

MHT CET 2026 PCM Question Paper April 15 Shift 2 with Solutions



General Instructions

- (i) This booklet contains 150 questions, each provided with a complete, step-by-step solution.
- (ii) It comprises 150 single-correct multiple-choice questions.
- (iii) Attempt each question on your own before reviewing the given solution.

Physics

Questions 1 to 50 | 1 mark each | 50 marks

1. Let $\vec{A}$ and $\vec{B}$ are two non-zero vectors of different magnitude. Which one of the following is the correct equation ?

- (A) $\vec{A} \cdot \vec{B} = -\vec{B} \cdot \vec{A}$
- (B) $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$
- (C) $\vec{A} + \vec{B} = \vec{B} + \vec{A}$
- (D) $\vec{A} - \vec{B} = \vec{B} - \vec{A}$

Correct Answer: (C) $\vec{A} + \vec{B} = \vec{B} + \vec{A}$

Solution:

Step 1: Understanding the Concept

Vector addition obeys $\vec{A} + \vec{B} = \vec{B} + \vec{A}$. The dot product is commutative, and the cross product reverses sign when the order is swapped.

Step 2: Check option (A)

$\vec{A} \cdot \vec{B} = AB \cos \theta = \vec{B} \cdot \vec{A}$. A minus sign is wrong, since both vectors are non-zero. So (A) is FALSE.

Step 3: Check option (B)

$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$, so the equality given is FALSE (unless they are parallel).

Step 4: Check option (C)

$\vec{A} + \vec{B} = \vec{B} + \vec{A}$ is the commutative law of addition, which is TRUE.

Step 5: Check option (D)

$\vec{A} - \vec{B} = -(\vec{B} - \vec{A})$, so equality would need $\vec{A} = \vec{B}$. They have different magnitudes, so (D) is FALSE.

Final Answer:

Only $\vec{A} + \vec{B} = \vec{B} + \vec{A}$ is always correct, option (C).

$$\boxed{\vec{A} + \vec{B} = \vec{B} + \vec{A}}$$

Quick Tip: Vector addition is commutative, but the dot product is symmetric and the cross product is antisymmetric.

2. A wire has mass (0.3 ± 0.003) gram, radius (0.5 ± 0.005) mm and length (6 ± 0.06) cm. The maximum error in the measurement of density is

- (A) 4%
- (B) 3%
- (C) 2%
- (D) 5%

Correct Answer: (A) 4%

Solution:

Step 1: Understanding the Concept

The wire is a cylinder, so $\rho = \frac{m}{\pi r^2 l}$. For a product or quotient of measured quantities, maximum fractional errors add, and a power multiplies the error of that quantity.

Step 2: Key Formula or Approach

$$\frac{\Delta \rho}{\rho} = \frac{\Delta m}{m} + 2 \frac{\Delta r}{r} + \frac{\Delta l}{l}$$

Step 3: Detailed Explanation

$$\begin{aligned} \frac{\Delta m}{m} &= \frac{0.003}{0.3} = 1\% \\ \frac{\Delta r}{r} &= \frac{0.005}{0.5} = 1\%, \text{ so } 2 \frac{\Delta r}{r} = 2\% \\ \frac{\Delta l}{l} &= \frac{0.06}{6} = 1\% \end{aligned}$$

$$\frac{\Delta \rho}{\rho} = 1 + 2 + 1 = 4\%$$

Option (B), 3%, results from forgetting that radius appears squared.

Final Answer:

The maximum error in density is 4%, option (A).

4%

Quick Tip: Density = $m / (\pi r^2 l)$, so the maximum fractional error is $dm/m + 2 dr/r + dl/l$.



3. A batsman hits a ball with a velocity 'v', making an angle of 60° with the vertical. After some time direction of velocity is making an angle of 60° with the horizontal. The speed of the ball at this instant is

$$[\cos(60^\circ) = \frac{1}{2}, \cos(30^\circ) = \frac{\sqrt{3}}{2}]$$

(A) $\frac{\sqrt{3}}{2}v$

(B) $\sqrt{3}v$

(C) $\frac{v}{2}$

(D) V

Correct Answer: (B) $\sqrt{3}v$

Solution:**Step 1: Understanding the Concept**

With no air resistance the horizontal velocity of a projectile never changes, only the vertical component does.

Step 2: Initial horizontal component

The ball is hit at 60° with the vertical, so at 30° with the horizontal. So:

$$v_x = v \cos 30^\circ = \frac{\sqrt{3}}{2}v$$

Step 3: Later instant

Now the velocity makes 60° with the horizontal, so $v_x = u \cos 60^\circ = \frac{u}{2}$, where u is the speed.

$$\frac{u}{2} = \frac{\sqrt{3}}{2}v \Rightarrow u = \sqrt{3}v$$

The speed is larger than v because the ball is lower than its launch height at that instant, so it has gained kinetic energy.

Final Answer:

The speed at that instant is $\sqrt{3}v$, option (B).

$$\boxed{\sqrt{3}v}$$

Quick Tip: The horizontal component of velocity stays constant in projectile motion.



4. A particle at rest starts moving with constant angular acceleration ' α ' in a circular path of radius ' r '. At certain instant, the magnitude of centripetal acceleration is $(\frac{1}{3})^{rd}$ the tangential acceleration. The relation between linear speed (V) and angular acceleration (α) is

- (A) $V = \frac{r\alpha}{3}$
- (B) $V = \alpha\sqrt{\frac{r}{2}}$
- (C) $V = r\sqrt{\frac{\alpha}{3}}$
- (D) $V = \sqrt{\frac{\alpha r}{3}}$

Correct Answer: (C) $V = r\sqrt{\frac{\alpha}{3}}$

Solution:

Step 1: Understanding the Concept

A particle moving in a circle with angular acceleration α has tangential acceleration $a_t = r\alpha$ and centripetal acceleration $a_c = \omega^2 r = V^2/r$.

Step 2: Apply the condition

$$a_c = \frac{1}{3}a_t \Rightarrow \omega^2 r = \frac{r\alpha}{3} \Rightarrow \omega^2 = \frac{\alpha}{3}$$

Step 3: Find V

$$V = \omega r = r\sqrt{\frac{\alpha}{3}}$$

Option (A) has no square root, option (B) has a 2 that does not appear, and (D) uses $\sqrt{\alpha r/3}$, which is dimensionally inconsistent with the above.

Final Answer:

The speed is $V = r\sqrt{\alpha/3}$, option (C).

$$V = r\sqrt{\frac{\alpha}{3}}$$

Quick Tip: Set centripetal acceleration equal to one third of tangential acceleration.

5. A car of mass m is crossing the convex bridge of radius of curvature R with speed V . At the highest point the thrust is (g = gravitational acceleration)

- (A) mg
- (B) $\frac{mv^2}{R}$
- (C) $mg - \frac{mv^2}{R}$
- (D) $mg + \frac{mv^2}{R}$

Correct Answer: (C) $mg - \frac{mv^2}{R}$

Solution:

Step 1: Understanding the Concept

On a convex bridge the centre of the circle is below the car. Net force toward the centre equals mv^2/R .

Step 2: Apply Newton second law

At the highest point, weight acts downward (toward the centre) and the normal reaction N acts upward:

$$mg - N = \frac{mV^2}{R} \Rightarrow N = mg - \frac{mV^2}{R}$$

The thrust on the bridge equals N . Option (D) has the wrong sign, which would apply at the bottom of a dip.

Final Answer:

The thrust is $mg - \frac{mV^2}{R}$, option (C).

$$mg - \frac{mV^2}{R}$$

Quick Tip: At the top, weight minus normal reaction provides the centripetal force.

6. A bucket containing water is revolved in vertical circle of radius r . To prevent the water from falling down, the period of revolution required is (g = gravitational acceleration)

- (A) $2\pi\sqrt{\frac{g}{r}}$
- (B) $2\pi\sqrt{\frac{r}{g}}$
- (C) $2\pi\sqrt{rg}$
- (D) $\frac{\sqrt{rg}}{2\pi}$

Correct Answer: (B) $2\pi\sqrt{\frac{r}{g}}$

Solution:

Step 1: Understanding the Concept

Water stays in the bucket at the top only if gravity alone is enough to supply the centripetal force, so the minimum speed there is $v = \sqrt{rg}$.

Step 2: Compute period

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{rg}} = 2\pi\sqrt{\frac{r}{g}}$$

This is the maximum period (a slower revolution would let the water fall). Option (A) has g/r inverted, and (D) is not a time.

Final Answer:

The period must be at most $2\pi\sqrt{r/g}$, option (B).

$$\boxed{2\pi\sqrt{\frac{r}{g}}}$$

Quick Tip: At the top, minimum speed satisfies $mg = mv^2/r$.

7. In case of perfectly elastic collision,

- (A) the total kinetic energy before collision is equal to the total kinetic energy after collision.
- (B) the total kinetic energy before collision is less than total kinetic energy after collision.
- (C) the total kinetic energy before collision is greater than the total kinetic energy after collision.
- (D) coefficient of restitution is equal to zero.

Correct Answer: (A) the total kinetic energy before collision is equal to the total kinetic energy after collision.

Solution:

Step 1: Understanding the Concept

In a perfectly elastic collision there is no loss of mechanical energy. Both momentum and kinetic energy are conserved.

Step 2: Detailed Explanation

So the total kinetic energy before the collision equals the total kinetic energy after it. This is (A).

(B) and (C) would describe a gain or loss of kinetic energy, which only happens in other types of collision. (D) is wrong because the coefficient of restitution is 1 for a perfectly elastic collision, not 0 (0 is a perfectly inelastic collision).

Final Answer:

Kinetic energy before equals kinetic energy after, option (A).

$$KE_{\text{before}} = KE_{\text{after}}$$

Quick Tip: In an elastic collision kinetic energy is conserved and the coefficient of restitution is 1.



8. The distance of the two planets A and B from the sun are ' r_A ' and ' r_B ' respectively such that $r_B = 100 r_A$. The ratio of the speed of planet A to that of planet B is (both the planets are revolving around the sun)

- (A) $\frac{1}{\sqrt{10}}$
(B) $\sqrt{10}$
(C) 10
(D) $10\sqrt{10}$

Correct Answer: (C) 10

Solution:

Step 1: Understanding the Concept

Gravity provides the centripetal force: $\frac{GMm}{r^2} = \frac{mv^2}{r}$, so $v = \sqrt{\frac{GM}{r}}$.

Step 2: Take the ratio

$$\frac{v_A}{v_B} = \sqrt{\frac{r_B}{r_A}} = \sqrt{100} = 10$$

The nearer planet moves faster. Option (A) and (B) involve $\sqrt{10}$, which would apply if $r_B = 10r_A$.

Final Answer:

The ratio is 10, option (C).

10

Quick Tip: For a circular orbit, $v = \sqrt{GM/r}$, so v is inversely proportional to the square root of r .

9. A square frame of each side L is dipped in a soap solution and taken out, the force acting on the film formed is (T = surface tension of soap solution)

- (A) $4TL$
- (B) $8TL$
- (C) $12TL$
- (D) $16TL$

Correct Answer: (B) $8TL$

Solution:

Step 1: Understanding the Concept

Surface tension force on a boundary of length ℓ is $T\ell$ per free surface. A soap film has two free surfaces (front and back).

Step 2: Compute

The frame has perimeter $4L$. Force:

$$F = 2T \times 4L = 8TL$$

Option (A), $4TL$, counts only one surface.

Final Answer:

The force on the film is $8TL$, option (B).

$8TL$

Quick Tip: A soap film has two surfaces, so force = $2 \times T \times$ total length of the frame.

10. Three liquids have same surface tension and densities ρ_1, ρ_2 and ρ_3 ($\rho_1 < \rho_2 < \rho_3$). In three identical capillaries rise of liquid is same. The corresponding angle of contact θ_1, θ_2 and θ_3 are related as

- (A) $\theta_1 < \theta_2 < \theta_3$
- (B) $\theta_1 > \theta_2 > \theta_3$
- (C) $\theta_1 = \theta_2 = \theta_3$
- (D) $\theta_1 > \theta_2 < \theta_3$

Correct Answer: (B) $\theta_1 > \theta_2 > \theta_3$

Solution:

Step 1: Understanding the Concept

The rise in a capillary is $h = \frac{2T \cos \theta}{r\rho g}$. Here h, T and r are the same for all three liquids.

Step 2: Relate cos theta to density

$$\cos \theta = \frac{hrg}{2T} \rho \Rightarrow \cos \theta \propto \rho$$

Since $\rho_1 < \rho_2 < \rho_3$, we get $\cos \theta_1 < \cos \theta_2 < \cos \theta_3$.

Step 3: Convert to angles

For angles between 0° and 90° , cosine decreases as the angle increases. A smaller cosine means a larger angle, so

$$\theta_1 > \theta_2 > \theta_3$$

This is option (B).

Final Answer:

The angles satisfy $\theta_1 > \theta_2 > \theta_3$, option (B).

$$\theta_1 > \theta_2 > \theta_3$$

Quick Tip: Capillary rise $h = 2T \cos(\theta) / (r \rho g)$, so $\cos(\theta)$ is proportional to density when h , T , r are fixed.



11. The excess pressure inside the first soap bubble of radius R_1 is three times that inside the second soap bubble of radius R_2 . The ratio of volumes of the first to second bubble is

- (A) 1 : 27
- (B) 1 : 18
- (C) 1 : 9
- (D) 1 : 3

Correct Answer: (A) 1 : 27

Solution:**Step 1: Understanding the Concept**

For a soap bubble with two surfaces, excess pressure $P = \frac{4T}{R}$. So $R \propto \frac{1}{P}$.

Step 2: Radius ratio

$P_1 = 3P_2$, so $R_1 = \frac{R_2}{3}$, that is $R_1 : R_2 = 1 : 3$.

Step 3: Volume ratio

Volume goes as R^3 :

$$\frac{V_1}{V_2} = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$$

So the ratio is 1:27. Option (D), 1:3, is the radius ratio.

Final Answer:

The volume ratio is 1:27, option (A).

1 : 27

Quick Tip: Excess pressure in a soap bubble is $4T/R$, so radius is inversely proportional to excess pressure.

12. The ratio of the thermal conductivity of two rods of different materials is 6:5. The two rods of same area of cross section and same thermal resistance will have the length in the ratio

- (A) 5 : 6
- (B) 6 : 5
- (C) 1 : 3
- (D) 3 : 1

Correct Answer: (B) 6 : 5

Solution:

Step 1: Understanding the Concept

Thermal resistance of a rod is $R = \frac{L}{kA}$, where k is the thermal conductivity and A the cross-sectional area.

Step 2: Equal resistance

Equal areas and equal resistance mean $\frac{L_1}{k_1} = \frac{L_2}{k_2}$, so $\frac{L_1}{L_2} = \frac{k_1}{k_2} = \frac{6}{5}$.

A better conductor must be longer to have the same resistance.

Option (A), 5:6, is the inverse.

Final Answer:

The length ratio is 6:5, option (B).

6 : 5

Quick Tip: Thermal resistance $R = L / (kA)$.

13. A black rectangular surface of area A emits energy E per second at 127°C . If length and breadth is reduced to half of initial value and temperature is raised to 527°C then energy emitted becomes

- (A) E
- (B) $2E$
- (C) $4E$
- (D) $8E$

Correct Answer: (C) $4E$

Solution:

Step 1: Understanding the Concept

By the Stefan-Boltzmann law, $E = \sigma AT^4$. Temperature must be in kelvin.

Step 2: Convert and compare

$T_1 = 127 + 273 = 400$ K and $T_2 = 527 + 273 = 800$ K, so $T_2/T_1 = 2$. Length and breadth are halved, so the area becomes $\frac{1}{4}$.

$$\frac{E'}{E} = \frac{A'}{A} \left(\frac{T_2}{T_1} \right)^4 = \frac{1}{4} \times 16 = 4$$

So the new energy is $4E$. Using Celsius directly would give a wrong ratio.

Final Answer:

The energy emitted becomes $4E$, option (C).

$$\boxed{4E}$$

Quick Tip: Power radiated = $\sigma A T^4$ with T in kelvin.



14. The energy spectrum of a black body exhibits a maximum around a wavelength λ . The temperature of a black body is now changed such that the energy is maximum at wavelength $2\lambda/3$. The power radiated by the black body will now increase by a factor of

- (A) $\frac{256}{81}$
 (B) $\frac{81}{16}$
 (C) $\frac{162}{41}$
 (D) $\frac{16}{81}$

Correct Answer: (B) $\frac{81}{16}$

Solution:

Step 1: Understanding the Concept

Wien displacement law: $\lambda_m T = \text{constant}$. Stefan law: power $\propto T^4$.

Step 2: Find the new temperature

The peak wavelength changes from λ to $\frac{2\lambda}{3}$, so $T' = T \times \frac{3}{2}$.

Step 3: Power ratio

$$\frac{P'}{P} = \left(\frac{T'}{T} \right)^4 = \left(\frac{3}{2} \right)^4 = \frac{81}{16}$$

Option (A), $256/81$, is $(4/3)^4$, which would be for a peak wavelength of $3\lambda/4$.

Final Answer:

The power increases by a factor $\frac{81}{16}$, option (B).

$$\boxed{\frac{81}{16}}$$

Quick Tip: Wien law gives T inversely proportional to the peak wavelength; power goes as T^4 .

15. In a cyclic process, work done by the system is

- (A) zero.
- (B) equal to the heat given to system.
- (C) more than the heat given to system.
- (D) independent of heat given to system.

Correct Answer: (B) equal to the heat given to system.

Solution:

Step 1: Understanding the Concept

Internal energy is a state function. After one complete cycle the system is back in its initial state, so $\Delta U = 0$.

Step 2: Apply the first law

$$Q = \Delta U + W = W$$

So the net work done by the system equals the net heat given to the system. This is (B).

It is not zero in general (A), and it cannot exceed the heat supplied (C), since that would violate energy conservation. It is not independent of the heat (D).

Final Answer:

Work done equals the heat given to the system, option (B).

$$W = Q$$

Quick Tip: In a cycle the internal energy change is zero, so $W = Q$.



16. The average translational kinetic energy of a molecule in a gas is E_1 . The kinetic energy of the electron (e) accelerated from rest through potential difference 'V' volt is E_2 . The temperature at which $E_1 = E_2$ possible is (N = number of molecules, R = gas constant)

- (A) $\frac{eVN}{2R}$
- (B) $\frac{2eVN}{3R}$
- (C) $\frac{eVN}{R}$
- (D) $\frac{3eVN}{4R}$

Correct Answer: (B) $\frac{2eVN}{3R}$

Solution:

Step 1: Understanding the Concept

Average translational kinetic energy of a gas molecule is $E_1 = \frac{3}{2}kT$, with Boltzmann constant $k = R/N$ (N being the Avogadro number here). An electron accelerated through V volts gains $E_2 = eV$.

Step 2: Equate

$$\frac{3}{2} \cdot \frac{R}{N} T = eV \Rightarrow T = \frac{2eVN}{3R}$$

Option (A) has the factor 2 in the denominator instead of the numerator, and (C) leaves out the 3/2.

Final Answer:

The temperature is $\frac{2eVN}{3R}$, option (B).

$$\frac{2eVN}{3R}$$

Quick Tip: Average translational energy per molecule is $(3/2) kT$, with $k = R/N$.



17. An ideal gas is expanded adiabatically. How many times has the gas to be expanded to reduce the r.m.s. speed of molecules 3 times ? ($\gamma = 1.5$)

- (A) 32 times
- (B) 16 times
- (C) 8 times
- (D) 81 times

Correct Answer: (D) 81 times

Solution:

Step 1: Understanding the Concept

The rms speed is $v_{rms} = \sqrt{3RT/M}$, so $v_{rms} \propto \sqrt{T}$. In an adiabatic process $TV^{\gamma-1} = \text{constant}$.

Step 2: Temperature ratio

If the rms speed falls three times, the temperature falls nine times: $T_2 = T_1/9$.

Step 3: Volume ratio

$$\frac{V_2}{V_1} = \left(\frac{T_1}{T_2} \right)^{\frac{1}{\gamma-1}} = 9^{\frac{1}{0.5}} = 9^2 = 81$$

The gas must expand 81 times, option (D).

Final Answer:

The gas must expand 81 times, option (D).

81

Quick Tip: v_{rms} is proportional to $\sqrt{T}$, and for adiabatic change $T V^{\gamma-1}$ is constant.



18. A body of mass 0.4 kg performs simple harmonic motion. It experiences a restoring force of 0.4 N when its displacement from the mean position is 4 cm. The force constant and magnitude of acceleration respectively are

- (A) $5\text{N/m}, 0.5\text{m/s}^2$
- (B) $10\text{N/m}, 1\text{m/s}^2$
- (C) $15\text{N/m}, 1.5\text{m/s}^2$
- (D) $20\text{N/m}, 2\text{m/s}^2$

Correct Answer: (B) $10\text{N/m}, 1\text{m/s}^2$

Solution:

Step 1: Understanding the Concept

The restoring force in SHM is $F = kx$, so $k = F/x$. The acceleration is $a = F/m$.

Step 2: Compute

$$k = \frac{0.4}{0.04} = 10 \text{ N/m}$$

$$a = \frac{0.4}{0.4} = 1 \text{ m/s}^2$$

The displacement 4 cm must first be converted to 0.04 m; leaving it as 4 gives $k = 0.1$, which is not an option.

Final Answer:

The force constant is 10 N/m and the acceleration is 1 m/s², option (B).

10 N/m, 1 m/s ²

Quick Tip: $k = F / x$ and $a = F / m$.

19. The minimum phase difference between two simple harmonic motions is

$$x_1 = \frac{1}{\sqrt{2}} \sin \omega t + \frac{1}{\sqrt{2}} \cos \omega t$$

$$x_2 = \sin \omega t + \cos \omega t \quad \left[\sin \frac{\pi}{4} = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} \right]$$

(A) zero

(B) $\frac{\pi^c}{3}$

(C) $\frac{\pi^c}{4}$

(D) $\frac{\pi^c}{5}$

Correct Answer: (A) zero

Solution:

Step 1: Understanding the Concept

Combine sine and cosine using $\sin \omega t + \cos \omega t = \sqrt{2} \sin \left(\omega t + \frac{\pi}{4} \right)$.

Step 2: Rewrite both

$$x_1 = \frac{1}{\sqrt{2}} (\sin \omega t + \cos \omega t) = \sin \left(\omega t + \frac{\pi}{4} \right)$$

$$x_2 = \sin \omega t + \cos \omega t = \sqrt{2} \sin \left(\omega t + \frac{\pi}{4} \right)$$

Step 3: Compare phases

Both have the same phase constant $\pi/4$. They differ only in amplitude (1 and $\sqrt{2}$). So the phase difference is zero.

Final Answer:

The phase difference is zero, option (A).

0

Quick Tip: Write each expression as a single sine with a phase constant.

20. A pendulum clock is running slow, In order to correct it, we should

- (A) reduce the amplitude of oscillation.
- (B) reduce the mass of the bob.
- (C) reduce the length of pendulum.
- (D) increase the length of pendulum.

Correct Answer: (C) reduce the length of pendulum.

Solution:

Step 1: Understanding the Concept

A clock that runs slow takes longer than it should for each swing, so its period is too large. The period of a simple pendulum is $T = 2\pi\sqrt{L/g}$.

Step 2: Detailed Explanation

To reduce T we must reduce L . So we shorten the pendulum. This is (C).

The period does not depend on the amplitude (for small swings) or the mass of the bob, so (A) and (B) will not help. Increasing the length (D) would make the clock slower.

Final Answer:

Reduce the length of the pendulum, option (C).

Reduce the length

Quick Tip: A slow clock has too long a time period; $T = 2\pi\sqrt{L/g}$ so decrease the length.

21. A simple harmonic progressive wave is represented by $y = A \sin(120\pi t + 3x)$. The distance between two points on the wave at a phase difference of $\frac{\pi}{3}$ radian is

- (A) $\frac{\pi}{6}$ m
- (B) $\frac{\pi}{9}$ m
- (C) $\frac{2\pi}{9}$ m
- (D) $\frac{\pi}{18}$ m

Correct Answer: (B) $\frac{\pi}{9}$ m

Solution:

Step 1: Understanding the Concept

Comparing $y = A \sin(120\pi t + 3x)$ with $y = A \sin(\omega t + kx)$ gives wave number $k = 3$ rad/m.

Step 2: Compute

The phase difference between two points at a distance Δx is $\Delta\phi = k \Delta x$:

$$\Delta x = \frac{\Delta\phi}{k} = \frac{\pi/3}{3} = \frac{\pi}{9} \text{ m}$$

Option (A) would come from using $k = 2$ instead of 3.

Final Answer:

The distance is $\frac{\pi}{9}$ m, option (B).

$\frac{\pi}{9} \text{ m}$

Quick Tip: Phase difference = $(2\pi / \lambda) \times \text{path difference} = k \times \text{path difference}$.



22. A pipe open at both ends has a fundamental frequency ' f ' in air. The pipe is dipped in water, so that $\frac{2}{3}$ rd length of pipe is in water, Now the fundamental frequency of the air column is

- (A) $\frac{1}{2}f$
- (B) $\frac{3}{2}f$
- (C) $\frac{5}{2}f$
- (D) $\frac{7}{2}f$

Correct Answer: (B) $\frac{3}{2}f$

Solution:

Step 1: Understanding the Concept

An open pipe of length L has fundamental frequency $f = \frac{v}{2L}$. A pipe dipped in water becomes a closed pipe whose length is the air column above the water, with fundamental $f' = \frac{v}{4\ell}$.

Step 2: Compute

Two thirds of the pipe is in water, so the air column is $\ell = L/3$.

$$f' = \frac{v}{4(L/3)} = \frac{3v}{4L} = \frac{3}{2} \cdot \frac{v}{2L} = \frac{3}{2}f$$

So the new fundamental frequency is $\frac{3}{2}f$.

Final Answer:

The new fundamental frequency is $\frac{3}{2}f$, option (B).

$$\frac{3}{2}f$$

Quick Tip: The part of the pipe above water behaves as a closed pipe of length $L/3$.



23. To increase the frequency of transverse oscillations of a stretched string by 40%, the tension must be increased by

- (A) 100%
- (B) 40%
- (C) 96%
- (D) 140%

Correct Answer: (C) 96%

Solution:

Step 1: Understanding the Concept

For a stretched string, $f = \frac{1}{2\ell} \sqrt{T/\mu}$, so $f \propto \sqrt{T}$.

Step 2: Compute

A 40% rise means $f' = 1.4f$. Then:

$$\frac{T'}{T} = \left(\frac{f'}{f} \right)^2 = 1.4^2 = 1.96$$

The tension must increase by $1.96 - 1 = 0.96$, that is 96%. Option (B) forgets the square, and (A) and (D) are not consistent with it.

Final Answer:

The tension must increase by 96%, option (C).

96%

Quick Tip: Frequency is proportional to the square root of tension.

24. When open pipe is closed from one end then second overtone of closed pipe is higher in frequency by 100 Hz than first overtone of open pipe. The fundamental frequency of open end pipe will be (Neglect end correction)

- (A) 200 Hz
- (B) 300 Hz
- (C) 100 Hz
- (D) 400 Hz

Correct Answer: (A) 200 Hz

Solution:**Step 1: Understanding the Concept**

Let the open pipe of length L have fundamental $f_0 = \frac{v}{2L}$. Its overtones are $2f_0, 3f_0, \dots$, so the first overtone is $2f_0$.

Step 2: Closed pipe

Closing one end of the same pipe gives a fundamental $\frac{v}{4L} = \frac{f_0}{2}$. Only odd harmonics exist: $\frac{f_0}{2}, \frac{3f_0}{2}, \frac{5f_0}{2}$. The second overtone is the 5th harmonic, $\frac{5f_0}{2}$.

Step 3: Use the 100 Hz difference

$$\frac{5f_0}{2} - 2f_0 = \frac{f_0}{2} = 100 \Rightarrow f_0 = 200 \text{ Hz}$$

Final Answer:

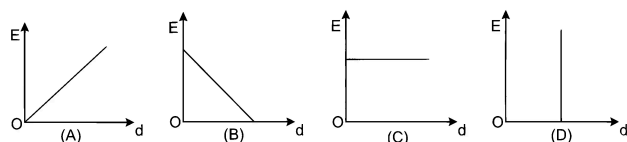
The fundamental frequency of the open pipe is 200 Hz, option (A).

200 Hz

Quick Tip: Open pipe: harmonics are $f_0, 2f_0, 3f_0$. Closed pipe of the same length: odd harmonics of $f_0/2$.



25. For a uniformly charged plane sheet, the variation of electric field (E) with distance (d) is correctly shown graphically in graph



- (A) A
- (B) B
- (C) C
- (D) D

Correct Answer: (C) C

Solution:

Step 1: Understanding the Concept

By Gauss law, an infinite uniformly charged plane sheet with surface charge density σ produces a field $E = \frac{\sigma}{2\epsilon_0}$ on either side of it.

Step 2: Read the graphs

The field does not contain the distance d , so the E-d graph is a horizontal straight line.

Graph (A) shows E rising linearly from zero, which is the field inside a uniformly charged solid sphere. Graph (B) shows E falling with distance, which does not happen for an infinite sheet. Graph (D) is a vertical line, which is not a function of d . Graph (C) is a horizontal line starting above the origin, the constant field.

Final Answer:

The field is constant with distance, graph (C).

Graph (C)

Quick Tip: The field of an infinite uniformly charged sheet is $\sigma/(2\epsilon_0)$, independent of distance.

26. A point charge 'Q' is placed at the centre of the line joining two equal charges '+q' and '+q'. The value of 'Q' when the system is in equilibrium is

- (A) $\frac{+q}{4}$
- (B) $\frac{+q}{2}$
- (C) $\frac{-q}{2}$
- (D) $\frac{-q}{4}$

Correct Answer: (D) $-\frac{q}{4}$

Solution:

Step 1: Understanding the Concept

Let the two $+q$ charges be a distance $2a$ apart, with Q at the centre, a distance a from each. For the whole system to be in equilibrium, every charge must have zero net force on it.

Step 2: Equilibrium of one end charge

Consider the right $+q$. The other $+q$ repels it with force $\frac{kq^2}{(2a)^2} = \frac{kq^2}{4a^2}$ to the right. The charge Q must pull it back with $\frac{kqQ}{a^2}$, so Q must be negative:

$$\frac{kq|Q|}{a^2} = \frac{kq^2}{4a^2} \Rightarrow |Q| = \frac{q}{4}$$

Step 3: Check the middle charge

By symmetry, the forces on Q from the two $+q$ charges cancel, so Q is also in equilibrium.

Therefore $Q = -\frac{q}{4}$. Option (B) and (C) have magnitude $q/2$, which would over-pull the end charges, and (A) is positive, which would repel them further.

Final Answer:

The charge is $Q = -\frac{q}{4}$, option (D).

$$\boxed{-\frac{q}{4}}$$

Quick Tip: For a +q charge to be in equilibrium, the force from the middle charge must cancel the force from the other +q.



27. Capacitors of capacities C_1 and C_2 are connected in series. If the combination is connected to a supply of V volt then the potential difference across capacitor C_2 is

- (A) $\frac{C_1+C_2}{C_1V}$
- (B) $\frac{C_1V}{C_1+C_2}$
- (C) $\frac{C_1+C_2}{C_2V}$
- (D) $\frac{C_2V}{C_1+C_2}$

Correct Answer: (B) $\frac{C_1V}{C_1+C_2}$

Solution:

Step 1: Understanding the Concept

In series, the charge Q is the same on both capacitors, and the supply voltage splits as $V = V_1 + V_2$, where $V_1 = Q/C_1$ and $V_2 = Q/C_2$.

Step 2: Compute

The equivalent capacitance is $C_s = \frac{C_1C_2}{C_1 + C_2}$, so $Q = C_sV = \frac{C_1C_2V}{C_1 + C_2}$.

$$V_2 = \frac{Q}{C_2} = \frac{C_1V}{C_1 + C_2}$$

So the voltage across C_2 contains C_1 in the numerator. Option (D) would be the voltage across C_1 .

Final Answer:

The potential difference across C_2 is $\frac{C_1 V}{C_1 + C_2}$, option (B).

$$\boxed{\frac{C_1 V}{C_1 + C_2}}$$

Quick Tip: In series each capacitor carries the same charge, so voltage is inversely proportional to capacitance.

28. Two parallel plate air capacitors are connected in parallel. Each capacitor has plate area $A/2$ and separation between the plates is d and $2d$ respectively. The equivalent capacity of the combination is

(ϵ_0 = absolute permittivity of free space)

- (A) $\frac{A\epsilon_0}{d}$
- (B) $\frac{3A\epsilon_0}{4d}$
- (C) $\frac{2A\epsilon_0}{3d}$
- (D) $\frac{A\epsilon_0}{4d}$

Correct Answer: (B) $\frac{3A\epsilon_0}{4d}$

Solution:**Step 1: Understanding the Concept**

A parallel plate air capacitor has $C = \frac{\epsilon_0 A_{\text{plate}}}{d}$. Capacitors connected in parallel add up.

Step 2: Compute

$$C_1 = \frac{\epsilon_0(A/2)}{d} = \frac{\epsilon_0 A}{2d}, \quad C_2 = \frac{\epsilon_0(A/2)}{2d} = \frac{\epsilon_0 A}{4d}$$

$$C = C_1 + C_2 = \frac{\epsilon_0 A}{2d} + \frac{\epsilon_0 A}{4d} = \frac{3\epsilon_0 A}{4d}$$

Option (D) is only C_2 , and (A) would need both capacitors to be as large as C_1 twice.

Final Answer:

The equivalent capacity is $\frac{3A\epsilon_0}{4d}$, option (B).

$$\boxed{\frac{3A\epsilon_0}{4d}}$$

Quick Tip: Add the two capacitances, each equal to ϵ_0 times area over separation.

29. In metre bridge experiment, the resistance in the left gap is $15 \, \Omega$ and in the right gap is $45 \, \Omega$. The bridge is balanced. The distance of the null point from the centre of the wire is

- (A) 80 cm
- (B) 60 cm
- (C) 40 cm
- (D) 25 cm

Correct Answer: (D) 25 cm

Solution:

Step 1: Understanding the Concept

In a metre bridge, at balance $\frac{R}{S} = \frac{l}{100 - l}$, where l is the length from the left end (in cm).

Step 2: Compute

$$\frac{15}{45} = \frac{l}{100 - l} \Rightarrow 100 - l = 3l \Rightarrow l = 25 \text{ cm}$$

The centre of the wire is at 50 cm, so the null point is $50 - 25 = 25$ cm from the centre. Option (C), 40 cm, and (A), 80 cm, are not the result of this balance.

Final Answer:

The null point is 25 cm from the centre, option (D).

25 cm

Quick Tip: Balance condition: $R_{\text{left}} / R_{\text{right}} = l / (100 - l)$; then subtract 50 cm.

30. A potentiometer wire is 4m long and potential difference of 2V is maintained between the ends. The e.m.f. of the cell which balances against a length of 80 cm of the potentiometer wire is

- (A) 0.25 V
- (B) 0.30 V
- (C) 0.40 V
- (D) 0.80 V

Correct Answer: (C) 0.40 V

Solution:

Step 1: Understanding the Concept

The potential gradient of a potentiometer wire is $k = \frac{V}{L}$. A cell balances at length ℓ when $E = k\ell$.

Step 2: Compute

$$k = \frac{2}{4} = 0.5 \text{ V/m}$$

$$E = 0.5 \times 0.8 = 0.4 \text{ V}$$

Using 80 cm without converting to metres gives 40, which is not an option, and 0.8 V would need the full 1.6 m of wire.

Final Answer:

The e.m.f. is 0.4 V, option (C).

0.4 V

Quick Tip: EMF = potential gradient x balancing length.

31. An iron rod is placed parallel to magnetic field intensity 1000 A/m. The magnetic flux through the rod is 3×10^{-4} Wb and its cross-sectional area is 1.5cm^2 . The magnetic permeability of rod in Wb/A-m is

- (A) 2×10^{-2}
- (B) 2×10^{-3}
- (C) 2×10^{-4}
- (D) 1×10^{-2}

Correct Answer: (B) 2×10^{-3}

Solution:

Step 1: Understanding the Concept

Magnetic flux density is $B = \frac{\phi}{A}$, and permeability is $\mu = \frac{B}{H}$.

Step 2: Compute

Area $A = 1.5\text{ cm}^2 = 1.5 \times 10^{-4}\text{ m}^2$.

$$B = \frac{3 \times 10^{-4}}{1.5 \times 10^{-4}} = 2\text{ T}$$

$$\mu = \frac{B}{H} = \frac{2}{1000} = 2 \times 10^{-3}\text{ Wb/(A m)}$$

Option (A) results if the area is not converted to square metres correctly.

Final Answer:

The permeability is 2×10^{-3} Wb/A-m, option (B).

$$2 \times 10^{-3}$$

Quick Tip: Flux density $B = \text{flux} / \text{area}$, and permeability $\mu = B / H$.

32. A charge moves in a circular path perpendicular to a magnetic field. The time period of revolution is independent of

- (A) mass of the particle
- (B) velocity of the particle
- (C) magnetic field
- (D) charge

Correct Answer: (B) velocity of the particle

Solution:

Step 1: Understanding the Concept

A charge moving perpendicular to a magnetic field follows a circle.

The magnetic force supplies the centripetal force: $qvB = \frac{mv^2}{r}$, so $r = \frac{mv}{qB}$.

Step 2: Period

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

The period depends on the mass, the charge and the field, but the speed v cancels out. So the period is independent of the velocity of the particle, option (B).

Final Answer:

The period is independent of the velocity, option (B).

$$T = \frac{2\pi m}{qB}$$

Quick Tip: Period $T = 2\pi m / (qB)$, independent of speed and radius.

33. A long wire is bent into a circular coil of one turn and then into a circular coil of smaller radius having n turns. If the same current is passed in both the cases, the ratio of magnetic field produced at the centre for one turn to that of n turns is

- (A) $1 : n$
- (B) $n : 1$
- (C) $1 : n^2$
- (D) $n^2 : 1$

Correct Answer: (C) $1 : n^2$

Solution:**Step 1: Understanding the Concept**

The field at the centre of a circular coil of N turns and radius r is $B = \frac{\mu_0 NI}{2r}$.

Step 2: Radius change

The same wire length $2\pi R$ is used. For n turns, $n \cdot 2\pi r = 2\pi R$, so $r = R/n$.

Step 3: Ratio

$$B_1 = \frac{\mu_0 I}{2R}, \quad B_n = \frac{\mu_0 n I}{2(R/n)} = \frac{\mu_0 n^2 I}{2R}$$

$$\frac{B_1}{B_n} = \frac{1}{n^2}$$

So the ratio is $1 : n^2$. Option (A) forgets that the radius also shrinks.

Final Answer:

The ratio is $1 : n^2$, option (C).

$$\boxed{1 : n^2}$$

Quick Tip: The same wire length is used, so the radius becomes R/n when there are n turns.

34. A thin ring of radius ' R ' metre has charge ' q ' coulomb uniformly spread on it. The ring rotates about its axis with a constant frequency of ' f ' revolution/s. The value of magnetic induction at the centre of the ring in Wb/m^2 is

(μ_0 = permeability of free space)

- (A) $\frac{\mu_0 q}{2fR}$
 (B) $\frac{\mu_0 q}{2\pi R}$
 (C) $\frac{\mu_0 q f}{2R}$
 (D) $\frac{\mu_0 q \pi}{2fR}$

Correct Answer: (C) $\frac{\mu_0 q f}{2R}$

Solution:

Step 1: Understanding the Concept

Charge q on the ring crosses any point f times each second, so it is equivalent to a current $I = qf$.

Step 2: Compute

The field at the centre of a circular current loop is $B = \frac{\mu_0 I}{2R}$.

$$B = \frac{\mu_0 q f}{2R}$$

Option (B) uses πR in the denominator, and (A) and (D) put f in the wrong place.

Final Answer:

The magnetic induction is $\frac{\mu_0 q f}{2R}$, option (C).

$$\frac{\mu_0 q f}{2R}$$

Quick Tip: A rotating charge forms a current $I = q f$.

35. Two concentric circular coils having radii r_1 and r_2 ($r_2 \ll r_1$) are placed co-axially with centres coinciding. The mutual inductance of the arrangement is (Both coils have single turn, μ_0 = permeability of free space)

(A) $\frac{\mu_0 \pi r_2}{2r_1}$

(B) $\frac{\mu_0 \pi r_1^2}{2r_2}$

(C) $\frac{\mu_0 \pi r_1}{2r_2}$

(D) $\frac{\mu_0 r_2^2 \pi}{2r_1}$

Correct Answer: (D) $\frac{\mu_0 r_2^2 \pi}{2r_1}$

Solution:

Step 1: Understanding the Concept

Mutual inductance is $M = \frac{\phi_2}{I_1}$, the flux linked with the small coil per unit current in the large coil.

Step 2: Compute

Current I in the large coil gives at its centre $B = \frac{\mu_0 I}{2r_1}$. Since $r_2 \ll r_1$, the field is almost uniform across the small coil.

$$\phi_2 = B \cdot \pi r_2^2 = \frac{\mu_0 I \pi r_2^2}{2r_1}$$

$$M = \frac{\phi_2}{I} = \frac{\mu_0 \pi r_2^2}{2r_1}$$

Final Answer:

The mutual inductance is $\frac{\mu_0 \pi r_2^2}{2r_1}$, option (D).

$$\boxed{\frac{\mu_0 \pi r_2^2}{2r_1}}$$

Quick Tip: Large coil gives a nearly uniform field $\mu_0 I/(2r_1)$ through the small coil.

36. Two solenoids A and B of equal number of turns have their lengths and radii in the same ratio 1 : 3. The ratio of the self-inductance of solenoid A to that of B will be

- (A) 1 : 1
- (B) 1 : 3
- (C) 1 : 9
- (D) 3 : 1

Correct Answer: (B) 1 : 3

Solution:**Step 1: Understanding the Concept**

Self-inductance of a long solenoid is $L = \frac{\mu_0 N^2 A}{\ell}$, with $A = \pi r^2$. With equal turns, $L \propto \frac{r^2}{\ell}$.

Step 2: Ratio

Lengths and radii of A and B are in the ratio 1 : 3.

$$\frac{L_A}{L_B} = \frac{r_A^2 / \ell_A}{r_B^2 / \ell_B} = \frac{1^2 / 1}{3^2 / 3} = \frac{1}{3}$$

The ratio is 1:3. Option (C), 1:9, forgets that length is also larger in B.

Final Answer:

The ratio is 1:3, option (B).

1 : 3

Quick Tip: $L = \mu_0 N^2 A / l$, so with the same N , L is proportional to r^2 / l .



37. An electric lamp connected in series with a capacitor and an a.c source is glowing with certain brightness. On reducing the frequency of source the brightness of the lamp

- (A) is increased
- (B) is reduced
- (C) remain the same
- (D) becomes zero

Correct Answer: (B) is reduced

Solution:

Step 1: Understanding the Concept

In an a.c. circuit with a lamp (resistance R) and a capacitor in series,

the current is $I = \frac{V}{\sqrt{R^2 + X_C^2}}$ with $X_C = \frac{1}{2\pi fC}$.

Step 2: Effect of lower frequency

When f decreases, X_C increases, so the impedance rises and the current falls. A smaller current means the lamp glows less brightly. The lamp does not become fully dark (D), because the circuit still carries a smaller a.c. current.

Final Answer:

The brightness is reduced, option (B).

Reduced

Quick Tip: Capacitive reactance is $1/(\omega C)$, which rises when frequency falls.



38. A series resonant circuit consists of an inductor 'L' and capacitor 'C' which produces resonant frequency 'f'. If 'L' is increased by '2L' and 'C' is changed to '9C' the resonant frequency will be

- (A) $\frac{f}{3}$
- (B) $\frac{f}{2}$
- (C) $\frac{f}{2\sqrt{2}}$
- (D) $\frac{f}{3\sqrt{3}}$

Correct Answer: (D) $\frac{f}{3\sqrt{3}}$

Solution:

Step 1: Understanding the Concept

The resonant frequency of a series LC circuit is $f = \frac{1}{2\pi\sqrt{LC}}$.

Step 2: New values

L is increased by 2L, so the new inductance is $L + 2L = 3L$. The capacitance becomes $9C$.

$$f' = \frac{1}{2\pi\sqrt{3L \cdot 9C}} = \frac{1}{2\pi\sqrt{27LC}} = \frac{f}{\sqrt{27}} = \frac{f}{3\sqrt{3}}$$

This is option (D).

Final Answer:

The new resonant frequency is $\frac{f}{3\sqrt{3}}$, option (D).

$$\boxed{\frac{f}{3\sqrt{3}}}$$

Quick Tip: Resonant frequency $f = 1 / (2 \pi \text{ sqrt}(LC))$.

39. In an LCR series circuit, at resonance,

- (A) the impedance is maximum
- (B) the current is minimum
- (C) the current leads the voltage by $\frac{\pi}{2}$
- (D) the current and voltage are in phase

Correct Answer: (D) the current and voltage are in phase

Solution:

Step 1: Understanding the Concept

In a series LCR circuit, impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$. At resonance $X_L = X_C$.

Step 2: Check each option

The reactive terms cancel, so $Z = R$, the minimum value. So (A) is wrong.

The current $I = V/R$ is maximum, not minimum, so (B) is wrong.

The phase angle is $\tan \phi = \frac{X_L - X_C}{R} = 0$, so current and voltage are in phase. (C), saying current leads by $\pi/2$, is wrong, and (D) is right.

Final Answer:

Current and voltage are in phase at resonance, option (D).

In phase

Quick Tip: At resonance $X_L = X_C$ so the impedance is minimum and equal to R .

40. The angle of minimum deviation produced by a thin prism in air is δ_1 . What will be the minimum deviation (δ_2) if the prism is immersed in liquid ?

$$[{}^a n_g = \frac{3}{2}, {}^a n_\ell = \frac{1}{3}]$$

- (A) $\delta_2 = 3\delta_1$
(B) $\delta_2 = 5\delta_1$
(C) $\delta_2 = 7\delta_1$
(D) $\delta_2 = 9\delta_1$

Correct Answer: (C) $\delta_2 = 7\delta_1$

Solution:

Step 1: Understanding the Concept

For a thin prism of angle A , the deviation is $\delta = (n - 1)A$, where n is the refractive index of the prism relative to its surroundings.

Step 2: In air

$$\delta_1 = \left(\frac{3}{2} - 1\right)A = \frac{A}{2}$$

Step 3: In the liquid

The relative index is $n = \frac{n_g}{n_\ell} = \frac{3/2}{1/3} = \frac{9}{2}$.

$$\delta_2 = \left(\frac{9}{2} - 1\right)A = \frac{7A}{2} = 7\delta_1$$

So $\delta_2 = 7\delta_1$, option (C).

Final Answer:

$\delta_2 = 7\delta_1$, option (C).

$\delta_2 = 7\delta_1$

Quick Tip: For a thin prism, deviation = $(n - 1) A$ where n is the refractive index relative to the surrounding medium.



41. In the phenomenon of refraction of light for the same angle of incidence

- (A) angle of refraction is same for different colours.
- (B) angle of refraction is different for different colours.
- (C) angle of refraction does not depend upon the colour of the light used.
- (D) angle of refraction depends on critical angle.

Correct Answer: (B) angle of refraction is different for different colours.

Solution:

Step 1: Understanding the Concept

By Snell law, $\sin i = n \sin r$. The refractive index n of a medium changes with wavelength (dispersion).

Step 2: Detailed Explanation

For the same angle of incidence, light of different colours has different n , so the angle of refraction r is different. This is why a prism separates white light into colours.

So (B) is right, and (A) and (C) are wrong. The critical angle (D) is a related result of n , not the cause of the refraction angle.

Final Answer:

The angle of refraction is different for different colours, option (B).

Different for different colours

Quick Tip: The refractive index depends on wavelength, so different colours refract by different amounts.



42. A double slit experiment is immersed in water of refractive index 1.33. The slit separation is 1 mm, distance between slit and screen is 1.33 m. The slits are illuminated by light of wavelength 6300 Å. The fringe width is

- (A) 4.9×10^{-4} m
- (B) 6.3×10^{-4} m
- (C) 8.6×10^{-4} m
- (D) 5.8×10^{-4} m

Correct Answer: (B) 6.3×10^{-4} m

Solution:

Step 1: Understanding the Concept

In a medium of index n , the wavelength becomes $\lambda_m = \lambda/n$, so the fringe width is $\beta = \frac{\lambda D}{n d}$. The wavelength given (6300 angstrom) is the one in air.

Step 2: Compute

$$\beta = \frac{6300 \times 10^{-10} \times 1.33}{1.33 \times 1 \times 10^{-3}} = 6.3 \times 10^{-4} \text{ m}$$

The 1.33 in D and the 1.33 in n cancel. Option (A) would result if the water factor were used twice.

Final Answer:

The fringe width is 6.3×10^{-4} m, option (B).

$$6.3 \times 10^{-4} \text{ m}$$

Quick Tip: Fringe width = $\lambda D / d$ with the wavelength in the medium λ / n .

43. Light of wavelength λ is incident on a single slit of width 'a' and the distance between slit and screen is 'D'. In diffraction pattern, if slit width is equal to the width of the central maximum then 'D' is equal to

- (A) $\frac{a}{\lambda}$
- (B) $\frac{a^2}{\lambda}$
- (C) $\frac{a}{2\lambda}$
- (D) $\frac{a^2}{2\lambda}$

Correct Answer: (D) $\frac{a^2}{2\lambda}$

Solution:

Step 1: Understanding the Concept

In single slit diffraction the width of the central maximum is $2\lambda D/a$, where D is the slit to screen distance.

Step 2: Equate

Setting this equal to the slit width a:

$$\frac{2\lambda D}{a} = a \Rightarrow D = \frac{a^2}{2\lambda}$$

Option (B), a^2/λ , would be the case if the central maximum width were $\lambda D/a$.

Final Answer:

$$D = \frac{a^2}{2\lambda}, \text{ option (D).}$$

$$\boxed{\frac{a^2}{2\lambda}}$$

Quick Tip: Width of the central maximum is $2\lambda D / a$.

44. When a metallic surface is illuminated with a radiation of wavelength ' λ ', the stopping potential is ' V '. If the same surface is illuminated with radiation of wavelength 6λ , the stopping potential is $(\frac{V}{12})$. The threshold wavelength for the surface is

- (A) 5λ
- (B) 10λ
- (C) 11λ
- (D) 12λ

Correct Answer: (C) 11λ

Solution:

Step 1: Understanding the Concept

Einstein equation: $eV_0 = \frac{hc}{\lambda} - \phi$, where $\phi = \frac{hc}{\lambda_0}$. Let $E = \frac{hc}{\lambda}$.

Step 2: Two equations

1. $eV = E - \phi$.

$$2. \frac{eV}{12} = \frac{E}{6} - \phi.$$

Multiply (2) by 12: $eV = 2E - 12\phi$. Equate with (1):

$$E - \phi = 2E - 12\phi \Rightarrow 11\phi = E \Rightarrow \phi = \frac{E}{11}$$

Step 3: Threshold wavelength

$$\frac{hc}{\lambda_0} = \frac{1}{11} \cdot \frac{hc}{\lambda} \Rightarrow \lambda_0 = 11\lambda$$

Final Answer:

The threshold wavelength is 11λ , option (C).

$$\boxed{11\lambda}$$

Quick Tip: Use $eV = hc/\lambda - \phi$ for both wavelengths and eliminate eV .

45. In photoelectric effect experiment, if the frequency of incident radiation (ν) is increased, keeping all other factors constant, the stopping potential, ($\nu > \nu_0$) (ν_0 = threshold frequency)

- (A) decreases
- (B) remains the same
- (C) increases
- (D) becomes zero

Correct Answer: (C) increases

Solution:

Step 1: Understanding the Concept

The stopping potential satisfies $eV_0 = h\nu - h\nu_0$.

Step 2: Detailed Explanation

As ν increases (with $\nu > \nu_0$), the maximum kinetic energy of the emitted electrons increases, so a larger stopping potential is needed.

V_0 is a linear function of frequency with positive slope h/e .

It does not stay the same (B) and it does not decrease (A). It would be zero only at the threshold frequency.

Final Answer:

The stopping potential increases, option (C).

Increases

Quick Tip: $eV_0 = h\nu - \phi$, so V_0 increases with frequency.

46. The electron in hydrogen atom is initially in the second excited state. When it finally moves to ground state, the maximum number of spectral lines emitted are

- (A) 2
- (B) 3
- (C) 4
- (D) 5

Correct Answer: (B) 3

Solution:

Step 1: Understanding the Concept

The ground state is $n = 1$, the first excited state is $n = 2$, and the second excited state is $n = 3$. An electron can come down in steps or directly.

Step 2: Count the lines

Possible transitions: $3 \rightarrow 2$, $2 \rightarrow 1$ and $3 \rightarrow 1$.

$$\frac{n(n-1)}{2} = \frac{3 \times 2}{2} = 3$$

So at most 3 spectral lines appear.

Final Answer:

A maximum of 3 lines are emitted, option (B).

3

Quick Tip: Second excited state means $n = 3$; lines = $n(n-1)/2$.

47. In the hydrogen atom, radii of the first four Bohr orbits are related as

(A) $1 : 2 : 3 : 4$

(B) $\frac{1}{2} : \frac{1}{3} : \frac{1}{4}$

(C) $1 : 4 : 9 : 16$

(D) $\frac{1}{4} : \frac{1}{9} : \frac{1}{16}$

Correct Answer: (C) 1 : 4 : 9 : 16

Solution:

Step 1: Understanding the Concept

For hydrogen, $r_n = r_1 n^2$, where $r_1 = 0.529$ angstrom.

Step 2: Ratio

$$r_1 : r_2 : r_3 : r_4 = 1^2 : 2^2 : 3^2 : 4^2 = 1 : 4 : 9 : 16$$

Option (A) is the ratio of n itself, and (B) and (D) are inverse ratios.

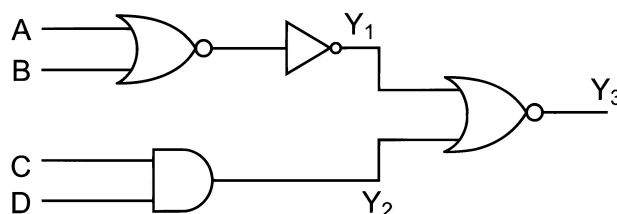
Final Answer:

The radii are in the ratio 1 : 4 : 9 : 16, option (C).

$$\boxed{1 : 4 : 9 : 16}$$

Quick Tip: Bohr radius r is proportional to n squared.

48. In the logic circuit diagram, when all the four inputs, A, B, C, D are 'one' the outputs Y_1 , Y_2 , Y_3 are respectively (1, 1, 0). When the inputs A and C are changed to zero and B and D are still 'one', then the outputs Y_1 , Y_2 , Y_3 are respectively change to



- (A) 1, 1, 1
- (B) 1, 0, 0
- (C) 0, 1, 0
- (D) 0, 0, 1

Correct Answer: (B) 1, 0, 0

Solution:

Step 1: Reading the circuit

The figure shows a NOR gate on inputs A and B feeding a NOT gate whose output is Y_1 . Inputs C and D go to an AND gate with output Y_2 . A final NOR gate takes Y_1 and Y_2 and gives Y_3 .

Step 2: Simplify the gates

$$Y_1 = \overline{\overline{A + B}} = A + B \text{ (OR)}. Y_2 = C \cdot D. Y_3 = \overline{Y_1 + Y_2}.$$

Check the given case: all inputs 1 gives $Y_1 = 1$, $Y_2 = 1$, $Y_3 = 0$, matching (1, 1, 0).

Step 3: New inputs

Now $A = 0$, $B = 1$, $C = 0$, $D = 1$.

$$Y_1 = 0 + 1 = 1$$

$$Y_2 = 0 \cdot 1 = 0$$

$$Y_3 = \overline{1 + 0} = 0$$

So the outputs are (1, 0, 0), option (B).

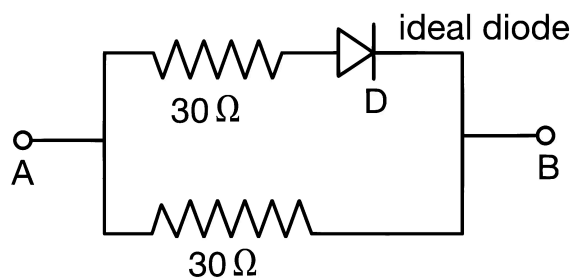
Final Answer:

The outputs are $Y_1, Y_2, Y_3 = 1, 0, 0$, option (B).

(1, 0, 0)

Quick Tip: $Y_1 = \text{NOT}(\text{NOR}(A, B)) = A \text{ OR } B$, $Y_2 = C \text{ AND } D$, $Y_3 = \text{NOR}(Y_1, Y_2)$.

49. In the diagram shown, the resistance between points A and B is ' R_1 ' when an ideal diode D is forward biased and is ' R_2 ' when ideal diode D is reverse biased. The ratio R_1/R_2 is



- (A) 2
- (B) 1
- (C) $\frac{1}{2}$
- (D) $\frac{1}{4}$

Correct Answer: (C) $\frac{1}{2}$

Solution:

Step 1: Reading the circuit

Between A and B there are two branches in parallel. The top branch has a 30 ohm resistor in series with an ideal diode D. The bottom branch has a 30 ohm resistor.

Step 2: Diode forward biased

An ideal forward biased diode has zero resistance, so both branches conduct:

$$R_1 = \frac{30 \times 30}{30 + 30} = 15 \Omega$$

Step 3: Diode reverse biased

An ideal reverse biased diode has infinite resistance, so the top branch carries no current. Only the bottom resistor remains:

$$R_2 = 30 \Omega$$

$$\frac{R_1}{R_2} = \frac{15}{30} = \frac{1}{2}$$

This is option (C).

Final Answer:

The ratio R_1/R_2 is $\frac{1}{2}$, option (C).

$$\boxed{\frac{1}{2}}$$

Quick Tip: Two 30 ohm resistors are in parallel; the diode is in series with one of them.

50. In common emitter amplifier, a change of 0.2 mA in the base current causes a change of 5 mA in the collector current. If input resistance is 2 k Ω and voltage gain is 75, the load resistance used in the circuit is

- (A) 2 k Ω
- (B) 3 k Ω
- (C) 4 k Ω
- (D) 6 k Ω

Correct Answer: (D) 6 k Ω

Solution:

Step 1: Understanding the Concept

The current gain is $\beta = \frac{\Delta I_C}{\Delta I_B}$, and the voltage gain of a CE amplifier is $A_V = \beta \frac{R_L}{R_{in}}$.

Step 2: Compute

$$\beta = \frac{5 \text{ mA}}{0.2 \text{ mA}} = 25$$

$$75 = 25 \times \frac{R_L}{2 \text{ k}\Omega} \Rightarrow R_L = \frac{75 \times 2}{25} = 6 \text{ k}\Omega$$

Option (B), 3 kilo-ohm, would give a voltage gain of only 37.5.

Final Answer:

The load resistance is 6 k Ω , option (D).

$$\boxed{6 \text{ k}\Omega}$$

Quick Tip: Voltage gain = beta x (R_L / R_{in}), with beta = delta I_c / delta I_b.

Chemistry

Questions 51 to 100 | 1 mark each | 50 marks

51. Find the mass of sodium carbonate in grams to form 100 ml 0.1 M solution in water?

(Molar mass of $\text{Na}_2\text{CO}_3 = 106 \text{ g/mol}$)

- (A) 1.06 g
- (B) 3.18 g
- (C) 2.12 g
- (D) 4.16 g

Correct Answer: (A) 1.06 g

Solution:

Step 1: Understanding the Concept

Molarity (M) is the number of moles of solute per litre of solution. So moles of solute = $M \times V$ (in litres), and mass = moles $\times$ molar mass.

Step 2: Key Formula or Approach

$$n = M \times V_{(L)}, \quad w = n \times M_{\text{molar}}$$

Step 3: Detailed Explanation

Volume $V = 100 \text{ mL} = 0.1 \text{ L}$. Moles of Na_2CO_3 :

$$n = 0.1 \times 0.1 = 0.01 \text{ mol}$$

Mass needed:

$$w = 0.01 \times 106 = 1.06 \text{ g}$$

The other options come from using a wrong volume or wrong molar mass. For example, 3.18 g would be 0.03 mol and 2.12 g would be 0.02 mol, which would give 0.3 M and 0.2 M in 100 mL.

Final Answer:

Weighing 1.06 g of sodium carbonate and making the solution up to 100 mL gives 0.1 M. This is option (A).

1.06 g

Quick Tip: Moles = molarity x volume in litres, then multiply by the molar mass.

52. What is number of hydrogen molecules needed to synthesize 3.4 g of ammonia by reaction with nitrogen?

- (A) 6.022×10^{23}
- (B) 1.2044×10^{23}
- (C) 1.8066×10^{23}
- (D) 1.8044×10^{24}

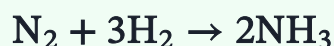
Correct Answer: (C) 1.8066×10^{23}

Solution:

Step 1: Understanding the Concept

The balanced reaction fixes the mole ratio of hydrogen to ammonia.

One mole of any gas has 6.022×10^{23} molecules (Avogadro number).

Step 2: Key Formula or Approach

So 3 mol H_2 gives 2 mol NH_3 .

Step 3: Detailed Explanation

Molar mass of $\text{NH}_3 = 14 + 3 = 17 \text{ g/mol}$.

Moles of ammonia:

$$n_{\text{NH}_3} = \frac{3.4}{17} = 0.2 \text{ mol}$$

Moles of hydrogen needed:

$$n_{\text{H}_2} = 0.2 \times \frac{3}{2} = 0.3 \text{ mol}$$

Number of molecules:

$$0.3 \times 6.022 \times 10^{23} = 1.8066 \times 10^{23}$$

Option (B) is the number for 0.2 mol, which wrongly uses a 1:1 ratio.

Option (A) is for 1 mol.

Final Answer:

About 1.8066×10^{23} hydrogen molecules are needed, option (C).

1.8066×10^{23}

Quick Tip: Write $\text{N}_2 + 3\text{H}_2 \rightarrow 2\text{NH}_3$, find moles of NH_3 and use the 3:2 ratio.



53. If the velocity of the electron in Bohr's first orbit is $2.19 \times 10^6 \text{ Ms}^{-1}$, Calculate the de Broglie wavelength associated with it.

[$h = 6.626 \times 10^{-34} \text{ J s}$ & Mass of electrons = $9.10938 \times 10^{-31} \text{ kg}$]

- (A) 332 pm
- (B) 313 pm
- (C) 342 pm
- (D) 323 pm

Correct Answer: (A) 332 pm

Solution:

Step 1: Understanding the Concept

Every moving particle has an associated wavelength given by de Broglie. For an electron of mass m and speed v , the wavelength is $\lambda = h/(mv)$.

Step 2: Key Formula or Approach

$$\lambda = \frac{h}{mv}$$

Step 3: Detailed Explanation

Put in the values:

$$\lambda = \frac{6.626 \times 10^{-34}}{9.10938 \times 10^{-31} \times 2.19 \times 10^6}$$

Denominator: $9.10938 \times 2.19 = 19.95$, so it is 1.995×10^{-24} .

$$\lambda = \frac{6.626 \times 10^{-34}}{1.995 \times 10^{-24}} = 3.32 \times 10^{-10} \text{ m}$$

Since $1 \text{ pm} = 10^{-12} \text{ m}$, $\lambda = 332 \text{ pm}$. This equals the circumference of the first Bohr orbit (2π times 52.9 pm), as expected for one full wave. The other options are nearby numbers that do not follow from the data.

Final Answer:

The de Broglie wavelength is 332 pm , option (A).

332 pm

Quick Tip: Use $\lambda = h/(m v)$ with the given speed and convert metres to picometres.

54. Identify the group of periodic table that contains element Astatine.

- (A) 17
- (B) 16
- (C) 18
- (D) 1

Correct Answer: (A) 17

Solution:

Step 1: Understanding the Concept

Elements are placed in groups by their outer electron configuration. The halogens (F, Cl, Br, I, At) all have the outer configuration ns^2np^5 .

Step 2: Detailed Explanation

Astatine (atomic number 85) has the configuration ending in $6s^26p^5$.

Seven valence electrons place it in group 17, below iodine.

Group 16 holds the chalcogens (O, S, Se, Te, Po), group 18 holds the noble gases, and group 1 holds the alkali metals. Astatine does not belong to any of these.

Final Answer:

Astatine is a halogen in group 17, option (A).

17

Quick Tip: Astatine is the heaviest halogen.

55. What type of bond and type of overlapping present in N_2 molecule ?

- (A) Single bond and s–s overlap
- (B) Double bond and s–p overlap
- (C) Triple bond and p–p overlap
- (D) Double bond and p–p overlap

Correct Answer: (C) Triple bond and p–p overlap

Solution:

Step 1: Understanding the Concept

Nitrogen has the configuration $2s^2 2p^3$, so each atom has three unpaired p electrons. Each atom needs three more electrons to complete the octet, so two atoms share three pairs.

Step 2: Detailed Explanation

The molecule is $N \equiv N$, a triple bond. Taking the molecular axis as z:

1. One σ bond forms by head-on (axial) overlap of $2p_z$ orbitals.
2. Two π bonds form by sidewise overlap of $2p_x$ and $2p_y$ orbitals.

All three bonds are made from p orbitals, so the overlap is p-p.

Option (A) is the s-s overlap seen in H_2 . Options (B) and (D) call it a double bond, which is wrong because N_2 has three shared pairs.

Option (B) also mentions s-p overlap, as in HF.

Final Answer:

N_2 has a triple bond formed by p-p overlap, option (C).

Triple bond, p-p overlap

Quick Tip: N_2 has a triple bond: one sigma from p-p axial overlap and two pi bonds from sidewise p-p overlap.

56. Calculate the new pressure of the gas enclosed in a cylinder when it compressed from 5 L to 2 L at 1 atm and 300 K. The final temperature in this process is 500 K.

- (A) 1.5 atm
- (B) 3.12 atm
- (C) 2.5 atm
- (D) 4.16 atm

Correct Answer: (D) 4.16 atm

Solution:

Step 1: Understanding the Concept

Here volume and temperature both change, so neither Boyle nor Charles alone applies. We use the combined gas law for a fixed amount of gas.

Step 2: Key Formula or Approach

$$\frac{P_1 V_1}{T_1} = \frac{P_2 V_2}{T_2}$$

Step 3: Detailed Explanation

Given $P_1 = 1 \text{ atm}$, $V_1 = 5 \text{ L}$, $T_1 = 300 \text{ K}$, $V_2 = 2 \text{ L}$, $T_2 = 500 \text{ K}$.

$$P_2 = P_1 \times \frac{V_1}{V_2} \times \frac{T_2}{T_1} = 1 \times \frac{5}{2} \times \frac{500}{300}$$

$$P_2 = 2.5 \times 1.667 = 4.16 \text{ atm}$$

Option (C), 2.5 atm, is what you get by using Boyle law alone and ignoring the temperature rise. The temperature rise from 300 K to 500 K pushes the pressure higher still.

Final Answer:

The new pressure is 4.16 atm, option (D).

4.16 atm

Quick Tip: Use the combined gas law $P_1V_1/T_1 = P_2V_2/T_2$ with the final temperature of 500 K.

57. For a certain reaction, ΔH^0 is -345 kJ and ΔS^0 is -123JK^{-1} . At what temperature the change over from spontaneous to nonspontaneous will occur?

- (A) 1052 K
- (B) 1956 K
- (C) 2568 K
- (D) 2805 K

Correct Answer: (D) 2805 K

Solution:**Step 1: Understanding the Concept**

A reaction is spontaneous when $\Delta G < 0$. Since $\Delta G = \Delta H - T\Delta S$, the switch between spontaneous and nonspontaneous happens where $\Delta G = 0$.

Step 2: Key Formula or Approach

$$T = \frac{\Delta H}{\Delta S}$$

Step 3: Detailed Explanation

Convert units first. $\Delta H = -345 \text{ kJ} = -345000 \text{ J}$, and $\Delta S = -123 \text{ J/K}$.

$$T = \frac{-345000}{-123} = 2804.9 \approx 2805 \text{ K}$$

Both ΔH and ΔS are negative. Below 2805 K the $-T\Delta S$ term is small, so ΔG is negative and the reaction is spontaneous. Above it, ΔG turns positive. Leaving ΔH in kJ would give a wrong value near 2.8 K.

Final Answer:

The changeover occurs at 2805 K, option (D).

2805 K

Quick Tip: At the changeover temperature, ΔG is zero, so $T = \Delta H / \Delta S$.

58. For a reaction to be spontaneous at all temperatures, the values of enthalpy change and entropy change should be

- (A) $\Delta H > 0$ and $\Delta S > 0$
- (B) $\Delta H < 0$ and $\Delta S > 0$
- (C) $\Delta H < 0$ and $\Delta S < 0$
- (D) $\Delta H > 0$ and $\Delta S < 0$

Correct Answer: (B) $\Delta H < 0$ and $\Delta S > 0$

Solution:

Step 1: Understanding the Concept

The Gibbs equation is $\Delta G = \Delta H - T\Delta S$. A reaction is spontaneous when ΔG is negative.

Step 2: Detailed Explanation

Check each sign combination:

(A) $\Delta H > 0, \Delta S > 0$: ΔG is negative only at high T. Not at all temperatures.

(B) $\Delta H < 0, \Delta S > 0$: both terms push ΔG negative, since $-T\Delta S$ is negative. This holds at every T.

(C) $\Delta H < 0, \Delta S < 0$: spontaneous only at low T.

(D) $\Delta H > 0, \Delta S < 0$: ΔG is always positive, so never spontaneous.

Final Answer:

Only option (B) gives a negative ΔG at all temperatures.

$$\Delta H < 0, \Delta S > 0$$

Quick Tip: We need delta G negative at every T, so enthalpy must fall and entropy must rise.

59. Find Δn when 1 mol of each $\text{NH}_{3(g)}$ and $\text{HCl}_{(g)}$, reacts to form solid NH_4Cl .

- (A) 1
- (B) -2
- (C) -1
- (D) -3

Correct Answer: (B) -2

Solution:

Step 1: Understanding the Concept

Δn_g is the change in moles of gas: moles of gaseous products minus moles of gaseous reactants. Solids and liquids are not counted.

Step 2: Detailed Explanation

The reaction is:



Gaseous reactants = 1 + 1 = 2 mol. Gaseous products = 0 (the product is solid).

$$\Delta n = 0 - 2 = -2$$

The value +1 or -1 would come from counting the solid or miscounting a reactant.

Final Answer:

$\Delta n = -2$, option (B).

$$\boxed{-2}$$

Quick Tip: Count only gaseous moles: products minus reactants.

60. What is the solubility product of binary sparingly soluble salt BA if 100 mL saturated solution of salt consist 10^{-4} moles at room temperature?

- (A) 1×10^{-4}
- (B) 1×10^{-6}
- (C) 1×10^{-8}
- (D) 1×10^{-10}

Correct Answer: (B) 1×10^{-6}

Solution:

Step 1: Understanding the Concept

A binary salt BA dissociates as $BA \rightleftharpoons B^+ + A^-$. If the solubility is S mol/L, then $[B^+] = [A^-] = S$ and $K_{sp} = S^2$.

Step 2: Detailed Explanation

100 mL of saturated solution contains 10^{-4} mol. For one litre:

$$S = \frac{10^{-4}}{0.1} = 10^{-3} \text{ mol/L}$$

$$K_{sp} = S^2 = (10^{-3})^2 = 10^{-6}$$

Option (A) forgets to convert the volume to one litre; (C) and (D) come from using the wrong power.

Final Answer:

$K_{sp} = 1 \times 10^{-6}$, option (B).

$$1 \times 10^{-6}$$

Quick Tip: Find solubility in mol/L from 1×10^{-4} mol in 100 mL, then $K_{sp} = S$ squared.

61. Calculate pOH of 0.001M HCl solution.

- (A) 11
- (B) 10
- (C) 12
- (D) 3

Correct Answer: (A) 11

Solution:**Step 1: Understanding the Concept**

HCl is a strong acid and ionises completely, so $[H^+]$ equals the acid concentration. At 25 degrees C, $pH + pOH = 14$.

Step 2: Detailed Explanation

$$[H^+] = 10^{-3} \text{ M} \Rightarrow pH = 3$$

$$pOH = 14 - 3 = 11$$

Option (D), 3, is the pH, not the pOH. Options (B) and (C) are not consistent with pH 3.

Final Answer:

The pOH is 11, option (A).

11

Quick Tip: HCl is a strong acid, so pH = 3 and pOH = 14 - 3.

62. Identify from following the conjugate base of $[\text{Zn}(\text{H}_2\text{O})_4]^{2+}$

- (A) $[\text{Zn}(\text{H}_2\text{O})_4]^{2-} \text{NH}_3$
- (B) $[\text{Zn}(\text{H}_2\text{O})_3]^{2-}$
- (C) $[\text{Zn}(\text{H}_2\text{O})_3\text{OH}]^+$
- (D) $[\text{Zn}(\text{H}_2\text{O})\text{H}]^{3+}$

Correct Answer: (C) $[\text{Zn}(\text{H}_2\text{O})_3\text{OH}]^+$

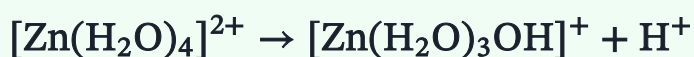
Solution:

Step 1: Understanding the Concept

In the Bronsted-Lowry idea, the conjugate base of an acid is what is left after the acid gives away one proton. The charge drops by 1 and one H is lost.

Step 2: Detailed Explanation

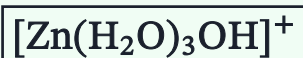
The acid is $[\text{Zn}(\text{H}_2\text{O})_4]^{2+}$. A coordinated water molecule loses H^+ and becomes OH^- :



So the conjugate base is $[\text{Zn}(\text{H}_2\text{O})_3\text{OH}]^+$. Option (B) has charge 2- and no hydroxide, which cannot arise by losing one proton, and (D) adds an H instead of removing it.

Final Answer:

The conjugate base is $[\text{Zn}(\text{H}_2\text{O})_3\text{OH}]^+$, option (C).



Quick Tip: A conjugate base forms by removing one H^+ from the acid, leaving charge one unit lower.

63. For the following redox reaction, find the correct statement.



- (A) Sn^{2+} is undergoing oxidation.
- (B) Fe^{3+} is undergoing oxidation.
- (C) It is not a redox reaction.
- (D) Both Sn^{2+} and Fe^{3+} are oxidized.

Correct Answer: (A) Sn^{2+} is undergoing oxidation.

Solution:

Step 1: Understanding the Concept

Oxidation is loss of electrons (oxidation number rises). Reduction is gain of electrons (oxidation number falls).

Step 2: Detailed Explanation

$\text{Sn}^{2+} \rightarrow \text{Sn}^{4+}$: oxidation number goes from +2 to +4, so it loses 2 electrons and is oxidised.

$\text{Fe}^{3+} \rightarrow \text{Fe}^{2+}$: oxidation number falls from +3 to +2, so it gains an electron and is reduced.

So (A) is correct. (B) reverses the roles, (C) is wrong because electrons are transferred, and (D) is wrong because Fe^{3+} is reduced.

Final Answer:

Only Sn^{2+} is oxidised, option (A).

Sn^{2+} is oxidised

Quick Tip: The species that loses electrons is oxidised; Sn goes from +2 to +4.

64. Which of the following is the most abundant element in universe?

- (A) H
- (B) O
- (C) Ne
- (D) C

Correct Answer: (A) H

Solution:

Step 1: Understanding the Concept

Abundance in the universe is set by how elements were formed. Stars and the early universe made mostly the lightest elements.

Step 2: Detailed Explanation

Hydrogen is by far the most abundant element, about 75 percent of the visible mass of the universe, followed by helium at about 24 percent. Oxygen, carbon and neon are far less abundant. Oxygen is the most abundant element in the Earth crust, which is a different question.

Final Answer:

Hydrogen is the most abundant element in the universe, option (A).

H

Quick Tip: Hydrogen makes up about three quarters of ordinary matter by mass.



65. What is the correct IUPAC name for a molecule that has two amino groups in opposing (para) locations around a benzene ring?

- (A) Benzenediamine
- (B) Benzene-1,4-diamine
- (C) p-Aminoaniline
- (D) 4-Aminobenzenamine

Correct Answer: (B) Benzene-1,4-diamine

Solution:**Step 1: Understanding the Concept**

When benzene carries two identical principal groups, the name uses

the suffix with a multiplying prefix and locants, such as benzene-1,4-diamine.

Step 2: Detailed Explanation

The para positions are carbon 1 and carbon 4. The parent is benzene, the groups are two -NH_2 groups, so the name is benzene-1,4-diamine.

(A) Benzenediamine has no locants, so it does not say which positions. (C) p-Aminoaniline is a common-style name, not IUPAC. (D) 4-Aminobenzenamine is not the preferred IUPAC form for two identical groups.

Final Answer:

The correct IUPAC name is benzene-1,4-diamine, option (B).

Benzene-1,4-diamine

Quick Tip: Number the ring so the two amino groups get the lowest locants: 1,4.

66. What is IUPAC name of the compound $\text{CH}_3\text{CH}=\text{CHCH}=\text{CHCOOH}$?

- (A) Hexanedioc acid
- (B) Hexa-2,4-dienoic acid
- (C) Penta-1,3-dienoic acid
- (D) Pentenedioc acid

Correct Answer: (B) Hexa-2,4-dienoic acid

Solution:

Step 1: Understanding the Concept

For a carboxylic acid the COOH carbon is C-1. The chain is the longest one containing it, and the suffix is -oic acid.

Step 2: Detailed Explanation

Count carbons: CH₃ (6), CH (5), CH (4), CH (3), CH (2), COOH (1).

So there are 6 carbons.

Double bonds are between C2-C3 and C4-C5, giving "hexa-2,4-diene" and the acid name hexa-2,4-dienoic acid.

(A) hexanedioic acid is a diacid, (C) and (D) have the wrong number of carbons.

Final Answer:

The name is hexa-2,4-dienoic acid, option (B).

Hexa-2,4-dienoic acid

Quick Tip: Number from the COOH carbon and place the two double bonds at 2 and 4.

67. Match List I with List II

List I

(A). Inductive effect

(B). Hyper-conjugation

(C). Resonance effect

List II

(I). Delocalization of π electrons

(II). Displacement of π electrons

(III). Delocalization of σ electrons

(IV). Displacement of σ electrons

Choose the **correct** answer from the options given below:

(A) (A) – (IV), (B) – (III), (C) – (I)

(B) (A) – (II), (B) – (IV), (C) – (I)

(C) (A) – (I), (B) – (II), (C) – (IV)

(D) (A) – (III), (B) – (IV), (C) – (I)

Correct Answer: (A) (A) – (IV), (B) – (III), (C) – (I)

Solution:

Step 1: Understanding the Concept

Electronic effects describe how electrons shift in a molecule. The key is whether sigma or pi electrons are involved, and whether they are displaced or delocalised.

Step 2: Match (A) Inductive effect

It is the permanent shift of sigma bond electrons along a chain due to electronegativity difference. So (A) matches (IV), displacement of sigma electrons.

Step 3: Match (B) Hyperconjugation

It is the delocalisation of sigma electrons of a C-H bond next to a pi

bond or empty p orbital. So (B) matches (III), delocalisation of sigma electrons.

Step 4: Match (C) Resonance effect

It is the delocalisation of pi electrons (or lone pairs) through conjugated pi systems. So (C) matches (I). Item (II), displacement of pi electrons, is the electromeric effect and is not asked.

Final Answer:

The correct matching is (A)-(IV), (B)-(III), (C)-(I), option (A).

$(A) - (IV), (B) - (III), (C) - (I)$

Quick Tip: Inductive effect moves sigma electrons, hyperconjugation delocalises sigma electrons, resonance delocalises pi electrons.

68. What are the reactants involved in Friedal Crafts alkylation?

- (A) $C_6H_6 + NH_3$
- (B) $C_6H_6 + CH_3Cl$
- (C) $C_6H_6 + CH_3COCl$
- (D) $C_6H_6 + CH_4$

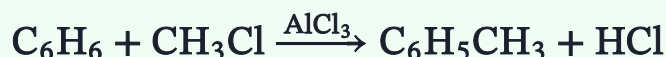
Correct Answer: (B) $C_6H_6 + CH_3Cl$

Solution:

Step 1: Understanding the Concept

Friedel-Crafts alkylation puts an alkyl group on a benzene ring using an alkyl halide and a Lewis acid, anhydrous $AlCl_3$.

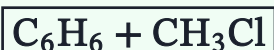
Step 2: Detailed Explanation



(A) with NH_3 gives no such reaction. (C) with CH_3COCl is the acylation reaction, giving acetophenone. (D) methane is not an alkylating agent here.

Final Answer:

The alkylation reactants are benzene and methyl chloride, option (B).



Quick Tip: Alkylation needs an alkyl halide with anhydrous AlCl_3 ; acylation needs an acyl chloride.

69. Identify the false statement regarding magnetic properties of substance.

- (A) Paramagnetic substances are weakly attracted.
- (B) Diamagnetic substances are weakly attracted.
- (C) Ferromagnetic substances strongly attracted.
- (D) Diamagnetic substance strongly attracted

- (A) (A) and (C) only
- (B) (A), (B), (C) and (D)
- (C) (B) and (D) only
- (D) (A) and (B) only

Correct Answer: (C) (B) and (D) only

Solution:

Step 1: Understanding the Concept

Substances respond to a magnetic field in three main ways.

Paramagnetic ones are weakly attracted, diamagnetic ones are weakly repelled, and ferromagnetic ones are strongly attracted.

Step 2: Check statement (A)

Paramagnetic substances have unpaired electrons and are weakly attracted. So (A) is TRUE.

Step 3: Check statement (B)

Diamagnetic substances are weakly repelled by a field, not attracted. So (B) is FALSE.

Step 4: Check statement (C)

Ferromagnetic substances such as iron are strongly attracted. So (C) is TRUE.

Step 5: Check statement (D)

Diamagnetic substances are not strongly attracted; they are weakly repelled. So (D) is FALSE.

Final Answer:

The false statements are (B) and (D) only, option (C).

(B) and (D) only

Quick Tip: Diamagnetic substances are weakly repelled, not attracted.

70. Calculate the number of atoms present in 1.625 g metal if it forms bcc unit cell structure.

$$[\rho \times a^3 = 3.25 \times 10^{-22} \text{g}]$$

(A) 2.0×10^{22}

(B) 1.5×10^{22}

(C) 2.5×10^{22}

(D) 1.0×10^{22}

Correct Answer: (D) 1.0×10^{22}

Solution:

Step 1: Understanding the Concept

Density is mass per volume, so for one unit cell, mass = $\rho \times a^3$. A bcc unit cell contains 2 atoms (8 corners at 1/8 each plus 1 body centre).

Step 2: Detailed Explanation

Mass of one unit cell = 3.25×10^{-22} g.

Number of unit cells in 1.625 g:

$$\frac{1.625}{3.25 \times 10^{-22}} = 0.5 \times 10^{22} = 5 \times 10^{21}$$

Atoms = cells $\times$ 2:

$$5 \times 10^{21} \times 2 = 1.0 \times 10^{22}$$

Option (C), 2.5×10^{22} , comes from assuming 5 atoms per cell, and (A) from 4 atoms (fcc) and a slip.

Final Answer:

The metal sample contains 1.0×10^{22} atoms, option (D).

$$1.0 \times 10^{22}$$

Quick Tip: Mass of one unit cell = $\rho \times a^3$, and a bcc cell holds 2 atoms.

71. Identify the covalent crystal from the following.

- (A) Fe
- (B) CH₄
- (C) SiO₂
- (D) KCl

Correct Answer: (C) SiO₂

Solution:**Step 1: Understanding the Concept**

Crystals are classified by the particles at lattice points and the forces between them: ionic, metallic, molecular, and covalent (network).

Step 2: Detailed Explanation

SiO₂ (quartz) is a giant network where each Si bonds to four O atoms, so it is a covalent crystal.

Fe is a metallic crystal, CH₄ is a molecular crystal held by weak van der Waals forces, and KCl is an ionic crystal.

Final Answer:

SiO₂ is the covalent crystal, option (C).



Quick Tip: Covalent (network) crystals have atoms joined by a continuous network of covalent bonds, such as SiO_2 .



72. If a 0.4 molal solution of a nonvolatile solute in organic solvent decreases its freezing point by 2.4 K., then the molal depression constant of solvent will be

- (A) $3.0 \text{ K kg mol}^{-1}$
- (B) $4.0 \text{ K kg mol}^{-1}$
- (C) $5.0 \text{ K kg mol}^{-1}$
- (D) $6.0 \text{ K kg mol}^{-1}$

Correct Answer: (D) $6.0 \text{ K kg mol}^{-1}$

Solution:

Step 1: Understanding the Concept

Depression of freezing point is a colligative property: $\Delta T_f = K_f m$, where m is molality.

Step 2: Detailed Explanation

$$K_f = \frac{\Delta T_f}{m} = \frac{2.4}{0.4} = 6.0 \text{ K kg mol}^{-1}$$

Options (A), (B), (C) would need a freezing point drop of 1.2 K, 1.6 K and 2.0 K respectively for a 0.4 molal solution, so they do not fit.

Final Answer:

$K_f = 6.0 \text{ K kg mol}^{-1}$, option (D).

$$6.0 \text{ K kg mol}^{-1}$$

Quick Tip: Use $\Delta T_f = K_f \times m$.

73. Which of the following colligative properties is useful to determine the molar masses of proteins, polymers or colloids, with the greatest precision?

- (A) Elevation in boiling point
- (B) Depression in freezing point
- (C) Osmotic pressure
- (D) Relative lowering of vapour pressure

Correct Answer: (C) Osmotic pressure

Solution:**Step 1: Understanding the Concept**

Large molecules such as proteins and polymers have low solubility and a very high molar mass, so their solutions have a tiny mole fraction.

Step 2: Detailed Explanation

For such solutions, boiling point elevation, freezing point depression and vapour pressure lowering are too small to measure accurately. Osmotic pressure $\pi = CRT$ is large even for very dilute solutions. It can be measured at room temperature, which also avoids heating that

could damage biomolecules. So osmotic pressure gives the greatest precision for molar mass of macromolecules.

Final Answer:

Osmotic pressure is used, option (C).

Osmotic pressure

Quick Tip: Osmotic pressure is large enough to measure at low concentrations and at room temperature.

74. Calculate the concentration of silver nitrate solution if molar conductivity and conductivity of silver nitrate at 25°C are respectively $120 \text{ ohm}^{-1} \text{ cm}^2 \text{ mol}^{-1}$ and $0.0024 \text{ ohm}^{-1} \text{ cm}^{-1}$.

- (A) 0.01 M
- (B) 0.02 M
- (C) 0.03 M
- (D) 0.04 M

Correct Answer: (B) 0.02 M

Solution:

Step 1: Understanding the Concept

Molar conductivity Λ_m relates the conductivity κ of a solution to its molar concentration c (in mol/L).

Step 2: Key Formula or Approach

$$\Lambda_m = \frac{\kappa \times 1000}{c}$$

Step 3: Detailed Explanation

$$c = \frac{\kappa \times 1000}{\Lambda_m} = \frac{0.0024 \times 1000}{120} = \frac{2.4}{120} = 0.02 \text{ M}$$

The factor 1000 converts cm^3 to litres. The other options would come from wrongly using 1000 or dividing by a different value.

Final Answer:

The concentration is 0.02 M, option (B).

0.02 M

Quick Tip: Molar conductivity = (conductivity x 1000) / concentration.

75. The E_{cell}^0 of $\text{Al}_{(s)}|\text{Al}^{3+}(1\text{M})||\text{Pb}^{2+}(1\text{M})|\text{Pb}_{(s)}$ cell is 1.5 V if E_{Pb}^0 is -0.14 V then E_{Al}^0 will be

- (A) 1.64 V
- (B) -1.64 V
- (C) 1.36 V
- (D) -1.36 V

Correct Answer: (B) -1.64 V

Solution:

Step 1: Understanding the Concept

In the cell notation anode then cathode, the left electrode is the anode (oxidation) and the right one is the cathode (reduction). $E_{cell}^0 = E_{cathode}^0 - E_{anode}^0$, using reduction potentials.

Step 2: Detailed Explanation

Here Al is the anode and Pb is the cathode.

$$1.5 = (-0.14) - E_{Al}^0$$

$$E_{Al}^0 = -0.14 - 1.5 = -1.64 \text{ V}$$

A positive value like +1.64 V (A) would be the oxidation potential, not the reduction potential asked for. Aluminium is more reactive than lead, so a negative reduction potential is expected.

Final Answer:

$E_{Al}^0 = -1.64 \text{ V}$, option (B).

-1.64 V

Quick Tip: $E_{cell} = E_{cathode} - E_{anode}$; lead is the cathode and aluminium the anode.

76. Which of the following is an electrolyte in dry cell ?

- (A) Potassium hydroxide
- (B) Sulphuric acid
- (C) Ammonium chloride and zinc chloride
- (D) Manganese dioxide

Correct Answer: (C) Ammonium chloride and zinc chloride

Solution:

Step 1: Understanding the Concept

A dry cell (Leclanche cell) has a zinc container as the anode and a graphite rod surrounded by manganese dioxide as the cathode. The electrolyte is the medium between them that carries ions.

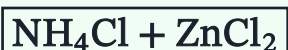
Step 2: Detailed Explanation

The electrolyte is a moist paste of ammonium chloride (NH_4Cl) and zinc chloride (ZnCl_2).

(A) Potassium hydroxide is the electrolyte of the alkaline cell and mercury cell, not the ordinary dry cell. (B) Sulphuric acid is used in the lead storage battery. (D) Manganese dioxide is a depolariser that mixes with carbon at the cathode, so it is not the electrolyte.

Final Answer:

The electrolyte is ammonium chloride with zinc chloride, option (C).



Quick Tip: The Leclanche dry cell uses a moist paste of NH_4Cl and ZnCl_2 .

77. The rate of reaction $A + B \rightarrow P$ is $4 \times 10^{-2} \text{ mol dm}^{-3} \text{ s}^{-1}$

When $[A] = 0.2 \text{ mole dm}^{-3}$ and $[B] = 0.1 \text{ mole dm}^{-3}$, What is the rate constant of reaction, if it is first order with respect to A and second order with respect to B ?

(A) $10 \text{ mol}^{-2} \text{ dm}^6 \text{ s}^{-1}$

(B) $20 \text{ mol}^{-2} \text{ dm}^6 \text{ s}^{-1}$

(C) $25 \text{ mol}^{-2} \text{ dm}^6 \text{ s}^{-1}$

(D) $40 \text{ mol}^{-2} \text{ dm}^6 \text{ s}^{-1}$

Correct Answer: (B) $20 \text{ mol}^{-2} \text{ dm}^6 \text{ s}^{-1}$

Solution:

Step 1: Understanding the Concept

The rate law with order 1 in A and order 2 in B is $r = k[A][B]^2$. The overall order is 3, so the unit of k is $\text{mol}^{-2} \text{ dm}^6 \text{ s}^{-1}$.

Step 2: Detailed Explanation

Given $r = 4 \times 10^{-2}$, $[A] = 0.2$, $[B] = 0.1$.

$$k = \frac{r}{[A][B]^2} = \frac{4 \times 10^{-2}}{0.2 \times (0.1)^2}$$

$$k = \frac{4 \times 10^{-2}}{0.2 \times 0.01} = \frac{4 \times 10^{-2}}{2 \times 10^{-3}} = 20$$

So $k = 20 \text{ mol}^{-2} \text{ dm}^6 \text{ s}^{-1}$. Option (A), 10, comes from forgetting to square [B] properly, and (C), 25, from a wrong denominator.

Final Answer:

The rate constant is $20 \text{ mol}^{-2} \text{ dm}^6 \text{ s}^{-1}$, option (B).

$$20 \text{ mol}^{-2} \text{ dm}^6 \text{ s}^{-1}$$

Quick Tip: Rate = $k [A][B]^2$, so $k = \text{rate} / ([A][B]^2)$.

78. For the reaction $2A \longrightarrow 3C + D$, the rate of reaction is represented by

- (A) $-\frac{d[A]}{dt} = -\frac{d[D]}{dt}$
- (B) $-\frac{d[C]}{dt} = -2\frac{d[A]}{dt}$
- (C) $-\frac{d[A]}{dt} = -\frac{d[D]}{dt}$
- (D) $-\frac{1}{2} \frac{d[A]}{dt} = +\frac{1}{3} \frac{d[C]}{dt}$

Correct Answer: (D) $-\frac{1}{2} \frac{d[A]}{dt} = +\frac{1}{3} \frac{d[C]}{dt}$

Solution:

Step 1: Understanding the Concept

For $aA \rightarrow bC + cD$, the reaction rate is $-\frac{1}{a} \frac{d[A]}{dt} = \frac{1}{b} \frac{d[C]}{dt} = \frac{1}{c} \frac{d[D]}{dt}$. The reactant concentration falls (negative sign) and products rise (positive sign).

Step 2: Detailed Explanation

Here $a = 2$, $b = 3$, $c = 1$:

$$\text{Rate} = -\frac{1}{2} \frac{d[A]}{dt} = +\frac{1}{3} \frac{d[C]}{dt} = +\frac{d[D]}{dt}$$

Option (D) states exactly the first equality. In options (A) and (C), $-\frac{d[A]}{dt}$ is a positive number but $-\frac{d[D]}{dt}$ is negative because D is formed, and the coefficient 2 is also missing. In option (B), $-\frac{d[C]}{dt}$ is negative while $-2 \frac{d[A]}{dt}$ is positive, and the ratio of coefficients is

also wrong.

Final Answer:

Option (D) is the correct form of the rate expression.

$$-\frac{1}{2} \frac{d[A]}{dt} = +\frac{1}{3} \frac{d[C]}{dt}$$

Quick Tip: Divide each rate by its stoichiometric coefficient; reactant rates carry a minus sign, products a plus sign.



79. Which of the following is applicable to thin film?

- (A) One dimensional nanostructure
- (B) Zero dimensional nanostructure
- (C) Two dimensional nanostructure
- (D) Three dimensional nanostructure

Correct Answer: (C) Two dimensional nanostructure

Solution:

Step 1: Understanding the Concept

Nanostructures are classified by how many dimensions are NOT confined to the nanoscale (below 100 nm).

Step 2: Detailed Explanation

A thin film has nanometre thickness but is large in length and width. Only one dimension is confined, so electrons move freely in two dimensions. It is a two dimensional (2D) nanostructure.

Nanowires and nanotubes are one dimensional, quantum dots and

nanoparticles are zero dimensional, and bulk materials are three dimensional.

Final Answer:

A thin film is a two dimensional nanostructure, option (C).

Two dimensional

Quick Tip: A thin film is confined in one direction only, so it is two dimensional.

80. What is the role of catalyst in catalytic reaction ?

- (A) It decreases the enthalpy of reaction.
- (B) It increases the enthalpy of reaction.
- (C) It decreases the energy of activation of reaction.
- (D) It increases the energy of activation of reaction.

Correct Answer: (C) It decreases the energy of activation of reaction.

Solution:

Step 1: Understanding the Concept

Activation energy is the minimum energy colliding molecules need to react. A catalyst changes the pathway, not the start and end states.

Step 2: Detailed Explanation

A catalyst offers a different route with a lower activation energy barrier. More molecules can cross it, so the rate increases.

The enthalpy of reaction depends only on reactants and products, so a

catalyst does not change it. Hence (A) and (B) are wrong. Raising the activation energy (D) would slow the reaction, the opposite of catalysis.

Final Answer:

A catalyst decreases the activation energy, option (C).

Decreases activation energy

Quick Tip: A catalyst provides an alternate path with lower activation energy.

81. Which of the following exhibits the minimum coagulating power for precipitating of positively charged ferric oxide sol ?

- (A) KNO_3
- (B) K_2SO_4
- (C) K_3PO_4
- (D) $\text{K}_4\text{Fe}(\text{CN})_6$

Correct Answer: (A) KNO_3

Solution:

Step 1: Understanding the Concept

Hardy-Schulze rule: the coagulating ion is the one with charge opposite to the sol, and its power rises sharply with its charge. Ferric oxide sol is positive, so anions coagulate it.

Step 2: Detailed Explanation

The anions in the options are NO_3^- (charge 1), SO_4^{2-} (2), PO_4^{3-} (3) and $[\text{Fe}(\text{CN})_6]^{4-}$ (4).

Order of power: $[\text{Fe}(\text{CN})_6]^{4-} > \text{PO}_4^{3-} > \text{SO}_4^{2-} > \text{NO}_3^-$.

So KNO_3 , with the monovalent nitrate, has the minimum coagulating power.

Final Answer:

KNO_3 has the least coagulating power, option (A).



Quick Tip: For a positive sol the coagulating ion is the anion; higher charge means higher power (Hardy-Schulze rule).

82. Identify weakest acid from following.

- (A) Hypochlorous acid
- (B) Chlorous acid
- (C) Chloric acid
- (D) Perchloric acid

Correct Answer: (A) Hypochlorous acid

Solution:

Step 1: Understanding the Concept

In oxoacids of the same element, more oxygen atoms pull electron density away from the O-H bond and stabilise the anion. So acidity rises with oxidation state of chlorine.

Step 2: Detailed Explanation

The acids are HOCl (+1), HClO_2 (+3), HClO_3 (+5) and HClO_4 (+7).

Order of acidity: $\text{HOCl} < \text{HClO}_2 < \text{HClO}_3 < \text{HClO}_4$.

Hypochlorous acid, with only one oxygen, is the weakest.

Final Answer:

Hypochlorous acid is the weakest, option (A).



Quick Tip: Acid strength rises with the number of oxygen atoms on chlorine: HClO is weakest.



83. Which of the following elements has completely filled 4 f orbital at the expected ground state configuration?

- (A) La
- (B) Yb
- (C) Eu
- (D) Gd

Correct Answer: (B) Yb

Solution:**Step 1: Understanding the Concept**

The 4f subshell holds 14 electrons. Lanthanoids fill it from La (4f⁰) to Lu (4f¹⁴).

Step 2: Detailed Explanation

La: $4f^0 5d^1 6s^2$. Eu: $4f^7 6s^2$. Gd: $4f^7 5d^1 6s^2$. Yb: $4f^{14} 6s^2$.

Only Yb ($Z = 70$) has the 4f subshell completely filled with 14 electrons.

Final Answer:

Ytterbium has a fully filled 4f subshell, option (B).

Yb

Quick Tip: Ytterbium is [Xe] $4f^{14} 6s^2$.

84. Which of the following elements in +3 oxidation state forms colourless compounds?

- (A) Sc
- (B) Ni
- (C) Cr
- (D) V

Correct Answer: (A) Sc

Solution:

Step 1: Understanding the Concept

Coloured compounds arise from d-d electron transitions. These need at least one d electron and at least one empty d orbital.

Step 2: Detailed Explanation

Sc^{3+} : [Ar] $3d^0$, no d electrons, so it is colourless.

Ni^{3+} : $3d^7$, Cr^{3+} : $3d^3$, V^{3+} : $3d^2$. All have partly filled d orbitals, so they

are coloured.

Final Answer:

Sc in +3 state gives colourless compounds, option (A).

Sc

Quick Tip: Colour comes from d-d transitions, which need partly filled d orbitals; Sc^{3+} has do.



85. Identify the complex having the highest number of unpaired electrons from following.

- (A) $[\text{Co}(\text{NH}_3)_6]^{3+}$
- (B) $[\text{CoF}_6]^{3-}$
- (C) $[\text{NiCl}_4]^{2-}$
- (D) $[\text{Ni}(\text{CN})_4]^{2-}$

Correct Answer: (B) $[\text{CoF}_6]^{3-}$

Solution:

Step 1: Understanding the Concept

Unpaired electrons depend on the metal d count and on whether the ligand is strong field (pairs electrons) or weak field.

Step 2: Check (A) and (B)

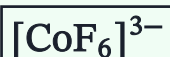
In $[\text{Co}(\text{NH}_3)_6]^{3+}$, Co is +3 (d^6). NH_3 is a strong field ligand, so it is low spin with zero unpaired electrons. In $[\text{CoF}_6]^{3-}$, F^- is weak field, so it is high spin $t_{2g}^4 e_g^2$ with 4 unpaired electrons.

Step 3: Check (C) and (D)

In $[\text{NiCl}_4]^{2-}$, Ni is +2 (d^8) with weak field Cl^- and tetrahedral shape, giving 2 unpaired electrons. In $[\text{Ni}(\text{CN})_4]^{2-}$, strong field CN^- gives square planar, with 0 unpaired.

Final Answer:

The highest is 4 unpaired electrons in $[\text{CoF}_6]^{3-}$, option (B).



Quick Tip: Compare Co^{3+} high spin d^6 in fluoride with Co^{3+} low spin and Ni^{2+} complexes.

86. Which of the following ligands splits d orbitals to maximum level?

- (A) CN^-
- (B) H_2O
- (C) Cl^-
- (D) NH_3

Correct Answer: (A) CN^-

Solution:**Step 1: Understanding the Concept**

The spectrochemical series arranges ligands by the crystal field splitting they cause. Strong field ligands give a large Δ_o .

Step 2: Detailed Explanation

The series runs: $I^- < Br^- < Cl^- < F^- < H_2O < NH_3 < en < CN^- < CO$.

Among the options, CN^- is the strongest, so it splits the d orbitals the most.

Final Answer:

Cyanide causes the largest splitting, option (A).



Quick Tip: Cyanide is at the strong field end of the spectrochemical series.

87. Which of the following is used as solvent for Grignard reagent?

- (A) Alcohol
- (B) Ether
- (C) Phenol
- (D) Ketone

Correct Answer: (B) Ether

Solution:

Step 1: Understanding the Concept

Grignard reagents ($RMgX$) are strongly basic and react with anything having an acidic hydrogen or a carbonyl group.

Step 2: Detailed Explanation

Dry diethyl ether is used. Its oxygen lone pairs coordinate with magnesium and stabilise the reagent, and it has no acidic H.

Alcohol and phenol have O-H hydrogens that destroy the reagent (giving alkane). A ketone has a carbonyl group that reacts with the reagent. So only ether is suitable.

Final Answer:

Dry ether is the solvent, option (B).

Ether

Quick Tip: Grignard reagents are made in dry ether, never in protic solvents.

88. Which of the following is a correct feature of the SN^2 mechanism?

- (A) The SN^2 mechanism proceeds mainly with racemization.
- (B) The SN^2 mechanism includes a transition state with penta coordinating carbon.
- (C) The SN^2 mechanism involves formation of planar carbocation intermediate in the first step.
- (D) The SN^2 mechanism involves formation of carbocation with SP^2 hybridized carbon.

Correct Answer: (B) The SN^2 mechanism includes a transition state with penta coordinating carbon.

Solution:

Step 1: Understanding the Concept

SN^2 is a bimolecular, one step substitution in which the nucleophile attacks from the back side as the leaving group departs.

Step 2: Detailed Explanation

The carbon passes through a transition state where it is partly bonded to both the nucleophile and the leaving group, so it is five-coordinate. This gives inversion of configuration, not racemisation. So (B) is correct.

(A) is wrong because racemisation is typical of SN_1 . (C) and (D) describe a planar sp^2 carbocation formed in the first step, which is the SN_1 mechanism.

Final Answer:

The SN_2 transition state has a pentacoordinate carbon, option (B).

Pentacoordinate transition state

Quick Tip: SN_2 is a one step process with a pentacoordinate transition state and inversion.

89. In benzylic halides, the halogen atom is bonded to

- (A) sp hybridized carbon atom.
- (B) sp^2 hybridized carbon atom.
- (C) sp^3 hybridized carbon atom.
- (D) dsp^2 hybridized carbon atom.

Correct Answer: (C) sp^3 hybridized carbon atom.

Solution:

Step 1: Understanding the Concept

A benzylic halide has the halogen on a carbon directly attached to a benzene ring, for example $\text{C}_6\text{H}_5\text{CH}_2\text{Cl}$.

Step 2: Detailed Explanation

The carbon bearing Cl has four single bonds (two H, one ring carbon, one Cl). Four sigma bonds mean sp^3 hybridisation.

An sp^2 carbon bearing the halogen would be an aryl or vinyl halide, which is a different class.

Final Answer:

The halogen is on an sp^3 carbon, option (C).



Quick Tip: In a benzylic halide the halogen is on a CH_2 carbon next to the ring, which is sp^3 .

90. Method of preparation of alkyl halide

This reaction is known as

- (A) Finkelstein reaction
- (B) Swartz reaction
- (C) Sandmeyer reaction
- (D) Wurtz reaction

Correct Answer: (B) Swartz reaction

Solution:

Step 1: Understanding the Concept

Alkyl fluorides cannot be made by direct halogenation of alkanes, so they are made by halogen exchange.

Step 2: Detailed Explanation

Swarts reaction: $\text{R-Br} + \text{AgF}$ (or Hg_2F_2 , CoF_2 , SbF_3) gives R-F .

Finkelstein reaction makes alkyl iodides with NaI in dry acetone.

Sandmeyer reaction converts diazonium salts into aryl halides with CuX . Wurtz reaction couples two alkyl halides with sodium to give alkanes.

Final Answer:

The reaction is the Swarts reaction, option (B).

Swarts reaction

Quick Tip: Heating alkyl bromide or chloride with a metallic fluoride such as AgF or Hg_2F_2 is the Swarts reaction.

91. Which of following on oxidation forms propanone?

- (A) 1-Propanol
- (B) 2-Propanol
- (C) 2-Butanol
- (D) 2-methylbutan-1-ol

Correct Answer: (B) 2-Propanol

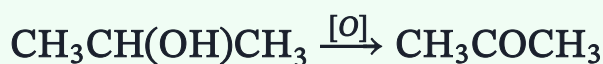
Solution:

Step 1: Understanding the Concept

Primary alcohols oxidise to aldehydes and acids, secondary alcohols oxidise to ketones.

Step 2: Detailed Explanation

Propanone is CH_3COCH_3 , made from 2-propanol:



1-Propanol gives propanal or propanoic acid. 2-Butanol gives butanone. 2-Methylbutan-1-ol is primary and gives an aldehyde or acid.

Final Answer:

2-Propanol gives propanone, option (B).

2-Propanol

Quick Tip: Oxidation of a secondary alcohol gives a ketone; 2-propanol gives acetone.

92. Identify the name of reaction when phenol reacts with chloroform in aqueous NaOH.

- (A) Kolbe's reaction
- (B) Reimer-Tiemann reaction
- (C) Williamson synthesis
- (D) Friedel-Crafts acylation

Correct Answer: (B) Reimer-Tiemann reaction

Solution:

Step 1: Understanding the Concept

Phenol is activated for electrophilic substitution at the ortho position.

Step 2: Detailed Explanation

With chloroform and aqueous NaOH, dichlorocarbene (:CCl_2) forms and attacks the phenoxide ring, giving salicylaldehyde after hydrolysis. This is the Reimer-Tiemann reaction.

Kolbe reaction uses CO_2 with sodium phenoxide to give salicylic acid. Williamson synthesis makes ethers. Friedel-Crafts acylation uses acyl chloride and AlCl_3 .

Final Answer:

The reaction is the Reimer-Tiemann reaction, option (B).

Reimer-Tiemann

Quick Tip: Phenol + CHCl_3 + NaOH gives salicylaldehyde: the Reimer-Tiemann reaction.

93. Which of the following compounds is obtained by Williamson synthesis ?

- (A) only simple ethers
- (B) only mixed ethers
- (C) simple and mixed ethers.
- (D) esters

Correct Answer: (C) simple and mixed ethers.

Solution:

Step 1: Understanding the Concept

In Williamson synthesis an alkoxide ion attacks an alkyl halide by $\text{S}_{\text{N}}2$: $\text{RONa} + \text{R}'\text{X} \rightarrow \text{ROR}' + \text{NaX}$.

Step 2: Detailed Explanation

If R and R' are the same, a simple (symmetrical) ether forms, for example diethyl ether from sodium ethoxide and ethyl bromide. If they differ, a mixed ether forms, for example ethyl methyl ether. It does not give esters, so (D) is wrong, and it works for both types, so (A) and (B) are incomplete.

Final Answer:

It gives both simple and mixed ethers, option (C).

Simple and mixed ethers

Quick Tip: Williamson synthesis uses an alkoxide and an alkyl halide, so it gives both simple and mixed ethers.

94. Which of the following is the strongest carboxylic acid?

- (A) CH_3COOH
- (B) HCOOH
- (C) $\text{CH}_3\text{CH}_2\text{-COOH}$
- (D) ClCH_2COOH

Correct Answer: (D) ClCH_2COOH

Solution:

Step 1: Understanding the Concept

Acid strength depends on how well the carboxylate anion is stabilised. Electron withdrawing groups stabilise it, electron donating alkyl groups destabilise it.

Step 2: Detailed Explanation

CH_3COOH and $\text{CH}_3\text{CH}_2\text{COOH}$ have electron donating alkyl groups, so they are weaker. HCOOH has no donor, so it is stronger than acetic acid.

In ClCH_2COOH , the strongly electronegative Cl shows a $-I$ effect and pulls electron density, stabilising the anion greatly. So chloroacetic acid is the strongest.

Final Answer:

Chloroacetic acid is the strongest, option (D).



Quick Tip: An electron withdrawing Cl on the alpha carbon increases acidity.

95. Which of the following amines is not represented by formula $\text{C}_3\text{H}_9\text{N}$?

- (A) 1° amine
- (B) 2° amine
- (C) 3° amine
- (D) quaternary ammonium salt

Correct Answer: (D) quaternary ammonium salt

Solution:

Step 1: Understanding the Concept

Amines are classified as 1, 2 or 3 degree by how many alkyl groups are on N. A quaternary ammonium salt has four groups and a positive charge, so it is not an amine.

Step 2: Detailed Explanation

Primary: $\text{CH}_3\text{CH}_2\text{CH}_2\text{NH}_2$ (propylamine) and isopropylamine, formula $\text{C}_3\text{H}_9\text{N}$.

Secondary: $\text{CH}_3\text{NHCH}_2\text{CH}_3$, also $\text{C}_3\text{H}_9\text{N}$.

Tertiary: $(\text{CH}_3)_3\text{N}$, also $\text{C}_3\text{H}_9\text{N}$.

A quaternary ammonium salt needs at least four carbons, as in tetramethylammonium, so it cannot match.

Final Answer:

A quaternary ammonium salt is not represented by $\text{C}_3\text{H}_9\text{N}$, option (D).

Quaternary ammonium salt

Quick Tip: $\text{C}_3\text{H}_9\text{N}$ can be a primary, secondary or tertiary amine, but a quaternary ammonium salt has four carbon groups on N.

96. Which of the following statements are correct?

A) α - amino acids present in protein is L- amino acid.

B) Amino acids contain NH_2 as well as COOH group.

C) Number of amino group and carboxylic group are same in amino acids always.

D) Trypsin was firstly obtained from cheese.

Choose the correct alternatives from below:

(A) A and B only

(B) A, B and C only

(C) B, C and D only

(D) A, B and D only

Correct Answer: (D) A, B and D only

Solution:

Step 1: Check statement A

The alpha amino acids found in proteins have the L configuration.
TRUE.

Step 2: Check statement B

An amino acid contains both an amino group and a carboxyl group.
TRUE.

Step 3: Check statement C

Amino acids may be acidic (more COOH , such as aspartic acid), basic (more NH_2 , such as lysine) or neutral. So the numbers are not always the same. FALSE.

Step 4: Check statement D

Tyrosine was first obtained from cheese (Greek tyros means cheese).
TRUE.

Final Answer:

Statements A, B and D are correct, option (D).

A, B and D only

Quick Tip: A, B and D are true; the numbers of amino and carboxyl groups are not always equal.

97. During denaturation, which level of protein structure remains unchanged?

- (A) primary
- (B) secondary
- (C) tertiary
- (D) quaternary

Correct Answer: (A) primary

Solution:

Step 1: Understanding the Concept

Proteins have primary (amino acid sequence), secondary (helix or sheet), tertiary (3D fold) and quaternary (subunit assembly) structure.

Step 2: Detailed Explanation

Denaturation by heat, acids or alkalis disrupts hydrogen bonds, ionic

interactions and other weak forces. So secondary, tertiary and quaternary structures change or are lost.

Covalent peptide bonds are not broken, so the primary structure remains intact.

Final Answer:

The primary structure remains unchanged, option (A).

Primary

Quick Tip: Denaturation breaks hydrogen bonds and folding but not the peptide bonds of the primary structure.

98. Identify the natural source of ascorbic acid from following.

- (A) Citrus fruits
- (B) Cinnamon
- (C) Turmeric
- (D) Clove

Correct Answer: (A) Citrus fruits

Solution:

Step 1: Understanding the Concept

Ascorbic acid is vitamin C. Its deficiency causes scurvy.

Step 2: Detailed Explanation

Vitamin C is plentiful in citrus fruits such as oranges and lemons, amla and guava. Cinnamon, turmeric and clove are spices and are not

major sources.

Final Answer:

Citrus fruits are a natural source, option (A).

Citrus fruits

Quick Tip: Ascorbic acid is vitamin C, found in citrus fruits.



99. Identify the monomers used to prepare bakelite from the following.

- (A) phenol , chloroform
- (B) formaldehyde , NaOH
- (C) phenol , methanal
- (D) aniline , urea

Correct Answer: (C) phenol , methanal

Solution:

Step 1: Understanding the Concept

Bakelite is a thermosetting phenol-formaldehyde resin, a cross linked condensation polymer.

Step 2: Detailed Explanation

Phenol reacts with methanal (formaldehyde) in the presence of an acid or base catalyst, first giving ortho and para hydroxymethylphenols, which then condense into a cross linked network.

Chloroform, NaOH, aniline and urea are not the monomers; aniline

and urea with methanal are different resins.

Final Answer:

The monomers are phenol and methanal, option (C).

Phenol and methanal

Quick Tip: Bakelite is a condensation polymer of phenol and formaldehyde (methanal).



100. Which among the following is NOT a polyester polymer ?

- (A) Glyptal
- (B) Novolac
- (C) PHBV
- (D) Dacron

Correct Answer: (B) Novolac

Solution:

Step 1: Understanding the Concept

A polyester has repeating ester ($-\text{COO}-$) links, formed from a diol with a diacid or a hydroxy acid.

Step 2: Detailed Explanation

Glyptal is made from ethylene glycol and phthalic acid, so it is a polyester. PHBV is a copolymer of 3-hydroxybutanoic and 3-hydroxypentanoic acids joined by ester links. Dacron (terylene) is made from ethylene glycol and terephthalic acid.

Novolac is a linear phenol-formaldehyde polymer with no ester groups, so it is not a polyester.

Final Answer:

Novolac is not a polyester, option (B).

Novolac

Quick Tip: Novolac is a phenol-formaldehyde polymer, not a polyester.

Mathematics

Questions 101 to 150 | 2 marks each | 100 marks

101. If x, y, z are the sides of a right angled triangle, where z is the largest side, then $\frac{1}{\log_{x+z} y} + \frac{1}{\log_{z-x} y} =$

- (A) 1
- (B) 2
- (C) 3
- (D) 4

Correct Answer: (B) 2

Solution:

Step 1: Understanding the Concept

The change of base rule says $\frac{1}{\log_a y} = \log_y a$. Also, the sum of two logs to the same base is the log of the product.

Step 2: Rewrite the expression

$$\frac{1}{\log_{x+z} y} + \frac{1}{\log_{z-x} y} = \log_y(x+z) + \log_y(z-x) = \log_y[(z+x)(z-x)]$$

Step 3: Use the right triangle

Since z is the hypotenuse, $x^2 + y^2 = z^2$, so $z^2 - x^2 = y^2$.

$$\log_y(y^2) = 2\log_y y = 2$$

Values 1, 3 and 4 would need the product to be y , y^3 or y^4 , which does

not follow from Pythagoras.

Final Answer:

The value is 2, option (B).

2

Quick Tip: Use $1/\log_a b = \log_b a$ to combine the two terms, then Pythagoras.



102. If ω is a complex cube root of unity, then the value of $\sin[\pi(\omega^{10} + \omega^{23}) - \frac{\pi}{4}] =$

(A) $-\frac{\sqrt{3}}{2}$

(B) $-\frac{1}{\sqrt{2}}$

(C) $\frac{1}{\sqrt{2}}$

(D) $\frac{\sqrt{3}}{2}$

Correct Answer: (C) $\frac{1}{\sqrt{2}}$

Solution:

Step 1: Understanding the Concept

For a complex cube root of unity, $\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$. So powers repeat every 3.

Step 2: Reduce the powers

$$\omega^{10} = \omega^9 \cdot \omega = \omega. \quad \omega^{23} = \omega^{21} \cdot \omega^2 = \omega^2.$$

$$\omega^{10} + \omega^{23} = \omega + \omega^2 = -1$$

Step 3: Evaluate the sine

$$\sin \left[\pi(-1) - \frac{\pi}{4} \right] = \sin \left(-\frac{5\pi}{4} \right) = -\sin \frac{5\pi}{4}$$

Since $\sin \frac{5\pi}{4} = -\frac{1}{\sqrt{2}}$, the value is $+\frac{1}{\sqrt{2}}$. The negative options come from a sign slip, and (D) is not a sine of a multiple of $\pi/4$.

Final Answer:

The value is $\frac{1}{\sqrt{2}}$, option (C).

$$\boxed{\frac{1}{\sqrt{2}}}$$

Quick Tip: Reduce powers of omega using omega cubed = 1, then use $1 + w + w^2 = 0$.

103. If 6 boys and 3 girls are to be seated in a row for a photograph, then the probability that the end seats are occupied by the girls and no two girls are side by side is

- (A) $\frac{5}{12}$
- (B) $\frac{1}{28}$
- (C) $\frac{1}{84}$
- (D) $\frac{5}{84}$

Correct Answer: (D) $\frac{5}{84}$

Solution:

Step 1: Understanding the Concept

Total arrangements of 9 people in a row are $9!$. We count favourable arrangements: girls at both end seats and no two girls adjacent.

Step 2: Count favourable cases

Choose which two girls take the ends: arrange 3 girls in 3 roles, the end seats and the third girl. The third girl cannot sit in seat 2 or seat 8, because that would put her next to an end girl. She can sit in seats 3, 4, 5, 6 or 7, which is 5 choices.

Girls: $3!$ ways to assign the three girls to the three chosen seats. Boys fill the other 6 seats in $6!$ ways.

$$\text{Favourable} = 5 \times 3! \times 6! = 5 \times 6 \times 720 = 21600$$

Step 3: Probability

$$P = \frac{21600}{9!} = \frac{21600}{362880} = \frac{5}{84}$$

Option (A), $5/12$, and (B), $1/28$, do not match the count. Option (C), $1/84$, would result if the 5 positions for the third girl were left out.

Final Answer:

The probability is $\frac{5}{84}$, option (D).

$$\boxed{\frac{5}{84}}$$

Quick Tip: Place girls at both ends, then count positions for the third girl so that no two girls touch.



104. A bag contains 23 balls, of which 7 are identical. The number of ways of selecting 12 balls from the bag is....

- (A) ${}^{18}C_6 + {}^{18}C_8$
- (B) ${}^{18}C_7 + {}^{19}C_8$
- (C) ${}^{16}C_8 + {}^{18}C_{15}$
- (D) ${}^{16}C_8 + {}^{18}C_9$

Correct Answer: (A) ${}^{18}C_6 + {}^{18}C_8$

Solution:

Step 1: Understanding the Concept

The bag has 7 identical balls and 16 other balls, all different. Identical balls count as one choice for each number taken.

Step 2: Set up the sum

If we take j identical balls ($j = 0, 1, \dots, 7$), the other $12 - j$ come from the 16 distinct balls. So the total is:

$$\sum_{j=0}^7 {}^{16}C_{12-j} = {}^{16}C_5 + {}^{16}C_6 + \dots + {}^{16}C_{12}$$

Step 3: Evaluate

Values: 4368, 8008, 11440, 12870, 11440, 8008, 4368, 1820. Their sum is 62322.

Check the options: ${}^{18}C_6 = 18564$ and ${}^{18}C_8 = 43758$, giving $18564 + 43758 = 62322$. That matches option (A). The other options give different totals, for example ${}^{16}C_8 + {}^{18}C_9 = 12870 + 48620 = 61490$.

Final Answer:

The number of ways is ${}^{18}C_6 + {}^{18}C_8$, option (A).

$$\boxed{{}^{18}C_6 + {}^{18}C_8}$$

Quick Tip: Take j identical balls (0 to 7) and the rest from the 16 distinct ones, then sum.

105. The approximate value of $(0.007)^{\frac{1}{3}}$ is

- (A) $\frac{21}{120}$
- (B) $\frac{23}{120}$
- (C) $\frac{29}{120}$
- (D) $\frac{31}{120}$

Correct Answer: (B) $\frac{23}{120}$

Solution:

Step 1: Understanding the Concept

For small h , $f(a - h) \approx f(a) - hf'(a)$. We choose a nearby perfect cube, $0.008 = (0.2)^3$.

Step 2: Apply the approximation

Take $f(x) = x^{1/3}$, $a = 0.008$, $h = 0.001$. Then $f(a) = 0.2$ and $f'(a)$

$$= \frac{1}{3a^{2/3}} = \frac{1}{3 \times 0.04} = \frac{25}{3}.$$

$$(0.007)^{1/3} \approx 0.2 - 0.001 \times \frac{25}{3} = 0.2 - \frac{1}{120}$$

$$= \frac{24}{120} - \frac{1}{120} = \frac{23}{120}$$

Step 3: Check options

Option (A) $21/120 = 0.175$ and (C) $29/120 = 0.2417$ are too far from 0.2. The exact cube root is about 0.1913, and $23/120 = 0.1917$ is the closest.

Final Answer:

The approximate value is $\frac{23}{120}$, option (B).

$\frac{23}{120}$

Quick Tip: Write 0.007 as 0.008 - 0.001 and use a small increment approximation.

106. If $33\theta = \pi$, then the value of $\cos\theta\cos2\theta\cos4\theta\cos8\theta\cos16\theta$ is

- (A) $\frac{1}{16}$
- (B) $\frac{1}{32}$
- (C) $\frac{1}{64}$
- (D) $\frac{-1}{32}$

Correct Answer: (B) $\frac{1}{32}$

Solution:

Step 1: Understanding the Concept

The identity $\sin 2A = 2 \sin A \cos A$ turns a chain of cosines with doubling angles into a single sine.

Step 2: Telescoping

Let $P = \cos \theta \cos 2\theta \cos 4\theta \cos 8\theta \cos 16\theta$. Multiply and divide by $\sin \theta$:

$$P = \frac{\sin \theta \cos \theta \cos 2\theta \cos 4\theta \cos 8\theta \cos 16\theta}{\sin \theta}$$

Each step uses $2 \sin A \cos A = \sin 2A$, so five steps give:

$$P = \frac{\sin 32\theta}{32 \sin \theta}$$

Step 3: Use the condition

Since $33\theta = \pi$, $32\theta = \pi - \theta$. Then $\sin 32\theta = \sin(\pi - \theta) = \sin \theta$.

$$P = \frac{\sin \theta}{32 \sin \theta} = \frac{1}{32}$$

Option (D) has the wrong sign, which would come from using $\sin(\pi + \theta)$.

Final Answer:

The value is $\frac{1}{32}$, option (B).

$$\boxed{\frac{1}{32}}$$

Quick Tip: Multiply and divide by $\sin(\theta)$ and use the double angle formula repeatedly.

107. If $\sec 4\theta - \sec 2\theta = 2$, then $\theta =$

- (A) $n\pi + \frac{\pi}{8}, \frac{n\pi}{5} + \frac{\pi}{6}, n \in \mathbb{Z}$
- (B) $n\pi + \frac{\pi}{6}, \frac{n\pi}{5} + \frac{\pi}{8}, n \in \mathbb{Z}$
- (C) $n\pi + \frac{\pi}{2}, \frac{n\pi}{5} + \frac{\pi}{10}, n \in \mathbb{Z}$
- (D) $n\pi + \frac{\pi}{3}, n\pi + \frac{\pi}{10}, n \in \mathbb{Z}$

Correct Answer: (C) $n\pi + \frac{\pi}{2}, \frac{n\pi}{5} + \frac{\pi}{10}, n \in \mathbb{Z}$

Solution:

Step 1: Understanding the Concept

Convert to cosines. Let $c = \cos 2\theta$, so $\cos 4\theta = 2c^2 - 1$.

Step 2: Form the equation

$$\frac{1}{\cos 4\theta} - \frac{1}{\cos 2\theta} = 2 \Rightarrow \cos 2\theta - \cos 4\theta = 2 \cos 4\theta \cos 2\theta$$

$$c - (2c^2 - 1) = 2c(2c^2 - 1) \Rightarrow 4c^3 + 2c^2 - 3c - 1 = 0$$

Try $c = -1$: $-4 + 2 + 3 - 1 = 0$. So $(c + 1)(4c^2 - 2c - 1) = 0$.

Step 3: Solve each factor

$c = -1$: $2\theta = (2n + 1)\pi$, so $\theta = n\pi + \frac{\pi}{2}$.

$4c^2 - 2c - 1 = 0$: $c = \frac{1 \pm \sqrt{5}}{4}$, which is $\cos 36^\circ$ or $\cos 108^\circ$. Combining $2\theta = 2n\pi \pm \frac{\pi}{5}$ and $2\theta = 2n\pi \pm \frac{3\pi}{5}$ gives $\theta = \frac{n\pi}{5} + \frac{\pi}{10}$.

So the solution is $\theta = n\pi + \frac{\pi}{2}, \frac{n\pi}{5} + \frac{\pi}{10}$. This is option (C). The other options have angles such as $\pi/8$ or $\pi/6$ that do not satisfy the equation.

Final Answer:

The solution set is option (C).

$$\boxed{n\pi + \frac{\pi}{2}, \frac{n\pi}{5} + \frac{\pi}{10}}$$

Quick Tip: Put $c = \cos 2\theta$, use $\cos 4\theta = 2c^2 - 1$ and factor the cubic.



108. The equation of the line passing through the point of intersection of the lines $x + 2y + 6 = 0$ and $2x - y = 2$ and making an intercept 5 on the y-axis is

- (A) $39x + 2y - 10 = 0$
- (B) $39x - 2y + 10 = 0$
- (C) $x + 4y - 20 = 0$
- (D) $x - 4y + 20 = 0$

Correct Answer: (B) $39x - 2y + 10 = 0$

Solution:

Step 1: Understanding the Concept

A line making y intercept 5 passes through (0, 5). It also passes through the intersection of the two given lines.

Step 2: Find the intersection

From $2x - y = 2$ we get $y = 2x - 2$. Put into $x + 2y + 6 = 0$:

$$x + 4x - 4 + 6 = 0 \Rightarrow x = -\frac{2}{5}, \quad y = -\frac{14}{5}$$

Step 3: Two-point form

Slope between $(0, 5)$ and $(-\frac{2}{5}, -\frac{14}{5})$:

$$m = \frac{5 + \frac{14}{5}}{0 + \frac{2}{5}} = \frac{39/5}{2/5} = \frac{39}{2}$$

Line: $y = \frac{39}{2}x + 5$, i.e. $2y = 39x + 10$, so $39x - 2y + 10 = 0$. All four options have y intercept 5, but only (B) passes through $(-\frac{2}{5}, -\frac{14}{5})$:
 $39(-\frac{2}{5}) - 2(-\frac{14}{5}) + 10 = -\frac{78}{5} + \frac{28}{5} + 10 = 0$.

Final Answer:

The line is $39x - 2y + 10 = 0$, option (B).

$39x - 2y + 10 = 0$

Quick Tip: Find the intersection point, then use the two-point form with $(0, 5)$.

109. The line $x + y = 3$ intersects the pair of straight lines $x^2 - 3xy + y^2 = 0$ at points A and B. Then the co-ordinates of the mid-point of AB are

- (A) $(\frac{5}{2}, \frac{3}{2})$
- (B) $(\frac{1}{2}, \frac{1}{2})$
- (C) $(-\frac{3}{2}, \frac{1}{2})$
- (D) $(\frac{3}{2}, \frac{3}{2})$

Correct Answer: (D) $(\frac{3}{2}, \frac{3}{2})$

Solution:

Step 1: Understanding the Concept

The pair of lines $x^2 - 3xy + y^2 = 0$ meets the line $x + y = 3$ at two points A and B. The midpoint x coordinate is half the sum of the roots of the resulting quadratic.

Step 2: Substitute

With $y = 3 - x$:

$$x^2 - 3x(3 - x) + (3 - x)^2 = 0$$

$$x^2 - 9x + 3x^2 + 9 - 6x + x^2 = 0 \Rightarrow 5x^2 - 15x + 9 = 0$$

Step 3: Midpoint

Sum of roots $x_1 + x_2 = 3$, so midpoint x is $\frac{3}{2}$. From the line, $y = 3 - \frac{3}{2} = \frac{3}{2}$.

The midpoint is $(\frac{3}{2}, \frac{3}{2})$. It lies on $x + y = 3$; option (A), $(\frac{5}{2}, \frac{3}{2})$, has sum 4 and (B) has sum 1, so they are not on the line.

Final Answer:

The midpoint of AB is $(\frac{3}{2}, \frac{3}{2})$, option (D).

$$\left(\frac{3}{2}, \frac{3}{2}\right)$$

Quick Tip: Substitute $y = 3 - x$ into the pair of lines and use the sum of the roots.

110. The angle between the tangents drawn from the origin to the circle $(x - 7)^2 + (y + 1)^2 = 25$ is

- (A) 45°
- (B) 90°
- (C) 60°
- (D) 30°

Correct Answer: (B) 90°

Solution:

Step 1: Understanding the Concept

From an external point at distance d from the centre of a circle of radius r , the angle 2α between the two tangents satisfies $\sin \alpha = r/d$.

Step 2: Compute

The circle has centre $(7, -1)$ and radius 5. Distance from the origin:

$$d = \sqrt{49 + 1} = \sqrt{50} = 5\sqrt{2}$$

$$\sin \alpha = \frac{5}{5\sqrt{2}} = \frac{1}{\sqrt{2}} \Rightarrow \alpha = 45^\circ$$

Step 3: Angle

The angle between the tangents is $2\alpha = 90^\circ$. The origin lies outside the circle because $d = 7.07 > 5$, so tangents exist.

Final Answer:

The angle is 90° , option (B).

90°

Quick Tip: Use $\sin(\text{half angle}) = \text{radius} / \text{distance of the point from the centre}$.

111. The equation of the common tangent touching the circle $(x - 3)^2 + y^2 = 9$ and the parabola $y^2 = 4x$ above the X-axis is

- (A) $\sqrt{3}y = 3x + 1$
- (B) $\sqrt{3}y = -(x + 3)$
- (C) $\sqrt{3}y = x + 3$
- (D) $\sqrt{3}y = -(3x + 1)$

Correct Answer: (C) $\sqrt{3}y = x + 3$

Solution:**Step 1: Understanding the Concept**

For $y^2 = 4x$, a tangent with slope m is $y = mx + \frac{1}{m}$. The same line must touch the circle with centre $(3, 0)$ and radius 3, so its distance from $(3, 0)$ is 3.

Step 2: Apply the condition

Write the tangent as $mx - y + \frac{1}{m} = 0$. Distance from $(3, 0)$:

$$\frac{|3m + \frac{1}{m}|}{\sqrt{m^2 + 1}} = 3 \Rightarrow \left(3m + \frac{1}{m}\right)^2 = 9(m^2 + 1)$$

$$9m^2 + 6 + \frac{1}{m^2} = 9m^2 + 9 \Rightarrow \frac{1}{m^2} = 3 \Rightarrow m = \pm \frac{1}{\sqrt{3}}$$

Step 3: Choose the one above the X-axis

The point of contact on the parabola is $(\frac{1}{m^2}, \frac{2}{m})$. It lies above the X-axis when $m > 0$. So $m = \frac{1}{\sqrt{3}}$ and $y = \frac{x}{\sqrt{3}} + \sqrt{3}$, that is $\sqrt{3}y = x + 3$.

Option (B), with negative slope, touches below the axis. Options (A) and (D) fail the distance condition.

Final Answer:

The tangent is $\sqrt{3}y = x + 3$, option (C).

$$\boxed{\sqrt{3}y = x + 3}$$

Quick Tip: Use $y = mx + 1/m$ as tangent to the parabola, then the distance from the circle centre must equal the radius.

112. If $\lim_{x \rightarrow \infty} \frac{(2x-1)^{19} \cdot (3x+2)^{11}}{(6x-5)^{30}} = 2^a \cdot 3^b$, then $a + b =$

- (A) -30
- (B) -11
- (C) -19
- (D) 30

Correct Answer: (A) -30

Solution:

Step 1: Understanding the Concept

When numerator and denominator have the same degree, the limit at infinity is the ratio of leading coefficients.

Step 2: Compute

The numerator has degree $19 + 11 = 30$ and the denominator has degree 30.

$$\lim_{x \rightarrow \infty} \frac{(2x - 1)^{19}(3x + 2)^{11}}{(6x - 5)^{30}} = \frac{2^{19} \cdot 3^{11}}{6^{30}}$$

$$= \frac{2^{19} 3^{11}}{2^{30} 3^{30}} = 2^{-11} \cdot 3^{-19}$$

Step 3: Read off

So $a = -11$ and $b = -19$, giving $a + b = -30$. Option (B) is only a , and (C) is only b .

Final Answer:

The value of $a + b$ is -30 , option (A).

Quick Tip: Compare leading coefficients since the degrees match: $19 + 11 = 30$.



113. The dual of the statement pattern $(p \wedge \sim q) \longrightarrow (q \wedge \sim p)$ is equivalent to

- (A) $\sim (p \rightarrow q) \wedge (q \rightarrow p)$
- (B) $(p \rightarrow q) \wedge \sim (q \rightarrow p)$
- (C) $(\sim p \rightarrow q) \wedge (q \rightarrow p)$
- (D) $(q \rightarrow p) \vee (\sim p \rightarrow \sim q)$

Correct Answer: (B) $(p \rightarrow q) \wedge \sim (q \rightarrow p)$

Solution:

Step 1: Understanding the Concept

The dual of a statement made with $\wedge$, $\vee$, $\sim$ is found by swapping $\wedge$ with $\vee$ (and t with f). An implication must first be written using these symbols.

Step 2: Rewrite

$$(p \wedge \sim q) \rightarrow (q \wedge \sim p) \equiv \sim (p \wedge \sim q) \vee (q \wedge \sim p) \equiv (\sim p \vee q) \vee (q \wedge \sim p)$$

Step 3: Take the dual

Swap $\vee$ and $\wedge$:

$$(\sim p \wedge q) \wedge (q \vee \sim p) \equiv \sim p \wedge q$$

Step 4: Match with the options

Option (B): $(p \rightarrow q) \wedge \sim (q \rightarrow p) = (\sim p \vee q) \wedge (q \wedge \sim p) = q \wedge \sim p$. This equals the dual.

Option (A): $\sim (p \rightarrow q) \wedge (q \rightarrow p) = (p \wedge \sim q) \wedge (p \vee \sim q) = p \wedge \sim q$, which is different.

Final Answer:

The dual is equivalent to $\sim p \wedge q$, which matches option (B).

$$(p \rightarrow q) \wedge \sim (q \rightarrow p)$$

Quick Tip: Rewrite the implication with or, take the dual by swapping and and or, then match.

114. Negation of the statement "If an integer is greater than 4 and less than 5, then it is a multiple of 3", is

- (A) An integer is not greater than 4 but less than 5 and it is a multiple of 3.
- (B) If an integer is not greater than 4 and less than 5 then it is not a multiple of 3.
- (C) An integer is greater than 4 and less than 5 but it is not a multiple of 3.
- (D) An integer is not greater than 4 and not less than 5 but it is not a multiple of 3.

Correct Answer: (C) An integer is greater than 4 and less than 5 but it is not a multiple of 3.

Solution:

Step 1: Understanding the Concept

A conditional $P \rightarrow Q$ is false only when P is true and Q is false. So $\sim (P \rightarrow Q) \equiv P \wedge \sim Q$.

Step 2: Identify P and Q

P: "an integer is greater than 4 and less than 5". Q: "it is a multiple of 3".

Step 3: Form the negation

The negation keeps P as it is and negates Q: "An integer is greater than 4 and less than 5 but it is not a multiple of 3."

Options (A) and (D) change P, option (B) is another conditional and not a negation of the original.

Final Answer:

The negation is option (C).

$$P \wedge \sim Q$$

Quick Tip: The negation of "if P then Q" is "P and not Q".

115. In a triangle ABC, with the usual notations, $\angle B = \frac{\pi}{3}$, $\angle C = \frac{\pi}{4}$. If D divides BC internally in the ratio 1:3, then $\frac{\sin \angle BAD}{\sin \angle CAD} =$

- (A) $\frac{1}{3}$
 (B) $\frac{1}{\sqrt{3}}$
 (C) $\frac{1}{\sqrt{6}}$
 (D) $\sqrt{\frac{2}{3}}$

Correct Answer: (C) $\frac{1}{\sqrt{6}}$

Solution:

Step 1: Understanding the Concept

Angle A is $180^\circ - 60^\circ - 45^\circ = 75^\circ$. The point D divides BC in the ratio $BD : DC = 1 : 3$.

Step 2: Sine rule in two triangles

$$\text{In } \triangle ABD: \frac{BD}{\sin \angle BAD} = \frac{AD}{\sin B}.$$

$$\text{In } \triangle ACD: \frac{DC}{\sin \angle CAD} = \frac{AD}{\sin C}.$$

Divide the first by the second:

$$\frac{BD \sin \angle CAD}{DC \sin \angle BAD} = \frac{\sin C}{\sin B} \Rightarrow \frac{\sin \angle BAD}{\sin \angle CAD} = \frac{BD}{DC} \cdot \frac{\sin B}{\sin C}$$

Step 3: Substitute

$$= \frac{1}{3} \times \frac{\sin 60^\circ}{\sin 45^\circ} = \frac{1}{3} \times \frac{\sqrt{3}/2}{1/\sqrt{2}} = \frac{1}{3} \sqrt{\frac{3}{2}} = \frac{1}{\sqrt{6}}$$

Option (B), $1/\sqrt{3}$, would result from leaving out the $\sin 45^\circ$ term.

Final Answer:

The ratio is $\frac{1}{\sqrt{6}}$, option (C).

$$\frac{1}{\sqrt{6}}$$

Quick Tip: Apply the sine rule in triangles ABD and ACD and use $BD : DC = 1 : 3$.



116. Let P_1, P_2, P_3 be the altitudes of a triangle ABC from the vertices A, B, C respectively. If Δ denotes the area of the triangle and s is the semi-perimeter of the triangle, then $\frac{\cos A}{P_1} + \frac{\cos B}{P_2} + \frac{\cos C}{P_3} =$

- (A) R
- (B) $\frac{1}{R}$
- (C) R^2
- (D) $\frac{1}{R^2}$

Correct Answer: (B) $\frac{1}{R}$

Solution:

Step 1: Understanding the Concept

The altitude from A is $P_1 = \frac{2\Delta}{a}$, and similarly for P_2, P_3 . Also $a = 2R \sin A$.

Step 2: Rewrite the sum

$$\sum \frac{\cos A}{P_1} = \frac{1}{2\Delta} \sum a \cos A = \frac{R}{2\Delta} \sum \sin 2A$$

Step 3: Evaluate

In a triangle, $\sin 2A + \sin 2B + \sin 2C = 4 \sin A \sin B \sin C$. Also

$$\sin A \sin B \sin C = \frac{abc}{8R^3} = \frac{4R\Delta}{8R^3} = \frac{\Delta}{2R^2}.$$

$$\sum \sin 2A = \frac{2\Delta}{R^2}$$

$$\text{Sum} = \frac{R}{2\Delta} \cdot \frac{2\Delta}{R^2} = \frac{1}{R}$$

Final Answer:

The value is $\frac{1}{R}$, option (B).

$$\boxed{\frac{1}{R}}$$

Quick Tip: Write each altitude as $2 \cdot \text{Area} / \text{side}$ and use $a = 2R \sin A$.

117. If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 7 & 3 \\ 0 & 6 \end{bmatrix}$ and $AB = C$, then the inverse of matrix B is

(A) $\frac{1}{42} \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$

(B) $\frac{1}{6} \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$

(C) $\frac{1}{42} \begin{bmatrix} 6 & 3 \\ -1 & 2 \end{bmatrix}$

(D) $\frac{1}{6} \begin{bmatrix} 7 & 3 \\ -1 & 3 \end{bmatrix}$

Correct Answer: (B) $\frac{1}{6} \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$

Solution:

Step 1: Understanding the Concept

From $AB = C$, $B = A^{-1}C$. The inverse of a product reverses the order:
 $B^{-1} = C^{-1}A$.

Step 2: Find C inverse

$|C| = 7 \times 6 - 3 \times 0 = 42$. So $C^{-1} = \frac{1}{42} \begin{bmatrix} 6 & -3 \\ 0 & 7 \end{bmatrix}$.

Step 3: Multiply

$$B^{-1} = \frac{1}{42} \begin{bmatrix} 6 & -3 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \frac{1}{42} \begin{bmatrix} 21 & 0 \\ -7 & 14 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$$

This is option (B). Option (A) forgets to divide the 7s out, (C) and (D) keep entries of C.

Final Answer:

The inverse is $\frac{1}{6} \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$, option (B).

$$\boxed{\frac{1}{6} \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}}$$

Quick Tip: Since $AB = C$, $B \text{ inverse} = C \text{ inverse times } A$.

118. The element in the third row and the second column in the inverse

matrix of a matrix $\begin{bmatrix} 1 & 3 & 3 \\ 3 & 1 & 3 \\ 3 & 3 & 4 \end{bmatrix}$ is

(A) $\frac{3}{2}$

(B) $\frac{2}{3}$

(C) $\frac{-3}{2}$

(D) $\frac{-2}{3}$

Correct Answer: (A) $\frac{3}{2}$

Solution:

Step 1: Understanding the Concept

$A^{-1} = \frac{\text{adj } A}{|A|}$, and the adjoint is the transpose of the cofactor matrix.

So the entry in row 3, column 2 of A^{-1} is $C_{23}/|A|$.

Step 2: Determinant

$$|A| = 1(4 - 9) - 3(12 - 9) + 3(9 - 3) = -5 - 9 + 18 = 4$$

Step 3: Cofactor

The minor M_{23} removes row 2 and column 3, leaving $\begin{vmatrix} 1 & 3 \\ 3 & 3 \end{vmatrix} = 3 - 9 = -6$.

$$C_{23} = (-1)^{2+3}(-6) = 6$$

$$\text{Entry} = \frac{6}{4} = \frac{3}{2}$$

Option (B) $2/3$ and (D) come from inverting the fraction, and (C) from missing the sign.

Final Answer:

The required element is $\frac{3}{2}$, option (A).

$$\boxed{\frac{3}{2}}$$

Quick Tip: The (3,2) entry of the inverse is the cofactor C_{23} divided by the determinant.



119. If $y = \tan^{-1} \sqrt{\frac{1+\sin 2x}{1-\sin 2x}}$, then $\frac{dy}{dx}$ at $x = \frac{\pi}{6}$ is

(A) 0

(B) 1

(C) $\frac{1}{2}$

(D) $\frac{\sqrt{3}}{2}$

Correct Answer: (B) 1

Solution:

Step 1: Understanding the Concept

Use $1 + \sin 2x = (\cos x + \sin x)^2$ and $1 - \sin 2x = (\cos x - \sin x)^2$.

Step 2: Simplify

$$\sqrt{\frac{1 + \sin 2x}{1 - \sin 2x}} = \frac{\cos x + \sin x}{\cos x - \sin x} = \frac{1 + \tan x}{1 - \tan x} = \tan\left(\frac{\pi}{4} + x\right)$$

For x near $\pi/6$, $\frac{\pi}{4} + x$ lies in $(-\frac{\pi}{2}, \frac{\pi}{2})$, so $y = \tan^{-1} \tan(\frac{\pi}{4} + x) = \frac{\pi}{4} + x$.

Step 3: Differentiate

$$\frac{dy}{dx} = 1$$

The derivative is 1 for all x in that range, including $x = \pi/6$.

Final Answer:

The derivative at $x = \frac{\pi}{6}$ is 1, option (B).

1

Quick Tip: Simplify the surd to $\tan(\pi/4 + x)$ before differentiating.

120. The value of $\tan^{-1}\left(\frac{\cos(\frac{19\pi}{4})-1}{\sin(\frac{\pi}{4})}\right)$ is equal to

- (A) $-\frac{\pi}{4}$
- (B) $-\frac{5\pi}{12}$
- (C) $-\frac{3\pi}{8}$
- (D) $-\frac{4\pi}{9}$

Correct Answer: (C) $-\frac{3\pi}{8}$

Solution:

Step 1: Understanding the Concept

Reduce the angle using a period of 2π : $\frac{19\pi}{4} = 4\pi + \frac{3\pi}{4}$.

Step 2: Evaluate the fraction

$$\cos \frac{19\pi}{4} = \cos \frac{3\pi}{4} = -\frac{1}{\sqrt{2}} \text{ and } \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}.$$

$$\frac{-\frac{1}{\sqrt{2}} - 1}{\frac{1}{\sqrt{2}}} = -1 - \sqrt{2} = -(1 + \sqrt{2})$$

Step 3: Inverse tangent

$\tan \frac{3\pi}{8} = \tan 67.5^\circ = 1 + \sqrt{2}$. Because $\tan^{-1}$ is odd and its range is $(-\frac{\pi}{2}, \frac{\pi}{2})$:

$$\tan^{-1}(-(1 + \sqrt{2})) = -\frac{3\pi}{8}$$

Option (A), $-\pi/4$, has $\tan$ equal to -1 , which is not the value.

Final Answer:

The value is $-\frac{3\pi}{8}$, option (C).

$$\boxed{-\frac{3\pi}{8}}$$

Quick Tip: Reduce $19\pi/4$ to a simple angle and recognise $\tan(3\pi/8) = 1 + \sqrt{2}$.

121. The value of $f(0)$ so that the function $f(x) = \frac{(256-8x)^{\frac{1}{4}}-4}{16-4(64+3x)^{\frac{1}{3}}}$, $x \neq 0$ is continuous at $x = 0$, is

- (A) $-\frac{1}{8}$
 (B) $\frac{1}{8}$
 (C) $\frac{1}{64}$
 (D) 8

Correct Answer: (B) $\frac{1}{8}$

Solution:

Step 1: Understanding the Concept

For continuity at $x = 0$, $f(0)$ must equal $\lim_{x \rightarrow 0} f(x)$. Both numerator and denominator tend to 0, so we expand with the binomial series.

Step 2: Numerator

$(256 - 8x)^{1/4} = 4\left(1 - \frac{x}{32}\right)^{1/4} \approx 4\left(1 - \frac{x}{128}\right) = 4 - \frac{x}{32}$. So the numerator is $\approx -\frac{x}{32}$.

Step 3: Denominator

$(64 + 3x)^{1/3} = 4\left(1 + \frac{3x}{64}\right)^{1/3} \approx 4\left(1 + \frac{x}{64}\right)$. So $16 - 4(64 + 3x)^{1/3} \approx 16 - 16 - \frac{x}{4} = -\frac{x}{4}$.

$$\lim_{x \rightarrow 0} f(x) = \frac{-x/32}{-x/4} = \frac{1}{8}$$

So $f(0) = \frac{1}{8}$, option (B). Option (A) has the wrong sign because both terms are negative.

Final Answer:

$f(0) = \frac{1}{8}$, option (B).

$\frac{1}{8}$

Quick Tip: Take the limit as x tends to 0 using binomial expansions, which equals the value of $f(0)$.

122. The derivative of $\sin(\log(\frac{x+3}{x}))$ is

- (A) $\frac{3}{x+3} \cos(\log(\frac{x+3}{x}))$
- (B) $\frac{3}{x(x+3)} \cos(\log(\frac{x+3}{x}))$
- (C) $\frac{-3}{x(x+3)} \cos(\log(\frac{x+3}{x}))$
- (D) $\frac{-1}{x(x+3)} \cos(\log(\frac{x+3}{x}))$

Correct Answer: (C) $\frac{-3}{x(x+3)} \cos(\log(\frac{x+3}{x}))$

Solution:

Step 1: Understanding the Concept

Let $u = \log \frac{x+3}{x}$. Then $\frac{d}{dx} \sin u = \cos u \cdot \frac{du}{dx}$.

Step 2: Differentiate u

$u = \log(x+3) - \log x$, so

$$\frac{du}{dx} = \frac{1}{x+3} - \frac{1}{x} = \frac{x - (x+3)}{x(x+3)} = \frac{-3}{x(x+3)}$$

Step 3: Combine

$$\frac{d}{dx} \sin \left(\log \frac{x+3}{x} \right) = \frac{-3}{x(x+3)} \cos \left(\log \frac{x+3}{x} \right)$$

Option (B) has the sign reversed, (A) uses only the first term, and (D) has the wrong constant.

Final Answer:

The derivative is option (C).

$$\frac{-3}{x(x+3)} \cos\left(\log \frac{x+3}{x}\right)$$

Quick Tip: Apply the chain rule three times and simplify the derivative of the inner fraction.

123. If $e^y + xy = e$, then the ordered pair $(\frac{dy}{dx}, \frac{d^2y}{dx^2})$ at $x = 0$ is equal to

- (A) $(\frac{1}{e}, \frac{-1}{e^2})$
- (B) $(\frac{-1}{e}, \frac{1}{e^2})$
- (C) $(\frac{1}{e}, \frac{1}{e^2})$
- (D) $(\frac{-1}{e}, \frac{-1}{e^2})$

Correct Answer: (B) $(\frac{-1}{e}, \frac{1}{e^2})$

Solution:**Step 1: Understanding the Concept**

Differentiate $e^y + xy = e$ implicitly twice, then evaluate at $x = 0$.

Step 2: Find y

At $x = 0$: $e^y = e$, so $y = 1$.

Step 3: First derivative

$$e^y y' + y + xy' = 0$$

At $x = 0, y = 1$: $e y' + 1 = 0$, so $y' = -\frac{1}{e}$.

Step 4: Second derivative

Differentiate again: $e^y(y')^2 + e^y y'' + y' + y' + xy'' = 0$.

At $x = 0$: $e \cdot \frac{1}{e^2} + e y'' - \frac{2}{e} = 0 \Rightarrow \frac{1}{e} + e y'' - \frac{2}{e} = 0$.

$$y'' = \frac{1}{e^2}$$

So the pair is $(-\frac{1}{e}, \frac{1}{e^2})$, option (B).

Final Answer:

The ordered pair is $(-\frac{1}{e}, \frac{1}{e^2})$, option (B).

$$\left(-\frac{1}{e}, \frac{1}{e^2}\right)$$

Quick Tip: Find y at $x = 0$, differentiate twice implicitly and substitute.

124. The surface area of a spherical ball is increasing at the rate of $4\pi\text{cm}^2/\text{second}$. The rate at which the radius is increasing when the surface area is $16\pi\text{cm}^2$ is

- (A) 0.5 cm/second
- (B) 0.25 cm/second
- (C) 0.125 cm/second
- (D) 1 cm/second

Correct Answer: (B) 0.25 cm/second

Solution:

Step 1: Understanding the Concept

Surface area of a sphere is $S = 4\pi r^2$. Differentiating with respect to time, $\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$.

Step 2: Find the radius

When $S = 16\pi$: $4\pi r^2 = 16\pi$, so $r = 2$ cm.

Step 3: Solve for dr/dt

$$4\pi = 8\pi(2) \frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{4\pi}{16\pi} = 0.25 \text{ cm/s}$$

The value 0.5 comes from forgetting the factor 2 in $8\pi r$, and 1 from using $r = 0.5$.

Final Answer:

The radius increases at 0.25 cm/s, option (B).

0.25 cm/s

Quick Tip: Differentiate $S = 4\pi r^2$ with respect to time.



125. The maximum value of $(\frac{1}{x})^x$, $x > 0$ is

- (A) e^e
(B) e^{-e}
(C) $e^{1/e}$
(D) $(\frac{1}{e})^{1/e}$

Correct Answer: (C) $e^{1/e}$

Solution:

Step 1: Understanding the Concept

Let $y = (1/x)^x$. Then $\ln y = -x \ln x$, so maximising y is the same as maximising $-x \ln x$.

Step 2: Find the critical point

$$\frac{d}{dx}(-x \ln x) = -\ln x - 1 = 0 \Rightarrow x = \frac{1}{e}$$

The second derivative is $-\frac{1}{x} < 0$ for $x > 0$, so this is a maximum.

Step 3: Maximum value

$$\ln y = -\frac{1}{e} \ln \frac{1}{e} = \frac{1}{e} \Rightarrow y_{\max} = e^{1/e}$$

Options (A) and (B) use e as the exponent, and (D) is less than 1, whereas the function reaches above 1.

Final Answer:

The maximum value is $e^{1/e}$, option (C).

$$\boxed{e^{1/e}}$$

Quick Tip: Take the log of the function and set the derivative to zero.

126. The point on the curve $9y^2 = x^3$ where the normal to the curve makes equal intercepts with the co-ordinate axes is

- (A) $(-4, \frac{8}{3})$
- (B) $(4, \frac{8}{3})$
- (C) $(-4, \frac{-8}{3})$
- (D) $(-4, \frac{3}{8})$

Correct Answer: (B) $(4, \frac{8}{3})$

Solution:

Step 1: Understanding the Concept

A line making equal intercepts with the coordinate axes has slope -1 (it is of the form $x + y = k$). So the normal to the curve must have slope -1 , which means the tangent has slope 1 .

Step 2: Slope of the tangent

From $9y^2 = x^3$: $18y \frac{dy}{dx} = 3x^2$, so $\frac{dy}{dx} = \frac{x^2}{6y}$.

Set it equal to 1: $x^2 = 6y$, so $y = \frac{x^2}{6}$.

Step 3: Locate the point

Substitute in the curve:

$$9 \cdot \frac{x^4}{36} = x^3 \Rightarrow \frac{x^4}{4} = x^3 \Rightarrow x = 4 \ (x \neq 0)$$

$$y = \frac{16}{6} = \frac{8}{3}$$

The point is $(4, \frac{8}{3})$, option (B). Check: $9 \cdot \frac{64}{9} = 64 = 4^3$. Options with $x = -4$ give $x^3 = -64 < 0$ while $9y^2 \geq 0$, so they are not on the curve.

Final Answer:

The point is $(4, \frac{8}{3})$, option (B).

$$\boxed{\left(4, \frac{8}{3}\right)}$$

Quick Tip: Equal intercepts on the axes means the normal has slope -1.



127. A spherical mothball has initial radius 3 cm. Due to evaporation, the radius of the ball reduces to 1 cm in 4 months. In how many months would the mothball evaporate completely if the volume is lost at a rate proportional to the surface area ?

- (A) 6 months
- (B) 8 months
- (C) 10 months
- (D) 12 months

Correct Answer: (A) 6 months

Solution:

Step 1: Understanding the Concept

Let the radius be r . Volume is $V = \frac{4}{3}\pi r^3$ and surface area is $S = 4\pi r^2$.

The volume decreases at a rate proportional to the surface area: $\frac{dV}{dt} = -kS$.

Step 2: Form the equation

$$4\pi r^2 \frac{dr}{dt} = -k 4\pi r^2 \Rightarrow \frac{dr}{dt} = -k$$

So the radius shrinks at a constant rate k per month.

Step 3: Use the data

The radius falls from 3 cm to 1 cm in 4 months, a drop of 2 cm. So $k = \frac{2}{4} = 0.5$ cm per month.

The remaining 1 cm takes $\frac{1}{0.5} = 2$ more months. So the total time for the radius to go from 3 cm to 0 is $4 + 2 = 6$ months.

This is option (A). The mothball therefore evaporates completely in 6 months.

Final Answer:

The mothball evaporates completely in 6 months, option (A).

6 months

Quick Tip: Volume loss proportional to surface area makes the radius shrink at a constant rate.

128. $\int \frac{x^2 dx}{(x^2+2)(x^2+5)} =$

(A) $-(\frac{\sqrt{2}}{3}\tan^{-1}\frac{x}{\sqrt{2}} + \frac{\sqrt{5}}{3}\tan^{-1}\frac{x}{\sqrt{5}}) + c$, where c is the constant of integration

(B) $(\frac{\sqrt{2}}{3}\tan^{-1}\frac{x}{\sqrt{2}} + \frac{\sqrt{5}}{3}\tan^{-1}\frac{x}{\sqrt{5}}) + c$, where c is the constant of integration

(C) $\frac{\sqrt{2}}{3}\tan^{-1}(\frac{x}{\sqrt{2}}) - \frac{\sqrt{5}}{3}\tan^{-1}(\frac{x}{\sqrt{5}}) + c$, where c is the constant of integration

(D) $-\frac{\sqrt{2}}{3}\tan^{-1}(\frac{x}{\sqrt{2}}) + \frac{\sqrt{5}}{3}\tan^{-1}(\frac{x}{\sqrt{5}}) + c$, where c is the constant of integration

Correct Answer: (D) $-\frac{\sqrt{2}}{3}\tan^{-1}(\frac{x}{\sqrt{2}}) + \frac{\sqrt{5}}{3}\tan^{-1}(\frac{x}{\sqrt{5}}) + c$, where c is the constant of integration

Solution:

Step 1: Understanding the Concept

Treat x^2 as a single variable and write the integrand as $\frac{A}{x^2 + 2} + \frac{B}{x^2 + 5}$.

Step 2: Find A and B

$$x^2 = A(x^2 + 5) + B(x^2 + 2)$$

Comparing coefficients: $A + B = 1$ and $5A + 2B = 0$. Solving gives $A = -\frac{2}{3}$ and $B = \frac{5}{3}$.

Step 3: Integrate

Using $\int \frac{dx}{x^2 + a^2} = \frac{1}{a}\tan^{-1}\frac{x}{a}$:

$$-\frac{2}{3} \cdot \frac{1}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}} + \frac{5}{3} \cdot \frac{1}{\sqrt{5}}\tan^{-1}\frac{x}{\sqrt{5}} + c$$

$$= -\frac{\sqrt{2}}{3} \tan^{-1} \frac{x}{\sqrt{2}} + \frac{\sqrt{5}}{3} \tan^{-1} \frac{x}{\sqrt{5}} + c$$

This is option (D). The signs matter: the $\sqrt{2}$ term is negative and the $\sqrt{5}$ term is positive.

Final Answer:

The integral is option (D).

$$-\frac{\sqrt{2}}{3} \tan^{-1} \frac{x}{\sqrt{2}} + \frac{\sqrt{5}}{3} \tan^{-1} \frac{x}{\sqrt{5}} + c$$

Quick Tip: Split into partial fractions in x^2 and integrate each as an inverse tangent.

129. The value of $\int \frac{\sin^6 x + \cos^6 x}{\sin^2 x \cdot \cos^2 x} dx$ is

- (A) $\cot x + \cot^2 x + c$, where c is the constant of integration
- (B) $\tan x - \cot x - 3x + c$, where c is the constant of integration
- (C) $2\sin^5 x + 2\cos^5 x + c$, where c is the constant of integration
- (D) $\sin^3 x + 2\cos^3 x + c$, where c is the constant of integration

Correct Answer: (B) $\tan x - \cot x - 3x + c$, where c is the constant of integration

Solution:

Step 1: Understanding the Concept

Use the identity $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$ with $a = \sin^2 x$ and $b = \cos^2 x$. Since $a + b = 1$, we get $\sin^6 x + \cos^6 x = 1 - 3 \sin^2 x \cos^2 x$.

Step 2: Split the integrand

$$\frac{1 - 3 \sin^2 x \cos^2 x}{\sin^2 x \cos^2 x} = \frac{1}{\sin^2 x \cos^2 x} - 3$$

$$\text{And } \frac{1}{\sin^2 x \cos^2 x} = \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} = \sec^2 x + \operatorname{cosec}^2 x.$$

Step 3: Integrate

$$\int (\sec^2 x + \operatorname{cosec}^2 x - 3) dx = \tan x - \cot x - 3x + c$$

This is option (B).

Final Answer:

The integral is $\tan x - \cot x - 3x + c$, option (B).

$$\boxed{\tan x - \cot x - 3x + c}$$

Quick Tip: Write $\sin^6 + \cos^6 = 1 - 3 \sin^2 \cos^2$ and split the fraction.

130. $\int \operatorname{cosec}^{-1}\left(\sqrt{\frac{a+x}{x}}\right) dx =$

(A) $a\theta \tan^2 \theta - a \tan \theta - a\theta + c$ (where $\theta = \tan^{-1}(\sqrt{\frac{x}{a}})$) and c is the constant of integration

(B) $a\theta \tan^2 \theta - a \tan \theta + a\theta + c$ (where $\theta = \tan^{-1}(\sqrt{\frac{x}{a}})$) and c is the constant of integration

(C) $a\theta \tan^2 \theta + a \tan \theta - a\theta + c$ (where $\theta = \tan^{-1}(\sqrt{\frac{x}{a}})$) and c is the constant of integration

(D) $a\theta \tan^2 \theta + a \tan \theta + a\theta + c$ (where $\theta = \tan^{-1}(\sqrt{\frac{x}{a}})$) and c is the constant of integration

Correct Answer: (B) $a\theta \tan^2 \theta - a \tan \theta + a\theta + c$ (where $\theta = \tan^{-1}(\sqrt{\frac{x}{a}})$) and c is the constant of integration

Solution:

Step 1: Understanding the Concept

Let $x = a \tan^2 \theta$, so $\theta = \tan^{-1} \sqrt{x/a}$. Then $\frac{a+x}{x} = \frac{\sec^2 \theta}{\tan^2 \theta} = \frac{1}{\sin^2 \theta}$.

Step 2: Simplify the integrand

$$\operatorname{cosec}^{-1} \sqrt{\frac{a+x}{x}} = \operatorname{cosec}^{-1} \left(\frac{1}{\sin \theta} \right) = \theta$$

Also $dx = 2a \tan \theta \sec^2 \theta d\theta$. So the integral is $\int 2a \theta \tan \theta \sec^2 \theta d\theta$.

Step 3: Integrate by parts

Take $u = \theta$, $dv = 2a \tan \theta \sec^2 \theta d\theta$, so $v = a \tan^2 \theta$.

$$= a\theta \tan^2 \theta - a \int \tan^2 \theta d\theta = a\theta \tan^2 \theta - a(\tan \theta - \theta)$$

$$= a\theta \tan^2 \theta - a \tan \theta + a\theta + c$$

This is option (B). The sign of the last term is plus because $\int \tan^2 \theta d\theta = \tan \theta - \theta$.

Final Answer:

The integral is $a\theta \tan^2 \theta - a \tan \theta + a\theta + c$, option (B).

$$a\theta \tan^2 \theta - a \tan \theta + a\theta + c$$

Quick Tip: Substitute $x = a \tan^2 \theta$ so the inverse cosecant collapses to θ , then integrate by parts.



131. $\int_0^\pi |\sin 2x| dx =$

- (A) 0
- (B) 1
- (C) 2
- (D) -1

Correct Answer: (C) 2

Solution:

Step 1: Understanding the Concept

On $[0, \pi/2]$, $2x \in [0, \pi]$ so $\sin 2x \geq 0$. On $[\pi/2, \pi]$, $2x \in [\pi, 2\pi]$ so $\sin 2x \leq 0$.

Step 2: Split and integrate

$$\int_0^{\pi} |\sin 2x| dx = \int_0^{\pi/2} \sin 2x dx + \int_{\pi/2}^{\pi} (-\sin 2x) dx$$

$$\int_0^{\pi/2} \sin 2x dx = \left[-\frac{\cos 2x}{2} \right]_0^{\pi/2} = \frac{1}{2} + \frac{1}{2} = 1$$

The second piece is also 1 because $|\sin 2x|$ repeats with period $\pi/2$.

$$\text{Total} = 1 + 1 = 2$$

Option (A), 0, is what you get for $\int_0^{\pi} \sin 2x dx$ without the absolute value.

Final Answer:

The value is 2, option (C).

2

Quick Tip: Split at $\pi/2$ where $\sin 2x$ changes sign and double the first piece by symmetry.

132. If $f(x)$ is an even function, then $\int_{-2}^2 (|x| + f(x)\sin x) dx$ is

- (A) 2
- (B) 4
- (C) 6
- (D) 8

Correct Answer: (B) 4

Solution:

Step 1: Understanding the Concept

For an odd function g , $\int_{-a}^a g(x) dx = 0$. For an even function, $\int_{-a}^a g(x) dx = 2 \int_0^a g(x) dx$.

Step 2: Split

$$\int_{-2}^2 (|x| + f(x) \sin x) dx = \int_{-2}^2 |x| dx + \int_{-2}^2 f(x) \sin x dx$$

Since f is even and $\sin x$ is odd, $f(x) \sin x$ is odd, so the second integral is 0.

Step 3: Evaluate

$$\int_{-2}^2 |x| dx = 2 \int_0^2 x dx = 2 \cdot \frac{4}{2} = 4$$

So the value is 4, option (B).

Final Answer:

The value is 4, option (B).

4

Quick Tip: f even and $\sin x$ odd make $f(x) \sin x$ an odd function, so its integral over $[-2, 2]$ is zero.

133. The area of the region bounded by the curves $y = |x - 4|$, $x = 3$ and $x = 5$, and the X-axis is

- (A) 5 sq. units
- (B) 3 sq. units
- (C) 6 sq. units
- (D) 1 sq. unit

Correct Answer: (D) 1 sq. unit

Solution:

Step 1: Understanding the Concept

$y = |x - 4|$ equals $4 - x$ for $x < 4$ and $x - 4$ for $x \geq 4$. The region lies between $x = 3$ and $x = 5$ above the X-axis.

Step 2: Integrate

$$A = \int_3^4 (4 - x) dx + \int_4^5 (x - 4) dx$$

$$\int_3^4 (4 - x) dx = \left[4x - \frac{x^2}{2} \right]_3^4 = (16 - 8) - (12 - 4.5) = \frac{1}{2}$$

The second integral is also $\frac{1}{2}$ by symmetry.

$$A = \frac{1}{2} + \frac{1}{2} = 1 \text{ sq. unit}$$

Option (A), 5, would come from integrating without splitting at the vertex.

Final Answer:

The area is 1 square unit, option (D).

1 sq. unit

Quick Tip: The graph is a V shape with vertex at (4, 0), so the area is two small triangles.

134. The solution of $\frac{dy}{dx} = \sin(x + y) + \cos(x + y)$ is

- (A) $\log[1 + \tan(\frac{x+y}{2})] = y + c$, where c is the constant of integration
(B) $\log[1 - \tan(\frac{x+y}{2})] = y + c$, where c is the constant of integration
(C) $\log[1 + \tan(\frac{x+y}{2})] = x + c$, where c is the constant of integration
(D) $\log[1 - \tan(\frac{x+y}{2})] = x + c$, where c is the constant of integration

Correct Answer: (C) $\log[1 + \tan(\frac{x+y}{2})] = x + c$, where c is the constant of integration

Solution:**Step 1: Understanding the Concept**

Put $v = x + y$. Then $\frac{dv}{dx} = 1 + \frac{dy}{dx} = 1 + \sin v + \cos v$, which is a variable separable form.

Step 2: Half angle substitution

Let $t = \tan \frac{v}{2}$, so $\sin v = \frac{2t}{1+t^2}$, $\cos v = \frac{1-t^2}{1+t^2}$.

$$1 + \sin v + \cos v = \frac{(1+t^2) + 2t + (1-t^2)}{1+t^2} = \frac{2(1+t)}{1+t^2}$$

Also $dv = \frac{2 dt}{1 + t^2}$.

Step 3: Integrate

$$dx = \frac{dv}{1 + \sin v + \cos v} = \frac{2 dt/(1 + t^2)}{2(1 + t)/(1 + t^2)} = \frac{dt}{1 + t}$$

$$x + c = \log(1 + t) \Rightarrow \log \left[1 + \tan \left(\frac{x + y}{2} \right) \right] = x + c$$

This is option (C). Option (A) has y on the right side, which would not follow since the left side was divided by dx .

Final Answer:

The solution is $\log \left[1 + \tan \frac{x+y}{2} \right] = x + c$, option (C).

$$\log \left[1 + \tan \left(\frac{x + y}{2} \right) \right] = x + c$$

Quick Tip: Substitute $v = x + y$ and use the tangent half angle substitution.

135. The order and degree of the differential equation $3 - \left(\frac{d^3y}{dx^3} \right)^{\frac{7}{3}} = \left(\frac{dy}{dx} \right)^5$ are respectively

- (A) 3, 7
- (B) 7, 3
- (C) 5, 7
- (D) 3, 5

Correct Answer: (A) 3, 7

Solution:

Step 1: Understanding the Concept

Order is the highest derivative present. Degree is the power of the highest order derivative after the equation is made a polynomial in the derivatives (free of fractional powers).

Step 2: Clear the fraction

Isolate the term with the fractional power:

$$\left(\frac{d^3y}{dx^3}\right)^{7/3} = 3 - \left(\frac{dy}{dx}\right)^5$$

Cube both sides:

$$\left(\frac{d^3y}{dx^3}\right)^7 = \left[3 - \left(\frac{dy}{dx}\right)^5\right]^3$$

Step 3: Read off

The highest derivative is the third derivative, so the order is 3. Its power is 7, so the degree is 7.

The pair is (3, 7), option (A). Option (D), (3, 5), takes the power of the first derivative, which is not the highest order.

Final Answer:

Order 3 and degree 7, option (A).

3, 7

Quick Tip: Clear the fractional power by isolating the radical term and raising both sides to a power.

136. The solution of the differential equation $e^{-x}(y+1)dy + (\cos^2 x - \sin 2x)y dx = 0$, given that $y = 1$ when $x = 0$ is

- (A) $\log y + \frac{1}{y} + e^x \cos^2 x = 1$
- (B) $\log y + y + e^x \cos^2 x = 2$
- (C) $(y+1) + e^x \cos^2 x = 2$
- (D) $\log(y + \frac{1}{y}) + e^x \cos^2 x = 1$

Correct Answer: (B) $\log y + y + e^x \cos^2 x = 2$

Solution:

Step 1: Understanding the Concept

The equation can be written in separated form, with all y terms on one side and all x terms on the other.

Step 2: Separate the variables

$$e^{-x}(y+1)dy = -(\cos^2 x - \sin 2x)y dx \Rightarrow \frac{y+1}{y} dy = -e^x(\cos^2 x - \sin 2x) dx$$

Step 3: Integrate

Left side: $\int \left(1 + \frac{1}{y}\right) dy = y + \log y$.

For the right side note that $\frac{d}{dx} (e^x \cos^2 x) = e^x \cos^2 x - e^x \cdot 2 \sin x \cos x = e^x(\cos^2 x - \sin 2x)$.

So the right side integrates to $-e^x \cos^2 x$, and

$$y + \log y + e^x \cos^2 x = C$$

Step 4: Use the condition

At $x = 0, y = 1$: $1 + 0 + 1 = C$, so $C = 2$.

$$\log y + y + e^x \cos^2 x = 2$$

This is option (B). Option (A) has $1/y$ instead of y and $C = 1$, which does not satisfy the given point.

Final Answer:

The solution is $\log y + y + e^x \cos^2 x = 2$, option (B).

$$\log y + y + e^x \cos^2 x = 2$$

Quick Tip: Separate variables and spot that $e^x \cos^2 x$ differentiates to $e^x(\cos^2 x - \sin 2x)$.

137. A unit vector coplanar with $\hat{i} + \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} + \hat{k}$ and perpendicular to $\hat{i} + \hat{j} + \hat{k}$ is

- (A) $\pm \frac{1}{\sqrt{3}}(\hat{i} + \hat{j} + \hat{k})$
- (B) $\pm \frac{1}{\sqrt{3}}(\hat{i} - \hat{j} + \hat{k})$
- (C) $\pm \frac{1}{\sqrt{2}}(\hat{j} - \hat{k})$
- (D) $\pm \frac{1}{\sqrt{2}}(\hat{j} + \hat{k})$

Correct Answer: (C) $\pm \frac{1}{\sqrt{2}}(\hat{j} - \hat{k})$

Solution:

Step 1: Understanding the Concept

A vector coplanar with $\vec{a}$ and $\vec{b}$ can be written as $\alpha\vec{a} + \beta\vec{b}$.

Step 2: Set up

$\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$. Their combination is $(\alpha + \beta)\hat{i} + (\alpha + 2\beta)\hat{j} + (2\alpha + \beta)\hat{k}$.

Perpendicular to $\hat{i} + \hat{j} + \hat{k}$:

$$(\alpha + \beta) + (\alpha + 2\beta) + (2\alpha + \beta) = 4\alpha + 4\beta = 0 \Rightarrow \beta = -\alpha$$

Step 3: Find the unit vector

The vector becomes $\alpha(0\hat{i} - \hat{j} + \hat{k})$, so the direction is $\hat{k} - \hat{j}$, of magnitude $\sqrt{2}$.

$$\pm \frac{1}{\sqrt{2}}(\hat{j} - \hat{k})$$

This is option (C). Option (A) is parallel to $\hat{i} + \hat{j} + \hat{k}$, not perpendicular, and (D) is not in the plane.

Final Answer:

The unit vector is $\pm \frac{1}{\sqrt{2}}(\hat{j} - \hat{k})$, option (C).

$$\boxed{\pm \frac{1}{\sqrt{2}}(\hat{j} - \hat{k})}$$

Quick Tip: Write the vector as a combination of the two given vectors and

force the dot product with (1,1,1) to be zero.



138. If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, $|\vec{a}| = |\vec{b}| = |\vec{c}| = 3$ and θ is the angle between $\vec{b}$ and $\vec{c}$ then $\tan^2 \theta + \cot^2 \theta =$

- (A) $\frac{2}{3}$
- (B) $\frac{5}{3}$
- (C) $\frac{8}{3}$
- (D) $\frac{10}{3}$

Correct Answer: (D) $\frac{10}{3}$

Solution:

Step 1: Understanding the Concept

Since $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, we have $\vec{a} = -(\vec{b} + \vec{c})$. Square both sides to get a relation with the dot product.

Step 2: Find cos theta

$$|\vec{a}|^2 = |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b} \cdot \vec{c} \Rightarrow 9 = 9 + 9 + 2 \cdot 9 \cos \theta$$

$$\cos \theta = -\frac{1}{2} \Rightarrow \theta = 120^\circ$$

Step 3: Evaluate

$\tan 120^\circ = -\sqrt{3}$, so $\tan^2 \theta = 3$, and $\cot^2 \theta = \frac{1}{3}$.

$$\tan^2 \theta + \cot^2 \theta = 3 + \frac{1}{3} = \frac{10}{3}$$

Option (C) $8/3$ would be 3 minus $1/3$, and (B) comes from a wrong angle.

Final Answer:

The value is $\frac{10}{3}$, option (D).

$$\boxed{\frac{10}{3}}$$

Quick Tip: Square the relation $a = -(b + c)$ to find the angle between b and c .

139. The altitude of the parallelepiped, whose coterminous edges are the vectors $\vec{a} = \hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} - \hat{k}$, $\vec{c} = \hat{i} + \hat{j} + 3\hat{k}$, where $\vec{a}$, $\vec{b}$ are the sides of the base of parallelepiped, is

- (A) $\frac{2\sqrt{2}}{\sqrt{19}}$ units
- (B) $\frac{\sqrt{2}}{\sqrt{19}}$ units
- (C) $\frac{\sqrt{19}}{\sqrt{2}}$ units
- (D) $\frac{\sqrt{19}}{2\sqrt{2}}$ units

Correct Answer: (A) $\frac{2\sqrt{2}}{\sqrt{19}}$ units

Solution:

Step 1: Understanding the Concept

Volume of the parallelepiped = base area $\times$ altitude. The base is formed by $\vec{a}$, $\vec{b}$, so the base area is $|\vec{a} \times \vec{b}|$ and the volume is $|(\vec{a} \times \vec{b}) \cdot \vec{c}|$

$\cdot \vec{c}|$.

Step 2: Base vector

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & 4 & -1 \end{vmatrix} = (-1 - 4)\hat{i} - (-1 - 2)\hat{j} + (4 - 2)\hat{k} = -5\hat{i} + 3\hat{j} + 2\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{25 + 9 + 4} = \sqrt{38}$$

Step 3: Volume and height

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = -5 + 3 + 6 = 4$$

$$h = \frac{4}{\sqrt{38}} = \frac{4}{\sqrt{2}\sqrt{19}} = \frac{2\sqrt{2}}{\sqrt{19}}$$

Options (B), (C) and (D) do not equal $4/\sqrt{38}$.

Final Answer:

The altitude is $\frac{2\sqrt{2}}{\sqrt{19}}$ units, option (A).

$$\boxed{\frac{2\sqrt{2}}{\sqrt{19}}}$$

Quick Tip: Altitude = volume divided by base area = $|\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})| / |\mathbf{a} \times \mathbf{b}|$.

140. Let ABCD be a quadrilateral with $\overrightarrow{AB} = \vec{a}$, $\overrightarrow{AD} = \vec{b}$ and $\overrightarrow{AC} = 3\vec{a} + 2\vec{b}$. If its area is α times the area of the parallelogram with AB, AD as adjacent sides, then the value of α is equal to

- (A) $\frac{3}{2}$
- (B) $\frac{5}{2}$
- (C) $\frac{1}{2}$
- (D) 1

Correct Answer: (B) $\frac{5}{2}$

Solution:

Step 1: Understanding the Concept

For quadrilateral ABCD, area = $\frac{1}{2} |\overrightarrow{AC} \times \overrightarrow{BD}|$. The parallelogram on AB and AD has area $|\vec{a} \times \vec{b}|$.

Step 2: Find BD

$$\overrightarrow{BD} = \overrightarrow{AD} - \overrightarrow{AB} = \vec{b} - \vec{a}.$$

Step 3: Cross product

$$\overrightarrow{AC} \times \overrightarrow{BD} = (3\vec{a} + 2\vec{b}) \times (\vec{b} - \vec{a}) = 3(\vec{a} \times \vec{b}) - 2(\vec{b} \times \vec{a}) = 3(\vec{a} \times \vec{b}) + 2(\vec{a} \times \vec{b}) = 5(\vec{a} \times \vec{b})$$

$$\text{Area} = \frac{1}{2} \cdot 5|\vec{a} \times \vec{b}| = \frac{5}{2} |\vec{a} \times \vec{b}|$$

So $\alpha = \frac{5}{2}$, option (B).

Final Answer:

$$\alpha = \frac{5}{2}, \text{ option (B).}$$

$$\boxed{\frac{5}{2}}$$

Quick Tip: Area of a quadrilateral is half the magnitude of the cross product of its diagonals.

141. Let $\vec{a} = \hat{i} + 2\hat{j} - 2\hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + \hat{k}$. If $\vec{c}$ is a vector such that $\vec{a} \cdot \vec{c} = |\vec{c}|$, $|\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $\vec{a} \times \vec{b}$ and $\vec{c}$ is 60° , then $|(\vec{a} \times \vec{b}) \times \vec{c}|$ is equal to

- (A) $\frac{3\sqrt{3}}{2}$
 (B) $\sqrt{\frac{3}{2}}$
 (C) $3\sqrt{\frac{3}{2}}$
 (D) $\frac{9\sqrt{3}}{2}$

Correct Answer: (C) $3\sqrt{\frac{3}{2}}$

Solution:**Step 1: Understanding the Concept**

We need $|\vec{c}|$. The magnitude of $(\vec{a} \times \vec{b}) \times \vec{c}$ equals $|\vec{a} \times \vec{b}| |\vec{c}| \sin 60^\circ$.

Step 2: Find $|\vec{c}|$

$|\vec{a}| = \sqrt{1 + 4 + 4} = 3$. From $|\vec{c} - \vec{a}|^2 = 8$:

$$|\vec{c}|^2 - 2\vec{a} \cdot \vec{c} + |\vec{a}|^2 = 8 \Rightarrow |\vec{c}|^2 - 2|\vec{c}| + 9 = 8 \Rightarrow (|\vec{c}| - 1)^2 = 0$$

So $|\vec{c}| = 1$.

Step 3: Find $|\vec{a} \times \vec{b}|$

$$\vec{a} \times \vec{b} = (2 \cdot 1 - (-2)(-1))\hat{i} - (1 \cdot 1 - (-2)(1))\hat{j} + (1 \cdot (-1) - 2 \cdot 1)\hat{k} = 0\hat{i} - 3\hat{j} - 3\hat{k}$$

$$|\vec{a} \times \vec{b}| = 3\sqrt{2}$$

Step 4: Final value

$$|(\vec{a} \times \vec{b}) \times \vec{c}| = 3\sqrt{2} \times 1 \times \sin 60^\circ = 3\sqrt{2} \cdot \frac{\sqrt{3}}{2} = \frac{3\sqrt{6}}{2} = 3\sqrt{\frac{3}{2}}$$

This is option (C).

Final Answer:

The value is $3\sqrt{\frac{3}{2}}$, option (C).

$$\boxed{3\sqrt{\frac{3}{2}}}$$

Quick Tip: Find $|c|$ from the two given conditions, then use $|\vec{p} \times \vec{c}| = |\vec{p}||\vec{c}|\sin(\text{angle})$.

142. The co-ordinates of the point where the line joining the points $(3, 5, -7)$ and $(-2, 1, 8)$ is intersected by the YOZ plane are

- (A) $(0, \frac{-13}{5}, \frac{2}{5})$
(B) $(0, \frac{13}{5}, \frac{2}{5})$
(C) $(0, \frac{13}{5}, 2)$
(D) $(0, \frac{-13}{5}, -2)$

Correct Answer: (C) $(0, \frac{13}{5}, 2)$

Solution:

Step 1: Understanding the Concept

The YOZ plane is $x = 0$. Points on the line joining $(3, 5, -7)$ and $(-2, 1, 8)$ are $(3 - 5t, 5 - 4t, -7 + 15t)$.

Step 2: Solve for t

$$3 - 5t = 0, \text{ so } t = \frac{3}{5}.$$

Step 3: Find y and z

$$y = 5 - \frac{12}{5} = \frac{13}{5}, \quad z = -7 + 9 = 2$$

The point is $(0, \frac{13}{5}, 2)$, option (C). Option (B) has $z = 2/5$, which does not match.

Final Answer:

The point is $(0, \frac{13}{5}, 2)$, option (C).

$$\boxed{\left(0, \frac{13}{5}, 2\right)}$$

Quick Tip: On the YOZ plane $x = 0$; write the line in parametric form and find the parameter.

143. If the lines $\vec{r} = (\hat{i} + m\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ and $\vec{r} = (4\hat{i} + \hat{j}) + \mu(5\hat{i} + m\hat{j} + \hat{k})$ intersect each other, then $m =$

- (A) 1
- (B) -1
- (C) 2
- (D) -2

Correct Answer: (C) 2

Solution:

Step 1: Understanding the Concept

Two lines intersect when there are values of λ and μ that give the same point.

Step 2: Equate coordinates

$$1 + 2\lambda = 4 + 5\mu \dots (i)$$

$$m + 3\lambda = 1 + m\mu \dots (ii)$$

$$3 + 4\lambda = \mu \dots (iii)$$

Put (iii) into (i): $1 + 2\lambda = 4 + 15 + 20\lambda$, so $-18\lambda = 18$ and $\lambda = -1$. Then $\mu = 3 - 4 = -1$.

Step 3: Use (ii)

$$m - 3 = 1 - m \Rightarrow 2m = 4 \Rightarrow m = 2$$

So $m = 2$, option (C).

Final Answer:

$m = 2$, option (C).

Quick Tip: Equate the position vectors of the two lines and solve for the parameters and m .

144. If the lines $x = -1 + s$, $y = 3 - \lambda s$, $z = 1 + \lambda s$ and $x = \frac{t}{2}$, $y = 1 + t$, $z = 2 - t$ with parameters s and t , are coplanar, then $\lambda =$

- (A) 2
- (B) 1
- (C) -2
- (D) -1

Correct Answer: (C) -2

Solution:

Step 1: Understanding the Concept

Two lines are coplanar when the vector joining a point on each and the two direction vectors have zero scalar triple product.

Step 2: Extract data

Line 1: point $P(-1, 3, 1)$, direction $\vec{d}_1 = (1, -\lambda, \lambda)$.

Line 2: point $Q(0, 1, 2)$, direction $(\frac{1}{2}, 1, -1)$, which is parallel to $\vec{d}_2 = (1, 2, -2)$.

$\vec{PQ} = (1, -2, 1)$.

Step 3: Evaluate the determinant

$$\begin{vmatrix} 1 & -2 & 1 \\ 1 & -\lambda & \lambda \\ 1 & 2 & -2 \end{vmatrix} = 1(2\lambda - 2\lambda) + 2(-2 - \lambda) + 1(2 + \lambda) = -2 - \lambda$$

Setting it to zero gives $\lambda = -2$. That is option (C).

Final Answer:

$\lambda = -2$, option (C).

-2

Quick Tip: Use the determinant condition for coplanarity of two lines in parametric form.

145. The equation of the plane passing through the point $(1, 2, 1)$ and perpendicular to the planes $x + 2y + 2z - 7 = 0$ and $3x + 3y + 2z - 5 = 0$ is

- (A) $2x + y - 2z - 2 = 0$
- (B) $2x - 4y + 3z + 3 = 0$
- (C) $2x + 4y - 5z - 5 = 0$
- (D) $x + 4y - 6z - 3 = 0$

Correct Answer: (B) $2x - 4y + 3z + 3 = 0$

Solution:

Step 1: Understanding the Concept

If a plane is perpendicular to two planes, its normal is perpendicular to both of their normals. So it is along $\vec{n}_1 \times \vec{n}_2$.

Step 2: Normal

$$\vec{n}_1 = (1, 2, 2), \vec{n}_2 = (3, 3, 2).$$

$$\vec{n}_1 \times \vec{n}_2 = (4 - 6)\hat{i} - (2 - 6)\hat{j} + (3 - 6)\hat{k} = -2\hat{i} + 4\hat{j} - 3\hat{k}$$

Step 3: Equation

The plane through (1, 2, 1):

$$-2(x - 1) + 4(y - 2) - 3(z - 1) = 0 \Rightarrow -2x + 4y - 3z - 3 = 0$$

$$2x - 4y + 3z + 3 = 0$$

Option (B). A quick check: substituting (1, 2, 1) gives $2 - 8 + 3 + 3 = 0$.

Final Answer:

The plane is $2x - 4y + 3z + 3 = 0$, option (B).

$$2x - 4y + 3z + 3 = 0$$

Quick Tip: The normal of the required plane is the cross product of the normals of the two given planes.

146. The distance of the point (1, 0, -3) from the plane $x - y - z = 9$ measured parallel to the line $\frac{x-2}{2} = \frac{y+2}{3} = \frac{z-6}{-6}$ is

- (A) 5 units
- (B) 7 units
- (C) 6 units
- (D) 8 units

Correct Answer: (B) 7 units

Solution:

Step 1: Understanding the Concept

To measure distance parallel to a line, travel from the point in the direction of the line. The line through $(1, 0, -3)$ with direction $(2, 3, -6)$ is $(1 + 2t, 3t, -3 - 6t)$.

Step 2: Find the meeting point

Substitute into $x - y - z = 9$:

$$(1 + 2t) - 3t - (-3 - 6t) = 9 \Rightarrow 1 + 2t - 3t + 3 + 6t = 9 \Rightarrow 4 + 5t = 9 \Rightarrow t = 1$$

Step 3: Distance

The meeting point is $(3, 3, -9)$. Distance from $(1, 0, -3)$:

$$\sqrt{(2)^2 + 3^2 + (-6)^2} = \sqrt{4 + 9 + 36} = 7$$

So the distance is 7 units, option (B). The perpendicular distance

$$\frac{|1-0+3-9|}{\sqrt{3}} = \frac{5}{\sqrt{3}} \text{ is not what is asked.}$$

Final Answer:

The distance is 7 units, option (B).

Quick Tip: Move from the point along the given direction until you reach the plane.

147. The difference between the maximum and minimum values of the objective function $Z = 3x + 5y$, subject to the constraints $x + 3y \leq 60$, $x + y \geq 10$, $x - y \leq 0$, $x, y \geq 0$ is

- (A) 50
- (B) 60
- (C) 70
- (D) 80

Correct Answer: (D) 80

Solution:

Step 1: Understanding the Concept

The optimum of a linear objective over a bounded feasible region occurs at a corner point. The constraints are $x + 3y \leq 60$, $x + y \geq 10$, $x \leq y$, $x, y \geq 0$.

Step 2: Find corner points

Intersections of the boundary lines:

1. $x = y$ with $x + y = 10$: (5, 5).
2. $x = y$ with $x + 3y = 60$: (15, 15).
3. $x = 0$ with $x + y = 10$: (0, 10).
4. $x = 0$ with $x + 3y = 60$: (0, 20).

The feasible region is the quadrilateral with these four vertices (it lies

above the line $y = x$ and above $x + y = 10$).

Step 3: Evaluate Z

$Z(5, 5) = 15 + 25 = 40$, $Z(15, 15) = 45 + 75 = 120$, $Z(0, 10) = 50$, $Z(0, 20) = 100$.

Maximum is 120 and minimum is 40. The difference is $120 - 40 = 80$, option (D).

Final Answer:

The difference is 80, option (D).

80

Quick Tip: Find the corner points of the feasible region and evaluate Z at each.

148. For the following probability distribution, the standard deviation of the random variable X is

X	2	3	4
$p(X=x)$	0.2	0.5	0.3

- (A) 0.5
- (B) 0.6
- (C) 0.61
- (D) 0.7

Correct Answer: (D) 0.7

Solution:

Step 1: Understanding the Concept

For a discrete random variable, $\text{Var}(X) = E(X^2) - [E(X)]^2$ and the standard deviation is its square root.

Step 2: Mean

$$E(X) = 2(0.2) + 3(0.5) + 4(0.3) = 0.4 + 1.5 + 1.2 = 3.1$$

Step 3: Variance and SD

$$E(X^2) = 4(0.2) + 9(0.5) + 16(0.3) = 0.8 + 4.5 + 4.8 = 10.1$$

$$\text{Var}(X) = 10.1 - (3.1)^2 = 10.1 - 9.61 = 0.49$$

$$\sigma = \sqrt{0.49} = 0.7$$

Option (C), 0.61, is not a value from this calculation, and (B) 0.6 would need variance 0.36.

Final Answer:

The standard deviation is 0.7, option (D).

0.7

Quick Tip: Compute $E(X)$ and $E(X^2)$, then variance = $E(X^2) - [E(X)]^2$.

149. A random variable X has the following probability distribution

X	1	2	3	4	5	6	7	8
P(X=x)	0.15	0.23	0.12	0.10	0.20	0.08	0.07	0.05

For the event $E = \{X \text{ is a prime number}\}$ and $F = \{X < 4\}$, $P(E \cup F) =$

- (A) 0.87
- (B) 0.77
- (C) 0.35
- (D) 0.50

Correct Answer: (B) 0.77

Solution:

Step 1: Understanding the Concept

$E = \{X \text{ is prime}\} = \{2, 3, 5, 7\}$. $F = \{X < 4\} = \{1, 2, 3\}$. The union rule is $P(E \cup F) = P(E) + P(F) - P(E \cap F)$.

Step 2: Probabilities

$$P(E) = 0.23 + 0.12 + 0.20 + 0.07 = 0.62.$$

$$P(F) = 0.15 + 0.23 + 0.12 = 0.50.$$

$$E \cap F = \{2, 3\}, \text{ so } P(E \cap F) = 0.23 + 0.12 = 0.35.$$

Step 3: Union

$$P(E \cup F) = 0.62 + 0.50 - 0.35 = 0.77$$

Option (C), 0.35, is only the probability of the common part E and F

both occurring.

Final Answer:

The probability is 0.77, option (B).

0.77

Quick Tip: Use $P(E \text{ union } F) = P(E) + P(F) - P(E \text{ intersect } F)$.

150. The probability that event A happens in a trial is 0.4. If three independent trials are made, then the probability that A happens at least once is.....

- (A) 0.216
- (B) 0.926
- (C) 0.064
- (D) 0.784

Correct Answer: (D) 0.784

Solution:

Step 1: Understanding the Concept

"At least once" is the complement of "never". Since the trials are independent, the probability of never is the product of the failure probabilities.

Step 2: Compute

$$P(\text{A fails in one trial}) = 1 - 0.4 = 0.6.$$

$$P(\text{never}) = 0.6^3 = 0.216$$

$$P(\text{at least once}) = 1 - 0.216 = 0.784$$

Option (A), 0.216, is the probability of never, and (C), 0.064, is the probability of three successes.

Final Answer:

The probability is 0.784, option (D).

0.784

Quick Tip: Use the complement: 1 minus the probability that it never happens.