

MHT CET 2026 PCM Question Paper April 15 Shift 1 with Solutions



General Instructions

- (i) This booklet contains 150 questions, each provided with a complete, step-by-step solution.
- (ii) It comprises 150 single-correct multiple-choice questions.
- (iii) Attempt each question on your own before reviewing the given solution.

Physics

Questions 1 to 50 | 1 mark each | 50 marks

1. The length of rod is measured by meter scale having least count 0.1 cm and diameter is measured by vernier callipers having least count 0.01 cm. Length of rod is 5.0 cm and radius 2.0 cm. The percentage error in the calculated value of the volume is

- (A) 1%
- (B) 2.5%
- (C) 5%
- (D) 7%

Correct Answer: (B) 2.5%

Solution:

Step 1: Understanding the Concept:

Volume of a cylinder is $V = \pi r^2 L$. Percentage errors add up with multiplication by the power of each quantity.

Step 2: Error formula:

$$\frac{\Delta V}{V} = 2 \frac{\Delta r}{r} + \frac{\Delta L}{L}$$

Step 3: Find the absolute errors:

The length has least count 0.1 cm, so $\Delta L = 0.1$ cm. The vernier measures the diameter with least count 0.01 cm, so $\Delta d = 0.01$ cm and hence $\Delta r = 0.005$ cm.

Step 4: Compute:

$$\frac{\Delta r}{r} = \frac{0.005}{2.0} = 0.25\% \text{ and } \frac{\Delta L}{L} = \frac{0.1}{5.0} = 2\%. \text{ So}$$

$$\frac{\Delta V}{V} \times 100 = 2(0.25) + 2 = 2.5\%$$

Step 5: Choose:

Option (B).

Final Answer:

The percentage error in volume is 2.5 percent.

2.5%

Quick Tip: Use $dV/V = 2 dr/r + dL/L$, with $dr = 0.005$ cm.

2. A vector $\vec{A}$ when added to the sum of the vectors $(\hat{i} - 2\hat{j} + 2\hat{k})$ and $(-2\hat{i} + \hat{j} - \hat{k})$ gives a unit vector along Y axis. The magnitude of vector $\vec{A}$ is

- (A) $\sqrt{3}$
- (B) $\sqrt{6}$
- (C) $\sqrt{8}$
- (D) $\sqrt{10}$

Correct Answer: (B) $\sqrt{6}$

Solution:

Step 1: Understanding the Concept:

Vector addition works by components. We want $\vec{A} + \vec{S} = \hat{j}$ where $\vec{S}$ is the sum of the two given vectors.

Step 2: Find the sum:

$$\vec{S} = (\hat{i} - 2\hat{j} + 2\hat{k}) + (-2\hat{i} + \hat{j} - \hat{k}) = -\hat{i} - \hat{j} + \hat{k}$$

Step 3: Find A:

$$\vec{A} = \hat{j} - \vec{S} = \hat{j} + \hat{i} + \hat{j} - \hat{k} = \hat{i} + 2\hat{j} - \hat{k}$$

Step 4: Magnitude:

$$|\vec{A}| = \sqrt{1 + 4 + 1} = \sqrt{6}$$

Step 5: Choose:

Option (B).

Final Answer:

The magnitude is $\sqrt{6}$.

$$\sqrt{6}$$

Quick Tip: Subtract the sum of the two vectors from $\mathbf{j}$.

3. A stone falls from the top of a tower of height 100 m and at the same time another stone is projected vertically upwards from the ground with a velocity 25 m/s. The two stones meet after

- (A) 2 s
- (B) 5 s
- (C) 4 s
- (D) 3 s

Correct Answer: (C) 4 s

Solution:

Step 1: Understanding the Concept:

Both stones move under the same acceleration g , so relative to each other they move at constant relative speed. This simplifies the meeting time.

Step 2: Relative motion:

The falling stone starts at rest, and the other starts at 25 m/s upward. Gravity affects both equally, so their separation shrinks at a constant

rate equal to the upward speed, 25 m/s.

Step 3: Compute:

Initial gap is 100 m. So

$$t = \frac{100}{25} = 4 \text{ s}$$

Step 4: Check with equations:

Stone 1 falls: $y_1 = 100 - \frac{1}{2}gt^2$. Stone 2 rises: $y_2 = 25t - \frac{1}{2}gt^2$. Setting $y_1 = y_2$ gives $100 = 25t$, so $t = 4 \text{ s}$.

Step 5: Choose:

Option (C).

Final Answer:

The stones meet after 4 s.

4 s

Quick Tip: Gravity cancels in the relative motion. $t = 100/25$.

4. A particle performing U.C.M. of radius $\frac{\pi}{2}$ m makes 'x' revolutions in time t. Its tangential velocity is

- (A) $\frac{\pi x}{t}$
 (B) $\frac{\pi^2 x}{t}$
 (C) $\frac{\pi}{xt}$
 (D) $\frac{xt}{\pi}$

Correct Answer: (B) $\frac{\pi^2 x}{t}$

Solution:

Step 1: Understanding the Concept:

In uniform circular motion, tangential speed is $v = \omega r$ and $\omega = \frac{2\pi}{T}$.

Step 2: Find the period:

x revolutions take time t , so one revolution takes $T = \frac{t}{x}$. Then ω
 $= \frac{2\pi x}{t}$.

Step 3: Find the speed:

$$v = \omega r = \frac{2\pi x}{t} \times \frac{\pi}{2} = \frac{\pi^2 x}{t}$$

Step 4: Choose:

Option (B).

Final Answer:

The tangential velocity is $\pi^2 x$ over t .

$$\boxed{\frac{\pi^2 x}{t}}$$

Quick Tip: Speed = distance per revolution times revolutions per time.

5. Two masses m_1 and m_2 moving with velocities V_1 and V_2 in opposite directions collide elastically and after collision m_1 and m_2 move with velocities V_2 and V_1 respectively. The ratio $\frac{m_2}{m_1}$ is

- (A) 0.25
- (B) 0.50
- (C) 0.75
- (D) 1.0

Correct Answer: (D) 1.0

Solution:

Step 1: Understanding the Concept:

In an elastic collision, momentum and kinetic energy are both conserved. The velocities after collision are given by standard formulas.

Step 2: Write the final velocity of m_1 :

With V_2 taken as opposite sign, the general result in one dimension is

$$v_1' = \frac{(m_1 - m_2)v_1 + 2m_2v_2}{m_1 + m_2}$$

We are told $v_1' = v_2$ (the velocity values are exchanged).

Step 3: Set up:

$$(m_1 - m_2)v_1 + 2m_2v_2 = (m_1 + m_2)v_2$$

$$(m_1 - m_2)v_1 = (m_1 - m_2)v_2$$

Step 4: Solve:

Because $v_1 \neq v_2$ (they move in opposite directions), $m_1 - m_2 = 0$, so $m_1 = m_2$.

Step 5: Choose:

$\frac{m_2}{m_1} = 1$, option (D).

Final Answer:

The masses must be equal, so the ratio is 1.

1.0

Quick Tip: Exchange of velocities needs equal masses.

6. The power (P) is supplied to rotating body having moment of inertia 'I' and angular acceleration ' α '. Its instantaneous angular velocity is

- (A) $\frac{I}{P\alpha}$
- (B) $\frac{\alpha}{PI}$
- (C) $\frac{P\alpha}{I}$
- (D) $\frac{P}{I\alpha}$

Correct Answer: (D) $\frac{P}{I\alpha}$

Solution:

Step 1: Understanding the Concept:

For a rotating body, power is torque times angular velocity: $P = \tau\omega$.
Torque is $\tau = I\alpha$.

Step 2: Combine:

$$P = I\alpha\omega$$

Step 3: Solve for omega:

$$\omega = \frac{P}{I\alpha}$$

Step 4: Choose:

Option (D). Options (A), (B) and (C) have the wrong dimensions. For example $\frac{I}{P\alpha}$ does not reduce to rad/s.

Final Answer:

Angular velocity is P divided by I alpha.

$$\omega = \frac{P}{I\alpha}$$

Quick Tip: $P = \text{torque} \times \text{omega}$ and $\text{torque} = I \alpha$.

7. If the earth suddenly contracts to $(\frac{1}{3})^{rd}$ of its present size without change in its mass, the ratio kinetic energy of the earth after and before contraction will be (Earth is assumed to be a rotating sphere about itself)

- (A) 9
- (B) 5
- (C) 7
- (D) 6

Correct Answer: (A) 9

Solution:

Step 1: Understanding the Concept:

There is no external torque on the earth, so angular momentum $L = I\omega$ is conserved. The rotational kinetic energy is $K = \frac{L^2}{2I}$.

Step 2: Moment of inertia:

For a solid sphere $I = \frac{2}{5}MR^2$. If $R \rightarrow R/3$ with M fixed, then $I' = \frac{I}{9}$.

Step 3: Energy ratio:

$$\frac{K'}{K} = \frac{L^2/2I'}{L^2/2I} = \frac{I}{I'} = 9$$

Step 4: Choose:

Option (A). The kinetic energy increases ninefold, with the extra energy coming from the work done by gravity during contraction.

Final Answer:

The kinetic energy becomes 9 times.

Quick Tip: L is conserved, so K is proportional to $1/I$.

8. What should be the angular velocity of earth due to rotation about its own axis so that the weight at equator becomes $(\frac{3}{5})^{th}$ of initial value? (g = acceleration due to gravity, R = radius of earth)

- (A) $(\frac{R}{3g})^{\frac{1}{2}}$
- (B) $(\frac{2g}{5R})^{\frac{1}{2}}$
- (C) $(\frac{5g}{9R})^{\frac{1}{2}}$
- (D) $(\frac{3g}{2R})^{\frac{1}{2}}$

Correct Answer: (B) $(\frac{2g}{5R})^{\frac{1}{2}}$

Solution:

Step 1: Understanding the Concept:

At the equator, the effective gravity is reduced by the centrifugal effect of rotation: $g' = g - \omega^2 R$.

Step 2: Weight condition:

Weight becomes $\frac{3}{5}$ of the initial, so $g' = \frac{3}{5}g$.

Step 3: Solve:

$$g - \omega^2 R = \frac{3}{5}g \Rightarrow \omega^2 R = \frac{2}{5}g \Rightarrow \omega = \left(\frac{2g}{5R}\right)^{1/2}$$

Step 4: Choose:

Option (B).

Final Answer:

$\omega = \sqrt{2g/5R}$.

$$\omega = \left(\frac{2g}{5R} \right)^{1/2}$$

Quick Tip: Apparent g at the equator is $g - \omega^2 R$.

9. Water rises to a height 3 cm in a capillary tube. If cross-sectional area of capillary tube is reduced to $1/3^{\text{rd}}$ of its initial area then water will rise to a height of

- (A) $3\sqrt{3}$ cm
- (B) $\sqrt{3}$ cm
- (C) 9 cm
- (D) 3 cm

Correct Answer: (A) $3\sqrt{3}$ cm

Solution:

Step 1: Understanding the Concept:

Height of capillary rise is $h = \frac{2T \cos \theta}{r \rho g}$, so $h \propto \frac{1}{r}$.

Step 2: Radius from area:

Area $A = \pi r^2$. If the area becomes $\frac{1}{3}$, then r^2 becomes $\frac{1}{3}$ and $r \rightarrow \frac{r}{\sqrt{3}}$.

Step 3: Height:

$$h' = h \times \sqrt{3} = 3\sqrt{3} \text{ cm}$$

Step 4: Choose:

Option (A). Option (C), 9 cm, would be for the radius reduced to one third, not the area.

Final Answer:

The water rises to $3\sqrt{3}$ cm.

$3\sqrt{3} \text{ cm}$

Quick Tip: h is inversely proportional to r , not to area.

10. A soap bubble of radius R is blown. After heating the solution, a second bubble of radius $2R$ is blown. The work required to blow the second bubble in comparison to that required for the first bubble is

- (A) slightly less than 4 times
- (B) exactly double
- (C) slightly less than double
- (D) slightly more than 4 times

Correct Answer: (A) slightly less than 4 times

Solution:

Step 1: Understanding the Concept:

The work done in blowing a bubble equals the increase in its surface energy. A soap bubble has two surfaces, so $W = T \times 2 \times 4\pi r^2 = 8\pi r^2 T$.

Step 2: Compare the radii:

If T were the same, $W \propto r^2$. The second bubble has radius $2R$, so the work would be 4 times.

Step 3: Effect of heating:

Heating the solution lowers the surface tension T . The second bubble is blown at a lower T , so the work is slightly less than exactly 4 times.

Step 4: Choose:

Option (A). Option (D) would need T to increase, and (B) and (C) ignore the square of the radius.

Final Answer:

Radius doubles, so the work is about 4 times, slightly less due to lower T .

Slightly less than 4 times

Quick Tip: Work is $8\pi r^2 T$. Heating reduces T .



11. The excess pressure inside the first soap bubble of radius R_1 is three times that inside the second soap bubble of radius R_2 . The ratio of volumes of the first bubble to second bubble is

- (A) 1 : 3
- (B) 1 : 6
- (C) 1 : 27
- (D) 1 : 9

Correct Answer: (C) 1 : 27

Solution:

Step 1: Understanding the Concept:

The excess pressure inside a soap bubble is $\Delta P = \frac{4T}{R}$, so $\Delta P \propto \frac{1}{R}$.

Step 2: Radius ratio:

$\Delta P_1 = 3\Delta P_2$ means $\frac{1}{R_1} = \frac{3}{R_2}$, so $R_1 = \frac{R_2}{3}$.

Step 3: Volume ratio:

Volume goes as R^3 :

$$\frac{V_1}{V_2} = \left(\frac{R_1}{R_2}\right)^3 = \frac{1}{27}$$

Step 4: Choose:

Option (C), ratio 1 : 27.

Final Answer:

The volume ratio is 1 : 27.

1 : 27

Quick Tip: Excess pressure goes as $1/R$ and volume as R^3 .

12. An ordinary body cools from 4θ to 3θ in ' t ' minutes. The temperature of the body after next ' t ' minutes is (Assume Newton's law of cooling and room temperature as θ)

- (A) $\frac{2\theta}{3}$
- (B) $\frac{7\theta}{3}$
- (C) $\frac{5\theta}{3}$
- (D) $\frac{8\theta}{3}$

Correct Answer: (B) $\frac{7\theta}{3}$

Solution:

Step 1: Understanding the Concept:

Newton's law of cooling says the rate of fall of temperature is proportional to the excess temperature over the surroundings. This leads to an exponential decay of the excess temperature.

Step 2: Excess temperatures:

Room temperature is θ . At the start the excess is $4\theta - \theta = 3\theta$. After t minutes it is $3\theta - \theta = 2\theta$.

Step 3: Decay factor:

In every interval of t minutes, the excess is multiplied by the same factor: $\frac{2\theta}{3\theta} = \frac{2}{3}$.

Step 4: Next interval:

After another t minutes, the excess is $2\theta \times \frac{2}{3} = \frac{4\theta}{3}$.

Step 5: Temperature:

$$T = \theta + \frac{4\theta}{3} = \frac{7\theta}{3}$$

Option (B).

Final Answer:

The temperature becomes $7\theta / 3$.

$$\boxed{\frac{7\theta}{3}}$$

Quick Tip: The excess temperature over the room falls by the same factor each interval.

13. A black disc has a radius 'R' and wavelength corresponding to the maximum intensity is ' λ '. The emissive power (E) for the different radii and maximum wavelengths is directly proportional to

- (A) R, λ
- (B) R^2, λ^{-4}
- (C) R^2, λ^{-2}
- (D) R^2, λ

Correct Answer: (B) R^2, λ^{-4}

Solution:

Step 1: Understanding the Concept:

A black body radiates energy at a rate given by Stefan's law: the power radiated per unit area is σT^4 . For a disc of radius R , the radiating area is proportional to R^2 .

Step 2: Link temperature to wavelength:

Wien's displacement law says $\lambda_m T = \text{constant}$, so $T \propto \frac{1}{\lambda}$. Hence $T^4 \propto \lambda^{-4}$.

Step 3: Combine:

The emitted power is

$$E \propto (\text{area}) \times T^4 \propto R^2 \lambda^{-4}$$

Step 4: Choose:

Option (B). The other options have a wrong power of λ or the wrong power of R .

Final Answer:

E is proportional to R squared and λ to the power -4.

$$\boxed{E \propto R^2, \lambda^{-4}}$$

Quick Tip: Stefan law gives T^4 , Wien law gives T proportional to $1/\lambda$.

14. About black body radiation, which one of the following is a WRONG statement?

- (A) For all wavelengths, power radiated is same
- (B) For shorter wavelengths, radiated power is more
- (C) for longer wavelengths, radiated power is less
- (D) All wavelengths are emitted by the black body

Correct Answer: (A) For all wavelengths, power radiated is same

Solution:

Step 1: Understanding the Concept:

A black body emits all wavelengths, but not equally. The graph of radiated power against wavelength rises to a peak at a wavelength fixed by the temperature (Wien law), and falls on both sides.

Step 2: Check statement (A):

It says power radiated is the same for all wavelengths. This is wrong, because the curve has a peak and is not flat. So (A) is the WRONG statement.

Step 3: Check statements (B), (C) and (D):

(B) and (C) describe the general trend the question intends: the power is small at long wavelengths and larger towards the shorter wavelengths near the peak. (D) is true because a black body emits every wavelength. All three are accepted as true.

Step 4: Choose:

The wrong statement is (A).

Final Answer:

Radiated power is not the same for all wavelengths.

A

Quick Tip: The black body curve has a peak, so power depends on wavelength.

15. In thermodynamic isobaric process

- (A) change in internal energy is zero
- (B) work done is zero
- (C) change in internal energy is not zero
- (D) heat supplied is zero

Correct Answer: (C) change in internal energy is not zero

Solution:**Step 1: Understanding the Concept:**

An isobaric process occurs at constant pressure. Gas is allowed to expand or contract, so work is done, and heat is exchanged.

Step 2: Apply the first law:

$Q = \Delta U + W$. In an isobaric process, $W = P\Delta V \neq 0$, and $Q = nC_p\Delta T$. The internal energy change is $\Delta U = nC_v\Delta T$, which is non-zero whenever the temperature changes.

Step 3: Check the options:

(A) $\Delta U = 0$ is true for an isothermal process. (B) $W = 0$ is true for an

isochoric process. (D) $Q = 0$ is true for an adiabatic process. (C) $\Delta U \neq 0$ is the isobaric case.

Step 4: Choose:

Option (C).

Final Answer:

In an isobaric process the internal energy changes.

C

Quick Tip: Pressure is constant, but temperature and volume change.

16. A monoatomic gas is stored in a thermally insulated container and the gas is suddenly compressed to $(\frac{1}{8})^{th}$ of its initial volume. The ratio of final pressure to initial pressure is

- (A) 16 : 1
- (B) 40 : 1
- (C) 32 : 1
- (D) 28 : 1

Correct Answer: (C) 32 : 1

Solution:

Step 1: Understanding the Concept:

A sudden compression in an insulated container leaves no time for heat exchange. So the process is adiabatic: $PV^\gamma = \text{constant}$.

Step 2: Find gamma:

For a monatomic gas, $\gamma = \frac{5}{3}$.

Step 3: Apply:

$$\frac{P_2}{P_1} = \left(\frac{V_1}{V_2} \right)^\gamma = 8^{5/3}$$

Step 4: Compute:

$$8^{5/3} = (8^{1/3})^5 = 2^5 = 32.$$

Step 5: Choose:

The ratio is 32 : 1, option (C).

Final Answer:

The pressure rises 32 times.

32 : 1

Quick Tip: Adiabatic: $P V^\gamma = \text{constant}$, $\gamma = 5/3$.

17. The average force applied on the walls of a closed container depends on temperature (T) as (T is the temperature of an ideal gas)

- (A) T^2
- (B) T^3
- (C) T^1
- (D) T^{-1}

Correct Answer: (C) T^1

Solution:

Step 1: Understanding the Concept:

The force on the walls is the pressure times the area, $F = PA$. For a closed container, the volume is fixed.

Step 2: Gas law:

At constant volume and amount of gas, $P \propto T$ (Gay-Lussac law).

Step 3: Conclude:

$F = PA \propto T^1$. Option (C).

Final Answer:

The force is proportional to T .

$$F \propto T^1$$

Quick Tip: Fixed volume means pressure is proportional to T .

18. A particle executes linear S.H.M. with amplitude 4 cm. The magnitude of velocity and acceleration is equal when it is at 3 cm from mean position. Time period is

- (A) $\frac{3\pi}{\sqrt{2}}$ s
- (B) $\frac{6\pi}{\sqrt{7}}$ s
- (C) $\frac{2\pi}{\sqrt{7}}$ s
- (D) $\frac{4\pi}{\sqrt{7}}$ s

Correct Answer: (B) $\frac{6\pi}{\sqrt{7}}$ s

Solution:

Step 1: Understanding the Concept:

In S.H.M. with angular frequency ω and amplitude A : velocity $v = \omega\sqrt{A^2 - x^2}$ and acceleration $a = \omega^2 x$ in magnitude.

Step 2: Equate them at $x = 3$ cm:

$$\omega\sqrt{16 - 9} = \omega^2 \cdot 3$$

$$\sqrt{7} = 3\omega \Rightarrow \omega = \frac{\sqrt{7}}{3} \text{ rad/s}$$

Step 3: Period:

$$T = \frac{2\pi}{\omega} = \frac{2\pi \cdot 3}{\sqrt{7}} = \frac{6\pi}{\sqrt{7}} \text{ s}$$

Step 4: Choose:

Option (B).

Final Answer:

The time period is $\frac{6\pi}{\sqrt{7}}$ seconds.

$$\boxed{\frac{6\pi}{\sqrt{7}} \text{ s}}$$

Quick Tip: Use $v = \omega \sqrt{A^2 - x^2}$ and $a = \omega^2 x$.

19. A particle starts from mean position and performs S.H.M. with period 6 second. At what time its kinetic energy is 50% of total energy? ($\cos 45^\circ = 1/\sqrt{2}$)

- (A) 0.75 s
- (B) 0.50 s
- (C) 0.25 s
- (D) 3 s

Correct Answer: (A) 0.75 s

Solution:

Step 1: Understanding the Concept:

Total energy is $\frac{1}{2}kA^2$ and potential energy at displacement x is $\frac{1}{2}kx^2$. Kinetic energy is half the total when the potential energy is also half.

Step 2: Find the displacement:

$$\frac{1}{2}kx^2 = \frac{1}{2} \cdot \frac{1}{2}kA^2 \text{ gives } x = \frac{A}{\sqrt{2}}.$$

Step 3: Use $x = A \sin(\omega t)$:

The particle starts from the mean position, so $x = A \sin \omega t$. Thus $\sin \omega t = \frac{1}{\sqrt{2}}$, so $\omega t = \frac{\pi}{4}$.

Step 4: Find time:

$$\omega = \frac{2\pi}{T}, \text{ so } t = \frac{T}{8} = \frac{6}{8} = 0.75 \text{ s.}$$

Step 5: Choose:

Option (A).

Final Answer:

The kinetic energy is 50 percent at $t = T/8 = 0.75 \text{ s}$.

0.75 s

Quick Tip: KE is half when $x = A/\sqrt{2}$, which is a phase of 45 degrees.

20. A small sphere oscillates simple harmonically in a watch glass whose radius of curvature is 1.6 m. The period of Oscillation of the sphere is (Acceleration due to gravity $g = 10 \text{ m/s}^2$)

- (A) 0.2π
- (B) 0.4π
- (C) 0.8π
- (D) π

Correct Answer: (C) 0.8π

Solution:

Step 1: Understanding the Concept:

A small sphere rolling in a watch glass (a shallow bowl of radius R) behaves like a simple pendulum of length R for small oscillations.

Step 2: Formula:

$$T = 2\pi\sqrt{\frac{R}{g}}$$

Step 3: Substitute:

$$T = 2\pi\sqrt{\frac{1.6}{10}} = 2\pi\sqrt{0.16} = 2\pi \times 0.4 = 0.8\pi \text{ s}$$

Step 4: Choose:

Option (C).

Final Answer:

The period is 0.8π s.

$$\boxed{0.8\pi}$$

Quick Tip: The sphere acts like a pendulum of length equal to the radius.

21. A wire is under tension of 2 kg wt and a wave is travelling through it with some speed. Tension in the wire is so increased that the wave travels through it with thrice the original speed. The increase in tension is (in kg wt)

- (A) 4
- (B) 8
- (C) 16
- (D) 12

Correct Answer: (C) 16

Solution:

Step 1: Understanding the Concept:

The speed of a transverse wave on a string is $v = \sqrt{T/\mu}$, so $v^2 \propto T$ for the same wire.

Step 2: Apply:

If the speed becomes 3 times, the tension becomes $3^2 = 9$ times. So

$$T' = 9 \times 2 = 18 \text{ kg wt}$$

Step 3: Increase:

$$18 - 2 = 16 \text{ kg wt}$$

Step 4: Choose:

Option (C). The value 12 would come from adding 6 instead of multiplying tension by 9.

Final Answer:

The tension must rise by 16 kg wt.

16 kg wt

Quick Tip: Wave speed is the square root of tension over mass per length.

22. A Pipe open at one end has length 0.8 m. At the open end of the tube a string 0.5 m long is vibrating in its first overtone and resonates with fundamental frequency of pipe. If tension in the string is 50 N, the mass of string is (Neglect end correction) (Speed of sound = 320 m/s)

- (A) 2 gram
- (B) 5 gram
- (C) 10 gram
- (D) 20 gram

Correct Answer: (C) 10 gram

Solution:

Step 1: Understanding the Concept:

A pipe open at one end has fundamental frequency $f = \frac{v}{4L}$. A string fixed at both ends vibrating in its first overtone is in its second

harmonic, so $f = \frac{2}{2l} \sqrt{\frac{T}{\mu}} = \frac{1}{l} \sqrt{\frac{T}{\mu}}$.

Step 2: Pipe frequency:

$$f = \frac{320}{4 \times 0.8} = 100 \text{ Hz}$$

Step 3: String condition:

The string is $l = 0.5$ m long, so

$$100 = \frac{1}{0.5} \sqrt{\frac{50}{\mu}} \Rightarrow \sqrt{\frac{50}{\mu}} = 50 \Rightarrow \mu = \frac{50}{2500} = 0.02 \text{ kg/m}$$

Step 4: Mass of string:

$$m = \mu l = 0.02 \times 0.5 = 0.01 \text{ kg} = 10 \text{ g}$$

Step 5: Choose:

Option (C).

Final Answer:

The mass of the string is 10 g.

10 g

Quick Tip: Pipe fundamental is 100 Hz, which sets the string frequency.



23. A sonometer wire resonates with 4 antinodes between the two bridges for a given tuning fork when 1 kg mass is suspended from the wire. Using same fork, when mass M is suspended, the wire resonates producing 2 antinodes between the two bridges. (Distance between the bridges as before) The value of M is

- (A) 2 kg
- (B) 3 kg
- (C) 4 kg
- (D) 5 kg

Correct Answer: (C) 4 kg

Solution:

Step 1: Understanding the Concept:

For a sonometer wire with p loops (antinodes) between the bridges, the frequency is $f = \frac{p}{2l} \sqrt{\frac{T}{\mu}}$. The tuning fork frequency, length and mass per length are the same in both cases.

Step 2: Relation:

So $p\sqrt{T}$ is constant: $4\sqrt{T_1} = 2\sqrt{T_2}$.

Step 3: Solve:

$\sqrt{T_2} = 2\sqrt{T_1}$, so $T_2 = 4T_1$. Tension is proportional to the suspended mass, so $M = 4 \times 1 = 4$ kg.

Step 4: Choose:

Option (C).

Final Answer:

$M = 4$ kg.

4 kg

Quick Tip: Frequency is proportional to p times $\sqrt{T}$.

24. The fundamental frequency of sonometer wire is 50 Hz for some length and tension. If the length is increased by 25% by keeping the tension same, then the frequency change of second harmonic is

- (A) increase by 20%
- (B) decrease by 20%
- (C) increase by 40%
- (D) decrease by 40%

Correct Answer: (B) decrease by 20%

Solution:

Step 1: Understanding the Concept:

For a wire fixed at both ends, the n th harmonic is $f_n = \frac{n}{2l} \sqrt{\frac{T}{\mu}}$. At a fixed tension, $f \propto \frac{1}{l}$.

Step 2: Second harmonic at the start:

$$f_2 = 2 \times 50 = 100 \text{ Hz.}$$

Step 3: After the change:

The length becomes $1.25l$, so the frequency becomes $\frac{100}{1.25} = 80 \text{ Hz.}$

Step 4: Percent change:

$$\frac{80 - 100}{100} \times 100 = -20\%$$

The frequency decreases by 20%. Option (B).

Final Answer:

The second harmonic falls from 100 Hz to 80 Hz, a 20 percent decrease.

Decrease by 20%

Quick Tip: Frequency is inversely proportional to length.

25. The number of lines of force originating from a point charge of $2 \times 8.85 \times 10^{-9}$ C in a medium of dielectric constant 10 is ($\epsilon_0 = 8.85 \times 10^{-12}$ SI unit)

- (A) 100
- (B) 200
- (C) 400
- (D) 2000

Correct Answer: (B) 200

Solution:

Step 1: Understanding the Concept:

Gauss law in a medium: the total number of lines of force (electric flux) from a point charge q is $N = \frac{q}{\epsilon_0 K}$, where K is the dielectric constant.

Step 2: Substitute:

$$N = \frac{2 \times 8.85 \times 10^{-9}}{8.85 \times 10^{-12} \times 10}$$

Step 3: Compute:

$$N = \frac{2 \times 10^3}{10} = 200$$

Step 4: Choose:

Option (B). The value 2000 would be the answer in air, without the factor 10.

Final Answer:

The number of lines of force is 200.

200

Quick Tip: Divide q/ϵ_0 by the dielectric constant.

26. The potential difference that must be applied across the parallel and series combination of three identical capacitors such that the energy stored in them becomes the same. The ratio of potential difference in parallel to series combination is

- (A) 1 : 3
- (B) 3 : 1
- (C) 9 : 1
- (D) 1 : 9

Correct Answer: (A) 1 : 3

Solution:

Step 1: Understanding the Concept:

Energy stored in a capacitor is $U = \frac{1}{2}C_{eq}V^2$. We find the equivalent capacitance in each arrangement.

Step 2: Equivalent capacitances:

Parallel: $C_p = 3C$. Series: $C_s = \frac{C}{3}$.

Step 3: Equal energy:

$$\frac{1}{2}(3C)V_p^2 = \frac{1}{2}\left(\frac{C}{3}\right)V_s^2$$

$$3V_p^2 = \frac{V_s^2}{3} \Rightarrow \frac{V_p^2}{V_s^2} = \frac{1}{9}$$

Step 4: Ratio:

$$\frac{V_p}{V_s} = \frac{1}{3}$$

The ratio is 1 : 3, option (A).

Final Answer:

The potential difference ratio is 1 : 3.

1 : 3

Quick Tip: Parallel capacitance is 9 times the series capacitance.

27. The function of a dielectric in a capacitor is to

- (A) reduce the plate area of the capacitor.
- (B) to decrease the capacitance.
- (C) reduce the effective potential on plates.
- (D) increase the effective potential on plates.

Correct Answer: (C) reduce the effective potential on plates.

Solution:

Step 1: Understanding the Concept:

When a dielectric is placed between the plates of a charged capacitor, the molecules polarise. The induced charges on the surface of the dielectric create a field opposite to the applied field.

Step 2: Effect:

The net field inside is reduced, so the potential difference for the same charge falls. This is why the capacitance $C = \frac{Q}{V}$ rises by the factor K .

Step 3: Check the options:

(A) Plate area is not changed by a dielectric. (B) It increases the capacitance, not decreases. (D) It reduces the effective potential, not increases it. So (C) is correct.

Final Answer:

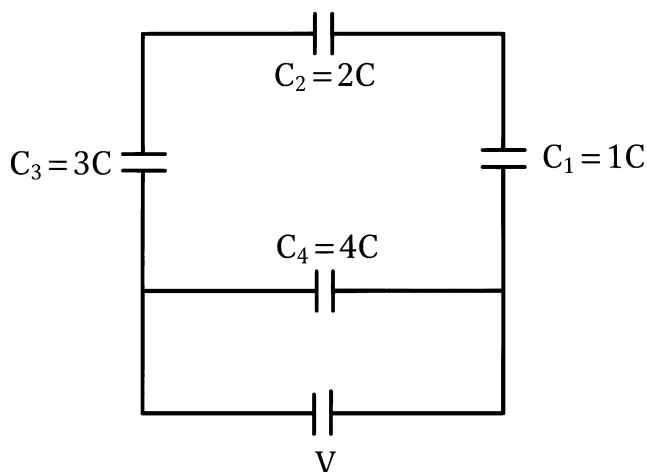
A dielectric reduces the effective potential across the plates.

C

Quick Tip: The dielectric field opposes the applied field, so the net potential falls.



28. Four capacitors are connected to a battery as shown in figure. The ratio of charges on capacitors C_2 and C_4 is



- (A) $\frac{1}{14}$
- (B) $\frac{3}{22}$
- (C) $\frac{2}{9}$
- (D) $\frac{4}{13}$

Correct Answer: (B) $\frac{3}{22}$

Solution:

Step 1: Read the circuit:

In the figure, C_3 ($3C$), C_2 ($2C$) and C_1 ($1C$) lie one after another in the outer loop, so they are in series. C_4 ($4C$) is on the middle wire joining the two sides, so it is in parallel with that series branch. The battery V sits across both.

Step 2: Series branch:

$$\frac{1}{C_s} = \frac{1}{C} + \frac{1}{2C} + \frac{1}{3C} = \frac{6+3+2}{6C} = \frac{11}{6C} \Rightarrow C_s = \frac{6C}{11}$$

Step 3: Charge on C₂:

Series capacitors carry the same charge, which equals the charge on the equivalent capacitor. The branch is across V, so $Q_2 = C_s V = \frac{6CV}{11}$.

Step 4: Charge on C₄:

C₄ is directly across V, so $Q_4 = 4CV$.

Step 5: Ratio:

$$\frac{Q_2}{Q_4} = \frac{6CV/11}{4CV} = \frac{6}{44} = \frac{3}{22}$$

Step 6: Check the options:

1/14, 2/9 and 4/13 do not reduce from 6/(11 × 4). A common slip is to give C₂ the full V, which would give a ratio of 1/2; that is not an option either.

Final Answer:

The ratio $Q_2 : Q_4 = 3/22$, option (B).

$$\boxed{\frac{3}{22}}$$

Quick Tip: Find the series equivalent of C₁, C₂, C₃ across V for the charge on C₂, and use Q = CV for C₄.

29. When a resistance of $200\ \Omega$ is connected in series with a galvanometer of resistance G , its range is V . To triple its range, a resistance of $2000\ \Omega$ is connected in series. The value of G is

- (A) $400\ \Omega$
- (B) $500\ \Omega$
- (C) $700\ \Omega$
- (D) $900\ \Omega$

Correct Answer: (C) $700\ \Omega$

Solution:

Step 1: Understanding the Concept:

To make a voltmeter, a high resistance is connected in series with a galvanometer. At full-scale deflection the current is I_g , so the range is $V = I_g(G + R)$.

Step 2: First case:

$$V = I_g(G + 200).$$

Step 3: Second case:

For triple range, $3V = I_g(G + 2000)$.

Step 4: Divide:

$$3 = \frac{G + 2000}{G + 200} \Rightarrow 3G + 600 = G + 2000 \Rightarrow 2G = 1400$$

$$G = 700\ \Omega$$

Step 5: Choose:

Option (C).

Final Answer:

The galvanometer resistance is 700 ohm.

$$\boxed{700 \, \Omega}$$

Quick Tip: Full scale current is fixed, so range is proportional to total resistance.



30. If a galvanometer is shunted by $(\frac{1}{n-1})^{th}$ of the value of its resistance, then the fraction of the total current passing through the galvanometer is

- (A) $\frac{1}{(n-1)}$
- (B) $\frac{1}{n}$
- (C) $\frac{n}{(n-1)}$
- (D) $\frac{1}{1-n}$

Correct Answer: (B) $\frac{1}{n}$

Solution:

Step 1: Understanding the Concept:

A shunt S is connected in parallel with the galvanometer G . The current divides in inverse proportion to the resistances.

Step 2: Shunt value:

$$S = \frac{G}{n-1}.$$

Step 3: Current through galvanometer:

$$I_g = I \cdot \frac{S}{S+G} = I \cdot \frac{\frac{G}{n-1}}{\frac{G}{n-1} + G} = I \cdot \frac{1}{1+(n-1)} = \frac{I}{n}$$

Step 4: Choose:

The fraction is $\frac{1}{n}$, option (B).

Final Answer:

The fraction of current through the galvanometer is $1/n$.

$$\boxed{\frac{1}{n}}$$

Quick Tip: Currents split in the ratio $1 : (n - 1)$.

31. The product of magnetic susceptibility (χ) and absolute temperature (T) is constant for a

- (A) diamagnetic substance.
- (B) paramagnetic substance.
- (C) ferromagnetic substance.
- (D) paramagnetic as well as ferromagnetic substance.

Correct Answer: (B) paramagnetic substance.

Solution:

Step 1: Understanding the Concept:

Curie's law for paramagnetic substances says the magnetic susceptibility is inversely proportional to absolute temperature: $\chi = \frac{C}{T}$.

Step 2: Product:

So $\chi T = C$, a constant. This is the Curie constant.

Step 3: Check the others:

Diamagnetic susceptibility is small, negative and almost independent of temperature. Ferromagnetic substances follow the Curie-Weiss law, $\chi = \frac{C}{T - T_c}$, so χT is not constant. Hence (A), (C) and (D) are wrong.

Step 4: Choose:

Option (B).

Final Answer:

Curie law holds for paramagnetic substances.

B : paramagnetic substance

Quick Tip: Curie law applies to paramagnetic substances.

32. A circular coil of 'N' turns and diameter 'd' carries current 'I'. It is unwound and rewound to make another coil of diameter '2d', current 'I' remaining the Same. The ratio of magnetic moments of the new coil to the original coil is

- (A) 2 : 1
- (B) 1 : 1
- (C) 1 : 2
- (D) 4 : 1

Correct Answer: (A) 2 : 1

Solution:

Step 1: Understanding the Concept:

The magnetic moment of a coil with N turns, current I and area A is $m = NIA$. Unwinding and rewinding keeps the total length of the wire the same.

Step 2: Wire length:

Original length = $N \cdot \pi d$. New coil has N' turns of diameter $2d$, so $N' \cdot \pi(2d) = N\pi d$. This gives $N' = \frac{N}{2}$.

Step 3: Magnetic moments:

Original: $m_1 = NI \cdot \frac{\pi d^2}{4}$.

New: $m_2 = \frac{N}{2}I \cdot \frac{\pi(2d)^2}{4} = \frac{N}{2}I \cdot \pi d^2 = 2 \cdot NI \frac{\pi d^2}{4}$.

Step 4: Ratio:

$\frac{m_2}{m_1} = 2$, so the ratio is 2 : 1, option (A).

Final Answer:

The new coil has twice the magnetic moment.

2 : 1

Quick Tip: The wire length is fixed, so turns halve when diameter doubles.

33. Lorentz magnetic force is acting on a particle of charge q moving with velocity $\vec{V}$ in magnetic field $\vec{B}$. The work done by this force on the charged particle is

- (A) zero
- (B) $\vec{V} \times \vec{B}$
- (C) $\vec{V} \times \vec{V}$
- (D) $q(\vec{V} \times \vec{B})$

Correct Answer: (A) zero

Solution:**Step 1: Understanding the Concept:**

Work done by a force is $W = \vec{F} \cdot \vec{s}$. The Lorentz magnetic force is $\vec{F} = q(\vec{V} \times \vec{B})$.

Step 2: Direction:

The force is perpendicular to both $\vec{V}$ and $\vec{B}$. So it is always perpendicular to the velocity, and hence to the displacement in a small time interval.

Step 3: Work:

$$dW = \vec{F} \cdot \vec{V} dt = q(\vec{V} \times \vec{B}) \cdot \vec{V} dt = 0$$

Step 4: Choose:

The work done is zero, option (A). Options (B), (C) and (D) are vectors, but work is a scalar.

Final Answer:

A magnetic force does no work on a moving charge.

0

Quick Tip: The force is perpendicular to the velocity.



34. A coil of 'n' turns and resistance $R \Omega$ is connected in series with a resistance $R/4$. The combination is moved for time 't' second through flux Φ_1 to Φ_2 . The induced current in the circuit is

- (A) $\frac{4n(\Phi_1 - \Phi_2)}{5RT}$
- (B) $\frac{5n^2(\Phi_1 - \Phi_2)}{4RT}$
- (C) $\frac{2n(\Phi_1 - \Phi_2)}{3RT}$
- (D) $\frac{3n(\Phi_1 - \Phi_2)}{4RT}$

Correct Answer: (A) $\frac{4n(\Phi_1 - \Phi_2)}{5RT}$

Solution:

Step 1: Understanding the Concept:

By Faraday's law, the induced emf in a coil of n turns is $\varepsilon = n \frac{\Delta\Phi}{t}$. By

Ohm's law, the current is $I = \frac{\varepsilon}{R_{total}}$.

Step 2: EMF:

The flux changes from Φ_1 to Φ_2 in time t , so the magnitude is

$$\varepsilon = \frac{n(\Phi_1 - \Phi_2)}{t}$$

Step 3: Total resistance:

The coil resistance R is in series with $\frac{R}{4}$, so $R_{total} = \frac{5R}{4}$.

Step 4: Current:

$$I = \frac{n(\Phi_1 - \Phi_2)/t}{5R/4} = \frac{4n(\Phi_1 - \Phi_2)}{5Rt}$$

Step 5: Choose:

Option (A). Option (B) has n^2 , which would be wrong since the emf has only one factor of n .

Final Answer:

The induced current is $4n(\text{flux change})/(5Rt)$.

$$I = \frac{4n(\Phi_1 - \Phi_2)}{5Rt}$$

Quick Tip: EMF is n times flux change over t ; resistance is $R + R/4$.

35. The coefficient of mutual induction is 3H and induced e.m.f. across secondary is 4 kV. Current in primary is reduced from 7A to 2A. The time required for the change of current is

- (A) $3.75 \times 10^{-3} \text{ s}$
- (B) $2.5 \times 10^{-3} \text{ s}$
- (C) $4.5 \times 10^{-3} \text{ s}$
- (D) $3.5 \times 10^{-3} \text{ s}$

Correct Answer: (A) $3.75 \times 10^{-3} \text{ s}$

Solution:

Step 1: Understanding the Concept:

The emf induced in the secondary due to a changing primary current

is $|e| = M \left| \frac{dI}{dt} \right|$.

Step 2: Substitute:

The current changes by $7 - 2 = 5 \text{ A}$ in time t .

$$4000 = 3 \times \frac{5}{t}$$

Step 3: Solve:

$$t = \frac{15}{4000} = 3.75 \times 10^{-3} \text{ s}$$

Step 4: Choose:

Option (A).

Final Answer:

The time is 3.75 ms.

$$3.75 \times 10^{-3} \text{ s}$$

Quick Tip: Use $\epsilon = M \, dI/dt$ with a 5 A change.

36. A bar magnet falls from a height 'h' through a metal pipe, its acceleration after coming out of pipe is

- (A) less than 9.8 m/s^2
- (B) greater than 9.8 m/s^2
- (C) equal to 9.8 m/s^2
- (D) equal to 'h' m/s^2

Correct Answer: (C) equal to 9.8 m/s^2

Solution:**Step 1: Understanding the Concept:**

When a magnet falls through a metal pipe, the changing flux induces eddy currents in the pipe. By Lenz's law they oppose the motion, so the magnet falls slower than free fall while it is inside the pipe.

Step 2: After leaving the pipe:

Once the magnet has come out, there is no conductor around it. So there are no eddy currents and nothing opposes its fall. Only gravity acts.

Step 3: Result:

Its acceleration is $g = 9.8 \text{ m/s}^2$. Option (C). Less than g applies only inside the pipe, and more than g is never possible here.

Final Answer:

Outside the pipe the magnet is in free fall.

$$9.8 \text{ m/s}^2$$

Quick Tip: Eddy current braking stops once the magnet leaves the pipe.

37. A series LCR circuit is connected to an a.c. source of 220 V, 50 Hz. The circuit contains a resistance $R = 80 \Omega$, an inductor of inductive reactance 70Ω and capacitor of capacitive reactance 130Ω . The power factor of the circuit is $\frac{x}{10}$. The value of x is

- (A) 8
- (B) 6
- (C) 4
- (D) 7

Correct Answer: (A) 8

Solution:

Step 1: Understanding the Concept:

In a series LCR circuit, the impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$ and the power factor is $\cos \phi = \frac{R}{Z}$.

Step 2: Net reactance:

$X_L - X_C = 70 - 130 = -60 \Omega$. Only its square matters.

Step 3: Impedance:

$$Z = \sqrt{80^2 + 60^2} = \sqrt{6400 + 3600} = 100 \, \Omega$$

Step 4: Power factor:

$$\cos \phi = \frac{80}{100} = 0.8 = \frac{8}{10}$$

So $x = 8$, option (A).

Final Answer:

The power factor is 0.8, so $x = 8$.

8

Quick Tip: $Z = \sqrt{R^2 + (X_L - X_C)^2}$; power factor = R / Z .

38. Determine the frequency for which a $10 \, \mu\text{F}$ capacitor has a reactance of $2 \times 10^{-3} \, \Omega$.

- (A) $\frac{25}{\pi} \, \text{MHz}$
- (B) $\frac{20}{\pi} \, \text{MHz}$
- (C) $2 \times 10^{-3} \, \text{MHz}$
- (D) $10\pi \, \text{MHz}$

Correct Answer: (A) $\frac{25}{\pi} \, \text{MHz}$

Solution:

Step 1: Understanding the Concept:

The capacitive reactance is $X_C = \frac{1}{2\pi fC}$.

Step 2: Rearrange:

$$f = \frac{1}{2\pi CX_C}$$

Step 3: Substitute:

$$f = \frac{1}{2\pi \times 10 \times 10^{-6} \times 2 \times 10^{-3}} = \frac{1}{4\pi \times 10^{-8}} = \frac{10^8}{4\pi} \text{ Hz}$$

Step 4: Convert:

$$\frac{10^8}{4\pi} = \frac{25 \times 10^6}{\pi} \text{ Hz} = \frac{25}{\pi} \text{ MHz. Option (A).}$$

Final Answer:

The frequency is $25/\pi$ MHz.

$\frac{25}{\pi} \text{ MHz}$

Quick Tip: Use $f = 1/(2\pi C X)$.

39. A coil has inductive reactance 3 H. The ratio of its reactance when it is first connected to an a.c. source and then to d.c. source is

- (A) zero
- (B) 3
- (C) infinite
- (D) 1.5

Correct Answer: (C) infinite

Solution:

Step 1: Understanding the Concept:

The question speaks of a coil of inductance 3 H. The inductive reactance is $X_L = \omega L = 2\pi fL$ in an a.c. circuit. For d.c., the frequency is zero.

Step 2: A.C. source:

$X_L = 2\pi f(3)$, which is a non-zero value.

Step 3: D.C. source:

For d.c., $f = 0$, so $X_L = 0$.

Step 4: Ratio:

$$\frac{X_L(\text{a.c.})}{X_L(\text{d.c.})} = \frac{2\pi f \cdot 3}{0} \rightarrow \infty$$

Option (C).

Final Answer:

The d.c. reactance is zero, so the ratio is infinite.

∞

Quick Tip: For d.c., the frequency is zero, so $X_L = 0$.

40. An equiconvex lens of focal length F is cut in to two equal parts along the vertical axis. The focal length of each part will be

- (A) $4F$
- (B) $\frac{F}{2}$
- (C) $2F$
- (D) F

Correct Answer: (C) $2F$

Solution:

Step 1: Understanding the Concept:

The lens maker's formula is $\frac{1}{F} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$. For an equiconvex lens, $R_1 = R$ and $R_2 = -R$.

Step 2: Original lens:

$$\frac{1}{F} = (\mu - 1) \frac{2}{R}$$

Step 3: Cut the lens:

The lens is cut by a vertical plane perpendicular to the principal axis, through its centre. Each half is then a plano-convex lens with one curved face of radius R and one flat face.

Step 4: Focal length of a half:

$$\frac{1}{f} = (\mu - 1) \left(\frac{1}{R} - \frac{1}{\infty} \right) = (\mu - 1) \frac{1}{R}$$

So $\frac{1}{f} = \frac{1}{2F}$, which gives $f = 2F$.

Step 5: Choose:

Option (C).

Final Answer:

Each half is plano-convex with focal length $2F$.

$$\boxed{2F}$$

Quick Tip: Each half is plano-convex, with half the power.

41. A ray of light travels from air to water to glass and again from glass to air. Refractive index of water with respect to air is 'x', glass with respect to water is 'y' and air with respect to glass is 'z'. Which one of the following is correct?

- (A) $xy = z$
- (B) $xyz = 1$
- (C) $xz = y$
- (D) $yz = x$

Correct Answer: (B) $xyz = 1$

Solution:

Step 1: Understanding the Concept:

The refractive index of medium 2 with respect to medium 1 is ${}_1n_2 = \frac{n_2}{n_1}$. For a closed path of media that returns to the start, the product of the successive relative indices is 1.

Step 2: Write each given index:

Water with respect to air: $x = \frac{n_w}{n_a}$.

Glass with respect to water: $y = \frac{n_g}{n_w}$.

Air with respect to glass: $z = \frac{n_a}{n_g}$.

Step 3: Multiply:

$$xyz = \frac{n_w}{n_a} \cdot \frac{n_g}{n_w} \cdot \frac{n_a}{n_g} = 1$$

Step 4: Choose:

Option (B). The other relations are not true identities.

Final Answer:

The three relative indices multiply to 1.

$$\boxed{xyz = 1}$$

Quick Tip: Multiply the ratios n_w/n_a , n_g/n_w , n_a/n_g .

42. When the distance of separation between the slit and screen is doubled, the angular separation between fringes in single slit diffraction pattern experiment

- (A) increases
- (B) decreases
- (C) remains same
- (D) first increases and then decreases

Correct Answer: (C) remains same

Solution:

Step 1: Understanding the Concept:

In a single slit diffraction pattern, the angular position of the first minimum is given by $a \sin \theta = \lambda$. The angular separation between fringes depends on λ and slit width a only.

Step 2: Apply:

$$\theta \approx \frac{\lambda}{a}$$

There is no screen distance D in this expression.

Step 3: Effect of doubling D :

The linear fringe width $\beta = \frac{\lambda D}{a}$ doubles, but the angle subtended at the slit stays the same.

Step 4: Choose:

Option (C), remains same.

Final Answer:

Angular separation does not depend on the screen distance.

C : remains same

Quick Tip: The fringe angle is λ/a , with no D.

43. Light of wavelength λ is incident on a single slit of width 'a' and the distance between slit and screen is D. In diffraction pattern, if slit width is equal to the width of the central maximum then D is equal to

- (A) $\frac{a^2}{2\lambda}$
- (B) $\frac{a}{2\lambda}$
- (C) $\frac{a^2}{\lambda}$
- (D) $\frac{a}{\lambda}$

Correct Answer: (A) $\frac{a^2}{2\lambda}$

Solution:**Step 1: Understanding the Concept:**

For single slit diffraction, the width of the central maximum on the screen is $2\frac{\lambda D}{a}$.

Step 2: Condition:

The slit width equals the width of the central maximum:

$$a = \frac{2\lambda D}{a}$$

Step 3: Solve:

$$D = \frac{a^2}{2\lambda}$$

Step 4: Choose:

Option (A).

Final Answer:

D = a squared over 2 lambda.

$$D = \frac{a^2}{2\lambda}$$

Quick Tip: Central maximum width is $2\lambda D / a$.

44. Light of wavelength ' λ ' which is less than threshold wavelength is incident on a photosensitive material. If incident wavelength is decreased so that emitted photoelectrons are moving with some velocity then stopping potential

- (A) becomes half
- (B) increases
- (C) decreases
- (D) is zero

Correct Answer: (B) increases

Solution:

Step 1: Understanding the Concept:

Einstein's photoelectric equation is $h\nu = \phi + K_{max}$, and the stopping potential V_0 satisfies $eV_0 = K_{max}$.

Step 2: Effect of smaller wavelength:

Decreasing λ increases the frequency $\nu = \frac{c}{\lambda}$, so the photon energy $h\nu$ increases. The work function ϕ is fixed.

Step 3: Result:

So $K_{max} = h\nu - \phi$ increases, and $V_0 = \frac{K_{max}}{e}$ increases. Option (B).

Final Answer:

The stopping potential increases.

B : increases

Quick Tip: Shorter wavelength means higher photon energy and higher V_0 .

45. When the electron orbiting in hydrogen atom in its ground state moves to the third excited state, the de-Broglie wavelength associated with it

- (A) will decrease
- (B) will increase
- (C) will remain same
- (D) will be zero

Correct Answer: (B) will increase

Solution:

Step 1: Understanding the Concept:

The third excited state is $n = 4$ (ground $n = 1$, first excited $n = 2$, second excited $n = 3$). In Bohr's model the orbit radius grows as n^2 and the speed falls as $\frac{1}{n}$.

Step 2: de Broglie wavelength:

$$\lambda = \frac{h}{mv}$$

Since $v \propto \frac{1}{n}$, $\lambda \propto n$. Equivalently $2\pi r = n\lambda$ gives $\lambda = \frac{2\pi r}{n} \propto \frac{n^2}{n} = n$.

Step 3: Compare:

From $n = 1$ to $n = 4$, λ becomes 4 times. So it will increase, option (B).

Final Answer:

The wavelength becomes 4 times larger.

B : will increase

Quick Tip: Speed falls with n , so $\lambda = h/mv$ rises.

46. If 'E' and 'L' denote the magnitude of total energy and angular momentum of revolving electron in n^{th} Bohr orbit then

- (A) $E \propto L$
(B) $E \propto L^{-1}$
(C) $E \propto L^{-2}$
(D) $E \propto L^2$

Correct Answer: (C) $E \propto L^{-2}$

Solution:

Step 1: Understanding the Concept:

In Bohr's model, the total energy in the n th orbit is $E = -\frac{13.6}{n^2}$ eV, so its magnitude is $|E| \propto \frac{1}{n^2}$. The angular momentum is $L = \frac{nh}{2\pi}$, so $L \propto n$.

Step 2: Combine:

Since $n \propto L$, we get

$$E \propto \frac{1}{n^2} \propto \frac{1}{L^2} = L^{-2}$$

Step 3: Choose:

Option (C).

Final Answer:

E is proportional to L to the power -2.

$$\boxed{E \propto L^{-2}}$$

Quick Tip: $|E|$ goes as $1/n^2$ and L goes as n.

47. Two different radioactive elements with half lives T_1 and T_2 have undecayed atoms N_1 and N_2 respectively, present at a given instant. The ratio of their activities at this instant is

- (A) $\frac{N_1 T_1}{N_2 T_2}$
(B) $\frac{N_1 T_2}{N_2 T_1}$
(C) $\frac{N_1 T_2}{T_1 T_2}$
(D) $\frac{T_1 T_2}{N_1 N_2}$

Correct Answer: (B) $\frac{N_1 T_2}{N_2 T_1}$

Solution:

Step 1: Understanding the Concept:

Activity is the number of decays per second: $A = \lambda N$, where λ is the decay constant. The decay constant is related to the half life by $\lambda = \frac{0.693}{T}$.

Step 2: Write each activity:

$$A_1 = \frac{0.693}{T_1} N_1, \quad A_2 = \frac{0.693}{T_2} N_2$$

Step 3: Ratio:

$$\frac{A_1}{A_2} = \frac{N_1/T_1}{N_2/T_2} = \frac{N_1 T_2}{N_2 T_1}$$

Step 4: Choose:

Option (B). Option (A) has the half lives the wrong way up, and

options (C) and (D) have unrelated combinations.

Final Answer:

The ratio of activities is $N_1 T_2 / (N_2 T_1)$.

$$\frac{N_1 T_2}{N_2 T_1}$$

Quick Tip: Activity = $0.693 N / T$.

48. Photodiode is a device

- (A) in which photocurrent is dependent on reverse bias.
- (B) in which photocurrent is independent of incident radiation.
- (C) which is always operated in forward bias.
- (D) which is always operated in reverse bias.

Correct Answer: (D) which is always operated in reverse bias.

Solution:

Step 1: Understanding the Concept:

A photodiode is a p-n junction diode that converts light into current. When light falls on the depletion region, it creates electron-hole pairs.

Step 2: Operating condition:

It works in reverse bias. In reverse bias, the dark current is very small, so the extra current caused by light is easy to notice. In forward bias, the large forward current would hide it.

Step 3: Check the options:

(A) The photocurrent depends on the intensity of light, and is nearly independent of the reverse bias value. (B) The photocurrent depends on the incident radiation. (C) It is not operated in forward bias. (D) It is always operated in reverse bias, which is correct.

Final Answer:

A photodiode is operated in reverse bias.

D : always operated in reverse bias

Quick Tip: It detects light in reverse bias.

49. For a p-n junction diode, breakdown voltage occurs when

- (A) reverse bias is decreased
- (B) reverse bias is not changed
- (C) reverse-bias is increased
- (D) forward bias is increased

Correct Answer: (C) reverse-bias is increased

Solution:

Step 1: Understanding the Concept:

In a p-n junction diode with reverse bias, only a small leakage current flows. As the reverse voltage is raised, the electric field across the depletion layer grows.

Step 2: Breakdown:

At a certain large reverse voltage, the field is so strong that it breaks

covalent bonds (Zener effect) or creates avalanche multiplication. The current then rises sharply. This is the breakdown voltage.

Step 3: Choose:

Breakdown occurs when the reverse bias is increased, option (C).

Final Answer:

Breakdown occurs on increasing the reverse bias.

C

Quick Tip: Breakdown needs a high reverse voltage.

50. If p-n junction diode is forward biased then

- (A) width of depletion layer decreases.
- (B) width of depletion layer increases.
- (C) barrier voltage increases
- (D) electric conduction is not possible

Correct Answer: (A) width of depletion layer decreases.

Solution:

Step 1: Understanding the Concept:

In forward bias, the p side is connected to the positive terminal and the n side to the negative terminal. The applied voltage opposes the built-in barrier potential.

Step 2: Effect:

The majority carriers are pushed towards the junction. The depletion region becomes narrower and the barrier voltage drops. Current can flow easily.

Step 3: Check the options:

(A) depletion layer width decreases, which is correct. (B) increases happens in reverse bias. (C) barrier voltage decreases, not increases. (D) conduction is possible above the knee voltage. So (A) is correct.

Final Answer:

Forward bias narrows the depletion layer.

A

Quick Tip: Forward bias reduces the barrier and the depletion layer.

Chemistry

Questions 51 to 100 | 1 mark each | 50 marks

51. What is the approximate mass of the precipitate formed when 50 mL of 16.9% solution of AgNO_3 is mixed with 50 mL of 7.45% KCl solution? (Molar mass of $\text{AgNO}_3 = 169 \text{ g/mol}$, $\text{KCl} = 74.5 \text{ g/mol}$, $\text{AgCl} = 143.3 \text{ g/mol}$)

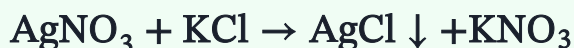
- (A) 3.5 g
- (B) 7 g
- (C) 14 g
- (D) 28 g

Correct Answer: (B) 7 g

Solution:

Step 1: Understanding the Concept:

Silver nitrate reacts with potassium chloride in a 1:1 ratio to give a precipitate of silver chloride.



Step 2: Find the amount of each reactant.

A 16.9 % solution means 16.9 g in 100 mL. So 50 mL holds 8.45 g of AgNO_3 , which is $\frac{8.45}{169} = 0.05 \text{ mol}$.

A 7.45 % solution means 7.45 g in 100 mL. So 50 mL holds 3.725 g of KCl, which is $\frac{3.725}{74.5} = 0.05 \text{ mol}$.

Step 3: Limiting reagent.

Both are 0.05 mol, which is exactly in the 1:1 ratio, so neither is in excess. 0.05 mol of AgCl forms.

Step 4: Mass of precipitate.

$$0.05 \times 143.3 = 7.165 \text{ g} \approx 7 \text{ g}$$

Final Answer:

The mass of AgCl formed is about 7 g, option (B).

$$\boxed{\approx 7 \text{ g}}$$

Quick Tip: Find the moles of each reactant from the mass percent in 50 mL, then use the limiting reagent.

52. Calculate the de Broglie wavelength of an electron in the first Bohr orbit of a hydrogen atom if the velocity of an electron in the first orbit is $2.2 \times 10^6 \text{ ms}^{-1}$. [mass of electron = $9.1 \times 10^{-31} \text{ kg}$, plank's constant (h) = $6.626 \times 10^{-34} \text{ J s}$]

- (A) $3.31 \times 10^{-10} \text{ m}$
- (B) $3.01 \times 10^{-10} \text{ m}$
- (C) $3.62 \times 10^{-10} \text{ m}$
- (D) $2.71 \times 10^{-10} \text{ m}$

Correct Answer: (A) $3.31 \times 10^{-10} \text{ m}$

Solution:

Step 1: Understanding the Concept:

The de Broglie wavelength of a particle is $\lambda = \frac{h}{mv}$.

Step 2: Substitute.

$$\lambda = \frac{6.626 \times 10^{-34}}{9.1 \times 10^{-31} \times 2.2 \times 10^6}$$

The denominator is $9.1 \times 2.2 \times 10^{-25} = 20.02 \times 10^{-25} = 2.002 \times 10^{-24}$.

Step 3: Calculate.

$$\lambda = \frac{6.626 \times 10^{-34}}{2.002 \times 10^{-24}} = 3.31 \times 10^{-10} \text{ m}$$

Step 4: Check.

This equals the circumference of the first orbit $2\pi a_0 = 2\pi \times 0.529 \text{ \AA} \approx 3.32 \text{ \AA}$, as the Bohr condition requires for $n = 1$.

Final Answer:

The wavelength is $3.31 \times 10^{-10} \text{ m}$, option (A).

$3.31 \times 10^{-10} \text{ m}$

Quick Tip: Use $\lambda = h / (m v)$ with the given velocity.

53. Which group elements have the smallest atomic radii in their respective periods?

- (A) group 15
- (B) group 16
- (C) group 17
- (D) group 18

Correct Answer: (C) group 17

Solution:

Step 1: Understanding the Concept:

Moving left to right in a period, the nuclear charge increases while the electrons are added to the same shell. The pull on the outer electrons grows, so the radius drops.

Step 2: Detailed Explanation:

Among the groups 15, 16, 17 and 18, group 17 (the halogens) shows the smallest covalent radius in each period. Group 18 is a special case: noble gases are listed by van der Waals radius, which is larger, so they are not counted as the smallest.

Step 3: Check the options.

Groups 15 and 16 come earlier in the period, so their radii are larger than those of group 17.

Final Answer:

Group 17 has the smallest atomic radii, option (C).

Group 17

Quick Tip: Atomic radius shrinks across a period as nuclear charge rises, and the halogens have the smallest radii before the noble gases.



54. What happens to the size of atoms in p-block elements when we move from left to right in the same period?

- (A) Size does not change
- (B) Size increases then decreases
- (C) Size increases
- (D) Size decreases

Correct Answer: (D) Size decreases

Solution:

Step 1: Understanding the Concept:

In a period, electrons enter the same shell while the nuclear charge increases by one each step.

Step 2: Detailed Explanation:

The stronger pull by the nucleus draws the electron cloud inward. The shielding by electrons in the same shell is poor. So the atomic size decreases from left to right.

Step 3: Check the options.

The size does not stay constant, does not increase, and does not first increase. It falls steadily.

Final Answer:

The size decreases, option (D).

decreases

Quick Tip: Atomic size decreases from left to right in a period.



55. Which of the following order of dipole moment for stated molecules is correct?

- (A) $\text{BF}_3 > \text{NF}_3 > \text{NH}_3$
- (B) $\text{NF}_3 > \text{BF}_3 > \text{NH}_3$
- (C) $\text{NH}_3 > \text{BF}_3 > \text{NF}_3$
- (D) $\text{NH}_3 > \text{NF}_3 > \text{BF}_3$

Correct Answer: (D) $\text{NH}_3 > \text{NF}_3 > \text{BF}_3$

Solution:

Step 1: Understanding the Concept:

The molecular dipole moment is the vector sum of bond dipoles and the lone pair contribution.

Step 2: Boron trifluoride.

BF_3 is trigonal planar. The three B-F bond dipoles cancel, so $\mu = 0$.

Step 3: Ammonia.

NH_3 is pyramidal. Nitrogen is more electronegative than hydrogen, so each N-H dipole points towards N, in the same direction as the lone pair. The dipoles add up, giving $\mu \approx 1.47 \text{ D}$.

Step 4: Nitrogen trifluoride.

NF_3 is also pyramidal, but fluorine is more electronegative than

nitrogen, so each N-F dipole points away from N, opposite to the lone pair. They partly cancel, giving $\mu \approx 0.24$ D.

Step 5: Order.



Final Answer:

The correct order is $\text{NH}_3 > \text{NF}_3 > \text{BF}_3$, option (D).



Quick Tip: BF_3 is planar and symmetric, so its dipole moment is zero. In NF_3 the bond dipoles oppose the lone pair, and in NH_3 they add to it.



56. Calculate the compressibility factor of 1 mole of a certain real gas if it occupies 0.4 dm^3 at 300 K and 40 atm. [$R = 0.082 \text{ atm dm}^3\text{K}^{-1}\text{mol}^{-1}$]

- (A) 0.45
- (B) 0.65
- (C) 0.85
- (D) 1.00

Correct Answer: (B) 0.65

Solution:

Step 1: Understanding the Concept:

For an ideal gas, $Z = 1$. The deviation of Z from 1 shows how far a real

gas departs from ideal behaviour.

Step 2: Key Formula or Approach:

$$Z = \frac{PV}{nRT}$$

Step 3: Calculate.

$P = 40 \text{ atm}$, $V = 0.4 \text{ dm}^3$, $n = 1$, $T = 300 \text{ K}$, $R = 0.082$.

$$PV = 40 \times 0.4 = 16$$

$$nRT = 0.082 \times 300 = 24.6$$

$$Z = \frac{16}{24.6} = 0.65$$

Step 4: Check the options.

$Z = 1$ would be an ideal gas. Here $Z < 1$, so attractive forces dominate.

Final Answer:

The compressibility factor is 0.65, option (B).

$$\boxed{Z = 0.65}$$

Quick Tip: Compressibility factor is $Z = PV / nRT$, which compares the real molar volume with the ideal one.



The value of enthalpy change (ΔH) for above reaction at 27°C is $-1366.5 \text{ kJ mol}^{-1}$. Then value of internal energy change for the same reaction at this temperature will be

- (A) $-1369.0 \text{ kJ mol}^{-1}$
- (B) $-1364.0 \text{ kJ mol}^{-1}$
- (C) $-1371.5 \text{ kJ mol}^{-1}$
- (D) $-1361.5 \text{ kJ mol}^{-1}$

Correct Answer: (B) $-1364.0 \text{ kJ mol}^{-1}$

Solution:

Step 1: Understanding the Concept:

The enthalpy and internal energy changes differ by the work of the change in the number of gas moles: $\Delta H = \Delta U + \Delta n_g RT$.

Step 2: Find Δn_g .

Reaction: $\text{C}_2\text{H}_5\text{OH}(l) + 3\text{O}_2(g) \rightarrow 2\text{CO}_2(g) + 3\text{H}_2\text{O}(l)$. Gaseous products: 2 mol. Gaseous reactants: 3 mol. Liquids are not counted.

$$\Delta n_g = 2 - 3 = -1$$

Step 3: Calculate.

$$T = 27 + 273 = 300 \text{ K and } R = 8.314 \times 10^{-3} \text{ kJ/(mol K)}.$$

$$\Delta U = \Delta H - \Delta n_g RT = -1366.5 - (-1)(8.314 \times 10^{-3})(300)$$

$$= -1366.5 + 2.494 = -1364.0 \text{ kJ/mol}$$

Step 4: Check the options.

Options (A) and (C) subtract the correction in the wrong direction, and (D) uses a wrong size of correction.

Final Answer:

The internal energy change is -1364.0 kJ/mol , option (B).

$$\boxed{-1364.0 \text{ kJ/mol}}$$

Quick Tip: Use $\Delta H = \Delta U + \Delta n(g) RT$, where $\Delta n(g)$ counts only the gas moles.

58. Which of the following processes has ΔS negative?

- (A) Dissolving Sugar in Water
- (B) Adsorption of a Gas on a Solid Surface
- (C) $2\text{NH}_3(g) \rightarrow \text{N}_2(g) + 3\text{H}_2(g)$
- (D) Sublimation of Dry Ice

Correct Answer: (B) Adsorption of a Gas on a Solid Surface

Solution:

Step 1: Understanding the Concept:

Entropy measures disorder. A process that makes the system more ordered has $\Delta S < 0$.

Step 2: Check option (A).

Sugar dissolving in water spreads out the sugar molecules, so disorder increases and $\Delta S > 0$.

Step 3: Check option (B).

In adsorption a gas molecule that moved freely gets held on a surface. Its freedom drops, so $\Delta S < 0$.

Step 4: Check option (C).

$2\text{NH}_3 \rightarrow \text{N}_2 + 3\text{H}_2$ increases gas moles from 2 to 4, so $\Delta S > 0$.

Step 5: Check option (D).

Sublimation of dry ice turns a solid into a gas, a huge rise in disorder, so $\Delta S > 0$.

Final Answer:

Only adsorption of a gas on a solid has a negative ΔS , option (B).

(B)

Quick Tip: Entropy falls when the disorder of the system decreases, such as a gas being held on a solid surface.

59. Which of the following properties of a system depends upon the amount of matter and path?

- (A) Heat
- (B) Free energy
- (C) Enthalpy
- (D) Entropy

Correct Answer: (A) Heat

Solution:

Step 1: Understanding the Concept:

Properties of a system are of two kinds. Extensive properties depend on the amount of matter present, and intensive properties do not. Separately, state functions depend only on the initial and final states, while path functions depend on how the change is carried out.

Step 2: Key Approach:

We need a property that is extensive and also a path function. So we first pick out the path function, then check that it scales with the amount of matter.

Step 3: Check Heat:

Heat q is energy that crosses the boundary because of a temperature difference. Its value is different for a reversible and an irreversible route between the same two states, so it is a path function. Doubling the amount of substance doubles the heat needed for the same temperature change, so it is extensive. So (A) satisfies both conditions.

Step 4: Why the other options are wrong:

Free energy G , enthalpy H and entropy S are all extensive, but each is a state function. Their change between two states is fixed, whatever

the path. So they fail the "depends on path" condition, and (B), (C) and (D) are wrong.

Final Answer:

Heat is both extensive and a path function.

Heat (option A)

Quick Tip: Heat is a path function and also extensive, while H, G and S are state functions.



60. The dissociation constant of weak acid HA is 1.5×10^{-5} . Find the percent dissociation, containing 0.2 moles per 2 liters of solution.

- (A) 1.22%
- (B) 1.18%
- (C) 1.14%
- (D) 1.26%

Correct Answer: (A) 1.22%

Solution:

Step 1: Understanding the Concept:

For a weak acid HA, only a small fraction splits into ions. Ostwald dilution law links the dissociation constant K_a , the concentration C and the degree of dissociation α .

Step 2: Key Formula:

For small α , $K_a = C\alpha^2$, so $\alpha = \sqrt{K_a/C}$. Percent dissociation is $\alpha \times 100$.

Step 3: Find the concentration:

The amount is 0.2 mol in 2 L. So

$$C = \frac{0.2}{2} = 0.1 \text{ M}$$

Step 4: Find the degree of dissociation:

$$\alpha = \sqrt{\frac{1.5 \times 10^{-5}}{0.1}} = \sqrt{1.5 \times 10^{-4}}$$

Since $\sqrt{1.5} = 1.2247$ and $\sqrt{10^{-4}} = 10^{-2}$,

$$\alpha = 1.2247 \times 10^{-2}$$

This is small compared with 1, so the approximation is fine.

Step 5: Convert to percent:

$$\text{Percent dissociation} = 1.2247 \times 10^{-2} \times 100 \approx 1.22\%$$

Options (B), (C) and (D) are close values that come from arithmetic slips in the square root, so they are wrong.

Final Answer:

The percent dissociation is about 1.22 percent, which is option (A).

$$\boxed{1.22\%}$$

Quick Tip: Use $C = 0.1 \text{ M}$ and $\alpha = \sqrt{K_a/C}$, then multiply by 100.

61. The solubility of sparingly soluble salts MX_1 , MX_2 and MX_3 is $1 \times 10^{-3} \text{ mol/L}$. Hence, their respective solubility products are

- (A) 1×10^{-6} , 4×10^{-9} and 27×10^{-12}
- (B) 1×10^{-9} , 4×10^{-9} and 32×10^{-12}
- (C) 1×10^{-9} , 8×10^{-8} and 32×10^{-12}
- (D) 1×10^{-6} , 8×10^{-8} and 27×10^{-12}

Correct Answer: (A) 1×10^{-6} , 4×10^{-9} and 27×10^{-12}

Solution:

Step 1: Understanding the Concept:

The solubility product K_{sp} is the product of ion concentrations raised to their coefficients at saturation. If the solubility is $s \text{ mol/L}$, each ion concentration is its coefficient times s .

Step 2: Key Formula:

For $\text{MX}_n \rightleftharpoons \text{M}^{n+} + n\text{X}^-$, we get $[\text{M}^{n+}] = s$ and $[\text{X}^-] = ns$. So $K_{sp} = s(ns)^n = n^n s^{n+1}$.

Step 3: MX ($n = 1$):

$$K_{sp} = s^2 = (10^{-3})^2 = 1 \times 10^{-6}$$

Step 4: MX₂ ($n = 2$):

$$K_{sp} = 4s^3 = 4 \times (10^{-3})^3 = 4 \times 10^{-9}$$

Step 5: MX_3 ($n = 3$):

$$K_{sp} = 27s^4 = 27 \times (10^{-3})^4 = 27 \times 10^{-12}$$

Step 6: Compare with options:

The sequence 1×10^{-6} , 4×10^{-9} , 27×10^{-12} matches option (A).

Options (B) and (C) use 10^{-9} for the first salt, which is wrong because s^2 gives 10^{-6} . Option (D) has 8×10^{-8} , which would come from squaring instead of cubing.

Final Answer:

The three values are $1e-6$, $4e-9$ and $27e-12$, option (A).

$(A) \ 1 \times 10^{-6}, \ 4 \times 10^{-9}, \ 27 \times 10^{-12}$

Quick Tip: Use $K_{sp} = n^n s^{(n+1)}$ for each salt with $s = 1e-3$.

62. Identify the strongest base according to Bronsted-Lowry theory from following.

- (A) Cl^-
- (B) CH_3COO^-
- (C) NO_3^-
- (D) HSO_4^-

Correct Answer: (B) CH_3COO^-

Solution:

Step 1: Understanding the Concept:

In Bronsted-Lowry theory a base accepts a proton. Every acid has a conjugate base. The weaker the acid, the stronger its conjugate base.

Step 2: Key Approach:

Write the parent acid of each ion, then compare acid strengths. The ion whose parent acid is weakest is the strongest base.

Step 3: Parent acids:

Cl^- comes from HCl , a very strong acid. NO_3^- comes from HNO_3 , a very strong acid. HSO_4^- comes from H_2SO_4 , a strong acid. CH_3COO^- comes from CH_3COOH , a weak acid with $K_a \approx 1.8 \times 10^{-5}$.

Step 4: Decide:

Only acetic acid is weak. So its conjugate base, acetate ion, holds on to a proton most strongly. The other three ions are very poor proton acceptors. So acetate is the strongest base, option (B).

Final Answer:

Acetate is the conjugate base of the only weak acid in the list.



Quick Tip: A weaker acid has a stronger conjugate base.

63. Identify the alkali metal having the smallest atomic size from following.

- (A) Sodium
- (B) Potassium
- (C) Rubidium
- (D) Caesium

Correct Answer: (A) Sodium

Solution:

Step 1: Understanding the Concept:

Down a group, new electron shells are added. So atomic size grows from top to bottom in alkali metals: $\text{Li} < \text{Na} < \text{K} < \text{Rb} < \text{Cs}$.

Step 2: Apply to the options:

Among sodium, potassium, rubidium and caesium, sodium is the highest in the group and has the fewest shells (three). Its atomic radius is about 186 pm, against about 227 pm for K, 248 pm for Rb and 265 pm for Cs.

Step 3: Conclusion:

So sodium has the smallest atomic size in this list, option (A). The other three have more shells, so they are larger.

Final Answer:

Sodium is the topmost element in the list, so it is the smallest.

A : Sodium

Quick Tip: Atomic size increases down a group, so pick the highest element listed.

64. Identify the compound formed when But-2-ene is treated with KMnO_4 in dilute H_2SO_4 .

- (A) Butanoic acid
- (B) Acetic acid
- (C) Butanal
- (D) Butanol

Correct Answer: (B) Acetic acid

Solution:

Step 1: Understanding the Concept:

Acidic KMnO_4 is a strong oxidising agent. It breaks the $\text{C} = \text{C}$ double bond of an alkene completely (oxidative cleavage). Each carbon of the double bond ends up as a carboxylic acid, or as a ketone if that carbon carries two alkyl groups.

Step 2: Structure of but-2-ene:

$\text{CH}_3 - \text{CH} = \text{CH} - \text{CH}_3$. Each double bond carbon carries one hydrogen and one methyl group.

Step 3: Cleavage:

Breaking the double bond gives two $\text{CH}_3 - \text{CH}$ fragments. Each is oxidised to CH_3COOH .



Step 4: Why the other options are wrong:

Butanoic acid and butanol would keep all four carbons, but the chain is cut in the middle. Butanal is an aldehyde, which would be oxidised further under these conditions. So only acetic acid, option (B), is formed.

Final Answer:

But-2-ene is cleaved into two molecules of acetic acid.

B : Acetic acid

Quick Tip: Acidic KMnO_4 cleaves the $\text{C}=\text{C}$ bond and oxidises each piece to an acid.

65. Which one of the following compound will show addition of HBr to alkene according to markownikoff's rule?

- (A) But-2-ene
- (B) Hex-3-ene
- (C) Ethene
- (D) 2-methyl propene

Correct Answer: (D) 2-methyl propene

Solution:

Step 1: Understanding the Concept:

Markovnikov rule: when HBr adds to an unsymmetrical alkene, the

hydrogen goes to the carbon that already has more hydrogens, and bromine goes to the carbon with fewer hydrogens. The rule only has meaning if the two double-bond carbons are different.

Step 2: Check each alkene:

But-2-ene $CH_3CH = CHCH_3$ is symmetrical. Hex-3-ene $CH_3CH_2CH = CHCH_2CH_3$ is symmetrical. Ethene $CH_2 = CH_2$ is symmetrical. In each, both ways of adding give the same product, so there is no choice to make.

Step 3: The unsymmetrical one:

2-Methylpropene is $(CH_3)_2C = CH_2$. One carbon has two methyl groups and no H, the other has two H atoms. So the rule applies.

Step 4: Product:

H adds to CH_2 and Br to the carbon with two methyls, forming the more stable tertiary carbocation:



So the answer is option (D).

Final Answer:

Only 2-methylpropene is unsymmetrical, so Markovnikov addition applies.

D : 2-methyl propene

Quick Tip: Look for the alkene whose two double-bond carbons are different.

66. When a mixture of two different alkyl halides reacts with metallic sodium in dry ether, the formation of a possible number of alkanes are-

- (A) Two
- (B) Three
- (C) Four
- (D) Five

Correct Answer: (B) Three

Solution:

Step 1: Understanding the Concept:

Wurtz reaction: two alkyl halides couple with sodium in dry ether to form an alkane with the two alkyl groups joined, $2R - X + 2Na \rightarrow R - R + 2NaX$.

Step 2: Apply to a mixture:

Let the two halides be $R - X$ and $R' - X$. Three couplings are possible.

Step 3: List the products:

1. R with R gives $R - R$.
2. R' with R' gives $R' - R'$.
3. R with R' gives $R - R'$.

Step 4: Conclusion:

Three different alkanes can form, so the answer is option (B). Two would be the count only if the cross product were missing, and four or five would need more distinct alkyl groups than we have.

Final Answer:

Two different halides give R-R, R'-R' and R-R', so three alkanes.

B : Three

Quick Tip: Count the pairs AA, BB and AB.

67. Identify the product of reforming reaction of n-hexane in the presence of V_2O_5 over alumina.

- (A) Benzene
- (B) Cyclohexane
- (C) Methyl benzene
- (D) Phenol

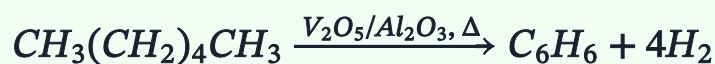
Correct Answer: (A) Benzene

Solution:**Step 1: Understanding the Concept:**

Reforming (aromatisation) converts straight-chain alkanes with six or more carbons into aromatic hydrocarbons. It uses a catalyst such as V_2O_5 , Mo_2O_3 or Cr_2O_3 on alumina at high temperature and pressure.

Step 2: Reaction:

n-Hexane loses hydrogen, closes into a ring, and loses more hydrogen to give a fully conjugated six-membered ring:



Step 3: Check the options:

Cyclohexane is only the ring without full dehydrogenation.

Methylbenzene needs a seventh carbon, so n-heptane would be needed. Phenol contains oxygen, which is absent from the reactant. So the product is benzene, option (A).

Final Answer:

Aromatisation of n-hexane gives benzene.

A : Benzene

Quick Tip: Reforming of n-hexane over a metal oxide catalyst gives an aromatic ring.

68. An element crystallises in fcc unit cell with cell edge length of 3.608×10^{-8} cm, the density of element is 8.92 gcm^{-3} . Calculate the atomic mass of element ($N_A = 6.022 \times 10^{23}$).

- (A) 60 g/mol
- (B) 65 g/mol
- (C) 63 g/mol
- (D) 108 g/mol

Correct Answer: (C) 63 g/mol

Solution:

Step 1: Understanding the Concept:

The density of a cubic crystal is linked to its unit cell edge and the atoms per cell. For fcc the number of atoms per unit cell is $Z = 4$.

Step 2: Key Formula:

$$\rho = \frac{Z M}{a^3 N_A} \Rightarrow M = \frac{\rho a^3 N_A}{Z}$$

Step 3: Compute the cell volume:

$$a^3 = (3.608 \times 10^{-8})^3 = 46.97 \times 10^{-24} = 4.697 \times 10^{-23} \text{ cm}^3$$

Step 4: Substitute:

$$M = \frac{8.92 \times 4.697 \times 10^{-23} \times 6.022 \times 10^{23}}{4}$$

The numerator is $8.92 \times 4.697 = 41.90$, and $41.90 \times 6.022 = 252.3$.
Dividing by 4 gives

$$M \approx 63.1 \text{ g/mol}$$

Step 5: Choose:

The closest option is 63 g/mol, option (C). The element is copper. If one wrongly used $Z = 2$ or $Z = 1$ the result would be 126 or 252, which are not offered.

Final Answer:

The atomic mass is about 63 g/mol, option (C).

$$63 \text{ g/mol}$$

Quick Tip: Use $M = \rho a^3 N_A / Z$ with $Z = 4$ for fcc.

69. Calculate the number of unit cells present in 1 g metal if the product of the density of metal and the volume of unit cell is 2.5×10^{-22} g.

- (A) 1.0×10^{21}
- (B) 2.0×10^{21}
- (C) 3.0×10^{21}
- (D) 4.0×10^{21}

Correct Answer: (D) 4.0×10^{21}

Solution:**Step 1: Understanding the Concept:**

The product $\rho \times V$ of density and unit-cell volume is the mass of one unit cell. The number of unit cells in a given mass is then total mass divided by this cell mass.

Step 2: Mass of one unit cell:

$$m_{\text{cell}} = \rho a^3 = 2.5 \times 10^{-22} \text{ g}$$

Step 3: Number of cells in 1 g:

$$N = \frac{1 \text{ g}}{2.5 \times 10^{-22} \text{ g}} = 0.4 \times 10^{22} = 4.0 \times 10^{21}$$

Step 4: Check the options:

1.0×10^{21} , 2.0×10^{21} and 3.0×10^{21} would need cell masses of 10^{-21} , 5×10^{-22} and 3.3×10^{-22} g, none of which is given. So the answer is (D).

Final Answer:

The metal has 4.0×10^{21} unit cells per gram, option (D).

$$4.0 \times 10^{21}$$

Quick Tip: Density times cell volume is the mass of one cell. Divide 1 g by it.

70. Which of the following is NOT a characteristic of Schottky defect?

- (A) After defect stoichiometry, the crystal remains unchanged.
- (B) After defect neutrality, the crystal's structure remains unchanged.
- (C) After defect density of crystal remains unchanged
- (D) It is a vacancy defect.

Correct Answer: (C) After defect density of crystal remains unchanged

Solution:

Step 1: Understanding the Concept:

Schottky defect is a vacancy defect in ionic solids. Equal numbers of cations and anions leave the lattice, so the crystal stays electrically

neutral. It is common in ionic compounds with similar cation and anion sizes, such as NaCl and CsCl.

Step 2: Effects:

1. Stoichiometry is unchanged, because cations and anions are missing in equal number.
2. The crystal stays neutral.
3. Because ions are missing, the mass per unit volume falls, so the density decreases.

Step 3: Check the options:

(A) is true. (B) is true. (D) is true. (C) says the density remains unchanged, which is false, since vacancies lower the density. So (C) is the statement that is NOT a characteristic.

Final Answer:

Schottky defect lowers density, so option (C) is the odd one.

C

Quick Tip: Missing ions mean the crystal gets less dense.

71. Calculate the van't Hoff factor of a centimolar solution of potassium ferrocyanide if it is 60% dissociated at 300 K.

- (A) 3.4
- (B) 3.9
- (C) 3.1
- (D) 3.7

Correct Answer: (A) 3.4

Solution:

Step 1: Understanding the Concept:

Potassium ferrocyanide $K_4[Fe(CN)_6]$ is an electrolyte. The van't Hoff factor i relates the real number of particles to the number expected without dissociation.

Step 2: Key Formula:

If one formula unit gives n ions and the degree of dissociation is α , then

$$i = 1 + (n - 1)\alpha$$

Step 3: Find n :

$K_4[Fe(CN)_6] \rightarrow 4K^+ + [Fe(CN)_6]^{4-}$. So $n = 5$.

Step 4: Calculate:

$$i = 1 + (5 - 1)(0.60) = 1 + 2.4 = 3.4$$

The concentration (0.01 M) and temperature do not enter the formula. The answer is option (A).

Final Answer:

The van't Hoff factor is 3.4.

3.4

Quick Tip: $K_4[Fe(CN)_6]$ gives 5 ions. Use $i = 1 + 4 \alpha$.

72. 5 g urea is dissolved in 100 g water. Find the amount of glucose to be dissolved in 120 g of water so that the boiling points of both solutions will be the same.

[Molar mass of urea = 60 g mol^{-1} & Molar mass of glucose = 180 g mol^{-1}]

- (A) 17 g
- (B) 19 g
- (C) 18 g
- (D) 20 g

Correct Answer: (C) 18 g

Solution:

Step 1: Understanding the Concept:

Elevation in boiling point is $\Delta T_b = K_b m$. Both solutes are non-electrolytes, so the boiling points are equal when the molalities are equal.

Step 2: Molality of urea solution:

Moles of urea = $5/60 = 0.0833$. Mass of water = 100 g = 0.1 kg.

$$m = \frac{0.0833}{0.1} = 0.833 \text{ mol/kg}$$

Step 3: Glucose solution:

Let the mass of glucose be w g. Moles = $w/180$, and water is 0.12 kg.

$$\frac{w/180}{0.12} = 0.833$$

Step 4: Solve:

$$w = 0.833 \times 0.12 \times 180 = 18 \text{ g}$$

Step 5: Choose:

The answer is 18 g, option (C).

Final Answer:

Equal molality gives 18 g of glucose.

18 g

Quick Tip: Set the two molalities equal and solve for the glucose mass.

73. Which of the following solutes has the ratio of theoretical molar mass to the experimentally observed molar mass is 3?

- (A) KCl
- (B) K_2SO_4
- (C) MgSO_4
- (D) $\text{Al}_2(\text{SO}_4)_3$

Correct Answer: (B) K_2SO_4

Solution:

Step 1: Understanding the Concept:

The van't Hoff factor is $i = \frac{\text{normal (theoretical) molar mass}}{\text{observed molar mass}}$. So the question asks for a solute with $i = 3$.

Step 2: Complete dissociation:

For complete dissociation, i equals the number of ions per formula unit.

Step 3: Count ions:

KCl gives 2 ions (K^+ , Cl^-), so $i = 2$.

K_2SO_4 gives $2K^+ + SO_4^{2-}$, which is 3 ions, so $i = 3$.

$MgSO_4$ gives 2 ions, so $i = 2$.

$Al_2(SO_4)_3$ gives $2Al^{3+} + 3SO_4^{2-}$, which is 5 ions, so $i = 5$.

Step 4: Select:

Only K_2SO_4 has $i = 3$, option (B).

Final Answer:

K_2SO_4 gives three ions, so the ratio is 3.

B : K_2SO_4

Quick Tip: Ratio of theoretical to observed molar mass equals i . Count ions.

74. The molar conductivity of 0.01 M monobasic acid at 25 °C is $15 \text{ ohm}^{-1} \text{ cm}^2 \text{ mol}^{-1}$. The molar conductivity of the same acid at infinite dilution is $375 \text{ ohm}^{-1} \text{ cm}^2 \text{ mol}^{-1}$. Calculate $[H^+]$ in solution.

- (A) $2.0 \times 10^{-4} \text{ M}$
- (B) $3.0 \times 10^{-4} \text{ M}$
- (C) $4.0 \times 10^{-4} \text{ M}$
- (D) $5.0 \times 10^{-4} \text{ M}$

Correct Answer: (C) $4.0 \times 10^{-4} \text{ M}$

Solution:

Step 1: Understanding the Concept:

For a weak monobasic acid, the degree of dissociation is the ratio of molar conductivity at the given concentration to that at infinite dilution.

Step 2: Key Formula:

$$\alpha = \frac{\Lambda_m}{\Lambda_m^\infty}, \quad [H^+] = C\alpha$$

Step 3: Calculate alpha:

$$\alpha = \frac{15}{375} = 0.04$$

Step 4: Calculate $[H^+]$:

$$[H^+] = 0.01 \times 0.04 = 4 \times 10^{-4} \text{ M}$$

Step 5: Choose:

Option (C). The other options come from using a wrong degree of dissociation.

Final Answer:

The hydrogen ion concentration is $4\text{e-}4 \text{ M}$.

$$4.0 \times 10^{-4} \text{ M}$$

Quick Tip: α = molar conductivity / limiting molar conductivity.



75. Which from following statements regarding mercury battery is NOT true?

- (A) It is a primary dry cell.
- (B) It consists of Zn anode amalgamated with mercury.
- (C) The electrolyte is strongly acidic.
- (D) In this Hg is obtained by reduction of HgO

Correct Answer: (C) The electrolyte is strongly acidic.

Solution:

Step 1: Understanding the Concept:

The mercury cell is a primary cell used in watches and hearing aids. It gives a steady voltage of about 1.35 V.

Step 2: Its parts:

Anode: zinc amalgamated with mercury. Cathode: a paste of HgO and carbon. Electrolyte: a paste of KOH and ZnO.

Cathode reaction: $\text{HgO} + \text{H}_2\text{O} + 2e^- \rightarrow \text{Hg} + 2\text{OH}^-$.

Step 3: Check each statement:

(A) It is a primary cell, true. (B) Zinc-mercury amalgam anode, true. (D) HgO is reduced to Hg at the cathode, true. (C) says the electrolyte is strongly acidic. The electrolyte is KOH, which is strongly basic, so (C) is not true.

Final Answer:

The electrolyte is KOH (basic), so option (C) is the false statement.

C

Quick Tip: The electrolyte in a mercury cell is basic KOH paste.

76. The molar conductivity of 0.04 M AB_2 type salt solution at 300 K is $200 \Omega^{-1}\text{cm}^2\text{mol}^{-1}$. Find the conductivity.

- (A) $0.006 \Omega^{-1}\text{cm}^{-1}$
- (B) $0.008 \Omega^{-1}\text{cm}^{-1}$
- (C) $0.01 \Omega^{-1}\text{cm}^{-1}$
- (D) $0.015 \Omega^{-1}\text{cm}^{-1}$

Correct Answer: (B) $0.008 \Omega^{-1}\text{cm}^{-1}$

Solution:

Step 1: Understanding the Concept:

Conductivity κ and molar conductivity Λ_m are linked by concentration. With C in mol/L and κ in $\Omega^{-1}\text{cm}^{-1}$,

$$\Lambda_m = \frac{1000 \kappa}{C}$$

Step 2: Rearrange:

$$\kappa = \frac{\Lambda_m C}{1000}$$

Step 3: Substitute:

$$\kappa = \frac{200 \times 0.04}{1000} = \frac{8}{1000} = 0.008 \Omega^{-1}\text{cm}^{-1}$$

Step 4: Note:

The salt type AB_2 and the temperature are not needed, because Λ_m is already given for the salt as a whole. The answer is option (B).

Final Answer:

The conductivity is 0.008 per ohm per cm.

$$\boxed{0.008 \Omega^{-1}\text{cm}^{-1}}$$

Quick Tip: $\kappa = \Lambda_m \times C / 1000$.

77. The time required for 90% completion of a certain first order reaction is 1 hour. Calculate the time required for 99.9% completion of the same reaction.

- (A) 2 hours.
- (B) 1 hour.
- (C) 3 hours.
- (D) 0.5 hour.

Correct Answer: (C) 3 hours.

Solution:

Step 1: Understanding the Concept:

For a first order reaction, $k = \frac{2.303}{t} \log \frac{[A]_0}{[A]}$. If $x\%$ has reacted, $[A] = (100 - x)\%$ of $[A]_0$.

Step 2: For 90% completion:

$[A]_0/[A] = 100/10 = 10$, so

$$k = \frac{2.303}{1 \text{ h}} \log 10 = 2.303 \text{ h}^{-1}$$

Step 3: For 99.9% completion:

$[A]_0/[A] = 100/0.1 = 1000$, so $\log 1000 = 3$.

$$t = \frac{2.303}{k} \times 3 = \frac{2.303 \times 3}{2.303} = 3 \text{ h}$$

Step 4: Select:

The answer is 3 hours, option (C). The time is not simply proportional

to the percentage.

Final Answer:

Time for 99.9 percent is three times the time for 90 percent, so 3 hours.

3 hours

Quick Tip: t is proportional to $\log(A_0/A)$. Compare $\log 10$ and $\log 1000$.

78. The rate of the reaction $2A + 3B \rightarrow 2C + D$ is $6 \times 10^{-4} \text{ mol dm}^{-3} \text{ s}^{-1}$, when $[A] = [B] = 0.3 \text{ mol dm}^{-3}$. If the reaction is of first order for A and zeroth order for B, then find the rate constant.

- (A) $1 \times 10^{-3} \text{ s}^{-1}$
- (B) $2 \times 10^{-3} \text{ s}^{-1}$
- (C) $3 \times 10^{-3} \text{ s}^{-1}$
- (D) $4 \times 10^{-3} \text{ s}^{-1}$

Correct Answer: (B) $2 \times 10^{-3} \text{ s}^{-1}$

Solution:

Step 1: Understanding the Concept:

The rate law is written from the given orders: first order in A and zeroth order in B. Zero order in B means B does not appear in the rate expression.

Step 2: Rate law:

$$\text{Rate} = k[A]^1[B]^0 = k[A]$$

Step 3: Substitute:

$$6 \times 10^{-4} = k \times 0.3$$

$$k = \frac{6 \times 10^{-4}}{0.3} = 2 \times 10^{-3} \text{ s}^{-1}$$

Step 4: Check units:

For a first order reaction the units of k are s^{-1} , which matches. The stoichiometric coefficients (2A and 3B) do not enter the rate law unless the reaction is elementary. So the answer is (B).

Final Answer:

$k = \text{rate} / [A] = 2 \times 10^{-3}$ per second.

$$\boxed{2 \times 10^{-3} \text{ s}^{-1}}$$

Quick Tip: Rate = $k[A]$, since B is zero order.



79. What is the value of the slope of a graph obtained by plotting the concentration of reactant $(A)_t$ versus time for zero order reaction?

- (A) $-k$
- (B) $1/2k$
- (C) $k/2.303$
- (D) $2.303/k$

Correct Answer: (A) $-k$

Solution:

Step 1: Understanding the Concept:

For a zero order reaction the rate does not depend on the concentration of the reactant. The concentration falls by a fixed amount in each unit of time.

Step 2: Key Formula:

The integrated rate law is $[A]_t = [A]_0 - kt$.

Step 3: Compare with a straight line:

A straight line is $y = mx + c$. Put $y = [A]_t$ and $x = t$. Then the slope is $m = -k$ and the intercept is $[A]_0$.

Step 4: Check the options:

The other values do not come from this equation. $k/2.303$ is a first order slope (for $\log[A]$ against t , with a negative sign), while $1/2k$ and $2.303/k$ are not slopes of any plot here. So the slope is $-k$, option (A).

Final Answer:

The slope of $[A]$ against t is $-k$.

$-k$

Quick Tip: For zero order, $[A] = [A]_0 - kt$, a straight line.

80. Which of the following nano structures includes nanotubes as an example?

- (A) Zero dimensional
- (B) One dimensional
- (C) Two dimensional
- (D) Three dimensional

Correct Answer: (B) One dimensional

Solution:

Step 1: Understanding the Concept:

Nanomaterials are grouped by how many of their dimensions lie outside the nanometre range (more than 100 nm). Zero dimensional means all three dimensions are in the nano range. One dimensional means two are nano and one is large. Two dimensional means one is nano and two are large.

Step 2: Examples:

Nanoparticles and quantum dots are zero dimensional. Nanotubes, nanowires and nanorods are one dimensional, because the diameter is nano but the length is large. Thin films and graphene sheets are two dimensional. Bulk crystals are three dimensional.

Step 3: Conclusion:

A nanotube has a nano-sized diameter and a long length, so it is one dimensional. The answer is option (B).

Final Answer:

Nanotubes are one dimensional nanostructures.

B : One dimensional

Quick Tip: A nanotube is thin in two directions and long in one.

81. Match List I with List II

List I	List II
(A). Brownian Motion	(I). Removal of impurities from the colloidal sols using a semipermeable membrane.
(B). Tyndall Effect	(II). Movement of colloidal particles under the influence of an electric field.
(C). Electrophoresis	(III). Random movement of colloidal particles due to kinetic energy.
(D). Dialysis	(IV). Scattering of light by colloidal particles

Choose the **correct** answer from the options given below:

- (A) (A) - (I), (B) - (II), (C) - (III), (D) - (IV)
(B) (A) - (II), (B) - (IV), (C) - (I), (D) - (III)
(C) (A) - (I), (B) - (II), (C) - (IV), (D) - (III)
(D) (A) - (III), (B) - (IV), (C) - (II), (D) - (I)

Correct Answer: (D) (A) - (III), (B) - (IV), (C) - (II), (D) - (I)

Solution:

Step 1: Understanding the Concept:

Each term in List I is a standard property or technique of colloids. We match it by its meaning.

Step 2: Brownian motion:

Colloidal particles keep moving in random zig-zag paths because the molecules of the dispersion medium hit them from all sides. So (A) matches (III).

Step 3: Tyndall effect:

A beam of light is scattered by colloidal particles and becomes visible. So (B) matches (IV).

Step 4: Electrophoresis:

Charged colloidal particles move towards one electrode under an electric field. So (C) matches (II).

Step 5: Dialysis:

Small ions and molecules pass through a semipermeable membrane and the colloid stays behind, which removes impurities. So (D) matches (I).

Step 6: Choose:

The matching is A-III, B-IV, C-II, D-I. This is option (D). The other options mix up at least two pairs, for instance (A) pairs Brownian motion with dialysis.

Final Answer:

The correct pairs are A-III, B-IV, C-II and D-I.

(D): A-III, B-IV, C-II, D-I

Quick Tip: Dialysis uses a membrane and electrophoresis uses an electric field.



82. Identify the factor responsible for the greater range of oxidation states in actinoids.

- (A) actinoid contraction
- (B) radioactive nature of actinoids
- (C) 5f, 6d and 7s levels having comparable energies
- (D) 4f and 5d levels being close in energies

Correct Answer: (C) 5f, 6d and 7s levels having comparable energies

Solution:

Step 1: Understanding the Concept:

Oxidation states of an element depend on how many electrons can be used for bonding. In transition and inner transition elements, electrons from nearby sub-shells can take part if the energy gap between those sub-shells is small.

Step 2: Apply to actinoids:

In actinoids, the 5f, 6d and 7s sub-shells have comparable energies. So electrons from all three can be used for bonding. That gives a wide range of oxidation states, from +3 up to +7 (for example in Np and Pu).

Step 3: Check the options:

(A) Actinoid contraction explains the gradual fall in size, not oxidation states. (B) Radioactivity affects stability of nuclei, not oxidation states. (D) $4f$ and $5d$ energies being close applies to lanthanoids. So (C) is correct.

Final Answer:

Close energies of $5f$, $6d$ and $7s$ allow many oxidation states.

C : $5f$, $6d$ and $7s$ levels of comparable energy

Quick Tip: More sub-shells of similar energy give more oxidation states.

83. Which of the following factors may be regarded as the main cause of lanthanide contraction?

- (A) Poor shielding of one of $4f$ electron by another in the sub-shell.
- (B) Effective shielding of one of $4f$ electrons by another in the subshell
- (C) Poorer shielding of $5d$ electrons by $4f$ electrons
- (D) Greater shielding of $5d$ electrons by $4f$ electrons

Correct Answer: (A) Poor shielding of one of $4f$ electron by another in the sub-shell.

Solution:

Step 1: Understanding the Concept:

Lanthanide contraction is the steady decrease in atomic and ionic radii from La to Lu. As we move across, one proton is added to the nucleus and one electron goes into the $4f$ sub-shell each time.

Step 2: Why the size falls:

The $4f$ orbitals have diffuse shapes, so one $4f$ electron shields another poorly from the nuclear charge. As a result the effective nuclear charge rises steadily and pulls the outer electrons in, which shrinks the size.

Step 3: Check the options:

(A) says poor shielding among $4f$ electrons, which is the cause. (B) says effective shielding, which would stop the contraction. (C) and (D) talk about $5d$ electrons being shielded by $4f$, which is not the main cause. So (A) is correct.

Final Answer:

Poor shielding by $4f$ electrons causes lanthanide contraction.

A

Quick Tip: The $4f$ electrons shield one another poorly.

84. What is the value of spin only magnetic moment for Cu^{2+} in BM?

- (A) 2.84
- (B) 3.87
- (C) 1.73
- (D) 0.0

Correct Answer: (C) 1.73

Solution:

Step 1: Understanding the Concept:

The spin-only magnetic moment depends on the number of unpaired electrons n : $\mu = \sqrt{n(n+2)}$ BM.

Step 2: Electronic configuration:

Cu has atomic number 29: $[Ar]3d^{10}4s^1$. Cu^{2+} loses the 4s electron and one 3d electron, giving $[Ar]3d^9$.

Step 3: Unpaired electrons:

In $3d^9$, four orbitals are full and one orbital holds a single electron. So $n = 1$.

Step 4: Calculate:

$$\mu = \sqrt{1 \times (1 + 2)} = \sqrt{3} = 1.73 \text{ BM}$$

Step 5: Check the options:

2.84 is for $n = 2$, 3.87 is for $n = 3$, and 0 is for $n = 0$. So the answer is (C).

Final Answer:

Cu^{2+} has one unpaired electron, so μ is 1.73 BM.

1.73 BM

Quick Tip: Find unpaired electrons in d9 and use $\sqrt{n(n+2)}$.

85. The sum of coordination number and oxidation number of M in $[M(en)_2C_2O_4]Cl$ is

- (A) 9
- (B) 8
- (C) 7
- (D) 6

Correct Answer: (A) 9

Solution:

Step 1: Understanding the Concept:

The coordination number is the total number of donor atoms attached to the central metal. The oxidation number is the charge the metal would carry if all ligands were removed with their pairs.

Step 2: Coordination number:

Ethylenediamine (en) is bidentate, so 2 en give 4 donor atoms. Oxalate ($C_2O_4^{2-}$) is also bidentate, so it gives 2. Total coordination number = $4 + 2 = 6$.

Step 3: Oxidation number:

The complex is $[M(en)_2C_2O_4]^+$, because one Cl^- is outside. Let M be x . en is neutral and oxalate is -2 .

$$x + 0 + (-2) = +1 \Rightarrow x = +3$$

Step 4: Sum:

$6 + 3 = 9$, option (A).

Final Answer:

Coordination number 6 plus oxidation number 3 gives 9.

9

Quick Tip: en and oxalate are both bidentate. Find CN and oxidation number.

86. In the coordination complex, Potassium hexacyanoferrate(II), the central atom/ion acts as,

- (A) Bronsted-Lowry acid
- (B) Lewis base
- (C) Lewis acid
- (D) Bronsted-Lowry base

Correct Answer: (C) Lewis acid

Solution:**Step 1: Understanding the Concept:**

In a coordination complex, the central metal ion accepts electron pairs from ligands into its empty orbitals. A species that accepts an electron pair is a Lewis acid.

Step 2: Apply:

Potassium hexacyanoferrate(II) is $K_4[Fe(CN)_6]$. The central ion is Fe^{2+} , which has empty d , s and p orbitals. Each CN^- ligand donates a lone pair to Fe^{2+} .

Step 3: Conclusion:

So Fe^{2+} is the electron pair acceptor, a Lewis acid, option (C). The ligands are Lewis bases. Bronsted-Lowry acid or base would involve proton transfer, which does not happen here.

Final Answer:

The metal ion accepts electron pairs, so it is a Lewis acid.

C : Lewis acid

Quick Tip: The central metal accepts lone pairs from ligands.

87. Which of the following halogen derivative forms a poisonous phosgene, when come in contact with air and light?

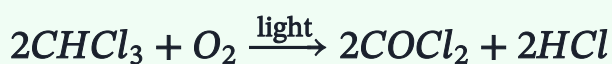
- (A) $CHCl_3$
- (B) PCl_3
- (C) C_6H_5Cl
- (D) CH_2Cl_2

Correct Answer: (A) $CHCl_3$

Solution:**Step 1: Understanding the Concept:**

Chloroform is slowly oxidised by air in the presence of light. It forms carbonyl chloride, which is called phosgene and is very poisonous.

Step 2: Reaction:



Step 3: Why chloroform is stored carefully:

To stop this, chloroform is kept in dark brown bottles filled to the brim, and about 1 percent ethanol is added. The ethanol turns phosgene into harmless ethyl carbonate.

Step 4: Check the others:

PCl_3 is not a halogen derivative of a hydrocarbon. Chlorobenzene and dichloromethane do not form phosgene this way. So the answer is CHCl_3 , option (A).

Final Answer:

Chloroform forms phosgene in air and light.

A : CHCl_3

Quick Tip: Chloroform is stored in dark bottles because it forms phosgene.

88. What is the general molecular formula of alkyl halides?

- (A) $\text{C}_n\text{H}_{2n+2}\text{X}$
- (B) $\text{C}_n\text{H}_{2n+1}\text{X}$
- (C) $\text{C}_n\text{H}_{2n}\text{X}$
- (D) $\text{C}_n\text{H}_{2n-2}\text{X}$

Correct Answer: (B) $\text{C}_n\text{H}_{2n+1}\text{X}$

Solution:

Step 1: Understanding the Concept:

An alkyl halide is an alkane in which one hydrogen is replaced by a halogen atom X. The alkane formula is C_nH_{2n+2} .

Step 2: Replace one H:

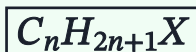
Removing one hydrogen gives an alkyl group C_nH_{2n+1} . Attaching X gives $C_nH_{2n+1}X$.

Step 3: Check:

Methyl chloride CH_3Cl : $n = 1$, so $2n + 1 = 3$ hydrogens. This matches. Option (A) has one too many hydrogens (that would be an alkane plus X). Options (C) and (D) belong to alkene and alkyne-type skeletons. So the answer is (B).

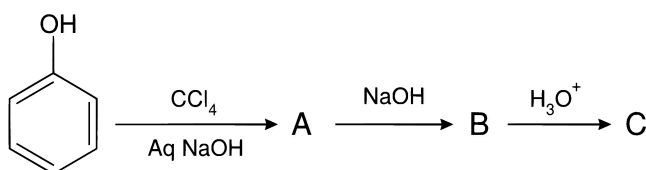
Final Answer:

Alkyl halides have the formula $C_nH_{2n+1}X$.

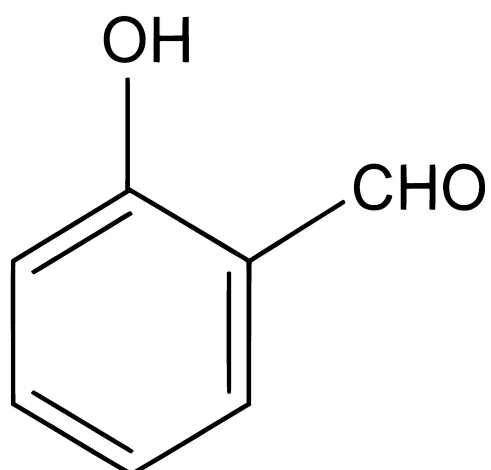


Quick Tip: Take an alkane C_nH_{2n+2} and replace one H with X.

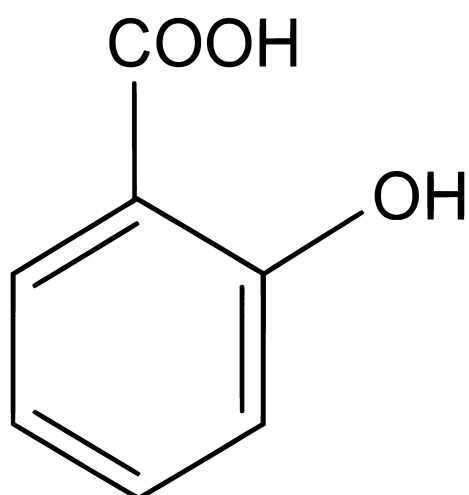
89. In the following sequence of the reaction, the final product "C" is-



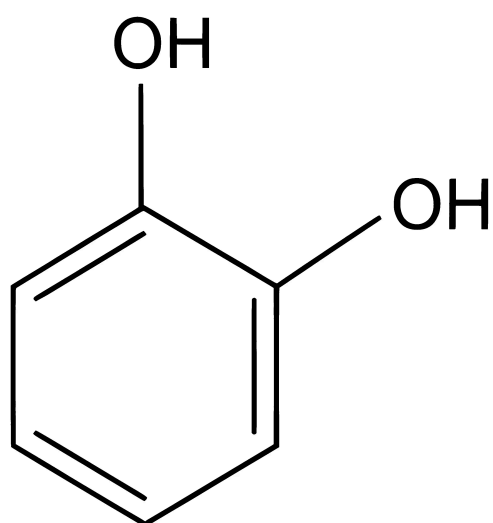
(A)



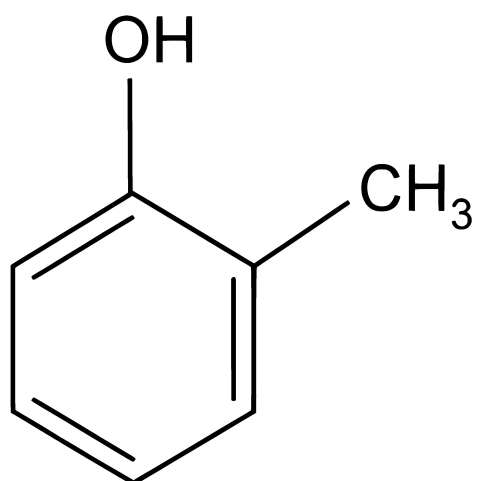
(B)



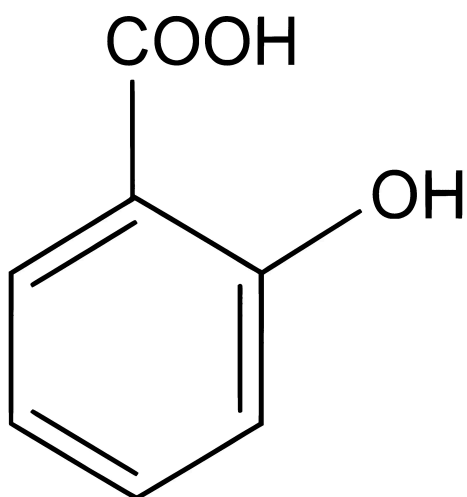
(C)



(D)



Correct Answer: (B)



Solution:

Step 1: Read the sequence:

Phenol is treated with CCl_4 and aqueous NaOH to give A. A is heated with NaOH to give B, and B is acidified with H_3O^+ to give C. This is a Reimer-Tiemann type reaction using CCl_4 .

Step 2: Formation of A:

In NaOH , phenol becomes the phenoxide ion, which is rich in electrons at the ortho position. It attacks the carbon of CCl_4 (through dichlorocarbene-like species) and puts a $-\text{CCl}_3$ group at the ortho position. A is o-trichloromethyl phenoxide.

Step 3: Formation of B:

On treatment with NaOH , the $-\text{CCl}_3$ group is hydrolysed. Each C-Cl bond is replaced and the group becomes $-\text{COO}^-\text{Na}^+$. B is sodium salicylate.

Step 4: Formation of C:

Acidification with H_3O^+ turns the carboxylate into $-\text{COOH}$ and the phenoxide into $-\text{OH}$. C is salicylic acid (2-hydroxybenzoic acid), with OH and COOH on adjacent ring carbons. This is option (B).

Step 5: Why the other options are wrong:

Option (A) has CHO instead of COOH . That is the product of $\text{CHCl}_3/\text{NaOH}$ (the normal Reimer-Tiemann reaction), not CCl_4 . Option (C) has two OH groups and option (D) has a methyl group, and neither can form from a CCl_3 group.

Final Answer:

The final product C is salicylic acid, option (B).

Salicylic acid (option B)

Quick Tip: CCl_4 in the Reimer-Tiemann reaction gives a COOH group ortho to OH after hydrolysis and acidification.

90. Which of the following is a dihydric alcohol?

- (A) Glycerol
- (B) Ethylene glycol
- (C) Sorbitol
- (D) Methanol

Correct Answer: (B) Ethylene glycol

Solution:

Step 1: Understanding the Concept:

Alcohols are named by the number of hydroxyl ($-\text{OH}$) groups: monohydric has one, dihydric has two, trihydric has three, and so on.

Step 2: Count OH groups:

Glycerol $\text{CH}_2\text{OH} - \text{CHOH} - \text{CH}_2\text{OH}$ has 3, so it is trihydric. Ethylene glycol $\text{CH}_2\text{OH} - \text{CH}_2\text{OH}$ has 2, so it is dihydric. Sorbitol has 6 OH groups. Methanol CH_3OH has 1.

Step 3: Conclusion:

Only ethylene glycol is dihydric, option (B).

Final Answer:

Ethylene glycol has two OH groups.

B : Ethylene glycol

Quick Tip: Count the OH groups in each compound.

91. What happens when phenol reacts with bromine water?

- (A) Brown coloured liquid is obtained
- (B) Colourless gas evolves
- (C) White precipitate formed
- (D) Pink colored solution is obtained

Correct Answer: (C) White precipitate formed

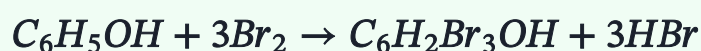
Solution:

Step 1: Understanding the Concept:

The OH group of phenol activates the benzene ring strongly towards electrophilic substitution at the ortho and para positions.

Step 2: Reaction:

With bromine water (no catalyst needed), all three positions are substituted at once. The product is 2,4,6-tribromophenol:



Step 3: Observation:

2,4,6-Tribromophenol is insoluble in water and appears as a white precipitate, and the brown colour of bromine water disappears. So the

answer is (C).

Step 4: Why the others are wrong:

No gas is released (HBr dissolves in water), and no pink colour forms. A brown liquid would mean no reaction.

Final Answer:

Phenol gives a white precipitate of 2,4,6-tribromophenol.

C : White precipitate

Quick Tip: Phenol plus bromine water gives 2,4,6-tribromophenol.

92. Which of the following statements is true regarding Cannizzaro reaction?

- (A) Some aldehydes are converted to the respective alcohols and the salt of the respective carboxylic acid.
- (B) Alcohol is converted into aldehyde
- (C) Primary amine is converted into isocyanide
- (D) Acid is converted into amine

Correct Answer: (A) Some aldehydes are converted to the respective alcohols and the salt of the respective carboxylic acid.

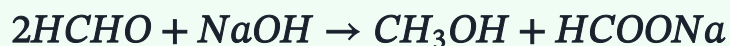
Solution:

Step 1: Understanding the Concept:

Cannizzaro reaction is the disproportionation of aldehydes that have no alpha hydrogen, when treated with concentrated alkali.

Step 2: What happens:

One molecule of the aldehyde is oxidised to a carboxylate salt, and another is reduced to an alcohol. For example

**Step 3: Check the options:**

(A) says some aldehydes give the alcohol and the salt of the acid, which is exactly this. (B) Alcohol to aldehyde is an oxidation, not Cannizzaro. (C) Amine to isocyanide is the carbylamine reaction. (D) Acid to amine is a Hofmann or Schmidt type change. So (A) is true.

Final Answer:

Cannizzaro gives an alcohol and a carboxylate salt.

A

Quick Tip: It is a disproportionation of aldehydes without alpha H.

93. Aldehydes are more reactive than ketone towards nucleophilic attack because of?

- (A) less steric hindrance in ketone
- (B) presence of two alkyl group on ketone decreases its electrophilicity
- (C) presence of one alkyl group on aldehyde decreases its electrophilicity
- (D) alkyl group have electron withdrawing effect.

Correct Answer: (B) presence of two alkyl group on ketone decreases its electrophilicity

Solution:

Step 1: Understanding the Concept:

Nucleophilic addition begins when a nucleophile attacks the partly positive carbonyl carbon. The more positive that carbon, and the less crowded it is, the faster the reaction.

Step 2: Aldehyde versus ketone:

An aldehyde has one alkyl group and one H on the carbonyl carbon. A ketone has two alkyl groups. Alkyl groups push electrons (+I effect) towards the carbon, which lowers its positive charge. Two alkyl groups push more than one, so a ketone carbon is less electrophilic.

Step 3: Check the options:

(A) wrongly says there is less hindrance in ketone, but ketones have more hindrance. (C) wrongly says one alkyl group on aldehyde decreases electrophilicity, while this is the smaller effect. (D) alkyl groups donate electrons, they are not withdrawing. So (B) is the reason.

Final Answer:

Two alkyl groups on a ketone reduce the electrophilicity of its carbonyl carbon.

B

Quick Tip: Alkyl groups donate electrons and crowd the carbonyl carbon.

94. Identify the product 'B' in the following reaction



- (A) $\text{CH}_3\text{-CH}_2\text{-CN}$
- (B) $\text{CH}_3\text{-CH}_2\text{-CH}_3$
- (C) $\text{CH}_3\text{-CH}_2\text{-NH}_2$
- (D) $\text{CH}_3\text{-NH-C}_2\text{H}_5$

Correct Answer: (C) $\text{CH}_3\text{-CH}_2\text{-NH}_2$

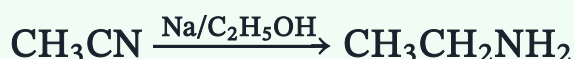
Solution:

Step 1: Step A:

Methyl iodide reacts with KCN by an $\text{S}_{\text{N}}2$ reaction. KCN is mostly ionic, so the carbon end of the cyanide ion attacks and the product is the nitrile CH_3CN (methyl cyanide). So A is CH_3CN .

Step 2: Step B:

Na with $\text{C}_2\text{H}_5\text{OH}$ is nascent hydrogen. It reduces a nitrile to a primary amine (Mendius reduction): $-\text{CN}$ becomes $-\text{CH}_2\text{NH}_2$.



Step 3: Why the other options are wrong:

Option (A), ethyl cyanide, has an extra carbon and is not formed from CH_3I plus KCN. Option (B), propane, would need loss of both the nitrogen and one carbon. Option (D), N-methylethanamine, is a secondary amine, and a nitrile gives a primary amine on this reduction.

Final Answer:

B is ethanamine (ethylamine), option (C).



Quick Tip: KCN replaces I with CN, and Na/C₂H₅OH reduces the nitrile to a primary amine with one extra CH₂.

95. Identify Hinsberg's reagent from following.

- (A) benzene sulfonic acid
- (B) benzene sulphonyl chloride
- (C) p-toluene sulphonyl chloride
- (D) chlorosulphuric acid

Correct Answer: (B) benzene sulphonyl chloride

Solution:**Step 1: Understanding the Concept:**

Hinsberg's test separates primary, secondary and tertiary amines. It uses a sulphonyl chloride that reacts with the N-H bonds.

Step 2: Reagent:

Hinsberg's reagent is benzene sulphonyl chloride, $\text{C}_6\text{H}_5\text{SO}_2\text{Cl}$. A primary amine forms a sulphonamide that dissolves in alkali. A secondary amine forms a sulphonamide that does not dissolve. A tertiary amine does not react.

Step 3: Check the options:

Benzene sulphonic acid has no leaving Cl. p-Toluene sulphonyl chloride is tosyl chloride, a different reagent. Chlorosulphuric acid is not used for this test. So the answer is (B).

Final Answer:

Hinsberg's reagent is benzene sulphonyl chloride.

B : Benzene sulphonyl chloride

Quick Tip: It is a sulphonyl chloride of benzene.

96. Which component of DNA carries genetic information?

- (A) Phosphate group
- (B) Sugar backbone
- (C) Sequence of nitrogenous bases
- (D) Hydrogen bonds

Correct Answer: (C) Sequence of nitrogenous bases

Solution:

Step 1: Understanding the Concept:

DNA has three parts: a phosphate group, a sugar (deoxyribose), and nitrogenous bases (A, T, G, C). The sugar and phosphate make the backbone.

Step 2: Which part carries information:

Genetic information is stored in the order in which the bases appear along the strand. Three bases form a codon that codes for an amino

acid.

Step 3: Check the options:

The phosphate group and sugar backbone are the same in every nucleotide, so they cannot differ to carry information. Hydrogen bonds hold the two strands together by pairing bases. So (C) is correct.

Final Answer:

The base sequence carries the genetic message.

C : Sequence of nitrogenous bases

Quick Tip: The backbone is identical everywhere. Look at what varies.

97. Which of the following is confirmed by the reaction of glucose with hydroxylamine?

- (A) Straight chain of six carbon atoms
- (B) Carbonyl group
- (C) Primary alcoholic group
- (D) Secondary alcoholic group

Correct Answer: (B) Carbonyl group

Solution:

Step 1: Understanding the Concept:

Hydroxylamine (NH_2OH) reacts with carbonyl compounds, aldehydes and ketones, to give oximes.

Step 2: Apply to glucose:

Glucose reacts with hydroxylamine to form glucose oxime. This shows that there is a carbonyl group, here an aldehyde group at C1, in glucose.

Step 3: Check the options:

Straight six-carbon chain is shown by reduction with HI. A primary alcohol and secondary alcohols are shown by acetylation or reaction with acid anhydrides. So (B) is correct.

Final Answer:

Reaction with hydroxylamine proves a carbonyl group.

B : Carbonyl group

Quick Tip: Hydroxylamine tests for a carbonyl group.

98. Which of the following polymers has ester linkage?

- (A) Orlon
- (B) Dacron
- (C) Teflon
- (D) Neoprene

Correct Answer: (B) Dacron

Solution:

Step 1: Understanding the Concept:

An ester linkage is —COO— . Polyesters are made by condensing a diacid with a diol.

Step 2: Check each polymer:

Orlon (polyacrylonitrile) is an addition polymer with nitrile groups. Dacron (terylene) is made from terephthalic acid and ethylene glycol, so it has ester links. Teflon is polytetrafluoroethylene with only C-F and C-C bonds. Neoprene is polychloroprene, a hydrocarbon rubber with Cl.

Step 3: Conclusion:

Only Dacron has ester linkages, option (B).

Final Answer:

Dacron is a polyester.

B : Dacron

Quick Tip: Polyester means ester linkage. Which one is it?

99. Which properties from following is improved by vulcanization of rubber?

- (A) Plasticity
- (B) Solubility in water
- (C) Tensile strength
- (D) Electrical conductivity

Correct Answer: (C) Tensile strength

Solution:

Step 1: Understanding the Concept:

Vulcanisation is heating natural rubber with sulphur. Sulphur forms cross-links between the long polymer chains through the double bonds.

Step 2: Effect:

The cross-links stop chains from sliding past each other. So rubber becomes harder, more elastic and stronger. Tensile strength rises, and plasticity (permanent change of shape) falls. It does not dissolve in water, and it is not conductive.

Step 3: Conclusion:

The improved property is tensile strength, option (C).

Final Answer:

Vulcanisation raises tensile strength and elasticity.

C : Tensile strength

Quick Tip: Sulphur cross-links tie the rubber chains together.

100. Which of the following cleansing agents contains cationic detergent?

- (A) Hair conditioner
- (B) Liquid detergent
- (C) Household detergent
- (D) Toothpaste

Correct Answer: (A) Hair conditioner

Solution:

Step 1: Understanding the Concept:

Cationic detergents have a positively charged head, usually a quaternary ammonium group, and a long hydrocarbon tail. They are mostly used as germicides and conditioners.

Step 2: Apply:

Hair conditioners contain cationic detergents such as cetyltrimethylammonium chloride. The positive head sticks to negatively charged hair and makes it smooth. Liquid detergent and household detergent use anionic ones (like sodium alkylbenzene sulphonate). Toothpaste uses mild anionic foaming agents such as sodium lauryl sulphate.

Step 3: Conclusion:

Option (A).

Final Answer:

Hair conditioner uses cationic detergent.

A : Hair conditioner

Quick Tip: Cationic detergents are used in hair conditioners.

Mathematics

Questions 101 to 150 | 2 marks each | 100 marks

101. If ω is a complex cube root of unity, then the value of the expression $2(1 + \frac{1}{\omega})(1 + \frac{1}{\omega^2}) + 3(2 + \frac{1}{\omega})(2 + \frac{1}{\omega^2}) + \dots + (n+1)(n + \frac{1}{\omega})(n + \frac{1}{\omega^2})$ is...

(A) $[\frac{n(n+1)}{2}]^2 + n$

(B) $[\frac{n(n+1)}{2}]^2 - n$

(C) $[\frac{n(n+1)}{2}]^2$

(D) $[\frac{n(n-1)}{2}]^2$

Correct Answer: (A) $[\frac{n(n+1)}{2}]^2 + n$

Solution:

Step 1: Understanding the Concept:

The k -th term of the series is $(k+1)(k + \frac{1}{\omega})(k + \frac{1}{\omega^2})$ for $k = 1$ to n .

For a cube root of unity, $\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$.

Step 2: Simplify the product:

Since $\frac{1}{\omega} = \omega^2$ and $\frac{1}{\omega^2} = \omega$, their sum is $\omega + \omega^2 = -1$ and their product is 1. So

$$(k + \frac{1}{\omega})(k + \frac{1}{\omega^2}) = k^2 + k(-1) + 1 = k^2 - k + 1$$

Step 3: Multiply by (k+1):

$$(k+1)(k^2 - k + 1) = k^3 + 1$$

Step 4: Sum the series:

$$\sum_{k=1}^n (k^3 + 1) = \left[\frac{n(n+1)}{2} \right]^2 + n$$

Step 5: Choose:

This matches option (A). Option (B) has $-n$ and option (C) leaves out n , which come from dropping the $+1$ or subtracting it.

Final Answer:

The sum equals $[n(n+1)/2]^2 + n$, option (A).

$$\boxed{\left[\frac{n(n+1)}{2} \right]^2 + n}$$

Quick Tip: Show $(k+1/w)(k+1/w^2) = k^2 - k + 1$, so the term is $k^3 + 1$.

102. The number of five-digit numbers formed using the digits 2, 3, 5, 7, 9 without repetition and which are greater than 24000 are

- (A) 120
- (B) 117
- (C) 114
- (D) 96

Correct Answer: (C) 114

Solution:

Step 1: Understanding the Concept:

We count five-digit numbers from the digits 2, 3, 5, 7, 9 (each used once) that exceed 24000. There are $5! = 120$ numbers in all.

Step 2: Case 1: first digit is 3, 5, 7 or 9:

Any of these four digits first makes the number at least 30000, which is above 24000. The remaining 4 digits can be arranged in $4! = 24$ ways. This gives $4 \times 24 = 96$ numbers.

Step 3: Case 2: first digit is 2:

The other digits are 3, 5, 7, 9. If the second digit is 3, the number is 23xxx, which is less than 24000. This happens in $3! = 6$ ways. If the second digit is 5, 7 or 9, the number is at least 25xxx, which is above 24000. This gives $3 \times 3! = 18$ numbers.

Step 4: Total:

$$96 + 18 = 114$$

Step 5: Choose:

Option (C). The value 120 counts every arrangement without applying the condition, and 117 would result from excluding only 3 numbers instead of 6.

Final Answer:

There are 114 such numbers, option (C).

114

Quick Tip: Count all 120 and subtract numbers starting with 23.



103. In $\triangle ABC$, with the usual notations, $\angle C = 90^\circ$, then $\sin(A - B)$ is equal to....

- (A) $\frac{a^2+b^2}{a^2-b^2}$
- (B) $\frac{a^2+c^2}{a^2-c^2}$
- (C) $\frac{b^2+c^2}{b^2-c^2}$
- (D) $\frac{a^2-b^2}{a^2+b^2}$

Correct Answer: (D) $\frac{a^2-b^2}{a^2+b^2}$

Solution:

Step 1: Understanding the Concept:

In a right triangle with $\angle C = 90^\circ$, the hypotenuse is c , so $a^2 + b^2 = c^2$, $\sin A = a/c$, $\cos A = b/c$, $\sin B = b/c$ and $\cos B = a/c$.

Step 2: Expand:

$$\sin(A - B) = \sin A \cos B - \cos A \sin B$$

Step 3: Substitute:

$$\sin(A - B) = \frac{a}{c} \cdot \frac{a}{c} - \frac{b}{c} \cdot \frac{b}{c} = \frac{a^2 - b^2}{c^2}$$

Step 4: Replace c squared:

Since $c^2 = a^2 + b^2$, we get $\sin(A - B) = \frac{a^2 - b^2}{a^2 + b^2}$, option (D).

Final Answer:

$$\sin(A-B) = (a^2 - b^2)/(a^2 + b^2).$$

$$\boxed{\frac{a^2 - b^2}{a^2 + b^2}}$$

Quick Tip: Use $\sin A = a/c$, $\cos A = b/c$ and $c^2 = a^2 + b^2$.

104. If $P = \tan 20^\circ$, then the value of $\frac{\tan 160^\circ - \tan 110^\circ}{1 + \tan 160^\circ \tan 110^\circ}$ in terms of P , is...

- (A) $\frac{1+P^2}{2P^2}$
- (B) $\frac{1+P^2}{2P}$
- (C) $\frac{1-P^2}{2P^2}$
- (D) $\frac{1-P^2}{2P}$

Correct Answer: (D) $\frac{1-P^2}{2P}$

Solution:

Step 1: Understanding the Concept:

The expression has the form $\frac{\tan x - \tan y}{1 + \tan x \tan y} = \tan(x - y)$.

Step 2: Apply:

Here $x = 160^\circ$ and $y = 110^\circ$. So the value is $\tan(160^\circ - 110^\circ) = \tan 50^\circ$.

Step 3: Write in terms of $\tan 20^\circ$:

Use $\tan 50^\circ = \tan(90^\circ - 40^\circ) = \cot 40^\circ = \frac{1}{\tan 40^\circ}$. With $\tan 40^\circ = \frac{2P}{1 - P^2}$ (double angle for $P = \tan 20^\circ$),

$$\tan 50^\circ = \frac{1 - P^2}{2P}$$

Step 4: Choose:

This is option (D). Option (B) has $1 + P^2$ and would be wrong because the double angle formula has $1 - P^2$ in the denominator of $\tan 40^\circ$.

Final Answer:

The value is $\tan 50^\circ = (1 - P^2)/(2P)$.

$$\boxed{\frac{1 - P^2}{2P}}$$

Quick Tip: Use $\tan(x - y)$ formula, or reduce each tangent to P .

105. If $\cos(p\theta) + \cos(q\theta) = 0$ and $p \neq q$, then the general value of θ (where n is any integer) is...

- (A) $\frac{(3n+1)\pi}{p-q}$
- (B) $\frac{(2n+1)\pi}{p \pm q}$
- (C) $\frac{(n \pm 1)\pi}{p \pm q}$
- (D) $\frac{(n+2)\pi}{p+q}$

Correct Answer: (B) $\frac{(2n+1)\pi}{p \pm q}$

Solution:

Step 1: Understanding the Concept:

Given $\cos p\theta + \cos q\theta = 0$, so $\cos p\theta = -\cos q\theta = \cos(\pi - q\theta)$. If $\cos \alpha = \cos \beta$ then $\alpha = 2n\pi \pm \beta$.

Step 2: Apply:

$$p\theta = 2n\pi \pm (\pi - q\theta)$$

Step 3: Take the plus sign:

$$p\theta = 2n\pi + \pi - q\theta, \text{ so } (p + q)\theta = (2n + 1)\pi \text{ and } \theta = \frac{(2n + 1)\pi}{p + q}.$$

Step 4: Take the minus sign:

$p\theta = 2n\pi - \pi + q\theta$, so $(p - q)\theta = (2n - 1)\pi$ and $\theta = \frac{(2n - 1)\pi}{p - q}$. As n runs over all integers, $2n - 1$ covers all odd numbers, the same as $2n + 1$.

Step 5: Combine:

$$\theta = \frac{(2n + 1)\pi}{p \pm q}, \text{ option (B).}$$

Final Answer:

The general solution is $(2n+1)\pi/(p \pm q)$.

$$\theta = \frac{(2n + 1)\pi}{p \pm q}$$

Quick Tip: Use the sum-to-product formula and set each cosine to zero.

106. If a triangle ABC has vertices $A(1, -6)$, $B(2, -3)$ and $C(3, -2)$, then the coordinates of its orthocenter are....

- (A) $(-6, 1)$
- (B) $(6, 1)$
- (C) $(-2, 3)$
- (D) $(3, 2)$

Correct Answer: (A) $(-6, 1)$

Solution:

Step 1: Understanding the Concept:

The orthocenter is the point where the three altitudes of a triangle meet. An altitude from a vertex is perpendicular to the opposite side. Two altitudes are enough to find the point.

Step 2: Altitude from A:

Slope of BC is $\frac{-2 - (-3)}{3 - 2} = 1$. So the altitude from A has slope -1 and passes through $A(1, -6)$:

$$y + 6 = -(x - 1) \Rightarrow y = -x - 5$$

Step 3: Altitude from B:

Slope of AC is $\frac{-2 + 6}{3 - 1} = 2$. So the altitude from B has slope $-\frac{1}{2}$ and passes through $B(2, -3)$:

$$y + 3 = -\frac{1}{2}(x - 2) \Rightarrow y = -\frac{x}{2} - 2$$

Step 4: Intersect:

Equate: $-x - 5 = -\frac{x}{2} - 2$, so $-\frac{x}{2} = 3$ and $x = -6$. Then $y = -(-6) - 5 = 1$.

Step 5: Choose:

The orthocenter is $(-6, 1)$, option (A). Option (B) swaps the sign of x and option (C) and (D) do not satisfy the altitude from A .

Final Answer:

The orthocenter is $(-6, 1)$.

$(-6, 1)$

Quick Tip: Find two altitudes using perpendicular slopes and intersect them.

107. If θ is the acute angle between the lines represented by the equation $x^2 - 3xy + 2y^2 = 0$, then $\frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta} =$

- (A) $-\frac{1}{2}$
- (B) $\frac{1}{2}$
- (C) -3
- (D) 3

Correct Answer: (C) -3

Solution:

Step 1: Understanding the Concept:

A homogeneous equation $ax^2 + 2hxy + by^2 = 0$ represents two lines through the origin. The acute angle between them satisfies $\tan \theta$

$$= \left| \frac{2\sqrt{h^2 - ab}}{a + b} \right|.$$

Step 2: Find the lines:

$x^2 - 3xy + 2y^2 = (x - y)(x - 2y) = 0$, so the lines are $y = x$ and $y = \frac{x}{2}$. Their slopes are 1 and $\frac{1}{2}$.

Step 3: Find tan theta:

$$\tan \theta = \left| \frac{1 - \frac{1}{2}}{1 + 1 \cdot \frac{1}{2}} \right| = \frac{1/2}{3/2} = \frac{1}{3}$$

Step 4: Evaluate the expression:

Divide numerator and denominator by $\cos \theta$:

$$\frac{3 \sin \theta + 2 \cos \theta}{3 \sin \theta - 2 \cos \theta} = \frac{3 \tan \theta + 2}{3 \tan \theta - 2} = \frac{1 + 2}{1 - 2} = -3$$

Step 5: Choose:

Option (C).

Final Answer:

$\tan \theta = 1/3$, so the expression equals -3.

$$\boxed{-3}$$

Quick Tip: Factor the pair of lines, find $\tan(\theta)$, then divide by $\cos(\theta)$.



108. The triangle formed by the lines $2x^2 - 3xy - 2y^2 = 0$ and $3x - y = 7$ is...

- (A) Right angled but not isosceles
- (B) isosceles with base angle 30°
- (C) Right angled with one angle 60°
- (D) Right angled isosceles

Correct Answer: (D) Right angled isosceles

Solution:

Step 1: Understanding the Concept:

We find the three vertices of the triangle, then check the side lengths and the angles.

Step 2: Split the pair of lines:

$2x^2 - 3xy - 2y^2 = (2x + y)(x - 2y) = 0$. So the lines are $y = -2x$ and $y = \frac{x}{2}$. The product of the slopes is $-2 \times \frac{1}{2} = -1$, so these two lines are perpendicular. The angle at the origin is 90° .

Step 3: Vertices:

With $3x - y = 7$, i.e. $y = 3x - 7$:

With $y = -2x$: $3x - 7 = -2x$, so $x = \frac{7}{5}$, $y = -\frac{14}{5}$. Point $P(\frac{7}{5}, -\frac{14}{5})$.

With $y = \frac{x}{2}$: $3x - 7 = \frac{x}{2}$, so $x = \frac{14}{5}$, $y = \frac{7}{5}$. Point $Q(\frac{14}{5}, \frac{7}{5})$.

Step 4: Lengths:

$$OP^2 = \frac{49}{25} + \frac{196}{25} = \frac{245}{25}, \text{ and } OQ^2 = \frac{196}{25} + \frac{49}{25} = \frac{245}{25}. \text{ So } OP = OQ.$$

Step 5: Conclusion:

The triangle has a right angle at O and two equal legs, so it is right angled isosceles, option (D).

Final Answer:

The triangle is right angled and isosceles.

Right angled isosceles (D)

Quick Tip: Check the angle between the two lines and compare the two legs.

109. The centre and radius of the circle $(a + 1)x^2 + 3y^2 - 6x + 9y + a + 4 = 0$ are respectively ...

- (A) $(-1, \frac{3}{2}), \frac{\sqrt{5}}{2}$
- (B) $(-1, -\frac{3}{2}), \frac{\sqrt{5}}{2}$
- (C) $(1, -\frac{3}{2}), \frac{\sqrt{5}}{2}$
- (D) $(1, \frac{3}{2}), \frac{\sqrt{5}}{2}$

Correct Answer: (C) $(1, -\frac{3}{2}), \frac{\sqrt{5}}{2}$

Solution:

Step 1: Understanding the Concept:

The general second degree equation represents a circle only if the coefficients of x^2 and y^2 are equal and there is no xy term.

Step 2: Find a:

Coefficients: $a + 1 = 3$, so $a = 2$. The equation is

$$3x^2 + 3y^2 - 6x + 9y + 6 = 0$$

Step 3: Divide by 3:

$$x^2 + y^2 - 2x + 3y + 2 = 0$$

Step 4: Centre and radius:

Compare with $x^2 + y^2 + 2gx + 2fy + c = 0$: $g = -1$, $f = \frac{3}{2}$, $c = 2$.

Centre = $(-g, -f) = (1, -\frac{3}{2})$.

$$r = \sqrt{g^2 + f^2 - c} = \sqrt{1 + \frac{9}{4} - 2} = \sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{2}$$

Step 5: Choose:

Option (C). Options (A), (B) and (D) have a wrong sign for one coordinate of the centre.

Final Answer:

Centre $(1, -3/2)$ and radius $\sqrt{5}/2$.

$$\left(1, -\frac{3}{2}\right), \frac{\sqrt{5}}{2}$$

Quick Tip: A circle needs equal x^2 and y^2 coefficients, so $a + 1 = 3$.

110. If the line $4x + 3y = 7$ touches the hyperbola $x^2 - y^2 = 7$, then the sum of the co-ordinates of the point of contact is...

- (A) 4
- (B) 7
- (C) 1
- (D) 0

Correct Answer: (C) 1

Solution:

Step 1: Understanding the Concept:

The tangent to $x^2 - y^2 = 7$ at the point (x_1, y_1) on it is $xx_1 - yy_1 = 7$. If the given line touches the hyperbola, it must be this same line.

Step 2: Compare:

Write $4x + 3y = 7$. Match with $x_1x - y_1y = 7$:

$$x_1 = 4, \quad -y_1 = 3 \Rightarrow y_1 = -3$$

Step 3: Check the point lies on the hyperbola:

$$x_1^2 - y_1^2 = 16 - 9 = 7. \text{ Yes.}$$

Step 4: Sum of coordinates:

$4 + (-3) = 1$, so the answer is (C).

Final Answer:

The point of contact is $(4, -3)$, sum 1.

Quick Tip: Compare the line with the tangent form $x x_1 - y y_1 = 7$.

111. If $\lim_{x \rightarrow 0} \frac{(4^x - 1)^3}{\tan(\frac{x}{4}) \log(1 + \frac{x^2}{3})} = 96(\log a)^b$, then $(a + b) =$

- (A) 5
- (B) 7
- (C) 3
- (D) 4

Correct Answer: (A) 5

Solution:

Step 1: Understanding the Concept:

Near $x = 0$ we use standard limits: $\frac{a^x - 1}{x} \rightarrow \ln a$, $\frac{\tan x}{x} \rightarrow 1$ and $\frac{\ln(1 + x)}{x} \rightarrow 1$.

Step 2: Rewrite:

$$\frac{(4^x - 1)^3}{\tan \frac{x}{4} \ln \left(1 + \frac{x^2}{3}\right)} = \frac{\left(\frac{4^x - 1}{x}\right)^3 x^3}{\frac{\tan(x/4)}{x/4} \cdot \frac{x}{4} \cdot \frac{\ln(1+x^2/3)}{x^2/3} \cdot \frac{x^2}{3}}$$

Step 3: Take the limit:

The numerator tends to $(\ln 4)^3 x^3$. The denominator tends to $1 \cdot \frac{x}{4} \cdot 1 \cdot \frac{x^2}{3} = \frac{x^3}{12}$. So

$$L = 12(\ln 4)^3 = 12(2 \ln 2)^3 = 96(\ln 2)^3$$

Step 4: Match:

Compare with $96(\log a)^b$: $a = 2$, $b = 3$. So $a + b = 5$, option (A).

Final Answer:

The limit is $96 (\ln 2)^3$, so $a = 2$, $b = 3$ and $a + b = 5$.

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Quick Tip: Use small x approximations for each factor.

112. The dual of the converse of the inverse of the logical statement $p \rightarrow (q \rightarrow r)$ is equivalent to...

- (A) $\sim [p \vee (r \rightarrow q)]$
- (B) $p \vee (r \rightarrow q)$
- (C) $\sim [p \vee (q \rightarrow r)]$
- (D) $p \vee (q \rightarrow r)$

Correct Answer: (A) $\sim [p \vee (r \rightarrow q)]$

Solution:

Step 1: Understanding the Concept:

For $p \rightarrow q$: the converse is $q \rightarrow p$, the inverse is $\sim p \rightarrow \sim q$. The dual of a statement made only with $\wedge, \vee, \sim$ is obtained by swapping $\wedge$ and $\vee$ (and T with F).

Step 2: Inverse:

The statement is $p \rightarrow (q \rightarrow r)$. Its inverse is $\sim p \rightarrow \sim (q \rightarrow r)$.

Step 3: Converse of the inverse:

$$\sim (q \rightarrow r) \rightarrow \sim p.$$

Step 4: Rewrite without implication:

$A \rightarrow B \equiv \sim A \vee B$. So this becomes $(q \rightarrow r) \vee \sim p$. Writing $q \rightarrow r \equiv \sim q \vee r$, we get $\sim q \vee r \vee \sim p$.

Step 5: Take the dual:

Swap $\vee$ with $\wedge$: $\sim q \wedge r \wedge \sim p$.

Step 6: Match with options:

Option (A): $\sim [p \vee (r \rightarrow q)] = \sim p \wedge \sim (r \rightarrow q) = \sim p \wedge (r \wedge \sim q)$. This equals $\sim p \wedge r \wedge \sim q$. So (A) is correct. Options (B) and (D) are not negated forms, and (C) has $q \rightarrow r$ instead of $r \rightarrow q$.

Final Answer:

The dual is not p and not q and r, which equals option (A).

$$\boxed{\sim [p \vee (r \rightarrow q)]}$$

Quick Tip: Do inverse, converse, then dual and rewrite implication as OR.

113. The statements p, q and r have truth values True, False and False respectively. The truth values of a logical statement $[\sim (p \wedge \sim q) \vee (q \vee \sim r)]$ and its dual are, respectively.....

- (A) True, True
- (B) True, False
- (C) False, True
- (D) False, False

Correct Answer: (B) True, False

Solution:

Step 1: Understanding the Concept:

The dual of a statement is formed by swapping $\wedge$ with $\vee$ (and T with F), keeping $\sim$ as is. We evaluate the statement and its dual with the given truth values.

Step 2: Given values:

$p = T, q = F, r = F$. So $\sim q = T$ and $\sim r = T$.

Step 3: Evaluate the statement:

$$S = \sim (p \wedge \sim q) \vee (q \vee \sim r).$$

$$p \wedge \sim q = T \wedge T = T, \text{ so } \sim (\dots) = F.$$

$$q \vee \sim r = F \vee T = T.$$

$$\text{So } S = F \vee T = T.$$

Step 4: Dual:

$$\text{Swap the connectives: } S^d = \sim (p \vee \sim q) \wedge (q \wedge \sim r).$$

$$p \vee \sim q = T \vee T = T, \text{ so } \sim (\dots) = F.$$

$$q \wedge \sim r = F \wedge T = F.$$

$$\text{So } S^d = F \wedge F = F.$$

Step 5: Choose:

True and False, option (B).

Final Answer:

The statement is True and its dual is False.

True, False (B)

Quick Tip: Evaluate each connective step by step for the dual too.

114. In $\triangle ABC$ with usual notations, if $1 + \tan\left(\frac{A}{2}\right)\tan\left(\frac{B}{2}\right) = \frac{k}{s}$ (where s is the semi-perimeter), then the value of k is...

- (A) 2
- (B) $a + b - c$
- (C) $a + b$
- (D) $s - c$

Correct Answer: (C) $a + b$

Solution:**Step 1: Understanding the Concept:**

In a triangle, $\tan \frac{A}{2} \tan \frac{B}{2} = \frac{s - c}{s}$ (half-angle formulae).

Step 2: Add 1:

$$1 + \tan \frac{A}{2} \tan \frac{B}{2} = 1 + \frac{s - c}{s} = \frac{2s - c}{s}$$

Step 3: Use the perimeter:

$2s = a + b + c$, so $2s - c = a + b$. Hence

$$1 + \tan \frac{A}{2} \tan \frac{B}{2} = \frac{a+b}{s}$$

Step 4: Compare with k/s:

So $k = a + b$, option (C). Option (B) $a + b - c$ is $2(s - c)$ and option (D) $s - c$ would give $\frac{s-c}{s}$ without the 1.

Final Answer:

k equals a + b.

$$k = a + b$$

Quick Tip: Use $\tan(A/2)\tan(B/2) = (s-c)/s$.

115. Let $A = \begin{bmatrix} 2k-1 & 1 & 1 \\ 0 & 2k-1 & 1 \\ 0 & 0 & 2k-1 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 2k-1 & 1 \\ 1-2k & 0 & k \\ -1 & -k & 0 \end{bmatrix}$

where k is a real number. If $\det(\text{adj}A) + \det(\text{adj}B) = 11^6$, then the value of $k - 5$ is equal to...

- (A) 1
- (B) 2
- (C) 4
- (D) 6

Correct Answer: (A) 1

Solution:

Step 1: Use the adjoint property:

For a square matrix of order n , $\det(\text{adj } M) = (\det M)^{n-1}$. Both matrices are 3 by 3, so $\det(\text{adj } M) = (\det M)^2$.

Step 2: Matrix A:

A is upper triangular, so its determinant is the product of the diagonal entries: $\det A = (2k - 1)^3$. So $\det(\text{adj } A) = (2k - 1)^6$.

Step 3: Matrix B:

Check B^T : the (1,2) entry is $2k - 1$ and the (2,1) entry is $1 - 2k$; the (1,3) entry is 1 and the (3,1) entry is -1 ; the (2,3) entry is k and the (3,2) entry is $-k$; the diagonal is 0. So $B^T = -B$, a skew-symmetric matrix of odd order, and its determinant is 0. So $\det(\text{adj } B) = 0$.

Step 4: Solve:

$$(2k - 1)^6 + 0 = 11^6 \Rightarrow 2k - 1 = \pm 11 \Rightarrow k = 6 \text{ or } k = -5$$

Then $k - 5 = 1$ or -10 . Only 1 is in the options.

Step 5: Check the options:

The values 2, 4 and 6 for $k - 5$ would need $k = 7, 9, 11$, which give $2k - 1 = 13, 17, 21$, none equal to ± 11 .

Final Answer:

$k - 5 = 1$, option (A).

1

Quick Tip: Use $\det(\text{adj } M) = (\det M)^2$ for a 3 by 3 matrix; B is skew-symmetric of odd order so its determinant is 0.

116. If $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$, then the matrix A^{-3} when $\theta = \frac{\pi}{6}$ is equal to...

(A) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

(B) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

(C) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Correct Answer: (A) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

Solution:

Step 1: Understanding the Concept:

The matrix $A = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix}$ is a rotation matrix. Multiplying such matrices adds the angles, so A^n is the rotation by $n\theta$, and A^{-1} is the rotation by $-\theta$.

Step 2: Apply:

A^{-3} is the rotation by $-3\theta = -3 \times \frac{\pi}{6} = -\frac{\pi}{2}$.

Step 3: Write the matrix:

$$A^{-3} = \begin{pmatrix} \cos(-\frac{\pi}{2}) & -\sin(-\frac{\pi}{2}) \\ \sin(-\frac{\pi}{2}) & \cos(-\frac{\pi}{2}) \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Step 4: Choose:

Option (A). Option (C) is the rotation by $+\frac{\pi}{2}$, which would be A^3 .

Final Answer:

A^{-3} is a rotation by -90 degrees.

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

Quick Tip: A is a rotation matrix, so A^n rotates by n theta.



117. If $\tan^{-1}\left[\frac{\sqrt{5-2\sqrt{6}}}{1+\sqrt{6}}\right] = \frac{\pi}{3} - \tan^{-1}(k)$, then $\sec^{-1}(k) = \dots$

- (A) $\frac{\pi}{6}$
- (B) $\frac{\pi}{4}$
- (C) $\frac{\pi}{3}$
- (D) $\frac{\pi}{2}$

Correct Answer: (B) $\frac{\pi}{4}$

Solution:**Step 1: Understanding the Concept:**

We use $\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}$ to split the left side.

Step 2: Simplify the root:

$$5 - 2\sqrt{6} = (\sqrt{3} - \sqrt{2})^2, \text{ so } \sqrt{5 - 2\sqrt{6}} = \sqrt{3} - \sqrt{2}.$$

Step 3: Rewrite the denominator:

$1 + \sqrt{6} = 1 + \sqrt{3} \cdot \sqrt{2}$. So

$$\tan^{-1} \frac{\sqrt{3} - \sqrt{2}}{1 + \sqrt{3}\sqrt{2}} = \tan^{-1} \sqrt{3} - \tan^{-1} \sqrt{2} = \frac{\pi}{3} - \tan^{-1} \sqrt{2}$$

Step 4: Compare:

So $k = \sqrt{2}$ and $\sec^{-1} \sqrt{2} = \frac{\pi}{4}$, because $\sec \frac{\pi}{4} = \sqrt{2}$.

Step 5: Choose:

Option (B).

Final Answer:

$k = \sqrt{2}$, so $\sec$ inverse of k is $\pi/4$.

$$\boxed{\frac{\pi}{4}}$$

Quick Tip: Write $5 - 2\sqrt{6}$ as $(\sqrt{3} - \sqrt{2})^2$ first.

118. If $f(x - y) + f(x + y) = 2f(x)f(y)$ for all $x, y \in R$, then $f(x)$ is

- (A) an odd function
- (B) an even function
- (C) neither even nor odd function
- (D) a periodic function

Correct Answer: (B) an even function

Solution:

Step 1: Understanding the Concept:

To decide whether f is even or odd, we compare $f(-x)$ with $f(x)$.

Step 2: Step A: find $f(0)$:

Put $x = y = 0$: $2f(0) = 2f(0)^2$, so $f(0) = 0$ or $f(0) = 1$.

Step 3: Step B: use $x = 0$:

$f(-y) + f(y) = 2f(0)f(y)$. If $f(0) = 1$ this gives $f(-y) = f(y)$, so f is even.

Step 4: Step C: the case $f(0) = 0$:

Then $f(-y) = -f(y)$ so f is odd. Interchange x and y in the given relation: $f(y - x) + f(x + y) = 2f(y)f(x)$. Comparing with the original gives $f(x - y) = f(y - x)$, so f is even. A function that is both even and odd is $f \equiv 0$, which is also even.

Step 5: Choose:

In every case f is even, option (B). A cosine function satisfies the relation and is periodic too, but periodicity is not forced (for example $f = \cosh x$ also works). Odd is not forced either.

Final Answer:

f is an even function in every case.

Even function (B)

Quick Tip: Put $x = 0$, then swap x and y .

119. If the function f is continuous at $x = 1$, where $f(x) = \frac{1+\cos(\pi x)}{\pi(1-x)^2}$, for $x \neq 1$, then the value of $f(1)$ is....

- (A) $\frac{\pi}{2}$
- (B) $\frac{\pi}{4}$
- (C) $\frac{\pi}{6}$
- (D) $\frac{\pi}{9}$

Correct Answer: (A) $\frac{\pi}{2}$

Solution:

Step 1: Understanding the Concept:

For f to be continuous at $x = 1$, $f(1)$ must equal $\lim_{x \rightarrow 1} f(x)$.

Step 2: Substitute:

Let $t = 1 - x$, so $t \rightarrow 0$ and $x = 1 - t$. Then $\cos \pi x = \cos(\pi - \pi t)$
 $= -\cos \pi t$.

$$f = \frac{1 - \cos \pi t}{\pi t^2}$$

Step 3: Use the identity:

$1 - \cos \pi t = 2 \sin^2 \frac{\pi t}{2}$. So

$$f = \frac{2 \sin^2(\pi t/2)}{\pi t^2} = \frac{2}{\pi} \cdot \frac{\pi^2}{4} \left(\frac{\sin(\pi t/2)}{\pi t/2} \right)^2$$

Step 4: Take the limit:

As $t \rightarrow 0$ the bracket tends to 1. So the limit is $\frac{2}{\pi} \cdot \frac{\pi^2}{4} = \frac{\pi}{2}$.

Step 5: Choose:

$$f(1) = \frac{\pi}{2}, \text{ option (A).}$$

Final Answer:

The limit and $f(1)$ equal $\pi/2$.

$$\boxed{\frac{\pi}{2}}$$

Quick Tip: Put $t = 1 - x$ and use $1 - \cos u = 2 \sin^2(u/2)$.



120. If $y = \log x^x$, then the value of $(\frac{dy}{dx})^2$ at $x = e^2$ is

- (A) 4
- (B) e
- (C) e^2
- (D) 9

Correct Answer: (D) 9

Solution:

Step 1: Understanding the Concept:

The logarithm here is natural. By the power rule of logs, $\log x^x = x \log x$.

Step 2: Differentiate:

$$\frac{dy}{dx} = \log x + x \cdot \frac{1}{x} = \log x + 1$$

Step 3: Evaluate at $x = e^2$:

$$\log e^2 = 2, \text{ so } \frac{dy}{dx} = 2 + 1 = 3.$$

Step 4: Square:

$$\left(\frac{dy}{dx}\right)^2 = 9, \text{ option (D).}$$

Final Answer:

$\frac{dy}{dx} = \log x + 1$ equals 3 at $x = e^2$, so its square is 9.

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Quick Tip: Rewrite $\log(x^x)$ as $x \log x$ and differentiate.



121. If $y = \cos^{-1}(\sin x)$, where $\frac{\pi}{2} < x < \pi$, then $\frac{dy}{dx} = \dots$

- (A) -1
- (B) 0
- (C) 1
- (D) 2

Correct Answer: (C) 1

Solution:

Step 1: Understanding the Concept:

$\cos^{-1}(\cos u) = |u|$ when $-\pi \leq u \leq \pi$, and equals u only for $0 \leq u \leq \pi$.

So we first write $\sin x$ as a cosine.

Step 2: Rewrite:

$\sin x = \cos\left(\frac{\pi}{2} - x\right)$. For $\frac{\pi}{2} < x < \pi$, the angle $u = \frac{\pi}{2} - x$ lies in $\left(-\frac{\pi}{2}, 0\right)$, which is negative.

Step 3: Simplify y:

$$y = \cos^{-1} \cos u = -u = x - \frac{\pi}{2}.$$

Step 4: Differentiate:

$\frac{dy}{dx} = 1$. Option (C). The value -1 would come from wrongly using u itself.

Final Answer:

On this interval $y = x - \pi/2$, so $dy/dx = 1$.

1

Quick Tip: Check the sign of $\cos x$ in the second quadrant.

122. If $y = \left(\frac{ax+b}{cx+d}\right)$, then $2 \frac{dy}{dx} \cdot \frac{d^3y}{dx^3}$ is equal to

- (A) $\left(\frac{d^2y}{dx^2}\right)^2$
- (B) $3 \frac{d^2y}{dx^2}$
- (C) $3\left(\frac{d^2y}{dx^2}\right)^2$
- (D) $3 \frac{d^2x}{dy^2}$

Correct Answer: (C) $3\left(\frac{d^2y}{dx^2}\right)^2$

Solution:

Step 1: Understanding the Concept:

A Mobius type function $y = \frac{ax + b}{cx + d}$ can be written as a constant plus $K(cx + d)^{-1}$.

Step 2: Rewrite:

$y = \frac{a}{c} + \frac{bc - ad}{c} \cdot \frac{1}{cx + d}$. Put $K = \frac{bc - ad}{c}$. Then $y = \frac{a}{c} + K(cx + d)^{-1}$.

Step 3: Derivatives:

$$y' = -Kc(cx + d)^{-2}$$

$$y'' = 2Kc^2(cx + d)^{-3}$$

$$y''' = -6Kc^3(cx + d)^{-4}$$

Step 4: Form the products:

$$2y'y''' = 2 \cdot (-Kc)(-6Kc^3)(cx + d)^{-6} = 12K^2c^4(cx + d)^{-6}.$$

$$(y'')^2 = 4K^2c^4(cx + d)^{-6}.$$

Step 5: Compare:

$2y'y''' = 3(y'')^2$. Option (C). Option (A) is smaller by a factor 3, and (B) mixes in a linear term.

Final Answer:

$$2y'y''' = 3(y'')^2.$$

$$\boxed{3 \left(\frac{d^2y}{dx^2} \right)^2}$$

Quick Tip: Write y as a constant plus $K/(cx+d)$ and differentiate three times.

123. Let $f(x) = 3x^3 + 4e^{\frac{x}{2}}$ and $g(x) = f^{-1}(x)$. Then the value of **Missing open brace for superscript** is equal to...

- (A) 4
- (B) 2
- (C) 1
- (D) $\frac{1}{2}$

Correct Answer: (D) $\frac{1}{2}$

Solution:

Step 1: Understanding the Concept:

If $g = f^{-1}$, then $g'(x) = \frac{1}{f'(g(x))}$.

Step 2: Find $g(4)$:

We need a with $f(a) = 4$. Try $a = 0$: $f(0) = 0 + 4e^0 = 4$. So $g(4) = 0$.

Step 3: Find $f'(0)$:

$f'(x) = 9x^2 + 2e^{x/2}$. So $f'(0) = 0 + 2 = 2$.

Step 4: Apply the rule:

$$g'(4) = \frac{1}{f'(0)} = \frac{1}{2}$$

Step 5: Choose:

Option (D).

Final Answer:

$$g'(4) = 1/f'(0) = 1/2.$$

$$\boxed{\frac{1}{2}}$$

Quick Tip: $g'(4) = 1 / f'(g(4))$, and $f(0) = 4$.

124. If the function $f(x) = ax^3 - bx^2 - 8x - 4$ satisfies Roll's theorem in $[1, 3]$, if **Missing open brace for superscript** then $a - b$ is equal to...

- (A) 2
- (B) -2
- (C) 0
- (D) 1

Correct Answer: (A) 2

Solution:**Step 1: Understanding the Concept:**

Rolle's theorem needs $f(1) = f(3)$ along with continuity and differentiability. The point c with $f'(c) = 0$ is given as $c = 2$.

Step 2: Use $f(1) = f(3)$:

$f(1) = a - b - 8 - 4 = a - b - 12$. $f(3) = 27a - 9b - 24 - 4 = 27a - 9b - 28$. Equate:

$$a - b - 12 = 27a - 9b - 28 \Rightarrow 13a - 4b = 8$$

Step 3: Use $f'(2) = 0$:

$f'(x) = 3ax^2 - 2bx - 8$, so $f'(2) = 12a - 4b - 8 = 0$, which gives $3a - b = 2$.

Step 4: Solve:

From $b = 3a - 2$: $13a - 4(3a - 2) = 8$, so $a + 8 = 8$ and $a = 0$. Then $b = -2$.

Step 5: Find $a - b$:

$a - b = 0 - (-2) = 2$, option (A).

Final Answer:

Solving the two conditions gives $a = 0$, $b = -2$, so $a - b = 2$.

2

Quick Tip: Use $f(1) = f(3)$ and $f'(2) = 0$ together.

125. The co-ordinates of the point on the curve $y = x \log x$ at which the normal is parallel to the line $2x - 2y = 3$ are...

- (A) $(0, 0)$
- (B) (e, e)
- (C) $(e^2, 2e^2)$
- (D) $(e^{-2}, -2e^{-2})$

Correct Answer: (D) $(e^{-2}, -2e^{-2})$

Solution:

Step 1: Understanding the Concept:

The normal at a point is perpendicular to the tangent. The line $2x - 2y = 3$ has slope 1, so the normal must have slope 1, which means the tangent has slope -1 .

Step 2: Slope of the curve:

$$y = x \ln x, \text{ so } \frac{dy}{dx} = \ln x + 1.$$

Step 3: Condition:

Tangent slope $= -1$: $\ln x + 1 = -1$, so $\ln x = -2$ and $x = e^{-2}$.

Step 4: Find y:

$$y = x \ln x = e^{-2} \cdot (-2) = -2e^{-2}.$$

Step 5: Choose:

The point is $(e^{-2}, -2e^{-2})$, option (D).

Final Answer:

The point is $(e^{-2}, -2e^{-2})$.

$$(e^{-2}, -2e^{-2})$$

Quick Tip: The normal is parallel to slope 1, so the tangent has slope -1 .

126. If $\int \frac{1}{\cos^3 x \sqrt{\sin 2x}} dx = p(\tan^2 x + q)\sqrt{\tan x} + c$, then the values of p and q respectively are

- (A) $p = \frac{\sqrt{2}}{5}, q = \frac{1}{5}$
 (B) $p = \frac{\sqrt{2}}{3}, q = 3$
 (C) $p = \frac{\sqrt{2}}{5}, q = 5$
 (D) $p = \frac{2}{\sqrt{5}}, q = \sqrt{5}$

Correct Answer: (C) $p = \frac{\sqrt{2}}{5}, q = 5$

Solution:

Step 1: Choose the substitution:

The integrand has $\cos^3 x$ and $\sin 2x$. Put $t = \tan x$, so $dx = \frac{dt}{1+t^2}$. Also $\sin 2x = \frac{2t}{1+t^2}$ and $\frac{1}{\cos^3 x} = \sec^3 x = (1+t^2)^{3/2}$.

Step 2: Simplify the integrand:

$$\int \frac{(1+t^2)^{3/2} \sqrt{1+t^2}}{\sqrt{2t}(1+t^2)} dt = \int \frac{(1+t^2)^2}{(1+t^2)\sqrt{2t}} dt = \frac{1}{\sqrt{2}} \int (t^{-1/2} + t^{3/2}) dt$$

Step 3: Integrate:

$$\frac{1}{\sqrt{2}} \left(2t^{1/2} + \frac{2}{5} t^{5/2} \right) + c = \frac{\sqrt{2}}{5} \sqrt{t} (5 + t^2) + c$$

Taking out the common factor $\frac{\sqrt{2}}{5} \sqrt{t}$ leaves $5 + t^2$. This works because $\frac{2}{\sqrt{2}} = \sqrt{2} = \frac{\sqrt{2}}{5} \cdot 5$ and $\frac{2}{5\sqrt{2}} = \frac{\sqrt{2}}{5}$.

Step 4: Compare with the given form:

Replace $t = \tan x$: the result is $\frac{\sqrt{2}}{5} (\tan^2 x + 5) \sqrt{\tan x} + c$. Comparing

with $p(\tan^2 x + q)\sqrt{\tan x} + c$ gives $p = \frac{\sqrt{2}}{5}$ and $q = 5$.

Step 5: Why the other options are wrong:

The ratio of the coefficients of $t^{1/2}$ and $t^{5/2}$ is $\frac{2}{2/5} = 5$, so q must be 5, not $\frac{1}{5}$ or 3. Option (A) has the right p but $q = \frac{1}{5}$ is upside down.

Options (B) and (D) have the wrong p as well, since the coefficient of $t^{5/2}$ is $\frac{\sqrt{2}}{5}$.

Final Answer:

We get $p = \frac{\sqrt{2}}{5}$ and $q = 5$, which is option (C).

$$p = \frac{\sqrt{2}}{5}, q = 5$$

Quick Tip: Substitute $t = \tan x$; the integrand turns into $(1 + t^2)$ divided by root of $2t$.

127. If $\int \sqrt{2}\sqrt{1 + \sin x} dx = -4\cos(ax + b) + c$, then the value of a, b respectively are...

- (A) $\frac{1}{2}, \frac{\pi}{2}$
- (B) $\frac{1}{2}, \frac{\pi}{4}$
- (C) $\frac{x}{2}, \frac{\pi}{4}$
- (D) $1, \frac{\pi}{2}$

Correct Answer: (B) $\frac{1}{2}, \frac{\pi}{4}$

Solution:

Step 1: Understanding the Concept:

We write $1 + \sin x$ as a perfect square using half angles.

Step 2: Rewrite:

$$1 + \sin x = \left(\sin \frac{x}{2} + \cos \frac{x}{2} \right)^2 = 2 \sin^2 \left(\frac{x}{2} + \frac{\pi}{4} \right)$$

So $\sqrt{1 + \sin x} = \sqrt{2} \sin \left(\frac{x}{2} + \frac{\pi}{4} \right)$ (taking the positive branch).

Step 3: Multiply by sqrt 2:

$$\sqrt{2}\sqrt{1 + \sin x} = 2 \sin \left(\frac{x}{2} + \frac{\pi}{4} \right).$$

Step 4: Integrate:

$$\int 2 \sin \left(\frac{x}{2} + \frac{\pi}{4} \right) dx = -4 \cos \left(\frac{x}{2} + \frac{\pi}{4} \right) + c$$

Step 5: Compare:

$$a = \frac{1}{2} \text{ and } b = \frac{\pi}{4}, \text{ option (B).}$$

Final Answer:

The integral is $-4 \cos(x/2 + \pi/4) + c$, so $a = 1/2$ and $b = \pi/4$.

$$\boxed{a = \frac{1}{2}, b = \frac{\pi}{4}}$$

Quick Tip: Write $1 + \sin x$ as $2 \sin^2(x/2 + \pi/4)$.

128. $\int \frac{\sqrt{x-2}}{x} dx =$

- (A) $2\sqrt{x-2} - 2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{x-2}}{\sqrt{2}}\right) + c$
- (B) $2\sqrt{x-2} + 2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{x-2}}{\sqrt{2}}\right) + c$
- (C) $2\sqrt{x-2} - 2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{x-2}}{2}\right) + c$
- (D) $2\sqrt{x-2} + 2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{x-2}}{2}\right) + c$

Correct Answer: (A) $2\sqrt{x-2} - 2\sqrt{2}\tan^{-1}\left(\frac{\sqrt{x-2}}{\sqrt{2}}\right) + c$

Solution:

Step 1: Understanding the Concept:

Substitution removes the square root. Let $t = \sqrt{x-2}$.

Step 2: Substitute:

Then $x = t^2 + 2$ and $dx = 2t dt$.

$$\int \frac{\sqrt{x-2}}{x} dx = \int \frac{t \cdot 2t}{t^2 + 2} dt = 2 \int \frac{t^2}{t^2 + 2} dt$$

Step 3: Split the fraction:

$$2 \int \left(1 - \frac{2}{t^2 + 2}\right) dt = 2t - 4 \int \frac{dt}{t^2 + 2}$$

Step 4: Integrate:

Using $\int \frac{dt}{t^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{t}{a}$ with $a = \sqrt{2}$:

$$2t - \frac{4}{\sqrt{2}} \tan^{-1} \frac{t}{\sqrt{2}} = 2t - 2\sqrt{2} \tan^{-1} \frac{t}{\sqrt{2}}$$

Step 5: Back-substitute:

$$2\sqrt{x-2} - 2\sqrt{2} \tan^{-1} \frac{\sqrt{x-2}}{\sqrt{2}} + c$$

This is option (A). Options (B) and (D) carry a plus sign, and (C) and (D) divide by 2 instead of $\sqrt{2}$ inside the tangent inverse.

Final Answer:

The result matches option (A).

$$2\sqrt{x-2} - 2\sqrt{2} \tan^{-1} \frac{\sqrt{x-2}}{\sqrt{2}} + c$$

Quick Tip: Let $t = \sqrt{x-2}$ and split $t^2 / (t^2 + 2)$.

129. If $f(x)$ and $g(x)$ are integrable functions then $[\int f(x)dx][\int g(x)dx] =$

- (A) Missing open brace for superscript
- (B) Missing open brace for superscript
- (C) $\int [f(x) \int g(x)dx + g(x) \int f(x)dx]dx$
- (D) $\int [f(x) \int g(x)dx - g(x) \int f(x)dx]dx$

Correct Answer: (C) $\int [f(x) \int g(x)dx + g(x) \int f(x)dx]dx$

Solution:

Step 1: Understanding the Concept:

Integration by parts: $\int u dv = uv - \int v du$. Applied with $u = \int f dx$ and $v = \int g dx$ gives a formula for the product of two integrals.

Step 2: Differentiate the product:

Let $F = \int f dx$ and $G = \int g dx$. Then

$$\frac{d}{dx}(FG) = F'G + FG' = fG + gF$$

Step 3: Integrate back:

$$FG = \int \left[f(x) \int g(x) dx + g(x) \int f(x) dx \right] dx$$

Step 4: Choose:

This is option (C). Options (A) and (B) put the derivatives f' and g' in, which is not the rule. Option (D) has a minus sign, which would be wrong because the product rule adds.

Final Answer:

The product rule gives option (C).

$$\boxed{\int \left[f \int g dx + g \int f dx \right] dx}$$

Quick Tip: Differentiate the product of the two integrals.

130. If an antiderivative of $f(x)$ is e^x and an antiderivative of $g(x)$ is $\cos x$, then $\int f(x)\cos x \, dx + \int g(x)e^x \, dx =$

- (A) $e^x \sin x + c$
- (B) $e^x(f(x) + g(x)) + c$
- (C) $e^x \cos x + c$
- (D) $e^x + c$

Correct Answer: (C) $e^x \cos x + c$

Solution:

Step 1: Understanding the Concept:

Let $F(x) = e^x$ with $F' = f$, and $G(x) = \cos x$ with $G' = g$. Then $f = e^x$ and $g = -\sin x$.

Step 2: Rewrite the integrals:

$\int f \cos x \, dx = \int F' G \, dx$. And $\int g e^x \, dx = \int G' F \, dx$.

Step 3: Add:

$$\int (F' G + F G') dx = \int (F G)' dx = F G + c$$

Step 4: Substitute:

$FG = e^x \cos x$. So the sum is $e^x \cos x + c$.

Step 5: Verify directly:

$\int e^x \cos x \, dx + \int (-\sin x)e^x \, dx$. The first is $\frac{e^x}{2}(\cos x + \sin x)$ and the second is $-\frac{e^x}{2}(\sin x - \cos x)$. The sum is $e^x \cos x$. Option (C).

Final Answer:

The sum is $e^x \cos x + c$.

$$e^x \cos x + c$$

Quick Tip: The two integrands are the parts of $d(e^x \cos x)$.

131. $\int_0^{\log 10} \left[\frac{e^x \sqrt{e^x - 1}}{e^x + 8} \right] dx =$

- (A) $\frac{3}{2}(4 + \pi)$
- (B) $\frac{3}{2}(4 - \pi)$
- (C) $\frac{3}{4}(4 - \pi)$
- (D) $\frac{3}{4}(4 + \pi)$

Correct Answer: (B) $\frac{3}{2}(4 - \pi)$

Solution:**Step 1: Choose a substitution:**

The root $\sqrt{e^x - 1}$ suggests $t = \sqrt{e^x - 1}$. Then $e^x = t^2 + 1$ and, by differentiating, $e^x dx = 2t dt$.

Step 2: Change the limits:

At $x = 0$: $t = \sqrt{1 - 1} = 0$. At $x = \log 10$: $t = \sqrt{10 - 1} = 3$. So the new limits are 0 to 3.

Step 3: Rewrite the integral:

The numerator $e^x \sqrt{e^x - 1} dx$ becomes $t \cdot 2t dt$, and the denominator $e^x + 8 = t^2 + 9$. So

$$I = \int_0^3 \frac{2t^2}{t^2 + 9} dt = 2 \int_0^3 \left(1 - \frac{9}{t^2 + 9}\right) dt$$

Step 4: Evaluate:

$$I = 2[t]_0^3 - 18 \cdot \frac{1}{3} \left[\tan^{-1} \frac{t}{3} \right]_0^3 = 6 - 6 \left(\frac{\pi}{4} - 0 \right) = 6 - \frac{3\pi}{2}$$

So $I = \frac{3}{2}(4 - \pi)$.

Step 5: Why the other options are wrong:

Options with $4 + \pi$ would come from adding the arctangent part instead of subtracting it. The minus sign comes from $t^2 = (t^2 + 9) - 9$. The factor $\frac{3}{4}$ in (C) and (D) would need $6 - \frac{3\pi}{2}$ to be halved, but the factor 2 in $2t dt$ was already used. A quick check: $\frac{3}{2}(4 - \pi) \approx 1.29$, which fits an integrand that stays below about 1.7 on a range of length about 2.3.

Final Answer:

The value of the integral is $\frac{3}{2}(4 - \pi)$, which is option (B).

$$\boxed{\frac{3}{2}(4 - \pi)}$$

Quick Tip: Put $t = \text{root of } (e^x - 1)$, so $e^x dx = 2t dt$ and the limits become 0 to 3.

132. The value of the integral $\int_{-\pi/2}^{\pi/2} \left(\frac{x^2 \cos x}{1+e^x} \right) dx$ is equal to $\left(\frac{\pi^2}{A} \right) - B$. Then $\left(\frac{A}{B} \right) =$

(A) -2

(B) 2

(C) 6

(D) -6

Correct Answer: (B) 2

Solution:

Step 1: Understanding the Concept:

For an even function $h(x)$, $\int_{-a}^a \frac{h(x)}{1+e^x} dx = \int_0^a h(x) dx$. Here $h(x) = x^2 \cos x$ is even.

Step 2: Reduce:

$$I = \int_0^{\pi/2} x^2 \cos x dx$$

Step 3: Integrate by parts:

$$\int x^2 \cos x dx = x^2 \sin x - \int 2x \sin x dx$$

$$\int x \sin x dx = -x \cos x + \sin x$$

Step 4: Evaluate:

$$I = [x^2 \sin x + 2x \cos x - 2 \sin x]_0^{\pi/2} = \frac{\pi^2}{4} + 0 - 2 = \frac{\pi^2}{4} - 2$$

Step 5: Read A and B:

$A = 4$ and $B = 2$, so $\frac{A}{B} = 2$. Option (B).

Final Answer:

The integral is $\pi^2/4 - 2$, so $A/B = 2$.

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Quick Tip: Use the even function trick with $1/(1+e^x)$, then integrate by parts.

133. The area of the smaller region bounded by the circle $x^2 + y^2 = 4$ and the line $x = 1$ is...

- (A) $\frac{4\pi}{3} - \sqrt{3}$
- (B) $\frac{\pi}{3} - \sqrt{3}$
- (C) $\frac{4\pi}{3} + \sqrt{3}$
- (D) $\frac{\pi}{3} + \sqrt{3}$

Correct Answer: (A) $\frac{4\pi}{3} - \sqrt{3}$

Solution:

Step 1: Understanding the Concept:

The circle $x^2 + y^2 = 4$ has radius 2. The line $x = 1$ cuts it into a small part (for $x \geq 1$) and a large part. The smaller region is the one with x

from 1 to 2.

Step 2: Set up:

By symmetry about the x -axis, the area is twice the area above the axis:

$$A = 2 \int_1^2 \sqrt{4 - x^2} \, dx$$

Step 3: Integrate:

Using $\int \sqrt{a^2 - x^2} \, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a}$ with $a = 2$:

$$\left[\frac{x}{2} \sqrt{4 - x^2} + 2 \sin^{-1} \frac{x}{2} \right]_1^2 = \pi - \left(\frac{\sqrt{3}}{2} + \frac{\pi}{3} \right) = \frac{2\pi}{3} - \frac{\sqrt{3}}{2}$$

Step 4: Double:

$$A = 2 \left(\frac{2\pi}{3} - \frac{\sqrt{3}}{2} \right) = \frac{4\pi}{3} - \sqrt{3}$$

Step 5: Choose:

Option (A).

Final Answer:

The smaller region has area $\frac{4\pi}{3} - \sqrt{3}$.

$$\frac{4\pi}{3} - \sqrt{3}$$

Quick Tip: Compute a circular segment with distance 1 from the centre.



134. If the area bounded by the curve $x^2 = by$ and the lines $y = 1, y = 4$ in the first quadrant is 28 sq. units, then the value of b is...

- (A) 36
- (B) 6
- (C) 9
- (D) 3

Correct Answer: (A) 36

Solution:

Step 1: Understanding the Concept:

In the first quadrant, the curve $x^2 = by$ gives $x = \sqrt{b}\sqrt{y}$. The area between the curve and the y -axis, from $y = 1$ to $y = 4$, is found by integrating x with respect to y .

Step 2: Set up:

$$A = \int_1^4 x \, dy = \sqrt{b} \int_1^4 y^{1/2} \, dy$$

Step 3: Integrate:

$$A = \sqrt{b} \left[\frac{2}{3} y^{3/2} \right]_1^4 = \sqrt{b} \cdot \frac{2}{3} (8 - 1) = \frac{14\sqrt{b}}{3}$$

Step 4: Use the given area:

$$\frac{14\sqrt{b}}{3} = 28, \text{ so } \sqrt{b} = 6 \text{ and } b = 36.$$

Step 5: Choose:

Option (A). The value 6 in option (B) is $\sqrt{b}$, not b .

Final Answer:

$$b = 36.$$

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Quick Tip: Integrate $x = \sqrt{b} \sqrt{y}$ from $y = 1$ to $y = 4$.

135. The solution of the differential equation $\frac{dy}{dx} = e^{x-y} + x^2e^{-y}$ is...

- (A) $y = e^x + c$
- (B) $e^y = e^x + x^3 + c$
- (C) $e^y = e^x + \frac{x^3}{3} + c$
- (D) $e^y = e^x + 2x + c$

Correct Answer: (C) $e^y = e^x + \frac{x^3}{3} + c$

Solution:

Step 1: Factor the right side:

Write $e^{x-y} = e^x e^{-y}$. Then

$$\frac{dy}{dx} = e^{-y} (e^x + x^2)$$

Both terms share the factor e^{-y} , so the variables can be separated.

Step 2: Separate the variables:

Multiply both sides by e^y and by dx :

$$e^y dy = (e^x + x^2) dx$$

Step 3: Integrate both sides:

The left side integrates to e^y . On the right, $\int e^x dx = e^x$ and $\int x^2 dx = \frac{x^3}{3}$. So

$$e^y = e^x + \frac{x^3}{3} + c$$

Step 4: Why the other options are wrong:

Check each option by differentiating: from $e^y = f(x)$ we get $e^y \frac{dy}{dx} = f'(x)$, and this must equal $e^x + x^2$. Option (B) gives $e^x + 3x^2$, because $\frac{d}{dx} x^3 = 3x^2$. Option (D) gives $e^x + 2$. Option (A), $y = e^x + c$, gives $\frac{dy}{dx} = e^x$, which does not satisfy the given equation since $e^{x-y} + x^2 e^{-y} \neq e^x$ in general.

Final Answer:

The solution is $e^y = e^x + \frac{x^3}{3} + c$, which is option (C).

$$e^y = e^x + \frac{x^3}{3} + c$$

Quick Tip: Take e^{-y} as a common factor, then multiply by e^y and integrate.

136. If the differential equation $ff'' - (f')^2 = 0$ for all x and $f(0) = 1$, then...

- (A) $f' = -f$
- (B) $f' = f$
- (C) $f' = 0$
- (D) $f' = 2f$

Correct Answer: (C) $f' = 0$

Solution:

Step 1: Expand the determinant:

The condition is $ff'' - (f')^2 = 0$ for all x , so $ff'' = (f')^2$.

Step 2: Recognise a derivative:

Divide by f^2 (f is not zero near 0 because $f(0) = 1$): $\frac{ff'' - (f')^2}{f^2} = 0$. The left side is exactly $\frac{d}{dx} \left(\frac{f'}{f} \right)$.

Step 3: Integrate:

So $\frac{f'}{f}$ is a constant c . Using $f(0) = 1$ and $f'(0) = 2$: $c = 2/1 = 2$.

Step 4: Conclude:

$$f'(x) = 2f(x)$$

The options $f' = -f$, $f' = f$ and $f' = 0$ would give $c = -1, 1, 0$, which

do not match $c = 2$.

Final Answer:

$f'(x) = 2f(x)$, option (C).

$$f'(x) = 2f(x)$$

Quick Tip: Show the determinant condition says d/dx of f prime over f is zero, then use the initial values.

137. The solution of the differential equation $x^2 \frac{dy}{dx} - xy = 1$ is...

- (A) $2xy - 2cx^2 - 1 = 0$
- (B) $2xy + 2cx^2 + 1 = 0$
- (C) $2xy - 2cx^2 + 1 = 0$
- (D) $2x^2y - 2cx + 1 = 0$

Correct Answer: (C) $2xy - 2cx^2 + 1 = 0$

Solution:

Step 1: Understanding the Concept:

The equation is first order linear. Divide by x^2 to write it as $\frac{dy}{dx} + Py = Q$.

Step 2: Standard form:

$$\frac{dy}{dx} - \frac{1}{x}y = \frac{1}{x^2}$$

So $P = -\frac{1}{x}$ and $Q = \frac{1}{x^2}$.

Step 3: Integrating factor:

$$\text{I.F.} = e^{\int -\frac{1}{x} dx} = e^{-\ln x} = \frac{1}{x}$$

Step 4: Solve:

$$y \cdot \frac{1}{x} = \int \frac{1}{x^2} \cdot \frac{1}{x} dx = \int x^{-3} dx = -\frac{1}{2x^2} + c$$

Step 5: Clear the fractions:

Multiply by $2x^2$: $2xy = -1 + 2cx^2$. So

$$2xy - 2cx^2 + 1 = 0$$

This is option (C). Options (A) and (B) have the wrong sign on the constant 1.

Final Answer:

The solution is $2xy - 2cx^2 + 1 = 0$.

$$\boxed{2xy - 2cx^2 + 1 = 0}$$

Quick Tip: Divide by x^2 and use the integrating factor $1/x$.

138. If $\vec{a}$ and $\vec{b}$ are unit vectors and θ ($0 < \theta < \pi$) is the angle between them, then the value of $\frac{|\vec{a}+\vec{b}|}{|\vec{a}-\vec{b}|}$ is equal to...

- (A) $\tan \frac{\theta}{2}$
- (B) $\sin \frac{\theta}{2}$
- (C) $\cos \frac{\theta}{2}$
- (D) $\cot \frac{\theta}{2}$

Correct Answer: (D) $\cot \frac{\theta}{2}$

Solution:

Step 1: Understanding the Concept:

For any vector, $|\vec{v}|^2 = \vec{v} \cdot \vec{v}$. For unit vectors, $\vec{a} \cdot \vec{a} = \vec{b} \cdot \vec{b} = 1$ and $\vec{a} \cdot \vec{b} = \cos \theta$.

Step 2: Square of the sum:

$$|\vec{a} + \vec{b}|^2 = 1 + 1 + 2 \cos \theta = 2(1 + \cos \theta) = 4 \cos^2 \frac{\theta}{2}$$

Step 3: Square of the difference:

$$|\vec{a} - \vec{b}|^2 = 2 - 2 \cos \theta = 4 \sin^2 \frac{\theta}{2}$$

Step 4: Ratio:

For $0 < \theta < \pi$, both $\cos \frac{\theta}{2}$ and $\sin \frac{\theta}{2}$ are positive. So

$$\frac{|\vec{a} + \vec{b}|}{|\vec{a} - \vec{b}|} = \frac{2 \cos \frac{\theta}{2}}{2 \sin \frac{\theta}{2}} = \cot \frac{\theta}{2}$$

Step 5: Choose:

Option (D). The tangent in option (A) is the reciprocal.

Final Answer:

The ratio is $\cot(\theta/2)$.

$$\cot \frac{\theta}{2}$$

Quick Tip: Square both modulus values and use $1 + \cos t = 2\cos^2(t/2)$.

139. If ABCDEF is a regular hexagon and $\overline{AB} + \overline{AC} + \overline{AD} + \overline{AE} + \overline{AF} = p\overline{AD} = q\overline{AO}$, where O is the center of the hexagon, then the values of p and q respectively are

- (A) 2, 3
- (B) 4, 6
- (C) 3, 6
- (D) 3, 5

Correct Answer: (C) 3, 6

Solution:

Step 1: Set up coordinates:

Take the centre O as the origin and let the radius (also the side) of the regular hexagon be a . Put $A = (a, 0)$, $B = (a/2, \frac{\sqrt{3}}{2}a)$, $C = (-a/2, \frac{\sqrt{3}}{2}a)$, $D = (-a, 0)$, $E = (-a/2, -\frac{\sqrt{3}}{2}a)$, $F = (a/2, -\frac{\sqrt{3}}{2}a)$.

Step 2: Find each vector:

$$\vec{AB} = \left(-\frac{a}{2}, \frac{\sqrt{3}a}{2}\right), \vec{AC} = \left(-\frac{3a}{2}, \frac{\sqrt{3}a}{2}\right), \vec{AD} = (-2a, 0), \vec{AE} = \left(-\frac{3a}{2}, -\frac{\sqrt{3}a}{2}\right), \\ \vec{AF} = \left(-\frac{a}{2}, -\frac{\sqrt{3}a}{2}\right).$$

Step 3: Add:

The y parts cancel in pairs. The x parts add to $-\frac{a}{2} - \frac{3a}{2} - 2a - \frac{3a}{2} - \frac{a}{2} = -6a$. So the sum is $(-6a, 0)$.

Step 4: Compare:

$\vec{AD} = (-2a, 0)$, so the sum is $3\vec{AD}$, giving $p = 3$. $\vec{AO} = (-a, 0)$, so the sum is $6\vec{AO}$, giving $q = 6$.

Step 5: Check the options:

Options (A) 2,3 and (B) 4,6 break the relation $AD = 2AO$, because $q = 2p$ must hold. Option (D) 3,5 also breaks $q = 2p$. Only (C) 3,6 keeps it.

Final Answer:

$p = 3$ and $q = 6$, option (C).

$p = 3, q = 6$

Quick Tip: Add the five vectors from A using position vectors from the centre; the six vertices sum to zero.

140. The vector $\vec{r}$ whose magnitude is $3\sqrt{2}$ units and which makes angles of $\frac{\pi}{4}$ and $\frac{\pi}{2}$ with the positive y- and z-axes respectively is....

- (A) $\hat{i} \pm 3\hat{j}$
- (B) $\hat{i} \pm \hat{j}$
- (C) $-\hat{i} \pm \hat{j}$
- (D) $\pm 3\hat{i} + 3\hat{j}$

Correct Answer: (D) $\pm 3\hat{i} + 3\hat{j}$

Solution:

Step 1: Understanding the Concept:

If a vector of magnitude r makes angles α, β, γ with the axes, its components are $r \cos \alpha, r \cos \beta, r \cos \gamma$ and $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$.

Step 2: Use the given angles:

With the y-axis: $\beta = \frac{\pi}{4}$. With the z-axis: $\gamma = \frac{\pi}{2}$. So $\cos \beta = \frac{1}{\sqrt{2}}$ and $\cos \gamma = 0$.

Step 3: Find cos alpha:

$$\cos^2 \alpha = 1 - \frac{1}{2} - 0 = \frac{1}{2}, \text{ so } \cos \alpha = \pm \frac{1}{\sqrt{2}}.$$

Step 4: Components:

$$x: 3\sqrt{2} \times \left(\pm \frac{1}{\sqrt{2}}\right) = \pm 3. \quad y: 3\sqrt{2} \times \frac{1}{\sqrt{2}} = 3. \quad z: 0.$$

Step 5: Choose:

$\vec{r} = \pm 3\hat{i} + 3\hat{j}$, option (D). The magnitude check: $\sqrt{9 + 9} = 3\sqrt{2}$. Option (B) has magnitude $\sqrt{2}$ and (A) has $\sqrt{10}$.

Final Answer:

The vector is plus or minus $3\hat{i} + 3\hat{j}$.

$$\pm 3\hat{i} + 3\hat{j}$$

Quick Tip: Use $\cos^2 \theta + \sin^2 \theta = 1$ to find the x direction cosine.

141. Let $(\vec{p} \wedge \vec{q})$ denote the angle between $\vec{p}$ and $\vec{q}$. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $|\vec{a}| = 7$, $|\vec{b}| = 5$ and $|\vec{c}| = 3$ then (take $\pi = \frac{22}{7}$)

- (A) $\sin(\vec{b} \wedge \vec{c}) = \frac{1}{2}$
- (B) $\cos(\vec{a} \wedge \vec{c}) = -\frac{\pi}{4}$
- (C) $\cos(\vec{b} \wedge \vec{c}) = -\frac{1}{2}$
- (D) $\sin(\vec{a} \wedge \vec{c}) = \frac{\pi}{4}$

Correct Answer: (B) $\cos(\vec{a} \wedge \vec{c}) = -\frac{\pi}{4}$

Solution:

Step 1: Understanding the Concept:

If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, the three vectors form a triangle. We can find angles between pairs by squaring one vector in terms of the other two.

Step 2: Angle between a and c:

$\vec{b} = -(\vec{a} + \vec{c})$. So $|\vec{b}|^2 = |\vec{a}|^2 + |\vec{c}|^2 + 2\vec{a} \cdot \vec{c}$:

$$25 = 49 + 9 + 2\vec{a} \cdot \vec{c} \Rightarrow \vec{a} \cdot \vec{c} = -\frac{33}{2}$$

Step 3: Cosine:

$$\cos(\vec{a} \wedge \vec{c}) = \frac{-33/2}{7 \times 3} = -\frac{33}{42} = -\frac{11}{14}$$

Step 4: Check option (B):

With $\pi = \frac{22}{7}$: $-\frac{\pi}{4} = -\frac{22}{28} = -\frac{11}{14}$. This matches, so (B) is true.

Step 5: Check the others:

For $\vec{b}$ and $\vec{c}$: $\vec{a} = -(\vec{b} + \vec{c})$ gives $49 = 25 + 9 + 2\vec{b} \cdot \vec{c}$, so $\vec{b} \cdot \vec{c} = \frac{15}{2}$ and $\cos(\vec{b} \wedge \vec{c}) = \frac{1}{2}$. So (C) with $-\frac{1}{2}$ is false and (A) with $\sin = \frac{1}{2}$ is false, since $\sin = \frac{\sqrt{3}}{2}$. For (D), $\sin(\vec{a} \wedge \vec{c}) = \sqrt{1 - \frac{121}{196}} = \frac{5\sqrt{3}}{14}$, not $\frac{11}{14}$. So (D) is false.

Final Answer:

$\cos(a,c) = -11/14 = -\pi/4$ with $\pi = 22/7$.

$$\cos(\vec{a} \wedge \vec{c}) = -\frac{\pi}{4}$$

Quick Tip: Square $b = -(a + c)$ to find $a \cdot c$.

142. If the lines $\frac{x-5}{5m+2} = \frac{2-y}{5} = \frac{1-z}{-1}$ and $x = \frac{2y+1}{4m} = \frac{1-z}{-3}$ are perpendicular to each other, then the value of m is ...

- (A) 1
- (B) 0
- (C) 2
- (D) -1

Correct Answer: (A) 1

Solution:

Step 1: Understanding the Concept:

Two lines are perpendicular when the dot product of their direction ratios is zero. First write each line in the form $\frac{x - x_1}{l} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$.

Step 2: Line 1:

$\frac{x - 5}{5m + 2} = \frac{2 - y}{5} = \frac{1 - z}{-1}$ can be written as $\frac{x - 5}{5m + 2} = \frac{y - 2}{-5} = \frac{z - 1}{1}$.
Direction ratios: $(5m + 2, -5, 1)$.

Step 3: Line 2:

$x = \frac{2y + 1}{4m} = \frac{1 - z}{-3}$ can be written as $\frac{x}{1} = \frac{y + \frac{1}{2}}{2m} = \frac{z - 1}{3}$. Direction ratios: $(1, 2m, 3)$.

Step 4: Perpendicular condition:

$$(5m + 2)(1) + (-5)(2m) + (1)(3) = 0$$

$$5m + 2 - 10m + 3 = 0 \Rightarrow 5 - 5m = 0 \Rightarrow m = 1$$

Step 5: Choose:

Option (A).

Final Answer:

$m = 1$.

Quick Tip: Get direction ratios $(5m+2, -5, 1)$ and $(1, 2m, 3)$.

143. For the line $\frac{x+1}{1} = \frac{y-2}{2} = \frac{z+3}{3}$, identify the incorrect statement among the following.

- (A) It can be represented by equation $\frac{x+2}{1} = \frac{y}{2} = \frac{z+6}{3}$
- (B) It lies in the plane $x - 2y + z + 8 = 0$
- (C) It is perpendicular to the plane $x - 2y + z = 0$
- (D) It passes through point $(0, 4, 0)$.

Correct Answer: (C) It is perpendicular to the plane $x - 2y + z = 0$

Solution:

Step 1: Understanding the Concept:

The given line passes through $(-1, 2, -3)$ and has direction ratios $(1, 2, 3)$. We test each statement.

Step 2: Statement (A):

Check whether $(-2, 0, -6)$ lies on the original line. Put $t = -1$: $(-1 - 1, 2 - 2, -3 - 3) = (-2, 0, -6)$. Yes. Direction ratios are the same $(1, 2, 3)$. So (A) is correct.

Step 3: Statement (B):

The plane $x - 2y + z + 8 = 0$ has normal $(1, -2, 1)$. The point $(-1, 2, -3)$ gives $-1 - 4 - 3 + 8 = 0$, so it lies on the plane. The direction dot normal is $1 - 4 + 3 = 0$, so the line is parallel to the plane. Hence the line lies in the plane. (B) is correct.

Step 4: Statement (C):

For the line to be perpendicular to the plane, its direction must be parallel to the normal $(1, -2, 1)$. But $(1, 2, 3)$ is not a multiple of $(1, -2, 1)$. So (C) is incorrect.

Step 5: Statement (D):

Put $t = 1$: $(0, 4, 0)$. Yes, it lies on the line. (D) is correct.

Step 6: Choose:

The incorrect statement is (C).

Final Answer:

Statement C is wrong, since the direction is not parallel to the normal.

C

Quick Tip: Perpendicular to a plane means parallel to its normal.

144. The direction cosines of a line which is perpendicular to the lines $\frac{x-7}{2} = \frac{y+17}{-3} = \frac{z-6}{1}$ and $\frac{x+5}{1} = \frac{y+3}{2} = \frac{z-6}{-2}$ are...

- (A) $\frac{\pm 4}{3\sqrt{10}}, \frac{\mp 5}{3\sqrt{10}}, \frac{\pm 7}{3\sqrt{10}}$
(B) $\frac{\pm 4}{3\sqrt{10}}, \frac{\pm 5}{3\sqrt{10}}, \frac{\mp 7}{3\sqrt{10}}$
(C) $\frac{\mp 4}{3\sqrt{10}}, \frac{\pm 5}{3\sqrt{10}}, \frac{\pm 7}{3\sqrt{10}}$
(D) $\frac{\pm 4}{3\sqrt{10}}, \frac{\pm 5}{3\sqrt{10}}, \frac{\pm 7}{3\sqrt{10}}$

Correct Answer: (D) $\frac{\pm 4}{3\sqrt{10}}, \frac{\pm 5}{3\sqrt{10}}, \frac{\pm 7}{3\sqrt{10}}$

Solution:

Step 1: Understanding the Concept:

A line perpendicular to two lines is parallel to the cross product of their direction vectors.

Step 2: Direction vectors:

$$\vec{d}_1 = (2, -3, 1) \text{ and } \vec{d}_2 = (1, 2, -2).$$

Step 3: Cross product:

$$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -3 & 1 \\ 1 & 2 & -2 \end{vmatrix} = \hat{i}(6 - 2) - \hat{j}(-4 - 1) + \hat{k}(4 + 3) = 4\hat{i} + 5\hat{j} + 7\hat{k}$$

Step 4: Magnitude:

$$\sqrt{16 + 25 + 49} = \sqrt{90} = 3\sqrt{10}.$$

Step 5: Direction cosines:

$$\pm \frac{4}{3\sqrt{10}}, \pm \frac{5}{3\sqrt{10}}, \pm \frac{7}{3\sqrt{10}}$$

The signs are all together plus or all together minus. Option (D).
Options (A), (B) and (C) change relative signs, so their ratios are not 4:5:7.

Final Answer:

The direction cosines are all plus or all minus 4, 5, 7 over 3 sqrt 10.

$$\pm \frac{4}{3\sqrt{10}}, \pm \frac{5}{3\sqrt{10}}, \pm \frac{7}{3\sqrt{10}}$$

Quick Tip: Take the cross product of the direction vectors.



145. The equation of the plane containing the lines $\frac{x-1}{2} = \frac{y+1}{\lambda} = \frac{z}{2}$ and $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{\lambda}$ is

- (A) $x \pm y + 1 = 0$
- (B) $y \pm z + 1 = 0$
- (C) $x \pm z + 1 = 0$
- (D) $y \pm z - 1 = 0$

Correct Answer: (B) $y \pm z + 1 = 0$

Solution:

Step 1: Understanding the Concept:

Two lines lie in one plane (coplanar) if the vector joining one point of each, and the two direction vectors, have zero scalar triple product.

Step 2: Data:

Line 1: point $(1, -1, 0)$, direction $(2, \lambda, 2)$. Line 2: point $(-1, -1, 0)$, direction $(5, 2, \lambda)$. Joining vector: $(-2, 0, 0)$.

Step 3: Coplanarity:

$$\begin{vmatrix} -2 & 0 & 0 \\ 2 & \lambda & 2 \\ 5 & 2 & \lambda \end{vmatrix} = -2(\lambda^2 - 4) = 0 \Rightarrow \lambda = \pm 2$$

Step 4: Case $\lambda = 2$:

$\vec{d}_1 = (2, 2, 2)$ and $\vec{d}_2 = (5, 2, 2)$. Normal $= \vec{d}_1 \times \vec{d}_2 = (0, 6, -6)$, so the plane is $y - z = \text{const}$. At $(1, -1, 0)$ the constant is -1 . So $y - z + 1 = 0$.

Step 5: Case $\lambda = -2$:

$\vec{d}_1 = (2, -2, 2)$ and $\vec{d}_2 = (5, 2, -2)$. Normal $= (0, 14, 14)$, so $y + z = \text{const} = -1$. So $y + z + 1 = 0$.

Step 6: Choose:

Both cases combine as $y \pm z + 1 = 0$, option (B).

Final Answer:

The plane is y plus or minus z plus $1 = 0$.

$$y \pm z + 1 = 0$$

Quick Tip: Coplanarity gives $\lambda = \text{plus or minus } 2$. Then cross the directions.

146. If a unit vector makes angles $\frac{\pi}{4}$ with $\hat{i}$, $\frac{\pi}{3}$ with $\hat{j}$ and $\theta \in (0, \pi)$ with $\hat{k}$, then a value of θ is equal to...

- (A) $\frac{\pi}{3}, \frac{2\pi}{3}$
- (B) $\frac{\pi}{6}, \frac{5\pi}{6}$
- (C) $\frac{\pi}{4}, \frac{5\pi}{4}$
- (D) $\frac{5\pi}{12}, \frac{7\pi}{12}$

Correct Answer: (A) $\frac{\pi}{3}, \frac{2\pi}{3}$

Solution:

Step 1: Understanding the Concept:

Direction cosines of any line satisfy $l^2 + m^2 + n^2 = 1$. For a unit vector, the direction cosines are its components.

Step 2: Substitute:

$$l = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}, m = \cos \frac{\pi}{3} = \frac{1}{2}, n = \cos \theta.$$

$$\frac{1}{2} + \frac{1}{4} + \cos^2 \theta = 1 \Rightarrow \cos^2 \theta = \frac{1}{4}$$

Step 3: Solve:

$$\cos \theta = \pm \frac{1}{2}. \text{ For } \theta \in (0, \pi): \theta = \frac{\pi}{3} \text{ or } \theta = \frac{2\pi}{3}.$$

Step 4: Choose:

Option (A). Option (C) has $\frac{5\pi}{4}$ outside $(0, \pi)$.

Final Answer:

theta is $\pi/3$ or $2\pi/3$.

$$\boxed{\frac{\pi}{3}, \frac{2\pi}{3}}$$

Quick Tip: Direction cosines squared add to 1.

147. The minimum value of $Z = 3x + y$, subject to the constraints $2x + 3y \leq 6, x + y \geq 1, x \geq 0, y \geq 0$ is....

- (A) 5
- (B) 2
- (C) 1
- (D) 9

Correct Answer: (C) 1

Solution:

Step 1: Understanding the Concept:

In graphical LPP, the optimal value of Z over a bounded feasible region lies at a corner point.

Step 2: Draw the constraints:

$2x + 3y \leq 6$ is the region below the line through $(3, 0)$ and $(0, 2)$. $x + y \geq 1$ is the region above the line through $(1, 0)$ and $(0, 1)$. With $x, y \geq 0$ we are in the first quadrant.

Step 3: Corner points:

The feasible region is a quadrilateral with corners $(1, 0)$, $(3, 0)$, $(0, 2)$ and $(0, 1)$.

Step 4: Evaluate $Z = 3x + y$:

$(1, 0)$: 3. $(3, 0)$: 9. $(0, 2)$: 2. $(0, 1)$: 1.

Step 5: Choose:

The minimum value is 1 at $(0, 1)$, option (C). The value 9 is the maximum.

Final Answer:

The minimum of Z is 1 at $(0, 1)$.

1

Quick Tip: Evaluate Z at all corners of the feasible region.

148. The centers for disease control have determined that when a person is given a vaccine, the probability that the person will develop immunity to a virus is 0.8. If 8 people are given this vaccine, then the probability that exactly 4 will develop immunity is...

- (A) $(0.2)(0.8)^4$
- (B) $(0.112)(0.8)^4$
- (C) $(0.112)(0.8)^6$
- (D) $(0.2)^4(0.8)^4$

Correct Answer: (B) $(0.112)(0.8)^4$

Solution:**Step 1: Understanding the Concept:**

Each person independently develops immunity with probability $p = 0.8$, and fails with probability $q = 0.2$. For $n = 8$ people, the number with immunity is binomial.

Step 2: Formula:

$$P(X = r) = \binom{n}{r} p^r q^{n-r}$$

Step 3: Substitute:

$$P(X = 4) = \binom{8}{4}(0.8)^4(0.2)^4$$

$\binom{8}{4} = 70$ and $(0.2)^4 = 0.0016$, so $70 \times 0.0016 = 0.112$.

Step 4: Result:

$$P(X = 4) = (0.112)(0.8)^4$$

Option (B).

Final Answer:

The probability is $0.112 \times (0.8)^4$.

$$(0.112)(0.8)^4$$

Quick Tip: Use binomial: $C(8,4)(0.8)^4(0.2)^4$.

149. The Variance of the following probability distribution of X is

X	0	1	2	3
P(X)	q^3	$3q^2p$	$3qp^2$	p^3

where $0 < p < 1, q = 1 - p$

- (A) $3p^2q$
- (B) $3pq$
- (C) $3pq^2$
- (D) $3q$

Correct Answer: (B) $3pq$

Solution:

Step 1: Understanding the Concept:

The probabilities q^3 , $3q^2p$, $3qp^2$, p^3 are those of a binomial distribution with $n = 3$ and success probability p .

Step 2: Recall:

For a binomial distribution the mean is np and the variance is npq .

Step 3: Apply:

With $n = 3$: variance = $3pq$.

Step 4: Verify directly:

Mean = $\sum xP = 3q^2p + 6qp^2 + 3p^3 = 3p(q^2 + 2qp + p^2) = 3p(q + p)^2 = 3p$. And $E(X^2) = 3q^2p + 12qp^2 + 9p^3 = 3p(q^2 + 4qp + 3p^2) = 3p(q + p)(q + 3p) = 3p(1 + 2p)$. So variance = $3p + 6p^2 - 9p^2 = 3p - 3p^2 = 3p(1 - p) = 3pq$.

Step 5: Choose:

Option (B).

Final Answer:

The variance is $3pq$.

$$3pq$$

Quick Tip: This is a binomial distribution with $n = 3$.

150. If A and B are two events such that $P(A) = \frac{2}{3}$, $P(B) = \frac{1}{2}$ and $P(A|B) = \frac{2}{3}$, then **Missing open brace for superscript**

(A) $\frac{2}{3}$

(B) 1

(C) $\frac{3}{2}$

(D) $\frac{5}{6}$

Correct Answer: (C) $\frac{3}{2}$

Solution:

Step 1: Understanding the Concept:

We need $P(A \cap B)$ first. From the conditional probability, $P(A|B)$
$$= \frac{P(A \cap B)}{P(B)}.$$

Step 2: Find the intersection:

$$P(A \cap B) = \frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$$

Step 3: First term:

$P(A' \cup B) = 1 - P(A \cap B')$. Now $P(A \cap B') = P(A) - P(A \cap B) = \frac{2}{3} - \frac{1}{3}$
 $= \frac{1}{3}$. So $P(A' \cup B) = \frac{2}{3}$.

Step 4: Second term:

$P(A \cup B') = 1 - P(A' \cap B)$. Now $P(A' \cap B) = P(B) - P(A \cap B) = \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$. So $P(A \cup B') = \frac{5}{6}$.

Step 5: Add:

$$\frac{2}{3} + \frac{5}{6} = \frac{4+5}{6} = \frac{3}{2}$$

Option (C).

Final Answer:

The sum is $\frac{2}{3} + \frac{5}{6} = \frac{3}{2}$.

$$\boxed{\frac{3}{2}}$$

Quick Tip: Find $P(A \text{ and } B) = \frac{1}{3}$ first.