MHT CET 2026 PCM Question Paper April 18 Shift 1 with Solutions

collegedunia

General Instructions

(i) This booklet contains 150 questions, each provided with a complete, step-by-step solution.

(ii) It comprises 150 single-correct multiple-choice questions.

(iii) Attempt each question on your own before reviewing the given solution.

Physics

Questions 1 to 50 | 1 mark each | 50 marks

1. The angle made by vector $\vec{A} = 2\hat{i} + 3\hat{j}$ with x-axis and that with y-axis are respectively.

(A) $tan^{-1}2\sqrt{13}$, $tan^{-1}3\sqrt{13}$

(B) $cos^{-1}\frac{2}{\sqrt{13}}$, $cos^{-1}\frac{3}{\sqrt{13}}$

(C) $cos^{-1}\frac{1}{\sqrt{2}}$, $cos^{-1}\frac{1}{\sqrt{3}}$

(D) $sin^{-1}\frac{1}{\sqrt{6}}$, $sin^{-1}\frac{1}{2\sqrt{3}}$

Correct Answer: (B) $cos^{-1}\frac{2}{\sqrt{13}}$, $cos^{-1}\frac{3}{\sqrt{13}}$

Solution:

Step 1: Understanding the Concept:

The angle a vector makes with an axis is found from its direction cosine. For the angle α with the x-axis, $\cos\alpha = \frac{A_x}{|\vec{A}|}$. For the angle β with the y-axis, $\cos\beta = \frac{A_y}{|\vec{A}|}$.

Step 2: Key Formula or Approach:

The magnitude is $|\vec{A}| = \sqrt{2^2 + 3^2} = \sqrt{13}$.

Step 3: Detailed Explanation:

With the x-axis: $\cos\alpha = \frac{2}{\sqrt{13}}$, so $\alpha = \cos^{-1}\frac{2}{\sqrt{13}}$.

With the y-axis: $\cos\beta = \frac{3}{\sqrt{13}}$, so $\beta = \cos^{-1}\frac{3}{\sqrt{13}}$.

Option A gives $\tan^{-1}(2\sqrt{13})$, which is a much larger angle than the vector makes with the x-axis. Option C would be right only for a vector with equal components. Option D uses sine values that do not match $\frac{3}{\sqrt{13}}$ or $\frac{2}{\sqrt{13}}$.

Final Answer:

The angles are $\cos^{-1}\frac{2}{\sqrt{13}}$ and $\cos^{-1}\frac{3}{\sqrt{13}}$, option (B).

$$\boxed{\cos^{-1}\frac{2}{\sqrt{13}},\ \cos^{-1}\frac{3}{\sqrt{13}}}$$

Quick Tip: The angle with an axis comes from the direction cosine, which is the component divided by the magnitude.

2. If two resistors of resistances have values $R_1 = (350 \pm 3)\Omega$ and $R_2 = (140 \pm 4)\Omega$. The percentage error for the sum and difference of R_1 and R_2 are respectively

(A) 1.43% , 3.33%

(B) 7% , 1%

(C) 1% , 7%

(D) 3.33% , 1.43%

Correct Answer: (A) 1.43% , 3.33%

Solution:

Step 1: Understanding the Concept:

When two measured quantities are added or subtracted, their absolute errors add. The percentage error is then found from the final value.

Step 2: Key Formula or Approach:

$R = R_1 \pm R_2$ has error $\Delta R = \Delta R_1 + \Delta R_2$, and percentage error $= \frac{\Delta R}{R} \times 100$.

Step 3: Detailed Explanation:

$\Delta R = 3 + 4 = 7\ \Omega$ for both cases.

Sum: $R = 350 + 140 = 490\ \Omega$, so the percentage error $= \frac{7}{490} \times 100 = 1.43\%$.

Difference: $R = 350 - 140 = 210\ \Omega$, so the percentage error $= \frac{7}{210} \times 100 = 3.33\%$.

The error for the difference is larger because the final value is smaller while the error is the same.

Final Answer:

The percentage errors are 1.43% and 3.33%, option (A).

$$\boxed{1.43\%,\ 3.33\%}$$

Quick Tip: For a sum or difference, absolute errors add; then divide by the result.

3. A body moves along a circular path of diameter 30cm. It starts from one end of diameter, moves along the circular path and reaches the other end of diameter in 3 second. The angular speed of the body in radian per second is

(A) $\frac{\pi}{5}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

Correct Answer: (C) $\frac{\pi}{3}$

Solution:

Step 1: Understanding the Concept:

Angular speed is the angle swept divided by the time taken: $\omega = \frac{\theta}{t}$.

Step 2: Key Formula or Approach:

The body goes from one end of a diameter to the other, which is half of a circle. The angle swept is π radians.

Step 3: Detailed Explanation:

$$\omega = \frac{\pi}{3} \text{ rad/s}$$

The diameter of 30 cm is extra information. The angular speed does not depend on the radius.

The values $\frac{\pi}{5}$, $\frac{\pi}{4}$ and $\frac{\pi}{2}$ would correspond to times of 5 s, 4 s and 2 s.

Final Answer:

The angular speed is $\frac{\pi}{3}$ rad/s, option (C).

$$\boxed{\frac{\pi}{3}\ \text{rad/s}}$$

Quick Tip: A trip between ends of a diameter covers an angle of pi radians.

4. A block of mass M is lying on a horizontal frictionless surface. One end of the uniform rope of half the mass of the block is fixed to the block which is pulled in the horizontal direction by applying a force F at the other end. The tension in the middle of the rope will be

(A) $\frac{F}{3}$

(B) $\frac{3F}{4}$

(C) $\frac{4F}{5}$

(D) $\frac{5F}{6}$

Correct Answer: (D) $\frac{5F}{6}$

Solution:

Step 1: Understanding the Concept:

The block and the rope move together with the same acceleration. The tension at a point in the rope is the force that pulls everything on the block side of that point.

Step 2: Key Formula or Approach:

Total mass $= M + \frac{M}{2} = \frac{3M}{2}$, so $a = \frac{F}{3M/2} = \frac{2F}{3M}$.

Step 3: Detailed Explanation:

At the middle of the rope, the part between the middle and the block is half of the rope, with mass $\frac{M}{4}$.

The tension T at the middle pulls the block together with this half rope:

$$T = \left(M + \frac{M}{4}\right)a = \frac{5M}{4} \times \frac{2F}{3M} = \frac{5F}{6}$$

Option A, $\frac{F}{3}$, would be the tension on the block alone, with no allowance for the rope mass. Option B, $\frac{3F}{4}$, does not follow from the mass split $M + \frac{M}{4}$.

Final Answer:

The tension in the middle is $\frac{5F}{6}$, option (D).

$$\boxed{\frac{5F}{6}}$$

Quick Tip: Find the common acceleration, then apply F = ma to what lies beyond the middle point.

5. The angular momentum of a rotating body is 'L'. When the frequency of rotating body is tripled and its kinetic energy is made one-third, the new angular momentum becomes

(A) $9L$

(B) $6L$

(C) $\frac{L}{3}$

(D) $\frac{L}{9}$

Correct Answer: (D) $\frac{L}{9}$

Solution:

Step 1: Understanding the Concept:

For rotation, $L = I\omega$ and $K = \frac{1}{2}I\omega^2 = \frac{1}{2}L\omega$. Both I and ω can change here, so use the form that links K, L and ω.

Step 2: Key Formula or Approach:

$K = \frac{1}{2}L\omega$, so $L = \frac{2K}{\omega}$. The frequency is proportional to ω.

Step 3: Detailed Explanation:

New $\omega = 3\omega$ and new $K = \frac{K}{3}$.

$$L' = \frac{2(K/3)}{3\omega} = \frac{1}{9} \cdot \frac{2K}{\omega} = \frac{L}{9}$$

If I stayed fixed, tripling ω would give $3L$ and $9K$, which does not match the stated $K' = \frac{K}{3}$. So the moment of inertia must have changed, and the answer is $\frac{L}{9}$.

Final Answer:

The new angular momentum is $\frac{L}{9}$, option (D).

$$\boxed{\frac{L}{9}}$$

Quick Tip: Use K = (1/2) L w and remember that the moment of inertia must change.

6. A solid cylinder and a solid sphere having the same mass and radius roll down on the same smooth inclined plane. The ratio of the acceleration of the cylinder (a_c) to that of the sphere (a_s) is

(A) $\frac{14}{15}$

(B) $\frac{15}{14}$

(C) $\frac{13}{14}$

(D) $\frac{11}{15}$

Correct Answer: (A) $\frac{14}{15}$

Solution:

Step 1: Understanding the Concept:

For a body that rolls down an incline without slipping, the acceleration is $a = \frac{g\sin\theta}{1+K^2/R^2}$, where K is the radius of gyration.

Step 2: Key Formula or Approach:

Solid cylinder: $\frac{K^2}{R^2} = \frac{1}{2}$. Solid sphere: $\frac{K^2}{R^2} = \frac{2}{5}$.

Step 3: Detailed Explanation:

$a_c = \frac{g\sin\theta}{1+\frac{1}{2}} = \frac{2}{3}g\sin\theta.$

$a_s = \frac{g\sin\theta}{1+\frac{2}{5}} = \frac{5}{7}g\sin\theta.$

$$\frac{a_c}{a_s} = \frac{2/3}{5/7} = \frac{14}{15}$$

The sphere is faster because it has less of its mass far from the axis.

Option B, $\frac{15}{14}$, is the inverse ratio.

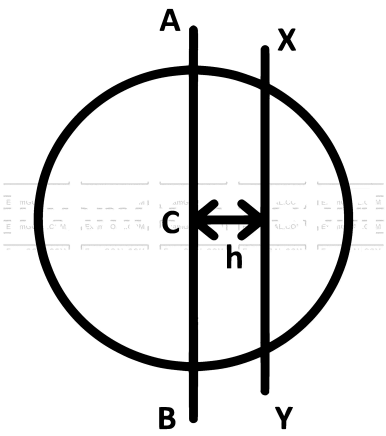
Final Answer:

The ratio $a_c : a_s$ is $\frac{14}{15}$, option (A).

$$\boxed{\frac{14}{15}}$$

Quick Tip: Use a = g sin(theta) / (1 + K squared over R squared).

7. The radius of gyration K of a hollow sphere of mass M and radius R about an axis XY is equal to R as shown in figure. The distance of that axis from the center of the sphere is h. The value of h is



(A) $\frac{R}{\sqrt{3}}$

(B) $\frac{R}{2}$

(C) $\frac{R}{\sqrt{2}}$

(D) $\frac{2R}{\sqrt{3}}$

Correct Answer: (A) $\frac{R}{\sqrt{3}}$

Solution:

Step 1: Understanding the Concept:

The figure shows an axis XY parallel to the diameter AB, at a distance h from the centre C. The moment of inertia about XY follows from the parallel axis theorem: $I = I_{cm} + Mh^2$.

Step 2: Key Formula or Approach:

For a thin hollow sphere, $I_{cm} = \frac{2}{3}MR^2$. The radius of gyration is defined by $I = MK^2$.

Step 3: Detailed Explanation:

$MK^2 = \frac{2}{3}MR^2 + Mh^2$, so $K^2 = \frac{2}{3}R^2 + h^2$.

Given $K = R$: $R^2 = \frac{2}{3}R^2 + h^2$, so $h^2 = \frac{R^2}{3}$.

$$h = \frac{R}{\sqrt{3}}$$

Option C, $\frac{R}{\sqrt{2}}$, would come from using $\frac{1}{2}MR^2$ for I_{cm}, which is not the hollow sphere value. Option D is larger than R and would give $K > R$.

Final Answer:

The distance is $h = \frac{R}{\sqrt{3}}$, option (A).

$$\boxed{\frac{R}{\sqrt{3}}}$$

Quick Tip: Use the parallel axis theorem on the moment of inertia of a thin hollow sphere.

8. The lengths of seconds pendulums on the surface of the earth and at an altitude 'h' from the surface of the earth are l_s and l_h respectively. The radius of the earth is

(A) $\frac{h\sqrt{l_h}}{\sqrt{l_s}-\sqrt{l_h}}$

(B) $\frac{h\sqrt{l_h}}{\sqrt{l_h}-\sqrt{l_s}}$

(C) $\frac{\sqrt{l_h}}{h(\sqrt{l_s}-\sqrt{l_h})}$

(D) $\frac{\sqrt{l_s}}{h(\sqrt{l_h}-\sqrt{l_s})}$

Correct Answer: (A) $\frac{h\sqrt{l_h}}{\sqrt{l_s}-\sqrt{l_h}}$

Solution:

Step 1: Understanding the Concept:

The period is $T = 2\pi\sqrt{\frac{l}{g}}$. For a seconds pendulum $T = 2$ s at both places, so $\frac{l}{g}$ is the same, which means $l \propto g$.

Step 2: Key Formula or Approach:

At height h, $g_h = g\left(\frac{R}{R+h}\right)^2$. So $\frac{l_h}{l_s} = \frac{g_h}{g_s} = \frac{R^2}{(R+h)^2}$.

Step 3: Detailed Explanation:

Take the square root: $\frac{\sqrt{l_h}}{\sqrt{l_s}} = \frac{R}{R+h}$.

Cross multiply: $R\sqrt{l_s} = (R+h)\sqrt{l_h}$.

So $R(\sqrt{l_s} - \sqrt{l_h}) = h\sqrt{l_h}$.

$$R = \frac{h\sqrt{l_h}}{\sqrt{l_s} - \sqrt{l_h}}$$

Since $l_s > l_h$, the denominator is positive, which fits option A. Option B has a negative denominator.

Final Answer:

The radius of the earth is $\frac{h\sqrt{l_h}}{\sqrt{l_s}-\sqrt{l_h}}$, option (A).

$$\boxed{\frac{h\sqrt{l_h}}{\sqrt{l_s}-\sqrt{l_h}}}$$

Quick Tip: A seconds pendulum has period 2 s, so its length is proportional to g.

9. Time period of a simple pendulum is T_1 when on the earth's surface and T_2 when taken to a height '2R' above the earth's surface, where 'R' is the radius of the earth. The ratio $T_1 : T_2$ is

(A) 1 : 2

(B) 1 : 3

(C) 1 : 4

(D) 1 : 5

Correct Answer: (B) 1 : 3

Solution:

Step 1: Understanding the Concept:

$T = 2\pi\sqrt{\frac{l}{g}}$, so $T \propto \frac{1}{\sqrt{g}}$. The value of g changes with distance r from the centre of the earth as $g \propto \frac{1}{r^2}$.

Step 2: Key Formula or Approach:

At the surface, $r = R$. At a height $2R$ above the surface, $r = 3R$.

Step 3: Detailed Explanation:

$g_2 = g_1\left(\frac{R}{3R}\right)^2 = \frac{g_1}{9}$.

$$\frac{T_1}{T_2} = \sqrt{\frac{g_2}{g_1}} = \sqrt{\frac{1}{9}} = \frac{1}{3}$$

So $T_1 : T_2 = 1 : 3$. The ratio $1 : 2$ would come from taking the distance as $2R$ instead of $3R$, forgetting that the height is measured from the surface.

Final Answer:

$T_1 : T_2 = 1 : 3$, option (B).

$$\boxed{1 : 3}$$

Quick Tip: Period is inversely proportional to the square root of g, and g falls with the square of distance from the centre.

10. The pressure at half the depth of a lake is equal to two-third pressure at the bottom of the lake. So the depth 'h' of the lake is (ρ = density of water in the lake, g = acceleration due to gravity, P_0 = atmospheric pressure).

(A) $\frac{2P_0}{\rho g}$

(B) $\frac{P_0}{\rho g}$

(C) $\frac{2P_0}{3\rho g}$

(D) $\frac{P_0}{2\rho g}$

Correct Answer: (A) $\frac{2P_0}{\rho g}$

Solution:

Step 1: Understanding the Concept:

The absolute pressure at depth d below the surface of a liquid is $P = P_0 + \rho g d$.

Step 2: Key Formula or Approach:

At half depth: $P_0 + \rho g \frac{h}{2}$. At the bottom: $P_0 + \rho g h$. The given condition is $P_{half} = \frac{2}{3} P_{bottom}$.

Step 3: Detailed Explanation:

$P_0 + \frac{\rho g h}{2} = \frac{2}{3}(P_0 + \rho g h)$.

Multiply by 6: $6P_0 + 3\rho g h = 4P_0 + 4\rho g h$.

So $2P_0 = \rho g h$.

$$h = \frac{2P_0}{\rho g}$$

Final Answer:

The depth is $h = \frac{2P_0}{\rho g}$, option (A).

$$\boxed{\frac{2P_0}{\rho g}}$$

Quick Tip: Write absolute pressure at half depth and at the bottom.

11. A spherical object is falling under gravity through a viscous fluid. The sphere attains the terminal velocity when

(A) viscous force is zero.

(B) buoyant force is equal to force due to gravity

(C) viscous force plus force of gravity becomes equal to buoyant force.

(D) buoyant force plus viscous force becomes equal to force due to gravity.

Correct Answer: (D) buoyant force plus viscous force becomes equal to force due to gravity.

Solution:

Step 1: Understanding the Concept:
A sphere falling through a viscous fluid feels three forces. Gravity acts downward. The buoyant force and the viscous drag both act upward.

Step 2: Key Formula or Approach:
At terminal velocity the acceleration is zero, so the net force is zero: downward force equals upward forces.

Step 3: Detailed Explanation:
$mg = F_{buoyant} + F_{viscous}$.
So the terminal velocity is reached when the buoyant force plus the viscous force equals the force due to gravity.
The viscous force is not zero at that instant, because it is the force that grows with speed until it balances the rest. Gravity must be on the opposite side of the equation from the other two forces.

Final Answer:
Terminal velocity is reached when buoyant force plus viscous force equals the force of gravity, option (D).

$$\boxed{\text{Buoyant force} + \text{viscous force} = \text{weight}}$$

Quick Tip: At terminal velocity the net force is zero.

12. A piston of cross-sectional area $2.5 \times 10^{-2}\text{m}^2$ is used in a hydraulic lift to exert a force of 250N on water. The cross-sectional area of the other piston which supports a car of mass 3000kg is ($g = 9.8\text{m/s}^2$)

(A) 1.96m^2

(B) 2.94m^2

(C) 3.92m^2

(D) 5.88m^2

Correct Answer: (B) 2.94m^2

Solution:

Step 1: Understanding the Concept:

By Pascal law, pressure applied to an enclosed liquid is transmitted equally, so $\frac{F_1}{A_1} = \frac{F_2}{A_2}$.

Step 2: Key Formula or Approach:

The load on the second piston is the weight of the car: $F_2 = mg = 3000 \times 9.8 = 29400$ N.

Step 3: Detailed Explanation:

Pressure in the liquid: $P = \frac{250}{2.5\times10^{-2}} = 10^4$ Pa.

$$A_2 = \frac{F_2}{P} = \frac{29400}{10^4} = 2.94\ \text{m}^2$$

The other options correspond to different car masses (2000, 4000 and 6000 kg).

Final Answer:

The area of the larger piston is 2.94 m^2, option (B).

$$\boxed{2.94\ m^2}$$

Quick Tip: Pressure is the same in the fluid, so F1/A1 = F2/A2.

13. Two spherical black bodies of radii R_1 and R_2 having surface temperature T_1 and T_2 respectively, radiate the same power. The ratio of R_1 to R_2 will be

(A) $(\frac{T_1}{T_2})^2$

(B) $(\frac{T_1}{T_2})^4$

(C) $(\frac{T_2}{T_1})^2$

(D) $(\frac{T_2}{T_1})^4$

Correct Answer: (C) $(\frac{T_2}{T_1})^2$

Solution:

Step 1: Understanding the Concept:

For a black body, the power radiated is $P = \sigma A T^4$, with $A = 4\pi R^2$ for a sphere.

Step 2: Key Formula or Approach:

Equal powers: $4\pi R_1^2 \sigma T_1^4 = 4\pi R_2^2 \sigma T_2^4$.

Step 3: Detailed Explanation:

$R_1^2 T_1^4 = R_2^2 T_2^4$.

$\frac{R_1^2}{R_2^2} = \frac{T_2^4}{T_1^4}$.

$$\frac{R_1}{R_2} = \left(\frac{T_2}{T_1}\right)^2$$

The hotter body must be smaller to radiate the same power. Options A and B have the temperatures in the wrong order, and option D has the wrong power.

Final Answer:

$\frac{R_1}{R_2} = \left(\frac{T_2}{T_1}\right)^2$, option (C).

$$\boxed{\left(\frac{T_2}{T_1}\right)^2}$$

Quick Tip: Use Stefan law: power is sigma times area times T to the fourth.

14. The ratio of thermal conductivity of two rods of different materials is 4 : 3. The two rods have same area of cross-section and same thermal resistance. The lengths of rods are in the ratio

(A) 5 : 4

(B) 4 : 3

(C) 1 : 5

(D) 7 : 1

Correct Answer: (B) 4 : 3

Solution:

Step 1: Understanding the Concept:

The thermal resistance of a rod is $R = \frac{L}{kA}$, where k is the thermal conductivity and A the cross-section.

Step 2: Key Formula or Approach:

Equal R and equal A give $\frac{L_1}{k_1} = \frac{L_2}{k_2}$.

Step 3: Detailed Explanation:

$$\frac{L_1}{L_2} = \frac{k_1}{k_2} = \frac{4}{3}$$

So the lengths are in the ratio 4 : 3.

A more conducting material needs a proportionally longer rod to give the same resistance.

Final Answer:

The lengths are in the ratio 4 : 3, option (B).

$$\boxed{4:3}$$

Quick Tip: Thermal resistance is length over (conductivity times area).

15. In thermodynamic process for free expansion, select the **'WRONG'** statement out of the following options.

(A) Free expansions are adiabatic expansions and there is no exchange of heat between system and environment.

(B) A free expansion can be plotted on a p-V diagram.

(C) There is no work done on the system or by the system.

(D) It is an instantaneous change and the system is not in thermodynamic equilibrium.

Correct Answer: (B) A free expansion can be plotted on a p-V diagram.

Solution:

Step 1: Understanding the Concept:
In free expansion, a gas expands suddenly into a vacuum. It is rapid and irreversible, so the gas passes through non-equilibrium states.

Step 2: Key Formula or Approach:
A p-V diagram shows only equilibrium states, so only quasi-static (slow) processes can be drawn as curves on it.

Step 3: Detailed Explanation:
Statement A is correct: the expansion is fast, so it is adiabatic and $Q = 0$.
Statement C is correct: the gas pushes against vacuum, so no work is done by or on the system.
Statement D is correct: the change is instantaneous and the intermediate states are not in equilibrium.
Statement B says that a free expansion can be plotted on a p-V diagram. This is the wrong one. Only the initial and final states have well defined p and V, and no curve joins them.

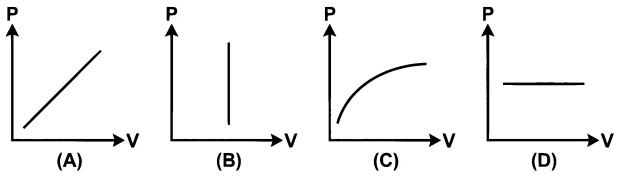
Final Answer:

The wrong statement is B, option (B).

$$\boxed{B}$$

Quick Tip: A free expansion is not a quasi-static process, so it has no path on a p-V diagram.

16. Which of the following graphs between pressure and volume of a gas correctly shows isochoric process in thermodynamics?



(A) (D)

(B) (A)

(C) (C)

(D) (B)

Correct Answer: (D) (B)

Solution:

Step 1: Understanding the Concept:

An isochoric process takes place at constant volume. On a P-V diagram the volume stays fixed while the pressure changes.

Step 2: Key Formula or Approach:

A fixed value of V with P varying is a vertical straight line on a graph

with V on the horizontal axis.

Step 3: Detailed Explanation:
In the figure, graph (A) is a sloping straight line, (B) is a vertical line, (C) is a curve rising to a plateau and (D) is a horizontal line.
Graph (D) is constant pressure, which is isobaric. Graph (B), the vertical line, shows V fixed, so it is isochoric.
In the option list, option D refers to graph (B).

Final Answer:
The isochoric process is the vertical line, graph (B), which is option (D).

Graph (B)

Quick Tip: Isochoric means constant volume.

17. The r.m.s. speed of hydrogen at S.T.P. is 'u' m/s. If the gas is heated at constant pressure till its volume becomes three times, the final temperature of the gas and the r.m.s. speed are respectively

(A) 1092K , $3u$m/s

(B) 1092K , $\frac{u}{3}$m/s

(C) 819K , $\sqrt{3}\,u$m/s

(D) 819K , $\frac{u}{\sqrt{3}}$m/s

Correct Answer: (C) 819K , $\sqrt{3}\,u$m/s

Solution:

Step 1: Understanding the Concept:

At constant pressure, Charles law gives $\frac{V}{T} = \text{constant}$. The r.m.s. speed is $v_{rms} = \sqrt{\frac{3RT}{M}}$, so $v_{rms} \propto \sqrt{T}$.

Step 2: Key Formula or Approach:

S.T.P. means $T_1 = 273$ K. The volume becomes $3V_1$.

Step 3: Detailed Explanation:

$T_2 = 3T_1 = 3 \times 273 = 819$ K.

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}} = \sqrt{3}$$

So the new r.m.s. speed is $\sqrt{3}u$.

Options A and B use 1092 K, which is 4×273, and options B and D divide the speed instead of increasing it.

Final Answer:

The final temperature is 819 K and the speed is $\sqrt{3}u$, option (C).

$$\boxed{819 \text{ K}, \sqrt{3}u}$$

Quick Tip: At constant pressure V is proportional to T, and r.m.s. speed is proportional to the square root of T.

18. An ideal gas is heated from 27°C to 627°C at constant pressure. If initial volume of gas is 4m^3, then the new volume of the gas will be

(A) $12m^3$

(B) $6m^3$

(C) $3m^3$

(D) $2m^3$

Correct Answer: (A) $12m^3$

Solution:

Step 1: Understanding the Concept:

At constant pressure the volume of a fixed amount of an ideal gas is proportional to the absolute temperature: $\frac{V_1}{T_1} = \frac{V_2}{T_2}$.

Step 2: Key Formula or Approach:

Convert to kelvin: $T_1 = 27 + 273 = 300$ K and $T_2 = 627 + 273 = 900$ K.

Step 3: Detailed Explanation:

$$V_2 = V_1 \frac{T_2}{T_1} = 4 \times \frac{900}{300} = 12 \text{ m}^3$$

Using the Celsius values directly would give $4 \times \frac{627}{27} = 92.9$, which is wrong, so the kelvin scale must be used. The values 6, 3 and 2 m^3 correspond to volume ratios of 1.5, 0.75 and 0.5, which do not match a temperature ratio of 3.

Final Answer:

The new volume is 12 m^3, option (A).

$$\boxed{12 \text{ m}^3}$$

Quick Tip: Use Charles law with temperatures in kelvin.

19.

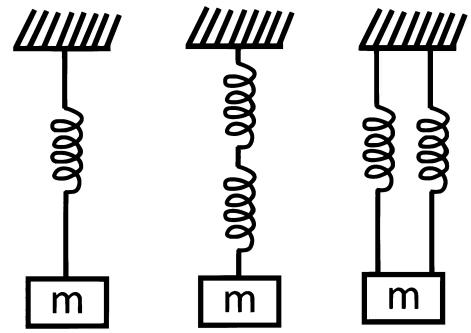


Fig.(a) Fig.(b) Fig.(c)

All the springs in fig (a), (b) and (c) are identical, each one having force constant K. Mass m is attached to each system. If T_a, T_b and T_c are the periodic time of oscillations of the three systems in fig (a), (b) and (c) respectively, then

(A) $T_a = \sqrt{2}\,T_b$

(B) $T_b = 2T_a$

(C) $T_a = \frac{T_c}{\sqrt{2}}$

(D) $T_b = 2T_c$

Correct Answer: (D) $T_b = 2T_c$

Solution:

Step 1: Understanding the Concept:

For a mass on a spring, $T = 2\pi\sqrt{\frac{m}{k_{eff}}}$. The figure shows (a) one

spring, (b) two springs one below the other (series) and (c) two springs side by side (parallel).

Step 2: Key Formula or Approach:

Series: $\frac{1}{k_{eff}} = \frac{1}{K} + \frac{1}{K}$, so $k_b = \frac{K}{2}$. Parallel: $k_c = K + K = 2K$. Single: $k_a = K$.

Step 3: Detailed Explanation:

$T_a = 2\pi\sqrt{\frac{m}{K}}$.

$T_b = 2\pi\sqrt{\frac{2m}{K}} = \sqrt{2}\,T_a$.

$T_c = 2\pi\sqrt{\frac{m}{2K}} = \frac{T_a}{\sqrt{2}}$.

Now $\frac{T_b}{T_c} = \frac{\sqrt{2}\,T_a}{T_a/\sqrt{2}} = 2$, so $T_b = 2T_c$.

Option A would need $T_a = \sqrt{2}\,T_b$, but T_b is larger. Option B would need $T_b = 2T_a$, but it is $\sqrt{2}\,T_a$. Option C would need $T_a = \frac{T_c}{\sqrt{2}}$, but $T_a = \sqrt{2}\,T_c$.

Final Answer:

$T_b = 2T_c$, option (D).

$$\boxed{T_b = 2T_c}$$

Quick Tip: Find the effective spring constant for each arrangement.

20. A particle is performing simple harmonic motion about $x = 0$ with an amplitude 'a' and periodic time T. The speed of the particle at $x = \frac{a}{3}$ will be

(A) $\frac{2\pi a}{T}$

(B) $\frac{4\pi a}{3T}$

(C) $\frac{4\sqrt{2}\,\pi a}{3T}$

(D) $\frac{\sqrt{3}\,\pi^2 a}{2T}$

Correct Answer: (C) $\frac{4\sqrt{2}\,\pi a}{3T}$

Solution:

Step 1: Understanding the Concept:

In SHM, the speed at displacement x is $v = \omega\sqrt{a^2 - x^2}$, with $\omega = \frac{2\pi}{T}$.

Step 2: Key Formula or Approach:

Put $x = \frac{a}{3}$.

Step 3: Detailed Explanation:

$a^2 - x^2 = a^2 - \frac{a^2}{9} = \frac{8a^2}{9}$, so $\sqrt{a^2 - x^2} = \frac{2\sqrt{2}\,a}{3}$.

$$v = \frac{2\pi}{T} \times \frac{2\sqrt{2}\,a}{3} = \frac{4\sqrt{2}\,\pi a}{3T}$$

Option A, $\frac{2\pi a}{T}$, is the maximum speed at the mean position.

Final Answer:

The speed is $\frac{4\sqrt{2}\pi a}{3T}$, option (C).

$$\boxed{\frac{4\sqrt{2}\,\pi a}{3T}}$$

Quick Tip: Use v = omega times the square root of (a squared minus x squared).

21. Two sources of sound are emitting progressive waves $y_1 = 4sin708\,\pi t$ and $y_2 = 3sin700\,\pi t$. The sources are placed close to each other. The number of beats heard per second and intensity ratio between waxing and wanning are respectively

(A) 4 , 16 : 9

(B) 8 , 16 : 9

(C) 8 , 49 : 1

(D) 4 , 49 : 1

Correct Answer: (D) 4 , 49 : 1

Solution:

Step 1: Understanding the Concept:

For $y = A\sin\omega t$, the frequency is $f = \frac{\omega}{2\pi}$. Beats per second equal the difference in frequency. Intensity is proportional to the square of the amplitude.

Step 2: Key Formula or Approach:

Maximum amplitude $= A_1 + A_2$ and minimum amplitude $= A_1 - A_2$.

Step 3: Detailed Explanation:

$f_1 = \frac{708\pi}{2\pi} = 354$ Hz and $f_2 = \frac{700\pi}{2\pi} = 350$ Hz.

Beats per second $= 354 - 350 = 4$.

$$\frac{I_{max}}{I_{min}} = \frac{(4+3)^2}{(4-3)^2} = \frac{49}{1}$$

The ratio 16 : 9 is the ratio of the intensities of the two sources, not of the maximum and minimum intensity.

Final Answer:

There are 4 beats per second and the intensity ratio is 49 : 1, option (D).

$$\boxed{4,\ 49:1}$$

Quick Tip: Frequencies come from the angular frequencies, and intensity goes as amplitude squared.

22. Fifth overtone of an open pipe of length L_0 is in unison with the fifth overtone of the pipe closed at one end of length L_c.
The ratio L_0 to L_c is

(A) 11 : 6

(B) 11 : 12

(C) 6 : 11

(D) 12 : 11

Correct Answer: (D) 12 : 11

Solution:

Step 1: Understanding the Concept:

An open pipe supports all harmonics: $f_n = \frac{nv}{2L}$, so the fifth overtone is the 6th harmonic. A closed pipe supports only odd harmonics: $f = \frac{(2n+1)v}{4L}$, so the fifth overtone is the 11th harmonic.

Step 2: Key Formula or Approach:

Open: $f_o = \frac{6v}{2L_o} = \frac{3v}{L_o}$. Closed: $f_c = \frac{11v}{4L_c}$.

Step 3: Detailed Explanation:

Set $f_o = f_c$: $\frac{3}{L_o} = \frac{11}{4L_c}$.

So $\frac{L_o}{L_c} = \frac{3\times4}{11} = \frac{12}{11}$.

Option C, 6 : 11, would result from comparing harmonics without including the factor of 4 for the closed pipe, and option A uses wrong harmonic numbers.

Final Answer:

The ratio $L_o : L_c$ is 12 : 11, option (D).

$$\boxed{12 : 11}$$

Quick Tip: Find the harmonic numbers for the fifth overtone in each type of pipe.

23. The lengths of the two pipes open at both ends are L and $(L + L_1)$. If they are sounded together, the beat frequency will be
(v = velocity of sound in air)

(A) $\frac{vL_1}{L(L+L_1)}$

(B) $\frac{2vL_1}{L(L+L_1)}$

(C) $\frac{2L(L+L_1)}{vL_1}$

(D) $\frac{vL_1}{2L(L+L_1)}$

Correct Answer: (D) $\frac{vL_1}{2L(L+L_1)}$

Solution:

Step 1: Understanding the Concept:

The fundamental frequency of an open pipe of length L is $f = \frac{v}{2L}$. Beat frequency is the difference between the two frequencies.

Step 2: Key Formula or Approach:

$f_1 = \frac{v}{2L}$ and $f_2 = \frac{v}{2(L+L_1)}$. The shorter pipe has the higher frequency.

Step 3: Detailed Explanation:

$$f_b = f_1 - f_2 = \frac{v}{2}\left[\frac{1}{L} - \frac{1}{L+L_1}\right] = \frac{v}{2} \cdot \frac{L_1}{L(L+L_1)} = \frac{vL_1}{2L(L+L_1)}$$

Option A is missing the factor of 2 that comes from the open pipe formula. Option B has an extra factor of 2 in the numerator, which does not come out of the algebra.

Final Answer:

The beat frequency is $\frac{vL_1}{2L(L+L_1)}$, option (D).

$$\boxed{\frac{vL_1}{2L(L+L_1)}}$$

Quick Tip: Fundamental frequency of an open pipe is v/2L; subtract the two frequencies.

24. The number of waves counted by an observer on the sea-coast in one minute is 54. If the wavelength of the waves is 5m then the velocity of the waves in m/s will be

(A) 4.5

(B) 5.4

(C) 6.3

(D) 7.2

Correct Answer: (A) 4.5

Solution:

Step 1: Understanding the Concept:

The speed of a wave is $v = f\lambda$, where f is the number of waves passing a point each second.

Step 2: Key Formula or Approach:

54 waves are counted in one minute, which is 60 s.

Step 3: Detailed Explanation:

$f = \frac{54}{60} = 0.9$ Hz.

$$v = f\lambda = 0.9 \times 5 = 4.5 \text{ m/s}$$

The wavelength is in metres and the frequency is in hertz, so the product comes out directly in m/s. Check by distance: $54 \times 5 = 270$ m of wave pattern passes in 60 s, which is 4.5 m each second.

Final Answer:

The wave speed is 4.5 m/s, option (A).

$$\boxed{4.5 \text{ m/s}}$$

Quick Tip: Wave speed is frequency times wavelength.

25. When a dipole placed parallel to electric field is rotating through π^c, the work done is W. The work done in rotating the dipole through $(\frac{\pi}{3})^c$ is $(cos0° = 1, cos60° = \frac{1}{2}, cos180° = -1)$

(A) $\frac{W}{4}$

(B) $\frac{W}{2}$

(C) W

(D) $2W$

Correct Answer: (A) $\frac{W}{4}$

Solution:

Step 1: Understanding the Concept:

The work done in rotating a dipole from angle θ_1 to θ_2 against the field is $W = pE(\cos\theta_1 - \cos\theta_2)$.

Step 2: Key Formula or Approach:

The dipole starts parallel to the field, so $\theta_1 = 0$ and $\cos\theta_1 = 1$.

Step 3: Detailed Explanation:

Rotation through π: $W = pE(\cos 0° - \cos 180°) = pE(1 + 1) = 2pE$.

Rotation through $\frac{\pi}{3}$: $W' = pE(\cos 0° - \cos 60°) = pE\left(1 - \frac{1}{2}\right) = \frac{pE}{2}$.

$$\frac{W'}{W} = \frac{pE/2}{2pE} = \frac{1}{4}$$

So $W' = \frac{W}{4}$.

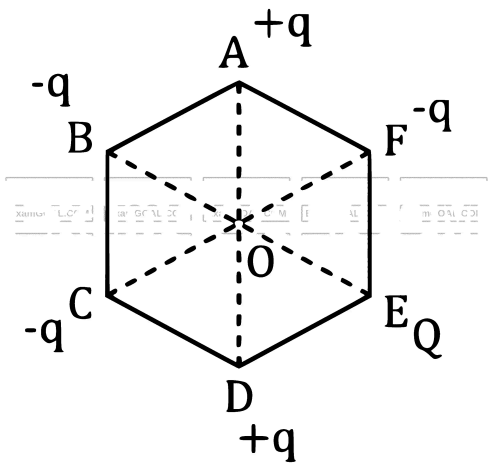
Final Answer:

The work done is $\frac{W}{4}$, option (A).

$$\boxed{\frac{W}{4}}$$

Quick Tip: Use W = pE(cos theta1 - cos theta2) with the dipole starting parallel to the field.

26. The point charges $+q$, $-q$, $-q$, $+q$, $+Q$ and $-q$ are placed at the vertices of a regular hexagon ABCDEF as shown in the figure.



The electric field at the centre of the hexagon 'O' due to the five charges at A, B, C, D and F is thrice the electric field at centre 'O' due to charge $+Q$ at E alone. The value of Q is

(A) $\frac{q}{3}$

(B) $\frac{q}{4}$

(C) $3q$

(D) $4q$

Correct Answer: (A) $\frac{q}{3}$

Solution:

Step 1: Understanding the Concept:

The six charges sit at the corners of a regular hexagon, all at the same distance r from the centre O. The field of each point charge at O has magnitude $\frac{kq}{r^2}$, pointing away from a positive charge and toward a negative charge.

Step 2: Key Formula or Approach:

From the figure: A $= +q$, B $= -q$, C $= -q$, D $= +q$, E $= +Q$, F $= -q$.

Corners A and D are opposite each other, as are B and E, and C and F.

Step 3: Detailed Explanation:

A and D are both $+q$ and opposite, so their fields at O are equal and opposite, and cancel.

C and F are both $-q$ and opposite, so they cancel too.

Only B is left from the five charges. Its field is $\frac{kq}{r^2}$, directed from O toward B.

The field of $+Q$ at E alone is $\frac{kQ}{r^2}$, directed away from E, which is also toward B.

Given: field of the five charges $= 3\times$ field of Q alone.

$$\frac{kq}{r^2} = 3 \cdot \frac{kQ}{r^2} \Rightarrow Q = \frac{q}{3}$$

Final Answer:

The value of Q is $\frac{q}{3}$, option (A).

$$\boxed{\frac{q}{3}}$$

Quick Tip: Charges at opposite corners of the hexagon cancel or add in pairs at the centre.

27. In an oscillating LC circuit, the maximum charge on the capacitor is Q. When the energy is stored equally between the electric and magnetic fields, the charge on the capacitor becomes

(A) $\frac{Q}{\sqrt{2}}$

(B) $Q\sqrt{3}$

(C) $\frac{Q}{2}$

(D) $\frac{Q}{4}$

Correct Answer: (A) $\frac{Q}{\sqrt{2}}$

Solution:

Step 1: Understanding the Concept:

In an LC circuit, energy moves between the capacitor (electric energy $\frac{q^2}{2C}$) and the inductor (magnetic energy $\frac{1}{2}Li^2$). The total is constant and equals the maximum electric energy $\frac{Q^2}{2C}$.

Step 2: Key Formula or Approach:

When the energy is shared equally, the electric energy is half the total.

Step 3: Detailed Explanation:

$\frac{q^2}{2C} = \frac{1}{2} \cdot \frac{Q^2}{2C}$.

So $q^2 = \frac{Q^2}{2}$.

$$q = \frac{Q}{\sqrt{2}}$$

Option C, $\frac{Q}{2}$, would give only a quarter of the maximum electric energy.

Final Answer:
The charge is $\frac{Q}{\sqrt{2}}$, option (A).

$$\boxed{\frac{Q}{\sqrt{2}}}$$

Quick Tip: Total energy is conserved; equal sharing means each field holds half.

28. Two circular plates each of radius 'r' are kept parallel to each other distance 'd' apart. The capacitance of the capacitor formed is 'C_1'. If the radius of each of the plates is increased to $\sqrt{3}$ times the earlier radius and their distance of separation decreased to half the initial value, the capacitance now becomes 'C_2'. The ratio $C_1 : C_2$ is

(A) 1 : 2

(B) 1 : 4

(C) 1 : 6

(D) 6 : 1

Correct Answer: (C) 1 : 6

Solution:

Step 1: Understanding the Concept:

For a parallel plate capacitor, $C = \frac{\varepsilon_0 A}{d}$. For circular plates, $A = \pi r^2$.

Step 2: Key Formula or Approach:

New radius $= \sqrt{3}r$, so the new area is $\pi \cdot 3r^2 = 3A$. New separation $= \frac{d}{2}$.

Step 3: Detailed Explanation:

$$C_2 = \frac{\varepsilon_0(3A)}{d/2} = 6 \cdot \frac{\varepsilon_0 A}{d} = 6C_1$$

So $\frac{C_1}{C_2} = \frac{1}{6}$, which is $1 : 6$.

Option D, $6 : 1$, is the inverse ratio.

Final Answer:

$C_1 : C_2 = 1 : 6$, option (C).

$$\boxed{1 : 6}$$

Quick Tip: Capacitance is proportional to area and inversely proportional to separation.

29. A capacitor of capacitance C has charge Q and energy stored in it is W. If the charge is increased to 3Q, the energy stored in the capacitor W' will be

(A) $9W$

(B) $3W$

(C) $\frac{W}{3}$

(D) $\frac{W}{6}$

Correct Answer: (A) $9W$

Solution:

Step 1: Understanding the Concept:

The energy stored in a capacitor is $W = \frac{Q^2}{2C}$. With C fixed, W is proportional to Q^2.

Step 2: Key Formula or Approach:

New charge $= 3Q$.

Step 3: Detailed Explanation:

$$W' = \frac{(3Q)^2}{2C} = 9 \cdot \frac{Q^2}{2C} = 9W$$

Option B, $3W$, would be the result if energy were proportional to Q, which is not so.

Since the capacitance C does not change, energy depends on charge alone. A common slip is to treat energy as proportional to Q (giving $3W$), but energy is quadratic in Q, so tripling the charge gives nine times the energy.

Final Answer:

The new energy is $9W$, option (A).

$\boxed{9W}$

Quick Tip: Energy in terms of charge is Q squared over 2C.

30. A potentiometer wire of length **4m** and resistance **5Ω** is connected in series with a resistance of **992Ω** and a cell of e.m.f. **4V** with internal resistance **3Ω**. The length of **0.75m** on potentiometer wire balances the e.m.f. of

(A) 2.50mV

(B) 3mV

(C) 3.75mV

(D) 4mV

Correct Answer: (C) 3.75mV

Solution:

Step 1: Understanding the Concept:

In a potentiometer the wire carries a steady current from the driving cell. The potential drop is uniform along the wire, so the balancing length gives the unknown emf.

Step 2: Key Formula or Approach:

Total resistance in the circuit $= 5 + 992 + 3 = 1000\ \Omega$ (wire, series resistor and internal resistance). Current $I = \frac{4}{1000} = 4$ mA.

Step 3: Detailed Explanation:

Potential drop across the whole wire: $V = I \times 5 = 4 \times 10^{-3} \times 5 = 20$

mV.

Potential gradient $= \frac{20\,\text{mV}}{4\,\text{m}} = 5\,\text{mV/m}$.

$$E = 5 \times 0.75 = 3.75\,\text{mV}$$

Final Answer:

The balanced emf is 3.75 mV, option (C).

$$\boxed{3.75\,\text{mV}}$$

Quick Tip: Find the current in the wire, then the potential gradient.

31. By connecting a resistance of 980Ω in series with a galvanometer, it is converted into a voltmeter of a certain range. When the resistance of 470Ω is connected in series, the range is halved. The resistance of the galvanometer is

(A) 30Ω

(B) 40Ω

(C) 50Ω

(D) 60Ω

Correct Answer: (B) 40Ω

Solution:

Step 1: Understanding the Concept:

A galvanometer of resistance G and full scale current I_g becomes a voltmeter of range $V = I_g(G + R)$ when a resistance R is put in series.

Step 2: Key Formula or Approach:

First case: $V_1 = I_g(G + 980)$. Second case: $V_2 = I_g(G + 470)$, with $V_2 = \frac{V_1}{2}$.

Step 3: Detailed Explanation:

$G + 470 = \frac{1}{2}(G + 980)$.

$2G + 940 = G + 980$.

$$G = 40\ \Omega$$

Check: $V_1 \propto 1020$ and $V_2 \propto 510$, which is half.

Final Answer:

The resistance of the galvanometer is $40\ \Omega$, option (B).

$$\boxed{40\ \Omega}$$

Quick Tip: The range V = Ig (G + R) halves when the series resistance changes from 980 to 470 ohm.

32. The region inside a current carrying toroid is filled with a material (Niobium) having susceptibility $\chi = 2.6 \times 10^{-5}$. The percentage increase in the magnetic field in the presence of Niobium over that without it, is

(A) 2.6×10^{-5}

(B) 2.6×10^{-4}

(C) 2.6×10^{-3}

(D) 2.6×10^{2}

Correct Answer: (C) 2.6×10^{-3}

Solution:

Step 1: Understanding the Concept:

Inside a toroid the field without a material is $B_0 = \mu_0 nI$. With a material of susceptibility χ, the field becomes $B = \mu_0(1 + \chi)nI = (1 + \chi)B_0$.

Step 2: Key Formula or Approach:

The fractional increase is $\frac{B-B_0}{B_0} = \chi$.

Step 3: Detailed Explanation:

$$\% \text{ increase} = \chi \times 100 = 2.6 \times 10^{-5} \times 100 = 2.6 \times 10^{-3}\%$$

Option A, 2.6×10^{-5}, is the fractional increase and has not been converted to a percentage.

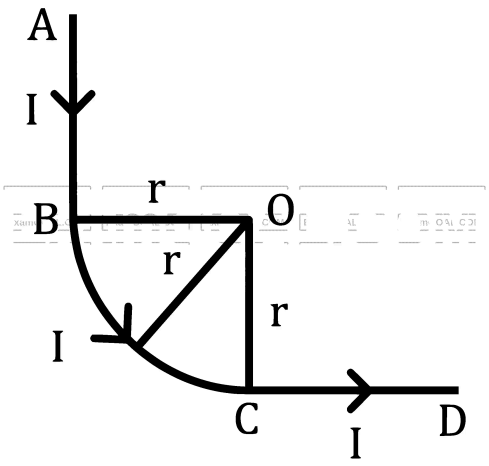
Final Answer:

The increase is $2.6 \times 10^{-3}\%$, option (C).

$$\boxed{2.6 \times 10^{-3}}$$

Quick Tip: The field with a material is B = muo (1 + chi) n I.

33. In the following figure, the magnitude of the magnetic field at point 'O' will be



(A) $\frac{\mu_0}{4\pi}\frac{I}{r}(\frac{2}{\pi}+2)$

(B) $\frac{\mu_0}{4\pi}\frac{I}{r}(\frac{2}{\pi}-2)$

(C) $\frac{\mu_0}{4\pi}\frac{I}{r}(2+\frac{\pi}{2})$

(D) $\frac{\mu_0}{4\pi}\frac{I}{r}(2-\frac{\pi}{2})$

Correct Answer: (C) $\frac{\mu_0}{4\pi}\frac{I}{r}(2+\frac{\pi}{2})$

Solution:

Step 1: Understanding the Concept:

The wire comes down along AB, turns through a quarter circle of radius r centred at O from B to C, and goes off along CD. The point O is at a distance r from both straight parts, and the foot of the perpendicular from O to AB is the end point B (and to CD it is the end point C).

Step 2: Key Formula or Approach:

A straight wire that ends at the foot of the perpendicular gives $B = \frac{\mu_0}{4\pi}\frac{I}{r}$. A circular arc subtending angle θ at the centre gives $B = \frac{\mu_0}{4\pi}\frac{I\theta}{r}$.

Step 3: Detailed Explanation:

Straight wire AB: $B_1 = \frac{\mu_0}{4\pi}\frac{I}{r}$.

Quarter circle BC with $\theta = \frac{\pi}{2}$: $B_2 = \frac{\mu_0}{4\pi}\frac{I}{r} \cdot \frac{\pi}{2}$.

Straight wire CD: $B_3 = \frac{\mu_0}{4\pi}\frac{I}{r}$.

By the right hand rule, all three fields at O point out of the page: the current goes down on the left of O, counterclockwise along the arc, and to the right below O.

$$B = \frac{\mu_0}{4\pi}\frac{I}{r}\left(1 + \frac{\pi}{2} + 1\right) = \frac{\mu_0}{4\pi}\frac{I}{r}\left(2 + \frac{\pi}{2}\right)$$

Final Answer:

The field at O is $\frac{\mu_0}{4\pi}\frac{I}{r}\left(2 + \frac{\pi}{2}\right)$, option (C).

$$\boxed{\frac{\mu_0}{4\pi}\frac{I}{r}\left(2 + \frac{\pi}{2}\right)}$$

Quick Tip: Add the fields from the two semi-infinite straight wires and the quarter circle arc.

34. The ratio of magnetic field at the centre of the current carrying circular loop and its magnetic moment is 'x'. When both the current and radius are tripled, then the ratio will be

(A) $\frac{x}{27}$

(B) $\frac{x}{9}$

(C) $\frac{x}{3}$

(D) $3x$

Correct Answer: (A) $\frac{x}{27}$

Solution:

Step 1: Understanding the Concept:

For a circular loop of radius R carrying current I: the field at the centre is $B = \frac{\mu_0 I}{2R}$, and the magnetic moment is $m = I\pi R^2$.

Step 2: Key Formula or Approach:

The ratio is $x = \frac{B}{m} = \frac{\mu_0 I/2R}{I\pi R^2} = \frac{\mu_0}{2\pi R^3}$.

Step 3: Detailed Explanation:

The ratio depends only on the radius, as $\frac{1}{R^3}$.

When the radius is tripled, the ratio becomes $\frac{1}{3^3} = \frac{1}{27}$ of its old value.

$$x' = \frac{x}{27}$$

The current is tripled too, but it cancels from the ratio, so it has no effect.

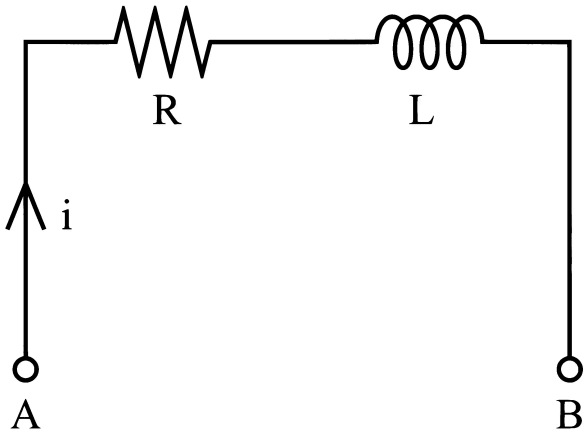
Final Answer:

The new ratio is $\frac{x}{27}$, option (A).

$$\boxed{\frac{x}{27}}$$

Quick Tip: Write both quantities in terms of I and R; the current cancels in the ratio.

35. In the circuit shown, when the current 'i' is 3A and increasing at the rate of 1A/s the measurement of the potential difference between A and B is 12V. But when the same current 3A is decreasing at the rate of 1A/s, the measured potential difference V_{AB} between A and B is 6V. The value of 'R' in the circuit is



(A) 3Ω

(B) 4Ω

(C) 6Ω

(D) 8Ω

Correct Answer: (A) 3Ω

Solution:

Step 1: Understanding the Concept:

The figure shows a resistor R and an inductor L in series between A and B, with current i flowing from A to B. The potential difference is $V_{AB} = iR + L\frac{di}{dt}$.

Step 2: Key Formula or Approach:

Case 1: $i = 3$ A, increasing at 1 A/s: $12 = 3R + L(1)$. Case 2: $i = 3$ A, decreasing at 1 A/s: $6 = 3R - L(1)$.

Step 3: Detailed Explanation:

Add the two equations: $18 = 6R$.

$$R = 3\ \Omega$$

Then $L = 12 - 9 = 3$ H, and the second equation gives $9 - 3 = 6$, which matches.

Final Answer:

The resistance is $3\ \Omega$, option (A).

$$\boxed{3\ \Omega}$$

Quick Tip: The voltage across R and L together is iR plus L di/dt, with the sign of di/dt flipping.

36. Two coils have a mutual inductance of 0.005H. The current changes in the first coil according to equation $I = I_0 sin\omega t$, where $I_0 = 10$A and $\omega = 60\pi$rad s^{-1}. The maximum values of e.m.f. in the second coil in volt will be

(A) 2π

(B) 3π

(C) 4π

(D) 6π

Correct Answer: (B) 3π

Solution:

Step 1: Understanding the Concept:

The emf induced in the second coil is $\varepsilon = M\frac{dI}{dt}$ in magnitude.

Step 2: Key Formula or Approach:

$I = I_0 \sin \omega t$ gives $\frac{dI}{dt} = I_0\omega \cos \omega t$, with maximum value $I_0\omega$.

Step 3: Detailed Explanation:

$$\varepsilon_{max} = MI_0\omega = 0.005 \times 10 \times 60\pi = 3\pi \text{ V}$$

Options A, C and D differ by a wrong factor in the arithmetic.

Final Answer:

The maximum emf is 3π V, option (B).

$$\boxed{3\pi \text{ V}}$$

Quick Tip: The induced emf is M times the rate of change of current.

37. The magnetic potential energy stored in a certain inductor is **49mJ** when the current in the inductor is **70mA**. The inductance of the inductor is

(A) 0.20H

(B) 2.0H

(C) 20H

(D) 200H

Correct Answer: (C) 20H

Solution:

Step 1: Understanding the Concept:

The energy stored in an inductor is $U = \frac{1}{2}LI^2$.

Step 2: Key Formula or Approach:

Rearrange: $L = \frac{2U}{I^2}$. Use SI units: $U = 49$ mJ $= 0.049$ J and $I = 70$ mA $= 0.07$ A.

Step 3: Detailed Explanation:

$$L = \frac{2 \times 0.049}{(0.07)^2} = \frac{0.098}{0.0049} = 20 \text{ H}$$

The options 0.2, 2.0 and 200 H differ by powers of 10 and come from slips in converting milli units.

Final Answer:

The inductance is 20 H, option (C).

$$\boxed{20 \text{ H}}$$

Quick Tip: Use U = (1/2) L I squared.

38. An a.c. source is applied to a series LR circuit with $X_L = 3R$ and power factor is X_1. Now a capacitor with $X_c = R$ is added in series to LR circuit and power factor is X_2. The ratio X_1 to X_2 is

(A) $2:1$

(B) $1:2$

(C) $\sqrt{2}:1$

(D) $1:\sqrt{2}$

Correct Answer: (D) $1:\sqrt{2}$

Solution:

Step 1: Understanding the Concept:

The power factor of a series circuit is $\cos\phi = \frac{R}{Z}$, where $Z = \sqrt{R^2 + X^2}$ and X is the net reactance.

Step 2: Key Formula or Approach:

LR circuit: $X = X_L = 3R$. LCR circuit: $X = X_L - X_C = 3R - R = 2R$.

Step 3: Detailed Explanation:

$X_1 = \frac{R}{\sqrt{R^2+9R^2}} = \frac{1}{\sqrt{10}}$.

$X_2 = \frac{R}{\sqrt{R^2+4R^2}} = \frac{1}{\sqrt{5}}$.

$$\frac{X_1}{X_2} = \frac{\sqrt{5}}{\sqrt{10}} = \frac{1}{\sqrt{2}}$$

So the ratio is $1:\sqrt{2}$.

Final Answer:

The ratio $X_1 : X_2$ is $1:\sqrt{2}$, option (D).

$$\boxed{1:\sqrt{2}}$$

Quick Tip: Power factor is R over impedance; the net reactance changes when the capacitor is added.

39. In an a.c. circuit, a resistance R is connected in series with an inductance 'L'. If phase angle between voltage and current is 45°, the value of inductive reactance will be

$(sin45° = \frac{1}{\sqrt{2}} = cos45°)$

(A) $2R$

(B) R

(C) $\sqrt{2}R$

(D) $\frac{1}{\sqrt{2}}R$

Correct Answer: (B) R

Solution:

Step 1: Understanding the Concept:

In a series LR circuit the phase angle between voltage and current satisfies $\tan\phi = \frac{X_L}{R}$.

Step 2: Key Formula or Approach:

Here $\phi = 45°$, and $\tan 45° = \frac{\sin 45°}{\cos 45°} = 1$.

Step 3: Detailed Explanation:

$$\frac{X_L}{R} = 1 \Rightarrow X_L = R$$

Option C, $\sqrt{2}R$, is the impedance $Z = \sqrt{R^2 + X_L^2}$ and not the

reactance.
In the phasor diagram, the voltage across R and the voltage across L are equal in size when $X_L = R$, so the resultant voltage leads the current by $45°$.

Final Answer:
The inductive reactance is R, option (B).

$$\boxed{R}$$

Quick Tip: For an LR series circuit, tan of the phase angle equals X_L over R.

40. A simple microscope is a combination of two lenses in contact. Their powers are $+20$D and -4D. The distance of distinct vision is 25cm. When seen through the microscope, the size of the image of an object 3mm high is

(A) 1.8cm

(B) 1.5cm

(C) 1.2cm

(D) 0.6cm

Correct Answer: (B) 1.5cm

Solution:

Step 1: Understanding the Concept:
Two thin lenses in contact have a combined power equal to the sum: $P = P_1 + P_2$. A simple microscope used with the image at the least

distance of distinct vision has magnification $m = 1 + \frac{D}{f}$.

Step 2: Key Formula or Approach:

$P = 20 - 4 = 16$ D, so $f = \frac{1}{16}$ m $= 6.25$ cm.

Step 3: Detailed Explanation:

$$m = 1 + \frac{25}{6.25} = 1 + 4 = 5$$

Image size $= 5 \times 3$ mm $= 15$ mm $= 1.5$ cm.

The value 1.8 cm would need $m = 6$, that is $f = 5$ cm, which does not match the combined power of 16 D.

Final Answer:

The image is 1.5 cm high, option (B).

$$\boxed{1.5 \text{ cm}}$$

Quick Tip: Add the powers to find the focal length, then use m = 1 + D/f for the image at the near point.

41. When a glass plate of refractive index 1.44 is introduced in the path of one of the interfering beams, the fringes are displaced by a distance 'y'. If this plate is replaced by another plate of same thickness but of refractive index 1.66, the fringes will be displaced by a distance

(A) $\frac{2y}{3}$

(B) $\frac{4y}{5}$

(C) $\frac{5y}{4}$

(D) $\frac{3y}{2}$

Correct Answer: (D) $\frac{3y}{2}$

Solution:

Step 1: Understanding the Concept:

When a thin transparent plate of thickness t and index μ covers one slit, the fringes shift by $\Delta y = \frac{D}{d}(\mu - 1)t$.

Step 2: Key Formula or Approach:

For the same thickness, geometry and wavelength, the shift is proportional to $(\mu - 1)$.

Step 3: Detailed Explanation:

First plate: $\mu - 1 = 0.44$, shift $= y$.

Second plate: $\mu - 1 = 0.66$.

$$y' = y \times \frac{0.66}{0.44} = \frac{3y}{2}$$

The options $\frac{2y}{3}$ and $\frac{4y}{5}$ are smaller than y, but a higher index must shift the fringes more.

Final Answer:

The new shift is $\frac{3y}{2}$, option (D).

$$\boxed{\frac{3y}{2}}$$

Quick Tip: Fringe displacement is proportional to (mu - 1) t.

42. In a certain region of the screen, number of fringes formed is observed to be 8 in Young's double slit experiment when light of wavelength 6600Å is used. If the light of wavelength 4400Å is used, the number of fringes observed in the same region of the screen will be

(A) 10

(B) 12

(C) 16

(D) 18

Correct Answer: (B) 12

Solution:

Step 1: Understanding the Concept:

The fringe width is $\beta = \frac{\lambda D}{d}$. In a fixed region of the screen of width W, the number of fringes is $N = \frac{W}{\beta}$.

Step 2: Key Formula or Approach:

So $N \propto \frac{1}{\lambda}$, or $N_1\lambda_1 = N_2\lambda_2$.

Step 3: Detailed Explanation:

$$N_2 = N_1\frac{\lambda_1}{\lambda_2} = 8 \times \frac{6600}{4400} = 8 \times 1.5 = 12$$

A shorter wavelength gives narrower fringes, so more of them fit in the same region.

Final Answer:

The number of fringes is 12, option (B).

$$\boxed{12}$$

Quick Tip: Fringe width is proportional to wavelength, so the number in a fixed region is inversely proportional to it.

43. In a Fraunhofer diffraction of a single slit, when a slit is illuminated by a light of wavelength 6480Å, angular width of central maximum is measured. When the slit is illuminated by light of another wavelength 'λ' the angular width decreases by 25%. The value of λ in Å units is

(A) 4220

(B) 4530

(C) 4860

(D) 5230

Correct Answer: (C) 4860

Solution:

Step 1: Understanding the Concept:

In single slit diffraction, the angular width of the central maximum is $2\theta = \frac{2\lambda}{a}$, so for the same slit it is proportional to λ.

Step 2: Key Formula or Approach:

A decrease of 25% means the new width is 75% of the old width.

Step 3: Detailed Explanation:

$\lambda' = 0.75\lambda = 0.75 \times 6480 = 4860$ angstrom.

$$\lambda' = 4860 \text{ angstrom}$$

The other options give decreases of about 35%, 30% and 19%, not 25%.

Final Answer:

The wavelength is 4860 angstrom, option (C).

$$\boxed{4860 \text{ angstrom}}$$

Quick Tip: Angular width of the central maximum is 2 lambda over a, so it is proportional to lambda.

44. If the kinetic energy of a particle is increased to 16 times, the percentage change in the de Broglie wavelength of a particle is

(A) 40%

(B) 60%

(C) 75%

(D) 80%

Correct Answer: (C) 75%

Solution:

Step 1: Understanding the Concept:

$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}}$, so $\lambda \propto \frac{1}{\sqrt{K}}$.

Step 2: Key Formula or Approach:

New kinetic energy $K' = 16K$.

Step 3: Detailed Explanation:

$\lambda' = \frac{\lambda}{\sqrt{16}} = \frac{\lambda}{4}$.

Change $= \lambda - \frac{\lambda}{4} = \frac{3\lambda}{4}$, which is a decrease of 75%.

$$\text{Percentage change} = 75\% \text{ (decrease)}$$

Final Answer:

The wavelength decreases by 75%, option (C).

$$\boxed{75\%}$$

Quick Tip: Wavelength is inversely proportional to the square root of kinetic energy.

45. In case of photoelectric emission from certain metal, the cutoff frequency is ν. If the radiation of frequency 3ν is incident on the metal plate, the maximum possible velocity of the emitted electrons will be (m = mass of electron, h = Planck's constant)

(A) $\sqrt{\frac{h\nu}{2m}}$

(B) $\sqrt{\frac{h\nu}{m}}$

(C) $2(\sqrt{\frac{h\nu}{m}})$

(D) $\sqrt{\frac{2h\nu}{m}}$

Correct Answer: (C) $2\left(\sqrt{\frac{h\nu}{m}}\right)$

Solution:

Step 1: Understanding the Concept:

Einstein photoelectric equation: $hf = \phi + K_{max}$, with $\phi = h\nu_0$. The cutoff frequency here is ν.

Step 2: Key Formula or Approach:

$K_{max} = h(3\nu) - h\nu = 2h\nu$, and $K_{max} = \frac{1}{2}mv^2$.

Step 3: Detailed Explanation:

$\frac{1}{2}mv^2 = 2h\nu$, so $v^2 = \frac{4h\nu}{m}$.

$$v = 2\sqrt{\frac{h\nu}{m}}$$

Option D, $\sqrt{\frac{2h\nu}{m}}$, would correspond to a kinetic energy of $h\nu$ only, which holds if the incident frequency were 2ν.

Final Answer:

The maximum speed is $2\sqrt{\frac{h\nu}{m}}$, option (C).

$$\boxed{2\sqrt{\frac{h\nu}{m}}}$$

Quick Tip: Use Einstein equation with the work function equal to h times the cutoff frequency.

46. The triply ionized beryllium (Be^{+++}) has the same electron orbital radius as that of the ground state of hydrogen. Hence, the energy state of triply ionized beryllium is (Given $Z = 4$ for beryllium)

(A) $n = 4$

(B) $n = 3$

(C) $n = 2$

(D) $n = 1$

Correct Answer: (C) $n = 2$

Solution:

Step 1: Understanding the Concept:

The radius of the nth Bohr orbit of a hydrogen-like ion is $r_n = \frac{n^2 a_0}{Z}$, where a_0 is the radius of the first orbit of hydrogen.

Step 2: Key Formula or Approach:

The ground state of hydrogen has $r = a_0$. For Be^{3+}, $Z = 4$.

Step 3: Detailed Explanation:

Set $\frac{n^2 a_0}{4} = a_0$.

$n^2 = 4$, so $n = 2$.

$$n = 2$$

The state $n = 1$ would have a radius of $\frac{a_0}{4}$, and $n = 3$ would have a radius of $\frac{9a_0}{4}$.

Final Answer:

The energy state is $n = 2$, option (C).

$$\boxed{n = 2}$$

Quick Tip: Bohr radius is proportional to n squared over Z.

47. The angular momentum of the electron in the third Bohr orbit of a hydrogen atom is 'l' so its angular momentum in the fourth Bohr orbit is

(A) $4l$

(B) $(\frac{5}{4})l$

(C) $(\frac{4}{3})l$

(D) $(\frac{3}{2})l$

Correct Answer: (C) $(\frac{4}{3})l$

Solution:

Step 1: Understanding the Concept:

Bohr quantization: $L = \frac{nh}{2\pi}$, so $L \propto n$.

Step 2: Key Formula or Approach:

$L_3 = l$ for $n = 3$. Find L_4 for $n = 4$.

Step 3: Detailed Explanation:

$$\frac{L_4}{L_3} = \frac{4}{3} \Rightarrow L_4 = \frac{4}{3}\, l$$

Option B, $\frac{5}{4}l$, would be wrong; the ratio is of the orbit numbers 4 and 3.

The orbit number n is the only quantity that changes, since h and 2π

are constants. So angular momentum grows in the same proportion as n, going from 3 units to 4 units, a ratio of $\frac{4}{3}$.

Final Answer:
The angular momentum is $\frac{4}{3}\,l$, option (C).

$$\boxed{\frac{4}{3}\,l}$$

Quick Tip: Angular momentum is n h over 2 pi, so it is proportional to n.

48. Which one of the following statements is 'WRONG' regarding LED?

(A) LED's have long lifetime if properly manufactured.

(B) Colours produced by LED's do not fade out.

(C) LED's are energy efficient.

(D) Brightness of light emitted by LED cannot be controlled.

Correct Answer: (D) Brightness of light emitted by LED cannot be controlled.

Solution:

Step 1: Understanding the Concept:
A light emitting diode (LED) is a forward biased p-n junction that gives out light. The brightness depends on the forward current.

Step 2: Key Formula or Approach:
Check each statement against the known properties of an LED.

Step 3: Detailed Explanation:

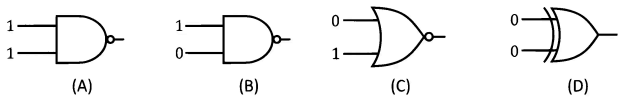
A: properly made LEDs last a long time. True.
B: the colour of an LED is fixed by the semiconductor material, so it does not fade. True.
C: LEDs turn most of the electrical energy into light, so they are efficient. True.
D: says that the brightness cannot be controlled. This is wrong, since changing the current changes the brightness, which is why LEDs can be dimmed.

Final Answer:
The wrong statement is D, option (D).

D

Quick Tip: The brightness of an LED can be controlled through the current.

49. Which of the following logic gates will have an output of '1' (one) for the given inputs?



(A) D

(B) B

(C) A

(D) C

Correct Answer: (B) B

Solution:

Step 1: Understanding the Concept:

From the figure: (A) is a NAND gate with inputs $1, 1$; (B) is a NAND gate with inputs $1, 0$; (C) is a NOR gate with inputs $0, 1$; (D) is an XOR gate with inputs $0, 0$.

Step 2: Key Formula or Approach:

NAND $= \overline{A \cdot B}$, NOR $= \overline{A + B}$, XOR $= A \oplus B$.

Step 3: Detailed Explanation:

(A): $\overline{1 \cdot 1} = \overline{1} = 0$.

(B): $\overline{1 \cdot 0} = \overline{0} = 1$.

(C): $\overline{0 + 1} = \overline{1} = 0$.

(D): $0 \oplus 0 = 0$.

Only gate (B) gives 1. In the answer list, option B is gate (B).

Final Answer:

Gate (B) gives output 1, option (B).

$$\boxed{\text{Gate (B)}}$$

Quick Tip: Evaluate each gate with its inputs.

50. A diode and resistance are connected as shown in figures.

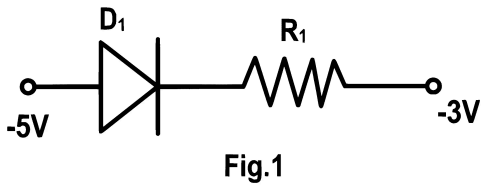


Fig.1

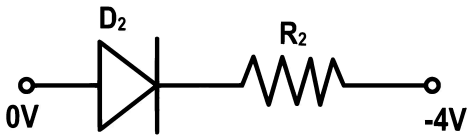


Fig.2

Out of the following statements, which one is true?

Diode in

A) fig (1) and fig (2) are both forward biased.

B) fig (1) and fig (2) are both reverse biased.

C) fig (1) is forward biased and in fig (2) is reverse biased.

D) fig (1) is reverse biased and in fig (2) is forward biased.

(A) D

(B) B

(C) A

(D) C

Correct Answer: (A) D

Solution:

Step 1: Understanding the Concept:

A p-n junction diode is forward biased when the p side (anode, the triangle base) is at a higher potential than the n side (cathode, the bar). Otherwise it is reverse biased.

Step 2: Key Formula or Approach:

In both figures the anode is on the left terminal and the cathode is joined to the resistor on the right.

Step 3: Detailed Explanation:

Fig. 1: the anode is at -5 V and the other end is at -3 V. The anode is at the lower potential ($-5 < -3$), so D_1 is reverse biased.

Fig. 2: the anode is at 0 V and the other end is at -4 V. The anode is at the higher potential ($0 > -4$), so D_2 is forward biased.

So the true statement is: the diode in Fig. 1 is reverse biased and the diode in Fig. 2 is forward biased. This is statement D, which is listed as option A.

Final Answer:

Fig. 1 is reverse biased and Fig. 2 is forward biased, option (A).

Fig. 1 reverse, Fig. 2 forward

Quick Tip: A diode conducts when its anode is at a higher potential than its cathode side.

Chemistry

Questions 51 to 100 | 1 mark each | 50 marks

51. How many moles of potassium chlorate are heated to produce 22.4lit of oxygen at STP ?

(A) $\frac{1}{2}$mole

(B) $\frac{1}{3}$mole

(C) $\frac{1}{4}$mole

(D) $\frac{2}{3}$mole

Correct Answer: (D) $\frac{2}{3}$mole

Solution:

Step 1: Understanding the Concept:

Potassium chlorate decomposes on heating to give potassium chloride and oxygen gas. We need the mole ratio from the balanced equation and the molar volume of a gas at STP.

Step 2: Key Formula or Approach:

1. Balanced equation: $2KClO_3 \rightarrow 2KCl + 3O_2$.
2. At STP, 1 mole of any gas occupies 22.4 L.

Step 3: Detailed Explanation:

Moles of oxygen formed $= \frac{22.4}{22.4} = 1$ mole.

From the equation, 3 moles of O_2 come from 2 moles of $KClO_3$.

So 1 mole of O_2 needs $\frac{2}{3}$ mole of $KClO_3$.

$$n(KClO_3) = 1 \times \frac{2}{3} = \frac{2}{3} \text{ mol}$$

Options A, B and C come from using a wrong mole ratio (for example 1:2 or 1:3 instead of 2:3), so they do not match the balanced equation.

Final Answer:
Heating $\frac{2}{3}$ mole of potassium chlorate gives 22.4 L of oxygen at STP, which is option (D).

$$\boxed{\frac{2}{3}\text{ mole}}$$

Quick Tip: Write the decomposition equation and note that 22.4 L of gas at STP is exactly 1 mole.

52. Which of the following colours of visible light has lowest energy?

(A) Violet

(B) Blue

(C) Yellow

(D) Red

Correct Answer: (D) Red

Solution:

Step 1: Understanding the Concept:
The energy of a photon is $E = h\nu = \frac{hc}{\lambda}$. Energy is directly proportional to frequency and inversely proportional to wavelength.

Step 2: Key Formula or Approach:
Visible light runs from violet (about 400 nm) to red (about 700 nm).

The colour with the longest wavelength has the lowest frequency and so the lowest energy.

Step 3: Detailed Explanation:
Order of wavelength: violet < blue < yellow < red.
So the order of energy is: violet > blue > yellow > red.
Violet and blue have short wavelengths and high frequencies, so they carry more energy per photon. Yellow sits in the middle of the spectrum. Red has the longest wavelength of the four, about 650 – 700 nm, so its photons carry the least energy.

Final Answer:
Red light has the lowest energy among the given colours, option (D).

$$\boxed{\text{Red}}$$

Quick Tip: Energy of a photon is proportional to frequency, so look for the longest wavelength in the visible range.

53. What is formal charge on sulphur in sulphuric acid molecule?

(A) −2

(B) +2

(C) −1

(D) 0

Correct Answer: (D) 0

Solution:

Step 1: Understanding the Concept:

Formal charge compares the electrons an atom owns in a Lewis structure with its valence electrons. The formula is $FC = V - N - \frac{B}{2}$, where V is valence electrons, N non-bonding electrons and B bonding electrons.

Step 2: Key Formula or Approach:

H_2SO_4 has sulphur at the centre, joined to two $-OH$ groups by single bonds and to two oxygen atoms by double bonds. Sulphur has no lone pair here.

Step 3: Detailed Explanation:

Sulphur: $V = 6, N = 0$, bonds $= 2$ single $+2$ double $= 6$ bond pairs, so $B = 12$.

$$FC = 6 - 0 - \frac{12}{2} = 0$$

The other choices need a different structure. A formal charge of $+2$ appears only if all four S-O bonds are drawn as single bonds, and the negative values would need lone pairs on sulphur, which this molecule does not have in its accepted structure.

Final Answer:

Sulphur carries zero formal charge in sulphuric acid, option (D).

$$\boxed{0}$$

Quick Tip: Use FC = valence electrons - non-bonding electrons - half of bonding electrons on the usual structure with two S=O bonds.

54. Which of the following compound has square pyramidal structure?

(A) XeF_4

(B) XeF_6

(C) XeO_3

(D) $XeOF_4$

Correct Answer: (D) $XeOF_4$

Solution:

Step 1: Understanding the Concept:

VSEPR theory gives the shape from the number of electron pairs around the central atom. Lone pairs also take up space, so they change the shape from the ideal one.

Step 2: Key Formula or Approach:

For $XeOF_4$: xenon has 8 valence electrons. Four go to F, two go to the Xe=O bond and two stay as one lone pair. That gives 5 bonded atoms and 1 lone pair, so sp^3d^2 hybridisation.

Step 3: Detailed Explanation:

Six electron regions give an octahedral arrangement. One position is a lone pair, so the molecule is square pyramidal.

Check the others.

XeF_4: 4 bonds and 2 lone pairs, square planar.

XeF_6: 6 bonds and 1 lone pair, distorted octahedral.

XeO_3: 3 bonds and 1 lone pair, trigonal pyramidal.

Final Answer:

Only $XeOF_4$ is square pyramidal, option (D).

$\boxed{XeOF_4}$

Quick Tip: Count bond pairs and lone pairs on xenon in each compound and use VSEPR.

55. Which of the following metal halides have more covalent character?

(A) $SnCl_2$

(B) $PbCl_4$

(C) $SbCl_3$

(D) $PbCl_2$

Correct Answer: (B) $PbCl_4$

Solution:

Step 1: Understanding the Concept:

Fajans rules say covalent character grows when the cation is small and highly charged, and when the anion is large and easily polarised. All four compounds here have chloride as the anion, so the comparison depends on the cation.

Step 2: Key Formula or Approach:

Compare the charge and size of Sn^{2+}, Pb^{4+}, Sb^{3+} and Pb^{2+}.

Step 3: Detailed Explanation:

Pb^{4+} has the highest charge in the list and is smaller than Pb^{2+}.

A high charge on a small ion gives a very high polarising power, so it distorts the chloride electron cloud strongly.

$PbCl_4$ is therefore a covalent liquid. The +2 chlorides $SnCl_2$ and $PbCl_2$

have a lower charge on the cation, so they are more ionic. $SbCl_3$ has a +3 cation, which still polarises less than a +4 cation.

Final Answer:

$PbCl_4$ has the most covalent character, option (B).

$$\boxed{PbCl_4}$$

Quick Tip: Apply Fajans rules: a smaller, more highly charged cation polarises the anion more.

56. What ratio by mass of 'Ne' and CH_4 should be mixed so that partial pressure exerted by each gas is same?

(A) 1 : 1

(B) 2 : 1

(C) 2 : 3

(D) 5 : 4

Correct Answer: (D) 5 : 4

Solution:

Step 1: Understanding the Concept:

In a mixture of gases in one container, partial pressure is $p_i = x_i P_{total}$, which depends on mole fraction. Equal partial pressures need equal mole fractions, so the two gases must have equal moles.

Step 2: Key Formula or Approach:

For n moles, mass $= n \times M$. Molar masses: Ne $= 20$ g/mol, $CH_4 = 16$

g/mol.

Step 3: Detailed Explanation:

Let each gas be 1 mole.

Mass of Ne = 20 g and mass of CH_4 = 16 g.

$$\frac{m_{Ne}}{m_{CH_4}} = \frac{20}{16} = \frac{5}{4}$$

A 1 : 1 mass ratio would give more moles of the lighter methane, so its partial pressure would be larger. The ratios 2 : 1 and 2 : 3 do not give equal moles either.

Final Answer:

The mass ratio of Ne to CH_4 is 5 : 4, option (D).

$$\boxed{5:4}$$

Quick Tip: At the same T and V, equal partial pressure means equal number of moles.

57. Calculate ΔH for the following reaction at 300K

$2C_{(s)} + 3H_{2(g)} \longrightarrow C_2H_{6(g)}$ if ΔU for the reaction is $-80kJ$ ($R = 8.314JK^{-1}mol^{-1}$)

(A) $-85.00kJ$

(B) $-43.00kJ$

(C) $-128.00kJ$

(D) $-170.00kJ$

Correct Answer: (A) -85.00kJ

Solution:

Step 1: Understanding the Concept:

Enthalpy and internal energy are related by $\Delta H = \Delta U + \Delta n_g RT$, where Δn_g is moles of gaseous products minus moles of gaseous reactants.

Step 2: Key Formula or Approach:

Reaction: $2\text{C}(s) + 3\text{H}_2(g) \rightarrow \text{C}_2\text{H}_6(g)$. Solids are ignored when counting gas moles.

Step 3: Detailed Explanation:

$\Delta n_g = 1 - 3 = -2$.

$$\Delta n_g RT = (-2)(8.314 \times 10^{-3}\ \text{kJ K}^{-1}\text{mol}^{-1})(300\ \text{K}) = -4.99\ \text{kJ}$$

$$\Delta H = -80 + (-4.99) = -84.99 \approx -85.00\ \text{kJ}$$

Options B, C and D are far from this value, because the $\Delta n_g RT$ correction is only about 5 kJ here.

Final Answer:

ΔH is about -85.00 kJ, option (A).

$$\boxed{-85.00\ \text{kJ}}$$

Quick Tip: Use delta H = delta U + delta n(g) R T, counting only gaseous

moles.

58. For a certain reaction, $\Delta H^\circ = 40 kJ$ and $\Delta S^\circ = 80 JK^{-1}$. Find the temperature so that $\Delta G^\circ = 0$

(A) 500K

(B) 400K

(C) 300K

(D) 600K

Correct Answer: (A) 500K

Solution:

Step 1: Understanding the Concept:

The Gibbs equation is $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$. A reaction sits at the borderline between spontaneous and non-spontaneous when $\Delta G^\circ = 0$.

Step 2: Key Formula or Approach:

Setting $\Delta G^\circ = 0$ gives $T = \frac{\Delta H^\circ}{\Delta S^\circ}$. Convert ΔH to joules because ΔS is in J/K.

Step 3: Detailed Explanation:

$\Delta H^\circ = 40$ kJ $= 40000$ J and $\Delta S^\circ = 80$ J/K.

$$T = \frac{40000}{80} = 500 \text{ K}$$

If the units are mixed (40 divided by 80) one gets 0.5, which matches none of the options. The values 300, 400 and 600 K would need ΔH of

24, 32 or 48 kJ for this ΔS.

Final Answer:

The temperature is 500 K, option (A).

$$\boxed{500\ \text{K}}$$

Quick Tip: At equilibrium delta G is zero, so T = delta H / delta S with consistent units.

59. Which from following is correct expression of first law of thermodynamics for isothermal process?

(A) $W = -Q$

(B) $-\Delta U = -W$

(C) $\Delta U = Q_v$

(D) $Q_p = \Delta U + P_{ext}\Delta V$

Correct Answer: (A) $W = -Q$

Solution:

Step 1: Understanding the Concept:

The first law is $\Delta U = Q + W$, where W is the work done on the system. In an isothermal process T is constant, so for an ideal gas $\Delta U = 0$.

Step 2: Key Formula or Approach:

Substitute $\Delta U = 0$ into $\Delta U = Q + W$.

Step 3: Detailed Explanation:

$0 = Q + W$, so $W = -Q$.

This is option (A): the work done on the system equals the heat given out, or the heat absorbed is converted fully into work by the system.

Option (B), $-\Delta U = -W$, means $\Delta U = W$ with no heat exchange, which is the adiabatic case.

Option (C), $\Delta U = Q_v$, holds for a constant volume process.

Option (D), $Q_p = \Delta U + P_{ext}\Delta V$, is the form for a constant pressure process.

Final Answer:

For an isothermal process $W = -Q$, option (A).

$$\boxed{W = -Q}$$

Quick Tip: In an isothermal ideal gas process the temperature does not change, so the internal energy does not change.

60. Which of the following is Lewis acid?

(A) $BaCl_2$

(B) KCl

(C) $BeCl_2$

(D) LiCl

Correct Answer: (C) $BeCl_2$

Solution:

Step 1: Understanding the Concept:
A Lewis acid is a species that can accept a pair of electrons. Electron deficient molecules with a vacant orbital on the central atom are typical Lewis acids.

Step 2: Key Formula or Approach:
In $BeCl_2$, beryllium has only 4 electrons in its valence shell after forming two bonds. It has empty orbitals and can accept electron pairs.

Step 3: Detailed Explanation:
$BeCl_2$ forms chain polymers in the solid state, using chlorine lone pairs to fill the empty orbitals of Be. It also reacts with Cl^- to form $[BeCl_4]^{2-}$. So it is a Lewis acid.
$BaCl_2$, KCl and LiCl are ionic salts. Their cations are large and have noble gas configurations, and they show no tendency to accept electron pairs, so they are not Lewis acids.

Final Answer:
$BeCl_2$ is the Lewis acid, option (C).

$$\boxed{BeCl_2}$$

Quick Tip: A Lewis acid accepts an electron pair, so look for the electron deficient molecule.

61. The solubility of a sparingly soluble a salt AB_2 is $1 \times 10^{-6} mol/dm^3$. Calculate its solubility product?

(A) 1×10^{-12}

(B) 2×10^{-12}

(C) 3×10^{-12}

(D) 4×10^{-18}

Correct Answer: (D) 4×10^{-18}

Solution:

Step 1: Understanding the Concept:

For a salt AB_2 the dissolution equilibrium is $AB_2 \rightleftharpoons A^{2+} + 2B^-$. The solubility product is the product of the ion concentrations, each raised to its coefficient.

Step 2: Key Formula or Approach:

If the solubility is s, then $[A^{2+}] = s$ and $[B^-] = 2s$, so $K_{sp} = s(2s)^2 = 4s^3$.

Step 3: Detailed Explanation:

Here $s = 1 \times 10^{-6}$ mol/dm^3.

$$K_{sp} = 4s^3 = 4 \times (10^{-6})^3 = 4 \times 10^{-18}$$

Options A, B and C are of order 10^{-12}. They come from treating the salt like an AB type salt (s^2) or from squaring instead of cubing, and so they are not correct for AB_2.

Final Answer:

$K_{sp} = 4 \times 10^{-18}$, option (D).

$$\boxed{4 \times 10^{-18}}$$

Quick Tip: Write the dissociation of AB2 and express Ksp as 4s cubed.

62. What is the molarity of H_2SO_4 solution having $pH = 4$?

(A) $4 \times 10^{-1}M$

(B) $4 \times 10^{-2}M$

(C) $5 \times 10^{-5}M$

(D) $1 \times 10^{-4}M$

Correct Answer: (C) $5 \times 10^{-5}M$

Solution:

Step 1: Understanding the Concept:

pH is $-\log[H^+]$. Sulphuric acid is a strong acid that gives two H^+ ions per molecule in dilute solution.

Step 2: Key Formula or Approach:

1. $[H^+] = 10^{-pH}$.
2. $H_2SO_4 \rightarrow 2H^+ + SO_4^{2-}$, so $[H_2SO_4] = \frac{[H^+]}{2}$.

Step 3: Detailed Explanation:

$[H^+] = 10^{-4}$ M.

$$[H_2SO_4] = \frac{10^{-4}}{2} = 5 \times 10^{-5}\ M$$

Option D (1×10^{-4} M) forgets that one molecule gives two protons. Options A and B are many times larger and would give a pH far below 4.

Final Answer:

The molarity is 5×10^{-5} M, option (C).

$$\boxed{5 \times 10^{-5}\ \text{M}}$$

Quick Tip: Find [H+] from pH, then divide by two because each H2SO4 gives two H+ ions.

63. Identify the conjugate bases of H_3PO_3 and H_2SO_4 respectively from following.

(A) HPO_3^{2-} and SO_4^{2-}

(B) HPO_3^{2-} and HSO_4^-

(C) $H_2PO_3^-$ and HSO_4^-

(D) PO_3^{3-} and SO_4^{2-}

Correct Answer: (C) $H_2PO_3^-$ and HSO_4^-

Solution:

Step 1: Understanding the Concept:

According to Bronsted-Lowry theory, the conjugate base of an acid is what remains after the acid loses exactly one proton (H^+).

Step 2: Key Formula or Approach:

$H_3PO_3 \rightarrow H_2PO_3^- + H^+$ and $H_2SO_4 \rightarrow HSO_4^- + H^+$.

Step 3: Detailed Explanation:

Removing one H^+ from H_3PO_3 gives $H_2PO_3^-$.

Removing one H^+ from H_2SO_4 gives HSO_4^-.

Options A, B and D contain species that have lost two or three protons, such as SO_4^{2-} or HPO_3^{2-}, or that pair a correct base with a wrong one. A conjugate base differs from its acid by just one proton.

Final Answer:

The conjugate bases are $H_2PO_3^-$ and HSO_4^-, option (C).

$$\boxed{H_2PO_3^- \text{ and } HSO_4^-}$$

Quick Tip: A conjugate base is formed by removing one proton, and H3PO3 is only dibasic.

64. What happens in the oxidation number of Mn during working of dry cell.

(A) Increases by 1

(B) decreases by 1

(C) Increases by 2

(D) decreases by 2

Correct Answer: (B) decreases by 1

Solution:

Step 1: Understanding the Concept:

In a dry (Leclanche) cell, zinc is the anode and is oxidised. Manganese dioxide mixed with carbon at the cathode is reduced.

Step 2: Key Formula or Approach:

Cathode reaction: $MnO_2 + NH_4^+ + e^- \rightarrow MnO(OH) + NH_3$. The overall

form can also be written as $2MnO_2 \rightarrow Mn_2O_3$.

Step 3: Detailed Explanation:
Oxidation number of Mn in MnO_2: $x + 2(-2) = 0$, so $x = +4$.
Oxidation number of Mn in Mn_2O_3: $2x + 3(-2) = 0$, so $x = +3$.

$$+4 \rightarrow +3$$

So the oxidation number of Mn decreases by 1. It is reduced by one electron per Mn atom. An increase would mean oxidation, but Mn is at the cathode and gains electrons.

Final Answer:
Mn goes from $+4$ to $+3$, a decrease of 1, option (B).

Decreases by 1

Quick Tip: In the Leclanche dry cell manganese dioxide is reduced to Mn2O3.

65. What is the formula of Tin (IV) oxide?

(A) Sn_2O_3

(B) SnO_2

(C) SnO_4

(D) SnO

Correct Answer: (B) SnO_2

Solution:

Step 1: Understanding the Concept:

The roman numeral in a name tells the oxidation state of the metal. Tin(IV) means Sn^{4+}. Oxide is O^{2-}.

Step 2: Key Formula or Approach:

The compound must be neutral, so the total positive charge equals the total negative charge. Use the criss-cross of charges.

Step 3: Detailed Explanation:

Sn^{4+} and O^{2-}: criss-cross gives Sn_2O_4, which reduces to SnO_2.

Check: $+4 + 2(-2) = 0$.

Sn_2O_3 would need Sn^{3+}, which is not the tin(IV) state. SnO_4 would need Sn^{8+}. SnO is tin(II) oxide.

Final Answer:

Tin(IV) oxide is SnO_2, option (B).

$$\boxed{SnO_2}$$

Quick Tip: The roman number gives the charge on tin, and oxygen is 2-.

66. Identify the name of second higher homologue of formic acid?

(A) Methanoic acid

(B) Ethanoic acid

(C) Propanoic acid

(D) Butanoic acidaa

Correct Answer: (C) Propanoic acid

Solution:

Step 1: Understanding the Concept:

A homologous series is a family of compounds that differ from the next member by a $-CH_2-$ unit. Formic acid, HCOOH, is the first member of the carboxylic acid series.

Step 2: Key Formula or Approach:

First higher homologue = formic acid $+CH_2$. Second higher homologue = formic acid $+2CH_2$.

Step 3: Detailed Explanation:

HCOOH (methanoic acid) $+CH_2 = CH_3COOH$ (ethanoic acid), the first higher homologue.

$CH_3COOH + CH_2 = CH_3CH_2COOH$ (propanoic acid), the second higher homologue.

Methanoic acid is formic acid itself, ethanoic acid is the first higher homologue, and butanoic acid is the third higher homologue.

Final Answer:

The second higher homologue is propanoic acid, option (C).

Propanoic acid

Quick Tip: Homologues differ by one CH2 unit, so count up from HCOOH twice.

67. What is the of number of chiral carbon atoms present in 2-Bromo-3,4,5-trimethylhexane ?

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (C) 3

Solution:

Step 1: Understanding the Concept:

A chiral centre is a carbon attached to four different groups. Draw the structure first: hexane chain with Br on C2 and methyl groups on C3, C4 and C5.

Step 2: Key Formula or Approach:

Structure: $C1H_3 - C2H(Br)$
$- C3H(CH_3) - C4H(CH_3) - C5H(CH_3) - C6H_3$.

Step 3: Detailed Explanation:

C2 has H, Br, CH_3 and the rest of the chain: four different groups, so it is chiral.

C3 has H, CH_3, C2 side (with Br) and C4 side: chiral.

C4 has H, CH_3, C3 side and C5 side. The two sides differ, so it is chiral.

C5 has H, a methyl substituent and the chain carbon C6, which is also a methyl group. Two groups are identical, so C5 is not chiral.

Total chiral carbons = 3.

Final Answer:

There are 3 chiral carbons, option (C).

$$\boxed{3}$$

Quick Tip: A carbon is chiral if it carries four different groups, so check each carbon of the main chain.

68. Which of the following alkyl chlorides on Wurtz reaction gives 2, 2, 5, 5- tetramethylhexane ?

(A) $(CH_3)_2CH - CH_2 - Cl$

(B) $(CH_3)_3C - CH_2 - Cl$

(C) $(CH_3)_2CH - \underset{\displaystyle CH_3}{\underset{|}{CH}} - Cl$

(D) $CH_3 - CH_2 - \underset{\displaystyle Cl}{\underset{|}{CH}} - CH_3$

Correct Answer: (B) $(CH_3)_3C - CH_2 - Cl$

Solution:

Step 1: Understanding the Concept:

In the Wurtz reaction, two molecules of an alkyl halide react with sodium in dry ether to form a symmetrical alkane: $2RX + 2Na \rightarrow R\text{-}R + 2NaX$.

Step 2: Key Formula or Approach:

Write the product 2,2,5,5-tetramethylhexane and cut it in the middle (between C3 and C4) to find the alkyl group R.

Step 3: Detailed Explanation:

2,2,5,5-Tetramethylhexane: $(CH_3)_3C - CH_2 - CH_2 - C(CH_3)_3$.

Cutting the central $CH_2 - CH_2$ bond gives two groups $(CH_3)_3C - CH_2 -$ (neopentyl).

So the halide is neopentyl chloride, $(CH_3)_3C - CH_2 - Cl$, which is option B.

Option A (isobutyl chloride) would give 2,5-dimethylhexane. Option D (2-chlorobutane) would give 3,4-dimethylhexane.

Final Answer:

The Wurtz reaction of neopentyl chloride gives the product, option (B).

$$\boxed{(CH_3)_3C - CH_2 - Cl}$$

Quick Tip: Wurtz reaction joins two alkyl groups, so split the product into two equal halves.

69. Calculate the number of atoms present in 1g metal that forms simple unit cell structure if product of density and volume of unit cell is 66.5 $\times 10^{-24}$g

(A) 1.504×10^{22}

(B) 3.419×10^{22}

(C) 2.516×10^{22}

(D) 2.018×10^{22}

Correct Answer: (A) 1.504×10^{22}

Solution:

Step 1: Understanding the Concept:

Mass of a unit cell $= \rho \times a^3 = \frac{Z \times M}{N_A}$. A simple cubic unit cell has $Z = 1$ atom.

Step 2: Key Formula or Approach:

Here the product of density and unit cell volume, which is the mass of the unit cell, is 66.5×10^{-24} g. With $Z = 1$ this is the mass of one atom.

Step 3: Detailed Explanation:

Mass of one atom $= 66.5 \times 10^{-24}$ g.

$$\text{Atoms in 1 g} = \frac{1}{66.5 \times 10^{-24}} = 1.504 \times 10^{22}$$

This is option A. The other options would result from using $Z = 2$ or $Z = 4$ (body or face centred cells), which do not apply to a simple unit cell.

Final Answer:

About 1.504×10^{22} atoms are present in 1 g, option (A).

$$\boxed{1.504 \times 10^{22}}$$

Quick Tip: Density times volume is the mass of the unit cell, and a simple cubic cell holds one atom.

70. Calculate the void volume in fcc unit cell if total volume of a unit cell is $6.4 \times 10^{-23} cm^3$?

(A) $3.172 \times 10^{-23} \text{cm}^3$

(B) $3.911 \times 10^{-23} \text{cm}^3$

(C) $1.664 \times 10^{-23} \text{cm}^3$

(D) $2.051 \times 10^{-23} \text{cm}^3$

Correct Answer: (C) $1.664 \times 10^{-23} \text{cm}^3$

Solution:

Step 1: Understanding the Concept:

Packing efficiency is the fraction of the unit cell volume actually filled by atoms. For a face centred cubic (fcc) lattice, it is 74%.

Step 2: Key Formula or Approach:

Void percentage $= 100 - 74 = 26\%$. Void volume $= 0.26 \times V_{cell}$.

Step 3: Detailed Explanation:

$$V_{void} = 0.26 \times 6.4 \times 10^{-23} = 1.664 \times 10^{-23} \text{ cm}^3$$

The occupied volume is $0.74 \times 6.4 \times 10^{-23} = 4.736 \times 10^{-23}$ cm^3, and this plus the void volume gives back the total 6.4×10^{-23} cm^3.

The other options do not match the 26% empty space of a close packed fcc lattice.

Final Answer:

The void volume is 1.664×10^{-23} cm^3, option (C).

$$\boxed{1.664 \times 10^{-23} \text{ cm}^3}$$

Quick Tip: fcc packing efficiency is 74 percent, so voids are 26 percent.

71. What is the total number of Bravais lattices in orthorhombic crystal system?

(A) 3

(B) 2

(C) 1

(D) 4

Correct Answer: (D) 4

Solution:

Step 1: Understanding the Concept:
Bravais lattices are the distinct ways of arranging lattice points in a crystal system. There are 14 in total across 7 crystal systems.

Step 2: Key Formula or Approach:
The orthorhombic system has $a \neq b \neq c$ and all angles 90°. It allows every centring type.

Step 3: Detailed Explanation:
The four types are: primitive (simple), end or base centred, body centred, and face centred.
So orthorhombic has 4 Bravais lattices.
By contrast, cubic has 3, tetragonal has 2, monoclinic has 2, rhombohedral has 1, hexagonal has 1 and triclinic has 1. These add up to 14 lattices in total.

Final Answer:

The orthorhombic system has 4 Bravais lattices, option (D).

$$\boxed{4}$$

Quick Tip: Orthorhombic has primitive, base centred, body centred and face centred lattices.

72. 40 gram nonelectrolyte solute having molar mass 180g mol^{-1} dissolved in water has osmotic pressure 2atm at 300K. Calculate the volume of solution. ($R = 0.0821\text{atm mol}^{-1}\text{K}^{-1}$)

(A) 2.10dm^3

(B) 2.34dm^3

(C) 2.74dm^3

(D) 3.40dm^3

Correct Answer: (C) 2.74dm^3

Solution:

Step 1: Understanding the Concept:

For a dilute solution of a nonelectrolyte, the osmotic pressure follows the van t Hoff equation $\pi V = nRT$, where n is the number of moles of solute.

Step 2: Key Formula or Approach:

$n = \frac{w}{M}$ and $V = \frac{nRT}{\pi}$.

Step 3: Detailed Explanation:

$n = \frac{40}{180} = 0.2222$ mol.

$$V = \frac{0.2222 \times 0.0821 \times 300}{2} = \frac{5.473}{2} = 2.74 \text{ L}$$

One litre equals one dm^3, so $V = 2.74\ dm^3$.

The other values come from using a wrong temperature or a wrong molar mass, and so they do not match.

Final Answer:

The volume is $2.74\ dm^3$, option (C).

$$\boxed{2.74\ dm^3}$$

Quick Tip: Use pi V = n R T with n found from mass and molar mass.

73. Calculate the molality of solution of nonvolatile solute if boiling point of solution, molal elevation constant for solvent are $319.8K$ and $2.5K\ kg\ mol^{-1}$ respectively [Boiling point of solvent = $319.5K$]

(A) $0.12 mol\ kg^{-1}$

(B) $0.14 mol\ kg^{-1}$

(C) $0.17 mol\ kg^{-1}$

(D) $0.21 mol\ kg^{-1}$

Correct Answer: (A) $0.12 mol\ kg^{-1}$

Solution:

Step 1: Understanding the Concept:

A nonvolatile solute raises the boiling point of a solvent. The elevation is proportional to the molality: $\Delta T_b = K_b m$.

Step 2: Key Formula or Approach:

$\Delta T_b = T_b(\text{solution}) - T_b(\text{solvent})$, then $m = \frac{\Delta T_b}{K_b}$.

Step 3: Detailed Explanation:

$\Delta T_b = 319.8 - 319.5 = 0.3$ K.

$$m = \frac{0.3}{2.5} = 0.12 \text{ mol kg}^{-1}$$

Options B, C and D would give ΔT_b of 0.35, 0.425 and 0.525 K, which do not match the data.

Final Answer:

The molality is 0.12 mol/kg, option (A).

$$\boxed{0.12 \text{ mol kg}^{-1}}$$

Quick Tip: Find delta Tb as the difference of boiling points, then divide by Kb.

74. Identify correct statements from following regarding 0.2M urea and 0.2M sucrose solutions.

(A) The urea solution is hypertonic to sucrose solution.

(B) The osmotic pressure of urea solution is higher than that of sucrose solution.

(C) These solutions exhibit same osmotic pressure.

(D) The sucrose solution is hypotonic to urea solution.

Correct Answer: (C) These solutions exhibit same osmotic pressure.

Solution:

Step 1: Understanding the Concept:

Osmotic pressure is a colligative property: $\pi = iCRT$. It depends on the concentration of dissolved particles, not on what they are.

Step 2: Key Formula or Approach:

Urea and sucrose are both molecular solutes that do not dissociate, so $i = 1$ for each.

Step 3: Detailed Explanation:

Both solutions are 0.2 M and at the same temperature, so $\pi_{urea} = \pi_{sucrose} = 0.2RT$.

Solutions with the same osmotic pressure are called isotonic.

Statements A and D call one solution hypertonic or hypotonic to the other, and statement B claims a higher pressure for urea. These need unequal osmotic pressures, so they are all false. Only statement C is correct.

Final Answer:

The two solutions have the same osmotic pressure, option (C).

Same osmotic pressure

Quick Tip: Osmotic pressure depends on the number of particles, and both solutes are nonelectrolytes.

75. For a certain redox reaction in galvanic cell $X_{(s)} + Y^{2+}_{(aq)} \longrightarrow X^{2+}_{(aq)} + Y_{(s)}$, E° cell is 0.0296V at 298K. What is equilibrium constant of reaction?

(A) 1

(B) 10

(C) 100

(D) 1000

Correct Answer: (B) 10

Solution:

Step 1: Understanding the Concept:

At equilibrium the cell potential is zero, which links the standard cell potential to the equilibrium constant: $E^\circ_{cell} = \frac{0.0592}{n} \log K$ at 298 K.

Step 2: Key Formula or Approach:

The reaction $X + Y^{2+} \rightarrow X^{2+} + Y$ transfers $n = 2$ electrons.

Step 3: Detailed Explanation:

$$\log K = \frac{nE^\circ}{0.0592} = \frac{2 \times 0.0296}{0.0592} = 1$$

$$K = 10^1 = 10$$

A value of $K = 1$ would need $E^\circ = 0$, and $K = 100$ or 1000 would need

E° of 0.0592 V or 0.0888 V.

Final Answer:

$K = 10$, option (B).

10

Quick Tip: Use Eo = (0.0592/n) log K with n = 2 for this reaction.

76. Which of the following net cell reactions occurs in a galvanic cell containing cadmium electrode and standard hydrogen electrode? $E^\circ_{(Cd^{2+}_{(aq)}|Cd_{(s)})} = -0.403V$.

(A) $H_{2(g)} + Cd^{2+}_{(aq)} \longrightarrow 2H^+_{(aq)} + Cd_{(s)}$

(B) $Cd_{(s)} + 2H^+_{(aq)} \longrightarrow Cd^{2+}_{(aq)} + H_{2(g)}$

(C) $2H_{2(g)} + Cd^{2+}_{(aq)} \longrightarrow 4H^+_{(aq)} + Cd_{(s)}$

(D) $2Cd_{(s)} + 2H^+_{(aq)} \longrightarrow 2Cd^{2+}_{(aq)} + H_{2(g)}$

Correct Answer: (B) $Cd_{(s)} + 2H^+_{(aq)} \longrightarrow Cd^{2+}_{(aq)} + H_{2(g)}$

Solution:

Step 1: Understanding the Concept:

In a galvanic cell the electrode with the lower (more negative) reduction potential loses electrons and acts as the anode. The standard hydrogen electrode (SHE) has $E^\circ = 0$ V by definition.

Step 2: Key Formula or Approach:

Compare $E^\circ(Cd^{2+}|Cd) = -0.403$ V with $E^\circ(H^+|H_2) = 0.000$ V. Then $E^\circ_{cell} = E^\circ_{cathode} - E^\circ_{anode}$ must be positive for a spontaneous reaction.

Step 3: Detailed Explanation:

Cadmium has the lower potential, so Cd is the anode: $Cd(s) \rightarrow Cd^{2+} + 2e^-$.

Hydrogen ions are reduced at the cathode: $2H^+ + 2e^- \rightarrow H_2(g)$.

Adding the two half reactions: $Cd(s) + 2H^+(aq) \rightarrow Cd^{2+}(aq) + H_2(g)$.

$$E^\circ_{cell} = 0 - (-0.403) = +0.403\ V$$

Option A is the reverse reaction, which would have $E^\circ = -0.403$ V and would not occur. Options C and D are not balanced in charge or in atoms.

Final Answer:

The net reaction is $Cd(s) + 2H^+ \rightarrow Cd^{2+} + H_2(g)$, option (B).

$$\boxed{Cd_{(s)} + 2H^+_{(aq)} \rightarrow Cd^{2+}_{(aq)} + H_{2(g)}}$$

Quick Tip: The electrode with the more negative reduction potential acts as the anode and is oxidised.

77. In a first order reaction 20 millimole of reactant is lowered to 10 millimole in 0.3010 minute. Find rate constant of the reaction?

(A) $3.010 minute^{-1}$

(B) $0.602 minute^{-1}$

(C) $2.303 minute^{-1}$

(D) $0.301 minute^{-1}$

Correct Answer: (C) 2.303minute^{-1}

Solution:

Step 1: Understanding the Concept:

In a first order reaction the time for the concentration to fall to half is a constant, called the half life: $t_{1/2} = \frac{0.693}{k}$.

Step 2: Key Formula or Approach:

The reactant goes from 20 to 10 millimole, which is exactly half, so $t_{1/2} = 0.3010$ min.

Step 3: Detailed Explanation:

$$k = \frac{0.693}{t_{1/2}} = \frac{0.693}{0.3010} = 2.303 \text{ min}^{-1}$$

Check with the integrated rate law: $k = \frac{2.303}{t} \log \frac{20}{10} = \frac{2.303 \times 0.3010}{0.3010}$ $= 2.303 \text{ min}^{-1}$.

Option D, 0.301, is just the value of $\log 2$ and not a rate constant.

Options A and B come from slips in the arithmetic.

Final Answer:

The rate constant is 2.303 min^{-1}, option (C).

$$\boxed{2.303 \text{ min}^{-1}}$$

Quick Tip: Reactant halves in 0.3010 min, so this is the half life.

78. For the reaction $2NOBr_{(g)} \longrightarrow 2NO_{(g)} + Br_{2(g)}$, rate law is $r = k[NOBr]^2$. Rate constant is $1.62 M\ s^{-1}$ and concentration of NOBr is $5 \times 10^{-3} M$,

What is rate of reaction?

(A) $4.05 \times 10^{-5} M\ s^{-1}$

(B) $6.48 \times 10^{-6} M\ s^{-1}$

(C) $2.12 \times 10^{-5} M\ s^{-1}$

(D) $8.63 \times 10^{-6} M\ s^{-1}$

Correct Answer: (A) $4.05 \times 10^{-5} M\ s^{-1}$

Solution:

Step 1: Understanding the Concept:

The rate law $r = k[NOBr]^2$ says the rate depends on the square of the concentration. We just substitute the numbers.

Step 2: Key Formula or Approach:

$r = k[NOBr]^2$ with $k = 1.62$ and $[NOBr] = 5 \times 10^{-3}$ M.

Step 3: Detailed Explanation:

$[NOBr]^2 = (5 \times 10^{-3})^2 = 25 \times 10^{-6} = 2.5 \times 10^{-5}$.

$$r = 1.62 \times 2.5 \times 10^{-5} = 4.05 \times 10^{-5}\ M\ s^{-1}$$

If the concentration was not squared we would get 8.1×10^{-3}, which is not among the options. The other values do not follow from the given numbers.

Final Answer:

The rate is 4.05×10^{-5} M/s, option (A).

$$\boxed{4.05 \times 10^{-5}\ \text{M s}^{-1}}$$

Quick Tip: Substitute concentration into rate = k[NOBr]^2.

79. Which of the following reactions has of overall order 1.5 ?

(A) $2H_2O_{2(g)} \longrightarrow 2H_2O_{(\ell)} + O_{2(g)}$

(B) $H_{2(g)} + I_{2(g)} \longrightarrow 2HI_{(g)}$

(C) $CH_3CHO_{(g)} \longrightarrow CH_{4(g)} + CO_{(g)}$

(D) $2NO_{(g)} + 2H_{2(g)} \longrightarrow N_{2(g)} + 2H_2O_{(\ell)}$

Correct Answer: (C) $CH_3CHO_{(g)} \longrightarrow CH_{4(g)} + CO_{(g)}$

Solution:

Step 1: Understanding the Concept:

The order of a reaction comes from the experimental rate law, not from the balanced equation. Fractional orders come from reactions that go by a multi step mechanism.

Step 2: Key Formula or Approach:

Known rate laws: H_2O_2 decomposition is first order; $H_2 + I_2 \rightarrow 2HI$ is second order (first in each); decomposition of acetaldehyde follows $r = k[CH_3CHO]^{3/2}$; $2NO + 2H_2$ is third order overall ($r = k[NO]^2[H_2]$).

Step 3: Detailed Explanation:

Option A: order 1.

Option B: order $1 + 1 = 2$.

Option C: order $\frac{3}{2} = 1.5$.

Option D: order $2 + 1 = 3$.

Only the thermal decomposition of acetaldehyde gives an overall order of 1.5.

Final Answer:
The reaction of overall order 1.5 is the decomposition of CH_3CHO, option (C).

$$\boxed{CH_3CHO \rightarrow CH_4 + CO}$$

Quick Tip: Recall the experimental rate laws of the standard reactions.

80. Which from following anions has greater power for coagulation of positive sol?

(A) Cl^-

(B) $[Fe(CN)_6]^{4-}$

(C) PO_4^{2-}

(D) SO_4^{2-}

Correct Answer: (B) $[Fe(CN)_6]^{4-}$

Solution:

Step 1: Understanding the Concept:
A sol is coagulated by ions carrying a charge opposite to the sol particles. The Hardy-Schulze rule says that the higher the charge on the coagulating ion, the greater its coagulating power.

Step 2: Key Formula or Approach:
A positive sol is coagulated by anions. Compare the charges: Cl^- (–1),

PO_4^{2-} and SO_4^{2-} (as written, -2) and $[Fe(CN)_6]^{4-}$ (-4).

Step 3: Detailed Explanation:
Order of coagulating power: $Cl^- < SO_4^{2-} \approx PO_4^{2-} < [Fe(CN)_6]^{4-}$.
The hexacyanoferrate(II) ion has the largest negative charge, $4-$, so it neutralises the positive charge on the sol particles most effectively and needs the smallest concentration.

Final Answer:
$[Fe(CN)_6]^{4-}$ has the greatest coagulating power, option (B).

$$\boxed{[Fe(CN)_6]^{4-}}$$

Quick Tip: Hardy-Schulze rule: coagulating power grows with the charge on the oppositely charged ion.

81. Which actinoid from following has smallest ionic size in $+3$ state?

(A) Pa

(B) Cm

(C) Fm

(D) Lr

Correct Answer: (D) Lr

Solution:

Step 1: Understanding the Concept:
Across the actinoid series, electrons enter the $5f$ subshell, which shields the nuclear charge poorly. The effective nuclear charge grows,

so the size of the M^{3+} ion decreases steadily. This is called the actinoid contraction.

Step 2: Key Formula or Approach:
Order the elements by atomic number: Pa (91), Cm (96), Fm (100), Lr (103).

Step 3: Detailed Explanation:
The element with the highest atomic number lies furthest along the series and has undergone the most contraction.
Lr ($Z = 103$) is the last member of the actinoids, so Lr^{3+} is the smallest.
Size order: $Pa^{3+} > Cm^{3+} > Fm^{3+} > Lr^{3+}$.

Final Answer:
Lr has the smallest +3 ion, option (D).

$$\boxed{\text{Lr}}$$

Quick Tip: Actinoid contraction: ionic size falls steadily across the series.

82. What is the total number of transition series of d-block elements?

(A) 3

(B) 4

(C) 10

(D) 12

Correct Answer: (B) 4

Solution:

Step 1: Understanding the Concept:

A transition series is a row of the d-block in which a d subshell is being filled. Each row has up to 10 elements.

Step 2: Key Formula or Approach:

The d-block covers the $3d$ (Sc to Zn), $4d$ (Y to Cd), $5d$ (La, Hf to Hg) and $6d$ (Ac, Rf to Cn) series.

Step 3: Detailed Explanation:

These are the first, second, third and fourth transition series: periods 4, 5, 6 and 7.

So the total number of d-block series is 4.

The number 10 is the number of elements in one full series, not the number of series.

Final Answer:

The d-block has 4 transition series, option (B).

$$\boxed{4}$$

Quick Tip: The d-block contains the 3d, 4d, 5d and 6d series.

83. Identify a ligand with highest field strength among the following.

(A) H_2O

(B) NH_3

(C) S^{2-}

(D) NCS^-

Correct Answer: (B) NH_3

Solution:

Step 1: Understanding the Concept:

Ligand field strength tells how strongly a ligand splits the d orbitals of the metal. It is ranked in the spectrochemical series, from weak to strong.

Step 2: Key Formula or Approach:

Spectrochemical series (part): $I^- < Br^- < S^{2-} < SCN^- < Cl^- < F^- < OH^- < H_2O < NCS^- < NH_3 < en < CN^- < CO$.

Step 3: Detailed Explanation:

Among the four given ligands, S^{2-} sits near the weak end. The N-bonded NCS^- and H_2O are in the middle, and NH_3 comes after both. So the strongest field ligand here is NH_3, a neutral N-donor that is also a good sigma donor.

Final Answer:

NH_3 is the strongest field ligand of the four, option (B).

$$\boxed{NH_3}$$

Quick Tip: Use the spectrochemical series.

84. Find EAN of Fe in $Fe(CO)_5$

(A) 35

(B) 36

(C) 26

(D) 30

Correct Answer: (B) 36

Solution:

Step 1: Understanding the Concept:

The effective atomic number (EAN) is the total number of electrons around the metal, counting those donated by ligands. $\text{EAN} = Z - \text{ox. state} + 2 \times (\text{number of ligands, for neutral donors})$.

Step 2: Key Formula or Approach:

In $Fe(CO)_5$, CO is a neutral ligand, so Fe is in the 0 oxidation state. Fe has $Z = 26$ and there are 5 CO ligands, each giving 2 electrons.

Step 3: Detailed Explanation:

$$\text{EAN} = 26 - 0 + 5 \times 2 = 36$$

This equals the atomic number of krypton, so the complex follows the EAN rule. The value 26 would be the bare atom, 30 would need only 2 ligands, and 35 is not a noble gas number.

Final Answer:

The EAN of Fe is 36, option (B).

36

Quick Tip: EAN = atomic number - oxidation state + 2 x coordination number.

85. The reaction of aryl halide with sodium metal in dry ether to give biphenyl is known as

(A) Swartz reaction

(B) Wurtz reaction

(C) Fittig reaction

(D) Wrutz-fittig reaction

Correct Answer: (C) Fittig reaction

Solution:

Step 1: Understanding the Concept:
Several named reactions couple halides using sodium metal in dry ether. Which name applies depends on the type of halide used.

Step 2: Key Formula or Approach:
Wurtz: two alkyl halides give an alkane. Wurtz-Fittig: an aryl halide plus an alkyl halide gives an alkyl benzene. Fittig: two aryl halides give a diaryl compound.

Step 3: Detailed Explanation:
Here the halide is an aryl halide, e.g. C_6H_5Br, and it reacts alone.

$$2C_6H_5Br + 2Na \xrightarrow{\text{dry ether}} C_6H_5 - C_6H_5 + 2NaBr$$

The product is biphenyl, so this is the Fittig reaction. Swarts reaction is halogen exchange using a metal fluoride, and it is unrelated to sodium coupling.

Final Answer:
The reaction is the Fittig reaction, option (C).

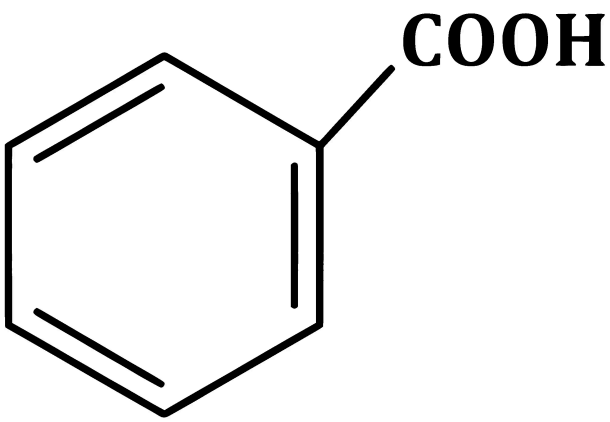
Fittig reaction

Quick Tip: Aryl halides coupling with sodium form the Fittig reaction.

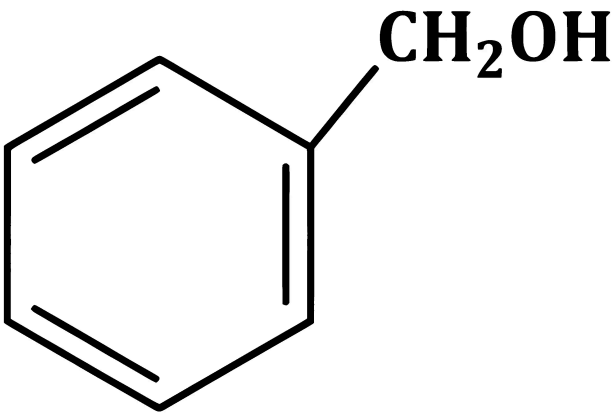
86. Identify the product P obtained in following reaction

$$C_6H_5OH + H-\underset{\displaystyle H}{\overset{\displaystyle |}{C}}=O \xrightarrow{\text{Acid}} P$$

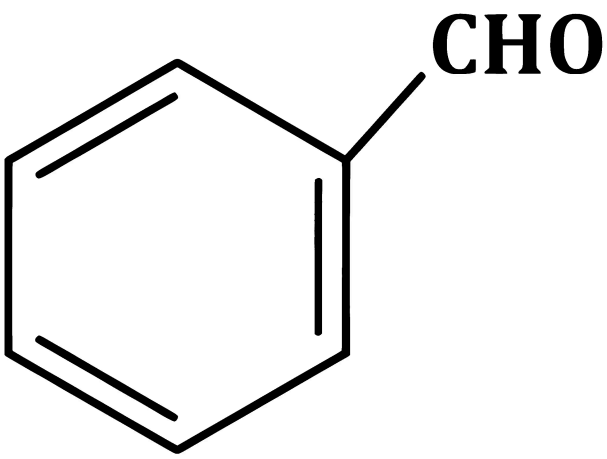
(A)



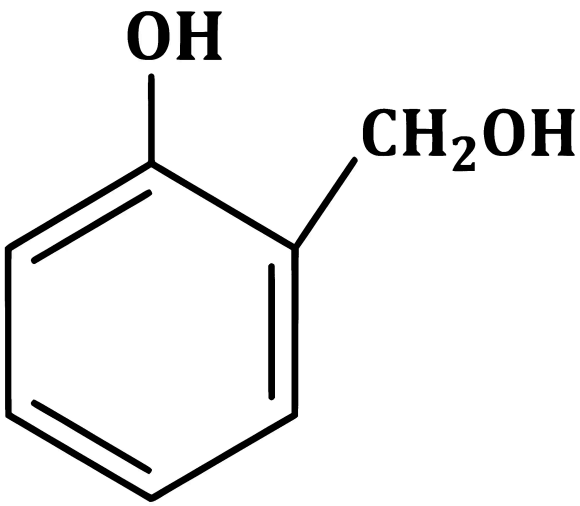
(B)



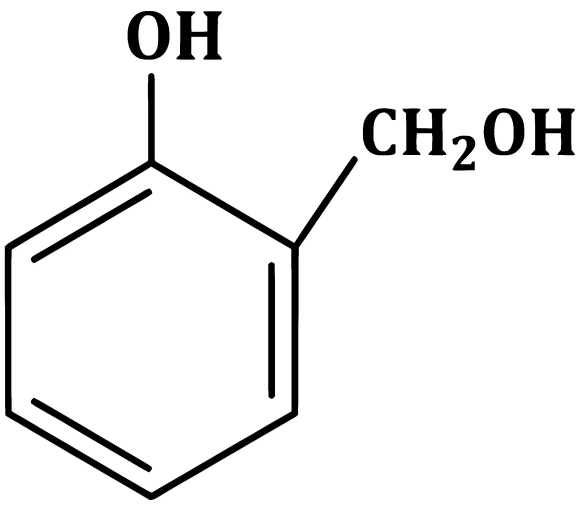
(C)



(D)



Correct Answer: (D)



Solution:

Step 1: Understanding the Concept:

Phenol reacts with formaldehyde (HCHO) in the presence of acid in an electrophilic substitution (Lederer-Manasse type) reaction. The $-OH$ group activates the ring and directs the new group to the ortho and para positions.

Step 2: Key Formula or Approach:

The electrophile is the protonated formaldehyde, $CH_2 = OH^+$. It

attacks the ring and introduces a $-CH_2OH$ group.

Step 3: Detailed Explanation:
The first product is o-hydroxybenzyl alcohol, a benzene ring with $-OH$ and $-CH_2OH$ on neighbouring carbons (the p-isomer forms too). This is option D.
Option A (benzoic acid) and option C (benzaldehyde) need oxidation, which is not present here. Option B (benzyl alcohol) has no $-OH$ on the ring, so the phenol group would have to be lost, which does not happen.

Final Answer:
Phenol and formaldehyde with acid give o-hydroxybenzyl alcohol, option (D).

o-Hydroxybenzyl alcohol

Quick Tip: Acid catalysed hydroxyalkylation of phenol puts CH2OH at the ortho or para position.

87. Which among the following has lowest boiling point?

(A) n - Butyl alcohol

(B) Isobutyl alcohol

(C) sec-Butyl alcohol

(D) tert-Butyl alcohol

Correct Answer: (D) tert-Butyl alcohol

Solution:

Step 1: Understanding the Concept:
Isomeric alcohols have the same molar mass and the same hydrogen bonding. Their boiling points differ because of the surface area: the more branched the molecule, the more compact it is and the weaker the van der Waals forces between molecules.

Step 2: Key Formula or Approach:
Boiling points: n-butyl alcohol 118 C, isobutyl alcohol 108 C, sec-butyl alcohol 99.5 C, tert-butyl alcohol 82 C.

Step 3: Detailed Explanation:
tert-Butyl alcohol is the most branched and almost spherical, so its surface area is the smallest.
With the weakest dispersion forces, it needs the least energy to boil.
The straight chain n-butyl alcohol has the largest surface contact and the highest boiling point.

Final Answer:
tert-Butyl alcohol has the lowest boiling point, option (D).

tert-Butyl alcohol

Quick Tip: More branching means a smaller surface area and weaker van der Waals forces.

88. Which from following is obtained by hydroboration - oxidation of propene?

(A) Propanal

(B) Propanone

(C) Propan-1-ol

(D) Propan-2-ol

Correct Answer: (C) Propan-1-ol

Solution:

Step 1: Understanding the Concept:

Hydroboration-oxidation adds water to an alkene with anti-Markovnikov orientation. Boron adds to the less substituted carbon and is later replaced by $-OH$.

Step 2: Key Formula or Approach:

Reagents: (i) B_2H_6 (diborane) in ether, (ii) H_2O_2/OH^-.

Step 3: Detailed Explanation:

$3CH_3CH{=}CH_2 + BH_3 \rightarrow (CH_3CH_2CH_2)_3B$.

Oxidation then gives $3CH_3CH_2CH_2OH$ (propan-1-ol).

Propan-2-ol would be the Markovnikov product obtained by acid catalysed hydration. Propanal and propanone are oxidation products, not formed here.

Final Answer:

The product is propan-1-ol, option (C).

$$\boxed{\text{Propan-1-ol}}$$

Quick Tip: Anti-Markovnikov addition of water gives the primary alcohol.

89. Identify the reagent 'R' used in the following reaction

$C_6H_5COCl \xrightarrow{R} C_6H_5CHO + HCl$

(A) CO, HCl

(B) $H_2, Pd - BaSO_4$

(C) $DIBAl - H$

(D) H_2O

Correct Answer: (B) $H_2, Pd - BaSO_4$

Solution:

Step 1: Understanding the Concept:

Converting an acid chloride to an aldehyde needs a gentle reduction that stops at the aldehyde stage. In Rosenmund reduction the catalyst is palladium on barium sulphate, which is partly poisoned so it does not reduce the aldehyde further.

Step 2: Key Formula or Approach:

$RCOCl + H_2 \xrightarrow{Pd\text{-}BaSO_4} RCHO + HCl.$

Step 3: Detailed Explanation:

Applying this: $C_6H_5COCl + H_2 \xrightarrow{Pd\text{-}BaSO_4} C_6H_5CHO + HCl$, which matches the given equation, including the HCl by-product.

CO with HCl is the Gattermann-Koch reagent, used to put a CHO group on benzene. Water would hydrolyse the acid chloride to benzoic acid.

Final Answer:

The reagent is H_2 with $Pd\text{-}BaSO_4$, option (B).

$\boxed{H_2,\ Pd\text{-}BaSO_4}$

Quick Tip: Rosenmund reduction uses H2 over poisoned palladium.

90. Identify the product 'B' in the following reaction

Isopropyl cyanide $\xrightarrow{SnCl_2,HCl}$ A $\xrightarrow{H_3O^+}$ B + NH_4Cl

(A) Propanal

(B) Propanone

(C) 2-Methylpropanal

(D) 2-Methylpropanoic acid

Correct Answer: (C) 2-Methylpropanal

Solution:

Step 1: Understanding the Concept:

In the Stephen reaction, a nitrile is reduced by $SnCl_2$ and HCl to an imine hydrochloride, which hydrolyses to an aldehyde.

Step 2: Key Formula or Approach:

$RCN + 2[H] + HCl \xrightarrow{SnCl_2} RCH{=}NH \cdot HCl$, then $RCH{=}NH \cdot HCl + H_2O \rightarrow RCHO + NH_4Cl$.

Step 3: Detailed Explanation:

Isopropyl cyanide is $(CH_3)_2CHCN$, so R = $(CH_3)_2CH-$.

A is the imine $(CH_3)_2CHCH{=}NH$.

Hydrolysis gives B = $(CH_3)_2CHCHO$, which is 2-methylpropanal, with NH_4Cl as the by-product.

Propanal has one carbon too few and the acid would be the product of a full hydrolysis, which does not occur here.

Final Answer:
B is 2-methylpropanal, option (C).

2-Methylpropanal

Quick Tip: Stephen reduction turns a nitrile into an aldehyde with the same carbon skeleton.

91. Benzonitrile on reaction with a reagent in dry ether followed by hydrolysis forms benzophenone along with ammonia and hydroxymagnesium bromide. Identify the reagent used in above transformation.

(A) C_6H_5MgCl

(B) C_2H_5MgBr

(C) C_6H_5MgI

(D) C_6H_5MgBr

Correct Answer: (D) C_6H_5MgBr

Solution:

Step 1: Understanding the Concept:
A nitrile reacts with a Grignard reagent to form an imine salt, which hydrolyses to a ketone: $RCN + R'MgX \rightarrow RC(=NMgX)R'$, then $\rightarrow RCOR'$.

Step 2: Key Formula or Approach:

Benzophenone is $C_6H_5COC_6H_5$. One phenyl comes from benzonitrile, so the Grignard must supply the second phenyl group.

Step 3: Detailed Explanation:

The by-product is hydroxymagnesium bromide, $Mg(OH)Br$, so the halide in the reagent is bromide.

That fits C_6H_5MgBr.

$$C_6H_5CN + C_6H_5MgBr \rightarrow C_6H_5C(=NMgBr)C_6H_5 \xrightarrow{H_2O} C_6H_5COC_6H_5 + NH_3 + Mg(OH)Br$$

The ethyl reagent would give propiophenone. The chloride and iodide reagents would give $Mg(OH)Cl$ and $Mg(OH)I$ instead.

Final Answer:

The reagent is phenylmagnesium bromide, option (D).

$$\boxed{C_6H_5MgBr}$$

Quick Tip: The Grignard reagent must supply a phenyl group, and bromide shows in the by-product.

92. Propyne on reaction with water in presence of 40% sulphuric acid and 1% mercuric sulphate forms

(A) Propan-1-ol

(B) Propan-2-ol

(C) Propanone

(D) Propanal

Correct Answer: (C) Propanone

Solution:

Step 1: Understanding the Concept:

Addition of water to an alkyne in dilute H_2SO_4 with $HgSO_4$ (Kucherov reaction) first gives an enol. The enol is unstable and rearranges to a carbonyl compound.

Step 2: Key Formula or Approach:

Water adds by Markovnikov rule: the $-OH$ goes to the more substituted triple bond carbon.

Step 3: Detailed Explanation:

$CH_3 - C \equiv CH + H_2O \rightarrow CH_3 - C(OH) = CH_2$ (enol).

The enol tautomerises to $CH_3 - CO - CH_3$, propanone (acetone).

Only ethyne gives an aldehyde (acetaldehyde). The alcohols in A and B are not obtained since the enol rearranges to a carbonyl.

Final Answer:

Propyne gives propanone, option (C).

Propanone

Quick Tip: Alkynes hydrate by Kucherov reaction to ketones, following Markovnikov.

93. Identify final products in the following sequence of reactions

$RCOOH + R'OH \xrightarrow{H^+}$ intermediate $\xrightarrow[\rightarrow]{Ni,\Delta}{}^{H_2}$ Products

(A) $R'COOH + RCH_2OH$

(B) $RCH_2OH + R' - OH$

(C) $R'CH_2OH + R - OH$

(D) $R - OH + R' - OH$

Correct Answer: (B) $RCH_2OH + R' - OH$

Solution:

Step 1: Understanding the Concept:

A carboxylic acid and an alcohol in the presence of H^+ give an ester: $RCOOH + R'OH \rightarrow RCOOR' + H_2O$. The ester is the intermediate.

Step 2: Key Formula or Approach:

Catalytic hydrogenation with H_2 over Ni on heating reduces an ester to two alcohols: $RCOOR' + 2H_2 \rightarrow RCH_2OH + R'OH$.

Step 3: Detailed Explanation:

The acyl part $RCO-$ is reduced to the primary alcohol RCH_2OH.

The alkoxy part $R'O-$ is released as the alcohol $R'OH$.

So the final products are RCH_2OH and $R'OH$. Option A swaps the groups, and the acid in A is not made. Options C and D do not reduce the carbonyl.

Final Answer:

The products are RCH_2OH and $R' - OH$, option (B).

$$\boxed{RCH_2OH + R'OH}$$

Quick Tip: Hydrogenation cleaves an ester into two alcohols.

94. Which from following amines does NOT form alkyl isocyanide when heated with alc. KOH and chloroform

(A) Methanamine

(B) Propan-2-amine

(C) N, N-Diethylethanamine

(D) Ethanamine

Correct Answer: (C) N, N-Diethylethanamine

Solution:

Step 1: Understanding the Concept:
The carbylamine (isocyanide) test works only for primary amines. When heated with chloroform and alcoholic KOH, $\mathbf{R\text{-}NH_2}$ gives the foul smelling isocyanide $\mathbf{R\text{-}NC}$.

Step 2: Key Formula or Approach:
Classify each amine: methanamine, ethanamine and propan-2-amine are primary. *N*,*N*-Diethylethanamine, $(C_2H_5)_3N$, is tertiary.

Step 3: Detailed Explanation:
The reaction needs two N-H hydrogens to be removed with the dichlorocarbene formed from $CHCl_3$ and KOH.

$$R\text{-}NH_2 + CHCl_3 + 3KOH \rightarrow R\text{-}NC + 3KCl + 3H_2O$$

A tertiary amine has no N-H, so it cannot form an isocyanide.

Final Answer:
N,*N*-Diethylethanamine does not form an isocyanide, option (C).

N, N-Diethylethanamine

Quick Tip: Only primary amines give isocyanides.

95. Which from following statements is NOT correct regarding Hofmann's exhaustive alkylation?

(A) In this quaternary ammmonium salts are obtained.

(B) Two moles of alkylhalide when heated with one mole primary amine forms tetraalkyl ammonium halide.

(C) Depending on the quantity of alkyl halides treated with primary amine, the mixture of secondary and tertiary amines obtained.

(D) The reaction is carried out in presence of $NaHCO_3$

Correct Answer: (B) Two moles of alkylhalide when heated with one mole primary amine forms tetraalkyl ammonium halide.

Solution:

Step 1: Understanding the Concept:
In Hofmann exhaustive alkylation, an amine is treated with excess alkyl halide so that all N-H hydrogens are replaced by alkyl groups, giving a quaternary ammonium salt. A base such as $NaHCO_3$ is used to take up the acid formed.

Step 2: Key Formula or Approach:
A primary amine, RNH_2, already has one alkyl group. A quaternary ammonium ion has four, so three more are needed, which means three moles of alkyl halide.

Step 3: Detailed Explanation:

Statement A: the final products are quaternary ammonium salts. This is correct.

Statement B: two moles of alkyl halide only add two alkyl groups, giving a tertiary amine, $\mathbf{R\text{-}NR'_2}$ (as its salt) and not a tetraalkyl ammonium halide. So B is NOT correct.

Statement C: with a limited amount of alkyl halide the product is a mixture of secondary and tertiary amines. This agrees with the textbook description.

Statement D: the reaction uses $\mathbf{NaHCO_3}$ as the base. This is correct.

Final Answer:

Statement B is the incorrect one, option (B).

$\boxed{B}$

Quick Tip: Count how many alkyl groups a primary amine needs to become a quaternary salt.

96. Which from following is NOT a globular protein?

(A) Serum albumin

(B) Insulin

(C) Egg albumin

(D) Keratin

Correct Answer: (D) Keratin

Solution:

Step 1: Understanding the Concept:
Proteins are divided by shape into fibrous proteins (long thread-like chains, insoluble in water) and globular proteins (coiled into a ball shape, soluble in water).

Step 2: Key Formula or Approach:
Fibrous examples: keratin (hair, nails, wool), myosin, collagen, fibroin. Globular examples: insulin, albumins, haemoglobin, enzymes.

Step 3: Detailed Explanation:
Serum albumin, insulin and egg albumin are all water soluble globular proteins.
Keratin has chains lying side by side, held by hydrogen bonds and disulphide bridges. It is insoluble in water, and it is a fibrous protein.

Final Answer:
Keratin is not a globular protein, option (D).

Keratin

Quick Tip: Keratin is a fibrous protein.

97. Which of the following has largest size?

(A) Antibody

(B) Virus

(C) Bacteria

(D) Cancer cell

Correct Answer: (D) Cancer cell

Solution:

Step 1: Understanding the Concept:

These four objects differ by several orders of magnitude in size. An antibody is a protein molecule, a virus is a particle, a bacterium is a single cell, and a cancer cell is a eukaryotic cell.

Step 2: Key Formula or Approach:

Typical sizes: antibody about 10 nm; virus about $20 - 300$ nm; bacterium about $1 - 5\ \mu$m; cancer cell (human) about $10 - 20\ \mu$m.

Step 3: Detailed Explanation:

Order: antibody < virus < bacteria < cancer cell.

Eukaryotic cells, such as a human cancer cell, are generally many times larger than bacterial cells, which are generally many times larger than viruses.

Final Answer:

The cancer cell is the largest, option (D).

Cancer cell

Quick Tip: Compare typical sizes in nanometres and micrometres.

98. What is the total mass of products obtained when one gram mole of sucrose is hydrolysed?

(A) 180g

(B) 342g

(C) 170g

(D) 360g

Correct Answer: (D) 360g

Solution:

Step 1: Understanding the Concept:

Sucrose hydrolyses in the presence of acid or the enzyme invertase to give glucose and fructose: $C_{12}H_{22}O_{11} + H_2O \rightarrow C_6H_{12}O_6 + C_6H_{12}O_6$.

Step 2: Key Formula or Approach:

Molar masses: sucrose = 342 g/mol, water = 18 g/mol, glucose = fructose = 180 g/mol.

Step 3: Detailed Explanation:

Mass of reactants = 342 + 18 = 360 g.

Mass of products = 180 + 180 = 360 g.

So one gram mole of sucrose gives 360 g of products in total (one mole of glucose plus one mole of fructose).

The value 180 g is the mass of only one of the two products, and 342 g is the mass of sucrose alone, which ignores the water added.

Final Answer:

The total mass of the products is 360 g, option (D).

$$\boxed{360\ \text{g}}$$

Quick Tip: Hydrolysis adds one water molecule per sucrose, and mass is conserved.

99. Which from following polymers is NOT obtained by addition polymerisation method?

(A) Teflon

(B) Polyacrolonitrile

(C) LDP

(D) PHBV

Correct Answer: (D) PHBV

Solution:

Step 1: Understanding the Concept:
Addition polymers form by joining unsaturated monomers through chain reaction without losing any small molecule. Condensation polymers form by joining monomers with the loss of small molecules such as water.

Step 2: Key Formula or Approach:
Teflon is from tetrafluoroethene, polyacrylonitrile from acrylonitrile and LDP (low density polythene) from ethene, all alkenes. PHBV comes from 3-hydroxybutanoic acid and 3-hydroxypentanoic acid.

Step 3: Detailed Explanation:
The first three are made from C=C monomers by addition.
PHBV (poly-β-hydroxybutyrate-co-β-hydroxyvalerate) is a polyester made by condensation of the two hydroxy acids, so it is not an

addition polymer.

Final Answer:
PHBV is not an addition polymer, option (D).

PHBV

Quick Tip: PHBV is a condensation copolymer of hydroxy acids.

100. Which medicinal property is exhibited by methylsalicylate?

(A) Analgesic

(B) Antioxident

(C) Antiseptic

(D) Antimicrobial

Correct Answer: (A) Analgesic

Solution:

Step 1: Understanding the Concept:
Methyl salicylate (oil of wintergreen) is the methyl ester of salicylic acid. It is used in liniments and sprays that are rubbed on the skin.

Step 2: Key Formula or Approach:
Analgesics are drugs that reduce or relieve pain without affecting consciousness.

Step 3: Detailed Explanation:
Methyl salicylate is applied to relieve muscle and joint pain, so it acts

as a topical analgesic.
It is not used as an antioxidant (a food preservative class), and it is not classed as an antiseptic or an antimicrobial in this context.

Final Answer:
Methyl salicylate is an analgesic, option (A).

Analgesic

Quick Tip: Methyl salicylate is the active ingredient of pain balms.

Mathematics

Questions 101 to 150 | 2 marks each | 100 marks

101. The value of $(\frac{-1+i\sqrt{3}}{2})^{18} + (\frac{-1-i\sqrt{3}}{2})^{18}$ is

(A) 2

(B) 35

(C) $\sqrt{3}$

(D) $\frac{\sqrt{3}}{2}$

Correct Answer: (A) 2

Solution:

Step 1: Understanding the Concept:

The numbers $\omega = \frac{-1+i\sqrt{3}}{2}$ and $\omega^2 = \frac{-1-i\sqrt{3}}{2}$ are the two non-real cube roots of unity. They satisfy $\omega^3 = 1$ and $1 + \omega + \omega^2 = 0$.

Step 2: Key Formula or Approach:

Write each power of ω as a power of ω^3 and use $\omega^3 = 1$.

Step 3: Detailed Explanation:

The first term is $\omega^{18} = (\omega^3)^6 = 1^6 = 1$.

The second term is $(\omega^2)^{18} = \omega^{36} = (\omega^3)^{12} = 1$.

$$\omega^{18} + \omega^{36} = 1 + 1 = 2$$

The values 35, $\sqrt{3}$ and $\frac{\sqrt{3}}{2}$ do not arise, because every power of a cube root of unity is again 1, ω or ω^2.

Final Answer:

The value is 2, option (A).

$$\boxed{2}$$

Quick Tip: The two numbers are the complex cube roots of unity, so use w cubed equals 1.

102. 11 players of the Indian cricket team are sitting at a circular table. The number of ways they can sit so that two players, the wicket keeper and the captain never sit together is ____

(A) $10!$

(B) $2 \times 9!$

(C) $8 \times 9!$

(D) $9 \times 9!$

Correct Answer: (C) $8 \times 9!$

Solution:

Step 1: Understanding the Concept:

The number of ways to seat n people around a circular table is $(n-1)!$. For "never together" problems, subtract the arrangements where the two are together from the total.

Step 2: Key Formula or Approach:

Total ways for 11 players: $10!$. Treat the captain and wicket keeper as one block: that gives 10 units, which can be arranged in $9!$ ways, and the two inside the block can swap in $2!$ ways.

Step 3: Detailed Explanation:

Together: $9! \times 2$.

Not together:

$$10! - 2 \times 9! = 10 \times 9! - 2 \times 9! = 8 \times 9!$$

Option A, $10!$, counts all arrangements with no restriction. Option B counts only the arrangements where they are together.

Final Answer:

The number of ways is $8 \times 9!$, option (C).

$$\boxed{8 \times 9!}$$

Quick Tip: Count all circular arrangements, then subtract those where the two players sit together.

103. In $\triangle ABC$, if $\angle C = \frac{\pi}{3}$, then the value of $cos^2A + cos^2B + cosAcosB$ is...

(A) $\frac{3}{4}$

(B) $\frac{5}{4}$

(C) $\frac{-3}{4}$

(D) $\frac{-5}{4}$

Correct Answer: (A) $\frac{3}{4}$

Solution:

Step 1: Understanding the Concept:

In a triangle $A+B+C=\pi$. With $C=\frac{\pi}{3}$ we get $A+B=\frac{2\pi}{3}$, so $\cos(A+B)=-\frac{1}{2}$.

Step 2: Key Formula or Approach:

$\cos(A+B)=\cos A\cos B-\sin A\sin B=-\frac{1}{2}$.

Step 3: Detailed Explanation:

Let $x=\cos A$ and $y=\cos B$. Then $\sin A\sin B=xy+\frac{1}{2}$.

Square $\sin A\sin B$: $(1-x^2)(1-y^2)=\left(xy+\frac{1}{2}\right)^2$.

Expand: $1-x^2-y^2+x^2y^2=x^2y^2+xy+\frac{1}{4}$.

So $x^2+y^2+xy=1-\frac{1}{4}=\frac{3}{4}$.

$$\cos^2 A+\cos^2 B+\cos A\cos B=\frac{3}{4}$$

Final Answer:

The value is $\frac{3}{4}$, option (A).

$$\boxed{\frac{3}{4}}$$

Quick Tip: Use A + B = 120 degrees and the identity for cos(A + B).

104. The line L_1 given by $\frac{x}{p}+\frac{y}{2}=1$ passes through the point $(5,0)$. The line L_2 given by $\frac{x}{10}+\frac{y}{q}=1$ is parallel to L_1. Then the distance between the lines L_1 and L_2 is...

(A) $\frac{10}{\sqrt{29}}$

(B) $\frac{19}{2\sqrt{29}}$

(C) $\frac{5}{\sqrt{41}}$

(D) $\frac{10}{\sqrt{41}}$

Correct Answer: (A) $\frac{10}{\sqrt{29}}$

Solution:

Step 1: Understanding the Concept:

Line L_1 passes through $(5, 0)$, which fixes p. Parallel lines have equal slopes, which fixes q. Then the distance between parallel lines $ax + by + c_1 = 0$ and $ax + by + c_2 = 0$ is $\frac{|c_1 - c_2|}{\sqrt{a^2+b^2}}$.

Step 2: Key Formula or Approach:

Substitute $(5, 0)$ into $\frac{x}{p} + \frac{y}{2} = 1$.

Step 3: Detailed Explanation:

$\frac{5}{p} = 1$, so $p = 5$. Then $L_1 : \frac{x}{5} + \frac{y}{2} = 1$, i.e. $2x + 5y = 10$.

For L_2: $\frac{x}{10} + \frac{y}{q} = 1$ has slope $-\frac{q}{10}$. It must equal the slope $-\frac{2}{5}$ of L_1, so $q = 4$.

$L_2 : 2x + 5y = 20$.

$$d = \frac{|20 - 10|}{\sqrt{2^2 + 5^2}} = \frac{10}{\sqrt{29}}$$

Final Answer:

The distance is $\frac{10}{\sqrt{29}}$, option (A).

$$\boxed{\frac{10}{\sqrt{29}}}$$

Quick Tip: Find p from the given point, find q from parallelism, then use the parallel line distance formula.

105. Two lines are given by $x^2 - 4xy + 4y^2 + kx - 2ky = 0$, then the value of k so that the distance between them is 3 is

(A) $\sqrt{5}$

(B) $3\sqrt{3}$

(C) $\sqrt{3}$

(D) $3\sqrt{5}$

Correct Answer: (D) $3\sqrt{5}$

Solution:

Step 1: Understanding the Concept:

The given second degree equation represents two lines. If the quadratic part is a perfect square, the two lines are parallel.

Step 2: Key Formula or Approach:

$x^2 - 4xy + 4y^2 = (x - 2y)^2$ and $kx - 2ky = k(x - 2y)$.

Step 3: Detailed Explanation:

The equation becomes $(x - 2y)^2 + k(x - 2y) = 0$, i.e. $(x - 2y)(x - 2y + k) = 0$.

The lines are $x - 2y = 0$ and $x - 2y + k = 0$, which are parallel.

$$d = \frac{|k|}{\sqrt{1^2 + 2^2}} = \frac{|k|}{\sqrt{5}} = 3$$

So $|k| = 3\sqrt{5}$.

Option A would give a distance of 1, option B would give $\frac{3\sqrt{3}}{\sqrt{5}}$ and option C would give $\sqrt{\frac{3}{5}}$, none of which equals 3.

Final Answer:

$k = 3\sqrt{5}$ (taking the positive value), option (D).

$$\boxed{3\sqrt{5}}$$

Quick Tip: Factorise the equation as a perfect square plus a linear term.

106. The area of the region (in sq. unit) bounded by x-axis, the tangent and normal to the circle $x^2 + y^2 = 4$, drawn at a point $(1, \sqrt{3})$ is

(A) 5

(B) $\sqrt{3}$

(C) 2

(D) $2\sqrt{3}$

Correct Answer: (D) $2\sqrt{3}$

Solution:

Step 1: Understanding the Concept:

The tangent to $x^2 + y^2 = 4$ at (x_1, y_1) is $xx_1 + yy_1 = 4$. The normal at any point of a circle passes through its centre, here the origin.

Step 2: Key Formula or Approach:

At $(1, \sqrt{3})$: tangent $x + \sqrt{3}y = 4$. Normal: the line through $(0, 0)$ and $(1, \sqrt{3})$, i.e. $y = \sqrt{3}x$.

Step 3: Detailed Explanation:

The tangent meets the x-axis ($y = 0$) at $(4, 0)$. The normal meets the x-axis at the origin $(0, 0)$. The tangent and the normal meet at the point $(1, \sqrt{3})$.

The region is a triangle with vertices $(0, 0)$, $(4, 0)$, $(1, \sqrt{3})$.

$$\text{Area} = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times 4 \times \sqrt{3} = 2\sqrt{3}$$

Final Answer:

The area is $2\sqrt{3}$ square units, option (D).

$$\boxed{2\sqrt{3}}$$

Quick Tip: Find the tangent and normal at the point, then the triangle they make with the x-axis.

107. The tangent to the circle $x^2 + y^2 = 10$ at the point $(3, 1)$ touches the circle $x^2 + y^2 - 2\sqrt{10}\,x - 20y + k = 0$, then the value of k is...

(A) -109

(B) 109

(C) -101

(D) 101

Correct Answer: (D) 101

Solution:

Step 1: Understanding the Concept:

A line touches a circle when the distance from the centre to the line equals the radius.

Step 2: Key Formula or Approach:

Tangent to $x^2 + y^2 = 10$ at $(3, 1)$: $3x + y = 10$. Second circle: centre $(\sqrt{10}, 10)$, radius$^2 = (\sqrt{10})^2 + 10^2 - k = 110 - k$.

Step 3: Detailed Explanation:

Distance of the centre from $3x + y - 10 = 0$:

$$d = \frac{|3\sqrt{10} + 10 - 10|}{\sqrt{9+1}} = \frac{3\sqrt{10}}{\sqrt{10}} = 3$$

Set $r = d$: $\sqrt{110 - k} = 3$, so $110 - k = 9$.

$$k = 101$$

Final Answer:

$k = 101$, option (D).

101

Quick Tip: The tangent is a tangent to the second circle too, so its distance from that centre equals its radius.

108. If the eccentricity of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, (a > b)$ is $\frac{2}{3}$ and its focal chord is $3x + 2y - 6 = 0$, then the value of $a^2 + b^2$ is...

(A) 11

(B) 12

(C) 13

(D) 14

Correct Answer: (D) 14

Solution:

Step 1: Understanding the Concept:

A focal chord is a chord that passes through a focus. For the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $a > b$, the foci are $(\pm ae, 0)$.

Step 2: Key Formula or Approach:

The line $3x + 2y - 6 = 0$ meets the x-axis at $(2, 0)$, so this point is a focus: $ae = 2$.

Step 3: Detailed Explanation:

Given $e = \frac{2}{3}$: $a \times \frac{2}{3} = 2$, so $a = 3$ and $a^2 = 9$.

$b^2 = a^2(1 - e^2) = 9\left(1 - \frac{4}{9}\right) = 5$.

$$a^2 + b^2 = 9 + 5 = 14$$

Final Answer:

$a^2 + b^2 = 14$, option (D).

$$\boxed{14}$$

Quick Tip: A focal chord passes through a focus, which lies at (ae, 0).

109. If $\lim_{x\to\infty}\left[\frac{x^2+x+1}{x+1} - ax - b\right] = 3$, then $a - b =$...

(A) 2

(B) 3

(C) -2

(D) 4

Correct Answer: (D) 4

Solution:

Step 1: Understanding the Concept:

For the limit to be finite as $x \to \infty$, the coefficient of x in the expression must vanish. Simplify the fraction by division.

Step 2: Key Formula or Approach:

$\frac{x^2+x+1}{x+1} = x + \frac{1}{x+1}$, since $x(x+1) = x^2 + x$ leaves remainder 1.

Step 3: Detailed Explanation:

The expression becomes $(1 - a)x - b + \frac{1}{x+1}$.

As $x \to \infty$, $\frac{1}{x+1} \to 0$. For a finite limit, $1 - a = 0$, so $a = 1$.
Then the limit is $-b = 3$, so $b = -3$.

$$a - b = 1 - (-3) = 4$$

Final Answer:
$a - b = 4$, option (D).

$\boxed{4}$

Quick Tip: Divide the polynomial first so the linear part can be matched.

110. The correct logical equivalence from the following is /are ____
(I) $p \to (q \to r) \equiv (p \wedge q) \to r$
(II) $(p \to q) \to r \equiv p \to (q \vee r)$
(III) $(p \to q) \to r \equiv (p \to r) \wedge (\sim q \to r)$
(IV) $p \to (q \to r) \equiv q \to (p \to r)$

(A) only (I) and (II)

(B) only (III) and (IV)

(C) only (II) and (IV)

(D) only (I) and (IV)

Correct Answer: (D) only (I) and (IV)

Solution:

Step 1: Understanding the Concept:
Use $p \to q \equiv \sim p \vee q$ to rewrite each statement and compare.

Step 2: Key Formula or Approach:

$p \to (q \to r) \equiv \sim p \vee \sim q \vee r$ and $(p \to q) \to r \equiv (p \wedge \sim q) \vee r$.

Step 3: Detailed Explanation:

Statement (I): $p \to (q \to r) \equiv \sim p \vee (\sim q \vee r)$, and $(p \wedge q) \to r \equiv \sim p \vee \sim q \vee r$. These are the same, so (I) is TRUE.

Statement (II): $(p \to q) \to r \equiv (p \wedge \sim q) \vee r$, but $p \to (q \vee r) \equiv \sim p \vee q \vee r$. They differ (take p false, r false: left is false, right is true), so (II) is FALSE.

Statement (III): $(p \to r) \wedge (\sim q \to r) \equiv (\sim p \vee r) \wedge (q \vee r) \equiv (\sim p \wedge q) \vee r$, which differs from $(p \wedge \sim q) \vee r$. So (III) is FALSE.

Statement (IV): both sides are $\sim p \vee \sim q \vee r$, so (IV) is TRUE.

Final Answer:

Only (I) and (IV) are correct, option (D).

$$\boxed{\text{only (I) and (IV)}}$$

Quick Tip: Convert each implication into an or statement and compare.

111. The contrapositive of the statement pattern $[p \vee (p \to q)] \to (p \wedge \sim q)$ is

(A) $(p \wedge \sim q) \to [p \wedge (p \to q)]$

(B) $(\sim p \wedge \sim q) \to [\sim p \wedge (p \to \sim q)]$

(C) $(\sim p \vee q) \wedge [\sim p \vee (p \wedge \sim q)]$

(D) $(\sim p \vee q) \to [\sim p \wedge (p \wedge \sim q)]$

Correct Answer: (D) $(\sim p \vee q) \rightarrow [\sim p \wedge (p \wedge \sim q)]$

Solution:

Step 1: Understanding the Concept:

For a conditional $A \rightarrow B$ the contrapositive is $\sim B \rightarrow \sim A$. Here $A = p \vee (p \rightarrow q)$ and $B = p \wedge \sim q$.

Step 2: Key Formula or Approach:

De Morgan: $\sim (p \wedge \sim q) \equiv \sim p \vee q$. And $\sim (p \vee X) \equiv \sim p \wedge \sim X$.

Step 3: Detailed Explanation:

$\sim B = \sim (p \wedge \sim q) = \sim p \vee q$.

$\sim A = \sim [p \vee (p \rightarrow q)] = \sim p \wedge \sim (p \rightarrow q)$.

Since $p \rightarrow q \equiv \sim p \vee q$, its negation is $p \wedge \sim q$. So $\sim A = \sim p \wedge (p \wedge \sim q)$.

$$\sim B \rightarrow \sim A : (\sim p \vee q) \rightarrow [\sim p \wedge (p \wedge \sim q)]$$

This is option D. Options A and B do not have $\sim p \vee q$ as the hypothesis, and C is not a conditional at all.

Final Answer:

The contrapositive is option (D).

$$\boxed{(\sim p \vee q) \rightarrow [\sim p \wedge (p \wedge \sim q)]}$$

Quick Tip: The contrapositive of A implies B is not-B implies not-A.

112. The negation of the converse of $p \vee q$ is

(A) $\sim p \vee q$

(B) $p \wedge q$

(C) $p \vee \sim q$

(D) $\sim p \wedge \sim q$

Correct Answer: (D) $\sim p \wedge \sim q$

Solution:

Step 1: Understanding the Concept:

The converse of a conditional $p \rightarrow q$ is $q \rightarrow p$. For a statement without an arrow, $p \vee q$, the converse is read with the or statement unchanged in form, because $p \vee q$ is symmetric.

Step 2: Key Formula or Approach:

Converse of $p \vee q$ is $q \vee p$, which is the same statement. Then use De Morgan: $\sim (p \vee q) \equiv \sim p \wedge \sim q$.

Step 3: Detailed Explanation:

Negation of the converse: $\sim (q \vee p) \equiv \sim q \wedge \sim p \equiv \sim p \wedge \sim q$.
Options A and C keep an or, so they are not the negation of an or.
Option B has no negations at all, so it cannot be the negation.

Final Answer:

The negation is $\sim p \wedge \sim q$, option (D).

$$\boxed{\sim p \wedge \sim q}$$

Quick Tip: Find the converse first, then negate using De Morgan.

113. In triangle ABC, with usual notations, if $(a+b+c)(a+b-c)=ab$, then the measure of angle C is...

(A) $\frac{\pi}{2}$

(B) $\frac{2\pi}{3}$

(C) $\frac{5\pi}{6}$

(D) $\frac{3\pi}{4}$

Correct Answer: (B) $\frac{2\pi}{3}$

Solution:

Step 1: Understanding the Concept:

The cosine rule states $c^2=a^2+b^2-2ab\cos C$.

Step 2: Key Formula or Approach:

Expand $(a+b+c)(a+b-c)=(a+b)^2-c^2$.

Step 3: Detailed Explanation:

$(a+b)^2-c^2=ab$, so $a^2+b^2+2ab-c^2=ab$.

This gives $a^2+b^2-c^2=-ab$.

$$\cos C=\frac{a^2+b^2-c^2}{2ab}=\frac{-ab}{2ab}=-\frac{1}{2}$$

So $C=\frac{2\pi}{3}$.

The angle $\frac{\pi}{2}$ would need $\cos C=0$, $\frac{5\pi}{6}$ would give $-\frac{\sqrt{3}}{2}$ and $\frac{3\pi}{4}$ would give $-\frac{1}{\sqrt{2}}$.

Final Answer:

$C=\frac{2\pi}{3}$, option (B).

$$\boxed{\frac{2\pi}{3}}$$

Quick Tip: Expand the product and use the cosine rule.

114. In $\triangle ABC$, with usual notation, if $a = 13, b = 14, c = 15$, then the sum of the values of $sin(\frac{A}{2})$ and $sinA$ is....

(A) $\frac{14}{5\sqrt{5}}$

(B) $\frac{\sqrt{5}+4}{5}$

(C) $\frac{5+\sqrt{5}}{5}$

(D) $\frac{2}{\sqrt{5}} + \frac{1}{2}$

Correct Answer: (B) $\frac{\sqrt{5}+4}{5}$

Solution:

Step 1: Understanding the Concept:

Angle A is opposite side $a = 13$. Use $\cos A = \frac{b^2+c^2-a^2}{2bc}$.

Step 2: Key Formula or Approach:

$\sin A = \sqrt{1 - \cos^2 A}$ and $\sin \frac{A}{2} = \sqrt{\frac{1-\cos A}{2}}$.

Step 3: Detailed Explanation:

$\cos A = \frac{196+225-169}{2\times14\times15} = \frac{252}{420} = \frac{3}{5}$.

$\sin A = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$.

$\sin \frac{A}{2} = \sqrt{\frac{1-\frac{3}{5}}{2}} = \sqrt{\frac{1}{5}} = \frac{1}{\sqrt{5}}$ (positive since $\frac{A}{2}$ is acute).

$$\sin\frac{A}{2}+\sin A=\frac{1}{\sqrt{5}}+\frac{4}{5}=\frac{\sqrt{5}}{5}+\frac{4}{5}=\frac{\sqrt{5}+4}{5}$$

Final Answer:

The sum is $\frac{\sqrt{5}+4}{5}$, option (B).

$$\boxed{\frac{\sqrt{5}+4}{5}}$$

Quick Tip: Use the cosine rule for cos A, then half angle formula.

115. Let $A=\begin{bmatrix}\cos\alpha & -\sin\alpha & 0\\ \sin\alpha & \cos\alpha & 0\\ 0 & 0 & 1\end{bmatrix}$. If $B=\text{adj}\,A$, then the matrix B^{-1} is equal to...

(A) I

(B) A^{-1}

(C) $-A$

(D) A

Correct Answer: (D) A

Solution:

Step 1: Understanding the Concept:

For a square matrix, $A\cdot\text{adj}A=|A|I$. If $|A|=1$, then $\text{adj}A=A^{-1}$.

Step 2: Key Formula or Approach:

Find $|A|$ by expanding along the third row.

Step 3: Detailed Explanation:

$|A| = 1 \times (\cos^2\alpha + \sin^2\alpha) = 1$.

So $B = \text{adj}A = |A|A^{-1} = A^{-1}$.

Then $B^{-1} = (A^{-1})^{-1} = A$.

Option A would need $A = I$, option B would mean $B^{-1} = B$, and option C has the wrong sign.

Final Answer:

$B^{-1} = A$, option (D).

$\boxed{A}$

Quick Tip: For a matrix with determinant 1, the adjoint equals the inverse.

116. Let $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & -13 \\ 5 & -10 \end{bmatrix}$ be two matrices. If the variables x and y satisfy the matrix equation $((A^{-1})^2 + B)\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, then the ordered pair $(x, y) =$

(A) $(3, 5)$

(B) $(10, 7)$

(C) $(4, 6)$

(D) $(5, 3)$

Correct Answer: (D) $(5, 3)$

Solution:

Step 1: Understanding the Concept:

We need the matrix $M = (A^{-1})^2 + B$ and then solve $M\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

Step 2: Key Formula or Approach:

$|A| = 3(-1) - (-4)(1) = 1$. For a 2×2 matrix, $A^{-1} = \frac{1}{|A|}\begin{pmatrix} -1 & 4 \\ -1 & 3 \end{pmatrix}$.

Step 3: Detailed Explanation:

$A^{-1} = \begin{pmatrix} -1 & 4 \\ -1 & 3 \end{pmatrix}$.

$(A^{-1})^2 = \begin{pmatrix} 1-4 & -4+12 \\ 1-3 & -4+9 \end{pmatrix} = \begin{pmatrix} -3 & 8 \\ -2 & 5 \end{pmatrix}$.

Add B: $M = \begin{pmatrix} 3 & -5 \\ 3 & -5 \end{pmatrix}$.

Both rows give $3x - 5y = 0$, so $\frac{x}{y} = \frac{5}{3}$.

Among the options, $(5, 3)$ satisfies this: $15 - 15 = 0$.

Checking the others: $(3, 5)$ gives $9 - 25$, $(10, 7)$ gives $30 - 35$, $(4, 6)$ gives $12 - 30$, none zero.

Final Answer:

The pair is $(5, 3)$, option (D).

$$\boxed{(5, 3)}$$

Quick Tip: Compute the inverse of A, square it, add B and solve the resulting homogeneous system.

117. If $0 \le x \le 1$, $I_1 = \int \sin^{-1}\sqrt{1-x^2}\,dx$ and $I_2 = \int \sin^{-1}x\,dx$, then which of the following is true?

(A) $I_1 = I_2$

(B) $I_1 = \frac{\pi}{2} I_2$

(C) $I_1 + I_2 = \frac{\pi}{2} x$

(D) $I_1 + I_2 = \frac{\pi}{2}$

Correct Answer: (C) $I_1 + I_2 = \frac{\pi}{2} x$

Solution:

Step 1: Understanding the Concept:

For $0 \le x \le 1$, put $x = \cos\theta$ with $\theta \in [0, \frac{\pi}{2}]$. Then $\sqrt{1 - x^2} = \sin\theta$ and $\sin^{-1}\sqrt{1 - x^2} = \theta = \cos^{-1} x$.

Step 2: Key Formula or Approach:

$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$.

Step 3: Detailed Explanation:

So $I_1 = \int \cos^{-1} x \, dx$ and $I_2 = \int \sin^{-1} x \, dx$.

$$I_1 + I_2 = \int (\cos^{-1} x + \sin^{-1} x) \, dx = \int \frac{\pi}{2} \, dx = \frac{\pi}{2} x$$

The options A and B would need $\cos^{-1} x$ and $\sin^{-1} x$ to be equal or proportional, which they are not. Option D would give a constant, but the integral gives a term in x.

Final Answer:

$I_1 + I_2 = \frac{\pi}{2} x$, option (C).

$$\boxed{I_1 + I_2 = \frac{\pi}{2} x}$$

Quick Tip: For x in [0,1], arcsin of the root of one minus x squared is arccos x.

118. Let $t \in (0,1)$ and $\alpha \in (0, \frac{\pi}{4})$. If $x = \text{cosec}^{-1}(\frac{1+t^2}{2t})$, $y = cot^{-1}(\frac{\sqrt{1-t^2}}{t})$ and $\frac{dy}{dx} = f(t)$, then the value of $f(tan\alpha)$ is

(A) $\frac{sec\alpha}{\sqrt{cos2\alpha}}$

(B) $\frac{sec\alpha}{2\sqrt{cos2\alpha}}$

(C) $\frac{cos\alpha}{2\sqrt{cos2\alpha}}$

(D) $\frac{cos\alpha}{\sqrt{cos2\alpha}}$

Correct Answer: (B) $\frac{sec\alpha}{2\sqrt{cos2\alpha}}$

Solution:

Step 1: Understanding the Concept:

Convert x and y to simpler forms. Then $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$.

Step 2: Key Formula or Approach:

$\text{cosec}\, x = \frac{1+t^2}{2t}$, so $\sin x = \frac{2t}{1+t^2}$, which means $x = 2\tan^{-1} t$ for $t \in (0,1)$.

For y, put $t = \sin\theta$: $\cot y = \frac{\cos\theta}{\sin\theta}$, so $y = \theta = \sin^{-1} t$.

Step 3: Detailed Explanation:

$\frac{dx}{dt} = \frac{2}{1+t^2}$ and $\frac{dy}{dt} = \frac{1}{\sqrt{1-t^2}}$.

$$f(t) = \frac{dy}{dx} = \frac{1+t^2}{2\sqrt{1-t^2}}$$

Now put $t = \tan\alpha$: $1 + t^2 = \sec^2\alpha$ and $\sqrt{1-\tan^2\alpha} = \frac{\sqrt{\cos 2\alpha}}{\cos\alpha}$.

$$f(\tan\alpha) = \frac{\sec^2\alpha \cdot \cos\alpha}{2\sqrt{\cos 2\alpha}} = \frac{\sec\alpha}{2\sqrt{\cos 2\alpha}}$$

Final Answer:

$f(\tan\alpha) = \frac{\sec\alpha}{2\sqrt{\cos 2\alpha}}$, option (B).

$$\boxed{\frac{\sec\alpha}{2\sqrt{\cos 2\alpha}}}$$

Quick Tip: Simplify x and y to simple inverse functions of t first.

119. If $y = tan^{-1}\left(\frac{log(\frac{e}{x^3})}{logex^3}\right) + tan^{-1}\left(\frac{log(e^4x^3)}{log(\frac{e}{x^{12}})}\right)$, $x \in (e^{-\frac{1}{3}}, e^{\frac{1}{12}})$ then $\frac{dy}{dx}$ is equal to...

(A) 1

(B) 0

(C) -1

(D) $\frac{1}{e}$

Correct Answer: (B) 0

Solution:

Step 1: Understanding the Concept:

Let $u = \log x^3 = 3\log x$ (natural log, so $\log e = 1$). Then $\log\frac{e}{x^3} = 1 - u$ and $\log(ex^3) = 1 + u$.

Step 2: Key Formula or Approach:

$\tan^{-1}\frac{1-u}{1+u} = \frac{\pi}{4} - \tan^{-1} u$ and $\tan^{-1}\frac{a+b}{1-ab} = \tan^{-1} a + \tan^{-1} b$.

Step 3: Detailed Explanation:

First term: $\tan^{-1}\frac{1-u}{1+u} = \frac{\pi}{4} - \tan^{-1} u$.

Second term: $\log(e^4 x^3) = 4 + u$ and $\log\frac{e}{x^{12}} = 1 - 4u$, so the fraction is $\frac{4+u}{1-4u}$, which gives $\tan^{-1} 4 + \tan^{-1} u$ for the given range.

$$y = \frac{\pi}{4} - \tan^{-1} u + \tan^{-1} 4 + \tan^{-1} u = \frac{\pi}{4} + \tan^{-1} 4$$

This is a constant, so $\frac{dy}{dx} = 0$.

Final Answer:

$\frac{dy}{dx} = 0$, option (B).

$$\boxed{0}$$

Quick Tip: Write each logarithm in terms of u = 3 log x and use the tan inverse addition identities.

120. If $0 \le x \le 1$ and $(sin^{-1}x)^3 + (cos^{-1}x)^3 = a\pi^3$ then

(A) $a \ge \frac{1}{32}$

(B) $a \ge \frac{1}{16}$

(C) $a \le \frac{1}{32}$

(D) $a \le \frac{1}{16}$

Correct Answer: (A) $a \ge \frac{1}{32}$

Solution:

Step 1: Understanding the Concept:

Let $s = \sin^{-1} x$ and $c = \cos^{-1} x$ with $s + c = \frac{\pi}{2}$. For $0 \le x \le 1$, $s \in [0, \frac{\pi}{2}]$.

Step 2: Key Formula or Approach:

$s^3 + c^3 = (s + c)^3 - 3sc(s + c) = \frac{\pi^3}{8} - \frac{3\pi}{2} s\left(\frac{\pi}{2} - s\right)$.

Step 3: Detailed Explanation:

The product $s\left(\frac{\pi}{2} - s\right)$ lies between 0 and its maximum $\frac{\pi^2}{16}$ (at $s = \frac{\pi}{4}$).

So $s^3 + c^3$ lies between $\frac{\pi^3}{8} - \frac{3\pi}{2} \cdot \frac{\pi^2}{16} = \frac{\pi^3}{32}$ and $\frac{\pi^3}{8}$.

Thus $a\pi^3 \ge \frac{\pi^3}{32}$, giving $a \ge \frac{1}{32}$ (and $a \le \frac{1}{8}$).

Option B, $a \ge \frac{1}{16}$, fails at $x = \frac{1}{\sqrt{2}}$ where $a = \frac{1}{32}$.

Final Answer:

$a \ge \frac{1}{32}$, option (A).

$$\boxed{a \ge \frac{1}{32}}$$

Quick Tip: Use s + c = pi/2 and find the minimum of the cubic expression.

121. If $\sum_{n=1}^{2026} tan^{-1}\left(\frac{1}{n^2+n+1}\right) = tan^{-1}\left(1 - \frac{1}{x}\right)$, where $x \ne 0$, then $x =$

(A) 2028

(B) 2026

(C) 1014

(D) 1013

Correct Answer: (C) 1014

Solution:

Step 1: Understanding the Concept:

Use $\tan^{-1} a - \tan^{-1} b = \tan^{-1} \frac{a-b}{1+ab}$. Take $a = n+1$ and $b = n$.

Step 2: Key Formula or Approach:

$\tan^{-1} \frac{(n+1)-n}{1+n(n+1)} = \tan^{-1} \frac{1}{n^2+n+1}$, so each term equals $\tan^{-1}(n+1) - \tan^{-1} n$.

Step 3: Detailed Explanation:

The sum telescopes: $\sum_{n=1}^{2026} [\tan^{-1}(n+1) - \tan^{-1} n] = \tan^{-1} 2027 - \tan^{-1} 1$.

$$= \tan^{-1} \frac{2027-1}{1+2027} = \tan^{-1} \frac{2026}{2028} = \tan^{-1} \frac{1013}{1014}$$

Set $1 - \frac{1}{x} = \frac{1013}{1014}$, so $\frac{1}{x} = \frac{1}{1014}$.

$$x = 1014$$

Final Answer:

$x = 1014$, option (C).

$$\boxed{1014}$$

Quick Tip: Write each term as a difference of two inverse tangents.

122. Let $f(x) = ax + b$ and $g(x) = cx + d$. The condition $f(g(x)) = g(f(x))$ holds for all x if and only if ...

(A) $f(a) = f(c)$

(B) $f(b) = g(b)$

(C) $f(d) = g(b)$

(D) $f(c) = g(a)$

Correct Answer: (C) $f(d) = g(b)$

Solution:

Step 1: Understanding the Concept:

Two functions commute when $f(g(x)) = g(f(x))$ for every x. For linear functions we compare both sides as linear expressions.

Step 2: Key Formula or Approach:

$f(g(x)) = a(cx + d) + b$ and $g(f(x)) = c(ax + b) + d$.

Step 3: Detailed Explanation:

$f(g(x)) = acx + ad + b$ and $g(f(x)) = acx + bc + d$.

The x terms are equal already, so the condition is $ad + b = bc + d$.

Now $f(d) = ad + b$ and $g(b) = cb + d$.

So the condition is $f(d) = g(b)$, option C.

Option A, $f(a) = f(c)$, gives $a^2 + b = ac + b$, which is not the same condition.

Final Answer:

The condition is $f(d) = g(b)$, option (C).

$$\boxed{f(d) = g(b)}$$

Quick Tip: Compute both compositions and compare constants.

123. The number of point / points where the function $f(x) = \frac{1}{x^2-5|x|+6}$ is discontinuous is.....

(A) 0

(B) 1

(C) 2

(D) 4

Correct Answer: (D) 4

Solution:

Step 1: Understanding the Concept:

A rational function is discontinuous at points where the denominator is zero. Here the denominator is $x^2 - 5|x| + 6$.

Step 2: Key Formula or Approach:

Since $x^2 = |x|^2$, put $t = |x|$: $t^2 - 5t + 6 = (t - 2)(t - 3)$.

Step 3: Detailed Explanation:

Zero when $|x| = 2$ or $|x| = 3$.

That gives $x = \pm 2$ and $x = \pm 3$, which are 4 different points.

No common factor cancels with the numerator, which is 1, so each of these four points is a real discontinuity.

The function is continuous everywhere else.

Final Answer:

There are 4 points of discontinuity, option (D).

$\boxed{4}$

Quick Tip: The function is discontinuous where the denominator is zero.

124. If $x\,e^{xy} = y + sin^2 x$, then the value of $\frac{dy}{dx}$ at $x = 0$ is equal to

(A) -1

(B) 1

(C) -2

(D) 2

Correct Answer: (B) 1

Solution:

Step 1: Understanding the Concept:

Use implicit differentiation, using the product rule on xe^{xy} and the chain rule on e^{xy} and $\sin^2 x$.

Step 2: Key Formula or Approach:

At $x = 0$: $0 \cdot e^0 = y + 0$, so $y(0) = 0$.

Step 3: Detailed Explanation:

Differentiate $xe^{xy} = y + \sin^2 x$ with respect to x:

$$e^{xy} + xe^{xy}\left(y + x\frac{dy}{dx}\right) = \frac{dy}{dx} + 2\sin x \cos x$$

Put $x = 0, y = 0$: $1 + 0 = \frac{dy}{dx} + 0$.

So $\frac{dy}{dx} = 1$.

Final Answer:

$\frac{dy}{dx} = 1$ at $x = 0$, option (B).

1

Quick Tip: Find y at x = 0 first, then differentiate both sides.

125. If $g(x) = (x^2 + 2x + 1) \cdot f(x)$ such that $f(0) = 5$ and $\lim_{x \to 0} \frac{f(x)-5}{x} = 4$ then

Missing open brace for superscript

(A) 20

(B) 12

(C) 18

(D) 14

Correct Answer: (D) 14

Solution:

Step 1: Understanding the Concept:

$\lim_{x \to 0} \frac{f(x)-f(0)}{x} = f'(0)$, so the given limit equals $f'(0) = 4$.

Step 2: Key Formula or Approach:

$g(x) = (x+1)^2 f(x)$, so $g'(x) = 2(x+1)f(x) + (x+1)^2 f'(x)$.

Step 3: Detailed Explanation:

At $x = 0$: $g'(0) = 2(1)f(0) + (1)^2 f'(0)$.

$$g'(0) = 2 \times 5 + 4 = 14$$

Option B (12) would result from forgetting the factor 2 in $2(x+1)$, and option A (20) from taking only the first term with $f(0) \times 4$.

Final Answer:

$g'(0) = 14$, option (D).

$$\boxed{14}$$

Quick Tip: Use the product rule and the definition of the derivative of f at 0.

126. If the line $x + By + C = 0$ is the normal to the curve given by $x = asin^3t$, $y = bcos^3t$, (where $a, b \neq 0$) at a point $t = \frac{\pi}{2}$, then $B - C =$

(A) a

(B) $2a$

(C) $-a$

(D) 0

Correct Answer: (A) a

Solution:

Step 1: Understanding the Concept:

For a parametric curve, $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$. The normal is perpendicular to the tangent, so its slope is $-\frac{dx}{dy}$ (or it is vertical if the tangent is horizontal).

Step 2: Key Formula or Approach:

$x = a\sin^3 t$ gives $\frac{dx}{dt} = 3a\sin^2 t\cos t$. $y = b\cos^3 t$ gives $\frac{dy}{dt} = -3b\cos^2 t\sin t$.

Step 3: Detailed Explanation:

$$\frac{dy}{dx} = \frac{-3b\cos^2 t \sin t}{3a\sin^2 t \cos t} = -\frac{b}{a}\cot t$$

At $t = \frac{\pi}{2}$, $\cot t = 0$, so the tangent is horizontal and the normal is vertical.

The point is $x = a\sin^3 \frac{\pi}{2} = a$, $y = b\cos^3 \frac{\pi}{2} = 0$, so the normal is $x = a$, i.e. $x + 0 \cdot y - a = 0$.

Comparing with $x + By + C = 0$ gives $B = 0$ and $C = -a$.

$$B - C = 0 - (-a) = a$$

Final Answer:

$B - C = a$, option (A).

$$\boxed{a}$$

Quick Tip: Find the slope of the tangent at t = pi/2 first; it turns out to be zero, so the normal is vertical.

127. If the tangent to the curve $xy + ax + by = 0$ at $(1, 1)$ makes an angle of $tan^{-1}2$ with positive direction of the x-axis, then the value of $\frac{ab}{a+b}$ is...

(A) 1

(B) -1

(C) 2

(D) -2

Correct Answer: (C) 2

Solution:

Step 1: Understanding the Concept:

The slope of the tangent at a point is $\frac{dy}{dx}$. The tangent makes an angle $\tan^{-1} 2$ with the positive x-axis, so its slope is $\tan(\tan^{-1} 2) = 2$.

Step 2: Key Formula or Approach:

The curve passes through $(1, 1)$, so $1 + a + b = 0$. Differentiate $xy + ax + by = 0$ implicitly.

Step 3: Detailed Explanation:

$y + x\frac{dy}{dx} + a + b\frac{dy}{dx} = 0$, so $\frac{dy}{dx} = -\frac{y+a}{x+b}$.

At $(1, 1)$: $-\frac{1+a}{1+b} = 2$, so $1 + a = -2 - 2b$, which gives $a + 2b = -3$.

From $a + b = -1$: subtract to get $b = -2$, and then $a = 1$.

$$\frac{ab}{a+b} = \frac{(1)(-2)}{-1} = 2$$

Final Answer:

The value is 2, option (C).

$$\boxed{2}$$

Quick Tip: Use the point to get one equation and the slope tan(angle) = 2 to get another.

128. If the side of an equilateral triangle increases at the rate of $\sqrt{3}$cm/sec, then the rate of change of increase of its area when the side is 12cm is

_____

(A) $18\text{cm}^2/\text{sec}$

(B) $10\text{cm}^2/\text{sec}$

(C) $12\text{cm}^2/\text{sec}$

(D) $3\sqrt{3}\text{cm}^2/\text{sec}$

Correct Answer: (A) $18\text{cm}^2/\text{sec}$

Solution:

Step 1: Understanding the Concept:

The area of an equilateral triangle of side s is $A = \frac{\sqrt{3}}{4}s^2$. Both A and s change with time, so use the chain rule.

Step 2: Key Formula or Approach:

$\frac{dA}{dt} = \frac{\sqrt{3}}{4} \cdot 2s \cdot \frac{ds}{dt} = \frac{\sqrt{3}}{2}s\frac{ds}{dt}$.

Step 3: Detailed Explanation:

Given $\frac{ds}{dt} = \sqrt{3}$ cm/s and $s = 12$ cm:

$$\frac{dA}{dt} = \frac{\sqrt{3}}{2} \times 12 \times \sqrt{3} = \frac{3 \times 12}{2} = 18 \text{ cm}^2/\text{s}$$

Option D ($3\sqrt{3}$) would be obtained by forgetting the factor s, and option C would be 12, from using only $\frac{ds}{dt}$ and s but not the $\sqrt{3} \cdot \sqrt{3}$

$= 3$ factor.

Final Answer:

The area increases at $18\ \text{cm}^2/\text{s}$, option (A).

$$\boxed{18\ \text{cm}^2/\text{sec}}$$

Quick Tip: Write area in terms of the side and differentiate with respect to time.

129. $\int \sin(\log x)\,dx =$

(A) $\frac{x}{2}[\sin(\log x) - \cos(\log x)] + c$

(B) $\frac{x}{2}[\sin(\log x) + \cos(\log x)] + c$

(C) $\frac{x}{2}[\cos(\log x) - \sin(\log x)] + c$

(D) $\frac{x}{4}[\cos(\log x) - \sin(\log x)] + c$

Correct Answer: (A) $\frac{x}{2}[\sin(\log x) - \cos(\log x)] + c$

Solution:

Step 1: Understanding the Concept:

Put $\log x = t$, so $x = e^t$ and $dx = e^t\,dt$. The integral becomes $\int e^t \sin t\,dt$, a standard form.

Step 2: Key Formula or Approach:

$\int e^t \sin t\,dt = \frac{e^t}{2}(\sin t - \cos t) + c$, found by applying integration by parts twice.

Step 3: Detailed Explanation:

Let $I = \int e^t \sin t \, dt$. By parts: $I = e^t \sin t - \int e^t \cos t \, dt$.
Apply parts again: $\int e^t \cos t \, dt = e^t \cos t + \int e^t \sin t \, dt = e^t \cos t + I$.
So $I = e^t \sin t - e^t \cos t - I$, hence $2I = e^t(\sin t - \cos t)$.

$$I = \frac{x}{2}[\sin(\log x) - \cos(\log x)] + c$$

Final Answer:
The integral is $\frac{x}{2}[\sin(\log x) - \cos(\log x)] + c$, option (A).

$$\boxed{\frac{x}{2}[\sin(\log x) - \cos(\log x)] + c}$$

Quick Tip: Substitute x = e^t, then use integration by parts twice.

130. The value of $\int \frac{n\sqrt{\text{cosec}^2 x^n - 1}}{x^{(1-n)}} \, dx$ is ...

(A) $log(sinx) + c$

(B) $log(sinx^n) + c$

(C) $log(cotx^n) + c$

(D) $log(cosx^n) + c$

Correct Answer: (B) $log(sinx^n) + c$

Solution:

Step 1: Understanding the Concept:
Since $\text{cosec}^2\theta - 1 = \cot^2\theta$, we have $\sqrt{\text{cosec}^2 x^n - 1} = \cot x^n$ for the usual range where $\cot x^n > 0$.

Step 2: Key Formula or Approach:

$\frac{n}{x^{1-n}} = nx^{n-1}$, and $\frac{d}{dx}(x^n) = nx^{n-1}$.

Step 3: Detailed Explanation:

The integral is $\int nx^{n-1}\cot(x^n)\,dx$.

Put $t = x^n$, so $dt = nx^{n-1}dx$.

$$\int \cot t\,dt = \log|\sin t| + c = \log(\sin x^n) + c$$

The answer is $\log(\sin x^n)$, not $\log(\sin x)$, because the substitution keeps the power inside the sine.

Final Answer:

The value is $\log(\sin x^n) + c$, option (B).

$$\boxed{\log(\sin x^n) + c}$$

Quick Tip: Use the identity cosec squared minus one equals cot squared and substitute t = x^n.

131. $\int \frac{(x+1)\,dx}{x(1+xe^x)} = \ldots\ldots$

(A) $\log\frac{xe^x}{1+xe^x} + c$

(B) $\log\frac{1+xe^x}{xe^x} + c$

(C) $\log\frac{(x+1)e^x}{xe^x} + c$

(D) $\log\frac{(x+1)e^x}{1+xe^x} + c$

Correct Answer: (A) $\log\frac{xe^x}{1+xe^x} + c$

Solution:

Step 1: Understanding the Concept:

Notice that $\frac{d}{dx}(xe^x) = (x+1)e^x$. This suggests the substitution $t = xe^x$.

Step 2: Key Formula or Approach:

Multiply numerator and denominator by e^x: $\frac{(x+1)e^x}{xe^x(1+xe^x)}$.

Step 3: Detailed Explanation:

With $t = xe^x$, $dt = (x+1)e^x\,dx$, so the integral is $\int \frac{dt}{t(1+t)}$.

Partial fractions: $\frac{1}{t(1+t)} = \frac{1}{t} - \frac{1}{1+t}$.

$$\int \left(\frac{1}{t} - \frac{1}{1+t}\right) dt = \log t - \log(1+t) + c = \log \frac{xe^x}{1+xe^x} + c$$

Final Answer:

The integral is $\log \frac{xe^x}{1+xe^x} + c$, option (A).

$$\boxed{\log \frac{xe^x}{1+xe^x} + c}$$

Quick Tip: Multiply the top and bottom by e^x so that the numerator is the derivative of x e^x.

132. The value of the definite integral $\int_0^{\log_e 5} \frac{e^x\sqrt{e^x-1}}{e^x+3}\,dx$ is equal to...

(A) $5+\pi$

(B) $5-\pi$

(C) $4+\pi$

(D) $4-\pi$

Correct Answer: (D) $4-\pi$

Solution:

Step 1: Understanding the Concept:

Put $t=\sqrt{e^x-1}$, so $t^2=e^x-1$ and $2t\,dt=e^x\,dx$. Also $e^x+3=t^2+4$.

Step 2: Key Formula or Approach:

Limits: $x=0\Rightarrow t=0$; $x=\log_e 5\Rightarrow t=\sqrt{5-1}=2$.

Step 3: Detailed Explanation:

$$I=\int_0^2 \frac{t\cdot 2t\,dt}{t^2+4}=2\int_0^2 \frac{t^2}{t^2+4}dt=2\int_0^2\left(1-\frac{4}{t^2+4}\right)dt$$

$$=2\left[t-2\tan^{-1}\frac{t}{2}\right]_0^2=2\left[2-2\cdot\frac{\pi}{4}\right]=4-\pi$$

Final Answer:

The value is $4-\pi$, option (D).

$$\boxed{4-\pi}$$

Quick Tip: Substitute t = sqrt(e^x - 1) to remove the square root.

133. If $[\cdot]$ denotes the greatest integer function, then $\int_0^{\pi/3}[tanx]\,dx =$

(A) $\frac{\pi}{12}$

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{3}$

Correct Answer: (A) $\frac{\pi}{12}$

Solution:

Step 1: Understanding the Concept:

$[\tan x]$ is the greatest integer not exceeding $\tan x$. On $[0, \frac{\pi}{3}]$, $\tan x$ rises from 0 to $\sqrt{3} \approx 1.73$.

Step 2: Key Formula or Approach:

$[\tan x] = 0$ when $0 \le \tan x < 1$, i.e. $0 \le x < \frac{\pi}{4}$. $[\tan x] = 1$ when $1 \le \tan x < 2$, i.e. $\frac{\pi}{4} \le x \le \frac{\pi}{3}$ (as $\sqrt{3} < 2$).

Step 3: Detailed Explanation:

$$\int_0^{\pi/3} [\tan x]\,dx = \int_0^{\pi/4} 0\,dx + \int_{\pi/4}^{\pi/3} 1\,dx = \frac{\pi}{3} - \frac{\pi}{4} = \frac{\pi}{12}$$

Final Answer:

The value is $\frac{\pi}{12}$, option (A).

$$\boxed{\frac{\pi}{12}}$$

Quick Tip: Find where tan x crosses an integer between 0 and pi/3.

134. The area (in square units) bounded by the line $y = x$, the X-axis and the lines $x = -2$ and $x = 4$ is...

(A) 6

(B) $\frac{15}{2}$

(C) $\frac{17}{2}$

(D) 10

Correct Answer: (D) 10

Solution:

Step 1: Understanding the Concept:

The line $y = x$ is below the x-axis for $x < 0$ and above it for $x > 0$. Area must be counted as a positive quantity for each part.

Step 2: Key Formula or Approach:

Area $= \int_{-2}^{0} |x|\,dx + \int_{0}^{4} x\,dx$.

Step 3: Detailed Explanation:

For x from -2 to 0: the triangle has base 2 and height 2, so area $= \frac{1}{2} \times 2 \times 2 = 2$.

For x from 0 to 4: base 4, height 4, area $= \frac{1}{2} \times 4 \times 4 = 8$.

$$\text{Total} = 2 + 8 = 10$$

If the integral $\int_{-2}^{4} x\,dx = 6$ is taken without the modulus, we get option A. That is wrong because the negative part cancels the area.

Final Answer:

The area is 10 square units, option (D).

$$\boxed{10}$$

Quick Tip: The region lies on both sides of the x-axis, so add the two triangles.

135. The equation of the curve whose slope is $\frac{y-1}{x^2+x}$ and which passes through the point $(1, 0)$ is

(A) $xy - x - y - 1 = 0$

(B) $(y - 1)(x + 1) = 2x$

(C) $xy + x + y - 1 = 0$

(D) $y(x + 1) - x + 1 = 0$

Correct Answer: (C) $xy + x + y - 1 = 0$

Solution:

Step 1: Understanding the Concept:

The slope of the curve at any point is $\frac{dy}{dx}$. Setting $\frac{dy}{dx} = \frac{y-1}{x^2+x}$ gives a differential equation with separable variables.

Step 2: Key Formula or Approach:

$\frac{dy}{y-1} = \frac{dx}{x(x+1)} = \left(\frac{1}{x} - \frac{1}{x+1}\right)dx.$

Step 3: Detailed Explanation:

Integrate: $\log|y - 1| = \log|x| - \log|x + 1| + \log|K|$, so $y - 1 = \frac{Kx}{x+1}$.

The curve passes through $(1,0)$: $-1 = \frac{K}{2}$, so $K = -2$.
$(y-1)(x+1) = -2x$, i.e. $xy + y - x - 1 + 2x = 0$.

$$xy + x + y - 1 = 0$$

Final Answer:
The curve is $xy + x + y - 1 = 0$, option (C).

$$\boxed{xy + x + y - 1 = 0}$$

Quick Tip: Separate the variables and use partial fractions on the x side.

136. A body cools from 100°C to 60°C in 20 minutes, the temperature of the surroundings being 20°C. The total time taken (in minutes) for the body to cool down to 40°C is ...

(A) 60

(B) 40

(C) 50

(D) 30

Correct Answer: (B) 40

Solution:

Step 1: Understanding the Concept:
By Newton law of cooling, $\frac{dT}{dt} = -k(T - T_s)$, so $T - T_s = (T_0 - T_s)e^{-kt}$.
Here $T_s = 20$ C and $T_0 = 100$ C.

Step 2: Key Formula or Approach:

Excess temperature: initially 80 C. After 20 min it is $60 - 20 = 40$ C.

Step 3: Detailed Explanation:

$40 = 80e^{-20k}$, so $e^{-20k} = \frac{1}{2}$ and $k = \frac{\ln 2}{20}$.

To reach 40 C the excess is 20 C: $20 = 80e^{-kt}$, so $e^{-kt} = \frac{1}{4}$, i.e. $kt = \ln 4 = 2\ln 2$.

$$t = \frac{2\ln 2}{\ln 2/20} = 40 \text{ min}$$

Final Answer:

The body takes 40 minutes in total, option (B).

$$\boxed{40}$$

Quick Tip: Temperature difference from the surroundings decays exponentially.

137. The general solution of the differential equation $xsinx\frac{dy}{dx} + (xcosx + sinx)y = sinx$ is

(A) $xsinx + ycosx = c$

(B) $ysinx + xcosx = c$

(C) $xysinx - cosx = c$

(D) $xysinx + cosx = c$

Correct Answer: (D) $xysinx + cosx = c$

Solution:

Step 1: Understanding the Concept:

Divide by $x \sin x$ to get the standard linear form $\frac{dy}{dx} + Py = Q$.

Step 2: Key Formula or Approach:

$\frac{dy}{dx} + \left(\cot x + \frac{1}{x}\right) y = \frac{1}{x}$. The integrating factor is $e^{\int P\,dx} = e^{\log \sin x + \log x}$ $= x \sin x$.

Step 3: Detailed Explanation:

Multiply by the integrating factor: $\frac{d}{dx}(y \cdot x \sin x) = \frac{1}{x} \cdot x \sin x = \sin x$.

Integrate: $xy \sin x = -\cos x + c$.

$$xy \sin x + \cos x = c$$

Final Answer:

The general solution is $xy \sin x + \cos x = c$, option (D).

$$\boxed{xy \sin x + \cos x = c}$$

Quick Tip: Write in the form dy/dx + Py = Q and find the integrating factor.

138. If $\bar{a} = \hat{i} - \hat{k}$, $\bar{b} = x\hat{i} + \hat{j} + (1 - x)\hat{k}$ and $\bar{c} = y\hat{i} + x\hat{j} + (1 + x - y)\hat{k}$ then $[\bar{a}\bar{b}\bar{c}]$ depends on

(A) only x

(B) neither x nor y

(C) either x or y

(D) only y

Correct Answer: (B) neither x nor y

Solution:

Step 1: Understanding the Concept:

The scalar triple product $[\bar{a}\ \bar{b}\ \bar{c}]$ equals the determinant of the components of the three vectors.

Step 2: Key Formula or Approach:

$$\begin{vmatrix} 1 & 0 & -1 \\ x & 1 & 1-x \\ y & x & 1+x-y \end{vmatrix}.$$

Step 3: Detailed Explanation:

Expand along the first row: $1 \cdot [1(1+x-y) - x(1-x)] - 0 + (-1)[x \cdot x - 1 \cdot y]$.

First bracket: $1 + x - y - x + x^2 = 1 - y + x^2$.

Second bracket: $x^2 - y$.

$$\text{Value} = (1 - y + x^2) - (x^2 - y) = 1$$

The result is the constant 1, so it depends on neither x nor y.

Final Answer:

The triple product is 1 and depends on neither x nor y, option (B).

neither x nor y

Quick Tip: Expand the determinant and see which variables remain.

139. Let $\bar{u}, \bar{v}, \bar{w}$ be three vectors such that $|\bar{u}| = 1, |\bar{v}| = 2, |\bar{w}| = 3$. If the projection of $\bar{v}$ along $\bar{u}$ is equal to the projection of $\bar{w}$ along $\bar{u}$ and $\bar{v}, \bar{w}$ are perpendicular to each other, then $|\bar{u} - \bar{v} + \bar{w}| =$...

(A) 4

(B) $\sqrt{7}$

(C) 2

(D) $\sqrt{14}$

Correct Answer: (D) $\sqrt{14}$

Solution:

Step 1: Understanding the Concept:

The projection of $\bar{v}$ along $\bar{u}$ is $\frac{\bar{u}\cdot\bar{v}}{|\bar{u}|}$. Since $|\bar{u}| = 1$, equal projections mean $\bar{u} \cdot \bar{v} = \bar{u} \cdot \bar{w}$.

Step 2: Key Formula or Approach:

$|\bar{u} - \bar{v} + \bar{w}|^2 = |\bar{u}|^2 + |\bar{v}|^2 + |\bar{w}|^2 - 2\bar{u} \cdot \bar{v} + 2\bar{u} \cdot \bar{w} - 2\bar{v} \cdot \bar{w}$.

Step 3: Detailed Explanation:

Since $\bar{u} \cdot \bar{v} = \bar{u} \cdot \bar{w}$, the terms $-2\bar{u} \cdot \bar{v} + 2\bar{u} \cdot \bar{w}$ cancel.

Since $\bar{v} \perp \bar{w}$, $\bar{v} \cdot \bar{w} = 0$.

$$|\bar{u} - \bar{v} + \bar{w}|^2 = 1 + 4 + 9 = 14$$

So the magnitude is $\sqrt{14}$.

Final Answer:
The magnitude is $\sqrt{14}$, option (D).

$$\boxed{\sqrt{14}}$$

Quick Tip: Square the magnitude and use the dot products; the unknown ones cancel.

140. Let $\bar{a}, \bar{b}, \bar{c}$ be three vectors of equal magnitude such that the angle between $\bar{a}$ and $\bar{b}$ is α, $\bar{b}$ and $\bar{c}$ is β, $\bar{c}$ and $\bar{a}$ is γ.
Then the minimum value of $cos\alpha + cos\beta + cos\gamma$ is ...

(A) $\frac{1}{2}$

(B) $-\frac{1}{2}$

(C) $\frac{3}{2}$

(D) $-\frac{3}{2}$

Correct Answer: (D) $-\frac{3}{2}$

Solution:

Step 1: Understanding the Concept:
Let each vector have magnitude m. Then $|\bar{a} + \bar{b} + \bar{c}|^2 \geq 0$.

Step 2: Key Formula or Approach:
$|\bar{a} + \bar{b} + \bar{c}|^2 = 3m^2 + 2m^2(\cos\alpha + \cos\beta + \cos\gamma)$.

Step 3: Detailed Explanation:

Since this is at least 0: $3 + 2(\cos\alpha + \cos\beta + \cos\gamma) \geq 0$.

So $\cos\alpha + \cos\beta + \cos\gamma \geq -\frac{3}{2}$.

Equality holds when $\bar{a} + \bar{b} + \bar{c} = \bar{0}$, for example three coplanar vectors at 120° to each other, where each cosine is $-\frac{1}{2}$.

$$\text{Minimum} = -\frac{3}{2}$$

Final Answer:

The minimum value is $-\frac{3}{2}$, option (D).

$$\boxed{-\frac{3}{2}}$$

Quick Tip: A square of a vector sum is never negative.

141. A vector which is orthogonal to the vector $\bar{a} = \hat{i} + 2\hat{j} + 3\hat{k}$ and coplanar with the vectors $\bar{b} = 3\hat{i} + 2\hat{j}$ and $\bar{c} = 2\hat{i} + \hat{j} + 3\hat{k}$ is

(A) $25\hat{i} + 19\hat{j} - 21\hat{k}$

(B) $-25\hat{i} + 19\hat{j} - 21\hat{k}$

(C) $-25\hat{i} + 19\hat{j} + 21\hat{k}$

(D) $25\hat{i} + 19\hat{j} + 21\hat{k}$

Correct Answer: (A) $25\hat{i} + 19\hat{j} - 21\hat{k}$

Solution:

Step 1: Understanding the Concept:

The vector $\bar{a} \times (\bar{b} \times \bar{c})$ is perpendicular to $\bar{a}$ and also lies in the plane of $\bar{b}$ and $\bar{c}$. This is exactly what the question asks for.

Step 2: Key Formula or Approach:

$\bar{a} \times (\bar{b} \times \bar{c}) = (\bar{a} \cdot \bar{c})\bar{b} - (\bar{a} \cdot \bar{b})\bar{c}$.

Step 3: Detailed Explanation:

$\bar{a} \cdot \bar{c} = 2 + 2 + 9 = 13$ and $\bar{a} \cdot \bar{b} = 3 + 4 + 0 = 7$.

$13\bar{b} - 7\bar{c} = 13(3, 2, 0) - 7(2, 1, 3) = (39 - 14,\ 26 - 7,\ -21)$

$= (25, 19, -21)$.

$$25\hat{i} + 19\hat{j} - 21\hat{k}$$

Check orthogonality with $\bar{a}$: $25 + 38 - 63 = 0$.

Final Answer:

The vector is $25\hat{i} + 19\hat{j} - 21\hat{k}$, option (A).

$$\boxed{25\hat{i} + 19\hat{j} - 21\hat{k}}$$

Quick Tip: Use the vector triple product, since a x (b x c) lies in the plane of b and c and is perpendicular to a.

142. The line ℓ passes through the point $(2, 1, 1)$ and is parallel to the plane $x + y + 2z = 18$. If line ℓ intersects the line $\frac{x+2}{3} = \frac{y+1}{-1} = \frac{z-2}{1}$, then equation of the line ℓ is...

(A) $\frac{x-2}{3} = \frac{y-1}{1} = \frac{z-1}{-2}$

(B) $\frac{x-3}{1} = \frac{y-2}{3} = \frac{z-2}{-2}$

(C) $\frac{x-2}{3} = \frac{y-1}{-1} = \frac{z-1}{-1}$

(D) $\frac{x-3}{1} = \frac{y-4}{3} = \frac{z+1}{-2}$

Correct Answer: (D) $\frac{x-3}{1} = \frac{y-4}{3} = \frac{z+1}{-2}$

Solution:

Step 1: Understanding the Concept:

A line parallel to the plane $x + y + 2z = 18$ has a direction vector perpendicular to the plane normal $(1, 1, 2)$. The line must also meet the given line.

Step 2: Key Formula or Approach:

A general point on the given line: $P = (-2 + 3s,\ -1 - s,\ 2 + s)$. The direction from $(2, 1, 1)$ to P is $(-4 + 3s,\ -2 - s,\ 1 + s)$.

Step 3: Detailed Explanation:

Parallel to the plane: $(-4 + 3s) + (-2 - s) + 2(1 + s) = 0$, so $-4 + 4s = 0$ and $s = 1$.

Then $P = (1, -2, 3)$ and the direction is $(-1, -3, 2)$, which is proportional to $(1, 3, -2)$.

The line is $\frac{x-2}{1} = \frac{y-1}{3} = \frac{z-1}{-2}$.

Option D, $\frac{x-3}{1} = \frac{y-4}{3} = \frac{z+1}{-2}$, has the same direction and passes through $(3, 4, -1) = (2, 1, 1) + (1, 3, -2)$, so it is the same line.

Final Answer:

The line is option (D).

$$\boxed{\frac{x-3}{1} = \frac{y-4}{3} = \frac{z+1}{-2}}$$

Quick Tip: Find the point on the given line using the parallel condition, then join it to the point.

143. The acute angle θ between the xy-plane and the plane passing through the point $(1, 2, 4)$ and parallel to the vectors with direction ratios $3, 2, -1$ and $1, -2, -2$ is...

(A) $cos^{-1}(\frac{8}{5\sqrt{5}})$

(B) $cos^{-1}(\frac{5}{8\sqrt{5}})$

(C) $sin^{-1}(\frac{8}{5\sqrt{5}})$

(D) $\frac{\pi}{4}$

Correct Answer: (A) $cos^{-1}(\frac{8}{5\sqrt{5}})$

Solution:

Step 1: Understanding the Concept:

A plane parallel to two vectors has normal along their cross product. The angle between two planes equals the angle between their normals. The xy-plane has normal $\hat{k}$.

Step 2: Key Formula or Approach:

$\bar{n} = (3, 2, -1) \times (1, -2, -2)$.

Step 3: Detailed Explanation:

$\bar{n} = \hat{i}[2(-2) - (-1)(-2)] - \hat{j}[3(-2) - (-1)(1)] + \hat{k}[3(-2) - 2(1)]$

$= \hat{i}(-4-2) - \hat{j}(-6+1) + \hat{k}(-6-2) = -6\hat{i} + 5\hat{j} - 8\hat{k}.$
$|\bar{n}| = \sqrt{36+25+64} = \sqrt{125} = 5\sqrt{5}.$

$$\cos\theta = \frac{|\bar{n} \cdot \hat{k}|}{|\bar{n}|} = \frac{8}{5\sqrt{5}}$$

The acute angle is $\theta = \cos^{-1}\frac{8}{5\sqrt{5}}$. The point $(1,2,4)$ only fixes the position of the plane, not its tilt.

Final Answer:
$\theta = \cos^{-1}\left(\frac{8}{5\sqrt{5}}\right)$, option (A).

$$\boxed{\cos^{-1}\left(\frac{8}{5\sqrt{5}}\right)}$$

Quick Tip: The normal of the plane is the cross product of the two direction vectors.

144. If the plane $\bar{r} = (\lambda+\mu)\hat{i} + (2+\mu)\hat{j} + (3\lambda+2\mu)\hat{k}$, where λ and μ are parameters, intersects coordinate axes at points $(a,0,0)$, $(0,b,0)$ and $(0,0,c)$ then $a+b+c$ =...

(A) $\frac{7}{2}$

(B) 5

(C) $\frac{8}{3}$

(D) $\frac{10}{3}$

Correct Answer: (D) $\frac{10}{3}$

Solution:

Step 1: Understanding the Concept:

The plane is $x = \lambda + \mu$, $y = 2 + \mu$, $z = 3\lambda + 2\mu$. Eliminating λ and μ gives a linear equation in x, y, z.

Step 2: Key Formula or Approach:

From y: $\mu = y - 2$. From x: $\lambda = x - \mu = x - y + 2$.

Step 3: Detailed Explanation:

$z = 3(x - y + 2) + 2(y - 2) = 3x - 3y + 6 + 2y - 4 = 3x - y + 2$.

So the plane is $3x - y - z = -2$, i.e. $\frac{x}{-2/3} + \frac{y}{2} + \frac{z}{2} = 1$.

Intercepts: $a = -\frac{2}{3}$, $b = 2$, $c = 2$.

$$a + b + c = -\frac{2}{3} + 4 = \frac{10}{3}$$

Final Answer:

The sum is $\frac{10}{3}$, option (D).

$$\boxed{\frac{10}{3}}$$

Quick Tip: Eliminate the parameters to get the Cartesian equation.

145. The coordinates of the point of intersection of the lines $\frac{x-3}{1} = \frac{y-5}{2} = \frac{z-1}{-1}$ and $\frac{x-4}{2} = \frac{y-2}{-1} = \frac{z-4}{2}$ are...

(A) $(2, 3, 2)$

(B) $(-2, 3, -2)$

(C) $(-2, -3, 2)$

(D) $(2, 3, -2)$

Correct Answer: (A) $(2, 3, 2)$

Solution:

Step 1: Understanding the Concept:

Two lines intersect when there are parameter values at which their points coincide. Write general points of each line.

Step 2: Key Formula or Approach:

Line 1: $(3 + s,\ 5 + 2s,\ 1 - s)$. Line 2: $(4 + 2t,\ 2 - t,\ 4 + 2t)$.

Step 3: Detailed Explanation:

Equate: $3 + s = 4 + 2t$ (i), $5 + 2s = 2 - t$ (ii), $1 - s = 4 + 2t$ (iii).

From (i): $s = 1 + 2t$. Put into (iii): $1 - 1 - 2t = 4 + 2t$, so $t = -1$ and $s = -1$.

Check (ii): $5 - 2 = 3$ and $2 + 1 = 3$. It holds.

Point: $(3 - 1,\ 5 - 2,\ 1 + 1) = (2, 3, 2)$.

Final Answer:

The point of intersection is $(2, 3, 2)$, option (A).

$$\boxed{(2, 3, 2)}$$

Quick Tip: Write both lines in parametric form and equate coordinates.

146. If the plane $2x + 3y + z = 6$ cuts coordinate axes at A, B and C, then the volume of tetrahedron OABC (where O is the origin) is .....cubic units.

(A) 30

(B) 6

(C) 36

(D) 180

Correct Answer: (B) 6

Solution:

Step 1: Understanding the Concept:

A plane meets the axes at $A(a, 0, 0)$, $B(0, b, 0)$, $C(0, 0, c)$. The tetrahedron $OABC$ has volume $\frac{1}{6}|abc|$.

Step 2: Key Formula or Approach:

Write $2x + 3y + z = 6$ as $\frac{x}{3} + \frac{y}{2} + \frac{z}{6} = 1$.

Step 3: Detailed Explanation:

Intercepts: $a = 3, b = 2, c = 6$.

$$V = \frac{1}{6} \times 3 \times 2 \times 6 = 6 \text{ cubic units}$$

The value 36 is the product abc, so it forgets the factor $\frac{1}{6}$. The value 30 and 180 come from wrong intercepts.

Final Answer:

The volume is 6 cubic units, option (B).

$$\boxed{6}$$

Quick Tip: Find the intercepts and use volume = one sixth of the product of the intercepts.

147. The maximum value of $Z = 4x + 5y$, subject to the constraints $3x + y \leq 15, 3x + 4y \leq 24, x \geq 0, y \geq 0$ is

(A) 31

(B) 30

(C) 42

(D) 47

Correct Answer: (A) 31

Solution:

Step 1: Understanding the Concept:

The maximum of a linear function over a bounded feasible region occurs at a corner point. Find all corner points of the region defined by the constraints.

Step 2: Key Formula or Approach:

Corner points come from the axes and from the intersection of $3x + y = 15$ and $3x + 4y = 24$.

Step 3: Detailed Explanation:

Subtract the equations: $3y = 9$, so $y = 3$ and $x = 4$. The point is $(4, 3)$. Other corners: $(0, 0)$, $(5, 0)$ (from $3x = 15$) and $(0, 6)$ (from $4y = 24$). $Z(0, 0) = 0$, $Z(5, 0) = 20$, $Z(0, 6) = 30$, $Z(4, 3) = 16 + 15 = 31$.

$$Z_{max} = 31$$

Final Answer:

The maximum value is 31, option (A).

$$\boxed{31}$$

Quick Tip: Check Z at each corner point of the feasible region.

148. A fair coin is tossed 9 times. On each toss, a man predicts that the outcome will be heads. The probability that the number of successful predictions is strictly greater than the number of unsuccessful predictions is...

(A) $\frac{3}{4}$

(B) $\frac{1}{6}$

(C) $\frac{1}{2}$

(D) $-\frac{1}{2}$

Correct Answer: (C) $\frac{1}{2}$

Solution:

Step 1: Understanding the Concept:

The number of heads X in 9 tosses of a fair coin follows $B(9, \frac{1}{2})$. A prediction of "heads" is successful when the toss is heads.

Step 2: Key Formula or Approach:

Successes exceed failures when $X > 9 - X$, i.e. $X \geq 5$.

Step 3: Detailed Explanation:

$P(X = r) = \binom{9}{r}\left(\frac{1}{2}\right)^9$, which is symmetric: $P(X = r) = P(X = 9 - r)$.

So $P(X \geq 5) = P(X \leq 4)$. Since there are no ties (9 is odd), these two events cover everything, and each has probability $\frac{1}{2}$.

$$P = \frac{1}{2}$$

Option D is negative, so it cannot be a probability.

Final Answer:

The probability is $\frac{1}{2}$, option (C).

$$\boxed{\frac{1}{2}}$$

Quick Tip: With 9 tosses, successes beat failures when there are at least 5 heads, and the distribution is symmetric.

149. Let $F(x)$ be the cumulative distribution function (c.d.f.) of a continuous random variable X. If $F(b) = 0.7$ and $P(X > a) = 0.4$, then the value of $P(a < X < b)$ is ...

(A) 0.1

(B) 0.2

(C) 0.3

(D) 0.5

Correct Answer: (A) 0.1

Solution:

Step 1: Understanding the Concept:

The c.d.f. is $F(x) = P(X \leq x)$. For a continuous variable, single points have zero probability, so $P(X > a) = 1 - F(a)$.

Step 2: Key Formula or Approach:

$P(a < X < b) = F(b) - F(a)$.

Step 3: Detailed Explanation:

$P(X > a) = 0.4$, so $F(a) = 1 - 0.4 = 0.6$.

$$P(a < X < b) = F(b) - F(a) = 0.7 - 0.6 = 0.1$$

For a continuous variable $P(X = a)$ and $P(X = b)$ are zero, so using strict or non-strict inequality does not change the answer. The other options (0.2, 0.3, 0.5) would need $F(a)$ to be 0.5, 0.4 or 0.2.

Final Answer:

$P(a < X < b) = 0.1$, option (A).

$$\boxed{0.1}$$

Quick Tip: For a continuous variable, P(a < X < b) = F(b) - F(a).

150. If the probabilities of a student succeeding in the entrance tests for institutes A, B and C are 0.6, 0.5 and 0.4 respectively, while the probability of succeeding in both A and B is 0.3, in both B and C is 0.2, in both A and C is 0.2, and in all three is 0.1, then the probability that the student succeeds in exactly one of these tests is.....

(A) 0.3

(B) 0.4

(C) 0.5

(D) 0.6

Correct Answer: (B) 0.4

Solution:

Step 1: Understanding the Concept:

Let A, B, C be the events of success in each test. Exactly one success means only one of the three happens.

Step 2: Key Formula or Approach:

$P(\text{exactly one}) = \sum P(A) - 2\sum P(A \cap B) + 3P(A \cap B \cap C)$.

Step 3: Detailed Explanation:

$\sum P(A) = 0.6 + 0.5 + 0.4 = 1.5$.

$\sum P(A \cap B) = 0.3 + 0.2 + 0.2 = 0.7$.

$P(A \cap B \cap C) = 0.1$.

$$P = 1.5 - 2(0.7) + 3(0.1) = 1.5 - 1.4 + 0.3 = 0.4$$

Final Answer:

The probability is 0.4, option (B).

$$\boxed{0.4}$$

Quick Tip: Use inclusion-exclusion for exactly one of three events.