MHT CET 2026 PCM Question Paper April 16 Shift 2 with Solutions

General Instructions

(i) This booklet contains 150 questions, each provided with a complete, step-by-step solution.

(ii) It comprises 150 single-correct multiple-choice questions.

(iii) Attempt each question on your own before reviewing the given solution.

Physics

Questions 1 to 50 | 1 mark each | 50 marks

1. The error in the measurement of length and mass of a cube is 3% and 4% respectively. The error in the measurement of its density will be

(A) 13%

(B) 6%

(C) 9%

(D) 15%

Correct Answer: (A) 13%

Solution:

Step 1: Write the formula

For a cube of side L and mass M, density $\rho = \frac{M}{L^3}$.

Step 2: Apply the error rule

For a quantity $x = \frac{a}{b^n}$, the maximum fractional error is $\frac{\Delta x}{x} = \frac{\Delta a}{a}$

$+ n\frac{\Delta b}{b}$. Errors add even for a quotient.

Step 3: Substitute

$$\frac{\Delta \rho}{\rho} \times 100 = 4\% + 3 \times 3\% = 13\%$$

Step 4: Why not the others

Option (B) 6% comes from adding $3 + 3$ and ignoring the cube. Option (C) 9% ignores the mass error. So 13 percent is correct.

Final Answer:

The error in density is 13 percent.

$$\boxed{\text{(A) } 13\%}$$

Quick Tip: Density is mass divided by length cubed, so percentage errors add with the power as weight.

2. Vectors $a\hat{i} + b\hat{j} + \hat{k}$ and $2\hat{i} - 3\hat{j} + 4\hat{k}$ are perpendicular to each other when $3a + 2b = 7$, the ratio of a to b is $x/2$. The value of x is

(A) 4

(B) 1

(C) 8

(D) 3

Correct Answer: (B) 1

Solution:

Step 1: Use the dot product

For perpendicular vectors, $(a\hat{i} + b\hat{j} + \hat{k}) \cdot (2\hat{i} - 3\hat{j} + 4\hat{k}) = 0$, so $2a - 3b + 4 = 0$.

Step 2: Use the given relation

We also have $3a + 2b = 7$. From the first, $a = \frac{3b-4}{2}$.

Step 3: Solve

$3 \cdot \frac{3b-4}{2} + 2b = 7$ gives $9b - 12 + 4b = 14$, so $b = 2$ and $a = 1$.

Step 4: Find x

$\frac{a}{b} = \frac{1}{2} = \frac{x}{2}$, so $x = 1$. Option (B).

Final Answer:

The value of x is 1.

$$\boxed{\text{(B) } 1}$$

Quick Tip: Perpendicular vectors have zero dot product; solve the two equations for a and b.

3. Two objects are projected with same velocity 'u' at different angles α and β with the horizontal. If $\alpha + \beta = 90°$, the ratio of horizontal range of the first object to the second object will be $[sin(\pi - \theta) = sin\theta]$

(A) 4 : 1

(B) 2 : 1

(C) 1 : 1

(D) 1 : 2

Correct Answer: (C) 1 : 1

Solution:

Step 1: Range formula

For a projectile, $R = \frac{u^2 \sin 2\theta}{g}$.

Step 2: Complementary angles

With $\beta = 90° - \alpha$, $2\beta = 180° - 2\alpha$, so $\sin 2\beta = \sin(180° - 2\alpha) = \sin 2\alpha$.

Step 3: Ratio

$\frac{R_1}{R_2} = \frac{\sin 2\alpha}{\sin 2\beta} = 1$. The ratio is 1 : 1, option (C).

Final Answer:

Both ranges are equal, so the ratio is 1:1.

$$\boxed{\text{(C) } 1 : 1}$$

Quick Tip: Range depends on sin 2 theta, and sin 2 alpha equals sin 2 beta when alpha plus beta is 90 degrees.

4. A body of mass 1kg begins to move under the action of a time dependent force $\vec{F} = (t\hat{i} + 3t^2\hat{j})$N where $\hat{i}$ and $\hat{j}$ are the unit vectors along x and y axis. The power developed by the above force at time $t = 2$s will be (in watt)

(A) 100

(B) 50

(C) 25

(D) 5

Correct Answer: (A) 100

Solution:

Step 1: Acceleration

With $m = 1$ kg, $\vec{a} = \vec{F} = t\hat{i} + 3t^2\hat{j}$.

Step 2: Velocity

Starting from rest, $\vec{v} = \frac{t^2}{2}\hat{i} + t^3\hat{j}$. At $t = 2$: $\vec{v} = 2\hat{i} + 8\hat{j}$.

Step 3: Force at t = 2

$\vec{F} = 2\hat{i} + 12\hat{j}$.

Step 4: Power

$$P = \vec{F} \cdot \vec{v} = 2 \times 2 + 12 \times 8 = 4 + 96 = 100 \text{ W}$$

Option (A).

Final Answer:

The power is 100 W.

$$\boxed{\text{(A) 100 W}}$$

Quick Tip: Find velocity by integrating acceleration, then use P = F dot v.

5. A solid sphere of mass M and a disc of mass $\frac{M}{2}$ have the same radius. The ratio of moment of inertia of the disc about a tangent in its plane to the moment of inertia of the sphere about its tangent will be

(A) 15 : 8

(B) 25 : 56

(C) 12 : 7

(D) 16 : 9

Correct Answer: (B) 25 : 56

Solution:

Step 1: Disc of mass M/2

About a diameter, $I = \frac{1}{4}\left(\frac{M}{2}\right)R^2$. A tangent in the plane is at distance R from the diameter, so $I_{disc} = \frac{MR^2}{8} + \frac{M}{2}R^2 = \frac{5MR^2}{8}$.

Step 2: Solid sphere

About a diameter, $I = \frac{2}{5}MR^2$. About a tangent: $I_{sphere} = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2$.

Step 3: Ratio

$$\frac{I_{disc}}{I_{sphere}} = \frac{5/8}{7/5} = \frac{25}{56}$$

Option (B).

Final Answer:

The ratio is 25:56.

(B) 25 : 56

Quick Tip: Use the parallel axis theorem for both bodies.

6. A uniform solid cylinder with radius R and length L has moment of inertia I_1 about the axis of the cylinder. A concentric solid cylinder of radius $R/2$ and length $L/2$ is carved out of the original cylinder. If I_2 is the moment of inertia of the carved out portion of the cylinder then I_1/I_2 is (Both I_1 and I_2 are about the axis of the cylinder)

(A) 4 : 1

(B) 8 : 1

(C) 16 : 1

(D) 32 : 1

Correct Answer: (D) 32 : 1

Solution:

Step 1: Mass of carved cylinder

Volume scales as r^2L. The small cylinder has radius $\frac{R}{2}$ and length $\frac{L}{2}$, so its volume is $\frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$ of the original. Its mass is $\frac{M}{8}$.

Step 2: Moments of inertia

$I_1 = \frac{1}{2}MR^2$. $I_2 = \frac{1}{2}\left(\frac{M}{8}\right)\left(\frac{R}{2}\right)^2 = \frac{MR^2}{64}$.

Step 3: Ratio

$\frac{I_1}{I_2} = \frac{1/2}{1/64} = 32$. So 32 : 1, option (D).

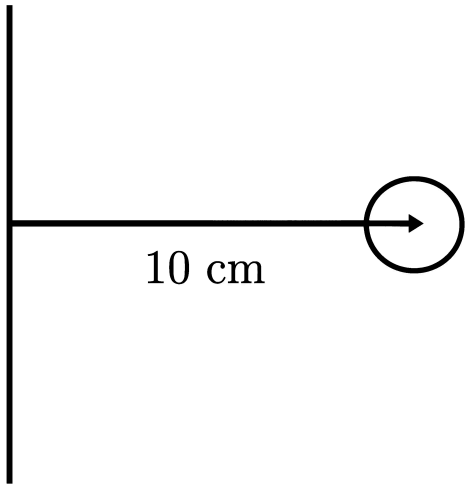
Final Answer:

The ratio is 32:1.

(D) 32 : 1

Quick Tip: Mass scales with volume, so find the mass of the small cylinder first.

7. Solid sphere is rotating about an axis as shown in figure. If the radius of the sphere is 5 cm, then radius of gyration about that axis will be $\sqrt{x}$ cm. The value of x is



(A) 100

(B) 110

(C) 120

(D) 140

Correct Answer: (B) 110

Solution:

Step 1: Read the figure

The figure shows a solid sphere of radius 5 cm whose centre is 10 cm from the axis of rotation.

Step 2: Moment of inertia

About the centre, $I_{cm} = \frac{2}{5}MR^2 = \frac{2}{5}M(25) = 10M$ (in cm squared). By the parallel axis theorem, $I = 10M + M(10)^2 = 110M$.

Step 3: Radius of gyration

$I = Mk^2$ gives $k^2 = 110\ \text{cm}^2$, so $k = \sqrt{110}\ \text{cm}$. Thus $x = 110$, option (B).

Final Answer:

The value of x is 110.

$$\boxed{\text{(B) } 110}$$

Quick Tip: Use the parallel axis theorem with the axis 10 cm from the centre.

8. A body of mass m is taken from earth surface to a height h equal to twice the radius of earth, the increase in potential energy will be (g = acceleration due to gravity on earth's surface) (R - radius of earth)

(A) $\frac{2}{3}$mgR

(B) $\frac{1}{3}$mgR

(C) $\frac{1}{2}$mgR

(D) 3mgR

Correct Answer: (A) $\frac{2}{3}$mgR

Solution:

Step 1: Potential energy

At distance r from the centre, $U = -\frac{GMm}{r}$. Also $GM = gR^2$.

Step 2: Initial and final

At the surface $r = R$: $U_i = -mgR$. At height $h = 2R$, $r = 3R$: $U_f = -\frac{mgR}{3}$.

Step 3: Increase

$$\Delta U = -\frac{mgR}{3} + mgR = \frac{2}{3}mgR$$

Option (A). Using $mgh = 2mgR$ is only valid for small heights.

Final Answer:

The increase in potential energy is 2/3 mgR.

$$\boxed{(A)\ \frac{2}{3}mgR}$$

Quick Tip: Use U = -GMm/r and compare the values at R and 3R.

9. One thousand small water drops of equal radii combine to form a big drop. The ratio of final surface energy to the total initial surface energy is

(A) 1 : 1

(B) 1 : 10

(C) 1 : 100

(D) 1 : 1000

Correct Answer: (B) 1 : 10

Solution:

Step 1: Find the radius

$\frac{4}{3}\pi R^3 = 1000 \cdot \frac{4}{3}\pi r^3$, so $R = 10r$.

Step 2: Surface areas

Final area $= 4\pi R^2 = 400\pi r^2$. Initial area $= 1000 \cdot 4\pi r^2 = 4000\pi r^2$.

Step 3: Ratio

Surface energy is surface tension times area, so the ratio is $\frac{400}{4000} = \frac{1}{10}$, i.e. 1 : 10. Option (B).

Final Answer:

The ratio is 1:10.

$$\boxed{\text{(B) } 1 : 10}$$

Quick Tip: Volume stays the same, so the big drop radius is 10 times the small one.

10. Under isothermal condition, two soap bubbles of radii r_1 and r_2 combine to form a single soap bubble of radius R. The surface tension of the soap solution is (P = outside pressure)

(A) $\frac{P(R^3-r_1^3-r_2^3)}{4(r_1^2+r_2^2-R^2)}$

(B) $\frac{P(R^3+r_1^3+r_2^3)}{2(r_1^2-r_2^2+R^2)}$

(C) $\frac{P(r_1^3-r_2^3-R^3)}{(R^2+r^2+r^2)}$

(D) $\frac{P(R^3-r_1^3+r_2^3)}{2(r_1^2+r_2^2-R^2)}$

Correct Answer: (A) $\frac{P(R^3-r_1^3-r_2^3)}{4(r_1^2+r_2^2-R^2)}$

Solution:

Step 1: Pressures

Inside a soap bubble of radius r the pressure is $P+\frac{4S}{r}$.

Step 2: Boyle law at constant temperature

Total PV before equals after:

$$\left(P+\frac{4S}{r_1}\right)\frac{4}{3}\pi r_1^3+\left(P+\frac{4S}{r_2}\right)\frac{4}{3}\pi r_2^3=\left(P+\frac{4S}{R}\right)\frac{4}{3}\pi R^3$$

Step 3: Simplify

Cancel $\frac{4}{3}\pi$: $Pr_1^3+4Sr_1^2+Pr_2^3+4Sr_2^2=PR^3+4SR^2$. So $4S(r_1^2+r_2^2-R^2)=P(R^3-r_1^3-r_2^3)$.

Step 4: Result

$$S=\frac{P(R^3-r_1^3-r_2^3)}{4(r_1^2+r_2^2-R^2)}$$

Option (A).

Final Answer:

The surface tension is given by option A.

$$\boxed{\text{(A) } S = \frac{P(R^3 - r_1^3 - r_2^3)}{4(r_1^2 + r_2^2 - R^2)}}$$

Quick Tip: Apply Boyle law to the total gas, using the excess pressure 4S/r inside a soap bubble.

11. A horizontal pipe carries water in a streamline flow. At point along the pipe, where the cross-sectional area is A_1, the velocity of water is V_1 and the pressure is P_1. What is the pressure of water at another point where the cross-sectional area is A_2?

(A) $P_1 - \frac{\rho V_2^2}{2A_1^2}(A_2^2 - A_1^2)$

(B) $P_1 + \frac{\rho V_1^2}{2A_2^2}(A_2^2 - A_1^2)$

(C) $P_1 \rho V_1^2 (A_1^2 - A_2^2)$

(D) $P/\rho V_1^2 A_2^2$

Correct Answer: (B) $P_1 + \frac{\rho V_1^2}{2A_2^2}(A_2^2 - A_1^2)$

Solution:

Step 1: Continuity

$A_1 V_1 = A_2 V_2$, so $V_2 = \frac{A_1 V_1}{A_2}$.

Step 2: Bernoulli

For a horizontal pipe, $P_1 + \frac{1}{2}\rho V_1^2 = P_2 + \frac{1}{2}\rho V_2^2$.

Step 3: Solve for P2

$$P_2 = P_1 + \frac{1}{2}\rho V_1^2 \left(1 - \frac{A_1^2}{A_2^2}\right) = P_1 + \frac{\rho V_1^2}{2A_2^2}(A_2^2 - A_1^2)$$

Option (B).

Step 4: Check sense

If $A_2 > A_1$, the water slows down and the pressure rises, which agrees with the sign above.

Final Answer:

The pressure at the second section is option B.

$$\boxed{\text{(B) } P_2 = P_1 + \frac{\rho V_1^2}{2A_2^2}(A_2^2 - A_1^2)}$$

Quick Tip: Use the equation of continuity to get V2, then Bernoulli for a horizontal pipe.

12. A bowl filled with very hot water cools from 98°C to 86°C in 2 minutes when the room temperature is 22°C. How long it will take to cool from 75°C to 69°C ?

(A) 0.5 minute

(B) 1.4 minute

(C) 1 minute

(D) 2 minute

Correct Answer: (B) 1.4 minute

Solution:

Step 1: Find k

Fall of $12°C$ in 2 min, mean temperature $92°C$, excess $70°C$. Newton's law: $\frac{12}{2} = k(70)$, so $k = \frac{3}{35}\ \text{min}^{-1}$.

Step 2: Second interval

Fall of $6°C$, mean temperature $72°C$, excess $50°C$. So $\frac{6}{t} = k(50)$.

Step 3: Solve

$$t = \frac{6}{\frac{3}{35} \times 50} = \frac{6 \times 35}{150} = 1.4\ \text{min}$$

Option (B).

Final Answer:

The time taken is 1.4 minutes.

(B) 1.4 minute

Quick Tip: The rate of fall equals k times the excess over room temperature, using the mean temperature.

13. A Carnot engine with efficiency 50% takes heat from a source at 600 K. To increase the efficiency by 20%, keeping temperature of the sink same, the new temperature of the source will be

(A) 300K

(B) 900K

(C) 1000K

(D) 360K

Correct Answer: (C) 1000K

Solution:

Step 1: Find sink temperature

$\eta = 1 - \frac{T_2}{T_1}$, so $0.5 = 1 - \frac{T_2}{600}$ gives $T_2 = 300$ K.

Step 2: New efficiency

The efficiency increases by 20 percentage points, from 50% to 70%.

Step 3: New source temperature

$0.7 = 1 - \frac{300}{T_1'}$ gives $\frac{300}{T_1'} = 0.3$, so $T_1' = 1000$ K. Option (C).

Final Answer:

The new source temperature is 1000 K.

$$\boxed{\text{(C) 1000 K}}$$

Quick Tip: Efficiency is 1 minus T2/T1. First find the sink temperature from the 50 percent efficiency.

14. One gram of a liquid is converted to vapour at 3×10^5 Pa pressure. If 10% of the heat supplied is used for increasing the volume by $1600 cm^3$ during this phase change, then the increase in internal energy in the process will be

(A) 4.32×10^8J

(B) 4800J

(C) 4320J

(D) 4.32×10^5J

Correct Answer: (C) 4320J

Solution:

Step 1: Work done

$W = P\Delta V = 3 \times 10^5 \times 1600 \times 10^{-6} = 480$ J.

Step 2: Heat supplied

This work is 10% of the heat, so $Q = 4800$ J.

Step 3: First law

$\Delta U = Q - W = 4800 - 480 = 4320$ J. Option (C).

Step 4: Check

Option (B) is the total heat, not the change in internal energy.

Final Answer:

The increase in internal energy is 4320 J.

$$\boxed{\text{(C) 4320 J}}$$

Quick Tip: Work done is P times the change in volume; it is 10 percent of the heat supplied.

15. The average transnational kinetic energy of a molecule in a gas is E_1. The kinetic energy of the electron (e) accelerated from rest through potential difference 'V' volt is E_2. The temperature at which $E_1 = E_2$ possible is (mass of molecule and electron is same) (N = number of molecules, V = Velocity, R = gas constant)

(A) $\frac{2VNe}{3R}$

(B) $\frac{VNe}{2R}$

(C) $\frac{3NeV}{2R}$

(D) $\frac{5NeV}{3R}$

Correct Answer: (A) $\frac{2VNe}{3R}$

Solution:

Step 1: Gas molecule

$E_1 = \frac{3}{2}kT = \frac{3}{2}\frac{R}{N}T$, where $k = \frac{R}{N}$ is the Boltzmann constant.

Step 2: Electron

The electron gains $E_2 = eV$.

Step 3: Equate

$$\frac{3RT}{2N} = eV \Rightarrow T = \frac{2NeV}{3R}$$

Option (A).

Final Answer:

The temperature is 2VNe/(3R).

$$\boxed{(A)\ T = \frac{2VNe}{3R}}$$

Quick Tip: Average translational KE is (3/2) kT per molecule, with k = R/N.

16. A rigid diatomic gas having molar mass m is contained in an insulated container. The container is moving with velocity V. If it is stopped suddenly, the change in temperature is (R - gas constant)

(A) $\frac{mV^2}{R}$

(B) $\frac{mV^2}{3R}$

(C) $\frac{mV^2}{5R}$

(D) $\frac{mV^2}{7R}$

Correct Answer: (C) $\frac{mV^2}{5R}$

Solution:

Step 1: Energy conversion

Let the gas be n moles with molar mass m, so its mass is nm. The bulk kinetic energy is $\frac{1}{2}nmV^2$.

Step 2: Internal energy gain

A rigid diatomic gas has $C_v = \frac{5}{2}R$. The container is insulated, so all energy goes into heating: $nC_v\Delta T = \frac{1}{2}nmV^2$.

Step 3: Solve

$$\Delta T = \frac{mV^2}{2C_v} = \frac{mV^2}{5R}$$

Option (C).

Final Answer:

The temperature change is mV^2/(5R).

$$\boxed{\text{(C) } \Delta T = \frac{mV^2}{5R}}$$

Quick Tip: The kinetic energy of the bulk motion becomes internal energy; use Cv = 5R/2.

17. If a gas is compressed isothermally then the r.m.s. velocity of the molecules

(A) increases

(B) remains the same

(C) decreases

(D) first increases and then decreases

Correct Answer: (B) remains the same

Solution:

Step 1: Formula

$v_{rms} = \sqrt{\frac{3RT}{M}}$.

Step 2: Isothermal

The temperature is constant during isothermal compression, so v_{rms} does not change. Pressure and density change, but not the speed.

Step 3: Result

The r.m.s. velocity remains the same. Option (B).

Final Answer:

The r.m.s. velocity remains the same.

(B) remains the same

Quick Tip: The r.m.s. speed depends only on temperature.

18. A mass m_1 performs S.H.M. with amplitude A which is connected to horizontal spring. While mass m_1 passing through mean position, another mass m_2 ($m_2 < m_1$) is placed on it so that both masses move together with amplitude A_1. The ratio of A/A_1 is

(A) $\left[\frac{m_1+m_2}{m_1}\right]^{\frac{1}{2}}$

(B) $\left[\frac{m_1}{m_1+m_2}\right]^{\frac{1}{2}}$

(C) $\frac{m_1}{m_1+m_2}$

(D) $\frac{m_2}{m_1+m_2}$

Correct Answer: (A) $\left[\frac{m_1+m_2}{m_1}\right]^{\frac{1}{2}}$

Solution:

Step 1: Before

At the mean position the speed is $v = A\omega_1$, with $\omega_1 = \sqrt{\frac{k}{m_1}}$.

Step 2: Momentum

After placing m_2, $m_1 v = (m_1 + m_2)v'$, so $v' = \frac{m_1 v}{m_1+m_2}$.

Step 3: New amplitude

New angular frequency $\omega_2 = \sqrt{\frac{k}{m_1+m_2}}$ and $A_1 = \frac{v'}{\omega_2}$.

Step 4: Ratio

$A_1 = \frac{m_1}{m_1+m_2} \cdot A\frac{\omega_1}{\omega_2} = \frac{m_1}{m_1+m_2} A\sqrt{\frac{m_1+m_2}{m_1}} = A\sqrt{\frac{m_1}{m_1+m_2}}$. So $\frac{A}{A_1}$ $= \left(\frac{m_1+m_2}{m_1}\right)^{1/2}$. Option (A).

Final Answer:

A/A1 equals root of (m1+m2)/m1.

$$\boxed{\text{(A) } \left[\frac{m_1+m_2}{m_1}\right]^{1/2}}$$

Quick Tip: Momentum is conserved when the mass is dropped on at the mean position.

19. The maximum potential energy of a block executing S.H.M. is E and amplitude of oscillation is 'A'. The kinetic energy of the block at amplitude A/2 is

(A) $\frac{3E}{4}$

(B) E

(C) $\frac{2E}{4}$

(D) $\frac{5E}{4}$

Correct Answer: (A) $\frac{3E}{4}$

Solution:

Step 1: Total energy

The maximum potential energy equals the total energy: $E = \frac{1}{2}kA^2$.

Step 2: PE at A/2

$U = \frac{1}{2}k\left(\frac{A}{2}\right)^2 = \frac{E}{4}$.

Step 3: KE

$K = E - \frac{E}{4} = \frac{3E}{4}$. Option (A).

Final Answer:

The kinetic energy is 3E/4.

$$\boxed{(A)\ \frac{3E}{4}}$$

Quick Tip: Total energy is E; subtract the potential energy at A/2.

20. The time taken by simple pendulum for one oscillation is T on earth's surface. Its time period becomes xT when taken to a height R (equal to earth's radius) above the earth's surface. The value of x is

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) 2

(D) 4

Correct Answer: (C) 2

Solution:

Step 1: g at height R

$g' = g\left(\frac{R}{R+h}\right)^2 = g\left(\frac{R}{2R}\right)^2 = \frac{g}{4}$.

Step 2: Period

$T = 2\pi\sqrt{\frac{l}{g}}$, so $T' = 2\pi\sqrt{\frac{l}{g/4}} = 2T$.

Step 3: Result

$x = 2$. Option (C).

Final Answer:

x = 2.

(C) 2

Quick Tip: The period varies inversely with the square root of g, and g falls to a quarter at distance 2R.

21. An air column in a pipe which is closed at one end will be in resonance with a vibrating tuning fork of frequency 415 Hz for various vibrating air columns. Which one of the following lengths is not in resonance ? (velocity of sound in air = 332 m/s) (Neglect end correction)

(A) 20cm

(B) 40cm

(C) 60cm

(D) 100cm

Correct Answer: (B) 40cm

Solution:

Step 1: Wavelength

$\lambda = \frac{v}{f} = \frac{332}{415} = 0.8$ m, so $\frac{\lambda}{4} = 20$ cm.

Step 2: Condition

Resonance in a closed pipe needs $l = (2n + 1)\frac{\lambda}{4}$, so l $= 20, 60, 100, 140$ cm ....

Step 3: Compare

20, 60 and 100 cm are odd multiples of 20. $40cm$ is an even multiple, so it is not in resonance. Option (B).

Final Answer:

40 cm is not in resonance.

$$\boxed{\text{(B) 40 cm}}$$

Quick Tip: A closed pipe resonates at odd multiples of a quarter wavelength.

22. Two waves $Y_1 = 0.25sin316t$ and $Y_2 = 0.25sin310t$ are propagating along the same direction. The number of beats produced per second are

(A) $\frac{3}{\pi}$

(B) $\frac{\pi}{3}$

(C) $\frac{2}{\pi}$

(D) $\frac{\pi}{2}$

Correct Answer: (A) $\frac{3}{\pi}$

Solution:

Step 1: Frequencies

$\omega = 2\pi f$, so $f_1 = \frac{316}{2\pi}$ and $f_2 = \frac{310}{2\pi}$.

Step 2: Beat frequency

$f_1 - f_2 = \frac{316-310}{2\pi} = \frac{6}{2\pi} = \frac{3}{\pi}$. Option (A).

Final Answer:

The number of beats per second is 3/pi.

$$\boxed{(A)\ \frac{3}{\pi}}$$

Quick Tip: Beat frequency is the difference of the frequencies, and each frequency is omega over 2 pi.

23. When open pipe is closed from one end then third overtone of closed pipe is higher in frequency by 150Hz than second overtone of open pipe. The fundamental frequency of open end pipe will be (Neglect end correction)

(A) 75Hz

(B) 150Hz

(C) 225Hz

(D) 300Hz

Correct Answer: (D) 300Hz

Solution:

Step 1: Open pipe

Let the fundamental of the open pipe be $n_0 = \frac{v}{2L}$. Its second overtone is $3n_0$.

Step 2: Closed pipe

Closing one end gives fundamental $\frac{v}{4L} = \frac{n_0}{2}$. The third overtone is the 7^{th} harmonic: $7 \times \frac{n_0}{2}$.

Step 3: Equation

$\frac{7n_0}{2} - 3n_0 = 150$, so $\frac{n_0}{2} = 150$ and $n_0 = 300$ Hz. Option (D).

Final Answer:

The fundamental of the open pipe is 300 Hz.

$$\boxed{\text{(D) 300 Hz}}$$

Quick Tip: Compare frequencies of the same pipe length, open and closed.

24. A person observes two moving trains. 'A' reaching the station and 'B' leaving the station with equal speed of 30m/s. If both trains emit sounds with frequency 300 Hz, (speed of sound = 330 m/s) difference of frequencies heard by the person will be

(A) 10Hz

(B) 33Hz

(C) 55Hz

(D) 80Hz

Correct Answer: (C) 55Hz

Solution:

Step 1: Approaching train

$f_A = f\frac{v}{v - v_s} = 300 \times \frac{330}{300} = 330$ Hz.

Step 2: Receding train

$f_B = f\frac{v}{v+v_s} = 300 \times \frac{330}{360} = 275$ Hz.

Step 3: Difference

$330 - 275 = 55$ Hz. Option (C).

Final Answer:

The difference is 55 Hz.

(C) 55 Hz

Quick Tip: One train approaches and one recedes, so use the Doppler formula for each.

25. Two spherical hollow spheres of radii 'R_1' and 'R_2' are charged with same charge. Let 'σ_1' and 'σ_2' are their respective charge densities, then the ratio σ_1 to σ_2 is

(A) $R_1 : R_2$

(B) $R_2 : R_1$

(C) $R_2^2 : R_1^2$

(D) $R_1^2 : R_2^2$

Correct Answer: (C) $R_2^2 : R_1^2$

Solution:

Step 1: Formula

$\sigma = \frac{q}{4\pi R^2}$.

Step 2: Ratio

With the same charge, $\frac{\sigma_1}{\sigma_2} = \frac{R_2^2}{R_1^2}$. Option (C).

Final Answer:

The ratio is R2 squared to R1 squared.

$$\boxed{(C)\ R_2^2 : R_1^2}$$

Quick Tip: Surface charge density is charge divided by surface area.

26. An electric dipole of length $0.5\mu m$ is placed with its axis making an angle of $30°$ with uniform electric field $10^4 V/m$. If it experiences a torque of $5 \times 10^{-9} Nm$, the magnitude of the charge on the dipole is $(sin 30° = 0.5)$

(A) $1\mu C$

(B) $1.5\mu C$

(C) $2\mu C$

(D) $2.5\mu C$

Correct Answer: (C) $2\mu C$

Solution:

Step 1: Formula

$\tau = pE\sin\theta = qLE\sin\theta$.

Step 2: Solve for q

$$q = \frac{\tau}{LE\sin\theta} = \frac{5 \times 10^{-9}}{(0.5 \times 10^{-6})(10^4)(0.5)} = \frac{5 \times 10^{-9}}{2.5 \times 10^{-3}}$$

Step 3: Result

$q = 2 \times 10^{-6}$ C $= 2\,\mu$C. Option (C).

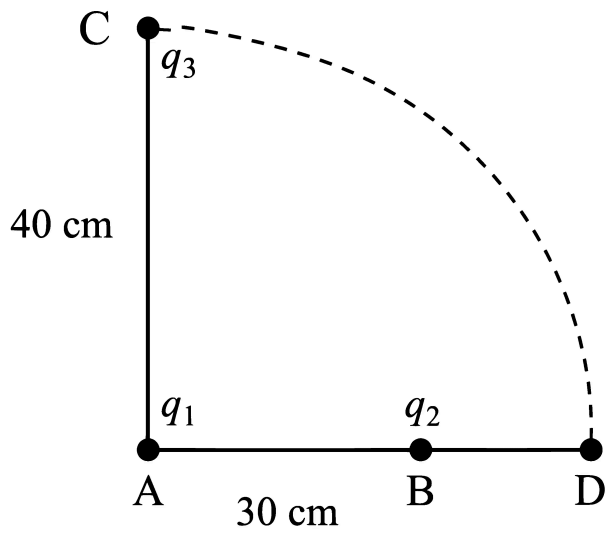
Final Answer:

The charge is 2 microcoulomb.

$$\boxed{\text{(C) } 2\,\mu\text{C}}$$

Quick Tip: Torque is p E sin theta, with dipole moment p equal to q times length.

27. Two charges q_1 and q_2 are separated by a distance of 30cm. A third charge q_3 initially at point C is shown in figure, is moved along the circular path of radius 40 cm from C to D. If the difference in potential energy due to movement of q_3 from C to D is $q_3k/4\pi\varepsilon_0$, the value of K is (ε_0 = permittivity of free space)



(A) $8q_2$

(B) $8q_1$

(C) $6q_2$

(D) $6q_1$

Correct Answer: (A) $8q_2$

Solution:

Step 1: Read the figure

Charge q_1 is at A and the path of q_3 is a circular arc of radius $40\ \text{cm}$ about A. So the distance to q_1 stays $0.4\ \text{m}$, and the energy due to q_1 does not change.

Step 2: Distances from q2

q_2 is at B, $30\ \text{cm}$ from A along AD. At C the distance $BC = \sqrt{0.3^2 + 0.4^2} = 0.5\ \text{m}$. At D the distance $BD = 0.4 - 0.3 = 0.1\ \text{m}$.

Step 3: Energy difference

$$U_D - U_C = \frac{q_3 q_2}{4\pi\varepsilon_0}\left(\frac{1}{0.1} - \frac{1}{0.5}\right) = \frac{q_3}{4\pi\varepsilon_0}(10 - 2)q_2$$

Step 4: Result

Comparing with $\frac{q_3 K}{4\pi\varepsilon_0}$, $K = 8q_2$ (with distances in metres). Option (A).

Final Answer:

The value of K is 8 q2.

$$\boxed{\text{(A) } K = 8q_2}$$

Quick Tip: Only q2 changes its distance to q3; q1 stays at a fixed 40 cm because the path is a circle about q1.

28. A capacitor has capacity C when it's parallel plates are separated by air medium of thickness 'd'. A slab of material of dielectric constant K having area equal to that of plates but thickness $\frac{d}{2}$ is inserted between the plates. Capacitance of the capacitor in the presence of slab will be

(A) KC

(B) 2KC

(C) $\frac{KC}{K+1}$

(D) $\frac{2KC}{K+1}$

Correct Answer: (D) $\frac{2KC}{K+1}$

Solution:

Step 1: Original capacitance

$C = \frac{\varepsilon_0 A}{d}$.

Step 2: Effective separation

With a slab of thickness $\frac{d}{2}$, the effective separation is $\frac{d}{2} + \frac{d}{2K}$ (air part plus slab thickness divided by K).

Step 3: New capacitance

$$C' = \frac{\varepsilon_0 A}{\frac{d}{2}\left(1 + \frac{1}{K}\right)} = \frac{2K}{K+1} \cdot \frac{\varepsilon_0 A}{d} = \frac{2KC}{K+1}$$

Option (D).

Final Answer:

The capacitance becomes 2KC/(K+1).

$$\boxed{\text{(D) } \frac{2KC}{K+1}}$$

Quick Tip: Treat the air gap and the slab as two capacitors in series.

29. A galvanometer has resistance G and range Vg. How much resistance is required to read voltage up to V volt?

(A) GV_g

(B) $G(\frac{V-V_g}{V})$

(C) $G(\frac{V}{V_g}-1)$

(D) $G(\frac{V+V_g}{V})$

Correct Answer: (C) $G(\frac{V}{V_g}-1)$

Solution:

Step 1: Full-scale current

The galvanometer gives full deflection at current $I_g = \frac{V_g}{G}$.

Step 2: Series resistance

For range V: $I_g(G+R) = V$, so $G+R = \frac{VG}{V_g}$.

Step 3: Solve

$$R = G\left(\frac{V}{V_g}-1\right)$$

Option (C).

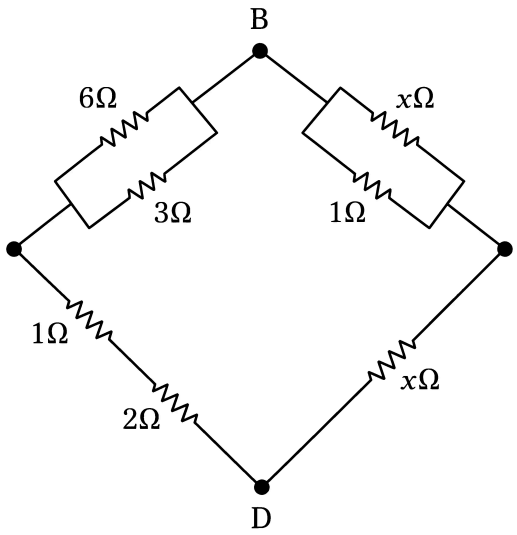
Final Answer:

The required resistance is G(V/Vg - 1).

$$\boxed{(C)\ G\left(\frac{V}{V_g}-1\right)}$$

Quick Tip: The total resistance must give full-scale current at voltage V.

30. If the potential difference between points B and D is zero in the given circuit, the value of x is $\frac{1}{n}\Omega$. The value n is



(A) 2

(B) 1

(C) 3

(D) 4

Correct Answer: (A) 2

Solution:

Step 1: Simplify each arm

Upper left: $6\Omega \parallel 3\Omega = \frac{18}{9} = 2\Omega$. Lower left: $1 + 2 = 3\Omega$. Upper right: $x \parallel 1 = \frac{x}{x+1}$. Lower right: $x\Omega$.

Step 2: Balance condition

For $V_B = V_D$, $\frac{2}{3} = \frac{x/(x+1)}{x} = \frac{1}{x+1}$.

Step 3: Solve

$x + 1 = \frac{3}{2}$, so $x = \frac{1}{2}\Omega = \frac{1}{n}\Omega$, giving $n = 2$. Option (A).

Final Answer:

The value of n is 2.

$$\boxed{\text{(A) 2}}$$

Quick Tip: Zero potential difference between B and D means the bridge ratio holds.

31. The magnetic field intensity (H) at the centre of a long solenoid having 'n' turns per unit length and carrying a current I, when no material is kept in it is (μ_0 = permeability of free space)

(A) n/I

(B) nI

(C) $\mu_0 I$

(D) μ_0/nI

Correct Answer: (B) nI

Solution:

Step 1: Definition

For a long solenoid the magnetic field is $B = \mu_0 nI$. The magnetic intensity is $H = \frac{B}{\mu_0} = nI$, for no material inside.

Step 2: Options

$\frac{n}{I}$ and $\frac{\mu_0}{nI}$ have wrong dimensions, and $\mu_0 I$ is missing n. Option (B).

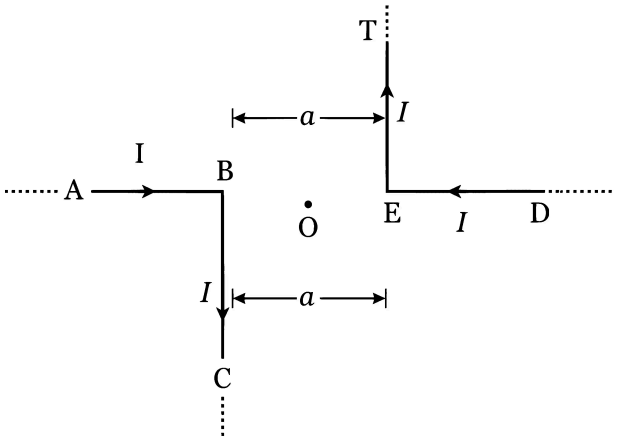
Final Answer:

H equals nI.

$$\boxed{\text{(B) } nI}$$

Quick Tip: H equals turns per unit length times current.

32. The magnitude of magnetic induction at mid point 'O' due to current arrangement as shown in figure will be (μ_0 = permeability of free space)



(A) $\frac{\mu_0 I}{2\pi a}$

(B) zero

(C) $\frac{\mu_0 I}{4\pi a}$

(D) $\frac{\mu_0 I}{\pi a}$

Correct Answer: (D) $\frac{\mu_0 I}{\pi a}$

Solution:

Step 1: Read the figure

Current runs along AB, turns down along BC, and along DE turns up along ET. The vertical wires are separated by a, so each is at distance $\frac{a}{2}$ from O. The horizontal wires lie along the line through O and give no field there.

Step 2: Field of one semi-infinite wire

The foot of the perpendicular from O is at the end of each vertical wire, so each gives $B = \frac{\mu_0 I}{4\pi\left(\frac{a}{2}\right)} = \frac{\mu_0 I}{2\pi a}$.

Step 3: Directions

Wire BC (current downward, to the left of O) and wire ET (current upward, to the right of O) both give a field out of the page at O by the right-hand rule. So the fields add.

Step 4: Total

$B = 2 \times \frac{\mu_0 I}{2\pi a} = \frac{\mu_0 I}{\pi a}$. Option (D).

Final Answer:

The field at O is muo I / (pi a).

$$\boxed{\text{(D) } \frac{\mu_0 I}{\pi a}}$$

Quick Tip: Only the two vertical semi-infinite wires matter at O; the horizontal wires point toward O.

33. A circular loop of radius R is carrying current I. The ratio of magnetic field at the center of circular loop and at a distance R from the center of the loop on its axis is

(A) $1 : \sqrt{2}$

(B) $1 : 3\sqrt{2}$

(C) $\sqrt{8} : 1$

(D) $3\sqrt{2} : 1$

Correct Answer: (C) $\sqrt{8} : 1$

Solution:

Step 1: Centre

$B_c = \frac{\mu_0 I}{2R}$.

Step 2: On axis at x = R

$B_x = \frac{\mu_0 I R^2}{2(R^2+x^2)^{3/2}} = \frac{\mu_0 I R^2}{2(2R^2)^{3/2}} = \frac{\mu_0 I}{4\sqrt{2}R}$.

Step 3: Ratio

$\frac{B_c}{B_x} = \frac{1/2}{1/(4\sqrt{2})} = 2\sqrt{2} = \sqrt{8}$. The ratio is $\sqrt{8} : 1$. Option (C).

Final Answer:

The ratio is root 8 : 1.

$\boxed{(C)\ \sqrt{8}:1}$

Quick Tip: Use the on-axis formula with x equal to R.

34. A circular coil of radius 'r' is placed on another circular coil whose radius is 'R' and the current flowing through it is changing and their centers coincide. ($R \gg r$). If both the coils are coplanar, then the mutual inductance between them is proportional to

(A) $\frac{r}{R}$

(B) $\frac{R}{r}$

(C) $\frac{R^2}{r}$

(D) $\frac{r^2}{R}$

Correct Answer: (D) $\frac{r^2}{R}$

Solution:

Step 1: Field of the big coil

At the centre, $B = \frac{\mu_0 I}{2R}$. It is almost uniform over the tiny area of the small coil.

Step 2: Flux through the small coil

$\Phi = B \cdot \pi r^2 = \frac{\mu_0 I \pi r^2}{2R}$.

Step 3: Mutual inductance

$M = \frac{\Phi}{I} = \frac{\mu_0 \pi r^2}{2R}$, so $M \propto \frac{r^2}{R}$. Option (D).

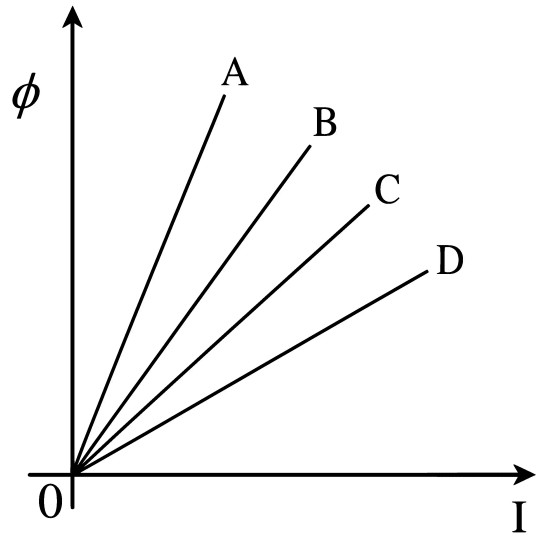
Final Answer:

M is proportional to r squared over R.

$$\boxed{(\text{D})\, M \propto \frac{r^2}{R}}$$

Quick Tip: The field of the big coil at its centre is nearly uniform over the small coil.

35. A graph of magnetic flux (Φ) versus current (I) is shown for four inductors A, B, C, D. The smallest value of self inductance is for inductor.



(A) A

(B) B

(C) C

(D) D

Correct Answer: (D) D

Solution:

Step 1: Relation

$\Phi = LI$, so the slope of each line is L.

Step 2: Compare slopes

From the graph, line A is steepest and D is the least steep. The smallest slope means the smallest L, so it is inductor D. Option (D).

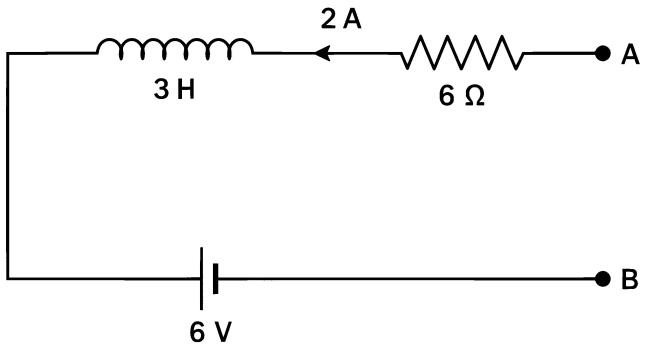
Final Answer:

Inductor D has the smallest self inductance.

$$\boxed{\text{(D) } D}$$

Quick Tip: Self inductance is the slope of the flux versus current line.

36. In the given circuit, if $dI/dt = -1\text{A/s}$ then the value of V_{AB} at this instant will be



(A) 30V

(B) 20V

(C) 10V

(D) 15V

Correct Answer: (D) 15V

Solution:

Step 1: Read the figure

From A the path goes through 6Ω with current 2 A toward the inductor 3 H, then through the battery of 6 V (positive terminal on the long plate, towards the inductor) and on to B.

Step 2: Resistor

Moving along the current, the potential falls by $IR = 2 \times 6 = 12$ V.

Step 3: Inductor

$V_{in} - V_{out} = L\frac{dI}{dt} = 3(-1) = -3$ V, so the potential rises by 3 V.

Step 4: Battery

Going from the positive plate to the negative plate the potential falls by 6 V.

Step 5: Total

$V_B = V_A - 12 + 3 - 6 = V_A - 15$, so $V_{AB} = 15$ V. Option (D).

Final Answer:

V_AB is 15 V.

$$\boxed{\text{(D) } 15 \text{ V}}$$

Quick Tip: Add the drops and rises along the path from A to B, including the induced emf of the inductor.

37. Alternating current of peak value ($\frac{2}{\pi}$) A flows through the primary coil of a transformer. The coefficient of mutual inductance between primary and secondary coils is 1H. The peak value of induced e.m.f. in the secondary coil is
(Frequency of a.c. = 50 Hz)

(A) 100V

(B) 300V

(C) 200V

(D) 400V

Correct Answer: (C) 200V

Solution:

Step 1: Formula
$e = M\frac{dI}{dt}$ with $I = I_0 \sin \omega t$, so the peak emf is $e_0 = M\omega I_0$.

Step 2: Substitute
$\omega = 2\pi f = 100\pi$ rad/s, $I_0 = \frac{2}{\pi}$ A, $M = 1$ H.

Step 3: Result
$e_0 = 1 \times 100\pi \times \frac{2}{\pi} = 200$ V. Option (C).

Final Answer:
The peak induced emf is 200 V.

$$\boxed{\text{(C) 200 V}}$$

Quick Tip: The peak emf equals M times omega times the peak current.

38. Series LCR circuit containing an a.c. source of 100V has a inductor and a capacitor of reactance 24Ω and 16Ω respectively. If a resistance of 6Ω is connected in series, then the potential difference across the series combination of inductor and capacitor only is

(A) 80V

(B) 8V

(C) 40V

(D) 20V

Correct Answer: (A) 80V

Solution:

Step 1: Impedance

Net reactance $X = 24 - 16 = 8\ \Omega$. With $R = 6\ \Omega$, $Z = \sqrt{6^2 + 8^2} = 10\ \Omega$.

Step 2: Current

$I = \frac{100}{10} = 10$ A.

Step 3: Voltage across L and C

The combination has net reactance 8 Ω, so $V = IX = 10 \times 8 = 80$ V.

Option (A).

Final Answer:

The voltage across the LC combination is 80 V.

$$\boxed{\text{(A) 80 V}}$$

Quick Tip: Find the current from the impedance, then multiply by the net reactance.

39. An inductor of inductance 2μH is connected in series with a resistance, a variable capacitor and an a.c. source of 10 kHz. The value of capacitance for which maximum current is drawn in to the circuit is $\frac{1}{x}$F, where the value of x is (Take $\pi^2 = 10$)

(A) 8000

(B) 600

(C) 400

(D) 1600

Correct Answer: (A) 8000

Solution:

Step 1: Resonance

Maximum current occurs when $X_L = X_C$, i.e. $f = \frac{1}{2\pi\sqrt{LC}}$.

Step 2: Solve for C

$C = \frac{1}{4\pi^2 f^2 L}$.

Step 3: Numbers

With $\pi^2 = 10$, $f = 10^4$ Hz and $L = 2 \times 10^{-6}$ H: $4\pi^2 f^2 L = 40 \times 10^8 \times 2 \times 10^{-6} = 8000$.

Step 4: Result

$C = \frac{1}{8000}$ F, so $x = 8000$. Option (A).

Final Answer:

x is 8000.

(A) 8000

Quick Tip: Maximum current needs resonance, so the frequency equals 1 over 2 pi root LC.

40. A ray of light passes from the vacuum into a medium of refractive index 'n'. If the angle of incidence is twice the angle of refraction, then the angle of incidence in terms of refractive index is

(A) $sin^{-1}(\frac{n}{2})$

(B) $2cos^{-1}(\frac{n}{2})$

(C) $2sin^{-1}(\frac{n}{2})$

(D) $cos^{-1}(\frac{n}{2})$

Correct Answer: (B) $2cos^{-1}(\frac{n}{2})$

Solution:

Step 1: Snell law

$\frac{\sin i}{\sin r} = n$ with $i = 2r$.

Step 2: Double angle

$\frac{\sin 2r}{\sin r} = 2\cos r = n$, so $\cos r = \frac{n}{2}$ and $r = \cos^{-1}\frac{n}{2}$.

Step 3: Incidence angle

$i = 2r = 2\cos^{-1}\left(\frac{n}{2}\right)$. Option (B).

Final Answer:

The angle of incidence is 2 cos^-1 (n/2).

$$\boxed{\text{(B) } 2\cos^{-1}\left(\frac{n}{2}\right)}$$

Quick Tip: Use Snell law and the double angle formula for the sine.

41. A convex lens having refractive index 1.6 has focal length 12 cm, when it is in air. The focal length of that lens when placed in water is (refractive index of water = 1.28)

(A) 655mm

(B) 288mm

(C) 555mm

(D) 355mm

Correct Answer: (B) 288mm

Solution:

Step 1: Lens maker formula

In a medium, $\frac{1}{f} = \left(\frac{n_l}{n_m} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$.

Step 2: Air and water

In air, $\frac{1}{12} = (1.6 - 1)K = 0.6K$. In water, $\frac{1}{f_w} = \left(\frac{1.6}{1.28} - 1\right)K = 0.25K$.

Step 3: Ratio

$\frac{f_w}{12} = \frac{0.6}{0.25} = 2.4$, so $f_w = 28.8\ \text{cm} = 288\ \text{mm}$. Option (B).

Final Answer:

The focal length in water is 288 mm.

(B) 288 mm

Quick Tip: Use the lens maker formula in both media and take the ratio.

42. Three identical polaroids P_1, P_2 and P_3 are placed one after another. The pass axis of P_2 and P_3 are inclined at angle of $60°$ and $90°$ with respect to axis of P_1. The source has an intensity I_0. The intensity of light finally coming out is

(A) $\frac{I_0}{2}cos^2 30° cos^2 90°$

(B) $\frac{I_0}{2}cos^2 60° cos^2 30°$

(C) $I_0 cos^2 60° cos^2 30°$

(D) zero

Correct Answer: (B) $\frac{I_0}{2}cos^2 60° cos^2 30°$

Solution:

Step 1: First polaroid

Unpolarised light of intensity I_0 becomes $\frac{I_0}{2}$ after P_1.

Step 2: Second polaroid

The axis of P_2 is $60°$ to P_1, so the intensity is $\frac{I_0}{2}\cos^2 60°$.

Step 3: Third polaroid

P_3 is at $90°$ to P_1, which is $30°$ to P_2. So the final intensity is $\frac{I_0}{2}\cos^2 60° \cos^2 30°$. Option (B).

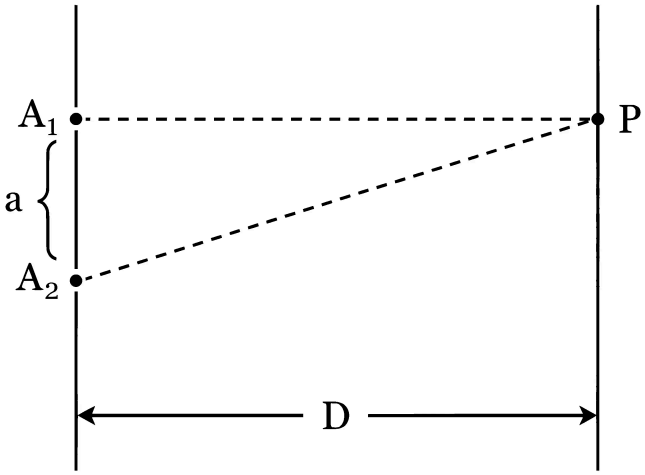
Final Answer:

The final intensity is (I0/2) cos^2 60 cos^2 30.

$$\boxed{\text{(B) } \frac{I_0}{2}\cos^2 60^\circ \cos^2 30^\circ}$$

Quick Tip: The first polaroid halves unpolarised light; Malus law applies after that.

43. In Young's double slit experiment, two slits are illuminated with a light of wavelength λ. The line joining A_1P is perpendicular to A_1A_2 as shown in figure. If the first minimum is detected at P, the value of slits separation 'a' will be
(D = distance between source and screen)



(A) λD
(B) $\sqrt{\lambda D}$
(C) $\sqrt{\frac{\lambda}{D}}$
(D) $\sqrt{\frac{D}{\lambda}}$

Correct Answer: (B) $\sqrt{\lambda D}$

Solution:

Step 1: Geometry

From the figure, $A_1P = D$ (perpendicular to the slit line) and $A_2P = \sqrt{D^2 + a^2}$.

Step 2: Condition for first minimum

Path difference $= \frac{\lambda}{2}$: $\sqrt{D^2 + a^2} - D = \frac{\lambda}{2}$.

Step 3: Solve

$D^2 + a^2 = D^2 + D\lambda + \frac{\lambda^2}{4}$, so $a^2 = D\lambda + \frac{\lambda^2}{4}$. Since $\lambda \ll D$, $a^2 \approx \lambda D$ and $a = \sqrt{\lambda D}$. Option (B).

Final Answer:

The slit separation is root of lambda D.

$$\boxed{\text{(B) } a = \sqrt{\lambda D}}$$

Quick Tip: The path difference at the first minimum is half a wavelength.

44. When a light of wavelength 'λ' falls on the emitter of a photosensitive surface, maximum speed of emitted photoelectrons is 'V'. If the incident wavelength is changed to '$2\lambda/3$', maximum speed of emitted photoelectrons will be

(A) less than $V(1.5)^{\frac{1}{2}}$

(B) greater than $V(1.5)^{\frac{1}{2}}$

(C) less than V

(D) less than $\frac{V}{2}$

Correct Answer: (B) greater than $V(1.5)^{\frac{1}{2}}$

Solution:

Step 1: First case

$\frac{1}{2}mV^2 = \frac{hc}{\lambda} - \phi$.

Step 2: Second case

The new wavelength is $\frac{2\lambda}{3}$, so the photon energy is $\frac{3hc}{2\lambda} = 1.5\frac{hc}{\lambda}$. Then $\frac{1}{2}mV'^2 = 1.5\frac{hc}{\lambda} - \phi$.

Step 3: Compare

Substitute $\frac{hc}{\lambda} = \frac{1}{2}mV^2 + \phi$: $\frac{1}{2}mV'^2 = 1.5 \cdot \frac{1}{2}mV^2 + 0.5\phi$. Because $\phi > 0$, $V'^2 > 1.5V^2$.

Step 4: Result

So the speed is greater than $V(1.5)^{1/2}$. Option (B).

Final Answer:

The new speed is greater than V times root 1.5.

$$\boxed{\text{(B) greater than } V(1.5)^{1/2}}$$

Quick Tip: Use Einstein photoelectric equation and keep the work function in the equation.

45. A photoemissive substance is illuminated with a radiation of wavelength λ_i so that it releases electrons with de-Broglie wavelength λ_e. The longest wavelength of radiation that can emit photoelectron is λ_0. Expression for de-Broglie wavelength is (m = mass of electron, h = Planck's constant, C = Speed of light)

(A) $(h\lambda_i/2mc)^{\frac{1}{2}}$

(B) $(h\lambda_0/2mc)^{\frac{1}{2}}$

(C) $[h/2mc(\frac{1}{\lambda_i} - \frac{1}{\lambda_0})]^{\frac{1}{2}}$

(D) $[h/[2mc(\frac{1}{\lambda_i} - \frac{1}{\lambda_0})]^{\frac{1}{2}}$

Correct Answer: (C) $[h/2mc(\frac{1}{\lambda_i} - \frac{1}{\lambda_0})]^{\frac{1}{2}}$

Solution:

Step 1: Kinetic energy

Work function $\phi = \frac{hc}{\lambda_0}$. So $K = \frac{hc}{\lambda_i} - \frac{hc}{\lambda_0} = hc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_0}\right)$.

Step 2: de Broglie wavelength

$\lambda_e = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mhc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_0}\right)}}$.

Step 3: Simplify

$$\lambda_e = \left[\frac{h}{2mc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_0}\right)}\right]^{1/2}$$

This matches option (C).

Final Answer:

The expression is option C.

$$\boxed{\text{(C)}\ \left[\frac{h}{2mc\left(\frac{1}{\lambda_i} - \frac{1}{\lambda_0}\right)}\right]^{1/2}}$$

Quick Tip: Kinetic energy is hc(1/lambda_i minus 1/lambda_0); then use

lambda = h over root(2mK).

46. An electron makes a transition from an excited state to the ground state of a hydrogen like atom. Out of the following statements which one is correct ?

(A) K.E, P. E. and T.E. decreases

(B) K.E. increases but P. E. and T.E. decreases

(C) K.E. and T.E. decreases but P.E. increases

(D) K.E. decreases, P.E. increases but T.E. remains the same

(A) A

(B) B

(C) C

(D) D

Correct Answer: (B) B

Solution:

Step 1: Energies in an orbit

$K = -E_n$, $U = 2E_n$, $E_n = -\frac{13.6Z^2}{n^2}$ eV.

Step 2: Transition to the ground state

The electron goes to a lower orbit, so E_n becomes more negative: total energy decreases.

Step 3: Effect on KE and PE

$K = -E_n$ increases. $U = 2E_n$ becomes more negative, so PE decreases.

Step 4: Result

KE increases while PE and TE decrease. Statement (B). Statement (A)

is wrong since KE rises.

Final Answer:

KE increases while PE and TE decrease.

$$\boxed{\text{(B) KE increases, PE and TE decrease}}$$

Quick Tip: In a Bohr orbit KE is the negative of total energy and PE is twice the total energy.

47. The ratio of the density of oxygen nucleus ($^{16}_{8}O$) and helium nucleus ($^{4}_{2}He$) is

(A) 1 : 1

(B) 2 : 1

(C) 4 : 1

(D) 8 : 1

Correct Answer: (A) 1 : 1

Solution:

Step 1: Nuclear radius

$R = R_0 A^{1/3}$, so the volume is proportional to A.

Step 2: Density

Mass is proportional to A as well, so density $\propto \frac{A}{A}$ is constant for all nuclei. The ratio is 1 : 1. Option (A).

Final Answer:

The densities are equal, ratio 1:1.

$$\boxed{\text{(A) } 1:1}$$

Quick Tip: Nuclear density does not depend on the mass number.

48. Electrical conductivity of insulators is

(A) exactly zero

(B) extremely large

(C) extremely small

(D) sometimes small and sometimes large

Correct Answer: (C) extremely small

Solution:

Step 1: Band picture

In insulators the energy gap between the valence and the conduction band is large, so almost no electrons reach the conduction band.

Step 2: Result

Conductivity is extremely small, but not exactly zero. Option (C).

Final Answer:

Conductivity of insulators is extremely small.

$$\boxed{\text{(C) extremely small}}$$

Quick Tip: Insulators have a very large energy gap, so very few carriers are available.

49. Which of the following statement is NOT correct in the case of LED?
(A) It is a heavily doped p-n junction diode.
(B) It emits light only when it is forward biased.
(C) It emits light only when it is reverse biased.
(D) The energy of the light emitted is equal to or slightly less than the energy gap of the semiconductor used.

(A) A

(B) B

(C) C

(D) D

Correct Answer: (C) C

Solution:

Step 1: Facts
An LED is a heavily doped p-n junction. It emits light when forward biased, because electron-hole recombination releases photons. The photon energy is close to, and a little less than, the band gap.

Step 2: Check
Statements (A), (B) and (D) are correct. Statement (C) says it emits light only when reverse biased, which is wrong. Option (C).

Final Answer:
Statement C is not correct.

(C) (C)

Quick Tip: An LED emits light only under forward bias.

50. In which of the logic gate the following statement is true? "The output is high when either of the inputs is high but not if both inputs are high"

(A) NAND

(B) NOR

(C) X-OR

(D) OR

Correct Answer: (C) X-OR

Solution:

Step 1: Truth table

The output is high when either input is high but low when both are high. For inputs $(0, 0)$: 0. $(0, 1)$: 1. $(1, 0)$: 1. $(1, 1)$: 0.

Step 2: Gate

This is the XOR gate, $Y = A\bar{B} + \bar{A}B$. OR gives 1 for $(1, 1)$, so it fails. NAND gives 1 for $(0, 0)$, and NOR gives 1 for $(0, 0)$. Option (C).

Final Answer:

The gate is XOR.

(C) X-OR

Quick Tip: This is the exclusive OR behaviour.

Chemistry

Questions 51 to 100 | 1 mark each | 50 marks

51. The ratio of mass of 'x' atoms of an element to mass of carbon atoms is 9:1. Find the mass of 1 mol of x if molar mass of carbon is 12 g/mol.

(A) 54g

(B) 108g

(C) 180g

(D) 216g

Correct Answer: (B) 108g

Solution:

Step 1: Understand the data

The question gives the ratio of the mass of an atom of x to the mass of a carbon atom as $9 : 1$.

So each atom of x is nine times as heavy as one carbon atom.

The molar mass of carbon is given as $12\ \text{g/mol}$.

Step 2: Use the ratio

Mass of one mole of atoms is proportional to the mass of one atom, because one mole always has the same number of atoms (Avogadro number).

So molar mass of x divided by molar mass of carbon equals $9/1$.

Step 3: Calculate

$$M_x = 9 \times 12 = 108\ \text{g/mol}$$

So 1 mol of x weighs 108 g.

Step 4: Check the options

54 g would come from a ratio of 4.5 : 1, and 180 g or 216 g would need ratios of 15 : 1 and 18 : 1. Only 108 g matches 9 : 1.

Final Answer:

One mole of x has a mass nine times that of one mole of carbon.

(B) 108 g

Quick Tip: Multiply the ratio by the molar mass of carbon taken as the reference for one atom.

52. What is the number of unpaired electrons present in ground state of Silicon and Chromium, respectively ?

(A) 2 and 4

(B) 2 and 6

(C) 0 and 6

(D) 4 and 5

Correct Answer: (B) 2 and 6

Solution:

Step 1: Write the configuration of Silicon

Silicon has atomic number 14, so its configuration is $1s^2 2s^2 2p^6 3s^2 3p^2$. The two $3p$ electrons occupy different orbitals with parallel spins

(Hund's rule).
So silicon has 2 unpaired electrons.

Step 2: Write the configuration of Chromium
Chromium has atomic number 24. The expected $[Ar]3d^4 4s^2$ is not the real one.
A half-filled $3d$ subshell is extra stable, so one $4s$ electron moves into $3d$.
The ground state is $[Ar]3d^5 4s^1$.

Step 3: Count the unpaired electrons in Chromium
The five $3d$ orbitals each hold one electron and the $4s$ orbital holds one electron.
That gives $5 + 1 = 6$ unpaired electrons.

Step 4: Compare with the options
Silicon has 2 and chromium has 6, which is option (B). Option (A) uses 4 for chromium, which follows the wrong configuration $3d^4 4s^2$. Option (C) gives silicon 0, which would need both $3p$ electrons paired. Option (D) gives silicon 4, which ignores that $3s$ is full.

Final Answer:
Silicon has 2 unpaired electrons and chromium has 6.

(B) 2 and 6

Quick Tip: Write the ground state configurations; Cr is an exception with a half-filled 3d and 4s.

53. Identify the most electronegative element by the following.

(A) Oxygen

(B) Nitrogen

(C) Fluorine

(D) Chlorine

Correct Answer: (C) Fluorine

Solution:

Step 1: Recall the trend

Electronegativity is the ability of an atom to attract the shared electrons in a bond. It increases from left to right in a period and decreases down a group.

Step 2: Compare the elements

Oxygen and nitrogen are in period 2 but are to the left of fluorine. Chlorine is in the same group as fluorine but lower down, so it is bigger and attracts electrons less.

On the Pauling scale: F = 4.0, O = 3.5, Cl = 3.0, N = 3.0.

Fluorine is the highest.

Step 3: Why the others are wrong

Oxygen and nitrogen have lower nuclear pull on the bonding pair than fluorine because they have fewer protons in the same shell. Chlorine has an extra shell, which weakens the pull.

Final Answer:

Fluorine is the most electronegative element of all.

$$\boxed{\text{(C) Fluorine}}$$

Quick Tip: Electronegativity rises across a period and falls down a group.

54. What is the total number of electrons present in bonding and antibonding orbitals respectively in F_2 molecule according to MO theory ?

(A) Bonding - 8, Antibonding - 10

(B) Bonding - 6, Antibonding - 12

(C) Bonding - 10, Antibonding - 8

(D) Bonding - 12, Antibonding - 6

Correct Answer: (C) Bonding - 10, Antibonding - 8

Solution:

Step 1: Count electrons

Each fluorine atom has 9 electrons, so F_2 has 18 electrons.

For F_2 the MO energy order is $\sigma 1s < \sigma^* 1s < \sigma 2s < \sigma^* 2s < \sigma 2p_z < \pi 2p_x = \pi 2p_y < \pi^* 2p_x = \pi^* 2p_y < \sigma^* 2p_z$.

Step 2: Fill the orbitals

$\sigma 1s^2, \sigma^* 1s^2, \sigma 2s^2, \sigma^* 2s^2, \sigma 2p_z^2, \pi 2p_x^2 \pi 2p_y^2, \pi^* 2p_x^2 \pi^* 2p_y^2$.

Step 3: Count

Bonding: $2 + 2 + 2 + 4 = 10$ electrons. Antibonding: $2 + 2 + 4 = 8$ electrons.

Check: $10 + 8 = 18$.

Step 4: Bond order

$\frac{10-8}{2} = 1$, which agrees with the single bond in fluorine. Option (C) is correct. The other options do not add to the correct pattern or do not

give bond order 1.

Final Answer:

Bonding electrons are 10 and antibonding electrons are 8.

(C) Bonding 10, Antibonding 8

Quick Tip: Fill 18 electrons into the MO energy order and count those in bonding and antibonding orbitals.

55. At constant temperature, a gas occupies a volume of 200 mL at a pressure of 500 mm Hg. What will be its volume at 800 mm Hg pressure?

(A) 280mL

(B) 125mL

(C) 100mL

(D) 350mL

Correct Answer: (B) 125mL

Solution:

Step 1: Identify the law

At constant temperature the volume of a fixed amount of gas is inversely proportional to its pressure. This is Boyle's law: $P_1V_1 = P_2V_2$.

Step 2: Substitute

$P_1 = 500$ mm Hg, $V_1 = 200$ mL, $P_2 = 800$ mm Hg.

$$V_2 = \frac{500 \times 200}{800} = \frac{100000}{800} = 125 \text{ mL}$$

Step 3: Check

Pressure has increased, so volume must fall below 200 mL. Only 125 mL and 100 mL are lower, and 100 mL would need a pressure of 1000 mm Hg. The values 280 and 350 mL are higher than 200 mL, so they are impossible.

Final Answer:

The gas occupies 125 mL at 800 mm Hg.

$$\boxed{\text{(B) 125 mL}}$$

Quick Tip: At constant temperature, pressure times volume stays constant.

56. Which of the following statements is not correct ?

(A) ΔG° is an extensive property.

(B) E° cell is an intensive property.

(C) For a chemical reaction to be spontaneous ΔG must be $-$ve.

(D) Electrical work is equal to ΔG

Correct Answer: (D) Electrical work is equal to ΔG

Solution:

Step 1: Recall the relations

The standard Gibbs energy and cell potential are linked by ΔG°

$= -nFE^\circ_{cell}$. The maximum electrical work obtained from a cell equals $-\Delta G$, not $+\Delta G$.

Step 2: Check statement (A)

ΔG° depends on the number of moles reacting, so it is extensive. So (A) is TRUE.

Step 3: Check statement (B)

E°_{cell} does not change when we multiply the cell reaction by a factor, because n in the equation changes with it. So it is intensive and (B) is TRUE.

Step 4: Check statement (C)

A process is spontaneous at constant temperature and pressure when $\Delta G < 0$. So (C) is TRUE.

Step 5: Check statement (D)

Electrical work done by the cell is $W_{elec} = -\Delta G$, so it is the negative of ΔG, not equal to it. So (D) is FALSE.

Final Answer:

Statement D is not correct, so D is the answer.

$$\boxed{\text{(D) } D}$$

Quick Tip: Gibbs energy depends on the amount of substance, while cell potential does not. Electrical work equals minus delta G.

57. The heat of combustion of carbon to CO_2 is $-393.5 kJ/mol$. The heat released upon formation of 35.2 g of CO_2 from carbon and oxygen gas is

(A) $+315 kJ$

(B) $-315 kJ$

(C) $-3.15 kJ$

(D) $-630 kJ$

Correct Answer: (B) $-315 kJ$

Solution:

Step 1: Understand the data

Burning one mole of carbon to CO_2 releases 393.5 kJ, written as $\Delta H = -393.5 \text{ kJ/mol}$. Forming CO_2 from carbon and oxygen is the same process, so the same value applies per mole of CO_2.

Step 2: Find the moles of CO2

Molar mass of CO_2 is $12 + 32 = 44 \text{ g/mol}$.

$$n = \frac{35.2}{44} = 0.8 \text{ mol}$$

Step 3: Find the heat

$$q = 0.8 \times (-393.5) = -314.8 \approx -315 \text{ kJ}$$

The negative sign shows that heat is released.

Step 4: Check the options

Option (A) has the wrong sign because formation by combustion is exothermic. Option (C) is off by a factor of 100. Option (D) would

correspond to 1.6 mol of CO_2, which is 70.4 g.

Final Answer:

Heat released is about 315 kJ, written as minus 315 kJ.

$$\boxed{\text{(B) } -315 \text{ kJ}}$$

Quick Tip: Find the moles of CO2 and multiply by the heat of combustion per mole.

58. Calculate the enthalpy change when 6 g. CO(g) reacts with sufficient $NO_2(g)$ according to the following reaction

$$4\,CO(g) + 2\,NO_2(g) \rightarrow 4\,CO_2(g) + 2N_2(g)\,;\Delta_r H^0 = -1200 kJ$$

(A) -16.08kJ.

(B) -32.15kJ.

(C) -64.29kJ

(D) -128.58kJ.

Correct Answer: (C) -64.29kJ

Solution:

Step 1: Read the reaction

The equation shows 4 mol of $CO(g)$ reacting, with $\Delta_r H^0 = -1200$ kJ.

So the enthalpy change per mole of CO is $\frac{-1200}{4} = -300$ kJ.

Step 2: Moles of CO in 6 g

Molar mass of CO is 28 g/mol.

$$n = \frac{6}{28} = 0.2143 \text{ mol}$$

Step 3: Enthalpy for 6 g

$$\Delta H = 0.2143 \times (-300) = -64.29 \text{ kJ}$$

Step 4: Check the options

Option (D) is double this value and (B) is half of it. Option (A) would result from dividing by 4 twice. Only -64.29 kJ matches.

Final Answer:

The enthalpy change is -64.29 kJ.

$$\boxed{\text{(C) } -64.29 \text{ kJ}}$$

Quick Tip: The given enthalpy is for 4 mol of CO, so scale it to the moles in 6 g.

59. Identify the process that proceeds with no heat exchange between system and surrounding.

(A) Isothermal

(B) Isobaric

(C) Isochoric

(D) Adiabatic

Correct Answer: (D) Adiabatic

Solution:

Step 1: Define each process

Isothermal means constant temperature, isobaric means constant pressure and isochoric means constant volume. Adiabatic means no heat enters or leaves the system.

Step 2: Match

In an adiabatic process $q = 0$, so any change in internal energy equals the work done, $\Delta U = w$.

In isothermal, isobaric and isochoric processes heat is usually exchanged to hold temperature, pressure or volume constant, so these are not the answer.

Final Answer:

The process with no heat exchange is adiabatic.

$$\boxed{\text{(D) Adiabatic}}$$

Quick Tip: Zero heat exchange means q equals zero.

60. The solubility product of sparingly soluble salt BA is 4×10^{-13}. Calculate the $[A^-]$ if $[B^+]$ is 1×10^{-6}M.

(A) 2×10^{-7}M

(B) 3×10^{-7}M

(C) 4×10^{-7}M

(D) 5×10^{-7}M

Correct Answer: (C) 4×10^{-7}M

Solution:

Step 1: Write the Ksp expression

The salt dissociates as $BA \rightleftharpoons B^+ + A^-$. So $K_{sp} = [B^+][A^-]$.

Step 2: Substitute

$$[A^-] = \frac{K_{sp}}{[B^+]} = \frac{4 \times 10^{-13}}{1 \times 10^{-6}} = 4 \times 10^{-7}\ \text{M}$$

Step 3: Check

Other options such as 2×10^{-7} would need $K_{sp} = 2 \times 10^{-13}$, which does not match the given data.

Final Answer:

The concentration of A- is 4 x 10^-7 M.

$$\boxed{(C)\ 4 \times 10^{-7}\ \text{M}}$$

Quick Tip: For BA, Ksp is the product of the two ion concentrations.

61. Which of the following salt undergoes anionic hydrolysis ?

(A) $CuSO_4$

(B) Na_2CO_3

(C) NH_4Cl

(D) $FeCl_3$

Correct Answer: (B) Na_2CO_3

Solution:

Step 1: Recall the idea

Anionic hydrolysis means the anion of the salt reacts with water and gives OH^-. This happens when the anion comes from a weak acid and the cation comes from a strong base. The solution becomes basic.

Step 2: Check each salt

Na_2CO_3 is made from strong base NaOH and weak acid H_2CO_3. The carbonate ion hydrolyses: $CO_3^{2-} + H_2O \rightleftharpoons HCO_3^- + OH^-$.

$CuSO_4$, NH_4Cl and $FeCl_3$ contain weak-base cations (Cu^{2+}, NH_4^+, Fe^{3+}) and anions of strong acids, so they undergo cationic hydrolysis and give acidic solutions.

Final Answer:

Only Na2CO3 undergoes anionic hydrolysis.

$$\boxed{\text{(B) } Na_2CO_3}$$

Quick Tip: Anionic hydrolysis happens for salts of a strong base and a weak acid.

62. Identify from the following salts an example of a salt of strong acid and weak base.

(A) Ammonium acetate.

(B) Sodium acetate.

(C) Copper (II) chloride.

(D) Sodium carbonate.

Correct Answer: (C) Copper (II) chloride.

Solution:

Step 1: Check each salt

Ammonium acetate comes from a weak base and a weak acid. Sodium acetate comes from a strong base and a weak acid. Sodium carbonate comes from a strong base and a weak acid.

Copper(II) chloride, $CuCl_2$, comes from HCl, a strong acid, and $Cu(OH)_2$, a weak base.

Step 2: Conclusion

So copper(II) chloride is the salt of a strong acid and a weak base. Its solution is acidic because Cu^{2+} hydrolyses.

Final Answer:

Copper(II) chloride is the required salt.

(C) Copper (II) chloride

Quick Tip: Look for a chloride or sulphate of a metal whose hydroxide is weak.

63. What is oxidation state of chlorine in chlorous acid?

(A) +2

(B) +4

(C) +3

(D) +7

Correct Answer: (C) +3

Solution:

Step 1: Write the formula

Chlorous acid is $HClO_2$. Hydrogen is +1 and oxygen is −2.

Step 2: Solve

Let the oxidation state of chlorine be x. Sum of oxidation numbers is zero:

$$(+1) + x + 2(-2) = 0$$

$$x = +3$$

Step 3: Check the options

+2 is not found in chlorine oxoacids. +4 is in ClO_2. +7 is in perchloric acid $HClO_4$, where $1 + x - 8 = 0$.

Final Answer:

Chlorine has oxidation state +3 in chlorous acid.

$$\boxed{\text{(C) } +3}$$

Quick Tip: Write the formula HClO2 and balance charges to zero.

64. Which of the following phenomena represents oxidation?

(A) Removal of oxygen

(B) Addition of electronegative element

(C) Addition of electropositive element

(D) Gain of electron

Correct Answer: (B) Addition of electronegative element

Solution:

Step 1: Recall the classical definition

Oxidation is the addition of oxygen or any electronegative element to a substance, or the removal of hydrogen or an electropositive element from it.

Step 2: Check the options

(A) Removal of oxygen is reduction. (B) Addition of an electronegative element is oxidation, for example $Mg + Cl_2 \rightarrow MgCl_2$. (C) Addition of an electropositive element is reduction. (D) Gain of electrons is reduction, in the electronic concept.

Final Answer:

Addition of an electronegative element is oxidation.

(B) Addition of electronegative element

Quick Tip: Oxidation is addition of oxygen or an electronegative element, or removal of hydrogen or electropositive element.

65. What is the role of glycerol in the decomposition of hydrogen peroxide?

(A) It increases the reaction rate.

(B) It reduces the reaction rate.

(C) It increases the enthalpy of the reaction.

(D) It increases the rate of collisions between reactant molecules.

Correct Answer: (B) It reduces the reaction rate.

Solution:

Step 1: Understand the situation

Hydrogen peroxide decomposes slowly into water and oxygen: $2H_2O_2 \rightarrow 2H_2O + O_2$. Traces of metals, light and rough surfaces speed up this decomposition.

Step 2: Role of glycerol

Glycerol is added as a negative catalyst (a stabiliser). It slows the decomposition and helps in storing the solution. So it reduces the reaction rate.

Step 3: Other options

A catalyst does not change the enthalpy of the reaction, so (C) is wrong. Glycerol does not increase rate or collision frequency, so (A) and (D) are wrong.

Final Answer:

Glycerol reduces the rate of decomposition of hydrogen peroxide.

(B) It reduces the reaction rate

Quick Tip: Glycerol is added to hydrogen peroxide as a stabiliser, which slows its decomposition.

66. Which of the following statements is NOT correct regarding dihydrogen?

(A) It is colorless and tasteless gas.

(B) It burns with a pale blue flame.

(C) It is a polar water-soluble gas.

(D) It is lighter than air.

Correct Answer: (C) It is a polar water-soluble gas.

Solution:

Step 1: Recall the properties

Dihydrogen is colourless, odourless and tasteless. It is the lightest gas, burns with a pale blue flame, and is a nonpolar molecule with very low solubility in water.

Step 2: Check the options

(A) is correct. (B) is correct. (D) is correct because it is lighter than air. (C) says it is a polar water-soluble gas, which is wrong, since H_2 is nonpolar and hardly dissolves.

Final Answer:

Option C is the incorrect statement.

(C) It is a polar water-soluble gas

Quick Tip: Dihydrogen is a nonpolar molecule and is almost insoluble in water.

67. Which of following has the highest nucleophilicity ?

(A) F^-

(B) OH^-

(C) CH_3^-

(D) NH_2^-

Correct Answer: (C) CH_3^-

Solution:

Step 1: Compare the species

All four have the attacking atom in period 2: C, N, O, F. The attacking atom holds its lone pair more tightly as electronegativity rises.

Step 2: Rank

Electronegativity order is C < N < O < F. Nucleophilicity runs in the opposite order, so $CH_3^- > NH_2^- > OH^- > F^-$.

Step 3: Conclusion

CH_3^- has the highest nucleophilicity because carbon is least electronegative and shares its pair most readily, so it is option (C).

Final Answer:

CH3- is the strongest nucleophile.

$$\boxed{\text{(C) } CH_3^-}$$

Quick Tip: Within a period, nucleophilicity falls as electronegativity of the attacking atom rises.

68. Identify the catalyst used in the Friedel crafts acylation reaction.

(A) Anhydrous $AlCl_3$

(B) Anhydrous $FeCl_3$

(C) $NaNO_2$

(D) Alcoholic KOH

Correct Answer: (A) Anhydrous $AlCl_3$

Solution:

Step 1: Recall the reaction

Friedel-Crafts acylation introduces an acyl group onto a benzene ring using an acid chloride or anhydride.

Step 2: Catalyst

Anhydrous $AlCl_3$ is a Lewis acid. It helps generate the acylium ion RCO^+ from $RCOCl$, which attacks the ring.

$FeCl_3$ is used in halogenation. $NaNO_2$ is used in diazotisation.

Alcoholic KOH is used for elimination.

Final Answer:

Anhydrous AlCl3 is the catalyst.

$$\boxed{\text{(A) Anhydrous } AlCl_3}$$

Quick Tip: A strong Lewis acid forms the acylium ion.

69. Identify the use of Baeyer's reagent by the following.

(A) Detecting the presence of carbon carbon double bond.

(B) Polymerisation

(C) Oxidation

(D) Reduction

Correct Answer: (A) Detecting the presence of carbon carbon double bond.

Solution:

Step 1: What it is

Baeyer's reagent is a cold, dilute, alkaline solution of $KMnO_4$.

Step 2: Use

It is decolourised (pink to brown ppt) by compounds with a carbon-carbon double or triple bond. This is the test for unsaturation. Polymerisation, general oxidation and reduction are not the specific use asked here.

Final Answer:

It detects a carbon-carbon double bond.

(A) Detecting C=C bond

Quick Tip: Baeyer reagent is cold dilute alkaline $KMnO_4$, used in the test for unsaturation.

70. What is the total contribution of all corner particles in bcc unit cell?

(A) 1 particle

(B) 2 particles

(C) 3 particles

(D) 4 particles

Correct Answer: (A) 1 particle

Solution:

Step 1: Count corner atoms

A cubic cell has 8 corners. Each corner atom is shared among 8 unit cells, so each contributes 1/8.

Step 2: Calculate

$$8 \times \frac{1}{8} = 1$$

Step 3: Note

The bcc cell has one extra atom at the body centre, giving two atoms per cell. The corners alone contribute 1.

Final Answer:

Corner atoms contribute 1 particle.

(A) 1 particle

Quick Tip: Each corner is shared by eight cells.

71. When silver crystallizes, it forms face centered cubic cells, if the volume of unit cells $6.84 \times 10^{-23} cm^3$. Calculate the density of silver. (Molar mass of silver is 108 g/mol, $N_A = 6.022 \times 10^{23}$)

(A) $12.49gm\ cm^{-3}$

(B) $10.49gm\ cm^{-3}$

(C) $16.89gm\ cm^{-3}$

(D) $20.49gm\ cm^{-3}$

Correct Answer: (B) $10.49gm\ cm^{-3}$

Solution:

Step 1: Write the formula

For a cubic cell, density is $\rho = \frac{ZM}{N_A V}$, where Z is atoms per cell, M is molar mass and V is cell volume.

Step 2: Find Z

An fcc cell has $8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 4$ atoms.

Step 3: Substitute

$$\rho = \frac{4 \times 108}{6.022 \times 10^{23} \times 6.84 \times 10^{-23}} = \frac{432}{41.19}$$

$$\rho = 10.49 \text{ g cm}^{-3}$$

Step 4: Check the options

If Z were 2 (bcc) we would get about 5.2, which is not an option.

Hence the answer is 10.49.

Final Answer:

The density of silver is 10.49 g per cubic centimetre.

$$\boxed{\text{(B) } 10.49 \text{ g cm}^{-3}}$$

Quick Tip: Density equals Z times molar mass divided by Avogadro number times cell volume, with Z = 4 for fcc.

72. What is the coordination number of Ag in a crystal lattice?

(A) 4

(B) 6

(C) 12

(D) 8

Correct Answer: (C) 12

Solution:

Step 1: Identify the lattice

Silver crystallises in a face centred cubic (cubic close packed) lattice.

Step 2: Coordination number

In ccp or hcp every atom touches 12 others, so the coordination number is 12. Simple cubic gives 6 and bcc gives 8, which are the wrong options for silver.

Final Answer:

The coordination number is 12.

(C) 12

Quick Tip: Silver crystallises in fcc (ccp), which has the highest packing.

73. The normal boiling point of Ethyl acetate is 77°C. A solution of non-volatile non-electrolyte solute in ethyl acetate boils at 78°C. K_b for ethyl acetate is 2.77°C kg mol^{-1}. What is the molality of the solution ?

(A) 0.361m

(B) 0.052m

(C) 0.075m

(D) 0.25m

Correct Answer: (A) 0.361m

Solution:

Step 1: Find elevation

Boiling point of the solution is $78°C$ and that of pure solvent is $77°C$.

So $\Delta T_b = 78 - 77 = 1$ K.

Step 2: Apply the formula

For a non-volatile non-electrolyte, $\Delta T_b = K_b m$.

$$m = \frac{\Delta T_b}{K_b} = \frac{1}{2.77} = 0.361 \text{ mol/kg}$$

Step 3: Check the options

0.052, 0.075 and 0.25 m would give elevations of 0.14, 0.21 and 0.69 K, none equal to 1 K.

Final Answer:

The molality is 0.361 m.

$$\boxed{\text{(A) } 0.361 \text{ m}}$$

Quick Tip: Use delta Tb = Kb times m.

74. 18 g of glucose (molar mass = 180 g/mol) is dissolved in water to prepare 500 ml solution at 15°C. Calculate the osmotic pressure of the solution. [$R = 0.0821 \text{L atm K}^{-1}\text{mol}^{-1}$]

(A) 1.65atm

(B) 4.73atm

(C) 5.57atm

(D) 2.34atm

Correct Answer: (B) 4.73atm

Solution:

Step 1: Find the molarity

Moles of glucose $= \frac{18}{180} = 0.1$ mol. Volume is 500 mL $= 0.5$ L.

$C = \frac{0.1}{0.5} = 0.2$ M.

Step 2: Convert temperature

$T = 15 + 273 = 288$ K.

Step 3: Apply the formula

$$\pi = CRT = 0.2 \times 0.0821 \times 288 = 4.73 \text{ atm}$$

Step 4: Check the options

Using $T = 15$ K would give a very small answer, and 298 K would give 4.89 atm. Neither is listed, so 4.73 atm is correct.

Final Answer:

The osmotic pressure is 4.73 atm.

$$\boxed{\text{(B) } 4.73 \text{ atm}}$$

Quick Tip: Use pi = CRT with temperature in kelvin.

75. What type of solution is a brass ?

(A) solid in solid

(B) liquid in solid

(C) gas in solid

(D) liquid in gas

Correct Answer: (A) solid in solid

Solution:

Step 1: What is brass

Brass is an alloy of copper and zinc. Zinc, the solute, is dissolved in copper, the solvent, in the solid state.

Step 2: Type

Both the solute and the solvent are solids, so it is a solid in solid solution.

Final Answer:

Brass is a solid in solid solution.

(A) Solid in solid

Quick Tip: Brass is an alloy of zinc dissolved in copper, both solids.

76. What mass of silver (Atomic mass = 108 g/mol) deposited by a quantity of electricity which displaces 5600 mL of O_2 at STP ?

(A) 5.4g

(B) 10.8g

(C) 54.0g

(D) 108.0g

Correct Answer: (D) 108.0g

Solution:

Step 1: Find moles of O2

At STP one mole of gas occupies 22400 mL. So $n(O_2) = \frac{5600}{22400}$ $= 0.25$ mol.

Step 2: Find the charge in moles of electrons

Oxygen is released by $2H_2O \rightarrow O_2 + 4H^+ + 4e^-$, so 1 mol of O_2 needs 4 mol of electrons.

$0.25 \times 4 = 1$ mol of electrons, which is 1 faraday.

Step 3: Silver deposited

$Ag^+ + e^- \rightarrow Ag$, so 1 mol of electrons deposits 1 mol of silver.

$$m = 1 \times 108 = 108 \text{ g}$$

Step 4: Check the options

54 g would need 0.5 F and 10.8 g would need 0.1 F. Only 108 g matches 1 F.

Final Answer:

The same one faraday of charge deposits 108 g of silver.

$$\boxed{\text{(D) } 108.0 \text{ g}}$$

Quick Tip: Same charge deposits equal equivalents; find equivalents of O2 first.

77. Which of the following reactions takes place at anode during the electrolysis of molten NaCl?

(A) $Na^{+}_{(l)} + e^{-} \rightarrow Na_{(l)}$

(B) $Na_{(s)} \rightarrow Na^{+}_{(l)} + e^{-}$

(C) $Cl_{2(g)} + 2e^{-} \rightarrow 2Cl^{-}_{(l)}$

(D) $2Cl^{-}_{(l)} \rightarrow Cl_{2(g)} + 2e^{-}$

Correct Answer: (D) $2Cl^{-}_{(l)} \rightarrow Cl_{2(g)} + 2e^{-}$

Solution:

Step 1: Recall the rule

At the anode oxidation occurs, which means a species loses electrons.

Step 2: Ions present

Molten NaCl has Na^{+} and Cl^{-}. Only Cl^{-} can lose electrons: $2Cl^{-}_{(l)} \rightarrow Cl_{2(g)} + 2e^{-}$.

Step 3: Why others are wrong

(A) is reduction of Na^{+} at the cathode. (B) is not the reaction in the cell, since solid sodium is not at the anode. (C) is the reverse of (D) and is a reduction.

Final Answer:

Chloride ions are oxidised at the anode.

$$\boxed{(D)\ 2Cl^{-}_{(l)} \rightarrow Cl_{2(g)} + 2e^{-}}$$

Quick Tip: Oxidation takes place at the anode, so chloride ions lose electrons.

78. The rate law for the reaction $A + B \rightarrow P$ is found to be rate $= k[A]^2[B]$. The rate constant of the reaction at 300 K is $6.0 M^{-2}s^{-1}$. Calculate the rate of the reaction when $[A] = 1M$ and $[B] = 0.2M$

(A) $0.6 M\ s^{-1}$.

(B) $1.2 M\ s^{-1}$.

(C) $1.8 M\ s^{-1}$.

(D) $2.4 M\ s^{-1}$.

Correct Answer: (B) $1.2 M\ s^{-1}$.

Solution:

Step 1: Write the rate law

rate $= k[A]^2[B]$ with $k = 6.0\ M^{-2}s^{-1}$.

Step 2: Substitute

$$\text{rate} = 6.0 \times (1)^2 \times 0.2 = 1.2\ M\ s^{-1}$$

Step 3: Check units

The overall order is 3, so k has units $M^{-2}s^{-1}$, giving rate in Ms^{-1}.

Option (B) is correct.

Final Answer:

The rate is 1.2 M per second.

$$\boxed{\text{(B) } 1.2\ M\ s^{-1}}$$

Quick Tip: Substitute concentrations into rate = k[A]^2[B].

79. If for a first-order reaction, $[A]_0 = 1.0\text{M}$ and $[A]_t = 0.25\text{M}$ after 276 s, find the value of the rate constant (k).

(A) 0.0021s^{-1}

(B) 0.0050s^{-1}

(C) 0.003s^{-1}

(D) 0.006s^{-1}

Correct Answer: (B) 0.0050s^{-1}

Solution:

Step 1: Write the formula

For first order, $k = \frac{2.303}{t}\log\frac{[A]_0}{[A]_t}$.

Step 2: Substitute

$\frac{[A]_0}{[A]_t} = \frac{1.0}{0.25} = 4$, and $\log 4 = 0.6021$.

$$k = \frac{2.303 \times 0.6021}{276} = \frac{1.3866}{276}$$

$$k = 5.02 \times 10^{-3}\ \text{s}^{-1} \approx 0.0050\ \text{s}^{-1}$$

Step 3: Check

Concentration drops to a quarter, i.e. two half-lives, so $t_{1/2} = 138$ s and $k = 0.693/138 = 0.0050$.

Step 4: Other options

0.0021, 0.003 and 0.006 would give the wrong time for a fall to a

quarter.

Final Answer:

The rate constant is 0.0050 per second.

$$\boxed{\text{(B) } 0.0050 \text{ s}^{-1}}$$

Quick Tip: Use k = (2.303/t) log (A0/At).

80. The equation for the rate constant is $k = Ae^{-E_a/RT}$.

A chemical reaction will proceed more rapidly if there is a decrease in

(A) k

(B) A

(C) E_a

(D) T

Correct Answer: (C) E_a

Solution:

Step 1: Study the equation

$k = Ae^{-E_a/RT}$. A larger k means a faster reaction.

Step 2: Effect of each factor

Decreasing E_a makes the exponent less negative, so the exponential term and k increase. So the reaction goes faster.

Decreasing k obviously slows the reaction. Decreasing A lowers k.

Decreasing T makes the exponent more negative, so k falls.

Final Answer:

A decrease in activation energy speeds up the reaction.

$$\boxed{\text{(C) } E_a}$$

Quick Tip: A lower activation energy makes the exponential term larger.

81. What is the oxidation state of chromium in the final product when KI reacts with acidified potassium dichromate solution?

(A) +2

(B) +3

(C) +6

(D) +4

Correct Answer: (B) +3

Solution:

Step 1: Write the reaction

Acidified $K_2Cr_2O_7$ oxidises I^- to I_2: $Cr_2O_7^{2-} + 14H^+ + 6I^- \rightarrow 2Cr^{3+} + 3I_2 + 7H_2O$.

Step 2: Oxidation state

In $Cr_2O_7^{2-}$ chromium is +6. The product Cr^{3+} has oxidation state +3, so each Cr gains 3 electrons.

Step 3: Other options

+6 is the starting value and +2 or +4 are not formed in this reaction.

Final Answer:

Chromium ends as Cr(III), oxidation state +3.

$$\boxed{\text{(B) } +3}$$

Quick Tip: Dichromate oxidises iodide to iodine and itself becomes Cr3+.

82. Which of the following series of transition elements has general electronic configuration, [Kr] $4d^{1-10}$ $5s^{0-2}$?

(A) 3d series elements

(B) 4d series elements

(C) 5d series elements

(D) 6d series elements

Correct Answer: (B) 4d series elements

Solution:

Step 1: Match the core

The core $[Kr]$ ends at atomic number 36, and filling $4d$ with $5s$ is the second row of d-block elements (Y to Cd). So this is the 4d series.

Step 2: Other series

The 3d series is $[Ar]3d^{1-10}4s^{0-2}$, the 5d series is $[Xe]4f^{14}5d^{1-10}6s^{0-2}$ and the 6d series uses $[Rn]$.

Final Answer:

It is the 4d series.

(B) 4d series elements

Quick Tip: Krypton core followed by 4d means the second transition series.

83. Identify the ionization isomer of $[Cr(H_2O)_4\,Cl(NO_2)]\,Cl$ from the following.

(A) $[Cr(H_2O)_4\,(O_2N)]\,Cl_2$

(B) $[Cr(H_2O)_4\,Cl_2]\,(NO_2)$

(C) $[Cr(H_2O)_4\,Cl(ONO)]\,Cl$

(D) $[Cr(H_2O)_4\,Cl_2(NO_2)]\,H_2O$

Correct Answer: (B) $[Cr(H_2O)_4\,Cl_2]\,(NO_2)$

Solution:

Step 1: Understand ionization isomerism
Ionization isomers have the same formula but give different ions in solution, because a ligand and the counter ion are exchanged.

Step 2: Original complex
$[Cr(H_2O)_4Cl(NO_2)]Cl$ has Cl^- and NO_2^- in the sphere and a free Cl^- outside.

Step 3: Exchange
Swap the outside Cl^- with the inside NO_2^-. The result is $[Cr(H_2O)_4Cl_2]NO_2$, option (B).

Step 4: Why others are wrong
(A) has a different formula, with a different count of chlorine. (C) is a

linkage isomer, because *ONO* is the nitrito form of the ligand. (D) has a different number of water molecules and a different formula.

Final Answer:
Option B is the ionization isomer.

$$\boxed{\text{(B) } [Cr(H_2O)_4Cl_2]NO_2}$$

Quick Tip: Ionization isomers swap an ion between the coordination sphere and the counter ion.

84. The number of ligands which are directly bonded to the metal is known as

(A) co-ordination number

(B) oxidation number

(C) co-ordination sphere

(D) valency

Correct Answer: (A) co-ordination number

Solution:

Step 1: Definition
The coordination number is the number of ligand donor atoms directly bonded to the central metal atom or ion.

Step 2: Others
Oxidation number is the charge on the metal. Coordination sphere is the metal plus its ligands in square brackets. Valency is the combining

capacity of an element.

Final Answer:
The correct term is coordination number.

(A) co-ordination number

Quick Tip: This is the definition of coordination number.

85. Identify the reagent used for replacement of $-N_2^+Cl^-$ from benzene diazonium chloride by iodine.

(A) HI

(B) KOI

(C) KI

(D) PI_3

Correct Answer: (C) KI

Solution:

Step 1: Reaction
Benzene diazonium chloride reacts with *KI* to give iodobenzene:
$C_6H_5N_2^+Cl^- + KI \rightarrow C_6H_5I + N_2 + KCl$.

Step 2: Why
Iodide is a good nucleophile and does not need a copper catalyst. *HI*, *KOI* and PI_3 are not used for this replacement.

Final Answer:

Potassium iodide, KI, is the reagent.

$$\boxed{\text{(C) } KI}$$

Quick Tip: Iodine is introduced with potassium iodide, no copper salt required.

86. What is the product P obtained in following reaction?

$CH_3CH_2Br + AgCN_{(alc)} \rightarrow P$

(A) CH_3CH_2NC

(B) CH_3CH_2CN

(C) $CH_3CH_2CONH_2$

(D) $CH_3CH_2NH_2$

Correct Answer: (A) CH_3CH_2NC

Solution:

Step 1: Nature of AgCN

Silver cyanide is largely covalent. The cyanide group attacks through the nitrogen lone pair, since carbon is tied up in bonding with silver.

Step 2: Product

$CH_3CH_2Br + AgCN \rightarrow CH_3CH_2NC + AgBr$. The product is ethyl isocyanide.

Step 3: Why others are wrong

CH_3CH_2CN forms with KCN, which is ionic and attacks through

carbon. The amide and the amine are not formed under these conditions.

Final Answer:
The product is ethyl isocyanide, CH3CH2NC.

$$\boxed{\text{(A) } CH_3CH_2NC}$$

Quick Tip: AgCN is covalent, so nitrogen attacks and an isocyanide forms.

87. What is action of CH_3ONa on CH_3CH_2Br
$CH_3ONa + CH_3CH_2Br \rightarrow ?$

(A) Formation of $CH_3OCH_2CH_3 + NaBr$

(B) Formation of $CH_3OH + NaBr$

(C) Formation of CH_3CH_2OH

(D) Formation of $(CH_3)_2O$

Correct Answer: (A) Formation of $CH_3OCH_2CH_3 + NaBr$

Solution:

Step 1: Recognise the reaction
Sodium methoxide is a strong nucleophile and ethyl bromide is a primary alkyl halide, which suits an SN2 reaction. This is the Williamson ether synthesis.

Step 2: Product
$CH_3ONa + CH_3CH_2Br \rightarrow CH_3OCH_2CH_3 + NaBr.$

Step 3: Other options

The alcohols and dimethyl ether would need the wrong halide or water, and do not form here.

Final Answer:

The product is ethyl methyl ether and NaBr.

$$\boxed{\text{(A) } CH_3OCH_2CH_3 + NaBr}$$

Quick Tip: This is the Williamson ether synthesis.

88. Which of the following reagents limits the oxidation of $R\text{-}CH_2OH$ up to R-CHO only?

(A) PCC

(B) CrO_3

(C) $K_2Cr_2O_7$ / dil. H_2SO_4

(D) $KMnO_4$ (acidic)

Correct Answer: (A) PCC

Solution:

Step 1: Idea

Strong oxidising agents convert a primary alcohol to aldehyde and then to carboxylic acid. A mild reagent is needed to stop at the aldehyde.

Step 2: Reagent

PCC (pyridinium chlorochromate) in dichloromethane oxidises

RCH_2OH to $RCHO$ only.
CrO_3, acidified $K_2Cr_2O_7$ and acidic $KMnO_4$ are strong and give $RCOOH$.

Final Answer:
PCC stops at the aldehyde.

(A) PCC

Quick Tip: PCC is a mild oxidising agent used in non-aqueous medium.

89. Which of the following ether, on hydrolysis, gives two different products that are successive members of the homologous series?

(A) Methoxymethane
(B) Methoxypropane
(C) Ethoxypropane
(D) 1-Methoxypropane

Correct Answer: (C) Ethoxypropane

Solution:

Step 1: Hydrolysis
Ether $R-O-R'$ with water gives $ROH + R'OH$.

Step 2: Check options
Methoxymethane gives two molecules of methanol, the same compound. Methoxypropane gives methanol (1 C) and propanol (3 C), which differ by two carbons. Ethoxypropane gives ethanol (2 C) and

propanol (3 C), which differ by one CH_2 unit, so they are successive homologues. 1-Methoxypropane is the same as methoxypropane, so it gives methanol and propanol.

Step 3: Conclusion

The answer is ethoxypropane, option (C).

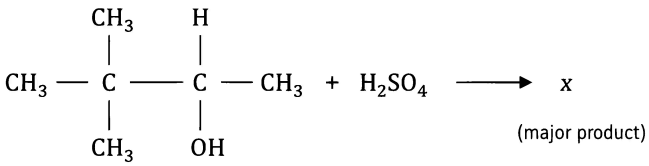
Final Answer:

Ethoxypropane gives ethanol and propan-1-ol.

(C) Ethoxypropane

Quick Tip: Hydrolysis of an ether gives two alcohols; compare their carbon counts.

90. Identify the product X in the following reaction.



(A) 2,3-dimethyl-2-butene

(B) 3,2-diethyl butene

(C) 3,3-dimethyl-1-butene

(D) 2,2-diethyl butene

Correct Answer: (A) 2,3-dimethyl-2-butene

Solution:

Step 1: Read the structure

The figure shows $(CH_3)_3C-CH(OH)-CH_3$, which is 3,3-dimethylbutan-2-ol, treated with H_2SO_4. The alcohol is secondary and has a quaternary neighbouring carbon.

Step 2: Form the carbocation

Protonation and loss of water give a secondary carbocation at C2. It is next to the $C(CH_3)_3$ carbon.

Step 3: Methyl shift

A methyl group moves from the neighbouring carbon to the cationic carbon. This gives a tertiary cation $(CH_3)_2C^+-CH(CH_3)_2$, which is more stable.

Step 4: Lose a proton

Loss of a proton from the carbon next to the cation follows Zaitsev's rule and gives the more substituted alkene $(CH_3)_2C=C(CH_3)_2$, which is 2,3-dimethyl-2-butene.

Step 5: Other options

3,3-dimethyl-1-butene would be the direct non-rearranged, less substituted product, so it is minor.

Final Answer:

The major product is 2,3-dimethyl-2-butene.

(A) 2,3-dimethyl-2-butene

Quick Tip: The carbocation rearranges by a methyl shift to a more stable tertiary cation before losing H+.

91. Acetone on heating with CrO_3 mainly forms,

(A) Acetic acid

(B) Ethyl alcohol

(C) Toluene

(D) Dimethyl ether

Correct Answer: (A) Acetic acid

Solution:

Step 1: Ketone oxidation

Ketones resist mild oxidants but under vigorous conditions, such as hot CrO_3, the C-C bond next to the carbonyl breaks.

Step 2: Acetone

Acetone CH_3COCH_3 breaks to give acetic acid (and carbon dioxide on further oxidation of the other fragment). So the main organic product is acetic acid.

Step 3: Other options

Ethyl alcohol, toluene and dimethyl ether are not oxidation products of a ketone.

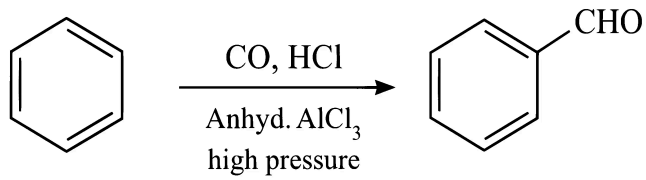
Final Answer:

Acetone on strong oxidation mainly gives acetic acid.

(A) Acetic acid

Quick Tip: Strong oxidation of a ketone breaks a C-C bond and gives acids.

92. Identify the name of the above reaction.



(Benzene) (Benzaldehyde)

(A) Etard reaction

(B) Gatterman-Koch formylation reaction

(C) Stephen reaction

(D) Rosenmund reaction

Correct Answer: (B) Gatterman-Koch formylation reaction

Solution:

Step 1: Read the figure

The figure shows benzene reacting with CO, HCl in the presence of anhydrous $AlCl_3$ at high pressure to give benzaldehyde, C_6H_5CHO.

Step 2: Name

This is the Gattermann-Koch formylation. The electrophile is formyl chloride, $HCOCl$, or the formyl cation, generated in situ.

Step 3: Other options

Etard uses CrO_2Cl_2 on toluene. Stephen uses $SnCl_2/HCl$ on a nitrile. Rosenmund reduces an acid chloride with $H_2/Pd - BaSO_4$.

Final Answer:

The reaction is Gattermann-Koch formylation.

(B) Gatterman-Koch formylation

Quick Tip: CO with HCl and anhydrous $AlCl_3$ under pressure is the Gattermann-Koch reaction.

93. When sodium bicarbonate is added to an organic compound, brisk effervescences is evolved. This indicate the presence of which functional group.

(A) $-CHO$

(B) $-C=O$

(C) $-COCl$

(D) $-COOH$

Correct Answer: (D) $-COOH$

Solution:

Step 1: Reaction

Carboxylic acids are stronger than carbonic acid, so they decompose bicarbonate: $RCOOH + NaHCO_3 \rightarrow RCOONa + H_2O + CO_2 \uparrow$.

Step 2: Meaning

The brisk effervescence is carbon dioxide. Aldehydes, ketones and acid chlorides do not give this test with bicarbonate.

Final Answer:

The group is carboxylic acid, -COOH.

(D) $-COOH$

Quick Tip: Only acids stronger than carbonic acid release CO_2 from bicarbonate.

94. Tertiary amines have the lowest boiling points because ,

(A) They possess polar N-C bonds

(B) They possess inter-molecular dipole-dipole attraction forces.

(C) They possess inter-molecular H-bonding

(D) They do not possess inter-molecular H- bonding

Correct Answer: (D) They do not possess inter-molecular H- bonding

Solution:

Step 1: Hydrogen bonding

Primary and secondary amines have N-H bonds and form intermolecular hydrogen bonds. Tertiary amines have no hydrogen on nitrogen.

Step 2: Result

Without intermolecular hydrogen bonding, only weaker dipole forces act, so the boiling point is the lowest.

Options (A) and (B) are true of tertiary amines but they do not explain the low boiling point.

Final Answer:

Tertiary amines lack intermolecular H-bonding.

(D) They do not possess inter-molecular H-bonding

Quick Tip: Tertiary amines have no N-H bond, so they cannot hydrogen bond with each other.

95. Which of the following molecules is capable of forming Zwitter ion?

(A) NH_2CH_2COOH

(B) CH_3COOH

(C) $CH_3CH_2NH_2$

(D) CCl_3NO_2

Correct Answer: (A) NH_2CH_2COOH

Solution:

Step 1: Idea

A zwitter ion forms when a proton moves from an acidic group to a basic group within the same molecule.

Step 2: Check

NH_2CH_2COOH (glycine) has both $-COOH$ and $-NH_2$, so it forms $^{+}NH_3CH_2COO^{-}$.

Acetic acid has only an acid group, ethylamine only a basic group, and CCl_3NO_2 has neither.

Final Answer:

Glycine forms a zwitter ion.

(A) NH_2CH_2COOH

Quick Tip: It needs both an acidic group and a basic group in the same molecule.

96. The letter 'D' in carbohydrates signifies

(A) dextrorotatory

(B) configuration

(C) diamagnetic nature

(D) mode of synthesis

Correct Answer: (B) configuration

Solution:

Step 1: Meaning

D and L describe the configuration at the highest-numbered chiral carbon, compared with glyceraldehyde. In D-sugars the OH on that carbon is on the right in the Fischer projection.

Step 2: Not related

It does not mean dextrorotatory. A D-sugar can rotate light either way. It has nothing to do with magnetism or synthesis.

Final Answer:

D denotes configuration.

(B) configuration

Quick Tip: D and L describe the spatial arrangement relative to glyceraldehyde.

97. Which of the following is found to raise the HDL level in the blood?

(A) Omega-6 fatty acids

(B) Omega-3 fatty acids

(C) Trans fats

(D) Cis fats

Correct Answer: (B) Omega-3 fatty acids

Solution:

Step 1: Fact

Omega-3 fatty acids (found in fish oil and flax) raise HDL, the good cholesterol, and lower triglycerides.

Step 2: Others

Trans fats raise LDL and lower HDL. Omega-6 does not specifically raise HDL. Cis fats as a general category do not raise HDL in this sense.

Final Answer:

Omega-3 fatty acids raise HDL.

(B) Omega-3 fatty acids

Quick Tip: Omega-3 fatty acids are known to raise good cholesterol.

98. Identify the polymer used to prepare plastic bottles needed to store cooking oil.

(A) PVC

(B) PS

(C) LDPE

(D) PET

Correct Answer: (D) PET

Solution:

Step 1: Fact

Polyethylene terephthalate, PET, is a strong, light and transparent polyester that resists oils, so it is used for bottles of cooking oil.

Step 2: Others

PVC is used for pipes, PS for foam and disposable cutlery, LDPE for films and squeeze bottles.

Final Answer:

PET is used for oil bottles.

(D) PET

Quick Tip: PET is clear, strong and used for oil and drink bottles.

99. Which polymer is commonly used in nonstick cookwares ?

(A) PVC

(B) Teflon

(C) SBR

(D) Buna -S

Correct Answer: (B) Teflon

Solution:

Step 1: Fact

Teflon, polytetrafluoroethylene (PTFE), has strong C-F bonds, is chemically inert and has a very low surface friction, so food does not stick.

Step 2: Others

PVC is for pipes. SBR and Buna-S are the same synthetic rubber, used for tyres.

Final Answer:

Teflon is used in nonstick cookware.

(B) Teflon

Quick Tip: Teflon is polytetrafluoroethylene.

100. Buna-S is prepared from

(A) 1, 3- butadiene and vinyl cyanide

(B) 1, 3- butadiene and styrene

(C) 1,2 -butadiene and vinyl cyanide

(D) 1, 2-butadiene and styrene

Correct Answer: (B) 1, 3- butadiene and styrene

Solution:

Step 1: Name

Buna-S: Bu for butadiene, na for sodium (the catalyst), S for styrene.

Step 2: Reaction

It is made by copolymerisation of $1,3-$ ***butadiene*** and ***styrene*** in a 3:1 ratio. Vinyl cyanide gives Buna-N instead.

Final Answer:

Buna-S is made from 1,3-butadiene and styrene.

(B) 1,3-butadiene and styrene

Quick Tip: Buna-S is a copolymer of butadiene and styrene.

Mathematics

Questions 101 to 150 | 2 marks each | 100 marks

101. If n is a positive integer, then $(1+i\sqrt{3})^{2n}+(1-i\sqrt{3})^{2n}$ is equal to .....

(A) $2^{2n+1}cos(\frac{2n\pi}{3})$

(B) $2^{2n}cos(\frac{n\pi}{3})$

(C) $2^{2n+1}cos(\frac{n\pi}{3})$

(D) $2^{n}cos(\frac{2n\pi}{3})$

Correct Answer: (A) $2^{2n+1}cos(\frac{2n\pi}{3})$

Solution:

Step 1: Convert to polar form

For $1+i\sqrt{3}$ the modulus is $\sqrt{1+3}=2$ and the argument is $\tan^{-1}\sqrt{3}=\frac{\pi}{3}$. So $1+i\sqrt{3}=2\left(\cos\frac{\pi}{3}+i\sin\frac{\pi}{3}\right)$. Its conjugate is $1-i\sqrt{3}=2\left(\cos\frac{\pi}{3}-i\sin\frac{\pi}{3}\right)$.

Step 2: Apply De Moivre

$$(1\pm i\sqrt{3})^{2n}=2^{2n}\left(\cos\frac{2n\pi}{3}\pm i\sin\frac{2n\pi}{3}\right)$$

Step 3: Add the two terms

The sine parts cancel, and the cosine parts double:

$$2^{2n}\cdot 2\cos\frac{2n\pi}{3}=2^{2n+1}\cos\frac{2n\pi}{3}$$

Step 4: Check the options

Option (A) matches. Option (C) has the right power of 2 but the wrong angle $\frac{n\pi}{3}$. Options (B) and (D) have the wrong power of 2. Test $n = 1$: $(1 + i\sqrt{3})^2 = -2 + 2i\sqrt{3}$, and the sum with its conjugate is -4. Option (A) gives $2^3 \cos \frac{2\pi}{3} = 8(-\frac{1}{2}) = -4$, which agrees.

Final Answer:

The expression equals 2^(2n+1) cos(2n pi / 3).

$$\boxed{(A)\ 2^{2n+1} \cos\left(\frac{2n\pi}{3}\right)}$$

Quick Tip: Write each number in polar form, apply De Moivre, and add the conjugate terms.

102. The number of 4-letter words formed from the English alphabet such that there are exactly 2 vowels and 2 consonants and no vowel is repeated, but consonants may be repeated is:

(A) ${}^4C_2 \times {}^5C_2 \times (21)^2$

(B) ${}^4C_2 \times {}^5C_2 \times {}^{21}C_2$

(C) ${}^4C_2 \times {}^5P_2 \times {}^{21}P_2$

(D) ${}^4C_2 \times {}^5P_2 \times (21)^2$

Correct Answer: (D) ${}^4C_2 \times {}^5P_2 \times (21)^2$

Solution:

Step 1: Count the letters available

The English alphabet has 5 vowels and 21 consonants.

Step 2: Choose positions

We need exactly two of the four places for vowels. This can be done in ${}^4C_2$ ways. The other two places are for consonants.

Step 3: Fill the vowel places

Vowels cannot repeat, so the two vowel places are filled in ${}^5P_2$ ways (choose and arrange).

Step 4: Fill the consonant places

Consonants may repeat, so each of the two places has 21 choices, giving 21^2.

Step 5: Multiply

$$\text{Total} = {}^4C_2 \times {}^5P_2 \times (21)^2$$

Step 6: Why other options fail

Options (A) and (B) use ${}^5C_2$, which does not arrange the vowels.
Options (B) and (C) use ${}^{21}C_2$ or ${}^{21}P_2$, which forbid repeated consonants, but the question allows them.

Final Answer:

The count is 4C2 x 5P2 x 21^2.

$$\boxed{\text{(D) } {}^4C_2 \times {}^5P_2 \times (21)^2}$$

Quick Tip: Choose positions for vowels, arrange distinct vowels, then fill consonant places with repetition allowed.

103. The value of $cos(60° - A) \cdot cosA \cdot cos(60° + A)$ is

(A) $\frac{1}{4}cos3A$

(B) $sin3A$

(C) $cos3A$

(D) $\frac{1}{4}sin3A$

Correct Answer: (A) $\frac{1}{4}cos3A$

Solution:

Step 1: Combine two factors

Use $\cos X \cos Y = \frac{1}{2}[\cos(X + Y) + \cos(X - Y)]$ on $\cos(60° - A)\cos(60° + A)$.

Step 2: Simplify

$$\cos(60° - A)\cos(60° + A) = \frac{1}{2}[\cos 120° + \cos 2A] = \frac{1}{2}\left[-\frac{1}{2} + 2\cos^2 A - 1\right]$$

$$= \cos^2 A - \frac{3}{4}$$

Step 3: Multiply by cos A

$$\cos A\left(\cos^2 A - \frac{3}{4}\right) = \frac{1}{4}(4\cos^3 A - 3\cos A) = \frac{1}{4}\cos 3A$$

Step 4: Check

Take $A = 0$: LHS $= \cos 60° \cdot 1 \cdot \cos 60° = \frac{1}{4}$. Option (A) gives $\frac{1}{4}\cos 0 = \frac{1}{4}$. Option (C) gives 1, which fails.

Final Answer:

The value is one quarter of cos 3A.

$$\boxed{(A)\ \frac{1}{4}\cos 3A}$$

Quick Tip: Combine the first and last factors using the product-to-sum formula.

104. The general solution of the equation $cot\theta \cdot cot2\theta = 1$ is...

(A) $\theta = n\pi \pm \frac{\pi}{6}, n \in Z$

(B) $\theta = n\pi \pm \frac{\pi}{3}, n \in Z$

(C) $\theta = n\pi \pm \frac{\pi}{4}, n \in Z$

(D) $\theta = n\pi \pm \frac{\pi}{8}, n \in Z$

Correct Answer: (A) $\theta = n\pi \pm \frac{\pi}{6}, n \in Z$

Solution:

Step 1: Rewrite the equation

$\cot\theta \cot 2\theta = 1$ gives $\cos\theta \cos 2\theta = \sin\theta \sin 2\theta$, so $\cos\theta \cos 2\theta - \sin\theta \sin 2\theta = 0$, that is $\cos 3\theta = 0$.

Step 2: Solve cos 3 theta = 0

$3\theta = (2m+1)\frac{\pi}{2}$, so $\theta = (2m+1)\frac{\pi}{6}$, with values $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{7\pi}{6}, \ldots$.

Step 3: Remove invalid values

At $\theta = \frac{\pi}{2} + k\pi$, $\cot 2\theta$ is undefined because $\sin 2\theta = 0$. These must be discarded.

Step 4: Combine the rest

The remaining values are $\frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}, \ldots$, which are $n\pi \pm \frac{\pi}{6}$. This is option (A). Options (B), (C) and (D) give angles such as $\frac{\pi}{3}$, where $\cot\theta \cot 2\theta = \frac{1}{\sqrt{3}} \cdot \left(-\frac{1}{\sqrt{3}}\right) \neq 1$.

Final Answer:

The general solution is theta = n pi plus or minus pi/6.

$$\boxed{(A)\ \theta = n\pi \pm \frac{\pi}{6}}$$

Quick Tip: Convert to cos3 theta = 0 and then remove the values where cot 2 theta is undefined.

105. The line $(2+k)x + (1+k)y = 5 + 7k$ passes through the fixed point for different values of k. If 'd' is the distance of a fixed point from the origin, then $d^2 = \ldots$

(A) 29

(B) 37

(C) 65

(D) 85

Correct Answer: (D) 85

Solution:

Step 1: Separate the parameter

Write $(2+k)x+(1+k)y=5+7k$ as $(2x+y-5)+k(x+y-7)=0$.

Step 2: Find the fixed point

For every k the line passes through the intersection of $2x+y=5$ and $x+y=7$. Subtracting the second from the first gives $x=-2$, then $y=9$.

Step 3: Distance

The fixed point is $(-2,9)$. Then $d^2=(-2)^2+9^2=4+81=85$.

Step 4: Check

Test with the fixed point: $(2+k)(-2)+(1+k)(9)=-4-2k+9+9k=5+7k$. It holds for all k. So option (D).

Final Answer:

d squared equals 85.

$$\boxed{\text{(D) } 85}$$

Quick Tip: Group the terms by k to get two lines whose intersection is the fixed point.

106. If θ is the acute angle between the lines $2x^2+7xy+3y^2=0$, then the value of $\frac{2\cos\theta-3\sin\theta}{4\sin\theta+5\cos\theta}=\ldots$

(A) 1

(B) $\frac{5}{9}$

(C) $-\frac{1}{9}$

(D) $\frac{1}{9}$

Correct Answer: (C) $-\frac{1}{9}$

Solution:

Step 1: Factor the equation

$2x^2 + 7xy + 3y^2 = (2x + y)(x + 3y)$.

Step 2: Slopes

The lines are $2x + y = 0$ with slope $m_1 = -2$ and $x + 3y = 0$ with slope $m_2 = -\frac{1}{3}$.

Step 3: Angle between them

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right| = \left|\frac{-2 + \frac{1}{3}}{1 + \frac{2}{3}}\right| = \frac{5/3}{5/3} = 1$$

So $\theta = 45°$ and $\sin\theta = \cos\theta = \frac{1}{\sqrt{2}}$.

Step 4: Evaluate

$$\frac{2\cos\theta - 3\sin\theta}{4\sin\theta + 5\cos\theta} = \frac{2 - 3}{4 + 5} = -\frac{1}{9}$$

Final Answer:

The value is -1/9.

$$\boxed{(C)\ -\frac{1}{9}}$$

Quick Tip: Factor the pair and find tan theta between the lines.

107. The combined equation of the pair of lines passing through the origin and making an angle $\frac{\pi}{4}$ with the line $3x + y - 6 = 0$ is...

(A) $x^2 + 3xy - 2y^2 = 0$

(B) $2x^2 - 3xy + y^2 = 0$

(C) $2x^2 - xy + 3y^2 = 0$

(D) $2x^2 + 3xy - 2y^2 = 0$

Correct Answer: (D) $2x^2 + 3xy - 2y^2 = 0$

Solution:

Step 1: Slope of the given line

The line $3x + y - 6 = 0$ has slope -3.

Step 2: Use the angle formula

For a line of slope m at $\frac{\pi}{4}$ to it, $\left|\frac{m+3}{1-3m}\right| = 1$.

Step 3: Solve both signs

Case 1: $m + 3 = 1 - 3m$ gives $m = -\frac{1}{2}$. Case 2: $m + 3 = -(1 - 3m)$ $= 3m - 1$ gives $m = 2$.

Step 4: Write the lines

Lines through the origin: $y = -\frac{x}{2}$, that is $x + 2y = 0$, and $y = 2x$, that is $2x - y = 0$.

Step 5: Combined equation

$$(x+2y)(2x-y) = 2x^2 - xy + 4xy - 2y^2 = 2x^2 + 3xy - 2y^2 = 0$$

Final Answer:

The combined equation is 2x^2 + 3xy - 2y^2 = 0.

$$\boxed{\text{(D) } 2x^2 + 3xy - 2y^2 = 0}$$

Quick Tip: Find the slopes of the two lines using the angle formula, then form the product of the two line equations.

108. Line $l : x + y = 4$ intersects the circle $x^2 + y^2 - 2x - 2y = 2$ at points A and B. If C is the center of the circle, then the area of $\triangle ABC$ is...

(A) $\sqrt{2}$

(B) 2

(C) $2\sqrt{2}$

(D) 4

Correct Answer: (B) 2

Solution:

Step 1: Centre and radius

$x^2 + y^2 - 2x - 2y = 2$ becomes $(x-1)^2 + (y-1)^2 = 4$. So $C = (1, 1)$ and $r = 2$.

Step 2: Distance from C to the line

$d = \frac{|1+1-4|}{\sqrt{2}} = \sqrt{2}.$

Step 3: Chord length

Half the chord is $\sqrt{r^2 - d^2} = \sqrt{4-2} = \sqrt{2}$, so $AB = 2\sqrt{2}$.

Step 4: Area

$$\text{Area} = \frac{1}{2} \times AB \times d = \frac{1}{2} \times 2\sqrt{2} \times \sqrt{2} = 2$$

Final Answer:

The area of the triangle is 2 square units.

$$\boxed{\text{(B) } 2}$$

Quick Tip: Find the centre and radius, then the perpendicular distance from the centre to the line.

109. If $\lim_{x \to 0} \frac{45^x - 9^x - 5^x + 1}{(k^x - 1)(3^x - 1)} = 2$, then the value of k is ...

(A) 45

(B) 9

(C) 5

(D) 3

Correct Answer: (C) 5

Solution:

Step 1: Factorise

$45^x - 9^x - 5^x + 1 = 9^x \cdot 5^x - 9^x - 5^x + 1 = (9^x - 1)(5^x - 1)$.

Step 2: Rewrite the limit

$$\lim_{x \to 0} \frac{(9^x - 1)(5^x - 1)}{(k^x - 1)(3^x - 1)}$$

Divide the numerator and the denominator by x^2.

Step 3: Use the standard limit

$\lim_{x \to 0} \frac{a^x - 1}{x} = \ln a$. The limit becomes $\frac{\ln 9 \cdot \ln 5}{\ln k \cdot \ln 3}$.

Step 4: Solve for k

$\ln 9 = 2 \ln 3$, so the value is $\frac{2 \ln 5}{\ln k}$. Setting this equal to 2 gives $\ln k = \ln 5$, so $k = 5$.

Step 5: Check

If $k = 3$ the value would be $2 \ln 5 / \ln 3$, not 2. So option (C) is right.

Final Answer:

The value of k is 5.

(C) 5

Quick Tip: Factorise the numerator as (9^x-1)(5^x-1) and use the limit of (a^x-1)/x.

110. If the truth value of the compound statement $[(p \leftrightarrow q) \wedge (q \rightarrow r) \wedge \sim r] \rightarrow (p \wedge \sim q)$ is false, then the truth values of the statement patterns $(p \rightarrow q) \leftrightarrow (q \rightarrow r)$ and $\sim (p \vee r) \rightarrow (q \wedge p)$ are, respectively ...

(A) (T, T)

(B) (T, F)

(C) (F, T)

(D) (F, F)

Correct Answer: (B) (T, F)

Solution:

Step 1: Find the truth values of p, q, r

The given implication is false, so the antecedent is true and $p \wedge \sim q$ is false. The antecedent is true only if $\sim r$ is true, so $r = F$.

Step 2: Use the other parts

$q \rightarrow r$ must be true with $r = F$, so $q = F$. Then $p \leftrightarrow q$ is true only if $p = F$. Check: $p \wedge \sim q = F \wedge T = F$, as required. So $p = q = r = F$.

Step 3: First pattern

$(p \rightarrow q) = T$ and $(q \rightarrow r) = T$. So $T \leftrightarrow T = T$.

Step 4: Second pattern

$p \vee r = F$, so $\sim (p \vee r) = T$. The consequent $q \wedge p = F$. $T \rightarrow F = F$.

Step 5: Result

The pair is (T, F), option (B).

Final Answer:

The truth values are T and F.

$\boxed{\text{(B) } (T, F)}$

Quick Tip: A conditional is false only when the antecedent is true and the consequent is false.

111. The negation of the contrapositive of the statement $(p \vee \sim q) \rightarrow (p \wedge \sim q)$ is

(A) $(p \wedge \sim q) \vee (\sim p \wedge \sim q)$

(B) $(\sim p \wedge q) \vee (p \wedge \sim q)$

(C) $(\sim p \vee \sim q) \wedge (p \vee q)$

(D) $(\sim p \vee q) \wedge (p \vee \sim q)$

Correct Answer: (D) $(\sim p \vee q) \wedge (p \vee \sim q)$

Solution:

Step 1: Write the contrapositive

The statement is $(p \vee \sim q) \rightarrow (p \wedge \sim q)$. Its contrapositive is $\sim (p \wedge \sim q) \rightarrow \sim (p \vee \sim q)$.

Step 2: Negate it

The negation of $X \rightarrow Y$ is $X \wedge \sim Y$. Here $X = \sim (p \wedge \sim q)$ and $Y = \sim (p \vee \sim q)$, so we get $\sim (p \wedge \sim q) \wedge (p \vee \sim q)$.

Step 3: Simplify

By De Morgan $\sim (p \wedge \sim q) = \sim p \vee q$. So the result is $(\sim p \vee q) \wedge (p \vee \sim q)$.

Final Answer:

The negation of the contrapositive is (~p or q) and (p or ~q).

$$\boxed{\text{(D) } (\sim p \vee q) \wedge (p \vee \sim q)}$$

Quick Tip: The negation of X implies Y is X and not Y.

112. In triangle ABC, with usual notations, if $a = 4, b = 5$ and $c = 6$, then angle C is equal to...

(A) A

(B) $2A$

(C) $3A$

(D) $\frac{A}{2}$

Correct Answer: (B) $2A$

Solution:

Step 1: Cosine of C

$\cos C = \frac{a^2+b^2-c^2}{2ab} = \frac{16+25-36}{40} = \frac{1}{8}$.

Step 2: Cosine of A

$\cos A = \frac{b^2+c^2-a^2}{2bc} = \frac{25+36-16}{60} = \frac{3}{4}$.

Step 3: Double angle

$\cos 2A = 2\cos^2 A - 1 = 2 \cdot \frac{9}{16} - 1 = \frac{1}{8}$.

Step 4: Compare

$\cos 2A = \cos C$ and both angles lie in the range where cosine is one-to-

one, so $C = 2A$. Option (B).

Final Answer:

Angle C equals 2A.

$$\boxed{\text{(B) } C = 2A}$$

Quick Tip: Find cos A and cos C by the cosine rule and compare cos 2A with cos C.

113. If $A = \begin{bmatrix} 2 & 3 \\ 5 & -2 \end{bmatrix}$, $B^{-1} = \begin{bmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{2}{5} & -\frac{1}{5} \end{bmatrix}$, then $(AB)^{-1} =$

(A) $\frac{1}{95}\begin{bmatrix} 8 & -1 \\ -1 & 12 \end{bmatrix}$

(B) $\frac{1}{95}\begin{bmatrix} 12 & -1 \\ -1 & 8 \end{bmatrix}$

(C) $\frac{1}{95}\begin{bmatrix} -12 & 1 \\ 1 & -8 \end{bmatrix}$

(D) $\frac{1}{95}\begin{bmatrix} -8 & 1 \\ 1 & -12 \end{bmatrix}$

Correct Answer: (B) $\frac{1}{95}\begin{bmatrix} 12 & -1 \\ -1 & 8 \end{bmatrix}$

Solution:

Step 1: Find A inverse

$\det A = 2(-2) - 3(5) = -19$. The adjoint of A is $\begin{pmatrix} -2 & -3 \\ -5 & 2 \end{pmatrix}$. So A^{-1} $= \frac{1}{19}\begin{pmatrix} 2 & 3 \\ 5 & -2 \end{pmatrix}$.

Step 2: Use the reversal law

$(AB)^{-1} = B^{-1}A^{-1} = \frac{1}{5}\begin{pmatrix}1 & 2\\ 2 & -1\end{pmatrix} \cdot \frac{1}{19}\begin{pmatrix}2 & 3\\ 5 & -2\end{pmatrix}$.

Step 3: Multiply

$$\frac{1}{95}\begin{pmatrix}2+10 & 3-4\\ 4-5 & 6+2\end{pmatrix} = \frac{1}{95}\begin{pmatrix}12 & -1\\ -1 & 8\end{pmatrix}$$

Step 4: Compare

This matches option (B). Option (A) is the product in the wrong order, $A^{-1}B^{-1}$, which has different entries.

Final Answer:

The inverse of AB is (1/95) times the matrix with rows (12, -1) and (-1, 8).

$$\boxed{\text{(B) } \frac{1}{95}\begin{pmatrix}12 & -1\\ -1 & 8\end{pmatrix}}$$

Quick Tip: The inverse of a product reverses the order: (AB)^-1 = B^-1 A^-1.

114. If $A = [a_{ij}]_{3\times3}$ is a matrix such that $a_{ij} = |2i - 5j|$, where $|.|$ denotes the modulus function, then the element in the 2^{nd} row and 3^{rd} column of A^{-1} is ...

(A) 3

(B) 1

(C) 0

(D) −1

Correct Answer: (D) −1

Solution:

Step 1: Build the matrix

$a_{ij} = |2i - 5j|$ gives $A = \begin{pmatrix} 3 & 8 & 13 \\ 1 & 6 & 11 \\ 1 & 4 & 9 \end{pmatrix}$.

Step 2: Determinant

$$|A| = 3(54 - 44) - 8(9 - 11) + 13(4 - 6) = 30 + 16 - 26 = 20$$

Step 3: Element of the inverse

The (2,3) element of A^{-1} equals $\frac{C_{32}}{|A|}$, where C_{32} is the cofactor of the element in row 3, column 2.

Step 4: Cofactor

$M_{32} = \begin{vmatrix} 3 & 13 \\ 1 & 11 \end{vmatrix} = 33 - 13 = 20$, so $C_{32} = (-1)^5 \cdot 20 = -20$.

Step 5: Answer

$\frac{-20}{20} = -1$. Option (D).

Final Answer:

The required element is -1.

(D) -1

Quick Tip: Build the matrix, find its determinant, and use the cofactor C32 for the (2,3) entry of the inverse.

115. If $A = \begin{bmatrix} 2 & -1 \\ 0 & 2 \end{bmatrix}$ and $A^2 + xA + yI_2 = O_2$, where I_2 and O_2 are the identity matrix and null matrix of order 2 respectively, then:

(A) $x = 4, y = 4$

(B) $x = -4, y = 4$

(C) $x = -4, y = -2$

(D) $x = 4, y = -4$

Correct Answer: (B) $x = -4, y = 4$

Solution:

Step 1: Compute A squared

$$A^2 = \begin{pmatrix} 2 & -1 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 4 & -4 \\ 0 & 4 \end{pmatrix}.$$

Step 2: Form the equation

$$A^2 + xA + yI = \begin{pmatrix} 4 + 2x + y & -4 - x \\ 0 & 4 + 2x + y \end{pmatrix} = O.$$

Step 3: Solve

From the (1,2) entry, $x = -4$. Then $4 + 2(-4) + y = 0$ gives $y = 4$.

Step 4: Check

Option (B), $x = -4, y = 4$. The characteristic equation of A is $\lambda^2 - 4\lambda$

$+4=0$, which agrees by the Cayley-Hamilton theorem.

Final Answer:

x = -4 and y = 4.

$$\boxed{\text{(B) } x=-4,\ y=4}$$

Quick Tip: Compute A squared and compare entries.

116. The value of $cot^{-1}[\frac{\sqrt{1-sinx}+\sqrt{1+sinx}}{\sqrt{1-sinx}-\sqrt{1+sinx}}]$, where $x \in (0, \frac{\pi}{2})$ is...

(A) $\pi - x$

(B) $2\pi - x$

(C) $\frac{\pi}{2} - \frac{x}{2}$

(D) $\pi - \frac{x}{2}$

Correct Answer: (D) $\pi - \frac{x}{2}$

Solution:

Step 1: Rewrite the roots

$1 \pm \sin x = \left(\cos \frac{x}{2} \pm \sin \frac{x}{2}\right)^2$. For $x \in (0, \frac{\pi}{2})$, $\frac{x}{2} \in (0, \frac{\pi}{4})$, so $\cos \frac{x}{2} > \sin \frac{x}{2}$. Hence $\sqrt{1+\sin x} = \cos \frac{x}{2} + \sin \frac{x}{2}$ and $\sqrt{1-\sin x} = \cos \frac{x}{2} - \sin \frac{x}{2}$.

Step 2: Evaluate the fraction

$$\frac{(c-s)+(c+s)}{(c-s)-(c+s)} = \frac{2c}{-2s} = -\cot \frac{x}{2}$$

Step 3: Apply the inverse

$\cot^{-1}(-y) = \pi - \cot^{-1} y$ because the principal range is $(0, \pi)$. So the result is $\pi - \frac{x}{2}$.

Step 4: Result

Option (D). Options (A) and (B) have the wrong coefficient of x. Option (C) forgets the negative sign.

Final Answer:

The value is pi - x/2.

$$\boxed{(D)\ \pi - \frac{x}{2}}$$

Quick Tip: Use half-angle forms: 1 plus or minus sin x equals (cos(x/2) plus or minus sin(x/2)) squared.

117. If $(tan^{-1}x)^2 + (cot^{-1}x)^2 = \frac{5\pi^2}{8}$, then the value of x is equal to...

(A) -1

(B) -2

(C) 1

(D) 2

Correct Answer: (A) -1

Solution:

Step 1: Substitute

Let $t = \tan^{-1} x$, so $t \in (-\frac{\pi}{2}, \frac{\pi}{2})$. Also $\cot^{-1} x = \frac{\pi}{2} - t$.

Step 2: Form the quadratic

$t^2 + \left(\frac{\pi}{2} - t\right)^2 = \frac{5\pi^2}{8}$ gives $2t^2 - \pi t + \frac{\pi^2}{4} - \frac{5\pi^2}{8} = 0$, that is $16t^2 - 8\pi t - 3\pi^2 = 0$.

Step 3: Solve

$t = \frac{8\pi \pm \sqrt{64\pi^2 + 192\pi^2}}{32} = \frac{8\pi \pm 16\pi}{32}$, so $t = \frac{3\pi}{4}$ or $t = -\frac{\pi}{4}$.

Step 4: Select

$t = \frac{3\pi}{4}$ is outside the range of $\tan^{-1}$, so $t = -\frac{\pi}{4}$ and $x = \tan(-\frac{\pi}{4}) = -1$. Option (A).

Final Answer:

The value of x is -1.

$$\boxed{\text{(A) } -1}$$

Quick Tip: Use cot^-1 x = pi/2 - tan^-1 x and solve the quadratic in tan^-1 x.

118. If $f(\frac{x-1}{x+1}) = x + 1$, then $\int f(x)\,dx = \cdots$

(A) $2log|1 - x| + c$

(B) $-2log|1 - x| + c$

(C) $2log|1 + x| + c$

(D) $-2log|1 + x| + c$

Correct Answer: (B) $-2log|1-x|+c$

Solution:

Step 1: Find f

Let $u=\frac{x-1}{x+1}$. Then $u(x+1)=x-1$, so $x(u-1)=-1-u$ and $x=\frac{1+u}{1-u}$.

Step 2: Write f(u)

$f(u)=x+1=\frac{1+u}{1-u}+1=\frac{2}{1-u}$. So $f(x)=\frac{2}{1-x}$.

Step 3: Integrate

$$\int \frac{2}{1-x}dx=-2\log|1-x|+c$$

Step 4: Why the sign

The derivative of $1-x$ is -1, so we get a minus sign. Option (B).

Option (A) misses this sign.

Final Answer:

The integral is -2 log|1-x| + c.

$$\boxed{\text{(B) } -2\log|1-x|+c}$$

Quick Tip: Find f(u) by substituting u = (x-1)/(x+1) and solving for x.

119. If $e^x+e^{f(x)}=e$, then the domain of $f(x)$ is

(A) $(1, \infty)$

(B) $(-\infty, 1)$

(C) $(-\infty, \infty)$

(D) $(-\infty, 0)$

Correct Answer: (B) $(-\infty, 1)$

Solution:

Step 1: Isolate

$e^{f(x)} = e - e^x$.

Step 2: Positivity

An exponential is always positive, so we need $e - e^x > 0$, that is $e^x < e$.

Step 3: Solve

Taking logarithm gives $x < 1$. So the domain is $(-\infty, 1)$, option (B).

Step 4: Why not the others

For $x \geq 1$, $e - e^x \leq 0$, so $f(x)$ does not exist. So the domain cannot include 1 or larger values.

Final Answer:

The domain is (-infinity, 1).

$$\boxed{\text{(B) } (-\infty, 1)}$$

Quick Tip: Since e^f(x) must be positive, e^x must be less than e.

120. If the function f is continuous at $x = \pi$, where $f(x) = \frac{1-\cos[7(x-\pi)]}{5(x-\pi)^2}$, for $x \neq \pi$, then $f(\pi) =$

(A) $\frac{49}{4}$

(B) $\frac{4}{49}$

(C) $\frac{49}{10}$

(D) $\frac{10}{49}$

Correct Answer: (C) $\frac{49}{10}$

Solution:

Step 1: Set up the limit

Let $h = x - \pi$. As $x \to \pi$, $h \to 0$. The limit is $\lim_{h\to 0} \frac{1-\cos 7h}{5h^2}$.

Step 2: Use half-angle

$1 - \cos 7h = 2\sin^2 \frac{7h}{2}$, so the limit is $\frac{2}{5} \lim \frac{\sin^2(7h/2)}{h^2} = \frac{2}{5} \cdot \frac{49}{4}$.

Step 3: Evaluate

$$\frac{2}{5} \cdot \frac{49}{4} = \frac{49}{10}$$

Step 4: Continuity

For continuity $f(\pi)$ equals this limit, so $f(\pi) = \frac{49}{10}$. Option (C).

Final Answer:

f(pi) is 49/10.

$$\boxed{(C)\ \frac{49}{10}}$$

Quick Tip: Continuity needs f(pi) to equal the limit; use 1 - cos t over t squared tends to one half.

121. Let f(x) be a twice differentiable function such that $f''(x) = -f(x)$, Missing open brace for superscript and $h(x) = \{f(x)\}^2 + \{g(x)\}^2$. If $h(5) = 11$, then $h(10)$ is equal to ...

(A) 11

(B) 22

(C) 0

(D) Not defined

Correct Answer: (A) 11

Solution:

Step 1: Differentiate h

$h(x) = f^2 + g^2$, so $h'(x) = 2ff' + 2gg'$. Since $f' = g$ and $g' = f'' = -f$, we get $h'(x) = 2fg + 2g(-f)$.

Step 2: Simplify

$h'(x) = 2fg - 2fg = 0$.

Step 3: Conclude

A function with zero derivative everywhere is constant, so $h(10) = h(5) = 11$. Option (A).

Step 4: Why not the others

Nothing in the data lets h change, so 22 or 0 are impossible. The function exists since it is twice differentiable, so h is defined.

Final Answer:

h is constant and h(10) = 11.

$$\boxed{\text{(A) } 11}$$

Quick Tip: Differentiate h and show it is zero, so h is constant.

122. If $cot[f(x)] = \frac{3x-x^3}{1-3x^2}$ and $sin[g(x)] = \frac{1-x^2}{1+x^2}$, then $\lim_{x \to t} \frac{f(x)-f(t)}{g(x)-g(t)} = \ldots$

(A) $\frac{3}{2(1+t^2)}$

(B) $\frac{3}{2}$

(C) $\frac{5}{2}$

(D) $-\frac{5}{2(1+t^2)}$

Correct Answer: (B) $\frac{3}{2}$

Solution:

Step 1: Find f

Put $x = \tan\theta$. Then $\frac{3x-x^3}{1-3x^2} = \tan 3\theta$. So $\cot f = \tan 3\theta$ gives $f = \frac{\pi}{2} - 3\theta$ $= \frac{\pi}{2} - 3\tan^{-1} x$.

Step 2: Find g

$\frac{1-x^2}{1+x^2} = \cos 2\theta$, so $\sin g = \cos 2\theta$ gives $g = \frac{\pi}{2} - 2\theta = \frac{\pi}{2} - 2\tan^{-1} x$.

Step 3: Derivatives

$f'(x) = -\frac{3}{1+x^2}$ and $g'(x) = -\frac{2}{1+x^2}$.

Step 4: Limit

The limit is $\frac{f'(t)}{g'(t)} = \frac{-3/(1+t^2)}{-2/(1+t^2)} = \frac{3}{2}$. Option (B). Option (A) keeps a stray factor of $1 + t^2$, which cancels.

Step 5: Note

The limit is the definition of the ratio of derivatives at t.

Final Answer:

The limit is 3/2.

$$\boxed{\text{(B) } \frac{3}{2}}$$

Quick Tip: Identify f and g with inverse tangent forms, then the limit is the ratio of derivatives.

123. If $y = cos^{-1}(\frac{1-4^x}{1+4^x})$, then $\frac{dy}{dx}$ at $x = 1$ is

(A) $\frac{4log2}{5}$

(B) $\frac{log2}{5}$

(C) $\frac{2log8}{5}$

(D) $\frac{5log8}{2}$

Correct Answer: (A) $\frac{4log2}{5}$

Solution:

Step 1: Simplify

Let $t = 2^x$ so $4^x = t^2$. With $t = \tan\phi$, $\frac{1-t^2}{1+t^2} = \cos 2\phi$. So $y = \cos^{-1}(\cos 2\phi) = 2\phi = 2\tan^{-1}(2^x)$ for $t > 0$.

Step 2: Differentiate

$\frac{dy}{dx} = 2 \cdot \frac{1}{1+4^x} \cdot 2^x \ln 2 = \frac{2 \cdot 2^x \ln 2}{1+4^x}$.

Step 3: At x = 1

$\frac{dy}{dx} = \frac{2 \cdot 2 \ln 2}{1+4} = \frac{4 \ln 2}{5}$. Option (A).

Step 4: Check

Option (C) $\frac{2 \log 8}{5} = \frac{6 \log 2}{5}$ is too large, since it contains an extra factor of 3/2.

Final Answer:

dy/dx at x = 1 is 4 log 2 / 5.

$$\boxed{(A)\ \frac{4 \log 2}{5}}$$

Quick Tip: Write 4^x as (2^x)^2 and use cos(2 theta) form to simplify before differentiating.

124. A cylindrical tank without a top lid is being manufactured to hold a fixed volume of 125π cubic cm. The minimum surface area required to construct this tank is .....square cm

(A) 75π

(B) 50π

(C) 25π

(D) 125π

Correct Answer: (A) 75π

Solution:

Step 1: Setup

Volume: $\pi r^2 h = 125\pi$, so $h = \frac{125}{r^2}$. The tank has no lid, so surface area $S = \pi r^2 + 2\pi rh$.

Step 2: Single variable

$S = \pi r^2 + \frac{250\pi}{r}$.

Step 3: Minimise

$S' = 2\pi r - \frac{250\pi}{r^2} = 0$ gives $r^3 = 125$, so $r = 5$. $S'' = 2\pi + \frac{500\pi}{r^3} > 0$, so this is a minimum.

Step 4: Value

$S = 25\pi + 50\pi = 75\pi$. Option (A).

Final Answer:

The minimum surface area is 75 pi square cm.

$$\boxed{\text{(A) } 75\pi}$$

Quick Tip: Express height from the volume, write surface area in radius, and minimise.

125. A spherical snow ball is melting so that its volume is decreasing at the rate of 8 c.c./sec then the rate of change of radius when the radius is 2 cm, is :

(A) The radius is increasing at the rate of $\frac{1}{2\pi}$ cm/s

(B) The radius is decreasing at the rate of $\frac{1}{2\pi}$ cm/s

(C) The radius is increasing at the rate of $\frac{1}{\pi}$ cm/s

(D) The radius is decreasing at the rate of $\frac{1}{\pi}$ cm/s

Correct Answer: (B) The radius is decreasing at the rate of $\frac{1}{2\pi}$ cm/s

Solution:

Step 1: Volume relation

$V = \frac{4}{3}\pi r^3$, so $\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$.

Step 2: Substitute

The volume is decreasing, so $\frac{dV}{dt} = -8$. With $r = 2$: $-8 = 4\pi(4)\frac{dr}{dt}$ $= 16\pi\frac{dr}{dt}$.

Step 3: Solve

$\frac{dr}{dt} = -\frac{1}{2\pi}$ cm/s. The minus sign shows the radius decreases. Option (B).

Step 4: Why not the others

Options with increasing radius have the wrong sign, and the option with $\frac{1}{\pi}$ has the wrong magnitude.

Final Answer:

The radius decreases at 1/(2 pi) cm per second.

$$\boxed{\text{(B) decreasing at } \frac{1}{2\pi} \text{ cm/s}}$$

Quick Tip: Differentiate V = 4/3 pi r^3 with respect to time.

126. The equation of tangent to the curves $x = 1 - 3t^2$ and $y = t - 3t^3$ at the point $(-2, 2)$ is...

(A) $4x + 3y + 2 = 0$

(B) $4x - 3y + 2 = 0$

(C) $3x + 4y + 2 = 0$

(D) $3x - 4y + 2 = 0$

Correct Answer: (A) $4x + 3y + 2 = 0$

Solution:

Step 1: Find the parameter

At $(-2, 2)$: $1 - 3t^2 = -2$ gives $t^2 = 1$, so $t = \pm 1$. Use $y = t - 3t^3 = 2$: for $t = 1$, $y = -2$; for $t = -1$, $y = -1 + 3 = 2$. So $t = -1$.

Step 2: Find the slope

$\frac{dx}{dt} = -6t$, $\frac{dy}{dt} = 1 - 9t^2$. So $\frac{dy}{dx} = \frac{1-9t^2}{-6t}$. At $t = -1$: $\frac{-8}{6} = -\frac{4}{3}$.

Step 3: Tangent line

$y - 2 = -\frac{4}{3}(x + 2)$ gives $3y - 6 = -4x - 8$, so $4x + 3y + 2 = 0$.

Option (A).

Final Answer:

The tangent is 4x + 3y + 2 = 0.

$$\boxed{\text{(A) } 4x + 3y + 2 = 0}$$

Quick Tip: Find the parameter value at the point, then dy/dx = (dy/dt)/(dx/dt).

127. If $f(x) = cosx$, then the value of $\int \frac{e^{f(x)}(xsin^3x+f(x))}{1-(f(x))^2}dx =$

(A) $e^{f(x)}(-\text{cosec}x - x) + c$

(B) $e^{f(x)}(\text{cosec}\, x + x) + c$

(C) $e^{f(x)}(cosx - x) + c$

(D) $e^{f(x)}(cot^2x + x) + c$

Correct Answer: (A) $e^{f(x)}(-\text{cosec}x - x) + c$

Solution:

Step 1: Simplify

Since $1 - \cos^2 x = \sin^2 x$, the integrand is $e^{\cos x}\left(x \sin x + \frac{\cos x}{\sin^2 x}\right)$.

Step 2: Guess the antiderivative

Try $F = e^{\cos x}(-\text{cosec}\, x - x)$. Differentiate with the product rule.

Step 3: Differentiate

$F' = -\sin x\, e^{\cos x}(-\text{cosec}\, x - x) + e^{\cos x}(\text{cosec}\, x \cot x - 1)$

$= e^{\cos x}(1 + x \sin x + \text{cosec}\, x \cot x - 1) = e^{\cos x}\left(x \sin x + \frac{\cos x}{\sin^2 x}\right)$.

Step 4: Result

This equals the integrand, so the integral is $e^{\cos x}(-\text{cosec}\, x - x) + c$.

Option (A).

Final Answer:

The integral is e^cos x (-cosec x - x) + c.

$$\boxed{(A)\ e^{f(x)}(-\text{cosec}\, x - x) + c}$$

Quick Tip: Split the integrand and recognise the derivative of e^cos x times (cosec x + x).

128. $\int \frac{x^4+1}{x^6+1}dx =$

(A) $tan^{-1}x - \frac{1}{3}tan^{-1}(x^3) + c$

(B) $tan^{-1}x + \frac{1}{3}tan^{-1}(x^3) + c$

(C) $tan^{-1}x - tan^{-1}(x^3) + c$

(D) $tan^{-1}x + tan^{-1}(x^3) + c$

Correct Answer: (B) $tan^{-1}x + \frac{1}{3}tan^{-1}(x^3) + c$

Solution:

Step 1: Factorise the denominator

$x^6 + 1 = (x^2 + 1)(x^4 - x^2 + 1)$.

Step 2: Split the numerator

$x^4 + 1 = (x^4 - x^2 + 1) + x^2$. So $\frac{x^4+1}{x^6+1} = \frac{1}{x^2+1} + \frac{x^2}{x^6+1}$.

Step 3: Integrate each

$\int \frac{dx}{1+x^2} = \tan^{-1} x$. For the second, put $u = x^3$, $du = 3x^2dx$, giving $\frac{1}{3}\int \frac{du}{1+u^2} = \frac{1}{3}\tan^{-1}(x^3)$.

Step 4: Result

$\tan^{-1} x + \frac{1}{3}\tan^{-1}(x^3) + c$, option (B).

Final Answer:

The integral is tan^-1 x + (1/3) tan^-1 (x^3) + c.

$$\boxed{\text{(B) } \tan^{-1} x + \frac{1}{3}\tan^{-1}(x^3) + c}$$

Quick Tip: Write x^6+1 as (x^2+1)(x^4-x^2+1) and split the numerator.

129. If $a > 0, b > 0$ and $\int \frac{1}{ax^2+b}dx = \frac{1}{\sqrt{6}}tan^{-1}(\frac{\sqrt{2}x}{\sqrt{3}}) + c$, then $\int \frac{1}{bx^2+a}dx$ = ...

(A) $-\frac{1}{\sqrt{6}}tan^{-1}(\frac{\sqrt{2}x}{\sqrt{3}}) + c$

(B) $\frac{1}{\sqrt{6}}tan^{-1}(\frac{\sqrt{3}x}{\sqrt{2}}) + c$

(C) $-\sqrt{6}\,tan^{-1}(\frac{\sqrt{2}x}{\sqrt{3}}) + c$

(D) $\sqrt{6}\,tan^{-1}(\frac{\sqrt{3}x}{\sqrt{2}}) + c$

Correct Answer: (B) $\frac{1}{\sqrt{6}}tan^{-1}(\frac{\sqrt{3}x}{\sqrt{2}}) + c$

Solution:

Step 1: Standard form

$\int \frac{dx}{ax^2+b} = \frac{1}{\sqrt{ab}}\tan^{-1}\left(x\sqrt{\frac{a}{b}}\right) + c.$

Step 2: Compare

So $\sqrt{ab} = \sqrt{6}$, giving $ab = 6$, and $\frac{a}{b} = \frac{2}{3}$. Multiplying, $a^2 = 4$, so $a = 2$ and $b = 3$.

Step 3: Required integral

$\int \frac{dx}{3x^2+2} = \frac{1}{\sqrt{6}}\tan^{-1}\left(x\sqrt{\frac{3}{2}}\right) + c = \frac{1}{\sqrt{6}}\tan^{-1}\left(\frac{\sqrt{3}x}{\sqrt{2}}\right) + c.$

Step 4: Result

Option (B). The sign cannot be negative since the integrand is positive, which removes (A) and (C).

Final Answer:

The integral is (1/root 6) tan^-1 (root3 x / root2) + c.

$$\boxed{(B)\ \frac{1}{\sqrt{6}}\tan^{-1}\left(\frac{\sqrt{3}x}{\sqrt{2}}\right)+c}$$

Quick Tip: Match the standard form to find a and b, then swap them.

130. The value of integral $\int_0^1 cot^{-1}(1+x^2-x)\,dx$ is...

(A) $\frac{\pi}{2}-log2$

(B) $\pi-log2$

(C) $\frac{\pi}{4}-log2$

(D) $2\pi-log2$

Correct Answer: (A) $\frac{\pi}{2}-log2$

Solution:

Step 1: Convert

$\cot^{-1}(1+x^2-x)=\tan^{-1}\frac{1}{1-x+x^2}$, because the argument is positive on $[0,1]$.

Step 2: Split

$\frac{1}{1-x(1-x)}=\frac{x+(1-x)}{1-x(1-x)}$, which is the tangent of a sum. So the term equals $\tan^{-1}x+\tan^{-1}(1-x)$.

Step 3: Use the property

Since $\int_0^1 \tan^{-1}(1-x)dx = \int_0^1 \tan^{-1} x\,dx$, the integral is $2\int_0^1 \tan^{-1} x\,dx$.

Step 4: Integrate by parts

$\int_0^1 \tan^{-1} x\,dx = \left[x\tan^{-1} x\right]_0^1 - \int_0^1 \frac{x}{1+x^2}dx = \frac{\pi}{4} - \frac{1}{2}\ln 2$.

Step 5: Result

Double: $\frac{\pi}{2} - \ln 2$. Option (A).

Final Answer:

The value is pi/2 - log 2.

$$\boxed{(A)\ \frac{\pi}{2} - \log 2}$$

Quick Tip: Write cot inverse as tan inverse of the reciprocal and split it as tan^-1 x + tan^-1(1-x).

131. The value of $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x}dx$ is...

(A) $-\sqrt{2}\pi^2$

(B) π^2

(C) $\frac{\pi^2}{\sqrt{2}}$

(D) $-\frac{\pi^2}{\sqrt{2}}$

Correct Answer: (B) π^2

Solution:

Step 1: Split

The integrand is $\frac{2x}{1+\cos^2 x} + \frac{2x \sin x}{1+\cos^2 x}$.

Step 2: Odd part

$\frac{2x}{1+\cos^2 x}$ is odd, so its integral over $[-\pi, \pi]$ is zero.

Step 3: Even part

$\frac{2x \sin x}{1+\cos^2 x}$ is even, so $I = 4\int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$.

Step 4: Use the reflection property

For $J = \int_0^{\pi} \frac{x \sin x}{1+\cos^2 x} dx$, using $x \to \pi - x$ gives $J = \frac{\pi}{2} \int_0^{\pi} \frac{\sin x}{1+\cos^2 x} dx$. Put $u = \cos x$: $\int_{-1}^{1} \frac{du}{1+u^2} = \frac{\pi}{2}$.

Step 5: Result

$J = \frac{\pi^2}{4}$ and $I = 4J = \pi^2$. Option (B).

Final Answer:

The value of the integral is pi squared.

$$\boxed{\text{(B) } \pi^2}$$

Quick Tip: Split into odd and even parts; the odd part vanishes.

132. The area of the region enclosed by the lines $y = 2x, 2y = x$ and $x = 2$ (in sq.units) is...

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (C) 3

Solution:

Step 1: Find the vertices

The lines $y = 2x$ and $y = \frac{x}{2}$ meet at $(0, 0)$. At $x = 2$ they give $(2, 4)$ and $(2, 1)$.

Step 2: Integrate

$$A = \int_0^2 \left(2x - \frac{x}{2}\right) dx = \frac{3}{2} \int_0^2 x\, dx = \frac{3}{2} \cdot 2 = 3$$

Step 3: Check with triangle formula

Base on $x = 2$ is $4 - 1 = 3$ and height is 2, so area $= \frac{1}{2} \times 3 \times 2 = 3$.

Option (C).

Final Answer:

The area is 3 square units.

$$\boxed{\text{(C) } 3}$$

Quick Tip: The region is a triangle with vertices at the origin, (2,4) and (2,1).

133. The area of the region common to the parabolas $4y^2 = 9x$ and $3x^2 = 16y$ is...

(A) 2 sq. units

(B) 4 sq. units

(C) 8 sq. units

(D) 16 sq. units

Correct Answer: (B) 4 sq. units

Solution:

Step 1: Write as functions

$4y^2 = 9x$ gives $y = \frac{3}{2}\sqrt{x}$ (upper branch) and $3x^2 = 16y$ gives $y = \frac{3x^2}{16}$.

Step 2: Intersection

Equating: $\frac{3}{2}\sqrt{x} = \frac{3x^2}{16}$ gives $x^{3/2} = 8$, so $x = 4$, and the other point is $x = 0$. The points are $(0, 0)$ and $(4, 3)$.

Step 3: Area

$$A = \int_0^4 \left(\frac{3}{2}\sqrt{x} - \frac{3x^2}{16}\right) dx = \left[x^{3/2} - \frac{x^3}{16}\right]_0^4 = 8 - 4 = 4$$

Step 4: Result

The area is 4 square units, option (B).

Final Answer:

The common area is 4 square units.

$$\boxed{\text{(B) 4 sq. units}}$$

Quick Tip: Find the intersection points, then integrate the upper curve minus the lower curve.

134. For the differential equation $\frac{d^3y}{dx^3} + cos(\frac{d^2y}{dx^2}) = 0$, which of the following is true ?

(A) Order = 3, degree = 1

(B) Order = 3, degree = 2

(C) Order = 3, degree is not defined

(D) Order = 2, degree is not defined

Correct Answer: (C) Order = 3, degree is not defined

Solution:

Step 1: Order

The highest derivative is $\frac{d^3y}{dx^3}$, so the order is 3.

Step 2: Degree

Degree is the power of the highest derivative once the equation is a polynomial in its derivatives. Here $\cos\left(\frac{d^2y}{dx^2}\right)$ is not a polynomial in the derivative, so it cannot be cleared.

Step 3: Result

The degree is not defined. Option (C). Options (A) and (B) assign a degree, and option (D) gives the wrong order.

Final Answer:

Order is 3 and the degree is not defined.

(C) Order = 3, degree not defined

Quick Tip: Degree is defined only if the equation is a polynomial in the derivatives.

135. If water at 100°C cools in 10 minutes to 80°C and to 65°C in the next 10 minutes, then the room temperature will be ...

(A) 30°C

(B) 15°C

(C) 25°C

(D) 20°C

Correct Answer: (D) 20°C

Solution:

Step 1: Set up

Let the room temperature be s. By Newton's law, the excess temperature falls by the same ratio in equal times: $\frac{80-s}{100-s} = \frac{65-s}{80-s}$.

Step 2: Solve

$(80 - s)^2 = (100 - s)(65 - s)$. Expand: $6400 - 160s + s^2 = 6500 - 165s + s^2$, so $5s = 100$ and $s = 20$.

Step 3: Check

Differences: $100 - 20 = 80$, $80 - 20 = 60$, $65 - 20 = 45$. The ratios are $\frac{60}{80} = \frac{3}{4}$ and $\frac{45}{60} = \frac{3}{4}$, equal as required. Option (D).

Final Answer:

The room temperature is 20 degrees Celsius.

$$\boxed{\text{(D) } 20^\circ\text{C}}$$

Quick Tip: Newton law gives a geometric progression of temperature differences over equal times.

136. The equation of the curve passing through the point (0, 1) and having a slope of the tangent at point $P(x, y)$ is equal to $\frac{y}{x+y}$, is

(A) $x = e^{-\frac{y}{x}}$

(B) $y = e^{-\frac{y}{x}} + 1$

(C) $x = e^{\frac{x}{y}} - 1$

(D) $y = e^{\frac{x}{y}}$

Correct Answer: (D) $y = e^{\frac{x}{y}}$

Solution:

Step 1: Form the equation

$\frac{dy}{dx} = \frac{y}{x+y}$, so $\frac{dx}{dy} = \frac{x+y}{y} = \frac{x}{y} + 1$.

Step 2: Make it linear

$\frac{dx}{dy} - \frac{1}{y}x = 1$. The integrating factor is $e^{-\int dy/y} = \frac{1}{y}$.

Step 3: Solve

$\frac{d}{dy}\left(\frac{x}{y}\right) = \frac{1}{y}$, so $\frac{x}{y} = \ln y + c$.

Step 4: Use the point

At $(0,1)$: $0 = 0 + c$, so $c = 0$. Then $\frac{x}{y} = \ln y$, that is $y = e^{x/y}$. Option (D).

Final Answer:

The curve is y = e^(x/y).

$$\boxed{\text{(D) } y = e^{\frac{x}{y}}}$$

Quick Tip: Treat x as a function of y to get a linear equation.

137. If $f(x)$ is a polynomial such that Missing open brace for superscript and $f(2) = 0$, then $f(-2) = \ldots$

(A) 1

(B) -1

(C) 4

(D) -4

Correct Answer: (C) 4

Solution:

Step 1: Degree

If f has degree n, then f' has degree $n - 1$, so $[f']^2$ has degree $2n - 2$. Equating with n gives $n = 2$.

Step 2: Write f

Let $f = ax^2 + bx + c$, so $f' = 2ax + b$ and $[f']^2 = 4a^2x^2 + 4abx + b^2$. Matching: $a = 4a^2$ gives $a = \frac{1}{4}$, $b = 4ab = b$ holds, and $c = b^2$.

Step 3: Use f(2) = 0

$f(2) = 1 + 2b + b^2 = (b+1)^2 = 0$, so $b = -1$, $c = 1$.

Step 4: Evaluate

$f(x) = \frac{x^2}{4} - x + 1 = \left(\frac{x}{2} - 1\right)^2$. Then $f(-2) = (-1-1)^2 = 4$. Option (C).

Final Answer:

f(-2) is 4.

$$\boxed{\text{(C) } 4}$$

Quick Tip: Compare degrees to see f is quadratic, then match coefficients.

138. The equation of a line in cartesian form passing through (0, 0, 0) and (4, 3, c) and parallel to $\vec{a} \times \vec{b}$ where $\vec{a} = 2\hat{i} + \hat{j} + 2\hat{k}$, $\vec{b} = 3\hat{i} - 4\hat{j}$ is

(A) $\frac{x-4}{8} = \frac{y-3}{6} = \frac{z+\frac{11}{2}}{11}$

(B) $\frac{x-4}{8} = \frac{y+3}{6} = \frac{z+\frac{11}{2}}{-11}$

(C) $\frac{x-4}{8} = \frac{y-3}{6} = \frac{z+\frac{11}{2}}{-11}$

(D) $\frac{x+4}{8} = \frac{y-3}{6} = \frac{z+\frac{11}{2}}{-11}$

Correct Answer: (C) $\frac{x-4}{8} = \frac{y-3}{6} = \frac{z+\frac{11}{2}}{-11}$

Solution:

Step 1: Direction

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 2 \\ 3 & -4 & 0 \end{vmatrix} = (0+8)\hat{i} - (0-6)\hat{j} + (-8-3)\hat{k} = 8\hat{i} + 6\hat{j} - 11\hat{k}.$$

Step 2: Find c

The line passes through the origin with direction $(8, 6, -11)$, so $(4, 3, c) = \frac{1}{2}(8, 6, -11)$. Thus $c = -\frac{11}{2}$.

Step 3: Write the line

Through $(4, 3, -\frac{11}{2})$ with direction $(8, 6, -11)$:

$$\frac{x-4}{8} = \frac{y-3}{6} = \frac{z+\frac{11}{2}}{-11}$$

Step 4: Result

Option (C). Option (A) has the wrong sign of the z direction, and (B) and (D) use the wrong points.

Final Answer:

The line is (x-4)/8 = (y-3)/6 = (z+11/2)/(-11).

$$\boxed{(C)\ \frac{x-4}{8} = \frac{y-3}{6} = \frac{z+\frac{11}{2}}{-11}}$$

Quick Tip: Compute a cross b, then find c so that (4,3,c) lies along it.

139. If $\bar{a} = 4\hat{i} + \hat{j} + \hat{k}$, $\bar{b} = 2\hat{i} + \hat{j} + 2\hat{k}$ and $\bar{c} = 3\hat{i} + 4\hat{j} + 5\hat{k}$, then $(\bar{a} + \bar{b}) \cdot (\bar{b} + \bar{c}) =$

(A) 30

(B) 21

(C) 61

(D) 10

Correct Answer: (C) 61

Solution:

Step 1: Add

$\vec{a} + \vec{b} = (4+2)\hat{i} + (1+1)\hat{j} + (1+2)\hat{k} = 6\hat{i} + 2\hat{j} + 3\hat{k}$.

Step 2: Add again

$\vec{b} + \vec{c} = (2+3)\hat{i} + (1+4)\hat{j} + (2+5)\hat{k} = 5\hat{i} + 5\hat{j} + 7\hat{k}$.

Step 3: Dot product

$6 \cdot 5 + 2 \cdot 5 + 3 \cdot 7 = 30 + 10 + 21 = 61$. Option (C).

Final Answer:

The value is 61.

$\boxed{\text{(C) } 61}$

Quick Tip: Add the vectors first, then take the dot product.

140. If $\bar{a} = 2\hat{i} + \hat{j} - \hat{k}$, $\bar{b} = \hat{i} + 3\hat{k}$ and $\bar{c}$ is a unit vector, then the maximum value of the scalar triple product $[\bar{a}\bar{b}\bar{c}]$ is

(A) $\sqrt{10} + \sqrt{6}$

(B) $\sqrt{10}$

(C) $\sqrt{6}$

(D) $\sqrt{59}$

Correct Answer: (D) $\sqrt{59}$

Solution:

Step 1: Compute a cross b

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 0 & 3 \end{vmatrix} = (3-0)\hat{i} - (6+1)\hat{j} + (0-1)\hat{k} = 3\hat{i} - 7\hat{j} - \hat{k}.$$

Step 2: Maximise

$[\vec{a}\,\vec{b}\,\vec{c}] = (\vec{a} \times \vec{b}) \cdot \vec{c} = |\vec{a} \times \vec{b}| \cos\theta$, which is largest for $\cos\theta = 1$.

Step 3: Magnitude

$|\vec{a} \times \vec{b}| = \sqrt{9+49+1} = \sqrt{59}$. Option (D).

Step 4: Why not the others

The sum $\sqrt{10} + \sqrt{6}$ or the individual lengths $\sqrt{10}$, $\sqrt{6}$ are not the magnitude of the cross product.

Final Answer:

The maximum value is root 59.

$$\boxed{\text{(D) } \sqrt{59}}$$

Quick Tip: The triple product equals (a x b) dot c, which is largest when c is along a x b.

141. Let $\bar{a}, \bar{b}$ and $\bar{c}$ be three coplanar unit vectors. A unit vector $\bar{d}$ is perpendicular to them. If $(\bar{a} \times \bar{b}) \times (\bar{c} \times \bar{d}) = \frac{3}{26}\hat{i} - \frac{2}{13}\hat{j} + \frac{6}{13}\hat{k}$ and the angle between $\bar{a}$ and $\bar{b}$ is $30°$, then $\bar{c}$ is equal to...

(A) $\frac{3}{13}\hat{i}-\frac{4}{13}\hat{j}+\frac{12}{13}\hat{k}$

(B) $\frac{3}{13}\hat{i}-\frac{2}{13}\hat{j}+\frac{6}{13}\hat{k}$

(C) $\frac{3}{26}\hat{i}-\frac{4}{13}\hat{j}+\frac{12}{13}\hat{k}$

(D) $\frac{3}{26}\hat{i}-\frac{3}{26}\hat{j}+\frac{5}{26}\hat{k}$

Correct Answer: (A) $\frac{3}{13}\hat{i}-\frac{4}{13}\hat{j}+\frac{12}{13}\hat{k}$

Solution:

Step 1: Expand

Let $\vec{u}=\vec{a}\times\vec{b}$. Then $\vec{u}\times(\vec{c}\times\vec{d})=\vec{c}(\vec{u}\cdot\vec{d})-\vec{d}(\vec{u}\cdot\vec{c})$.

Step 2: Use coplanarity

The vectors are coplanar, so $\vec{u}\perp\vec{c}$ and $\vec{u}\cdot\vec{c}=0$. Also $\vec{d}$ is perpendicular to the plane, so $\vec{d}$ is parallel to $\vec{u}$.

Step 3: Magnitude

$|\vec{u}|=\sin 30^\circ=\frac{1}{2}$, so $\vec{u}\cdot\vec{d}=\pm\frac{1}{2}$. The expression becomes $\pm\frac{1}{2}\vec{c}$.

Step 4: Solve

$\vec{c}=2\left(\frac{3}{26}\hat{i}-\frac{2}{13}\hat{j}+\frac{6}{13}\hat{k}\right)=\frac{3}{13}\hat{i}-\frac{4}{13}\hat{j}+\frac{12}{13}\hat{k}$ (taking the positive sign).

Step 5: Check

Its length: $\frac{9+16+144}{169}=1$, so it is a unit vector. Option (A). The other options are not unit vectors.

Final Answer:

c = (3/13) i - (4/13) j + (12/13) k.

$$\boxed{(A)\ \frac{3}{13}\hat{i}-\frac{4}{13}\hat{j}+\frac{12}{13}\hat{k}}$$

Quick Tip: Expand u cross (c cross d) with u = a cross b and use that c lies in the plane of a and b.

142. The volume of a parallelopiped with coterminous edges $\bar{a}, \bar{b}, \bar{c}$ is 3 cubic units. The volume (in cubic units) of a tetrahedron with coterminous edges $(\bar{a} \times \bar{b}), (\bar{a} \times 2\bar{c}), (\bar{b} \times 2\bar{c})$ is...

(A) 6

(B) 12

(C) 24

(D) 36

Correct Answer: (A) 6

Solution:

Step 1: Rewrite the edges

The edges are $\vec{a} \times \vec{b}$, $2(\vec{a} \times \vec{c})$ and $2(\vec{b} \times \vec{c})$.

Step 2: Triple product

The scalar triple product is $4\,[\vec{a} \times \vec{b},\ \vec{a} \times \vec{c},\ \vec{b} \times \vec{c}]$.

Step 3: Use the identity

$[\vec{a} \times \vec{b},\ \vec{b} \times \vec{c},\ \vec{c} \times \vec{a}] = [\vec{a}\ \vec{b}\ \vec{c}]^2 = 9$. Our triple uses $\vec{a} \times \vec{c} = -(\vec{c} \times \vec{a})$, and its factors are in the order $\vec{a} \times \vec{b},\ \vec{c} \times \vec{a},\ \vec{b} \times \vec{c}$ after that, which is one swap from the cyclic order. The two sign changes cancel, so the triple product is 9. With the factors of 2, the triple product is $4 \times 9 = 36$.

Step 4: Tetrahedron

The volume of the tetrahedron is $\frac{1}{6} \times 36 = 6$. Option (A).

Final Answer:

The tetrahedron volume is 6 cubic units.

$$\boxed{\text{(A) } 6}$$

Quick Tip: Use [a x b, b x c, c x a] = [a b c] squared, then adjust for the factors of 2.

143. If the lines $2x = ky = -z$ and $6x = -y = -4z$ are perpendicular to each other then the value of k is ...

(A) 16

(B) 5

(C) 10

(D) 3

Correct Answer: (D) 3

Solution:

Step 1: First line

$2x = ky = -z = t$ gives $x = \frac{t}{2}, y = \frac{t}{k}, z = -t$. Direction ratios $\left(\frac{1}{2}, \frac{1}{k}, -1\right)$, or multiplying by $2k$, $(k, 2, -2k)$.

Step 2: Second line

$6x = -y = -4z = s$ gives $x = \frac{s}{6}, y = -s, z = -\frac{s}{4}$. Multiply by 12: $(2, -12, -3)$.

Step 3: Perpendicular condition

$k \cdot 2 + 2(-12) + (-2k)(-3) = 0$ gives $2k - 24 + 6k = 0$, so $8k = 24$ and $k = 3$. Option (D).

Final Answer:

The value of k is 3.

(D) 3

Quick Tip: Write each line in symmetric form to read its direction ratios, then set their dot product to zero.

144. The vector equation of plane in parametric form, passing through the points (-1, 2, 0), (2, 2, -1) and parallel to the line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$ is

(A) $\bar{r} = (-\hat{i} + 2\hat{j}) + \lambda(3\hat{i} - \hat{k}) + \mu(\hat{i} + \hat{j} - \hat{k})$

(B) $\bar{r} = (-\hat{i} + 2\hat{j}) + \lambda(3\hat{i} - \hat{k}) + \mu(\hat{i} + 2\hat{j} - \hat{k})$

(C) $\bar{r} = (\hat{i} - 2\hat{j}) + \lambda(3\hat{i} + \hat{k}) + \mu(\hat{i} + \hat{j} - \hat{k})$

(D) $\bar{r} = (-\hat{i} + 2\hat{j}) + \lambda(3\hat{i} - \hat{k}) + \mu(\hat{i} - \hat{j} + \hat{k})$

Correct Answer: (A) $\bar{r} = (-\hat{i} + 2\hat{j}) + \lambda(3\hat{i} - \hat{k}) + \mu(\hat{i} + \hat{j} - \hat{k})$

Solution:

Step 1: Direction vectors

The line joining $(-1, 2, 0)$ and $(2, 2, -1)$ has direction $3\hat{i} - \hat{k}$.

Step 2: Given line

$\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$ means $\frac{x-1}{1} = \frac{y+\frac{1}{2}}{1} = \frac{z+1}{-1}$, so its direction is $\hat{i} + \hat{j} - \hat{k}$.

Step 3: Write the plane

With position vector $-\hat{i}+2\hat{j}$: $\vec{r}=(-\hat{i}+2\hat{j})+\lambda(3\hat{i}-\hat{k})+\mu(\hat{i}+\hat{j}-\hat{k})$.

Option (A).

Step 4: Common mistake

Option (B) uses $\hat{i}+2\hat{j}-\hat{k}$, which forgets to divide $2y$ by 2.

Final Answer:

The plane is r = (-i + 2j) + lambda(3i - k) + mu(i + j - k).

$$\boxed{(A)\ \vec{r}=(-\hat{i}+2\hat{j})+\lambda(3\hat{i}-\hat{k})+\mu(\hat{i}+\hat{j}-\hat{k})}$$

Quick Tip: Use one point as position vector, the line joining the two points as one direction, and the given line as the other.

145. If d is the distance of point (2, 5, 10) from the plane containing the lines $\bar{r}=(4\hat{j}-\hat{k})+\lambda(\hat{i}+2\hat{j}-2\hat{k})$ and $\bar{r}=(2\hat{i}+\hat{j})+\mu(\hat{i}+2\hat{j}-2\hat{k})$, then d^2 =

(A) 145

(B) 13

(C) 90

(D) 79

Correct Answer: (C) 90

Solution:

Step 1: Data

Both lines have direction $\vec{d} = \hat{i} + 2\hat{j} - 2\hat{k}$. Points on them: $P_1(0, 4, -1)$ and $P_2(2, 1, 0)$.

Step 2: Normal

$\overrightarrow{P_1P_2} = (2, -3, 1)$. $\vec{n} = \vec{d} \times \overrightarrow{P_1P_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -2 \\ 2 & -3 & 1 \end{vmatrix} = (2 - 6)\hat{i} - (1 + 4)\hat{j}$ $+ (-3 - 4)\hat{k} = (-4, -5, -7)$.

Step 3: Plane

Use $(4, 5, 7)$ and the point $(0, 4, -1)$: $4x + 5(y - 4) + 7(z + 1) = 0$, that is $4x + 5y + 7z - 13 = 0$.

Step 4: Distance

For $(2, 5, 10)$: $8 + 25 + 70 - 13 = 90$. The norm is $\sqrt{16 + 25 + 49} = \sqrt{90}$. So $d = \frac{90}{\sqrt{90}} = \sqrt{90}$ and $d^2 = 90$. Option (C).

Final Answer:

d squared is 90.

(C) 90

Quick Tip: The two lines are parallel, so the normal is the cross product of their common direction and the vector joining one point on each line.

146. The values of p and q so that the line joining the points (7, p, 2) and (q, -2, 5) may be parallel to the line joining the points (2, -3, 5) and (-6, -15, 11) are

(A) $p = 4, q = -3$

(B) $p = 4, q = 3$

(C) $p = -4, q = 3$

(D) $p = -4, q = -3$

Correct Answer: (B) $p = 4, q = 3$

Solution:

Step 1: Second line

Joining $(2, -3, 5)$ and $(-6, -15, 11)$ gives direction ratios $(-8, -12, 6)$, proportional to $(4, 6, -3)$.

Step 2: First line

Joining $(7, p, 2)$ and $(q, -2, 5)$ gives $(q - 7, \ -2 - p, \ 3)$.

Step 3: Proportion

Compare the z components: $3 : 6 = \frac{1}{2}$, so each component of the first is half of the corresponding one: $q - 7 = -4$ gives $q = 3$, and $-2 - p = -6$ gives $p = 4$. Option (B).

Final Answer:

p = 4 and q = 3.

$$\boxed{\text{(B) } p = 4, \ q = 3}$$

Quick Tip: Direction ratios of parallel lines must be proportional.

147. The minimum value of $z = 3x + 5y$, subject to constraints $x \leq 80$, $y \geq 60$, $x + y \leq 200$ & $x, y \geq 0$ occurs at the point...

(A) $(0, 200)$

(B) $(60, 0)$

(C) $(0, 60)$

(D) $(80, 60)$

Correct Answer: (C) $(0, 60)$

Solution:

Step 1: Feasible region

The constraints are $x \leq 80$, $y \geq 60$, $x + y \leq 200$, $x, y \geq 0$. The corner points are $(0, 60)$, $(80, 60)$, $(80, 120)$ and $(0, 200)$.

Step 2: Evaluate z

$(0, 60) : 300$, $(80, 60) : 240 + 300 = 540$, $(80, 120) : 240 + 600 = 840$, $(0, 200) : 1000$.

Step 3: Minimum

The smallest value is 300 at $(0, 60)$. Option (C). The point $(60, 0)$ is not feasible because $y \geq 60$ fails.

Final Answer:

The minimum occurs at (0, 60).

$$\boxed{\text{(C) } (0, 60)}$$

Quick Tip: Find the corner points of the feasible region and compare z at each.

148. If a fair coin is tossed 8 times, then the probability of getting at most 2 heads is...

(A) $\frac{37}{64}$

(B) $\frac{37}{256}$

(C) $\frac{37}{128}$

(D) $\frac{37}{512}$

Correct Answer: (B) $\frac{37}{256}$

Solution:

Step 1: Setup

$n = 8$, $p = \frac{1}{2}$. $P(X = r) = {}^8C_r \left(\frac{1}{2}\right)^8$.

Step 2: Add up

${}^8C_0 + {}^8C_1 + {}^8C_2 = 1 + 8 + 28 = 37$.

Step 3: Result

$P(X \leq 2) = \frac{37}{256}$. Option (B).

Final Answer:

The probability is 37/256.

$$\boxed{\text{(B) } \frac{37}{256}}$$

Quick Tip: Add the binomial probabilities for 0, 1 and 2 heads.

149. If a random variable X has a probability mass function $P(x) = \begin{cases} kx^2, & \text{for } x = 1, 2, 3, 4 \\ 0, & \text{otherwise} \end{cases}$, then the mean of X is ...

(A) 3.232

(B) 2.221

(C) 3.333

(D) 2.222

Correct Answer: (C) 3.333

Solution:

Step 1: Find k

$k(1 + 4 + 9 + 16) = 1$ gives $30k = 1$, so $k = \frac{1}{30}$.

Step 2: Mean

$E(X) = \sum xP(x) = k(1 + 8 + 27 + 64) = \frac{100}{30}$.

Step 3: Value

$\frac{100}{30} = 3.333$. Option (C).

Final Answer:

The mean is 3.333.

(C) 3.333

Quick Tip: Find k from the total probability, then compute the sum of x P(x).

150. If E_1 and E_2 are equally likely, mutually exclusive and exhaustive events and $P(A|E_1) = 0.2$, $P(A|E_2) = 0.3$, then $P(E_1|A)$ equal to...

(A) $\frac{1}{5}$

(B) $\frac{4}{5}$

(C) $\frac{2}{3}$

(D) $\frac{2}{5}$

Correct Answer: (D) $\frac{2}{5}$

Solution:

Step 1: Setup

$P(E_1) = P(E_2) = \frac{1}{2}$, $P(A|E_1) = 0.2$, $P(A|E_2) = 0.3$.

Step 2: Bayes theorem

$$P(E_1|A) = \frac{P(E_1)P(A|E_1)}{P(E_1)P(A|E_1) + P(E_2)P(A|E_2)} = \frac{\frac{1}{2}(0.2)}{\frac{1}{2}(0.2) + \frac{1}{2}(0.3)}$$

Step 3: Simplify

$\frac{0.2}{0.5} = \frac{2}{5}$. Option (D).

Final Answer:

The probability is 2/5.

$$\boxed{\text{(D) } \frac{2}{5}}$$

Quick Tip: Apply Bayes theorem with equal priors, so the priors cancel.