

MHT CET 2026 PCM Question Paper April 17 Shift 2 with Solutions



General Instructions

- (i) This booklet contains 150 questions, each provided with a complete, step-by-step solution.
- (ii) It comprises 150 single-correct multiple-choice questions.
- (iii) Attempt each question on your own before reviewing the given solution.

Physics

Questions 1 to 50 | 1 mark each | 50 marks

1. There are two vectors $\vec{A} = 6\hat{i} + 9\hat{j} - \hat{k}$ and $\vec{B} = 2\hat{i} + 3\hat{j} - p\hat{k}$ which have the same direction. The value of 'p' is

- (A) 3
- (B) $\frac{1}{3}$
- (C) $\frac{2}{3}$
- (D) $-\frac{1}{3}$

Correct Answer: (B) $\frac{1}{3}$

Solution:

Step 1: Understanding the Concept:

For any two vectors, vector addition and the scalar (dot) product are commutative, while the vector (cross) product changes sign when the order is reversed. The magnitudes being different makes no difference to these rules.

Step 2: Check (A):

$\vec{A} \cdot \vec{B} = AB \cos \theta = \vec{B} \cdot \vec{A}$, not $-\vec{B} \cdot \vec{A}$ (unless it is zero). So (A) is false.

Step 3: Check (B):

$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$. So (B) is false for non-parallel vectors.

Step 4: Check (C) and (D):

$\vec{A} + \vec{B} = \vec{B} + \vec{A}$ always holds, so (C) is true. Subtraction is not commutative: $\vec{A} - \vec{B} = -(\vec{B} - \vec{A})$, so (D) is false.

Final Answer:

The correct equation is (C), $\vec{A} + \vec{B} = \vec{B} + \vec{A}$.

$$\boxed{\vec{A} + \vec{B} = \vec{B} + \vec{A}}$$

Quick Tip: Addition and dot product are commutative; the cross product is anti-commutative.

2. A force 'F' is applied on a square plate of side 'L'. If the percentage error in determining 'L' is 3% and that in 'F' is 2% then the percentage error in determining the pressure is

- (A) 5%
- (B) 6%
- (C) 7%
- (D) 8%

Correct Answer: (D) 8%

Solution:

Step 1: Understanding the Concept:

The wire is a cylinder, so volume is $\pi r^2 l$ and density is $\rho = \frac{m}{\pi r^2 l}$.

Step 2: Key Formula or Approach:

$$\frac{\Delta \rho}{\rho} = \frac{\Delta m}{m} + 2 \frac{\Delta r}{r} + \frac{\Delta l}{l}$$

Step 3: Calculation:

$$\frac{\Delta m}{m} = \frac{0.003}{0.3} = 1\%, \quad \frac{\Delta r}{r} = \frac{0.005}{0.5} = 1\%, \quad \frac{\Delta l}{l} = \frac{0.06}{6} = 1\%.$$

$$\frac{\Delta \rho}{\rho} \times 100 = 1 + 2(1) + 1 = 4\%$$

The error in radius counts twice, because r appears squared.

Final Answer:

The maximum error in density is 4 percent, option (A).

4%

Quick Tip: Density is mass divided by volume $\pi r^2 l$, so add relative errors with the radius error doubled.

3. A ball P is projected at an angle of 60° with the vertical with certain initial speed. Another ball Q of the same mass as that of ball P is projected vertically upwards with the same initial speed as that of P. At the highest point, the ratio of potential energy of ball P to that of ball Q is

($\sin 30^\circ = 0.5$)

(A) 1 : 4

(B) 4 : 1

(C) 2 : 3

(D) 3 : 2

Correct Answer: (A) 1 : 4

Solution:

Step 1: Understanding the Concept:

In projectile motion with no air resistance, the horizontal component of velocity does not change. Only the vertical component changes.

Step 2: Initial velocity:

The ball is hit at 60° to the vertical, so it makes 30° with the horizontal. Horizontal component = $v \cos 30^\circ = \frac{\sqrt{3}}{2}v$.

Step 3: Later instant:

Now the velocity makes 60° with the horizontal. If the speed is v' , the horizontal component is $v' \cos 60^\circ = \frac{v'}{2}$.

$$\frac{v'}{2} = \frac{\sqrt{3}}{2}v \Rightarrow v' = \sqrt{3}v$$

Check: the vertical component at this instant is $\sqrt{3}v \sin 60^\circ = \frac{3}{2}v$, larger than the initial $\frac{v}{2}$, so the ball is at a lower height than the start

on its way down.

Final Answer:

The speed is $\sqrt{3} v$, option (B).

$$\boxed{\sqrt{3} v}$$

Quick Tip: The horizontal component of velocity stays constant in projectile motion.

4. A machine gun fires a bullet of mass 40g with a speed of 600 ms^{-1} . The person holding the gun can bare maximum force of 168N on it. The number of bullets that can be fired from the gun per second is

- (A) 8
- (B) 7
- (C) 6
- (D) 5

Correct Answer: (B) 7

Solution:

Step 1: Understanding the Concept:

Tangential acceleration $a_t = r\alpha$. Centripetal acceleration $a_c = \frac{v^2}{r}$.

Step 2: Apply the condition:

Given $a_c = \frac{1}{3} a_t$:

$$\frac{v^2}{r} = \frac{r\alpha}{3} \Rightarrow v^2 = \frac{r^2\alpha}{3}$$

Step 3: Take the root:

$$v = r\sqrt{\frac{\alpha}{3}}$$

Option (A), $\frac{r\alpha}{3}$, is an acceleration, not a speed. Options (B) and (D) put r inside the root, which does not give the units of speed when α is in rad/s^2 .

Final Answer:

The speed is $v = r\sqrt{\frac{\alpha}{3}}$, option (C).

$$v = r\sqrt{\frac{\alpha}{3}}$$

Quick Tip: Tangential acceleration is r times α and centripetal acceleration is v^2 over r .

5. Radius of gyration of a thin uniform circular disc about the axis passing through its centre and perpendicular to its plane is K_c . Radius of gyration of the same disc about a diameter of the disc is K_d . the ratio $K_d : K_c$ is

- (A) 1 : 4
- (B) $1 : \sqrt{2}$
- (C) $\sqrt{2} : 1$
- (D) 2 : 1

Correct Answer: (B) $1 : \sqrt{2}$

Solution:

Step 1: Understanding the Concept:

At the highest point of a convex bridge the centre of curvature is below the car. The net force toward the centre must supply the centripetal force $\frac{mv^2}{R}$.

Step 2: Write the equation:

Weight mg acts downward (toward the centre) and the normal reaction N acts upward:

$$mg - N = \frac{mv^2}{R} \Rightarrow N = mg - \frac{mv^2}{R}$$

Step 3: Thrust:

The thrust on the bridge equals N by Newton's third law, so it is $mg - \frac{mv^2}{R}$. It is less than the weight, which is why we feel lighter at the top of a hump. Option (D) would apply at the bottom of a dip.

Final Answer:

The thrust is $mg - \frac{mv^2}{R}$, option (C).

$$\boxed{mg - \frac{mv^2}{R}}$$

Quick Tip: At the highest point the net force toward the centre is mg minus the normal reaction.

6. A solid sphere of radius R and mass M is rotating about its diameter. The moment of inertia of the solid sphere rotating about an axis at a distance $\frac{R}{3}$ from the centre and parallel to that diameter is

- (A) $\frac{11}{15}MR^2$
- (B) $\frac{13}{20}MR^2$
- (C) $\frac{23}{45}MR^2$
- (D) $\frac{47}{45}MR^2$

Correct Answer: (C) $\frac{23}{45}MR^2$

Solution:

Step 1: Understanding the Concept:

The critical position is the top of the circle. The water does not fall if at least the weight provides the centripetal force: $mg \leq m\omega^2 r$.

Step 2: Minimum condition:

At the limiting case $mg = m\omega^2 r$, so $\omega = \sqrt{\frac{g}{r}}$. The period is

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{r}{g}}$$

Step 3: Meaning:

The period must be at most this value. Option (A) has g over r inside the root, which is the inverse of what we need, and (C) and (D) have wrong dimensions for a time.

Final Answer:

The required period is $2\pi\sqrt{\frac{r}{g}}$, option (B).

$$2\pi\sqrt{\frac{r}{g}}$$

Quick Tip: At the top, the water stays if gravity alone can supply the centripetal force.



7. A mass tied to a string is whirled in a horizontal circular path with a constant angular velocity and its angular momentum is 'L'. If the length of the string is now halved, keeping angular velocity same then the angular momentum will be

- (A) $\frac{L}{4}$
- (B) $\frac{L}{2}$
- (C) L
- (D) $2L$

Correct Answer: (A) $\frac{L}{4}$

Solution:

Step 1: Understanding the Concept:

In a perfectly elastic collision, both momentum and total kinetic energy are conserved. No energy is lost as heat or sound or deformation.

Step 2: Check the options:

(A) Kinetic energy before equals kinetic energy after. This is the definition, so (A) is true.

(B) Kinetic energy cannot increase in an ordinary collision, so (B) is wrong.

(C) Kinetic energy decreases only in an inelastic collision, so (C) is wrong here.

(D) The coefficient of restitution is 1 for an elastic collision (zero for a perfectly inelastic one), so (D) is wrong.

Final Answer:

Option (A) is correct.

(A)

Quick Tip: In an elastic collision kinetic energy is conserved and the coefficient of restitution is 1.

8. Two planets A and B are orbiting around the sun. The distances of the two planets A and B from the sun are r_A and r_B respectively. Also $r_B = 225 r_A$. If the orbital speed of the planet A is 'V' then the orbital speed of planet B will be

(A) $\frac{V}{3}$

(B) $\frac{V}{5}$

(C) $\frac{V}{15}$

(D) $\sqrt{15}V$

Correct Answer: (C) $\frac{V}{15}$

Solution:

Step 1: Understanding the Concept:

For a planet in a circular orbit, gravity supplies the centripetal force:

$$\frac{GMm}{r^2} = \frac{mv^2}{r}, \text{ so } v = \sqrt{\frac{GM}{r}}.$$

Step 2: Ratio:

$$\frac{v_A}{v_B} = \sqrt{\frac{r_B}{r_A}} = \sqrt{100} = 10$$

The planet closer to the sun moves faster, which is consistent with the ratio being greater than 1.

Final Answer:

The ratio of the speeds is 10, option (C).

10

Quick Tip: Orbital speed around the sun is inversely proportional to the square root of the orbit radius.

9. When a capillary tube of radius 'r' is immersed in water, rise of water is upto height 'h'. The mass of water in capillary tube is 'm'. When another capillary tube of radius 'xr' is immersed in water, the mass of water that will rise in this tube is

- (A) $x \cdot m$
- (B) $\frac{m}{x}$
- (C) $x^2 m$
- (D) $(x + 1)m$

Correct Answer: (A) $x \cdot m$

Solution:

Step 1: Understanding the Concept:

A soap film has two surfaces (inner and outer), and each pulls with a force T per unit length on the frame.

Step 2: Calculation:

The total boundary length of the square frame is $4L$.

$$F = 2T \times 4L = 8TL$$

The values $4TL$ would apply to a single surface, and 12 and 16 do not follow from any counting of surfaces here.

Final Answer:

The force on the film is $8TL$, option (B).

$$\boxed{8TL}$$

Quick Tip: A soap film has two free surfaces, so the force is $2T$ times the length of the boundary.

10. The excess pressure inside the first soap bubble is three times that inside the second soap bubble. The ratio of the volume of the first soap bubble to the volume of the second soap bubble is

- (A) 3 : 4
- (B) 1 : 3
- (C) 1 : 9
- (D) 1 : 27

Correct Answer: (D) 1 : 27

Solution:

Step 1: Understanding the Concept:

Capillary rise: $h = \frac{2T \cos \theta}{r \rho g}$. Here T, r, g and h are the same for all three liquids.

Step 2: Relate theta and density:

Equal h means $\frac{\cos \theta}{\rho}$ is constant, that is, $\cos \theta \propto \rho$.

Step 3: Order the angles:

Since $\rho_1 < \rho_2 < \rho_3$, we have $\cos \theta_1 < \cos \theta_2 < \cos \theta_3$. Cosine decreases as the angle increases in 0 to 90° , so $\theta_1 > \theta_2 > \theta_3$.

Final Answer:

The angles of contact satisfy $\theta_1 > \theta_2 > \theta_3$, option (B).

$$\theta_1 > \theta_2 > \theta_3$$

Quick Tip: Capillary rise is $2T \cos(\theta)$ divided by $r \rho g$; with equal rise, $\cos(\theta)$ is proportional to density.

11. Two soap bubbles, A and B, have radii in the ratio 3 : 2. The excess pressure inside the bubble A to that of B is in the ratio

- (A) 2 : 3
- (B) 3 : 2
- (C) 1 : 5
- (D) 4 : 9

Correct Answer: (A) 2 : 3

Solution:

Step 1: Understanding the Concept:

A soap bubble has two surfaces, so the excess pressure is $\Delta P = \frac{4T}{R}$, giving $R \propto \frac{1}{\Delta P}$.

Step 2: Radius ratio:

$$\Delta P_1 = 3\Delta P_2, \text{ so } \frac{R_1}{R_2} = \frac{\Delta P_2}{\Delta P_1} = \frac{1}{3}.$$

Step 3: Volume ratio:

Volume $\propto R^3$, so $\frac{V_1}{V_2} = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$. So the ratio is 1 : 27.

Final Answer:

The ratio of volumes is 1 : 27, option (A).

1 : 27

Quick Tip: Excess pressure inside a soap bubble is $4T/R$, so radius is inversely proportional to pressure.

12. A black body emits radiation of maximum intensity of wavelength ' λ ' at temperature 'T' K. Its corresponding wavelength at temperature $(2.5T)$ K will be

- (A) $\frac{2\lambda}{3}$
- (B) $\frac{2\lambda}{5}$
- (C) $\frac{4\lambda}{5}$
- (D) $\frac{16\lambda}{81}$

Correct Answer: (B) $\frac{2\lambda}{5}$

Solution:

Step 1: Understanding the Concept:

Thermal resistance of a rod is $R_{th} = \frac{L}{KA}$.

Step 2: Equate:

Same A and same R_{th} give $\frac{L_1}{K_1} = \frac{L_2}{K_2}$, so $\frac{L_1}{L_2} = \frac{K_1}{K_2} = \frac{6}{5}$.

Final Answer:

The lengths are in the ratio 6:5, option (B).

6 : 5

Quick Tip: Thermal resistance is L over KA , so equal area and resistance make L proportional to K .

13. In a composite slab there are two materials having coefficients of thermal conductivity K and $2K$, thickness x and $4x$ respectively. The temperature of the two outer surfaces of a composite slab are T_2 and T_1 ($T_2 > T_1$). T_2 is on side K and T_1 is on side $2K$. The rate of heat transfer through the slab in a steady state is $\left[\frac{A(T_2 - T_1)K}{x} \right] \cdot f$, where f is equal to

- (A) 1
- (B) $\frac{1}{2}$
- (C) $\frac{2}{3}$
- (D) $\frac{1}{3}$

Correct Answer: (D) $\frac{1}{3}$

Solution:

Step 1: Understanding the Concept:

Stefan's law: $E = \sigma AT^4$, with T in kelvin.

Step 2: Change in area:

Length and breadth are both halved, so the area becomes $\frac{1}{4}$ of the original.

Step 3: Change in temperature:

$T_1 = 127 + 273 = 400$ K and $T_2 = 527 + 273 = 800$ K, so T doubles and T^4 becomes 16 times.

Step 4: New energy:

$$E' = E \times \frac{1}{4} \times 16 = 4E$$

Final Answer:

The new emitted energy is $4E$, option (C).

$$\boxed{4E}$$

Quick Tip: Energy emitted per second is proportional to area and the fourth power of absolute temperature.

14. A carnot engine having efficiency $\frac{1}{6}$, operates between the source temperature T_H and the sink temperature T_C . Its efficiency increases to $\frac{1}{3}$, when T_C is decreased by 64 K. The temperatures T_H and T_C are repectively

- (A) 374 K , 310 K
- (B) 384 K , 320 K
- (C) 384 K , 340 K
- (D) 320 K , 256 K

Correct Answer: (B) 384 K , 320 K

Solution:

Step 1: Understanding the Concept:

Wien's law: $\lambda_m T = \text{constant}$. Stefan's law: power $\propto T^4$.

Step 2: New temperature:

The peak wavelength changes from λ to $\frac{2\lambda}{3}$, so $T' = T \times \frac{\lambda}{2\lambda/3} = \frac{3}{2} T$.

Step 3: Power ratio:

$$\frac{P'}{P} = \left(\frac{3}{2}\right)^4 = \frac{81}{16}$$

Final Answer:

The power increases by the factor $\frac{81}{16}$, option (B).

$\frac{81}{16}$

Quick Tip: Wien law gives temperature from peak wavelength; power goes as T to the fourth.

15. A thermodynamic system is compressed adiabatically, then its temperature

- (A) becomes zero.
- (B) remains constant.
- (C) increases.
- (D) decreases.

Correct Answer: (C) increases.

Solution:

Step 1: Understanding the Concept:

Internal energy is a state function. After a complete cycle the system returns to its initial state, so $\Delta U = 0$.

Step 2: First law:

$Q = \Delta U + W = W$. So the work done by the system equals the net heat given to the system.

Step 3: Other options:

It cannot be zero in general (a heat engine does work in each cycle), it cannot exceed the heat given (that would violate energy conservation), and it is not independent of heat.

Final Answer:

Work done equals the heat given to the system, option (B).

$$W = Q$$

Quick Tip: In a cycle the internal energy returns to its initial value, so work

equals net heat.



16. Two cylinders A and B fitted with pistons contain equal amount of an ideal diatomic gas at 303 K. The piston of cylinder A is free to move and that of cylinder B is held fixed. The same amount of heat is given to the gas in each cylinder. If the rise in temperature of the gas in cylinder B is 63 K then the rise in temperature of the gas in A is

- (A) 35 K
- (B) 40 K
- (C) 45 K
- (D) 50 K

Correct Answer: (C) 45 K

Solution:

Step 1: Understanding the Concept:

Average translational kinetic energy of a gas molecule is $E_1 = \frac{3}{2}kT$, where $k = \frac{R}{N}$. The electron gains $E_2 = eV$.

Step 2: Equate:

$$\frac{3}{2} \cdot \frac{R}{N} T = eV \Rightarrow T = \frac{2eVN}{3R}$$

Here N is the Avogadro-type number of molecules per mole and R is the gas constant, so $\frac{R}{N}$ is the Boltzmann constant.

Final Answer:

The temperature is $\frac{2eVN}{3R}$, option (B).

$$T = \frac{2eVN}{3R}$$

Quick Tip: Average translational energy of a molecule is $3/2 kT$; an electron accelerated through V gets eV .



17. The temperature at which r.m.s. velocity of hydrogen molecules is 4.5 times that of an oxygen molecule at 47°C is (molecular weight of hydrogen and oxygen are 2 and 32 respectively)

- (A) 47°C
- (B) 132°C
- (C) 320°C
- (D) 405°C

Correct Answer: (B) 132°C

Solution:

Step 1: Understanding the Concept:

$v_{rms} \propto \sqrt{T}$. To reduce v_{rms} by a factor of 3, the temperature must reduce by $3^2 = 9$.

Step 2: Adiabatic relation:

$TV^{\gamma-1} = \text{constant}$. With $\gamma = 1.5$, $\gamma - 1 = 0.5$:

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1} \right)^{0.5}$$

Step 3: Solve:

$9 = \left(\frac{V_2}{V_1}\right)^{1/2}$, so $\frac{V_2}{V_1} = 81$. The gas must expand 81 times.

Final Answer:

The gas must be expanded 81 times, option (D).

81 times

Quick Tip: In an adiabatic change $TV^{(\gamma-1)}$ is constant and the r.m.s. speed goes as the square root of T .



18. The displacement of a particle performing linear S.H.M. is given by $y = A\cos[\pi(t + \phi)]$. If at $t = 0$, the displacement is $y = 2$ cm and velocity is 2π cm/s, the value of amplitude A in cm is

- (A) $\frac{1}{\sqrt{2}}$
- (B) $\sqrt{2}$
- (C) 2
- (D) $2\sqrt{2}$

Correct Answer: (D) $2\sqrt{2}$

Solution:**Step 1: Force constant:**

Restoring force $F = kx$. With $F = 0.4$ N and $x = 4$ cm = 0.04 m:

$$k = \frac{0.4}{0.04} = 10 \text{ N/m}$$

Step 2: Acceleration:

$$a = \frac{F}{m} = \frac{0.4}{0.4} = 1 \text{ m/s}^2$$

So the force constant is 10 N/m and the acceleration is 1 m/s² at that displacement.

Final Answer:

The force constant is 10 N/m and the acceleration is 1 m/s², option (B).

10 N/m, 1 m/s ²

Quick Tip: The restoring force equals k times displacement, and acceleration equals force over mass.

19. A light spring is suspended with mass ' m_1 ' at its lower end and its upper end is fixed to a rigid support. The mass is pulled down a short distance and then released. The period of oscillation is T second. When a mass ' m_2 ' is added to ' m_1 ' and the system is made to oscillate the period is found to be $\frac{3}{2}$ T. The ratio ($\frac{m_1}{m_2}$) is

- (A) 2 : 3
- (B) 3 : 4
- (C) 4 : 5
- (D) 5 : 6

Correct Answer: (C) 4 : 5

Solution:

Step 1: Rewrite x_1 :

$$x_1 = \frac{1}{\sqrt{2}} \sin \omega t + \frac{1}{\sqrt{2}} \cos \omega t = \sin \omega t \cos \frac{\pi}{4} + \cos \omega t \sin \frac{\pi}{4} = \sin \left(\omega t + \frac{\pi}{4} \right).$$

Amplitude 1, phase $\frac{\pi}{4}$.

Step 2: Rewrite x_2 :

$$x_2 = \sin \omega t + \cos \omega t = \sqrt{2} \left[\frac{1}{\sqrt{2}} \sin \omega t + \frac{1}{\sqrt{2}} \cos \omega t \right] = \sqrt{2} \sin \left(\omega t + \frac{\pi}{4} \right).$$

Amplitude $\sqrt{2}$, phase $\frac{\pi}{4}$.

Step 3: Phase difference:

Both have phase $\frac{\pi}{4}$, so the phase difference is 0. Only the amplitudes differ.

Final Answer:

The phase difference is zero, option (A).

0

Quick Tip: Write each in the form $A \sin(\omega t + \phi)$ and compare the phase angles.

20. For a particle performing linear S.H.M. of amplitude 'r', the potential energy is ' λ ' times its total energy. The displacement of particle is

- (A) $r\lambda$
- (B) $\frac{r}{\lambda}$
- (C) $r\sqrt{\lambda}$
- (D) $\frac{r}{\sqrt{\lambda}}$

Correct Answer: (C) $r\sqrt{\lambda}$

Solution:

Step 1: Understanding the Concept:

Time period of a simple pendulum $T = 2\pi\sqrt{\frac{l}{g}}$. It depends on length and g , not on mass or (for small swings) amplitude.

Step 2: Interpret:

A slow clock takes longer for each oscillation, so its period is too large. We need to reduce the period.

Step 3: Decide:

Reduce the length, since $T \propto \sqrt{l}$. Changing mass does nothing, and reducing amplitude has almost no effect for small swings. Increasing length makes the clock even slower.

Final Answer:

Reduce the length of the pendulum, option (C).

Reduce the length

Quick Tip: A slow clock has too large a time period, so the pendulum length must be reduced.

21. In a sonometer experiment, the fundamental frequency of vibration of wire is ' n_1 ' when wire is stretched by hanging a metal bob. If the bob is completely immersed in water, the frequency of vibration of wire becomes ' n_2 '. The relative density of the metal of the bob is

- (A) $\frac{n_1}{n_1 - n_2}$
 (B) $\frac{n_1 - n_2}{n_2}$
 (C) $\frac{n_1^2}{n_1^2 - n_2^2}$
 (D) $\frac{n_2^2}{n_1^2 - n_2^2}$

Correct Answer: (C) $\frac{n_1^2}{n_1^2 - n_2^2}$

Solution:

Step 1: Read the wave number:

Comparing $y = A \sin(120\pi t + 3x)$ with $y = A \sin(\omega t + kx)$, the wave number is $k = 3 \text{ rad/m}$.

Step 2: Relation:

Phase difference $\Delta\phi = k \Delta x$, so $\Delta x = \frac{\Delta\phi}{k}$.

Step 3: Calculate:

$$\Delta x = \frac{\pi/3}{3} = \frac{\pi}{9} \text{ m}$$

Final Answer:

The distance is $\frac{\pi}{9} \text{ m}$, option (B).

$$\boxed{\frac{\pi}{9} \text{ m}}$$

Quick Tip: Phase difference equals wave number times path difference.

22. The beats are produced when there is a superposition of two sound waves which will have

- (A) same amplitude and same frequency.
- (B) different amplitude but same frequency.
- (C) same amplitude but slightly different frequencies.
- (D) different amplitude and different frequencies.

Correct Answer: (C) same amplitude but slightly different frequencies.

Solution:

Step 1: Open pipe:

For a pipe of length L open at both ends, $f = \frac{v}{2L}$.

Step 2: After dipping:

Two thirds of the pipe is in water, so the air column has length $\frac{L}{3}$. It is closed at the water end and open at the other end, so the fundamental is

$$f' = \frac{v}{4(L/3)} = \frac{3v}{4L}$$

Step 3: Compare:

$$\frac{f'}{f} = \frac{3v/4L}{v/2L} = \frac{3}{2}, \text{ so } f' = \frac{3}{2}f.$$

Final Answer:

The new fundamental frequency is $\frac{3}{2}f$, option (B).

$$\boxed{\frac{3}{2}f}$$

Quick Tip: The pipe becomes closed at the water end, and the air column is one third of the original length.



23. In a pipe closed at one end, air column is vibrating in the fourth overtone so the vibrating air column has ' x ' nodes and ' y ' antinodes. The values of x and y are respectively

- (A) 4, 4
- (B) 4, 5
- (C) 5, 4
- (D) 5, 5

Correct Answer: (D) 5, 5

Solution:

Step 1: Understanding the Concept:

$$f = \frac{1}{2l} \sqrt{\frac{T}{m}}, \text{ so } f \propto \sqrt{T} \text{ and } T \propto f^2.$$

Step 2: Calculation:

The frequency rises by 40 percent, so $f' = 1.4f$. Then $T' = (1.4)^2 T = 1.96T$.

Step 3: Percentage:

The tension increases by $1.96 - 1 = 0.96$, that is 96 percent. Choosing 40 percent would ignore the square, and 100 percent would be a doubling.

Final Answer:

Tension must be increased by 96 percent, option (C).

Quick Tip: Frequency of a stretched string goes as the square root of tension.

24. Two sounding waves send waves at certain temperature in air of wavelength 50 cm and 50.5 cm respectively. The frequency of sources differ by 6 Hz. The velocity of sound in air at same temperature is

- (A) 330 m/s
- (B) 313 m/s
- (C) 303 m/s
- (D) 300 m/s

Correct Answer: (C) 303 m/s

Solution:

Step 1: Open pipe:

Let the open pipe have fundamental $f_0 = \frac{v}{2L}$. Its first overtone is the second harmonic, $2f_0$.

Step 2: Closed pipe of the same length:

Closing one end gives a fundamental of $\frac{v}{4L} = \frac{f_0}{2}$. A closed pipe has only odd harmonics, so its second overtone is the fifth harmonic: $5 \times \frac{f_0}{2} = 2.5f_0$.

Step 3: Difference:

$$2.5f_0 - 2f_0 = 0.5f_0 = 100 \text{ Hz} \Rightarrow f_0 = 200 \text{ Hz}$$

Final Answer:

The fundamental frequency of the open pipe is 200 Hz, option (A).

200 Hz

Quick Tip: Compare the first overtone of an open pipe with the second overtone of a closed pipe of the same length.

25. Three point charges $+q$, $+2q$ and $+Q$ are placed at the vertices of an equilateral triangle. If the potential energy of the system of three charges is zero, the value of Q in terms of q is

- (A) $-\frac{q}{3}$
- (B) $\frac{-2q}{3}$
- (C) $\frac{q}{2}$
- (D) $\frac{3q}{2}$

Correct Answer: (B) $\frac{-2q}{3}$

Solution:**Step 1: Understanding the Concept:**

For a uniformly charged infinite plane sheet, Gauss's law gives $E = \frac{\sigma}{2\epsilon_0}$. The field does not depend on the distance from the sheet.

Step 2: Look at the graphs:

Graph (A) is a straight line rising from the origin, so E grows with d .
Graph (B) is a straight line falling to zero, so E decreases with d .

Graph (D) is a vertical line, which is not a function of d . Graph (C) is a horizontal straight line, a constant E at all d .

Step 3: Pick:

A constant field matches the horizontal line of graph (C).

Final Answer:

The correct graph is (C).

(C)

Quick Tip: For an infinite sheet the field is $\sigma / 2\epsilon_0$, independent of distance.



26. Two point charges $+e$ and $+4e$ are kept at a distance ' d ' units apart. Third point charge $+q$ is placed between the two charges at a distance ' x ' units from charge $+e$ so as to be in equilibrium. The value of ' x ' is

- (A) $\frac{d}{3}$ units
- (B) $\frac{d}{2}$ units
- (C) $\frac{2d}{5}$ units
- (D) $\frac{3d}{4}$ units

Correct Answer: (A) $\frac{d}{3}$ units

Solution:

Step 1: Understanding the Concept:

Take two charges $+q$ at a separation L , with Q at the midpoint. The system is in equilibrium if each charge feels zero net force. By

symmetry it is enough to check one +q.

Step 2: Forces on the right-hand +q:

Repulsion from the other +q at distance L: $F_1 = \frac{kq^2}{L^2}$ (outward).

Force from Q at distance $\frac{L}{2}$: $F_2 = \frac{kqQ}{(L/2)^2} = \frac{4kqQ}{L^2}$. To balance, this force must point inward, so Q must be opposite in sign to q (attractive).

Step 3: Balance:

$$\frac{kq^2}{L^2} = \frac{4kq|Q|}{L^2} \Rightarrow |Q| = \frac{q}{4}$$

So $Q = -\frac{q}{4}$. A positive Q would push the end charges outward, so options (A) and (B) cannot give equilibrium, and $-\frac{q}{2}$ gives too large an attraction.

Final Answer:

The charge at the centre is $-\frac{q}{4}$, option (D).

$$\boxed{-\frac{q}{4}}$$

Quick Tip: Force on one end charge from the other end charge must balance the pull from the middle charge.

27. Two identical metal plates are given charges q_1 and q_2 ($q_2 < q_1$) respectively. They are brought close together to form a parallel plate capacitor with capacitance 'C'. The potential difference 'V' between the plates is

- (A) $\frac{q_1 - q_2}{C}$
 (B) $\frac{q_1 + q_2}{C}$
 (C) $\frac{q_1 - q_2}{2C}$
 (D) $\frac{q_1 + q_2}{2C}$

Correct Answer: (C) $\frac{q_1 - q_2}{2C}$

Solution:

Step 1: Understanding the Concept:

In a series combination, the same charge Q is on each capacitor. The equivalent capacitance is $C_s = \frac{C_1 C_2}{C_1 + C_2}$.

Step 2: Charge:

$$Q = C_s V = \frac{C_1 C_2 V}{C_1 + C_2}.$$

Step 3: Voltage across C_2 :

$$V_2 = \frac{Q}{C_2} = \frac{C_1 V}{C_1 + C_2}$$

The smaller capacitor takes the larger share of the voltage, and the answer has C_1 in the numerator for V_2 . Option (D), $\frac{C_2 V}{C_1 + C_2}$, is the voltage across C_1 .

Final Answer:

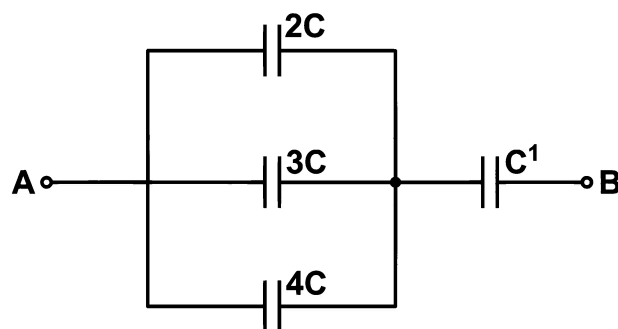
The voltage across C_2 is $\frac{C_1 V}{C_1 + C_2}$, option (B).

$$\boxed{\frac{C_1 V}{C_1 + C_2}}$$

Quick Tip: In series the charge is the same, so voltage divides inversely with capacitance.



28. If the equivalent capacitance between points A and B of the combination of capacitors shown in figure, is $6C$, the capacitor C^1 is



- (A) $9C$
- (B) $15C$
- (C) $18C$
- (D) $24C$

Correct Answer: (C) $18C$

Solution:

Step 1: Capacitance of each:

Each plate area is $\frac{A}{2}$. $C_1 = \frac{\epsilon_0(A/2)}{d} = \frac{\epsilon_0 A}{2d}$ and $C_2 = \frac{\epsilon_0(A/2)}{2d} = \frac{\epsilon_0 A}{4d}$.

Step 2: Parallel combination:

$$C = C_1 + C_2 = \frac{\epsilon_0 A}{2d} + \frac{\epsilon_0 A}{4d} = \frac{3\epsilon_0 A}{4d}$$

Using series by mistake would give $\frac{\epsilon_0 A}{6d}$, which is not an option, so the

parallel rule is the right one.

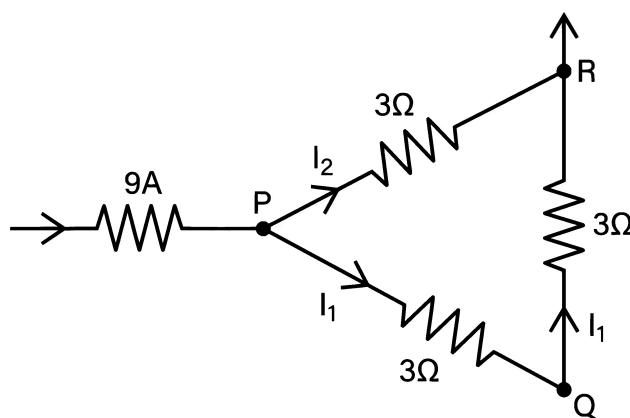
Final Answer:

The equivalent capacity is $\frac{3A\epsilon_0}{4d}$, option (B).

$$\boxed{\frac{3A\epsilon_0}{4d}}$$

Quick Tip: Find each capacitance with epsilon A over d and add them for a parallel connection.

29. A current of 9A enters point P of an equilateral triangle PQR having three wires of 3Ω each and leaves by point R. The currents I_1 and I_2 are respectively



- (A) 2A, 7A
- (B) 3A, 6A
- (C) 5A, 4A
- (D) 6A, 3A

Correct Answer: (B) 3A, 6A

Solution:

Step 1: Understanding the Concept:

In a metre bridge at balance, $\frac{R_{left}}{R_{right}} = \frac{l}{100-l}$, where l is the length measured from the left end.

Step 2: Find the balance length:

$$\frac{15}{45} = \frac{l}{100-l} \Rightarrow 100-l = 3l \Rightarrow l = 25 \text{ cm}$$

Step 3: Distance from the centre:

The centre of the wire is at 50 cm, so the null point is $50 - 25 = 25$ cm from the centre. Option (C), 40 cm, would be the balance length for a ratio of 2:3.

Final Answer:

The null point is 25 cm from the centre, option (D).

25 cm

Quick Tip: At balance, the resistances are in the ratio of the lengths of the wire.

30. The current passing through the galvanometer is 4 % of the total current in the circuit. If the resistance of the galvanometer is 'G', the shunt resistance 'S' connected across the galvanometer is

- (A) $25 G$
(B) $24 G$
(C) $\frac{G}{24}$
(D) $\frac{G}{25}$

Correct Answer: (C) $\frac{G}{24}$

Solution:

Step 1: Potential gradient:

$$k = \frac{V}{L} = \frac{2}{4} = 0.5 \text{ V/m.}$$

Step 2: EMF:

The balancing length is $80 \text{ cm} = 0.8 \text{ m}$. So $E = k l = 0.5 \times 0.8 = 0.4 \text{ V}$.

Final Answer:

The emf of the cell is 0.4 V , option (C).

0.4 V

Quick Tip: The potential gradient is the potential difference divided by the wire length.

31. Two wires of same length are bent to form the circular loop with two turns and a square loop of one turn. Both of them carry same current. The ratio of magnetic moment of circular loop of two turns to that of square loop is

- (A) $\pi : 4$
- (B) $4 : \pi$
- (C) $\pi : 1$
- (D) $2 : \pi$

Correct Answer: (D) $2 : \pi$

Solution:

Step 1: Flux density:

$$B = \frac{\Phi}{A} = \frac{3 \times 10^{-4}}{1.5 \times 10^{-4}} = 2 \text{ T, since } 1.5 \text{ cm}^2 = 1.5 \times 10^{-4} \text{ m}^2.$$

Step 2: Permeability:

$$\mu = \frac{B}{H} = \frac{2}{1000} = 2 \times 10^{-3} \text{ Wb/A-m}$$

The area conversion is the key step: using cm^2 directly would give a value that is off by a factor of 10^4 .

Final Answer:

The permeability is $2 \times 10^{-3} \text{ Wb/A-m}$, option (B).

$2 \times 10^{-3} \text{ Wb/A-m}$

Quick Tip: Flux density is flux over area, and permeability is B over H.

32. The magnetic field at a distance 'r' from a long straight wire carrying current I is 0.4 tesla. The magnetic field at a distance 2r will be

- (A) 0.1 tesla
- (B) 0.2 tesla
- (C) 0.8 tesla
- (D) 1.6 tesla

Correct Answer: (B) 0.2 tesla

Solution:

Step 1: Understanding the Concept:

A charge moving perpendicular to a field follows a circle. The magnetic force provides the centripetal force: $qvB = \frac{mv^2}{r}$, so $r = \frac{mv}{qB}$.

Step 2: Period:

$$T = \frac{2\pi r}{v} = \frac{2\pi m}{qB}$$

The velocity cancels out. The period depends on mass, charge and the field, but not on the speed.

Final Answer:

The period is independent of the velocity of the particle, option (B).

velocity of the particle

Quick Tip: The period is $2\pi m / qB$, which has no velocity in it.

33. In the cyclotron, as radius of the circular path of the charged particle increases, (ω = angular velocity, V = linear velocity)

- (A) V increases , ω decreases.
- (B) V increases , ω remains constant.
- (C) only ω increases , V remains constant.
- (D) both ω and V increase.

Correct Answer: (B) V increases , ω remains constant.

Solution:

Step 1: Radius of the n-turn coil:

Let the single loop have radius R , so the wire length is $2\pi R$. Forming n turns of radius r gives $n \cdot 2\pi r = 2\pi R$, so $r = \frac{R}{n}$.

Step 2: Fields at the centre:

One turn: $B_1 = \frac{\mu_0 I}{2R}$. n turns: $B_n = \frac{n\mu_0 I}{2r} = \frac{n\mu_0 I n}{2R} = \frac{n^2 \mu_0 I}{2R}$.

Step 3: Ratio:

$\frac{B_1}{B_n} = \frac{1}{n^2}$, so the ratio is $1 : n^2$. The factor is n^2 , not n , because both the number of turns and the smaller radius raise the field.

Final Answer:

The ratio is $1 : n^2$, option (C).

$$\boxed{1 : n^2}$$

Quick Tip: The wire length is fixed, so n turns have radius R over n .

34. Two planar concentric rings of metal wire having radii ' r_1 ' and ' r_2 ' (with $r_1 > r_2$) are placed in air. The current 'I' is flowing through the coil of larger radius. The mutual inductance between the coils is given by (μ_0 = permeability of free space)

- (A) $\frac{\mu_0 \pi r_1^2}{2 r_2}$
 (B) $\frac{\mu_0 \pi r_2^2}{2 r_1}$
 (C) $\frac{\mu_0 \pi (r_1 + r_2)^2}{2 r_1}$
 (D) $\frac{\mu_0 \pi (r_1 - r_2)^2}{2 r_2}$

Correct Answer: (B) $\frac{\mu_0 \pi r_2^2}{2 r_1}$

Solution:

Step 1: Equivalent current:

The ring completes f revolutions per second, so charge q passes any point f times each second. The equivalent current is $I = qf$.

Step 2: Field at the centre:

$$B = \frac{\mu_0 I}{2R} = \frac{\mu_0 qf}{2R}$$

Options with f in the denominator are wrong, since a faster rotation gives a larger current and a larger field.

Final Answer:

The magnetic induction is $\frac{\mu_0 qf}{2R}$, option (C).

$$\boxed{\frac{\mu_0 qf}{2R}}$$

Quick Tip: A rotating charge is equivalent to a current $q f$, and a circular loop gives $\mu_0 I$ over $2R$ at the centre.

35. Two coils having self-inductance $L_1 = 75 \text{ mH}$ and $L_2 = 48 \text{ mH}$ are coupled with each other. If the mutual inductance of the coils is 37.2 mH , then coefficient of coupling will be

- (A) 0.58
- (B) 0.60
- (C) 0.62
- (D) 0.64

Correct Answer: (C) 0.62

Solution:

Step 1: Field at the centre:

The big coil (radius r_1 , one turn) carries a current I and produces a field at the centre $B = \frac{\mu_0 I}{2r_1}$.

Step 2: Flux through the small coil:

Since $r_2 \ll r_1$, the field is nearly uniform over the small coil: $\Phi = B \cdot \pi r_2^2 = \frac{\mu_0 I \pi r_2^2}{2r_1}$.

Step 3: Mutual inductance:

$$M = \frac{\Phi}{I} = \frac{\mu_0 \pi r_2^2}{2r_1}$$

Final Answer:

The mutual inductance is $\frac{\mu_0 \pi r_2^2}{2r_1}$, option (D).

$$M = \frac{\mu_0 \pi r_2^2}{2r_1}$$

Quick Tip: Find the flux of the large coil field through the small coil, treating the field as uniform.



36. A straight line conductor of length 0.4 m is moved with a speed of 7.0 ms^{-1} perpendicular to magnetic field of intensity 0.8 Wb m^{-2} . The induced e.m.f. across the conductor is

- (A) $2 \cdot 24 \text{ V}$
- (B) $2 \cdot 80 \text{ V}$
- (C) $3 \cdot 20 \text{ V}$
- (D) $5 \cdot 60 \text{ V}$

Correct Answer: (A) $2 \cdot 24 \text{ V}$

Solution:**Step 1: Understanding the Concept:**

$$L = \frac{\mu_0 N^2 A}{l} = \frac{\mu_0 N^2 \pi r^2}{l}. \text{ With equal turns, } L \propto \frac{r^2}{l}.$$

Step 2: Ratio:

The lengths and radii of A and B are in the ratio 1 : 3. So $\frac{L_A}{L_B} = \frac{r_A^2/l_A}{r_B^2/l_B}$
 $= \frac{1/1}{9/3} = \frac{1}{3}.$

Final Answer:

The ratio of self-inductances is 1 : 3, option (B).

$$\boxed{1 : 3}$$

Quick Tip: Self-inductance goes as N^2 times area over length.

37. In LCR circuit, at a particular angular frequency the capacitive reactance and inductive reactance is same. If the angular frequency is doubled, the ratio of the reactance of the capacitor to that of the inductor will be

- (A) $\frac{1}{4}$
- (B) $\frac{1}{2}$
- (C) 1
- (D) 4

Correct Answer: (A) $\frac{1}{4}$

Solution:**Step 1: Understanding the Concept:**

In an AC circuit with a lamp (resistance R) and a capacitor, the impedance is $Z = \sqrt{R^2 + X_C^2}$ with $X_C = \frac{1}{2\pi fC}$.

Step 2: Effect of lowering f :

When f decreases, X_C increases, so Z increases and the current $I = \frac{V}{Z}$ decreases. The lamp's power I^2R falls, so the lamp becomes dimmer. It does not become completely dark, because the current is still non-zero at any finite frequency.

Final Answer:

The brightness of the lamp is reduced, option (B).

is reduced

Quick Tip: Capacitive reactance rises as frequency falls, so the current decreases.

38. In an a.c. circuit with pure capacitance 'C' and a.c. source $E = E_0 \sin \omega t$, the equation of instantaneous current is given by

- (A) $I = E_0 \omega C \cdot \sin(\omega t)$
- (B) $I = E_0 \omega C \sin(\omega t + \frac{\pi}{2})$
- (C) $I = \frac{E_0}{\omega C} \sin(\omega t)$
- (D) $I = \frac{E_0}{\omega C} \sin(\omega t + \frac{\pi}{2})$

Correct Answer: (B) $I = E_0 \omega C \sin(\omega t + \frac{\pi}{2})$

Solution:

Step 1: Original frequency:

$$f = \frac{1}{2\pi\sqrt{LC}}.$$

Step 2: New values:

L is increased by 2L, so the new inductance is $L + 2L = 3L$, and C becomes 9C. Then

$$f' = \frac{1}{2\pi\sqrt{3L \times 9C}} = \frac{1}{2\pi\sqrt{27LC}} = \frac{1}{3\sqrt{3}} \cdot \frac{1}{2\pi\sqrt{LC}}$$

Step 3: Result:

$$f' = \frac{f}{3\sqrt{3}}.$$

Final Answer:

The new resonant frequency is $\frac{f}{3\sqrt{3}}$, option (D).

$$\boxed{\frac{f}{3\sqrt{3}}}$$

Quick Tip: Resonant frequency is $\frac{1}{2\pi\sqrt{LC}}$.



39. In a series LCR circuit alternating e.m.f. and current are given by the equations $V = V_0 \sin(\omega t)$ and $I = I_0 \sin(\omega t + \frac{\pi}{3})$ respectively. The average power dissipated in the circuit over one cycle of a.c. is ($\cos 60^\circ = 0.5$)

- (A) zero
- (B) $\frac{V_0 I_0}{2}$
- (C) $\frac{\sqrt{3}}{2} V_0 I_0$
- (D) $\frac{V_0 I_0}{4}$

Correct Answer: (D) $\frac{V_0 I_0}{4}$

Solution:

Step 1: Understanding the Concept:

At resonance $X_L = X_C$, so the net reactance is zero and $Z = R$.

Step 2: Check the options:

(A) The impedance is a minimum (equal to R), not a maximum, so (A) is false.

(B) The current is a maximum $\frac{V}{R}$, not a minimum, so (B) is false.

(C) The current leads the voltage only in a capacitive circuit. At resonance the phase angle is zero, so (C) is false.

(D) The phase difference is zero, so the current and voltage are in phase. (D) is true.

Final Answer:

At resonance the current and voltage are in phase, option (D).

(D)

Quick Tip: At resonance the reactances cancel, so the circuit behaves as a pure resistance.



40. A ray of light incident on one face of an equilateral glass prism having refractive index $\sqrt{2}$, produces the emergent ray which just grazes along the adjacent face. The value of angle of incidence is ($\sin 90^\circ = 1$)

$$(\sin 30^\circ = \frac{1}{2})(\sin 45^\circ = \frac{1}{\sqrt{2}})$$

(A) $\sin^{-1}(\sqrt{2}\sin 15^\circ)$

(B) $\sin^{-1}(\frac{1}{\sqrt{2}}\sin 15^\circ)$

(C) $\sin^{-1}(\sqrt{2}\sin 30^\circ)$

(D) $\sin^{-1}(\frac{1}{\sqrt{2}}\sin 45^\circ)$

Correct Answer: (A) $\sin^{-1}(\sqrt{2}\sin 15^\circ)$

Solution:

Step 1: Understanding the Concept:

For a thin prism of angle A , the minimum deviation is $\delta = (n - 1)A$, where n is the refractive index of the prism relative to the surrounding medium.

Step 2: In air:

$$\delta_1 = \left(\frac{3}{2} - 1\right)A = \frac{A}{2}.$$

Step 3: In the liquid (using the values given):

The relative index is $\frac{n_g}{n_l} = \frac{3/2}{1/3} = \frac{9}{2}$.

$$\delta_2 = \left(\frac{9}{2} - 1\right)A = \frac{7}{2}A$$

Step 4: Ratio:

$$\frac{\delta_2}{\delta_1} = \frac{7A/2}{A/2} = 7, \text{ so } \delta_2 = 7\delta_1.$$

Final Answer:

The new deviation is $7\delta_1$, option (C).

$$\boxed{\delta_2 = 7\delta_1}$$

Quick Tip: For a thin prism the deviation is $(n_{\text{rel}} - 1)A$, where n_{rel} is the index of the prism relative to the surrounding medium.

41. An unpolarised light of intensity 64 Wm^{-2} passes through three polarizers successively such that the transmission axis of last polarizer is crossed with the first. The intensity of the emerging light is 6 Wm^{-2} . The angle between the transmission axis of the first two polarizers is

(A) $\frac{1}{2} \sin^{-1}(\frac{\sqrt{3}}{2})$

(B) $\frac{1}{2} \cos^{-1}(\frac{\sqrt{3}}{2})$

(C) $\frac{1}{2} \sin^{-1}(\frac{1}{\sqrt{3}})$

(D) $\frac{1}{2} \cos^{-1}(\frac{1}{\sqrt{3}})$

Correct Answer: (A) $\frac{1}{2} \sin^{-1}(\frac{\sqrt{3}}{2})$

Solution:

Step 1: Understanding the Concept:

By Snell's law, $\sin i = n \sin r$. The refractive index n of a medium depends on the wavelength (colour) of light.

Step 2: Result:

For the same angle of incidence, different colours have different n , and therefore different angles of refraction. Violet bends the most and red the least. This is the cause of dispersion.

So (B) is true, while (A) and (C) say the opposite. The critical angle in (D) is a separate idea for total internal reflection and does not decide the angle of refraction.

Final Answer:

The angle of refraction differs for different colours, option (B).

(B)

Quick Tip: The refractive index depends on wavelength, so different colours bend by different amounts.



42. In Young's experiment, the intensity ratio of the maxima and minima in an interference pattern produced by two coherent sources is $36 : 1$. The ratio of the amplitudes of the two individual sources will be

- (A) $4 : 7$
- (B) $5 : 7$
- (C) $7 : 5$
- (D) $7 : 4$

Correct Answer: (C) $7 : 5$

Solution:

Step 1: Wavelength in water:

$$\lambda' = \frac{\lambda}{n} = \frac{6300 \text{ \AA}}{1.33}.$$

Step 2: Fringe width:

$$\beta = \frac{\lambda' D}{d} = \frac{6300 \times 10^{-10}}{1.33} \times \frac{1.33}{1 \times 10^{-3}} = 6.3 \times 10^{-4} \text{ m}$$

The 1.33 in the wavelength and in the distance cancel exactly.

Final Answer:

The fringe width is $6.3 \times 10^{-4} \text{ m}$, option (B).

$6.3 \times 10^{-4} \text{ m}$

Quick Tip: In a medium the wavelength is divided by the refractive index.

43. For obtaining maximum contrast between the bright and dark fringes in an interference pattern, the intensities of the two light coherent sources should be

- (A) large
- (B) small
- (C) equal
- (D) in the ratio 2 : 1

Correct Answer: (C) equal

Solution:

Step 1: Understanding the Concept:

In single slit diffraction, the width of the central maximum is $\frac{2\lambda D}{a}$.

Step 2: Equate to the slit width:

$$\frac{2\lambda D}{a} = a \Rightarrow D = \frac{a^2}{2\lambda}$$

This is option (D). Option (B), $\frac{a^2}{\lambda}$, would come from using only the half width of the central maximum.

Final Answer:

The distance is $D = \frac{a^2}{2\lambda}$, option (D).

$$D = \frac{a^2}{2\lambda}$$

Quick Tip: Width of the central maximum is $2 \lambda D$ over a .

44. The ratio of the accelerating potentials required to accelerate (i) an α -particle and (ii) a proton to have the same de Broglie wavelength associated with them is

(mass of α - particle = 6.4×10^{-27} kg, mass of proton = 1.6×10^{-27} kg)

(A) 1 : 8

(B) 1 : 4

(C) 8 : 1

(D) 4 : 1

Correct Answer: (A) 1 : 8

Solution:

Step 1: Einstein's equation:

$$\frac{hc}{\lambda} = \phi + eV_0. \text{ Write } E = \frac{hc}{\lambda}.$$

Step 2: Two equations:

$$\text{For } \lambda: E = \phi + eV. \text{ For } 6\lambda: \frac{E}{6} = \phi + \frac{eV}{12}.$$

Step 3: Eliminate:

$$\text{Subtract: } E - \frac{E}{6} = eV - \frac{eV}{12}, \text{ so } \frac{5E}{6} = \frac{11eV}{12} \text{ and } eV = \frac{10E}{11}.$$

$$\text{Then } \phi = E - eV = E - \frac{10E}{11} = \frac{E}{11}.$$

Step 4: Threshold wavelength:

$$\phi = \frac{hc}{\lambda_0} = \frac{E}{11} = \frac{hc}{11\lambda}, \text{ so } \lambda_0 = 11\lambda.$$

Final Answer:

The threshold wavelength is 11λ , option (C).

$$11\lambda$$

Quick Tip: Use Einstein photoelectric equation for the two wavelengths and eliminate the work function.

45. In photoelectric effect, keeping the frequency of incident radiation and accelerating potential fixed, if the intensity of incident light is increased,

- (A) the photoelectric current increases.
- (B) the kinetic energy of emitted photoelectrons increases.
- (C) photoelectric current decreases.
- (D) kinetic energy of emitted photoelectrons decreases.

Correct Answer: (A) the photoelectric current increases.

Solution:**Step 1: Understanding the Concept:**

Einstein's equation gives $eV_0 = h\nu - h\nu_0$, so $V_0 = \frac{h}{e}\nu - \frac{h\nu_0}{e}$.

Step 2: Effect:

The stopping potential increases linearly with the frequency (slope $\frac{h}{e}$), and does not depend on intensity. So when ν increases with all else constant, V_0 increases.

Final Answer:

The stopping potential increases, option (C).

increases

Quick Tip: Stopping potential rises linearly with the frequency above threshold.

46. In Bohr's atomic model, the energy of the electron is 'E' in the second orbit of hydrogen atom. So the energy of the electron in the third orbit of helium atom ($Z = 2$) will be

- (A) $\frac{16E}{3}$
- (B) $\frac{16E}{9}$
- (C) $\frac{4E}{3}$
- (D) $\frac{4E}{9}$

Correct Answer: (B) $\frac{16E}{9}$

Solution:**Step 1: Understanding the Concept:**

The second excited state is $n = 3$ (ground state $n = 1$, first excited $n = 2$).

Step 2: Transitions:

The electron can go $3 \rightarrow 2$, $3 \rightarrow 1$ and $2 \rightarrow 1$. That is 3 different lines.

Using the formula $\frac{n(n-1)}{2} = \frac{3 \times 2}{2} = 3$.

Final Answer:

The maximum number of spectral lines is 3, option (B).

3

Quick Tip: Count the possible transitions from $n = 3$ to lower levels.

47. The activity of a radioactive sample is measured as N_0 counts per minute at time $t = 0$ and $\frac{N_0}{e}$ counts per minute at time $t = 3$ minute. The time (in minute) in which the activity reduces to half the value, is

- (A) $\frac{1}{3} \log_e 2$
- (B) $3 \log_e 2$
- (C) $3 \log_{10} 2$
- (D) $\frac{3}{\log_e 2}$

Correct Answer: (B) $3 \log_e 2$

Solution:**Step 1: Understanding the Concept:**

The radius of the n th Bohr orbit is $r_n = 0.529 n^2$ angstrom, so $r_n \propto n^2$.

Step 2: Ratio:

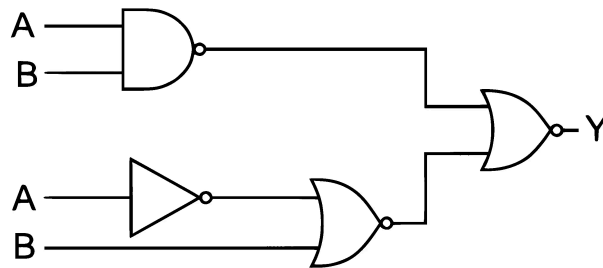
For $n = 1, 2, 3, 4$: $r_1 : r_2 : r_3 : r_4 = 1 : 4 : 9 : 16$. Options with $\frac{1}{n}$ or $\frac{1}{n^2}$ would describe velocity or energy, not radius.

Final Answer:

The radii are in the ratio $1 : 4 : 9 : 16$, option (C).

Quick Tip: The radius of the n th Bohr orbit is proportional to n squared.

48. In the following digital circuit, the output 'Y' will be '1' (one) for inputs 'A' and 'B' having values



- (A) $A = 0, B = 0$
- (B) $A = 0, B = 1$
- (C) $A = 1, B = 0$
- (D) $A = 1, B = 1$

Correct Answer: (D) $A = 1, B = 1$

Solution:

Step 1: Understanding the Circuit:

From the figure: inputs A and B go to a NOR gate, and its output goes to a NOT gate to give Y_1 . Inputs C and D go to an AND gate to give Y_2 . Y_1 and Y_2 go to a NOR gate to give Y_3 .

Step 2: Write the outputs:

NOR followed by NOT is an OR gate, so $Y_1 = A + B$. Also $Y_2 = C \cdot D$

and $Y_3 = \overline{Y_1 + Y_2}$.

Check with the given case, $A = B = C = D = 1$: $Y_1 = 1$, $Y_2 = 1$, $Y_3 = 0$.
This matches (1, 1, 0).

Step 3: New inputs:

Now $A = 0$, $B = 1$, $C = 0$, $D = 1$:

$$Y_1 = 0 + 1 = 1. Y_2 = 0 \cdot 1 = 0. Y_3 = \overline{1 + 0} = 0.$$

So the outputs are (1, 0, 0).

Final Answer:

The outputs are $Y_1, Y_2, Y_3 = 1, 0, 0$, option (B).

1, 0, 0

Quick Tip: Trace each gate: a NOR followed by a NOT, an AND gate, and a final NOR.

49. In common emitter configuration of transistor amplifier, r_i , R_L and β represent the input resistance, load resistance and the a.c. current gain respectively. The voltage gain A_V and power gain A_P are represented in magnitude respectively by

- (A) $\beta(\frac{r_i}{R_L}), \beta^2(\frac{r_i}{R_L})$
- (B) $\beta(\frac{R_L}{r_i}), \beta(\frac{R_L}{r_i})^2$
- (C) $\beta(\frac{R_L}{r_i}), \beta^2(\frac{R_L}{r_i})$
- (D) $\beta(\frac{r_i}{R_L}), \beta(\frac{r_i}{R_L})^2$

Correct Answer: (C) $\beta(\frac{R_L}{r_i}), \beta^2(\frac{R_L}{r_i})$

Solution:

Step 1: Understanding the Circuit:

Between A and B there are two parallel branches. The upper branch has a 30 ohm resistor in series with the ideal diode D. The lower branch has a 30 ohm resistor.

Step 2: Forward biased:

An ideal forward biased diode has zero resistance, so both branches carry current: $R_1 = \frac{30 \times 30}{30 + 30} = 15 \Omega$.

Step 3: Reverse biased:

An ideal reverse biased diode has infinite resistance, so the upper branch carries no current. Only the lower resistor is left: $R_2 = 30 \Omega$.

Step 4: Ratio:

$$\frac{R_1}{R_2} = \frac{15}{30} = \frac{1}{2}$$

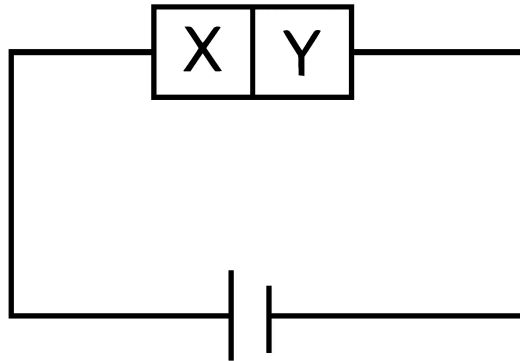
Final Answer:

The ratio $\frac{R_1}{R_2}$ is $\frac{1}{2}$, option (C).

$$\boxed{\frac{1}{2}}$$

Quick Tip: A forward biased ideal diode acts as a wire and a reverse biased one acts as an open switch.

50. A semiconductor X is made by doping a germanium crystal with indium ($Z = 49$). A second semiconductor Y is made by doping germanium crystal with arsenic ($Z = 33$). Both are joined end to end and connected to a battery as shown. Which of the following statements is correct?



- (A) X is p-type, Y is n-type and the junction is forward biased.
- (B) X is p-type, Y is n-type and the junction is reverse biased.
- (C) X is n-type, Y is p-type and the junction is forward biased.
- (D) X is n-type, Y is p-type and the junction is reverse biased.

Correct Answer: (A) X is p-type, Y is n-type and the junction is forward biased.

Solution:

Step 1: Current gain:

$$\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{5 \text{ mA}}{0.2 \text{ mA}} = 25.$$

Step 2: Voltage gain:

$$A_v = \beta \frac{R_L}{R_i} \Rightarrow 75 = 25 \times \frac{R_L}{2 \text{ k}\Omega}$$

Step 3: Solve:

$$R_L = \frac{75 \times 2}{25} = 6 \text{ k}\Omega$$

Final Answer:

The load resistance is 6 k Ω , option (D).

6 k Ω

Quick Tip: Current gain is the ratio of collector to base current change, and voltage gain is beta times load over input resistance.

Chemistry

Questions 51 to 100 | 1 mark each | 50 marks

51. How many moles of potassium chlorate are heated to produce 11.2 L oxygen at STP ?

- (A) $\frac{1}{2}$ mole
- (B) $\frac{1}{3}$ mole
- (C) $\frac{1}{4}$ mole
- (D) $\frac{2}{3}$ mole

Correct Answer: (B) $\frac{1}{3}$ mole

Solution:

Step 1: Understanding the Concept:

Heating potassium chlorate decomposes it into potassium chloride and oxygen. We need the moles of KClO_3 that give 11.2 L of O_2 at STP. At STP one mole of any gas occupies 22.4 L .

Step 2: Key Formula or Approach:



So 2 mol of KClO_3 give 3 mol of O_2 .

Step 3: Detailed Explanation:

Moles of O_2 formed:

$$n(\text{O}_2) = \frac{11.2}{22.4} = 0.5\text{ mol}$$

From the equation, $n(\text{KClO}_3) = \frac{2}{3} \times n(\text{O}_2)$.

$$n(\text{KClO}_3) = \frac{2}{3} \times 0.5 = \frac{1}{3} \text{ mol}$$

The value $\frac{1}{2}$ would come from a 1:1 ratio and $\frac{2}{3}$ from inverting the ratio, so neither fits the balanced equation.

Final Answer:

Moles of potassium chlorate needed are one third of a mole, option (B).

$$\boxed{\frac{1}{3} \text{ mol}}$$

Quick Tip: Write the decomposition equation and use 22.4 L per mole at STP.

52. Find the energy of third stationary orbit of Bohr's model of hydrogen atom if the energy of ground state is $-E \text{ J}$

- (A) $-\frac{E}{2} \text{ J}$
- (B) $-\frac{E}{4} \text{ J}$
- (C) $-\frac{E}{9} \text{ J}$
- (D) $-\frac{E}{16} \text{ J}$

Correct Answer: (C) $-\frac{E}{9} \text{ J}$

Solution:

Step 1: Understanding the Concept:

The energy of an electron in the n^{th} Bohr orbit of hydrogen is inversely proportional to n^2 . The ground state is $n = 1$ and the third orbit is $n = 3$.

Step 2: Key Formula or Approach:

$$E_n = -\frac{13.6}{n^2} \text{ eV} = \frac{E_1}{n^2}$$

Here $E_1 = -E$ J is the ground state energy.

Step 3: Detailed Explanation:

For the third stationary orbit, $n = 3$.

$$E_3 = \frac{E_1}{3^2} = \frac{-E}{9}$$

Option (A) corresponds to $n^2 = 2$, option (B) to $n = 2$ (second orbit) and option (D) to $n = 4$ (fourth orbit). None of these is the third orbit.

Final Answer:

The energy of the third orbit is one ninth of the ground state energy, so option (C) is correct.

$$\boxed{-\frac{E}{9} \text{ J}}$$

Quick Tip: In a hydrogen atom the orbit energy varies as $1/n$ squared.

53. Which of the following statements regarding Pi (π) bond is NOT true ?

- (A) It is weaker than sigma bond.
- (B) It is formed by lateral overlapping of two half filled P orbitals.
- (C) It is formed after the formation of sigma bond.
- (D) The presence of Pi (π) bond increases the bond length of covalent bond.

Correct Answer: (D) The presence of Pi (π) bond increases the bond length of covalent bond.

Solution:

Step 1: Understanding the Concept:

A pi (π) bond forms when two half filled p orbitals overlap sideways (laterally). It always forms after a sigma bond has already been formed along the internuclear axis.

Step 2: Check statement (A):

Lateral overlap is less effective than head-on overlap, so the pi bond is weaker than the sigma bond. (A) is TRUE.

Step 3: Check statements (B) and (C):

Pi bonds form by lateral overlap of two half filled p orbitals, so (B) is TRUE. The sigma bond forms first and the pi bond follows, so (C) is TRUE.

Step 4: Check statement (D):

Adding a pi bond between two atoms increases the electron density between the nuclei and pulls them closer. For example, carbon-carbon bond length falls from 154 pm (single) to 134 pm (double) to 120 pm (triple). So the pi bond decreases the bond length and (D) is

NOT true.

Final Answer:

The statement that is not true is (D).

(D)

Quick Tip: A pi bond adds to bonding between two atoms, so it pulls them closer.

54. Identify the number of lone pair and bond pair electrons present in the valence shell of central atom in BrF_3 .

- (A) 1 _ Lone pair and 2 _ Bond pair of electrons
- (B) 2 _ Lone pair and 2 _ Bond pair of electrons
- (C) 2 _ Lone pair and 3 _ Bond pair of electrons
- (D) 1 _ Lone pair and 3 _ Bond pair of electrons

Correct Answer: (C) 2 _ Lone pair and 3 _ Bond pair of electrons

Solution:

Step 1: Understanding the Concept:

In BrF_3 the central atom is bromine. It has 7 valence electrons (group 17). Three of them pair with fluorine atoms to form three Br-F bonds.

Step 2: Key Formula or Approach:

Bond pairs = number of atoms bonded to the central atom = 3.

Lone pairs = $\frac{7-3}{2} = 2$.

Step 3: Detailed Explanation:

Bromine uses 3 of its 7 electrons for bonding, leaving 4 electrons, which form 2 lone pairs.

So the valence shell has 3 bond pairs and 2 lone pairs, five electron pairs in total. The shape is T-shaped (trigonal bipyramidal arrangement with the two lone pairs in equatorial positions).

Options with only 1 lone pair, or with only 2 bond pairs, do not match three Br-F bonds and seven valence electrons.

Final Answer:

Central bromine carries 2 lone pairs and 3 bond pairs, option (C).

2 lone pairs and 3 bond pairs

Quick Tip: Count valence electrons of Br and subtract those used for three Br-F bonds.



55. A gas at 10°C occupies a volume of 283 mL. if it is heated to 20°C keeping the pressure constant. What is the new volume of gas ?

- (A) 566 mL
- (B) 450 mL
- (C) 350 mL
- (D) 293 mL

Correct Answer: (D) 293 mL

Solution:

Step 1: Understanding the Concept:

At constant pressure, the volume of a fixed amount of gas is directly proportional to its absolute (Kelvin) temperature. This is Charles' law.

Step 2: Key Formula or Approach:

$$\frac{V_1}{T_1} = \frac{V_2}{T_2}$$

Convert the temperatures: $T_1 = 10 + 273 = 283 \text{ K}$ and $T_2 = 20 + 273 = 293 \text{ K}$.

Step 3: Detailed Explanation:

$$V_2 = V_1 \times \frac{T_2}{T_1} = 283 \times \frac{293}{283} = 293 \text{ mL}$$

Doubling the Celsius temperature would wrongly give 566 mL (option A), but Charles' law needs Kelvin. The volume rises by only about 3.5 percent, so 450 and 350 mL are far too large.

Final Answer:

The new volume is 293 mL, option (D).

293 mL

Quick Tip: At constant pressure the volume is proportional to the Kelvin temperature.

56. Calculate the quantity of heat released from system when 2 moles of ideal gas compressed isothermally from volume 25 dm^3 to 10 dm^3 at constant external pressure 4 bar.

(A) 5.5 kJ

(B) 5.0 kJ

(C) 6.5 kJ

(D) 6.0 kJ

Correct Answer: (D) 6.0 kJ

Solution:

Step 1: Understanding the Concept:

The compression is isothermal, and for an ideal gas the internal energy depends only on temperature. So $\Delta U = 0$ and the first law gives $q = -w$.

Step 2: Key Formula or Approach:

Work done on the gas against a constant external pressure: w
 $= -p_{ex}(V_2 - V_1)$.

Use $1 \text{ bar} \times 1 \text{ dm}^3 = 100 \text{ J}$.

Step 3: Detailed Explanation:

$$V_2 - V_1 = 10 - 25 = -15 \text{ dm}^3.$$

$$w = -4 \times (-15) = +60 \text{ bar dm}^3 = 60 \times 100 = 6000 \text{ J} = +6.0 \text{ kJ}$$

The sign is positive, so 6.0 kJ of work is done on the system. Since $\Delta U = q + w = 0$:

$$q = -6.0 \text{ kJ}$$

The negative sign means 6.0 kJ of heat is released by the system. The number of moles (2) is not needed because the pressure is constant and the volumes are given.

Final Answer:

Heat released is 6.0 kJ, option (D).

6.0 kJ

Quick Tip: For an ideal gas at constant temperature the internal energy does not change, so $q = -w$.



57. Calculate the standard enthalpy of combustion of carbon monoxide if

$$\Delta_f H^\circ(\text{CO}) = -110 \text{ kJ mol}^{-1}$$

$$\Delta_f H^\circ(\text{CO}_2) = -393 \text{ kJ mol}^{-1}$$

(A) -503 kJ mol^{-1}

(B) -110 kJ mol^{-1}

(C) -283 kJ mol^{-1}

(D) -383 kJ mol^{-1}

Correct Answer: (C) -283 kJ mol^{-1}

Solution:

Step 1: Understanding the Concept:

Combustion of carbon monoxide is $\text{CO} + \frac{1}{2}\text{O}_2 \rightarrow \text{CO}_2$. The enthalpy of formation of O_2 (an element in its standard state) is zero.

Step 2: Key Formula or Approach:

$$\Delta_c H^\circ = \sum \Delta_f H^\circ(\text{products}) - \sum \Delta_f H^\circ(\text{reactants})$$

Step 3: Detailed Explanation:

$$\Delta_c H^\circ = \Delta_f H^\circ(\text{CO}_2) - [\Delta_f H^\circ(\text{CO}) + \frac{1}{2} \Delta_f H^\circ(\text{O}_2)]$$

$$= -393 - (-110 + 0) = -283 \text{ kJ mol}^{-1}$$

Option (A), -503, adds the two values instead of subtracting. Option (B) is the formation enthalpy of CO itself, and (D), -383, comes from a slip in arithmetic.

Final Answer:

The enthalpy of combustion of CO is -283 kJ per mole, option (C).

-283 kJ mol^{-1}

Quick Tip: Combustion of CO gives CO₂, so subtract the enthalpy of formation of CO from that of CO₂.

58. Which of the following processes is an endothermic ?

- (A) The aqueous NaOH and HCl solutions mixed together.
- (B) Formation of ammonia by Haber process.
- (C) Ice melts at room temperature.
- (D) The freezing of liquid water.

Correct Answer: (C) Ice melts at room temperature.

Solution:

Step 1: Understanding the Concept:

An endothermic process absorbs heat from the surroundings, so ΔH is positive. An exothermic process releases heat.

Step 2: Check option (A):

Mixing aqueous NaOH and HCl is neutralisation, which releases about 57 kJ per mole of water formed. It is exothermic.

Step 3: Check option (B):

Formation of ammonia by the Haber process, $\text{N}_2 + 3\text{H}_2 \rightarrow 2\text{NH}_3$, has $\Delta H = -92 \text{ kJ}$. It is exothermic.

Step 4: Check options (C) and (D):

Ice melting needs heat to break hydrogen bonds in the solid (fusion), so it is endothermic. Freezing of water is the reverse and releases heat, so it is exothermic.

Final Answer:

Only melting of ice absorbs heat, so option (C) is correct.

(C) Ice melts at room temperature

Quick Tip: An endothermic process absorbs heat from the surroundings.

59. The solubility of sparingly soluble salt AB_2 is $18.78 \times 10^{-4} \text{ g/dm}^3$. What is its solubility product? (Molar mass of $AB_2 = 187.8 \text{ g mol}^{-1}$)

- (A) 2×10^{-15}
- (B) 4×10^{-15}
- (C) 6×10^{-15}
- (D) 8×10^{-15}

Correct Answer: (B) 4×10^{-15}

Solution:

Step 1: Understanding the Concept:

AB_2 dissociates as $AB_2 \rightleftharpoons A^{2+} + 2B^-$. If the solubility is $S \text{ mol dm}^{-3}$, then $[A^{2+}] = S$ and $[B^-] = 2S$.

Step 2: Convert solubility to molar units:

$$S = \frac{18.78 \times 10^{-4}}{187.8} = 1 \times 10^{-5} \text{ mol dm}^{-3}$$

Step 3: Find the solubility product:

$$K_{sp} = [A^{2+}][B^-]^2 = S \cdot (2S)^2 = 4S^3$$

$$K_{sp} = 4 \times (10^{-5})^3 = 4 \times 10^{-15}$$

Forgetting the factor 4 gives 1×10^{-15} , which is not an option, so the factor 4 from $(2S)^2$ matters.

Final Answer:

The solubility product is 4×10^{-15} , option (B).

$$4 \times 10^{-15}$$

Quick Tip: Convert g per dm³ to mol per dm³ and use $K_{sp} = 4S^3$ cubed.

60. What is the pH of resulting solution when 20 mL $\frac{M}{10}$ NaOH and 10 mL $\frac{M}{10}$ H₂SO₄ are mixed together ?

- (A) 0
- (B) 2
- (C) 7
- (D) 10

Correct Answer: (C) 7

Solution:

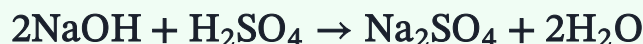
Step 1: Understanding the Concept:

NaOH is a monoacidic base and H₂SO₄ is a dibasic acid. We must compare the moles of OH⁻ with those of H⁺.

Step 2: Moles of each:

Moles of NaOH = $20 \times 0.1 = 2$ millimol, so OH⁻ = 2 millimol.

Moles of H₂SO₄ = $10 \times 0.1 = 1$ millimol, giving H⁺ = 2 millimol.

Step 3: Neutralisation:

The OH^- and H^+ are exactly equal, so both are used up completely. The resulting solution is sodium sulphate, a salt of a strong acid and strong base, which does not hydrolyse. So $[\text{H}^+] = 10^{-7}$ and $\text{pH} = 7$. pH 0, 2 or 10 would need leftover acid or base, which does not happen here.

Final Answer:

The solution is neutral, so $\text{pH} = 7$, option (C).

$$\boxed{\text{pH} = 7}$$

Quick Tip: Compare the moles of OH^- with the moles of H^+ before finding pH.

61. Which of the following is correct relationship between solubility and solubility product for silver oxalate ?

(A) $S = \sqrt[3]{K_{sp} \times 4}$

(B) $S = \sqrt[3]{\frac{4}{K_{sp}}}$

(C) $S = \sqrt[3]{\frac{K_{sp}}{4}}$

(D) $S = \sqrt[4]{\frac{K_{sp}}{27}}$

Correct Answer: (C) $S = \sqrt[3]{\frac{K_{sp}}{4}}$

Solution:

Step 1: Understanding the Concept:

Silver oxalate is $\text{Ag}_2\text{C}_2\text{O}_4$. It dissociates as $\text{Ag}_2\text{C}_2\text{O}_4 \rightleftharpoons 2\text{Ag}^+ + \text{C}_2\text{O}_4^{2-}$.

Step 2: Express ions in solubility S:

$[\text{Ag}^+] = 2S$ and $[\text{C}_2\text{O}_4^{2-}] = S$.

Step 3: Detailed Explanation:

$$K_{sp} = (2S)^2 \cdot S = 4S^3$$

$$S^3 = \frac{K_{sp}}{4} \Rightarrow S = \sqrt[3]{\frac{K_{sp}}{4}}$$

Option (A) multiplies instead of dividing, option (B) inverts the fraction, and option (D) has a fourth root, which does not follow from $K_{sp} = 4S^3$.

Final Answer:

The correct relation is $S = \sqrt[3]{K_{sp}/4}$, option (C).

$$S = \sqrt[3]{\frac{K_{sp}}{4}}$$

Quick Tip: Write the dissociation of $\text{Ag}_2\text{C}_2\text{O}_4$ and express K_{sp} in terms of S .

62. Identify oxidation number of Sulphur respectively in SO_2 and SO_4^{2-} .

(A) +6, +4

(B) +4, +6

(C) +4, +3

(D) +3, +4

Correct Answer: (B) +4, +6

Solution:

Step 1: Understanding the Concept:

The sum of oxidation numbers of all atoms in a species equals its charge. Oxygen is -2 here.

Step 2: Oxidation number in sulphur dioxide:

Let the oxidation number of S be x : $x + 2(-2) = 0$, so $x = +4$.

Step 3: Oxidation number in the sulphate ion:

$x + 4(-2) = -2$, so $x = +6$.

Final Answer:

Sulphur has oxidation numbers +4 and +6, option (B). Option (A) reverses the order, while +3 does not occur in either.

+4, +6

Quick Tip: Fix oxygen at -2 and make the sum equal the total charge.

63. Identify colour change when NaCl solution is titrated against AgNO_3 solution using fluorescein indicator.

- (A) Colourless to pale yellow
- (B) Reddish pink to pale yellow
- (C) Pale yellow to reddish pink
- (D) Colourless to reddish pink

Correct Answer: (C) Pale yellow to reddish pink

Solution:

Step 1: Understanding the Concept:

Titration of chloride with silver nitrate using fluorescein is the Fajans method. Fluorescein is an adsorption indicator, and the colour change occurs on the surface of the AgCl precipitate.

Step 2: Before the end point:

Chloride ions are in excess, so AgCl adsorbs Cl^- and becomes negatively charged. The fluorescein anion is repelled, so the solution stays pale yellow (yellowish green).

Step 3: At the end point:

After all chloride is used, one extra drop of Ag^+ makes the precipitate positively charged. The fluorescein anions are now adsorbed on it, forming a reddish pink coloured surface complex.

So the colour changes from pale yellow to reddish pink. A change from colourless would not apply, because the fluorescein solution is already yellow.

Final Answer:

The colour change is pale yellow to reddish pink, option (C).

Pale yellow to reddish pink

Quick Tip: Fluorescein is an adsorption indicator, so the colour change happens on the precipitate surface.

64. Which from the following metal chlorides does not contain water of crystallisation ?

- (A) LiCl
- (B) MgCl_2
- (C) CsCl
- (D) BeCl_2

Correct Answer: (C) CsCl

Solution:

Step 1: Understanding the Concept:

Salts of small cations with high charge density hydrate easily because the cation strongly attracts water molecules. A large cation with low charge density, such as Cs, holds water poorly.

Step 2: Look at each chloride:

LiCl crystallises as $\text{LiCl} \cdot 2\text{H}_2\text{O}$ (Li is small). MgCl_2 crystallises as $\text{MgCl}_2 \cdot 6\text{H}_2\text{O}$. BeCl_2 forms a hydrate, $\text{BeCl}_2 \cdot 4\text{H}_2\text{O}$.

Step 3: Caesium chloride:

Cs is the largest of these cations and has the lowest charge density, so CsCl crystallises in the anhydrous form. So CsCl is the chloride with no water of crystallisation.

Final Answer:

CsCl is anhydrous, so option (C) is correct.



Quick Tip: Small, highly polarising cations hold water of hydration; large cations do not.

65. What is the name of second lower homologue of $\text{CH}_3(\text{CH}_2)_2\text{COOH}$?

- (A) Methanoic acid
- (B) Ethanoic acid
- (C) Propanoic acid
- (D) Pentanoic acid

Correct Answer: (B) Ethanoic acid

Solution:**Step 1: Understanding the Concept:**

Homologues in a series differ by a $-\text{CH}_2-$ group. The given compound $\text{CH}_3(\text{CH}_2)_2\text{COOH}$ has 4 carbons, so it is butanoic acid.

Step 2: Go down the series:

First lower homologue: remove one CH_2 , giving propanoic acid (3 carbons).

Second lower homologue: remove another CH_2 , giving ethanoic acid CH_3COOH (2 carbons).

Step 3: Check options:

Methanoic acid is the third lower homologue, propanoic acid is the first lower, and pentanoic acid is a higher homologue.

Final Answer:

The second lower homologue is ethanoic acid, option (B).

Ethanoic acid

Quick Tip: Each lower homologue has one CH₂ group fewer.

66. How many chiral carbon atoms are present in 2-Iodo-3, 4, 5-trimethylhexane ?

- (A) 2
- (B) 3
- (C) 4
- (D) 1

Correct Answer: (B) 3

Solution:**Step 1: Understanding the Concept:**

A chiral carbon is sp³ hybridised and bonded to four different groups.

First draw the structure: CH₃ – CH(I)

– CH(CH₃) – CH(CH₃) – CH(CH₃) – CH₃ (carbons 1 to 6).

Step 2: Test carbons one by one:

C-1 and C-6 are CH₃ groups, so they are not chiral.

C-2 has H, I, CH_3 and the rest of the chain: chiral.

C-3 has H, CH_3 , C-2 side and C-4 side, all different: chiral.

C-4 has H, CH_3 , C-3 side and C-5 side, all different: chiral.

C-5 has H, CH_3 (the substituent), CH_3 (C-6) and C-4 side. Two groups are the same methyl, so it is not chiral.

Step 3: Count:

Chiral carbons are C-2, C-3 and C-4, which is 3 in total.

Final Answer:

Three chiral carbons, option (B).

3

Quick Tip: A chiral carbon has four different groups attached.

67. Identify the product 'B' in the following reaction



- (A) Benzal chloride
- (B) Benzaldehyde
- (C) Benzyl alcohol
- (D) Benzoic acid

Correct Answer: (B) Benzaldehyde

Solution:

Step 1: Understanding the Concept:

Toluene reacts with chromyl chloride (CrO_2Cl_2) in carbon disulphide.

This is the Etard reaction.

Step 2: Reaction steps:



A is the brown chromium complex (Etard complex), and its hydrolysis gives benzaldehyde.

Step 3: Why not the others:

The reaction stops at the aldehyde stage because the complex protects it from further oxidation. So benzoic acid is not formed. Benzal chloride and benzyl alcohol are not products of this reaction.

Final Answer:

B is benzaldehyde, option (B).

Benzaldehyde

Quick Tip: Chromyl chloride in CS₂ oxidises the methyl group to a chromium complex, which hydrolyses to an aldehyde.

68. Identify the 'A' in the following reaction.



- (A) Ethene
- (B) Ethyne
- (C) Propene
- (D) Propyne

Correct Answer: (B) Ethyne

Solution:

Step 1: Understanding the Concept:

Calcium carbide contains the carbide ion C_2^{2-} . It is hydrolysed by water, taking two protons.

Step 2: Reaction:



So A is ethyne (acetylene).

Step 3: Check options:

Ethene and propene need double bonds, which carbide does not give. Propyne forms from magnesium carbide Mg_2C_3 , not from calcium carbide.

Final Answer:

A is ethyne, option (B).

Ethyne

Quick Tip: Calcium carbide gives the simplest alkyne with water.

69. Calculate the number of atoms in 1 g metal that forms bcc crystal structure [$\rho \times a^3 = 6 \cdot 6 \times 10^{-22}$ g]

- (A) $4 \cdot 05 \times 10^{21}$
 (B) $5 \cdot 12 \times 10^{21}$
 (C) $3 \cdot 03 \times 10^{21}$
 (D) $2 \cdot 47 \times 10^{21}$

Correct Answer: (C) $3 \cdot 03 \times 10^{21}$

Solution:

Step 1: Understanding the Concept:

A bcc unit cell has $Z = 2$ atoms. Density of the metal is $\rho = \frac{ZM}{N_A a^3}$, so we can get the molar mass from ρa^3 .

Step 2: Find the molar mass:

$$M = \frac{\rho a^3 N_A}{Z} = \frac{6.6 \times 10^{-22} \times 6.022 \times 10^{23}}{2} \approx 198.7 \text{ g mol}^{-1}$$

Step 3: Atoms in 1 g:

$$N = \frac{N_A}{M} = \frac{6.022 \times 10^{23}}{198.7} = 3.03 \times 10^{21}$$

This is option (C). The other values do not follow from $Z = 2$.

Final Answer:

1 g of the metal contains 3.03×10^{21} atoms, option (C).

3.03×10^{21}

Quick Tip: Use $\text{density} = \frac{Z M}{N_A a^3}$ with $Z = 2$ for bcc to get M , then divide N_A by M .



70. Which of the following dopants is used in germanium to form n-type semiconductor ?

- (A) B
- (B) In
- (C) Ga
- (D) P

Correct Answer: (D) P

Solution:

Step 1: Understanding the Concept:

Germanium is in group 14 with four valence electrons. For an n-type semiconductor the dopant must give extra electrons, so it must be a group 15 element.

Step 2: Options:

B (boron), In (indium) and Ga (gallium) are group 13 elements with three valence electrons. They leave holes and give a p-type semiconductor.

P (phosphorus) is in group 15 with five valence electrons. Four form bonds with Ge and the fifth is free, giving n-type.

Final Answer:

The dopant is phosphorus, option (D).

Quick Tip: n-type needs a group 15 dopant with five valence electrons.



71. Calculate the void volume of bcc unit cell if volume of unit cell is $8 \cdot 0 \times 10^{-23} \text{ cm}^3$

- (A) $3 \cdot 08 \times 10^{-23} \text{ cm}^3$
- (B) $4 \cdot 16 \times 10^{-23} \text{ cm}^3$
- (C) $5 \cdot 44 \times 10^{-23} \text{ cm}^3$
- (D) $2 \cdot 56 \times 10^{-23} \text{ cm}^3$

Correct Answer: (D) $2 \cdot 56 \times 10^{-23} \text{ cm}^3$

Solution:

Step 1: Understanding the Concept:

The packing efficiency of a bcc structure is 68 percent, so 32 percent of the unit cell volume is void space.

Step 2: Calculation:

$$V_{\text{void}} = 0.32 \times 8.0 \times 10^{-23} = 2.56 \times 10^{-23} \text{ cm}^3$$

The value $5.44 \times 10^{-23} \text{ cm}^3$ is the volume occupied by atoms (68 percent), so it is not the void volume.

Final Answer:

The void volume is $2.56 \times 10^{-23} \text{ cm}^3$, option (D).

$$2.56 \times 10^{-23} \text{ cm}^3$$

Quick Tip: Packing efficiency of bcc is 68 percent, so 32 percent is empty.

72. Calculate the molar mass of a nonvolatile solute if 6.4 g of it dissolved in 100 g water produces a relative lowering in vapour pressure of 0.016 at 300 K.

- (A) 60 gmol^{-1}
- (B) 66 gmol^{-1}
- (C) 72 gmol^{-1}
- (D) 84 gmol^{-1}

Correct Answer: (C) 72 gmol^{-1}

Solution:

Step 1: Understanding the Concept:

Raoult's law for a non-volatile solute: the relative lowering of vapour pressure equals the mole fraction of the solute, $\frac{p^0 - p}{p^0} = x_2 \approx \frac{n_2}{n_1}$ for a dilute solution.

Step 2: Moles of solvent:

$$n_1 = \frac{100}{18} = 5.556 \text{ mol of water.}$$

Step 3: Solve for the molar mass:

$$0.016 = \frac{w_2/M_2}{n_1} = \frac{6.4}{M_2 \times 5.556}$$

$$M_2 = \frac{6.4}{0.016 \times 5.556} = \frac{6.4}{0.0889} = 72 \text{ g mol}^{-1}$$

This is option (C). The temperature of 300 K is not needed for this calculation.

Final Answer:

The molar mass of the solute is about 72 g/mol, option (C).

$$72 \text{ g mol}^{-1}$$

Quick Tip: Relative lowering equals the mole fraction of solute, which is about n_2/n_1 for a dilute solution.



73. Calculate the cryoscopic constant of a solvent if depression in freezing point of 0.3 m solution of nonelectrolyte is 0.48 K

- (A) 1.1 K kg mol⁻¹
- (B) 1.6 K kg mol⁻¹
- (C) 2.2 K kg mol⁻¹
- (D) 2.6 K kg mol⁻¹

Correct Answer: (B) 1.6 K kg mol⁻¹

Solution:

Step 1: Understanding the Concept:

For a non-electrolyte, the depression in freezing point is directly proportional to molality: $\Delta T_f = K_f m$.

Step 2: Calculation:

$$K_f = \frac{\Delta T_f}{m} = \frac{0.48}{0.3} = 1.6 \text{ K kg mol}^{-1}$$

Only option (B) satisfies this relation.

Final Answer:

The cryoscopic constant is 1.6 K kg/mol , option (B).

$1.6 \text{ K kg mol}^{-1}$

Quick Tip: Use the depression equal to K_f times molality.



74. Identify the correct statement from following.

- (A) The vapour pressure of a solvent increases by dissolving nonvolatile solute into it.
- (B) The boiling point of solvent decreases by dissolving a nonvolatile solute into it.
- (C) The osmotic pressure of electrolytic solution is greater than nonelectrolytic solution of the same concentration.
- (D) The freezing point of solvent is a colligative property.

Correct Answer: (C) The osmotic pressure of electrolytic solution is greater than nonelectrolytic solution of the same concentration.

Solution:

Step 1: Understanding the Concept:

Colligative properties depend on the number of solute particles, not their nature. Electrolytes dissociate into ions, so they give more particles.

Step 2: Check (A):

A non-volatile solute lowers the vapour pressure of the solvent, so (A) is false.

Step 3: Check (B):

A non-volatile solute raises the boiling point (elevation), so (B) is false.

Step 4: Check (C):

$\pi = iCRT$ with $i > 1$ for electrolytes. So an electrolyte solution has a greater osmotic pressure than a non-electrolyte of the same concentration. (C) is true.

Step 5: Check (D):

The freezing point itself is a property of the solvent. Its depression is the colligative property, so (D) as stated is false.

Final Answer:

Only statement (C) is correct.

(C)

Quick Tip: Electrolytes give more particles, so they have a larger colligative effect.

75. What is the change in oxidation number of Pb at positive electrode of lead accumulator acting as galvanic cell ?

- (A) increases by 1
- (B) decreases by 1
- (C) increases by 2
- (D) decreases by 2

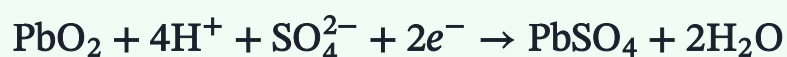
Correct Answer: (D) decreases by 2

Solution:

Step 1: Understanding the Concept:

In a lead accumulator working as a galvanic cell (discharging), the positive electrode is the cathode, where reduction occurs. It is made of PbO_2 .

Step 2: Half reaction:



Step 3: Oxidation numbers:

In PbO_2 , Pb is +4. In PbSO_4 , Pb is +2. So the oxidation number decreases by 2.

At the negative electrode Pb goes from 0 to +2, which is an increase, but that is not the electrode asked about.

Final Answer:

Pb changes from +4 to +2, a decrease of 2, option (D).

decreases by 2

Quick Tip: The positive plate is PbO_2 , which is reduced to PbSO_4 .

76. The graphical variation of molar conductivity (Λ) against square root of molar concentration ($\sqrt{c}$) of certain electrolyte 'X' is linear with intercept on y axis. Identify X from following.

- (A) CH_3COOH
- (B) NH_4OH
- (C) HCOOH
- (D) CH_3COONa

Correct Answer: (D) CH_3COONa

Solution:

Step 1: Understanding the Concept:

For a strong electrolyte the Debye-Huckel-Onsager equation applies:
 $\Lambda_m = \Lambda_m^0 - A\sqrt{c}$. Plotting Λ_m against $\sqrt{c}$ gives a straight line whose intercept on the y axis is Λ_m^0 .

Step 2: Weak electrolytes:

CH_3COOH , NH_4OH and HCOOH are weak electrolytes. Their degree of dissociation rises sharply on dilution, so Λ_m rises steeply at low concentration and the curve cannot be extrapolated to a limiting value.

Step 3: Strong electrolyte:

CH_3COONa is a salt that dissociates completely, so it is a strong

electrolyte and gives the linear plot with an intercept.

Final Answer:

The electrolyte X is sodium acetate, option (D).



Quick Tip: Only strong electrolytes give a straight line with a finite intercept on the molar conductivity axis.

77. If standard reduction potential (E^0) of $(\text{Al}_{(aq)}^{+3}|\text{Al}_{(s)})$, $(\text{Fe}_{(aq)}^{+2}|\text{Fe}_{(s)})$, $(\text{Cu}_{(aq)}^{+2}|\text{Cu}_{(s)})$ and $(\text{Ag}_{(aq)}^{+1}|\text{Ag}_{(s)})$ are -1.66 V , -0.44 V , $+0.34\text{ V}$ and $+0.79\text{ V}$ respectively. Which of the following reaction is non spontaneous ?

- (A) $2\text{Ag}_{(s)} + \text{Fe}_{(aq)}^{+2} \rightarrow 2\text{Ag}_{(aq)}^{+1} + \text{Fe}_{(s)}$
(B) $2\text{Al}_{(s)} + 3\text{Cu}_{(aq)}^{+2} \rightarrow 2\text{Al}_{(aq)}^{+3} + 3\text{Cu}_{(s)}$
(C) $\text{Fe}_{(s)} + \text{Cu}_{(aq)}^{+2} \rightarrow \text{Fe}_{(aq)}^{+2} + \text{Cu}_{(s)}$
(D) $2\text{Al}_{(s)} + 3\text{Fe}_{(aq)}^{+2} \rightarrow 2\text{Al}_{(aq)}^{+3} + 3\text{Fe}_{(s)}$

Correct Answer: (A) $2\text{Ag}_{(s)} + \text{Fe}_{(aq)}^{+2} \rightarrow 2\text{Ag}_{(aq)}^{+1} + \text{Fe}_{(s)}$

Solution:

Step 1: Understanding the Concept:

A redox reaction is spontaneous if $E_{\text{cell}}^0 = E_{\text{reduction (cathode)}}^0 - E_{\text{reduction (anode)}}^0 > 0$.

Step 2: Check (A):

$2\text{Ag} + \text{Fe}^{2+} \rightarrow 2\text{Ag}^+ + \text{Fe}$. Silver is oxidised (anode, 0.79 V) and Fe^{2+} is reduced (cathode, -0.44 V). $E_{\text{cell}}^0 = -0.44 - 0.79 = -1.23\text{ V}$. This is

negative, so (A) is NON spontaneous.

Step 3: Check (B), (C), (D):

(B) Al anode, Cu cathode: $0.34 - (-1.66) = +2.00$ V, spontaneous.

(C) Fe anode, Cu cathode: $0.34 - (-0.44) = +0.78$ V, spontaneous.

(D) Al anode, Fe cathode: $-0.44 - (-1.66) = +1.22$ V, spontaneous.

Final Answer:

Only reaction (A) has a negative cell potential, so it is non spontaneous.

(A)

Quick Tip: A reaction is spontaneous when the cell EMF, $E(\text{cathode})$ minus $E(\text{anode})$, is positive.

78. For the reaction $2\text{NOBr}_{(g)} \rightarrow 2\text{NO}_{(g)} + \text{Br}_{2(g)}$ rate law is $r = k[\text{NOBr}]^2$, if rate constant is $1 \cdot 62 \text{ mol dm}^{-3} \text{ s}^{-1}$ and concentration of NOBr is $2 \times 10^{-3} \text{ mol L}^{-1}$. What is the rate of reaction ?

(A) $4 \cdot 05 \times 10^{-5} \text{ mol dm}^{-3} \text{ s}^{-1}$

(B) $6 \cdot 48 \times 10^{-6} \text{ mol dm}^{-3} \text{ s}^{-1}$

(C) $2 \cdot 12 \times 10^{-5} \text{ mol dm}^{-3} \text{ s}^{-1}$

(D) $8 \cdot 63 \times 10^{-6} \text{ mol dm}^{-3} \text{ s}^{-1}$

Correct Answer: (B) $6 \cdot 48 \times 10^{-6} \text{ mol dm}^{-3} \text{ s}^{-1}$

Solution:

Step 1: Understanding the Concept:

The rate law is $r = k[\text{NOBr}]^2$. The unit of k is $\text{mol dm}^{-3} \text{s}^{-1}$ as given, and $1 \text{ L} = 1 \text{ dm}^3$, so the concentration $2 \times 10^{-3} \text{ mol L}^{-1}$ is $2 \times 10^{-3} \text{ mol dm}^{-3}$.

Step 2: Calculation:

$$r = 1.62 \times (2 \times 10^{-3})^2 = 1.62 \times 4 \times 10^{-6} = 6.48 \times 10^{-6} \text{ mol dm}^{-3} \text{s}^{-1}$$

The other options come from errors in squaring (for example, not squaring the concentration gives 3.24×10^{-3} , which is not listed).

Final Answer:

The rate of the reaction is $6.48 \times 10^{-6} \text{ mol dm}^{-3} \text{s}^{-1}$, option (B).

$6.48 \times 10^{-6} \text{ mol dm}^{-3} \text{s}^{-1}$

Quick Tip: Convert the concentration to the same units as k and substitute into the rate law.

79. In first order reaction 20 millimole of reactant is reduced to 10 millimole in 1.151 minute. Find rate constant.

- (A) $0.6023 \text{ minute}^{-1}$
- (B) $6.120 \text{ minute}^{-1}$
- (C) $0.3010 \text{ minute}^{-1}$
- (D) $2.010 \text{ minute}^{-1}$

Correct Answer: (A) $0.6023 \text{ minute}^{-1}$

Solution:

Step 1: Understanding the Concept:

Reactant falls from 20 to 10 millimole, which is half of the initial amount. So 1.151 minute is the half-life of this first order reaction.

Step 2: Key Formula or Approach:

$$k = \frac{0.693}{t_{1/2}} = \frac{2.303 \log 2}{t_{1/2}}$$

Step 3: Calculation:

$$k = \frac{0.693}{1.151} = 0.602 \text{ min}^{-1}$$

Using $2.303 \times 0.3010 = 0.6932$, we get $\frac{0.6932}{1.151} = 0.6023 \text{ min}^{-1}$, option (A).

Final Answer:

The rate constant is 0.6023 per minute, option (A).

$$\boxed{0.6023 \text{ min}^{-1}}$$

Quick Tip: Time taken for 50 percent change is the half-life, and $k = 0.693 / t_{\text{half}}$.

80. Which of the following is an example of second order reaction ?

- (A) $2\text{H}_2\text{O}_{2(l)} \rightarrow 2\text{H}_2\text{O}_{(l)} + \text{O}_{2(g)}$
- (B) $\text{CH}_3\text{CHO}_{(g)} \rightarrow \text{CH}_{4(g)} + \text{CO}_{(g)}$
- (C) $2\text{NO}_{2(g)} + \text{F}_{2(g)} \rightarrow 2\text{NO}_2\text{F}_{(g)}$
- (D) $2\text{NO}_{(g)} + 2\text{H}_{2(g)} \rightarrow \text{N}_{2(g)} + 2\text{H}_2\text{O}_{(l)}$

Correct Answer: (C) $2\text{NO}_{2(g)} + \text{F}_{2(g)} \rightarrow 2\text{NO}_2\text{F}_{(g)}$

Solution:

Step 1: Understanding the Concept:

The order of a reaction comes from experiments, not from the balanced equation. Known results: hydrogen peroxide decomposition is first order, acetaldehyde decomposition is $3/2$ order, and $2\text{NO}_2 + \text{F}_2 \rightarrow 2\text{NO}_2\text{F}$ follows $r = k[\text{NO}_2][\text{F}_2]$.

Step 2: Go through the options:

- (A) $r = k[\text{H}_2\text{O}_2]$, first order.
- (B) $r = k[\text{CH}_3\text{CHO}]^{3/2}$, order 1.5.
- (C) $r = k[\text{NO}_2][\text{F}_2]$, order $1 + 1 = 2$, second order.
- (D) $r = k[\text{NO}]^2[\text{H}_2]$, third order.

Final Answer:

Reaction (C) is of second order.

(C)

Quick Tip: Second order means the rate depends on the sum of powers of concentration equal to 2.

81. Which of the following noble gases exhibits higher oxidation states ?

- (A) Ne
- (B) Ar
- (C) Kr
- (D) Xe

Correct Answer: (D) Xe

Solution:

Step 1: Understanding the Concept:

Noble gases are chemically inert because of closed shells and high ionisation enthalpy. Xenon, the largest of the common ones, has the lowest ionisation enthalpy and vacant d orbitals.

Step 2: Reasoning:

Xenon forms compounds like XeF_2 (+2), XeF_4 (+4), XeF_6 (+6) and XeO_3 (+6). Krypton forms only KrF_2 (+2). Neon and argon form no stable compounds.

Final Answer:

Xenon shows higher oxidation states, option (D).

Xe

Quick Tip: Xenon has low ionisation enthalpy and can use its empty d orbitals.



82. Which from following pair of elements in their respective oxidation states develops same value of calculated spin only magnetic moment ?

- (A) Ti^{3+} and Zn^{2+}
- (B) Cr^{2+} and Fe^{2+}
- (C) Cr^{3+} and Ti^{3+}
- (D) Zn^{2+} and Cu^{2+}

Correct Answer: (B) Cr^{2+} and Fe^{2+}

Solution:

Step 1: Key Formula or Approach:

$\mu = \sqrt{n(n+2)}$ BM, where n is the number of unpaired electrons.
Same n gives same μ .

Step 2: Find n for each ion:

Ti^{3+} : $3d^1$, n = 1. Zn^{2+} : $3d^{10}$, n = 0.

Cr^{2+} : $3d^4$, n = 4. Fe^{2+} : $3d^6$, n = 4.

Cr^{3+} : $3d^3$, n = 3. Cu^{2+} : $3d^9$, n = 1.

Step 3: Match the pairs:

(A) 1 and 0, different. (B) 4 and 4, same. (C) 3 and 1, different. (D) 0 and 1, different. So (B) is correct, with $\mu = \sqrt{24} = 4.9$ BM.

Final Answer:

Cr^{2+} and Fe^{2+} both have four unpaired electrons, option (B).

Cr^{2+} and Fe^{2+}

Quick Tip: Count unpaired electrons: equal n gives equal spin only moment.

83. What is the atomic number of first post actinoid element ?

- (A) 101
- (B) 108
- (C) 104
- (D) 110

Correct Answer: (C) 104

Solution:

Step 1: Understanding the Concept:

The actinoid series runs from actinium (89) to lawrencium (103). The element after Lr is the first post actinoid (a 6d series element).

Step 2: Identify:

The next atomic number is 104, rutherfordium (Rf). It sits below hafnium in group 4. The values 101 (Md) belongs to actinoids, while 108 and 110 are further along the 6d series.

Final Answer:

The first post actinoid element has atomic number 104, option (C).

104

Quick Tip: Actinoids end at lawrencium (103), so the next element is the first post actinoid.

84. What is the EAN of metal ion in $[\text{Cu}(\text{NH}_3)_4]^{2+}$?

- (A) 35
- (B) 27
- (C) 29
- (D) 36

Correct Answer: (A) 35

Solution:

Step 1: Key Formula or Approach:

$$EAN = Z - (\text{oxidation state}) + 2 \times (\text{number of ligands})$$

Each NH_3 is a monodentate ligand donating 2 electrons.

Step 2: Calculation:

For $[\text{Cu}(\text{NH}_3)_4]^{2+}$: Cu has $Z = 29$ and oxidation state +2.

$$EAN = 29 - 2 + 2 \times 4 = 27 + 8 = 35$$

The value 27 is the electron count of Cu^{2+} alone, 29 is the atomic number, and 36 would be krypton, which this complex does not reach.

Final Answer:

EAN of copper is 35, option (A).

35

Quick Tip: Effective atomic number = atomic number minus oxidation

state plus 2 times number of ligands.

85. Identify a ligand with lowest field strength among the following.

- (A) Br^-
- (B) OH^-
- (C) en
- (D) H_2O

Correct Answer: (A) Br^-

Solution:

Step 1: Understanding the Concept:

The spectrochemical series arranges ligands by crystal field splitting power: $\text{I}^- < \text{Br}^- < \text{Cl}^- < \text{F}^- < \text{OH}^- < \text{H}_2\text{O} < \text{NH}_3 < \text{en} < \text{CN}^-$.

Step 2: Compare:

Of the four ligands given, bromide is at the weak end, OH^- comes after the halides, H_2O is stronger and en is stronger still. So Br^- has the lowest field strength.

Final Answer:

Bromide is the weakest field ligand here, option (A).



Quick Tip: In the spectrochemical series, halides are weak field ligands and Br^- is weaker than OH^- .

86. Identify the product obtained when 2-chlorobutane is heated with aqueous solution of potassium hydroxide.

- (A) But - 1 - ene
- (B) But - 2 - ene
- (C) Butan - 1 - ol
- (D) Butan - 2 - ol

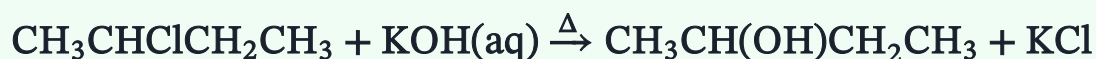
Correct Answer: (D) Butan - 2 - ol

Solution:

Step 1: Understanding the Concept:

With aqueous KOH, the hydroxide ion acts as a nucleophile and replaces halogen (substitution). With alcoholic KOH it acts as a base and removes HX (elimination).

Step 2: Reaction:



Chlorine on C-2 is replaced by OH on C-2, giving butan-2-ol. The alkenes (but-1-ene and but-2-ene) would need alcoholic KOH, and butan-1-ol would need the halogen on C-1.

Final Answer:

The product is butan-2-ol, option (D).

Butan-2-ol

Quick Tip: Aqueous KOH causes substitution; alcoholic KOH causes

elimination.

87. Identify the product obtained when 2-Chloro - 2 - methylbutane is reacted with sodium metal in presence of dry ether?

- (A) 2, 2, 3, 3– tetramethyloctane
- (B) 3, 3, 4, 4– tetramethyloctane
- (C) 2, 2, 3, 3– tetramethylhexane
- (D) 3, 3, 4, 4– tetramethylhexane

Correct Answer: (D) 3, 3, 4, 4– tetramethylhexane

Solution:

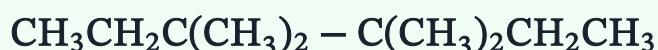
Step 1: Understanding the Concept:

In the Wurtz reaction, two alkyl halide molecules react with sodium metal in dry ether to give an alkane with double the carbon skeleton:
$$2R-X + 2Na \rightarrow R-R + 2NaX.$$

Step 2: Structure of R:

2-Chloro-2-methylbutane is $(CH_3)_2C(Cl)CH_2CH_3$. The alkyl group R is $(CH_3)_2C-CH_2CH_3$ (tert-pentyl).

Step 3: Coupling:



The longest chain has 6 carbons: $C_1H_3-C_2H_2-C_3(CH_3)_2-C_4(CH_3)_2-C_5H_2-C_6H_3$. Methyl groups sit on C-3 (two) and C-4 (two), so the name is 3,3,4,4-tetramethylhexane. Octane-based names have an 8 carbon chain and do not fit, because the product has only 10 carbons

in total, 6 in the chain plus 4 methyls.

Final Answer:

The product is 3,3,4,4-tetramethylhexane, option (D).

3, 3, 4, 4-tetramethylhexane

Quick Tip: Wurtz coupling joins two alkyl groups at the carbons that carried the halogen.

88. Which among the following has highest melting point ?

- (A) Phenol
- (B) o-Nitrophenol
- (C) p-Nitrophenol
- (D) p-Cresol

Correct Answer: (C) p-Nitrophenol

Solution:

Step 1: Understanding the Concept:

Melting point depends on intermolecular forces and on how well molecules pack in the crystal.

Step 2: o-Nitrophenol:

In o-nitrophenol the -OH and -NO₂ groups are close, so they form intramolecular hydrogen bonds. This leaves little hydrogen bonding between molecules, so its melting point is low (about 45 C).

Step 3: p-Nitrophenol:

In p-nitrophenol the groups are far apart, so only intermolecular hydrogen bonds form, linking many molecules together. It also packs well. Its melting point is about 114 C, the highest here.

Step 4: Others:

Phenol melts at about 41 C and p-cresol at about 35 C, both lower because they lack the strongly polar nitro group.

Final Answer:

p-Nitrophenol has the highest melting point, option (C).

p-Nitrophenol

Quick Tip: Strong intermolecular hydrogen bonding and good packing raise the melting point.



89. Which of the following alcohols on addition of Lucas reagent reacts fast and turns the reagent turbid instantly.

- (A) n - Butyl alcohol
- (B) sec - Butyl alcohol
- (C) iso - Butyl alcohol
- (D) tert - Butyl alcohol

Correct Answer: (D) tert - Butyl alcohol

Solution:

Step 1: Understanding the Concept:

Lucas reagent is anhydrous ZnCl_2 in concentrated HCl . The alcohol is converted to an alkyl chloride, which is insoluble and makes the solution turbid.

Step 2: Reactivity:

The rate depends on the stability of the carbocation formed: tertiary > secondary > primary. Tertiary alcohols give turbidity instantly, secondary in about 5 minutes, and primary only on heating.

Step 3: Options:

n-Butyl alcohol and iso-butyl alcohol are primary. sec-Butyl alcohol is secondary. tert-Butyl alcohol is tertiary, so it reacts fastest.

Final Answer:

tert-Butyl alcohol, option (D).

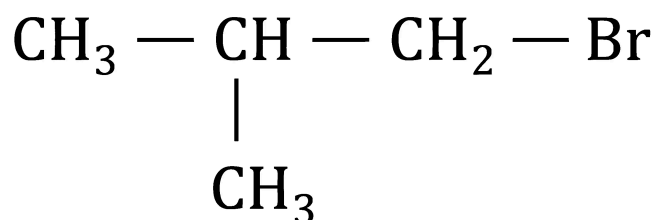
tert-Butyl alcohol

Quick Tip: Tertiary alcohols give the most stable carbocation, so the turbidity appears at once.

90. Which among the following does NOT undergo Williamson's synthesis?

(A) $\text{C}_2\text{H}_5\text{Br}$

(B)



(C) $\text{C}_6\text{H}_5 - \text{Br}$

(D) $\text{C}_6\text{H}_5 - \text{CH}_2 - \text{Br}$

Correct Answer: (C) $\text{C}_6\text{H}_5 - \text{Br}$

Solution:

Step 1: Understanding the Concept:

Williamson synthesis forms an ether by an $\text{S}_{\text{N}}2$ reaction of an alkoxide with a primary alkyl halide: $\text{R-ONa} + \text{R}'\text{-X} \rightarrow \text{R-O-R}' + \text{NaX}$.

Step 2: Check the options:

(A) $\text{C}_2\text{H}_5\text{Br}$ is a primary alkyl halide, so it works.

(B) The structure shown is $(\text{CH}_3)_2\text{CH-CH}_2\text{Br}$, isobutyl bromide, a primary halide, so it works.

(C) $\text{C}_6\text{H}_5\text{-Br}$ is bromobenzene. The C-Br bond has partial double bond character from resonance, and backside attack at an sp^2 carbon is blocked by the ring, so $\text{S}_{\text{N}}2$ cannot occur.

(D) $\text{C}_6\text{H}_5\text{CH}_2\text{Br}$ is benzylic and reacts readily.

Final Answer:

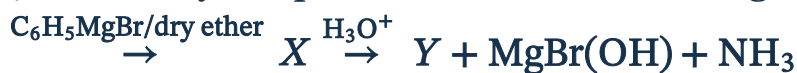
Bromobenzene does not undergo Williamson's synthesis, option (C).



Quick Tip: Williamson synthesis needs an alkyl halide that can undergo $\text{S}_{\text{N}}2$; aryl halides cannot.



91. Identify the product 'Y' in the following reaction **Benzonitrile**



- (A) Benzophenone
- (B) Benzylalcohol
- (C) Benzaldehyde
- (D) Benzoic acid

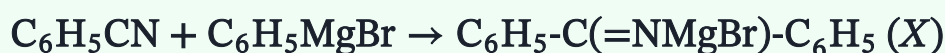
Correct Answer: (A) Benzophenone

Solution:

Step 1: Understanding the Concept:

A Grignard reagent attacks the electrophilic carbon of a nitrile group, forming an imine salt. Acid hydrolysis then converts it to a ketone.

Step 2: Reaction:



Y is benzophenone. The ammonia released matches the equation given in the question, which shows that the imine nitrogen was lost during hydrolysis.

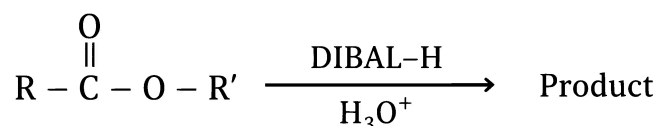
Final Answer:

Y is benzophenone, option (A).

Benzophenone

Quick Tip: Grignard reagents add to the nitrile carbon, and hydrolysis of the imine gives a ketone.

92. Identify the main product of following reaction



- (A) $\text{R} - \text{COOH}$
(B) Missing open brace for superscript
(C) RCHO
(D) RCH_2OH

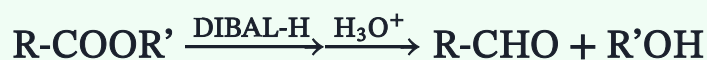
Correct Answer: (C) RCHO

Solution:**Step 1: Understanding the Concept:**

The reactant in the figure is an ester, $\text{R-CO-OR}'$, treated with DIBAL-H (diisobutylaluminium hydride) and then H_3O^+ .

Step 2: Reaction:

DIBAL-H is a bulky reducing agent. It delivers one hydride to the ester carbonyl and gives a stable tetrahedral intermediate. Hydrolysis of this intermediate releases the aldehyde and the alcohol R'OH .



The acid R-COOH would need oxidation, the ketone has no source for a second R group, and the primary alcohol needs a stronger reagent like LiAlH_4 .

Final Answer:

The main product is the aldehyde RCHO , option (C).



Quick Tip: DIBAL-H partially reduces esters to aldehydes at low temperature.

93. Which from following amines when heated with ethanolic KOH and chloroform forms aryl isocyanide ?

- (A) Benzenamine
- (B) N -methylaniline
- (C) N, N - Dimethylaniline
- (D) Ethylethanamine

Correct Answer: (A) Benzenamine

Solution:

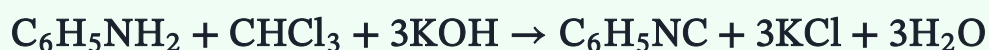
Step 1: Understanding the Concept:

Heating a primary amine with chloroform and alcoholic KOH gives an isocyanide (carbylamine), which has a foul smell. This is the carbylamine reaction, a test for primary amines.

Step 2: Check the options:

Benzenamine (aniline) is $\text{C}_6\text{H}_5\text{NH}_2$, a primary aromatic amine, so it gives phenyl isocyanide.

N-methylaniline is secondary, N,N-dimethylaniline is tertiary, and ethylethanamine (diethylamine) is secondary. They do not give this reaction.



Final Answer:

Benzenamine is the primary amine, option (A).

Benzenamine

Quick Tip: Only primary amines give the carbylamine reaction.

94. Identify false statement from following regarding preparation of amines by Hofmann degradation method.

- (A) It is useful to convert amides to primary amines.
- (B) In this -CO-group is reduced to $-\text{CH}_2-$ group.
- (C) Molar mass of the amine obtained is always less than the amide.
- (D) The reagent used is Br_2 and $\text{KOH}_{(aq)}$.

Correct Answer: (B) In this -CO-group is reduced to $-\text{CH}_2-$ group.

Solution:

Step 1: Understanding the Concept:

In Hofmann bromamide degradation: $\text{R-CONH}_2 + \text{Br}_2 + 4\text{KOH} \rightarrow \text{R-NH}_2 + \text{K}_2\text{CO}_3 + 2\text{KBr} + 2\text{H}_2\text{O}$.

Step 2: Check (A), (C), (D):

(A) An amide is converted to a primary amine, true.

(C) The amine has one carbon fewer than the amide, so its molar mass is lower, true.

(D) The reagent is Br_2 with aqueous KOH , true.

Step 3: Check (B):

The carbonyl carbon is lost as carbonate through an isocyanate intermediate. It is not reduced to $-\text{CH}_2-$. Reduction of the carbonyl to CH_2 happens when amide is reduced with LiAlH_4 , giving an amine with the same number of carbons. So (B) is false.

Final Answer:

The false statement is (B).

(B)

Quick Tip: Hofmann bromamide reaction removes the carbonyl carbon as carbonate; it does not reduce it.

95. Identify medicinally important compound extracted from citrus fruits?

- (A) Antimicrobials
- (B) Vit. C
- (C) Analgesics
- (D) Antiseptics

Correct Answer: (B) Vit. C

Solution:

Step 1: Understanding the Concept:

Citrus fruits (lemon, orange, amla-like fruits) are a major natural source of vitamin C, also called ascorbic acid. It is used to prevent and treat scurvy.

Step 2: Options:

Antimicrobials, analgesics and antiseptics are classes of drugs, not substances taken from citrus fruits. Vitamin C is a specific compound obtained from them.

Final Answer:

The compound is vitamin C, option (B).

Vitamin C

Quick Tip: Citrus fruits such as lemon and orange are rich in ascorbic acid.

◆

96. Which from following is a protein of muscles ?

- (A) Keratin
- (B) Myosin
- (C) Legumelin
- (D) Insulin

Correct Answer: (B) Myosin

Solution:

Step 1: Understanding the Concept:

Muscles contract because of two proteins, actin and myosin. Myosin is the main structural and motor protein of muscle.

Step 2: Options:

Keratin is found in hair, nails and skin. Legumelin is a protein from legumes (pulses). Insulin is a hormone made by the pancreas. So myosin is the muscle protein.

Final Answer:

The muscle protein is myosin, option (B).

Myosin

Quick Tip: Myosin is the contractile protein of muscle.

◆

97. Which from following enzymes cleaves glycosidic bond in sucrose?

- (A) Amylose
- (B) Trypsin
- (C) Invertase
- (D) Maltase

Correct Answer: (C) Invertase

Solution:

Step 1: Understanding the Concept:

Sucrose is a disaccharide made of glucose and fructose joined by a glycosidic bond. Hydrolysis of this bond is catalysed by invertase (sucrase).

Step 2: Options:

Amylose is a starch component, not an enzyme. Trypsin digests proteins. Maltase splits maltose into two glucose units. Invertase splits sucrose into glucose and fructose, and the mixture is called invert sugar.

Final Answer:

Invertase, option (C).

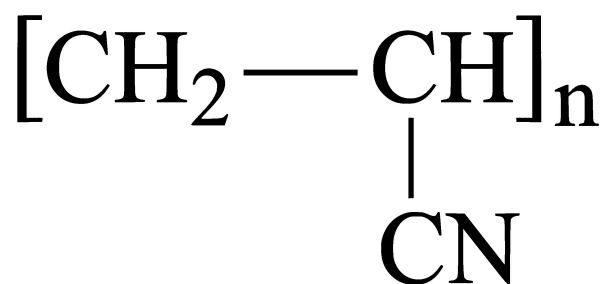
Invertase

Quick Tip: Invertase hydrolyses sucrose to glucose and fructose.

◆

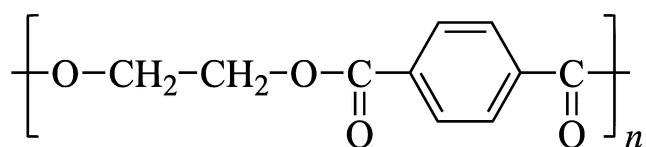
98. Which from following polymer is obtained by addition polymerization?

(A)

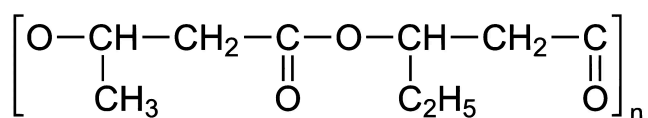


(B) $[\text{NH} - (\text{CH}_2)_5 - \text{CO}]_n$

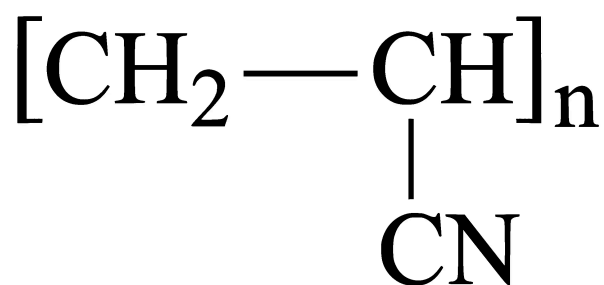
(C)



(D)



Correct Answer: (A)



Solution:

Step 1: Understanding the Concept:

Addition polymerisation joins monomers that contain double bonds, without loss of any small molecule. Condensation polymerisation

joins bifunctional monomers with loss of water, ammonia or similar.

Step 2: Option (A):

The structure $[\text{CH}_2\text{-CH}(\text{CN})]_n$ is polyacrylonitrile, made from acrylonitrile $\text{CH}_2 = \text{CH-CN}$ by addition polymerisation.

Step 3: Other options:

(B) $[\text{NH}(\text{CH}_2)_5\text{CO}]_n$ is nylon-6, made from caprolactam by ring-opening, a condensation type polyamide.

(C) The ester chain with $-\text{O-CH}_2\text{CH}_2\text{-O-CO-C}_6\text{H}_4\text{-CO-}$ is terylene (PET), formed from ethylene glycol and terephthalic acid by condensation.

(D) The chain with $-\text{O-CH}(\text{CH}_3)\text{-CH}_2\text{-CO-O-CH}(\text{C}_2\text{H}_5)\text{-CH}_2\text{-CO-}$ is PHBV, a polyester made by condensation of hydroxy acids.

Final Answer:

Only polyacrylonitrile is an addition polymer, option (A).

Polyacrylonitrile

Quick Tip: Addition polymers form from unsaturated monomers without loss of small molecules.

99. Identify the polymer used to prepare disposable cups and plates.

- (A) Polypropylene
- (B) Polystyrene
- (C) Polyethylene terephthalate
- (D) High density polyethylene

Correct Answer: (B) Polystyrene

Solution:

Step 1: Understanding the Concept:

Polystyrene (from styrene, $\text{C}_6\text{H}_5\text{CH}=\text{CH}_2$) is a light, rigid, brittle thermoplastic that can be moulded easily or expanded into foam.

Step 2: Uses:

It is used for disposable cups, plates, packing material (thermocol) and insulation. Polypropylene is used for ropes and containers, PET for bottles and fibres, and HDPE for pipes and toys.

Final Answer:

The polymer is polystyrene, option (B).

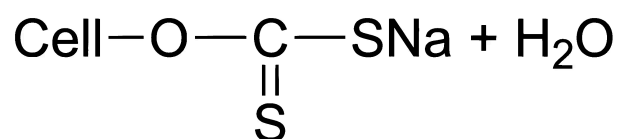
Polystyrene

Quick Tip: Polystyrene is light, rigid and cheap, so it is moulded into disposable ware.

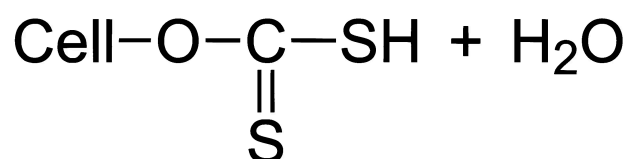
100. Which from following is a first stage in preparation of viscose rayon ?



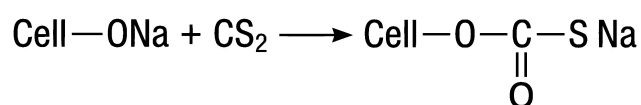
(B)



(C)



(D)



Correct Answer: (A) $\text{Cell} - \text{OH} + \text{NaOH}$

Solution:

Step 1: Understanding the Concept:

Viscose rayon is regenerated cellulose. The process has three stages: alkali cellulose formation, xanthate formation, and regeneration of cellulose.

Step 2: Stage by stage:

Stage 1: $\text{Cell-OH} + \text{NaOH} \rightarrow \text{Cell-ONa} + \text{H}_2\text{O}$ (alkali cellulose).

Stage 2: $\text{Cell-ONa} + \text{CS}_2 \rightarrow \text{Cell-O-C(=S)-SNa}$ (cellulose xanthate), shown in the figure for option (D).

Stage 3: The xanthate is converted to xanthic acid (Cell-O-C(=S)-SH , figure of option C), which decomposes to regenerate cellulose when

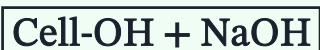
extruded into an acid bath.

Step 3: Match:

Option (A) is the first stage. The figures in (B), (C) and (D) are xanthate and xanthic acid species of later stages, so they are not first.

Final Answer:

The first stage is $\text{Cell-OH} + \text{NaOH}$, option (A).



Quick Tip: Viscose rayon begins by treating cellulose with sodium hydroxide to make alkali cellulose.

Mathematics

Questions 101 to 150 | 2 marks each | 100 marks

101. If $\tan A$ and $\tan B$ are the roots of the equation $5x^2 - 4x + 1 = 0$, then the value of $A + B$ is...

- (A) $\frac{\pi}{6}$
- (B) $\frac{\pi}{4}$
- (C) $\frac{\pi}{3}$
- (D) $\frac{\pi}{2}$

Correct Answer: (B) $\frac{\pi}{4}$

Solution:

Step 1: Understanding the Concept:

For a right angled triangle with hypotenuse z , Pythagoras gives $x^2 + y^2 = z^2$, so $y^2 = z^2 - x^2 = (z - x)(z + x)$.

Step 2: Key Formula or Approach:

$$\frac{1}{\log_a b} = \log_b a, \quad \log_b m + \log_b n = \log_b(mn)$$

Step 3: Detailed Explanation:

$$\begin{aligned} \frac{1}{\log_{x+z} y} + \frac{1}{\log_{z-x} y} &= \log_y(x + z) + \log_y(z - x) \\ &= \log_y[(z + x)(z - x)] = \log_y(z^2 - x^2) = \log_y y^2 = 2 \end{aligned}$$

The result does not depend on the actual sides, only on the relation $y^2 = z^2 - x^2$. The other options (1, 3, 4) would need a different relation between the sides.

Final Answer:

The value of the expression is 2, option (B).

2

Quick Tip: Convert $1/\log_a b$ into $\log_b a$ and use the Pythagoras relation.

102. $\frac{(\cos 2\theta + i \sin 2\theta)^7}{(\cos 4\theta + i \sin 4\theta)^3} =$

- (A) $\cos 2\theta + i \sin 2\theta$
- (B) $(\cos 2\theta + i \sin 2\theta)^4$
- (C) $\cos 4\theta + i \sin 4\theta$
- (D) $(\cos 4\theta + i \sin 4\theta)^{-2}$

Correct Answer: (A) $\cos 2\theta + i \sin 2\theta$

Solution:

Step 1: Understanding the Concept:

If ω is a complex cube root of unity, then $\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$.

Step 2: Reduce the powers:

$$\omega^{10} = \omega^9 \cdot \omega = \omega \text{ and } \omega^{23} = \omega^{21} \cdot \omega^2 = \omega^2.$$

$$\text{So } \omega^{10} + \omega^{23} = \omega + \omega^2 = -1.$$

Step 3: Evaluate the sine:

$$\sin \left[\pi(-1) - \frac{\pi}{4} \right] = -\sin \left(\pi + \frac{\pi}{4} \right) = - \left(-\sin \frac{\pi}{4} \right) = \frac{1}{\sqrt{2}}$$

Here $\sin(\pi + \theta) = -\sin \theta$ was used. A negative answer would result if the sign of the inner angle were mishandled.

Final Answer:

The value is $\frac{1}{\sqrt{2}}$, option (C).

$$\boxed{\frac{1}{\sqrt{2}}}$$

Quick Tip: Reduce the powers of omega using omega cubed = 1 and 1 + omega + omega squared = 0.

103. In a test, there are 5 questions of the 'true or false' type. No student has got all the answers correct and the sequence of answers for every student is unique. The maximum number of students who could have appeared for the test is....

- (A) 5
- (B) 31
- (C) 15
- (D) 89

Correct Answer: (B) 31

Solution:

Step 1: Understanding the Concept:

Total seating arrangements of 9 people in a row: $9!$. We count the arrangements where the two end seats hold girls and no two girls are adjacent.

Step 2: Arrange the boys first:

The 6 boys can be arranged in $6!$ ways. They create 7 gaps, including the 2 end gaps. Girls placed in separate gaps are never adjacent.

Step 3: Place the girls:

Two girls must sit in the two end gaps and the third girl in one of the 5 inner gaps (5 ways). The three girls can be assigned to these three gaps in $3!$ ways.

Favourable arrangements = $6! \times 5 \times 3!$.

Step 4: Probability:

$$P = \frac{6! \times 5 \times 3!}{9!} = \frac{5 \times 6}{9 \times 8 \times 7} = \frac{30}{504} = \frac{5}{84}$$

A girl at each end with the third girl in an inner gap automatically means no two girls are adjacent, because each is separated by at least one boy.

Final Answer:

The required probability is $\frac{5}{84}$, option (D).

$$\boxed{\frac{5}{84}}$$

Quick Tip: Seat the boys first, then place girls in the gaps between boys.

104. The line $y = 2x + c$ passes through a point that is equidistant from both the axes and lies in the first quadrant ($x > 0, y > 0$). Then the value of c is...

- (A) 0
- (B) 1
- (C) 2
- (D) -2

Correct Answer: (D) -2

Solution:

Step 1: Understanding the Concept:

There are 7 identical balls and 16 distinct balls. If r identical balls are taken ($0 \leq r \leq 7$), there is only 1 way to choose them, and the other $12 - r$ balls come from the 16 distinct ones.

Step 2: Add up the cases:

$$N = \sum_{r=0}^7 {}^{16}C_{12-r} = {}^{16}C_{12} + {}^{16}C_{11} + \dots + {}^{16}C_5$$

The values are 1820, 4368, 8008, 11440, 12870, 11440, 8008 and 4368, and their sum is 62322.

Step 3: Match with the options:

Option (A): ${}^{18}C_6 + {}^{18}C_8 = 18564 + 43758 = 62322$. This equals the

count. Other options give $75582 + 31824 = 107406$, $12870 + 816 = 13686$ and $12870 + 48620 = 61490$, which do not match.

Final Answer:

The number of selections is ${}^{18}C_6 + {}^{18}C_8 = 62322$, option (A).

$${}^{18}C_6 + {}^{18}C_8$$

Quick Tip: The 7 identical balls count as one kind; select any number of them and fill the rest from 16 distinct balls.

105. If the equation $ax^2 + 4xy - 2y^2 + 4x + 8y + 1 = 0$ represents a pair of straight lines, then the coordinates of their point of intersection are.....

- (A) $(\frac{1}{2}, -\frac{3}{2})$
- (B) $(-\frac{3}{2}, \frac{1}{2})$
- (C) $(\frac{1}{2}, \frac{3}{2})$
- (D) $(-\frac{1}{2}, \frac{3}{2})$

Correct Answer: (B) $(-\frac{3}{2}, \frac{1}{2})$

Solution:

Step 1: Understanding the Concept:

Use $f(x + \Delta x) \approx f(x) + f'(x)\Delta x$ with $f(x) = x^{1/3}$, taking $x = 0.008$ and $\Delta x = -0.001$.

Step 2: Compute:

$$f(0.008) = 0.2 \text{ and } f'(x) = \frac{1}{3}x^{-2/3}, \text{ so } f'(0.008) = \frac{1}{3} \cdot \frac{1}{(0.2)^2} = \frac{1}{3 \times 0.04} = \frac{25}{3}.$$

$$(0.007)^{1/3} \approx 0.2 + \frac{25}{3}(-0.001) = 0.2 - \frac{1}{120} = \frac{24-1}{120} = \frac{23}{120}$$

This is about 0.1917.

Final Answer:

The approximate value is $\frac{23}{120}$, option (B).

$\frac{23}{120}$

Quick Tip: Use the differential about the nearby perfect cube 0.008.



106. If the circles $x^2 + y^2 = 16$ and $x^2 + y^2 + 2ax + 4y + 4 = 0$ touch each other internally then $a =$

- (A) $\frac{2}{3}$
- (B) $\frac{1}{56}$
- (C) $\frac{-2}{3}$
- (D) $\frac{3}{2}$

Correct Answer: (D) $\frac{3}{2}$

Solution:

Step 1: Key Formula or Approach:

$$\cos \theta \cos 2\theta \cos 4\theta \cdots \cos 2^{n-1}\theta = \frac{\sin 2^n \theta}{2^n \sin \theta}$$

This comes from repeated use of $2 \sin A \cos A = \sin 2A$.

Step 2: Apply for five factors:

Here the factors go up to $\cos 16\theta$, so $n = 5$:

$$\cos \theta \cos 2\theta \cos 4\theta \cos 8\theta \cos 16\theta = \frac{\sin 32\theta}{32 \sin \theta}$$

Step 3: Use the given condition:

Since $32\theta = \pi$, $32\theta = \pi - \theta$ and $\sin 32\theta = \sin(\pi - \theta) = \sin \theta$.

$$\text{Product} = \frac{\sin \theta}{32 \sin \theta} = \frac{1}{32}$$

The sign is positive since $\sin \theta \neq 0$ and the identity holds without any sign change.

Final Answer:

The value is $\frac{1}{32}$, option (B).

$$\boxed{\frac{1}{32}}$$

Quick Tip: Multiply and divide by $\sin \theta$ and use the double angle formula repeatedly.



107. If the line $3x + 4y + k = 0$ touches the ellipse $9x^2 + 16y^2 = 144$, then the value of k is.

- (A) $\pm 3\sqrt{2}$
 (B) $\pm 4\sqrt{2}$
 (C) $\mp 8\sqrt{2}$
 (D) $\mp 12\sqrt{2}$

Correct Answer: (D) $\mp 12\sqrt{2}$

Solution:

Step 1: Understanding the Concept:

Let $c = \cos 2\theta$. Then $\cos 4\theta = 2c^2 - 1$. The equation $\sec 4\theta - \sec 2\theta = 2$ becomes $\frac{1}{2c^2-1} - \frac{1}{c} = 2$.

Step 2: Form the cubic:

$$c - (2c^2 - 1) = 2c(2c^2 - 1) \Rightarrow 4c^3 + 2c^2 - 3c - 1 = 0$$

Check $c = -1$: $-4 + 2 + 3 - 1 = 0$. So $(c + 1)(4c^2 - 2c - 1) = 0$.

$$c = -1 \quad \text{or} \quad c = \frac{1 \pm \sqrt{5}}{4}$$

Step 3: Solve for theta:

$$c = -1: 2\theta = (2n + 1)\pi, \text{ so } \theta = n\pi + \frac{\pi}{2}.$$

$$c = \frac{1+\sqrt{5}}{4} = \cos 36^\circ: \theta = n\pi \pm \frac{\pi}{10}.$$

$$c = \frac{1-\sqrt{5}}{4} = \cos 108^\circ: \theta = n\pi \pm \frac{3\pi}{10}.$$

The last two families together give all odd multiples of $\frac{\pi}{10}$, which is $\theta = \frac{n\pi}{5} + \frac{\pi}{10}$.

Step 4: Match with the options:

Option (C) lists $n\pi + \frac{\pi}{2}$ and $\frac{n\pi}{5} + \frac{\pi}{10}$, which are exactly our two

families.

Final Answer:

The solution is $\theta = n\pi + \frac{\pi}{2}, \frac{n\pi}{5} + \frac{\pi}{10}$, option (C).

$$\boxed{n\pi + \frac{\pi}{2}, \frac{n\pi}{5} + \frac{\pi}{10}}$$

Quick Tip: Write everything in $\cos 2\theta$ and solve the resulting cubic.

108. Which of the following statements is/are False?

$S_1 : \exists n \in N$, such that $n^2 + n + 2$ is divisible by 4.

$S_2 : \exists x \in N$, such that $x - 17 < 20$.

$S_3 : \forall n \in N, x^2 + 3x - 10 = 0$.

$S_4 : \forall n \in N, n^2 \geq 1$.

(A) S_1 and S_2 .

(B) S_1 and S_3 .

(C) Only S_3 .

(D) S_2 and S_4 .

Correct Answer: (C) Only S_3 .

Solution:

Step 1: Find the intersection:

From $2x - y = 2$, $y = 2x - 2$. Put it in $x + 2y + 6 = 0$: $x + 4x - 4 + 6 = 0$, so $x = -\frac{2}{5}$ and $y = -\frac{14}{5}$.

Step 2: Line with y intercept 5:

Use $y = mx + 5$ and require it to pass through $(-\frac{2}{5}, -\frac{14}{5})$:

$$-\frac{14}{5} = -\frac{2m}{5} + 5 \Rightarrow -14 = -2m + 25 \Rightarrow m = \frac{39}{2}$$

Step 3: Equation:

$$y = \frac{39}{2}x + 5 \Rightarrow 2y = 39x + 10 \Rightarrow 39x - 2y + 10 = 0$$

Check with option (A): it has a y intercept of 5 but passes the wrong sign of the slope (it gives $y = -\frac{39}{2}x + 5$), so it does not pass through the point.

Final Answer:

The required line is $39x - 2y + 10 = 0$, option (B).

$$\boxed{39x - 2y + 10 = 0}$$

Quick Tip: Find the intersection point, then use slope-intercept form with the given intercept.

109. The negation of $(p \wedge q) \rightarrow ((p \vee r) \rightarrow \sim q)$ is equivalent to ...

- (A) $p \wedge q$
- (B) $p \wedge \sim r$
- (C) $q \wedge (p \vee r)$
- (D) $\sim p \wedge \sim q$

Correct Answer: (A) $p \wedge q$

Solution:

Step 1: Understanding the Concept:

The points A and B lie on both the line $x + y = 3$ and the pair $x^2 - 3xy + y^2 = 0$. Substitute $y = 3 - x$ into the pair.

Step 2: Form the quadratic:

$$x^2 - 3x(3 - x) + (3 - x)^2 = 0$$

$$x^2 - 9x + 3x^2 + 9 - 6x + x^2 = 0 \Rightarrow 5x^2 - 15x + 9 = 0$$

Step 3: Midpoint:

The sum of the roots is $x_1 + x_2 = 3$, so the x coordinate of the midpoint is $\frac{3}{2}$.

Then $y = 3 - \frac{3}{2} = \frac{3}{2}$. The midpoint is $(\frac{3}{2}, \frac{3}{2})$.

Final Answer:

The midpoint of AB is $(\frac{3}{2}, \frac{3}{2})$, option (D).

$$\left(\frac{3}{2}, \frac{3}{2}\right)$$

Quick Tip: Substitute the line into the pair of lines to get a quadratic and use the sum of roots.

110. The statement pattern $[(p \wedge q) \rightarrow (\sim p \vee r)] \vee [(\sim p \vee r) \rightarrow (p \wedge q)]$ is

- (A) a contradiction
- (B) a tautology
- (C) equivalent to $(p \wedge q) \vee r$.
- (D) equivalent to $(p \vee q)$.

Correct Answer: (B) a tautology

Solution:

Step 1: Understanding the Concept:

The circle has centre $C(7, -1)$ and radius 5. The distance from the origin O to the centre is $OC = \sqrt{49 + 1} = \sqrt{50}$.

Step 2: Half angle:

If 2α is the angle between the tangents, then in the right triangle formed by O, C and the point of contact, $\sin \alpha = \frac{r}{OC} = \frac{5}{\sqrt{50}} = \frac{1}{\sqrt{2}}$. So $\alpha = 45^\circ$.

Step 3: Angle between tangents:

$2\alpha = 90^\circ$. This also agrees with the power of the origin: $S_{11} = 49 + 1 - 25 = 25 = r^2$, the condition for perpendicular tangents.

Final Answer:

The angle between the tangents is 90° , option (B).

90°

Quick Tip: Find the length of tangent and radius, then use the half angle sine.

111. In $\triangle ABC$, with usual notation, if $\cot A, \cot B, \cot C$ are in arithmetic progression, then

- (A) $\sin A, \sin B, \sin C$ are in arithmetic progression.
- (B) a^2, b^2, c^2 are in arithmetic progression.
- (C) $\cos A, \cos B, \cos C$ are in arithmetic progression.
- (D) a, b, c are in arithmetic progression.

Correct Answer: (B) a^2, b^2, c^2 are in arithmetic progression.

Solution:

Step 1: Tangent to the parabola:

For $y^2 = 4x$ (with $a = 1$), the tangent with slope m is $y = mx + \frac{1}{m}$, that is, $m^2x - my + 1 = 0$.

Step 2: Condition for the circle:

The circle $(x - 3)^2 + y^2 = 9$ has centre $(3, 0)$ and radius 3. The distance from the centre to the tangent must be 3:

$$\frac{|3m + \frac{1}{m}|}{\sqrt{1 + m^2}} = 3 \Rightarrow 9m^2 + 6 + \frac{1}{m^2} = 9 + 9m^2 \Rightarrow m^2 = \frac{1}{3}$$

Step 3: Choose the tangent above the axis:

$m = \pm \frac{1}{\sqrt{3}}$. The point of contact on the parabola is $(\frac{1}{m^2}, \frac{2}{m})$, which lies above the X-axis only when $m > 0$. So $m = \frac{1}{\sqrt{3}}$.

$$y = \frac{x}{\sqrt{3}} + \sqrt{3} \Rightarrow \sqrt{3}y = x + 3$$

Option (C).

Final Answer:

The common tangent is $\sqrt{3}y = x + 3$, option (C).

$$\sqrt{3}y = x + 3$$

Quick Tip: A tangent to the parabola is $y = mx + 1/m$; set its distance from the circle centre equal to the radius.

112. If $A = \begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ then $A^{-1}B$ is equal to

- (A) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$
- (B) $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$
- (C) $\begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$
- (D) $\begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$

Correct Answer: (A) $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

Solution:**Step 1: Understanding the Concept:**

The numerator has degree $19 + 11 = 30$, the same as the denominator $(6x - 5)^{30}$. The limit at infinity is the ratio of leading coefficients.

Step 2: Leading coefficients:

$$\lim_{x \rightarrow \infty} \frac{(2x)^{19}(3x)^{11}}{(6x)^{30}} = \frac{2^{19} \cdot 3^{11}}{6^{30}} = \frac{2^{19} \cdot 3^{11}}{2^{30} \cdot 3^{30}} = 2^{-11} \cdot 3^{-19}$$

Step 3: Compare with $2^a 3^b$:

So $a = -11$ and $b = -19$, and $a + b = -30$.

Final Answer:

$a + b = -30$, option (A).

-30

Quick Tip: Compare the leading coefficients of numerator and denominator, since degrees match.

◆

113. If $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -5 & 0 \\ 0 & 0 & 7 \end{bmatrix}$; $B = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ then, $(2A + 3B)^{-1} =$

$$(A) \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & -\frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{26} \end{bmatrix}$$

$$(B) \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{26} \end{bmatrix}$$

$$(C) \begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & -\frac{1}{26} \end{bmatrix}$$

$$(D) \begin{bmatrix} -\frac{1}{3} & 0 & 0 \\ 0 & \frac{1}{4} & 0 \\ 0 & 0 & -\frac{1}{26} \end{bmatrix}$$

Correct Answer: (A) $\begin{bmatrix} \frac{1}{3} & 0 & 0 \\ 0 & -\frac{1}{4} & 0 \\ 0 & 0 & \frac{1}{26} \end{bmatrix}$

Solution:

Step 1: Understanding the Concept:

The dual of a statement is obtained by interchanging $\wedge$ and $\vee$ (and t and c), but only after the statement uses only $\wedge, \vee, \sim$. So we first remove the implication.

Step 2: Rewrite:

$$(p \wedge \sim q) \rightarrow (q \wedge \sim p) \equiv \sim (p \wedge \sim q) \vee (q \wedge \sim p) \equiv (\sim p \vee q) \vee (q \wedge \sim p)$$

Step 3: Take the dual:

Swap the connectives: $(\sim p \wedge q) \wedge (q \vee \sim p)$. By absorption, this simplifies to $\sim p \wedge q$.

Step 4: Compare the options:

Option (B): $(p \rightarrow q) \wedge \sim (q \rightarrow p) = (\sim p \vee q) \wedge (q \wedge \sim p) = q \wedge \sim p$. This is the same as $\sim p \wedge q$.

Option (A) simplifies to $p \wedge \sim q$, option (C) to p , and option (D) to a tautology, none equal to $\sim p \wedge q$.

Final Answer:

The dual is equivalent to option (B).

$$(p \rightarrow q) \wedge \sim (q \rightarrow p)$$

Quick Tip: First rewrite the implication using and/or, then swap and with or.

114. If $\sin^{-1}(x-2) + \cos^{-1}(x) + \tan^{-1}(x+2) + \cot^{-1}(x+4) = \sec^{-1}(\sqrt{k}) - \frac{\pi}{2}$, then $\cos(2 \operatorname{cosec}^{-1} \sqrt{k-1}) = \dots$

- (A) $\frac{15}{16}$
- (B) $\frac{31}{32}$
- (C) $\frac{63}{64}$
- (D) $\frac{7}{8}$

Correct Answer: (B) $\frac{31}{32}$

Solution:**Step 1: Understanding the Concept:**

The statement is "if P then Q" where P is "an integer is greater than 4 and less than 5" and Q is "it is a multiple of 3". The negation of $P \rightarrow Q$

is $P \wedge \sim Q$.

Step 2: Apply:

Keep P as it is and negate Q: "An integer is greater than 4 and less than 5 but it is not a multiple of 3".

Step 3: Check the options:

Option (A) changes P, option (B) is another conditional, and option (D) negates P as well as Q. Only (C) keeps P and negates Q.

Final Answer:

Option (C) is the negation.

(C)

Quick Tip: The negation of if P then Q is P and not Q.

115. $\sin(3\sin^{-1}(\frac{1}{5})) =$

- (A) $\frac{74}{125}$
- (B) $\frac{71}{125}$
- (C) $\frac{3}{5}$
- (D) $\frac{1}{2}$

Correct Answer: (B) $\frac{71}{125}$

Solution:

Step 1: Understanding the Concept:

Angles: $B = 60^\circ$, $C = 45^\circ$. D lies on BC with $BD : DC = 1 : 3$. In triangle

ABC, by the sine rule, $\frac{AB}{AC} = \frac{\sin C}{\sin B} = \frac{\sin 45^\circ}{\sin 60^\circ} = \frac{\sqrt{2}}{\sqrt{3}}$.

Step 2: Sine rule in sub-triangles:

In triangle ABD: $\frac{BD}{\sin \angle BAD} = \frac{AB}{\sin \angle ADB}$. In triangle ADC: $\frac{DC}{\sin \angle CAD} = \frac{AC}{\sin \angle ADC}$. Since $\sin \angle ADB = \sin \angle ADC$, dividing gives

$$\frac{BD}{DC} \cdot \frac{\sin \angle CAD}{\sin \angle BAD} = \frac{AB}{AC}$$

Step 3: Solve:

$$\frac{\sin \angle BAD}{\sin \angle CAD} = \frac{BD}{DC} \cdot \frac{AC}{AB} = \frac{1}{3} \cdot \frac{\sqrt{3}}{\sqrt{2}} = \frac{1}{\sqrt{3}\sqrt{2}} = \frac{1}{\sqrt{6}}$$

Final Answer:

The ratio is $\frac{1}{\sqrt{6}}$, option (C).

$$\boxed{\frac{1}{\sqrt{6}}}$$

Quick Tip: Apply the sine rule in two sub-triangles and use the ratio BD to DC.

116. $\cos^{-1}(\cos \frac{4\pi}{3}) + \sin^{-1}(\sin \frac{4\pi}{3}) = \dots$

- (A) $\frac{4\pi}{3}$
 (B) $\frac{8\pi}{3}$
 (C) $\frac{\pi}{3}$
 (D) $\frac{3\pi}{2}$

Correct Answer: (C) $\frac{\pi}{3}$

Solution:

Step 1: Express the altitudes:

$$P_1 = \frac{2\Delta}{a}, P_2 = \frac{2\Delta}{b}, P_3 = \frac{2\Delta}{c}. \text{ So } \frac{\cos A}{P_1} = \frac{a \cos A}{2\Delta}.$$

Step 2: Sum the terms:

$$\sum \frac{a \cos A}{2\Delta} = \frac{a \cos A + b \cos B + c \cos C}{2\Delta}$$

Using $a = 2R \sin A$, we have $a \cos A = R \sin 2A$, so the numerator is $R(\sin 2A + \sin 2B + \sin 2C) = 4R \sin A \sin B \sin C$.

Step 3: Use the area formula:

$\Delta = 2R^2 \sin A \sin B \sin C$, so

$$\frac{4R \sin A \sin B \sin C}{2 \times 2R^2 \sin A \sin B \sin C} = \frac{1}{R}$$

The dimension also checks out, because a ratio of a cosine to a length must have dimension 1/length, as does $\frac{1}{R}$.

Final Answer:

The sum equals $\frac{1}{R}$, option (B).

$$\frac{1}{R}$$

Quick Tip: Express altitudes with area and use the cosine and sine rules.

117. If $f(x) = \frac{1-x}{1+x}$, then $f(f(\cos x)) =$

- (A) $\cos x$
- (B) x
- (C) $\tan \frac{x}{2}$
- (D) $\cos \frac{x}{2}$

Correct Answer: (A) $\cos x$

Solution:

Step 1: Understanding the Concept:

$$AB = C \Rightarrow B = A^{-1}C \Rightarrow B^{-1} = C^{-1}A.$$

Step 2: Find the inverse of C:

$$|C| = 7 \times 6 - 0 = 42. \quad C^{-1} = \frac{1}{42} \begin{bmatrix} 6 & -3 \\ 0 & 7 \end{bmatrix}.$$

Step 3: Multiply:

$$\begin{aligned} C^{-1}A &= \frac{1}{42} \begin{bmatrix} 6 & -3 \\ 0 & 7 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \frac{1}{42} \begin{bmatrix} 18+3 & 6-6 \\ -7 & 14 \end{bmatrix} = \frac{1}{42} \begin{bmatrix} 21 & 0 \\ -7 & 14 \end{bmatrix} \\ &= \frac{1}{6} \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix} \end{aligned}$$

which matches option (B).

Final Answer:

$$B^{-1} = \frac{1}{6} \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}, \text{ option (B).}$$

$$\frac{1}{6} \begin{bmatrix} 3 & 0 \\ -1 & 2 \end{bmatrix}$$

Quick Tip: From $AB = C$, the inverse of B is C inverse times A.

118. If the function $f(x) = \frac{4\sqrt{2}(\sin 3x + \sin x)}{2\sin 2x \sin \frac{3x}{2} + \cos \frac{5x}{2} - \cos \frac{3x}{2}}$ for $x \neq \frac{\pi}{2}$ is continuous at $x = \frac{\pi}{2}$, then the value of $f(\frac{\pi}{2})$ is equal to

- (A) $(2)^2$
- (B) $(3)^2$
- (C) $4\sqrt{2}$
- (D) $2\sqrt{2}$

Correct Answer: (A) $(2)^2$

Solution:

Step 1: Determinant:

$$|A| = 1(4 - 9) - 3(12 - 9) + 3(9 - 3) = -5 - 9 + 18 = 4$$

Step 2: Needed cofactor:

The inverse is $\frac{\text{adj}A}{|A|}$, and $\text{adj}A$ is the transpose of the cofactor matrix.

So the element in row 3, column 2 of A^{-1} is $\frac{C_{23}}{|A|}$, where C_{23} is the cofactor of the element in row 2, column 3.

Step 3: Compute:

Delete row 2 and column 3: the minor is $\begin{vmatrix} 1 & 3 \\ 3 & 3 \end{vmatrix} = 3 - 9 = -6$.

$$C_{23} = (-1)^{2+3}(-6) = 6.$$

$$(A^{-1})_{32} = \frac{6}{4} = \frac{3}{2}$$

The negative value in option (C) would result from missing the sign $(-1)^5$.

Final Answer:

The required element is $\frac{3}{2}$, option (A).

$$\boxed{\frac{3}{2}}$$

Quick Tip: The (3,2) element of the inverse is the cofactor of the (2,3) element divided by the determinant.

119. If $f(x) = \cos x \cos 2x \cos 4x \cos 8x \cos 16x$, then

Missing open brace for superscript

- (A) $\operatorname{cosec}\left(\frac{\pi}{4}\right)$
- (B) $\cos\left(\frac{\pi}{4}\right)$
- (C) $\tan\left(\frac{\pi}{4}\right)$
- (D) 0

Correct Answer: (A) $\operatorname{cosec}(\frac{\pi}{4})$

Solution:

Step 1: Simplify the root:

$1 + \sin 2x = (\sin x + \cos x)^2$ and $1 - \sin 2x = (\cos x - \sin x)^2$. So

$$\sqrt{\frac{1 + \sin 2x}{1 - \sin 2x}} = \left| \frac{\cos x + \sin x}{\cos x - \sin x} \right| = \left| \frac{1 + \tan x}{1 - \tan x} \right| = \left| \tan \left(\frac{\pi}{4} + x \right) \right|$$

Step 2: Remove the modulus near the given point:

At $x = \frac{\pi}{6}$, $\cos x > \sin x$ so the quantity is positive. Then $y = \tan^{-1} \tan \left(x + \frac{\pi}{4} \right) = x + \frac{\pi}{4}$, because $x + \frac{\pi}{4} = \frac{5\pi}{12}$ lies in $(-\frac{\pi}{2}, \frac{\pi}{2})$.

Step 3: Differentiate:

$\frac{dy}{dx} = 1$, which holds at $x = \frac{\pi}{6}$.

Final Answer:

The derivative at $x = \frac{\pi}{6}$ is 1, option (B).

1

Quick Tip: Simplify the root to $\tan(x + \pi/4)$ using $\sin 2x = 2 \sin x \cos x$.

120. If $y = \frac{1}{3x+5}$, then the value of $\frac{d^9 y}{dx^9}$ is..

- (A) $\frac{9! \times 3^9}{(3x+5)^9}$
 (B) $\frac{(-1)^8 \times 9! \times 3^9}{(3x+5)^9}$
 (C) $\frac{(-1)^9 \times 9! \times 3^9}{(3x+5)^{10}}$
 (D) $\frac{(-1)^8 \times 8! \times 3^9}{(3x+5)^{10}}$

Correct Answer: (C) $\frac{(-1)^9 \times 9! \times 3^9}{(3x+5)^{10}}$

Solution:

Step 1: Reduce the angles:

$$\cos \frac{19\pi}{4} = \cos \left(4\pi + \frac{3\pi}{4} \right) = \cos \frac{3\pi}{4} = -\frac{1}{\sqrt{2}}. \text{ And } \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}.$$

Step 2: Compute the ratio:

$$\frac{-\frac{1}{\sqrt{2}} - 1}{\frac{1}{\sqrt{2}}} = -1 - \sqrt{2} = -(1 + \sqrt{2})$$

Step 3: Take the inverse tangent:

$\tan \frac{3\pi}{8} = \tan 67.5^\circ = 1 + \sqrt{2}$. So $\tan^{-1}(-(1 + \sqrt{2})) = -\frac{3\pi}{8}$, because $\tan^{-1}$ is odd and its range is $(-\frac{\pi}{2}, \frac{\pi}{2})$.

Final Answer:

The value is $-\frac{3\pi}{8}$, option (C).

$$\boxed{-\frac{3\pi}{8}}$$

Quick Tip: Reduce the angle 19π by 4, then use $\tan$ of 3π by 8.

121. The derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ with respect to $\tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$ at $x = \frac{1}{2}$ is

- (A) $\frac{\sqrt{3}}{2}$
- (B) $\frac{1}{2}$
- (C) $\frac{\sqrt{3}}{5}$
- (D) $\frac{1}{4}$

Correct Answer: (C) $\frac{\sqrt{3}}{5}$

Solution:

Step 1: Understanding the Concept:

For continuity at $x = 0$, $f(0)$ must equal $\lim_{x \rightarrow 0} f(x)$. The limit is of the form $\frac{0}{0}$.

Step 2: Expand the numerator:

$(256 - 8x)^{1/4} = 4\left(1 - \frac{x}{32}\right)^{1/4} \approx 4\left(1 - \frac{x}{128}\right)$. So the numerator is about $-\frac{x}{32}$.

Step 3: Expand the denominator:

$(64 + 3x)^{1/3} = 4\left(1 + \frac{3x}{64}\right)^{1/3} \approx 4\left(1 + \frac{x}{64}\right)$. So $16 - 4(64 + 3x)^{1/3} \approx 16 - 16 - \frac{x}{4} = -\frac{x}{4}$.

Step 4: Limit:

$$\lim_{x \rightarrow 0} f(x) = \frac{-x/32}{-x/4} = \frac{4}{32} = \frac{1}{8}$$

Final Answer:

$f(0) = \frac{1}{8}$, option (B).

$$\frac{1}{8}$$

Quick Tip: The limit at 0 is of the form 0/0, so expand using the binomial approximation.



122. If $y = \sqrt{\cos x^2 + \sqrt{\cos x^2 + \sqrt{\cos x^2 + \dots \infty}}}$ and $\frac{dy}{dx} = \frac{f(x)}{2y-1}$ then, $\int f(x) dx = \dots$

- (A) $\sin x^2 + c$
- (B) $-\sin x^2 + c$
- (C) $\cos x^2 + c$
- (D) $-\cos x^2 + c$

Correct Answer: (C) $\cos x^2 + c$

Solution:

Step 1: Chain rule:

Let $u = \log \frac{x+3}{x} = \log(x+3) - \log x$. Then $\frac{d}{dx} \sin u = \cos u \cdot \frac{du}{dx}$.

Step 2: Derivative of u:

$$\frac{du}{dx} = \frac{1}{x+3} - \frac{1}{x} = \frac{x - (x+3)}{x(x+3)} = \frac{-3}{x(x+3)}$$

Step 3: Result:

$$\frac{d}{dx} \sin \left(\log \frac{x+3}{x} \right) = \frac{-3}{x(x+3)} \cos \left(\log \frac{x+3}{x} \right)$$

The negative sign is present because the ratio $\frac{x+3}{x}$ decreases as x increases.

Final Answer:

The derivative is $\frac{-3}{x(x+3)} \cos\left(\log \frac{x+3}{x}\right)$, option (C).

$$\boxed{\frac{-3}{x(x+3)} \cos\left(\log \frac{x+3}{x}\right)}$$

Quick Tip: Use the chain rule and the derivative of $\log((x+3)/x)$.

123. If $f(x) = \cot^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$ and $g(x) = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, then $\lim_{x \rightarrow a} \frac{f(x)-f(a)}{g(x)-g(a)}$, ($0 < a < \frac{1}{2}$) is

- (A) $\frac{3}{2}$
- (B) $\frac{1}{2}$
- (C) $-\frac{3}{2}$
- (D) $-\frac{1}{2}$

Correct Answer: (C) $-\frac{3}{2}$

Solution:

Step 1: Find y at $x = 0$:

Put $x = 0$: $e^y = e$, so $y = 1$.

Step 2: First derivative:

Differentiate $e^y + xy = e$: $e^y y' + y + xy' = 0$. At $(0, 1)$: $e y' + 1 = 0$, so $y' = -\frac{1}{e}$.

Step 3: Second derivative:

Differentiate again: $e^y(y')^2 + e^y y'' + y' + y' + xy'' = 0$.

At $x = 0$: $e \cdot \frac{1}{e^2} + e y'' - \frac{2}{e} = 0$, so $\frac{1}{e} + e y'' - \frac{2}{e} = 0$. Therefore $y'' = \frac{1}{e^2}$.

The ordered pair is $(-\frac{1}{e}, \frac{1}{e^2})$.

Final Answer:

The pair is $(-\frac{1}{e}, \frac{1}{e^2})$, option (B).

$$\left(-\frac{1}{e}, \frac{1}{e^2}\right)$$

Quick Tip: Differentiate implicitly twice and substitute $x = 0, y = 1$.

124. The coordinates of the points on the curve $4y = x^2$ that are nearest to the point $(0, 5)$ are ...

- (A) $(-2\sqrt{3}, 3)$
- (B) $(2\sqrt{3}, -3)$
- (C) $(3, 2\sqrt{3})$
- (D) $(2\sqrt{3}, 2)$

Correct Answer: (A) $(-2\sqrt{3}, 3)$

Solution:**Step 1: Setup:**

Surface area $S = 4\pi r^2$. Differentiating with time, $\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$.

Step 2: Find r:

When $S = 16\pi$, $4\pi r^2 = 16\pi$, so $r = 2$ cm.

Step 3: Find dr/dt :

$$4\pi = 8\pi(2)\frac{dr}{dt} \Rightarrow \frac{dr}{dt} = \frac{4\pi}{16\pi} = 0.25 \text{ cm/s}$$

Here the area grows at a steady $4\pi \text{ cm}^2/\text{s}$, but as the ball gets bigger, the same increase in area needs a smaller increase in radius. That is why the answer is smaller than 1 cm/s . The values 0.5 and 0.125 would come from using $r = 1$ or $r = 4$ by mistake.

Final Answer:

The radius increases at 0.25 cm/s , option (B).

0.25 cm/s

Quick Tip: Relate dS/dt to dr/dt using $S = 4\pi r^2$.

125. The equation of the tangent to the curve $y = \sqrt{9 - 3x^2}$ at the point where the ordinate and abscissa equal is...

- (A) $x - 3y + 3 = 0$
- (B) $3x - y - 3 = 0$
- (C) $x + 3y - 6 = 0$
- (D) $3x + y - 6 = 0$

Correct Answer: (D) $3x + y - 6 = 0$

Solution:

Step 1: Take the logarithm:

Let $y = \left(\frac{1}{x}\right)^x$. Then $\ln y = -x \ln x$, so maximising y means maximising $g(x) = -x \ln x$ for $x > 0$.

Step 2: Find the critical point:

$g'(x) = -\ln x - 1 = 0 \Rightarrow x = \frac{1}{e}$. $g''(x) = -\frac{1}{x} < 0$, so this is a maximum.

Step 3: Maximum value:

$g\left(\frac{1}{e}\right) = -\frac{1}{e} \ln \frac{1}{e} = \frac{1}{e}$. So $y_{\max} = e^{1/e}$.

Final Answer:

The maximum value is $e^{1/e}$, option (C).

$$\boxed{e^{1/e}}$$

Quick Tip: Take logarithms and find where the derivative of $-x \ln x$ vanishes.

126. If the line $y = 4x - 5$ is tangent to the curve $y^2 = ax^3 + b$ at the point $(2, 3)$, then the value of $7a - 2b$ is...

- (A) 0
- (B) 7
- (C) 14
- (D) 28

Correct Answer: (D) 28

Solution:

Step 1: Understanding the Concept:

A line making equal intercepts on the axes has slope -1. So the normal to the curve at the required point has slope -1, and the tangent has slope +1.

Step 2: Slope of the tangent:

Differentiate $9y^2 = x^3$: $18y y' = 3x^2$, so $y' = \frac{x^2}{6y}$. Set $y' = 1$: $x^2 = 6y$, that is $y = \frac{x^2}{6}$.

Step 3: Use the curve equation:

$$9\left(\frac{x^2}{6}\right)^2 = x^3 \Rightarrow \frac{x^4}{4} = x^3 \Rightarrow x = 4 \ (x \neq 0)$$

Then $y = \frac{16}{6} = \frac{8}{3}$. The point is $(4, \frac{8}{3})$.

The points with $x = -4$ do not lie on the curve, since $x^3 = -64$ cannot equal $9y^2 \geq 0$.

Final Answer:

The point is $(4, \frac{8}{3})$, option (B).

$$\boxed{\left(4, \frac{8}{3}\right)}$$

Quick Tip: Equal intercepts means the normal has slope -1.

127. If $\int \frac{\sin x}{\sin 4x} dx = \alpha \log \frac{1+\sin x}{1-\sin x} + \beta \log \frac{1+\sqrt{2}\sin x}{1-\sqrt{2}\sin x} + c$, then the value of $32(\alpha + \beta^2) =$

- (A) 5
- (B) -1
- (C) 9
- (D) -3

Correct Answer: (D) -3

Solution:

Step 1: Setup:

Volume $V = \frac{4}{3}\pi r^3$ and surface area $S = 4\pi r^2$. The condition is $\frac{dV}{dt} = -kS$.

Step 2: Simplify:

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$. So $4\pi r^2 \frac{dr}{dt} = -k \cdot 4\pi r^2$, giving $\frac{dr}{dt} = -k$. The radius shrinks at a constant rate.

Step 3: Use the data:

The radius falls from 3 cm to 1 cm in 4 months, a drop of 2 cm, so the rate is 0.5 cm per month. To lose the full 3 cm at this rate takes $\frac{3}{0.5} = 6$ months.

Final Answer:

The mothball evaporates completely in 6 months, option (A).

6 months

Quick Tip: If volume loss is proportional to surface area, the radius decreases at a constant rate.

128. If $\int f(x) dx = g(x)$ then $\int x^3 f(x^2) dx$ is equal to

- (A) $\frac{1}{2}[x^2 g(x^2) - \int g(x^2) d(x^2)]$
- (B) $\frac{1}{2}[x^2 [f(x)]^2 - \int [g(x)]^2 dx]$
- (C) $\frac{1}{2}[x^2 g(x) - \int g(x) d(x)]$
- (D) $\frac{1}{2}[x^2 g(x^2) + \int g(x^2) d(x^2)]$

Correct Answer: (A) $\frac{1}{2}[x^2 g(x^2) - \int g(x^2) d(x^2)]$

Solution:

Step 1: Partial fractions:

Write $\frac{x^2}{(x^2+2)(x^2+5)} = \frac{A}{x^2+2} + \frac{B}{x^2+5}$. Then $x^2 = A(x^2 + 5) + B(x^2 + 2)$.

Step 2: Find A and B:

Compare coefficients: $A + B = 1$ and $5A + 2B = 0$. So $A = -\frac{2}{3}$ and $B = \frac{5}{3}$.

Step 3: Integrate:

$$\begin{aligned}\int \frac{x^2 dx}{(x^2 + 2)(x^2 + 5)} &= -\frac{2}{3} \cdot \frac{1}{\sqrt{2}} \tan^{-1} \frac{x}{\sqrt{2}} + \frac{5}{3} \cdot \frac{1}{\sqrt{5}} \tan^{-1} \frac{x}{\sqrt{5}} + c \\ &= -\frac{\sqrt{2}}{3} \tan^{-1} \frac{x}{\sqrt{2}} + \frac{\sqrt{5}}{3} \tan^{-1} \frac{x}{\sqrt{5}} + c\end{aligned}$$

since $\frac{2}{3\sqrt{2}} = \frac{\sqrt{2}}{3}$ and $\frac{5}{3\sqrt{5}} = \frac{\sqrt{5}}{3}$. The signs differ because A is negative and B is positive.

Final Answer:

The result matches option (D).

$$-\frac{\sqrt{2}}{3} \tan^{-1} \frac{x}{\sqrt{2}} + \frac{\sqrt{5}}{3} \tan^{-1} \frac{x}{\sqrt{5}} + c$$

Quick Tip: Split $x^2/((x^2+2)(x^2+5))$ into partial fractions in x^2 .



129. If $\int \frac{3x+7}{x^2-3x+2} dx = m \log\left(\frac{x-2}{x-1}\right) + n \log(x-2) + c$, where $m, n \in R$ and c is an integration constant, then $m + n =$

- (A) 6
- (B) 7
- (C) 3
- (D) 13

Correct Answer: (D) 13

Solution:

Step 1: Simplify the numerator:

$$\sin^6 x + \cos^6 x = (\sin^2 x + \cos^2 x)^3 - 3 \sin^2 x \cos^2 x (\sin^2 x + \cos^2 x) = 1 - 3 \sin^2 x \cos^2 x.$$

Step 2: Split the fraction:

$$\frac{\sin^6 x + \cos^6 x}{\sin^2 x \cos^2 x} = \frac{1}{\sin^2 x \cos^2 x} - 3 = \sec^2 x + \operatorname{cosec}^2 x - 3$$

$$\text{because } \frac{1}{\sin^2 x \cos^2 x} = \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} = \sec^2 x + \operatorname{cosec}^2 x.$$

Step 3: Integrate:

$$\int (\sec^2 x + \operatorname{cosec}^2 x - 3) dx = \tan x - \cot x - 3x + c$$

Final Answer:

The integral is $\tan x - \cot x - 3x + c$, option (B).

$$\tan x - \cot x - 3x + c$$

Quick Tip: Use $\sin^6 + \cos^6 = 1 - 3 \sin^2 \cos^2$.

130. If $f(x) = \frac{\sin^{-1}x}{\sqrt{1-x^2}}$ and $g(x) = e^{\sin^{-1}x}$, then the value of $\int f(x)g(x) dx = \dots$

- (A) $e^{\sin^{-1}x}(\sin^{-1}x - 1) + c$
- (B) $e^{\sin^{-1}x}(1 - \sin^{-1}x) + c$
- (C) $e^{\sin^{-1}x}(\sin^{-1}x + 1) + c$
- (D) $e^{\sin^{-1}x}(-\sin^{-1}x - 1) + c$

Correct Answer: (A) $e^{\sin^{-1}x}(\sin^{-1}x - 1) + c$

Solution:

Step 1: Substitution:

Let $x = a \tan^2 \theta$, so $\theta = \tan^{-1} \sqrt{\frac{x}{a}}$. Then $\frac{a+x}{x} = \frac{\sec^2 \theta}{\tan^2 \theta} = \frac{1}{\sin^2 \theta}$, so $\sqrt{\frac{a+x}{x}} = \operatorname{cosec} \theta$. Hence $\operatorname{cosec}^{-1} \sqrt{\frac{a+x}{x}} = \theta$.

Also $dx = 2a \tan \theta \sec^2 \theta d\theta$.

Step 2: Integrate by parts:

$$I = \int \theta \cdot 2a \tan \theta \sec^2 \theta d\theta$$

Take $u = \theta$ and $dv = 2a \tan \theta \sec^2 \theta d\theta$, so $v = a \tan^2 \theta$.

$$I = a\theta \tan^2 \theta - a \int \tan^2 \theta d\theta = a\theta \tan^2 \theta - a(\tan \theta - \theta) + c$$

Step 3: Simplify:

$$I = a\theta \tan^2 \theta - a \tan \theta + a\theta + c$$

This matches option (B), which has $+a\theta$. Options with $-a\theta$ come from a sign error in $\int \tan^2 \theta d\theta = \tan \theta - \theta$.

Final Answer:

The integral is $a\theta \tan^2 \theta - a \tan \theta + a\theta + c$, option (B).

$$\boxed{a\theta \tan^2 \theta - a \tan \theta + a\theta + c}$$

Quick Tip: Substitute $x = a \tan^2 \theta$, which makes the inverse cosecant equal to θ , then integrate by parts.

131. The function $f(x) = \int_0^x \frac{dt}{1+\cos t}$ satisfies which of the following differential equations?

- (A) $2\frac{df}{dx} = 1 + [f(x)]^2$
 (B) $\frac{df}{dx} = 1 + [f(x)]^2$
 (C) $2\frac{df}{dx} = 1 - [f(x)]^2$
 (D) $\frac{df}{dx} = 1 - [f(x)]^2$

Correct Answer: (A) $2\frac{df}{dx} = 1 + [f(x)]^2$

Solution:

Step 1: Sign of $\sin 2x$:

$\sin 2x \geq 0$ for $0 \leq x \leq \frac{\pi}{2}$ and $\sin 2x \leq 0$ for $\frac{\pi}{2} \leq x \leq \pi$.

Step 2: Split the integral:

$$\int_0^{\pi} |\sin 2x| dx = \int_0^{\pi/2} \sin 2x dx - \int_{\pi/2}^{\pi} \sin 2x dx$$

$$\int_0^{\pi/2} \sin 2x dx = \left[-\frac{\cos 2x}{2} \right]_0^{\pi/2} = \frac{1}{2} + \frac{1}{2} = 1$$

$$- \int_{\pi/2}^{\pi} \sin 2x dx = \left[\frac{\cos 2x}{2} \right]_{\pi/2}^{\pi} = \frac{1}{2} + \frac{1}{2} = 1$$

Step 3: Total:

The sum is $1 + 1 = 2$. Without the modulus, the integral would be 0 (option A), since the two halves cancel.

Final Answer:

The value is 2, option (C).

Quick Tip: Split the interval where $\sin 2x$ changes sign and use symmetry.

132. If $\int_a^b (x^2 - x) dx = 18$, $\int_a^b x^3 dx = 0$, then $a + b$ is equal to

- (A) 3
- (B) 6
- (C) 0
- (D) 9

Correct Answer: (C) 0

Solution:

Step 1: Split the integrand:

$$\int_{-2}^2 (|x| + f(x) \sin x) dx = \int_{-2}^2 |x| dx + \int_{-2}^2 f(x) \sin x dx.$$

Step 2: Second integral:

Since f is even and $\sin x$ is odd, $f(x) \sin x$ is an odd function. The integral of an odd function over $[-2, 2]$ is 0.

Step 3: First integral:

$$|x| \text{ is even, so } \int_{-2}^2 |x| dx = 2 \int_0^2 x dx = 2 \cdot \frac{4}{2} = 4.$$

$$\text{Total} = 4 + 0 = 4.$$

Final Answer:

The value is 4, option (B).

Quick Tip: Use the odd function property of $f(x) \sin x$ over a symmetric interval.

133. The value of the definite integral $\int_0^\pi \frac{1}{5+4\cos x} dx$ is equal to ...

- (A) π
- (B) $\frac{\pi}{2}$
- (C) $\frac{\pi}{3}$
- (D) $\frac{\pi}{4}$

Correct Answer: (C) $\frac{\pi}{3}$

Solution:

Step 1: Split the region:

For $3 \leq x \leq 4$, $y = 4 - x$. For $4 \leq x \leq 5$, $y = x - 4$.

Step 2: Integrate:

$$\int_3^4 (4 - x) dx = \left[4x - \frac{x^2}{2} \right]_3^4 = (16 - 8) - (12 - 4.5) = \frac{1}{2}$$

$$\int_4^5 (x - 4) dx = \left[\frac{x^2}{2} - 4x \right]_4^5 = (12.5 - 20) - (8 - 16) = \frac{1}{2}$$

Step 3: Total:

Total area = $\frac{1}{2} + \frac{1}{2} = 1$ square unit. The shape is two small right triangles, each with legs 1 and 1.

Final Answer:

The area is 1 square unit, option (D).

1 sq. unit

Quick Tip: Split at $x = 4$ where the absolute value changes form.

134. Let $f(x) = x - [x]$ for every real number x , where $[x]$ is integral part of x . then $\int_{-1}^1 f(x) dx$ is

- (A) 1
- (B) 2
- (C) 1/2
- (D) 0

Correct Answer: (A) 1

Solution:

Step 1: Substitution:

Let $u = x + y$. Then $\frac{du}{dx} = 1 + \frac{dy}{dx} = 1 + \sin u + \cos u$. Separate variables: $\frac{du}{1 + \sin u + \cos u} = dx$.

Step 2: Half angle substitution:

Let $t = \tan \frac{u}{2}$. Then $\sin u = \frac{2t}{1+t^2}$, $\cos u = \frac{1-t^2}{1+t^2}$ and $du = \frac{2 dt}{1+t^2}$.

$$1 + \sin u + \cos u = \frac{(1+t^2) + 2t + (1-t^2)}{1+t^2} = \frac{2(1+t)}{1+t^2}$$

Step 3: Integrate:

$$\int \frac{2 dt/(1+t^2)}{2(1+t)/(1+t^2)} = \int \frac{dt}{1+t} = \log(1+t) = x + c$$

So $\log \left[1 + \tan \frac{x+y}{2} \right] = x + c$. The right hand side has x (not y), because we integrated with respect to x , which matches option (C).

Final Answer:

The solution is $\log \left[1 + \tan \frac{x+y}{2} \right] = x + c$, option (C).

$$\boxed{\log \left[1 + \tan \frac{x+y}{2} \right] = x + c}$$

Quick Tip: Substitute $u = x + y$ and use the half angle tangent substitution.

135. The area of the region bounded by curves $y = \sin x$, $y = \cos x$ and the lines $x = 0$, $x = \frac{\pi}{4}$ is

- (A) $2\sqrt{2} - 1$
- (B) $\sqrt{2} - 1$
- (C) $2 - \sqrt{2}$
- (D) $\frac{1-\sqrt{2}}{2}$

Correct Answer: (B) $\sqrt{2} - 1$

Solution:

Step 1: Order:

The highest derivative present is $\frac{d^3y}{dx^3}$, so the order is 3.

Step 2: Remove the fractional power:

Rewrite as $\left(\frac{d^3y}{dx^3}\right)^{7/3} = 3 - \left(\frac{dy}{dx}\right)^5$. Cube both sides:

$$\left(\frac{d^3y}{dx^3}\right)^7 = \left[3 - \left(\frac{dy}{dx}\right)^5\right]^3$$

Step 3: Degree:

The equation is now a polynomial in the derivatives, and the highest derivative $\frac{d^3y}{dx^3}$ has power 7. So the degree is 7.

Final Answer:

Order 3 and degree 7, option (A).

3, 7

Quick Tip: Clear the fractional power by cubing before reading the degree.

136. The general solution of the differential equation $\sec y + (x - e^{\sin y}) \frac{dy}{dx} = 0$ is...

- (A) $e^{\sin y} = x + c$
- (B) $xe^{\sin y} = \frac{e^{2\sin y}}{2} + c$
- (C) $2x \cos y = e^x + c$
- (D) $\sin y - e^{\sin y} = c$

Correct Answer: (B) $xe^{\sin y} = \frac{e^{2\sin y}}{2} + c$

Solution:

Step 1: Separate the variables:

$e^{-x}(y+1)dy = -(\cos^2 x - \sin 2x)y dx$. Divide by y and multiply by e^x :

$$\frac{y+1}{y} dy = -e^x(\cos^2 x - \sin 2x) dx$$

Step 2: Recognise the derivative:

$$\frac{d}{dx}(e^x \cos^2 x) = e^x \cos^2 x - e^x \cdot 2 \sin x \cos x = e^x(\cos^2 x - \sin 2x).$$

So the right hand side is $-d(e^x \cos^2 x)$.

Step 3: Integrate:

$$y + \log y = -e^x \cos^2 x + C \Rightarrow \log y + y + e^x \cos^2 x = C$$

At $x = 0, y = 1$: $0 + 1 + 1 = 2 = C$. So $\log y + y + e^x \cos^2 x = 2$, option (B).

Final Answer:

The solution is $\log y + y + e^x \cos^2 x = 2$, option (B).

$\log y + y + e^x \cos^2 x = 2$

Quick Tip: Separate variables after noticing that $d(e^x \cos^2 x)$ appears.

137. The general solution of the differential equations $\frac{dy}{dx} = (9x + y + 5)^2$ is...

(A) $\tan^{-1}\left(\frac{9x+y+5}{2}\right) = -2x + c$

(B) $\tan^{-1}\left(\frac{9x+y+5}{2}\right) = 2x + c$

(C) $\tan^{-1}\left(\frac{9x+y+5}{3}\right) = -3x + c$

(D) $\tan^{-1}\left(\frac{9x+y+5}{3}\right) = 3x + c$

Correct Answer: (D) $\tan^{-1}\left(\frac{9x+y+5}{3}\right) = 3x + c$

Solution:

Step 1: Setup:

Let $\vec{a} = \hat{i} + \hat{j} + 2\hat{k}$, $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$, $\vec{c} = \hat{i} + \hat{j} + \hat{k}$. Any vector coplanar with a and b is $\vec{v} = \alpha\vec{a} + \beta\vec{b}$.

Step 2: Perpendicular to c:

$\vec{a} \cdot \vec{c} = 1 + 1 + 2 = 4$ and $\vec{b} \cdot \vec{c} = 1 + 2 + 1 = 4$. So $\vec{v} \cdot \vec{c} = 4\alpha + 4\beta = 0$, giving $\beta = -\alpha$.

Step 3: Unit vector:

$\vec{v} \propto \vec{a} - \vec{b} = -\hat{j} + \hat{k}$. Its magnitude is $\sqrt{2}$. So the unit vector is $\pm \frac{1}{\sqrt{2}}(\hat{j} - \hat{k})$.

Check: $(\hat{j} - \hat{k}) \cdot (\hat{i} + \hat{j} + \hat{k}) = 1 - 1 = 0$.

Final Answer:

The unit vector is $\pm \frac{1}{\sqrt{2}}(\hat{j} - \hat{k})$, option (C).

$$\boxed{\pm \frac{1}{\sqrt{2}}(\hat{j} - \hat{k})}$$

Quick Tip: A vector coplanar with a and b is a combination of them; impose the perpendicularity.



138. Let $\vec{a}$ and $\vec{b}$ be linearly independent vectors such that

$$|\vec{a}| = \sqrt{3}, |\vec{b}| = 3 \text{ and } |\vec{a} - \vec{b}| = 4.$$

If $\vec{a} \times (2\hat{i} + 2\hat{j} - \hat{k}) = (2\hat{i} + 2\hat{j} - \hat{k}) \times \vec{b}$ and $|(\vec{a} + \vec{b}) \cdot (3\hat{i} + 4\hat{j} + 2\hat{k})| = \sqrt{\lambda}$, then $\lambda = \dots$

- (A) 32
- (B) 64
- (C) 256
- (D) 128

Correct Answer: (D) 128

Solution:

Step 1: Use the sum condition:

$$\vec{a} = -(\vec{b} + \vec{c}), \text{ so } |\vec{a}|^2 = |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{b} \cdot \vec{c}.$$

Step 2: Find cos theta:

$$9 = 9 + 9 + 2 \cdot 9 \cos \theta, \text{ so } \cos \theta = -\frac{1}{2} \text{ and } \theta = 120^\circ.$$

Step 3: Evaluate:

$$\tan^2 \theta = (\sqrt{3})^2 = 3 \text{ and } \cot^2 \theta = \frac{1}{3}. \text{ So } \tan^2 \theta + \cot^2 \theta = 3 + \frac{1}{3} = \frac{10}{3}.$$

Final Answer:

The value is $\frac{10}{3}$, option (D).

$$\frac{10}{3}$$

Quick Tip: Use $a + b + c = 0$ to get $b \cdot c$ from the magnitudes.

139. The value of b such that the scalar product of the vector $\hat{i} + \hat{j} + \hat{k}$ with the unit vector parallel to the sum of the vectors $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $b\hat{i} + 2\hat{j} + 3\hat{k}$ is one, is...

- (A) -2
- (B) -1
- (C) 0
- (D) 1

Correct Answer: (D) 1

Solution:

Step 1: Understanding the Concept:

Volume of the parallelepiped = $|\vec{a} \cdot \vec{b} \times \vec{c}|$. Base area = $|\vec{a} \times \vec{b}|$. Altitude h
 $= \frac{\text{volume}}{\text{base area}} \cdot$

Step 2: Base area:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ 2 & 4 & -1 \end{vmatrix} = (-1 - 4)\hat{i} - (-1 - 2)\hat{j} + (4 - 2)\hat{k} = -5\hat{i} + 3\hat{j} + 2\hat{k}.$$

$$|\vec{a} \times \vec{b}| = \sqrt{25 + 9 + 4} = \sqrt{38}.$$

Step 3: Volume and altitude:

$$(\vec{a} \times \vec{b}) \cdot \vec{c} = -5 + 3 + 6 = 4.$$

$$h = \frac{4}{\sqrt{38}} = \frac{4}{\sqrt{2}\sqrt{19}} = \frac{2\sqrt{2}}{\sqrt{19}}$$

Final Answer:

The altitude is $\frac{2\sqrt{2}}{\sqrt{19}}$ units, option (A).

$\frac{2\sqrt{2}}{\sqrt{19}} \text{ units}$

Quick Tip: Altitude equals the volume divided by the base area.

140. If $|\vec{a} \cdot \vec{b}| = |\vec{a} \times \vec{b}|$, $\vec{a} \cdot \vec{b} < 0$ and θ is the angle between $\vec{a}$ and $\vec{b}$, then the value of $\sin\theta + \tan\theta$ is...

- (A) $\frac{\sqrt{2}-2}{2}$
- (B) $\frac{\sqrt{2}+2}{2}$
- (C) $\frac{1+\sqrt{2}}{2}$
- (D) $\frac{2-\sqrt{2}}{2}$

Correct Answer: (A) $\frac{\sqrt{2}-2}{2}$

Solution:

Step 1: Understanding the Concept:

For a quadrilateral ABCD, area = $\frac{1}{2} |\vec{AC} \times \vec{BD}|$.

Step 2: Find BD:

$$\vec{BD} = \vec{AD} - \vec{AB} = \vec{b} - \vec{a}.$$

Step 3: Cross product:

$$\overrightarrow{AC} \times \overrightarrow{BD} = (3\vec{a} + 2\vec{b}) \times (\vec{b} - \vec{a}) = 3\vec{a} \times \vec{b} - 2\vec{b} \times \vec{a} = 3\vec{a} \times \vec{b} + 2\vec{a} \times \vec{b} = 5\vec{a} \times \vec{b}$$

Area of ABCD = $\frac{5}{2}|\vec{a} \times \vec{b}|$. The parallelogram on AB and AD has area $|\vec{a} \times \vec{b}|$. So $\alpha = \frac{5}{2}$.

Final Answer:

$\alpha = \frac{5}{2}$, option (B).

$$\boxed{\frac{5}{2}}$$

Quick Tip: Area of a quadrilateral is half the magnitude of the cross product of its diagonals.

141. Let $\vec{a} = \lambda\hat{i} + \hat{j} + \hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} + 4\hat{k}$, $\vec{c} = \hat{i} + \mu\hat{j} + \hat{k}$
If $\vec{a}$ is parallel to $\vec{b}$ and $\vec{b}$ is perpendicular to $\vec{c}$ then $\lambda - \mu = \dots$

- (A) -2
- (B) -1
- (C) 2
- (D) 1

Correct Answer: (C) 2

Solution:

Step 1: Find $|\vec{c}|$:

$|\vec{a}| = \sqrt{1 + 4 + 4} = 3$. From $|\vec{c} - \vec{a}|^2 = 8$: $|\vec{c}|^2 - 2\vec{a} \cdot \vec{c} + 9 = 8$. Using $\vec{a}$

$$\cdot \vec{c} = |\vec{c}|: |\vec{c}|^2 - 2|\vec{c}| + 1 = 0, \text{ so } |\vec{c}| = 1.$$

Step 2: Find $|\mathbf{a} \times \mathbf{b}|$:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -2 \\ 1 & -1 & 1 \end{vmatrix} = (2 - 2)\hat{i} - (1 + 2)\hat{j} + (-1 - 2)\hat{k} = -3\hat{j} - 3\hat{k}.$$

$$\text{So } |\vec{a} \times \vec{b}| = 3\sqrt{2}.$$

Step 3: Magnitude of the double product:

$$|(\vec{a} \times \vec{b}) \times \vec{c}| = |\vec{a} \times \vec{b}||\vec{c}| \sin 60^\circ = 3\sqrt{2} \cdot 1 \cdot \frac{\sqrt{3}}{2} = \frac{3\sqrt{6}}{2} = 3\sqrt{\frac{3}{2}}$$

Final Answer:

The magnitude is $3\sqrt{\frac{3}{2}}$, option (C).

$$\boxed{3\sqrt{\frac{3}{2}}}$$

Quick Tip: Find $|c|$ from the first two conditions, then use the angle between $\mathbf{a} \times \mathbf{b}$ and $\mathbf{c}$.

142. The equation of the perpendicular line from the point $(2, -3, 1)$ to the line $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z+2}{-1}$ is

(A) $\frac{x-2}{24} = \frac{y+3}{13} = \frac{z-1}{9}$

(B) $\frac{x-2}{24} = \frac{y-3}{-13} = \frac{z-1}{9}$

(C) $\frac{x+2}{-24} = \frac{y+3}{13} = \frac{z+1}{-9}$

(D) $\frac{x-2}{-24} = \frac{y+3}{13} = \frac{z-1}{-9}$

Correct Answer: (D) $\frac{x-2}{-24} = \frac{y+3}{13} = \frac{z-1}{-9}$

Solution:

Step 1: Equation of the line:

Points $(3, 5, -7)$ and $(-2, 1, 8)$. A general point is $(3 - 5t, 5 - 4t, -7 + 15t)$.

Step 2: YOZ plane:

On the YOZ plane $x = 0$: $3 - 5t = 0$, so $t = \frac{3}{5}$.

Step 3: Coordinates:

$y = 5 - \frac{12}{5} = \frac{13}{5}$ and $z = -7 + 9 = 2$. The point is $(0, \frac{13}{5}, 2)$.

Final Answer:

The point is $(0, \frac{13}{5}, 2)$, option (C).

$$\left(0, \frac{13}{5}, 2\right)$$

Quick Tip: On the YOZ plane $x = 0$, so find the parameter where the x coordinate vanishes.

143. The angle between the line $x - 1 = 2 - y = \frac{2z-6}{4}$ and the plane $\vec{r} \cdot (2\hat{i} + \hat{j} + \hat{k}) = 10$ is

- (A) $\frac{\pi}{3}$
- (B) $\frac{\pi}{6}$
- (C) $\frac{\pi}{4}$
- (D) $\frac{\pi}{12}$

Correct Answer: (B) $\frac{\pi}{6}$

Solution:

Step 1: Equate components:

Line 1: $(1 + 2\lambda, m + 3\lambda, 3 + 4\lambda)$. Line 2: $(4 + 5\mu, 1 + m\mu, \mu)$.

Step 2: Solve for lambda and mu:

From the z components: $\mu = 3 + 4\lambda$. From the x components: $1 + 2\lambda = 4 + 5(3 + 4\lambda) = 19 + 20\lambda$, so $\lambda = -1$ and $\mu = -1$.

Step 3: Use the y components:

$$m + 3\lambda = 1 + m\mu \Rightarrow m - 3 = 1 - m \Rightarrow 2m = 4 \Rightarrow m = 2.$$

Final Answer:

$m = 2$, option (C).

2

Quick Tip: Equate the coordinates of the two lines and solve for the parameters and m.

144. The sum of the coordinates of one of the points on the line $\frac{x-2}{1} = \frac{y+3}{-2} = \frac{z+5}{2}$ which is at a distance of 3 units from the point $(2, -3, -5)$ is ...

- (A) 7
- (B) -7
- (C) 11
- (D) -11

Correct Answer: (B) -7

Solution:

Step 1: Points and directions:

Line 1 passes through $P(-1, 3, 1)$ with direction $\vec{d}_1 = (1, -\lambda, \lambda)$. Line 2 passes through $Q(0, 1, 2)$ with direction $\vec{d}_2 = (\frac{1}{2}, 1, -1)$, proportional to $(1, 2, -2)$.

Step 2: Coplanarity condition:

$\vec{PQ} = (1, -2, 1)$. The lines are coplanar if

$$\begin{vmatrix} 1 & -2 & 1 \\ 1 & -\lambda & \lambda \\ 1 & 2 & -2 \end{vmatrix} = 0$$

Step 3: Expand:

$$1(2\lambda - 2\lambda) + 2(-2 - \lambda) + 1(2 + \lambda) = 0 - 4 - 2\lambda + 2 + \lambda = -2 - \lambda$$

So $-2 - \lambda = 0$, giving $\lambda = -2$.

Final Answer:

$\lambda = -2$, option (C).

$$\boxed{-2}$$

Quick Tip: Use the determinant of the joining vector and the two direction vectors.

145. If A and B are the feet of the perpendiculars drawn from (1, 2, 3) to planes YZ and ZX, then the equation of the plane passing through the points A, B and the origin is

- (A) $6x + 3y + 2z = 0$
- (B) $6x - 3y - 2z = 0$
- (C) $6x + 3y - 2z = 0$
- (D) $3x + 6y + 2z = 0$

Correct Answer: (C) $6x + 3y - 2z = 0$

Solution:

Step 1: Normal vector:

Normals of the given planes: $\vec{n}_1 = (1, 2, 2)$ and $\vec{n}_2 = (3, 3, 2)$. The required plane is perpendicular to both, so its normal is $\vec{n} = \vec{n}_1 \times \vec{n}_2$.

Step 2: Cross product:

$$\vec{n} = (2 \cdot 2 - 2 \cdot 3, 2 \cdot 3 - 1 \cdot 2, 1 \cdot 3 - 2 \cdot 3) = (-2, 4, -3)$$

So the plane is $2x - 4y + 3z + d = 0$.

Step 3: Use the point:

(1, 2, 1): $2 - 8 + 3 + d = 0$, so $d = 3$. The plane is $2x - 4y + 3z + 3 = 0$.

Final Answer:

The required plane is $2x - 4y + 3z + 3 = 0$, option (B).

$2x - 4y + 3z + 3 = 0$

Quick Tip: The normal of the required plane is the cross product of the two plane normals.



146. If the distance of point $B(2, 1, -3)$ from the line passing through the point $A(4, -2, 2)$, and parallel to the vector $\vec{c} = -4\hat{i} - 6\hat{j} - 2\hat{k}$ is x , then $x^4 + x^2 + 541 =$

- (A) 2026
- (B) 2025
- (C) 2024
- (D) 2023

Correct Answer: (D) 2023

Solution:

Step 1: Parametric line:

The line through $(1, 0, -3)$ parallel to $(2, 3, -6)$ is $(1 + 2t, 3t, -3 - 6t)$.

Step 2: Meet the plane:

Substitute in $x - y - z = 9$: $1 + 2t - 3t + 3 + 6t = 9$, so $4 + 5t = 9$ and $t = 1$. The point is $(3, 3, -9)$.

Step 3: Distance:

Distance $= |t|\sqrt{2^2 + 3^2 + 6^2} = 1 \times 7 = 7$ units.

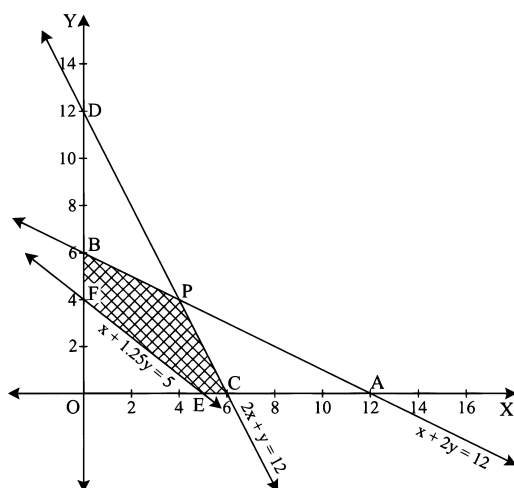
Final Answer:

The distance is 7 units, option (B).

7 units

Quick Tip: Move along the line direction until the plane is reached.

147. In the following figure, the shaded region represents the system of constraints:



- (A) $2x + y \leq 12, x + 2y \leq 12, x + 1.25y \geq 5, x \leq 0, y \geq 0$
- (B) $2x + y \leq 12, x + 2y \leq 12, x + 1.25y \geq 5, x \geq 0, y \leq 0$
- (C) $2x + y \leq 12, x + 2y \leq 12, x + 1.25y \leq 5, x \geq 0, y \geq 0$
- (D) $2x + y \leq 12, x + 2y \leq 12, x + 1.25y \geq 5, x \geq 0, y \geq 0$

Correct Answer: (D) $2x + y \leq 12, x + 2y \leq 12, x + 1.25y \geq 5, x \geq 0, y \geq 0$

Solution:

Step 1: Feasible region:

Constraints: $x + 3y \leq 60, x + y \geq 10, x \leq y, x, y \geq 0$.

Step 2: Corner points:

Intersections of boundary lines: $x = y$ with $x + y = 10$ gives (5, 5). $x = y$ with $x + 3y = 60$ gives (15, 15). The y axis meets $x + y = 10$ at (0, 10) and $x + 3y = 60$ at (0, 20).

Step 3: Evaluate Z:

$$Z(5, 5) = 40, Z(15, 15) = 120, Z(0, 10) = 50, Z(0, 20) = 100.$$

Maximum = 120 and minimum = 40. Difference = 80.

Final Answer:

The difference is 80, option (D).

80

Quick Tip: Find the corner points of the feasible region and evaluate Z at each.

148. On average, one out of 10 persons is busy. If six persons are selected at random, then the probability that at least 5 of them will be busy is...

- (A) $\frac{25}{(10)^6}$
- (B) $\frac{35}{(10)^6}$
- (C) $\frac{45}{(10)^6}$
- (D) $\frac{55}{(10)^6}$

Correct Answer: (D) $\frac{55}{(10)^6}$

Solution:

Step 1: Mean:

$$E(X) = 2(0.2) + 3(0.5) + 4(0.3) = 0.4 + 1.5 + 1.2 = 3.1.$$

Step 2: Mean of squares:

$$E(X^2) = 4(0.2) + 9(0.5) + 16(0.3) = 0.8 + 4.5 + 4.8 = 10.1.$$

Step 3: Standard deviation:

$$\text{Var}(X) = E(X^2) - [E(X)]^2 = 10.1 - 9.61 = 0.49$$

$$\sigma = \sqrt{0.49} = 0.7$$

Final Answer:

The standard deviation is 0.7, option (D).

0.7

Quick Tip: Compute the mean and then the variance from $E(X^2)$ minus mean squared.

149. If a discrete random variable X takes the values 1, 2, 3, 4 such that $2P(X = 1) = 3P(X = 2) = P(X = 3) = 5P(X = 4)$, then $P(X = 4) = \dots$

- (A) $\frac{15}{61}$
- (B) $\frac{30}{61}$
- (C) $\frac{10}{61}$
- (D) $\frac{6}{61}$

Correct Answer: (D) $\frac{6}{61}$

Solution:

Step 1: List the outcomes:

E (prime): 2, 3, 5, 7. F (X less than 4): 1, 2, 3. The union $E \cup F = \{1, 2, 3, 5, 7\}$. The outcomes 2 and 3 are in both, so they are counted only once.

Step 2: Add the probabilities:

$$P(E \cup F) = P(1) + P(2) + P(3) + P(5) + P(7) = 0.15 + 0.23 + 0.12 + 0.20 + 0.07 = 0.77.$$

Step 3: Check by the formula:

$$P(E) = 0.23 + 0.12 + 0.20 + 0.07 = 0.62, P(F) = 0.50, P(E \cap F) = 0.35.$$

Then $0.62 + 0.50 - 0.35 = 0.77$.

Final Answer:

$$P(E \cup F) = 0.77, \text{ option (B).}$$

0.77

Quick Tip: Identify the outcomes in E or F, then add their probabilities once each.

150. Three numbers are chosen from 1 to 30. The probability that minimum is 10 and maximum is 26 is _____

- (A) $\frac{1}{406}$
- (B) $\frac{3}{812}$
- (C) $\frac{5}{812}$
- (D) $\frac{7}{812}$

Correct Answer: (B) $\frac{3}{812}$

Solution:

Step 1: Understanding the Concept:

$P(A) = 0.4$, so $P(\text{not } A) = 0.6$. The trials are independent.

Step 2: Complement:

The probability that A never happens in three trials is $0.6^3 = 0.216$.

Step 3: At least once:

$$P = 1 - 0.216 = 0.784$$

The value 0.216 in option (A) is only the probability of no occurrence, so it is not the answer.

Final Answer:

The probability is 0.784, option (D).

0.784

Quick Tip: Use the complement: 1 minus the probability that it never happens.