

MHT CET 2026 PCM Question Paper April 16 Shift 1 with Solutions



General Instructions

- (i) This booklet contains 150 questions, each provided with a complete, step-by-step solution.
- (ii) It comprises 150 single-correct multiple-choice questions.
- (iii) Attempt each question on your own before reviewing the given solution.

Physics

Questions 1 to 50 | 1 mark each | 50 marks

1. A wire has mass (0.3 ± 0.003) gram, radius (0.5 ± 0.005) cm and length (6 ± 0.06) cm. The maximum percentage error in the measurement of density is

- (A) 2 %
- (B) 3 %
- (C) 4 %
- (D) 5 %

Correct Answer: (C) 4 %

Solution:

Step 1: Understanding the Concept:

The wire is a cylinder, so its density is $\rho = \frac{m}{\pi r^2 l}$. In a product or quotient the maximum fractional errors add, and a power multiplies the fractional error by that power.

Step 2: Key Formula or Approach:

$$\frac{\Delta \rho}{\rho} = \frac{\Delta m}{m} + 2 \frac{\Delta r}{r} + \frac{\Delta l}{l}$$

Step 3: Detailed Explanation:

$$\frac{\Delta m}{m} = \frac{0.003}{0.3} = 1\%.$$

$$\frac{\Delta r}{r} = \frac{0.005}{0.5} = 1\%, \text{ so the radius contributes } 2 \times 1 = 2\%.$$

$$\frac{\Delta l}{l} = \frac{0.06}{6} = 1\%.$$

$$\frac{\Delta \rho}{\rho} \times 100 = 1 + 2 + 1 = 4\%$$

Option (A) 2% and (B) 3% come from forgetting the square on the radius or adding only some terms. Option (D) 5% adds an extra term.

Step 4: Final Answer:

The maximum percentage error in density is 4%.

Final Answer:

Density error is $1 + 2 + 1 = 4$ percent.

(C) 4%

Quick Tip: Density is $m / (\pi r^2 l)$, so add the mass error, twice the radius error and the length error.

2. Let $\vec{P} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{Q} = -(\hat{i} + \hat{j} + \hat{k})$. The angle between $(\vec{P} - \vec{Q})$ and $\vec{P}$ is

- (A) 90°
- (B) 60°
- (C) 0°
- (D) 30°

Correct Answer: (C) 0°

Solution:

Step 1: Understanding the Concept:

The angle between two vectors follows from $\cos \theta = \frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}$. Two vectors in the same direction have an angle of 0.

Step 2: Compute P - Q:

$$\vec{Q} = -\vec{P}, \text{ so } \vec{P} - \vec{Q} = \vec{P} + \vec{P} = 2\vec{P} = 2(\hat{i} + \hat{j} + \hat{k}).$$

Step 3: Find the angle:

$$\cos \theta = \frac{2\vec{P} \cdot \vec{P}}{|2\vec{P}||\vec{P}|} = \frac{2|\vec{P}|^2}{2|\vec{P}|^2} = 1 \Rightarrow \theta = 0^\circ$$

Step 4: Check:

Numerically: $2\vec{P} = (2, 2, 2)$, $\vec{P} = (1, 1, 1)$, dot product is 6, magnitudes $2\sqrt{3}$ and $\sqrt{3}$, product 6. So $\cos \theta = 1$. Option (C).

Final Answer:

P - Q is twice P, so the angle is 0 degrees.

(C) 0°

Quick Tip: Since $Q = -P$, the vector $P - Q$ equals $2P$, which points along P .



3. A ball is released from height ' h ' which makes perfectly elastic collision with ground. The frequency of periodic vibratory motion is (g =acceleration due to gravity)

- (A) $\frac{1}{2} \sqrt{\frac{g}{2h}}$
- (B) $\frac{1}{2} \sqrt{\frac{2h}{g}}$
- (C) $\frac{1}{2\pi} \sqrt{\frac{g}{2h}}$
- (D) $\frac{1}{2\pi} \sqrt{\frac{2h}{g}}$

Correct Answer: (A) $\frac{1}{2} \sqrt{\frac{g}{2h}}$

Solution:

Step 1: Understanding the Concept:

A ball dropped from height h hits the ground and rebounds elastically to the same height. This repeats, so the motion is periodic.

Step 2: Time of one fall:

$$h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}}$$

Step 3: Period and frequency:

One full cycle consists of a fall and a rise of equal time: $T = 2\sqrt{\frac{2h}{g}}$.

$$f = \frac{1}{T} = \frac{1}{2}\sqrt{\frac{g}{2h}}$$

Step 4: Check the options:

Option (A) matches. Option (B) is $\frac{1}{2}\sqrt{2h/g}$, which is half the fall time and not a frequency. Options (C) and (D) contain 2π , which belongs to SHM and not to this motion.

Final Answer:

The period is twice the fall time.

$(A) \frac{1}{2}\sqrt{\frac{g}{2h}}$

Quick Tip: One full oscillation is a fall and a rise, so the period is twice the fall time.

4. Two boys are standing at points A and B on ground where distance AB = a. The boy at point B starts running perpendicular to line AB with velocity ' V_1 '. The boy at point A starts running simultaneously with velocity ' V ' and catches the other boy in time ' t '. The value of ' t ' is

(A) $\left[\frac{a^2}{(V^2 - V_1^2)} \right]^{\frac{1}{2}}$

(B) $\left[\frac{a^2}{(V_1^2 - V^2)} \right]^{\frac{1}{2}}$

(C) $\left[\frac{a^2}{(V^2 - V_1^2)} \right]$

(D) $\left[\frac{a^2}{(V_1^2 - V^2)} \right]$

Correct Answer: (A) $\left[\frac{a^2}{(V^2 - V_1^2)} \right]^{\frac{1}{2}}$

Solution:

Step 1: Understanding the Concept:

B runs perpendicular to AB, so in time t he moves $V_1 t$ from B. A catches him at that point, so A's path is the hypotenuse of a right triangle with sides a and $V_1 t$.

Step 2: Write the condition:

A's distance is Vt . By Pythagoras:

$$(Vt)^2 = a^2 + (V_1 t)^2$$

Step 3: Solve for t:

$$t^2(V^2 - V_1^2) = a^2 \Rightarrow t = \left[\frac{a^2}{V^2 - V_1^2} \right]^{1/2}$$

Step 4: Check the options:

Option (A) matches, and it needs $V > V_1$ so that A can catch B. Option (B) has $V_1^2 - V^2$, which would be negative when $V > V_1$. Options (C) and (D) are missing the square root, so they would be t^2 and not t .

Final Answer:

Pythagoras gives $t = a / \sqrt{V^2 - V_1^2}$.

$$(A) \left[\frac{a^2}{V^2 - V_1^2} \right]^{1/2}$$

Quick Tip: In time t , A covers Vt along a hypotenuse of a right triangle with sides a and $V_1 t$.

5. A particle is moving with constant angular acceleration 4 rad/s^2 in circular path. At what time the magnitudes of its tangential acceleration and centripetal acceleration will be equal ?

- (A) 0.2 s
- (B) 0.4 s
- (C) 0.5 s
- (D) 0.6 s

Correct Answer: (C) 0.5 s

Solution:**Step 1: Understanding the Concept:**

For circular motion with constant angular acceleration α starting from rest, $\omega = \alpha t$. Tangential acceleration is $a_t = r\alpha$ and centripetal acceleration is $a_c = \omega^2 r$.

Step 2: Set them equal:

$$r\alpha = \omega^2 r = \alpha^2 t^2 r \Rightarrow \alpha t^2 = 1$$

Step 3: Solve:

$$t = \frac{1}{\sqrt{\alpha}} = \frac{1}{\sqrt{4}} = 0.5 \text{ s}$$

Step 4: Check:

At $t = 0.5 \text{ s}$, $\omega = 4 \times 0.5 = 2 \text{ rad/s}$, so $a_c = 4r$ and $a_t = 4r$. They are equal. At 0.2 s , $a_c = 0.64r < a_t$, so option (A) is not it. Option (C).

Final Answer:

Equating $r\alpha$ with $\alpha^2 t^2 r$ gives $t = 0.5 \text{ s}$.

(C) 0.5 s

Quick Tip: Tangential is $r\alpha$, centripetal is $\omega^2 r$ with $\omega = \alpha t$.

6. A particle describes a horizontal circle of radius 'r' in conical funnel with smooth inner surface with a speed of 0.5 m/s. The height of the plane of the circle from the vertex of the funnel is (acceleration due to gravity, $g = 10 \text{ m/s}^2$)

- (A) 1.5 cm
- (B) 2.5 cm
- (C) 3.5 cm
- (D) 0.5 cm

Correct Answer: (B) 2.5 cm

Solution:

Step 1: Understanding the Concept:

The particle moves in a horizontal circle on the smooth inner surface of a cone. The forces are gravity mg and the normal reaction N , perpendicular to the surface. Let the surface make angle θ with the horizontal.

Step 2: Force balance:

Vertical: $N \cos \theta = mg$. Horizontal (centripetal): $N \sin \theta = \frac{mv^2}{r}$.

Dividing: $\tan \theta = \frac{v^2}{rg}$.

Step 3: Relate to the height:

The height of the circle above the vertex is $h = r \tan \theta$. So

$$v^2 = gr \tan \theta = gh \Rightarrow h = \frac{v^2}{g}$$

Step 4: Compute:

$$h = \frac{(0.5)^2}{10} = 0.025 \text{ m} = 2.5 \text{ cm}$$

Option (B). The other values (1.5, 3.5, 0.5 cm) do not satisfy $h = v^2/g$.

Final Answer:

For a smooth cone, v squared equals $g h$, so $h = 2.5 \text{ cm}$.

(B) 2.5 cm

Quick Tip: Resolve the normal force: $v^2 = gh$ for a smooth cone.

7. A block of mass 'm' moving on a frictionless horizontal surface collides with a spring of spring constant 'K' and compresses it through a distance 'x'. The maximum momentum of the block after collision is

- (A) zero
- (B) $\sqrt{kmx}$
- (C) mx^2/K
- (D) $Kx^2/2m$

Correct Answer: (B) $\sqrt{kmx}$

Solution:

Step 1: Understanding the Concept:

On a frictionless surface the total mechanical energy is conserved. All the kinetic energy of the block goes into the spring at maximum compression x .

Step 2: Energy balance:

$$\frac{1}{2}mv^2 = \frac{1}{2}Kx^2 \Rightarrow v = x\sqrt{\frac{K}{m}}$$

Step 3: Momentum:

$$p = mv = m x \sqrt{\frac{K}{m}} = x \sqrt{Km}$$

This is the largest momentum the block carries, since its speed only falls once the spring is compressed.

Step 4: Check the options:

Option (B) $\sqrt{km} x$ matches. Option (A) is the momentum at the instant of maximum compression. Options (C) and (D) have units that do not match momentum: mx^2/K and $Kx^2/2m$ are not kg m/s.

Final Answer:

Momentum is m times x times root of K over m.

(B) $\sqrt{Km} x$

Quick Tip: The block's kinetic energy before contact equals the spring energy at maximum compression.

8. A Solid sphere has mass M and radius R. Its moment of inertia about a parallel axis passing through a point at a distance R/3 from its centre is

- (A) $8MR^2/11$
- (B) $11MR^2/15$
- (C) $23MR^2/45$
- (D) $13MR^2/20$

Correct Answer: (C) $23MR^2/45$

Solution:

Step 1: Understanding the Concept:

The moment of inertia of a solid sphere about an axis through its centre is $\frac{2}{5}MR^2$. For a parallel axis at distance d from the centre, $I = I_{cm} + Md^2$.

Step 2: Apply the theorem:

With $d = \frac{R}{3}$:

$$I = \frac{2}{5}MR^2 + M\left(\frac{R}{3}\right)^2 = \frac{2}{5}MR^2 + \frac{1}{9}MR^2$$

Step 3: Add the fractions:

$$\frac{2}{5} + \frac{1}{9} = \frac{18 + 5}{45} = \frac{23}{45}$$

$$I = \frac{23}{45}MR^2$$

Step 4: Check:

Option (C), $\frac{23}{45}MR^2$, is the value that results.

Final Answer:

$$I = \frac{2}{5}MR^2 + M(R/3)^2 = \frac{23}{45}MR^2$$

$(C) \frac{23}{45}MR^2$

Quick Tip: Use the parallel axis theorem with $I_{cm} = 2MR^2/5$.

9. The masses and radii of the earth and moon are M_1, R_1 and M_2, R_2 and respectively, Their centres are at a distance 'd' apart. The minimum speed with which body of mass 'm' should be projected from a distance $2d/3$ from the centre of M_1 so as to escape to infinity is

(A) $\sqrt{\frac{6G}{d}(2M_1 + M_2)}$

(B) $\sqrt{\frac{6G}{d}(M_1 - 2M_2)}$

(C) $\sqrt{\frac{3G}{d}(M_1 + 2M_2)}$

(D) $\sqrt{\frac{8G}{d}(M_1 - 2M_2)}$

Correct Answer: (C) $\sqrt{\frac{3G}{d}(M_1 + 2M_2)}$

Solution:

Step 1: Understanding the Concept:

To escape to infinity the total energy at launch must be at least zero:

$\frac{1}{2}mv^2 + mV = 0$, where V is the total gravitational potential from both bodies.

Step 2: Distances:

The point is $\frac{2d}{3}$ from M_1 . Along the line towards M_2 the distance to M_2 is $d - \frac{2d}{3} = \frac{d}{3}$.

Step 3: Potential at the point:

$$V = -\frac{GM_1}{2d/3} - \frac{GM_2}{d/3} = -\frac{3G}{d} \left(\frac{M_1}{2} + M_2 \right)$$

Step 4: Solve for v:

$$\frac{1}{2}v^2 = \frac{3G}{d} \left(\frac{M_1}{2} + M_2 \right) \Rightarrow v^2 = \frac{3G}{d} (M_1 + 2M_2)$$

$$v = \sqrt{\frac{3G}{d} (M_1 + 2M_2)}$$

Option (C). Options (A), (B), (D) have wrong numerical factors or a subtraction of the moon's potential.

Final Answer:

Add both potentials at $2d/3$ and $d/3$ to get option C.

$$(C) \sqrt{\frac{3G}{d} (M_1 + 2M_2)}$$

Quick Tip: Add the potentials of both bodies at the launch point and set total energy to zero.

10. A rectangular block of mass 'm' and cross-sectional area 'A' floats on a liquid of density ' ρ '. It is given a small vertical displacement from equilibrium, it starts oscillating with frequency (g=acceleration due to gravity)

- (A) $2\pi\sqrt{\frac{m}{A\rho g}}$
 (B) $2\pi\sqrt{\frac{A\rho g}{m}}$
 (C) $\frac{1}{2\pi}\sqrt{\frac{A\rho g}{m}}$
 (D) $\frac{1}{2\pi}\sqrt{\frac{m}{A\rho g}}$

Correct Answer: (C) $\frac{1}{2\pi}\sqrt{\frac{A\rho g}{m}}$

Solution:

Step 1: Understanding the Concept:

At equilibrium, weight equals buoyant force. If the block is pushed down a small distance y , it displaces extra liquid volume Ay .

Step 2: Restoring force:

$$F = -A\rho g y$$

This has the form $F = -ky$ with $k = A\rho g$, so the motion is simple harmonic.

Step 3: Frequency:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{A\rho g}{m}}, \quad f = \frac{1}{2\pi}\sqrt{\frac{A\rho g}{m}}$$

Step 4: Check the options:

Option (C) matches. Options (A) and (B) are time periods with 2π in the numerator, and (D) has the ratio upside down.

Final Answer:

The extra buoyant force acts like a spring of constant $A\rho g$.

$$(C) \frac{1}{2\pi} \sqrt{\frac{A\rho g}{m}}$$

Quick Tip: The extra buoyant force is $A\rho g$ times the displacement, like a spring of constant $A\rho g$.



11. A closed pipe containing liquid showed a pressure P_1 by gauge. When the valve was opened, pressure was reduced to P_2 . The speed of water flowing out of the pipe is (ρ =density of water)

- (A) $\left[\frac{(P_1-P_2)}{\rho}\right]^{\frac{1}{2}}$
- (B) $\left[\frac{2(P_1-P_2)}{\rho}\right]^{\frac{1}{2}}$
- (C) $\left[\frac{(P_2-P_1)}{\rho}\right]^{\frac{1}{2}}$
- (D) $\left[\frac{2(P_2-P_1)}{\rho}\right]^{\frac{1}{2}}$

Correct Answer: (B) $\left[\frac{2(P_1-P_2)}{\rho}\right]^{\frac{1}{2}}$

Solution:**Step 1: Understanding the Concept:**

Bernoulli's equation for a streamline at the same level: $P + \frac{1}{2}\rho v^2 = \text{constant}$.

Step 2: Apply it:

Inside the closed pipe the water is at rest at pressure P_1 . At the outlet the pressure is P_2 and the speed is v :

$$P_1 = P_2 + \frac{1}{2}\rho v^2$$

Step 3: Solve for v:

$$v = \sqrt{\frac{2(P_1 - P_2)}{\rho}}$$

Step 4: Check the options:

Option (B) matches. Option (A) misses the factor 2. Options (C) and (D) have $P_2 - P_1$, which is negative because $P_2 < P_1$.

Final Answer:

Pressure difference turns into $\frac{1}{2} \rho v^2$.

$$\boxed{(B) \left[\frac{2(P_1 - P_2)}{\rho} \right]^{1/2}}$$

Quick Tip: Apply Bernoulli's equation: the pressure drop becomes kinetic energy.

12. Water flows through a horizontal pipe at a speed 'V'. Internal diameter of the pipe is 'd'. If the water is emerging at a speed ' V_1 ' then the diameter of the nozzle is

(A) $d\sqrt{\frac{V}{V_1}}$

(B) $d\sqrt{\frac{V_1}{V}}$

(C) $\frac{dV}{V_1}$

(D) $\frac{dV_1}{V}$

Correct Answer: (A) $d\sqrt{\frac{V}{V_1}}$

Solution:

Step 1: Understanding the Concept:

For an incompressible liquid the equation of continuity is $A_1v_1 = A_2v_2$. The area of a circular section is $\frac{\pi d^2}{4}$.

Step 2: Apply:

$$\frac{\pi d^2}{4} V = \frac{\pi d_1^2}{4} V_1$$

Step 3: Solve:

$$d_1^2 = d^2 \frac{V}{V_1} \Rightarrow d_1 = d\sqrt{\frac{V}{V_1}}$$

Step 4: Check:

Option (A) matches. Since the nozzle makes the water faster ($V_1 > V$), $d_1 < d$, which makes sense. Option (B) has the ratio inverted, and (C) and (D) forget the square root.

Final Answer:

Area times speed is constant, so $d_1 = d\sqrt{\frac{V}{V_1}}$.

$(A) d\sqrt{\frac{V}{V_1}}$

Quick Tip: Flow rate $A v$ is the same at the pipe and nozzle.

13. A liquid is at rest in a container. In a sphere of influence, the liquid molecule at its centre is

- (A) attracted by other molecules in the sphere of influence.
- (B) attracted by other molecules outside sphere of influence.
- (C) repelled by other molecules outside the sphere of influence.
- (D) repelled by other molecules in the sphere of influence.

Correct Answer: (A) attracted by other molecules in the sphere of influence.

Solution:

Step 1: Understanding the Concept:

Cohesive forces between liquid molecules are short ranged. The sphere of influence is the sphere of about 10^{-9} m radius around a molecule, outside of which the force is negligible.

Step 2: Detailed Explanation:

A molecule at the centre of the sphere is attracted by the other molecules that lie inside the sphere. Since it is surrounded from all sides, the pulls are balanced and the net force on it is zero.

Molecules outside the sphere are too far away to exert a noticeable attraction, so options (B) and (C) are not the case. The force between liquid molecules at this range is attractive, so repulsion in (D) is not what acts.

Step 3: Final Answer:

Option (A): the molecule is attracted by the other molecules within the sphere of influence.

Final Answer:

A centre molecule is pulled by neighbours within its sphere.

(A) attracted by molecules inside the sphere

Quick Tip: The molecule at the centre is pulled by neighbours within the sphere of influence.

14. Rate of flow of heat through a cylindrical rod is ' H_1 '. The temperature at the ends of the rod are T_1 and T_2 . If all the dimensions of the rod become double and the temperature difference remains the same and if the rate of flow of heat becomes ' H_2 ' then $H_2 =$

- (A) $\frac{H_1}{2}$
- (B) $4H_1$
- (C) $2H_1$
- (D) $\frac{H_1}{4}$

Correct Answer: (C) $2H_1$

Solution:

Step 1: Understanding the Concept:

Rate of heat conduction through a rod is $H = \frac{kA \Delta T}{L}$, where A is the cross-sectional area and L is the length.

Step 2: Effect of doubling dimensions:

Doubling the radius makes the area $A \rightarrow 4A$. Doubling the length gives $L \rightarrow 2L$. The temperature difference and material are unchanged.

Step 3: Compute:

$$H_2 = \frac{k(4A)\Delta T}{2L} = 2 \frac{kA\Delta T}{L} = 2H_1$$

Step 4: Check:

Option (C). Option (B) $4H_1$ forgets the longer length, and option (A) $H_1/2$ takes the area as unchanged and the length as doubled.

Final Answer:

Area grows 4 times, length 2 times, so H doubles.

(C) $2H_1$

Quick Tip: H is proportional to A / L for a fixed temperature difference.

15. The energy spectrum of a black body exhibits a maximum around a wavelength ' λ '. The temperature of the black body is now changed such that the energy is maximum around a wavelength $2\lambda/3$. The power radiated by the black body will now increase by a factor

- (A) $\frac{9}{4}$
- (B) $\frac{81}{16}$
- (C) $\frac{3}{2}$
- (D) $\frac{27}{8}$

Correct Answer: (B) $\frac{81}{16}$

Solution:

Step 1: Understanding the Concept:

Wien's displacement law: $\lambda_m T = \text{constant}$. Stefan-Boltzmann law: power radiated $P \propto T^4$.

Step 2: New temperature:

$$\lambda_1 T_1 = \lambda_2 T_2 \text{ with } \lambda_2 = \frac{2\lambda}{3}:$$

$$T_2 = T_1 \cdot \frac{\lambda}{2\lambda/3} = \frac{3}{2} T_1$$

Step 3: Power ratio:

$$\frac{P_2}{P_1} = \left(\frac{T_2}{T_1} \right)^4 = \left(\frac{3}{2} \right)^4 = \frac{81}{16}$$

Step 4: Check:

Option (B). Option (A) $\frac{9}{4}$ is only $(T_2/T_1)^2$, and (C) is T_2/T_1 itself.

Option (D) $\frac{27}{8}$ is $(T_2/T_1)^3$.

Final Answer:

T becomes $3/2$ times, so power rises by $(3/2)^4$.

$$(B) \frac{81}{16}$$

Quick Tip: Wien's law gives the new temperature; power goes as T^4 .

16. A black rectangular surface of area 'A' emits energy 'E' per second at 27°C . If length and breadth is reduced to half of its initial value and temperature is raised to 327°C then energy emitted per second becomes

- (A) $2E$
- (B) $4E$
- (C) $8E$
- (D) $16E$

Correct Answer: (B) $4E$

Solution:

Step 1: Understanding the Concept:

Stefan's law for a black body: $E = \sigma AT^4$, with T in kelvin.

Step 2: Convert temperatures and area:

$T_1 = 27 + 273 = 300 \text{ K}$ and $T_2 = 327 + 273 = 600 \text{ K}$, so $T_2/T_1 = 2$.

Length and breadth are both halved, so the area becomes $\frac{A}{4}$.

Step 3: Compute the new energy:

$$\frac{E_2}{E_1} = \frac{A/4}{A} \left(\frac{600}{300} \right)^4 = \frac{1}{4} \times 16 = 4$$

Step 4: Check:

Option (B) $4E$. Option (D) $16E$ forgets the area reduction. Option (A) and (C) do not follow from the factors $1/4$ and 16 .

Final Answer:

Area drops by 4 and T^4 rises by 16, so energy is $4E$.

(B) $4E$

Quick Tip: Energy per second is proportional to area times T^4 with T in kelvin.

17. A monoatomic gas at pressure P having volume V expands isothermally to a volume $3V$ and then adiabatically to a volume $81V$. The final pressure of the gas is ($\gamma = 5/3$).

- (A) $\frac{P}{3}$
- (B) $\frac{P}{(3)^2}$
- (C) $\frac{P}{(3)^4}$
- (D) $\frac{P}{(3)^6}$

Correct Answer: (D) $\frac{P}{(3)^6}$

Solution:

Step 1: Understanding the Concept:

Isothermal: $PV = \text{constant}$. Adiabatic: $PV^\gamma = \text{constant}$ with $\gamma = \frac{5}{3}$.

Step 2: Isothermal step:

$$P \cdot V = P_1 \cdot 3V \Rightarrow P_1 = \frac{P}{3}.$$

Step 3: Adiabatic step:

$$P_1(3V)^{5/3} = P_2(81V)^{5/3} \Rightarrow P_2 = P_1 \left(\frac{3}{81} \right)^{5/3} = P_1 \left(\frac{1}{27} \right)^{5/3}$$

$$27^{5/3} = (3^3)^{5/3} = 3^5, \text{ so } P_2 = \frac{P_1}{3^5}.$$

Step 4: Result:

$$P_2 = \frac{P}{3} \cdot \frac{1}{3^5} = \frac{P}{3^6}$$

Option (D). Options (A), (B), (C) have too small a power of 3.

Final Answer:

$P_1 = P/3$, then dividing by 3^5 gives $P/3^6$.

$(D) \frac{P}{3^6}$

Quick Tip: Use Boyle's law for the isothermal step and PV^γ for the adiabatic step.

18. An ideal gas at 27°C is compressed adiabatically to $(\frac{8}{27})$ of its original volume. If $\gamma = 5/3$, then the rise in temperature of a gas is

- (A) 300 K
- (B) 675 K
- (C) 375 K
- (D) 27 K

Correct Answer: (C) 375 K

Solution:

Step 1: Understanding the Concept:

For an adiabatic change of an ideal gas, $TV^{\gamma-1}$ stays constant.

Step 2: Apply:

$$T_1 = 27 + 273 = 300 \text{ K and } V_2 = \frac{8}{27} V_1, \gamma - 1 = \frac{2}{3}.$$

$$\frac{T_2}{T_1} = \left(\frac{V_1}{V_2} \right)^{2/3} = \left(\frac{27}{8} \right)^{2/3} = \frac{9}{4}$$

Step 3: Final temperature and rise:

$$T_2 = 300 \times \frac{9}{4} = 675 \text{ K.}$$

$$\text{Rise in temperature} = 675 - 300 = 375 \text{ K.}$$

Step 4: Check:

Option (C). Option (B) 675 K is the final temperature itself, not the rise.

Final Answer:

$T_2 = 675 \text{ K}$, so the rise is 375 K.

(C) 375 K

Quick Tip: Use $TV^{(\gamma - 1)} = \text{constant}$.

19. The translational kinetic energy of the molecules of a gas at absolute temperature (T) can be doubled by

- (A) increasing T to $4T$
- (B) increasing T to $\sqrt{2}T$
- (C) decreasing T to $T/2$
- (D) increasing T to $2T$

Correct Answer: (D) increasing T to $2T$

Solution:

Step 1: Understanding the Concept:

The average translational kinetic energy per molecule is $\frac{3}{2}k_B T$. It is directly proportional to the absolute temperature.

Step 2: Detailed Explanation:

To double the energy, the absolute temperature must double: $T \rightarrow 2T$. Raising T to $4T$ would give four times the energy. Raising to $\sqrt{2}T$ gives only $\sqrt{2}$ times the energy. Decreasing to $T/2$ halves the energy.

Step 3: Final Answer:

Option (D), increasing T to $2T$.

Final Answer:

Kinetic energy is proportional to T , so double T .

$$\boxed{(D) \ T \rightarrow 2T}$$

Quick Tip: Average translational energy is $(3/2) kT$, directly proportional to T .



20. For a particle P performing S.H.M., when displacement is 'x', potential energy and restoring force acting on it is denoted by 'E' and 'F' respectively. The relation between x, E and F is

(A) $\frac{2E}{F} - x = 0$

(B) $\frac{2E}{F} + x = 0$

(C) $\frac{2F}{x} + F = 0$

(D) $\frac{2E}{x} - F = 0$

Correct Answer: (B) $\frac{2E}{F} + x = 0$

Solution:

Step 1: Understanding the Concept:

In SHM the restoring force is $F = -kx$ and the potential energy is $E = \frac{1}{2}kx^2$.

Step 2: Divide the two:

$$\frac{E}{F} = \frac{\frac{1}{2}kx^2}{-kx} = -\frac{x}{2}$$

Step 3: Rearrange:

$$\frac{2E}{F} = -x \Rightarrow \frac{2E}{F} + x = 0$$

Step 4: Check:

Option (B). Option (A) has the wrong sign. Options (C) and (D) are not consistent: in (D), $\frac{2E}{x} = kx$ while $F = -kx$, so $\frac{2E}{x} - F$ would be $2kx$, not zero.

Final Answer:

Dividing E by F gives $2E/F = -x$.

$$(B) \frac{2E}{F} + x = 0$$

Quick Tip: Use $E = (1/2) k x^2$ and $F = -k x$.

21. A progressive wave of frequency 50 Hz is travelling with velocity 350 m/s through a medium. The change in phase at given time interval of 0.01s is

- (A) $\frac{\pi}{4}$ rad
- (B) $\frac{\pi}{2}$ rad
- (C) π rad
- (D) $\frac{3\pi}{2}$ rad

Correct Answer: (C) π rad

Solution:**Step 1: Understanding the Concept:**

At a given point, the phase of a travelling wave changes with time as $\Delta\phi = \omega\Delta t = 2\pi f\Delta t$. The wave speed is not needed here.

Step 2: Compute:

$$\Delta\phi = 2\pi \times 50 \times 0.01 = \pi \text{ rad}$$

Step 3: Check:

The period is $T = \frac{1}{50} = 0.02 \text{ s}$, so 0.01 s is half a period. Half a period corresponds to a phase change of π . Option (C). The other options are $\frac{1}{4}$, $\frac{1}{2}$ and $\frac{3}{2}$ of that.

Final Answer:

0.01 s is half a period, a phase change of π .

(C) $\pi \text{ rad}$

Quick Tip: Phase change equals angular frequency times the time interval.

22. When a sonometer wire vibrates in the third overtone, the number of antinodes and nodes formed on the wire are respectively

- (A) 4, 5
- (B) 5, 4
- (C) 3, 4
- (D) 4, 3

Correct Answer: (A) 4, 5

Solution:

Step 1: Understanding the Concept:

A sonometer wire is fixed at both ends, so both ends are nodes. The fundamental has 1 loop, the first overtone has 2 loops, and so on.

Step 2: Third overtone:

Third overtone means the 4th harmonic, so there are 4 loops. Each loop contains one antinode, so antinodes = 4.

Step 3: Count nodes:

With both ends as nodes, nodes = antinodes + 1 = 5.

Step 4: Check:

Option (A): 4 antinodes and 5 nodes. Option (B) swaps the numbers. Options (C) and (D) correspond to the 2nd and 3rd harmonic or other modes.

Final Answer:

The 4th harmonic has 4 antinodes and 5 nodes.

(A) 4, 5

Quick Tip: Third overtone is the 4th harmonic of a string fixed at both ends.

23. When the observer moves towards a stationary source with velocity ' V_1 ', the apparent frequency of the emitted note is ' F_1 '. When the observer moves away from the source with velocity ' V_1 ', the apparent frequency is ' F_2 '. If ' V ' is the speed of sound in air and $F_1/F_2 = 2$, then $V/V_1 =$

- (A) 2
- (B) 3
- (C) 6
- (D) 8

Correct Answer: (B) 3

Solution:

Step 1: Understanding the Concept:

For a stationary source and a moving observer, apparent frequency is $f' = f \frac{V \pm V_o}{V}$, with a plus sign for approach and a minus sign for recession.

Step 2: Write F1 and F2:

$$F_1 = f \frac{V + V_1}{V} \text{ and } F_2 = f \frac{V - V_1}{V}.$$

Step 3: Ratio:

$$\frac{F_1}{F_2} = \frac{V + V_1}{V - V_1} = 2 \Rightarrow V + V_1 = 2V - 2V_1 \Rightarrow V = 3V_1$$

Step 4: Result:

$$\frac{V}{V_1} = 3, \text{ option (B).}$$

Final Answer:

$$(V + V_1)/(V - V_1) = 2 \text{ gives } V = 3 V_1.$$

(B) 3

Quick Tip: Write apparent frequencies for approach and recession, then take the ratio.



24. The equation of wave motion is $Y = 6\sin(12\pi t - 0.02\pi x + \frac{\pi}{3})$ where x is in metre and time in second. The velocity of the wave is

- (A) 200 m/s
- (B) 300 m/s
- (C) 400 m/s
- (D) 600 m/s

Correct Answer: (D) 600 m/s

Solution:

Step 1: Understanding the Concept:

Compare with the standard form $y = A \sin(\omega t - kx + \phi)$. The wave speed is $v = \frac{\omega}{k}$.

Step 2: Read off the values:

$\omega = 12\pi$ rad/s and $k = 0.02\pi$ rad/m.

Step 3: Compute:

$$v = \frac{12\pi}{0.02\pi} = 600 \text{ m/s}$$

Step 4: Check:

Option (D). The wave travels in the $+x$ direction because the time and position terms have opposite signs. Options (A), (B), (C) would need

ω/k of 200, 300 or 400.

Final Answer:

$v = \omega \text{ over } k = 12 \pi / 0.02 \pi$.

(D) 600 m/s

Quick Tip: Wave speed equals omega divided by k.

25. The electric field in the region is $\vec{E} = a\hat{i} + b\hat{j}$ where 'a' and 'b' are constants. The net electric flux passing through a square area of side 'l' parallel to Y-Z plane is

- (A) al
- (B) al^4
- (C) al^6
- (D) al^2

Correct Answer: (D) al^2

Solution:

Step 1: Understanding the Concept:

Flux through a flat area is $\Phi = \vec{E} \cdot \vec{A}$. A square in the Y-Z plane has its normal along the x-axis, so $\vec{A} = l^2\hat{i}$.

Step 2: Compute:

$$\Phi = (a\hat{i} + b\hat{j}) \cdot (l^2\hat{i}) = al^2$$

The $b\hat{j}$ part is parallel to the surface and gives no flux.

Step 3: Check:

Option (D) al^2 . Options (B) and (C) have wrong powers of l , and (A) al has the units of a length rather than an area.

Final Answer:

Only the i component passes through the surface.

$$(D) \ al^2$$

Quick Tip: Only the component of E along the normal to the area gives flux.

26. Two point charges $q_1 = 6\mu\text{C}$ and $q_2 = 4\mu\text{C}$ are kept at points A and B in air where distance $AB = 10$ cm. What is the increase in potential energy of the system when q_2 is moved towards q_1 , by 2 cm ? ($\frac{1}{4\pi\epsilon_0} = 9 \times 10^9 \text{SI units}$)

- (A) 21.6 J
- (B) 216 J
- (C) 0.54 J
- (D) 54 J

Correct Answer: (C) 0.54 J

Solution:

Step 1: Understanding the Concept:

The potential energy of a pair of point charges is $U = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r}$.

Step 2: Initial and final energies:

$$q_1 q_2 = 6 \times 10^{-6} \times 4 \times 10^{-6} = 24 \times 10^{-12} \text{ C}^2.$$

$$\text{Initially } r = 0.10 \text{ m: } U_1 = \frac{9 \times 10^9 \times 24 \times 10^{-12}}{0.10} = 2.16 \text{ J.}$$

$$\text{After moving 2 cm closer } r = 0.08 \text{ m: } U_2 = \frac{9 \times 10^9 \times 24 \times 10^{-12}}{0.08} = 2.70 \text{ J.}$$

Step 3: Change:

$$\Delta U = 2.70 - 2.16 = 0.54 \text{ J}$$

Step 4: Check:

Option (C). Options (A), (B), (D) are off by powers of ten or use the wrong separation.

Final Answer:

U changes from 2.16 J to 2.70 J.

(C) 0.54 J

Quick Tip: Potential energy of two charges is $k q_1 q_2 / r$; find the change from 10 cm to 8 cm.

27. The electric field between the plates of a parallel plate capacitor is 'E'. If the charge on the plates is Q then the force on each plate is

- (A) QE^2
- (B) QE
- (C) $\frac{QE}{2}$
- (D) $\frac{QE^2}{2}$

Correct Answer: (C) $\frac{QE}{2}$

Solution:

Step 1: Understanding the Concept:

The field E between the plates is the sum of the fields from the two plates, each contributing $E/2$.

Step 2: Force on one plate:

The charge Q on one plate feels only the field of the other plate, not its own. So

$$F = Q \cdot \frac{E}{2} = \frac{QE}{2}$$

Step 3: Check:

Option (C). Option (B) QE would use the total field, which includes the plate's own field that exerts no net force on itself. Options (A) and (D) have E^2 , which does not fit the unit of force here.

Final Answer:

The other plate gives $E/2$, so $F = QE/2$.

$(C) \frac{QE}{2}$

Quick Tip: The field of one plate is $E/2$, and it acts on the charge of the other plate.



28. Two condensers of capacities, ' $2C$ ' and ' C ' are joined in parallel and charged upto potential ' V '. The battery is then disconnected and condenser of capacity ' C ' is filled completely with a medium of dielectric constant ' K '. The potential difference across the capacitors in the second case is

- (A) $\frac{3V}{(K+2)}$
- (B) $\frac{V}{(K+2)}$
- (C) $\frac{5V}{(K+2)}$
- (D) $\frac{2V}{(K+2)}$

Correct Answer: (A) $\frac{3V}{(K+2)}$

Solution:

Step 1: Understanding the Concept:

Two capacitors $2C$ and C in parallel have a total capacitance $3C$. Charged to V , the total charge is $Q = 3CV$. After disconnecting the battery, Q cannot change.

Step 2: New capacitance:

Filling C with a dielectric of constant K makes it KC . The parallel combination now has $2C + KC = (K + 2)C$.

Step 3: New potential:

$$V' = \frac{Q}{C_{new}} = \frac{3CV}{(K + 2)C} = \frac{3V}{K + 2}$$

Step 4: Check:

Option (A). For $K = 1$ (no dielectric), $V' = V$, as expected. Options (B), (C), (D) do not give V at $K = 1$.

Final Answer:

Total charge is fixed, capacitance becomes $(K + 2)C$.

$$(A) \frac{3V}{K + 2}$$

Quick Tip: The battery is disconnected, so total charge stays constant.



29. A potentiometer wire is 4m long and potential difference of 3V is maintained between the ends. The e.m.f. of the cell which balances against a length of 100 cm of the potentiometer wire is

- (A) 0.75 V
- (B) 0.5 V
- (C) 0.25 V
- (D) 1.5 V

Correct Answer: (A) 0.75 V

Solution:**Step 1: Understanding the Concept:**

In a potentiometer the potential difference along the wire is proportional to length. The cell balances when its emf equals the potential drop over the balancing length.

Step 2: Potential gradient:

$$k = \frac{3 \text{ V}}{4 \text{ m}} = 0.75 \text{ V/m}$$

Step 3: EMF:

Balancing length = 100 cm = 1 m, so

$$E = k l = 0.75 \times 1 = 0.75 \text{ V}$$

Step 4: Check:

Option (A). Option (B) 0.5 V would be for 2/3 m, and (D) 1.5 V would need 2 m.

Final Answer:

Potential gradient 0.75 V/m times 1 m.

(A) 0.75 V

Quick Tip: Potential gradient is 3 V per 4 m; multiply by 1 m.



30. A metre bridge experiment, the resistance in the left gap is 20Ω and in the right gap is 60Ω . The bridge is balanced. The distance of the null point from the centre of the wire is

- (A) 50 cm
- (B) 75 cm
- (C) 25 cm
- (D) 40 cm

Correct Answer: (C) 25 cm

Solution:

Step 1: Understanding the Concept:

Metre bridge balance condition: $\frac{R_{left}}{R_{right}} = \frac{l}{100 - l}$, where l is measured from the left end.

Step 2: Find the null point:

$$\frac{20}{60} = \frac{l}{100 - l} \Rightarrow 60l = 20(100 - l) \Rightarrow 80l = 2000 \Rightarrow l = 25 \text{ cm}$$

Step 3: Distance from the centre:

The centre of the wire is at 50 cm. So the null point is $50 - 25 = 25$ cm from the centre.

Step 4: Check:

Option (C). The value 25 cm from the left end happens to also be 25 cm from the centre in this case. Option (A) 50 cm and (B) 75 cm confuse the position with the position of the centre or the right end.

Final Answer:

Null point at 25 cm, which is 25 cm from the centre.

(C) 25 cm

Quick Tip: At balance, $R_1/R_2 = l/(100 - l)$.

31. The magnetic moment produced in a sample of 2 gram is 8×10^{-7} A/m². If its density is 4g/cm³ then the magnetization of the sample is

- (A) 1.6
- (B) 1.2
- (C) 1.4
- (D) 1.8

Correct Answer: (A) 1.6

Solution:

Step 1: Understanding the Concept:

Magnetisation is $M = \frac{m_{\text{moment}}}{V}$, the magnetic moment per unit volume.

Step 2: Volume:

$$V = \frac{\text{mass}}{\text{density}} = \frac{2 \text{ g}}{4 \text{ g/cm}^3} = 0.5 \text{ cm}^3 = 0.5 \times 10^{-6} \text{ m}^3$$

Step 3: Magnetisation:

$$M = \frac{8 \times 10^{-7}}{0.5 \times 10^{-6}} = 1.6 \text{ A/m}$$

Step 4: Check:

Option (A). Other options would need a different volume, such as $0.667 \times 10^{-6} \text{ m}^3$ for 1.2.

Final Answer:

Volume is 0.5 cm^3 , so $M = 1.6 \text{ A/m}$.

(A) 1.6 A/m

Quick Tip: Magnetisation is magnetic moment per unit volume; find the volume from mass and density.

32. A wire carrying current 'I' along x axis has length 'L' and it is kept in a magnetic field $B(\hat{i} + 2\hat{j} - 2\hat{k}) \text{ T}$. The magnitude of magnetic force acting on the wire is

- (A) $\sqrt{8} ILB$
- (B) $2 ILB$
- (C) $4 ILB$
- (D) $\sqrt{2} ILB$

Correct Answer: (A) $\sqrt{8} ILB$

Solution:

Step 1: Understanding the Concept:

The force on a straight wire is $\vec{F} = I \vec{L} \times \vec{B}$. The wire is along the x-axis, so $\vec{L} = L\hat{i}$.

Step 2: Cross product:

$$\hat{i} \times (\hat{i} + 2\hat{j} - 2\hat{k}) = 0 + 2\hat{k} + 2\hat{j} = 2\hat{j} + 2\hat{k}$$

(using $\hat{i} \times \hat{j} = \hat{k}$ and $\hat{i} \times \hat{k} = -\hat{j}$).

Step 3: Magnitude:

$$|\vec{F}| = ILB\sqrt{2^2 + 2^2} = \sqrt{8} ILB$$

Step 4: Check:

Option (A). The component of $\vec{B}$ along the wire (the $\hat{i}$ part) gives no force. Option (C) $4ILB$ would be the sum of the two perpendicular components added directly, but they are in different directions.

Final Answer:

Only the $2\hat{j} - 2\hat{k}$ part of B matters, giving root 8 ILB.

$(A) \sqrt{8} ILB$

Quick Tip: Force is $I L \text{ cross } B$; only the components of B perpendicular to the wire matter.

33. A charged particle is moving in a uniform magnetic field in a circular path of radius 'R'. When the energy of the particle becomes 3 times the original, the new radius will be

- (A) R
- (B) $3R$
- (C) $\sqrt{3}R$
- (D) $R/3$

Correct Answer: (C) $\sqrt{3}R$

Solution:

Step 1: Understanding the Concept:

A charged particle in a uniform magnetic field moves in a circle of radius $R = \frac{mv}{qB} = \frac{p}{qB}$.

Step 2: Express in terms of energy:

$$p = \sqrt{2mE} \Rightarrow R = \frac{\sqrt{2mE}}{qB} \propto \sqrt{E}$$

Step 3: New radius:

$$\frac{R'}{R} = \sqrt{\frac{3E}{E}} = \sqrt{3} \Rightarrow R' = \sqrt{3}R$$

Step 4: Check:

Option (C). Option (B) $3R$ would be right if R were proportional to E , and (D) $R/3$ would be right if it went inversely.

Final Answer:

R is proportional to root of E , so R becomes root 3 times R .

$$(C) \sqrt{3} R$$

Quick Tip: Radius is proportional to momentum, so it goes as the root of kinetic energy.

34. Torque acting on a coil carrying current 'I' situated parallel to magnetic field of induction 'B' having of turns 'n' and area 'A' is

- (A) $nI(\vec{A} \cdot \vec{B})$
- (B) $nI(\vec{A} \times \vec{B})$
- (C) IBA/n
- (D) nBA/I

Correct Answer: (B) $nI(\vec{A} \times \vec{B})$

Solution:

Step 1: Understanding the Concept:

A coil with n turns, current I and area vector $\vec{A}$ has magnetic moment $\vec{m} = nI\vec{A}$.

Step 2: Torque:

$$\vec{\tau} = \vec{m} \times \vec{B} = nI(\vec{A} \times \vec{B})$$

When the coil plane is parallel to the field, $\vec{A}$ is perpendicular to $\vec{B}$, so the magnitude is $nIAB$, the maximum torque.

Step 3: Check the options:

Option (B) is the vector form. Option (A) uses a dot product, which would give zero here. Options (C) and (D) divide by n or I , which does not agree with torque being proportional to both.

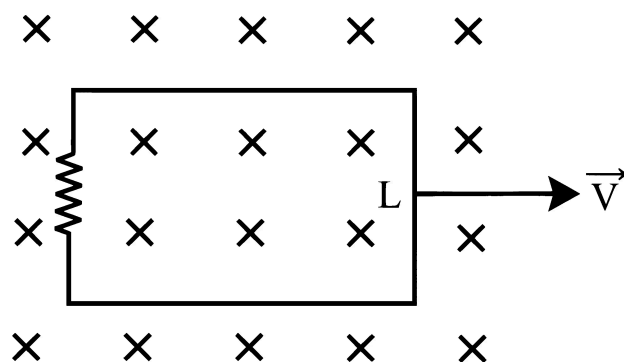
Final Answer:

Torque is $n I (\vec{A} \times \vec{B})$.

$$(B) \ nI(\vec{A} \times \vec{B})$$

Quick Tip: Torque on a coil is $n I A \sin \theta$.

35. A metal rod of length ' L ' completes the circuit as shown. The area of the circuit is perpendicular to magnetic field ' B '. Total resistance of the circuit is ' R '. The force needed to move the rod in the direction as shown with constant speed ' V ' is



- (A) $\frac{BVL}{R}$
- (B) $\frac{B^2 L^2 V}{R}$
- (C) $\frac{B^2 L^2 V^2}{R}$
- (D) $\frac{BLV^2}{R}$

Correct Answer: (B) $\frac{B^2 L^2 V}{R}$

Solution:

Step 1: Understanding the Concept:

The figure shows a rectangular circuit with a resistor on the left and a rod of length L on the right moving with speed V in a field B pointing into the page. The flux through the loop grows, so an emf appears.

Step 2: Emf and current:

$$\varepsilon = BLV, \quad I = \frac{\varepsilon}{R} = \frac{BLV}{R}$$

Step 3: Magnetic force on the rod:

The current in the rod feels a force $F = BIL$, directed opposite to the motion (Lenz's law).

$$F = B \cdot \frac{BLV}{R} \cdot L = \frac{B^2 L^2 V}{R}$$

Step 4: Result:

To keep a constant speed the applied force must equal this magnetic force, option (B). Option (A) BVL/R is not a force, since it equals the current in amperes. Options (C) and (D) have an extra factor of V , which does not fit a force.

Final Answer:

The applied force must match $B I L = B^2 L^2 V / R$.

$(B) \frac{B^2 L^2 V}{R}$

Quick Tip: The rod's emf drives a current, and the magnetic force on that current opposes the motion.



36. When a capacitor is connected in series LR circuit, the alternating current flowing in the circuit

- (A) increases
- (B) decreases
- (C) remains constant
- (D) is zero

Correct Answer: (A) increases

Solution:

Step 1: Understanding the Concept:

In an LR circuit, the impedance is $Z = \sqrt{R^2 + X_L^2}$. The inductive reactance X_L and capacitive reactance X_C act in opposite senses in a series circuit.

Step 2: Add the capacitor:

With a capacitor in series, $Z = \sqrt{R^2 + (X_L - X_C)^2}$.

The LR circuit is inductive, with X_L greater than X_C , so $|X_L - X_C| < X_L$.

Step 3: Effect on the current:

The impedance falls, so $I = \frac{V}{Z}$ rises. The capacitor partly cancels the inductor's opposition.

Step 4: Final Answer:

The current increases, option (A).

Final Answer:

The capacitor cancels part of the inductive reactance.

(A) increases

Quick Tip: Capacitive reactance works against inductive reactance and reduces the net reactance.



37. With gradual increase in frequency of an a.c. supply, the impedance of an LCR series circuit

- (A) increases
- (B) decreases
- (C) remains constant
- (D) first decreases, becomes minimum and then increases

Correct Answer: (D) first decreases, becomes minimum and then increases

Solution:

Step 1: Understanding the Concept:

For a series LCR circuit, $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$.

Step 2: Low frequency:

At low frequency $\frac{1}{\omega C}$ is large, so the reactance is large and capacitive. Z is large.

Step 3: At resonance:

As ω increases, $\frac{1}{\omega C}$ falls and ωL rises until $\omega L = \frac{1}{\omega C}$, where $Z = R$, the minimum value.

Step 4: Beyond resonance:

For higher ω , ωL dominates and Z rises again.

So the impedance first decreases, becomes minimum and then increases, option (D). The other options describe only one part of this behaviour.

Final Answer:

Z dips to R at resonance, then increases again.

(D) first decreases, becomes minimum and then increases

Quick Tip: Impedance falls to its minimum at resonance and then rises.

38. An alternating e.m.f. is given by $e = e_0 \sin \omega t$. In what time the e.m.f. will have half its maximum value, if 'e' Starts from zero?

- (A) $\frac{T}{4}$
- (B) $\frac{T}{8}$
- (C) $\frac{T}{12}$
- (D) $\frac{T}{16}$

Correct Answer: (C) $\frac{T}{12}$

Solution:

Step 1: Understanding the Concept:

The emf $e = e_0 \sin \omega t$ starts at zero. We need the first time $e = \frac{e_0}{2}$.

Step 2: Solve:

$$\sin \omega t = \frac{1}{2} \Rightarrow \omega t = \frac{\pi}{6}$$

With $\omega = \frac{2\pi}{T}$:

$$\frac{2\pi}{T}t = \frac{\pi}{6} \Rightarrow t = \frac{T}{12}$$

Step 3: Check:

Option (C). At $T/4$ the emf is at its peak, and at $T/8$ it is $e_0 \sin 45^\circ = 0.707e_0$, not half.

Final Answer:

$\sin(\omega t) = 1/2$ gives $\omega t = \pi/6$, so $t = T/12$.

$(C) \frac{T}{12}$

Quick Tip: Set $\sin(\omega t) = 1/2$ and use $\omega = 2\pi / T$.



39. A step down transformer has turns ratio 20:1. If 8V are applied across 0.4Ω secondary then the primary current will be

- (A) 0.5 A
- (B) 1 A
- (C) 2 A
- (D) 4 A

Correct Answer: (B) 1 A

Solution:

Step 1: Understanding the Concept:

In an ideal transformer, $\frac{I_p}{I_s} = \frac{N_s}{N_p}$. A step down transformer with ratio 20:1 has $N_p : N_s = 20 : 1$.

Step 2: Secondary current:

The 8 V is across the 0.4Ω secondary:

$$I_s = \frac{8}{0.4} = 20 \text{ A}$$

Step 3: Primary current:

$$I_p = I_s \cdot \frac{N_s}{N_p} = 20 \times \frac{1}{20} = 1 \text{ A}$$

Step 4: Check:

Option (B). Power check: secondary power = $8 \times 20 = 160 \text{ W}$, primary voltage = 160 V, and $160 \times 1 = 160 \text{ W}$.

Final Answer:

Secondary current 20 A, divided by 20 gives 1 A.

(B) 1 A

Quick Tip: Find the secondary current, then divide by the turns ratio.



40. A simple microscope is used to see the object first in blue light and then in red light. Due to the change from blue to red light, what is the effect on its magnifying power?

- (A) decreases
- (B) increases
- (C) remains constant
- (D) first increase and then decreases

Correct Answer: (A) decreases

Solution:

Step 1: Understanding the Concept:

The magnifying power of a simple microscope (image at the near point) is $M = 1 + \frac{D}{f}$, where D is the least distance of distinct vision.

Step 2: Effect of colour:

The refractive index of glass is smaller for red light than for blue. By the lens maker's formula $\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$, a smaller μ gives a larger f .

Step 3: Result:

For red light f is larger, so $\frac{D}{f}$ is smaller and M decreases.

Final Answer:

Red light gives a longer focal length, so M decreases.

(A) decreases

Quick Tip: The focal length is longer for red light, so $M = 1 + D/f$ is smaller.



41. In Young's double slit experiment, the fringe width is 0.4 mm. What is the distance between 4th dark band and 6th bright band on the same side of the interference pattern?

- (A) 0.5 mm
- (B) 0.75 mm
- (C) 1.0 mm
- (D) 1.5 mm

Correct Answer: (C) 1.0 mm

Solution:**Step 1: Understanding the Concept:**

On the same side of the central fringe, the n -th bright fringe is at $n\beta$ and the n -th dark fringe is at $(n - \frac{1}{2})\beta$.

Step 2: Positions:

4th dark: $3.5\beta = 3.5 \times 0.4 = 1.4$ mm.

6th bright: $6\beta = 6 \times 0.4 = 2.4$ mm.

Step 3: Separation:

$$2.4 - 1.4 = 1.0 \text{ mm}$$

Step 4: Check:

Option (C). In terms of fringe width the separation is $6 - 3.5 = 2.5\beta = 1.0 \text{ mm}$.

Final Answer:

Positions are 1.4 mm and 2.4 mm, a gap of 1.0 mm.

(C) 1.0 mm

Quick Tip: Dark fringes are at $(n - 1/2)\lambda$ and bright fringes at $n\lambda$.

42. The ratio of intensities at two points on the screen in Young's double slit experiment when waves from the two slits have a path difference of zero and $\lambda/4$ is (λ is the wavelength of light used) ($\cos 0^\circ = 1$, $\cos \pi/2 = 0$)

- (A) 1 : 2
- (B) 2 : 1
- (C) 1 : 4
- (D) 4 : 1

Correct Answer: (B) 2 : 1

Solution:

Step 1: Understanding the Concept:

Intensity from two equal sources is $I = 4I_0 \cos^2 \frac{\phi}{2}$, where $\phi = \frac{2\pi}{\lambda} \Delta x$.

Step 2: Path difference zero:

$\phi = 0$, so $I_1 = 4I_0 \cos^2 0 = 4I_0$.

Step 3: Path difference $\lambda/4$:

$\phi = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$, so $I_2 = 4I_0 \cos^2 \frac{\pi}{4} = 4I_0 \cdot \frac{1}{2} = 2I_0$.

Step 4: Ratio:

$$I_1 : I_2 = 4I_0 : 2I_0 = 2 : 1$$

Option (B).

Final Answer:

$I_1 = 4 I_0$ and $I_2 = 2 I_0$, a ratio of 2 to 1.

(B) 2 : 1

Quick Tip: Phase difference is 2π over λ times path difference; intensity goes as $\cos^2$ of half the phase.

43. In Young's double slit experiment, the distance between the slits is 3 mm and the slits are 2m away from the screen. Two interference patterns can be obtained on the screen due to light of wavelength 480 nm and 600 nm respectively. The separation on the screen between the 5th order bright fringes on two interference patterns is

- (A) $2 \times 10^{-4} \text{ m}$
- (B) $1 \times 10^{-4} \text{ m}$
- (C) $8 \times 10^{-4} \text{ m}$
- (D) $4 \times 10^{-4} \text{ m}$

Correct Answer: (D) $4 \times 10^{-4} \text{ m}$

Solution:

Step 1: Understanding the Concept:

The n -th bright fringe is at $y_n = \frac{n\lambda D}{d}$ from the central maximum.

Step 2: Find each position:

$D = 2 \text{ m}$, $d = 3 \times 10^{-3} \text{ m}$, $n = 5$.

$$y_{600} = \frac{5 \times 600 \times 10^{-9} \times 2}{3 \times 10^{-3}} = 2 \times 10^{-3} \text{ m}.$$

$$y_{480} = \frac{5 \times 480 \times 10^{-9} \times 2}{3 \times 10^{-3}} = 1.6 \times 10^{-3} \text{ m}.$$

Step 3: Separation:

$$2 \times 10^{-3} - 1.6 \times 10^{-3} = 4 \times 10^{-4} \text{ m}$$

Step 4: Check:

Option (D). The formula can also be applied to the wavelength difference of 120 nm directly.

Final Answer:

The two 5th fringes are $4 \times 10^{-4} \text{ m}$ apart.

(D) $4 \times 10^{-4} \text{ m}$

Quick Tip: Position of the n th bright fringe is $n \lambda D / d$.

44. For a photosensitive material, work function is ' W_0 ' and stopping potential is ' V '. The wavelength of the incident radiation is (h =Planck's constant, c = velocity of light, e = electronic charge)

(A) $\frac{hc}{(W_0 - eV)}$

(B) $\frac{hc}{(W_0 + eV)}$

(C) $hc(W_0 - eV)$

(D) $hc(W_0 + eV)$

Correct Answer: (B) $\frac{hc}{(W_0 + eV)}$

Solution:

Step 1: Understanding the Concept:

Einstein's photoelectric equation: $h\nu = W_0 + K_{max}$. The maximum kinetic energy is eV , where V is the stopping potential.

Step 2: Write the equation:

$$\frac{hc}{\lambda} = W_0 + eV$$

Step 3: Solve for lambda:

$$\lambda = \frac{hc}{W_0 + eV}$$

Step 4: Check:

Option (B). Option (A) has a minus sign, which would apply if the radiation had less energy than the work function plus kinetic energy. Options (C) and (D) multiply by hc instead of dividing, which gives the wrong units for a wavelength.

Final Answer:

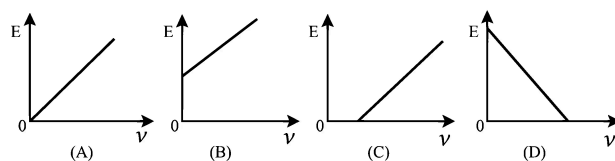
Photon energy = $W_0 + eV$, so $\lambda = hc / (W_0 + eV)$.

$$(B) \frac{hc}{W_0 + eV}$$

Quick Tip: Einstein's equation: photon energy equals work function plus maximum kinetic energy.



45. Using Einstein's photoelectric equation, the graphical representation between the kinetic energy (E) of emitted Photoelectrons and the frequency of incident radiation (ν) is shown correctly in figure



- (A) A
- (B) B
- (C) C
- (D) D

Correct Answer: (C) C

Solution:

Step 1: Understanding the Concept:

Einstein's equation: $E_k = h\nu - W_0 = h(\nu - \nu_0)$. For $\nu < \nu_0$ no photoelectron is emitted, so there is no kinetic energy.

Step 2: Shape of the graph:

The graph is a straight line with slope h that meets the frequency axis at ν_0 . To the left of ν_0 there is no emission.

Step 3: Compare the four graphs:

(A) A line through the origin would mean $W_0 = 0$, which is not true for a metal.

(B) A line with a positive intercept on the E axis would give emission with $\nu = 0$.

(C) A line that starts from the frequency axis at ν_0 and rises: matches.

(D) A falling line would mean kinetic energy decreases as frequency rises, which is the opposite of the equation.

Step 4: Final Answer:

Graph (C) is correct.

Final Answer:

A straight line starting at the threshold frequency.

(C) Graph (C)

Quick Tip: No electrons are emitted below the threshold frequency; above it, E rises linearly.

46. The force acting on the electron in hydrogen atom (Bohr's theory) is related to the principal quantum number n as

- (A) n^4
- (B) n^{-4}
- (C) n^2
- (D) n^{-2}

Correct Answer: (B) n^{-4}

Solution:

Step 1: Understanding the Concept:

In Bohr's theory the electrostatic force on the electron is $F = \frac{ke^2}{r^2}$, and the radius of the n -th orbit is $r_n = n^2 r_1$.

Step 2: Substitute:

$$F \propto \frac{1}{r^2} = \frac{1}{(n^2)^2} = n^{-4}$$

Step 3: Check:

Option (B). The speed goes as $1/n$ and the radius as n^2 , so $\frac{mv^2}{r} \propto \frac{n^{-2}}{n^2} = n^{-4}$, which agrees.

Final Answer:

r goes as n squared, so F goes as n to the minus 4.

(B) n^{-4}

Quick Tip: Force is $k e^2 / r^2$ and r is proportional to n^2 .

47. Using Bohr's atomic model, the orbital period of electron in hydrogen atom in the n^{th} orbit is (ϵ_0 =permittivity of free space, h =Planck's constant, m = mass of electron, e = electronic charge)

(A) $\frac{4 \epsilon_0^2 n^3 h^3}{m e^4}$

(B) $\frac{2 \epsilon_0^2 n^2 h^3}{m e^2}$

(C) $\frac{4 \epsilon_0^2 n^2 h^3}{m e^2}$

(D) $\frac{2 \epsilon_0 n^3 h^3}{m e^4}$

Correct Answer: (A) $\frac{4 \epsilon_0^2 n^3 h^3}{m e^4}$

Solution:

Step 1: Understanding the Concept:

Period $T = \frac{2\pi r_n}{v_n}$. From Bohr's theory:

$$r_n = \frac{\epsilon_0 n^2 h^2}{\pi m e^2} \text{ and } v_n = \frac{e^2}{2 \epsilon_0 n h}.$$

Step 2: Substitute:

$$T = 2\pi \cdot \frac{\epsilon_0 n^2 h^2}{\pi m e^2} \cdot \frac{2 \epsilon_0 n h}{e^2}$$

Step 3: Simplify:

The factor π cancels:

$$T = \frac{4 \epsilon_0^2 n^3 h^3}{m e^4}$$

Step 4: Check:

Option (A). The power of n is 3, as expected since $T \propto n^3$ (radius n^2 over speed $1/n$). Options (B) and (C) have n^2 and e^2 , which do not fit.

Final Answer:

$T = 2\pi r / v$ with the Bohr radius and speed.

$$(A) \frac{4\epsilon_0^2 n^3 h^3}{me^4}$$

Quick Tip: Period equals orbit circumference over speed; use Bohr radius and speed formulas.

48. In a transistor amplifier, a change of 0.2 mA in the base current causes a change of 5 mA in the collector current. If input resistance is $2k\Omega$ and voltage gain is 75, the load resistance used in the circuit is

- (A) $4k\Omega$
- (B) $6k\Omega$
- (C) $8k\Omega$
- (D) $2k\Omega$

Correct Answer: (B) $6k\Omega$

Solution:

Step 1: Understanding the Concept:

Current gain $\beta = \frac{\Delta I_C}{\Delta I_B}$ and voltage gain $A_v = \beta \frac{R_L}{R_{in}}$.

Step 2: Current gain:

$$\beta = \frac{5 \text{ mA}}{0.2 \text{ mA}} = 25$$

Step 3: Load resistance:

$$75 = 25 \times \frac{R_L}{2 \text{ k}\Omega} \Rightarrow R_L = \frac{75 \times 2}{25} = 6 \text{ k}\Omega$$

Step 4: Check:

Option (B). With 4 k Ω the gain would be 50, with 8 k Ω it would be 100, and with 2 k Ω it would be 25.

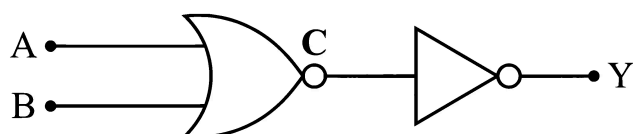
Final Answer:

Beta is 25, and $75 = 25 \text{ times } R_L / 2 \text{ k}$.

(B) 6 k Ω

Quick Tip: Voltage gain equals current gain times load over input resistance.

49. The resultant gate and its Boolean expression in the given circuit is



- (A) AND, $\overline{A + B}$
- (B) NAND, $A + B$
- (C) OR, $A + B$
- (D) NOR, $A - B$

Correct Answer: (C) OR, $A + B$

Solution:

Step 1: Reading the circuit:

The figure shows inputs A and B going into a gate with an OR shape and a small circle at its output (a NOR gate). Its output C then passes into a triangle with a circle (a NOT gate), giving output Y.

Step 2: Write the Boolean expressions:

$$C = \overline{A + B}, \quad Y = \overline{C} = \overline{\overline{A + B}} = A + B$$

Step 3: Truth table check:

A = 0, B = 0: C = 1, Y = 0. A = 0, B = 1: C = 0, Y = 1. A = 1, B = 0: C = 0, Y = 1. A = 1, B = 1: C = 0, Y = 1.

This is the OR truth table.

Step 4: Final Answer:

The combination is an OR gate with $Y = A + B$, option (C).

Final Answer:

NOR followed by NOT gives an OR gate.

(C) OR, $A + B$

Quick Tip: A NOR output goes through an inverter, which cancels the bubble.



50. When a small amount of impurity atoms are added to a semiconductor then generally its resistivity

- (A) increases
- (B) decreases
- (C) does not change
- (D) may increase or decrease depending upon the percentage of doping

Correct Answer: (B) decreases

Solution:

Step 1: Understanding the Concept:

A pure (intrinsic) semiconductor has few free charge carriers at room temperature. Doping adds donor or acceptor atoms.

Step 2: Detailed Explanation:

A pentavalent impurity adds free electrons (n-type). A trivalent impurity adds holes (p-type). In both cases the number of charge carriers goes up by a large factor, even for a very small doping fraction.

Conductivity $\sigma = e(n\mu_e + p\mu_h)$ rises, so resistivity $\rho = 1/\sigma$ falls.

Step 3: Final Answer:

The resistivity generally decreases, option (B).

Final Answer:

Doping adds carriers, so resistivity falls.

(B) decreases

Quick Tip: Impurity atoms add charge carriers, so conductivity rises and resistivity falls.

Chemistry

Questions 51 to 100 | 1 mark each | 50 marks

51. Calculate the amount of aluminium present in $\text{Al}_2(\text{SO}_4)_3$ if the compound contains 4 g of sulfur.

[Molar mass of sulfur = 32 g/mol , Molar mass of Al = 27 g/mol]

- (A) 2.25 g
- (B) 2.50 g
- (C) 2.0 g
- (D) 2.75 g

Correct Answer: (A) 2.25 g

Solution:

Step 1: Understanding the Concept:

One formula unit of $\text{Al}_2(\text{SO}_4)_3$ holds 2 aluminium atoms and 3 sulfur atoms. So the mole ratio of Al to S in the compound is always 2 : 3.

Step 2: Key Formula or Approach:

Moles = mass / molar mass. Convert the 4 g of sulfur to moles, scale by 2/3 to get moles of Al, then convert back to grams.

Step 3: Detailed Explanation:

Moles of S = $\frac{4}{32} = 0.125$ mol.

Moles of Al = $0.125 \times \frac{2}{3} = 0.0833$ mol.

Mass of Al = $0.0833 \times 27 = 2.25$ g.

Option (B) 2.50 g, (C) 2.0 g and (D) 2.75 g do not match this ratio. They come from using a wrong ratio such as 1 : 1 or from rounding

errors.

Final Answer:

The compound holds 2.25 g of aluminium.

(A) 2.25 g

Quick Tip: Find moles of sulfur, then use the 2 Al : 3 S ratio in the formula.

52. What is the observed electronic configuration of Cu ?

- (A) $[\text{Ar}]3d^94s^2$
- (B) $[\text{Ar}]3d^{10}4s^1$
- (C) $[\text{Ar}]3d^54s^2$
- (D) $[\text{Ar}]3d^64s^2$

Correct Answer: (B) $[\text{Ar}]3d^{10}4s^1$

Solution:

Step 1: Understanding the Concept:

Filling by the Aufbau order would give $[\text{Ar}] 3d^94s^2$ for copper ($Z = 29$). But a completely filled d subshell has extra stability from symmetry and exchange energy.

Step 2: Detailed Explanation:

One electron from 4s moves into 3d. This makes the 3d subshell full ($3d^{10}$) and leaves 4s half filled ($4s^1$).

The observed configuration is therefore $[\text{Ar}] 3d^{10}4s^1$.

Option (A) is the expected Aufbau configuration, not the observed

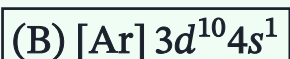
one. Options (C) and (D) do not add up to 29 electrons in total: $18 + 5 + 2 = 25$ and $18 + 6 + 2 = 26$ belong to Mn and Fe.

Step 3: Check:

Total electrons in option (B): $18 + 10 + 1 = 29$, which matches Cu.

Final Answer:

Copper is an exception with a fully filled 3d subshell.



Quick Tip: Copper takes a completely filled 3d subshell by moving one 4s electron into 3d.



53. What is the energy of an electron in a hydrogen atom in a stationary state corresponding to $n = 2$?

- (A) $-5.45 \times 10^{-19} \text{ J}$
- (B) $-2.40 \times 10^{-19} \text{ J}$
- (C) $-4.35 \times 10^{-18} \text{ J}$
- (D) $-6.70 \times 10^{-18} \text{ J}$

Correct Answer: (A) $-5.45 \times 10^{-19} \text{ J}$

Solution:

Step 1: Understanding the Concept:

In Bohr's model the energy of the electron in a hydrogen atom depends only on the principal quantum number n . It is negative because the electron is bound to the nucleus.

Step 2: Key Formula or Approach:

$$E_n = -\frac{2.18 \times 10^{-18}}{n^2} \text{ J}$$

Step 3: Detailed Explanation:

For $n = 2$:

$$E_2 = -\frac{2.18 \times 10^{-18}}{4} = -5.45 \times 10^{-19} \text{ J}$$

Option (C) $-4.35 \times 10^{-18} \text{ J}$ is twice the ground state energy, which would need n below 1, so it is not possible. Option (D) is even larger in size than that. Option (B) does not follow from any integer n in the formula.

Step 4: Check:

The ground state energy is $-2.18 \times 10^{-18} \text{ J}$, and the $n = 2$ level must lie one quarter of that value, which matches option (A).

Final Answer:

The energy at $n = 2$ is $-5.45 \times 10^{-19} \text{ J}$.

(A) $-5.45 \times 10^{-19} \text{ J}$

Quick Tip: Use $E_n = -2.18 \times 10^{-18} \text{ J} / n^2$ and put $n = 2$.

54. Identify increasing order of acidity for following diprotic acids in aqueous solutions.

- (A) $\text{H}_2\text{S} < \text{H}_2\text{Se} < \text{H}_2\text{Te}$
- (B) $\text{H}_2\text{Se} < \text{H}_2\text{S} < \text{H}_2\text{Te}$
- (C) $\text{H}_2\text{Te} < \text{H}_2\text{S} < \text{H}_2\text{Se}$
- (D) $\text{H}_2\text{Se} < \text{H}_2\text{Te} < \text{H}_2\text{S}$

Correct Answer: (A) $\text{H}_2\text{S} < \text{H}_2\text{Se} < \text{H}_2\text{Te}$

Solution:

Step 1: Understanding the Concept:

Acidity of hydrides of one group depends on how easily the H-E bond breaks to release H^+ . A weaker bond means a stronger acid.

Step 2: Key Formula or Approach:

Going down group 16 from S to Se to Te, the atom gets bigger, so the E-H bond gets longer and its bond enthalpy falls.

Step 3: Detailed Explanation:

Bond enthalpy order: $\text{S-H} > \text{Se-H} > \text{Te-H}$.

The weaker the bond, the easier the release of H^+ in water.

So acid strength rises as $\text{H}_2\text{S} < \text{H}_2\text{Se} < \text{H}_2\text{Te}$.

Options (B), (C) and (D) place Te or Se at the wrong position and so reverse or scramble this bond-strength trend.

Step 4: Check:

Typical pK_{a1} values are about 7.0 for H_2S , 3.9 for H_2Se and 2.6 for H_2Te . A smaller pK_a means a stronger acid, which agrees with option (A).

Final Answer:

Acidity increases down the group from H_2S to H_2Te .



Quick Tip: Down the group the H-E bond gets longer and weaker, so acidity rises.



55. Which of the following molecules is paramagnetic?

- (A) Li_2
- (B) N_2
- (C) O_2
- (D) F_2

Correct Answer: (C) O_2

Solution:**Step 1: Understanding the Concept:**

A molecule is paramagnetic when its molecular orbital configuration has at least one unpaired electron.

Step 2: Key Formula or Approach:

Fill the MO energy levels. For O_2 and F_2 , the order is $\sigma 2p_z$ below $\pi 2p$.

Step 3: Detailed Explanation:

Li_2 : $\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2$. All paired, diamagnetic.

N_2 : $\dots \pi 2p_x^2 \pi 2p_y^2 \sigma 2p_z^2$. All paired, diamagnetic.

O_2 : 16 electrons, with the last two going singly into the two

degenerate π^*2p_x and π^*2p_y orbitals. Two unpaired electrons, so paramagnetic.

F_2 : the π^* orbitals are completely filled. All paired, diamagnetic.

Step 4: Final Check:

Only O_2 , which is option (C), has unpaired electrons.

Final Answer:

Only O_2 has unpaired electrons in its MO diagram.

(C) O_2

Quick Tip: Write the MO configuration and look for unpaired electrons.

56. Which of the following statements is true about ideal gas ?

- (A) It shows deviation from Boyle's law and Charles' law.
- (B) Its molecules are not perfectly elastic.
- (C) It does not exist.
- (D) Intermolecular attraction is present in it, hence a collision takes place with loss of kinetic energy.

Correct Answer: (C) It does not exist.

Solution:

Step 1: Understanding the Concept:

An ideal gas follows $PV = nRT$ at every pressure and temperature. Its molecules have negligible volume and no mutual attraction, and all collisions are perfectly elastic.

Step 2: Detailed Explanation:

Real gases deviate from this behaviour, because their molecules have finite volume and attract one another. A gas that obeys the gas laws exactly under all conditions is only a theoretical model.

So the statement "It does not exist" is the true one here.

Option (A) describes a real gas, since an ideal gas obeys Boyle's and Charles' laws exactly. Option (B) does not describe an ideal gas, because its collisions are perfectly elastic. Option (D) also does not describe it, because an ideal gas has no intermolecular attraction and loses no kinetic energy in collisions.

Step 3: Final Answer:

Option (C) is the statement that holds.

Final Answer:

An ideal gas is only a hypothetical model.

(C) It does not exist

Quick Tip: An ideal gas is a theoretical model that no real gas fully matches.

57. What is the change in entropy of surrounding for the reaction,
 $\text{H}_{2(g)} + 1/2\text{O}_{2(g)} \rightarrow \text{H}_2\text{O}_{(l)}$
at 298 K if standard enthalpy of formation of water is -286 kJ ?

- (A) 959.7JK^{-1}
- (B) 801.5JK^{-1}
- (C) 850.7JK^{-1}
- (D) 980.0JK^{-1}

Correct Answer: (A) 959.7JK^{-1}

Solution:

Step 1: Understanding the Concept:

The heat given out by the reaction is absorbed by the surroundings at constant temperature and pressure. So the entropy change of the surroundings is tied to ΔH of the reaction.

Step 2: Key Formula or Approach:

$$\Delta S_{surr} = -\frac{\Delta H}{T}$$

Step 3: Detailed Explanation:

The reaction forms 1 mol of liquid water, so $\Delta H = \Delta_f H^\circ = -286\text{ kJ} = -286000\text{ J}$.

$$\Delta S_{surr} = -\frac{-286000}{298} = +959.7\text{ J K}^{-1}$$

The value is positive because the surroundings absorb heat from an exothermic reaction. Options (B), (C) and (D) do not result from dividing 286000 J by 298 K.

Step 4: Check:

$286000/298 = 959.73$, which matches option (A).

Final Answer:

Surroundings gain 286 kJ of heat at 298 K.

$$(A) \ 959.7 \text{ J K}^{-1}$$

Quick Tip: Surroundings gain the heat released: $\Delta S_{\text{surr}} = -\Delta H / T$.

58. In a particular reaction, 4 kJ heat is released by the system and 12 kJ work done on the system. Calculate the ΔH and ΔU .

- (A) $\Delta H = 4 \text{ kJ}$ and $\Delta U = 16 \text{ kJ}$
- (B) $\Delta H = -4 \text{ kJ}$ and $\Delta U = 8 \text{ kJ}$
- (C) $\Delta H = -4 \text{ kJ}$ and $\Delta U = -16 \text{ kJ}$
- (D) $\Delta H = 4 \text{ kJ}$ and $\Delta U = -16 \text{ kJ}$

Correct Answer: (B) $\Delta H = -4 \text{ kJ}$ and $\Delta U = 8 \text{ kJ}$

Solution:**Step 1: Understanding the Concept:**

The first law states $\Delta U = q + w$, with the sign taken from the system's point of view. Heat released by the system is negative, and work done on the system is positive.

Step 2: Key Formula or Approach:

$\Delta U = q + w$, and $\Delta H = q_p$ (heat exchanged at constant pressure).

Step 3: Detailed Explanation:

$q = -4 \text{ kJ}$ and $w = +12 \text{ kJ}$.

$$\Delta U = -4 + 12 = +8 \text{ kJ}$$

The heat change at constant pressure is the enthalpy change, so $\Delta H = -4 \text{ kJ}$.

Options (A) and (D) give $\Delta H = +4 \text{ kJ}$, which wrongly treats the heat as absorbed. Option (C) gives $\Delta U = -16 \text{ kJ}$, which would be $-4 - 12$, wrongly treating the work as done by the system.

Step 4: Final Answer:

$\Delta H = -4 \text{ kJ}$ and $\Delta U = 8 \text{ kJ}$.

Final Answer:

$\Delta H = -4 \text{ kJ}$ and $\Delta U = 8 \text{ kJ}$.

$(B) \Delta H = -4 \text{ kJ}, \Delta U = 8 \text{ kJ}$

Quick Tip: $\Delta U = q + w$ with $q = -4$ and $w = +12$; ΔH equals q at constant pressure.

59. Which of the following relations is correct for a reaction $S_{(s)} + O_{2(g)} \rightarrow SO_{2(g)}$?

- (A) $\Delta H > \Delta U$
- (B) $\Delta H < \Delta U$
- (C) $\Delta H = 0$
- (D) $\Delta H = \Delta U$

Correct Answer: (D) $\Delta H = \Delta U$

Solution:

Step 1: Understanding the Concept:

ΔH and ΔU differ by the work term due to change in the number of moles of gas.

Step 2: Key Formula or Approach:

$$\Delta H = \Delta U + \Delta n_g RT$$

Step 3: Detailed Explanation:

In $S_{(s)} + O_{2(g)} \rightarrow SO_{2(g)}$, only gases are counted: reactant gas = 1 mol of O_2 , product gas = 1 mol of SO_2 .

So $\Delta n_g = 1 - 1 = 0$.

Then $\Delta H = \Delta U$.

Options (A) and (B) need a nonzero Δn_g . Option (C) says $\Delta H = 0$, which has no basis because combustion releases heat.

Step 4: Final Answer:

$\Delta H = \Delta U$, option (D).

Final Answer:

Delta n g is zero, so delta H equals delta U.

$(D) \Delta H = \Delta U$

Quick Tip: Count gaseous moles: one mole of gas on each side, so delta n g is zero.

60. The solubility product of a sparingly soluble salt BA is 6.4×10^{-13} . Calculate its solubility in g dm^{-3} .

Molar mass of salt is 190 g mol^{-1} .

(A) 1.52×10^{-4}

(B) 1.25×10^{-4}

(C) 2.1×10^{-4}

(D) 1.9×10^{-4}

Correct Answer: (A) 1.52×10^{-4}

Solution:

Step 1: Understanding the Concept:

BA dissociates as $\text{BA} \rightleftharpoons \text{B}^+ + \text{A}^-$. If solubility is $S \text{ mol/L}$, both ion concentrations equal S .

Step 2: Key Formula or Approach:

$$K_{sp} = S \times S = S^2$$

Step 3: Detailed Explanation:

$$S = \sqrt{6.4 \times 10^{-13}} = 8 \times 10^{-7} \text{ mol dm}^{-3}$$

Solubility in grams per $\text{dm}^3 = 8 \times 10^{-7} \times 190 = 1.52 \times 10^{-4} \text{ g dm}^{-3}$.

Options (B), (C) and (D) do not come from this calculation.

Step 4: Final Answer:

Solubility is $1.52 \times 10^{-4} \text{ g dm}^{-3}$, option (A).

Final Answer:

Solubility is 1.52×10^{-4} g per dm^3 .

$$(A) \ 1.52 \times 10^{-4} \text{ g dm}^{-3}$$

Quick Tip: For a 1:1 salt, $S = \sqrt{K_{sp}}$; multiply by molar mass for g per dm^3 .

61. The solubility of AgCl in 0.1 M NaCl is S mol/L. If the solubility product of AgCl is 1.8×10^{-10} , then S is approximately:

- (A) 1.8×10^{-9} M
- (B) 1.8×10^{-10} M
- (C) 1.8×10^{-11} M
- (D) 1.8×10^{-12} M

Correct Answer: (A) 1.8×10^{-9} M

Solution:**Step 1: Understanding the Concept:**

NaCl is fully ionised and gives $[\text{Cl}^-] = 0.1$ M. This common ion suppresses the dissolving of AgCl.

Step 2: Key Formula or Approach:

$$K_{sp} = [\text{Ag}^+][\text{Cl}^-]$$

Chloride from AgCl itself is tiny next to 0.1 M, so we take $[\text{Cl}^-] \approx 0.1$.

Step 3: Detailed Explanation:

$$S = [\text{Ag}^+] = \frac{1.8 \times 10^{-10}}{0.1} = 1.8 \times 10^{-9} \text{ M}$$

In pure water the solubility would be $\sqrt{1.8 \times 10^{-10}} = 1.34 \times 10^{-5} \text{ M}$. The common ion effect reduces it by about four orders of magnitude. Options (B), (C) and (D) are smaller by a further factor of 10 or more and do not obey the K_{sp} expression.

Step 4: Final Answer:

$S = 1.8 \times 10^{-9} \text{ M}$, option (A).

Final Answer:

S is $1.8 \times 10^{-9} \text{ M}$ because of the common ion effect.

(A) $1.8 \times 10^{-9} \text{ M}$

Quick Tip: The common ion Cl^- from NaCl is 0.1 M , so $S = K_{sp} / 0.1$.

62. Identify from following a weak electrolyte based on their dissociation nature in aqueous medium.

- (A) NH_3
- (B) NaOH
- (C) KOH
- (D) HI

Correct Answer: (A) NH_3

Solution:

Step 1: Understanding the Concept:

A strong electrolyte ionises almost completely in water. A weak electrolyte ionises only slightly and sets up an equilibrium.

Step 2: Detailed Explanation:

NaOH and KOH are strong bases and dissociate completely into metal ions and OH^- .

HI is a strong acid and ionises fully into H^+ and I^- .

NH_3 reacts with water only to a small extent:



Its K_b is about 1.8×10^{-5} , so very few molecules ionise.

Step 3: Final Answer:

NH_3 is the weak electrolyte, option (A).

Final Answer:

Ammonia is a weak base and a weak electrolyte.

(A) NH_3

Quick Tip: Strong bases and strong acids ionise fully; ammonia ionises only slightly.

63. What is the oxidation number of oxygen in KO_2 ?

- (A) -2
- (B) $-1/2$
- (C) -1
- (D) $+2$

Correct Answer: (B) $-1/2$

Solution:

Step 1: Understanding the Concept:

In KO_2 , potassium is an alkali metal with oxidation state $+1$. The compound contains the superoxide ion O_2^- .

Step 2: Key Formula or Approach:

The sum of oxidation numbers in a neutral compound is zero.

Step 3: Detailed Explanation:

Let the oxidation number of each O be x .

$$(+1) + 2x = 0 \Rightarrow x = -\frac{1}{2}$$

A fractional value arises because two oxygen atoms share one negative charge in the superoxide ion.

Option (A) -2 is for oxides, option (C) -1 is for peroxides like Na_2O_2 , and option (D) $+2$ is found only in OF_2 .

Step 4: Final Answer:

The oxidation number of oxygen is $-1/2$, option (B).

Final Answer:

Superoxide oxygen has oxidation state $-1/2$.

$$(B) - \frac{1}{2}$$

Quick Tip: KO_2 is a superoxide with the O_2 minus ion.

64. The most abundant element in the universe is

- (A) Hydrogen
- (B) Sodium
- (C) Magnesium
- (D) Potassium

Correct Answer: (A) Hydrogen

Solution:

Step 1: Understanding the Concept:

Stars and interstellar gas are made mainly of the lightest element, formed first after the Big Bang.

Step 2: Detailed Explanation:

Hydrogen makes up about 75 percent of the universe by mass, and helium makes up most of the rest. All heavier elements together are only a small fraction.

Sodium, magnesium and potassium are present only in trace amounts on cosmic scales. They are more common on Earth, but not in the universe as a whole.

Step 3: Final Answer:

Hydrogen is the most abundant element, option (A).

Final Answer:

Hydrogen is the most abundant element.

(A) Hydrogen

Quick Tip: Hydrogen makes up about 75 percent of the mass of the universe.

65. Which of the following compounds is formed when chlorine reacts with dry slake lime Ca(OH)_2 ?

- (A) CaOCl_2
- (B) CaO_2Cl_2
- (C) CaOCl
- (D) CaCl_2

Correct Answer: (A) CaOCl_2

Solution:**Step 1: Understanding the Concept:**

Chlorine reacts with dry slaked lime to form bleaching powder. It is a mixed salt of hypochlorite and chloride.

Step 2: Detailed Explanation:

The product has the formula CaOCl_2 , often written Ca(OCl)Cl . The other formulae CaO_2Cl_2 and CaOCl are not the product of this

reaction. CaCl_2 alone would form only if the hypochlorite part were absent.

Step 3: Final Answer:

The compound formed is CaOCl_2 , option (A).

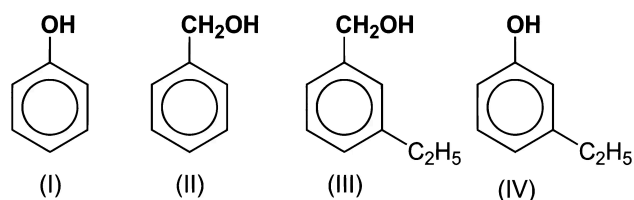
Final Answer:

Bleaching powder, CaOCl_2 , forms.

(A) CaOCl_2

Quick Tip: Chlorine and dry slaked lime give bleaching powder, CaOCl_2 .

66. Which of the following is an aromatic alcohol?



- (A) I Only
(B) II and III Only
(C) I, II, III and IV
(D) I and IV Only

Correct Answer: (B) II and III Only

Solution:

Step 1: Understanding the Concept:

An aromatic alcohol (like benzyl alcohol) has the $-\text{OH}$ group attached

to an sp^3 carbon of a side chain, and not directly to the benzene ring. If —OH sits on the ring, the compound is a phenol.

Step 2: Detailed Explanation:

The figure shows four structures.

(I) A benzene ring with —OH directly on the ring: phenol.

(II) A benzene ring with $\text{—CH}_2\text{OH}$: benzyl alcohol, an aromatic alcohol.

(III) A ring with $\text{—CH}_2\text{OH}$ and an ethyl group in the meta position: also an aromatic alcohol.

(IV) A ring with —OH directly on the ring and an ethyl group: a substituted phenol.

So only (II) and (III) are aromatic alcohols.

Step 3: Final Answer:

II and III only, option (B).

Final Answer:

Structures II and III have CH_2OH on the ring.

(B) II and III only

Quick Tip: An aromatic alcohol has OH on a side chain carbon, not directly on the ring.

67. What type of following reactions, the oxidative rancidity is?

- (A) addition
- (B) substitution
- (C) free radical chain reaction
- (D) displacement

Correct Answer: (C) free radical chain reaction

Solution:

Step 1: Understanding the Concept:

Rancidity is the spoiling of fats and oils. In oxidative rancidity, oxygen from air attacks unsaturated fatty acid chains.

Step 2: Detailed Explanation:

The process starts when a hydrogen next to a double bond is removed to form a free radical. The radical reacts with O_2 to give a peroxy radical, and that takes a hydrogen from another molecule to continue the chain. Steps are initiation, propagation and termination.

So it is a free radical chain reaction. It is not a simple addition, substitution or displacement reaction.

Step 3: Final Answer:

Option (C), free radical chain reaction.

Final Answer:

Oxidative rancidity is a free radical chain reaction.

(C) free radical chain reaction

Quick Tip: Oxidative rancidity of fats goes through a radical chain mechanism.

68. Find the number of hyperconjugation structures in the isopropyl carbocation.

- (A) 3
- (B) 6
- (C) 9
- (D) 12

Correct Answer: (B) 6

Solution:

Step 1: Understanding the Concept:

Hyperconjugation counts the number of α -hydrogens, i.e. the C-H bonds on carbons next to the positively charged carbon. Each such C-H bond gives one hyperconjugative structure.

Step 2: Detailed Explanation:

The isopropyl cation is $(\text{CH}_3)_2\text{CH}^+$. The cationic carbon is attached to two CH_3 groups.

Each CH_3 carries 3 hydrogens, so total α -hydrogens = $3 + 3 = 6$.

The H on the cationic carbon itself is not an α -hydrogen.

So there are 6 hyperconjugation structures. The value 3 counts one methyl only, and 9 is for the tert-butyl cation.

Step 3: Final Answer:

Six structures, option (B).

Final Answer:

There are 6 alpha hydrogens, so 6 structures.

Quick Tip: Count the alpha C-H bonds next to the positive carbon.

69. Which from following alkenes is highly stabilized ?

- (A) $R_2C = CR_2$
- (B) $R_2C = CH_2$
- (C) $RCH = CHR$
- (D) $RCH = CH_2$

Correct Answer: (A) $R_2C = CR_2$

Solution:

Step 1: Understanding the Concept:

Stability of an alkene rises with the number of alkyl groups on the $C=C$ carbons. Alkyl groups donate electron density and add hyperconjugation.

Step 2: Detailed Explanation:

Count alkyl groups on the double bond:

$R_2C = CR_2$: 4 groups (tetra-substituted).

$R_2C = CH_2$: 2 groups.

$RCH = CHR$: 2 groups.

$RCH = CH_2$: 1 group.

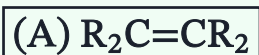
The tetra-substituted alkene has the most α -hydrogens and the lowest heat of hydrogenation, so it is the most stable.

Step 3: Final Answer:

Option (A), $R_2C=CR_2$.

Final Answer:

The tetra-substituted alkene is the most stable.



Quick Tip: More alkyl groups on the double bond give more hyperconjugation and higher stability.



70. Which of the following alkanes is NOT obtained when a mixture of ethyl iodide and n-propyl iodide is subjected to Wurtz reaction?

- (A) n-Butane
- (B) Propane
- (C) n-Pentane
- (D) n-Hexane

Correct Answer: (B) Propane

Solution:

Step 1: Understanding the Concept:

In the Wurtz reaction, two alkyl halides react with sodium in dry ether to join their alkyl groups: $R-X + R'-X + 2Na \rightarrow R-R' + 2NaX$.

Step 2: Detailed Explanation:

With C_2H_5I (ethyl) and $n-C_3H_7I$ (propyl), three couplings are possible:

Ethyl + ethyl gives butane (4 carbons).

Propyl + propyl gives n-hexane (6 carbons).

Ethyl + propyl gives n-pentane (5 carbons).

Propane has only 3 carbons, and no combination of 2 + 3 or 2 + 2 or 3 + 3 carbons gives 3. So propane is not formed.

Step 3: Final Answer:

Option (B), propane.

Final Answer:

Coupling gives C₄, C₅ and C₆ alkanes, never C₃.

(B) Propane

Quick Tip: Wurtz couples two alkyl groups: C₂+C₂, C₃+C₃ and C₂+C₃ are the only products.

71. A compound forms hcp structure. What is the number of tetrahedral voids formed in 0.8 mol of it.

(A) 4.818×10^{23}

(B) 9.635×10^{23}

(C) 7.227×10^{23}

(D) 2.410×10^{23}

Correct Answer: (B) 9.635×10^{23}

Solution:

Step 1: Understanding the Concept:

In a close-packed structure (hcp or ccp), every atom has 2 tetrahedral

voids and 1 octahedral void.

Step 2: Detailed Explanation:

Number of atoms in 0.8 mol = $0.8 \times 6.022 \times 10^{23} = 4.818 \times 10^{23}$.

Number of tetrahedral voids = $2 \times 4.818 \times 10^{23} = 9.635 \times 10^{23}$.

Option (A) is the number of atoms (one void per atom). Option (C) equals 1.5 voids per atom and (D) is a half void per atom, neither matches the 2 per atom rule.

Step 3: Final Answer:

Option (B), 9.635×10^{23} .

Final Answer:

Tetrahedral voids are twice the number of atoms.

(B) 9.635×10^{23}

Quick Tip: There are two tetrahedral voids per atom in hcp.

72. Calculate the number of atoms present in 90 g of metal if it forms bcc structure.

$$[\rho \times a^3 = 1.8 \times 10^{-22} \text{ g}]$$

(A) 4.0×10^{24}

(B) 1.0×10^{24}

(C) 2.0×10^{24}

(D) 3.0×10^{24}

Correct Answer: (B) 1.0×10^{24}

Solution:

Step 1: Understanding the Concept:

For a cubic cell, density $\rho = \frac{ZM}{N_A a^3}$, where Z is the number of atoms per cell. For bcc, $Z = 2$.

Step 2: Key Formula or Approach:

$$M = \frac{\rho a^3 N_A}{Z}$$

Step 3: Detailed Explanation:

$$M = \frac{1.8 \times 10^{-22} \times 6.022 \times 10^{23}}{2} = 54.2 \text{ g/mol}$$

Moles in 90 g = $90/54.2 = 1.66$ mol.

Atoms = $1.66 \times 6.022 \times 10^{23} \approx 1.0 \times 10^{24}$.

Step 4: Check:

Alternatively, mass per atom = $\rho a^3 / Z = 0.9 \times 10^{-22}$ g, so atoms in 90 g = $90 / 0.9 \times 10^{-22} = 1.0 \times 10^{24}$. The other options would need 2, 3 or 4 times as many atoms and do not fit.

Final Answer:

The 90 g sample holds 1.0×10^{24} atoms.

(B) 1.0×10^{24}

Quick Tip: Use density formula to get molar mass, then moles and atoms.

73. In the fcc structure, the number of tetrahedral holes per sphere is

- (A) 1
- (B) 4
- (C) 2
- (D) 8

Correct Answer: (C) 2

Solution:

Step 1: Understanding the Concept:

An fcc (ccp) cell has 4 atoms per unit cell, and tetrahedral holes sit inside the cell at 8 positions.

Step 2: Detailed Explanation:

Tetrahedral holes per cell = 8. Atoms per cell = 4.

$$\frac{8}{4} = 2$$

So each sphere has 2 tetrahedral holes. Option (A) 1 is the octahedral count per sphere, and 4 and 8 are not ratios in this lattice.

Step 3: Final Answer:

Two tetrahedral holes per sphere, option (C).

Final Answer:

fcc has 8 tetrahedral holes for 4 spheres.

(C) 2

Quick Tip: An fcc cell has 4 spheres and 8 tetrahedral holes.

74. For a binary solution, when a nonvolatile nonelectrolyte solute is dissolved in solvent, the vapour pressure of solvent decreases by 20 % . what is mole fraction of solvent in solution ?

- (A) 0.02
- (B) 0.08
- (C) 0.2
- (D) 0.8

Correct Answer: (D) 0.8

Solution:

Step 1: Understanding the Concept:

Raoult's law for a nonvolatile solute says $\frac{p^0 - p}{p^0} = x_{\text{solute}}$.

Step 2: Detailed Explanation:

A 20 percent decrease means $\frac{p^0 - p}{p^0} = 0.20$, so $x_{\text{solute}} = 0.20$.

$$x_{\text{solvent}} = 1 - 0.20 = 0.80$$

Option (C) 0.2 is the solute fraction. Options (A) and (B) are not related to the data.

Step 3: Final Answer:

Mole fraction of solvent is 0.8, option (D).

Final Answer:

Solvent mole fraction is 1 minus the lowering.

(D) 0.8

Quick Tip: Relative lowering of vapour pressure equals the mole fraction of solute.

75. Calculate the boiling point of an aqueous solution containing 18 g glucose in 100 g water if the molal elevation constant of water is 0.5K kg mol^{-1} .

[Molar mass of glucose = 180g mol^{-1} and boiling point of water = 100°C]

- (A) 100°C
- (B) 100.5°C
- (C) 101.0°C
- (D) 101.5°C

Correct Answer: (B) 100.5°C

Solution:**Step 1: Understanding the Concept:**

Boiling point elevation is $\Delta T_b = K_b m$, where m is molality (mol per kg of solvent).

Step 2: Detailed Explanation:

Moles of glucose = $18/180 = 0.1$ mol. Mass of water = $100\text{ g} = 0.1\text{ kg}$.

$$m = \frac{0.1}{0.1} = 1\text{ mol/kg}$$

$$\Delta T_b = 0.5 \times 1 = 0.5 \text{ K}$$

Boiling point of solution = $100 + 0.5 = 100.5 \text{ }^\circ\text{C}$.

Option (A) ignores the elevation. Options (C) and (D) would need a molality of 2 or 3.

Step 3: Final Answer:

100.5 $^\circ\text{C}$, option (B).

Final Answer:

The boiling point becomes 100.5 C.

(B) 100.5 $^\circ\text{C}$

Quick Tip: $\Delta T_b = K_b \times \text{molality}$; add to 100 C.



76. Which of the following is the colligative property of a solution?

- (A) vapour pressure
- (B) boiling point
- (C) osmotic pressure
- (D) Freezing point

Correct Answer: (C) osmotic pressure

Solution:

Step 1: Understanding the Concept:

A colligative property depends on the number of solute particles in a fixed amount of solvent, not on what the solute is. The four standard ones are relative lowering of vapour pressure, elevation of boiling point, depression of freezing point and osmotic pressure.

Step 2: Detailed Explanation:

Osmotic pressure follows $\pi = CRT$. It depends only on the molar concentration of particles, so it is a colligative property in the direct sense.

Vapour pressure, boiling point and freezing point are properties of the solution that change when a solute is added, but the colligative quantity is the lowering, elevation or depression and not the raw value itself.

Step 3: Final Answer:

Option (C), osmotic pressure, is the colligative property.

Final Answer:

Osmotic pressure depends only on particle number.

(C) Osmotic pressure

Quick Tip: Colligative properties depend on the number of solute particles; osmotic pressure is the standard example.

77. Identify the reaction from following so that the standard potential of Cu^{+2}/Cu electrode is 0.34 V. with respect to SHE ?

- (A) $\text{Cu} \rightarrow \text{Cu}^{+2} + 2e^{-}$
(B) $\text{Cu}^{+2} + 2e^{-} \rightarrow \text{Cu}$
(C) $\text{Cu}^{+} \rightarrow \text{Cu}^{+2} + e^{-}$
(D) $\text{Cu}^{+3} \rightarrow \text{Cu}^{+2} + e^{-}$

Correct Answer: (B) $\text{Cu}^{+2} + 2e^{-} \rightarrow \text{Cu}$

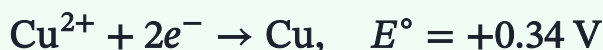
Solution:

Step 1: Understanding the Concept:

By IUPAC convention, a standard electrode potential refers to the reduction half reaction measured against the standard hydrogen electrode (SHE).

Step 2: Detailed Explanation:

For the Cu^{2+}/Cu electrode, the reduction is



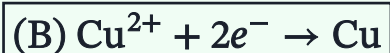
Option (A) is the reverse (oxidation), so its potential would be -0.34 V. Options (C) and (D) involve Cu^{+} and Cu^{3+} , which are not the Cu^{2+}/Cu couple, and are oxidations.

Step 3: Final Answer:

Option (B) is the reaction for $E^{\circ} = 0.34 \text{ V}$.

Final Answer:

Standard potentials are for reduction half reactions.



Quick Tip: Standard electrode potentials are written for reduction half reactions.



78. The cells A and B contain aqueous solutions of ferrous chloride and ferric chloride respectively. The mass of iron deposited by the same quantity of electricity in cells A and B is in the ratio.

- (A) 1 : 1
- (B) 2 : 1
- (C) 3 : 1
- (D) 3 : 2

Correct Answer: (D) 3 : 2

Solution:

Step 1: Understanding the Concept:

Faraday's law: for the same charge, the mass deposited is proportional to the equivalent mass, i.e. M/z , where z is the number of electrons needed per ion.

Step 2: Detailed Explanation:

Cell A (ferrous chloride): $\text{Fe}^{2+} + 2e^- \rightarrow \text{Fe}$, $z = 2$.

Cell B (ferric chloride): $\text{Fe}^{3+} + 3e^- \rightarrow \text{Fe}$, $z = 3$.

$$\frac{m_A}{m_B} = \frac{M/2}{M/3} = \frac{3}{2}$$

The mass of iron in cell A is 1.5 times that in cell B. Options (A), (B) and (C) come from ignoring the charge on the ion or inverting the ratio.

Step 3: Final Answer:

The ratio is 3 : 2, option (D).

Final Answer:

Mass deposited goes as $1/z$, so the ratio is 3 to 2.

(D) 3 : 2

Quick Tip: Moles of Fe deposited per faraday are $1/2$ for Fe^{2+} and $1/3$ for Fe^{3+} .



79. What happens during the discharge of a lead storage battery?

- (A) SO_2 is evolved
- (B) Pb is formed
- (C) H_2SO_4 is consumed
- (D) PbSO_4 is consumed

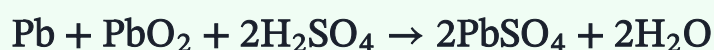
Correct Answer: (C) H_2SO_4 is consumed

Solution:**Step 1: Understanding the Concept:**

In a lead storage battery the anode is Pb and the cathode is PbO_2 , both dipping in dilute H_2SO_4 .

Step 2: Detailed Explanation:

Overall discharge reaction:



Sulfuric acid is used up, which also lowers the density of the electrolyte.

No SO_2 gas is produced. Pb is a reactant, not a product, during discharge. PbSO_4 is formed on discharge, and consumed only during recharging.

Step 3: Final Answer:

H_2SO_4 is consumed, option (C).

Final Answer:

Discharge uses up sulfuric acid.

(C) H_2SO_4 is consumed

Quick Tip: On discharge both plates turn to PbSO_4 and sulfuric acid is used up.

80. The rate constant of reaction is $1.5 \times 10^7 \text{s}^{-1}$ at 300 K and $3.0 \times 10^7 \text{s}^{-1}$ at 330 K. What is the activation energy for the reaction? [$R \times 2.303 = 19.15 \text{JK}^{-1} \text{mol}^{-1}$]

- (A) 18.02kJ mol^{-1}
- (B) 20.1kJ mol^{-1}
- (C) 19.02kJ mol^{-1}
- (D) 21.5kJ mol^{-1}

Correct Answer: (C) 19.02kJ mol^{-1}

Solution:

Step 1: Understanding the Concept:

The Arrhenius equation compares rate constants at two temperatures and links them to the activation energy E_a .

Step 2: Key Formula or Approach:

$$\log \frac{k_2}{k_1} = \frac{E_a}{2.303R} \left(\frac{T_2 - T_1}{T_1 T_2} \right)$$

Step 3: Detailed Explanation:

$k_2/k_1 = 3.0 \times 10^7 / 1.5 \times 10^7 = 2$, so $\log 2 = 0.301$.

$$\frac{T_2 - T_1}{T_1 T_2} = \frac{30}{300 \times 330} = \frac{1}{3300}$$

$$E_a = \frac{0.301 \times 19.15 \times 3300}{1} = 19022 \text{ J mol}^{-1} \approx 19.02 \text{ kJ mol}^{-1}$$

Options (A), (B) and (D) differ from this result in the third digit or more.

Step 4: Final Answer:

$E_a = 19.02 \text{ kJ mol}^{-1}$, option (C).

Final Answer:

Rate doubles from 300 K to 330 K, so E_a is 19.02 kJ/mol.

(C) 19.02 kJ mol⁻¹

Quick Tip: Use the two-temperature Arrhenius equation with 2.303R given.

81. The rate constant for the reaction $2\text{N}_2\text{O}_5 \rightarrow 4\text{NO}_2 + \text{O}_2$ is $3.0 \times 10^{-5}\text{s}^{-1}$. If the rate is $2.4 \times 10^{-5}\text{mole L}^{-1}\text{s}^{-1}$ then the concentration of N_2O_5 (mol L^{-1}) is

- (A) 1.4
- (B) 1.2
- (C) 0.04
- (D) 0.8

Correct Answer: (D) 0.8

Solution:

Step 1: Understanding the Concept:

The unit of k is s^{-1} , which tells us the reaction is first order in N_2O_5 .
So rate = $k[\text{N}_2\text{O}_5]$.

Step 2: Detailed Explanation:

$$2.4 \times 10^{-5} = 3.0 \times 10^{-5} \times [\text{N}_2\text{O}_5]$$

$$[\text{N}_2\text{O}_5] = \frac{2.4 \times 10^{-5}}{3.0 \times 10^{-5}} = 0.8 \text{ mol L}^{-1}$$

Options (A) and (B) are greater than 0.8 and would give a larger rate.
Option (C) 0.04 would give a rate of only $1.2 \times 10^{-6} \text{ mol L}^{-1}\text{s}^{-1}$.

Step 3: Final Answer:

$[\text{N}_2\text{O}_5] = 0.8 \text{ mol L}^{-1}$, option (D).

Final Answer:

Units of k show first order: rate = k times concentration.

$$(D) 0.8 \text{ mol L}^{-1}$$

Quick Tip: The unit of k shows a first order reaction, so rate = $k[\text{N}_2\text{O}_5]$.

82. What is order of radioactive decay of a substance ?

- (A) 0
- (B) 1
- (C) 1.5
- (D) 2

Correct Answer: (B) 1

Solution:**Step 1: Understanding the Concept:**

Radioactive decay rate depends on how many unstable nuclei remain:

$$-\frac{dN}{dt} = \lambda N.$$

Step 2: Detailed Explanation:

The rate is directly proportional to the first power of N . This is the rate law of a first order reaction.

$$N = N_0 e^{-\lambda t}, \quad t_{1/2} = \frac{0.693}{\lambda}$$

The half life does not depend on the starting amount, which is a clear sign of first order. Zero, 1.5 and second order laws would give

different dependence of rate on N .

Step 3: Final Answer:

Order = 1, option (B).

Final Answer:

Decay rate is proportional to N , so first order.

(B) 1

Quick Tip: Radioactive decay rate is proportional to the number of nuclei present.

83. Nano materials not applied in

- (A) Manufacturing of scratch proof glasses
- (B) Electronic devices
- (C) Conventional water treatment
- (D) self cleaning material

Correct Answer: (C) Conventional water treatment

Solution:

Step 1: Understanding the Concept:

Nano materials have particles of 1-100 nm. Their size gives them unusual strength, optical and electrical properties, so they are used in speciality applications.

Step 2: Detailed Explanation:

Scratch-proof glass uses nano coatings, so (A) is an application.

Electronic devices use nano scale transistors and quantum dots, so (B) is an application.

Self-cleaning materials use nano titanium dioxide coatings, so (D) is an application.

Conventional water treatment relies on ordinary processes such as sedimentation, filtration and chlorination, without nano materials.

Step 3: Final Answer:

Option (C) is the one where nano materials are not applied.

Final Answer:

Conventional water treatment does not use nano materials.

(C) Conventional water treatment

Quick Tip: Nano materials suit special uses like glass coatings, electronics and self cleaning surfaces.

84. Which of the following properties is responsible for the development of colour to the colloids?

- (A) Tyndall effect
- (B) Brownian motion
- (C) Electrophoresis
- (D) Electroosmosis

Correct Answer: (A) Tyndall effect

Solution:

Step 1: Understanding the Concept:

Colloidal particles are 1-1000 nm, comparable to the wavelength of visible light. They therefore scatter light strongly.

Step 2: Detailed Explanation:

The Tyndall effect is the scattering of light by colloidal particles. The colour a colloid shows depends on which wavelengths are scattered and which pass through. Small particles scatter blue light more, so a sol often looks bluish from the side and reddish when viewed against the light.

Brownian motion, electrophoresis and electrokinetic effects describe motion and charge of particles, not how they treat light.

Step 3: Final Answer:

The colour is linked to the Tyndall effect, option (A).

Final Answer:

Light scattering by particles (Tyndall effect) gives colour.

(A) Tyndall effect

Quick Tip: Colloidal particles scatter light (Tyndall effect), which gives the coloured appearance.

85. Identify the factor responsible for colour of transition metal compounds.

- (A) d-d transitions
- (B) s-p mixing
- (C) Ionisation
- (D) Shielding effect

Correct Answer: (A) d-d transitions

Solution:

Step 1: Understanding the Concept:

In a compound of a transition metal ion, the five d orbitals split into two energy groups because of surrounding ligands.

Step 2: Detailed Explanation:

An electron can jump from a lower d level to a higher d level by absorbing a photon of visible light. This is a d-d transition. The colour we see is the complement of the colour absorbed.

This needs partly filled d orbitals. Ions with d^0 or d^{10} configurations, such as Sc^{3+} and Zn^{2+} , are colourless.

s-p mixing, ionisation and shielding effect do not produce absorption in the visible range.

Step 3: Final Answer:

d-d transitions, option (A).

Final Answer:

d-d transitions absorb visible light.

(A) d-d transitions

Quick Tip: Partly filled d orbitals split in a ligand field and absorb visible light for d-d jumps.



86. Identify the number of unpaired electrons present and geometry respectively of $[\text{Co}(\text{NH}_3)_6]^{3+}$ complex.

- (A) 0, square planar
- (B) 2, square planar
- (C) 4, octahedral
- (D) 0, octahedral

Correct Answer: (D) 0, octahedral

Solution:

Step 1: Understanding the Concept:

Count the d electrons of the metal ion, then decide spin state from the ligand strength. Six ligands around the metal give an octahedral geometry.

Step 2: Detailed Explanation:

Co has atomic number 27 and $[\text{Ar}]3d^74s^2$. So Co^{3+} is $3d^6$.

NH_3 is a strong field ligand, so the crystal field splitting is large and electrons pair up: $t_{2g}^6e_g^0$.

Unpaired electrons = 0, so the complex is diamagnetic.

Six NH_3 ligands give an octahedral shape (d^2sp^3 hybridisation).

Options (A) and (B) call it square planar, which needs 4 ligands.

Option (C) gives 4 unpaired electrons, which is the high spin case and does not occur with ammonia.

Step 3: Final Answer:

0 unpaired electrons and octahedral, option (D).

Final Answer:

Low spin d6 gives 0 unpaired electrons, octahedral.

(D) 0, octahedral

Quick Tip: Co^{3+} is d6; ammonia is a strong field ligand giving a low spin octahedral complex.

87. Identify oxidation state of cobalt ion in the complex, $[\text{Co}(\text{NH}_3)_5\text{Br}]\text{SO}_4$.

(A) +2

(B) +3

(C) +1

(D) +4

Correct Answer: (B) +3

Solution:**Step 1: Understanding the Concept:**

The charge on the complex ion is set by the counter ion outside the brackets.

Step 2: Detailed Explanation:

Sulfate is SO_4^{2-} , so the complex ion $[\text{Co}(\text{NH}_3)_5\text{Br}]$ has charge +2.

Let the oxidation state of Co be x . NH_3 is neutral (0), and Br^- contributes -1 .

$$x + 5(0) + (-1) = +2 \Rightarrow x = +3$$

So cobalt is in the $+3$ state.

Step 3: Final Answer:

Option (B), $+3$.

Final Answer:

Co is $+3$ in the complex.

(B) $+3$

Quick Tip: Sulfate carries -2 , so the complex ion is $+2$; ammonia is neutral and bromide is -1 .



88. Which of the following alkyl halides is hydrolyzed most rapidly by the S_N^2 mechanism ?

- (A) $\text{CH}_3\text{-Br}$
- (B) $(\text{CH}_3)_3\text{C-Br}$
- (C) $\text{CH}_3\text{-CH}_2\text{-Br}$
- (D) $(\text{CH}_3)_2\text{CH-Br}$

Correct Answer: (A) $\text{CH}_3\text{-Br}$

Solution:

Step 1: Understanding the Concept:

In an SN^2 reaction the nucleophile attacks from the back side of the carbon in a single step. Bulky groups near that carbon block the attack.

Step 2: Detailed Explanation:

Order of reactivity in SN^2 : methyl > primary > secondary > tertiary. $\text{CH}_3\text{-Br}$ is methyl with three small H atoms, so the carbon is easily approached.

$\text{CH}_3\text{CH}_2\text{Br}$ is primary, slower. $(\text{CH}_3)_2\text{CHBr}$ is secondary, slower still. $(\text{CH}_3)_3\text{CBr}$ is tertiary, so crowded that SN^2 is almost blocked.

Step 3: Final Answer:

Option (A), $\text{CH}_3\text{-Br}$.

Final Answer:

Least hindered carbon reacts fastest in SN^2 .

(A) $\text{CH}_3\text{-Br}$

Quick Tip: SN^2 is fastest where the carbon is least crowded: methyl halide.

89. Which of the following alkyl halides undergoes SN^1 reaction most readily?

- (A) $(\text{CH}_3)_3\text{C-F}$
- (B) $(\text{CH}_3)_3\text{C-Cl}$
- (C) $(\text{CH}_3)_3\text{C-Br}$
- (D) $(\text{CH}_3)_3\text{C-I}$

Correct Answer: (D) $(\text{CH}_3)_3\text{C-I}$

Solution:

Step 1: Understanding the Concept:

In SN^1 the slow step is ionisation of the C-X bond to a carbocation. A better leaving group means faster ionisation.

Step 2: Detailed Explanation:

All four compounds give the same tert-butyl carbocation, so only the leaving group differs.

C-X bond strength falls in the order $\text{C-F} > \text{C-Cl} > \text{C-Br} > \text{C-I}$.

Leaving group ability rises as $\text{F}^- < \text{Cl}^- < \text{Br}^- < \text{I}^-$, because the larger halide ion carries its charge better.

So $(\text{CH}_3)_3\text{C-I}$ ionises most readily.

Step 3: Final Answer:

Option (D), the tert-butyl iodide.

Final Answer:

Iodide is the best leaving group in SN^1 .

(D) $(\text{CH}_3)_3\text{C-I}$

Quick Tip: All four share the same tert-butyl carbocation; the best leaving group goes fastest.

90. What is the nature of alcohols in aqueous medium?

- (A) Acidic
- (B) Basic
- (C) Neutral
- (D) Amphoteric

Correct Answer: (C) Neutral

Solution:

Step 1: Understanding the Concept:

Alcohols have an -OH group, but in water they ionise only to a negligible extent.

Step 2: Detailed Explanation:

Alcohols are very weak acids (pK_a about 16-18 for simple alcohols, close to water) and equally feeble bases. A solution of an alcohol in water does not change the pH noticeably and does not change the colour of litmus.

So, in aqueous medium, alcohols behave as neutral compounds.

Step 3: Final Answer:

Option (C), neutral.

Final Answer:

Alcohols do not change the pH of water.

(C) Neutral

Quick Tip: Alcohols are neutral to litmus: both their acidic and basic character are extremely weak.

91. Which of following observations is found in Tollens test for aldehydes?

- (A) The formation of pink or magenta color upon the addition of the reagent to the sample solution.
- (B) Formation of an Ag layer on sides of test tube upon addition and warming of the reagent.
- (C) Formation of brick-red precipitate upon heating with reagent.
- (D) Appearance of red color upon shaking with reagent.

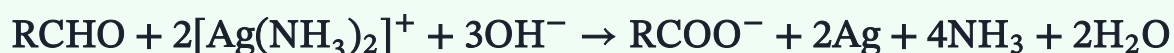
Correct Answer: (B) Formation of an Ag layer on sides of test tube upon addition and warming of the reagent.

Solution:

Step 1: Understanding the Concept:

Tollens reagent is ammoniacal silver nitrate, $[\text{Ag}(\text{NH}_3)_2]^+$. Aldehydes are oxidised by it and silver ions are reduced to metallic silver.

Step 2: Detailed Explanation:



The silver deposits as a shining layer (silver mirror) on the wall of the test tube on gentle warming.

A pink or magenta colour is seen in Schiff's test. A brick-red precipitate is seen in Fehling's test.

Step 3: Final Answer:

A silver mirror, option (B).

Final Answer:

Tollens test gives a silver mirror.

(B) Ag layer on the test tube

Quick Tip: Tollens reagent deposits a silver mirror with aldehydes on warming.

92. An organic compound $\text{CH}_3\text{-CH=CH-CH}_2\text{-CHO}$ is taken in two different containers A and B. Sample in A is treated with H_2 / Ni forming new compound P. Sample in B is treated with LiAlH_4 and hydrolyzed further forming new compound Q. Identify P and Q.

- (A) $\text{P} = \text{Q} = \text{CH}_3\text{-CH=CH-CH}_2\text{-CH}_2\text{OH}$
(B) $\text{P} = \text{Q} = \text{CH}_3(\text{CH}_2)_3\text{CH}_2\text{OH}$
(C) $\text{P} = \text{CH}_3(\text{CH}_2)_3\text{CHO}$ and $\text{Q} = \text{CH}_3(\text{CH}_2)_3\text{CH}_2\text{OH}$
(D) $\text{P} = \text{CH}_3(\text{CH}_2)_3\text{CH}_2\text{OH}$ and $\text{Q} = \text{CH}_3\text{-CH=CH-CH}_2\text{-CH}_2\text{OH}$

Correct Answer: (D) $\text{P} = \text{CH}_3(\text{CH}_2)_3\text{CH}_2\text{OH}$ and $\text{Q} = \text{CH}_3\text{-CH=CH-CH}_2\text{-CH}_2\text{OH}$

Solution:**Step 1: Understanding the Concept:**

The compound $\text{CH}_3\text{CH=CH-CH}_2\text{CHO}$ has two reducible groups: a C=C bond and an aldehyde. The two reagents differ in what they reduce.

Step 2: Detailed Explanation:

In A, catalytic hydrogenation with H_2/Ni adds hydrogen to the double bond and also reduces $-\text{CHO}$ to $-\text{CH}_2\text{OH}$.



In B, LiAlH_4 reduces $-\text{CHO}$ to $-\text{CH}_2\text{OH}$ but does not touch an isolated $\text{C}=\text{C}$.



So P is pentan-1-ol and Q is pent-3-en-1-ol.

Step 3: Final Answer:

Option (D) matches both products.

Final Answer:

H_2/Ni reduces both groups; LiAlH_4 only the aldehyde.

(D) P = pentan-1-ol, Q = pent-3-en-1-ol

Quick Tip: H_2/Ni reduces $\text{C}=\text{C}$ and CHO ; LiAlH_4 reduces only CHO .

93. What is the common name for the simplest carboxylic acid, HCOOH ?

- (A) Acetic acid
- (B) Formic acid
- (C) Propanoic acid
- (D) Butanoic acid

Correct Answer: (B) Formic acid

Solution:

Step 1: Understanding the Concept:

Common names of many carboxylic acids come from their natural sources.

Step 2: Detailed Explanation:

HCOOH was first isolated from red ants, and Latin for ant is formica. So it is formic acid. Its IUPAC name is methanoic acid.

Acetic acid is CH_3COOH , propanoic acid is $\text{C}_2\text{H}_5\text{COOH}$, and butanoic acid is $\text{C}_3\text{H}_7\text{COOH}$.

Step 3: Final Answer:

Formic acid, option (B).

Final Answer:

HCOOH is formic acid (methanoic acid).

(B) Formic acid

Quick Tip: HCOOH is named after ants, Latin formica.

94. Which of the following reactions uses diazonium salt ?

- (A) Sandmeyer reaction
- (B) Mendius reaction
- (C) Hofmann rearrangement reaction
- (D) carbylamine reaction

Correct Answer: (A) Sandmeyer reaction

Solution:

Step 1: Understanding the Concept:

A diazonium salt ArN_2^+X^- is made by diazotisation of a primary aromatic amine. It is a useful intermediate.

Step 2: Detailed Explanation:

In the Sandmeyer reaction, the diazonium group is replaced by $-\text{Cl}$, $-\text{Br}$ or $-\text{CN}$ using CuCl , CuBr or CuCN . So it uses a diazonium salt. Mendius reaction reduces nitriles to primary amines. Hofmann rearrangement (bromamide) converts an amide to an amine with one fewer carbon. The carbylamine reaction converts a primary amine and chloroform into an isocyanide. None of these involve a diazonium salt.

Step 3: Final Answer:

Sandmeyer reaction, option (A).

Final Answer:

The Sandmeyer reaction replaces the diazonium group.

(A) Sandmeyer reaction

Quick Tip: Sandmeyer converts aryl diazonium salt to aryl halide using CuX .

95. Which of the following statements is NOT correct?

- (A) Primary amines show intermolecular hydrogen bonds.
- (B) Tertiary butylamine is a primary amine.
- (C) Tertiary amines do not show inter molecular hydrogen bonds.
- (D) iso-propylamine is a secondary amine.

Correct Answer: (D) iso-propylamine is a secondary amine.

Solution:

Step 1: Understanding the Concept:

Amines are classed as primary, secondary or tertiary by the number of carbon groups attached to the nitrogen atom.

Step 2: Detailed Explanation:

(A) Primary amines have N-H bonds, so they form intermolecular hydrogen bonds. This is correct.

(B) Tertiary butylamine is $(\text{CH}_3)_3\text{C-NH}_2$. The nitrogen carries only one alkyl group, so it is a primary amine. The word "tertiary" refers to the carbon. This is correct.

(C) Tertiary amines have no N-H bond, so no intermolecular hydrogen bonding. This is correct.

(D) Isopropylamine is $(\text{CH}_3)_2\text{CH-NH}_2$. Nitrogen again carries only one alkyl group, so it is primary. Calling it secondary is wrong.

Step 3: Final Answer:

The incorrect statement is (D).

Final Answer:

Isopropylamine is a primary amine, not secondary.

(D) (D) is not correct

Quick Tip: Classify amines by the number of alkyl groups on nitrogen, not on carbon.



96. Identify the linkage in backbone of single strand of DNA ?

- (A) Base- phosphate -sugar
- (B) Phosphate -base -sugar
- (C) Sugar -phosphate- base
- (D) Phosphate -sugar-phosphate

Correct Answer: (D) Phosphate -sugar-phosphate

Solution:

Step 1: Understanding the Concept:

A nucleotide is made of a base, a pentose sugar and a phosphate group. Nucleotides join through phosphodiester bonds to form a strand.

Step 2: Detailed Explanation:

The phosphate on one nucleotide links the 3' carbon of one sugar to the 5' carbon of the next sugar. So the backbone reads sugar - phosphate - sugar - phosphate.

Bases hang off the sugars and are not part of the backbone itself.

Written from the phosphate end, the repeating pattern is phosphate - sugar - phosphate, which is what option (D) states.

Step 3: Final Answer:

Option (D), phosphate - sugar - phosphate.

Final Answer:

The DNA backbone alternates phosphate and sugar.

(D) Phosphate-sugar-phosphate

Quick Tip: The backbone alternates sugar and phosphate; the phosphate links two sugars.



97. Which of the following ionic species is present in hemoglobin and responsible for transformation of oxygen ?

- (A) Ca^{+2}
- (B) Co^{+3}
- (C) Fe^{+2}
- (D) Mg^{+2}

Correct Answer: (C) Fe^{+2}

Solution:**Step 1: Understanding the Concept:**

Haemoglobin is a protein that carries oxygen in blood. Each heme group holds one metal ion at the centre of a porphyrin ring.

Step 2: Detailed Explanation:

The metal ion is Fe^{2+} . It binds oxygen reversibly and carries it from the lungs to the tissues.

Mg^{2+} is present in chlorophyll, Co^{3+} is in vitamin B_{12} , and Ca^{2+} is found in bones and in blood clotting, but none is in haemoglobin.

Step 3: Final Answer:

Fe^{2+} , option (C).

Final Answer:

Haemoglobin contains Fe^{2+} .

(C) Fe^{2+}

Quick Tip: Haemoglobin contains iron in the +2 state inside a heme group.

98. Which of the following processes develops cross-links in elastomer?

- (A) galvanization
- (B) vulcanization
- (C) polymerization
- (D) hydration

Correct Answer: (B) vulcanization

Solution:

Step 1: Understanding the Concept:

Natural rubber is a soft elastomer. Its chains slide over each other easily, so it is sticky and weak.

Step 2: Detailed Explanation:

In vulcanisation, rubber is heated with sulfur at 373-415 K. Sulfur forms bridges between neighbouring chains at the double bonds. These cross-links give strength, stiffness and elastic recovery. Galvanisation is coating iron with zinc. Polymerisation builds the

chains but does not specifically add cross-links to an elastomer. Hydration is the addition of water.

Step 3: Final Answer:

Vulcanisation, option (B).

Final Answer:

Vulcanisation makes sulfur cross-links in rubber.

(B) Vulcanization

Quick Tip: Vulcanisation adds sulfur cross-links to rubber.

99. Which of the following is not a basis of classification of polymers ?

- (A) Source
- (B) Number of monomers
- (C) Method of preparation
- (D) Structure

Correct Answer: (B) Number of monomers

Solution:

Step 1: Understanding the Concept:

There are several standard ways to classify polymers.

Step 2: Detailed Explanation:

Source: natural, semi-synthetic, synthetic. Structure: linear, branched, cross-linked. Mode of polymerisation: addition or

condensation. Molecular forces: elastomers, fibres, thermoplastics, thermosetting.

The "number of monomers" in a chain is not a standard classification basis. Homopolymer versus copolymer depends on the type of monomers, not on how many units there are.

Step 3: Final Answer:

Option (B) is not a basis.

Final Answer:

Number of monomers is not a classification basis.

(B) Number of monomers

Quick Tip: Polymers are classified by source, structure, mode of polymerisation and intermolecular forces.

100. Which of the following detergents is used as germicide?

- (A) Anionic detergent
- (B) Non ionic detergent (an ester)
- (C) Cationic detergent
- (D) Non ionic detergents (an ether)

Correct Answer: (C) Cationic detergent

Solution:

Step 1: Understanding the Concept:

Cationic detergents have a positively charged head, usually a

quaternary ammonium group, with a long hydrocarbon tail.

Step 2: Detailed Explanation:

Because of the positive charge, they kill bacteria and are used as germicides, for example cetyltrimethylammonium bromide. They are also used in hair conditioners.

Anionic detergents (like sodium dodecylbenzene sulfonate) are mainly cleaning agents for laundry. Non-ionic detergents are used in dishwashing liquids and are mild.

Step 3: Final Answer:

Cationic detergent, option (C).

Final Answer:

Cationic detergents act as germicides.

(C) Cationic detergent

Quick Tip: Cationic detergents are quaternary ammonium salts with germicidal action.

Mathematics

Questions 101 to 150 | 2 marks each | 100 marks

101. A ball is thrown in the air. Its height at any time t is given by $h = 3 + 14t - 5t^2$, then the maximum height it can reach

- (A) 12.9
- (B) 12.8
- (C) 12.7
- (D) 12.6

Correct Answer: (B) 12.8

Solution:

Step 1: Understanding the Concept:

The height is a downward-opening quadratic in t . Its largest value occurs where the rate of change of height, dh/dt , is zero.

Step 2: Finding the time of maximum height:

$$\frac{dh}{dt} = 14 - 10t = 0 \Rightarrow t = 1.4$$

The second derivative is $\frac{d^2h}{dt^2} = -10 < 0$, so this is a maximum.

Step 3: Computing the height:

$$h(1.4) = 3 + 14(1.4) - 5(1.4)^2 = 3 + 19.6 - 9.8 = 12.8$$

Options (A), (C) and (D) are 12.9, 12.7 and 12.6, which come from

arithmetic slips such as using $t = 1.5$ or $t = 1.3$.

Step 4: Check:

Using $t = 1.3$: $3 + 18.2 - 8.45 = 12.75$. Using $t = 1.5$: $3 + 21 - 11.25 = 12.75$. Both are below 12.8, confirming the peak.

Final Answer:

The maximum height is 12.8.

(B) 12.8

Quick Tip: Differentiate h , set the derivative to zero and put that time back into h .

102. If $(1 + i) \cdot (1 + 2i) \dots \dots \dots (1 + ni) = x + iy$ (Where $i = \sqrt{-1}$), then the value of $(2) \cdot (5) \cdot (10) \dots \dots \dots (1 + n^2)$

- (A) $\frac{\sqrt{x} + \sqrt{y}}{2}$
- (B) $x^2 + y^2$
- (C) $\frac{\sqrt{x} + \sqrt{y}}{\sqrt{x} - \sqrt{y}}$
- (D) $x^2 - y^2$

Correct Answer: (B) $x^2 + y^2$

Solution:

Step 1: Understanding the Concept:

For a complex number $z = p + iq$, the product $z\bar{z} = |z|^2 = p^2 + q^2$.

Also, $|z_1 z_2| = |z_1| |z_2|$.

Step 2: Take the modulus squared:

Given $(1 + i)(1 + 2i) \cdots (1 + ni) = x + iy$.

Taking the squared modulus on both sides:

$$|1 + i|^2 |1 + 2i|^2 \cdots |1 + ni|^2 = x^2 + y^2$$

Step 3: Evaluate each factor:

$|1 + ki|^2 = 1 + k^2$. So the left side is

$$(1 + 1)(1 + 4)(1 + 9) \cdots (1 + n^2) = 2 \cdot 5 \cdot 10 \cdots (1 + n^2)$$

This is exactly the product asked for.

Step 4: Result:

The required product equals $x^2 + y^2$. Option (D), $x^2 - y^2$, would be the real part of the square, not the modulus squared. Options (A) and (C) involve square roots of x and y , which do not arise.

Final Answer:

The product equals $x^2 + y^2$.

(B) $x^2 + y^2$

Quick Tip: Take the modulus squared on both sides of the given product.

103. The number of arrangements of the numbers strictly between 10 and 1000 formed with the digits 0, 1, 2, 3, 4, 5, 6 without repetition is

- (A) 180
- (B) 215
- (C) 216
- (D) 222

Correct Answer: (B) 215

Solution:

Step 1: Understanding the Concept:

Numbers strictly between 10 and 1000 have either 2 or 3 digits, with no digit repeated and digits taken from {0, 1, 2, 3, 4, 5, 6}. A number cannot begin with 0.

Step 2: Two-digit numbers:

Tens digit: 6 choices (1 to 6). Units digit: any of the remaining 6 digits (including 0).

$$\text{Count} = 6 \times 6 = 36.$$

The number 10 is among these but is not strictly greater than 10, so remove it: $36 - 1 = 35$.

Step 3: Three-digit numbers:

Hundreds digit: 6 choices. Tens digit: 6 remaining choices. Units digit: 5 remaining choices.

$$\text{Count} = 6 \times 6 \times 5 = 180.$$

Step 4: Total:

$$35 + 180 = 215$$

Option (C) 216 would result from forgetting to exclude 10 itself.

Option (A) counts only three-digit numbers.

Final Answer:

There are 35 two-digit and 180 three-digit numbers.

(B) 215

Quick Tip: Count two-digit and three-digit numbers separately, then remove 10 itself.

104. The value of $\frac{\sin^2 3A}{\sin^2 A} - \frac{\cos^2 3A}{\cos^2 A}$ is

- (A) $\cos 2A$
- (B) $8\cos 2A$
- (C) $\frac{1}{8}\cos 2A$
- (D) $\frac{1}{2}\sin 2A$

Correct Answer: (B) $8\cos 2A$

Solution:

Step 1: Understanding the Concept:

Triple angle formulas: $\sin 3A = 3 \sin A - 4 \sin^3 A$ and $\cos 3A = 4 \cos^3 A - 3 \cos A$.

Step 2: Rewrite the ratios:

$$\frac{\sin 3A}{\sin A} = 3 - 4 \sin^2 A, \quad \frac{\cos 3A}{\cos A} = 4 \cos^2 A - 3$$

Step 3: Use difference of squares:

The expression is $(3 - 4 \sin^2 A)^2 - (4 \cos^2 A - 3)^2$.

$$= [(3 - 4 \sin^2 A) - (4 \cos^2 A - 3)][(3 - 4 \sin^2 A) + (4 \cos^2 A - 3)]$$

First bracket: $6 - 4(\sin^2 A + \cos^2 A) = 2$.

Second bracket: $4(\cos^2 A - \sin^2 A) = 4 \cos 2A$.

Step 4: Result:

$$2 \times 4 \cos 2A = 8 \cos 2A$$

Options (A), (C) and (D) are off by a constant factor or use $\sin 2A$.

Final Answer:

The expression simplifies to $8 \cos 2A$.

(B) $8 \cos 2A$

Quick Tip: Use $\sin 3A / \sin A = 3 - 4 \sin^2 A$ and $\cos 3A / \cos A = 4 \cos^2 A - 3$.

105. If $0 \leq x \leq \frac{\pi}{2}$, then the number of values of x for which $\sin x - \sin 2x + \sin 3x = 0$ is

- (A) 1
- (B) 2
- (C) 3
- (D) 4

Correct Answer: (C) 3

Solution:

Step 1: Understanding the Concept:

Group $\sin 3x + \sin x$ with the sum-to-product formula: $\sin 3x + \sin x = 2 \sin 2x \cos x$.

Step 2: Factor the equation:

$$2 \sin 2x \cos x - \sin 2x = 0 \Rightarrow \sin 2x (2 \cos x - 1) = 0$$

Step 3: Solve each factor in the interval:

$\sin 2x = 0$: $2x = 0, \pi, \dots$ so $x = 0$ or $x = \frac{\pi}{2}$ in $[0, \pi/2]$.

$$\cos x = \frac{1}{2}: x = \frac{\pi}{3}.$$

Step 4: Count:

The values are $0, \frac{\pi}{3}, \frac{\pi}{2}$, so there are 3 values. Both endpoints are allowed because the interval is closed. Option (B) 2 would drop one endpoint, and (D) 4 has no extra root.

Final Answer:

Roots are 0, $\pi/3$ and $\pi/2$.

(C) 3

Quick Tip: Combine $\sin 3x + \sin x$ into $2 \sin 2x \cos x$, then factor $\sin 2x$.

106. The number of lines passing through A (3, 4) and the sum of whose non-zero intercepts is zero is (are)

- (A) 1
- (B) 2
- (C) 3
- (D) 4

Correct Answer: (A) 1

Solution:

Step 1: Understanding the Concept:

If the nonzero intercepts on the axes are a and b , the line is $\frac{x}{a} + \frac{y}{b} = 1$. The sum is zero when $b = -a$.

Step 2: Form the equation:

$$\frac{x}{a} + \frac{y}{-a} = 1 \Rightarrow x - y = a$$

Step 3: Use the point (3, 4):

$$3 - 4 = a \Rightarrow a = -1$$

The line is $x - y = -1$, with intercepts -1 and 1 , which sum to zero. The line through (3, 4) and the origin has both intercepts equal to zero, so it has no nonzero intercepts and does not count here.

Step 4: Final count:

Only one such line exists.

Final Answer:

Exactly one line, $x - y = -1$, fits.

(A) 1

Quick Tip: Write the intercept form with intercepts a and $-a$ and use the point.

107. If $4ab = 3h^2$, then the ratio of the slopes of the lines represented by $ax^2 + 2hxy + by^2 = 0$ is...

- (A) $\sqrt{2} : 1$
- (B) $2 : 1$
- (C) $\sqrt{3} : 1$
- (D) $1 : 3$

Correct Answer: (D) $1 : 3$

Solution:**Step 1: Understanding the Concept:**

For $ax^2 + 2hxy + by^2 = 0$, the slopes m_1, m_2 satisfy $m_1 + m_2 = -\frac{2h}{b}$ and $m_1 m_2 = \frac{a}{b}$.

Step 2: Form an expression free of a, b, h :

$$\frac{(m_1 + m_2)^2}{m_1 m_2} = \frac{4h^2}{ab}$$

Since $4ab = 3h^2$, we have $h^2 = \frac{4ab}{3}$, so $\frac{4h^2}{ab} = \frac{16}{3}$.

Step 3: Solve for the ratio:

Let $r = m_1/m_2$. Then $\frac{(r+1)^2}{r} = \frac{16}{3}$.

$$3r^2 + 6r + 3 = 16r \Rightarrow 3r^2 - 10r + 3 = 0 \Rightarrow r = 3 \text{ or } \frac{1}{3}$$

Step 4: Result:

The slopes are in the ratio 3 : 1, which is the same pair as 1 : 3 read the other way round. Among the options, 1 : 3 is listed as (D).

Final Answer:

The slopes are in the ratio 1 : 3.

(D) 1 : 3

Quick Tip: Use the sum and product of slopes and solve for the ratio.

108. The equations of the tangent to the curve $x^2 + y^2 = 10$, where the tangent is parallel to the line $2x + y - 1 = 0$, are

- (A) $2x + y = \pm\sqrt{2}$
- (B) $2x + y = \pm5\sqrt{2}$
- (C) $2x + y = \pm2\sqrt{2}$
- (D) $2x + y = \pm3\sqrt{2}$

Correct Answer: (B) $2x + y = \pm 5\sqrt{2}$

Solution:

Step 1: Understanding the Concept:

The circle $x^2 + y^2 = 10$ has centre at the origin and radius $\sqrt{10}$. A tangent parallel to $2x + y - 1 = 0$ has slope $m = -2$.

Step 2: Condition for tangency:

For $y = mx + c$ to touch $x^2 + y^2 = r^2$, we need $c^2 = r^2(1 + m^2)$.

$$c^2 = 10(1 + 4) = 50 \Rightarrow c = \pm 5\sqrt{2}$$

Step 3: Write the tangents:

$$y = -2x \pm 5\sqrt{2} \Rightarrow 2x + y = \pm 5\sqrt{2}$$

Options (A), (C), (D) with $\sqrt{2}, 2\sqrt{2}, 3\sqrt{2}$ give distances from the origin of $\sqrt{2/5}, \sqrt{8/5}$ and $\sqrt{18/5}$, not $\sqrt{10}$.

Step 4: Check:

Distance from origin to $2x + y - 5\sqrt{2} = 0$ is $\frac{5\sqrt{2}}{\sqrt{5}} = \sqrt{10}$, equal to the radius, as needed.

Final Answer:

The tangents are $2x + y = \text{plus or minus } 5 \text{ root } 2$.

$(B) \ 2x + y = \pm 5\sqrt{2}$

Quick Tip: A tangent $y = mx + c$ to a circle of radius r needs $c^2 = r^2(1 + m^2)$.



109. If the line $y = 2x + \lambda$ is a tangent to the hyperbola $36x^2 - 25y^2 = 3600$, then $\lambda =$

- (A) ± 36
- (B) ± 25
- (C) ± 16
- (D) ± 9

Correct Answer: (C) ± 16

Solution:

Step 1: Understanding the Concept:

Divide $36x^2 - 25y^2 = 3600$ by 3600 to get $\frac{x^2}{100} - \frac{y^2}{144} = 1$. So $a^2 = 100$ and $b^2 = 144$.

Step 2: Tangency condition:

The line $y = mx + c$ touches $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ when $c^2 = a^2m^2 - b^2$.

Step 3: Apply:

Here $m = 2$, so

$$c^2 = 100(4) - 144 = 256 \Rightarrow c = \pm 16$$

So $\lambda = \pm 16$. The values 36, 25 and 9 do not satisfy $c^2 = 256$.

Step 4: Check:

Substituting $y = 2x + 16$: $36x^2 - 25(4x^2 + 64x + 256) = 3600$ gives $-64x^2 - 1600x - 10000 = 0$, i.e. $x^2 + 25x + 156.25 = 0$, with discriminant $625 - 625 = 0$. This confirms tangency.

Final Answer:

The tangent condition gives $\lambda = \text{plus or minus } 16$.

$$\boxed{(C) \pm 16}$$

Quick Tip: Reduce the hyperbola to standard form and use $c^2 = a^2 + b^2$.



110. $\lim_{x \rightarrow 0} \frac{(5^x - 1)^4 \operatorname{cosec}(x \log 5)}{\tan(x \log 5) \cdot \log(1 + x^2 \log 25)} = \dots$

- (A) $5 \log 5$
- (B) $\log \sqrt{5}$
- (C) $(\log 5)^2$
- (D) $\frac{1}{4} \log 5$

Correct Answer: (B) $\log \sqrt{5}$

Solution:**Step 1: Understanding the Concept:**

Near $x = 0$: $5^x - 1 \approx x \log 5$, $\tan(x \log 5) \approx x \log 5$, $\operatorname{cosec}(x \log 5) \approx \frac{1}{x \log 5}$, and $\log(1 + u) \approx u$.

Step 2: Replace each part:

Numerator: $(x \log 5)^4 \cdot \frac{1}{x \log 5} = (x \log 5)^3$.

Denominator: $x \log 5 \cdot x^2 \log 25 = x \log 5 \cdot x^2 \cdot 2 \log 5 = 2x^3(\log 5)^2$.

Step 3: Divide:

$$\frac{x^3(\log 5)^3}{2x^3(\log 5)^2} = \frac{\log 5}{2}$$

Step 4: Match the option:

$$\frac{\log 5}{2} = \log 5^{1/2} = \log \sqrt{5}, \text{ which is option (B).}$$

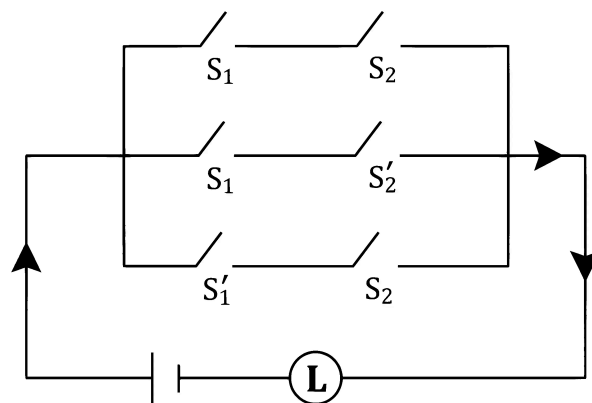
Final Answer:

The limit equals half of $\log 5$.

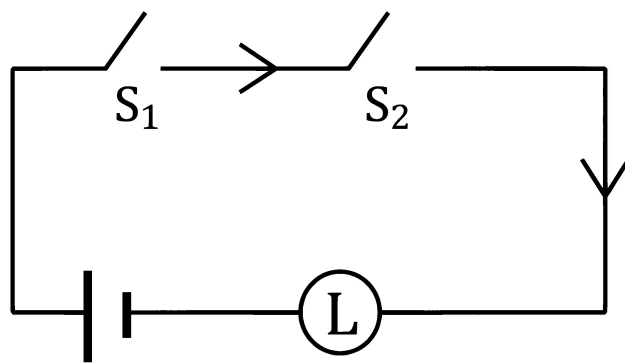
(B) $\log \sqrt{5}$

Quick Tip: Replace each factor by its small- x approximation.

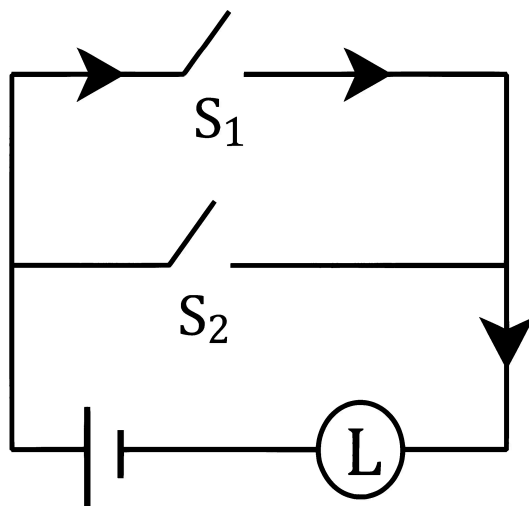
111. Simplest form of the following switching circuit is



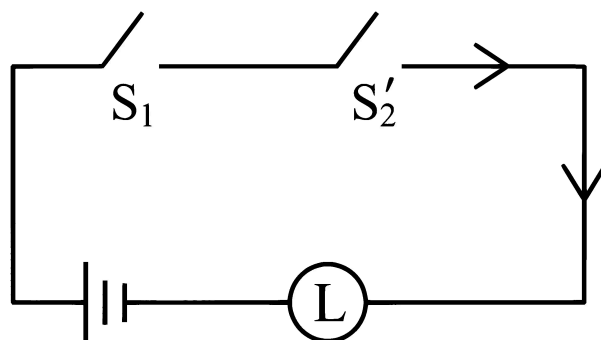
(A)



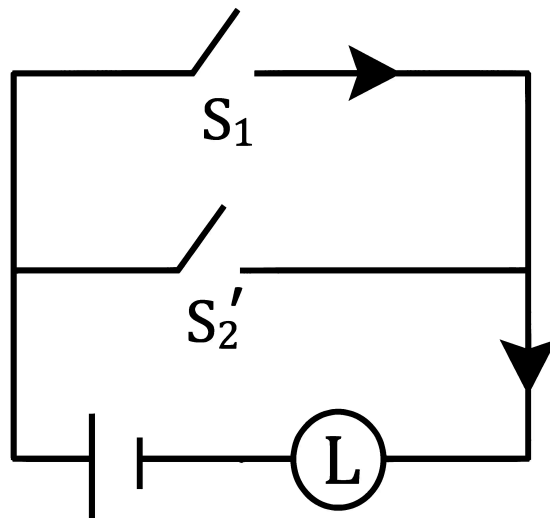
(B)



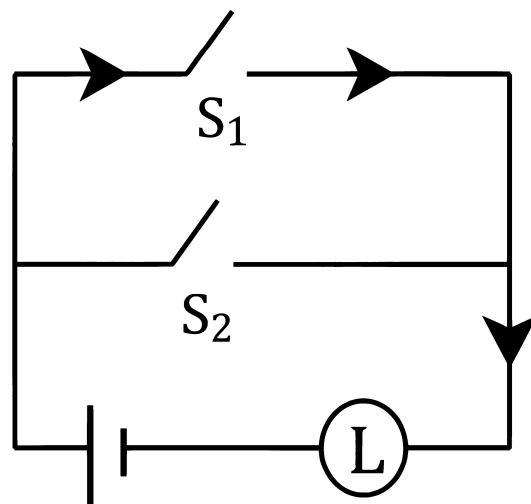
(C)



(D)



Correct Answer: (B)



Solution:

Step 1: Reading the circuit:

The figure shows three parallel branches between the battery and the lamp. Branch 1 has S_1 and S_2 in series, branch 2 has S_1 and S_2' in series, and branch 3 has S_1' and S_2 in series.

Step 2: Boolean expression:

Series means AND and parallel means OR:

$$(S_1 \wedge S_2) \vee (S_1 \wedge S'_2) \vee (S'_1 \wedge S_2)$$

Step 3: Simplify:

Combine the first two terms: $S_1 \wedge (S_2 \vee S'_2) = S_1 \wedge T = S_1$.

The expression becomes $S_1 \vee (S'_1 \wedge S_2)$.

By the absorption identity $p \vee (\sim p \wedge q) = p \vee q$, this is $S_1 \vee S_2$.

Step 4: Final circuit:

$S_1 \vee S_2$ is a circuit with S_1 and S_2 in parallel. This matches option (B), where two switches are drawn on parallel branches. Option (A) is a series circuit, (C) is $S_1 \wedge S'_2$, and (D) is $S_1 \vee S'_2$.

Final Answer:

The circuit reduces to S_1 or S_2 , two switches in parallel.

(B) S_1 and S_2 in parallel

Quick Tip: Write the Boolean expression of the circuit and simplify.



112. With usual notations in $\triangle ABC$, if $b\cos^2 \frac{C}{2} + c\cos^2 \frac{B}{2} = \frac{3a}{2}$ then

- (A) a, b, c are in A.P.
- (B) a, c, b are in A.P.
- (C) b, a, c are in A.P.
- (D) a, b, c are in G.P.

Correct Answer: (C) b, a, c are in A.P.

Solution:

Step 1: Understanding the Concept:

Half-angle identity: $\cos^2 \frac{\theta}{2} = \frac{1 + \cos \theta}{2}$. Projection formula: $a = b \cos C + c \cos B$.

Step 2: Rewrite the given condition:

$$\frac{b(1 + \cos C)}{2} + \frac{c(1 + \cos B)}{2} = \frac{3a}{2}$$

$$b + c + (b \cos C + c \cos B) = 3a$$

Step 3: Use the projection formula:

$b \cos C + c \cos B = a$, so $b + c + a = 3a$, giving $b + c = 2a$.

Step 4: Interpret:

$a = \frac{b + c}{2}$ means a is the arithmetic mean of b and c , so b, a, c are in A.P. This is option (C). Option (A) would need $b = \frac{a+c}{2}$, and (B) would need $c = \frac{a+b}{2}$.

Final Answer:

$b + c = 2a$, so b, a, c are in A.P.

(C) b, a, c in A.P.

Quick Tip: Use $\cos^2(x/2) = (1 + \cos x)/2$ and the projection formula $a = b \cos C + c \cos B$.

113. In a triangle ABC, with usual notations $a = \sqrt{3} + 1$, $b = \sqrt{3} - 1$ and $\angle C = 60^\circ$ then the values of $\angle A$ and $\angle B$ respectively are

- (A) $105^\circ, 15^\circ$
- (B) $100^\circ, 20^\circ$
- (C) $90^\circ, 30^\circ$
- (D) $110^\circ, 10^\circ$

Correct Answer: (A) $105^\circ, 15^\circ$

Solution:

Step 1: Understanding the Concept:

Napier's analogy: $\tan \frac{A-B}{2} = \frac{a-b}{a+b} \cot \frac{C}{2}$.

Step 2: Compute:

$a - b = 2$ and $a + b = 2\sqrt{3}$, so $\frac{a-b}{a+b} = \frac{1}{\sqrt{3}}$.

$\cot 30^\circ = \sqrt{3}$.

$$\tan \frac{A-B}{2} = \frac{1}{\sqrt{3}} \cdot \sqrt{3} = 1 \Rightarrow \frac{A-B}{2} = 45^\circ$$

So $A - B = 90^\circ$.

Step 3: Use the angle sum:

$A + B = 180^\circ - 60^\circ = 120^\circ$. Solving $A - B = 90^\circ$ and $A + B = 120^\circ$:

$$A = 105^\circ, \quad B = 15^\circ$$

Step 4: Check:

Option (A) satisfies both conditions. Options (B) and (D) have $A - B = 80^\circ$ and 100° , and (C) has $A - B = 60^\circ$, so none fits.

Final Answer:

A is 105 degrees and B is 15 degrees.

(A) $105^\circ, 15^\circ$

Quick Tip: Use the tangent rule: $\tan((A-B)/2) = ((a-b)/(a+b)) \cot(C/2)$.

114. Inverse of $\begin{bmatrix} 3 & -2 \\ 1 & 4 \end{bmatrix}$ is

(A) $\begin{bmatrix} \frac{2}{7} & \frac{1}{7} \\ -\frac{1}{14} & \frac{3}{14} \end{bmatrix}$

(B) $\begin{bmatrix} \frac{3}{14} & -\frac{1}{7} \\ \frac{1}{14} & \frac{2}{7} \end{bmatrix}$

(C) $\begin{bmatrix} \frac{2}{7} & -\frac{1}{7} \\ \frac{1}{14} & \frac{3}{14} \end{bmatrix}$

(D) $\begin{bmatrix} -\frac{3}{14} & \frac{1}{7} \\ \frac{1}{14} & \frac{2}{7} \end{bmatrix}$

Correct Answer: (A) $\begin{bmatrix} \frac{2}{7} & \frac{1}{7} \\ -\frac{1}{14} & \frac{3}{14} \end{bmatrix}$

Solution:**Step 1: Understanding the Concept:**

For $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$, $A^{-1} = \frac{1}{ps - qr} \begin{bmatrix} s & -q \\ -r & p \end{bmatrix}$.

Step 2: Determinant:

$$|A| = 3 \cdot 4 - (-2)(1) = 12 + 2 = 14$$

Step 3: Adjoint and inverse:

Swap the diagonal entries and change the signs of the off-diagonal entries:

$$\text{adj } A = \begin{bmatrix} 4 & 2 \\ -1 & 3 \end{bmatrix}$$

$$A^{-1} = \frac{1}{14} \begin{bmatrix} 4 & 2 \\ -1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{2}{7} & \frac{1}{7} \\ -\frac{1}{14} & \frac{3}{14} \end{bmatrix}$$

Step 4: Check:

Row 1 times column 1 of A : $\frac{2}{7} \cdot 3 + \frac{1}{7} \cdot 1 = \frac{6+1}{7} = 1$. This matches option (A); the other options have wrong signs or entries.

Final Answer:

The inverse is $(1/14)$ times $[[4, 2], [-1, 3]]$.

$$(A) \begin{bmatrix} 2/7 & 1/7 \\ -1/14 & 3/14 \end{bmatrix}$$

Quick Tip: Use $A \text{ inverse} = \text{adj } A / |A|$ for a 2 by 2 matrix.

115. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A(\text{adj } A) = AA^T$, Then $5a + b =$

(A) 2

(B) 3

(C) 5

(D) $\frac{15}{2}$

Correct Answer: (C) 5

Solution:

Step 1: Understanding the Concept:

$A(\text{adj } A) = |A| I$. Here $|A| = 10a + 3b$.

Step 2: Compute AA^T :

$$AA^T = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix} \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix} = \begin{bmatrix} 25a^2 + b^2 & 15a - 2b \\ 15a - 2b & 13 \end{bmatrix}$$

Step 3: Compare with $(10a + 3b) I$:

Off-diagonal: $15a - 2b = 0$, so $b = \frac{15a}{2}$.

Entry (2,2): $13 = 10a + 3b$.

Substitute: $10a + \frac{45a}{2} = \frac{65a}{2} = 13 \Rightarrow a = \frac{2}{5}$, so $b = 3$.

Check entry (1,1): $25 \cdot \frac{4}{25} + 9 = 13$. Correct.

Step 4: Compute:

$5a + b = 2 + 3 = 5$. Options (A), (B) and (D) do not match.

Final Answer:

$a = \frac{2}{5}$ and $b = 3$, so $5a + b = 5$.

Quick Tip: $A \operatorname{adj} A$ equals the determinant times identity; compare entries with $A A^T$.

116. If $y = \tan^{-1} \left[\frac{x - \sqrt{1-x^2}}{x + \sqrt{1-x^2}} \right]$, then $\frac{dy}{dx} =$

- (A) $\frac{-1}{\sqrt{1-x^2}}$
- (B) $\frac{1}{\sqrt{1-x^2}}$
- (C) 1
- (D) -1

Correct Answer: (B) $\frac{1}{\sqrt{1-x^2}}$

Solution:

Step 1: Understanding the Concept:

Substitute $x = \sin \theta$, so $\sqrt{1-x^2} = \cos \theta$ with $\theta = \sin^{-1} x$.

Step 2: Simplify the argument:

$$\frac{\sin \theta - \cos \theta}{\sin \theta + \cos \theta} = \frac{\tan \theta - 1}{\tan \theta + 1} = \tan \left(\theta - \frac{\pi}{4} \right)$$

Step 3: Reduce y:

$$y = \tan^{-1} \left[\tan \left(\theta - \frac{\pi}{4} \right) \right] = \theta - \frac{\pi}{4} = \sin^{-1} x - \frac{\pi}{4}$$

Step 4: Differentiate:

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}}$$

The constant $\pi/4$ drops out. Option (A) has the wrong sign, and (C) and (D) are constants.

Final Answer:

y equals arcsin x minus $\pi/4$, so $dy/dx = 1/\sqrt{1-x^2}$.

(B) $\frac{1}{\sqrt{1-x^2}}$

Quick Tip: Put $x = \sin(\theta)$ so the fraction becomes $\tan(\theta - \pi/4)$.



117. $\cos(\cos^{-1}(-\frac{1}{2}) + \frac{\pi}{3}) =$

- (A) 0
- (B) -1
- (C) 1
- (D) $\frac{1}{2}$

Correct Answer: (B) -1

Solution:

Step 1: Understanding the Concept:

The principal value of $\cos^{-1}$ lies in $[0, \pi]$.

Step 2: Evaluate the inner term:

$$\cos^{-1}\left(-\frac{1}{2}\right) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Step 3: Add and evaluate:

$$\cos\left(\frac{2\pi}{3} + \frac{\pi}{3}\right) = \cos \pi = -1$$

Step 4: Check the options:

Option (A) 0 would need an angle of $\pi/2$, (C) 1 would need 0 or 2π , and (D) $\frac{1}{2}$ would need $\pi/3$. Only -1 matches the angle π .

Final Answer:

The angle is pi, so the cosine is -1.

(B) -1

Quick Tip: $\arccos(-1/2)$ is $2\pi/3$, so the angle becomes pi.

118. $\sin^{-1}\left(\frac{12}{13}\right) + \cos^{-1}\left(\frac{4}{5}\right) + \tan^{-1}\left(\frac{63}{16}\right) =$

- (A) $\frac{\pi}{2}$
- (B) $\frac{3\pi}{2}$
- (C) π
- (D) 2π

Correct Answer: (C) π

Solution:

Step 1: Understanding the Concept:

Convert to a common form: $\sin^{-1} \frac{12}{13} = \tan^{-1} \frac{12}{5}$ (3-4-5 style triangle 5, 12, 13) and $\cos^{-1} \frac{4}{5} = \tan^{-1} \frac{3}{4}$.

Step 2: Add the first two:

Since $\frac{12}{5} \cdot \frac{3}{4} = \frac{9}{5} > 1$, the formula for $\tan^{-1} p + \tan^{-1} q$ gains a π :

$$\tan^{-1} \frac{12}{5} + \tan^{-1} \frac{3}{4} = \pi + \tan^{-1} \frac{\frac{12}{5} + \frac{3}{4}}{1 - \frac{9}{5}} = \pi + \tan^{-1} \left(-\frac{63}{16} \right) = \pi - \tan^{-1} \frac{63}{16}$$

Step 3: Add the third:

$$\left(\pi - \tan^{-1} \frac{63}{16} \right) + \tan^{-1} \frac{63}{16} = \pi$$

Step 4: Check:

Numerically: $67.38^\circ + 36.87^\circ + 75.75^\circ = 180^\circ$. The value is π , option (C).

Final Answer:

The three angles add up to pi.

(C) π

Quick Tip: Convert each term to an arctangent and add using the tangent formula.

119. If $f(x) = \frac{4x+3}{6x-4}$, $x \neq \frac{2}{3}$ and $(f \circ f)(x) = g(x)$ where $g : \mathbb{R} - \{\frac{2}{3}\} \rightarrow \mathbb{R} - \{\frac{2}{3}\}$, then $(g \circ g \circ g \circ g \circ g)(3) =$

- (A) 3
- (B) $\frac{1}{3}$
- (C) 3^5
- (D) $\frac{1}{3^5}$

Correct Answer: (A) 3

Solution:

Step 1: Understanding the Concept:

$g = f \circ f$. If $g(x) = x$, then repeating g any number of times leaves the input unchanged.

Step 2: Compute $f(f(x))$:

$$f(f(x)) = \frac{4 \cdot \frac{4x+3}{6x-4} + 3}{6 \cdot \frac{4x+3}{6x-4} - 4}$$

Multiply top and bottom by $6x - 4$:

Numerator: $16x + 12 + 18x - 12 = 34x$.

Denominator: $24x + 18 - 24x + 16 = 34$.

Step 3: Identify g :

$g(x) = \frac{34x}{34} = x$, the identity function on $\mathbb{R} - \{\frac{2}{3}\}$.

Step 4: Apply five times:

$(g \circ g \circ g \circ g \circ g)(3) = 3$. Options (B), (C) and (D) are not obtained from an identity map.

Final Answer:

g is the identity, so the answer is 3.

(A) 3

Quick Tip: Compute $f \circ f$ first; it turns out to be the identity.

120. If the derivative of the function $f(x) = \begin{cases} ax^2 + b & \text{if } x < -1 \\ bx^2 + ax + 4 & \text{if } x \geq -1 \end{cases}$ is continuous everywhere then

- (A) $a = 2, b = 3$
- (B) $a = 3, b = 2$
- (C) $a = -2, b = 3$
- (D) $a = -3, b = -2$

Correct Answer: (A) $a = 2, b = 3$

Solution:**Step 1: Understanding the Concept:**

For the derivative to exist and be continuous, f must be continuous at -1 , differentiable at -1 , and f' must have equal left and right limits there.

Step 2: Continuity of f at $x = -1$:

Left value: $a(1) + b = a + b$. Right value: $b(1) + a(-1) + 4 = b - a + 4$.

$$a + b = b - a + 4 \Rightarrow a = 2$$

Step 3: Matching derivatives:

Left derivative: $2ax$ at -1 gives $-2a$. Right derivative: $2bx + a$ at -1 gives $-2b + a$.

$$-2a = -2b + a \Rightarrow 3a = 2b \Rightarrow b = 3$$

Step 4: Check:

With $a = 2, b = 3$: left value 5, right value $3 - 2 + 4 = 5$. Left slope -4 , right slope $-6 + 2 = -4$. Both agree. Option (A).

Final Answer:

Continuity gives $a = 2$ and matching slopes gives $b = 3$.

$$\boxed{(A) \ a = 2, \ b = 3}$$

Quick Tip: Match function values and derivative values at $x = -1$.

121. The derivative of $\log_8(\log_5 x)$ w. r. t. x is

- (A) $\frac{1}{\log_5 8 \log x}$
- (B) $\frac{1}{x \log 5 \log x}$
- (C) $\frac{1}{x \log x \log 8 \log 5}$
- (D) $\frac{1}{x \log 8 \log x}$

Correct Answer: (D) $\frac{1}{x \log 8 \log x}$

Solution:

Step 1: Understanding the Concept:

Change of base: $\log_8 u = \frac{\log u}{\log 8}$ and $\log_5 x = \frac{\log x}{\log 5}$.

Step 2: Write the function:

$$y = \frac{1}{\log 8} \log \left(\frac{\log x}{\log 5} \right)$$

Step 3: Differentiate:

The derivative of $\log u$ is u'/u . Here $u = \frac{\log x}{\log 5}$, so $u' = \frac{1}{x \log 5}$.

$$\frac{dy}{dx} = \frac{1}{\log 8} \cdot \frac{1/(x \log 5)}{(\log x)/\log 5} = \frac{1}{x \log 8 \log x}$$

Step 4: Match:

The $\log 5$ cancels completely, so option (D) is the answer. Options (A), (B), (C) keep a stray $\log 5$ factor.

Final Answer:

The $\log 5$ factor cancels, leaving $1/(x \log 8 \log x)$.

(D) $\frac{1}{x \log 8 \log x}$

Quick Tip: Convert to natural logs and use the chain rule.

122. If $y^{\frac{1}{m}} + y^{\frac{-1}{m}} = 2x$, then $(x^2 - 1)y_1^2 =$

- (A) my^2
- (B) m^2y^2
- (C) m^2y
- (D) my

Correct Answer: (B) m^2y^2

Solution:

Step 1: Understanding the Concept:

Let $t = y^{1/m}$. The equation $t + \frac{1}{t} = 2x$ is a quadratic in t .

Step 2: Solve for y:

$t^2 - 2xt + 1 = 0$ gives $t = x + \sqrt{x^2 - 1}$ (taking the root with the plus sign).

$$y = (x + \sqrt{x^2 - 1})^m$$

Step 3: Differentiate:

$$y_1 = m(x + \sqrt{x^2 - 1})^{m-1} \left(1 + \frac{x}{\sqrt{x^2 - 1}}\right) = m(x + \sqrt{x^2 - 1})^{m-1} \frac{\sqrt{x^2 - 1} + x}{\sqrt{x^2 - 1}}$$

$$y_1 = \frac{m y}{\sqrt{x^2 - 1}}$$

Step 4: Square and rearrange:

$(x^2 - 1)y_1^2 = m^2 y^2$. This is option (B). The other options do not carry the factor m^2 with y^2 .

Final Answer:

The result is m squared times y squared.

$$\boxed{(B) \ m^2 y^2}$$

Quick Tip: Solve for $y^{(1/m)}$ and write y as a power of $x + \sqrt{x^2 - 1}$.

123. A triangle has two fixed vertices $A(a, 0)$ and $B(0, b)$. Let its third vertex C is moving along the line $x = y$. If s is the area of triangle ABC , then $\frac{ds}{dx} =$

- (A) $a + b$
- (B) $-\left(\frac{a+b}{2}\right)$
- (C) $\frac{a-b}{2}$
- (D) $\frac{a}{2}$

Correct Answer: (B) $-\left(\frac{a+b}{2}\right)$

Solution:

Step 1: Understanding the Concept:

Area of a triangle with vertices $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ is $\frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$.

Step 2: Set up with $C = (x, x)$:

$A = (a, 0), B = (0, b), C = (x, x)$:

$$s = \frac{1}{2} [a(b - x) + 0 \cdot (x - 0) + x(0 - b)] = \frac{1}{2} [ab - (a + b)x]$$

This is the area when the vertices are listed in the order that gives a positive determinant.

Step 3: Differentiate with respect to x:

$$\frac{ds}{dx} = -\frac{a + b}{2}$$

Step 4: Match:

This is option (B). Options (A), (C), (D) have the wrong coefficient or sign for x.

Final Answer:

The area is linear in x with slope $-(a + b)/2$.

$$\boxed{(B) - \frac{a + b}{2}}$$

Quick Tip: Write the area using the determinant formula with $C = (x, x)$.

124. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in the interval.....

- (A) $(0, \frac{\pi}{2})$
 (B) $(-\frac{\pi}{2}, \frac{\pi}{2})$
 (C) $(\frac{\pi}{4}, \frac{\pi}{2})$
 (D) $(-\frac{\pi}{2}, \frac{\pi}{4})$

Correct Answer: (D) $(-\frac{\pi}{2}, \frac{\pi}{4})$

Solution:

Step 1: Understanding the Concept:

$\tan^{-1} u$ is an increasing function of u . So f increases exactly where $u = \sin x + \cos x$ increases.

Step 2: Derivative:

$$\frac{d}{dx}(\sin x + \cos x) = \cos x - \sin x$$

We need $\cos x - \sin x > 0$, i.e. $\sqrt{2} \cos\left(x + \frac{\pi}{4}\right) > 0$.

Step 3: Solve:

$$x + \frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right), \text{ so } x \in \left(-\frac{3\pi}{4}, \frac{\pi}{4}\right).$$

Step 4: Match with the options:

The interval $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$ lies inside this range, so f increases there:

option (D). Options (A), (B), (C) all contain points beyond $\pi/4$ where $\cos x < \sin x$.

Final Answer:

f increases where $\cos x$ exceeds $\sin x$.

$$(D) \left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$$

Quick Tip: The inverse tangent is increasing, so we need $\sin x + \cos x$ increasing.



125. The minimum value of $\frac{\log x}{x}$ in the interval $(2, \infty)$ is

- (A) 0
- (B) e
- (C) $\frac{1}{e}$
- (D) Does not exist

Correct Answer: (D) Does not exist

Solution:

Step 1: Understanding the Concept:

Let $f(x) = \frac{\log x}{x}$. Its stationary points come from $f'(x) = 0$.

Step 2: Differentiate:

$$f'(x) = \frac{1 - \log x}{x^2}$$

$f' = 0$ at $x = e$. For $x < e$, $f' > 0$ (increasing). For $x > e$, $f' < 0$ (decreasing).

Step 3: Analyse the interval $(2, \infty)$:

$e \approx 2.718$ lies in $(2, \infty)$, and it gives a maximum value $\frac{1}{e}$, not a

minimum.

After $x = e$ the function keeps decreasing and approaches 0 as $x \rightarrow \infty$, but never reaches 0, because $\log x > 0$ there.

Step 4: Conclusion:

There is no smallest value attained. So the minimum does not exist, option (D). Option (C) $1/e$ is the maximum.

Final Answer:

The function peaks at $x = e$ and has no minimum.

(D) Does not exist

Quick Tip: Differentiate: the function has a maximum at $x = e$ and decreases afterwards.

126. $\int \frac{(x+1)(x+\log x)^2}{x} dx =$

- (A) $\left(\frac{x+\log x}{x}\right)^2 + c$, where c is the constant of integration
(B) $\frac{(x+\log x)^2}{x} + c$, where c is the constant of integration
(C) $\frac{(x+\log x)^3}{3} + c$, where c is the constant of integration
(D) $\frac{(x+\log x)^3}{3x} + c$, where c is the constant of integration

Correct Answer: (C) $\frac{(x+\log x)^3}{3} + c$, where c is the constant of integration

Solution:

Step 1: Understanding the Concept:

If the integrand has the form $g(u)u'$, substitute u as the inner function. Look at the derivative of $x + \log x$.

Step 2: Spot the substitution:

$$u = x + \log x \Rightarrow du = \left(1 + \frac{1}{x}\right) dx = \frac{x+1}{x} dx$$

The factor $\frac{x+1}{x} dx$ in the integrand is exactly du .

Step 3: Integrate:

$$\int u^2 du = \frac{u^3}{3} + c = \frac{(x + \log x)^3}{3} + c$$

Step 4: Check:

Differentiating the answer gives $(x + \log x)^2 \left(1 + \frac{1}{x}\right) = \frac{(x+1)(x + \log x)^2}{x}$, the integrand. Options (A), (B) and (D) divide by x , which would add a factor that is not in the derivative.

Final Answer:

The integral is $(x + \log x)^3 / 3$ plus c .

(C) $\frac{(x + \log x)^3}{3} + c$

Quick Tip: Notice that $(x+1)/x dx$ is the differential of $x + \log x$.

127. $\int \cot^4 x dx$ is equal to

- (A) $\frac{\cot^3 x}{3} - \cot x + x + c$
 (B) $\frac{\cot^3 x}{3} + \cot x + x + c$
 (C) $-\frac{\cot^3 x}{3} + \cot x + x + c$
 (D) $\frac{\cot^3 x}{3} - 2\cot x + x + c$

Correct Answer: (C) $-\frac{\cot^3 x}{3} + \cot x + x + c$

Solution:

Step 1: Understanding the Concept:

Use $\cot^2 x = \operatorname{cosec}^2 x - 1$ to reduce the power.

Step 2: Split the integrand:

$$\cot^4 x = \cot^2 x (\operatorname{cosec}^2 x - 1) = \cot^2 x \operatorname{cosec}^2 x - \cot^2 x$$

Step 3: Integrate each part:

First part: let $t = \cot x$, $dt = -\operatorname{cosec}^2 x dx$, so $\int \cot^2 x \operatorname{cosec}^2 x dx$
 $= -\frac{\cot^3 x}{3}$.

Second part: $-\int \cot^2 x dx = -\int (\operatorname{cosec}^2 x - 1) dx = \cot x + x$.

Step 4: Combine:

$$\int \cot^4 x dx = -\frac{\cot^3 x}{3} + \cot x + x + c$$

Options (A), (B) and (D) have the wrong sign on $\cot^3 x/3$ or the wrong coefficient of $\cot x$.

Final Answer:

The integral is $-\cot^3 x/3 + \cot x + x + c$.

$$(C) - \frac{\cot^3 x}{3} + \cot x + x + c$$

Quick Tip: Write $\cot^4 x$ as $\cot^2 x (\operatorname{cosec}^2 x - 1)$ and integrate in two parts.



128. $\int \frac{\cos^3 x}{\sin^2 x + \sin x} dx =$

- (A) $\log(\cos x) + \cos x + c$, where c is the constant of integration
- (B) $\log(\sin x) - \sin x + c$, where c is the constant of integration
- (C) $\log(\sin x) + \sin x + c$, where c is the constant of integration
- (D) $\log(\cos x) - \cos x + c$, where c is the constant of integration

Correct Answer: (B) $\log(\sin x) - \sin x + c$, where c is the constant of integration

Solution:**Step 1: Understanding the Concept:**

Write $\cos^3 x \, dx = \cos^2 x \cdot \cos x \, dx = (1 - \sin^2 x) d(\sin x)$.

Step 2: Substitute:

Let $t = \sin x$, $dt = \cos x \, dx$.

$$\int \frac{1 - t^2}{t^2 + t} dt = \int \frac{(1 - t)(1 + t)}{t(t + 1)} dt = \int \frac{1 - t}{t} dt$$

Step 3: Integrate:

$$\int \left(\frac{1}{t} - 1 \right) dt = \log |t| - t + c = \log(\sin x) - \sin x + c$$

Step 4: Check the options:

Option (B) matches. Options (A) and (D) use $\cos x$ as the variable, and option (C) has the wrong sign on $\sin x$.

Final Answer:

The integral is $\log(\sin x) - \sin x + c$.

(B) $\log(\sin x) - \sin x + c$

Quick Tip: Substitute $t = \sin x$ and cancel the common factor $1 + t$.



129. $\int_{\pi/6}^{\pi/3} \frac{\sin x - \cos x}{1 + \sin x \cos x} dx =$

(A) 0

(B) $\frac{\pi}{12}$

(C) $\frac{\pi}{24}$

(D) $\frac{\pi^2}{2}$

Correct Answer: (A) 0

Solution:

Step 1: Understanding the Concept:

The limits $\pi/6$ and $\pi/3$ add up to $\pi/2$. So we can use $\int_a^b f(x) dx = \int_a^b f(a + b - x) dx$ with $a + b = \pi/2$.

Step 2: Apply the reflection:

Let $f(x) = \frac{\sin x - \cos x}{1 + \sin x \cos x}$. Then

$$f\left(\frac{\pi}{2} - x\right) = \frac{\cos x - \sin x}{1 + \cos x \sin x} = -f(x)$$

Step 3: Conclude:

So $I = \int f(x)dx = \int f\left(\frac{\pi}{2} - x\right)dx = -I$, giving $2I = 0$ and $I = 0$.

Step 4: Check:

The function is negative for $x < \pi/4$ and positive for $x > \pi/4$, and the two areas are equal in size. So the integral is 0, option (A).

Final Answer:

The integrand is odd about $\pi/4$, so the integral is 0.

(A) 0

Quick Tip: Use the reflection $x \rightarrow \pi/2 - x$ on a symmetric interval.

130. $\int_0^{\log 5} \frac{e^x \sqrt{e^x - 1}}{e^x + 3} dx =$

- (A) $2 - \frac{\pi}{4}$
- (B) $4 - \frac{\pi}{4}$
- (C) $2 - \pi$
- (D) $4 - \pi$

Correct Answer: (D) $4 - \pi$

Solution:

Step 1: Understanding the Concept:

Remove the square root with $u = \sqrt{e^x - 1}$, so $e^x = u^2 + 1$ and $e^x dx = 2u du$.

Step 2: Change the limits:

At $x = 0$: $u = 0$. At $x = \log 5$: $u = \sqrt{4} = 2$.

Also $e^x + 3 = u^2 + 4$.

Step 3: Rewrite and integrate:

$$\begin{aligned} I &= \int_0^2 \frac{u \cdot 2u du}{u^2 + 4} = 2 \int_0^2 \frac{u^2}{u^2 + 4} du = 2 \int_0^2 \left(1 - \frac{4}{u^2 + 4}\right) du \\ &= 2 \left[u - 2 \tan^{-1} \frac{u}{2} \right]_0^2 = 2 \left[2 - 2 \cdot \frac{\pi}{4} \right] \end{aligned}$$

Step 4: Result:

$$I = 4 - \pi$$

Options (A) and (B) have $\pi/4$ instead of π , and (C) is $2 - \pi$, which is negative and cannot be right for a positive integrand.

Final Answer:

The substitution gives $4 - \pi$.

(D) $4 - \pi$

Quick Tip: Let $u = \sqrt{e^x - 1}$ so that $e^x dx = 2u du$.

131. The integrating factor of the differential equation $(1 + t^2) + (x - e^{\tan^{-1}t}) \frac{dt}{dx} = 0$ is

- (A) $e^{\tan^{-1}t}$
- (B) $-e^{\tan^{-1}t}$
- (C) $e^{-\tan^{-1}t}$
- (D) $-e^{-\tan^{-1}t}$

Correct Answer: (A) $e^{\tan^{-1}t}$

Solution:

Step 1: Understanding the Concept:

The equation is $(1 + t^2) + (x - e^{\tan^{-1}t}) \frac{dt}{dx} = 0$. Rewrite to get $\frac{dx}{dt}$.

Step 2: Rearrange:

$$(x - e^{\tan^{-1}t}) \frac{dt}{dx} = -(1 + t^2) \Rightarrow \frac{dx}{dt} = -\frac{x - e^{\tan^{-1}t}}{1 + t^2}$$

$$\frac{dx}{dt} + \frac{x}{1 + t^2} = \frac{e^{\tan^{-1}t}}{1 + t^2}$$

This is linear in x with $P = \frac{1}{1 + t^2}$.

Step 3: Integrating factor:

$$\text{I.F.} = e^{\int P dt} = e^{\int \frac{dt}{1+t^2}} = e^{\tan^{-1}t}$$

Step 4: Result:

Option (A). Options (B) and (D) carry a minus sign that cannot come from an exponential, and (C) has the wrong sign inside the exponent.

Final Answer:

The integrating factor is $e^{\tan^{-1} t}$.

$$(A) e^{\tan^{-1} t}$$

Quick Tip: Rewrite so x is the dependent variable: $dx/dt + P x = Q$.

132. The degree of the differential equation obtained from the equation $(y - a)^2 = 4(x - b)$ [where a and b are arbitrary constants] is

- (A) 1
- (B) 2
- (C) 3
- (D) not defined

Correct Answer: (A) 1

Solution:**Step 1: Understanding the Concept:**

Two arbitrary constants mean the differential equation is of order 2. Degree is the power of the highest order derivative after clearing radicals and fractions.

Step 2: Differentiate twice:

$$2(y - a)y' = 4 \Rightarrow (y - a)y' = 2$$

$$y'^2 + (y - a)y'' = 0$$

Step 3: Eliminate a:

From the first, $y - a = \frac{2}{y'}$. Substitute:

$$y'^2 + \frac{2y''}{y'} = 0 \Rightarrow y'^3 + 2y'' = 0$$

Step 4: Read order and degree:

The highest derivative is y'' (order 2), and it appears to the first power. So the degree is 1. Options (B) and (C) use the power 3 of y' , which is a lower-order derivative.

Final Answer:

The equation $y'^3 + 2y'' = 0$ has degree 1.

(A) 1

Quick Tip: Eliminate both constants by differentiating twice.

133. The solution of the differential equation $(x + 2y^3)\frac{dy}{dx} - y = 0$ is

- (A) $x = (c + y^2)y$, where c is the constant of integration
- (B) $y = (c + y^2)x$, where c is the constant of integration
- (C) $x = (c + y)y$, where c is the constant of integration
- (D) $y = (c + x^2)$, where c is the constant of integration

Correct Answer: (A) $x = (c + y^2)y$, where c is the constant of integration

Solution:

Step 1: Understanding the Concept:

The equation is not linear in y , but it is linear in x when written with $\frac{dx}{dy}$.

Step 2: Rewrite:

$$\frac{dx}{dy} = \frac{x + 2y^3}{y} \Rightarrow \frac{dx}{dy} - \frac{1}{y}x = 2y^2$$

Here $P = -\frac{1}{y}$ and $Q = 2y^2$.

Step 3: Integrating factor:

$$\text{I.F.} = e^{-\int \frac{dy}{y}} = e^{-\log y} = \frac{1}{y}$$

Step 4: Solve:

$$\frac{x}{y} = \int \frac{1}{y} \cdot 2y^2 dy = \int 2y dy = y^2 + c$$

$$x = (c + y^2)y$$

This matches option (A). The others mix x and y in forms that do not satisfy the equation.

Final Answer:

The solution is $x = (c + y^2)y$.

$$(A) \ x = (c + y^2)y$$

Quick Tip: Treat x as a function of y and use the linear form $dx/dy + Px = Q$.

134. The rate of reduction is proportional to the square root of a persons existing assets at that moment. If his assets at the beginning are 10000 and they dwindle down to 5625 in 2 years, then the person will be bankrupt in

- (A) 4 years
- (B) 6 years
- (C) 8 years
- (D) 10 years

Correct Answer: (C) 8 years

Solution:

Step 1: Understanding the Concept:

Let A be the assets at time t . The rate of reduction is proportional to $\sqrt{A}$, so $\frac{dA}{dt} = -k\sqrt{A}$.

Step 2: Solve:

$$\frac{dA}{\sqrt{A}} = -k dt \Rightarrow 2\sqrt{A} = -kt + C$$

So $\sqrt{A}$ falls linearly with time.

Step 3: Use the data:

At $t = 0$: $\sqrt{10000} = 100$. At $t = 2$: $\sqrt{5625} = 75$.

$\sqrt{A}$ drops by 25 in 2 years, so the rate is 12.5 per year.

Step 4: Bankruptcy time:

Assets are zero when $\sqrt{A} = 0$: $t = \frac{100}{12.5} = 8$ years. Option (C).

Final Answer:

The root of assets falls by 12.5 a year, reaching 0 at 8 years.

(C) 8 years

Quick Tip: Separate variables: $dA/dt = -k \sqrt{A}$, then use the two data points.

135. If a vector $3\hat{i} + 4\hat{j} - 5\hat{k}$ is rotated through a certain angle about the origin in the anti-clockwise direction, then the components of the new vector are $a + 1, -3, 5$. The possible values of a is

- (A) 5 or 3
- (B) 5 or -3
- (C) 4 or -2
- (D) -5 or 3

Correct Answer: (D) -5 or 3

Solution:

Step 1: Understanding the Concept:

Rotating a vector about the origin changes its direction but not its magnitude.

Step 2: Original magnitude squared:

$$|3\hat{i} + 4\hat{j} - 5\hat{k}|^2 = 9 + 16 + 25 = 50$$

Step 3: New vector:

The new vector is $(a + 1)\hat{i} - 3\hat{j} + 5\hat{k}$, with magnitude squared $(a + 1)^2 + 9 + 25$.

$$(a + 1)^2 + 34 = 50 \Rightarrow (a + 1)^2 = 16 \Rightarrow a + 1 = \pm 4$$

Step 4: Values of a:

$a = 3$ or $a = -5$. This is option (D). Check: $a = 3$ gives $(4, -3, 5)$ and $a = -5$ gives $(-4, -3, 5)$, both of squared length $16 + 9 + 25 = 50$.

Final Answer:

Length is preserved, so $(a + 1)^2 = 16$.

(D) – 5 or 3

Quick Tip: A rotation keeps the length of the vector unchanged.

136. Let A, B, C, D be the points in the plane with position vectors $-2\hat{i} - \hat{j}$, $4\hat{i}$, $3\hat{i} + 3\hat{j}$ and $-3\hat{i} + 2\hat{j}$ respectively, then $\square ABCD$ is

- (A) a parallelogram which is neither a rhombus nor a rectangle
- (B) a square
- (C) a rectangle but not a square
- (D) a rhombus but not a square

Correct Answer: (A) a parallelogram which is neither a rhombus nor a rectangle

Solution:**Step 1: Understanding the Concept:**

Points: A(-2, -1), B(4, 0), C(3, 3), D(-3, 2). Check vectors of opposite sides to test for a parallelogram.

Step 2: Opposite sides:

$\vec{AB} = (6, 1)$, $\vec{DC} = (6, 1)$. So AB and DC are equal and parallel, hence ABCD is a parallelogram.

Step 3: Adjacent sides:

$\vec{AD} = (-1, 3)$. $|AB| = \sqrt{37}$, $|AD| = \sqrt{10}$, which are different, so it is not a rhombus.

$\vec{AB} \cdot \vec{AD} = -6 + 3 = -3 \neq 0$, so the angle is not a right angle and it is not a rectangle.

Step 4: Result:

ABCD is a parallelogram that is neither a rhombus nor a rectangle, option (A). Since it is not a rectangle or rhombus, it cannot be a square either.

Final Answer:

Opposite sides match, but sides and angle are unequal.

(A) Parallelogram, not rhombus or rectangle

Quick Tip: Compare opposite sides, then adjacent side lengths and angle.

137. Let $\vec{a} = \hat{i} + \hat{j}$, $\vec{c} = \hat{i} - \hat{j}$ and a vector $\vec{b}$ be such that $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{a} \cdot \vec{b} = 3$ then $|\vec{b}| =$

- (A) $\frac{11}{\sqrt{2}}$
- (B) $\frac{11}{\sqrt{3}}$
- (C) $\sqrt{\frac{11}{2}}$
- (D) $\sqrt{\frac{11}{3}}$

Correct Answer: (C) $\sqrt{\frac{11}{2}}$

Solution:

Step 1: Understanding the Concept:

Use the triple product identity $\vec{a} \times (\vec{a} \times \vec{b}) = (\vec{a} \cdot \vec{b})\vec{a} - (\vec{a} \cdot \vec{a})\vec{b}$.

Step 2: Compute the left side with the given data:

$\vec{a} \times \vec{c} = (\hat{i} + \hat{j}) \times (\hat{i} - \hat{j}) = -\hat{k} - \hat{k} = -2\hat{k}$. Here $\vec{a} \cdot \vec{a} = 2$ and $\vec{a} \cdot \vec{b} = 3$.

$$3(\hat{i} + \hat{j}) - 2\vec{b} = -2\hat{k}$$

Step 3: Solve for b:

$$\vec{b} = \frac{3(\hat{i} + \hat{j}) + 2\hat{k}}{2} = \frac{3}{2}\hat{i} + \frac{3}{2}\hat{j} + \hat{k}$$

Step 4: Magnitude:

$$|\vec{b}|^2 = \frac{9}{4} + \frac{9}{4} + 1 = \frac{11}{2} \Rightarrow |\vec{b}| = \sqrt{\frac{11}{2}}$$

Check: $\vec{a} \times \vec{b} = \hat{i} - \hat{j} = \vec{c}$ and $\vec{a} \cdot \vec{b} = 3$. Option (C).

Final Answer:

$\vec{b} = (3/2, 3/2, 1)$, so $|\vec{b}| = \sqrt{11/2}$.

(C) $\sqrt{\frac{11}{2}}$

Quick Tip: Take the cross product of $\vec{a}$ with both sides to get an expression for $\vec{b}$.

138. The acute angle between the vector $2\hat{i} + \hat{j} - 3\hat{k}$ and the plane containing the vectors $2\hat{i} + 3\hat{j} - \hat{k}$ and $\hat{i} - \hat{j} + 2\hat{k}$ is

(A) $\sin^{-1}\left(\frac{4}{\sqrt{42}}\right)$

(B) $\cos^{-1}\left(\frac{4}{\sqrt{42}}\right)$

(C) $\sin^{-1}\left(\frac{3}{\sqrt{42}}\right)$

(D) $\cos^{-1}\left(\frac{3}{\sqrt{42}}\right)$

Correct Answer: (A) $\sin^{-1}\left(\frac{4}{\sqrt{42}}\right)$

Solution:

Step 1: Understanding the Concept:

The normal to the plane containing $\vec{p} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{q} = \hat{i} - \hat{j} + 2\hat{k}$ is $\vec{n} = \vec{p} \times \vec{q}$. The angle θ between a vector and a plane satisfies $\sin \theta = \frac{|\vec{v} \cdot \vec{n}|}{|\vec{v}||\vec{n}|}$.

Step 2: Normal vector:

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 1 & -1 & 2 \end{vmatrix} = (6 - 1)\hat{i} - (4 + 1)\hat{j} + (-2 - 3)\hat{k} = 5\hat{i} - 5\hat{j} - 5\hat{k}$$

Divide by 5: $\vec{n} \parallel (1, -1, -1)$.

Step 3: Dot product:

$$\vec{v} = (2, 1, -3). \vec{v} \cdot \vec{n} = 2 - 1 + 3 = 4. |\vec{v}| = \sqrt{14}, |\vec{n}| = \sqrt{3}.$$

Step 4: Angle:

$$\sin \theta = \frac{4}{\sqrt{14}\sqrt{3}} = \frac{4}{\sqrt{42}} \Rightarrow \theta = \sin^{-1} \frac{4}{\sqrt{42}}$$

Option (B) and (D) use cosine, which would give the angle with the normal, not with the plane. Option (A).

Final Answer:

The sine of the angle is 4 over root 42.

(A) $\sin^{-1} \frac{4}{\sqrt{42}}$

Quick Tip: Find the plane normal as a cross product and use $\sin(\theta) = |\mathbf{v} \cdot \mathbf{n}| / (|\mathbf{v}||\mathbf{n}|)$.



139. A parallelogram is constructed on $5\vec{a} + 2\vec{b}$ and $\vec{a} - 3\vec{b}$ as its adjacent sides, with $|\vec{a}| = 2\sqrt{2}$, $|\vec{b}| = 3$. The angle between $\vec{a}$ and $\vec{b}$ is $\frac{\pi}{4}$. Then the length of the diagonals of the parallelogram are

- (A) $15, \sqrt{593}$
- (B) $15, 593$
- (C) $225, 593$
- (D) $20, 593$

Correct Answer: (A) $15, \sqrt{593}$

Solution:

Step 1: Understanding the Concept:

For adjacent sides $\vec{u}$ and $\vec{v}$, the diagonals are $\vec{u} + \vec{v}$ and $\vec{u} - \vec{v}$.

Step 2: Dot product of a and b:

$$\vec{a} \cdot \vec{b} = 2\sqrt{2} \cdot 3 \cos \frac{\pi}{4} = 6\sqrt{2} \cdot \frac{1}{\sqrt{2}} = 6$$

$$|\vec{a}|^2 = 8, |\vec{b}|^2 = 9.$$

Step 3: First diagonal:

$$\vec{u} + \vec{v} = 6\vec{a} - \vec{b}:$$

$$|6\vec{a} - \vec{b}|^2 = 36(8) + 9 - 12(6) = 288 + 9 - 72 = 225 \Rightarrow 15$$

Step 4: Second diagonal:

$$\vec{u} - \vec{v} = 4\vec{a} + 5\vec{b}:$$

$$|4\vec{a} + 5\vec{b}|^2 = 16(8) + 25(9) + 40(6) = 128 + 225 + 240 = 593 \Rightarrow \sqrt{593}$$

The lengths are 15 and $\sqrt{593}$, option (A). Options (B), (C), (D) give squared values instead of lengths.

Final Answer:

The diagonals are 15 and root 593.

(A) 15, $\sqrt{593}$

Quick Tip: The diagonals are the sum and the difference of the adjacent sides.

140. The lines $\frac{x-2}{1} = \frac{y-3}{1} = \frac{z-4}{-k}$ and $\frac{x-1}{k} = \frac{y-4}{2} = \frac{z-5}{1}$ are coplanar if

- (A) $k = 0, -3$
- (B) $k = -1, 3$
- (C) $k = 1, 2$
- (D) $k = 2, 4$

Correct Answer: (A) $k = 0, -3$

Solution:

Step 1: Understanding the Concept:

Two lines through (x_1, y_1, z_1) and (x_2, y_2, z_2) with direction ratios (l_1, m_1, n_1) and (l_2, m_2, n_2) are coplanar if the determinant of the connecting vector and the two directions is zero.

Step 2: Data:

Line 1 passes through $(2, 3, 4)$ with direction $(1, 1, -k)$. Line 2 passes through $(1, 4, 5)$ with direction $(k, 2, 1)$. Connecting vector: $(-1, 1, 1)$.

Step 3: Expand the determinant:

$$\begin{vmatrix} -1 & 1 & 1 \\ 1 & 1 & -k \\ k & 2 & 1 \end{vmatrix} = -1(1 + 2k) - 1(1 + k^2) + 1(2 - k) = -k^2 - 3k$$

Step 4: Solve:

$-k(k + 3) = 0$, so $k = 0$ or $k = -3$. Option (A). Options (B), (C), (D) fail the check: for $k = 1$, the determinant is $-4 \neq 0$.

Final Answer:

The determinant gives $k^2 + 3k = 0$.

$$(A) k = 0, -3$$

Quick Tip: Use the determinant condition for coplanar lines.

141. The direction ratios of the normal to the plane passing through $(1, 0, 0)$, $(0, 1, 0)$ which makes an angle of measure 45° with the plane $2x + 3y = 7$ are....

- (A) $\sqrt{6}, 1, 1$
- (B) $1, 1, \sqrt{6}$
- (C) $\sqrt{13}, \sqrt{13}, \sqrt{6}$
- (D) $\sqrt{13}, \sqrt{13}, 2\sqrt{6}$

Correct Answer: (D) $\sqrt{13}, \sqrt{13}, 2\sqrt{6}$

Solution:**Step 1: Understanding the Concept:**

A plane $ax + by + cz = d$ through $(1, 0, 0)$ gives $d = a$. Through $(0, 1, 0)$ gives $d = b$. So $a = b$ and the normal is (a, a, c) .

Step 2: Apply the angle formula:

Normal of $2x + 3y = 7$ is $(2, 3, 0)$, with magnitude $\sqrt{13}$.

$$\cos 45^\circ = \frac{|2a + 3a|}{\sqrt{2a^2 + c^2}\sqrt{13}} = \frac{5a}{\sqrt{13}\sqrt{2a^2 + c^2}}$$

Step 3: Solve for c:

Squaring: $\frac{1}{2} = \frac{25a^2}{13(2a^2 + c^2)}$, so $13(2a^2 + c^2) = 50a^2$, giving $13c^2 = 24a^2$, so $c = a\sqrt{\frac{24}{13}}$.

Step 4: Direction ratios:

Scale by $\sqrt{13}$: $(a, a, c) \rightarrow (\sqrt{13}, \sqrt{13}, \sqrt{24}) = (\sqrt{13}, \sqrt{13}, 2\sqrt{6})$. Option (D).

Final Answer:

The normal has direction ratios root 13, root 13, 2 root 6.

$$\boxed{(D) \sqrt{13}, \sqrt{13}, 2\sqrt{6}}$$

Quick Tip: The plane through (1,0,0) and (0,1,0) has normal (a, a, c); use the angle formula.

142. The lines $\frac{x-1}{-1} = \frac{y+2}{1} = \frac{z-3}{-2}$ and $\frac{x-1}{1} = \frac{y+2}{1} = \frac{z+1}{-2}$ are

- (A) intersecting but not perpendicular
- (B) parallel
- (C) perpendicular
- (D) skew lines

Correct Answer: (D) skew lines

Solution:

Step 1: Understanding the Concept:

Line 1: point $P_1(1, -2, 3)$, direction $\vec{d}_1 = (-1, 1, -2)$. Line 2: point $P_2(1, -2, -1)$, direction $\vec{d}_2 = (1, 1, -2)$.

Step 2: Parallel or perpendicular?:

The directions are not proportional, so the lines are not parallel.

$\vec{d}_1 \cdot \vec{d}_2 = -1 + 1 + 4 = 4 \neq 0$, so they are not perpendicular.

Step 3: Intersecting or skew?:

$\overrightarrow{P_1P_2} = (0, 0, -4)$.

$$\begin{vmatrix} 0 & 0 & -4 \\ -1 & 1 & -2 \\ 1 & 1 & -2 \end{vmatrix} = -4[(-1)(1) - (1)(1)] = -4(-2) = 8 \neq 0$$

Step 4: Result:

A nonzero triple product means the lines do not lie in one plane. They do not meet and are not parallel, so they are skew. Option (D).

Final Answer:

The scalar triple product is 8, not zero, so they are skew.

(D) skew lines

Quick Tip: Check parallel, perpendicular, then test the scalar triple product for intersection.

143. If the product of the distances of the point $(1, 2, 3)$ from the origin and the plane $2x - 3y + z + k = 0$ is 7, then the value of k is

- (A) 8
- (B) 10
- (C) 7
- (D) 5

Correct Answer: (A) 8

Solution:

Step 1: Understanding the Concept:

Distance of $(1, 2, 3)$ from the origin: $\sqrt{1 + 4 + 9} = \sqrt{14}$.

Step 2: Distance from the plane:

For $2x - 3y + z + k = 0$: numerator $|2 - 6 + 3 + k| = |k - 1|$;
denominator $\sqrt{4 + 9 + 1} = \sqrt{14}$.

$$d = \frac{|k - 1|}{\sqrt{14}}$$

Step 3: Use the product:

$$\sqrt{14} \cdot \frac{|k - 1|}{\sqrt{14}} = |k - 1| = 7$$

So $k = 8$ or $k = -6$.

Step 4: Match:

Among the options, only $k = 8$ appears. Options (B), (C), (D) give $|k - 1| = 9, 6, 4$, not 7. Option (A).

Final Answer:

$|k - 1| = 7$ gives $k = 8$ among the options.

(A) 8

Quick Tip: Distance from origin is root 14; the plane distance is $|k - 1| / \text{root } 14$.

144. The vector equation of the line whose cartesian equations are $x = 2, 2y - 3z + 7 = 0$

(A) $\vec{r} = (2\hat{i} + 2\hat{j} - 3\hat{k}) + \lambda(2\hat{j} - 3\hat{k})$

(B) $\vec{r} = (2\hat{i} + 2\hat{j} - 3\hat{k}) + \lambda(2\hat{i} - 3\hat{j})$

(C) $\vec{r} = (2\hat{i} + \frac{7}{3}\hat{k}) + \lambda(3\hat{j} + 2\hat{k})$

(D) $\vec{r} = (-2\hat{i} + \frac{7}{3}\hat{k}) + \lambda(3\hat{j} + 2\hat{k})$

Correct Answer: (C) $\vec{r} = (2\hat{i} + \frac{7}{3}\hat{k}) + \lambda(3\hat{j} + 2\hat{k})$

Solution:**Step 1: Understanding the Concept:**

The line is the intersection of the planes $x = 2$ and $2y - 3z + 7 = 0$. Its direction is perpendicular to both normals.

Step 2: Direction:

Normals: $\vec{n}_1 = (1, 0, 0)$ and $\vec{n}_2 = (0, 2, -3)$.

$$\vec{n}_1 \times \vec{n}_2 = (0 \cdot (-3) - 0 \cdot 2, 0 \cdot 0 - 1 \cdot (-3), 1 \cdot 2 - 0) = (0, 3, 2)$$

Step 3: A point on the line:

Take $y = 0$: $-3z + 7 = 0$, so $z = \frac{7}{3}$. The point is $(2, 0, \frac{7}{3})$.

Step 4: Write the equation:

$$\vec{r} = 2\hat{i} + \frac{7}{3}\hat{k} + \lambda(3\hat{j} + 2\hat{k})$$

This is option (C). Option (A) and (B) use a point like $(2, 2, -3)$, which gives $4 + 9 + 7 \neq 0$ in $2y - 3z + 7$, so it is not on the line.

Final Answer:

Point $(2, 0, 7/3)$ with direction $(0, 3, 2)$.

$$(C) \vec{r} = \left(2\hat{i} + \frac{7}{3}\hat{k}\right) + \lambda(3\hat{j} + 2\hat{k})$$

Quick Tip: Find a point on the line and the direction as the cross product of the two plane normals.

145. The feasible region represented by the constraints $y - 2x \leq 4, x + y \geq 5, x \leq 4, y \geq 2, x, y \geq 0$ is

- (A) a convex bounded region with 4 corner points
- (B) an unbounded region
- (C) a convex bounded region with 5 corner points
- (D) no feasible region

Correct Answer: (A) a convex bounded region with 4 corner points

Solution:

Step 1: Understanding the Concept:

Constraints: $y - 2x \leq 4$, $x + y \geq 5$, $x \leq 4$, $y \geq 2$, $x, y \geq 0$. The feasible region is the intersection of all these half planes.

Step 2: Corner points:

$y = 2$ with $x + y = 5$: $(3, 2)$.

$y = 2$ with $x = 4$: $(4, 2)$.

$x = 4$ with $y = 2x + 4$: $(4, 12)$.

$y = 2x + 4$ with $x + y = 5$: $x = \frac{1}{3}$, $y = \frac{14}{3}$: $(\frac{1}{3}, \frac{14}{3})$.

Points on $x = 0$ are excluded: there $y \leq 4$ by the first constraint, but $x + y \geq 5$ needs $y \geq 5$.

Step 3: Shape:

The region is enclosed by four lines: a quadrilateral with vertices $(3, 2)$, $(4, 2)$, $(4, 12)$, $(\frac{1}{3}, \frac{14}{3})$. It is bounded and convex.

Step 4: Result:

A convex bounded region with 4 corner points, option (A).

Final Answer:

The region is a quadrilateral with four corners.

(A) convex bounded region with 4 corner points

Quick Tip: Find the corner points by pairing boundary lines.

146. A fair die is thrown at random. Let A be the event that the number obtained on the die is a non-even prime number and B be the event that the number obtained on the die is an odd number.

Let $p : P(A) = \frac{1}{3}$, $q : A$ and B are independent events.

Then the truth values of statements p and q respectively are.....

(A) T , T

(B) T , F

(C) F , T

(D) F , F

Correct Answer: (B) T , F

Solution:

Step 1: Understanding the Concept:

A = non-even prime numbers on a die = {3, 5} (2 is prime but even). B = odd numbers = {1, 3, 5}.

Step 2: Statement p:

$P(A) = \frac{2}{6} = \frac{1}{3}$, so p is TRUE.

Step 3: Statement q:

$A \cap B = \{3, 5\}$, so $P(A \cap B) = \frac{1}{3}$.

$$P(A)P(B) = \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{6}.$$

Since $\frac{1}{3} \neq \frac{1}{6}$, the events are not independent, so q is FALSE.

Step 4: Result:

The truth values are T, F. Option (B).

Final Answer:

p is true, but A and B are not independent.

(B) T, F

Quick Tip: List the favourable outcomes of A and B, then test $P(A \text{ and } B) = P(A)P(B)$.

147. Three ships A, B, C sail from England to India. If the odds in favour of their safe arrival are 2:5, 3:7 and 6:11 respectively, then the probability that the exactly two ships arrive safely is

- (A) $\frac{24}{119}$
- (B) $\frac{24}{1190}$
- (C) $\frac{66}{1190}$
- (D) $\frac{84}{1190}$

Correct Answer: (A) $\frac{24}{119}$

Solution:**Step 1: Understanding the Concept:**

Odds in favour $m : n$ mean probability $\frac{m}{m+n}$.

Step 2: Individual probabilities:

A: $\frac{2}{7}$, not safe $\frac{5}{7}$. B: $\frac{3}{10}$, not safe $\frac{7}{10}$. C: $\frac{6}{17}$, not safe $\frac{11}{17}$.

Step 3: Three cases for exactly two:

A and B safe, C not: $\frac{2 \cdot 3 \cdot 11}{1190} = \frac{66}{1190}$.

$$\begin{aligned} \text{A and C safe, B not: } & \frac{2 \cdot 7 \cdot 6}{1190} = \frac{84}{1190} \\ \text{B and C safe, A not: } & \frac{5 \cdot 3 \cdot 6}{1190} = \frac{90}{1190} \end{aligned}$$

Step 4: Add:

$$\frac{66 + 84 + 90}{1190} = \frac{240}{1190} = \frac{24}{119}$$

Option (A). Options (B), (C), (D) are individual terms or arithmetic slips.

Final Answer:

The three cases add up to $240/1190 = 24/119$.

$$\boxed{(A) \frac{24}{119}}$$

Quick Tip: Convert odds to probabilities and add the three two-safe cases.

148. A random variable X has the probability distribution

$X = x$	1	2	3	4	5	6	7	8
$P(X = x)$	0.23	0.15	0.12	0.10	0.20	0.07	0.08	0.05

for the events $E = \{X \text{ is a prime number}\}$ and $F = \{X \leq 3\}$, then $P(E \cup F) =$

- (A) 0.55
- (B) 0.70
- (C) 0.87
- (D) 0.78

Correct Answer: (D) 0.78

Solution:

Step 1: Understanding the Concept:

$E = \{2, 3, 5, 7\}$ (primes) and $F = \{1, 2, 3\}$ ($X \leq 3$). Read the probabilities from the table.

Step 2: Compute each part:

$$P(E) = 0.15 + 0.12 + 0.20 + 0.08 = 0.55.$$

$$P(F) = 0.23 + 0.15 + 0.12 = 0.50.$$

$$E \cap F = \{2, 3\}, \text{ so } P(E \cap F) = 0.15 + 0.12 = 0.27.$$

Step 3: Apply the formula:

$$P(E \cup F) = 0.55 + 0.50 - 0.27 = 0.78$$

Step 4: Check by listing:

$E \cup F = \{1, 2, 3, 5, 7\}$: $0.23 + 0.15 + 0.12 + 0.20 + 0.08 = 0.78$. Option (D).

Final Answer:

$$P(E \text{ or } F) = 0.55 + 0.50 - 0.27 = 0.78.$$

(D) 0.78

Quick Tip: Use $P(E \text{ union } F) = P(E) + P(F) - P(E \text{ and } F)$.

149. The probability distribution of a random variable X is given by

$$\begin{array}{c} X = x \quad 1 \quad 2 \quad 3 \quad 4 \\ P(X = x) \quad k \quad 2k \quad 3k \quad 4k \end{array}$$

Then the c.d.f. of X is given by

(A)

$$\begin{array}{c} X = x \quad 1 \quad 2 \quad 3 \quad 4 \\ F(X = x) \quad \frac{1}{10} \quad \frac{3}{10} \quad \frac{6}{10} \quad 1 \end{array}$$

(B)

$$\begin{array}{c} X = x \quad 1 \quad 2 \quad 3 \quad 4 \\ F(X = x) \quad \frac{3}{10} \quad \frac{1}{10} \quad \frac{6}{10} \quad 1 \end{array}$$

(C)

$$\begin{array}{c} X = x \quad 1 \quad 2 \quad 3 \quad 4 \\ F(X = x) \quad \frac{1}{10} \quad \frac{3}{10} \quad \frac{5}{10} \quad \frac{1}{10} \end{array}$$

(D)

$$\begin{array}{c} X = x \quad 1 \quad 2 \quad 3 \quad 4 \\ F(X = x) \quad \frac{1}{10} \quad \frac{6}{10} \quad \frac{3}{10} \quad 1 \end{array}$$

Correct Answer: (A)

$$X = x \quad 1 \quad 2 \quad 3 \quad 4$$

$$F(X = x) \quad \frac{1}{10} \quad \frac{3}{10} \quad \frac{6}{10} \quad 1$$

Solution:

Step 1: Understanding the Concept:

The probabilities sum to 1, and the c.d.f. is $F(x) = P(X \leq x)$.

Step 2: Find k:

$$k + 2k + 3k + 4k = 10k = 1 \Rightarrow k = \frac{1}{10}$$

So the probabilities are $\frac{1}{10}, \frac{2}{10}, \frac{3}{10}, \frac{4}{10}$.

Step 3: Cumulative sums:

$$F(1) = \frac{1}{10}, F(2) = \frac{1}{10} + \frac{2}{10} = \frac{3}{10}, F(3) = \frac{3}{10} + \frac{3}{10} = \frac{6}{10}, F(4) = \frac{6}{10} + \frac{4}{10} = 1.$$

Step 4: Match:

The table with $\frac{1}{10}, \frac{3}{10}, \frac{6}{10}, 1$ is option (A). Option (B) has a decreasing step, and (C) and (D) do not end in 1 or increase steadily.

Final Answer:

The c.d.f. values are $1/10, 3/10, 6/10$ and 1.

$(A) F = 1/10, 3/10, 6/10, 1$

Quick Tip: Find k from the total probability, then add up the probabilities.

150. Two dice are thrown successively 4 times and getting a doublet is called a success. The probability of getting at least 1 success is

- (A) $\frac{50}{1296}$
- (B) $\frac{1246}{1296}$
- (C) $\frac{625}{1296}$
- (D) $\frac{671}{1296}$

Correct Answer: (D) $\frac{671}{1296}$

Solution:

Step 1: Understanding the Concept:

A throw of two dice has 36 outcomes, and 6 of them are doublets. So $p = \frac{1}{6}$ and $q = \frac{5}{6}$ per throw. Four independent throws make a binomial experiment with $n = 4$.

Step 2: Use the complement:

$$P(\text{at least one}) = 1 - P(\text{none}) = 1 - \left(\frac{5}{6}\right)^4$$

Step 3: Compute:

$$\left(\frac{5}{6}\right)^4 = \frac{625}{1296}$$

$$1 - \frac{625}{1296} = \frac{671}{1296}$$

Step 4: Result:

Option (D). Option (C) $\frac{625}{1296}$ is the probability of no success at all.

Final Answer:

The probability is $1 - (5/6)^4 = 671/1296$.

$$\boxed{(D) \frac{671}{1296}}$$

Quick Tip: Use the complement: 1 minus the probability of no doublet in four throws.