases heat.

Step 4: Why the other options are wrong.

The values -242 and -431 are just single bond energies of Cl-Cl and H-Cl, not a difference between broken and formed totals. The value -246 does not follow from the data at all. Option (A) is the only value that follows the formula.

Final Answer:

Energy released on forming 2 H-Cl bonds exceeds the energy needed to break H-H and Cl-Cl by 184 kJ.

-184 kJ/mol

Quick Tip: Enthalpy change equals bond energy of bonds broken minus bond energy of bonds formed.

59. A weak monobasic acid is 0.04 % dissociated in 0.25 M solution. What is pH of the solution?

- (A) 2.5
- (B) 4
- (C) 5
- (D) 10

Correct Answer: (B) 4

Solution:

Step 1: Understanding the Concept:

For a weak monobasic acid HA, each molecule that dissociates gives one H^+ . So $[\text{H}^+] = C\alpha$, where C is the initial concentration and α is the degree of dissociation.

Step 2: Key Formula or Approach:

$$[\text{H}^+] = C\alpha, \quad \text{pH} = -\log[\text{H}^+]$$

Step 3: Detailed Explanation:

Percentage dissociation is 0.04 %, so $\alpha = \frac{0.04}{100} = 4 \times 10^{-4}$.

$$[\text{H}^+] = 0.25 \times 4 \times 10^{-4} = 1 \times 10^{-4} \text{ M}$$

$$\text{pH} = -\log(10^{-4}) = 4$$

Step 4: Why the other options are wrong.

pH 2.5 and pH 5 would need $[\text{H}^+]$ of about $3.2 \times 10^{-3} \text{ M}$ and 10^{-5} M .
pH 10 is basic, which cannot be true for an acid solution. Only pH 4

matches 10^{-4} M.

Final Answer:

The hydrogen ion concentration is 10^{-4} M, so the pH is 4, option (B).

4

Quick Tip: Convert the percentage to a degree of dissociation, multiply by the concentration to get the hydrogen ion concentration, then take the negative log.

60. What is the relationship between the degree of dissociation of a weak base and its concentration?

- (A) The degree of dissociation of a weak base is directly proportional to square root of its concentration.
- (B) The degree of dissociation of a weak base is inversely proportional to square root of its concentration.
- (C) The degree of dissociation of a weak base doesn't depend on concentration.
- (D) The degree of dissociation of a weak base is directly proportional to its concentration

Correct Answer: (B) The degree of dissociation of a weak base is inversely proportional to square root of its concentration.

Solution:

Step 1: Understanding the Concept:

A weak base only partly ionises. How far it ionises depends on its

concentration. The more dilute the solution, the larger the fraction of ionised molecules.

Step 2: Key Formula or Approach:

Ostwald dilution law for a weak base BOH of concentration C :

$$K_b = \frac{C\alpha^2}{1 - \alpha} \approx C\alpha^2 \quad (\alpha \ll 1)$$

Step 3: Detailed Explanation:

Solve for α :

$$\alpha = \sqrt{\frac{K_b}{C}}$$

So $\alpha \propto \frac{1}{\sqrt{C}}$. If the concentration becomes 4 times, α halves.

Step 4: Why the other options are wrong.

(A) says directly proportional to $\sqrt{C}$, which is the opposite trend. (C) is wrong because α clearly changes with dilution. (D) says directly proportional to C , again the wrong direction.

Final Answer:

The degree of dissociation is inversely proportional to the square root of concentration, option (B).

$$\boxed{\alpha \propto \frac{1}{\sqrt{C}}}$$

Quick Tip: Use Ostwald dilution law: the degree of dissociation varies as the inverse square root of concentration.

61. Which of the following sentences is true for a basic solution?

- (A) For basic solution, $[\text{H}_3\text{O}^+] = [\text{OH}^-]$
- (B) In basic solution, the excess of OH^- ions are present.
- (C) In the basic solution, there is excess of H_3O^+ ions.
- (D) The pOH of a basic solution can be calculated by $\text{pOH} = -\log[\text{H}^+]$

Correct Answer: (B) In basic solution, the excess of OH^- ions are present.

Solution:

Step 1: Understanding the Concept:

Acidic, neutral and basic solutions are told apart by comparing $[\text{H}_3\text{O}^+]$ with $[\text{OH}^-]$.

Step 2: Check statement (A).

$[\text{H}_3\text{O}^+] = [\text{OH}^-]$ describes a neutral solution, such as pure water at 25 degrees C. So (A) is FALSE.

Step 3: Check statement (B).

In a basic solution $[\text{OH}^-] > [\text{H}_3\text{O}^+]$, so excess hydroxide ions are present. So (B) is TRUE.

Step 4: Check statement (C).

Excess H_3O^+ is the definition of an acidic solution. So (C) is FALSE.

Step 5: Check statement (D).

The correct relation is $\text{pOH} = -\log[\text{OH}^-]$. The expression $-\log[\text{H}^+]$ is pH, not pOH. So (D) is FALSE.

Final Answer:

Only statement (B) is correct for a basic solution.

(B)

Quick Tip: A basic solution has more hydroxide ions than hydronium ions.

62. Which of the following is NOT a redox reaction?

- (A) $\text{Zn} + \text{H}_2\text{SO}_4 \rightarrow \text{ZnSO}_4 + \text{H}_2$
- (B) $\text{Al}(\text{OH})_3 + 3\text{HCl} \rightarrow \text{AlCl}_3 + 3\text{H}_2\text{O}$
- (C) $\text{Mg} + 1/2\text{O}_2 \rightarrow \text{Mg}^{2+} + \text{O}^{2-}$
- (D) $\text{Fe}_{(s)} + \text{Cu}_{(aq)}^{2+} \rightarrow \text{Fe}_{(aq)}^{2+} + \text{Cu}_{(s)}$

Correct Answer: (B) $\text{Al}(\text{OH})_3 + 3\text{HCl} \rightarrow \text{AlCl}_3 + 3\text{H}_2\text{O}$

Solution:

Step 1: Understanding the Concept:

A redox reaction involves a change in oxidation number of at least one element, which means electron transfer. If no element changes its oxidation number, it is not redox.

Step 2: Check option (A).

Zn goes from 0 to +2 and H goes from +1 to 0. Oxidation numbers change, so it is redox.

Step 3: Check option (B).

$\text{Al}(\text{OH})_3 + 3\text{HCl} \rightarrow \text{AlCl}_3 + 3\text{H}_2\text{O}$. Al stays +3, Cl stays -1, H stays +1 and O stays -2. Nothing changes. This is an acid-base neutralisation, so it is NOT redox.

Step 4: Check option (C).

Mg goes from 0 to +2 and O goes from 0 to -2. This is redox.

Step 5: Check option (D).

Fe goes from 0 to +2 and Cu^{2+} goes to Cu(0). This is redox, a displacement reaction.

Final Answer:

Only reaction (B) has no change in oxidation numbers.

(B)

Quick Tip: A redox reaction must show a change in oxidation number, and a neutralisation does not.

63. Identify the false statement regarding chirality from the following.

- (A) SN^1 reaction yields 1:1 mixture of both enantiomers
- (B) A racemic mixture shows zero dipole rotation
- (C) Enantiomers are superimposable mirror images of each other
- (D) The product obtained by SN^2 reaction of haloalkane having chirality at the reactive site shows inversion of configuration

Correct Answer: (C) Enantiomers are superimposable mirror images of each other

Solution:

Step 1: Understanding the Concept:

Chirality describes molecules that cannot be placed on top of their mirror image. The substitution mechanisms S_N1 and S_N2 affect the configuration of a chiral carbon in different ways.

Step 2: Check statement (A).

In S_N1 , a planar carbocation forms first. The nucleophile attacks from either face with equal chance, so a 1:1 mixture of enantiomers forms. So (A) is TRUE.

Step 3: Check statement (B).

A racemic mixture holds equal amounts of both enantiomers. Their optical rotations cancel, so net rotation is zero. So (B) is TRUE.

Step 4: Check statement (C).

Enantiomers are mirror images that are NON-superimposable. Superimposable mirror images belong to an achiral molecule. So (C) is FALSE.

Step 5: Check statement (D).

In S_N2 the nucleophile attacks from the back side, so the configuration at the carbon is inverted (Walden inversion). So (D) is TRUE.

Final Answer:

The false statement is (C).

Quick Tip: Enantiomers are non-superimposable mirror images, so the statement calling them superimposable is false.



64. Which of the following statements are TRUE?

- (I) Inductive effect operates through the sigma bond
 - (II) Resonance effect operates through the pi bond
 - (III) Inductive effects operate through the pi bond
 - (IV) Resonance effect operates through the sigma bond
- (A) (I) and (II)
 - (B) (II) and (III)
 - (C) (III) and (IV)
 - (D) (I) and (IV)

Correct Answer: (A) (I) and (II)

Solution:

Step 1: Understanding the Concept:

Electron displacement effects decide how charge is spread in an organic molecule. Two key ones are the inductive effect and the resonance (mesomeric) effect.

Step 2: Inductive effect.

It is the permanent shift of electrons along a chain of sigma bonds because of an electronegative atom or group. It works through sigma bonds only. So (I) is TRUE and (III) is FALSE.

Step 3: Resonance effect.

It is the delocalisation of pi electrons or lone pairs through a conjugated system. It works through pi bonds and not sigma bonds. So (II) is TRUE and (IV) is FALSE.

Step 4: Pick the pair.

The true statements are (I) and (II), which is option (A). Options (B), (C) and (D) each include (III) or (IV), which are false.

Final Answer:

Inductive effect travels via sigma bonds, resonance effect via pi bonds.

(I) and (II)

Quick Tip: The inductive effect moves sigma electrons, while the resonance effect moves pi electrons.

65. The cyclic π -molecular orbital formed by overlap of p-orbitals must contain,

- (A) $(4n + 2)$ p electrons.
- (B) $(2n + 2)$ p electrons.
- (C) $(4n - 2)$ p electrons.
- (D) $(2n - 2)$ p electrons.

Correct Answer: (A) $(4n + 2)$ p electrons.

Solution:

Step 1: Understanding the Concept:

A cyclic, planar, fully conjugated system is aromatic and specially stable when its pi electron count obeys the Huckel rule.

Step 2: Key Formula or Approach:

$$\text{number of } \pi \text{ electrons} = 4n + 2, \quad n = 0, 1, 2, \dots$$

Step 3: Detailed Explanation:

For $n = 0, 1, 2$ the count is 2, 6, 10. Benzene has 6 pi electrons ($n = 1$) and is aromatic. A count of $4n$ electrons (such as 4 or 8) gives an anti-aromatic, unstable ring.

Step 4: Check the options.

(B) $2n + 2$ gives 2, 4, 6, 8, which includes anti-aromatic counts. (C) $4n - 2$ and (D) $2n - 2$ are not the Huckel expression. Only (A) matches.

Final Answer:

The cyclic pi system needs $(4n + 2)$ p electrons, option (A).

$(4n + 2)$

Quick Tip: Huckel rule for aromatic stability needs $4n+2$ pi electrons.

66. Identify the reagent used in Etard reaction

- (A) Acetic anhydride
- (B) Anhydrous aluminum chloride
- (C) DIBAL-H
- (D) Chromyl chloride

Correct Answer: (D) Chromyl chloride

Solution:

Step 1: Understanding the Concept:

The Etard reaction is a named method that converts an aromatic methyl group into an aldehyde group without going on to the acid.

Step 2: Key Formula or Approach:

Toluene reacts with chromyl chloride (CrO_2Cl_2) in CS_2 to give a chromium complex. Hydrolysis with water gives benzaldehyde.

Step 3: Detailed Explanation:

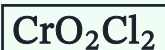


Step 4: Why the other options are wrong.

Chromic oxide in acetic anhydride is a different oxidation of toluene that gives benzylidene diacetate. Anhydrous AlCl_3 is the catalyst in Friedel-Crafts and Gattermann-Koch reactions. DIBAL-H reduces esters and nitriles to aldehydes.

Final Answer:

The Etard reaction uses chromyl chloride, option (D).



Quick Tip: Etard reaction oxidises toluene to benzaldehyde using chromyl chloride followed by hydrolysis.



67. An element with a molar mass 27g mol^{-1} forms a cubic unit cell. Calculate the number of atoms present in a unit cell if the density of metal is 2.7g cm^{-3} .

$$[a^3 \times N_A = 40\text{ cm}^3\text{ mol}^{-1}]$$

- (A) 1
- (B) 2
- (C) 4
- (D) 6

Correct Answer: (C) 4

Solution:

Step 1: Understanding the Concept:

The density of a crystal is the mass of the atoms in one unit cell divided by the volume of that cell. From that we can count how many atoms Z the cell holds.

Step 2: Key Formula or Approach:

$$d = \frac{Z \times M}{a^3 \times N_A} \Rightarrow Z = \frac{d \times (a^3 N_A)}{M}$$

Step 3: Detailed Explanation:

Given $d = 2.7 \text{ g cm}^{-3}$, $M = 27 \text{ g mol}^{-1}$ and $a^3 N_A = 40 \text{ cm}^3 \text{ mol}^{-1}$.

$$Z = \frac{2.7 \times 40}{27} = \frac{108}{27} = 4$$

Four atoms per cubic unit cell means the structure is face-centred cubic (8 corners $\times 1/8 + 6$ faces $\times 1/2 = 4$).

Step 4: Why the other options are wrong.

$Z = 1$ is simple cubic, $Z = 2$ is body-centred cubic. With $Z = 1$ the density would be only 0.675 g/cm^3 and with $Z = 2$ only 1.35 g/cm^3 , not 2.7. $Z = 6$ is not a common cubic arrangement.

Final Answer:

Each unit cell holds 4 atoms, option (C).

$$\boxed{Z = 4}$$

Quick Tip: Use density = $Z \times M / (a^3 \times N_A)$ and the given value of $a^3 \times N_A$ to find Z .

68. A compound formed by elements A and B crystallizes in a cubic structure where A atoms are at the corner of the cube and B atoms are at the center of the cube. What is the formula of the compound?

- (A) AB
- (B) A_2B
- (C) AB_2
- (D) A_2B_2

Correct Answer: (A) AB

Solution:

Step 1: Understanding the Concept:

In a cubic unit cell, a corner atom is shared by 8 cells, so only one-eighth of it belongs to the cell. A body-centre atom lies wholly inside.

Step 2: Detailed Explanation:

A atoms at 8 corners: $8 \times \frac{1}{8} = 1$.

B atom at the body centre: 1.

Ratio A : B = 1 : 1, so the formula is AB.

Step 3: Why the other options are wrong.

A₂B and AB₂ would need a 2 : 1 or 1 : 2 ratio. A₂B₂ is just AB written as a multiple, not the simplest formula.

Final Answer:

The simplest formula is AB, option (A).

AB

Quick Tip: Count each corner atom as one-eighth and the body centre atom as one whole.

69. Which of the following dopant is NOT used in Ge to obtain n-type semiconductor?

- (A) P
- (B) As
- (C) Sb
- (D) B

Correct Answer: (D) B

Solution:

Step 1: Understanding the Concept:

Germanium is in group 14 with four valence electrons. Doping adds a small amount of another element. A group 15 dopant has an extra electron, giving an n-type semiconductor.

Step 2: Detailed Explanation:

Phosphorus (P), arsenic (As) and antimony (Sb) are all group 15 elements with 5 valence electrons. Each uses 4 to bond with Ge and leaves one free electron. This makes n-type material.

Boron (B) is group 13 with 3 valence electrons. It leaves a missing electron, called a hole, so it makes a p-type semiconductor.

Step 3: Pick the odd one.

P, As and Sb are valid n-type dopants. Boron is not.

Final Answer:

Boron is not used for n-type doping, option (D).

B

Quick Tip: N-type doping uses group 15 elements with five valence electrons, while boron gives a p-type semiconductor.

70. 2 g nonelectrolyte solute when dissolved in 50 g water increases its boiling point by 0.13 K.

Calculate the molar mass of solute if molal elevation constant of water is $0.52 \text{ K kg mol}^{-1}$

- (A) 140 g/mol
- (B) 150 g/mol
- (C) 160 g/mol
- (D) 170 g/mol

Correct Answer: (C) 160 g/mol

Solution:

Step 1: Understanding the Concept:

When a non-volatile solute dissolves in a solvent, the boiling point rises in proportion to the molality of the solution.

Step 2: Key Formula or Approach:

$$\Delta T_b = K_b \times m, \quad m = \frac{w_2 \times 1000}{M_2 \times w_1 \text{ (in g)}}$$

Step 3: Detailed Explanation:

Given $w_2 = 2 \text{ g}$, $w_1 = 50 \text{ g}$, $\Delta T_b = 0.13 \text{ K}$ and $K_b = 0.52 \text{ K kg mol}^{-1}$.

$$M_2 = \frac{K_b \times w_2 \times 1000}{\Delta T_b \times w_1} = \frac{0.52 \times 2 \times 1000}{0.13 \times 50}$$

$$M_2 = \frac{1040}{6.5} = 160 \text{ g/mol}$$

Step 4: Why the other options are wrong.

Putting 140, 150 or 170 back into the formula gives ΔT_b of 0.149 K, 0.139 K and 0.122 K. None equals 0.13 K. Only 160 g/mol does.

Final Answer:

The molar mass of the solute is 160 g/mol, option (C).

160 g/mol

Quick Tip: Use the elevation formula with mass of solvent in kilograms, then rearrange it for molar mass.

71. Which of the following aqueous solutions will show maximum vapour pressure at 300K?

- (A) 1 M NaCl
- (B) 1 M CaCl_2
- (C) 1 M AlCl_3
- (D) 1 M $\text{C}_{12}\text{H}_{22}\text{O}_{11}$

Correct Answer: (D) 1 M $\text{C}_{12}\text{H}_{22}\text{O}_{11}$

Solution:

Step 1: Understanding the Concept:

Raoult's law says the vapour pressure of a solution is $p = x_{\text{solvent}} p^\circ$. Adding a non-volatile solute lowers the mole fraction of the solvent, so the vapour pressure falls. This is a colligative property: it depends

on the number of solute particles, not on what they are.

Step 2: Key Formula or Approach:

More particles means a bigger fall in vapour pressure. So the solution with the fewest particles shows the highest vapour pressure. The number of particles from one formula unit is given by the van't Hoff factor i .

Step 3: Detailed Explanation:

All solutions are 1 M, so we compare i .

$\text{NaCl} \rightarrow \text{Na}^+ + \text{Cl}^-$, so $i = 2$.

$\text{CaCl}_2 \rightarrow \text{Ca}^{2+} + 2\text{Cl}^-$, so $i = 3$.

$\text{AlCl}_3 \rightarrow \text{Al}^{3+} + 3\text{Cl}^-$, so $i = 4$.

Sucrose, $\text{C}_{12}\text{H}_{22}\text{O}_{11}$, is a molecular solid that does not ionise in water, so $i = 1$.

Effective particle concentrations are about 2 M, 3 M, 4 M and 1 M.

Step 4: Why the other options are wrong:

The solution of AlCl_3 has the most particles and the greatest lowering, so it has the lowest vapour pressure. CaCl_2 and NaCl also remove more solvent from the surface than sucrose does. So (A), (B) and (C) all show a smaller vapour pressure than (D).

Final Answer:

The 1 M sucrose solution has the fewest particles, so it has the maximum vapour pressure. This is option (D).

Option (D)

Quick Tip: Vapour pressure is highest for the solution with the fewest dissolved particles.



72. Identify a pair of gases from following that does not obey Henry's law.

- (A) NH_3 and CO_2
- (B) CO_2 and O_2
- (C) NH_3 and N_2
- (D) N_2 and CH_4

Correct Answer: (A) NH_3 and CO_2

Solution:

Step 1: Understanding the Concept:

Henry law says the solubility of a gas in a liquid is proportional to its partial pressure. It holds only when the gas does not react with the solvent.

Step 2: Detailed Explanation:

Ammonia reacts with water to give NH_4^+ and OH^- . Carbon dioxide reacts with water to give carbonic acid. Their solubility is far more than the law predicts.

Oxygen, nitrogen and methane dissolve physically in water without reacting, so they follow Henry law.

Step 3: Check the options.

(A) NH_3 and CO_2 : both react, so the pair does not obey. (B) CO_2 with O_2 : O_2 obeys, so not both disobey. (C) NH_3 with N_2 and (D) N_2 with CH_4 each contain a gas that obeys.

Final Answer:

Both gases in option (A) react with water, so they deviate from Henry law.

(A)

Quick Tip: Gases that react with water, such as ammonia and carbon dioxide, do not follow Henry law.



73. Calculate the molar conductivity of 0.2 M KCl solution at 298 K ($k = 0.0248\Omega^{-1}\text{cm}^{-1}$)

- (A) $143\Omega^{-1}\text{cm}^2\text{mol}^{-1}$
- (B) $98\Omega^{-1}\text{cm}^2\text{mol}^{-1}$
- (C) $124\Omega^{-1}\text{cm}^2\text{mol}^{-1}$
- (D) $87\Omega^{-1}\text{cm}^2\text{mol}^{-1}$

Correct Answer: (C) $124\Omega^{-1}\text{cm}^2\text{mol}^{-1}$

Solution:**Step 1: Understanding the Concept:**

Molar conductivity is the conductivity of all the ions produced by one mole of electrolyte. It is found by dividing conductivity by concentration, with a factor to convert litres into cm^3 .

Step 2: Key Formula or Approach:

$$\Lambda_m = \frac{\kappa \times 1000}{C}$$

where κ is in $\Omega^{-1}\text{cm}^{-1}$ and C is in mol/L.

Step 3: Detailed Explanation:

$$\Lambda_m = \frac{0.0248 \times 1000}{0.2} = \frac{24.8}{0.2} = 124 \Omega^{-1}\text{cm}^2\text{mol}^{-1}$$

Step 4: Why the other options are wrong.

Options 143, 98 and 87 do not follow from $24.8/0.2$. They come from using a wrong concentration or forgetting the 1000 factor.

Final Answer:

The molar conductivity is $124 \Omega^{-1}\text{cm}^2\text{mol}^{-1}$, option (C).

$$124 \Omega^{-1}\text{cm}^2\text{mol}^{-1}$$

Quick Tip: Molar conductivity equals conductivity times 1000 divided by molarity.



74. Identify from following cell reactions that is spontaneous under standard state of conditions.

(A) $\text{Ca(s)} + \text{Cd}^{2+}(\text{aq}) \rightarrow \text{Ca}^{2+}(\text{aq}) + \text{Cd(s)}$, [$E_{\text{Ca}}^0 = -2.866 \text{ V}$ & $E_{\text{Cd}}^0 = -0.403 \text{ V}$]

(B) $2\text{Br}^-(\text{s}) + \text{Sn}^{2+}(\text{aq}) \rightarrow \text{Br}_2(\text{l}) + \text{Sn(s)}$, [$E_{\text{Br}}^0 = 1.08 \text{ V}$ & $E_{\text{Sn}}^0 = -0.136 \text{ V}$]

(C) $2\text{Ag(s)} + \text{Ni}^{2+}(\text{aq}) \rightarrow 2\text{Ag}^+(\text{aq}) + \text{Ni(s)}$, [$E_{\text{Ag}}^0 = 0.799 \text{ V}$ & $E_{\text{Ni}}^0 = -0.257 \text{ V}$]

(D) $2\text{Au(s)} + \text{Zn}^{2+}(\text{aq}) \rightarrow 2\text{Au}^+(\text{aq}) + \text{Zn(s)}$, [$E_{\text{Au}}^0 = 1.68 \text{ V}$ & $E_{\text{Zn}}^0 = -0.763 \text{ V}$]

Correct Answer: (A) $\text{Ca(s)} + \text{Cd}^{2+}(\text{aq}) \rightarrow \text{Ca}^{2+}(\text{aq}) + \text{Cd(s)}$, [$E_{\text{Ca}}^0 = -2.866 \text{ V}$ & $E_{\text{Cd}}^0 = -0.403 \text{ V}$]

Solution:

Step 1: Understanding the Concept:

A cell reaction is spontaneous under standard conditions when $E_{\text{cell}}^0 > 0$, because $\Delta G^0 = -nFE_{\text{cell}}^0$ is then negative.

Step 2: Key Formula or Approach:

$$E_{\text{cell}}^0 = E_{\text{cathode}}^0 - E_{\text{anode}}^0$$

The species that is reduced is the cathode, the species that is oxidised is the anode.

Step 3: Check option (A).

Ca is oxidised (anode), Cd^{2+} is reduced (cathode). $E_{\text{cell}}^0 = -0.403 - (-2.866) = +2.463 \text{ V}$. Positive, so spontaneous.

Step 4: Check option (B).

Br^- is oxidised, Sn^{2+} is reduced. $E_{\text{cell}}^0 = -0.136 - 1.08 = -1.216 \text{ V}$. Negative, so not spontaneous.

Step 5: Check option (C).

Ag is oxidised, Ni^{2+} is reduced. $E_{\text{cell}}^0 = -0.257 - 0.799 = -1.056 \text{ V}$. Not spontaneous.

Step 6: Check option (D).

Au is oxidised, Zn^{2+} is reduced. $E_{\text{cell}}^0 = -0.763 - 1.68 = -2.443 \text{ V}$. Not spontaneous.

Final Answer:

Only the reaction in option (A) has a positive cell potential.

$$E_{\text{cell}}^0 = +2.463 \text{ V}$$

Quick Tip: A cell reaction is spontaneous when its standard cell EMF is positive.

75. Identify the correct name of law.

"At infinite dilution each ion migrates independent of co-ion and contributes to total molar conductivity of an electrolyte irrespective of the nature of other ion to which it is associated."

- (A) Henry's law
- (B) Raoult's law
- (C) Nernst derivative law
- (D) Kohlrausch's law of independent migration of ions

Correct Answer: (D) Kohlrausch's law of independent migration of ions

Solution:

Step 1: Understanding the Concept:

At infinite dilution, ions are far apart and do not affect each other. So each ion adds a fixed amount to the molar conductivity of the electrolyte.

Step 2: Detailed Explanation:

This statement is Kohlrausch law of independent migration of ions. In symbols $\Lambda_m^0 = \lambda_+^0 + \lambda_-^0$.

Step 3: Why the other options are wrong.

Henry law relates gas solubility to pressure. Raoult law relates vapour pressure of a solution to mole fraction. There is no accepted "Nernst derivative law" for this statement.

Final Answer:

The law is Kohlrausch law, option (D).

Kohlrausch law

Quick Tip: At infinite dilution each ion contributes independently to molar conductivity, which is the Kohlrausch law.



76. For the reaction, $A + B \rightarrow P$, the rate law is $\text{rate} = k[A][B]^2$. The rate of reaction is 0.25 Ms^{-1} when $[A] = 1 \text{ M}$, $[B] = 0.2 \text{ M}$ at 25°C . Calculate the rate constant of the reaction at the same temperature.

- (A) $6.25 \text{ M}^{-2}\text{s}^{-1}$
- (B) $6.0 \text{ M}^{-2}\text{s}^{-1}$
- (C) $6.5 \text{ M}^{-2}\text{s}^{-1}$
- (D) $6.75 \text{ M}^{-2}\text{s}^{-1}$

Correct Answer: (A) $6.25 \text{ M}^{-2}\text{s}^{-1}$

Solution:

Step 1: Understanding the Concept:

The rate law links the rate of a reaction with the concentrations of the reactants. The constant of proportionality k is the rate constant.

Step 2: Key Formula or Approach:

$$\text{rate} = k[A][B]^2 \Rightarrow k = \frac{\text{rate}}{[A][B]^2}$$

Step 3: Detailed Explanation:

$$k = \frac{0.25}{(1)(0.2)^2} = \frac{0.25}{0.04} = 6.25$$

The reaction is third order overall ($1 + 2$), so the unit of k is $\text{M}^{-2}\text{s}^{-1}$.

Step 4: Check the options.

All four options have the correct unit. Only 6.25 equals $0.25/0.04$.

Final Answer:

The rate constant is $6.25 \text{ M}^{-2}\text{s}^{-1}$, option (A).

$$6.25 \text{ M}^{-2}\text{s}^{-1}$$

Quick Tip: Substitute the given rate and concentrations into the rate law and solve for k.



77. In first order reaction, 20 % concentration remains after 10 min. What would be the rate constant of reaction if $\log_{10}(5) = 0.6989$?

- (A) 1.609 min^{-1}
- (B) 6.989 min^{-1}
- (C) 16.09 min^{-1}
- (D) 0.1609 min^{-1}

Correct Answer: (D) 0.1609 min^{-1}

Solution:

Step 1: Understanding the Concept:

For a first order reaction the concentration falls exponentially, and the rate constant follows from the integrated rate law.

Step 2: Key Formula or Approach:

$$k = \frac{2.303}{t} \log_{10} \frac{[A]_0}{[A]}$$

Step 3: Detailed Explanation:

20 % remains, so $[A] = 0.2[A]_0$ and $\frac{[A]_0}{[A]} = 5$. Time $t = 10 \text{ min}$.

$$k = \frac{2.303}{10} \times \log_{10} 5 = 0.2303 \times 0.6989$$

$$k = 0.1609 \text{ min}^{-1}$$

Step 4: Why the other options are wrong.

1.609, 6.989 and 16.09 are ten or more times too large. They come from dropping the division by 10 or a decimal slip. A first order reaction that loses 80 % in 10 minutes cannot have k above 1 per minute.

Final Answer:

The rate constant is 0.1609 min^{-1} , option (D).

$$0.1609 \text{ min}^{-1}$$

Quick Tip: Use the first order integrated rate equation with $[A]_0/[A] = 5$.

78. Identify a type of order of reaction that its unit of rate constant is $\text{mol dm}^{-3}\text{s}^{-1}$ from the following.

- (A) Zero-order reaction
- (B) First-order reaction
- (C) Second-order reaction
- (D) Third-order reaction

Correct Answer: (A) Zero-order reaction

Solution:

Step 1: Understanding the Concept:

The rate constant k connects the rate of a reaction with the concentrations of reactants. Rate is always measured as concentration change per unit time, so its unit is $\text{mol dm}^{-3}\text{s}^{-1}$ for every reaction.

Step 2: Key Formula or Approach:

For a reaction of order n , $\text{Rate} = k[\text{A}]^n$. So

$$k = \frac{\text{Rate}}{[\text{A}]^n} \Rightarrow \text{unit of } k = (\text{mol dm}^{-3})^{1-n} \text{s}^{-1}$$

Step 3: Detailed Explanation:

Put $n = 0$: the unit is $(\text{mol dm}^{-3})^1\text{s}^{-1} = \text{mol dm}^{-3}\text{s}^{-1}$. This is the unit given in the question.

For $n = 1$: $(\text{mol dm}^{-3})^0\text{s}^{-1} = \text{s}^{-1}$.

For $n = 2$: $(\text{mol dm}^{-3})^{-1}\text{s}^{-1} = \text{mol}^{-1}\text{dm}^3\text{s}^{-1}$.

For $n = 3$: $(\text{mol dm}^{-3})^{-2}\text{s}^{-1} = \text{mol}^{-2}\text{dm}^6\text{s}^{-1}$.

So the first-order, second-order and third-order reactions each have a different unit, and only the zero-order reaction has the same unit as the rate.

Final Answer:

A zero-order reaction has $\text{Rate} = k$, so k has the unit of rate. This is option (A).

Zero-order reaction (A)

Quick Tip: Find the order n for which the unit of k is the same as the unit of rate.



79. Identify the dimensional nature of nanoparticles by the following.

- (A) one dimensional
- (B) two dimensional
- (C) four dimensional
- (D) zero dimensional

Correct Answer: (D) zero dimensional

Solution:

Step 1: Understanding the Concept:

Nanomaterials are classified by how many of their dimensions are NOT confined to the nanometre scale (1 to 100 nm).

Step 2: Detailed Explanation:

In a zero dimensional nanomaterial, all three dimensions (length, width, height) are in the nano range. Nanoparticles and quantum dots are examples.

One dimensional materials, such as nanowires and nanotubes, have one dimension outside the nano range. Two dimensional materials, such as thin films, have two dimensions outside it.

Step 3: Check the options.

Four dimensional is not a classification used for nanomaterials. Nanoparticles are zero dimensional.

Final Answer:

Nanoparticles are zero dimensional, option (D).

zero dimensional

Quick Tip: Nanoparticles are small in all three dimensions, so they are called zero dimensional.

80. Which of the following statements is correct for the spontaneous adsorption of a gas?

- (A) ΔS is negative and, therefore ΔH should be highly positive
- (B) ΔS is negative and therefore, ΔH should be highly negative.
- (C) ΔS is positive and therefore, ΔH should be negative.
- (D) ΔS is positive and therefore, ΔH should also be highly positive.

Correct Answer: (B) ΔS is negative and therefore, ΔH should be highly negative.

Solution:**Step 1: Understanding the Concept:**

Spontaneity needs $\Delta G < 0$. When a gas sticks to a solid surface, its molecules lose freedom of movement, so ΔS is negative.

Step 2: Key Formula or Approach:

$$\Delta G = \Delta H - T\Delta S$$

Step 3: Detailed Explanation:

With $\Delta S < 0$ the term $-T\Delta S$ is positive. This works against spontaneity. To still get $\Delta G < 0$, ΔH must be negative with a large magnitude. Adsorption is exothermic.

Step 4: Check the options.

(A) says ΔH should be highly positive, which would make ΔG even more positive. (C) and (D) claim ΔS is positive, which is wrong because the gas is confined to the surface. Only (B) fits.

Final Answer:

ΔS is negative and ΔH is highly negative, option (B).

$$\Delta S < 0, \Delta H \ll 0$$

Quick Tip: Adsorption reduces the freedom of gas molecules, so entropy falls and the enthalpy change must be strongly negative.

81. Identify the correct statement regarding lyophilic colloid from the following.

- (A) It is irreversible.
- (B) It is solvent hating colloid.
- (C) It is formed only by special methods.
- (D) It is solvent loving colloid.

Correct Answer: (D) It is solvent loving colloid.

Solution:

Step 1: Understanding the Concept:

Colloids are classed by the attraction between dispersed particles and the dispersion medium. "Lyo" means solvent and "philic" means loving.

Step 2: Check statement (A).

Lyophilic colloids are reversible. After the medium is removed, adding it back brings the colloid again. So (A) is FALSE.

Step 3: Check statement (B).

Solvent hating is the meaning of lyophobic, so (B) is FALSE.

Step 4: Check statement (C).

Lyophilic sols form easily by direct mixing, for example starch in water. Special methods are needed for lyophobic sols. So (C) is FALSE.

Step 5: Check statement (D).

Lyophilic colloid is solvent loving. So (D) is TRUE.

Final Answer:

Only (D) is correct.

(D)

Quick Tip: Lyophilic means solvent loving, and such colloids are stable and reversible.

82. In the Charring process, concentrated sulfuric acid reacts with sugar to give sugar charcoal. What is the role of conc. H_2SO_4 in this reaction?

- (A) An oxidizing agent
- (B) A sulfonating agent
- (C) A dehydrating agent
- (D) A decomposing agent

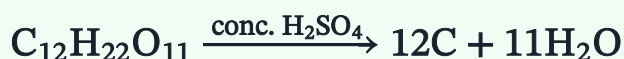
Correct Answer: (C) A dehydrating agent

Solution:

Step 1: Understanding the Concept:

Sugar is $\text{C}_{12}\text{H}_{22}\text{O}_{11}$, a carbohydrate with hydrogen and oxygen in the ratio of water. Concentrated sulphuric acid has a strong attraction for water.

Step 2: Detailed Explanation:



The acid removes water from sugar and leaves black carbon (charcoal). So it acts as a dehydrating agent.

Step 3: Why the other options are wrong.

It may also oxidise carbon later, but the main charring action is water removal. It is not sulfonating sugar, and "decomposing agent" is not a recognised role here.

Final Answer:

The acid acts as a dehydrating agent, option (C).

dehydrating agent

Quick Tip: Concentrated sulphuric acid removes the elements of water from sugar, so it acts as a dehydrating agent.

83. Which of the following is the correct decreasing order of oxidizing power of perhalate ions?

- (A) $\text{BrO}_4^- < \text{IO}_4^- < \text{ClO}_4^-$
- (B) $\text{IO}_4^- > \text{BrO}_4^- > \text{ClO}_4^-$
- (C) $\text{IO}_4^- < \text{BrO}_4^- < \text{ClO}_4^-$
- (D) $\text{BrO}_4^- > \text{IO}_4^- > \text{ClO}_4^-$

Correct Answer: (D) $\text{BrO}_4^- > \text{IO}_4^- > \text{ClO}_4^-$

Solution:

Step 1: Understanding the Concept:

The oxidising power of perhalate ions XO_4^- (halogen in +7) depends on how readily the +7 state is reduced.

Step 2: Detailed Explanation:

Bromine in +7 state is unusually unstable because of the poor shielding by the filled 3d electrons (the d-block contraction). So BrO_4^- is the strongest oxidising agent.

Iodine in +7 is more stable due to its larger size, and chlorine in +7 is kinetically slow to react. So ClO_4^- is the weakest oxidiser of the three. Order: $\text{BrO}_4^- > \text{IO}_4^- > \text{ClO}_4^-$.

Step 3: Check the options.

(A) and (C) put ClO_4^- as the strongest, which is incorrect. (B) puts IO_4^- first and is incorrect. Only (D) matches.

Final Answer:

The decreasing order is $\text{BrO}_4^- > \text{IO}_4^- > \text{ClO}_4^-$, option (D).



Quick Tip: Perbromate is the strongest oxidiser among perhalates because of the poor stability of Br in the +7 state.

84. Which of the following elements is doped to a fiber amplifier in an optical fiber communication system?

- (A) Tm
- (B) Yb
- (C) Er
- (D) Nd

Correct Answer: (C) Er

Solution:

Step 1: Understanding the Concept:

Some lanthanoid ions have energy levels that let them amplify light when they are put into the glass of an optical fibre.

Step 2: Detailed Explanation:

Erbium (Er) is doped into the fibre in an erbium doped fibre amplifier (EDFA). It amplifies light near 1550 nm, the wavelength where silica

fibre has the lowest loss. This makes long distance communication possible.

Step 3: Why the other options are wrong.

Thulium (Tm) is used in other wavelength bands, ytterbium (Yb) is used as a sensitiser or in high power lasers, and neodymium (Nd) is mostly used in solid state lasers.

Final Answer:

The dopant is erbium, option (C).

Er

Quick Tip: Erbium doped fibre amplifiers are the standard amplifiers in optical fibre communication.

85. Which of the following comparisons is correct for Δ_0 of the following complexes?



(A) $\text{I} < \text{II}$

(B) $\text{I} < \text{III}$

(C) $\text{IV} < \text{II}$

(D) $\text{II} < \text{I}$

Correct Answer: (A) $\text{I} < \text{II}$

Solution:

Step 1: Understanding the Concept:

The crystal field splitting energy Δ_o is the gap between the t_{2g} and e_g levels in an octahedral complex. A larger gap means a stronger crystal field. The gap grows with (i) a stronger ligand and (ii) a higher charge on the metal ion.

Step 2: Key Formula or Approach:

We compare the complexes using two rules.

1. For the same metal and ligand, a higher oxidation state gives a larger Δ_o , because the ligands sit closer to a more highly charged ion.
2. For the same metal ion, the spectrochemical series puts $\text{CN}^- \gg \text{H}_2\text{O}$.

Step 3: Compare I and II.

Both have the same metal, cobalt, and the same ligand, water.

Complex I has Co^{2+} and complex II has Co^{3+} .

The Co^{3+} ion is smaller and more highly charged, so water ligands are pulled in closer and split the d orbitals more.

So Δ_o of II is larger than Δ_o of I. That means $\text{I} < \text{II}$, which is option (A).

Step 4: Check the remaining options.

Option (D) says $\text{II} < \text{I}$. This is the reverse of what we just found, so it is wrong.

Option (C) says $\text{IV} < \text{II}$. Complex IV has CN^- , a very strong field ligand, on Fe^{3+} , and it has a much larger splitting than the water complex of Co^{3+} . So IV is greater than II, and (C) is wrong.

Final Answer:

The comparison $I < II$ is correct, because Co^{3+} gives a larger Δ_o than Co^{2+} with the same ligand.

(A) $I < II$

Quick Tip: With the same ligand (water), the metal ion with the higher charge gives the larger splitting, so the Co^{3+} complex is above the Co^{2+} complex.

86. Among the given ligands, the negative ligand is

- (A) Ammonia
- (B) Methylamine
- (C) Hydrazinium
- (D) Nitrate

Correct Answer: (D) Nitrate

Solution:

Step 1: Understanding the Concept:

Ligands are neutral, negative or positive species that donate electron pairs to a metal. Their charge must be known to find the charge on a complex.

Step 2: Detailed Explanation:

Ammonia NH_3 is neutral. Methylamine CH_3NH_2 is neutral.

Hydrazinium N_2H_5^+ carries a positive charge. Nitrate NO_3^- carries a negative charge.

Final Answer:

Nitrate is the negative ligand, option (D).



Quick Tip: Count the charge on each ligand, and pick the one carrying a negative charge.

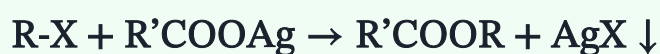
87. Which of the following compounds is obtained when Bromoethane is reacted with silver acetate?

- (A) propanal
- (B) acetic acid
- (C) ethyl acetate
- (D) methyl acetate

Correct Answer: (C) ethyl acetate

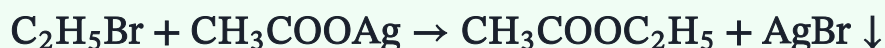
Solution:**Step 1: Understanding the Concept:**

Bromoethane is a primary alkyl halide. Silver salts of carboxylic acids react with alkyl halides in a nucleophilic substitution. The halogen leaves as silver bromide, which is insoluble.

Step 2: Key Formula or Approach:

Step 3: Detailed Explanation:

Here R = ethyl and R' = methyl.



The product is an ester made of the acetyl part (from acetate) and the ethyl part (from bromoethane). Its name is ethyl acetate.

Step 4: Why the other options are wrong.

Propanal is an aldehyde and acetic acid is a free acid, and neither forms by simple substitution. Methyl acetate would need a methyl halide, but we started with an ethyl halide.

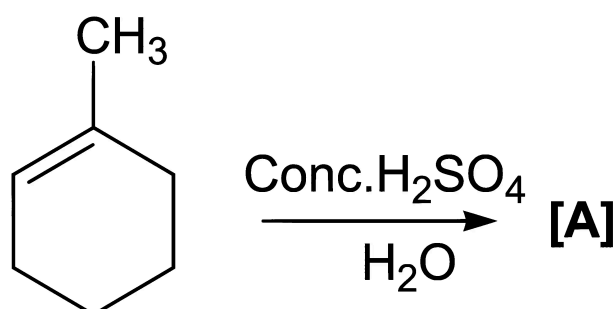
Final Answer:

The product is ethyl acetate, option (C).

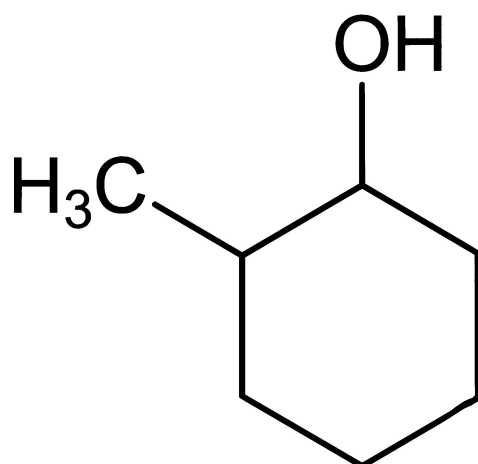


Quick Tip: Silver acetate is a source of the acetate ion, which replaces bromine in a substitution reaction and gives an ester.

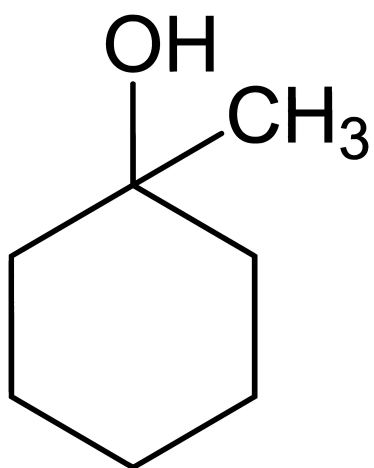
88. Find the structure of A in following reaction



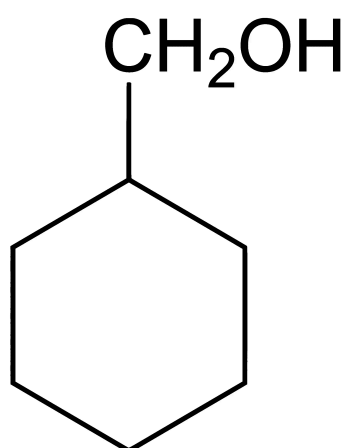
(A)



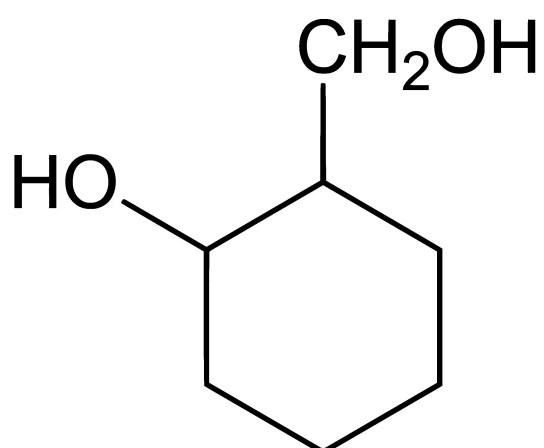
(B)



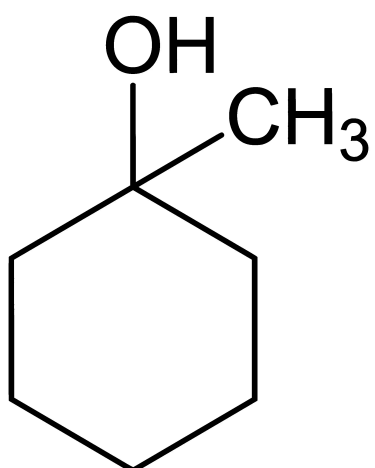
(C)



(D)



Correct Answer: (B)



Solution:

Step 1: Read the reaction:

The starting compound is 1-methylcyclohexene (a cyclohexene ring with a CH_3 on one of the double-bond carbons). The reagent is conc. H_2SO_4 with H_2O , which carries out acid-catalysed hydration of an alkene.

Step 2: Protonation (Markovnikov rule):

H^+ adds to the double-bond carbon that already has more hydrogens. This puts the positive charge on the carbon that carries the methyl group, giving a tertiary carbocation, the most stable one.

Step 3: Attack by water:

A water molecule attacks the tertiary carbocation and forms an oxonium ion. The oxonium ion then loses H^+ to give the alcohol, and the acid is regenerated.

Step 4: Product:

The OH group and the CH_3 group end up on the same ring carbon, so A is 1-methylcyclohexanol. This is the structure in option (B).

Step 5: Why the other options are wrong:

Option (A) has OH on the carbon next to the CH_3 -carbon (2-methylcyclohexanol), which would need a less stable secondary carbocation. Options (C) and (D) contain a CH_2OH group, which needs a carbon skeleton change or oxidation of the methyl group; plain hydration does neither.

Final Answer:

Hydration of 1-methylcyclohexene gives 1-methylcyclohexanol, option (B).

1-Methylcyclohexanol (option B)

Quick Tip: Acid-catalysed hydration follows Markovnikov: OH goes to the carbon that holds the methyl group.

89. Identify the alcohol obtained when aldehyde other than formaldehyde is treated with Grignard's reagent followed by hydrolysis.

- (A) Primary alcohol
- (B) Secondary alcohol
- (C) Tertiary alcohol
- (D) Ketone

Correct Answer: (B) Secondary alcohol

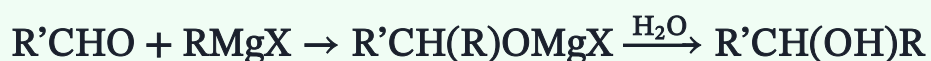
Solution:

Step 1: Understanding the Concept:

A Grignard reagent R-MgX adds across the C=O bond. The R group goes to carbon and MgX goes to oxygen. Hydrolysis then gives the alcohol.

Step 2: Detailed Explanation:

For an aldehyde R'CHO :



The carbon bearing the OH group is now attached to two carbon groups (R and R') and one hydrogen. An alcohol with two alkyl groups on the carbinol carbon is a secondary alcohol.

Step 3: Compare with other carbonyls.

Formaldehyde HCHO gives a primary alcohol. A ketone gives a tertiary alcohol. So an aldehyde other than formaldehyde gives a secondary alcohol, option (B). A ketone (D) is not an alcohol.

Final Answer:

The product is a secondary alcohol.

Secondary alcohol

Quick Tip: Formaldehyde gives a primary alcohol, other aldehydes give secondary alcohols and ketones give tertiary alcohols.

90. Identify the reagent used in the transformation of Ketone into semicarbazone.

- (A) $\text{NH}_2 - \text{OH}$
- (B) $\text{NH}_2 - \text{NH}_2$
- (C) $\text{NH}_2 - \text{NH} - \text{CONH}_2$
- (D) $\text{NH}_2 - \text{NH} - \text{C}_6\text{H}_5$

Correct Answer: (C) $\text{NH}_2 - \text{NH} - \text{CONH}_2$

Solution:

Step 1: Understanding the Concept:

Aldehydes and ketones react with ammonia derivatives $\text{NH}_2\text{-Z}$ in a condensation reaction. Water is lost and a C=N compound forms. The name of the product depends on Z.

Step 2: Detailed Explanation:

NH_2OH , hydroxylamine, gives an oxime. NH_2NH_2 , hydrazine, gives a hydrazone. $\text{NH}_2\text{NH-CONH}_2$, semicarbazide, gives a semicarbazone. $\text{NH}_2\text{NH-C}_6\text{H}_5$, phenylhydrazine, gives a phenylhydrazone.

Final Answer:

The reagent is semicarbazide, option (C).



Quick Tip: Semicarbazide is $\text{NH}_2\text{-NH-CONH}_2$.

91. Identify the product obtained when ketone reacts with $\text{NH}_2\text{-NH}_2$.

- (A) Aniline
- (B) Hydrazone
- (C) Phenyl hydrazone
- (D) Anisole

Correct Answer: (B) Hydrazone

Solution:**Step 1: Understanding the Concept:**

Ketones react with $\text{NH}_2\text{-Z}$ type reagents by addition followed by loss

of water, forming a C=N-Z group.

Step 2: Detailed Explanation:

With Z = NH₂ the reagent is hydrazine.



The product is a hydrazone.

Step 3: Why the other options are wrong.

Aniline and anisole are not formed from a ketone and hydrazine. A phenylhydrazone needs phenylhydrazine, not plain hydrazine.

Final Answer:

The product is a hydrazone, option (B).

Hydrazone

Quick Tip: A carbonyl compound plus hydrazine gives a hydrazone.

92. Which of the following reagents is used to convert -CN group to -CH₂NH₂ group?

- (A) CrO₃
- (B) Na + C₂H₅OH
- (C) H₃PO₄
- (D) Al₂O₃

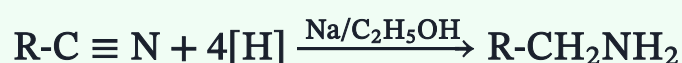
Correct Answer: (B) Na + C₂H₅OH

Solution:

Step 1: Understanding the Concept:

Reducing a nitrile means adding hydrogen to the carbon-nitrogen triple bond until a primary amine forms. This is how many primary amines are made.

Step 2: Detailed Explanation:



This is called the Mendius reduction.

Step 3: Why the other options are wrong.

CrO_3 is an oxidising agent. H_3PO_4 is an acid with no reducing power. Al_2O_3 is a dehydrating or catalyst material, not a source of hydrogen.

Final Answer:

Sodium with ethanol reduces $-\text{CN}$ to $-\text{CH}_2\text{NH}_2$, option (B).



Quick Tip: Nitriles are reduced to primary amines by nascent hydrogen from sodium and ethanol or by catalytic hydrogenation.

93. Which of the following reactions is exhibited only by primary amines?

- (A) acetylation
- (B) alkylation
- (C) reaction with nitrous acid
- (D) carbyl amine test

Correct Answer: (D) carbyl amine test

Solution:

Step 1: Understanding the Concept:

Primary, secondary and tertiary amines differ in how many hydrogens sit on nitrogen. A reaction needing two N-H hydrogens can be given only by a primary amine.

Step 2: Check option (A).

Acetylation replaces an N-H hydrogen with an acetyl group. Primary and secondary amines both do this. Not unique.

Step 3: Check option (B).

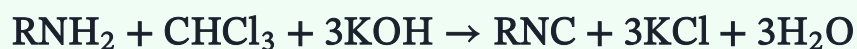
Alkylation of the nitrogen works with primary, secondary and even tertiary amines (to give quaternary salts). Not unique.

Step 4: Check option (C).

Nitrous acid reacts with all three classes, although the products differ. Not unique.

Step 5: Check option (D).

In the carbylamine test, a primary amine is heated with chloroform and alcoholic KOH to give a foul smelling isocyanide.



Secondary and tertiary amines do not give it. So (D) is the answer.

Final Answer:

Only primary amines give the carbylamine test, option (D).

Carbylamine test

Quick Tip: The carbylamine test works only with primary amines.

94. Which from following reagents is used to prepare alkyl isocyanides from RX?

- (A) AgCN (alco)
- (B) NH_3 (alco)
- (C) KNO_2
- (D) AgNO_2

Correct Answer: (A) AgCN (alco)

Solution:

Step 1: Understanding the Concept:

The cyanide ion is an ambident nucleophile, meaning it can attack through carbon or through nitrogen. Ionic cyanides (KCN) attack through carbon and give nitriles. Covalent AgCN attacks through nitrogen.

Step 2: Detailed Explanation:



So alcoholic silver cyanide gives alkyl isocyanide.

Step 3: Why the other options are wrong.

Alcoholic NH_3 gives amines. KNO_2 gives alkyl nitrite mainly, and AgNO_2 gives nitroalkane.

Final Answer:

The reagent is alcoholic AgCN, option (A).

AgCN (alco)

Quick Tip: Silver cyanide is covalent, so the nitrogen end attacks the carbon and gives the isocyanide.

95. Which carbon atoms of glucose, numbered from 1 to 6 forms a hemiacetal structure between -CHO and -OH groups.?

- (A) C-2
- (B) C-3
- (C) C-4
- (D) C-5

Correct Answer: (D) C-5

Solution:

Step 1: Understanding the Concept:

Glucose has an aldehyde at C-1 and hydroxyl groups on C-2 to C-6. In

solution it forms a ring because the aldehyde reacts with one of its own -OH groups.

Step 2: Detailed Explanation:

The -OH on C-5 attacks the -CHO group on C-1. This forms a hemiacetal (an -O- linking C-1 and C-5). The ring has five carbon atoms and one oxygen, so it is a six-membered ring called pyranose.

Step 3: Why the other options are wrong.

C-2, C-3 or C-4 hydroxyls would give rings of 3, 4 or 5 members, which are strained or not formed in glucose.

Final Answer:

The hemiacetal forms with the C-5 hydroxyl, option (D).

C-5

Quick Tip: The aldehyde carbon C-1 reacts with the hydroxyl on C-5 to form the six-membered pyranose ring.

96. Which one of the following is a globular protein?

- (A) Insulin
- (B) Myosin
- (C) Keratin in hair
- (D) Keratin in wool

Correct Answer: (A) Insulin

Solution:

Step 1: Understanding the Concept:

Proteins are grouped into fibrous (long threads, insoluble in water) and globular (folded and roughly spherical, soluble in water).

Step 2: Detailed Explanation:

Fibrous proteins include keratin (in hair, wool, skin, nails), myosin (in muscles) and collagen. Globular proteins include insulin, albumin and haemoglobin.

Step 3: Check the options.

Myosin and both forms of keratin are fibrous. Insulin is globular.

Final Answer:

The globular protein is insulin, option (A).

Insulin

Quick Tip: Insulin is globular, while keratin and myosin are fibrous proteins.

97. Identify the cation present in chlorophyll in green plants.

- (A) Mg^{2+}
- (B) Ca^{2+}
- (C) Fr^{+}
- (D) Na^{+}

Correct Answer: (A) Mg^{2+}

Solution:

Step 1: Understanding the Concept:

Chlorophyll is the green pigment that captures light in plants. A metal ion sits at the centre of a ring structure called porphyrin.

Step 2: Detailed Explanation:

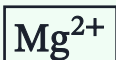
The metal ion at the centre is magnesium, as Mg^{2+} . About 2.7 percent of chlorophyll is magnesium by mass.

Step 3: Why the other options are wrong.

Ca^{2+} is found in bones, teeth and plant cell walls. Na^+ is an electrolyte in animal cells. Fr^+ is a radioactive element which is not biological.

Final Answer:

The cation is magnesium, option (A).



Quick Tip: Chlorophyll has magnesium at the centre of its porphyrin ring.

98. Identify the monomer used to prepare orlon.

- (A) vinyl chloride
- (B) acrylonitrile
- (C) styrene
- (D) tetrafluoroethylene

Correct Answer: (B) acrylonitrile

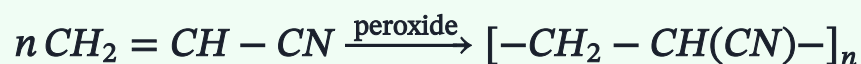
Solution:

Step 1: Understanding the Concept:

Orlon (also called acrilan) is a synthetic fibre. Its chemical name is polyacrylonitrile. It is made by addition polymerisation, so we only need the monomer unit that repeats in the chain.

Step 2: Key Approach:

Orlon is the polymer of the monomer with formula $CH_2 = CH - CN$. This monomer is acrylonitrile (vinyl cyanide). Its double bond opens up and many units join in a chain.

Step 3: Reaction:

The repeating unit carries a nitrile group on every second carbon.

Step 4: Why the other options are wrong:

Vinyl chloride ($CH_2 = CHCl$) gives PVC. Styrene ($C_6H_5CH = CH_2$) gives polystyrene. Tetrafluoroethylene ($CF_2 = CF_2$) gives teflon. None of these has a $-CN$ group, so none gives orlon.

Final Answer:

Orlon is made from acrylonitrile, which is option (B).

B: acrylonitrile

Quick Tip: Orlon is the commercial name of polyacrylonitrile, an addition polymer.

99. What type of polymer the natural rubber is?

- (A) Fibre
- (B) Elastomer
- (C) Thermoplastic
- (D) Thermosetting plastic

Correct Answer: (B) Elastomer

Solution:

Step 1: Understanding the Concept:

Polymers are grouped by the strength of forces between their chains. This gives elastomers, fibres, thermoplastics and thermosetting plastics.

Step 2: Detailed Explanation:

Natural rubber is cis-1,4-polyisoprene. Its chains are held by weak van der Waals forces and are coiled, so it stretches and then returns to its shape. Polymers that behave like this are elastomers.

Step 3: Why the other options are wrong.

Fibres have strong hydrogen bonds or dipole forces (nylon). Thermoplastics soften on heating (polythene). Thermosetting plastics harden on heating (bakelite).

Final Answer:

Natural rubber is an elastomer, option (B).

Elastomer

Quick Tip: Natural rubber is a polymer with weak forces between chains that can stretch, so it is an elastomer.



100. Identify the use of antipyretic drugs.

- (A) To kill microorganism.
- (B) To relieve pain.
- (C) To reduce fever.
- (D) To prevent pregnancy.

Correct Answer: (C) To reduce fever.

Solution:

Step 1: Understanding the Concept:

"Pyretic" refers to fever, and "anti" means against, so an antipyretic works against fever.

Step 2: Detailed Explanation:

Paracetamol is a typical antipyretic. It lowers the body temperature in fever.

Step 3: Why the other options are wrong.

Killing microorganisms is the job of antibiotics, antiseptics or disinfectants. Pain relief is done by analgesics. Preventing pregnancy is done by antifertility drugs.

Final Answer:

Antipyretic drugs reduce fever, option (C).

Reduce fever

Quick Tip: Antipyretics bring down a raised body temperature.

Mathematics

Questions 101 to 150 | 2 marks each | 100 marks

101. The polar form of the complex number $z = \frac{1}{1+i}$, (where $i = \sqrt{-1}$) is

(A) $\frac{1}{\sqrt{2}}(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4})$

(B) $\frac{1}{\sqrt{2}}(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})$

(C) $\frac{1}{2}(\cos \frac{\pi}{4} + i \sin \frac{\pi}{4})$

(D) $\frac{1}{2}(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4})$

Correct Answer: (A) $\frac{1}{\sqrt{2}}(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4})$

Solution:

Step 1: Understanding the Concept:

The polar form of $z = x + iy$ is $r(\cos \theta + i \sin \theta)$, where $r = |z|$ and θ is the argument, chosen according to the quadrant of (x, y) .

Step 2: Step to a + ib form.

Multiply top and bottom by the conjugate of the denominator.

$$z = \frac{1}{1+i} \times \frac{1-i}{1-i} = \frac{1-i}{2} = \frac{1}{2} - \frac{1}{2}i$$

Step 3: Find the modulus.

$$r = \sqrt{\left(\frac{1}{2}\right)^2 + \left(-\frac{1}{2}\right)^2} = \sqrt{\frac{1}{2}} = \frac{1}{\sqrt{2}}$$

Step 4: Find the argument.

The real part is positive and the imaginary part is negative, so z lies in the fourth quadrant. The reference angle is $\tan^{-1}(1) = \pi/4$, so $\theta = 2\pi - \pi/4 = 7\pi/4$ (or $-\pi/4$).

Step 5: Check the options.

Options (B) and (C) use $\pi/4$, which is the first quadrant. Options (C) and (D) use the modulus $1/2$, which is wrong. Only (A) has both $1/\sqrt{2}$ and $7\pi/4$.

Final Answer:

In polar form $z = \frac{1}{\sqrt{2}} \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right)$, option (A).

$$\boxed{\frac{1}{\sqrt{2}} \left(\cos \frac{7\pi}{4} + i \sin \frac{7\pi}{4} \right)}$$

Quick Tip: Rationalise to get the real and imaginary parts, then find modulus and argument, remembering the quadrant.

102. The number of cubic equations that can be formed with the coefficients 0, 3, 2, 4, 5, 6 when repetition of coefficients is allowed, is....

- (A) 720
- (B) 1296
- (C) 864
- (D) 1080

Correct Answer: (D) 1080

Solution:

Step 1: Understanding the Concept:

A cubic equation has the form $ax^3 + bx^2 + cx + d = 0$ with four coefficients. For it to be cubic, $a \neq 0$. The other three can be any of the six numbers because repetition is allowed.

Step 2: Key Formula or Approach:

Use the fundamental counting principle: multiply the number of choices for each coefficient.

Step 3: Detailed Explanation:

Choices for a : any of the 5 non-zero numbers (3, 2, 4, 5, 6).

Choices for each of b, c, d : 6 each.

$$\text{Total} = 5 \times 6 \times 6 \times 6 = 5 \times 216 = 1080$$

Step 4: Why the other options are wrong.

$6^4 = 1296$ counts the cases with $a = 0$, which are not cubic. 720 and 864 do not come from any valid counting here.

Final Answer:

The number of cubic equations is 1080, option (D).

1080

Quick Tip: The leading coefficient cannot be zero, so fill that place first and the remaining places freely.

103. $\tan 105^\circ = \dots$

(A) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

(B) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

(C) $\frac{1+\sqrt{3}}{1-\sqrt{3}}$

(D) $\frac{2\sqrt{2}}{\sqrt{3}+1}$

Correct Answer: (C) $\frac{1+\sqrt{3}}{1-\sqrt{3}}$

Solution:

Step 1: Understanding the Concept:

We can find $\tan 105^\circ$ by splitting the angle into two angles whose tangent is known.

Step 2: Key Formula or Approach:

$$\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

Step 3: Detailed Explanation:

Take $A = 60^\circ$ and $B = 45^\circ$, with $\tan 60^\circ = \sqrt{3}$ and $\tan 45^\circ = 1$.

$$\tan 105^\circ = \frac{\sqrt{3} + 1}{1 - \sqrt{3} \cdot 1} = \frac{1 + \sqrt{3}}{1 - \sqrt{3}}$$

This matches option (C). The value is negative, as expected for an angle in the second quadrant.

Step 4: Why the other options are wrong.

(A) and (B) are both positive numbers, but $\tan 105^\circ$ is negative. (D) is also positive.

Final Answer:

$$\tan 105^\circ = \frac{1 + \sqrt{3}}{1 - \sqrt{3}}, \text{ option (C).}$$

$$\boxed{\frac{1 + \sqrt{3}}{1 - \sqrt{3}}}$$

Quick Tip: Write 105 degrees as 60 plus 45 degrees and use the tangent addition formula.

104. The number of solutions in $[0, \frac{\pi}{2})$ of the equation $\cos 3x \cdot \tan 5x = \sin 7x$ is

- (A) 4
- (B) 5
- (C) 7
- (D) 6

Correct Answer: (D) 6

Solution:

Step 1: Understanding the Concept:

To solve a mixed trigonometric equation we write everything in sines and cosines and use product-to-sum formulas to reduce it to an equation of the form $\sin A = \sin B$.

Step 2: Rewrite the equation.

$\cos 3x \cdot \frac{\sin 5x}{\cos 5x} = \sin 7x$, with $\cos 5x \neq 0$. Multiply by $\cos 5x$:

$$\cos 3x \sin 5x = \sin 7x \cos 5x$$

Step 3: Apply product to sum.

$$\frac{1}{2}[\sin 8x + \sin 2x] = \frac{1}{2}[\sin 12x + \sin 2x]$$

So $\sin 8x = \sin 12x$.

Step 4: Solve $\sin 8x = \sin 12x$.

Either $12x = 8x + 2k\pi$, giving $x = k\pi/2$, or $12x = \pi - 8x + 2k\pi$, giving $x = (2k + 1)\pi/20$.

Step 5: Count in $[0, \pi/2)$.

From the first family, only $x = 0$ lies in the interval. From the second family, $(2k + 1)\pi/20 < \pi/2$ means $2k + 1 < 10$, so $2k + 1 = 1, 3, 5, 7, 9$. This gives 5 values.

Step 6: Check $\cos 5x \neq 0$.

$\cos 5x = 0$ at $x = \pi/10, 3\pi/10, \dots$, which are $2\pi/20, 6\pi/20, \dots$, even multiples of $\pi/20$. None of our 6 solutions is one of these, and $x = 0$ works since both sides are 0.

Final Answer:

The total number of solutions is $1 + 5 = 6$, option (D).

6

Quick Tip: Convert tan to sine over cosine, use product-to-sum formulas to get $\sin 8x = \sin 12x$, then count the solutions in the interval.



105. The area of the triangle formed by the co-ordinate axes and a line $px + qy = r$ (where p, q and r are positive real numbers) is $\frac{1}{54}$ sq.units, then

- (A) $p, 3r, q$ are in G.P.
- (B) $p^2 + q^2 = r^2$
- (C) $p, 3\sqrt{3}r, q$ are in G.P.
- (D) $p, 27r, q$ are in G.P.

Correct Answer: (C) $p, 3\sqrt{3}r, q$ are in G.P.

Solution:

Step 1: Understanding the Concept:

A line meeting the two axes at $(a, 0)$ and $(0, b)$ forms a right triangle with the axes. Its area is $\frac{1}{2}ab$.

Step 2: Find the intercepts.

Put $y = 0$: $x = r/p$. Put $x = 0$: $y = r/q$. All of p, q, r are positive.

Step 3: Write the area.

$$\frac{1}{2} \cdot \frac{r}{p} \cdot \frac{r}{q} = \frac{r^2}{2pq} = \frac{1}{54} \Rightarrow pq = 27r^2$$

Step 4: Test G.P. conditions.

If p, m, q are in G.P., then $m^2 = pq = 27r^2$, so $m = \sqrt{27}r = 3\sqrt{3}r$.

Step 5: Check the options.

(A) needs $m = 3r$, which gives $9r^2$, not $27r^2$. (D) needs $m = 27r$, which gives $729r^2$. (B) $p^2 + q^2 = r^2$ does not follow from the area. Only (C) with $m = 3\sqrt{3}r$ matches.

Final Answer:

$p, 3\sqrt{3}r, q$ are in G.P., option (C).

$p, 3\sqrt{3}r, q \text{ in G.P.}$

Quick Tip: The intercepts of the line are r/p and r/q , so the area is r squared over $2pq$. Set it equal to $1/54$.

106. The joint equation of the pair of lines through the origin and making an angle of $\frac{\pi}{4}$ with the line $3x + y - 6 = 0$ is

- (A) $2x^2 + 3xy - 2y^2 = 0$
- (B) $2x^2 + 3xy + 2y^2 = 0$
- (C) $x^2 + 3xy - y^2 = 0$
- (D) $x^2 - 3xy + y^2 = 0$

Correct Answer: (A) $2x^2 + 3xy - 2y^2 = 0$

Solution:

Step 1: Understanding the Concept:

Lines through the origin that make a given angle with a given line are found from the angle formula between two lines, which yields two possible slopes.

Step 2: Key Formula or Approach:

$$\tan \theta = \left| \frac{m - m_1}{1 + mm_1} \right|$$

Here $\theta = \pi/4$, so $\tan \theta = 1$. The slope of $3x + y - 6 = 0$ is $m_1 = -3$.

Step 3: Solve for the slopes.

$$\left| \frac{m + 3}{1 - 3m} \right| = 1$$

Case 1: $m + 3 = 1 - 3m$, so $4m = -2$ and $m = -\frac{1}{2}$.

Case 2: $m + 3 = -(1 - 3m) = 3m - 1$, so $2m = 4$ and $m = 2$.

Step 4: Write the lines and combine.

The lines through the origin are $y = 2x$ and $y = -\frac{x}{2}$, that is $2x - y = 0$ and $x + 2y = 0$.

$$(2x - y)(x + 2y) = 2x^2 + 4xy - xy - 2y^2 = 2x^2 + 3xy - 2y^2 = 0$$

Step 5: Check the options.

Only option (A) matches. Option (B) has $+2y^2$ and options (C) and (D) have no coefficient 2 in front of x^2 and y^2 .

Final Answer:

The pair of lines is $2x^2 + 3xy - 2y^2 = 0$, option (A).

$$\boxed{2x^2 + 3xy - 2y^2 = 0}$$

Quick Tip: Find the slopes of lines making 45 degrees with a given slope using the angle formula, then multiply the line equations.



107. If a circle with center $(-1, 1)$ touches the line $x + 2y + 4 = 0$, then the co-ordinates of the point of contact are

- (A) $(-2, -1)$
- (B) $(8, -2)$
- (C) $(2, -3)$
- (D) $(4, -4)$

Correct Answer: (A) $(-2, -1)$

Solution:

Step 1: Understanding the Concept:

A tangent touches a circle at the foot of the perpendicular dropped from the centre to the line.

Step 2: Find the foot of the perpendicular.

For a line $ax + by + c = 0$ and a point (x_0, y_0) , the foot (x, y) satisfies

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = -\frac{ax_0 + by_0 + c}{a^2 + b^2}.$$

Step 3: Detailed Explanation:

Here $a = 1, b = 2, c = 4$ and $(x_0, y_0) = (-1, 1)$. Then $ax_0 + by_0 + c = -1 + 2 + 4 = 5$ and $a^2 + b^2 = 5$, so the common ratio is -1 .

$$x = -1 + 1(-1) = -2, \quad y = 1 + 2(-1) = -1$$

Step 4: Check.

The point $(-2, -1)$ lies on the line: $-2 - 2 + 4 = 0$. The distance from the centre to this point is $\sqrt{1 + 4} = \sqrt{5}$, which equals the radius $\frac{|-1 + 2 + 4|}{\sqrt{5}} = \sqrt{5}$. So the circle touches the line at this point.

Final Answer:

The point of contact is $(-2, -1)$, option (A).

$$\boxed{(-2, -1)}$$

Quick Tip: The point of contact is the foot of the perpendicular from the centre to the tangent line.

108. A point on the parabola $y^2 = \frac{36}{5}x$ at which the ordinate increases at thrice the rate of the abscissa is

- (A) $(\frac{6}{5}, \frac{1}{5})$
- (B) $(\frac{1}{5}, \frac{6}{5})$
- (C) $(5, 6)$
- (D) $(\frac{3}{5}, \frac{9}{5})$

Correct Answer: (B) $(\frac{1}{5}, \frac{6}{5})$

Solution:**Step 1: Understanding the Concept:**

"Ordinate increases at thrice the rate of abscissa" means $\frac{dy}{dt} = 3 \frac{dx}{dt}$,
so $\frac{dy}{dx} = 3$.

Step 2: Differentiate the curve.

$$y^2 = \frac{36}{5}x \Rightarrow 2y \frac{dy}{dx} = \frac{36}{5} \Rightarrow \frac{dy}{dx} = \frac{18}{5y}$$

Step 3: Apply the condition.

$$\frac{18}{5y} = 3 \Rightarrow y = \frac{6}{5}$$

Find x from the curve: $x = \frac{5}{36}y^2 = \frac{5}{36} \cdot \frac{36}{25} = \frac{1}{5}$.

Step 4: Check the options.

The point is $(\frac{1}{5}, \frac{6}{5})$, which is option (B). Option (A) has the coordinates swapped and does not satisfy the curve.

Final Answer:

The required point is $(\frac{1}{5}, \frac{6}{5})$.

$$\boxed{\left(\frac{1}{5}, \frac{6}{5}\right)}$$

Quick Tip: Differentiate with respect to time and set dy/dt equal to $3 \, dx/dt$.



109. The area of the triangle formed by the lines joining the vertex of the parabola $x^2 = 48y$ to the ends of its latus rectum, is

- (A) 144 sq. units
- (B) 288 sq. units
- (C) 72 sq. units
- (D) 576 sq. units

Correct Answer: (B) 288 sq. units

Solution:

Step 1: Understanding the Concept:

For a parabola $x^2 = 4ay$, the vertex is the origin, the focus is $(0, a)$ and the latus rectum is the chord through the focus at $y = a$. Its length is $4a$.

Step 2: Detailed Explanation:

Compare $x^2 = 48y$ with $x^2 = 4ay$: $4a = 48$, so $a = 12$.

The ends of the latus rectum are $(\pm 2a, a) = (\pm 24, 12)$.

Step 3: Find the area.

The triangle has the vertex $(0, 0)$ and the two ends $(24, 12)$ and $(-24, 12)$. Its base is 48 (the latus rectum) and its height is 12.

$$\text{Area} = \frac{1}{2} \times 48 \times 12 = 288 \text{ sq. units}$$

Final Answer:

The area is 288 square units, option (B).

288 sq. units

Quick Tip: Find a from $4a = 48$, then the ends of the latus rectum are (plus or minus $2a, a$), and use the triangle area.

110. $\lim_{x \rightarrow 0} \left[\frac{\log|2+x| - \log|2-x|}{\tan x} \right] = \dots\dots$

- (A) 2
- (B) 0
- (C) -1
- (D) 1

Correct Answer: (D) 1

Solution:

Step 1: Understanding the Concept:

At $x = 0$, the numerator is $\log 2 - \log 2 = 0$ and the denominator is $\tan 0 = 0$. So the form is $0/0$ and we can use L'Hopital rule.

Step 2: Detailed Explanation:

Differentiate the top and bottom (near 0, $2 + x$ and $2 - x$ are positive so the modulus can be dropped).

$$\frac{d}{dx} [\log(2 + x) - \log(2 - x)] = \frac{1}{2 + x} + \frac{1}{2 - x}$$

$$\frac{d}{dx} \tan x = \sec^2 x$$

$$\lim_{x \rightarrow 0} \frac{\frac{1}{2+x} + \frac{1}{2-x}}{\sec^2 x} = \frac{\frac{1}{2} + \frac{1}{2}}{1} = 1$$

Step 3: Check the options.

The limit is 1, which is not 0, 2 or -1 .

Final Answer:

The limit equals 1, option (D).

1

Quick Tip: Both numerator and denominator tend to zero, so use series or L Hopital rule, or write the standard limits.

111. Consider the statement patterns

- A. $(q \rightarrow p) \vee (p \rightarrow q)$
- B. $(\sim p \vee \sim q) \leftrightarrow \sim (p \wedge q)$
- C. $[(p \vee q) \wedge \sim p] \wedge \sim q$
- D. $(p \wedge q) \wedge (\sim p \vee \sim q)$

then statement patterns

- (A) A and B are contradictions, C and D are tautologies.
- (B) A and B are tautologies, C and D are contradictions.
- (C) A, B, C, D all are tautologies.
- (D) A, B, C, D all are contradictions.

Correct Answer: (B) A and B are tautologies, C and D are contradictions.

Solution:

Step 1: Understanding the Concept:

A tautology is true for every truth value assignment and a contradiction is false for every assignment. We simplify each pattern. In pattern B the connective between the two brackets is not shown clearly in print, and we read it so that it is the De Morgan type equivalence (an OR or an equivalence between $\sim p \vee \sim q$ and the negation of $p \wedge q$).

Step 2: Pattern A.

$(q \rightarrow p) \vee (p \rightarrow q)$. Write $p \rightarrow q \equiv \sim p \vee q$ and $q \rightarrow p \equiv \sim q \vee p$. Then $A \equiv \sim q \vee p \vee \sim p \vee q$. This contains $p \vee \sim p$, so it is always true. A is a tautology.

Step 3: Pattern B.

$\sim p \vee \sim q \equiv \sim (p \wedge q)$, so joining it with $p \wedge q$ by OR gives $\sim (p \wedge q) \vee (p \wedge q)$, which is of the form $r \vee \sim r$. B is a tautology.

Step 4: Pattern C.

$[(p \vee q) \wedge \sim p] \wedge \sim q$. If p is false then $p \vee q$ needs q true, but $\sim q$ needs q false. Both cannot hold. C is a contradiction.

Step 5: Pattern D.

$(p \wedge q) \wedge (\sim p \vee \sim q)$. The second bracket is $\sim (p \wedge q)$, so D is $r \wedge \sim r$. D is a contradiction.

Final Answer:

A and B are tautologies, and C and D are contradictions, option (B).

A, B tautologies; C, D contradictions

Quick Tip: Simplify each pattern using standard laws and decide whether it is always true or always false.



112. The statement pattern $(p \wedge q) \rightarrow (r \vee \sim s)$ is false. Then the truth values of p, q, r and s are respectively

- (A) T, F, T, T
- (B) T, T, F, T
- (C) T, T, T, F
- (D) T, T, F, F

Correct Answer: (B) T, T, F, T

Solution:

Step 1: Understanding the Concept:

An implication $A \rightarrow B$ is false in exactly one case: A is true and B is false. All other combinations make it true.

Step 2: Apply to the statement:

Here $A = (p \wedge q)$ and $B = (r \vee \sim s)$. So $p \wedge q$ must be true and $r \vee \sim s$ must be false.

Step 3: Find p and q:

A conjunction is true only when both parts are true. So $p = T$ and $q = T$.

Step 4: Find r and s:

A disjunction is false only when both parts are false. So $r = F$ and $\sim s = F$, which means $s = T$.

Step 5: Match the options:

The values are $p, q, r, s = T, T, F, T$. Option (A) has $q = F$, so $p \wedge q$ is false. Option (C) has $r = T$, so B is true. Option (D) has $s = F$, so $\sim s = T$ and B is true. In all three the whole statement would be true.

Final Answer:

The truth values are T, T, F, T, which is option (B).

B

Quick Tip: An implication is false only when the first part is true and the second part is false.

113. In a triangle ABC, with usual notations if $\frac{s-a}{11} = \frac{s-b}{12} = \frac{s-c}{13}$ and $\lambda \tan^2 \frac{A}{2} = 455$, then $\lambda =$

- (A) 1155
- (B) 1255
- (C) 1355
- (D) 1055

Correct Answer: (A) 1155

Solution:**Step 1: Understanding the Concept:**

In triangle ABC, s is the semi-perimeter. The half-angle formula is

$$\tan^2 \frac{A}{2} = \frac{(s-b)(s-c)}{s(s-a)}.$$

Step 2: Find the ratios.

Let $s - a = 11k$, $s - b = 12k$, $s - c = 13k$. Adding gives $3s - 2s = s = 36k$.

Step 3: Apply the formula.

$$\tan^2 \frac{A}{2} = \frac{12k \times 13k}{36k \times 11k} = \frac{156}{396} = \frac{13}{33}$$

Step 4: Solve for λ .

$$\lambda \cdot \frac{13}{33} = 455 \Rightarrow \lambda = \frac{455 \times 33}{13} = 35 \times 33 = 1155$$

Final Answer:

$\lambda = 1155$, option (A).

1155

Quick Tip: Let $s-a$, $s-b$, $s-c$ be proportional to 11, 12, 13 and use the half-angle formula for $\tan(A/2)$.

114. With usual notations, in $\triangle ABC$, if $2a^2 = b^2 + c^2$, then $\frac{\cos 3A}{\cos A} + 2 =$

- (A) $\frac{b^2 - c^2}{2bc}$
- (B) $\left(\frac{b^2 - c^2}{2bc}\right)^2$
- (C) $\left(\frac{c^2 - b^2}{bc}\right)^2$
- (D) $\frac{c^2 - b^2}{bc}$

Correct Answer: (B) $\left(\frac{b^2-c^2}{2bc}\right)^2$

Solution:

Step 1: Understanding the Concept:

The triple angle formula gives $\cos 3A = 4 \cos^3 A - 3 \cos A$. Dividing by $\cos A$ turns the ratio into an expression in $\cos^2 A$ only.

Step 2: Simplify the ratio.

$$\frac{\cos 3A}{\cos A} + 2 = 4 \cos^2 A - 3 + 2 = 4 \cos^2 A - 1$$

Step 3: Find $\cos A$.

Cosine rule: $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$. Since $a^2 = \frac{b^2 + c^2}{2}$, the numerator is $\frac{b^2 + c^2}{2}$ and

$$\cos A = \frac{b^2 + c^2}{4bc}$$

Step 4: Evaluate.

$$\begin{aligned} 4 \cos^2 A - 1 &= \frac{(b^2 + c^2)^2}{4b^2c^2} - 1 = \frac{(b^2 + c^2)^2 - 4b^2c^2}{4b^2c^2} = \frac{(b^2 - c^2)^2}{4b^2c^2} \\ &= \left(\frac{b^2 - c^2}{2bc}\right)^2 \end{aligned}$$

Step 5: Check the options.

Option (B) matches. Option (C) has bc instead of $2bc$ in the denominator. (A) and (D) are not squared.

Final Answer:

The value is $\left(\frac{b^2 - c^2}{2bc}\right)^2$, option (B).

$$\boxed{\left(\frac{b^2 - c^2}{2bc}\right)^2}$$

Quick Tip: Use $\cos 3A = 4\cos^3 A - 3\cos A$ to simplify the ratio, then use the cosine rule with the given relation.

115. With usual notations, in $\triangle ABC$, $(b - c)^2 \cos^2 \frac{A}{2} + (b + c)^2 \sin^2 \frac{A}{2} =$

- (A) a^2
- (B) $b^2 - c^2$
- (C) $a^2 + b^2 + c^2$
- (D) 0

Correct Answer: (A) a^2

Solution:

Step 1: Understanding the Concept:

We need to combine the two terms using the half-angle identities and the cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$.

Step 2: Expand.

$$(b - c)^2 \cos^2 \frac{A}{2} + (b + c)^2 \sin^2 \frac{A}{2} = (b^2 + c^2) \left(\cos^2 \frac{A}{2} + \sin^2 \frac{A}{2} \right) - 2bc \left(\cos^2 \frac{A}{2} - \sin^2 \frac{A}{2} \right)$$

Step 3: Simplify.

The first bracket is 1 and the second bracket is $\cos A$.

$$= b^2 + c^2 - 2bc \cos A = a^2$$

Step 4: Check the options.

The result is a^2 , so (A). The other options do not equal $b^2 + c^2 - 2bc \cos A$.

Final Answer:

The expression equals a^2 , option (A).

$$\boxed{a^2}$$

Quick Tip: Expand both squares and use $\cos A = \cos^2(A/2) - \sin^2(A/2)$, then the cosine rule.

116. Let $A = \begin{bmatrix} -5 & -3 \\ 2 & 1 \end{bmatrix}$. The Row transformation $R_1 \rightarrow R_1 + 3R_2$ will transform matrix A into

- (A) an upper triangular matrix
- (B) a lower triangular matrix
- (C) an identity matrix
- (D) a singular matrix

Correct Answer: (B) a lower triangular matrix

Solution:

Step 1: Understanding the Concept:

In the operation $R_1 \rightarrow R_1 + 3R_2$ we replace the first row by the first row plus three times the second row. The second row stays the same.

Step 2: Detailed Explanation:

$R_1 = (-5, -3)$ and $3R_2 = (6, 3)$.

New $R_1 = (-5 + 6, -3 + 3) = (1, 0)$.

$$A \rightarrow \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$$

Step 3: Classify.

The entry above the diagonal is 0, so this is a lower triangular matrix. It is not upper triangular (the entry 2 below the diagonal is non-zero), not an identity (because of the 2), and its determinant is 1, so it is not singular.

Final Answer:

The matrix becomes lower triangular, option (B).

Lower triangular matrix

Quick Tip: Carry out the row operation and read off the shape of the new matrix.

117. If $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}_{3 \times 3}$, and $A^{-1} = \begin{bmatrix} \gamma & -1 & 1 \\ \alpha & 6 & -5 \\ \beta & -2 & 2 \end{bmatrix}_{3 \times 3}$, then $|\alpha \cdot \beta \cdot \gamma| = \underline{\hspace{2cm}}$ (where $|\cdot|$ denotes the absolute value)

- (A) 125
- (B) 220
- (C) 225
- (D) -225

Correct Answer: (C) 225

Solution:

Step 1: Idea:

We are given A^{-1} , so $AA^{-1} = I$. Taking only the first column of A^{-1} , which is $(\gamma, \alpha, \beta)^T$, we must get the first column of I , which is $(1, 0, 0)^T$.

Step 2: Multiply:

$$\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} \begin{bmatrix} \gamma \\ \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

This gives three equations:

$$2\gamma - \beta = 1, 5\gamma + \alpha = 0, \alpha + 3\beta = 0.$$

Step 3: Solve:

From the second, $\alpha = -5\gamma$. From the third, $\alpha = -3\beta$, so $\beta = \frac{5\gamma}{3}$. Put this in the first: $2\gamma - \frac{5\gamma}{3} = 1$, so $\frac{\gamma}{3} = 1$ and $\gamma = 3$. Then $\alpha = -15$ and $\beta = 5$.

Step 4: Check with the determinant:

$|A| = 2(3 - 0) - 0 + (-1)(5 - 0) = 1$. The (1,1) entry of the adjoint is the cofactor 3, matching $\gamma = 3$.

Step 5: Compute the product:

$$|\alpha\beta\gamma| = |(-15)(5)(3)| = 225$$

Option (D) is -225 , which is wrong because an absolute value cannot be negative. Options (A) 125 and (B) 220 do not come from these values.

Final Answer:

$|\alpha\beta\gamma| = 225$, option (C).

225

Quick Tip: Multiply A by the first column of A inverse and set it equal to the first column of the identity matrix.

118. If $y = \tan^{-1}\left\{\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right\}$, where $|x| < 1$, then $\frac{dy}{dx}$ is equal to

- (A) $\frac{-x}{\sqrt{1-x^4}}$
- (B) $\frac{x}{\sqrt{1-x^4}}$
- (C) $\frac{-2x}{\sqrt{1-x^4}}$
- (D) $\frac{2x}{\sqrt{1-x^4}}$

Correct Answer: (B) $\frac{x}{\sqrt{1-x^4}}$

Solution:

Step 1: Understanding the Concept:

Expressions containing $\sqrt{1+x^2}$ and $\sqrt{1-x^2}$ can be simplified by substituting $x^2 = \cos 2\theta$.

Step 2: Substitute.

Let $x^2 = \cos 2\theta$. Then $1+x^2 = 2\cos^2 \theta$ and $1-x^2 = 2\sin^2 \theta$, so $\sqrt{1+x^2} = \sqrt{2}\cos \theta$ and $\sqrt{1-x^2} = \sqrt{2}\sin \theta$ (for θ in the first quadrant).

$$y = \tan^{-1} \left(\frac{\cos \theta - \sin \theta}{\cos \theta + \sin \theta} \right) = \tan^{-1} \left(\frac{1 - \tan \theta}{1 + \tan \theta} \right) = \tan^{-1} \tan \left(\frac{\pi}{4} - \theta \right) = \frac{\pi}{4} - \theta$$

Step 3: Go back to x.

Since $2\theta = \cos^{-1}(x^2)$, we get $y = \frac{\pi}{4} - \frac{1}{2}\cos^{-1}(x^2)$.

Step 4: Differentiate.

$$\frac{dy}{dx} = -\frac{1}{2} \cdot \left(-\frac{1}{\sqrt{1-x^4}} \right) \cdot 2x = \frac{x}{\sqrt{1-x^4}}$$

Final Answer:

$$\frac{dy}{dx} = \frac{x}{\sqrt{1-x^4}}, \text{ option (B).}$$

$$\boxed{\frac{x}{\sqrt{1-x^4}}}$$

Quick Tip: Put x squared equal to cos 2 theta so the fraction becomes tan of (pi/4 - theta).



119. The value of $\tan^{-1}(\sqrt{3}) + \sec^{-1}(-2) - \sin^{-1}(-\frac{1}{2})$ is

- (A) $\frac{5\pi}{6}$
- (B) $\frac{2\pi}{3}$
- (C) $\frac{7\pi}{6}$
- (D) $\frac{4\pi}{3}$

Correct Answer: (C) $\frac{7\pi}{6}$

Solution:

Step 1: Understanding the Concept:

Each inverse function must be evaluated in its principal range. For $\tan^{-1}$ the range is $(-\frac{\pi}{2}, \frac{\pi}{2})$, for $\sin^{-1}$ it is $[-\frac{\pi}{2}, \frac{\pi}{2}]$, and for $\sec^{-1}$ it is $[0, \pi]$ except $\frac{\pi}{2}$.

Step 2: Evaluate each term:

Since $\tan \frac{\pi}{3} = \sqrt{3}$ and $\frac{\pi}{3}$ is in range, $\tan^{-1}(\sqrt{3}) = \frac{\pi}{3}$.

For $\sec^{-1}(-2)$ we need $\cos \theta = -\frac{1}{2}$ with $\theta \in [0, \pi]$. That gives $\theta = \frac{2\pi}{3}$.

Since $\sin(-\frac{\pi}{6}) = -\frac{1}{2}$ and $-\frac{\pi}{6}$ is in range, $\sin^{-1}(-\frac{1}{2}) = -\frac{\pi}{6}$.

Step 3: Add the values:

$$\frac{\pi}{3} + \frac{2\pi}{3} - \left(-\frac{\pi}{6}\right) = \pi + \frac{\pi}{6} = \frac{7\pi}{6}$$

Step 4: Why the other options are wrong:

Option (A) $\frac{5\pi}{6}$ comes from taking $\sin^{-1}(-\frac{1}{2})$ with the wrong sign, which subtracts $\frac{\pi}{6}$ instead of adding it. Option (B) $\frac{2\pi}{3}$ and option (D) $\frac{4\pi}{3}$ come from wrong values for $\sec^{-1}(-2)$ or a lost term. None of these match the sum above.

Final Answer:

The three principal values add up to $\frac{7\pi}{6}$, which is option (C).

$$\boxed{\frac{7\pi}{6}}$$

Quick Tip: Use the principal value ranges: $\sec^{-1} x$ lies in $[0, \pi]$ except $\pi/2$, and $\sin^{-1} x$ lies in $[-\pi/2, \pi/2]$.

120. The domain of the function $f(x) = e^{\sqrt{5x-3-2x^2}}$ is

- (A) $(1, 2)$
- (B) $(-1, \frac{3}{2})$
- (C) $[1, \frac{3}{2}]$
- (D) $(-1, 0)$

Correct Answer: (C) $[1, \frac{3}{2}]$

Solution:

Step 1: Understanding the Concept:

e^t is defined for every real t . The only restriction comes from $\sqrt{}$, which needs a non-negative quantity inside.

Step 2: Set up the inequality.

$$5x - 3 - 2x^2 \geq 0 \Rightarrow 2x^2 - 5x + 3 \leq 0$$

Step 3: Solve.

Factorise: $2x^2 - 5x + 3 = (2x - 3)(x - 1)$. The product is ≤ 0 between the roots $x = 1$ and $x = 3/2$.

$$1 \leq x \leq \frac{3}{2}$$

Step 4: Check the options.

The end points are included because the square root of zero is defined. So the domain is the closed interval $[1, 3/2]$. Options (A), (B), (D) are open intervals or do not match.

Final Answer:

The domain is $\left[1, \frac{3}{2}\right]$, option (C).

$$\left[1, \frac{3}{2}\right]$$

Quick Tip: The exponential is defined everywhere, so only the expression under the square root must be non-negative.

121. If $f(x) = \begin{cases} \frac{(8-2x)^{\frac{1}{3}}-2}{3-(243+5x)^{\frac{1}{3}}}, & \text{if } x \neq 0 \\ k, & \text{if } x = 0 \end{cases}$ is continuous at $x = 0$, then $k =$

- (A) $\frac{5}{2}$
- (B) $-\frac{5}{2}$
- (C) $\frac{27}{2}$
- (D) $-\frac{27}{2}$

Correct Answer: (C) $\frac{27}{2}$

Solution:

Step 1: Continuity condition:

For f to be continuous at $x = 0$, $k = \lim_{x \rightarrow 0} \frac{(8-2x)^{1/3}-2}{3-(243+5x)^{1/5}}$. Both top and bottom go to 0, so this is a 0/0 form.

Step 2: Use the derivative idea:

Near $x = 0$, the top is $g(x) - g(0)$ with $g(x) = (8 - 2x)^{1/3}$, and the bottom is $-(h(x) - h(0))$ with $h(x) = (243 + 5x)^{1/5}$. Divide both by x and let $x \rightarrow 0$, so the limit becomes $\frac{g'(0)}{-h'(0)}$.

Step 3: Find the derivatives:

$$g'(x) = \frac{1}{3}(8 - 2x)^{-2/3}(-2), \text{ so } g'(0) = \frac{1}{3} \cdot \frac{1}{4} \cdot (-2) = -\frac{1}{6}.$$

$$h'(x) = \frac{1}{5}(243 + 5x)^{-4/5}(5), \text{ so } h'(0) = 243^{-4/5} = \frac{1}{81}.$$

Step 4: Combine:

$$k = \frac{-1/6}{-1/81} = \frac{81}{6} = \frac{27}{2}$$

Step 5: Check the options:

The negative values come from losing one of the two minus signs. $5/2$ would need wrong powers. Only $27/2$ fits.

Final Answer:

$$k = \frac{27}{2}, \text{ option (C).}$$

$$\boxed{\frac{27}{2}}$$

Quick Tip: Both parts tend to 0, so use the derivative of each root expression at 0 and divide.

122. If $y = x + e^x$, then $\frac{d^2x}{dy^2}$ is equal to

- (A) $\frac{e^x}{(1+e^x)^2}$
- (B) $\frac{-e^x}{(1+e^x)^2}$
- (C) $\frac{e^x}{(1+e^x)^3}$
- (D) $\frac{-e^x}{(1+e^x)^3}$

Correct Answer: (D) $\frac{-e^x}{(1+e^x)^3}$

Solution:**Step 1: Understanding the Concept:**

To get $\frac{d^2x}{dy^2}$ we treat x as a function of y . We first find $\frac{dx}{dy} = \frac{1}{dy/dx}$ and then differentiate this again with respect to y .

Step 2: First derivative.

$$\frac{dy}{dx} = 1 + e^x \Rightarrow \frac{dx}{dy} = \frac{1}{1 + e^x}$$

Step 3: Second derivative.

$$\frac{d^2x}{dy^2} = \frac{d}{dx} \left(\frac{1}{1+e^x} \right) \cdot \frac{dx}{dy}$$

$$\frac{d}{dx} \left(\frac{1}{1+e^x} \right) = \frac{-e^x}{(1+e^x)^2}$$

$$\frac{d^2x}{dy^2} = \frac{-e^x}{(1+e^x)^2} \cdot \frac{1}{1+e^x} = \frac{-e^x}{(1+e^x)^3}$$

Step 4: Why the other options are wrong.

Option (B) forgets the extra factor $\frac{dx}{dy}$, which gives only a square in the denominator. Options (A) and (C) have the wrong sign.

Final Answer:

$$\frac{d^2x}{dy^2} = \frac{-e^x}{(1+e^x)^3}, \text{ option (D).}$$

$$\boxed{\frac{-e^x}{(1+e^x)^3}}$$

Quick Tip: First find dx/dy as the reciprocal of dy/dx , then differentiate again with respect to y using the chain rule.

123. Let $f(x) = x - 5$.

If $g(x) = [f(4h(x) + 3)]^2$ and Missing open brace for superscript, then Missing open brace for superscript

(A) -242

(B) -224

(C) -112

(D) -640

Correct Answer: (B) -224

Solution:

Step 1: Understanding the Concept:

We need the derivative of a composite function at $x = 1$. First simplify f inside, then use the chain rule.

Step 2: Simplify g .

$f(4h + 3) = (4h + 3) - 5 = 4h - 2$. So $g(x) = (4h(x) - 2)^2$.

Step 3: Differentiate.

$$g'(x) = 2(4h(x) - 2) \cdot 4h'(x) = 8(4h(x) - 2)h'(x)$$

Step 4: Evaluate at $x = 1$.

With $h(1) = 4$: $4h(1) - 2 = 14$. With $h'(1) = -2$:

$$g'(1) = 8 \times 14 \times (-2) = -224$$

Step 5: Check the options.

The value -224 is option (B). The others come from arithmetic slips such as using $h(1)$ in place of $4h(1) - 2$.

Final Answer:

$g'(1) = -224$, option (B).

-224

Quick Tip: Simplify f first, then use the chain rule at $x = 1$ with the given values of $h(1)$ and $h'(1)$.



124. The equation of the normal to the curve $xy + 7 = 0$ is $Ax + By + C = 0$, then

- (A) $A > 0, B > 0$ or $A < 0, B < 0$
- (B) $A > 0, B < 0$
- (C) $A < 0, B > 0$
- (D) $C = 0$

Correct Answer: (A) $A > 0, B > 0$ or $A < 0, B < 0$

Solution:

Step 1: Understanding the Concept:

The normal is perpendicular to the tangent. Its slope is the negative reciprocal of the tangent slope. A line $Ax + By + C = 0$ has slope $-A/B$.

Step 2: Tangent slope.

$xy = -7$, so $y = -\frac{7}{x}$ and $\frac{dy}{dx} = \frac{7}{x^2}$. This is positive at every point of

the curve (where $x \neq 0$).

Step 3: Normal slope.

$$m_{normal} = -\frac{x^2}{7} < 0$$

Step 4: Link to A and B.

$-\frac{A}{B} < 0 \Rightarrow \frac{A}{B} > 0$, so A and B have the same sign. That means $A > 0, B > 0$ or $A < 0, B < 0$.

Step 5: Why the other options are wrong.

(B) and (C) say A and B have opposite signs, which would give a positive slope. (D) says $C = 0$, which holds only if the normal passes through the origin, and that is not true for every normal.

Final Answer:

A and B have the same sign, option (A).

$$A > 0, B > 0 \text{ or } A < 0, B < 0$$

Quick Tip: Find the slope of the tangent, then the slope of the normal, and compare it with the slope of the line $Ax + By + C = 0$.

125. A wire 40 metre in length is to be cut into two pieces. One piece is formed into a square and the other piece into a circle. The lengths of the two pieces so that the combined area of the square and the circle is minimum, are respectively

- (A) $\frac{160}{\pi+4}, \frac{40\pi}{\pi+4}$
 (B) $\frac{\pi+4}{160}, \frac{40\pi}{\pi+4}$
 (C) $\frac{160}{\pi+4}, \frac{\pi+4}{40\pi}$
 (D) 30, 10

Correct Answer: (A) $\frac{160}{\pi+4}, \frac{40\pi}{\pi+4}$

Solution:

Step 1: Understanding the Concept:

We have a wire of length 40 m cut into two pieces. One piece makes a square and the other makes a circle. We need the cut that makes the total area smallest, using calculus.

Step 2: Set up the area.

Let the square use x metres. Then its side is $x/4$ and its area is $x^2/16$. The circle uses $40 - x$ metres, so its radius is $r = \frac{40 - x}{2\pi}$ and its area is $\pi r^2 = \frac{(40 - x)^2}{4\pi}$.

$$A(x) = \frac{x^2}{16} + \frac{(40 - x)^2}{4\pi}$$

Step 3: Find the critical point.

$$A'(x) = \frac{x}{8} - \frac{40 - x}{2\pi} = 0 \Rightarrow \pi x = 4(40 - x) \Rightarrow x = \frac{160}{\pi + 4}$$

Step 4: Check it is a minimum.

$$A''(x) = \frac{1}{8} + \frac{1}{2\pi} > 0, \text{ so the area is minimum at this } x.$$

Step 5: Find the other piece.

$$40 - x = 40 - \frac{160}{\pi + 4} = \frac{40\pi}{\pi + 4}$$

Step 6: Check the options.

Option (A) gives $\frac{160}{\pi + 4}$ for the square and $\frac{40\pi}{\pi + 4}$ for the circle.

Options (B) and (C) have lengths of the wrong form. Option (D) does not add up to the minimising split.

Final Answer:

The pieces are $\frac{160}{\pi + 4}$ m for the square and $\frac{40\pi}{\pi + 4}$ m for the circle, option (A).

$\frac{160}{\pi + 4}, \frac{40\pi}{\pi + 4}$
--

Quick Tip: Let one piece be x metres, write the total area in x , then set the derivative to zero.

126. If a spherical balloon has a variable diameter $3x + \frac{9}{2}$ units, then the rate of change of its volume with respect to x is

- (A) $\frac{27\pi}{4}(2x + 3)^2$
- (B) $\frac{2\pi}{3}(2x + 3)^2$
- (C) $\frac{27\pi}{8}(2x + 3)^2$
- (D) $\frac{27\pi}{2}(2x + 3)^2$

Correct Answer: (C) $\frac{27\pi}{8}(2x + 3)^2$

Solution:

Step 1: Understanding the Concept:

The volume of a sphere is $V = \frac{4}{3}\pi r^3$. We need $\frac{dV}{dx}$, so we express V in terms of x and differentiate.

Step 2: Find the radius.

Diameter = $3x + \frac{9}{2}$, so $r = \frac{3x}{2} + \frac{9}{4} = \frac{3}{4}(2x + 3)$.

Step 3: Write the volume.

$$V = \frac{4}{3}\pi \cdot \frac{27}{64}(2x + 3)^3 = \frac{9\pi}{16}(2x + 3)^3$$

Step 4: Differentiate.

$$\frac{dV}{dx} = \frac{9\pi}{16} \cdot 3(2x + 3)^2 \cdot 2 = \frac{27\pi}{8}(2x + 3)^2$$

Step 5: Check the options.

Option (C) matches. Option (A) has a coefficient $\frac{27\pi}{4}$, which is double the correct one, and option (D) has $\frac{27\pi}{2}$, which is four times it.

Final Answer:

$$\frac{dV}{dx} = \frac{27\pi}{8}(2x + 3)^2, \text{ option (C).}$$

$$\frac{27\pi}{8}(2x+3)^2$$

Quick Tip: Write the radius from the diameter, find the volume in terms of x , then differentiate.



127. $\int \frac{30x^{14} + 15^x \log 225}{x^{15} + 15^x} dx =$

- (A) $x^{15} + 15x^{14} + 30x^{13} + c$, where c is the constant of integration
- (B) $\frac{\log(x^{15} + 15^x)}{2} + c$, where c is the constant of integration
- (C) $\log(x^{15} + 15^x)^2 + c$, where c is the constant of integration
- (D) $\log(x^{15} + 15^x) + c$, where c is the constant of integration

Correct Answer: (C) $\log(x^{15} + 15^x)^2 + c$, where c is the constant of integration

Solution:

Step 1: Understanding the Concept:

If the numerator is a multiple of the derivative of the denominator, the integral is that multiple of the log of the denominator: $\int \frac{f'(x)}{f(x)} dx = \log |f(x)| + c$.

Step 2: Find the derivative of the denominator.

Let $f(x) = x^{15} + 15^x$. Then $f'(x) = 15x^{14} + 15^x \log 15$.

Step 3: Compare with the numerator.

$\log 225 = \log 15^2 = 2 \log 15$. So the numerator is $30x^{14} + 2 \cdot 15^x \log 15 = 2(15x^{14} + 15^x \log 15) = 2f'(x)$.

Step 4: Integrate.

$$\int \frac{2f'(x)}{f(x)} dx = 2 \log(x^{15} + 15^x) + c = \log(x^{15} + 15^x)^2 + c$$

Step 5: Check the options.

Option (D) misses the factor 2. Option (B) divides by 2 instead of multiplying. Option (A) is a polynomial and not a logarithm.

Final Answer:

The integral is $\log(x^{15} + 15^x)^2 + c$, option (C).

$$\boxed{\log(x^{15} + 15^x)^2 + c}$$

Quick Tip: The numerator is a constant multiple of the derivative of the denominator.

128. If $\int \frac{\sin 2x}{\sin^4 x + \cos^4 x} dx = f(x) + c$ where c is constant of integration, then the value of $\tan(f(x))$ is

- (A) $\tan x$
- (B) $\tan^2 x$
- (C) $\tan^4 x$
- (D) $\cot^4 x$

Correct Answer: (B) $\tan^2 x$

Solution:

Step 1: Understanding the Concept:

Integrals with $\sin^4 x + \cos^4 x$ in the denominator are usually done by dividing by $\cos^4 x$, which turns everything into $\tan x$ and $\sec^2 x$.

Step 2: Rewrite the integrand.

$\sin 2x = 2 \sin x \cos x = 2 \tan x \cos^2 x$. Divide the numerator and denominator by $\cos^4 x$:

$$\frac{\sin 2x}{\sin^4 x + \cos^4 x} = \frac{2 \tan x \sec^2 x}{\tan^4 x + 1}$$

Step 3: Substitute.

Let $u = \tan^2 x$, so $du = 2 \tan x \sec^2 x dx$. Also $\tan^4 x = u^2$.

$$\int \frac{du}{u^2 + 1} = \tan^{-1} u + c = \tan^{-1}(\tan^2 x) + c$$

Step 4: Find $\tan f(x)$.

So $f(x) = \tan^{-1}(\tan^2 x)$ and $\tan f(x) = \tan^2 x$.

Step 5: Check the options.

Option (B) matches. The others do not come from this substitution.

Final Answer:

$\tan(f(x)) = \tan^2 x$, option (B).

$\tan^2 x$

Quick Tip: Divide the numerator and denominator by $\cos^4 x$ and substitute $u = \tan^2 x$.

129. $\int e^{2x} \frac{2(\sin 2x \cos 2x - 1)}{2\sin^2 2x} dx = A e^{2x} \cot 2x + c$

(Where c is the constant of integration.), then $A^3 =$

- (A) $\frac{1}{8}$
- (B) 64
- (C) 8
- (D) $\frac{1}{64}$

Correct Answer: (A) $\frac{1}{8}$

Solution:

Step 1: Understanding the Concept:

The integrand has the form e^{2x} [something]. We try to write the bracket as $g(x) + \frac{g'(x)}{2}$ with $g = \cot 2x$.

Step 2: Simplify the fraction.

$$\frac{2(\sin 2x \cos 2x - 1)}{2\sin^2 2x} = \frac{\sin 2x \cos 2x - 1}{\sin^2 2x} = \cot 2x - \operatorname{cosec}^2 2x$$

Step 3: Check the form.

Let $g(x) = \cot 2x$. Then $g'(x) = -2 \operatorname{cosec}^2 2x$, so $\frac{g'(x)}{2} = -\operatorname{cosec}^2 2x$.

The bracket is exactly $g + \frac{g'}{2}$.

Step 4: Integrate.

Since $\frac{d}{dx} [e^{2x}g] = e^{2x}(2g + g') = 2e^{2x} \left(g + \frac{g'}{2}\right)$, we get

$$\int e^{2x} \left(g + \frac{g'}{2}\right) dx = \frac{1}{2} e^{2x} \cot 2x + c$$

Step 5: Find A^3 .

So $A = \frac{1}{2}$ and $A^3 = \frac{1}{8}$.

Final Answer:

$A^3 = \frac{1}{8}$, option (A).

$\frac{1}{8}$

Quick Tip: Simplify the fraction to $\cot 2x - \operatorname{cosec}^2 2x$ and use the result that $e^{(ax)}[f(x) + f'(x)/a]$ integrates to $e^{(ax)} f(x)/a$.

130. If $\int_0^a \frac{dx}{1+4x^2} = \frac{\pi}{8}$, then $a =$

- (A) $\frac{1}{3}$
- (B) $\frac{1}{2}$
- (C) $\frac{1}{4}$
- (D) 1

Correct Answer: (B) $\frac{1}{2}$

Solution:

Step 1: Understanding the Concept:

The standard form is $\int \frac{dx}{1+k^2x^2} = \frac{1}{k} \tan^{-1}(kx)$.

Step 2: Integrate.

Here $k = 2$:

$$\int_0^a \frac{dx}{1+4x^2} = \frac{1}{2} \tan^{-1}(2x) \Big|_0^a = \frac{1}{2} \tan^{-1}(2a)$$

Step 3: Solve.

$$\frac{1}{2} \tan^{-1}(2a) = \frac{\pi}{8} \Rightarrow \tan^{-1}(2a) = \frac{\pi}{4} \Rightarrow 2a = 1 \Rightarrow a = \frac{1}{2}$$

Step 4: Check the options.

Putting $a = 1/3, 1/4$ or 1 gives $\frac{1}{2} \tan^{-1}(2/3), \frac{1}{2} \tan^{-1}(1/2)$ and $\frac{1}{2} \tan^{-1} 2$, none equal to $\pi/8$.

Final Answer:

$a = \frac{1}{2}$, option (B).

$a = \frac{1}{2}$

Quick Tip: Integrate using the arctan formula and solve for the upper limit.

131. The value of the integral $\int_{-\sqrt{3}}^{\sqrt{3}} \frac{(2x^9+3x^8-5x^7+9x^6+4x^3-x+3)}{x^2+3} dx$ is

- (A) $\frac{162}{7} + \sqrt{3} \frac{\pi}{4}$
 (B) $\frac{162\sqrt{3}}{7} + \sqrt{3} \frac{\pi}{2}$
 (C) $\frac{162\sqrt{3}}{7} + \frac{\pi}{2}$
 (D) $\frac{162\sqrt{3}}{7} - \frac{\pi}{2}$

Correct Answer: (B) $\frac{162\sqrt{3}}{7} + \sqrt{3} \frac{\pi}{2}$

Solution:

Step 1: Understanding the Concept:

Over the symmetric limits $-\sqrt{3}$ to $\sqrt{3}$, the integral of an odd function is 0 and the integral of an even function is twice the integral from 0 to $\sqrt{3}$.

Step 2: Split the numerator.

Odd terms: $2x^9 - 5x^7 + 4x^3 - x$. Even terms: $3x^8 + 9x^6 + 3$. The denominator $x^2 + 3$ is even, so the odd terms give an odd integrand, whose integral is 0.

Step 3: Simplify the even part.

$$\frac{3x^8 + 9x^6 + 3}{x^2 + 3} = \frac{3x^6(x^2 + 3) + 3}{x^2 + 3} = 3x^6 + \frac{3}{x^2 + 3}$$

Step 4: Integrate.

$$I = 2 \int_0^{\sqrt{3}} \left(3x^6 + \frac{3}{x^2 + 3} \right) dx = 2 \left[\frac{3x^7}{7} + \sqrt{3} \tan^{-1} \frac{x}{\sqrt{3}} \right]_0^{\sqrt{3}}$$

Since $(\sqrt{3})^7 = 27\sqrt{3}$ and $\tan^{-1} 1 = \pi/4$:

$$I = 2 \left[\frac{81\sqrt{3}}{7} + \sqrt{3} \cdot \frac{\pi}{4} \right] = \frac{162\sqrt{3}}{7} + \sqrt{3} \frac{\pi}{2}$$

Step 5: Check the options.

Option (B) matches. Option (A) misses a factor of $\sqrt{3}$ in the first term, and (C) and (D) have a wrong second term.

Final Answer:

The value of the integral is $\frac{162\sqrt{3}}{7} + \sqrt{3} \frac{\pi}{2}$, option (B).

$$\boxed{\frac{162\sqrt{3}}{7} + \sqrt{3} \frac{\pi}{2}}$$

Quick Tip: Split into odd and even parts over the symmetric interval, so the odd part vanishes, then simplify the even part by division.



132. The area of the region (in sq.units) bounded by the curve $y = 2\sqrt{1-x^2}$ and the X-axis is _____

- (A) $\frac{\pi}{2}$
- (B) $\frac{\pi^2}{2}$
- (C) π
- (D) $\frac{2\pi}{3}$

Correct Answer: (C) π

Solution:

Step 1: Understanding the Concept:

The curve exists only when $1 - x^2 \geq 0$, so x runs from -1 to 1 . The curve lies above the X -axis and meets it at $x = \pm 1$.

Step 2: Set up the integral:

$$A = \int_{-1}^1 2\sqrt{1 - x^2} \, dx$$

Step 3: Evaluate:

The graph of $y = \sqrt{1 - x^2}$ is the upper half of the unit circle. Its area is $\frac{1}{2}\pi(1)^2 = \frac{\pi}{2}$. The factor 2 doubles it:

$$A = 2 \times \frac{\pi}{2} = \pi$$

Step 4: Why the other options are wrong:

Option (A) $\frac{\pi}{2}$ forgets the factor 2. Option (B) $\frac{\pi^2}{2}$ has a wrong power of π , since an area of this type is a rational multiple of π . Option (D) $\frac{2\pi}{3}$ does not come from any correct step.

Final Answer:

The area is π square units, option (C).

$$\boxed{\pi}$$

Quick Tip: Square the curve to see it is half of an ellipse, or use the area of a semicircle times 2.

133. If $\frac{dy}{dx} = y + 5$ and $y(0) = 4$ then $y(\log 2)$ is equal to

- (A) 2
- (B) 5
- (C) 7
- (D) 13

Correct Answer: (D) 13

Solution:

Step 1: Understanding the Concept:

The equation is first order with separable variables. We move all y terms to one side and all x terms to the other, then integrate.

Step 2: Separate and integrate:

$$\frac{dy}{y+5} = dx \Rightarrow \log |y+5| = x + c$$

So $y + 5 = Ce^x$.

Step 3: Use the initial condition:

At $x = 0$, $y = 4$, so $9 = C$. Hence $y = 9e^x - 5$.

Step 4: Evaluate at $x = \log 2$:

Since $e^{\log 2} = 2$, we get $y(\log 2) = 9(2) - 5 = 13$.

Step 5: Why the other options are wrong:

Option (A) 2 and option (B) 5 come from dropping the constant or the factor 9. Option (C) 7 comes from forgetting to subtract or add the shift of 5 correctly. Only 13 follows from $C = 9$ and $y = 9e^x - 5$.

Final Answer:

The value is 13, option (D).

13

Quick Tip: Separate variables: $dy/(y+5) = dx$, integrate, then use $y(0)=4$.

134. The solution of the differential equation $\frac{dy}{dx} = \frac{x+y}{x-y}$ is

- (A) $c(x^2 + y^2)^{\frac{1}{2}} + e^{\tan^{-1}(\frac{y}{x})} = 0$, where c is an arbitrary constant
(B) $c(x^2 + y^2)^{\frac{1}{2}} = e^{\tan^{-1}(\frac{y}{x})}$, where c is an arbitrary constant
(C) $c(x^2 - y^2) = e^{\tan^{-1}(\frac{y}{x})}$, where c is an arbitrary constant
(D) $c(x^2 + y^2) = e^{\tan^{-1}(\frac{y}{x})}$, where c is an arbitrary constant

Correct Answer: (B) $c(x^2 + y^2)^{\frac{1}{2}} = e^{\tan^{-1}(\frac{y}{x})}$, where c is an arbitrary constant

Solution:**Step 1: Identify the type of equation:**

The right side $\frac{x+y}{x-y}$ has the same degree in x and y on top and bottom. Dividing both by x gives $\frac{1+y/x}{1-y/x}$, a function of y/x only. So the equation is homogeneous and the substitution $y = vx$ works. Then $\frac{dy}{dx} = v + x \frac{dv}{dx}$.

Step 2: Substitute and separate the variables:

Putting $y = vx$ gives $v + x \frac{dv}{dx} = \frac{1+v}{1-v}$. Move v to the right side:

$$x \frac{dv}{dx} = \frac{1+v}{1-v} - v = \frac{1+v^2}{1-v}$$

Now separate the variables:

$$\frac{1-v}{1+v^2} dv = \frac{dx}{x}$$

Step 3: Integrate both sides:

Split the left side into two standard integrals:

$$\int \frac{dv}{1+v^2} - \int \frac{v dv}{1+v^2} = \int \frac{dx}{x}$$

$$\tan^{-1} v - \frac{1}{2} \ln(1+v^2) = \ln|x| + C$$

Step 4: Go back to x and y:

Replace $v = \frac{y}{x}$. Since $\frac{1}{2} \ln\left(1 + \frac{y^2}{x^2}\right) = \frac{1}{2} \ln(x^2 + y^2) - \ln|x|$, the $\ln|x|$ terms cancel on both sides. This leaves

$$\tan^{-1}\left(\frac{y}{x}\right) = \frac{1}{2} \ln(x^2 + y^2) + C$$

Take the exponential of both sides: $e^{\tan^{-1}(y/x)} = e^C(x^2 + y^2)^{1/2}$. Writing the constant as c gives $c(x^2 + y^2)^{1/2} = e^{\tan^{-1}(y/x)}$.

Step 5: Why the other options are wrong:

Option (A) has $\frac{y^2}{x}$ inside the inverse tangent and a plus sign, which does not come out of the working. Option (C) has $x^2 - y^2$, but we always got $x^2 + y^2$ from $1 + v^2$. Option (D) has $x^2 + y^2$ without the square root. That would need the coefficient 1 instead of $\frac{1}{2}$ in front of

the logarithm, which our integration does not give.

Final Answer:

The general solution is $c(x^2 + y^2)^{1/2} = e^{\tan^{-1}(y/x)}$, which is option (B).

$$c(x^2 + y^2)^{\frac{1}{2}} = e^{\tan^{-1}(y/x)}$$

Quick Tip: The right side is homogeneous, so put $y = vx$ and separate the variables.



135. A spherical balloon expands at a rate proportional to its surface area. Initially its radius is 2 cm and 5 minutes later it increases upto 7 cm, then surface area of spherical balloon after 12 minutes will be

- (A) 2480 sq.cm.
- (B) 2460 sq.cm.
- (C) 2464 sq.cm.
- (D) 2400 sq.cm.

Correct Answer: (C) 2464 sq.cm.

Solution:

Step 1: Understanding the Concept:

For a sphere, $V = \frac{4}{3}\pi r^3$ and the surface area is $S = 4\pi r^2$. The statement says the volume grows at a rate proportional to the surface area.

Step 2: Form the differential equation.

$$\frac{dV}{dt} = kS \Rightarrow 4\pi r^2 \frac{dr}{dt} = k \cdot 4\pi r^2 \Rightarrow \frac{dr}{dt} = k$$

So the radius increases at a constant rate: $r = kt + C$.

Step 3: Use the data.

At $t = 0$, $r = 2$, so $C = 2$. At $t = 5$, $r = 7$, so $5k = 5$ and $k = 1$ cm per minute.

Step 4: Find the radius at 12 min.

$$r = 2 + 12 = 14 \text{ cm.}$$

Step 5: Surface area.

$$S = 4\pi r^2 = 4 \times \frac{22}{7} \times 196 = 4 \times 22 \times 28 = 2464 \text{ cm}^2$$

Final Answer:

The surface area after 12 minutes is 2464 square cm, option (C).

2464 sq. cm

Quick Tip: The volume rate is proportional to surface area, which makes the radius grow at a constant rate.



136. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors having magnitudes 1, 1 and 2 respectively. If $\vec{a} \times (\vec{a} \times \vec{c}) + \vec{b} = \vec{0}$, then the acute angle between $\vec{a}$ and $\vec{c}$ is

- (A) $\frac{\pi}{4}$
- (B) $\frac{\pi}{6}$
- (C) $\frac{\pi}{3}$
- (D) $\frac{\pi}{8}$

Correct Answer: (B) $\frac{\pi}{6}$

Solution:

Step 1: Understanding the Concept:

The vector triple product rule is $\vec{a} \times (\vec{a} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{a} - (\vec{a} \cdot \vec{a})\vec{c}$.

Step 2: Expand.

With $|\vec{a}| = 1$, we have $\vec{a} \cdot \vec{a} = 1$, so

$$(\vec{a} \cdot \vec{c})\vec{a} - \vec{c} + \vec{b} = \vec{0} \Rightarrow \vec{b} = \vec{c} - (\vec{a} \cdot \vec{c})\vec{a}$$

Step 3: Take the magnitude squared.

$$|\vec{b}|^2 = |\vec{c}|^2 - 2(\vec{a} \cdot \vec{c})^2 + (\vec{a} \cdot \vec{c})^2 |\vec{a}|^2 = 4 - (\vec{a} \cdot \vec{c})^2$$

Since $|\vec{b}| = 1$: $(\vec{a} \cdot \vec{c})^2 = 3$.

Step 4: Find the angle.

$\vec{a} \cdot \vec{c} = |\vec{a}||\vec{c}| \cos \theta = 2 \cos \theta$. So $4 \cos^2 \theta = 3$, giving $\cos \theta = \pm \frac{\sqrt{3}}{2}$. The acute angle has $\cos \theta = \frac{\sqrt{3}}{2}$, so $\theta = \frac{\pi}{6}$.

Final Answer:

The acute angle is $\frac{\pi}{6}$, option (B).

$$\frac{\pi}{6}$$

Quick Tip: Use the vector triple product expansion to relate b with a and c , then take magnitudes.

137. The volume of a tetrahedron with vertices $5\hat{i} - \hat{j} + \hat{k}$, $7\hat{i} - 4\hat{j} + p\hat{k}$, $\hat{i} - 6\hat{j} + 10\hat{k}$ and $-\hat{i} - 3\hat{j} + 7\hat{k}$ is 11 cubic units, then one of the values of p is

- (A) 1
- (B) 2
- (C) 0
- (D) 3

Correct Answer: (A) 1

Solution:

Step 1: Understanding the Concept:

For a tetrahedron with vertices A, B, C, D , the volume is

$$\frac{1}{6} |[\vec{AB} \vec{AC} \vec{AD}]|.$$

Step 2: Edge vectors.

Take $A(5, -1, 1)$. Then $\vec{AB} = (2, -3, p-1)$, $\vec{AC} = (-4, -5, 9)$, $\vec{AD} = (-6, -2, 6)$.

Step 3: Compute the determinant.

$$\begin{vmatrix} 2 & -3 & p-1 \\ -4 & -5 & 9 \\ -6 & -2 & 6 \end{vmatrix} = 2(-30 + 18) + 3(-24 + 54) + (p-1)(8 - 30)$$

$$= -24 + 90 - 22(p - 1) = 66 - 22(p - 1)$$

Step 4: Use the volume.

$$\frac{1}{6}|66 - 22(p - 1)| = 11 \Rightarrow |66 - 22(p - 1)| = 66.$$

Case 1: $66 - 22(p - 1) = 66$, so $p = 1$.

Case 2: $66 - 22(p - 1) = -66$, so $p - 1 = 6$ and $p = 7$.

Step 5: Check the options.

One of the values of p is 1, which is option (A). The value 7 is not listed.

Final Answer:

One value is $p = 1$, option (A).

$$\boxed{p = 1}$$

Quick Tip: The volume is one sixth of the absolute value of the scalar triple product of the three edge vectors from one vertex.

138. Let $\vec{a}, \vec{b}, \vec{c}$ be unit vectors such that $\vec{a}$ is perpendicular to the plane of $\vec{b}$ and $\vec{c}$. If the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$, then $|\vec{a} + \vec{b} + \vec{c}| =$

- (A) 1
- (B) 2
- (C) 4
- (D) 16

Correct Answer: (B) 2

Solution:

Step 1: Understanding the Concept:

The vector $\vec{a}$ is perpendicular to the plane that holds $\vec{b}$ and $\vec{c}$. So $\vec{a} \cdot \vec{b} = 0$ and $\vec{a} \cdot \vec{c} = 0$.

Step 2: Expand the square.

$$|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{a} \cdot \vec{b} + 2\vec{b} \cdot \vec{c} + 2\vec{c} \cdot \vec{a}$$

Step 3: Put the values.

All three are unit vectors, so the squares add to 3. The terms with $\vec{a}$ vanish. And $\vec{b} \cdot \vec{c} = \cos \frac{\pi}{3} = \frac{1}{2}$.

$$= 3 + 0 + 2 \cdot \frac{1}{2} + 0 = 4$$

Step 4: Take the root.

$$|\vec{a} + \vec{b} + \vec{c}| = \sqrt{4} = 2.$$

Final Answer:

The magnitude is 2, option (B).

2

Quick Tip: Square the sum and use the dot products: $\vec{a}$ is perpendicular to both $\vec{b}$ and $\vec{c}$, and $\vec{b} \cdot \vec{c}$ is $\cos 60$ degrees.

139. If a parallelogram is constructed on the vectors $\vec{a} = 3\vec{p} - \vec{q}$, $\vec{b} = \vec{p} + 3\vec{q}$ and $|\vec{p}| = 3$, $|\vec{q}| = 2$ and angle between $\vec{p}$ and $\vec{q}$ is $\frac{\pi}{3}$, then the ratio of the lengths of adjacent sides $\vec{a}$ and $\vec{b}$ of the parallelogram is

(A) $\sqrt{57} : \sqrt{54}$

(B) $\sqrt{67} : \sqrt{63}$

(C) $\sqrt{63} : \sqrt{47}$

(D) $\sqrt{57} : \sqrt{52}$

Correct Answer: (B) $\sqrt{67} : \sqrt{63}$

Solution:

Step 1: Understanding the Concept:

The length of a vector squared is the dot product with itself. For this we need $\vec{p} \cdot \vec{q} = |\vec{p}||\vec{q}| \cos \frac{\pi}{3} = 3 \times 2 \times \frac{1}{2} = 3$.

Step 2: Length of $\vec{a}$.

$$|\vec{a}|^2 = |3\vec{p} - \vec{q}|^2 = 9|\vec{p}|^2 - 6\vec{p} \cdot \vec{q} + |\vec{q}|^2 = 81 - 18 + 4 = 67$$

Step 3: Length of $\vec{b}$.

$$|\vec{b}|^2 = |\vec{p} + 3\vec{q}|^2 = |\vec{p}|^2 + 6\vec{p} \cdot \vec{q} + 9|\vec{q}|^2 = 9 + 18 + 36 = 63$$

Step 4: Ratio.

$$|\vec{a}| : |\vec{b}| = \sqrt{67} : \sqrt{63}, \text{ which is option (B).}$$

Final Answer:

The ratio of the sides is $\sqrt{67} : \sqrt{63}$.

$$\boxed{\sqrt{67} : \sqrt{63}}$$

Quick Tip: Find the square of each side using the dot product with $\mathbf{p} \cdot \mathbf{q} = 3$.

140. A vector $\vec{r}$ of magnitude $3\sqrt{2}$ units which makes angles of $\frac{\pi}{4}$ and $\frac{\pi}{2}$ respectively with Y and Z axes is

- (A) $\vec{r} = \pm 3\hat{i} + 3\hat{j}$
- (B) $\vec{r} = \hat{i} + \hat{j}$
- (C) $\vec{r} = \pm 2\hat{i} + 3\hat{j}$
- (D) $\vec{r} = \pm 5\hat{i} + \hat{j}$

Correct Answer: (A) $\vec{r} = \pm 3\hat{i} + 3\hat{j}$

Solution:**Step 1: Understanding the Concept:**

A vector of magnitude r with direction cosines l, m, n is $\vec{r} = r(l\hat{i} + m\hat{j} + n\hat{k})$, where $l^2 + m^2 + n^2 = 1$.

Step 2: Find m and n.

The angle with the Y axis is $\pi/4$, so $m = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$. The angle with the Z axis is $\pi/2$, so $n = 0$.

Step 3: Find l.

$$l^2 + \frac{1}{2} + 0 = 1, \text{ so } l = \pm \frac{1}{\sqrt{2}}.$$

Step 4: Build the vector.

$$\vec{r} = 3\sqrt{2} \left(\pm \frac{1}{\sqrt{2}} \hat{i} + \frac{1}{\sqrt{2}} \hat{j} \right) = \pm 3\hat{i} + 3\hat{j}$$

Step 5: Check.

Its magnitude is $\sqrt{9 + 9} = 3\sqrt{2}$. Options (B), (C) and (D) have a different magnitude.

Final Answer:

$$\vec{r} = \pm 3\hat{i} + 3\hat{j}, \text{ option (A).}$$

$$\boxed{\vec{r} = \pm 3\hat{i} + 3\hat{j}}$$

Quick Tip: Use direction cosines: l squared plus m squared plus n squared equals 1, then multiply by the magnitude.

141. The point having position vector $4\hat{i} - 11\hat{j} + 2\hat{k}$ lies on the line

- (A) $\vec{r} = (6\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(\hat{i} + 7\hat{j} + 3\hat{k})$
- (B) $\vec{r} = (6\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + \hat{j} + 3\hat{k})$
- (C) $\vec{r} = (6\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + \hat{k})$
- (D) $\vec{r} = (6\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 7\hat{j} + 3\hat{k})$

Correct Answer: (D) $\vec{r} = (6\hat{i} - 4\hat{j} + 5\hat{k}) + \lambda(2\hat{i} + 7\hat{j} + 3\hat{k})$

Solution:

Step 1: Understanding the Concept:

A point P lies on the line $\vec{r} = \vec{a} + \lambda\vec{d}$ if $\vec{AP} = \vec{P} - \vec{a}$ is a scalar multiple of $\vec{d}$.

Step 2: Compute the difference.

All four lines have the same base point $(6, -4, 5)$. The difference is $(4 - 6, -11 + 4, 2 - 5) = (-2, -7, -3)$.

Step 3: Compare with each direction.

$(-2, -7, -3) = -1 \times (2, 7, 3)$. This matches the direction in option (D).

Step 4: Why the other options are wrong.

(A) $(1, 7, 3)$: ratios $-2, -1, -1$ are not equal. (B) $(2, 1, 3)$: ratios $-1, -7, -1$ are not equal. (C) $(2, 3, 1)$: ratios $-1, -7/3, -3$ are not equal.

Final Answer:

The point lies on the line in option (D) with $\lambda = -1$.

(D)

Quick Tip: Subtract the base point of each line from the given point and check whether the difference is a multiple of the direction vector.

142. The shortest distance between the lines, where the first line passes through $(0, 0, 0)$ and $(2, 0, 3)$ and the second line passes through $(2, 5, 0)$ and $(0, 4, 0)$ is

- (A) $\frac{24}{7}$ units
 (B) $\frac{9}{7}$ units
 (C) $\frac{1}{7}$ units
 (D) $\frac{36}{7}$ units

Correct Answer: (A) $\frac{24}{7}$ units

Solution:

Step 1: Understanding the Concept:

The shortest distance between two skew lines is d

$$= \frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{d}_1 \times \vec{d}_2)|}{|\vec{d}_1 \times \vec{d}_2|}.$$

Step 2: Find the directions.

Line 1 through $(0, 0, 0)$ and $(2, 0, 3)$: $\vec{d}_1 = (2, 0, 3)$. Line 2 through $(2, 5, 0)$ and $(0, 4, 0)$: $\vec{d}_2 = (-2, -1, 0)$.

Step 3: Cross product.

$$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 0 & 3 \\ -2 & -1 & 0 \end{vmatrix} = (0 + 3)\hat{i} - (0 + 6)\hat{j} + (-2 - 0)\hat{k} = (3, -6, -2)$$

Its magnitude is $\sqrt{9 + 36 + 4} = 7$.

Step 4: Joining vector.

$\vec{a}_2 - \vec{a}_1 = (2, 5, 0)$. The dot product with the cross product is $6 - 30 + 0 = -24$.

Step 5: Distance.

$$d = \frac{|-24|}{7} = \frac{24}{7} \text{ units}$$

Final Answer:

The shortest distance is $\frac{24}{7}$ units, option (A).

$$\boxed{\frac{24}{7}}$$

Quick Tip: Find direction vectors of both lines, take their cross product, and project the vector joining points on the lines.

143. The co-ordinates of the foot of the perpendicular from the origin to the plane $2x - 3y - 6z = 49$ are...

- (A) $(2, -3, -6)$
- (B) $(\frac{2}{7}, \frac{-3}{7}, \frac{-6}{7})$
- (C) $(-2, 3, 6)$
- (D) $(\frac{-2}{7}, \frac{3}{7}, \frac{6}{7})$

Correct Answer: (A) $(2, -3, -6)$

Solution:

Step 1: Understanding the Concept:

The perpendicular from the origin to a plane runs along the normal vector of the plane. So the foot is a point on the line through the origin with that direction.

Step 2: Write the foot:

The normal is $(2, -3, -6)$. Take the foot as $(2\lambda, -3\lambda, -6\lambda)$.

Step 3: Use the plane equation:

The foot lies on the plane, so

$$2(2\lambda) - 3(-3\lambda) - 6(-6\lambda) = 49$$

$$4\lambda + 9\lambda + 36\lambda = 49 \Rightarrow 49\lambda = 49 \Rightarrow \lambda = 1$$

Step 4: Find the point:

Put $\lambda = 1$: the foot is $(2, -3, -6)$.

Step 5: Why the other options are wrong:

Option (C) $(-2, 3, 6)$ gives -49 on the left side, so it is not on the plane.

Options (B) and (D) are the points with $\lambda = \frac{1}{7}$ and $\lambda = -\frac{1}{7}$. Putting them in the plane gives 7 and -7 , not 49, so they do not lie on it.

Final Answer:

The foot of the perpendicular is $(2, -3, -6)$, option (A).

$$\boxed{(2, -3, -6)}$$

Quick Tip: The foot lies on the line through the origin along the normal $(2, -3, -6)$; put that point in the plane.

144. Let a plane P pass through the point $(3, 7, -7)$ and contain the line $\frac{x-2}{-3} = \frac{y-3}{2} = \frac{z+2}{1}$. If the distance of the plane P from the origin is d, then d^2 is

- (A) 2
- (B) 3
- (C) 4
- (D) 6

Correct Answer: (B) 3

Solution:

Step 1: Understanding the Concept:

A plane that contains a line and a given point has a normal perpendicular to both the line direction and the vector from a point on the line to the given point.

Step 2: Collect the data.

The line has direction $\vec{d} = (-3, 2, 1)$ and passes through $A(2, 3, -2)$. The given point is $B(3, 7, -7)$, so $\vec{AB} = (1, 4, -5)$.

Step 3: Find the normal.

$$\vec{n} = \vec{d} \times \vec{AB} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 2 & 1 \\ 1 & 4 & -5 \end{vmatrix} = (-14, -14, -14)$$

So the normal is along $(1, 1, 1)$.

Step 4: Equation of the plane.

The plane is $x + y + z = c$. Put $B(3, 7, -7)$: $c = 3$. Check with A : $2 + 3$

$-2 = 3$, which agrees.

Step 5: Distance from the origin.

$$d = \frac{|3|}{\sqrt{3}} = \sqrt{3} \Rightarrow d^2 = 3$$

Final Answer:

$d^2 = 3$, option (B).

3

Quick Tip: Find the normal as the cross product of the line direction and a vector joining a point on the line to the given point.

145. The plane $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$ cuts the co-ordinate axes at the points A, B, C respectively. Then the area of triangle ABC is

- (A) $\sqrt{61}$ sq. units
- (B) $\frac{\sqrt{61}}{2}$ sq. units
- (C) $\frac{\sqrt{61}}{4}$ sq. units
- (D) $\frac{\sqrt{71}}{2}$ sq. units

Correct Answer: (A) $\sqrt{61}$ sq. units

Solution:

Step 1: Understanding the Concept:

The plane meets the axes at $A(2, 0, 0)$, $B(0, 3, 0)$ and $C(0, 0, 4)$. The area

of triangle ABC is $\frac{1}{2}|\vec{AB} \times \vec{AC}|$.

Step 2: Find the sides.

$\vec{AB} = (-2, 3, 0)$ and $\vec{AC} = (-2, 0, 4)$.

Step 3: Cross product.

$$\vec{AB} \times \vec{AC} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 3 & 0 \\ -2 & 0 & 4 \end{vmatrix} = (12, 8, 6)$$

Step 4: Area.

$$\text{Area} = \frac{1}{2}\sqrt{144 + 64 + 36} = \frac{1}{2}\sqrt{244} = \frac{1}{2} \cdot 2\sqrt{61} = \sqrt{61}$$

Step 5: Check the options.

Option (A) matches. Option (B) is half of it, and option (C) is a quarter of it.

Final Answer:

The area is $\sqrt{61}$ square units, option (A).

$\sqrt{61}$ sq. units

Quick Tip: Find the intercept points A, B and C, then compute half the magnitude of the cross product of two sides.

146. In L.P.P., the corner points of the feasible region for the constraints $3x - y \geq 6, x \leq 3, y \leq 2, y \geq 0, x \geq 0$ are

- (A) $(3, 2), (3, 0), (2, 0)$
- (B) $(\frac{8}{3}, 2), (3, 2), (3, 0), (2, 0)$
- (C) $(0, 0), (2, 0), (\frac{8}{3}, 2), (0, 2)$
- (D) $(3, 2), (0, 3), (0, 2)$

Correct Answer: (B) $(\frac{8}{3}, 2), (3, 2), (3, 0), (2, 0)$

Solution:

Step 1: Understanding the Concept:

The corner points of the feasible region are intersections of the boundary lines that satisfy all the constraints.

Step 2: Study the constraints.

$3x - y \geq 6, x \leq 3, y \leq 2, x \geq 0, y \geq 0$. The line $3x - y = 6$ meets $y = 0$ at $x = 2$, and meets $y = 2$ at $x = \frac{8}{3}$.

Step 3: Find the corners.

The region is the area to the right of the line $3x - y = 6$, between $y = 0$ and $y = 2$, and to the left of $x = 3$.

Corner 1: line with $y = 0$: $(2, 0)$.

Corner 2: $x = 3$ with $y = 0$: $(3, 0)$.

Corner 3: $x = 3$ with $y = 2$: $(3, 2)$.

Corner 4: line with $y = 2$: $(\frac{8}{3}, 2)$.

Step 4: Check the options.

Option (B) lists exactly these four points. Option (A) misses $(\frac{8}{3}, 2)$.

Options (C) and (D) contain points such as $(0, 0)$ and $(0, 2)$, which fail

$$3x - y \geq 6.$$

Final Answer:

The corner points are $\left(\frac{8}{3}, 2\right), (3, 2), (3, 0), (2, 0)$, option (B).

$$\left(\frac{8}{3}, 2\right), (3, 2), (3, 0), (2, 0)$$

Quick Tip: Draw each constraint, find the boundary intersections that lie inside every other constraint, and list them.

147. The odds in favour of A winning a game of table tennis against B are 1:2. If 3 games are to be played, then the probability of A winning at least two games out of the three is.....

- (A) $\frac{1}{27}$
- (B) $\frac{2}{27}$
- (C) $\frac{5}{27}$
- (D) $\frac{7}{27}$

Correct Answer: (D) $\frac{7}{27}$

Solution:**Step 1: Understanding the Concept:**

Odds in favour of 1:2 mean 1 favourable outcome for every 2 unfavourable. So $p = \frac{1}{1+2} = \frac{1}{3}$ and $q = \frac{2}{3}$.

Step 2: Key Formula or Approach:

For $n = 3$ games, $P(X = r) = {}^3C_r p^r q^{3-r}$. We need $P(X \geq 2) = P(X = 2)$

+ $P(X = 3)$.

Step 3: Calculate.

$$P(X = 2) = 3 \cdot \left(\frac{1}{3}\right)^2 \cdot \frac{2}{3} = \frac{6}{27}$$

$$P(X = 3) = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$$

$$P(X \geq 2) = \frac{6}{27} + \frac{1}{27} = \frac{7}{27}$$

Step 4: Check the options.

Option (C), $\frac{5}{27}$, and the others come from forgetting one of the two cases or from using $p = 1/2$.

Final Answer:

The probability is $\frac{7}{27}$, option (D).

$$\boxed{\frac{7}{27}}$$

Quick Tip: Convert the odds to a probability of winning, then use the binomial formula for 2 or 3 wins.

148. A random variable X has the following probability distribution

$X = x$	1	2	3	4	5	6	7	8
$P(X = x)$	0.15	0.23	0.10	0.12	0.20	0.08	0.07	0.05

For the events $E = \{X \text{ is a prime number}\}$, $F = \{X < 4\}$, $P(E \cup F)$ is

- (A) 0.5
- (B) 0.77
- (C) 0.35
- (D) 0.75

Correct Answer: (D) 0.75

Solution:

Step 1: Understanding the Concept:

For two events, $P(E \cup F) = P(E) + P(F) - P(E \cap F)$.

Step 2: Find the events.

E (prime numbers in 1 to 8): $\{2, 3, 5, 7\}$. F ($X < 4$): $\{1, 2, 3\}$. $E \cap F = \{2, 3\}$.

Step 3: Add the probabilities.

$$P(E) = 0.23 + 0.10 + 0.20 + 0.07 = 0.60.$$

$$P(F) = 0.15 + 0.23 + 0.10 = 0.48.$$

$$P(E \cap F) = 0.23 + 0.10 = 0.33.$$

Step 4: Combine.

$$P(E \cup F) = 0.60 + 0.48 - 0.33 = 0.75$$

Final Answer:

$P(E \cup F) = 0.75$, option (D).

0.75

Quick Tip: List the values in each event, add their probabilities and subtract the overlap.

149. A random variable X takes the values 0, 1, 2, 3 and its mean is 1.3. If $P(X = 3) = 2P(X = 1)$ and $P(X = 2) = 0.3$, then $P(X = 0)$ is

- (A) 0.3
- (B) 0.4
- (C) 0.2
- (D) 0.5

Correct Answer: (B) 0.4

Solution:**Step 1: Understanding the Concept:**

Let $P(X = 1) = a$. Then $P(X = 3) = 2a$, $P(X = 2) = 0.3$, and let $P(X = 0) = b$.

Step 2: Use the mean.

$$E(X) = 0 \cdot b + 1 \cdot a + 2(0.3) + 3(2a) = 7a + 0.6 = 1.3 \Rightarrow a = 0.1$$

Step 3: Use the total.

$$b + a + 0.3 + 2a = 1, \text{ so } b = 1 - 0.3 - 3(0.1) = 0.4.$$

Step 4: Check.

The probabilities are 0.4, 0.1, 0.3, 0.2, and they add to 1. The mean is $0.1 + 0.6 + 0.6 = 1.3$.

Final Answer:

$$P(X = 0) = 0.4, \text{ option (B).}$$

0.4

Quick Tip: Use the total probability being 1 and the mean equation to find the unknown probabilities.

150. Let $X \sim B(6, \frac{1}{2})$. Then the maximum probability occurs at

- (A) $X = 1$
- (B) $X = 2$
- (C) $X = 3$
- (D) $X = 4$

Correct Answer: (C) $X = 3$

Solution:

Step 1: Understanding the Concept:

For $X \sim B(n, p)$, $P(X = r) = {}^nC_r p^r q^{n-r}$. Here $p = q = \frac{1}{2}$, so $P(X = r)$

$= {}^6C_r \left(\frac{1}{2}\right)^6$. The probability is largest where ${}^6C_r$ is largest.

Step 2: Compare the coefficients.

$${}^6C_0 = 1, {}^6C_1 = 6, {}^6C_2 = 15, {}^6C_3 = 20, {}^6C_4 = 15, {}^6C_5 = 6, {}^6C_6 = 1.$$

Step 3: Pick the maximum.

The largest is 20 at $r = 3$. So the maximum probability $\frac{20}{64}$ occurs at $X = 3$.

Final Answer:

The maximum probability occurs at $X = 3$, option (C).

$$\boxed{X = 3}$$

Quick Tip: Compare the binomial probabilities, or use the mode formula for a binomial distribution.