

MHT CET 2026 PCM Question Paper April 17 Shift 1 with Solutions



General Instructions

- (i) This booklet contains 150 questions, each provided with a complete, step-by-step solution.
- (ii) It comprises 150 single-correct multiple-choice questions.
- (iii) Attempt each question on your own before reviewing the given solution.

Physics

Questions 1 to 50 | 1 mark each | 50 marks

1. Resultant of two vectors $\vec{P}$ and $\vec{Q}$ is of magnitude A. If $\vec{Q}$ is reversed, then the resultant is of magnitude B. The value of $A^2 + B^2$ is

- (A) $P^2 + Q^2$
- (B) $P^2 - Q^2$
- (C) $2(P^2 + Q^2)$
- (D) $2(P^2 - Q^2)$

Correct Answer: (C) $2(P^2 + Q^2)$

Solution:

Step 1: Understanding the Concept

The magnitude of the resultant of $\vec{P}$ and $\vec{Q}$ at angle θ is given by the parallelogram law: $R^2 = P^2 + Q^2 + 2PQ \cos \theta$.

Step 2: Key Formula or Approach

Reversing $\vec{Q}$ changes the angle to $180^\circ - \theta$, so $\cos \theta$ changes sign.

Step 3: Detailed Explanation

$$A^2 = P^2 + Q^2 + 2PQ \cos \theta$$

$$B^2 = P^2 + Q^2 - 2PQ \cos \theta$$

Adding, the cross terms cancel:

$$A^2 + B^2 = 2(P^2 + Q^2)$$

Final Answer:

$A^2 + B^2 = 2(P^2 + Q^2)$, option (C).

$$\boxed{2(P^2 + Q^2) \text{ (C)}}$$

Quick Tip: Write A^2 and B^2 with the cosine law and add.

2. Mass and volume of a body are found to be (6.00 ± 0.03) kg and (2.00 ± 0.02) m³ respectively. Then the density of a body is

- (A) (3.00 ± 0.010) kg/m³
- (B) (3.00 ± 0.015) kg/m³
- (C) (3.00 ± 0.020) kg/m³
- (D) (3.00 ± 0.045) kg/m³

Correct Answer: (D) $(3.00 \pm 0.045) \text{ kg/m}^3$

Solution:

Step 1: Understanding the Concept

Density is $\rho = m/V$. In a quotient the maximum fractional errors add up.

Step 2: Key Formula or Approach

$$\frac{\Delta\rho}{\rho} = \frac{\Delta m}{m} + \frac{\Delta V}{V}$$

Step 3: Detailed Explanation

$$\rho = \frac{6.00}{2.00} = 3.00 \text{ kg/m}^3.$$

$$\frac{\Delta m}{m} = \frac{0.03}{6.00} = 0.005 \text{ and } \frac{\Delta V}{V} = \frac{0.02}{2.00} = 0.01.$$

$$\frac{\Delta\rho}{\rho} = 0.015 \Rightarrow \Delta\rho = 3.00 \times 0.015 = 0.045$$

So $\rho = (3.00 \pm 0.045) \text{ kg/m}^3$.

Final Answer:

The density is $(3.00 \pm 0.045) \text{ kg/m}^3$, option (D).

$(3.00 \pm 0.045) \text{ kg/m}^3 \text{ (D)}$

Quick Tip: For a quotient, the fractional errors add.

3. Two balls A and B are projected at an angle of 45° and 60° respectively, so that the maximum heights reached are same for both. The ratio of initial velocity of projection of ball A to that for ball B is

$$(\sin 30^\circ = \cos 60^\circ = \frac{1}{2}, \sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}, \sin 60^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2})$$

(A) $2 : \sqrt{3}$

(B) $\sqrt{3} : 2$

(C) $\sqrt{2} : \sqrt{3}$

(D) $\sqrt{3} : \sqrt{2}$

Correct Answer: (D) $\sqrt{3} : \sqrt{2}$

Solution:

Step 1: Understanding the Concept

The maximum height depends on the vertical component of initial

velocity: $H = \frac{(u \sin \theta)^2}{2g}$.

Step 2: Key Formula or Approach

Equal heights mean equal vertical components: $u_A \sin 45^\circ = u_B \sin 60^\circ$.

Step 3: Detailed Explanation

$$\frac{u_A}{u_B} = \frac{\sin 60^\circ}{\sin 45^\circ} = \frac{\sqrt{3}/2}{1/\sqrt{2}} = \frac{\sqrt{3}\sqrt{2}}{2} = \frac{\sqrt{3}}{\sqrt{2}}$$

The ratio is $\sqrt{3} : \sqrt{2}$.

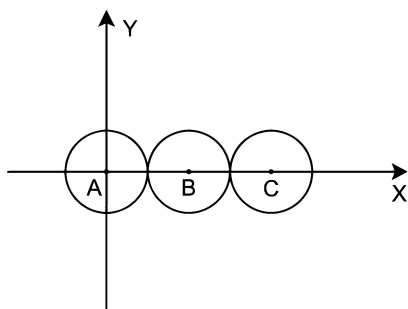
Final Answer:

The ratio $u_A : u_B$ is $\sqrt{3} : \sqrt{2}$, option (D).

$$\sqrt{3} : \sqrt{2} \text{ (D)}$$

Quick Tip: Maximum height is $H = \frac{u^2 \sin^2 \theta}{2g}$; equate for both balls.

4. Three identical spheres, each of mass 'm' kg, are kept as shown in the figure, touching each other, with their centers on a straight line. If their centers are marked as A, B, C respectively, the distance of center of mass of the system from A is



- (A) $\frac{AB+BC}{3}$
(B) $\frac{AB+AC}{3}$
(C) $\frac{AC+BC}{2}$
(D) $\frac{AB+BC+AC}{3}$

Correct Answer: (B) $\frac{AB+AC}{3}$

Solution:

Step 1: Understanding the Concept

The figure shows three identical spheres in a row, touching, with centres A, B and C on a straight line. Each has the same mass m .

Step 2: Key Formula or Approach

With A at the origin, B is at distance AB and C is at distance AC along the line.

$$x_{cm} = \frac{m(0) + m(AB) + m(AC)}{3m}$$

Step 3: Detailed Explanation

$$x_{cm} = \frac{AB + AC}{3}$$

The spheres touch, so $AB = 2r$ and $AC = 4r$, giving $x_{cm} = 2r$, which is the centre B, as expected from symmetry.

Final Answer:

The centre of mass is at $\frac{AB+AC}{3}$ from A, option (B).

$$\boxed{\frac{AB + AC}{3} \text{ (B)}}$$

Quick Tip: Take A at the origin; the centre of mass is the average of the positions of the centres.

5. Four point masses m , $2m$, $3m$ and $4m$ are kept at the corners A, B, C and D respectively of a square ABCD of side ' b '. The moment of inertia of the system about an axis perpendicular to the plane of the square and passing through the point D is

- (A) $12mb^2$
- (B) $10mb^2$
- (C) $8mb^2$
- (D) $5mb^2$

Correct Answer: (C) $8mb^2$

Solution:

Step 1: Understanding the Concept

For point masses, $I = \sum m_i r_i^2$, where r_i is the perpendicular distance from the axis. The axis is perpendicular to the square through corner D.

Step 2: Key Formula or Approach

Distances from D: to A and C it is b (adjacent corners), to B it is the diagonal $b\sqrt{2}$, to D it is 0.

Step 3: Detailed Explanation

A (mass m): mb^2 .

B (mass $2m$): $2m(b\sqrt{2})^2 = 4mb^2$.

C (mass $3m$): $3mb^2$.

D (mass $4m$): 0.

$$I = mb^2 + 4mb^2 + 3mb^2 = 8mb^2$$

Final Answer:

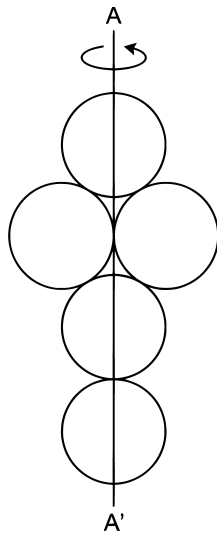
The moment of inertia is $8mb^2$, option (C).

$8mb^2$ (C)

Quick Tip: Add mr^2 for each mass, with r measured from the axis through D.



6. A system of five solid spheres each of mass ' m ' and radius ' r ' are rotating about an axis AA' as shown in figure. Hence the moment of inertia of the system about the axis of rotation AA' is



- (A) $6mr^2$
- (B) $5mr^2$
- (C) $4mr^2$
- (D) $3mr^2$

Correct Answer: (C) $4mr^2$

Solution:

Step 1: Understanding the Concept

In the figure, three spheres have their centres on the axis AA' (top, one low, and the bottom one). The two spheres at the sides each touch the axis, so their centres are at distance r from it.

Step 2: Key Formula or Approach

Solid sphere about a diameter: $\frac{2}{5}mr^2$. For the side spheres use the parallel axis theorem: $I = \frac{2}{5}mr^2 + mr^2 = \frac{7}{5}mr^2$.

Step 3: Detailed Explanation

Three spheres on the axis: $3 \times \frac{2}{5}mr^2 = \frac{6}{5}mr^2$.

Two spheres at the side: $2 \times \frac{7}{5}mr^2 = \frac{14}{5}mr^2$.

$$I = \frac{6}{5}mr^2 + \frac{14}{5}mr^2 = \frac{20}{5}mr^2 = 4mr^2$$

Final Answer:

The moment of inertia is $4mr^2$, option (C).

$4mr^2$ (C)

Quick Tip: Spheres on the axis contribute $\frac{2}{5}mr^2$ each; the two touching the axis also need the parallel axis term.

7. Two bodies A and B have moment of inertia I_A and I_B , and angular momenta L_A and L_B respectively. Both of them have same kinetic energy of rotation. So the ratio L_A to L_B is

- (A) $\frac{I_A}{I_B}$
- (B) $\frac{I_A^2}{I_B^2}$
- (C) $\sqrt{\frac{I_A}{I_B}}$
- (D) $\sqrt{\frac{I_B}{I_A}}$

Correct Answer: (C) $\sqrt{\frac{I_A}{I_B}}$

Solution:

Step 1: Understanding the Concept

Rotational kinetic energy is $K = \frac{1}{2}I\omega^2$ and angular momentum is $L = I\omega$.

Step 2: Key Formula or Approach

Eliminate ω : $K = \frac{L^2}{2I}$, so $L = \sqrt{2IK}$.

Step 3: Detailed Explanation

Both bodies have the same K , so $L \propto \sqrt{I}$.

$$\frac{L_A}{L_B} = \sqrt{\frac{I_A}{I_B}}$$

Final Answer:

The ratio is $\sqrt{I_A/I_B}$, option (C).

$$\boxed{\sqrt{\frac{I_A}{I_B}} \text{ (C)}}$$

Quick Tip: Use $K = \frac{L^2}{2I}$ so $L = \sqrt{2IK}$.

8. The speed with which the earth would have to rotate about its axis so that a person on the equator would weigh $\frac{3}{5}$ th as much as at present is (g = gravitational acceleration, R = equatorial radius of the earth.)

- (A) $\sqrt{\frac{3}{5}gR}$
 (B) $\sqrt{\frac{2g}{5R}}$
 (C) $\sqrt{\frac{3g}{5R}}$
 (D) $\sqrt{\frac{5R}{2g}}$

Correct Answer: (B) $\sqrt{\frac{2g}{5R}}$

Solution:

Step 1: Understanding the Concept

Because of rotation, the apparent weight at the equator is reduced by the centripetal term. $g' = g - \omega^2 R$.

Step 2: Key Formula or Approach

We want $g' = \frac{3}{5}g$.

Step 3: Detailed Explanation

$$g - \omega^2 R = \frac{3}{5}g \Rightarrow \omega^2 R = \frac{2}{5}g$$

$$\omega = \sqrt{\frac{2g}{5R}}$$

Final Answer:

The required angular speed is $\sqrt{\frac{2g}{5R}}$, option (B).

$$\boxed{\sqrt{\frac{2g}{5R}} \text{ (B)}}$$

Quick Tip: At the equator, effective gravity is $g' = g - \omega^2 R$.

9. The excess pressure inside a spherical water drop A is four times that of another water drop B. Then, the ratio of the mass of water drop A to that of drop B is

- (A) 1 : 8
- (B) 1 : 16
- (C) 1 : 32
- (D) 1 : 64

Correct Answer: (D) 1 : 64

Solution:

Step 1: Understanding the Concept

A water drop has one free surface, so the excess pressure inside it is

$$\Delta P = \frac{2T}{r}.$$

Step 2: Key Formula or Approach

$$\Delta P \propto \frac{1}{r} \text{ and mass } m \propto r^3.$$

Step 3: Detailed Explanation

$$\Delta P_A = 4\Delta P_B \text{ gives } r_A = \frac{r_B}{4}.$$

$$\frac{m_A}{m_B} = \left(\frac{r_A}{r_B}\right)^3 = \left(\frac{1}{4}\right)^3 = \frac{1}{64}$$

Final Answer:

The mass ratio is 1 : 64, option (D).

$$1 : 64 \text{ (D)}$$

Quick Tip: Excess pressure in a drop is $\frac{2T}{r}$ and mass is proportional to r^3 .

10. In a capillary tube of area of cross-section ' a ' water rises to height ' h '. To what height will water rise in a capillary tube of area of cross-section $4a$?

- (A) $\frac{h}{4}$
- (B) $\frac{h}{2}$
- (C) $2h$
- (D) $4h$

Correct Answer: (B) $\frac{h}{2}$

Solution:**Step 1: Understanding the Concept**

In a capillary of radius r , $h = \frac{2T \cos \theta}{r \rho g}$, so $h \propto \frac{1}{r}$.

Step 2: Key Formula or Approach

Area $a = \pi r^2$, so area $4a$ means radius $2r$.

Step 3: Detailed Explanation

The radius doubles, so the height halves:

$$h' = \frac{h}{2}$$

Final Answer:

The new height is $h/2$, option (B).

$$\boxed{\frac{h}{2} \text{ (B)}}$$

Quick Tip: Rise is inversely proportional to the radius, $h \propto \frac{1}{r}$.

11. A soap bubble of radius $\frac{1}{\sqrt{\pi}}$ cm is expanded to radius $\frac{3}{\sqrt{\pi}}$ cm. Surface tension of soap solution is 25 dyne/cm. The work done during expansion in erg is

- (A) 800
- (B) 1200
- (C) 1600
- (D) 2400

Correct Answer: (C) 1600

Solution:**Step 1: Understanding the Concept**

A soap bubble has an inner and an outer surface. The work done equals surface tension times the increase in total surface area.

Step 2: Key Formula or Approach

$$W = T \cdot 2 \cdot 4\pi (r_2^2 - r_1^2)$$

Step 3: Detailed Explanation

$$r_1^2 = \frac{1}{\pi} \text{ cm}^2 \text{ and } r_2^2 = \frac{9}{\pi} \text{ cm}^2.$$

$$W = 25 \times 2 \times 4\pi \times \frac{8}{\pi} = 25 \times 64 = 1600 \text{ erg}$$

Final Answer:

The work done is 1600 erg, option (C).

1600 erg (C)

Quick Tip: A soap bubble has two surfaces, so $W = T \times 2 \times \Delta A$.

12. In an external environment of temperature (T) kelvin, a sphere at temperature ($3T$) kelvin has cooling rate R_1 . When the temperature of that sphere falls to ($2T$) kelvin, the cooling rate R_2 of the sphere will become

- (A) $\frac{15}{16} R_1$
- (B) $\frac{11}{16} R_1$
- (C) $\frac{7}{16} R_1$
- (D) $\frac{3}{16} R_1$

Correct Answer: (D) $\frac{3}{16} R_1$

Solution:

Step 1: Understanding the Concept

A hot sphere in a surrounding at temperature T loses heat by radiation. By Stefan's law the net rate is proportional to $T_s^4 - T_0^4$.

Step 2: Key Formula or Approach

$R \propto T_s^4 - T^4$ with the surroundings at T .

Step 3: Detailed Explanation

At $3T$: $R_1 \propto (3T)^4 - T^4 = 80T^4$.

At $2T$: $R_2 \propto (2T)^4 - T^4 = 15T^4$.

$$R_2 = \frac{15}{80}R_1 = \frac{3}{16}R_1$$

Final Answer:

The cooling rate becomes $\frac{3}{16}R_1$, option (D).

$\frac{3}{16}R_1 \text{ (D)}$

Quick Tip: Radiation rate $\propto T^4 - T_0^4$ for a body in surroundings at T_0 .

13. 'x' joule of heat is incident on a body. Out of it, the total heat reflected and transmitted by it is 'y' joule. The absorption coefficient of the body is

- (A) $\frac{x}{y}$
- (B) $\frac{x-y}{x}$
- (C) $\frac{y}{x-y}$
- (D) $\frac{x}{y-x}$

Correct Answer: (B) $\frac{x-y}{x}$

Solution:

Step 1: Understanding the Concept

Incident heat splits into reflected, transmitted and absorbed parts: $x = Q_r + Q_t + Q_a$.

Step 2: Key Formula or Approach

The absorption coefficient is $a = \frac{Q_a}{x}$.

Step 3: Detailed Explanation

Reflected plus transmitted is y , so absorbed heat is $x - y$.

$$a = \frac{x - y}{x}$$

Final Answer:

The absorption coefficient is $\frac{x-y}{x}$, option (B).

$\frac{x - y}{x} \text{ (B)}$

Quick Tip: Absorptance equals absorbed heat divided by incident heat.

14. Consider two rods 1 and 2 of same length. They have different specific heats (C_1, C_2), thermal conductivities (K_1, K_2) and area of cross-section (A_1, A_2) respectively. Both the rods have temperatures (T_1, T_2) at their ends. If their rate of loss of heat due to conduction is equal, then

(A) $A_1 K_2 = A_2 K_1$

(B) $A_1 K_1 = A_2 K_2$

(C) $\frac{A_1 K_1}{C_1} = \frac{A_2 K_2}{C_2}$

(D) $\frac{A_1 K_2}{C_1} = \frac{A_2 K_1}{C_2}$

Correct Answer: (B) $A_1 K_1 = A_2 K_2$

Solution:

Step 1: Understanding the Concept

The rate of heat flow through a rod is $\frac{Q}{t} = \frac{KA(T_1 - T_2)}{L}$.

Step 2: Key Formula or Approach

Both rods have the same length and the same end temperatures, so L and ΔT cancel.

Step 3: Detailed Explanation

Equal rates give $K_1 A_1 = K_2 A_2$.

The specific heat does not appear in the steady conduction rate, so it does not enter the relation.

Final Answer:

The relation is $A_1 K_1 = A_2 K_2$, option (B).

$$A_1 K_1 = A_2 K_2 \text{ (B)}$$

Quick Tip: Rate of conduction is $\frac{KA \Delta T}{L}$; specific heat plays no role.

15. A Carnot engine, whose efficiency is 40% takes heat from a source maintained at temperature 600K. To have an efficiency 60%, the intake temperature for the same exhaust temperature should be

- (A) 1800 K
- (B) 900 K
- (C) 720 K
- (D) 360 K

Correct Answer: (B) 900 K

Solution:

Step 1: Understanding the Concept

The efficiency of a Carnot engine is $\eta = 1 - \frac{T_2}{T_1}$, where T_2 is the sink (exhaust) temperature.

Step 2: Key Formula or Approach

First find T_2 from the 40% case, then the new source temperature for 60%.

Step 3: Detailed Explanation

$$0.4 = 1 - \frac{T_2}{600}, \text{ so } T_2 = 360 \text{ K.}$$

$$\text{For 60\%: } 0.6 = 1 - \frac{360}{T'_1}, \text{ so } \frac{360}{T'_1} = 0.4.$$

$$T'_1 = \frac{360}{0.4} = 900 \text{ K}$$

Final Answer:

The source temperature must be 900 K, option (B).

$$900 \text{ K (B)}$$

Quick Tip: Use $\eta = 1 - \frac{T_2}{T_1}$ with the same exhaust temperature.

16. The heat energy that must be supplied to 14 g of nitrogen at room temperature to raise its temperature by 48°C at constant pressure is (R = gas constant, Molecular weight of nitrogen (N_2) = 28)

- (A) $72R$
- (B) $84R$
- (C) $96R$
- (D) $108R$

Correct Answer: (B) $84R$

Solution:

Step 1: Understanding the Concept

Nitrogen is a diatomic gas, so $C_p = \frac{7}{2}R$.

Step 2: Key Formula or Approach

$$n = \frac{14}{28} = 0.5 \text{ mol and } Q = nC_p\Delta T.$$

Step 3: Detailed Explanation

$$Q = 0.5 \times \frac{7}{2}R \times 48 = 0.5 \times 3.5 \times 48 R = 84R$$

Final Answer:

The heat needed is $84R$, option (B).

$$84R \text{ (B)}$$

Quick Tip: $Q = nC_p\Delta T$ with $C_p = \frac{7}{2}R$ for a diatomic gas.

17. The speeds of the seven molecules are 1, 3, 5, 7, 2, 4 and 6 km/s respectively. The ratio of their r.m.s. velocity and average velocity will be

(A) $2\sqrt{5} : 1$

(B) $\sqrt{5} : 1$

(C) $1 : 2\sqrt{5}$

(D) $\sqrt{5} : 2$

Correct Answer: (D) $\sqrt{5} : 2$

Solution:**Step 1: Understanding the Concept**

Average speed is the plain mean. The rms speed is the square root of the mean of squares.

Step 2: Key Formula or Approach

Speeds: 1, 2, 3, 4, 5, 6, 7 km/s.

Step 3: Detailed Explanation

Mean: $\frac{1 + 2 + 3 + 4 + 5 + 6 + 7}{7} = \frac{28}{7} = 4.$

Mean of squares: $\frac{1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2 + 7^2}{7} = \frac{140}{7} = 20$, so rms
 $= \sqrt{20} = 2\sqrt{5}.$

$$\frac{v_{rms}}{v_{avg}} = \frac{2\sqrt{5}}{4} = \frac{\sqrt{5}}{2}$$

Final Answer:

The ratio is $\sqrt{5} : 2$, option (D).

$$\boxed{\sqrt{5} : 2 \text{ (D)}}$$

Quick Tip: Compute rms and mean of the given list.



18. Two simple pendulums of lengths L_1 and L_2 have periodic time T_1 and T_2 respectively ($T_1 > T_2$). The time period of the pendulum of length $(L_1 - L_2)$ is

$$[(L_1 - L_2) > 60 \text{ cm}]$$

(A) $\sqrt{T_1^2 + T_2^2}$

(B) $\sqrt{T_1^2 - T_2^2}$

(C) $T_1 + T_2$

(D) $T_1 - T_2$

Correct Answer: (B) $\sqrt{T_1^2 - T_2^2}$

Solution:

Step 1: Understanding the Concept

For a simple pendulum, $T = 2\pi\sqrt{L/g}$, so $T^2 = \frac{4\pi^2}{g}L$.

Step 2: Key Formula or Approach

$$T_1^2 = \frac{4\pi^2 L_1}{g} \text{ and } T_2^2 = \frac{4\pi^2 L_2}{g}.$$

Step 3: Detailed Explanation

$$T_1^2 - T_2^2 = \frac{4\pi^2}{g}(L_1 - L_2) = T^2$$

$$T = \sqrt{T_1^2 - T_2^2}$$

Final Answer:

The time period is $\sqrt{T_1^2 - T_2^2}$, option (B).

$$\boxed{\sqrt{T_1^2 - T_2^2} \text{ (B)}}$$

Quick Tip: Use $T^2 \propto L$ and subtract.

19. For a particle executing S.H.M., its potential energy is 8 times its kinetic energy at a certain displacement ' x ' from the mean position. If ' A ' is the amplitude of S.H.M., the value of ' x ' is

- (A) $\frac{2}{\sqrt{3}}A$
- (B) $\frac{\sqrt{2}}{3}A$
- (C) $\frac{2\sqrt{2}}{3}A$
- (D) $\frac{3}{\sqrt{2}}A$

Correct Answer: (C) $\frac{2\sqrt{2}}{3}A$

Solution:

Step 1: Understanding the Concept

At displacement x , $PE = \frac{1}{2}m\omega^2x^2$ and $KE = \frac{1}{2}m\omega^2(A^2 - x^2)$.

Step 2: Key Formula or Approach

$PE = 8 KE$ gives $x^2 = 8(A^2 - x^2)$.

Step 3: Detailed Explanation

$$9x^2 = 8A^2 \Rightarrow x = \frac{2\sqrt{2}}{3}A$$

Final Answer:

The displacement is $\frac{2\sqrt{2}}{3}A$, option (C).

$$\boxed{\frac{2\sqrt{2}}{3}A \text{ (C)}}$$

Quick Tip: Use $PE = \frac{1}{2}m\omega^2x^2$ and $KE = \frac{1}{2}m\omega^2(A^2 - x^2)$.

20. A particle executing simple harmonic motion starts from mean position with amplitude ' A ' and periodic time ' T '. At what displacement is its speed one-fourth of the maximum speed?

- (A) $\frac{A}{\sqrt{15}}$
(B) $\frac{A}{4}$
(C) $\frac{4A}{\sqrt{15}}$
(D) $\frac{A\sqrt{15}}{4}$

Correct Answer: (D) $\frac{A\sqrt{15}}{4}$

Solution:

Step 1: Understanding the Concept

In SHM, speed at displacement x is $v = \omega\sqrt{A^2 - x^2}$ and the maximum speed is ωA .

Step 2: Key Formula or Approach

We need $v = \frac{\omega A}{4}$.

Step 3: Detailed Explanation

$$\sqrt{A^2 - x^2} = \frac{A}{4} \Rightarrow A^2 - x^2 = \frac{A^2}{16}$$

$$x^2 = \frac{15A^2}{16} \Rightarrow x = \frac{A\sqrt{15}}{4}$$

Final Answer:

The displacement is $\frac{A\sqrt{15}}{4}$, option (D).

$$\boxed{\frac{A\sqrt{15}}{4} \text{ (D)}}$$

Quick Tip: Use $v = \omega\sqrt{A^2 - x^2}$ and $v_{max} = \omega A$.



21. A and B are two wires whose fundamental frequencies are 256 Hz and 382 Hz respectively. When third harmonic of A and the second harmonic of B are sounded together, the number of beats heard in two second will be

- (A) 8
- (B) 6
- (C) 4
- (D) 2

Correct Answer: (A) 8

Solution:

Step 1: Understanding the Concept

The n th harmonic of a stretched wire has frequency n times the fundamental.

Step 2: Key Formula or Approach

Third harmonic of A: $3 \times 256 = 768$ Hz. Second harmonic of B: $2 \times 382 = 764$ Hz.

Step 3: Detailed Explanation

Beat frequency = $768 - 764 = 4$ Hz, so 4 beats per second.

In two seconds the number of beats is $4 \times 2 = 8$.

Final Answer:

There are 8 beats in two seconds, option (A).

Quick Tip: Find the two frequencies, subtract to get beats per second, then multiply by 2 s.

22. In case of stationary wave pattern, which of the following statement is CORRECT ?

- (A) The distance between consecutive antinodes is equal to the wavelength.
- (B) In a pipe open at both ends when an air column is vibrated, only even harmonics are present.
- (C) When an air column is vibrated in a pipe closed at one end, all harmonics are present.
- (D) In the case of a stretched string, when vibrated frequency of the first overtone is same as the second harmonic.

Correct Answer: (D) In the case of a stretched string, when vibrated frequency of the first overtone is same as the second harmonic.

Solution:

Step 1: Understanding the Concept

A harmonic is a multiple of the fundamental frequency. The first overtone is the first frequency above the fundamental.

Step 2: Detailed Explanation

(A) Consecutive antinodes are separated by $\lambda/2$, not by a full wavelength.

(B) A pipe open at both ends supports all harmonics, both even and

odd.

(C) A pipe closed at one end supports only odd harmonics.

(D) For a stretched string, the fundamental is the first harmonic, and the next frequency, the first overtone, is $2f_0$, the second harmonic.

This statement is correct.

Final Answer:

Statement (D) is correct, option (D).

(D)

Quick Tip: For a string fixed at both ends, the first overtone is the second harmonic.



23. Out of the following musical instruments which is ' NOT ' a percussion instrument?

- (A) Drum
- (B) Saxophone
- (C) Daphali
- (D) Xylophone

Correct Answer: (B) Saxophone

Solution:

Step 1: Understanding the Concept

Percussion instruments produce sound when struck or shaken. Wind instruments produce sound by blowing air through them.

Step 2: Detailed Explanation

A drum, a daphali (a hand-held frame drum) and a xylophone are all struck, so they are percussion instruments.

A saxophone is blown with a reed, so it is a wind instrument.

Final Answer:

The saxophone is not a percussion instrument, option (B).

Saxophone (B)

Quick Tip: Percussion instruments are struck; a saxophone is a wind instrument.



24. The enclosed air inside a closed box of rigid walls has pressure P_1 . The density of air inside this box is constant. On heating the gas, the pressure of the enclosed air is increased from P_1 to P_2 . Now it is observed that the sound travels 1.4 times faster than at pressure P_1 . The ratio P_2 by P_1 is

- (A) 2.25
- (B) 1.96
- (C) 1.5
- (D) 1.4

Correct Answer: (B) 1.96

Solution:**Step 1: Understanding the Concept**

For a gas, $v = \sqrt{\frac{\gamma P}{\rho}}$. In a rigid closed box the density stays constant.

Step 2: Key Formula or Approach

So $v \propto \sqrt{P}$ and $\frac{v_2}{v_1} = \sqrt{\frac{P_2}{P_1}}$.

Step 3: Detailed Explanation

$$1.4 = \sqrt{\frac{P_2}{P_1}} \Rightarrow \frac{P_2}{P_1} = 1.4^2 = 1.96$$

Final Answer:

The ratio P_2/P_1 is 1.96, option (B).

1.96 (B)

Quick Tip: Speed of sound is $v = \sqrt{\gamma P/\rho}$ and the density is fixed.

25. Select the WRONG statement about polar molecules.

- (A) Polar molecules have permanent dipole moment.
- (B) A molecule in which centre of mass of positive charges does not coincide with the centre of mass of negative charges is a polar molecule.
- (C) Polar molecules have symmetrical shape.
- (D) Polar molecules act as tiny electric dipoles.

Correct Answer: (C) Polar molecules have symmetrical shape.

Solution:

Step 1: Understanding the Concept

A polar molecule has a permanent dipole moment because the centres of positive and negative charge do not coincide.

Step 2: Detailed Explanation

(A) True: polar molecules have a permanent dipole moment.

(B) True: this is the definition.

(D) True: they behave like tiny electric dipoles.

(C) A symmetrical shape tends to make the charge centres coincide, giving zero dipole moment. So a polar molecule cannot be described as symmetrical, and (C) is the wrong statement.

Final Answer:

Statement (C) is the wrong one, option (C).

(C)

Quick Tip: A polar molecule has an asymmetric charge distribution, not a symmetrical shape.



26. An electric dipole of length 3cm is placed with its axis making an angle of 60° with a uniform electric field of 5×10^4 N/C. It experiences a torque of $9\sqrt{3}$ Nm. The magnitude of charge on the dipole is ($\sin 60^\circ = \frac{\sqrt{3}}{2}$)

(A) 0.6×10^{-2} C

(B) 1.2×10^{-2} C

(C) 1.8×10^{-2} C

(D) 2.4×10^{-2} C

Correct Answer: (B) $1.2 \times 10^{-2} \text{ C}$

Solution:

Step 1: Understanding the Concept

A dipole in a uniform field feels a torque $\tau = pE \sin \theta$, where $p = q \times \ell$ and ℓ is the length of the dipole.

Step 2: Key Formula or Approach

$$\tau = q \ell E \sin \theta \Rightarrow q = \frac{\tau}{\ell E \sin \theta}$$

Step 3: Detailed Explanation

$$\ell = 3 \text{ cm} = 0.03 \text{ m}, E = 5 \times 10^4 \text{ N/C}, \sin 60^\circ = \frac{\sqrt{3}}{2}.$$

$$\ell E \sin \theta = 0.03 \times 5 \times 10^4 \times \frac{\sqrt{3}}{2} = 750\sqrt{3}$$

$$q = \frac{9\sqrt{3}}{750\sqrt{3}} = 0.012 = 1.2 \times 10^{-2} \text{ C}$$

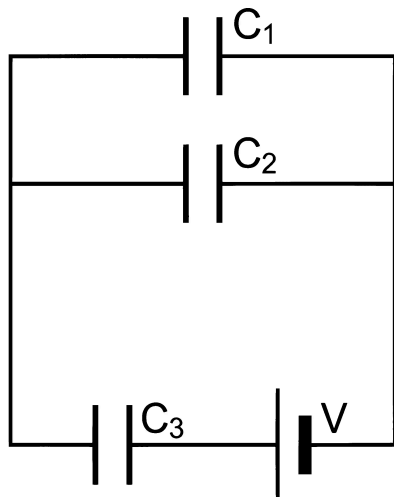
Final Answer:

The charge on the dipole is $1.2 \times 10^{-2} \text{ C}$, option (B).

$$\boxed{1.2 \times 10^{-2} \text{ C (B)}}$$

Quick Tip: Torque $\tau = pE \sin \theta$ with $p = q \times 2l$ (the dipole length).

27. Three capacitors C_1 , C_2 and C_3 are connected to voltage source V as shown in figure. The voltage across C_3 will be



- (A) $\frac{C_3 V}{(C_1 + C_2 + C_3)}$
(B) $\frac{(C_1 + C_2) V}{C_3}$
(C) $\frac{(C_2 + C_3) V}{C_1 + C_2}$
(D) $\frac{(C_1 + C_2) V}{(C_1 + C_2 + C_3)}$

Correct Answer: (D) $\frac{(C_1 + C_2) V}{(C_1 + C_2 + C_3)}$

Solution:

Step 1: Understanding the Concept

From the figure, C_1 and C_2 are connected in parallel, and this combination is in series with C_3 and the battery V .

Step 2: Key Formula or Approach

Parallel pair: $C_{12} = C_1 + C_2$. In a series chain, all capacitors hold the same charge Q and the voltage divides inversely with capacitance.

Step 3: Detailed Explanation

$$C_{eq} = \frac{(C_1 + C_2)C_3}{C_1 + C_2 + C_3}, \quad Q = C_{eq}V$$

$$V_3 = \frac{Q}{C_3} = \frac{(C_1 + C_2)V}{C_1 + C_2 + C_3}$$

Final Answer:

The voltage across C_3 is $\frac{(C_1+C_2)V}{C_1+C_2+C_3}$, option (D).

$$\boxed{\frac{(C_1 + C_2)V}{C_1 + C_2 + C_3} \text{ (D)}}$$

Quick Tip: C_1 and C_2 are in parallel, and that pair is in series with C_3 and the source.

28. Two capacitors, C_1 and C_2 have their capacitances in the ratio 1:2. V_s and V_p are the potential differences applied across the series and parallel combination of C_1 and C_2 respectively, so that the energy stored in the two cases becomes same. The ratio V_s to V_p is

- (A) $\sqrt{2} : 3$
- (B) $3 : \sqrt{2}$
- (C) $2 : \sqrt{3}$
- (D) $\sqrt{3} : 2$

Correct Answer: (B) $3 : \sqrt{2}$

Solution:

Step 1: Understanding the Concept

Let $C_1 = C$ and $C_2 = 2C$. Series: $C_s = \frac{C \cdot 2C}{3C} = \frac{2C}{3}$. Parallel: $C_p = 3C$.

Step 2: Key Formula or Approach

Energy stored is $U = \frac{1}{2}C_{eq}V^2$.

Step 3: Detailed Explanation

Equal energies: $\frac{1}{2}C_s V_s^2 = \frac{1}{2}C_p V_p^2$.

$$\frac{V_s^2}{V_p^2} = \frac{C_p}{C_s} = \frac{3C}{2C/3} = \frac{9}{2}$$

$$\frac{V_s}{V_p} = \frac{3}{\sqrt{2}}$$

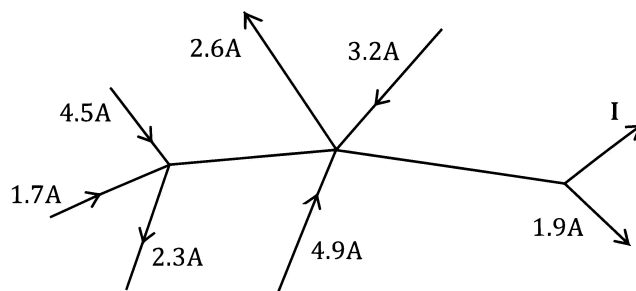
Final Answer:

The ratio $V_s : V_p$ is $3 : \sqrt{2}$, option (B).

$3 : \sqrt{2} \text{ (B)}$

Quick Tip: Energy is $\frac{1}{2}C_{eq}V^2$; equate for the two combinations.

29. In the following figure, the current I is equal to



- (A) 2.9 A
- (B) 3.9 A
- (C) 6.2 A
- (D) 7.5 A

Correct Answer: (D) 7.5 A

Solution:

Step 1: Understanding the Concept

At any junction, the current entering equals the current leaving.

Step 2: Key Formula or Approach

Move from left to right through the three junctions of the figure.

Step 3: Detailed Explanation

Left junction: in $4.5 + 1.7 = 6.2$ A, out 2.3 A. So $6.2 - 2.3 = 3.9$ A goes along the connecting wire.

Middle junction: in $3.9 + 3.2 + 4.9 = 12.0$ A, out 2.6 A. So $12.0 - 2.6 = 9.4$ A goes to the right.

Right junction: $9.4 = 1.9 + I$.

$$I = 9.4 - 1.9 = 7.5 \text{ A}$$

Final Answer:

The current I is 7.5 A, option (D).

$$7.5 \text{ A (D)}$$

Quick Tip: Apply Kirchhoff's current law at each junction in turn.

30. With a resistance ' X ' connected in series with a galvanometer of resistance 100Ω , it acts as a voltmeter of range 0 - 15 V. To double the range, a resistance of 1500Ω is to be connected in series with ' X '. The value of ' X ' in Ω is

- (A) 1000
- (B) 1200
- (C) 1400
- (D) 1600

Correct Answer: (C) 1400

Solution:**Step 1: Understanding the Concept**

A galvanometer with series resistance acts as a voltmeter. The full-scale current I_g is the same for both ranges.

Step 2: Key Formula or Approach

Range 1: $15 = I_g(100 + X)$. Range 2: $30 = I_g(100 + X + 1500)$.

Step 3: Detailed Explanation

Divide: $\frac{30}{15} = \frac{100 + X + 1500}{100 + X}$.

$$2(100 + X) = 1600 + X \Rightarrow 200 + 2X = 1600 + X \Rightarrow X = 1400 \, \Omega$$

Final Answer:

$X = 1400 \, \Omega$, option (C).

1400 Ω (C)

Quick Tip: The full-scale current of the galvanometer is fixed: $V = I_g(G + R_{series})$.

31. The magnetic field at the center of circular coil carrying current ' I ' for a single turn of a given length of wire is ' B '. The same wire is bent in a circular coil having two turns. When the same current passes through it, the value of magnetic field becomes

- (A) $4B$
- (B) $2B$
- (C) $\frac{B}{2}$
- (D) $\frac{B}{4}$

Correct Answer: (A) $4B$

Solution:

Step 1: Understanding the Concept

Field at the centre of a coil is $B = \frac{\mu_0 NI}{2r}$.

Step 2: Key Formula or Approach

Wire length $\ell = 2\pi rN$ is fixed, so $r \propto \frac{1}{N}$.

Step 3: Detailed Explanation

One turn: $B = \frac{\mu_0 I}{2r}$.

Two turns: radius $= \frac{r}{2}$, so $B' = \frac{\mu_0(2)I}{2(r/2)} = \frac{4\mu_0 I}{2r} = 4B$.

Final Answer:

The field becomes $4B$, option (A).

$4B \text{ (A)}$

Quick Tip: Same wire length: with N turns the radius becomes r/N , so $B \propto N^2$.

32. The magnetic field at perpendicular distance ' r ' from a long wire carrying current ' I ' is ' B '. The magnetic field at perpendicular distance ' $2r$ ' from the same wire is

- (A) $\frac{B}{4}$
- (B) $\frac{B}{2}$
- (C) $2B$
- (D) $4B$

Correct Answer: (B) $\frac{B}{2}$

Solution:

Step 1: Understanding the Concept

The field of a long straight current-carrying wire at distance r is B

$$= \frac{\mu_0 I}{2\pi r}.$$

Step 2: Detailed Explanation

$$\text{At } 2r: B' = \frac{\mu_0 I}{2\pi(2r)} = \frac{B}{2}.$$

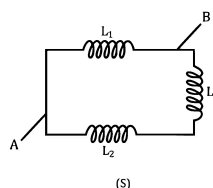
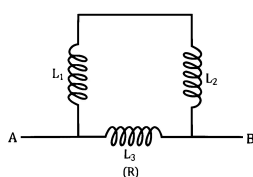
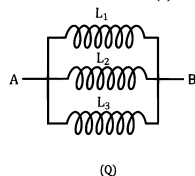
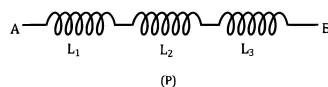
Final Answer:

The field becomes $B/2$, option (B).

$$\boxed{\frac{B}{2} \text{ (B)}}$$

Quick Tip: $B = \frac{\mu_0 I}{2\pi r}$, so $B \propto \frac{1}{r}$.

33. Three coils of inductance, $L_1 = 2 \text{ H}$, $L_2 = 3 \text{ H}$ and $L_3 = 6 \text{ H}$ are connected so that they are separated from each other. To obtain the effective inductance of $\frac{18}{11} \text{ H}$ between points A and B, out of the following figures, the correct one is



- (A) P
- (B) S
- (C) Q
- (D) R

Correct Answer: (B) S

Solution:

Step 1: Understanding the Concept

Inductors follow the same rules as resistors when there is no mutual coupling: series adds, parallel adds reciprocals. $L_1 = 2 \text{ H}$, $L_2 = 3 \text{ H}$, $L_3 = 6 \text{ H}$.

Step 2: Key Formula or Approach

Read each figure carefully and compute its effective value.

Step 3: Detailed Explanation

(P) All three in series: $2 + 3 + 6 = 11 \text{ H}$.

(Q) All three in parallel: $\frac{1}{L} = \frac{1}{2} + \frac{1}{3} + \frac{1}{6} = 1$, so $L = 1 \text{ H}$.

(R) L_1 and L_2 in series (5 H) joined across L_3 in parallel: $\frac{5 \times 6}{11} = \frac{30}{11} \text{ H}$.

(S) L_1 is directly between A... In this figure, L_1 connects the two terminals A and B on one path, while L_2 and L_3 in series (9 H) form the other path. So $L_1 \parallel (L_2 + L_3)$: $\frac{2 \times 9}{2 + 9} = \frac{18}{11} \text{ H}$.

Final Answer:

Figure (S) gives $\frac{18}{11} \text{ H}$, option (B).

$S \text{ (B)}$

Quick Tip: Evaluate the effective inductance of each arrangement (coils are far apart, so no mutual inductance).



34. Two coils have self inductance L_1 and L_2 . The current through them is increasing at constant rate. If the power dissipated in both the coils is same, then the ratio of energy stored in the coil having inductance L_1 to that in L_2 is

- (A) $\frac{L_1^2}{L_2^2}$
- (B) $\frac{L_2^2}{L_1^2}$
- (C) $\frac{L_1}{L_2}$
- (D) $\frac{L_2}{L_1}$

Correct Answer: (D) $\frac{L_2}{L_1}$

Solution:

Step 1: Understanding the Concept

With a steady rate of change of current, the power in an inductor is $P = LI \frac{dI}{dt}$. The stored energy is $U = \frac{1}{2}LI^2$.

Step 2: Key Formula or Approach

Equal power with the same dI/dt gives $L_1 I_1 = L_2 I_2$, so $I \propto \frac{1}{L}$.

Step 3: Detailed Explanation

$$\frac{U_1}{U_2} = \frac{L_1 I_1^2}{L_2 I_2^2} = \frac{L_1}{L_2} \cdot \frac{L_2^2}{L_1^2} = \frac{L_2}{L_1}$$

Final Answer:

The energy ratio is $\frac{L_2}{L_1}$, option (D).

$$\frac{L_2}{L_1} \text{ (D)}$$

Quick Tip: Power in an inductor is $P = LI \frac{dI}{dt}$, and energy is $U = \frac{1}{2} LI^2$.

35. The resistor ' R ' inductance ' L ' and capacitance ' C ' are connected in series with an a.c. source. When ' L ' is removed from the circuit, the phase difference between voltage and current in the circuit is $\frac{\pi}{3}$. If instead, C is removed from the circuit, the phase difference is again $\frac{\pi}{3}$. The power factor of the circuit is

- (A) $\frac{\sqrt{3}}{2}$
- (B) $\frac{1}{2}$
- (C) $\frac{1}{\sqrt{2}}$
- (D) 1

Correct Answer: (D) 1

Solution:**Step 1: Understanding the Concept**

With L removed, the circuit is R and C : $\tan \phi = \frac{X_C}{R}$. With C removed, it is R and L : $\tan \phi = \frac{X_L}{R}$.

Step 2: Key Formula or Approach

Both give $\tan \frac{\pi}{3} = \sqrt{3}$, so $X_C = X_L = \sqrt{3}R$.

Step 3: Detailed Explanation

In the full series circuit, the net reactance is $X_L - X_C = 0$. This is the condition for resonance.

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = R$$

$$\cos \phi = \frac{R}{Z} = 1$$

Final Answer:

The power factor is 1, option (D).

1 (D)

Quick Tip: Both phase differences are $\frac{\pi}{3}$, so $X_L = X_C$.

36. The LC series resonant circuit produces a resonant frequency ' f '. If L is tripled and ' C ' is increased by $3C$, the resonant frequency will be

- (A) $\frac{f}{3}$
- (B) $\frac{f}{2\sqrt{3}}$
- (C) $6f$
- (D) $\frac{f}{3\sqrt{2}}$

Correct Answer: (B) $\frac{f}{2\sqrt{3}}$

Solution:

Step 1: Understanding the Concept

The resonant frequency of an LC circuit is $f = \frac{1}{2\pi\sqrt{LC}}$.

Step 2: Key Formula or Approach

New inductance $3L$. The capacitance is increased by $3C$, so it becomes $C + 3C = 4C$.

Step 3: Detailed Explanation

$$f' = \frac{1}{2\pi\sqrt{3L \cdot 4C}} = \frac{1}{2\pi\sqrt{12LC}} = \frac{f}{\sqrt{12}} = \frac{f}{2\sqrt{3}}$$

Final Answer:

The new frequency is $\frac{f}{2\sqrt{3}}$, option (B).

$\frac{f}{2\sqrt{3}} \text{ (B)}$

Quick Tip: $f = \frac{1}{2\pi\sqrt{LC}}$; here $L \rightarrow 3L$ and $C \rightarrow C + 3C = 4C$.

37. In LCR series circuit at resonance

- (A) the current is minimum.
- (B) the impedance is maximum.
- (C) the current leads the voltage by $\frac{\pi}{2}$
- (D) the current and voltage are in phase.

Correct Answer: (D) the current and voltage are in phase.

Solution:

Step 1: Understanding the Concept

At resonance the inductive and capacitive reactances are equal in size and opposite in effect: $X_L = X_C$.

Step 2: Detailed Explanation

The impedance is then $Z = R$, which is the minimum value, so the current $I = V/R$ is maximum.

The net reactance is zero, so the phase difference is zero: current and voltage are in phase.

This matches statement (D).

Final Answer:

At resonance the current and voltage are in phase, option (D).

(D)

Quick Tip: At resonance $X_L = X_C$, so impedance equals R .

38. A transformer has 120 turns in the primary coil and carries 5A current. Input power is one kilowatt. To have 560 V output, the number of turns in secondary coil will be

- (A) 168
- (B) 200
- (C) 336
- (D) 400

Correct Answer: (C) 336

Solution:

Step 1: Understanding the Concept

For a transformer $\frac{V_s}{V_p} = \frac{N_s}{N_p}$.

Step 2: Key Formula or Approach

Input power $P = V_p I_p$ gives $V_p = \frac{1000}{5} = 200 \text{ V}$.

Step 3: Detailed Explanation

$$N_s = N_p \frac{V_s}{V_p} = 120 \times \frac{560}{200} = 336$$

Final Answer:

The secondary has 336 turns, option (C).

336 (C)

Quick Tip: Find the primary voltage from $P = V_p I_p$, then use the turns ratio.

39. In telescopes for a given wavelength, the objectives with large aperture are used for

- (A) greater resolution.
- (B) reducing lens aberration.
- (C) greater magnification.
- (D) ease of manufacture.

Correct Answer: (A) greater resolution.

Solution:

Step 1: Understanding the Concept

The limit of resolution of a telescope objective of aperture D is $\Delta\theta$
$$= \frac{1.22\lambda}{D}.$$

Step 2: Detailed Explanation

A larger aperture means a smaller $\Delta\theta$ for the same wavelength, so two close stars can be separated. This is greater resolution.

A bigger lens also gathers more light, but the question asks for the reason given for a fixed wavelength, which is resolution.

Final Answer:

Large apertures give greater resolution, option (A).

Greater resolution (A)

Quick Tip: Resolving power of a telescope is $\frac{D}{1.22\lambda}$, so it grows with aperture D .

40. A lens of refractive index ' μ ' has focal length ' f '. When the lens is immersed in a liquid of refractive index ' μ_0 ', its focal length becomes ' f_0 '. Then ' f_0 ' is given by

- (A) $\frac{(\mu_0 - \mu)f}{\mu(\mu_0 - 1)}$
 (B) $\frac{\mu(\mu_0 - 1)f}{(\mu_0 - \mu)}$
 (C) $\frac{(\mu - \mu_0)f}{\mu_0(\mu - 1)}$
 (D) $\frac{\mu_0(\mu - 1)f}{(\mu - \mu_0)}$

Correct Answer: (D) $\frac{\mu_0(\mu - 1)f}{(\mu - \mu_0)}$

Solution:

Step 1: Understanding the Concept

In air, $\frac{1}{f} = (\mu - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$. In a liquid, the relative index is $\frac{\mu}{\mu_0}$.

Step 2: Key Formula or Approach

$$\frac{1}{f_0} = \left(\frac{\mu}{\mu_0} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

Step 3: Detailed Explanation

Divide the two equations:

$$\frac{f_0}{f} = \frac{\mu - 1}{\frac{\mu}{\mu_0} - 1} = \frac{\mu_0(\mu - 1)}{\mu - \mu_0}$$

$$f_0 = \frac{\mu_0(\mu - 1)f}{\mu - \mu_0}$$

Final Answer:

The focal length in the liquid is $\frac{\mu_0(\mu-1)f}{\mu-\mu_0}$, option (D).

$$\boxed{\frac{\mu_0(\mu-1)f}{\mu-\mu_0} \text{ (D)}}$$

Quick Tip: Use the lens maker's formula in air and in liquid and divide.

41. In Young's double slit experiment, width of the second slit is double the width of first slit, consequently the amplitude of the light from two slits. ' I_m ' is the maximum intensity. The resultant intensity ' I ' when they interfere with the phase difference of ϕ is given by

- (A) $\frac{I_m}{9}(1 + 8\cos^2 \frac{\phi}{2})$
- (B) $\frac{I_m}{7}(3 + 5\cos^2 \frac{\phi}{2})$
- (C) $\frac{I_m}{5}(1 + 2\cos^2 \frac{\phi}{2})$
- (D) $\frac{I_m}{3}(1 + 6\cos^2 \frac{\phi}{2})$

Correct Answer: (A) $\frac{I_m}{9}(1 + 8\cos^2 \frac{\phi}{2})$

Solution:**Step 1: Understanding the Concept**

The stem is garbled, so we take the amplitude of the second slit to be twice that of the first: $a_1 = a$, $a_2 = 2a$. This is the reading under which the maximum intensity is $I_m = (a + 2a)^2 = 9a^2$.

Step 2: Key Formula or Approach

$$I = a_1^2 + a_2^2 + 2a_1a_2 \cos \phi$$

Step 3: Detailed Explanation

$$I = a^2 + 4a^2 + 4a^2 \cos \phi = a^2 (5 + 4 \cos \phi)$$

Use $\cos \phi = 2 \cos^2 \frac{\phi}{2} - 1$:

$$I = a^2 \left(1 + 8 \cos^2 \frac{\phi}{2} \right)$$

With $a^2 = \frac{I_m}{9}$:

$$I = \frac{I_m}{9} \left(1 + 8 \cos^2 \frac{\phi}{2} \right)$$

Final Answer:

Under the stated assumption, $I = \frac{I_m}{9} \left(1 + 8 \cos^2 \frac{\phi}{2} \right)$, option (A).

$$\boxed{\frac{I_m}{9} \left(1 + 8 \cos^2 \frac{\phi}{2} \right) \text{ (A)}}$$

Quick Tip: Assumption: the amplitudes from the two slits are in the ratio 1 : 2, with $I_m = 9a^2$ as the maximum.

42. In a single slit diffraction experiment, for wavelength ' λ ', half angular width of the principal maxima is ' θ '. Also for wavelength of light ' $p\lambda$ ', the half angular width of the principal maxima is ' $q\theta$ '. The ratio of the half angular widths of the first secondary maxima in the first case to second case will be

- (A) $p : 1$
- (B) $1 : q$
- (C) $p : q$
- (D) $q : p$

Correct Answer: (B) $1 : q$

Solution:

Step 1: Understanding the Concept

In single slit diffraction, the first minimum is at $\sin \theta = \frac{\lambda}{a}$, which gives the half angular width θ of the central maximum. The first secondary maximum lies at about $\frac{3\lambda}{2a}$.

Step 2: Key Formula or Approach

So the angle of the first secondary maximum is 1.5 times the half angular width of the principal maximum.

Step 3: Detailed Explanation

Case 1: half angular width = θ , so the first secondary maximum is at 1.5θ .

Case 2: half angular width = $q\theta$, so the first secondary maximum is at $1.5q\theta$.

$$\text{Ratio} = \frac{1.5\theta}{1.5q\theta} = \frac{1}{q}$$

Final Answer:

The ratio is $1 : q$, option (B).

$$1 : q \text{ (B)}$$

Quick Tip: Half angular width of the principal maximum is $\theta = \frac{\lambda}{a}$; the first secondary maximum lies near $\frac{3\lambda}{2a}$.

43. Photoelectric emission is observed from a metallic surface for frequencies ν_1 and ν_2 of the incident light rays ($\nu_1 > \nu_2$). If the ratio of maximum value of kinetic energy of the photoelectron emitted in first case to that in second case $3 : K$, then the threshold frequency of the metallic surface is

- (A) $\frac{K\nu_1 - \nu_2}{K - 1}$
- (B) $\frac{K - 1}{K\nu_1 - \nu_2}$
- (C) $\frac{K - 3}{K\nu_1 - 3\nu_2}$
- (D) $\frac{K\nu_1 - 3\nu_2}{K - 3}$

Correct Answer: (D) $\frac{K\nu_1 - 3\nu_2}{K - 3}$

Solution:**Step 1: Understanding the Concept**

Einstein's photoelectric equation: $K_{max} = h\nu - h\nu_0 = h(\nu - \nu_0)$.

Step 2: Key Formula or Approach

$$\frac{K_1}{K_2} = \frac{\nu_1 - \nu_0}{\nu_2 - \nu_0} = \frac{3}{K}$$

Step 3: Detailed Explanation

Cross-multiply: $K(\nu_1 - \nu_0) = 3(\nu_2 - \nu_0)$.

$K\nu_1 - K\nu_0 = 3\nu_2 - 3\nu_0$, so $K\nu_1 - 3\nu_2 = (K - 3)\nu_0$.

$$\nu_0 = \frac{K\nu_1 - 3\nu_2}{K - 3}$$

Final Answer:

The threshold frequency is $\frac{K\nu_1 - 3\nu_2}{K - 3}$, option (D).

$$\boxed{\frac{K\nu_1 - 3\nu_2}{K - 3} \text{ (D)}}$$

Quick Tip: Use $K_{max} = h(\nu - \nu_0)$ for both frequencies.

44. The kinetic energy of a free electron increases to 3 times the previous K.E. The ratio of new de-Broglie wavelength to previous de-Broglie wavelength is

- (A) $\frac{1}{\sqrt{3}}$
- (B) $\frac{1}{3}$
- (C) 3
- (D) $\sqrt{3}$

Correct Answer: (A) $\frac{1}{\sqrt{3}}$

Solution:

Step 1: Understanding the Concept

The de Broglie wavelength of a particle of kinetic energy K is $\lambda = \frac{h}{p}$
 $= \frac{h}{\sqrt{2mK}}$.

Step 2: Detailed Explanation

If K becomes $3K$, then

$$\frac{\lambda'}{\lambda} = \sqrt{\frac{K}{3K}} = \frac{1}{\sqrt{3}}$$

Final Answer:

The wavelength ratio is $\frac{1}{\sqrt{3}}$, option (A).

$$\boxed{\frac{1}{\sqrt{3}} \text{ (A)}}$$

Quick Tip: $\lambda = \frac{h}{\sqrt{2mK}}$, so $\lambda \propto \frac{1}{\sqrt{K}}$.

45. For wavelength of visible radiation of Hydrogen spectrum, Balmer gave an equation as $\lambda = \frac{xm^2}{m^2-4}$, where m is the integer value. The value of x in terms of Rydberg's constant R is

- (A) $\frac{R}{4}$
(B) $\frac{4}{R}$
(C) $2R$
(D) $4R$

Correct Answer: (B) $\frac{4}{R}$

Solution:

Step 1: Understanding the Concept

The Balmer series arises from transitions to $n = 2$ in hydrogen: $\frac{1}{\lambda}$
 $= R \left(\frac{1}{4} - \frac{1}{m^2} \right).$

Step 2: Key Formula or Approach

Combine the fractions in the bracket.

Step 3: Detailed Explanation

$$\frac{1}{\lambda} = R \frac{m^2 - 4}{4m^2}$$

$$\lambda = \frac{4}{R} \cdot \frac{m^2}{m^2 - 4}$$

Comparing with $\lambda = \frac{xm^2}{m^2 - 4}$ gives $x = \frac{4}{R}.$

Final Answer:

The constant is $x = \frac{4}{R}$, option (B).

$$\boxed{\frac{4}{R} \text{ (B)}}$$

Quick Tip: Start from $\frac{1}{\lambda} = R \left(\frac{1}{2^2} - \frac{1}{m^2} \right)$ and invert.



46. The orbital magnetic moment (m_{orb}) of a revolving electron around the nucleus varies with the principal quantum number (n) as

- (A) $m_{orb} \propto n^2$
- (B) $m_{orb} \propto n$
- (C) $m_{orb} \propto \frac{1}{n^2}$
- (D) $m_{orb} \propto \frac{1}{n}$

Correct Answer: (B) $m_{orb} \propto n$

Solution:

Step 1: Understanding the Concept

For an electron in orbit, the magnetic moment is $m_{orb} = \frac{e}{2m_e} L$, where L is the orbital angular momentum.

Step 2: Key Formula or Approach

In Bohr's model, $L = \frac{nh}{2\pi}$.

Step 3: Detailed Explanation

$$m_{orb} = \frac{e}{2m_e} \cdot \frac{nh}{2\pi} = n \frac{eh}{4\pi m_e}$$

So $m_{orb} \propto n$.

Final Answer:

The magnetic moment is proportional to n , option (B).

$$m_{orb} \propto n (B)$$

Quick Tip: $m_{orb} = \frac{e}{2m} L$ with $L = \frac{nh}{2\pi}$.

47. The half-life of the isotope ${}_{11}\text{Na}^{24}$ is 15 hr. How much time does it take for $(\frac{7}{8})^{\text{th}}$ of a sample of this isotope to decay ?

- (A) 45 hour
- (B) 60 hour
- (C) 75 hour
- (D) 90 hour

Correct Answer: (A) 45 hour

Solution:**Step 1: Understanding the Concept**

After each half-life, half of the remaining nuclei are left: $N = N_0 \left(\frac{1}{2}\right)^n$.

Step 2: Key Formula or Approach

$\frac{7}{8}$ decayed means $\frac{N}{N_0} = \frac{1}{8} = \left(\frac{1}{2}\right)^3$, so $n = 3$.

Step 3: Detailed Explanation

$$t = n t_{1/2} = 3 \times 15 = 45 \text{ hours}$$

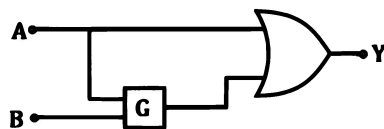
Final Answer:

The time required is 45 hours, option (A).

45 hours (A)

Quick Tip: If $\frac{7}{8}$ decays, $\frac{1}{8}$ remains, which takes three half-lives.

48. To get the truth table shown from the following logic circuit, the gate G should be



A	B	Y
0	0	0
0	1	0
1	0	1
1	1	1

- (A) NAND gate.
- (B) NOR gate.
- (C) AND gate.
- (D) OR gate.

Correct Answer: (C) AND gate.

Solution:**Step 1: Understanding the Concept**

From the figure, input A goes straight to an OR gate, and also into gate G together with B. The OR gate then combines A with the output of G. So $Y = A + G$.

Step 2: Key Formula or Approach

The truth table shows $Y = 0$ for $A = 0$ (for both values of B) and $Y = 1$ for $A = 1$. So $Y = A$.

Step 3: Detailed Explanation

When $A = 0$: $Y = 0 + G(0, B)$. This must be 0 for $B = 0$ and $B = 1$, so $G(0, 0) = 0$ and $G(0, 1) = 0$.

When $A = 1$: $Y = 1 + G = 1$ whatever G is.

An AND gate gives $G(0, 0) = 0$ and $G(0, 1) = 0$, which matches. Then $Y = A + AB = A(1 + B) = A$.

Final Answer:

Gate G is an AND gate, option (C).

AND (C)

Quick Tip: The output is $Y = A + G(A, B)$ and the table shows $Y = A$.

49. When the Zener diode is used as a voltage regulator, it is connected in

- (A) forward bias and in parallel with load.
- (B) reverse bias and in series with load.
- (C) forward bias and in series with load.
- (D) reverse bias and in parallel with load.

Correct Answer: (D) reverse bias and in parallel with load.

Solution:

Step 1: Understanding the Concept

A Zener diode holds a nearly constant voltage across it when it is

reverse biased in the breakdown region.

Step 2: Detailed Explanation

To hold the load voltage constant, the Zener is connected in parallel with the load, and it works in reverse bias at its breakdown (Zener) voltage. A series resistor limits the current.

So it is in reverse bias and parallel with the load.

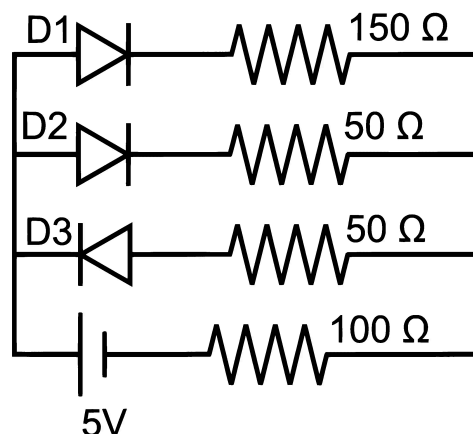
Final Answer:

The Zener is connected in reverse bias, in parallel with the load, option (D).

Reverse bias, parallel (D)

Quick Tip: A Zener regulator uses reverse bias and is connected in parallel with the load.

50. In the circuit shown in figure all the three diodes D1, D2, D3 have forward resistance 50Ω each and infinite backward resistance. If the battery voltage is $5V$, the current through 100Ω resistance is



- (A) 60 mA
- (B) 30 mA
- (C) 20 mA
- (D) 10 mA

Correct Answer: (B) 30 mA

Solution:

Step 1: Understanding the Concept

A forward-biased diode here has resistance $50\ \Omega$. A reverse-biased diode has infinite resistance and blocks current.

Step 2: Key Formula or Approach

From the figure, the battery's positive terminal feeds D1 and D2 in the forward direction, while D3 points the opposite way, so it is reverse biased and carries no current.

Step 3: Detailed Explanation

Branch 1: $D_1 (50) + 150 = 200\ \Omega$.

Branch 2: $D_2 (50) + 50 = 100\ \Omega$.

Parallel: $\frac{200 \times 100}{300} = \frac{200}{3}\ \Omega$.

Add the 100 ohm series resistor: $R_{tot} = 100 + \frac{200}{3} = \frac{500}{3}\ \Omega$.

$$I = \frac{5}{500/3} = 0.03\ \text{A} = 30\ \text{mA}$$

The same current flows through the 100 ohm resistor.

Final Answer:

The current through the 100 ohm resistor is 30 mA, option (B).

30 mA (B)

Quick Tip: Find which diodes conduct, then combine the resistances.

Chemistry

Questions 51 to 100 | 1 mark each | 50 marks

51. Calculate the number of molecules present in 2.24 dm^3 of carbon dioxide at STP ?

- (A) 2.011×10^{22}
- (B) 3.011×10^{22}
- (C) 6.022×10^{22}
- (D) 0.1×10^{23}

Correct Answer: (C) 6.022×10^{22}

Solution:

Step 1: Understanding the Concept

At STP, one mole of an ideal gas occupies 22.4 dm^3 . One mole contains 6.022×10^{23} molecules (Avogadro's number).

So we first convert the given volume into moles, then into molecules.

Step 2: Key Formula or Approach

Moles $n = \frac{V}{22.4 \text{ dm}^3}$ and number of molecules $N = n \times N_A$.

Step 3: Detailed Explanation

Moles of CO_2 :

$$n = \frac{2.24}{22.4} = 0.1 \text{ mol}$$

Number of molecules:

$$N = 0.1 \times 6.022 \times 10^{23} = 6.022 \times 10^{22}$$

Options (A) and (B) come from slips in the multiplication, and (D) 0.1×10^{23} equals 1×10^{22} , which is not the product above. Only 6.022×10^{22} matches the calculation.

Final Answer:

2.24 dm³ of CO₂ at STP is 0.1 mol, which holds 6.022×10^{22} molecules. This is option (C).

$$6.022 \times 10^{22} \text{ (C)}$$

Quick Tip: At STP one mole of any gas occupies 22.4 dm³, so find the moles first and multiply by Avogadro's number.

52. What is uncertainty in velocity of an electron if uncertainty in measurement of position is 50 pm ? ($m_e = 9.1 \times 10^{-31}$ kg, $h = 6.63 \times 10^{-34}$ Js, $\pi = 3.142$)

- (A) $0.98 \times 10^6 \text{ ms}^{-1}$
- (B) $1.16 \times 10^6 \text{ ms}^{-1}$
- (C) $2.61 \times 10^6 \text{ ms}^{-1}$
- (D) $3.77 \times 10^6 \text{ ms}^{-1}$

Correct Answer: (B) $1.16 \times 10^6 \text{ ms}^{-1}$

Solution:

Step 1: Understanding the Concept

Heisenberg's uncertainty principle says the position and momentum

of a particle cannot both be known exactly. The product of their uncertainties has a minimum value.

Step 2: Key Formula or Approach

$$\Delta x \cdot m \Delta v \geq \frac{h}{4\pi} \Rightarrow \Delta v = \frac{h}{4\pi m \Delta x}$$

Step 3: Detailed Explanation

Convert the position uncertainty: $\Delta x = 50 \text{ pm} = 5 \times 10^{-11} \text{ m}$.

Denominator: $4\pi m \Delta x = 4(3.142)(9.1 \times 10^{-31})(5 \times 10^{-11}) = 5.718 \times 10^{-40}$.

$$\Delta v = \frac{6.63 \times 10^{-34}}{5.718 \times 10^{-40}} = 1.16 \times 10^6 \text{ m s}^{-1}$$

The other options do not follow from this calculation.

Final Answer:

The minimum uncertainty in the electron's velocity is $1.16 \times 10^6 \text{ m/s}$, option (B).

$$\boxed{1.16 \times 10^6 \text{ m s}^{-1} \text{ (B)}}$$

Quick Tip: Use Heisenberg's relation $\Delta v = h/(4\pi m \Delta x)$ with $\Delta x = 5 \times 10^{-11} \text{ m}$.

53. Which of the following elements belongs to group 2 and sixth period of periodic table?

- (A) Rubidium
- (B) Strontium
- (C) Caesium
- (D) Barium

Correct Answer: (D) Barium

Solution:

Step 1: Understanding the Concept

Group 2 holds the alkaline earth metals: Be, Mg, Ca, Sr, Ba, Ra, going down periods 2 to 7.

Step 2: Detailed Explanation

Counting down the group: Be (period 2), Mg (3), Ca (4), Sr (5), Ba (6), Ra (7).

So the group 2, sixth period element is barium (Ba).

Rubidium (Rb) and caesium (Cs) are group 1 alkali metals, in periods 5 and 6. Strontium is group 2 but sits in period 5.

So only barium fits both conditions.

Final Answer:

Barium is the group 2 element of period 6. This is option (D).

Barium (D)

Quick Tip: Group 2 elements are alkaline earth metals; the one in the sixth period is barium.

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54. Identify a molecule having highest dipole moment from following.

- (A) NH_3
- (B) H_2O
- (C) CHCl_3
- (D) BeF_2

Correct Answer: (B) H_2O

Solution:

Step 1: Understanding the Concept

Dipole moment depends on bond polarity and on molecular shape. Bond dipoles can cancel in a symmetric shape.

Step 2: Detailed Explanation

BeF_2 is linear, so the two Be-F dipoles cancel and the moment is zero. CHCl_3 is tetrahedral and the three C-Cl dipoles partly cancel, giving about 1.0 D.

NH_3 is pyramidal with a lone pair, giving about 1.47 D.

H_2O is bent with two lone pairs and very polar O-H bonds, giving about 1.85 D, the highest of the four.

Final Answer:

Water has the highest dipole moment, option (B).

H_2O (B)

Quick Tip: Compare dipole moments: BeF_2 is linear and zero, and water has the largest value among the rest.

55. V_0 and V_t are volumes of an ideal gas at 0°C and $t^\circ\text{C}$ respectively, find ratio $\frac{V_t}{V_0}$

(A) $\frac{1}{273} \times t$

(B) $1 + 273 \times t$

(C) $1 + \frac{t}{273}$

(D) $\frac{1}{273+t}$

Correct Answer: (C) $1 + \frac{t}{273}$

Solution:

Step 1: Understanding the Concept

Charles's law: at constant pressure, the volume of a fixed amount of gas changes by $\frac{1}{273}$ of its volume at 0°C for each degree Celsius.

Step 2: Detailed Explanation

$$V_t = V_0 + \frac{V_0}{273} t = V_0 \left(1 + \frac{t}{273} \right)$$

Dividing by V_0 :

$$\frac{V_t}{V_0} = 1 + \frac{t}{273}$$

Check: at $t = 0$ the ratio is 1, as it should be. Option (A) gives 0 at $t = 0$, so it cannot be right.

Final Answer:

The ratio is $1 + \frac{t}{273}$, option (C).

$$1 + \frac{t}{273} \text{ (C)}$$

Quick Tip: Charles's law gives $V_t = V_0 \left(1 + \frac{t}{273}\right)$ at constant pressure.

56. Which of the following equations is used to obtain enthalpy change of a reaction?

- (A) $\Delta H = \sum \Delta H_P - \sum \Delta H_R$
- (B) $\Delta H = \sum \Delta H_R - \sum \Delta H_P$
- (C) $\Delta H = \sum \Delta H_P + \sum \Delta H_R$
- (D) $\Delta H = \frac{\sum \Delta H_P}{\sum \Delta H_R}$

Correct Answer: (A) $\Delta H = \sum \Delta H_P - \sum \Delta H_R$

Solution:

Step 1: Understanding the Concept

Enthalpy is a state function, so the change for a reaction depends only on the initial (reactants) and final (products) states.

Step 2: Detailed Explanation

Change = final minus initial:

$$\Delta H = \sum \Delta H_P - \sum \Delta H_R$$

A positive value means heat is absorbed, and a negative value means heat is released. Option (B) reverses the sign, (C) adds the terms, and (D) divides them, which has no physical meaning.

Final Answer:

The correct relation is products minus reactants, option (A).

$$\Delta H = \sum \Delta H_P - \sum \Delta H_R \text{ (A)}$$

Quick Tip: Enthalpy change is products minus reactants.



57. A system is provided 50 J of heat and work done on the system is 10 J. What is change in internal energy ?

- (A) 40 J
- (B) 60 J
- (C) 30 J
- (D) 50 J

Correct Answer: (B) 60 J

Solution:**Step 1: Understanding the Concept**

First law of thermodynamics: $\Delta U = q + w$, where q is heat absorbed by the system and w is work done on the system.

Step 2: Detailed Explanation

Heat is given to the system, so $q = +50 \text{ J}$.

Work is done on the system, so $w = +10 \text{ J}$.

$$\Delta U = 50 + 10 = 60 \text{ J}$$

Option (A) subtracts the work, which would be right only if the system

did the work, so it does not apply here.

Final Answer:

The internal energy rises by 60 J, option (B).

60 J (B)

Quick Tip: Use the first law $\Delta U = q + w$ with both terms positive here.

58. At 25°C standard enthalpy of combustion of H₂, Cyclohexene and cyclohexane are -241, -3800 and -3920 kJ mol⁻¹ respectively. What is the heat of hydrogenation of cyclohexene?

- (A) -121 kJ
- (B) -242 kJ
- (C) 121 kJ
- (D) 363 kJ

Correct Answer: (A) -121 kJ

Solution:

Step 1: Understanding the Concept

Hydrogenation of cyclohexene is $\text{C}_6\text{H}_{10} + \text{H}_2 \rightarrow \text{C}_6\text{H}_{12}$. By Hess's law we can build it from the combustion equations.

Step 2: Key Formula or Approach

Add the combustion of cyclohexene and of H₂, then reverse the combustion of cyclohexane:

$$\Delta H_{hyd} = \Delta H_c(H_2) + \Delta H_c(C_6H_{10}) - \Delta H_c(C_6H_{12})$$

Step 3: Detailed Explanation

$$\Delta H_{hyd} = (-241) + (-3800) - (-3920)$$

$$= -4041 + 3920 = -121 \text{ kJ}$$

The negative sign means the reaction is exothermic. The value -242 would double the H_2 term, and 121 or 363 have wrong signs or terms.

Final Answer:

The heat of hydrogenation is -121 kJ , option (A).

$$\boxed{-121 \text{ kJ (A)}}$$

Quick Tip: Apply Hess's law: $\Delta H_{hyd} = \Delta H_c(H_2) + \Delta H_c(\text{cyclohexene}) - \Delta H_c(\text{cyclohexane})$.

59. Calculate the solubility in mol dm^{-3} of sparingly soluble salt BA at 293 K if its solubility product is 8.56×10^{-5} at same temperature.

- (A) 8.123×10^{-3}
- (B) 8.780×10^{-3}
- (C) 9.252×10^{-3}
- (D) 7.756×10^{-3}

Correct Answer: (C) 9.252×10^{-3}

Solution:

Step 1: Understanding the Concept

BA dissociates as $BA \rightleftharpoons B^+ + A^-$. If the solubility is $S \text{ mol dm}^{-3}$, each ion has concentration S .

Step 2: Key Formula or Approach

$$K_{sp} = [B^+][A^-] = S^2$$

Step 3: Detailed Explanation

$$S = \sqrt{8.56 \times 10^{-5}} = \sqrt{85.6 \times 10^{-6}} = 9.252 \times 10^{-3} \text{ mol dm}^{-3}$$

Check: $(9.252 \times 10^{-3})^2 = 85.6 \times 10^{-6}$. The other values, when squared, give 6.6×10^{-5} , 7.7×10^{-5} and 6.0×10^{-5} , which do not match the given K_{sp} .

Final Answer:

The solubility is $9.252 \times 10^{-3} \text{ mol dm}^{-3}$, option (C).

$9.252 \times 10^{-3} \text{ mol dm}^{-3} \text{ (C)}$

Quick Tip: For a 1:1 salt, $K_{sp} = S^2$, so $S = \sqrt{K_{sp}}$.

60. Which from following salts does not undergo hydrolysis ?

- (A) Ammonium nitrate
- (B) Sodium acetate
- (C) Ammonium cyanide
- (D) Sodium nitrate

Correct Answer: (D) Sodium nitrate

Solution:

Step 1: Understanding the Concept

A salt hydrolyses when at least one of its ions comes from a weak acid or a weak base. Ions from strong acids and strong bases do not react with water.

Step 2: Detailed Explanation

Ammonium nitrate: NH_4^+ comes from a weak base, so it hydrolyses.

Sodium acetate: CH_3COO^- comes from a weak acid, so it hydrolyses.

Ammonium cyanide: both ions are weak, so both hydrolyse.

Sodium nitrate: Na^+ is from strong NaOH and NO_3^- is from strong HNO_3 , so neither reacts with water and the solution stays neutral.

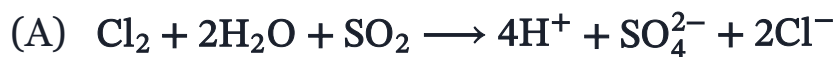
Final Answer:

Sodium nitrate is the salt of a strong acid and a strong base, so it does not hydrolyse. This is option (D).

Sodium nitrate (D)

Quick Tip: A salt of a strong acid and strong base does not hydrolyse.

61. Which of the following reactions is NOT an example of redox reaction?



Correct Answer: (D) $\text{HCl} + \text{H}_2\text{O} \longrightarrow \text{H}_3\text{O}^+ + \text{Cl}^-$

Solution:

Step 1: Understanding the Concept

A redox reaction has a change in oxidation number of at least one element. If no oxidation numbers change, the reaction is not redox.

Step 2: Detailed Explanation

(A) Cl_2 goes from 0 to -1 , and S in SO_2 goes from $+4$ to $+6$: redox.

(B) Zn goes from 0 to $+2$ and Cu goes from $+2$ to 0: redox.

(C) H goes from 0 to $+1$ and O goes from 0 to -2 : redox.

(D) $\text{HCl} + \text{H}_2\text{O} \rightarrow \text{H}_3\text{O}^+ + \text{Cl}^-$: H stays $+1$, Cl stays -1 , O stays -2 . It is only a proton transfer (acid-base).

Final Answer:

Reaction (D) involves no change in oxidation number, so it is not a redox reaction. Option (D).

(D)

Quick Tip: In a redox reaction oxidation numbers change; ionisation of HCl in water has no such change.

62. Which from the following is NOT polymorphic form of silica ?

- (A) Cristobalite
- (B) α - quartz
- (C) β - quartz
- (D) Aragonite

Correct Answer: (D) Aragonite

Solution:

Step 1: Understanding the Concept

Silica (SiO_2) exists in several crystalline forms, called polymorphs: quartz (α and β), tridymite and cristobalite.

Step 2: Detailed Explanation

Cristobalite, α -quartz and β -quartz are all forms of SiO_2 .

Aragonite is a form of calcium carbonate, CaCO_3 , and has nothing to do with silica.

So it is the one that is not a polymorph of silica.

Final Answer:

Aragonite is not a form of silica, so the answer is option (D).

Aragonite (D)

Quick Tip: Silica forms quartz, tridymite and cristobalite; aragonite is calcium carbonate.

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63. Which among the following is haloalkyne?

- (A) Chloroethyne
- (B) 3 - Chloropropyne
- (C) 1 - Chloropent - 2 - yne
- (D) 4 - Chloropent - 2 - yne

Correct Answer: (A) Chloroethyne

Solution:

Step 1: Understanding the Concept

A haloalkyne (alkynyl halide) is an alkyne in which the halogen is attached to an sp carbon, that is, to a carbon of the $C\equiv C$ bond.

Step 2: Detailed Explanation

Chloroethyne is $HC \equiv C-Cl$. The chlorine is bonded directly to a triple-bonded carbon, so it fits the definition exactly.

In the other three names the chlorine sits on a saturated sp^3 carbon next to or away from the triple bond, so the halogen is not on the alkyne carbon itself.

Final Answer:

Chloroethyne has chlorine on the sp carbon itself, so it is the haloalkyne asked for. This is option (A).

Chloroethyne (A)

Quick Tip: A haloalkyne has the halogen directly on a triple-bonded carbon.

64. Which from following statements is NOT true about homologous series?

- (A) Each member of a series has same functional group.
- (B) Each member has same type of carbon skeleton.
- (C) Each member differs by ethylene group from next member.
- (D) All members of series possess similar chemical properties.

Correct Answer: (C) Each member differs by ethylene group from next member.

Solution:

Step 1: Understanding the Concept

A homologous series is a family of compounds with the same functional group and similar chemical properties, whose neighbouring members differ by a fixed unit.

Step 2: Detailed Explanation

- (A) Same functional group: true.
- (B) Same type of carbon skeleton: true.
- (D) Similar chemical properties: true.
- (C) Neighbouring members differ by a methylene group, $-\text{CH}_2-$ (14 mass units), not an ethylene group, which is C_2H_4 . So (C) is not true.

Final Answer:

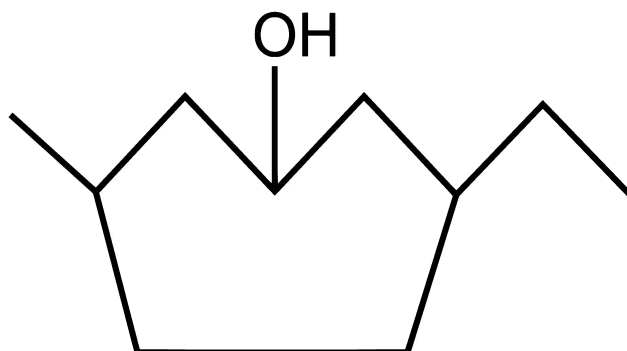
The difference is a CH_2 group, so statement (C) is the false one.

(C)

Quick Tip: Successive members of a homologous series differ by a

methylene (CH_2) group.

65. What is IUPAC name of following compound?



- (A) 3 - Methyl - 6 - ethyl cycloheptanol
- (B) 3 - Ethylcyclooctanol
- (C) 3 - Ethyl - 6 - methylcycloheptanol
- (D) 1 - Ethyl - 5 - methylcycloheptan - 3 - ol

Correct Answer: (C) 3 - Ethyl - 6 - methylcycloheptanol

Solution:

Step 1: Understanding the Concept

The figure shows a seven-membered ring with an OH on one ring carbon, a methyl group and an ethyl group on other ring carbons. The parent is cycloheptanol, and the carbon with OH is C-1.

Step 2: Key Formula or Approach

Rules: OH gets locant 1. Then choose the direction giving the lowest set of locants to substituents. If the sets tie, the alphabetically first substituent gets the lower number.

Step 3: Detailed Explanation

From the OH carbon, going one way the ethyl-bearing carbon is third and the methyl-bearing carbon is sixth. Going the other way, it is the methyl-bearing carbon that is third and the ethyl-bearing carbon that is sixth. The locant set is {3, 6} either way.

Ethyl comes before methyl alphabetically, so ethyl takes 3 and methyl takes 6.

Name: 3-ethyl-6-methylcycloheptanol.

Option (A) puts methyl at 3 and ethyl at 6, which breaks the alphabetical tie rule.

Final Answer:

The compound is 3-ethyl-6-methylcycloheptanol, option (C).

3-Ethyl-6-methylcycloheptanol (C)

Quick Tip: Number from the carbon with OH, then give the substituents the lowest locants, alphabetical order breaks ties.

66. Which from following elements is NOT present in mustard gas?

- (A) Cl
- (B) N
- (C) S
- (D) C

Correct Answer: (B) N

Solution:

Step 1: Understanding the Concept

Mustard gas is the chemical warfare agent bis(2-chloroethyl) sulphide.

Step 2: Detailed Explanation

Its formula is $(\text{ClCH}_2\text{CH}_2)_2\text{S}$. It contains carbon, hydrogen, chlorine and sulphur.

There is no nitrogen in the molecule. (Nitrogen mustards are different compounds.)

Final Answer:

Nitrogen is absent from mustard gas, option (B).

N (B)

Quick Tip: Mustard gas is $\text{ClCH}_2\text{CH}_2\text{-S-CH}_2\text{CH}_2\text{Cl}$, so it has no nitrogen.

67. When excess of chlorine is used in the chlorination of methane find the major product obtained.

- (A) Chloromethane
- (B) Dichloromethane
- (C) Trichloromethane
- (D) Tetrachloromethane

Correct Answer: (D) Tetrachloromethane

Solution:

Step 1: Understanding the Concept

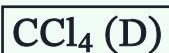
Chlorination of methane is a free radical substitution. It goes step by step: $\text{CH}_4 \rightarrow \text{CH}_3\text{Cl} \rightarrow \text{CH}_2\text{Cl}_2 \rightarrow \text{CHCl}_3 \rightarrow \text{CCl}_4$.

Step 2: Detailed Explanation

Each step replaces one H by Cl. When chlorine is in excess and the reaction continues, substitution does not stop at the first products. All four hydrogens are replaced, so the major product is CCl_4 (tetrachloromethane).

Final Answer:

Excess chlorine replaces all four hydrogens, giving tetrachloromethane, option (D).



Quick Tip: With excess chlorine every hydrogen is replaced and the final product is carbon tetrachloride.

68. Alkenes on oxidation with KMnO_4 in dil H_2SO_4 forms

- (A) Alkanol
- (B) Alkanal
- (C) Alkanone
- (D) Alkanoic acid

Correct Answer: (D) Alkanoic acid

Solution:

Step 1: Understanding the Concept

Acidified KMnO_4 is a strong oxidant. It breaks the double bond completely, and any aldehyde formed is oxidised further.

Step 2: Detailed Explanation

The alkene splits into two fragments at the double bond. A $=\text{CH}_2$ end becomes CO_2 , and a $=\text{CHR}$ end becomes RCOOH .

Since aldehydes cannot survive the oxidising medium, they go on to the carboxylic acid.

The class of product asked in the question is alkanolic acid.

Final Answer:

Alkenes give carboxylic acids (alkanoic acids), option (D).

Alkanoic acid (D)

Quick Tip: Acidified KMnO_4 cleaves the $\text{C}=\text{C}$ bond and oxidises the fragments to carboxylic acids.

69. Find the volume occupied by a particle in a simple cubic unit cell if the radius of a particle in it is 190 pm.

- (A) $6.410 \times 10^{-23} \text{cm}^3$
- (B) $3.625 \times 10^{-23} \text{cm}^3$
- (C) $5.715 \times 10^{-23} \text{cm}^3$
- (D) $2.873 \times 10^{-23} \text{cm}^3$

Correct Answer: (D) $2.873 \times 10^{-23} \text{cm}^3$

Solution:

Step 1: Understanding the Concept

A simple cubic unit cell contains one atom. The volume occupied by the particle is the volume of one sphere of radius r .

Step 2: Key Formula or Approach

$$V = \frac{4}{3}\pi r^3$$

Step 3: Detailed Explanation

Convert: $190 \text{ pm} = 190 \times 10^{-10} \text{ cm} = 1.9 \times 10^{-8} \text{ cm}$.

$$r^3 = (1.9 \times 10^{-8})^3 = 6.859 \times 10^{-24} \text{ cm}^3$$

$$V = 4.189 \times 6.859 \times 10^{-24} = 2.873 \times 10^{-23} \text{ cm}^3$$

Final Answer:

The volume occupied by the particle is $2.873 \times 10^{-23} \text{ cm}^3$, option (D).

$2.873 \times 10^{-23} \text{ cm}^3 \text{ (D)}$

Quick Tip: Volume of one sphere is $\frac{4}{3}\pi r^3$, with $r = 190 \text{ pm} = 1.9 \times 10^{-8} \text{ cm}$.



70. Calculate the number of unit cells in 1 cm^3 of metal if it forms simple cubic structure with unit cell edge length 500 pm .

- (A) 4.0×10^{21}
- (B) 2.0×10^{21}
- (C) 8.0×10^{21}
- (D) 3.0×10^{21}

Correct Answer: (C) 8.0×10^{21}

Solution:

Step 1: Understanding the Concept

The number of unit cells equals total volume divided by the volume of one unit cell. For a cube of edge a , the cell volume is a^3 .

Step 2: Detailed Explanation

Edge: $500 \text{ pm} = 5 \times 10^{-8} \text{ cm}$.

$$a^3 = (5 \times 10^{-8})^3 = 125 \times 10^{-24} = 1.25 \times 10^{-22} \text{ cm}^3$$

$$N = \frac{1}{1.25 \times 10^{-22}} = 8.0 \times 10^{21}$$

The other options come from a wrong cube or a wrong conversion of picometres.

Final Answer:

There are 8.0×10^{21} unit cells in 1 cm^3 , option (C).

$8.0 \times 10^{21} \text{ (C)}$

Quick Tip: Divide 1 cm^3 by the volume of one cell, a^3 with $a = 500 \text{ pm} = 5$

$\times 10^{-8}$ cm.



71. Which of the following concentrations of solutions of urea in water exhibits minimum freezing point depression?

- (A) 0.12 m
- (B) 0.18 m
- (C) 0.08 m
- (D) 0.06 m

Correct Answer: (D) 0.06 m

Solution:

Step 1: Understanding the Concept

Depression in freezing point is a colligative property. It depends on the number of solute particles per kg of solvent, not on what the solute is.

Step 2: Key Formula or Approach

$$\Delta T_f = i K_f m$$

Urea does not ionise, so $i = 1$ for every solution, and the water is the same, so K_f is fixed.

Step 3: Detailed Explanation

Then $\Delta T_f \propto m$.

The molalities are 0.12, 0.18, 0.08 and 0.06 m. The smallest is 0.06 m.

So the 0.06 m solution has the smallest depression, $\Delta T_f = 1.86 \times 0.06$

= 0.11 K, and its freezing point is closest to that of pure water.

Final Answer:

The 0.06 m solution shows the minimum depression, option (D).

0.06 m (D)

Quick Tip: Freezing point depression is proportional to molality, so pick the lowest molality.

72. Calculate van't Hoff factor for aqueous solution of 0.02 m formic acid if it freezes at -0.045°C .

[$K_f = 1.86 \text{ K kg mol}^{-1}$ and freezing point of water = 0°C]

- (A) 1.21
- (B) 1.46
- (C) 1.68
- (D) 1.05

Correct Answer: (A) 1.21

Solution:

Step 1: Understanding the Concept

Formic acid partly ionises in water, so the number of particles is a little more than the number of moles dissolved. The van't Hoff factor i measures this ratio.

Step 2: Key Formula or Approach

$$\Delta T_f = i K_f m \Rightarrow i = \frac{\Delta T_f}{K_f m}$$

Step 3: Detailed Explanation

Freezing point of solution is -0.045°C , so $\Delta T_f = 0 - (-0.045) = 0.045$ K.

$$i = \frac{0.045}{1.86 \times 0.02} = \frac{0.045}{0.0372} = 1.21$$

A value above 1 but well below 2 fits a weak acid that ionises only slightly.

Final Answer:

The van't Hoff factor is 1.21, option (A).

1.21 (A)

Quick Tip: Use $\Delta T_f = i K_f m$ with $\Delta T_f = 0.045$ K.



73. Calculate the molar mass of nonvolatile solute when 4 g of it is dissolved in 100 g solvent that boils at 319.4 K [$K_b = 2.4 \text{ K kg mol}^{-1}$; B.P of pure solvent = 319 K]

- (A) 220 g mol^{-1}
- (B) 230 g mol^{-1}
- (C) 240 g mol^{-1}
- (D) 250 g mol^{-1}

Correct Answer: (C) 240g mol^{-1}

Solution:

Step 1: Understanding the Concept

Dissolving a nonvolatile solute raises the boiling point. The rise depends on the molality of the solution.

Step 2: Key Formula or Approach

$$\Delta T_b = K_b m = K_b \frac{w_2 \times 1000}{M_2 w_1}$$

Step 3: Detailed Explanation

$$\Delta T_b = 319.4 - 319 = 0.4 \text{ K}, w_2 = 4 \text{ g}, w_1 = 100 \text{ g}.$$

$$M_2 = \frac{K_b w_2 \times 1000}{\Delta T_b w_1} = \frac{2.4 \times 4 \times 1000}{0.4 \times 100} = \frac{9600}{40} = 240 \text{ g mol}^{-1}$$

Final Answer:

The molar mass of the solute is 240 g/mol , option (C).

$$\boxed{240 \text{ g mol}^{-1} \text{ (C)}}$$

Quick Tip: Use $\Delta T_b = K_b \times \frac{w_2 \times 1000}{M_2 \times w_1}$.

74. If the emf of cell, $\text{Cu}_{(s)}|\text{Cu}_{(1M)}^{2+}|\text{Ag}_{(1M)}^{+}|\text{Ag}$ is 0.463 V at 25°C and standard potential of Cu electrode is 0.337 V. Find the standard potential of Ag electrode

- (A) 0.128V
- (B) -0.128V
- (C) 0.8V
- (D) -0.8V

Correct Answer: (C) 0.8V

Solution:

Step 1: Understanding the Concept

The cell is $\text{Cu} | \text{Cu}^{2+} (1 \text{ M}) || \text{Ag}^{+} (1 \text{ M}) | \text{Ag}$. Copper is the anode (oxidation) and silver is the cathode (reduction).

Step 2: Key Formula or Approach

Both concentrations are 1 M, so the cell is at standard conditions:

$$E_{\text{cell}} = E_{\text{Ag}}^{\circ} - E_{\text{Cu}}^{\circ}$$

Step 3: Detailed Explanation

$$0.463 = E_{\text{Ag}}^{\circ} - 0.337$$

$$E_{\text{Ag}}^{\circ} = 0.463 + 0.337 = 0.800 \text{ V}$$

A negative value cannot come out, because the cell emf is positive and silver is above copper in the series.

Final Answer:

The standard potential of the silver electrode is 0.8 V, option (C).

$$0.8 \text{ V (C)}$$

Quick Tip: At 1 M both ions, $E_{\text{cell}} = E_{\text{cathode}}^{\circ} - E_{\text{anode}}^{\circ}$.



75. Which of the following hot aqueous solutions is used to dip carbon rods of $\text{H}_2 - \text{O}_2$ fuel cell ?

- (A) KCl
- (B) KOH
- (C) H_2SO_4
- (D) NH_4Cl

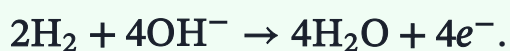
Correct Answer: (B) KOH

Solution:**Step 1: Understanding the Concept**

In the $\text{H}_2\text{-O}_2$ fuel cell, porous carbon electrodes carrying a catalyst dip into a hot aqueous solution of an alkali.

Step 2: Detailed Explanation

The electrolyte is aqueous KOH. At the anode, hydrogen is oxidised:



At the cathode, oxygen is reduced: $\text{O}_2 + 2\text{H}_2\text{O} + 4e^- \rightarrow 4\text{OH}^-$.

The OH^- ions carry current through the solution, so a hydroxide solution is needed.

Final Answer:

The carbon rods dip in hot aqueous KOH, option (B).

KOH (B)

Quick Tip: The H₂-O₂ fuel cell uses a hot concentrated alkaline electrolyte, KOH.



76. The conductivity of 0.02 M of KCl solution at 298 K is 0.0123 ohm⁻¹cm⁻¹. If the resistance of the cell containing this solution is 120 ohms, what is the value of the cell constant ?

- (A) 1.050cm⁻¹
- (B) 1.025cm⁻¹
- (C) 1.376cm⁻¹
- (D) 1.476cm⁻¹

Correct Answer: (D) 1.476cm⁻¹

Solution:**Step 1: Understanding the Concept**

Cell constant is the ratio l/A of the conductivity cell. Conductivity is κ

$$= \frac{1}{R} \cdot \frac{l}{A}.$$

Step 2: Key Formula or Approach

$$G^* = \frac{l}{A} = \kappa R$$

Step 3: Detailed Explanation

$$G^* = 0.0123 \text{ ohm}^{-1}\text{cm}^{-1} \times 120 \text{ ohm} = 1.476 \text{ cm}^{-1}$$

The molarity 0.02 M is not needed for this step, because the conductivity already contains that information.

Final Answer:

The cell constant is 1.476 cm^{-1} , option (D).

$1.476 \text{ cm}^{-1} \text{ (D)}$

Quick Tip: Cell constant = $\kappa \times R$.

77. Half life of zero order reaction is 1 hour. If initial concentration of reactant is 2.0 mol L^{-1} , find the time required to decrease concentration from 0.50 to 0.25 mol L^{-1}

- (A) 0.25 hour
- (B) 0.50 hour
- (C) 1.00 hour
- (D) 4.00 hour

Correct Answer: (A) 0.25 hour

Solution:

Step 1: Understanding the Concept

A zero order reaction has a constant rate, so the concentration falls by the same amount in each equal time. The half life is $t_{1/2} = \frac{[A]_0}{2k}$.

Step 2: Key Formula or Approach

$$k = \frac{[A]_0}{2t_{1/2}} = \frac{2.0}{2 \times 1} = 1.0 \text{ mol L}^{-1}\text{h}^{-1}$$

Step 3: Detailed Explanation

The concentration falls from 0.50 to 0.25, a drop of 0.25 mol/L.

$$t = \frac{0.25}{1.0} = 0.25 \text{ h}$$

For a zero order reaction the time does not depend on the starting point of the drop, only on the size of the drop.

Final Answer:

The time needed is 0.25 hour, option (A).

0.25 hour (A)

Quick Tip: For zero order, $k = [A]_0/(2t_{1/2})$ and $t = \Delta[A]/k$.



78. What is the term used for the minimum kinetic energy required for the reactant molecules to undergo reaction?

- (A) Potential energy
- (B) Bond energy
- (C) Thermal energy
- (D) Activation energy

Correct Answer: (D) Activation energy

Solution:

Step 1: Understanding the Concept

Colliding molecules react only if they carry at least a certain minimum energy. That threshold is the activation energy E_a .

Step 2: Detailed Explanation

Activation energy is the minimum energy the reactant molecules must have to cross the energy barrier and form products.

Potential energy and thermal energy are general forms of energy, and bond energy is the energy to break a bond. None of these names the threshold for reaction.

Final Answer:

The term is activation energy, option (D).

Activation energy (D)

Quick Tip: The minimum energy that reactants must have to react is the activation energy.

79. If time required for 90 % completion of a first order reaction is ' t '. What is the time required for 99.9 % completion of reaction at same temperature?

- (A) $3t$
- (B) $2t$
- (C) t
- (D) $\frac{3t}{2}$

Correct Answer: (A) $3t$

Solution:

Step 1: Understanding the Concept

For a first order reaction $t = \frac{2.303}{k} \log \frac{[A]_0}{[A]}$.

Step 2: Key Formula or Approach

For x percent completion, $[A]_0/[A] = 100/(100 - x)$.

Step 3: Detailed Explanation

90 percent: $t = \frac{2.303}{k} \log 10 = \frac{2.303}{k}$.

99.9 percent: $t' = \frac{2.303}{k} \log 1000 = \frac{2.303}{k} \times 3$.

So $t' = 3t$.

Final Answer:

The time for 99.9 percent completion is $3t$, option (A).

$3t$ (A)

Quick Tip: For first order, $t = \frac{2.303}{k} \log \frac{100}{100-x}$.

80. Identify a nanomaterial having all three dimensions less than 100 nm?

- (A) Fibers
- (B) Thin films
- (C) Quantum dots
- (D) Nanotubes

Correct Answer: (C) Quantum dots

Solution:

Step 1: Understanding the Concept

Nanomaterials are classed by how many dimensions lie in the nanoscale (1 to 100 nm).

Step 2: Detailed Explanation

Thin films have one dimension (thickness) in the nanoscale.

Fibres and nanotubes have two dimensions (diameter) in the nanoscale and a long length.

Quantum dots have all three dimensions in the nanoscale, so they are zero-dimensional nanomaterials.

Final Answer:

Quantum dots have all three dimensions below 100 nm, option (C).

Quantum dots (C)

Quick Tip: Quantum dots are zero-dimensional nanomaterials, with all dimensions in the nanoscale.

81. Which from following is NOT an example of oil in water emulsion?

- (A) Paint
- (B) Butter
- (C) Vanishing cream
- (D) Milk

Correct Answer: (B) Butter

Solution:

Step 1: Understanding the Concept

In an O/W emulsion, oil droplets are dispersed in water. In a W/O emulsion, water droplets are dispersed in oil.

Step 2: Detailed Explanation

Milk: fat droplets in water, O/W. Vanishing cream: O/W. Paint (as an emulsion paint): O/W.

Butter: tiny water droplets in a fat phase, so it is W/O.

So butter is the one that is not O/W.

Final Answer:

Butter is a water in oil emulsion, so the answer is option (B).

Butter (B)

Quick Tip: Butter is a water in oil emulsion; milk, paint and vanishing cream are oil in water type.

82. Which from following catalysts is used in contact process of industrial production of sulfuric acid using SO_2 and O_2 (air).

- (A) Ni (finely divided)
- (B) Platinised asbestos
- (C) Co-Th
- (D) Fe-Cr

Correct Answer: (B) Platinised asbestos

Solution:

Step 1: Understanding the Concept

The contact process makes sulphuric acid. The key step is $2\text{SO}_2 + \text{O}_2 \rightleftharpoons 2\text{SO}_3$ on a solid catalyst.

Step 2: Detailed Explanation

Among the options, platinised asbestos is the catalyst used for this step (V_2O_5 is the other common one).

Finely divided nickel is used in hydrogenation. Co-Th is used in Fischer-Tropsch type synthesis. Fe-Cr is used for the water gas shift reaction.

Final Answer:

The catalyst is platinised asbestos, option (B).

Platinised asbestos (B)

Quick Tip: The contact process oxidises SO_2 to SO_3 over a catalyst of platinised asbestos or V_2O_5 .

83. Which from following compounds has lowest thermal stability?

- (A) IBr
- (B) BrCl
- (C) BrF
- (D) ClF

Correct Answer: (A) IBr

Solution:

Step 1: Understanding the Concept

Thermal stability of an interhalogen XX' depends on the strength of the $X-X'$ bond. Bond strength decreases as the difference in size from the bonded atoms leads to poorer overlap and as the atoms get larger.

Step 2: Detailed Explanation

ClF, BrF and BrCl contain smaller halogens, giving shorter bonds. In IBr, the large iodine atom makes a longer bond with poor overlap, so the bond is the weakest.

The weakest bond breaks most easily on heating, so IBr is the least stable.

Final Answer:

IBr has the lowest thermal stability, option (A).

IBr (A)

Quick Tip: Stability falls as the bond becomes longer and weaker; IBr has the largest, weakest bond.

84. Why transition elements have more tendency to form interstitial compounds?

- (A) Presence of interstitial sites (voids) in the crystal lattice
- (B) They have reducing property
- (C) They have low ionization enthalpy
- (D) They have same atomic size.

Correct Answer: (A) Presence of interstitial sites (voids) in the crystal lattice

Solution:

Step 1: Understanding the Concept

Interstitial compounds form when small atoms such as H, B, C or N are trapped inside the lattice of a metal.

Step 2: Detailed Explanation

Transition metals have crystal lattices with vacant spaces (voids) between the atoms, and these spaces are the right size for small non-metal atoms.

So the tendency comes from the presence of interstitial sites in the lattice, which is statement (A).

Final Answer:

The cause is the presence of interstitial voids in the lattice, option (A).

(A)

Quick Tip: Small atoms like H, C, N fit into the voids in the metal lattice.

85. What is EAN of Co in $[\text{Co}(\text{NH}_3)_6]^{3+}$?

- (A) 24
- (B) 27
- (C) 36
- (D) 30

Correct Answer: (C) 36

Solution:

Step 1: Understanding the Concept

Effective atomic number (EAN) is the total number of electrons around the central metal after accepting electron pairs from ligands.

Step 2: Key Formula or Approach

$$\text{EAN} = Z - (\text{oxidation state}) + 2 \times (\text{coordination number})$$

Step 3: Detailed Explanation

Co has $Z = 27$. The complex is $[\text{Co}(\text{NH}_3)_6]^{3+}$, so the oxidation state is +3 and the coordination number is 6.

$$\text{EAN} = 27 - 3 + 2(6) = 36$$

This equals the atomic number of krypton, a noble gas.

Final Answer:

The EAN of cobalt is 36, option (C).

Quick Tip: EAN = atomic number - oxidation state + 2 x coordination number.

86. Identify the total number of complexes from following list having monodentate ligands in them

- (a) Tetracyanonickelate (II) ion
- (b) Potassium hexafluoroaluminate (III)
- (c) Tetraamminecopper(II) ion
- (d) Potassium trioxalatoaluminate (III)

- (A) 4
- (B) 3
- (C) 2
- (D) 1

Correct Answer: (B) 3

Solution:

Step 1: Understanding the Concept

A monodentate ligand binds the metal through one donor atom. Bidentate ligands bind through two.

Step 2: Detailed Explanation

- (a) Tetracyanonickelate(II): CN^- is monodentate.
- (b) Potassium hexafluoroaluminate(III): F^- is monodentate.
- (c) Tetraamminecopper(II): NH_3 is monodentate.
- (d) Potassium trioxalatoaluminate(III): oxalate is bidentate, so this

one is excluded.

Three complexes, (a), (b) and (c), have only monodentate ligands.

Final Answer:

Three of the listed complexes have monodentate ligands, option (B).

3 (B)

Quick Tip: Cyanide, fluoride and ammine are monodentate; oxalate is bidentate.



87. Which of the following reactions is used for the preparation of alkyl fluorides from alkyl chlorides?

- (A) Sandmeyer reaction
- (B) Fittig reaction
- (C) Swartz reaction
- (D) Finkelstein reaction

Correct Answer: (C) Swartz reaction

Solution:

Step 1: Understanding the Concept

Halogen exchange reactions swap one halogen for another.

Step 2: Detailed Explanation

Swarts reaction: $\text{R-Cl} + \text{AgF} \rightarrow \text{R-F} + \text{AgCl}$. This is the fluoride preparation.

Finkelstein reaction gives alkyl iodides using NaI in dry acetone.

Sandmeyer reaction converts diazonium salts to aryl halides with Cu(I) salts. Fittig reaction joins aryl halides with sodium to give biaryls.

Final Answer:

The alkyl fluoride preparation is the Swarts reaction, option (C).

Swarts reaction (C)

Quick Tip: Swarts reaction converts alkyl chlorides or bromides to fluorides using metallic fluorides like AgF.

88. Which among the following is allylic halide?

- (A) $\text{CH}_3 - \text{CH}_2 - \text{CH}_2 - \text{X}$
- (B) $\text{C}_6\text{H}_5 - \text{CH}_2 - \text{X}$
- (C) $\text{CH}_2 = \text{CH} - \text{CH}_2 - \text{X}$
- (D) $\text{CH}_3 - \text{CH} = \text{CH} - \text{X}$

Correct Answer: (C) $\text{CH}_2 = \text{CH} - \text{CH}_2 - \text{X}$

Solution:

Step 1: Understanding the Concept

In an allylic halide the halogen is attached to an sp^3 carbon that is next to a carbon-carbon double bond.

Step 2: Detailed Explanation

- (A) $\text{CH}_3\text{CH}_2\text{CH}_2\text{X}$ has no double bond: ordinary alkyl halide.
- (B) $\text{C}_6\text{H}_5\text{CH}_2\text{X}$: halogen on the carbon next to a benzene ring, a

benzylic halide.

(C) $\text{CH}_2 = \text{CH}-\text{CH}_2\text{X}$: halogen on the sp^3 carbon next to the double bond, so allylic.

(D) $\text{CH}_3\text{CH}=\text{CH}-\text{X}$: halogen on the double bond carbon itself, a vinylic halide.

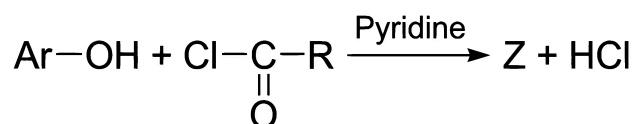
Final Answer:

Compound (C) is the allylic halide, option (C).

(C)

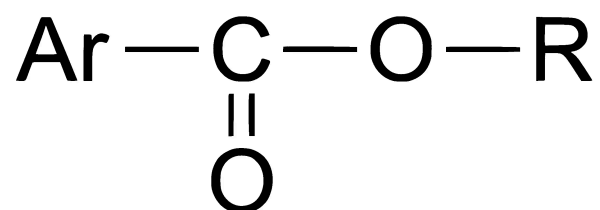
Quick Tip: An allylic halide has the halogen on a carbon adjacent to a $\text{C}=\text{C}$ double bond.

89. Identify 'Z' in the following reaction.

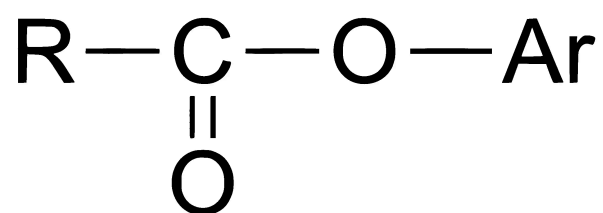




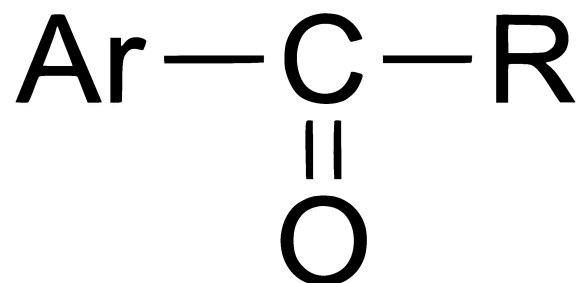
(B)



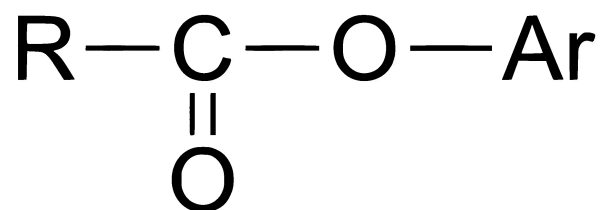
(C)



(D)



Correct Answer: (C)



Solution:

Step 1: Understanding the Concept

The figure shows Ar-OH reacting with an acyl chloride Cl-CO-R in the presence of pyridine, giving Z and HCl. This is acylation of phenol.

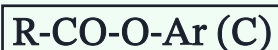
Step 2: Detailed Explanation

The OH oxygen attacks the carbonyl carbon and chloride leaves. The acyl group replaces the hydrogen of OH, forming an ester in which the aryl group is on the oxygen: R-CO-O-Ar.

Pyridine takes up the HCl formed, which pushes the reaction forward. The option that shows R-C(=O)-O-Ar is the third structure (C). The structure Ar-CO-OR (B) has the aryl group on carbon and so is an ester of an aromatic acid, and Ar-CO-R (D) is a ketone.

Final Answer:

Z is the phenyl ester R-CO-O-Ar, option (C).



Quick Tip: Phenol with an acid chloride in pyridine gives a phenyl ester, RCOOAr.

90. Identify trihydric phenol from following.

- (A) Hydroquinone
- (B) Catechol
- (C) O-Cresol
- (D) Pyrogallol

Correct Answer: (D) Pyrogallol

Solution:

Step 1: Understanding the Concept

Phenols are classed by the number of OH groups on the ring: monohydric (1), dihydric (2), trihydric (3).

Step 2: Detailed Explanation

Hydroquinone (1,4-dihydroxybenzene) has two OH groups. Catechol (1,2-dihydroxybenzene) has two. O-cresol (2-methylphenol) has one. Pyrogallol (1,2,3-trihydroxybenzene) has three OH groups, so it is trihydric.

Final Answer:

Pyrogallol is the trihydric phenol, option (D).

Pyrogallol (D)

Quick Tip: Pyrogallol has three OH groups on the benzene ring.

91. Which from the following is a correct decreasing order of boiling points for following organic compounds ?

- (A) Aldehyde > Ketone > Alcohol > Carboxylic acid
- (B) Carboxylic acid > Alcohol > Ketone > Aldehyde
- (C) Ketone > Aldehyde > Carboxylic acid > Alcohol
- (D) Alcohol > Aldehyde > Ketone > Carboxylic acid

Correct Answer: (B) Carboxylic acid > Alcohol > Ketone > Aldehyde

Solution:

Step 1: Understanding the Concept

For compounds of similar molar mass, boiling points depend on intermolecular forces. Hydrogen bonding is the strongest, then dipole-dipole forces.

Step 2: Detailed Explanation

Carboxylic acids form strong hydrogen bonds and exist as dimers, giving the highest boiling points.

Alcohols form hydrogen bonds, but weaker than the acid dimers.

Ketones and aldehydes have no hydrogen bonding between their own molecules, only dipole-dipole forces. Ketones have a slightly larger dipole due to two electron-donating alkyl groups, so ketones boil higher than aldehydes.

Order: acid > alcohol > ketone > aldehyde.

Final Answer:

The correct decreasing order is acid, alcohol, ketone, aldehyde, option (B).

Acid > Alcohol > Ketone > Aldehyde (B)

Quick Tip: Boiling point order follows hydrogen bonding: acid, then alcohol, then ketone, then aldehyde.

92. When will be the acidic character of carboxylic acid higher in following cases ?

- (A) Lower K_a and pK_a values
- (B) Lower K_a value and higher pK_a value
- (C) Higher K_a value and lower pK_a value
- (D) Higher K_a and pK_a values

Correct Answer: (C) Higher K_a value and lower pK_a value

Solution:

Step 1: Understanding the Concept

K_a measures how far an acid ionises. $pK_a = -\log K_a$, so the two move in opposite directions.

Step 2: Detailed Explanation

A larger K_a means more ionisation and hence a stronger acid. Since $pK_a = -\log K_a$, a larger K_a gives a smaller pK_a .

So higher acidic character goes with higher K_a and lower pK_a , which is option (C).

Options (A), (B) and (D) pair the values in ways that cannot hold, because K_a and pK_a cannot both be lower or both be higher.

Final Answer:

Acidity is greater when K_a is higher and pK_a is lower, option (C).

High K_a , low pK_a (C)

Quick Tip: A stronger acid has a larger K_a and a smaller pK_a .

93. Identify the major product obtained when ethyl amine is reacted with excess of methyl iodide ?

- (A) Tetramethylammonium iodide
- (B) Ethyltrimethylammonium iodide
- (C) Ethyldimethylamine
- (D) Ethylmethylanine

Correct Answer: (B) Ethyltrimethylammonium iodide

Solution:

Step 1: Understanding the Concept

Amines react with alkyl halides by nucleophilic substitution. With excess alkyl halide the reaction goes on until the nitrogen carries four groups (exhaustive alkylation).

Step 2: Detailed Explanation



Each step adds one methyl group. After three methylations the nitrogen has the ethyl group and three methyl groups.

The final product is ethyltrimethylammonium iodide. The ethyl group stays, so tetramethylammonium iodide cannot form.

Final Answer:

The major product is ethyltrimethylammonium iodide, option (B).

Ethyltrimethylammonium iodide (B)

Quick Tip: Excess CH_3I alkylates the amine fully, giving a quaternary ammonium salt.

94. Identify correct decreasing order of basic strength of amines

- (A) $\text{CH}_3\text{NH}_2 > (\text{CH}_3)_2\text{NH} > \text{C}_6\text{H}_5\text{NH}_2 > \text{NH}_3$
(B) $(\text{CH}_3)_2\text{NH} > \text{CH}_3\text{NH}_2 > \text{NH}_3 > \text{C}_6\text{H}_5\text{NH}_2$
(C) $\text{NH}_3 > \text{CH}_3\text{NH}_2 > (\text{CH}_3)_2\text{NH} > \text{C}_6\text{H}_5\text{NH}_2$
(D) $\text{C}_6\text{H}_5\text{NH}_2 > (\text{CH}_3)_2\text{NH} > \text{CH}_3\text{NH}_2 > \text{NH}_3$

Correct Answer: (B) $(\text{CH}_3)_2\text{NH} > \text{CH}_3\text{NH}_2 > \text{NH}_3 > \text{C}_6\text{H}_5\text{NH}_2$

Solution:

Step 1: Understanding the Concept

Basic strength depends on the availability of the nitrogen lone pair. Alkyl groups push electrons and make the lone pair more available. In aniline the lone pair is delocalised into the ring.

Step 2: Detailed Explanation

Dimethylamine has two electron-donating methyl groups, so it is more basic than methylamine.

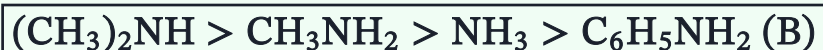
Methylamine is more basic than ammonia, which has no alkyl group.

Aniline is the weakest base, as the lone pair is spread over the benzene ring by resonance.

Order: $(\text{CH}_3)_2\text{NH} > \text{CH}_3\text{NH}_2 > \text{NH}_3 > \text{C}_6\text{H}_5\text{NH}_2$.

Final Answer:

The order is dimethylamine, methylamine, ammonia, aniline, option (B).



Quick Tip: Aliphatic amines are stronger bases than ammonia, which is stronger than aniline.

95. Which from following compounds is used to prepare adipic acid, enzymatically in Green technology developed by Darth and Frost?

- (A) Sucrose
- (B) Glucose
- (C) Lactose
- (D) Maltose

Correct Answer: (B) Glucose

Solution:

Step 1: Understanding the Concept

Conventional adipic acid synthesis uses benzene, a carcinogenic feedstock. Draths and Frost developed an alternative route.

Step 2: Detailed Explanation

In their method, glucose, a renewable carbohydrate, is converted by a genetically modified microbe (*E. coli*) to cis,cis-muconic acid.

Catalytic hydrogenation of this acid then gives adipic acid.

So glucose is the starting compound.

Final Answer:

Glucose is used in the green synthesis of adipic acid, option (B).

Glucose (B)

Quick Tip: Draths and Frost developed a green route to adipic acid from glucose using genetically modified bacteria.

96. Which carbon atom (numbered from 1' to 5') of ribose lacks the oxygen to form deoxyribose ?

- (A) 5'
- (B) 3'
- (C) 2'
- (D) 1'

Correct Answer: (C) 2'

Solution:

Step 1: Understanding the Concept

Ribose is the sugar of RNA and deoxyribose is the sugar of DNA. The name means a sugar with one oxygen removed.

Step 2: Detailed Explanation

In ribose the carbons 2' and 3' both carry an OH group. In deoxyribose the OH at the 2' position is replaced by H.

The 3' OH remains (it forms the phosphodiester link) and the 5' carbon carries a CH₂OH group.

Final Answer:

The 2' carbon lacks the oxygen in deoxyribose, option (C).

2' (C)

Quick Tip: Deoxyribose has no OH on the 2' carbon.

97. Which from following amino acids does NOT contain chiral carbon?

- (A) Valine
- (B) Alanine
- (C) Glycine
- (D) Isoleucine

Correct Answer: (C) Glycine

Solution:

Step 1: Understanding the Concept

A chiral carbon is bonded to four different groups.

Step 2: Detailed Explanation

The alpha carbon of an amino acid carries NH_2 , COOH , H and the side chain R.

In glycine, R is H, so the alpha carbon has two identical H atoms and is not chiral.

Valine, alanine and isoleucine have different R groups (isopropyl, methyl, sec-butyl), so their alpha carbons are chiral. Isoleucine has one more chiral carbon in its side chain.

Final Answer:

Glycine contains no chiral carbon, option (C).

Glycine (C)

Quick Tip: Glycine has two hydrogens on its alpha carbon, so it is not chiral.

98. Which from following oils is mainly a saturated fats?

- (A) Coconut oil
- (B) Olive oil
- (C) Safflower oil
- (D) Fish oil

Correct Answer: (A) Coconut oil

Solution:

Step 1: Understanding the Concept

Oils rich in saturated fatty acids are solid or semi-solid at cool temperature, while oils rich in unsaturated fatty acids stay liquid.

Step 2: Detailed Explanation

Coconut oil contains about 90 percent saturated fatty acids, mainly lauric acid.

Olive oil is mainly monounsaturated (oleic acid). Safflower oil is mainly polyunsaturated (linoleic acid). Fish oil is rich in polyunsaturated omega-3 acids.

Final Answer:

Coconut oil is mainly saturated, option (A).

Coconut oil (A)

Quick Tip: Coconut oil is rich in saturated fatty acids like lauric acid.

99. Which from following is a homopolymer ?

- (A) Polyacrylonitrile
- (B) Buna - N
- (C) Urea formaldehyde resin
- (D) Polycarbonate

Correct Answer: (A) Polyacrylonitrile

Solution:

Step 1: Understanding the Concept

Homopolymers have one kind of repeating unit. Copolymers are made from two or more monomers.

Step 2: Detailed Explanation

Polyacrylonitrile is made from acrylonitrile alone, so it is a homopolymer.

Buna-N is made from butadiene and acrylonitrile, so it is a copolymer.

Urea-formaldehyde resin is made from urea and formaldehyde, and polycarbonate is made from bisphenol-A and a carbonate source, so both come from two monomers.

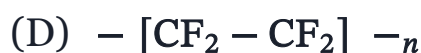
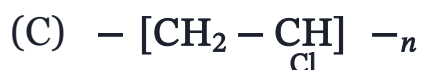
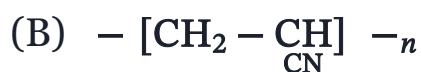
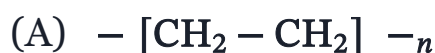
Final Answer:

Polyacrylonitrile is the homopolymer, option (A).

Polyacrylonitrile (A)

Quick Tip: A homopolymer is made from one type of monomer; polyacrylonitrile comes from acrylonitrile only.

100. Which from following polymers is used to obtain articles like rain coats?



Correct Answer: (C) $-\text{[CH}_2-\underset{\text{Cl}}{\text{CH}}\text{]}-_n$

Solution:

Step 1: Understanding the Concept

Each option is the repeating unit of a different polymer.

Step 2: Detailed Explanation

(A) Polythene: bags, bottles. (B) Polyacrylonitrile (orlon): fibres. (C) PVC, with Cl on alternate carbons: raincoats, hand bags, vinyl flooring. (D) Teflon: non-stick coating.

So raincoats are made from PVC, option (C).

Final Answer:

PVC is used for raincoats, option (C).

PVC (C)

Quick Tip: Polyvinyl chloride (PVC) is used for raincoats, pipes and similar items.

Mathematics

Questions 101 to 150 | 2 marks each | 100 marks

101. If α and β are the distinct roots of the equation $x^2 - x + 1 = 0$, then the value of $\alpha^{200} + \beta^{206} + 2$ is equal to

- (A) 1
- (B) -1
- (C) 0
- (D) 2

Correct Answer: (A) 1

Solution:

Step 1: Understanding the Concept

The roots of $x^2 - x + 1 = 0$ are $\alpha, \beta = \frac{1 \pm i\sqrt{3}}{2} = e^{\pm i\pi/3}$. Their sixth power is $e^{\pm 2\pi i} = 1$.

Step 2: Key Formula or Approach

From the equation: $\alpha + \beta = 1$ and $\alpha\beta = 1$. Also $\alpha^6 = \beta^6 = 1$, so only the remainder of the power on division by 6 matters.

Step 3: Detailed Explanation

$200 = 6 \times 33 + 2$, so $\alpha^{200} = \alpha^2$.

$206 = 6 \times 34 + 2$, so $\beta^{206} = \beta^2$.

$$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = 1 - 2 = -1$$

$$\alpha^{200} + \beta^{206} + 2 = -1 + 2 = 1$$

Final Answer:

The value is 1, option (A).

1 (A)

Quick Tip: The roots satisfy $\alpha^6 = \beta^6 = 1$, so reduce each power modulo 6.



102. The number of arrangements of the letters in the word SOLAPUR, so that consonants and vowels are placed alternately is

- (A) 100
- (B) 129
- (C) 144
- (D) 121

Correct Answer: (C) 144

Solution:**Step 1: Understanding the Concept**

The word SOLAPUR has 7 distinct letters. Vowels: O, A, U (3).

Consonants: S, L, P, R (4).

Step 2: Key Formula or Approach

For alternate placement with 4 consonants and 3 vowels, the only pattern is C V C V C V C, with consonants in positions 1, 3, 5, 7 and

vowels in 2, 4, 6.

Step 3: Detailed Explanation

Arrange 4 consonants in their 4 places: $4! = 24$ ways.

Arrange 3 vowels in their 3 places: $3! = 6$ ways.

$$\text{Total} = 24 \times 6 = 144$$

The pattern V C V C V C V is impossible, since it would need 4 vowels.

Final Answer:

There are 144 such arrangements, option (C).

144 (C)

Quick Tip: SOLAPUR has 4 consonants and 3 vowels, so the pattern must be C V C V C V C.

103. If $\cos 43^\circ + \sin 43^\circ = k^3$, then $\cos 2^\circ = \dots$

- (A) $-\frac{k^3}{\sqrt{2}}$
- (B) $\frac{k^3}{\sqrt{2}}$
- (C) $-\frac{k^3}{\sqrt{3}}$
- (D) $\frac{k^3}{\sqrt{3}}$

Correct Answer: (B) $\frac{k^3}{\sqrt{2}}$

Solution:

Step 1: Understanding the Concept

A sum $\cos \theta + \sin \theta$ can be written as $\sqrt{2} \cos(\theta - 45^\circ)$.

Step 2: Key Formula or Approach

$$\cos \theta + \sin \theta = \sqrt{2} \left(\cos \theta \cdot \frac{1}{\sqrt{2}} + \sin \theta \cdot \frac{1}{\sqrt{2}} \right) = \sqrt{2} \cos(\theta - 45^\circ)$$

Step 3: Detailed Explanation

With $\theta = 43^\circ$: $\cos 43^\circ + \sin 43^\circ = \sqrt{2} \cos(-2^\circ) = \sqrt{2} \cos 2^\circ$.

So $\sqrt{2} \cos 2^\circ = k^3$, which gives

$$\cos 2^\circ = \frac{k^3}{\sqrt{2}}$$

Final Answer:

$$\cos 2^\circ = \frac{k^3}{\sqrt{2}}, \text{ option (B).}$$

$$\boxed{\frac{k^3}{\sqrt{2}} \text{ (B)}}$$

Quick Tip: Write $\cos 43^\circ + \sin 43^\circ = \sqrt{2} \cos(43^\circ - 45^\circ)$.



104. If the equation $\sin 3\theta - \cos^2 \theta = \frac{1}{4}$ and $\theta \in [0, \pi]$, then the number of solutions is...

- (A) 0
- (B) 2
- (C) 4
- (D) 6

Correct Answer: (C) 4

Solution:

Step 1: Understanding the Concept

We need the number of roots of $\sin 3\theta = \cos^2 \theta + \frac{1}{4}$ in $[0, \pi]$. The right side lies between $\frac{1}{4}$ and $\frac{5}{4}$, so a root needs $\sin 3\theta \geq \frac{1}{4}$.

Step 2: Key Formula or Approach

Compare the curves $y = \sin 3\theta$ and $y = \cos^2 \theta + \frac{1}{4}$ on $[0, \pi]$. The sine curve has 3 half-waves, positive on $(0, \pi/3)$, negative on $(\pi/3, 2\pi/3)$ and positive on $(2\pi/3, \pi)$.

Step 3: Detailed Explanation

On $(0, \pi/3)$: at $\theta = 0$, $f = -1.25$. At $\theta = \pi/6$, $\sin 3\theta = 1$ and $\cos^2 \theta + \frac{1}{4} = 1$, so $f = 0$ exactly, and the maximum of f is only slightly above zero near here. The curve rises through zero and comes back through zero, giving 2 roots in this lobe.

On $(\pi/3, 2\pi/3)$ the sine is negative, so there is no root.

On $(2\pi/3, \pi)$ the same pattern repeats, since $\cos^2 \theta$ is symmetric about $\pi/2$ and $\sin 3\theta$ is symmetric under $\theta \rightarrow \pi - \theta$ in magnitude. This gives 2 more roots.

Total roots: 4.

Final Answer:

The equation has 4 solutions in $[0, \pi]$, option (C).

Quick Tip: Study $f(\theta) = \sin 3\theta - \cos^2 \theta - \frac{1}{4}$ on $[0, \pi]$ and count where it crosses zero.

105. If $A(2, -3)$ and $C(-6, 7)$ are opposite vertices of a rhombus ABCD, then the equation of diagonal BD is

- (A) $5x - 4y + 2 = 0$
- (B) $5x + 4y + 2 = 0$
- (C) $4x - 5y + 18 = 0$
- (D) $4x + 5y + 18 = 0$

Correct Answer: (C) $4x - 5y + 18 = 0$

Solution:

Step 1: Understanding the Concept

In a rhombus, diagonals bisect each other perpendicularly. So BD passes through the midpoint of AC and is perpendicular to AC.

Step 2: Key Formula or Approach

Midpoint of AC, slope of AC, then slope of BD is the negative reciprocal.

Step 3: Detailed Explanation

Midpoint of $A(2, -3)$ and $C(-6, 7)$: $(-2, 2)$.

Slope of AC: $\frac{7 - (-3)}{-6 - 2} = -\frac{5}{4}$, so the slope of BD is $\frac{4}{5}$.

Line through $(-2, 2)$: $y - 2 = \frac{4}{5}(x + 2)$, so $5y - 10 = 4x + 8$, which gives

$$4x - 5y + 18 = 0$$

Final Answer:

The diagonal BD is $4x - 5y + 18 = 0$, option (C).

$$4x - 5y + 18 = 0 \text{ (C)}$$

Quick Tip: The diagonals of a rhombus bisect each other at right angles.



106. The measure of the acute angle between the pair of lines $2x^2 + xy - y^2 - x + 2y - 1 = 0$ is:

- (A) $\tan^{-1} \frac{1}{3}$
- (B) $\tan^{-1} 1$
- (C) $\cos^{-1} \frac{1}{\sqrt{10}}$
- (D) $\cos^{-1} \sqrt{10}$

Correct Answer: (C) $\cos^{-1} \frac{1}{\sqrt{10}}$

Solution:

Step 1: Understanding the Concept

The equation represents a pair of lines. Factorise to get the two lines and their slopes.

Step 2: Key Formula or Approach

The equation factorises as $(2x - y + 1)(x + y - 1) = 0$. Check: the product gives $2x^2 + xy - y^2 - x + 2y - 1$.

Step 3: Detailed Explanation

Lines: $2x - y + 1 = 0$ with slope 2, and $x + y - 1 = 0$ with slope -1 .

$$\tan \theta = \left| \frac{2 - (-1)}{1 + 2(-1)} \right| = \left| \frac{3}{-1} \right| = 3$$

With $\tan \theta = 3$, $\cos \theta = \frac{1}{\sqrt{1+9}} = \frac{1}{\sqrt{10}}$.

So $\theta = \cos^{-1} \frac{1}{\sqrt{10}}$.

Final Answer:

The acute angle is $\cos^{-1} \frac{1}{\sqrt{10}}$, option (C).

$\cos^{-1} \frac{1}{\sqrt{10}} \text{ (C)}$

Quick Tip: Factor into two lines, find slopes and use the angle formula.



107. The joint equation of a pair of lines passing through point $(1, 4)$, one of which is parallel to X-axis and the other makes an angle of 45° with the positive direction of X-axis, is

- (A) $x^2 - xy - x + 4y - 12 = 0$
(B) $xy - y^2 - 4x + 7y - 12 = 0$
(C) $x^2 + 2xy - y^2 + 7 = 0$
(D) $xy - 2y^2 + 3x + 2y + 17 = 0$

Correct Answer: (B) $xy - y^2 - 4x + 7y - 12 = 0$

Solution:

Step 1: Understanding the Concept

The joint equation of two lines is the product of their equations set equal to zero.

Step 2: Key Formula or Approach

Line 1: parallel to X-axis through (1, 4), so $y - 4 = 0$. Line 2: slope $\tan 45^\circ = 1$ through (1, 4), so $y - 4 = x - 1$, i.e. $x - y + 3 = 0$.

Step 3: Detailed Explanation

$$(y - 4)(x - y + 3) = 0$$

$$xy - y^2 + 3y - 4x + 4y - 12 = 0$$

$$xy - y^2 - 4x + 7y - 12 = 0$$

Final Answer:

The joint equation is $xy - y^2 - 4x + 7y - 12 = 0$, option (B).

$$xy - y^2 - 4x + 7y - 12 = 0 \text{ (B)}$$

Quick Tip: One line has slope 0, the other slope 1; multiply the two line equations.

108. If a circle passes through the points (2, 3) and (4, 5) and its center lies on the straight line $y - 4x + 3 = 0$, then its equation is.....

- (A) $x^2 + y^2 - 4x - 10y + 25 = 0$
- (B) $x^2 + y^2 - 4x - 10y - 25 = 0$
- (C) $x^2 + y^2 - 4x + 10y - 25 = 0$
- (D) $x^2 + y^2 + 25 = 0$

Correct Answer: (A) $x^2 + y^2 - 4x - 10y + 25 = 0$

Solution:

Step 1: Understanding the Concept

The centre is equidistant from (2, 3) and (4, 5), so it lies on the perpendicular bisector of the segment joining them. It also lies on $y = 4x - 3$.

Step 2: Key Formula or Approach

Midpoint (3, 4), slope of the chord = 1, so the bisector is $x + y = 7$.

Step 3: Detailed Explanation

Solve $y = 4x - 3$ and $x + y = 7$: $5x - 3 = 7$, $x = 2$, $y = 5$. Centre (2, 5).

Radius squared: distance to (2, 3): $r^2 = 0 + 4 = 4$.

Equation: $(x - 2)^2 + (y - 5)^2 = 4$, i.e.

$$x^2 + y^2 - 4x - 10y + 25 = 0$$

Final Answer:

The circle is $x^2 + y^2 - 4x - 10y + 25 = 0$, option (A).

$$x^2 + y^2 - 4x - 10y + 25 = 0 \text{ (A)}$$

Quick Tip: Centre lies on the perpendicular bisector of the chord and on the given line.

109. The equations of the tangents to the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$ making an inclination of 30° with the major axis are

- (A) $x + \sqrt{3}y \pm \sqrt{43} = 0$
- (B) $x - \sqrt{3}y \pm \sqrt{43} = 0$
- (C) $\sqrt{3}x - y \pm \sqrt{43} = 0$
- (D) $x - \sqrt{3}y \pm \sqrt{3} = 0$

Correct Answer: (B) $x - \sqrt{3}y \pm \sqrt{43} = 0$

Solution:

Step 1: Understanding the Concept

Tangents of slope m to $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ are $y = mx \pm \sqrt{a^2m^2 + b^2}$. Here $a^2 = 16$ and $b^2 = 9$.

Step 2: Key Formula or Approach

Inclination 30° means $m = \tan 30^\circ = \frac{1}{\sqrt{3}}$.

Step 3: Detailed Explanation

$$a^2m^2 + b^2 = \frac{16}{3} + 9 = \frac{43}{3}$$

$$y = \frac{x}{\sqrt{3}} \pm \sqrt{\frac{43}{3}}$$

Multiply by $\sqrt{3}$: $\sqrt{3}y = x \pm \sqrt{43}$, so

$$x - \sqrt{3}y \pm \sqrt{43} = 0$$

Final Answer:

The tangents are $x - \sqrt{3}y \pm \sqrt{43} = 0$, option (B).

$$\boxed{x - \sqrt{3}y \pm \sqrt{43} = 0 \text{ (B)}}$$

Quick Tip: Use $y = mx \pm \sqrt{a^2m^2 + b^2}$ with $m = \tan 30^\circ$.

110. If $\lim_{x \rightarrow 1} \frac{\sin(3x^2 - 4x + 1) - x^2 + 1}{2x^3 - 7x^2 + ax + b} = -2$, then the quadratic equation having roots a and b is

- (A) $x^2 + 5x - 24 = 0$
- (B) $x^2 - 5x + 24 = 0$
- (C) $x^2 - 5x - 24 = 0$
- (D) $x^2 + 5x + 24 = 0$

Correct Answer: (C) $x^2 - 5x - 24 = 0$

Solution:

Step 1: Understanding the Concept

At $x = 1$ the numerator $\sin(3 - 4 + 1) - 1 + 1 = 0$. For a finite limit the denominator must also be 0, giving a first condition on a and b .

Step 2: Key Formula or Approach

Use L'Hopital repeatedly until the limit is determined.

Step 3: Detailed Explanation

Denominator zero at $x = 1$: $2 - 7 + a + b = 0$, so $a + b = 5$.

First derivatives: numerator $\cos(3x^2 - 4x + 1)(6x - 4) - 2x$ equals $2 - 2 = 0$ at $x = 1$. Denominator $6x^2 - 14x + a$ equals $a - 8$. For the ratio not to be $0/(a - 8) = 0$ (we need -2), we need $a = 8$, so the denominator derivative also vanishes. Then $b = -3$.

Second derivatives at $x = 1$: numerator $= -\sin(0)(2)^2 + 6 \cos 0 - 2 = 4$. Denominator $= 12 - 14 = -2$. Ratio $= -2$, which matches the given value.

Quadratic with roots 8 and -3 : sum 5, product -24 :

$$x^2 - 5x - 24 = 0$$

Final Answer:

The roots are $a = 8$ and $b = -3$, so the equation is $x^2 - 5x - 24 = 0$, option (C).

$x^2 - 5x - 24 = 0 \text{ (C)}$

Quick Tip: The denominator must vanish at $x = 1$ twice for the limit to be finite and nonzero.



111. If the statements

p : If voltage increases then current decreases.

q : If voltage does not increase then current does not decrease.

r : If current decreases then voltage increases.

s : If current does not decrease then voltage does not increase.

then which of the following pairs of statements having same meaning

(A) p , q and r , s

(B) p , r and q , s

(C) p , s and q , r

(D) no pair having same meaning

Correct Answer: (C) p , s and q , r

Solution:

Step 1: Understanding the Concept

Let A : voltage increases, and B : current decreases. Then $p : A \rightarrow B$, $q : \sim A \rightarrow \sim B$, $r : B \rightarrow A$, $s : \sim B \rightarrow \sim A$.

Step 2: Key Formula or Approach

A statement $X \rightarrow Y$ is equivalent to its contrapositive $\sim Y \rightarrow \sim X$.

Step 3: Detailed Explanation

The contrapositive of $p : A \rightarrow B$ is $\sim B \rightarrow \sim A$, which is s . So p and s have the same meaning.

The contrapositive of $r : B \rightarrow A$ is $\sim A \rightarrow \sim B$, which is q . So q and r

have the same meaning.

The pairs are (p, s) and (q, r) .

Final Answer:

The equivalent pairs are p, s and q, r , option (C).

(p, s) and (q, r) (C)

Quick Tip: A conditional is equivalent to its contrapositive.

112. If $p \rightarrow (q \vee \sim r)$ is false, then the truth values of $(p \leftrightarrow q) \wedge r$ and $\sim p \rightarrow \sim q$ are :

- (A) T, T
- (B) T, F
- (C) F, T
- (D) F, F

Correct Answer: (C) F, T

Solution:

Step 1: Understanding the Concept

$p \rightarrow (q \vee \sim r)$ is false only when p is T and $q \vee \sim r$ is F.

Step 2: Key Formula or Approach

$q \vee \sim r$ is false only when both q and $\sim r$ are false, so $q = F$ and $r = T$.

Step 3: Detailed Explanation

So $p = T, q = F, r = T$.

$$(p \leftrightarrow q) \wedge r: p \leftrightarrow q = T \leftrightarrow F = F. F \wedge T = F.$$

$$\sim p \rightarrow \sim q: F \rightarrow T = T.$$

The truth values are (F, T) .

Final Answer:

The values are F and T, option (C).

$$\boxed{F, T (C)}$$

Quick Tip: A conditional is false only when the antecedent is true and the consequent false.



113. With usual notations, in $\triangle ABC$, if $\cos C = \frac{\sin A}{2\sin B}$, then which of the following is true ?

- (A) $a = c$
- (B) $a = b$
- (C) $b = c$
- (D) $a^2 = b^2 + c^2$

Correct Answer: (C) $b = c$

Solution:

Step 1: Understanding the Concept

In any triangle $A + B + C = \pi$, so $\sin A = \sin(B + C)$.

Step 2: Key Formula or Approach

The given condition gives $2 \sin B \cos C = \sin A$.

Step 3: Detailed Explanation

$$2 \sin B \cos C = \sin B \cos C + \cos B \sin C$$

$$\sin B \cos C - \cos B \sin C = 0 \Rightarrow \sin(B - C) = 0$$

Since $B - C$ lies between $-\pi$ and π , $B = C$. Equal angles mean equal opposite sides, so $b = c$.

Final Answer:

The triangle has $b = c$, option (C).

$$b = c \text{ (C)}$$

Quick Tip: Write $\sin A = \sin(B + C)$ and simplify.

114. In $\triangle ABC$, if $a = 13, b = 14, c = 15$, then the value of $\sin A + \cos A$ is...

- (A) $\frac{3}{5}$
- (B) $\frac{7}{5}$
- (C) $\frac{4}{5}$
- (D) $\frac{12}{5}$

Correct Answer: (B) $\frac{7}{5}$

Solution:

Step 1: Understanding the Concept

With three sides known, the cosine rule gives the angle A opposite to a .

Step 2: Key Formula or Approach

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

Step 3: Detailed Explanation

$$\cos A = \frac{196 + 225 - 169}{2 \times 14 \times 15} = \frac{252}{420} = \frac{3}{5}$$

$$\text{Then } \sin A = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}.$$

$$\sin A + \cos A = \frac{4}{5} + \frac{3}{5} = \frac{7}{5}$$

Final Answer:

The value is $7/5$, option (B).

$$\boxed{\frac{7}{5} \text{ (B)}}$$

Quick Tip: Find $\cos A$ by the cosine rule, then $\sin A$.

115. If the matrix $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$ is expressed as the sum of a symmetric matrix B and a skew symmetric matrix C then which of the following relations is correct?

(A) $|A| = |B| \times |C|$

(B) $|A| = |B| + |C|$

(C) $|C| = 0$

(D) $|A| = |B|$

Correct Answer: (B) $|A| = |B| + |C|$

Solution:

Step 1: Understanding the Concept

Every square matrix is the sum of a symmetric matrix $B = \frac{1}{2}(A + A^T)$ and a skew-symmetric matrix $C = \frac{1}{2}(A - A^T)$.

Step 2: Key Formula or Approach

For $A = \begin{bmatrix} 4 & 1 \\ 3 & 2 \end{bmatrix}$, $A^T = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$.

Step 3: Detailed Explanation

$B = \begin{bmatrix} 4 & 2 \\ 2 & 2 \end{bmatrix}$ and $C = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$.

$|A| = 8 - 3 = 5$, $|B| = 8 - 4 = 4$, $|C| = 0 + 1 = 1$.

So $|A| = |B| + |C|$ since $5 = 4 + 1$.

The product $4 \times 1 = 4 \neq 5$, and $|C| = 1 \neq 0$.

Final Answer:

The correct relation is $|A| = |B| + |C|$, option (B).

$$\boxed{|A| = |B| + |C| \text{ (B)}}$$

Quick Tip: Find $B = \frac{1}{2}(A + A^T)$ and $C = \frac{1}{2}(A - A^T)$ and compare determinants.



116. If A is a non-singular matrix and $A^2 - A + I = 0$, then $A^{-1} = \dots$

- (A) A
- (B) $A - I$
- (C) $I - A$
- (D) $A + I$

Correct Answer: (C) $I - A$

Solution:

Step 1: Understanding the Concept

A non-singular matrix has an inverse A^{-1} with $AA^{-1} = A^{-1}A = I$.

Step 2: Key Formula or Approach

Multiply both sides of $A^2 - A + I = 0$ by A^{-1} .

Step 3: Detailed Explanation

$$A^{-1}A^2 - A^{-1}A + A^{-1}I = 0$$

$$A - I + A^{-1} = 0$$

$$A^{-1} = I - A$$

Final Answer:

The inverse is $I - A$, option (C).

$$A^{-1} = I - A \text{ (C)}$$

Quick Tip: Multiply the given relation by A^{-1} .

117. The value of $3\tan^{-1}(\frac{1}{2}) = \dots$

- (A) $\tan^{-1}(\frac{5}{2})$
- (B) $\tan^{-1}(\frac{2}{5})$
- (C) $\cot^{-1}(\frac{11}{2})$
- (D) $\tan^{-1}(\frac{11}{2})$

Correct Answer: (D) $\tan^{-1}(\frac{11}{2})$

Solution:**Step 1: Understanding the Concept**

Use $2 \tan^{-1} x = \tan^{-1} \frac{2x}{1-x^2}$ for $|x| < 1$, and $\tan^{-1} a + \tan^{-1} b = \tan^{-1} \frac{a+b}{1-ab}$ for $ab < 1$.

Step 2: Key Formula or Approach

$$2 \tan^{-1} \frac{1}{2} = \tan^{-1} \frac{1}{1 - \frac{1}{4}} = \tan^{-1} \frac{4}{3}$$

Step 3: Detailed Explanation

$$\begin{aligned} 3 \tan^{-1} \frac{1}{2} &= \tan^{-1} \frac{4}{3} + \tan^{-1} \frac{1}{2} = \tan^{-1} \frac{\frac{4}{3} + \frac{1}{2}}{1 - \frac{2}{3}} \\ &= \tan^{-1} \frac{11/6}{1/3} = \tan^{-1} \frac{11}{2} \end{aligned}$$

The product $\frac{4}{3} \cdot \frac{1}{2} = \frac{2}{3} < 1$, so the formula applies directly.

Final Answer:

The value is $\tan^{-1} \frac{11}{2}$, option (D).

$$\boxed{\tan^{-1} \frac{11}{2} \text{ (D)}}$$

Quick Tip: Use the double angle formula for $\tan^{-1}$, then add one more $\tan^{-1} \frac{1}{2}$.

118. If $f(x) = \frac{x+2}{x^2-3x+1}$, then the values of x for which $f(x)$ is not defined are

- (A) $x = \frac{3+\sqrt{5}}{2}, x = \frac{3-\sqrt{5}}{2}$
- (B) $x = \frac{-3+\sqrt{5}}{2}, x = \frac{-3-\sqrt{5}}{2}$
- (C) $x = \frac{3+\sqrt{3}}{2}, x = \frac{3-\sqrt{3}}{2}$
- (D) $x = \frac{2+\sqrt{5}}{2}, x = \frac{2-\sqrt{5}}{2}$

Correct Answer: (A) $x = \frac{3+\sqrt{5}}{2}, x = \frac{3-\sqrt{5}}{2}$

Solution:

Step 1: Understanding the Concept

A rational function is undefined at the zeros of its denominator.

Step 2: Key Formula or Approach

Solve $x^2 - 3x + 1 = 0$ with the quadratic formula.

Step 3: Detailed Explanation

$$x = \frac{3 \pm \sqrt{9 - 4}}{2} = \frac{3 \pm \sqrt{5}}{2}$$

The numerator $x + 2$ is not zero at either value, so both are genuine points where f is undefined.

Final Answer:

The function is undefined at $x = \frac{3 \pm \sqrt{5}}{2}$, option (A).

$$x = \frac{3 \pm \sqrt{5}}{2} \text{ (A)}$$

Quick Tip: The function is undefined where the denominator is zero.



119. If $f : R \rightarrow R$ and $g : R \rightarrow R$ are defined as $f(x) = 2x - |x|$ and $g(x) = 2x + |x|$, then

- (A) $(f \circ g)(2) + (g \circ f)(2) = 0$
 (B) $(f \circ g)(2) - (g \circ f)(-2) = 0$
 (C) $(f \circ g)(2) - (f \circ g)(-2) = 0$
 (D) $(g \circ f)(2) + (g \circ f)(-2) = 0$

Correct Answer: (D) $(g \circ f)(2) + (g \circ f)(-2) = 0$

Solution:

Step 1: Understanding the Concept

$f(x) = x$ for $x \geq 0$ and $3x$ for $x < 0$. $g(x) = 3x$ for $x \geq 0$ and x for $x < 0$.

Step 2: Key Formula or Approach

Evaluate $f \circ g$ and $g \circ f$ at ± 2 .

Step 3: Detailed Explanation

$$(f \circ g)(2) = f(6) = 6. (g \circ f)(2) = g(2) = 6.$$

$$(f \circ g)(-2) = f(-2) = -6. (g \circ f)(-2) = g(-6) = -6.$$

$$(A) 6 + 6 = 12 \neq 0. (B) 6 - (-6) = 12 \neq 0. (C) 6 - (-6) = 12 \neq 0.$$

$$(D) (g \circ f)(2) + (g \circ f)(-2) = 6 - 6 = 0, \text{ which is true.}$$

Final Answer:

Statement (D) holds, option (D).

(D)

Quick Tip: Write f and g piecewise, then evaluate the compositions.

120. Which of the following function is discontinuous at $x = 0$?

$$(A) \ f(x) = (1+x)^{\frac{2}{x}}, \text{ for } x \neq 0$$

$$= e^2, \text{ for } x = 0$$

$$(B) \ f(x) = \sin x - \cos x, \text{ for } x \neq 0$$

$$= -1, \text{ for } x = 0$$

$$(C) \ f(x) = \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}, \text{ for } x \neq 0$$

$$= -1, \text{ for } x = 0$$

$$(D) \ f(x) = \frac{e^{5x} - e^{2x}}{\sin 3x}, \text{ for } x \neq 0$$

$$= 1, \text{ for } x = 0$$

Correct Answer: (C) $f(x) = \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}, \text{ for } x \neq 0$
 $= -1, \text{ for } x = 0$

Solution:

Step 1: Understanding the Concept

A function is continuous at 0 if the left limit, right limit and value are all equal.

Step 2: Key Formula or Approach

Compute each limit using standard results.

Step 3: Detailed Explanation

(A) $(1+x)^{2/x} \rightarrow e^2 = f(0)$: continuous.

(B) $\sin x - \cos x \rightarrow -1 = f(0)$: continuous.

(D) $\frac{e^{5x} - e^{2x}}{\sin 3x} = \frac{e^{2x}(e^{3x} - 1)}{\sin 3x} \rightarrow 1 \cdot 1 = 1 = f(0)$: continuous.

(C) As $x \rightarrow 0^+$, $e^{1/x} \rightarrow \infty$ so the ratio tends to 1. As $x \rightarrow 0^-$, $e^{1/x} \rightarrow 0$ so the ratio tends to -1 . The two limits differ, so f is discontinuous at 0.

Final Answer:

Function (C) is discontinuous at $x = 0$, option (C).

(C)

Quick Tip: Compare left and right limits at $x = 0$ for each piece.



121. If $y = x \cdot 7^x$, then the value of $\frac{dx}{dy}$ when $x = 1$ is

(A) $7(\log 7 + 1)$

(B) $\log 7 + 1$

(C) $\frac{1}{7(\log 7 + 1)}$

(D) $\frac{1}{\log 7 + 1}$

Correct Answer: (C) $\frac{1}{7(\log 7 + 1)}$

Solution:

Step 1: Understanding the Concept

$$\frac{dx}{dy} = \frac{1}{dy/dx} \text{ whenever } dy/dx \neq 0.$$

Step 2: Key Formula or Approach

$$\frac{dy}{dx} = 7^x + x 7^x \log 7$$

Step 3: Detailed Explanation

$$\text{At } x = 1: \frac{dy}{dx} = 7 + 7 \log 7 = 7(1 + \log 7).$$

$$\frac{dx}{dy} = \frac{1}{7(\log 7 + 1)}$$

Final Answer:

The value is $\frac{1}{7(\log 7 + 1)}$, option (C).

$$\boxed{\frac{1}{7(\log 7 + 1)} \text{ (C)}}$$

Quick Tip: Find $\frac{dy}{dx}$ by the product rule and take the reciprocal.

122. If $y = [(x + 1)(2x + 1)(3x + 1) \dots \dots (nx + 1)]^4$, where $n \in N$ and $\frac{dy}{dx}$ at $x = 0$ is $2k$, then the value of k is

- (A) $\frac{n(n+1)}{2}$
- (B) $n(n + 1)$
- (C) $2n(n + 1)$
- (D) $4n(n + 1)$

Correct Answer: (B) $n(n + 1)$

Solution:**Step 1: Understanding the Concept**

Let $P(x) = \prod_{k=1}^n (kx + 1)$, so $y = P^4$.

Step 2: Key Formula or Approach

$$\frac{dy}{dx} = 4P^3 P'.$$

Step 3: Detailed Explanation

$P(0) = 1$. By logarithmic differentiation, $\frac{P'}{P} = \sum \frac{k}{kx + 1}$, so $P'(0)$
 $= \sum_{k=1}^n k = \frac{n(n+1)}{2}$.

$$\left. \frac{dy}{dx} \right|_{x=0} = 4 \cdot 1 \cdot \frac{n(n+1)}{2} = 2n(n+1)$$

Given that this equals $2k$, we get $k = n(n+1)$.

Final Answer:

$k = n(n+1)$, option (B).

$$\boxed{n(n+1) \text{ (B)}}$$

Quick Tip: Use $y = P^4$ and evaluate $P(0)$ and $P'(0)$.

123. If $f(x) = \cos x \cos 2x \cos 4x \cos 8x \cos 16x$, then

Missing open brace for superscript is equal to

- (A) 1
- (B) 0
- (C) $\sqrt{2}$
- (D) $\frac{1}{\sqrt{2}}$

Correct Answer: (C) $\sqrt{2}$

Solution:

Step 1: Understanding the Concept

Repeated use of $\sin 2\theta = 2 \sin \theta \cos \theta$ gives

$$\cos x \cos 2x \cos 4x \cos 8x \cos 16x = \frac{\sin 32x}{32 \sin x}.$$

Step 2: Key Formula or Approach

Differentiate by the quotient rule:

$$f'(x) = \frac{32 \cos 32x \sin x - \sin 32x \cos x}{32 \sin^2 x}$$

Step 3: Detailed Explanation

At $x = \pi/4$: $32x = 8\pi$, so $\sin 32x = 0$ and $\cos 32x = 1$.

$$\sin x = \frac{1}{\sqrt{2}}, \sin^2 x = \frac{1}{2}.$$

$$f'\left(\frac{\pi}{4}\right) = \frac{32 \cdot 1 \cdot \frac{1}{\sqrt{2}} - 0}{32 \cdot \frac{1}{2}} = \frac{32/\sqrt{2}}{16} = \sqrt{2}$$

Using the product form directly would meet $\cos 2x = 0$ at $\pi/4$, so the compact form is easier.

Final Answer:

The derivative is $\sqrt{2}$, option (C).

$$\boxed{\sqrt{2} \text{ (C)}}$$

Quick Tip: Use $\prod \cos(2^k x) = \frac{\sin 32x}{32 \sin x}$ before differentiating.

124. The derivative of the function $f(x) = \cos^4 x + \sin^4 x$, $0 \leq x \leq 2\pi$ is positive for

- (A) $0 < x < \frac{\pi}{8}$
 (B) $\frac{\pi}{4} < x < \frac{\pi}{2}$
 (C) $\frac{\pi}{2} < x < \frac{5\pi}{8}$
 (D) $\frac{5\pi}{8} < x < \frac{3\pi}{4}$

Correct Answer: (B) $\frac{\pi}{4} < x < \frac{\pi}{2}$

Solution:

Step 1: Understanding the Concept

Write $\cos^4 x + \sin^4 x = 1 - 2 \sin^2 x \cos^2 x = 1 - \frac{1}{2} \sin^2 2x$.

Step 2: Key Formula or Approach

$\sin^2 2x = \frac{1 - \cos 4x}{2}$, so $f(x) = \frac{3}{4} + \frac{1}{4} \cos 4x$ and $f'(x) = -\sin 4x$.

Step 3: Detailed Explanation

$f'(x) > 0$ means $\sin 4x < 0$, i.e. $4x \in (\pi, 2\pi)$ (within the first period), so $x \in \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$.

Check (A): $4x \in (0, \pi/2)$, sine positive, so $f' < 0$. (C): $4x \in (2\pi, 5\pi/2)$, sine positive. (D): $4x \in (5\pi/2, 3\pi)$, sine positive. So only (B) works.

Final Answer:

The derivative is positive on $(\pi/4, \pi/2)$, option (B).

$$\boxed{\left(\frac{\pi}{4}, \frac{\pi}{2}\right) \text{ (B)}}$$

Quick Tip: Simplify $f(x) = \frac{3}{4} + \frac{1}{4} \cos 4x$ and find where $f' > 0$.

125. The tangent to the curve intersects the Y-axis at point P. A line drawn through point P is perpendicular to this tangent and passes through another point (1, 0). The differential equation of the curve is...

(A) $y \frac{dy}{dx} - x \left(\frac{dy}{dx} \right)^2 = 1$

(B) $x \frac{dy}{dx} - y \left(\frac{dy}{dx} \right)^2 = 1$

(C) $y \frac{dy}{dx} + x = 1$

(D) $x \frac{dy}{dx} + y = 1$

Correct Answer: (A) $y \frac{dy}{dx} - x \left(\frac{dy}{dx} \right)^2 = 1$

Solution:

Step 1: Understanding the Concept

Let the curve be $y = y(x)$ with slope $m = \frac{dy}{dx}$ at a point (x, y) .

Step 2: Key Formula or Approach

Tangent at (x, y) meets the Y-axis where its equation $Y - y = m(X - x)$ gives $X = 0$: $P = (0, y - xm)$.

Step 3: Detailed Explanation

The line through P and $(1, 0)$ has slope $\frac{0 - (y - xm)}{1 - 0} = xm - y$.

It is perpendicular to the tangent, so $m(xm - y) = -1$.

$$xm^2 - ym = -1 \Rightarrow ym - xm^2 = 1$$

$$y \frac{dy}{dx} - x \left(\frac{dy}{dx} \right)^2 = 1$$

Final Answer:

The differential equation is $y \frac{dy}{dx} - x \left(\frac{dy}{dx} \right)^2 = 1$, option (A).

$$\boxed{y \frac{dy}{dx} - x \left(\frac{dy}{dx} \right)^2 = 1 \text{ (A)}}$$

Quick Tip: Find the Y-intercept P of the tangent, then use the perpendicular through P and $(1, 0)$.

126. The function $f(x) = \int \frac{x+3}{x^2-9x+20} dx$, then $f(x)$ is

- (A) increases on $\mathbb{R}$
- (B) decreases on $\mathbb{R} - (4, 5)$
- (C) decreases on $(-\infty, -3] \cup (4, 5)$
- (D) increases on $(-3, \infty)$

Correct Answer: (C) decreases on $(-\infty, -3] \cup (4, 5)$

Solution:**Step 1: Understanding the Concept**

If f is an antiderivative of g , then $f'(x) = g(x)$, so monotonicity follows the sign of g .

Step 2: Key Formula or Approach

$$f'(x) = \frac{x+3}{x^2-9x+20} = \frac{x+3}{(x-4)(x-5)}. \text{ Critical points: } -3, 4, 5.$$

Step 3: Detailed Explanation

For $x < -3$: numerator negative, denominator positive, so $f' < 0$.

For $-3 < x < 4$: numerator positive, denominator positive, so $f' > 0$.

For $4 < x < 5$: numerator positive, denominator negative, so $f' < 0$.

For $x > 5$: both positive, so $f' > 0$.

Hence f decreases on $(-\infty, -3] \cup (4, 5)$.

Final Answer:

The function decreases on $(-\infty, -3] \cup (4, 5)$, option (C).

$$(-\infty, -3] \cup (4, 5) \text{ (C)}$$

Quick Tip: The sign of $f'(x) = \frac{x+3}{(x-4)(x-5)}$ decides where f increases or decreases.

127. If $f(x) = \log(1+x) - \frac{x}{1+x}$, then the values of x for which $f(x)$ is monotonically increasing and monotonically decreasing are respectively.....

- (A) $(-\infty, 0), (0, \infty)$
- (B) $(0, \infty), (-\infty, 0)$
- (C) $(-1, 0), (0, \infty)$
- (D) $(0, \infty), (-1, 0)$

Correct Answer: (D) $(0, \infty), (-1, 0)$

Solution:

Step 1: Understanding the Concept

The domain is $x > -1$. f increases where $f' > 0$ and decreases where $f' < 0$.

Step 2: Key Formula or Approach

$$f'(x) = \frac{1}{1+x} - \frac{1}{(1+x)^2} = \frac{x}{(1+x)^2}$$

Step 3: Detailed Explanation

$(1+x)^2 > 0$ on the domain, so the sign of f' is the sign of x .

For $x > 0$, $f' > 0$: increasing on $(0, \infty)$.

For $-1 < x < 0$, $f' < 0$: decreasing on $(-1, 0)$.

Final Answer:

Increasing on $(0, \infty)$ and decreasing on $(-1, 0)$, option (D).

$(0, \infty), (-1, 0)$ (D)

Quick Tip: Differentiate and study the sign on the domain $x > -1$.

128. The number 28 is divided into two positive parts such that the sum of the cube of one part and the square of the other part is minimum, then the absolute difference between the two parts is

- (A) 24
- (B) 12
- (C) 8
- (D) 20

Correct Answer: (D) 20

Solution:

Step 1: Understanding the Concept

Let the two positive parts be x and $y = 28 - x$. Minimise $S = x^3 + y^2$.

Step 2: Key Formula or Approach

$$S(x) = x^3 + (28 - x)^2, \text{ so } S'(x) = 3x^2 - 2(28 - x) = 3x^2 + 2x - 56.$$

Step 3: Detailed Explanation

Set $S' = 0$: $3x^2 + 2x - 56 = 0$, giving $x = \frac{-2 \pm 26}{6}$, so $x = 4$ (the positive root).

$S'' = 6x + 2 > 0$, so this is a minimum.

The parts are 4 and 24, and the absolute difference is $24 - 4 = 20$.

Final Answer:

The difference between the parts is 20, option (D).

20 (D)

Quick Tip: Let the parts be x and $28 - x$ and minimise $x^3 + (28 - x)^2$.

129. The value of $\int \frac{\sin^3 x}{(\cos^4 x + 3\cos^2 x + 1)\tan^{-1}(\sec x + \cos x)} dx$ is

- (A) $\log(\tan^{-1}(\sec x + \cos x)) + c$
- (B) $2\log(\tan^{-1}(\sec x + \cos x)) + c$
- (C) $\frac{(\tan^{-1}(\sec x + \cos x))^2}{2} + c$
- (D) $\tan^{-1}(\sec x + \cos x) + c$

Correct Answer: (A) $\log(\tan^{-1}(\sec x + \cos x)) + c$

Solution:

Step 1: Understanding the Concept

The integrand has the form $\frac{u'}{u}$ with $u = \tan^{-1}(\sec x + \cos x)$. We show this by substitution.

Step 2: Key Formula or Approach

Let $t = \cos x$, so $\sin^3 x dx = -(1 - t^2) dt$. Let $w = \sec x + \cos x = t + \frac{1}{t}$.

Step 3: Detailed Explanation

$$\frac{dw}{dt} = 1 - \frac{1}{t^2} = \frac{t^2 - 1}{t^2}.$$

$$\frac{d}{dt} \tan^{-1} w = \frac{(t^2 - 1)/t^2}{1 + (t^2 + 1)^2/t^2} = \frac{t^2 - 1}{t^4 + 3t^2 + 1}$$

The integral becomes

$$\int \frac{(t^2 - 1) dt}{(t^4 + 3t^2 + 1) \tan^{-1} w} = \int \frac{d(\tan^{-1} w)}{\tan^{-1} w} = \log(\tan^{-1} w) + c$$

Final Answer:

The integral is $\log(\tan^{-1}(\sec x + \cos x)) + c$, option (A).

$\log(\tan^{-1}(\sec x + \cos x)) + c \text{ (A)}$

Quick Tip: Substitute $t = \cos x$ and notice the numerator is the derivative of $\tan^{-1}(\sec x + \cos x)$.

130. The value of $\int 2x^{\frac{1}{3}} \sin^3 \sqrt{x^2} dx$ is

- (A) $(x^{\frac{2}{3}} \cos x^{\frac{2}{3}} + \sin x^{\frac{2}{3}}) + c$
 (B) $3[x^{\frac{2}{3}} \cos x^{\frac{2}{3}} - \sin x^{\frac{2}{3}}] + c$
 (C) $3[-x^{\frac{2}{3}} \cos x^{\frac{2}{3}} - \sin x^{\frac{2}{3}}] + c$
 (D) $3[-x^{\frac{2}{3}} \cos x^{\frac{2}{3}} + \sin x^{\frac{2}{3}}] + c$

Correct Answer: (D) $3[-x^{\frac{2}{3}} \cos x^{\frac{2}{3}} + \sin x^{\frac{2}{3}}] + c$

Solution:

Step 1: Understanding the Concept

Let $t = x^{2/3}$. Then $dt = \frac{2}{3}x^{-1/3}dx$, so $dx = \frac{3}{2}x^{1/3}dt$.

Step 2: Key Formula or Approach

The integrand becomes $2x^{1/3} \sin t \cdot \frac{3}{2}x^{1/3}dt = 3x^{2/3} \sin t dt = 3t \sin t dt$.

Step 3: Detailed Explanation

By parts: $\int t \sin t dt = -t \cos t + \sin t$.

$$\int 2x^{1/3} \sin x^{2/3} dx = 3[-t \cos t + \sin t] + c$$

$$= 3[-x^{2/3} \cos x^{2/3} + \sin x^{2/3}] + c$$

Final Answer:

The integral is $3[-x^{2/3} \cos x^{2/3} + \sin x^{2/3}] + c$, option (D).

$$\boxed{3[-x^{2/3} \cos x^{2/3} + \sin x^{2/3}] + c \text{ (D)}}$$

Quick Tip: Put $t = x^{2/3}$ and integrate by parts.

131. $\int x^3 \log x \, dx =$

- (A) $\frac{x^4}{16} [4 \log x - 1] + c$
(B) $\frac{x^4}{16} [4 \log x + 1] + c$
(C) $\frac{x^4}{16} [-4 \log x - 1] + c$
(D) $\frac{x^4}{16} [-4 \log x + 1] + c$

Correct Answer: (A) $\frac{x^4}{16} [4 \log x - 1] + c$

Solution:

Step 1: Understanding the Concept

Integration by parts: $\int u \, dv = uv - \int v \, du$.

Step 2: Key Formula or Approach

Take $u = \log x$, $dv = x^3 dx$, so $du = dx/x$ and $v = x^4/4$.

Step 3: Detailed Explanation

$$\begin{aligned} \int x^3 \log x \, dx &= \frac{x^4}{4} \log x - \int \frac{x^3}{4} \, dx = \frac{x^4}{4} \log x - \frac{x^4}{16} + c \\ &= \frac{x^4}{16} [4 \log x - 1] + c \end{aligned}$$

Final Answer:

The integral is $\frac{x^4}{16} [4 \log x - 1] + c$, option (A).

$$\frac{x^4}{16} [4 \log x - 1] + c \text{ (A)}$$

Quick Tip: Use integration by parts with $u = \log x$ and $dv = x^3 dx$.

132. If Missing open brace for superscript and $f(0) = e$ then $f(1) = \dots \dots \dots$

- (A) e
- (B) $2e$
- (C) $3e$
- (D) $4e$

Correct Answer: (C) $3e$

Solution:

Step 1: Understanding the Concept

$$f'(x) = \frac{(\sqrt{x} + 1)e^{\sqrt{x}}}{\sqrt{x}} = e^{\sqrt{x}} + \frac{e^{\sqrt{x}}}{\sqrt{x}}.$$

Step 2: Key Formula or Approach

Put $t = \sqrt{x}$, so $dx = 2t dt$.

Step 3: Detailed Explanation

$$\int \frac{e^{\sqrt{x}}}{\sqrt{x}} dx = \int 2e^t dt = 2e^{\sqrt{x}}.$$

$$\int e^{\sqrt{x}} dx = 2 \int te^t dt = 2(t - 1)e^t = 2(\sqrt{x} - 1)e^{\sqrt{x}}.$$

$$\text{Add: } f(x) = 2\sqrt{x}e^{\sqrt{x}} + C.$$

$$f(0) = C = e. \text{ Then } f(1) = 2e + e = 3e.$$

Final Answer:

$f(1) = 3e$, option (C).

$$3e \text{ (C)}$$

Quick Tip: Split f' and integrate each term using $t = \sqrt{x}$.

133. The value of $\int_0^\pi \frac{1}{1+2^{\cos x}} dx$ is _____

- (A) $\frac{\pi}{2}$
- (B) π
- (C) 2π
- (D) $\frac{3\pi}{2}$

Correct Answer: (A) $\frac{\pi}{2}$

Solution:**Step 1: Understanding the Concept**

Replace x by $\pi - x$. Since $\cos(\pi - x) = -\cos x$, the integrand changes form.

Step 2: Key Formula or Approach

$$I = \int_0^\pi \frac{dx}{1+2^{\cos x}} = \int_0^\pi \frac{dx}{1+2^{-\cos x}} = \int_0^\pi \frac{2^{\cos x} dx}{2^{\cos x} + 1}$$

Step 3: Detailed Explanation

Add the two forms:

$$2I = \int_0^{\pi} \frac{1 + 2^{\cos x}}{1 + 2^{\cos x}} dx = \int_0^{\pi} dx = \pi$$

$$I = \frac{\pi}{2}$$

Final Answer:

The value is $\pi/2$, option (A).

$$\boxed{\frac{\pi}{2} \text{ (A)}}$$

Quick Tip: Use $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ and add.



134. If $I_n = \int_1^e (\log_e x)^n dx$ where $n \in \mathbb{Z}^+$ and $I_m + mI_{2026} = e$, then $m =$

- (A) 2024
- (B) 2025
- (C) 2026
- (D) 2027

Correct Answer: (D) 2027

Solution:

Step 1: Understanding the Concept

$I_n = \int_1^e (\log x)^n dx$. Use integration by parts with $u = (\log x)^n$ and $dv = dx$.

Step 2: Key Formula or Approach

$$I_n = [x(\log x)^n]_1^e - n \int_1^e (\log x)^{n-1} dx = e - nI_{n-1}$$

Step 3: Detailed Explanation

So $I_n + nI_{n-1} = e$.

Put $n = 2027$: $I_{2027} + 2027 I_{2026} = e$.

Compare with $I_m + mI_{2026} = e$. Matching terms gives $m = 2027$.

Final Answer:

The value of m is 2027, option (D).

2027 (D)

Quick Tip: Integrate by parts to get $I_n = e - nI_{n-1}$.



135. The area of the region bounded by the lines $2x - y + 1 = 0$, $y = -1$, $y = 3$ and the y-axis is _____

- (A) 2
- (B) 5
- (C) 3
- (D) 4

Correct Answer: (A) 2

Solution:

Step 1: Understanding the Concept

From $2x - y + 1 = 0$, $x = \frac{y-1}{2}$. The line crosses the y-axis at $y = 1$, so the region lies on both sides of the y-axis.

Step 2: Key Formula or Approach

$$\text{Area} = \int_{-1}^3 \left| \frac{y-1}{2} \right| dy$$

Step 3: Detailed Explanation

From $y = -1$ to 1 , $x \leq 0$: area $= \int_{-1}^1 \frac{1-y}{2} dy = \frac{1}{2} \left[y - \frac{y^2}{2} \right]_{-1}^1 = \frac{1}{2} \cdot 2 = 1$.

From $y = 1$ to 3 , $x \geq 0$: area $= \int_1^3 \frac{y-1}{2} dy = \frac{1}{2} \cdot 2 = 1$.

Total $= 1 + 1 = 2$ sq. units.

Final Answer:

The area is 2 square units, option (A).

2 (A)

Quick Tip: Integrate $|x|$ with respect to y where $x = \frac{y-1}{2}$.

136. Area enclosed by curve $y = 2x^2$ and lines $x \geq 1, y \leq 4$ is sq. units

- (A) $\frac{8\sqrt{2}-10}{3}$
 (B) $\frac{8(\sqrt{2}-1)}{3}$
 (C) $\frac{4\sqrt{2}-5}{3}$
 (D) $\frac{4(\sqrt{2}-1)}{3}$

Correct Answer: (A) $\frac{8\sqrt{2}-10}{3}$

Solution:

Step 1: Understanding the Concept

The region is bounded above by $y = 4$, below by the parabola $y = 2x^2$, and on the left by $x = 1$.

Step 2: Key Formula or Approach

$2x^2 = 4$ gives $x = \sqrt{2}$. So the limits are $x = 1$ to $x = \sqrt{2}$.

Step 3: Detailed Explanation

$$\text{Area} = \int_1^{\sqrt{2}} (4 - 2x^2) dx = \left[4x - \frac{2x^3}{3} \right]_1^{\sqrt{2}}$$

$$\text{At } \sqrt{2}: 4\sqrt{2} - \frac{2 \cdot 2\sqrt{2}}{3} = \frac{8\sqrt{2}}{3}. \text{ At } 1: 4 - \frac{2}{3} = \frac{10}{3}.$$

$$\text{Area} = \frac{8\sqrt{2} - 10}{3}$$

Final Answer:

The area is $\frac{8\sqrt{2} - 10}{3}$ square units, option (A).

$$\frac{8\sqrt{2} - 10}{3} \text{ (A)}$$

Quick Tip: The curve meets $y = 4$ at $x = \sqrt{2}$, so integrate $4 - 2x^2$ from 1 to $\sqrt{2}$.



137. If $y = e^{-mx}$ is a solution of the differential equation $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 3y = 0$, then the values of m are

- (A) 1, 3
- (B) -1, 3
- (C) -1, -3
- (D) 1, -3

Correct Answer: (A) 1, 3

Solution:

Step 1: Understanding the Concept

If $y = e^{-mx}$ then $y' = -me^{-mx}$ and $y'' = m^2e^{-mx}$.

Step 2: Key Formula or Approach

Substitute into $y'' + 4y' + 3y = 0$.

Step 3: Detailed Explanation

$$m^2e^{-mx} - 4me^{-mx} + 3e^{-mx} = 0 \Rightarrow m^2 - 4m + 3 = 0$$

$$(m - 1)(m - 3) = 0 \Rightarrow m = 1, 3$$

Final Answer:

The values of m are 1 and 3, option (A).

$$m = 1, 3 \text{ (A)}$$

Quick Tip: Substitute $y = e^{-mx}$ and solve for m .

138. For the differential equation $(x^2 + y^2) dy = xy dx$, it is given that $y(1) = 1$ and $y(x_0) = e$, then the value of x_0 is _____

- (A) e
- (B) $\pm\sqrt{3}e$
- (C) $3e^2$
- (D) e^2

Correct Answer: (B) $\pm\sqrt{3}e$

Solution:**Step 1: Understanding the Concept**

$\frac{dy}{dx} = \frac{xy}{x^2 + y^2}$ is homogeneous of degree 0, so put $y = vx$.

Step 2: Key Formula or Approach

$$v + x \frac{dv}{dx} = \frac{v}{1 + v^2}, \text{ so } x \frac{dv}{dx} = -\frac{v^3}{1 + v^2}.$$

Step 3: Detailed Explanation

Separate: $\frac{1 + v^2}{v^3} dv = -\frac{dx}{x}$. Integrate: $-\frac{1}{2v^2} + \log v = -\log x + C$.

Since $v = y/x$: $\log y - \frac{x^2}{2y^2} = C$.

Using $y(1) = 1$: $0 - \frac{1}{2} = C$.

At $y = e$: $1 - \frac{x_0^2}{2e^2} = -\frac{1}{2}$, so $x_0^2 = 3e^2$ and $x_0 = \pm\sqrt{3}e$.

Final Answer:

The value of x_0 is $\pm\sqrt{3}e$, option (B).

$$\boxed{\pm\sqrt{3}e \text{ (B)}}$$

Quick Tip: Substitute $y = vx$ and separate the variables.

139. In $\triangle OAB$, $O(0, 0, 0)$, $A(6, 2, -3)$ and $B(4, 0, 3)$ are the vertices. Let $\vec{a}$ and $\vec{b}$ be position vectors of points A and B respectively and OM is the projection of $\vec{a}$ on $\vec{b}$ then $l(AM)$ is equal to...

- (A) $\sqrt{10}$ units
- (B) $2\sqrt{10}$ units
- (C) 10units
- (D) 40units

Correct Answer: (B) $2\sqrt{10}$ units

Solution:

Step 1: Understanding the Concept

OM is the component of $\vec{a}$ along $\vec{b}$, so M is the foot of the perpendicular from A to OB . Then AM is the perpendicular distance.

Step 2: Key Formula or Approach

$$OM = \frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} \text{ and } AM^2 = |\vec{a}|^2 - OM^2.$$

Step 3: Detailed Explanation

$$\vec{a} \cdot \vec{b} = 24 + 0 - 9 = 15 \text{ and } |\vec{b}| = \sqrt{16 + 0 + 9} = 5, \text{ so } OM = 3.$$

$$|\vec{a}| = \sqrt{36 + 4 + 9} = 7.$$

$$AM^2 = 49 - 9 = 40 \Rightarrow AM = 2\sqrt{10}$$

Final Answer:

The length $AM = 2\sqrt{10}$ units, option (B).

$2\sqrt{10}$ units (B)

Quick Tip: AM is the perpendicular from A to line OB .



140. Let $\vec{a} = 2\hat{i} - 3\hat{j} + \hat{k}$, $\vec{b} = 3\hat{i} + 2\hat{j} + 2\hat{k}$, $\vec{c} = 4\hat{i} - 3\hat{j} + \hat{k}$, then the vectors $\vec{a}, \vec{b}, \vec{c}$ are

- (A) Linearly dependent
- (B) Orthogonal
- (C) Linearly independent
- (D) Coplanar

Correct Answer: (C) Linearly independent

Solution:

Step 1: Understanding the Concept

Three vectors are coplanar (linearly dependent) exactly when their scalar triple product is zero.

Step 2: Key Formula or Approach

$$[\vec{a} \ \vec{b} \ \vec{c}] = \begin{vmatrix} 2 & -3 & 1 \\ 3 & 2 & 2 \\ 4 & -3 & 1 \end{vmatrix}$$

Step 3: Detailed Explanation

Expand along the first row: $2(2 - (-6)) - (-3)(3 - 8) + 1(-9 - 8) = 16 - 15 - 17 = -16$.

The determinant is nonzero, so the vectors are linearly independent and not coplanar.

Also $\vec{a} \cdot \vec{b} = 6 - 6 + 2 = 2 \neq 0$, so $\vec{a}, \vec{b}$ are not orthogonal.

Final Answer:

The vectors are linearly independent, option (C).

Linearly independent (C)

Quick Tip: Compute the scalar triple product.

141. Let $\vec{a} = (a_1\hat{i} + a_2\hat{j} + a_3\hat{k})$, $\vec{b} = (b_1\hat{i} + b_2\hat{j} + b_3\hat{k})$, $\vec{c} = (c_1\hat{i} + c_2\hat{j} + c_3\hat{k})$ be three non-zero vectors such that $\vec{a}$ is a unit vector perpendicular to both $\vec{b}$

and $\vec{c}$. If the angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{3}$ then

$$\frac{a_1}{b_1} \frac{a_2}{b_2} \frac{a_3}{c_3} = \frac{a_1^2}{b_1^2} \frac{a_2^2}{b_2^2} \frac{a_3^2}{c_3^2}$$

- (A) $\frac{3}{4}|\vec{b}|^2|\vec{c}|^2$
 (B) 1
 (C) 0
 (D) $\frac{1}{4}|\vec{b}|^2|\vec{c}|^2$

Correct Answer: (A) $\frac{3}{4}|\vec{b}|^2|\vec{c}|^2$

Solution:

Step 1: Understanding the Concept

The determinant of the three rows equals $\vec{a} \cdot (\vec{b} \times \vec{c})$.

Step 2: Key Formula or Approach

$\vec{a}$ is perpendicular to both $\vec{b}$ and $\vec{c}$, so it is parallel to $\vec{b} \times \vec{c}$.

Step 3: Detailed Explanation

$$|\vec{b} \times \vec{c}| = |\vec{b}||\vec{c}| \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}|\vec{b}||\vec{c}|.$$

$\vec{a}$ is a unit vector along $\vec{b} \times \vec{c}$, so $\vec{a} \cdot (\vec{b} \times \vec{c}) = \pm \frac{\sqrt{3}}{2}|\vec{b}||\vec{c}|$.

Squaring:

$$\det^2 = \frac{3}{4}|\vec{b}|^2|\vec{c}|^2$$

Final Answer:

The square of the determinant is $\frac{3}{4}|\vec{b}|^2|\vec{c}|^2$, option (A).

$\frac{3}{4}|\vec{b}|^2|\vec{c}|^2 \text{ (A)}$

Quick Tip: The determinant is the scalar triple product $\vec{a} \cdot (\vec{b} \times \vec{c})$.



142. Two adjacent sides of a parallelogram ABCD are given by $\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\overrightarrow{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes AD'. If AD' makes a right angle with the side AB, then the cosine of the angle α is given by

- (A) $\frac{8}{9}$
- (B) $\frac{\sqrt{17}}{9}$
- (C) $\frac{1}{9}$
- (D) $\frac{4\sqrt{5}}{9}$

Correct Answer: (B) $\frac{\sqrt{17}}{9}$

Solution:

Step 1: Understanding the Concept

Let φ be the angle between $\vec{AB}$ and $\vec{AD}$. The new side AD' is in the same plane, perpendicular to AB , and obtained by an acute rotation.

Step 2: Key Formula or Approach

$$\cos \varphi = \frac{\vec{AB} \cdot \vec{AD}}{|\vec{AB}| |\vec{AD}|}$$

Step 3: Detailed Explanation

$$\vec{AB} \cdot \vec{AD} = -2 + 20 + 22 = 40, |\vec{AB}| = \sqrt{4 + 100 + 121} = 15, |\vec{AD}| = 3.$$

$$\cos \varphi = \frac{40}{45} = \frac{8}{9}, \text{ so } \sin \varphi = \frac{\sqrt{17}}{9}.$$

To reach a direction at 90° from AB , the acute rotation is $\alpha = 90^\circ - \varphi$.

$$\cos \alpha = \sin \varphi = \frac{\sqrt{17}}{9}$$

Final Answer:

$$\cos \alpha = \frac{\sqrt{17}}{9}, \text{ option (B).}$$

$$\boxed{\frac{\sqrt{17}}{9} \text{ (B)}}$$

Quick Tip: AD' lies in the plane and is perpendicular to AB ; find the angle between AD and AB .

143. If θ is the angle between the lines $\frac{x-1}{2} = \frac{2y+3}{4}; z = -2$ and $x = 1; \frac{y-1}{2} = \frac{z+1}{2}$, then

- (A) $\theta = \frac{\pi}{6}$
- (B) $\theta = \frac{\pi}{3}$
- (C) $\theta = \frac{\pi}{4}$
- (D) $\theta = \frac{\pi}{2}$

Correct Answer: (B) $\theta = \frac{\pi}{3}$

Solution:

Step 1: Understanding the Concept

Rewrite each line in standard form to read its direction ratios.

Step 2: Key Formula or Approach

Line 1: $\frac{x-1}{2} = \frac{y+3/2}{2}$, $z = -2$, direction $(2, 2, 0) \sim (1, 1, 0)$. Line 2: $x = 1$, $\frac{y-1}{2} = \frac{z+1}{2}$, direction $(0, 2, 2) \sim (0, 1, 1)$.

Step 3: Detailed Explanation

$$\cos \theta = \frac{|1 \cdot 0 + 1 \cdot 1 + 0 \cdot 1|}{\sqrt{2} \cdot \sqrt{2}} = \frac{1}{2}$$

$$\text{So } \theta = \frac{\pi}{3}.$$

Final Answer:

The angle is $\pi/3$, option (B).

$$\boxed{\frac{\pi}{3} \text{ (B)}}$$

Quick Tip: Find the direction ratios and use the dot product formula.

144. If for some $m \in R$ the lines $L_1 : \frac{x+1}{m} = \frac{y-m}{-1} = \frac{z-1}{1}$ and $L_2 : \frac{x+2}{-4} = \frac{y+1}{9} = \frac{z+1}{1}$ are coplanar, then line L_1 passes through the point

- (A) $(-7, 2, -5)$
- (B) $(7, -2, 5)$
- (C) $(7, 2, 5)$
- (D) $(7, -2, -5)$

Correct Answer: (B) $(7, -2, 5)$

Solution:

Step 1: Understanding the Concept

Two lines are coplanar when the scalar triple product of the joining vector and the two directions is zero.

Step 2: Key Formula or Approach

$P_1(-1, m, 1)$ with direction $(m, -1, 1)$; $P_2(-2, -1, -1)$ with direction $(-4, 9, 1)$. Joining vector $\overrightarrow{P_1P_2} = (-1, -1 - m, -2)$.

Step 3: Detailed Explanation

$$\begin{vmatrix} -1 & -1 - m & -2 \\ m & -1 & 1 \\ -4 & 9 & 1 \end{vmatrix} = m^2 - 13m + 22 = 0$$

So $m = 2$ or $m = 11$.

Take $m = 2$: $L_1 : \frac{x+1}{2} = \frac{y-2}{-1} = \frac{z-1}{1}$. At the point $(7, -2, 5)$: $\frac{8}{2} = 4$, $\frac{-4}{-1} = 4$, $\frac{4}{1} = 4$. All equal, so $(7, -2, 5)$ lies on L_1 .

Final Answer:

The line L_1 passes through $(7, -2, 5)$, option (B).

$$\boxed{(7, -2, 5) \text{ (B)}}$$

Quick Tip: Use the coplanarity determinant to find m , then test which point lies on L_1 .

145. If the lines $\vec{r} = \hat{i} + \hat{j} - \hat{k} + \lambda(q\hat{i} - 2\hat{j} + \hat{k})$ and $\vec{r} = p\hat{i} - 3\hat{j} + 2\hat{k} + \mu(\hat{i} - 2\hat{j} + 2\hat{k})$ intersect each other and $q\hat{i} - 2\hat{j} + \hat{k}$ is collinear to $4\hat{i} - 4\hat{j} + 2\hat{k}$, then the values of p and q are

- (A) $p = 4, q = 3$
- (B) $p = 2, q = 3$
- (C) $p = 4, q = 2$
- (D) $p = 4, q = 1$

Correct Answer: (C) $p = 4, q = 2$

Solution:

Step 1: Understanding the Concept

$q\hat{i} - 2\hat{j} + \hat{k}$ is collinear with $4\hat{i} - 4\hat{j} + 2\hat{k}$, so the ratio of components must be equal.

Step 2: Key Formula or Approach

$$\frac{q}{4} = \frac{-2}{-4} = \frac{1}{2}, \text{ so } q = 2.$$

Step 3: Detailed Explanation

Line 1: $(1 + 2\lambda, 1 - 2\lambda, -1 + \lambda)$. Line 2: $(p + \mu, -3 - 2\mu, 2 + 2\mu)$.

y: $1 - 2\lambda = -3 - 2\mu \Rightarrow \lambda = \mu + 2$.

z: $-1 + \lambda = 2 + 2\mu \Rightarrow -1 + \mu + 2 = 2 + 2\mu \Rightarrow \mu = -1, \lambda = 1$.

x: $1 + 2 = p + (-1) \Rightarrow p = 4$.

So $p = 4, q = 2$.

Final Answer:

$p = 4$ and $q = 2$, option (C).

$p = 4, q = 2 \text{ (C)}$

Quick Tip: Collinearity fixes q , and intersection gives p .

146. The angle between a diagonal and one of its edges of a cube is

.....

(A) $\tan^{-1}(\frac{1}{\sqrt{2}})$

(B) $\cos^{-1}(\frac{1}{3})$

(C) $\cos^{-1}(\frac{1}{2\sqrt{2}})$

(D) $\tan^{-1}(\sqrt{2})$

Correct Answer: (D) $\tan^{-1}(\sqrt{2})$

Solution:

Step 1: Understanding the Concept

Place the cube with one vertex at the origin. A body diagonal has direction ratios 1, 1, 1, and an edge from the same vertex has direction 1, 0, 0.

Step 2: Key Formula or Approach

$$\cos \theta = \frac{1 \cdot 1 + 1 \cdot 0 + 1 \cdot 0}{\sqrt{3} \cdot 1} = \frac{1}{\sqrt{3}}$$

Step 3: Detailed Explanation

Then $\sin \theta = \sqrt{1 - \frac{1}{3}} = \sqrt{\frac{2}{3}}$.

$$\tan \theta = \frac{\sqrt{2/3}}{1/\sqrt{3}} = \sqrt{2}$$

So $\theta = \tan^{-1} \sqrt{2}$, equivalent to $\cos^{-1} \frac{1}{\sqrt{3}}$.

Final Answer:

The angle is $\tan^{-1} \sqrt{2}$, option (D).

$$\tan^{-1} \sqrt{2} \text{ (D)}$$

Quick Tip: Take the cube with edge 1, diagonal (1, 1, 1) and edge (1, 0, 0).



147. For the linear programming problem, $x + 2y \leq 10$, $3x + y \leq 12$, $x, y \geq 0$, the maximum value of $z = 5x + 10y$ occurs at every point on the line segment joining the points..

- (A) (0, 0) and (4, 0)
- (B) (0, 0) and (0, 5)
- (C) (4, 0) and $(\frac{14}{5}, \frac{18}{5})$
- (D) (0, 5) and $(\frac{14}{5}, \frac{18}{5})$

Correct Answer: (D) (0, 5) and $(\frac{14}{5}, \frac{18}{5})$

Solution:

Step 1: Understanding the Concept

$z = 5x + 10y = 5(x + 2y)$. The constraint $x + 2y \leq 10$ then limits $z \leq 50$.

Step 2: Key Formula or Approach

Find the corner points of the feasible region.

Step 3: Detailed Explanation

Corners: $(0, 0)$, $(4, 0)$, $(0, 5)$ and the intersection of $x + 2y = 10$, $3x + y = 12$: $x = \frac{14}{5}$, $y = \frac{18}{5}$.

$z(0, 0) = 0$, $z(4, 0) = 20$, $z(0, 5) = 50$, $z\left(\frac{14}{5}, \frac{18}{5}\right) = 14 + 36 = 50$.

The maximum 50 occurs at both $(0, 5)$ and $\left(\frac{14}{5}, \frac{18}{5}\right)$, so it holds at every point on the segment joining them.

Final Answer:

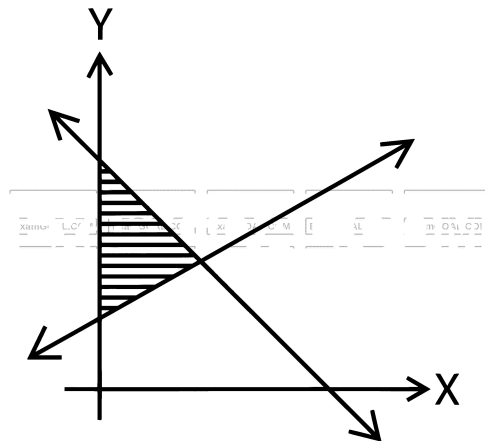
The maximum occurs along the segment joining $(0, 5)$ and $\left(\frac{14}{5}, \frac{18}{5}\right)$, option (D).

$$(0, 5) \text{ and } \left(\frac{14}{5}, \frac{18}{5}\right) \text{ (D)}$$

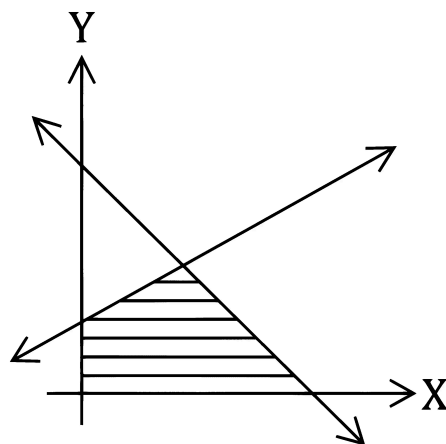
Quick Tip: The objective is parallel to the constraint $x + 2y \leq 10$, so the maximum occurs along an edge.

148. The region satisfying the inequalities $y - x \geq 2$, $x + y \leq 5$, $x \geq 0$ and $y \geq 0$ is

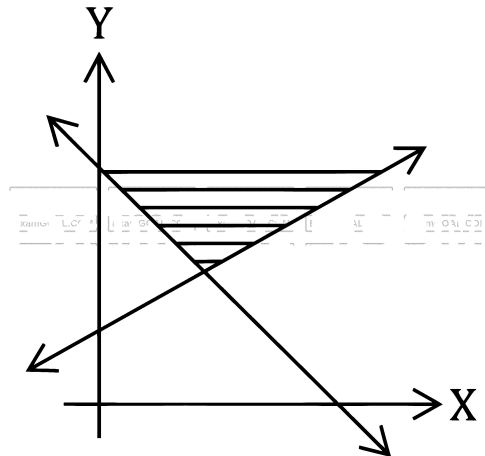
(A)



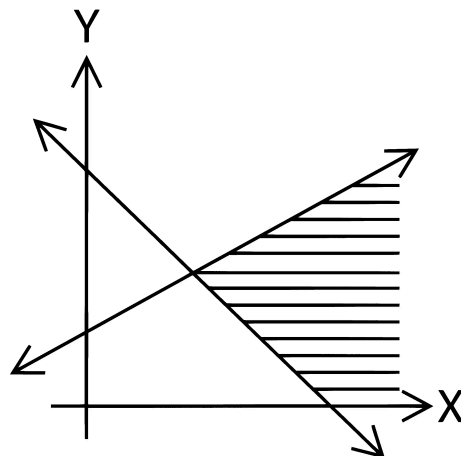
(B)



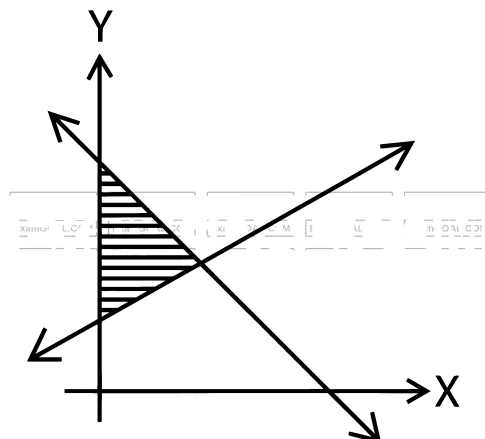
(C)



(D)



Correct Answer: (A)



Solution:

Step 1: Understanding the Concept

$y - x \geq 2$ means on or above the line $y = x + 2$. $x + y \leq 5$ means on or below $y = 5 - x$. $x \geq 0, y \geq 0$ keep the region in the first quadrant.

Step 2: Key Formula or Approach

Lines meet where $x + 2 = 5 - x$: $x = \frac{3}{2}, y = \frac{7}{2}$. The line $y = x + 2$ cuts the y-axis at 2 and $y = 5 - x$ cuts it at 5.

Step 3: Detailed Explanation

The region is a triangle with vertices $(0, 2)$, $(0, 5)$ and $(\frac{3}{2}, \frac{7}{2})$: bounded on the left by the y-axis, below by the rising line and above by the falling line.

The figure in option (A) shows this small triangle shaded against the y-axis between the two lines, ending where the lines meet.

Final Answer:

Figure (A) shows the required region.

(A)

Quick Tip: Draw the lines $y = x + 2$ and $y = 5 - x$ and find the region on the correct sides.

149. If the mean and the variance of a binomial variate X are 1 and 0.75 respectively, then which of the following is true?

- (A) $P(X = 0) = 3P(X = 4)$
 (B) $P(X = 1) = 2P(X = 2)$
 (C) $P(X = 3) = 3P(X = 4)$
 (D) $3P(X = 0) = 4P(X = 1)$

Correct Answer: (B) $P(X = 1) = 2P(X = 2)$

Solution:

Step 1: Understanding the Concept

For a binomial variate, mean = np and variance = npq .

Step 2: Key Formula or Approach

$$q = \frac{npq}{np} = 0.75, \text{ so } p = 0.25 \text{ and } n = \frac{1}{0.25} = 4.$$

Step 3: Detailed Explanation

$$P(X = r) = \binom{4}{r} \left(\frac{1}{4}\right)^r \left(\frac{3}{4}\right)^{4-r} = \frac{\binom{4}{r} 3^{4-r}}{256}.$$

$$P(0) = \frac{81}{256}, P(1) = \frac{4 \cdot 27}{256} = \frac{108}{256}, P(2) = \frac{6 \cdot 9}{256} = \frac{54}{256}, P(3) = \frac{12}{256},$$

$$P(4) = \frac{1}{256}.$$

Now $P(1) = 108$ and $2P(2) = 108$, so $P(X = 1) = 2P(X = 2)$, which is option (B).

Final Answer:

The true relation is $P(X = 1) = 2P(X = 2)$, option (B).

$P(X = 1) = 2P(X = 2) \text{ (B)}$

Quick Tip: Find n and p from $np = 1$ and $npq = 0.75$, then compare probabilities.

150. A box contains 8 red and N green balls. Two balls are drawn at random from it. If X is the random variable representing the number of green balls drawn and $E(X) = 1.2$, then $N = \dots$

- (A) 4
- (B) 8
- (C) 12
- (D) 16

Correct Answer: (C) 12

Solution:

Step 1: Understanding the Concept

X counts the green balls in two draws. For each draw the chance of green is $\frac{N}{N+8}$ by symmetry, so the expectation adds up.

Step 2: Key Formula or Approach

$$E(X) = 2 \cdot \frac{N}{N+8}$$

Step 3: Detailed Explanation

Set $\frac{2N}{N+8} = 1.2$: $2N = 1.2N + 9.6$, so $0.8N = 9.6$ and $N = 12$.

Check with the distribution: $P(X = 1) = \frac{8 \cdot 12}{\binom{20}{2}} = \frac{96}{190}$, $P(X = 2) = \frac{66}{190}$, and $E(X) = \frac{96 + 132}{190} = 1.2$.

Final Answer:

$N = 12$, option (C).

12 (C)

Quick Tip: $E(X)$ for drawing 2 balls is $2 \times \frac{N}{N+8}$.