

MHT CET 2026 PCM Question Paper April 18 Shift 2 with Solutions



General Instructions

- (i) This booklet contains 150 questions, each provided with a complete, step-by-step solution.
- (ii) It comprises 150 single-correct multiple-choice questions.
- (iii) Attempt each question on your own before reviewing the given solution.

Physics

Questions 1 to 50 | 1 mark each | 50 marks

1. If a unit vector is represented by $\vec{U} = 0.9\hat{i} - 0.2\hat{j} + m\hat{k}$, then the value of m is

- (A) 0.85
- (B) $\sqrt{0.15}$
- (C) 1
- (D) $\sqrt{0.77}$

Correct Answer: (B) $\sqrt{0.15}$

Solution:

Step 1: Understanding the Concept:

A unit vector is a vector of magnitude exactly one. The magnitude of a vector is the square root of the sum of the squares of its components.

Step 2: Key Formula or Approach:

$$|\vec{U}| = \sqrt{(0.9)^2 + (-0.2)^2 + m^2} = 1$$

Step 3: Detailed Explanation:

Square both sides:

$$0.81 + 0.04 + m^2 = 1$$

$$m^2 = 1 - 0.85 = 0.15$$

$$m = \sqrt{0.15}$$

Check the other options: $m = 0.85$ would give $m^2 = 0.7225$, and the total would be about 1.57. The choice $m = 1$ gives total 1.85. The choice $m = \sqrt{0.77}$ gives total 1.62. All of these exceed 1, so none of them is a unit vector.

Final Answer:

The value of m is $\sqrt{0.15}$, option (B).

$$\boxed{\sqrt{0.15} \text{ (B)}}$$

Quick Tip: A unit vector has magnitude 1, so the squares of its components add up to 1.

2. The maximum error in the measurement of the density and mass of the uniform cube are 8% and 2% respectively. Hence, the maximum error in the measurement of length will be

- (A) 2%
- (B) 4%
- (C) 6%
- (D) 8%

Correct Answer: (A) 2%

Solution:

Step 1: Understanding the Concept:

For a cube of side L and mass M , the density is $\rho = \frac{M}{L^3}$. In the maximum error method, the percentage errors of the factors add up, each multiplied by the power of its quantity.

Step 2: Key Formula or Approach:

$$\frac{\Delta\rho}{\rho} = \frac{\Delta M}{M} + 3 \frac{\Delta L}{L}$$

Step 3: Detailed Explanation:

Given $\frac{\Delta\rho}{\rho} = 8\%$ and $\frac{\Delta M}{M} = 2\%$.

$$8 = 2 + 3x \Rightarrow 3x = 6 \Rightarrow x = 2\%$$

So the maximum error in the measurement of the length is 2 percent. Option (B) 4 % would need the density error to be 14 %. Option (C) 6 % and (D) 8 % ignore the power 3.

Final Answer:

The maximum error in length is 2 %, option (A).

$$2\% \text{ (A)}$$

Quick Tip: Density = mass / L^3 , so the percentage error in density is that of mass plus 3 times that of length.

3. For a projectile motion, the range R is 'n' times the maximum height H . So the angle of projection is

- (A) $\sin^{-1}(\frac{2}{n})$
- (B) $\cos^{-1}(\frac{4}{n})$
- (C) $\tan^{-1}(\frac{4}{n})$
- (D) $\tan^{-1}(\frac{2}{n})$

Correct Answer: (C) $\tan^{-1}(\frac{4}{n})$

Solution:**Step 1: Understanding the Concept:**

For a projectile launched with speed u at angle θ , the range and the maximum height are fixed by the same two quantities.

Step 2: Key Formula or Approach:

$$R = \frac{u^2 \sin 2\theta}{g}, \quad H = \frac{u^2 \sin^2 \theta}{2g}$$

Step 3: Detailed Explanation:

Divide range by height:

$$\frac{R}{H} = \frac{u^2 \cdot 2 \sin \theta \cos \theta / g}{u^2 \sin^2 \theta / 2g} = \frac{4 \cos \theta}{\sin \theta} = \frac{4}{\tan \theta}$$

It is given that $R = nH$, so

$$\frac{4}{\tan \theta} = n \Rightarrow \tan \theta = \frac{4}{n} \Rightarrow \theta = \tan^{-1} \left(\frac{4}{n} \right)$$

Check: for $n = 4$ the angle is 45° , which is the angle at which $R = 4H$ for a projectile. The options in (A) and (D) have 2 in place of 4, and option (B) uses cosine, none of which gives this check.

Final Answer:

The angle of projection is $\tan^{-1}(4/n)$, option (C).

$$\tan^{-1} \left(\frac{4}{n} \right) \text{ (C)}$$

Quick Tip: Divide $R = u^2 \sin 2\theta / g$ by $H = u^2 \sin^2 \theta / 2g$ to get $R/H = 4 / \tan \theta$.



4. An inextensible string passing over a smooth pulley connects two blocks of masses m_1 and m_2 ($m_2 > m_1$) vertically. If the acceleration of the system is $(\frac{g}{n})$, then the ratio of masses $(\frac{m_2}{m_1})$ is

- (A) $\frac{n}{n+1}$
 (B) $\frac{n}{n-1}$
 (C) $\frac{n+1}{n-1}$
 (D) $\frac{n+1}{n}$

Correct Answer: (C) $\frac{n+1}{n-1}$

Solution:

Step 1: Understanding the Concept:

Two masses hang on either side of a smooth pulley. The heavier mass m_2 goes down and the lighter mass m_1 goes up with the same acceleration a .

Step 2: Key Formula or Approach:

For m_2 : $m_2g - T = m_2a$. For m_1 : $T - m_1g = m_1a$. Add them to remove T :

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2}$$

Step 3: Detailed Explanation:

Set $a = \frac{g}{n}$:

$$\frac{m_2 - m_1}{m_2 + m_1} = \frac{1}{n} \Rightarrow n(m_2 - m_1) = m_2 + m_1$$

$$nm_2 - m_2 = nm_1 + m_1 \Rightarrow m_2(n - 1) = m_1(n + 1)$$

$$\frac{m_2}{m_1} = \frac{n+1}{n-1}$$

Check with $n = 3$: ratio 2. Then $a = \frac{(2-1)g}{3} = \frac{g}{3}$, which fits. Options (A) and (D) give ratios below 1 for $n > 1$ or not matching, and (B) gives $\frac{3}{2}$ for $n = 3$, which would make $a = g/5$.

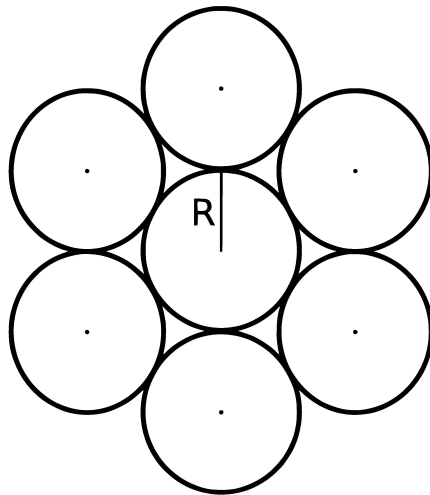
Final Answer:

The ratio $m_2/m_1 = \frac{n+1}{n-1}$, option (C).

$$\boxed{\frac{n+1}{n-1} \text{ (C)}}$$

Quick Tip: Atwood acceleration is $a = (m_2 - m_1)g / (m_1 + m_2)$. Set $a = g/n$ and solve for m_2/m_1 .

5. Given identical rings are arranged in a hexagonal plane pattern so as to touch each neighbouring ring as shown in figure. Each ring has mass M and radius R . The moment of inertia of the system of seven rings about an axis passing through the centre of central ring and normal to plane of all rings is



- (A) $31MR^2$
- (B) $19MR^2$
- (C) $11MR^2$
- (D) $7MR^2$

Correct Answer: (A) $31MR^2$

Solution:

Step 1: Understanding the Concept:

The figure shows one ring at the centre and six identical rings around it. Each outer ring touches the central ring and its two neighbours, so the centres of the outer rings lie on a regular hexagon around the central ring. The axis passes through the centre of the central ring, perpendicular to the plane.

Step 2: Key Formula or Approach:

1. A ring about its own axis: $I_{cm} = MR^2$.
2. Parallel axis theorem: $I = I_{cm} + Md^2$.

Step 3: Detailed Explanation:

Because the outer ring touches the central ring, the distance between their centres is $R + R = 2R$, so $d = 2R$.

Central ring: $I_0 = MR^2$.

Each outer ring:

$$I_1 = MR^2 + M(2R)^2 = MR^2 + 4MR^2 = 5MR^2$$

Six outer rings:

$$6I_1 = 30MR^2$$

Total:

$$I = MR^2 + 30MR^2 = 31MR^2$$

Option (B) 19 and (C) 11 would arise from using $d = R$ or fewer rings.
Option (D) 7 is the moment of inertia of seven rings with no shift.

Final Answer:

The moment of inertia of the system is $31MR^2$, option (A).

$31MR^2 \text{ (A)}$

Quick Tip: Use the parallel axis theorem for the six outer rings, each at distance $2R$ from the central axis.

6. A kathak dancer is standing on horizontal surface with folded hands. In the beginning dancer is rotating about his central axis and his kinetic energy is 'K' at that time. The kathak dancer now stretches his arms so that the moment of inertia of the dancer becomes three times and the angular velocity becomes one-third. The kinetic energy of the dancer now is

- (A) $\frac{K}{6}$
- (B) $\frac{K}{3}$
- (C) $3K$
- (D) $6K$

Correct Answer: (B) $\frac{K}{3}$

Solution:

Step 1: Understanding the Concept:

The dancer's rotational kinetic energy depends on both the moment of inertia I and the angular velocity ω . No external torque acts, so angular momentum $I\omega$ stays constant.

Step 2: Key Formula or Approach:

$$K = \frac{1}{2}I\omega^2$$

Step 3: Detailed Explanation:

Initially $K = \frac{1}{2}I\omega^2$.

After stretching the arms, $I' = 3I$ and $\omega' = \frac{\omega}{3}$. Check that the angular

momentum is conserved: $I'\omega' = 3I \cdot \frac{\omega}{3} = I\omega$. It is.

New kinetic energy:

$$K' = \frac{1}{2}(3I)\left(\frac{\omega}{3}\right)^2 = \frac{1}{2} \cdot 3I \cdot \frac{\omega^2}{9} = \frac{1}{3} \left(\frac{1}{2}I\omega^2\right) = \frac{K}{3}$$

The kinetic energy decreases, because the dancer's arm muscles do negative work when stretching out against the spin. Option (A) $K/6$, (C) $3K$ and (D) $6K$ do not follow from this calculation.

Final Answer:

The new kinetic energy is $\frac{K}{3}$, option (B).

$$\boxed{\frac{K}{3} \text{ (B)}}$$

Quick Tip: Angular momentum is conserved. Kinetic energy is $(1/2) I \omega^2$, so I times 3 and ω divided by 3 gives $K/3$.

7. A solid sphere rolls without slipping on an inclined plane at an angle θ . The ratio of total kinetic energy to its rotational kinetic energy is

- (A) $\frac{5}{2}$
- (B) $\frac{7}{2}$
- (C) $\frac{5}{4}$
- (D) $\frac{5}{7}$

Correct Answer: (B) $\frac{7}{2}$

Solution:

Step 1: Understanding the Concept:

A sphere rolling without slipping has two kinetic energies: translation of its centre of mass and rotation about the centre. The condition for rolling is $v = \omega R$.

Step 2: Key Formula or Approach:

For a solid sphere $I = \frac{2}{5}mR^2$.

Step 3: Detailed Explanation:

Translational KE:

$$K_T = \frac{1}{2}mv^2$$

Rotational KE:

$$K_R = \frac{1}{2}I\omega^2 = \frac{1}{2} \cdot \frac{2}{5}mR^2 \cdot \frac{v^2}{R^2} = \frac{1}{5}mv^2$$

Total KE:

$$K = K_T + K_R = \frac{1}{2}mv^2 + \frac{1}{5}mv^2 = \frac{7}{10}mv^2$$

Ratio:

$$\frac{K}{K_R} = \frac{7/10}{1/5} = \frac{7}{2}$$

Option (A) $\frac{5}{2}$ is K_T/K_R ... in fact $K_T/K_R = \frac{1/2}{1/5} = \frac{5}{2}$. That is the ratio of translational to rotational energy, which is not what is asked. Option (C) and (D) do not match any combination.

Final Answer:

The ratio of total kinetic energy to rotational kinetic energy is $\frac{7}{2}$, option (B).

$$\boxed{\frac{7}{2} \text{ (B)}}$$

Quick Tip: Rolling without slipping: translational KE is $(1/2) m v^2$ and rotational KE is $(1/5) m v^2$ for a solid sphere.

8. The acceleration due to gravity at a height 'h' above the surface of earth is ' g_h '. At the depth 90 km below the earth's surface the acceleration due to gravity is also ' g_h '. The value of 'h' is

- (A) 180 km
- (B) 120 km
- (C) 90 km
- (D) 45 km

Correct Answer: (D) 45 km

Solution:**Step 1: Understanding the Concept:**

The acceleration due to gravity decreases both above the surface and below the surface, but at different rates. For small heights compared with the Earth's radius R , the first order formulas hold.

Step 2: Key Formula or Approach:

1. Height h : $g_h = g \left(1 - \frac{2h}{R}\right)$.

2. Depth d : $g_d = g \left(1 - \frac{d}{R} \right)$.

Step 3: Detailed Explanation:

Set the two values equal, with $d = 90$ km:

$$\frac{2h}{R} = \frac{d}{R} \Rightarrow 2h = d \Rightarrow h = \frac{d}{2} = \frac{90}{2} = 45 \text{ km}$$

The height does not depend on the value of R here, because R cancels. Since g falls twice as fast with height as with depth, we need only half the distance above the surface. Options (A) 180 km would need a depth of 360 km, and option (C) 90 km would give a larger drop than the depth does.

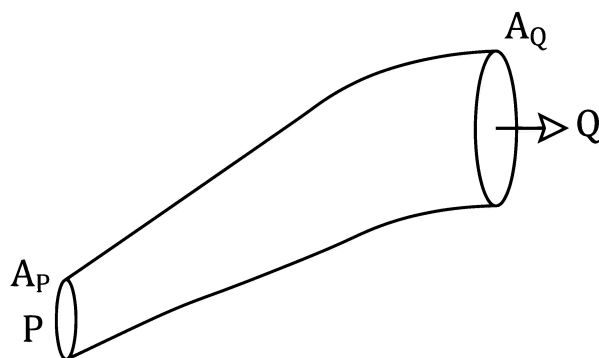
Final Answer:

The height is 45 km, option (D).

45 km (D)

Quick Tip: Use $g_h = g(1 - 2h/R)$ for small h and $g_d = g(1 - d/R)$ for depth d . Equate them.

9. An incompressible fluid (ideal fluid) is flowing through non uniform cross-sectional tube PQ as shown in the figure from end P to end Q. If K_P and K_Q are the kinetic energy per unit volume of the fluid at end P and end Q respectively, then



- (A) $K_P = \frac{1}{2}K_Q$
- (B) $K_P = K_Q$
- (C) $K_P < K_Q$
- (D) $K_P > K_Q$

Correct Answer: (D) $K_P > K_Q$

Solution:

Step 1: Understanding the Concept:

In the figure, the tube is narrow at end P (area A_P) and wider at end Q (area A_Q), and the fluid flows from P to Q. The fluid is incompressible and ideal.

Step 2: Key Formula or Approach:

1. Equation of continuity: $A_P v_P = A_Q v_Q$.
2. Kinetic energy per unit volume: $K = \frac{1}{2} \rho v^2$.

Step 3: Detailed Explanation:

Since $A_P < A_Q$, the continuity equation gives

$$v_P = \frac{A_Q}{A_P} v_Q > v_Q$$

The density ρ is the same at both ends, so K depends only on the square of the speed:

$$K_P = \frac{1}{2} \rho v_P^2 > \frac{1}{2} \rho v_Q^2 = K_Q$$

So $K_P > K_Q$. The ratio is $\left(\frac{A_Q}{A_P}\right)^2$, which is greater than 1 and is not a fixed number like $\frac{1}{2}$. So option (A) is not a general result, and option (B) would need equal speeds, which needs equal areas.

Final Answer:

$K_P > K_Q$, option (D).

$K_P > K_Q \text{ (D)}$

Quick Tip: Continuity $A v = \text{constant}$. The narrow end P has the larger speed, so $K = (1/2) \rho v^2$ is larger at P.



10. In case of fluid flowing on horizontal surface the flow of fluid becomes unsteady when Reynolds number (Rn) is

- (A) less than 499
- (B) between 500 - 999
- (C) between 1000 - 1999
- (D) greater than 2000

Correct Answer: (C) between 1000 - 1999

Solution:

Step 1: Understanding the Concept:

The Reynolds number $R_n = \frac{\rho v D}{\eta}$ compares inertial forces with viscous forces. Its value tells us the type of flow in a pipe.

Step 2: Key Formula or Approach:

In the Maharashtra Board Std XI classification:

1. $R_n < 1000$: the flow is streamline (steady).
2. $1000 < R_n < 2000$: the flow is unsteady.
3. $R_n > 2000$: the flow is turbulent.

Step 3: Detailed Explanation:

The question asks when the flow becomes unsteady, which is the intermediate band between streamline and turbulent flow.

Option (A) less than 499 and option (B) 500 to 999 lie below 1000, so the flow there is streamline.

Option (D) greater than 2000 is the turbulent range.

Option (C), between 1000 and 1999, is the unsteady range.

Final Answer:

The flow is unsteady for R_n between 1000 and 1999, option (C).

$$1000 < R_n < 2000 \text{ (C)}$$

Quick Tip: In the Std XI classification, flow is streamline below 1000, unsteady between 1000 and 2000, and turbulent above 2000.

11. If the shape of the liquid surface is curved, then

- (A) the pressure on convex side is equal to atmospheric pressure.
- (B) the pressure on the convex side is equal to pressure on concave side.
- (C) the pressure on the concave side is greater than that on the convex side.
- (D) the pressure on the concave side is lesser than that on the convex side.

Correct Answer: (C) the pressure on the concave side is greater than that on the convex side.

Solution:

Step 1: Understanding the Concept:

Because of surface tension, a curved liquid surface acts like a stretched skin. This skin pulls on the surface and produces a pressure difference across it.

Step 2: Key Formula or Approach:

The excess pressure on the concave side of a curved surface over the convex side is

$$\Delta P = \frac{2T}{R} \quad (\text{for a spherical surface})$$

Step 3: Detailed Explanation:

For a drop or a bubble, the pressure inside (the concave side) is more than outside (the convex side). The same idea works for any curved liquid surface: the centre of curvature lies on the concave side, and the pressure is higher there.

So the pressure on the concave side is greater than the pressure on the convex side, which is option (C). Option (B) would hold for a flat surface, where there is no curvature and no pressure difference.

Option (D) states the reverse of the actual result. Option (A) does not apply, because the convex side has the lower pressure by an amount $2T/R$.

Final Answer:

The pressure on the concave side is greater than on the convex side, option (C).

$$P_{concave} > P_{convex} \text{ (C)}$$

Quick Tip: Surface tension makes the concave side of a curved surface the high pressure side.

12. A black body radiates maximum energy at wavelength ' λ ' at temperature T_1 and its emissive power is E . When the temperature of the body is changed to T_2 , it radiates maximum energy at wavelength $\frac{2\lambda}{3}$, then the emissive power will become

- (A) $\frac{99}{16} E$
- (B) $\frac{81}{16} E$
- (C) $\frac{63}{16} E$
- (D) $\frac{45}{16} E$

Correct Answer: (B) $\frac{81}{16} E$

Solution:

Step 1: Understanding the Concept:

The wavelength of maximum emission depends inversely on temperature (Wien's displacement law), and the emissive power of a black body goes as the fourth power of temperature (Stefan-Boltzmann law).

Step 2: Key Formula or Approach:

1. $\lambda_m T = \text{constant}$.
2. $E \propto T^4$.

Step 3: Detailed Explanation:

The wavelength changes from λ to $\frac{2\lambda}{3}$:

$$\lambda T_1 = \frac{2\lambda}{3} T_2 \Rightarrow \frac{T_2}{T_1} = \frac{3}{2}$$

Then the emissive power ratio is

$$\frac{E_2}{E_1} = \left(\frac{T_2}{T_1} \right)^4 = \left(\frac{3}{2} \right)^4 = \frac{81}{16}$$

So $E_2 = \frac{81}{16} E$. The other options have numerators 99, 63 and 45, which are not fourth powers of simple ratios.

Final Answer:

The new emissive power is $\frac{81}{16} E$, option (B).

$$\frac{81}{16} E (B)$$

Quick Tip: Wien's law: λ_m times T is constant. Stefan's law: E is proportional to T^4 .



13. A body cools from a temperature 3θ to 2θ in 10 minute. The room temperature is θ . The temperature of the body at the end of the next 10 minute is ' x '. Assuming that Newton's law of cooling is applicable, the value of ' x ' will be

- (A) $\frac{9}{5}\theta$
- (B) $\frac{7}{4}\theta$
- (C) $\frac{3}{2}\theta$
- (D) $\frac{4}{3}\theta$

Correct Answer: (C) $\frac{3}{2}\theta$

Solution:

Step 1: Understanding the Concept:

Newton's law of cooling says the rate of fall of temperature is proportional to the excess temperature over the surroundings. This gives an exponential decay of the excess temperature.

Step 2: Key Formula or Approach:

$$\theta_{\text{excess}}(t) = \theta_{\text{excess}}(0) e^{-kt}$$

Equal time intervals give equal ratios of the excess temperature.

Step 3: Detailed Explanation:

The room temperature is θ .

At the start the excess is $3\theta - \theta = 2\theta$. After 10 minutes the body is at 2θ , so the excess is $2\theta - \theta = \theta$.

The ratio over 10 minutes is $\frac{\theta}{2\theta} = \frac{1}{2}$.

In the next 10 minutes the excess halves again, from θ to $\frac{\theta}{2}$.

So the body temperature is

$$x = \theta + \frac{\theta}{2} = \frac{3}{2}\theta$$

Option (A) $\frac{9}{5}\theta$ and (B) $\frac{7}{4}\theta$ would result from the approximate average temperature method, which does not apply exactly when the fall is large. Option (D) $\frac{4}{3}\theta$ gives a ratio of 3 and not 2.

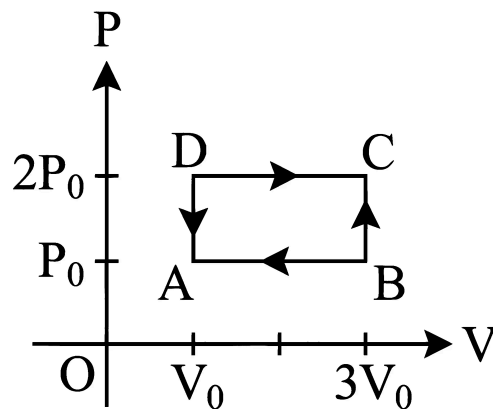
Final Answer:

The temperature after the next 10 minutes is $\frac{3}{2}\theta$, option (C).

$$\boxed{\frac{3}{2}\theta \text{ (C)}}$$

Quick Tip: Excess temperature over the room decays exponentially. It halves in the first 10 minutes, so it halves again in the next 10 minutes.

14. An ideal monoatomic gas is taken around the cycle ABCDA as shown in the P.V diagram. The work done during the cycle is



- (A) $-P_0V_0$
- (B) $2P_0V_0$
- (C) $-2P_0V_0$
- (D) $6P_0V_0$

Correct Answer: (C) $-2P_0V_0$

Solution:

Step 1: Understanding the Concept:

In a P-V diagram, the work done by the gas in a cycle equals the area enclosed. It is positive for a clockwise cycle and negative for an anticlockwise cycle.

Step 2: Key Formula or Approach:

$$W = \int P dV$$

On a path of constant pressure, $W = P\Delta V$. On a path of constant volume, $W = 0$.

Step 3: Detailed Explanation:

The figure shows a rectangle with A at (V_0, P_0) , B at $(3V_0, P_0)$, C at $(3V_0, 2P_0)$ and D at $(V_0, 2P_0)$. The cycle is taken in the order A to B to C to D to A.

A to B: pressure P_0 , volume rises from V_0 to $3V_0$:

$$W_{AB} = P_0(3V_0 - V_0) = 2P_0V_0$$

B to C: constant volume, so $W_{BC} = 0$.

C to D: pressure $2P_0$, volume falls from $3V_0$ to V_0 :

$$W_{CD} = 2P_0(V_0 - 3V_0) = -4P_0V_0$$

D to A: constant volume, so $W_{DA} = 0$.

Total:

$$W = 2P_0V_0 - 4P_0V_0 = -2P_0V_0$$

This equals minus the area of the rectangle, $2V_0 \times P_0$. Option (A) uses half the area, option (B) has the wrong sign, and option (D) is far too big.

Final Answer:

The work done during the cycle is $-2P_0V_0$, option (C).

$-2P_0V_0 \text{ (C)}$

Quick Tip: Work in a cycle equals the signed area enclosed. The direction



15. An ideal gas with pressure P , volume V and temperature T is expanded isothermally to volume $2V$ and a final pressure P_i . The same gas is expanded adiabatically to a volume $2V$ then the final pressure is P_a . In terms of γ , the ratio of the two specific heats for the gas, the ratio P_i/P_a is

- (A) 2^γ
- (B) $2^{1-\gamma}$
- (C) $2^{\gamma-1}$
- (D) 2γ

Correct Answer: (C) $2^{\gamma-1}$

Solution:

Step 1: Understanding the Concept:

The gas starts at (P, V) and expands to $2V$ by two different processes. In the isothermal case, temperature stays fixed. In the adiabatic case, no heat flows in or out.

Step 2: Key Formula or Approach:

1. Isothermal: $PV = P_i(2V)$.
2. Adiabatic: $PV^\gamma = P_a(2V)^\gamma$.

Step 3: Detailed Explanation:

Isothermal final pressure:

$$P_i = \frac{P}{2}$$

Adiabatic final pressure:

$$P_a = P \left(\frac{V}{2V} \right)^\gamma = \frac{P}{2^\gamma}$$

Ratio:

$$\frac{P_i}{P_a} = \frac{P/2}{P/2^\gamma} = 2^{\gamma-1}$$

Since $\gamma > 1$, this ratio is above 1, which means the isothermal pressure is higher. That is right, because the adiabatic expansion also cools the gas. Option (B) is the inverse of the correct ratio, and option (A) forgets the factor $\frac{1}{2}$ in P_i .

Final Answer:

$P_i/P_a = 2^{\gamma-1}$, option (C).

$$\boxed{2^{\gamma-1} \text{ (C)}}$$

Quick Tip: Isothermal: PV constant. Adiabatic: $P V^\gamma$ constant. Compare the two final pressures at volume $2V$.

16. At a pressure P and temperature T , 7 gram of oxygen occupies a volume V . The equation of state will be (Molecular weight of Oxygen = 32)

- (A) $PV = (\frac{2}{7})RT$
(B) $PV = (\frac{7}{2})RT$
(C) $PV = (\frac{7}{16})RT$
(D) $PV = (\frac{7}{32})RT$

Correct Answer: (D) $PV = (\frac{7}{32})RT$

Solution:

Step 1: Understanding the Concept:

The ideal gas equation for a gas of n moles is $PV = nRT$. We need the number of moles in 7 g of oxygen.

Step 2: Key Formula or Approach:

$$n = \frac{\text{mass}}{\text{molecular weight}}$$

Step 3: Detailed Explanation:

The molecular weight of oxygen is given as 32 g/mol.

$$n = \frac{7}{32}$$

Substitute into the ideal gas equation:

$$PV = \frac{7}{32}RT$$

Option (A) $\frac{2}{7}$ and (B) $\frac{7}{2}$ are not ratios of the mass and the molar mass. Option (C) $\frac{7}{16}$ would result from dividing by the atomic weight 16 instead of the molecular weight 32, which is the standard slip when

the gas is diatomic.

Final Answer:

The equation of state is $PV = \frac{7}{32}RT$, option (D).

$$PV = \frac{7}{32}RT \text{ (D)}$$

Quick Tip: Number of moles is mass divided by molar mass, so $n = 7/32$.
Then $PV = nRT$.



17. When the r.m.s. velocity of a gas is denoted by ' v ' at temperature ' T '.
Out of the following relations, the true one is

- (A) $v \cdot T^2 = \text{constant}$
- (B) $\frac{v}{T^2} = \text{constant}$
- (C) $v^2 T = \text{constant}$
- (D) $\frac{v^2}{T} = \text{constant}$

Correct Answer: (D) $\frac{v^2}{T} = \text{constant}$

Solution:

Step 1: Understanding the Concept:

The root mean square speed of gas molecules depends on the temperature and the molar mass. For a given gas the molar mass is fixed.

Step 2: Key Formula or Approach:

$$v = \sqrt{\frac{3RT}{M}}$$

Step 3: Detailed Explanation:

Square both sides:

$$v^2 = \frac{3R}{M} T$$

For a given gas, $\frac{3R}{M}$ is a constant. So

$$\frac{v^2}{T} = \text{constant}$$

This is option (D). Options (A), (B) and (C) have the wrong powers: for instance $v T^2$ constant would mean that v falls as the temperature rises, which is not the case.

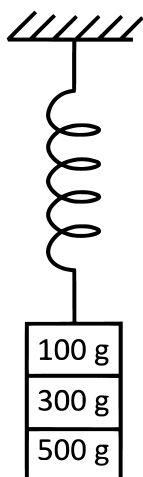
Final Answer:

The true relation is $\frac{v^2}{T} = \text{constant}$, option (D).

$$\boxed{\frac{v^2}{T} = \text{constant (D)}}$$

Quick Tip: $v_{\text{rms}} = \sqrt{3RT/M}$, so v squared is proportional to T .

18. Three masses 100g, 300g and 500g are suspended at the end of a spring as shown in figure and are in equilibrium. When the 500 g mass is removed, the system oscillates with a period of 3 second. When the 300 g mass is also removed, it will oscillate with a period of



- (A) 1.0 s
- (B) 1.5 s
- (C) 2.0 s
- (D) 2.5 s

Correct Answer: (B) 1.5 s

Solution:

Step 1: Understanding the Concept:

In the figure, a spring hangs from a support, and three blocks of 100 g, 300 g and 500 g are stacked one below the other at its lower end. The time period of a mass on a spring depends on the mass attached.

Step 2: Key Formula or Approach:

$$T = 2\pi\sqrt{\frac{m}{k}} \Rightarrow T \propto \sqrt{m}$$

Step 3: Detailed Explanation:

When the 500 g block is removed, the blocks of 100 g and 300 g stay, so the oscillating mass is 400 g and $T_1 = 3$ s.

When the 300 g block is also removed, only the 100 g block remains.

$$\frac{T_2}{T_1} = \sqrt{\frac{100}{400}} = \frac{1}{2}$$

$$T_2 = \frac{3}{2} = 1.5 \text{ s}$$

Option (A) 1.0 s would need a mass ratio of 1/9, and (C) 2.0 s would need 4/9. Option (D) 2.5 s would need a mass larger than 100 g.

Final Answer:

The period with only the 100 g block is 1.5 s, option (B).

1.5 s (B)

Quick Tip: $T = 2\pi \sqrt{m/k}$, so T is proportional to $\sqrt{m}$. Compare 400 g with 100 g.

19. Two particles A and B of equal masses are suspended from two massless springs of spring constants K_A and K_B respectively. The maximum velocities of the particles during oscillations are equal. The ratio of the amplitude of 'B' to that of 'A' is

- (A) $K_A : K_B$
(B) $\sqrt{K_A} : \sqrt{K_B}$
(C) $K_B : K_A$
(D) $\sqrt{K_B} : \sqrt{K_A}$

Correct Answer: (B) $\sqrt{K_A} : \sqrt{K_B}$

Solution:

Step 1: Understanding the Concept:

In SHM the maximum velocity occurs at the mean position and equals amplitude times angular frequency. For a spring-mass system, $\omega = \sqrt{K/m}$.

Step 2: Key Formula or Approach:

$$v_{max} = A\omega = A\sqrt{\frac{K}{m}}$$

Step 3: Detailed Explanation:

The masses of A and B are equal, so write for each particle:

$$A_A\sqrt{\frac{K_A}{m}} = A_B\sqrt{\frac{K_B}{m}}$$

Cancel m :

$$A_A\sqrt{K_A} = A_B\sqrt{K_B}$$

$$\frac{A_B}{A_A} = \frac{\sqrt{K_A}}{\sqrt{K_B}}$$

So the ratio is $\sqrt{K_A} : \sqrt{K_B}$. The stiffer spring gives the smaller amplitude. Option (D) is the inverse of this ratio, and options (A) and (C) forget the square root.

Final Answer:

The ratio of the amplitude of B to that of A is $\sqrt{K_A} : \sqrt{K_B}$, option (B).

$$\boxed{\sqrt{K_A} : \sqrt{K_B} \text{ (B)}}$$

Quick Tip: $v_{\text{max}} = A \omega = A \sqrt{K/m}$. Equal masses and equal v_{max} give A proportional to $1/\sqrt{K}$.

20. A particle of mass ' m ' is executing S.H.M. about the origin on x-axis with frequency $\sqrt{\frac{Ka}{\pi m}}$, where K is a constant and a is the amplitude of S.H.M. If ' x ' is the displacement of a particle at time ' t ', the potential energy of a particle will be

- (A) $\frac{1}{2}Kax^2$
- (B) πKax^2
- (C) $2\pi Kax^2$
- (D) $2Kax^2$

Correct Answer: (C) $2\pi Kax^2$

Solution:

Step 1: Understanding the Concept:

The potential energy of a particle in SHM at displacement x is $U = \frac{1}{2}m\omega^2x^2$. We are given the frequency f , so we first get $\omega = 2\pi f$.

Step 2: Key Formula or Approach:

$$\omega = 2\pi f, \quad U = \frac{1}{2}m\omega^2x^2$$

Step 3: Detailed Explanation:

$$f = \sqrt{\frac{Ka}{\pi m}} \Rightarrow \omega = 2\pi\sqrt{\frac{Ka}{\pi m}}$$

$$\omega^2 = 4\pi^2 \cdot \frac{Ka}{\pi m} = \frac{4\pi Ka}{m}$$

Substitute:

$$U = \frac{1}{2}m \cdot \frac{4\pi Ka}{m} x^2 = 2\pi Ka x^2$$

Mass cancels out. Option (A) $\frac{1}{2}Kax^2$ forgets the factor 4π . Option (B) πKax^2 has half the right factor, which appears if ω^2 is taken as $2\pi Ka/m$. Option (D) lacks the factor π .

Final Answer:

The potential energy is $2\pi Ka x^2$, option (C).

$2\pi Kax^2 \text{ (C)}$

Quick Tip: Find ω from the frequency, then use $U = (1/2) m \omega^2 x^2$.



21. In an organ pipe closed at one end produces a fundamental note of frequency ' ν '. The pipe is cut into two pipes of equal length. The fundamental frequencies produced in the two pipes are

- (A) $\nu, 2\nu$
- (B) $\frac{\nu}{2}, \nu$
- (C) $2\nu, 4\nu$
- (D) $\frac{\nu}{2}, 2\nu$

Correct Answer: (C) $2\nu, 4\nu$

Solution:

Step 1: Understanding the Concept:

A pipe closed at one end has a node at the closed end and an antinode at the open end. Its fundamental frequency is $\nu = \frac{v}{4L}$. An open pipe of length l has a fundamental frequency $\frac{v}{2l}$.

Step 2: Key Formula or Approach:

Cut the pipe into two halves, each of length $\frac{L}{2}$.

Step 3: Detailed Explanation:

The first half still has the original closed end, so it is a closed pipe of length $\frac{L}{2}$:

$$\nu_1 = \frac{v}{4(L/2)} = \frac{v}{2L} = 2\nu$$

The second half has two open ends, so it is an open pipe of length $\frac{L}{2}$:

$$\nu_2 = \frac{v}{2(L/2)} = \frac{v}{L} = 4\nu$$

The fundamental frequencies are 2ν and 4ν . Options (A), (B) and (D) all contain a frequency of ν , or $\nu/2$, which could only be right if the length had not been shortened.

Final Answer:

The two pipes give 2ν and 4ν , option (C).

$2\nu, 4\nu$ (C)

Quick Tip: A closed pipe has fundamental $v/4L$. Cut in half, one half is still closed and the other is open at both ends.

22. The fundamental frequency ' n ' of the tuning fork is 288 Hz, it will not resonate with the frequency

- (A) 288 Hz
- (B) 576 Hz
- (C) 844 Hz
- (D) 864 Hz

Correct Answer: (C) 844 Hz

Solution:

Step 1: Understanding the Concept:

A tuning fork can set up resonance in another body or fork if the driving frequency is the fork's fundamental frequency or one of its overtones. For a vibrating fork the overtones are integer multiples of the fundamental.

Step 2: Key Formula or Approach:

Check whether each given frequency is an integer multiple of 288 Hz.

Step 3: Detailed Explanation:

(A) $288 = 1 \times 288$, so it is the fundamental. It resonates.

(B) $576 = 2 \times 288$, the second harmonic. It resonates.

(D) $864 = 3 \times 288$, the third harmonic. It resonates.

(C) $844 \div 288 = 2.93$, which is not an integer, so 844 Hz is not in the harmonic series. It will not resonate.

Final Answer:

The fork will not resonate with 844 Hz, option (C).

844 Hz (C)

Quick Tip: A fork resonates with its fundamental and its integer multiples. 844 is not a multiple of 288.

23. A wire of length 'L' and linear density 'm' is stretched between two rigid supports with tension 'T'. It is observed that wire resonates in the P^{th} harmonic at a frequency of 320 Hz and resonates again at next higher frequency of 400 Hz in two successive modes. The value of 'P' is

- (A) 2
- (B) 4
- (C) 8
- (D) 10

Correct Answer: (B) 4

Solution:

Step 1: Understanding the Concept:

A wire fixed at both ends resonates at $f_p = p f_1$, where $f_1 = \frac{1}{2L} \sqrt{\frac{T}{m}}$ is the fundamental and p is a whole number.

Step 2: Key Formula or Approach:

The gap between two successive resonances is $f_{p+1} - f_p = f_1$.

Step 3: Detailed Explanation:

$$f_1 = 400 - 320 = 80 \text{ Hz}$$

For the lower resonance:

$$p = \frac{320}{80} = 4$$

Check: the next harmonic is $5 \times 80 = 400$ Hz. This agrees with the given frequency.

Option (A) 2 would give a fundamental of 160 Hz and a next resonance at 480 Hz. Options (C) and (D) give fundamental frequencies of 40 Hz and 32 Hz, which do not give a gap of 80 Hz.

Final Answer:

The wire resonates in the 4th harmonic at 320 Hz, option (B).

$$P = 4 \text{ (B)}$$

Quick Tip: Successive harmonics of a stretched wire differ by the fundamental frequency. So $f_1 = 80 \text{ Hz}$ and $P = 320/80$.

24. A person standing between two parallel cliffs fires a gun and hears two echoes, first echo after 1 second and second echo after 3 second. The distance between the two cliffs is (The velocity of sound = 330 m/s)

- (A) 330 m
- (B) 660 m
- (C) 990 m
- (D) 1320 m

Correct Answer: (B) 660 m

Solution:**Step 1: Understanding the Concept:**

An echo is sound that goes to a cliff and comes back. So the distance to each cliff is half of the distance travelled by the sound before the echo is heard.

Step 2: Key Formula or Approach:

$$d = \frac{vt}{2}$$

Step 3: Detailed Explanation:

The person stands between two parallel cliffs. The first echo comes from the nearer cliff, and the second echo from the farther cliff.

Nearer cliff:

$$d_1 = \frac{330 \times 1}{2} = 165 \text{ m}$$

Farther cliff:

$$d_2 = \frac{330 \times 3}{2} = 495 \text{ m}$$

The distance between the two cliffs is the sum:

$$d = d_1 + d_2 = 165 + 495 = 660 \text{ m}$$

Option (A) 330 m is only the distance travelled in 1 s. Option (C) 990 m would be 330×3 , the path length of the second echo, and (D) 1320 m is twice the correct value.

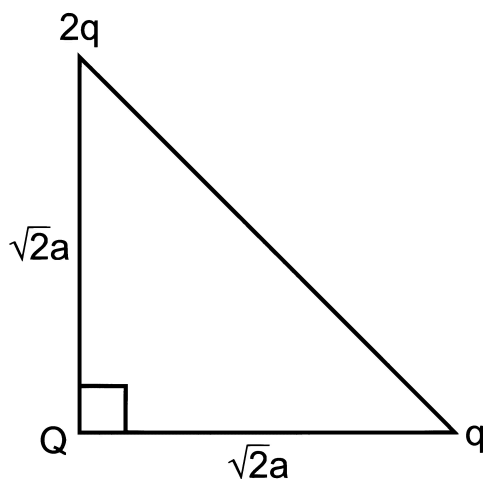
Final Answer:

The distance between the cliffs is 660 m, option (B).

660 m (B)

Quick Tip: Each echo travels to a cliff and back, so distance = $v t / 2$ for each cliff. Add the two distances.

25. Three point charges Q , $+2q$ and $+q$ are placed at the vertices of a right-angled isosceles triangle of length $\sqrt{2}a$ as shown in figure. The net electrostatic potential energy of the configuration is zero, if Q is equal to



- (A) $-\frac{1}{2\sqrt{3}}q$
- (B) $-\frac{\sqrt{2}}{3}q$
- (C) $-\frac{\sqrt{3}}{2}q$
- (D) $-\sqrt{\frac{2}{3}}q$

Correct Answer: (B) $-\frac{\sqrt{2}}{3}q$

Solution:

Step 1: Understanding the Concept:

The electrostatic potential energy of a set of point charges is the sum, over every pair, of $\frac{1}{4\pi\epsilon_0} \frac{q_i q_j}{r_{ij}}$.

Step 2: Key Formula or Approach:

In the figure, the charge Q is at the right-angle corner. The charge $2q$ is on one leg at distance $\sqrt{2}a$, and the charge q is on the other leg, also at distance $\sqrt{2}a$. The two legs are equal, so the distance between $2q$ and q is the hypotenuse, $\sqrt{2} \times \sqrt{2}a = 2a$.

Step 3: Detailed Explanation:

$$U = k \left[\frac{Q(2q)}{\sqrt{2}a} + \frac{Qq}{\sqrt{2}a} + \frac{(2q)(q)}{2a} \right]$$

$$U = \frac{k}{a} \left[\frac{3Qq}{\sqrt{2}} + q^2 \right]$$

Set $U = 0$:

$$\frac{3Qq}{\sqrt{2}} = -q^2 \Rightarrow Q = -\frac{\sqrt{2}}{3} q$$

The sign is negative, so Q attracts the other two charges and balances the repulsion between $2q$ and q . Option (A) $-\frac{1}{2\sqrt{3}}q$ and (C) $-\frac{\sqrt{3}}{2}q$ do not satisfy the equation, and (D) $-\sqrt{\frac{2}{3}}q$ is larger in magnitude by a factor $\sqrt{3}$.

Final Answer:

$$Q = -\frac{\sqrt{2}}{3}q, \text{ option (B).}$$

$$\boxed{-\frac{\sqrt{2}}{3}q \text{ (B)}}$$

Quick Tip: Add the potential energy of each of the three pairs: Q with $2q$, Q with q , and $2q$ with q . Set the sum to zero.

26. Two equal point charges exert a force F on each other when they are placed distance ' d ' apart in air. When they are placed distance ' D ' apart in a medium of dielectric constant K , they exert the same force. The distance D is equal to

- (A) $\frac{d}{\sqrt{K}}$
- (B) $d\sqrt{K}$
- (C) d^2K
- (D) $\frac{K}{d^2}$

Correct Answer: (A) $\frac{d}{\sqrt{K}}$

Solution:

Step 1: Understanding the Concept:

In air, the force between two point charges is given by Coulomb's law. In a medium of dielectric constant K , the force is reduced by the factor K .

Step 2: Key Formula or Approach:

1. In air: $F = \frac{1}{4\pi\epsilon_0} \frac{q^2}{d^2}$.
2. In the medium: $F = \frac{1}{4\pi\epsilon_0 K} \frac{q^2}{D^2}$.

Step 3: Detailed Explanation:

The force is the same in both cases, so we equate the two expressions and cancel the common factors:

$$\frac{1}{d^2} = \frac{1}{KD^2}$$

$$KD^2 = d^2 \Rightarrow D^2 = \frac{d^2}{K}$$

$$D = \frac{d}{\sqrt{K}}$$

Since $K > 1$ for any dielectric, $D < d$, which makes sense: the medium weakens the force, so the charges must be closer to feel the same force. Options (B) and (C) give $D > d$, and option (D) has the wrong dimensions for a length.

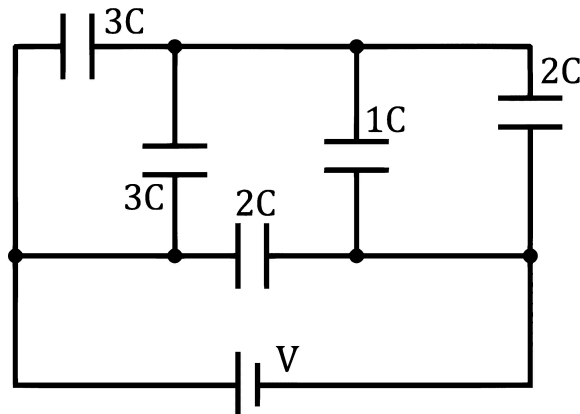
Final Answer:

The distance is $D = \frac{d}{\sqrt{K}}$, option (A).

$$\boxed{\frac{d}{\sqrt{K}} \text{ (A)}}$$

Quick Tip: A dielectric of constant K divides the force by K , so the distance must shrink by the square root of K to keep F the same.

27. Figure shows a network of five capacitors connected to a supply voltage 'V'. The equivalent capacitance and the energy stored in the network is respectively



- (A) $4C, 2CV^2$
- (B) $6C, 3CV^2$
- (C) $9C, 4CV^2$
- (D) $11C, 9CV^2$

Correct Answer: (A) $4C, 2CV^2$

Solution:

Step 1: Understanding the Concept:

The figure shows five capacitors: two of $3C$, one of $2C$ on the right branch, one of $1C$, and one of $2C$ in the lower middle, connected across a supply of voltage V at the bottom. First we need to see which junctions are the same point electrically.

Step 2: Key Formula or Approach:

Parallel: $C = C_1 + C_2$. Series: $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2}$. Energy: $U = \frac{1}{2} C_{eq} V^2$.

Step 3: Detailed Explanation:

The bottom wire joins the left battery terminal to the bottom end of the vertical $3C$ capacitor, and the right battery terminal to the bottom end of both the $1C$ and the right $2C$ capacitors. So call the left terminal L , the right terminal R , and the common top wire T .

Between T and L : the top-left $3C$ and the vertical $3C$ are in parallel, giving $3C + 3C = 6C$.

Between T and R : the $1C$ and the right $2C$ are in parallel, giving $1C + 2C = 3C$.

These two groups are in series from L to R through T :

$$\frac{6C \times 3C}{6C + 3C} = 2C$$

The lower $2C$ capacitor lies directly between L and R , so it is in parallel with this combination:

$$C_{eq} = 2C + 2C = 4C$$

Energy:

$$U = \frac{1}{2}(4C)V^2 = 2CV^2$$

Final Answer:

The equivalent capacitance is $4C$ and the energy is $2CV^2$, option (A).

$$\boxed{4C, 2CV^2 \text{ (A)}}$$

Quick Tip: Trace the nodes. The bottom wires join the left end of the $2C$ capacitor to the battery's left terminal and its right end to the right terminal.

28. Three identical capacitors connected in series have net capacitance ' x '. Then they are connected in parallel. The ratio of energy stored in series configuration to that in parallel configuration is (if both configurations are connected to the same source)

- (A) 1 : 3
- (B) 3 : 1
- (C) 1 : 9
- (D) 9 : 1

Correct Answer: (C) 1 : 9

Solution:

Step 1: Understanding the Concept:

Three identical capacitors, each of capacitance C , are arranged once in series and once in parallel, and each arrangement is connected to the same source of voltage V .

Step 2: Key Formula or Approach:

1. Series: $C_s = \frac{C}{3}$.
2. Parallel: $C_p = 3C$.
3. Energy at fixed voltage: $U = \frac{1}{2} C_{eq} V^2$.

Step 3: Detailed Explanation:

$$\frac{U_s}{U_p} = \frac{C_s}{C_p} = \frac{C/3}{3C} = \frac{1}{9}$$

So the energy in the series arrangement is one ninth of that in the parallel arrangement.

Option (D) 9:1 is the inverse ratio. Option (A) 1:3 and (B) 3:1 would arise from comparing $C/3$ with C , or $3C$ with C , instead of comparing the two arrangements with each other.

Final Answer:

The ratio of energies is 1:9, option (C).

$$1 : 9 \text{ (C)}$$

Quick Tip: Same voltage: $U = (1/2) C V^2$, so the energy ratio equals the ratio of the equivalent capacitances.



29. When cell of E.M.F. ' E_1 ' is connected to potentiometer wire the balancing length is ' l_1 '. Another cell of E.M.F. ' E_2 ' ($E_1 > E_2$) is connected along with E_1 so as two cells oppose each other, the balancing length is ' l_2 '. The ratio $E_1 : E_2$ is

- (A) $(l_1) : (l_1 + l_2)$
- (B) $(l_1) : (l_1 - l_2)$
- (C) $(l_1 + l_2) : (l_1)$
- (D) $(l_1 + l_2) : (l_1 - l_2)$

Correct Answer: (B) $(l_1) : (l_1 - l_2)$

Solution:

Step 1: Understanding the Concept:

In a potentiometer, the emf of a cell is proportional to the balancing length on the wire: $E = \phi l$, where ϕ is the potential gradient.

Step 2: Key Formula or Approach:

1. Cell E_1 alone: $E_1 = \phi l_1$.
2. Cells in opposition: the net emf is $E_1 - E_2 = \phi l_2$.

Step 3: Detailed Explanation:

Divide the first by the second:

$$\frac{E_1}{E_1 - E_2} = \frac{l_1}{l_2}$$

Cross-multiply and rearrange:

$$E_1 l_2 = l_1 E_1 - l_1 E_2 \Rightarrow l_1 E_2 = E_1 (l_1 - l_2)$$

$$\frac{E_1}{E_2} = \frac{l_1}{l_1 - l_2}$$

Since $E_1 > E_2$, the net emf is positive and $l_2 < l_1$, so the denominator is positive. Options (C) and (D) have $l_1 + l_2$, which would apply to cells that help each other, where the net emf is $E_1 + E_2$.

Final Answer:

$E_1 : E_2 = l_1 : (l_1 - l_2)$, option (B).

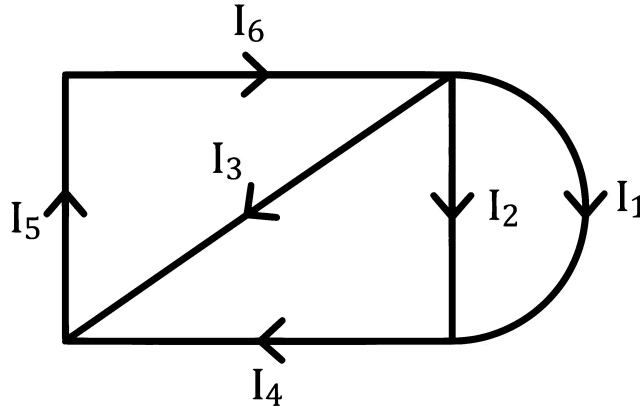
$$\boxed{l_1 : (l_1 - l_2) \text{ (B)}}$$

Quick Tip: With the cells opposing, the net emf is $E_1 - E_2$, which balances at l_2 . The emf is proportional to the balancing length.

30. In the following network,

$$I_1 = -0.4 \text{ A}, I_4 = 1 \text{ A}, I_5 = 0.4 \text{ A}$$

The values of I_2 , I_3 and I_6 are respectively



(A) $1.4 \text{ A}, 0.4 \text{ A}, -0.6 \text{ A}$

(B) $0.4 \text{ A}, -0.6 \text{ A}, 1.4 \text{ A}$

(C) $1.4 \text{ A}, -0.6 \text{ A}, 0.4 \text{ A}$

(D) $-0.6 \text{ A}, 1.4 \text{ A}, 0.4 \text{ A}$

Correct Answer: (C) $1.4 \text{ A}, -0.6 \text{ A}, 0.4 \text{ A}$

Solution:

Step 1: Understanding the Concept:

Kirchhoff's junction rule says the total current entering a junction equals the total current leaving it. In the figure, the network has four corners: top-left, top-right, bottom-left and bottom-right.

Step 2: Key Formula or Approach:

Directions from the figure: I_6 flows left to right along the top. I_3 flows along the diagonal from the top-right corner to the bottom-left corner. I_2 and I_1 flow downward from the top-right corner to the bottom-right corner. I_4 flows leftwards along the bottom, and I_5 flows upward on the left side.

Step 3: Step A: the bottom-right corner:

Entering: I_1 and I_2 . Leaving: I_4 .

$$I_1 + I_2 = I_4 \Rightarrow -0.4 + I_2 = 1 \Rightarrow I_2 = 1.4 \text{ A}$$

Step 4: Step B: the bottom-left corner:

Entering: I_3 and I_4 . Leaving: I_5 .

$$I_3 + I_4 = I_5 \Rightarrow I_3 + 1 = 0.4 \Rightarrow I_3 = -0.6 \text{ A}$$

Step 5: Step C: the top-left corner:

Entering: I_5 . Leaving: I_6 .

$$I_6 = I_5 = 0.4 \text{ A}$$

Check at the top-right corner: entering $I_6 = 0.4$, leaving $I_1 + I_2 + I_3 = -0.4 + 1.4 - 0.6 = 0.4$. The rule holds.

Final Answer:

$I_2 = 1.4 \text{ A}$, $I_3 = -0.6 \text{ A}$ and $I_6 = 0.4 \text{ A}$, option (C).

$1.4 \text{ A}, -0.6 \text{ A}, 0.4 \text{ A (C)}$
--

Quick Tip: Apply Kirchhoff's current law at each corner of the network, using the arrow directions in the figure.

31. A particle of charge ' q ' and mass ' m ' moves in a circular orbit of radius ' r ' with angular speed ' ω '. The ratio of the magnitude of its magnetic moment to that of its angular momentum depends upon

- (A) q and m
- (B) ω and m
- (C) ω and q
- (D) ω , q and m

Correct Answer: (A) q and m

Solution:

Step 1: Understanding the Concept:

A charge q moving in a circle is a current loop. Its magnetic moment is the product of current and area, and its angular momentum is mvr .

Step 2: Key Formula or Approach:

Current: $I = \frac{q}{T} = \frac{q\omega}{2\pi}$. Area: $A = \pi r^2$.

Step 3: Detailed Explanation:

Magnetic moment:

$$M = IA = \frac{q\omega}{2\pi} \cdot \pi r^2 = \frac{q\omega r^2}{2}$$

Angular momentum:

$$L = mvr = m\omega r^2$$

Ratio:

$$\frac{M}{L} = \frac{q\omega r^2/2}{m\omega r^2} = \frac{q}{2m}$$

The factors ω and r cancel. So the ratio depends only on q and m .

Options (B), (C) and (D) include ω , which does not appear in the final ratio.

Final Answer:

The ratio depends only on q and m , option (A).

$$\boxed{\frac{M}{L} = \frac{q}{2m} \text{ (A)}}$$

Quick Tip: For a charge in a circular orbit, $M/L = q/2m$, which depends on q and m only.



32. A straight long wire is carrying current I . The ratio of magnetic field due to this wire at perpendicular distance 2 cm and 5 cm respectively from the wire is

- (A) 2 : 5
- (B) 3 : 5
- (C) 7 : 3
- (D) 5 : 2

Correct Answer: (D) 5 : 2

Solution:

Step 1: Understanding the Concept:

The magnetic field around a long straight current-carrying wire

decreases with distance. The field lines are circles around the wire.

Step 2: Key Formula or Approach:

$$B = \frac{\mu_0 I}{2\pi r} \Rightarrow B \propto \frac{1}{r}$$

Step 3: Detailed Explanation:

$$\frac{B_1}{B_2} = \frac{r_2}{r_1} = \frac{5}{2}$$

Here $r_1 = 2$ cm is the nearer point, so its field is larger. The ratio is 5:2.

Option (A) 2:5 is the inverse ratio and would be the answer if B were proportional to r . Options (B) and (C) are not ratios of 2 and 5.

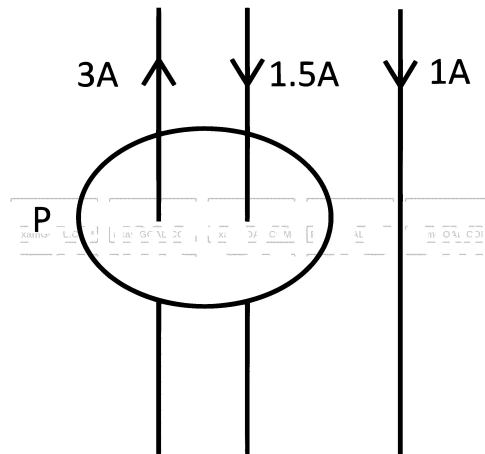
Final Answer:

The ratio is 5:2, option (D).

5 : 2 (D)

Quick Tip: $B = \mu_0 I / (2\pi r)$, so B is inversely proportional to distance.

33. Two wires with current 3A and 1.5A are enclosed in a circular loop P. Third parallel wire with current 1A is situated outside the loop as shown. All the wires are perpendicular to the plane of the circular loop. The value of $\oint \vec{B} \cdot d\vec{l}$ around the loop is (μ_0 = permeability of free space)



- (A) $5.5\mu_0$
- (B) $2.5\mu_0$
- (C) $1.5\mu_0$
- (D) $0.5\mu_0$

Correct Answer: (C) $1.5\mu_0$

Solution:

Step 1: Understanding the Concept:

Ampere's circuital law states that the line integral of $\vec{B}$ around a closed loop equals μ_0 times the net current that passes through the loop.

Step 2: Key Formula or Approach:

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I_{\text{enclosed}}$$

Currents in opposite directions through the loop have opposite signs.

Step 3: Detailed Explanation:

In the figure, the loop P encloses two wires. The 3 A wire carries current upward and the 1.5 A wire carries current downward. The third wire, with 1 A, lies outside the loop.

Net enclosed current (taking upward as positive):

$$I_{\text{enc}} = 3 - 1.5 = 1.5 \text{ A}$$

The wire outside the loop does not contribute to the line integral, even though it does contribute to the field at points on the loop.

$$\oint \vec{B} \cdot d\vec{l} = 1.5 \mu_0$$

Option (A) 5.5 adds all three currents. Option (B) 2.5 adds 1.5 and 1, and (D) 0.5 uses $1.5 - 1$.

Final Answer:

The value is $1.5 \mu_0$, option (C).

$1.5 \mu_0 \text{ (C)}$

Quick Tip: Ampere's law counts only the net current passing through the loop. The 1 A wire outside contributes nothing.

34. Two coils A and B have 180 and 360 turns respectively. The current of 1A flows through both the coils. Due to current of 1A in coil A, flux per turn of 0.8×10^{-3} Wb is linked with coil A. Due to current of 1A in coil B, flux per turn of 1×10^{-3} Wb is linked with coil B. The self inductance of coil A is L_A and the self inductance of coil B is L_B . The ratio L_A to L_B is

- (A) $\frac{1}{5}$
- (B) $\frac{2}{5}$
- (C) $\frac{3}{2}$
- (D) $\frac{5}{2}$

Correct Answer: (B) $\frac{2}{5}$

Solution:

Step 1: Understanding the Concept:

The self inductance of a coil is its total flux linkage per unit current: $L = \frac{N\phi}{I}$, where ϕ is the flux per turn.

Step 2: Key Formula or Approach:

Both coils carry 1 A, so $L = N\phi$ numerically.

Step 3: Detailed Explanation:

Coil A:

$$L_A = \frac{180 \times 0.8 \times 10^{-3}}{1} = 0.144 \text{ H}$$

Coil B:

$$L_B = \frac{360 \times 1 \times 10^{-3}}{1} = 0.36 \text{ H}$$

Ratio:

$$\frac{L_A}{L_B} = \frac{0.144}{0.36} = 0.4 = \frac{2}{5}$$

Option (A) $\frac{1}{5}$ results from using only the turns ratio $\frac{180}{360}$ times $\frac{1}{2}$.

Option (D) $\frac{5}{2}$ is the inverse of the correct ratio.

Final Answer:

$L_A : L_B = 2 : 5$, option (B).

$$\boxed{\frac{2}{5} \text{ (B)}}$$

Quick Tip: Self inductance $L = N$ times flux per turn divided by current.

35. If current of 4 A produces magnetic flux of 3×10^{-3} Wb through a coil of 400 turns, the energy stored in the coil will be

- (A) 1.2 J
- (B) 2.4 J
- (C) 24 J
- (D) 240 J

Correct Answer: (B) 2.4 J

Solution:

Step 1: Understanding the Concept:

The energy stored in the magnetic field of a coil is $U = \frac{1}{2}LI^2$. We first

find the self inductance from the flux.

Step 2: Key Formula or Approach:

$$L = \frac{N\phi}{I}, \quad U = \frac{1}{2}LI^2$$

Step 3: Detailed Explanation:

$$L = \frac{400 \times 3 \times 10^{-3}}{4} = \frac{1.2}{4} = 0.3 \text{ H}$$

$$U = \frac{1}{2} \times 0.3 \times 4^2 = \frac{1}{2} \times 0.3 \times 16 = 2.4 \text{ J}$$

Option (A) 1.2 J is the total flux linkage, $N\phi = 1.2$ Wb-turns, with the wrong unit. Option (C) and (D) are ten times and a hundred times larger than the correct value.

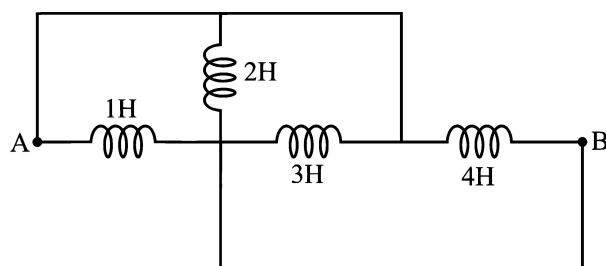
Final Answer:

The energy stored is 2.4 J, option (B).

2.4 J (B)

Quick Tip: Find $L = N\phi / I$, then $U = (1/2) L I^2$.

36. The equivalent inductance between A and B is equal to



(A) 10 H

(B) 5 H

(C) $\frac{12}{25}$ H

(D) $\frac{2}{5}$ H

Correct Answer: (C) $\frac{12}{25}$ H

Solution:

Step 1: Understanding the Concept:

The figure has four inductors, 1 H, 2 H, 3 H and 4 H, joined by wires. We need to find which points are the same electrical node, because wire segments of zero resistance join the same potential.

Step 2: Key Formula or Approach:

For inductors in parallel (no mutual inductance): $\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \frac{1}{L_4}$.

Step 3: Detailed Explanation:

Point A is joined by a wire, over the top of the figure, to the right-hand end of the 3 H inductor and to the left end of the 4 H inductor, and also to the top of the 2 H inductor.

The bottom wire joins point B to the junction between the 1 H and 3

H inductors, which is also the lower end of the 2 H inductor.
 So the 1 H inductor has one end at A and the other end at the node connected to B. The 2 H inductor has one end at the node connected to A and the other at the node connected to B. The 3 H inductor has one end at the node connected to B and the other at the node connected to A. The 4 H inductor has one end at the node connected to A and its other end at B.

All four inductors are connected between A and B in parallel:

$$\frac{1}{L_{eq}} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{12 + 6 + 4 + 3}{12} = \frac{25}{12}$$

$$L_{eq} = \frac{12}{25} \text{ H}$$

Final Answer:

The equivalent inductance is $\frac{12}{25}$ H, option (C).

$\frac{12}{25} \text{ H (C)}$

Quick Tip: Trace the wires: all four inductors end up connected directly between A and B, so they are in parallel.

37. A series combination of resistor 'R' and capacitor 'C' is connected to an a.c. source of angular frequency ' ω '. Keeping the voltage same, if the frequency is changed to $(\frac{\omega}{3})$ the current becomes half of the original current. Then the ratio of capacitive reactance and resistance is

- (A) $\sqrt{6}$
- (B) $\sqrt{0.3}$
- (C) $\sqrt{3}$
- (D) $\sqrt{0.6}$

Correct Answer: (D) $\sqrt{0.6}$

Solution:

Step 1: Understanding the Concept:

In an RC series circuit the impedance is $Z = \sqrt{R^2 + X_C^2}$, with $X_C = \frac{1}{\omega C}$. With the same voltage, the current is inversely proportional to the impedance.

Step 2: Key Formula or Approach:

At frequency ω : $Z_1 = \sqrt{R^2 + X_C^2}$. At $\omega/3$, the reactance becomes $3X_C$, so $Z_2 = \sqrt{R^2 + 9X_C^2}$.

Step 3: Detailed Explanation:

The current halves, so $Z_2 = 2Z_1$:

$$R^2 + 9X_C^2 = 4(R^2 + X_C^2)$$

$$9X_C^2 - 4X_C^2 = 4R^2 - R^2$$

$$5X_C^2 = 3R^2$$

$$\frac{X_C}{R} = \sqrt{\frac{3}{5}} = \sqrt{0.6}$$

Option (A) $\sqrt{6}$ and (C) $\sqrt{3}$ are too large. Option (B) $\sqrt{0.3}$ would arise from taking $5X_C^2 = 1.5R^2$.

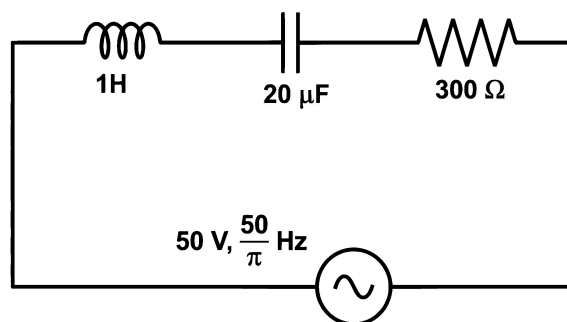
Final Answer:

The ratio X_C/R is $\sqrt{0.6}$, option (D).

$$\boxed{\sqrt{0.6} \text{ (D)}}$$

Quick Tip: Lowering the frequency to one third makes X_C three times larger. Current halving means impedance doubles.

38. In the series LCR circuit shown in figure the impedance is



- (A) 300Ω
- (B) 500Ω
- (C) 700Ω
- (D) 900Ω

Correct Answer: (B) 500Ω

Solution:

Step 1: Understanding the Concept:

The figure shows a series circuit with an inductor of 1 H, a capacitor of 20 microfarad and a resistor of 300 ohm, driven by a 50 V source of frequency $\frac{50}{\pi}$ Hz.

Step 2: Key Formula or Approach:

$$\omega = 2\pi f, \quad X_L = \omega L, \quad X_C = \frac{1}{\omega C}, \quad Z = \sqrt{R^2 + (X_L - X_C)^2}$$

Step 3: Detailed Explanation:

Angular frequency:

$$\omega = 2\pi \cdot \frac{50}{\pi} = 100 \text{ rad/s}$$

Inductive reactance:

$$X_L = 100 \times 1 = 100 \, \Omega$$

Capacitive reactance:

$$X_C = \frac{1}{100 \times 20 \times 10^{-6}} = \frac{1}{2 \times 10^{-3}} = 500 \, \Omega$$

Impedance:

$$Z = \sqrt{300^2 + (100 - 500)^2} = \sqrt{90000 + 160000} = \sqrt{250000} = 500 \, \Omega$$

Option (A) $300\ \Omega$ would be the resistance alone. Option (C) $700\ \Omega$ is $R + |X_L - X_C|$ minus something, and (D) $900\ \Omega$ adds $R + X_L + X_C$, which wrongly adds the reactances.

Final Answer:

The impedance is $500\ \Omega$, option (B).

500 Ω (B)

Quick Tip: Find ω from the frequency, then X_L and X_C , then $Z = \sqrt{R^2 + (X_L - X_C)^2}$.

39. In an LCR series circuit, the potential difference across the terminals of the inductor, capacitor and resistor is 60 V, 30 V and 40 V respectively. Then supply voltage will be equal to

- (A) 10 V
- (B) 50 V
- (C) 70 V
- (D) 130 V

Correct Answer: (B) 50 V

Solution:

Step 1: Understanding the Concept:

In a series LCR circuit, the voltage across the resistor is in phase with the current. The voltage across the inductor leads by 90 degrees, and the voltage across the capacitor lags by 90 degrees. So V_L and V_C

point in opposite directions and partly cancel.

Step 2: Key Formula or Approach:

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

Step 3: Detailed Explanation:

$$V = \sqrt{40^2 + (60 - 30)^2} = \sqrt{1600 + 900} = \sqrt{2500} = 50 \text{ V}$$

Option (D) 130 V simply adds all three voltages, which ignores the phase differences. Option (C) 70 V adds V_R and $V_L - V_C$ directly, and option (A) 10 V is not obtained by any correct combination.

Final Answer:

The supply voltage is 50 V, option (B).

50 V (B)

Quick Tip: The inductor and capacitor voltages are opposite in phase, so $V = \sqrt{V_R^2 + (V_L - V_C)^2}$.

40. Refractive index of a glass convex lens is 1.5. The radius of curvature of each of the two surfaces of the lens is 40 cm. The ratio of the power of the lens when immersed in a liquid of refractive index 1.25 to that when placed in air is

- (A) 2 : 3
(B) 2 : 5
(C) 3 : 4
(D) 5 : 2

Correct Answer: (B) 2 : 5

Solution:

Step 1: Understanding the Concept:

The power of a thin lens depends on the refractive index of the lens relative to the medium around it, and on the radii of curvature.

Step 2: Key Formula or Approach:

$$P = \frac{1}{f} = \left(\frac{n_{lens}}{n_{medium}} - 1 \right) \left(\frac{1}{R_1} - \frac{1}{R_2} \right)$$

The radii do not change when the lens goes into the liquid, so the second bracket is the same.

Step 3: Detailed Explanation:

In air: $n_{rel} = 1.5$, so the first bracket is $1.5 - 1 = 0.5$.

In the liquid: $n_{rel} = \frac{1.5}{1.25} = 1.2$, so the first bracket is $1.2 - 1 = 0.2$.

Ratio of power in liquid to power in air:

$$\frac{P_{liquid}}{P_{air}} = \frac{0.2}{0.5} = \frac{2}{5}$$

Option (D) 5:2 is the inverse ratio. Options (A) and (C) do not follow from 0.2 and 0.5.

Final Answer:

The ratio is 2:5, option (B).

$$2 : 5 \text{ (B)}$$

Quick Tip: Power is proportional to $(n_{\text{lens}} / n_{\text{medium}} - 1)$. Compare $1.5/1.25 - 1$ with $1.5 - 1$.



41. In a biprism experiment, fifth dark fringe is obtained at a point. A thin transparent film of refractive index ' μ ' is placed in one of the interfering paths. Now 7th bright fringe is obtained at the same point. If ' λ ' is the wavelength of light used, the thickness of film is equal to

(A) $1.5(\mu - 1)\lambda$

(B) $\frac{1.5\lambda}{(\mu-1)}$

(C) $2.5(\mu - 1)\lambda$

(D) $\frac{2.5\lambda}{(\mu-1)}$

Correct Answer: (D) $\frac{2.5\lambda}{(\mu-1)}$

Solution:**Step 1: Understanding the Concept:**

In a biprism (or double slit) experiment, a point is dark when the path difference is an odd multiple of $\lambda/2$, and bright when it is a whole multiple of λ . A thin film of index μ and thickness t adds an extra optical path of $(\mu - 1)t$ in the beam it covers.

Step 2: Key Formula or Approach:

1. n th dark fringe: $\Delta = (2n - 1)\frac{\lambda}{2}$.

2. n th bright fringe: $\Delta = n\lambda$.

Step 3: Detailed Explanation:

Before the film, the point has the fifth dark fringe:

$$\Delta_1 = (2 \times 5 - 1) \frac{\lambda}{2} = \frac{9\lambda}{2} = 4.5\lambda$$

After the film, the same point has the seventh bright fringe:

$$\Delta_2 = 7\lambda$$

The film changes the path difference by

$$(\mu - 1)t = 7\lambda - 4.5\lambda = 2.5\lambda$$

$$t = \frac{2.5\lambda}{\mu - 1}$$

Options (A) and (C) have t proportional to $(\mu - 1)$, which has the wrong dependence: a thicker film needs a lower index, not a higher one, to produce the same shift. Option (B) 1.5 would result from $7 - 5.5$.

Final Answer:

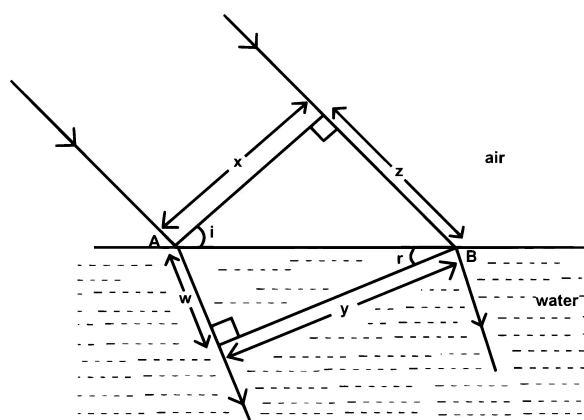
The thickness of the film is $\frac{2.5\lambda}{\mu - 1}$, option (D).

$$\boxed{\frac{2.5\lambda}{\mu - 1} \text{ (D)}}$$

Quick Tip: Fifth dark fringe has path difference 4.5λ . Seventh bright fringe has 7λ . The film adds $(\mu - 1)t$.



42. A plane wavefront of width x , is incident on an air-water interface and the corresponding refracted wavefront has a width y as shown in figure. The refractive index of air with respect to water in terms of distances w and z is



- (A) $\frac{w}{z}$
- (B) $\frac{z}{w}$
- (C) $\sqrt{\frac{w}{z}}$
- (D) $\sqrt{\frac{z}{w}}$

Correct Answer: (A) $\frac{w}{z}$

Solution:

Step 1: Understanding the Concept:

The figure shows a plane wavefront of width x meeting the air-water surface between points A and B. By Huygens' construction, the point of the wavefront that touches A first sends a wavelet into the water, while the other end of the wavefront is still travelling in air to reach B.

Step 2: Key Formula or Approach:

The refractive index of air with respect to water is the ratio of the speed of light in water to the speed in air:

$${}_w n_a = \frac{v_{\text{water}}}{v_{\text{air}}}$$

Step 3: Detailed Explanation:

Let t be the time in which the far end of the wavefront travels the distance z in air to reach B. Then

$$z = v_{\text{air}} t$$

In the same time t , the wavelet from A travels the distance w in water:

$$w = v_{\text{water}} t$$

Divide:

$$\frac{w}{z} = \frac{v_{\text{water}}}{v_{\text{air}}}$$

This is the refractive index of air with respect to water, which is less than 1 because light is slower in water. The inverse ratio z/w in option (B) is the refractive index of water with respect to air. Options (C) and (D) with square roots have no basis in this construction.

Final Answer:

The refractive index of air with respect to water is $\frac{w}{z}$, option (A).

$$\frac{w}{z} (A)$$

Quick Tip: In the same time t the wave travels z in air and w in water. So $v_{\text{water}} / c = w / z$.

43. In an optical instrument- microscope, the wavelength of light used are $\lambda_1 = 6800 \text{ \AA}$ and $\lambda_2 = 5100 \text{ \AA}$. The ratio of the resolving power of microscope corresponding to ' λ_1 ' to that for ' λ_2 ' is

- (A) 9 : 16
- (B) 5 : 4
- (C) 4 : 3
- (D) 3 : 4

Correct Answer: (D) 3 : 4

Solution:

Step 1: Understanding the Concept:

The resolving power of a microscope is its ability to show two close points as separate. A shorter wavelength gives better resolving power.

Step 2: Key Formula or Approach:

$$\text{R.P.} = \frac{2\mu \sin \theta}{1.22\lambda} \Rightarrow \text{R.P.} \propto \frac{1}{\lambda}$$

Step 3: Detailed Explanation:

$$\frac{(\text{R.P.})_1}{(\text{R.P.})_2} = \frac{\lambda_2}{\lambda_1} = \frac{5100}{6800} = \frac{3}{4}$$

So the resolving power at 6800 Å is three fourths of the resolving power at 5100 Å. Option (C) 4:3 is the inverse ratio, which would be right if the resolving power was proportional to λ . Options (A) 9:16 and (B) 5:4 do not match.

Final Answer:

The ratio of resolving powers is 3:4, option (D).

3 : 4 (D)

Quick Tip: Resolving power of a microscope is proportional to $1 / \lambda$.

44. The maximum kinetic energies of photoelectrons emitted are K_1 and K_2 when light of wavelength λ_1 and λ_2 respectively are incident on a metallic surface. If $\lambda_1 = 3\lambda_2$ then

- (A) $K_1 = \frac{K_2}{3}$
- (B) $K_1 < \frac{K_2}{3}$
- (C) $K_1 = 3K_2$
- (D) $K_1 = \frac{2}{3}K_2$

Correct Answer: (B) $K_1 < \frac{K_2}{3}$

Solution:

Step 1: Understanding the Concept:

Einstein's photoelectric equation gives the maximum kinetic energy of

the emitted electrons as the photon energy minus the work function ϕ .

Step 2: Key Formula or Approach:

$$K = \frac{hc}{\lambda} - \phi$$

Step 3: Detailed Explanation:

For the two wavelengths:

$$K_1 = \frac{hc}{\lambda_1} - \phi, \quad K_2 = \frac{hc}{\lambda_2} - \phi$$

With $\lambda_1 = 3\lambda_2$:

$$K_1 = \frac{hc}{3\lambda_2} - \phi = \frac{1}{3} (K_2 + \phi) - \phi$$

$$K_1 = \frac{K_2}{3} + \frac{\phi}{3} - \phi = \frac{K_2}{3} - \frac{2\phi}{3}$$

Since the work function is positive, $\frac{2\phi}{3} > 0$, so

$$K_1 < \frac{K_2}{3}$$

Option (A) $K_1 = K_2/3$ would be true only for zero work function.

Option (C) and (D) do not hold, because K_1 is smaller than K_2 (longer wavelength, lower photon energy) by more than a factor of 3.

Final Answer:

$$K_1 < \frac{K_2}{3}, \text{ option (B).}$$

$$K_1 < \frac{K_2}{3} \text{ (B)}$$

Quick Tip: Write Einstein's equation for both wavelengths and eliminate the work function.

45. The de Broglie wavelength of the electron in the ground state is λ_1 and that in the $n = 3$ level is λ_3 then λ_3 is given by

- (A) $\frac{\lambda_1}{3}$
- (B) $\frac{\lambda_1}{2}$
- (C) $2\lambda_1$
- (D) $3\lambda_1$

Correct Answer: (D) $3\lambda_1$

Solution:**Step 1: Understanding the Concept:**

In Bohr's model, the electron's orbit holds a whole number of de Broglie wavelengths, so $2\pi r_n = n\lambda_n$.

Step 2: Key Formula or Approach:

$$\lambda_n = \frac{2\pi r_n}{n}, \quad r_n \propto n^2$$

Step 3: Detailed Explanation:

$$\lambda_n \propto \frac{n^2}{n} = n$$

So the wavelength is proportional to the orbit number:

$$\frac{\lambda_3}{\lambda_1} = \frac{3}{1} \Rightarrow \lambda_3 = 3\lambda_1$$

Alternative check: the speed in orbit n is $v_n \propto \frac{1}{n}$, so the momentum mv falls as $1/n$, and $\lambda = \frac{h}{mv}$ rises as n . The result is the same.

Final Answer:

$\lambda_3 = 3\lambda_1$, option (D).

$3\lambda_1$ (D)

Quick Tip: Bohr's quantisation says $2\pi r = n\lambda$, and r is proportional to n^2 , so λ is proportional to n .

46. If the difference between $(n + 1)^{th}$ Bohr radius and n^{th} Bohr radius is given by $(n - 1)^{th}$ Bohr radius, then the value of n is

- (A) 4
- (B) 3
- (C) 2
- (D) 1

Correct Answer: (A) 4

Solution:

Step 1: Understanding the Concept:

The radius of the n th Bohr orbit of hydrogen is $r_n = n^2 r_1$, where r_1 is the first Bohr radius.

Step 2: Key Formula or Approach:

The condition is $r_{n+1} - r_n = r_{n-1}$.

Step 3: Detailed Explanation:

$$(n+1)^2 - n^2 = (n-1)^2$$

Expand the left side: $n^2 + 2n + 1 - n^2 = 2n + 1$.

Expand the right side: $n^2 - 2n + 1$.

$$2n + 1 = n^2 - 2n + 1 \Rightarrow n^2 - 4n = 0 \Rightarrow n(n - 4) = 0$$

The root $n = 0$ is not allowed, so $n = 4$.

Check: $r_5 - r_4 = 25 - 16 = 9 = r_3$. It works.

Final Answer:

The value of n is 4, option (A).

4 (A)

Quick Tip: The Bohr radius of orbit n is proportional to n squared. Set $(n+1)^2 - n^2 = (n-1)^2$.

47. Two samples A and B contain equal amount of radioactive substances. If $(\frac{1}{8})^{th}$ of sample A and $(\frac{1}{128})^{th}$ of sample B, remain after 9 hours, then the ratio of half life period of B to that of A is

- (A) 9 : 7
- (B) 7 : 3
- (C) 3 : 7
- (D) 3 : 1

Correct Answer: (C) 3 : 7

Solution:

Step 1: Understanding the Concept:

After n half lives, the fraction of a radioactive sample that remains is $(\frac{1}{2})^n$, where $n = \frac{t}{T_{1/2}}$.

Step 2: Key Formula or Approach:

Find the number of half lives for each sample in 9 hours, then divide 9 hours by that number.

Step 3: Detailed Explanation:

Sample A: $\frac{1}{8} = (\frac{1}{2})^3$, so $n_A = 3$ and

$$T_A = \frac{9}{3} = 3 \text{ h}$$

Sample B: $\frac{1}{128} = (\frac{1}{2})^7$, so $n_B = 7$ and

$$T_B = \frac{9}{7} \text{ h}$$

Ratio:

$$\frac{T_B}{T_A} = \frac{9/7}{3} = \frac{9}{21} = \frac{3}{7}$$

Option (B) 7:3 is the inverse ratio of the half lives (it is the ratio of numbers of half lives, $n_B : n_A$). Option (A) 9:7 is $T_A : \dots$ using the 9 hours without dividing by 3.

Final Answer:

$T_B : T_A = 3 : 7$, option (C).

3 : 7 (C)

Quick Tip: Fraction left is $(1/2)^{(t/T)}$. $1/8$ means 3 half lives and $1/128$ means 7 half lives in the same 9 hours.

48. In an NPN transistor, the collector current is 28 mA. If 80% electrons reach the collector, its base current in mA is

- (A) 7 mA
- (B) 14 mA
- (C) 28 mA
- (D) 35 mA

Correct Answer: (A) 7 mA

Solution:

Step 1: Understanding the Concept:

In a transistor, the emitter current splits into the base current and the collector current: $I_E = I_B + I_C$. The fraction of carriers that reach the collector is $\alpha = \frac{I_C}{I_E}$.

Step 2: Key Formula or Approach:

$$\alpha = 0.8 = \frac{I_C}{I_E}, \quad I_B = I_E - I_C$$

Step 3: Detailed Explanation:

$$I_E = \frac{I_C}{\alpha} = \frac{28}{0.8} = 35 \text{ mA}$$

$$I_B = 35 - 28 = 7 \text{ mA}$$

Option (B) 14 mA and (C) 28 mA are not consistent with an alpha of 0.8, and option (D) 35 mA is the emitter current, not the base current.

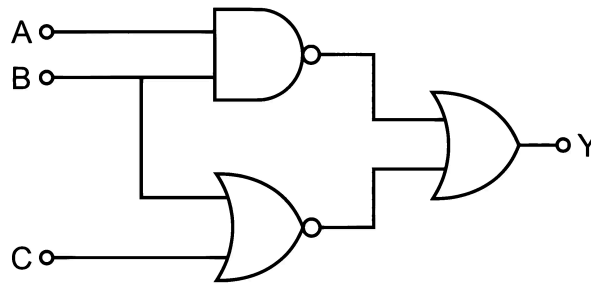
Final Answer:

The base current is 7 mA, option (A).

7 mA (A)

Quick Tip: Collector current is 80 percent of the emitter current. Then $I_B = I_E - I_C$.

49. The output Y of the given logic circuit is



- (A) $\overline{(A \cdot B)} + \overline{(C \cdot B)}$
- (B) $\overline{(A + B)} \cdot \overline{(B \cdot C)}$
- (C) $\overline{(A \cdot B)} + \overline{(B + C)}$
- (D) $(A + B) + (B + C)$

Correct Answer: (C) $\overline{(A \cdot B)} + \overline{(B + C)}$

Solution:

Step 1: Understanding the Concept:

The figure shows three gates. The upper gate is an AND gate with a small circle on its output, so it is a NAND gate with inputs A and B. The lower gate is an OR gate with a circle on its output, so it is a NOR gate with inputs B and C. The right-hand gate is an OR gate that takes both outputs.

Step 2: Key Formula or Approach:

NAND: $\overline{A \cdot B}$. NOR: $\overline{B + C}$. OR: sum of its inputs.

Step 3: Detailed Explanation:

Output of the NAND gate: $\overline{A \cdot B}$.

Output of the NOR gate: $\overline{B + C}$.

The final OR gate combines them:

$$Y = \overline{A \cdot B} + \overline{B + C}$$

Option (A) has a NAND of C and B in place of the NOR. Option (B) multiplies the two outputs, which would need an AND gate at the end. Option (D) has no inversions at all.

Final Answer:

The output is $Y = \overline{A \cdot B} + \overline{B + C}$, option (C).

$$\boxed{\overline{A \cdot B} + \overline{B + C} \text{ (C)}}$$

Quick Tip: Read the figure gate by gate: a NAND of A and B, a NOR of B and C, and an OR of the two outputs.

50. For intrinsic semiconductor, if n_h and n_e represent the number of holes per unit volume and the number of free electrons per unit volume respectively then

- (A) $n_h < n_e$
- (B) $n_h > n_e$
- (C) $n_h = n_e$
- (D) $n_h \neq n_e$

Correct Answer: (C) $n_h = n_e$

Solution:

Step 1: Understanding the Concept:

An intrinsic semiconductor is a pure crystal with no doping. At

temperatures above absolute zero, thermal energy breaks some covalent bonds.

Step 2: Key Formula or Approach:

Every broken bond frees one electron into the conduction band and leaves one vacancy, a hole, in the valence band. Electrons and holes are created in pairs.

Step 3: Detailed Explanation:

Since the electrons and holes are made in equal numbers, the number of free electrons per unit volume equals the number of holes per unit volume:

$$n_e = n_h$$

This equality is lost only after doping. In an n-type semiconductor $n_e > n_h$, and in a p-type semiconductor $n_h > n_e$. Options (A), (B) and (D) describe those doped cases or a general inequality, none of which holds for a pure crystal.

Final Answer:

For an intrinsic semiconductor $n_h = n_e$, option (C).

$$n_h = n_e \text{ (C)}$$

Quick Tip: In a pure semiconductor each electron that leaves its bond leaves behind one hole, so the two numbers are equal.

Chemistry

Questions 51 to 100 | 1 mark each | 50 marks

51. Find the number of molecules present in 70 g dinitrogen.

- (A) 1.5055×10^{24}
- (B) 1.5055×10^{23}
- (C) 3.011×10^{24}
- (D) 3.011×10^{23}

Correct Answer: (A) 1.5055×10^{24}

Solution:

Step 1: Understanding the Concept:

The number of molecules in a sample equals the number of moles times Avogadro's number. So we first convert the given mass of dinitrogen into moles.

Dinitrogen is the diatomic molecule N_2 .

Step 2: Key Formula or Approach:

1. Molar mass of $N_2 = 2 \times 14 = 28 \text{ g mol}^{-1}$.
2. Moles $n = \frac{\text{mass}}{\text{molar mass}}$.
3. Number of molecules $= n \times N_A$, where $N_A = 6.022 \times 10^{23} \text{ mol}^{-1}$.

Step 3: Detailed Explanation:

Moles of dinitrogen:

$$n = \frac{70}{28} = 2.5 \text{ mol}$$

Number of molecules:

$$N = 2.5 \times 6.022 \times 10^{23} = 15.055 \times 10^{23} = 1.5055 \times 10^{24}$$

Option (B) is ten times too small, which would happen if the moles were taken as 0.25. Option (C) would need 5 mol (140 g) and option (D) would need 0.5 mol (14 g), so neither matches 70 g.

Final Answer:

70 g of N_2 is 2.5 mol, which holds 1.5055×10^{24} molecules. This is option (A).

$$1.5055 \times 10^{24} \text{ molecules (A)}$$

Quick Tip: Convert 70 g to moles using the molar mass 28 g/mol, then multiply by Avogadro's number.

52. Which from following colours has highest energy if wave lengths of violet, blue, yellow and red light are 410 nm, 470 nm, 580 nm and 750 nm respectively ?

- (A) Blue
- (B) Violet
- (C) Yellow
- (D) Red

Correct Answer: (B) Violet

Solution:

Step 1: Understanding the Concept:

Light of a given colour has a photon energy fixed by its wavelength. The shorter the wavelength, the higher the frequency and the higher the energy of each photon.

Step 2: Key Formula or Approach:

$$E = h\nu = \frac{hc}{\lambda}$$

Here h and c are constants, so $E \propto \frac{1}{\lambda}$. The colour with the smallest wavelength has the largest energy.

Step 3: Detailed Explanation:

The given wavelengths are violet 410 nm, blue 470 nm, yellow 580 nm and red 750 nm.

Violet has the smallest wavelength, 410 nm, so it has the largest value of $1/\lambda$.

Energy ratio of violet to red: $\frac{750}{410} \approx 1.83$, so a violet photon carries almost twice the energy of a red photon.

Blue (470 nm) is higher in energy than yellow and red but lower than violet. Yellow (580 nm) and red (750 nm) have longer wavelengths, so their energies are smaller still.

Final Answer:

Violet light has the shortest wavelength, so it has the highest energy. This is option (B).

Quick Tip: Energy of a photon is inversely proportional to wavelength, so pick the shortest wavelength.



53. Select the incorrect statement from the following.

- (A) NH_3 and NF_3 both are polar molecules
- (B) NH_3 and NF_3 both have pyramidal shape
- (C) In NH_3 and NF_3 the orbital dipole due to lone pair is in the same direction as that of resultant dipole moment of other three bonds
- (D) Dipole moment of NH_3 is greater than NF_3

Correct Answer: (C) In NH_3 and NF_3 the orbital dipole due to lone pair is in the same direction as that of resultant dipole moment of other three bonds

Solution:

Step 1: Understanding the Concept:

Both NH_3 and NF_3 have nitrogen with three bond pairs and one lone pair. The shape is pyramidal and both molecules are polar. What differs is the direction of the bond dipoles compared with the lone-pair dipole.

Step 2: Key Formula or Approach:

Net dipole moment is the vector sum of the bond dipoles and the lone-pair (orbital) dipole. The lone pair on N points away from the nitrogen, along the axis of the pyramid.

Step 3: Check statement (A):

Both molecules are pyramidal with unequal charge distribution, so both are polar. (A) is TRUE.

Step 4: Check statement (B):

Both have sp^3 nitrogen with one lone pair, giving a trigonal pyramidal shape. (B) is TRUE.

Step 5: Check statement (C):

In NH_3 , N is more electronegative than H, so the N-H bond dipoles point towards N. Their resultant points the same way as the lone-pair dipole, so the two add up.

In NF_3 , F is more electronegative than N, so the N-F dipoles point towards F. Their resultant is opposite to the lone-pair dipole and the two partly cancel.

So the claim that the lone-pair dipole is in the same direction as the bond-dipole resultant in both molecules is FALSE.

Step 6: Check statement (D):

Because of the addition in NH_3 (about 1.46 D) and the opposition in NF_3 (about 0.24 D), the dipole moment of NH_3 is larger. (D) is TRUE.

Final Answer:

Statement (C) is the incorrect one, because in NF_3 the lone-pair dipole opposes the resultant of the bond dipoles.

(C)

Quick Tip: In NH_3 the lone-pair moment adds to the bond moments, but in NF_3 it opposes them.

54. Which from following gases is least soluble in water at same temperature and pressure ?

- (A) NH_3
- (B) CO_2
- (C) HCl
- (D) N_2

Correct Answer: (D) N_2

Solution:

Step 1: Understanding the Concept:

Solubility of a gas in water depends on how strongly the gas interacts with water. Gases that react with water, or can hydrogen bond with it, dissolve very well. Nonpolar, unreactive gases dissolve very little.

Step 2: Key Formula or Approach:

Compare the interaction of each gas with water at the same temperature and pressure.

Step 3: Detailed Explanation:

NH_3 forms hydrogen bonds with water and reacts to form NH_4^+ and OH^- , so it is extremely soluble.

HCl ionises completely in water to give H_3O^+ and Cl^- , so it is also extremely soluble.

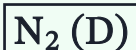
CO_2 is nonpolar but reacts partly with water to form carbonic acid, so it dissolves much more than an unreactive gas (about one volume per volume of water).

N_2 is nonpolar, has a very stable triple bond and does not react with water. Only weak dipole-induced dipole forces act, so its solubility is

the lowest (about 0.02 volume per volume).

Final Answer:

Dinitrogen is the least soluble gas, which is option (D).



Quick Tip: Gases that react with water or ionise in it dissolve far more than an inert nonpolar gas like nitrogen.



55. Which of the following ethers is gaseous at room temperature ?

- (A) 1 - Methoxypropane
- (B) 1 - Ethoxypropane
- (C) Methoxyethane
- (D) Ethoxyethane

Correct Answer: (C) Methoxyethane

Solution:

Step 1: Understanding the Concept:

Ethers have no hydrogen bonding between their own molecules, so their boiling points depend mainly on molecular mass. A small ether has a boiling point close to or below room temperature and exists as a gas.

Step 2: Key Formula or Approach:

Count the carbon atoms of each ether. Fewer carbons means a lower molecular mass and a lower boiling point.

Step 3: Detailed Explanation:

1-Methoxypropane, $\text{CH}_3\text{OCH}_2\text{CH}_2\text{CH}_3$, has 4 carbons and boils at about 39 degrees C, so it is a liquid.

1-Ethoxypropane has 5 carbons and boils at about 64 degrees C, so it is a liquid.

Ethoxyethane (diethyl ether) has 4 carbons and boils at about 35 degrees C, so it is a volatile liquid.

Methoxyethane, $\text{CH}_3\text{OCH}_2\text{CH}_3$, has only 3 carbons and a molar mass of 60 g/mol. It boils at about 7 degrees C, which is below room temperature, so it is a gas.

Final Answer:

Only methoxyethane boils below room temperature, so it is the gaseous ether. This is option (C).

Methoxyethane (C)

Quick Tip: Fewer carbon atoms means a smaller molecular mass and a lower boiling point. Pick the smallest ether.

56. A container contains mixture of He and Ne gases at certain temperature. He is 20 % by mass of mixture. What is the ratio of partial pressure of Helium to Neon ?

- (A) 1 : 2
- (B) 2 : 1
- (C) 5 : 4
- (D) 3 : 2

Correct Answer: (C) 5 : 4

Solution:

Step 1: Understanding the Concept:

In an ideal gas mixture at one temperature and volume, the partial pressure of each gas is proportional to its number of moles. So the ratio of partial pressures equals the ratio of moles.

Step 2: Key Formula or Approach:

$$p_i = x_i P_{\text{total}}, \quad \frac{p_{\text{He}}}{p_{\text{Ne}}} = \frac{n_{\text{He}}}{n_{\text{Ne}}}$$

Molar masses: He = 4 g/mol, Ne = 20 g/mol.

Step 3: Detailed Explanation:

Take 100 g of the mixture. Then He is 20 g and Ne is 80 g.

$$n_{\text{He}} = \frac{20}{4} = 5 \text{ mol}$$

$$n_{\text{Ne}} = \frac{80}{20} = 4 \text{ mol}$$

Ratio of partial pressures:

$$\frac{p_{\text{He}}}{p_{\text{Ne}}} = \frac{5}{4}$$

Using the mass ratio 20:80 = 1:4 directly would be a mistake, since pressure depends on moles and not on mass. Options (A) 1:2 and (B) 2:1 do not follow from these moles, and option (D) 3:2 would need 6

mol He for 4 mol Ne.

Final Answer:

The ratio of the partial pressure of helium to neon is 5:4, option (C).

5 : 4 (C)

Quick Tip: Partial pressure ratio equals the mole ratio, so convert 20 g He and 80 g Ne into moles.



57. 1 mole of an ideal gas is compressed isothermally and reversibly from initial pressure x kPa to final pressure $2x$ kPa at 300 K. Find the work done ($R = 8.314 \text{ J K}^{-1} \text{ mol}^{-1}$)

- (A) 1432 J
- (B) 1865 J
- (C) 1296 J
- (D) 1729 J

Correct Answer: (D) 1729 J

Solution:

Step 1: Understanding the Concept:

For an isothermal reversible process on an ideal gas, the work depends only on the temperature and the ratio of pressures.

Compression means work is done on the gas, so the work is positive in the convention where work done on the system is positive.

Step 2: Key Formula or Approach:

$$W = nRT \ln \frac{P_2}{P_1} = 2.303 nRT \log \frac{P_2}{P_1}$$

Here $n = 1$, $T = 300$ K, $P_2/P_1 = 2x/x = 2$.

Step 3: Detailed Explanation:

$$W = 2.303 \times 1 \times 8.314 \times 300 \times \log 2$$

$$\log 2 = 0.3010$$

$$2.303 \times 8.314 \times 300 = 5744.3$$

$$W = 5744.3 \times 0.3010 = 1729 \text{ J}$$

The sign is positive because the gas is compressed. The other values come from wrong arithmetic: 1432 J, 1296 J and 1865 J do not match $nRT \ln 2$.

Final Answer:

The magnitude of work done is about 1729 J, option (D).

$$\boxed{1729 \text{ J (D)}}$$

Quick Tip: Use $w = nRT \ln(P_2/P_1)$ for isothermal reversible compression with $P_2/P_1 = 2$.

58. Which of the following is the correct expression of the first law of thermodynamics for adiabatic processes ?

- (A) $W = -Q$
- (B) $-\Delta U = -W$
- (C) $\Delta U = QV$
- (D) $\Delta U = Q_P - P_{\text{ext}} \Delta V$

Correct Answer: (B) $-\Delta U = -W$

Solution:

Step 1: Understanding the Concept:

The first law of thermodynamics says the change in internal energy equals heat absorbed plus work done on the system. An adiabatic process is one in which no heat is exchanged with the surroundings.

Step 2: Key Formula or Approach:

$$\Delta U = Q + W$$

For an adiabatic process, $Q = 0$.

Step 3: Detailed Explanation:

Put $Q = 0$ into the first law:

$$\Delta U = W$$

Multiply both sides by -1 :

$$-\Delta U = -W$$

This is exactly option (B). Option (A), $W = -Q$, would mean $\Delta U = 0$, which describes an isothermal process of an ideal gas. Option (C) is not a valid statement of the first law. Option (D) is the general form at constant pressure with heat flowing, so it is not specific to adiabatic change.

Final Answer:

For an adiabatic process $\Delta U = W$, which is written as $-\Delta U = -W$ in option (B).

$$-\Delta U = -W \text{ (B)}$$

Quick Tip: In an adiabatic process $Q = 0$, so the first law gives $\Delta U = W$.



59. Calculate the standard enthalpy change for the following reaction,



Given, $\Delta_f H^\circ(\text{C}_2\text{H}_6) = -85\text{kJ mol}^{-1}$

$\Delta_f H^\circ(\text{CO}_2) = -390\text{kJ mol}^{-1}$

$\Delta_f H^\circ(\text{H}_2\text{O}) = -285\text{kJ mol}^{-1}$

(A) -2900kJ

(B) -3100kJ

(C) -3000kJ

(D) -3200kJ

Correct Answer: (B) -3100kJ

Solution:

Step 1: Understanding the Concept:

The standard enthalpy change of a reaction equals the total formation enthalpy of the products minus that of the reactants, each multiplied by its coefficient. Elements in their standard states, such as O₂, have zero formation enthalpy.

Step 2: Key Formula or Approach:

$$\Delta_r H^\circ = \sum n \Delta_f H^\circ(\text{products}) - \sum n \Delta_f H^\circ(\text{reactants})$$

Step 3: Detailed Explanation:

Products:

$$4(-390) + 6(-285) = -1560 - 1710 = -3270 \text{ kJ}$$

Reactants:

$$2(-85) + 7(0) = -170 \text{ kJ}$$

Reaction enthalpy:

$$\Delta_r H^\circ = -3270 - (-170) = -3270 + 170 = -3100 \text{ kJ}$$

The other options are all off by round amounts. For example -3270 alone would result from forgetting the ethane term, and none of the other values follows from a correct application of the formula.

Final Answer:

The standard enthalpy change for the reaction is -3100 kJ, option (B).

$$\boxed{-3100 \text{ kJ (B)}}$$

Quick Tip: Use $\Delta H = \text{sum of formation enthalpies of products minus reactants}$. O_2 has zero formation enthalpy.

60. The solubility of salt AX_2 is $6 \times 10^{-12} \text{ mol dm}^{-3}$. What is solubility product of the salt ?

- (A) 2.06×10^{-24}
- (B) 2.48×10^{-24}
- (C) 4.32×10^{-34}
- (D) 8.64×10^{-34}

Correct Answer: (D) 8.64×10^{-34}

Solution:

Step 1: Understanding the Concept:

A salt of type AX_2 dissolves as $\text{AX}_2 \rightleftharpoons \text{A}^{2+} + 2\text{X}^-$. If the solubility is s , then $[\text{A}^{2+}] = s$ and $[\text{X}^-] = 2s$.

Step 2: Key Formula or Approach:

$$K_{sp} = [\text{A}^{2+}][\text{X}^-]^2 = s(2s)^2 = 4s^3$$

Step 3: Detailed Explanation:

Here $s = 6 \times 10^{-12} \text{ mol dm}^{-3}$.

$$s^3 = (6 \times 10^{-12})^3 = 216 \times 10^{-36}$$

$$K_{sp} = 4 \times 216 \times 10^{-36} = 864 \times 10^{-36} = 8.64 \times 10^{-34}$$

Options (A) and (B) have the exponent -24, which would arise from squaring instead of cubing. Option (C), 4.32×10^{-34} , is half of the correct value, which comes from using $2s^3$ instead of $4s^3$.

Final Answer:

The solubility product is 8.64×10^{-34} , option (D).

$$8.64 \times 10^{-34} \text{ (D)}$$

Quick Tip: For AX_2 , $K_{sp} = 4s^3$, where s is the molar solubility.

61. An organic monobasic acid has dissociation constant 1.96×10^{-8} . What is its percentage dissociation in 0.01 M solution ?

- (A) 1.40 %
- (B) 0.14 %
- (C) 2.4 %
- (D) 0.19 %

Correct Answer: (B) 0.14 %

Solution:

Step 1: Understanding the Concept:

For a weak monobasic acid, the degree of dissociation is small, so Ostwald's dilution law can be used in its approximate form.

Step 2: Key Formula or Approach:

$$K_a = C\alpha^2 \Rightarrow \alpha = \sqrt{\frac{K_a}{C}}$$

Step 3: Detailed Explanation:

Given $K_a = 1.96 \times 10^{-8}$ and $C = 0.01$ M.

$$\alpha = \sqrt{\frac{1.96 \times 10^{-8}}{10^{-2}}} = \sqrt{1.96 \times 10^{-6}} = 1.4 \times 10^{-3}$$

Percentage dissociation:

$$\alpha \times 100 = 1.4 \times 10^{-3} \times 100 = 0.14 \%$$

The value is small, so the approximation $1 - \alpha \approx 1$ is valid. Option (A) 1.40 % comes from missing a factor of ten. Options (C) 2.4 % and (D) 0.19 % do not match $\sqrt{K_a/C}$.

Final Answer:

The acid is 0.14 % dissociated, option (B).

0.14 % (B)

Quick Tip: For a weak acid $\alpha = \sqrt{K_a/C}$. Then multiply by 100.

62. Which of the following aqueous salt solutions has the least pH value ?

- (A) Na_2CO_3
- (B) CuCl_2
- (C) CH_3COONa
- (D) NaNO_3

Correct Answer: (B) CuCl_2

Solution:

Step 1: Understanding the Concept:

The pH of a salt solution depends on which ions undergo hydrolysis. A salt of a strong acid and a weak base is acidic. A salt of a weak acid and a strong base is basic. A salt of a strong acid and a strong base is neutral.

Step 2: Key Formula or Approach:

Classify each salt by its parent acid and base.

Step 3: Detailed Explanation:

Na_2CO_3 : strong base NaOH and weak acid H_2CO_3 . The carbonate ion hydrolyses and gives a basic solution, pH above 7.

CuCl_2 : weak base $\text{Cu}(\text{OH})_2$ and strong acid HCl . The Cu^{2+} ion hydrolyses to give H^+ , so the solution is acidic, pH below 7.

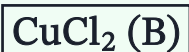
CH_3COONa : strong base and weak acid, so it is basic.

NaNO_3 : strong base and strong acid, so it is neutral, pH about 7.

The lowest pH belongs to the acidic salt.

Final Answer:

CuCl_2 gives the most acidic solution, so it has the least pH. This is option (B).



Quick Tip: A salt of a strong acid and a weak base gives an acidic solution, so CuCl_2 has the lowest pH.

63. What is the oxidation number of carbon in oxalic acid ?

- (A) +2
- (B) +3
- (C) +4
- (D) +6

Correct Answer: (B) +3

Solution:

Step 1: Understanding the Concept:

The oxidation number of an atom in a neutral compound is found by making the sum of all oxidation numbers equal to zero. Hydrogen is +1 and oxygen is -2 here.

Step 2: Key Formula or Approach:

Oxalic acid is $\text{H}_2\text{C}_2\text{O}_4$, that is HOOC-COOH . Let the oxidation number of C be x .

$$2(+1) + 2x + 4(-2) = 0$$

Step 3: Detailed Explanation:

$$2 + 2x - 8 = 0$$

$$2x = 6$$

$$x = +3$$

Each carbon is bonded to two oxygens (one double bond and one single bond), so both carbons have the same state, +3.

Option (A) +2 is carbon in carbon monoxide, option (C) +4 is carbon in CO_2 , and option (D) +6 is not possible for carbon since it has only four valence electrons.

Final Answer:

The oxidation number of carbon in oxalic acid is +3, option (B).

+3 (B)

Quick Tip: Oxalic acid is $\text{H}_2\text{C}_2\text{O}_4$. Set the sum of all oxidation numbers to zero and solve for carbon.

64. What is the general formula of halide salts of alkali metals if M is a metal atom ?

- (A) MX
- (B) MX_2
- (C) M_3X_2
- (D) M_2X_3

Correct Answer: (A) MX

Solution:

Step 1: Understanding the Concept:

Alkali metals belong to group 1 and each has one electron in the outer shell. They lose that electron easily to form the M^+ ion.

Step 2: Key Formula or Approach:

Halogens are in group 17. Each gains one electron to form X^- . A compound must be electrically neutral, so the charges must cancel.

Step 3: Detailed Explanation:

One M^+ ion carries +1 charge and one X^- ion carries -1 charge.

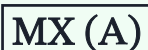
$$(+1) + (-1) = 0$$

So they combine in a 1:1 ratio and the formula is MX . Examples are NaCl , KBr and LiF .

Option (B) MX_2 would be formed by a +2 metal such as an alkaline earth, for example CaCl_2 . Options (C) and (D) would need charges like +3 or mixed charges, which alkali metals never show.

Final Answer:

Alkali metal halides have the general formula MX , option (A).



Quick Tip: Alkali metals form M^+ ions and halogens form X^- ions, so the halide is MX.

65. Which from following compounds is NOT optically active?

- (A) 3 - Bromohexane
- (B) 2 - Bromo - 2 - methylbutane
- (C) 2 - Bromopentane
- (D) 2 - Bromo - 3 - methylbutane

Correct Answer: (B) 2 - Bromo - 2 - methylbutane

Solution:

Step 1: Understanding the Concept:

A compound is optically active when it is chiral, which usually means it contains a carbon bonded to four different groups (a chiral centre).

Step 2: Key Formula or Approach:

Draw each structure and check every carbon that carries Br or a branch for four different substituents.

Step 3: Check 3-bromohexane:

$\text{CH}_3\text{CH}_2\text{CHBrCH}_2\text{CH}_2\text{CH}_3$: C_3 carries H, Br, ethyl and propyl. All four are different, so it is chiral and optically active.

Step 4: Check 2-bromo-2-methylbutane:

$(\text{CH}_3)_2\text{CBrCH}_2\text{CH}_3$: C_2 carries Br, two identical methyl groups and an

ethyl group. Two groups are the same, so there is no chiral centre and the molecule is optically inactive.

Step 5: Check the other two:

2-Bromopentane: C2 carries H, Br, methyl and propyl, all different, so it is chiral.

2-Bromo-3-methylbutane, $\text{CH}_3\text{CHBrCH}(\text{CH}_3)_2$: C2 carries H, Br, methyl and isopropyl, all different, so it is chiral.

Final Answer:

Only 2-bromo-2-methylbutane has no chiral carbon, so it is not optically active. This is option (B).

2-Bromo-2-methylbutane (B)

Quick Tip: A compound is optically active only if it has a carbon bonded to four different groups.

66. Which from following compounds does NOT contain double bond in it ?

- (A) Tetrahydrofuran
- (B) 4 H - Pyran
- (C) Pyrrole
- (D) Thiophene

Correct Answer: (A) Tetrahydrofuran

Solution:

Step 1: Understanding the Concept:

Heterocyclic compounds are ring compounds containing an atom other than carbon. Some are aromatic with C=C bonds in the ring, and some are saturated.

Step 2: Key Formula or Approach:

Recall the structure of each ring and count the double bonds.

Step 3: Detailed Explanation:

Tetrahydrofuran (THF) is C_4H_8O , a five-membered ring of four CH_2 groups and one oxygen. All bonds are single, so it has no double bond.

4H-Pyran is a six-membered ring with one oxygen and two C=C bonds, with one CH_2 at position 4.

Pyrrole is a five-membered aromatic ring with nitrogen and two C=C bonds.

Thiophene is a five-membered aromatic ring with sulfur and two C=C bonds.

Final Answer:

Only tetrahydrofuran is fully saturated and has no double bond, option (A).

Tetrahydrofuran (A)

Quick Tip: Tetrahydrofuran is the fully saturated form of furan, so it has no C=C bond.

67. Which of the following is NOT terminal alkyne ?

- (A) Ethyne
- (B) Propyne
- (C) But - 1 - yne
- (D) But - 2 - yne

Correct Answer: (D) But - 2 - yne

Solution:

Step 1: Understanding the Concept:

A terminal alkyne has the triple bond at the end of the carbon chain, so one triple-bonded carbon carries a hydrogen. Its general form is $R-C \equiv C-H$.

Step 2: Key Formula or Approach:

Write each structure and see whether the triple bond is at the chain end.

Step 3: Detailed Explanation:

Ethyne, $HC \equiv CH$: both carbons carry H. It is terminal.

Propyne, $CH_3C \equiv CH$: one triple-bonded carbon carries H. It is terminal.

But-1-yne, $CH_3CH_2C \equiv CH$: the triple bond is between C1 and C2, with H on C1. It is terminal.

But-2-yne, $CH_3C \equiv CCH_3$: the triple bond sits between C2 and C3, and both triple-bonded carbons carry methyl groups. There is no acidic alkyne hydrogen, so it is an internal alkyne.

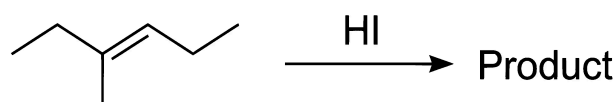
Final Answer:

But-2-yne is not a terminal alkyne, option (D).

But-2-yne (D)

Quick Tip: A terminal alkyne has its triple bond at the end of the chain, so it has a C-H on a triple-bonded carbon.

68. The major product of the following reaction is



- (A) 3 - iodo - 3 - methylhexane
- (B) 4 - iodo - 4 - methylhexane
- (C) 3 - iodo - 4 - methylhexane
- (D) 4 - iodo - 3 - methylhexane

Correct Answer: (A) 3 - iodo - 3 - methylhexane

Solution:

Step 1: Understanding the Concept:

The reactant in the figure is a six-carbon chain with a double bond between C3 and C4 and a methyl group on C3. This is 3-methylhex-3-ene. Hydrogen iodide adds across the double bond by electrophilic addition and follows Markovnikov's rule.

Step 2: Key Formula or Approach:

The proton of HI adds first to give the more stable carbocation. Then iodide attacks the cation.

Stability order: tertiary > secondary > primary.

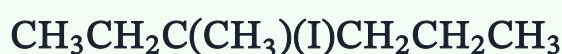
Step 3: Detailed Explanation:

The double bond carbons are C3 (bonded to a methyl, an ethyl and C4) and C4 (bonded to an ethyl and H).

If H^+ adds to C4, the positive charge sits on C3, which is a tertiary carbocation (three alkyl groups).

If H^+ adds to C3, the positive charge sits on C4, a secondary carbocation.

The tertiary cation is more stable, so it forms mainly. Iodide then bonds to C3:



The chain is still hexane with iodine and a methyl on C3, so the name is 3-iodo-3-methylhexane. Option (C) and (D) put iodine on C4 or carry extra substituents that do not fit, and (B) puts a methyl on C4, which the reactant does not have.

Final Answer:

The major product is 3-iodo-3-methylhexane, option (A).

3-iodo-3-methylhexane (A)

Quick Tip: The figure shows 3-methylhex-3-ene. Apply Markovnikov: iodine goes to the more substituted carbon.

69. Calculate the number of unit cell in 0.5 g metal if the product of volume and density of unit cell is 6.65×10^{-23} g

- (A) 6.340×10^{21}
(B) 7.518×10^{21}
(C) 5.625×10^{21}
(D) 8.492×10^{21}

Correct Answer: (B) 7.518×10^{21}

Solution:

Step 1: Understanding the Concept:

Density is mass divided by volume. So for one unit cell, volume times density gives the mass of that unit cell.

Step 2: Key Formula or Approach:

$$\text{mass of one unit cell} = V \times d = 6.65 \times 10^{-23} \text{ g}$$

$$\text{number of unit cells} = \frac{\text{total mass}}{\text{mass of one unit cell}}$$

Step 3: Detailed Explanation:

$$N = \frac{0.5}{6.65 \times 10^{-23}}$$

$$\frac{0.5}{6.65} = 0.07519$$

$$N = 0.07519 \times 10^{23} = 7.518 \times 10^{21}$$

The other options come from slips in this division. The correct quotient is 0.0752, so only option (B) matches.

Final Answer:

There are 7.518×10^{21} unit cells in 0.5 g of the metal, option (B).

$$7.518 \times 10^{21} \text{ (B)}$$

Quick Tip: Volume times density of a unit cell is the mass of one unit cell. Divide 0.5 g by this mass.



70. What is the total number of adjacent unit cells shared by each face particle of fcc unit cell ?

- (A) 1
- (B) 2
- (C) 4
- (D) 8

Correct Answer: (B) 2

Solution:

Step 1: Understanding the Concept:

In a face-centred cubic (fcc) unit cell, atoms sit at the 8 corners and at the centres of the 6 faces. Each type of position is shared with neighbouring cells.

Step 2: Key Formula or Approach:

A corner atom is shared by 8 unit cells, so it contributes $\frac{1}{8}$. A face-centre atom lies on a face that belongs to exactly two cubes, so it is shared by 2 unit cells and contributes $\frac{1}{2}$.

Step 3: Detailed Explanation:

Place one fcc cell. The atom in the middle of its front face is also in the middle of the back face of the cell placed right in front. No other cell touches that point.

So each face particle is shared by 2 unit cells.

Total atoms in fcc: $8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 1 + 3 = 4$.

Option (D) 8 refers to a corner particle. Option (C) 4 is the number of atoms per fcc cell, which is a different quantity.

Final Answer:

Each face-centre particle is shared by 2 unit cells, option (B).

2 (B)

Quick Tip: A particle in the centre of a face is shared between two adjacent unit cells.

71. A metal forms fcc structure. Calculate the volume of fcc unit cell in cm^3 if void volume is $1.66 \times 10^{-23} \text{cm}^3$.

- (A) $4.912 \times 10^{-23} \text{cm}^3$
- (B) $8.151 \times 10^{-23} \text{cm}^3$
- (C) $9.346 \times 10^{-23} \text{cm}^3$
- (D) $6.385 \times 10^{-23} \text{cm}^3$

Correct Answer: (D) $6.385 \times 10^{-23} \text{ cm}^3$

Solution:

Step 1: Understanding the Concept:

The packing efficiency of an fcc lattice is 74 percent, so 26 percent of the unit cell volume is void space.

Step 2: Key Formula or Approach:

$$V_{\text{void}} = 0.26 V_{\text{cell}} \Rightarrow V_{\text{cell}} = \frac{V_{\text{void}}}{0.26}$$

Step 3: Detailed Explanation:

Given void volume = $1.66 \times 10^{-23} \text{ cm}^3$.

$$V_{\text{cell}} = \frac{1.66 \times 10^{-23}}{0.26} = 6.385 \times 10^{-23} \text{ cm}^3$$

Check: the occupied volume is $0.74 \times 6.385 \times 10^{-23} = 4.72 \times 10^{-23}$ and the void plus occupied volumes add to 6.385×10^{-23} .

The other options correspond to dividing by wrong fractions, none of which is the void fraction of fcc.

Final Answer:

The volume of the fcc unit cell is $6.385 \times 10^{-23} \text{ cm}^3$, option (D).

$6.385 \times 10^{-23} \text{ cm}^3 \text{ (D)}$

Quick Tip: fcc packing efficiency is 74 percent, so voids occupy 26 percent of the cell volume.

72. Calculate the mole fraction of solute in solution if vapour pressure of a solution and pure solvent are 27 mm Hg and 30 mm Hg respectively at 298 K.

- (A) 0.9
- (B) 0.1
- (C) 0.4
- (D) 0.6

Correct Answer: (B) 0.1

Solution:

Step 1: Understanding the Concept:

For a solution of a nonvolatile solute, the relative lowering of vapour pressure equals the mole fraction of the solute (Raoult's law).

Step 2: Key Formula or Approach:

$$\frac{p^{\circ} - p}{p^{\circ}} = x_{\text{solute}}$$

Step 3: Detailed Explanation:

Given $p^{\circ} = 30$ mm Hg and $p = 27$ mm Hg.

$$x_{\text{solute}} = \frac{30 - 27}{30} = \frac{3}{30} = 0.1$$

Check: the mole fraction of the solvent is $27/30 = 0.9$, so the solute is $1 - 0.9 = 0.1$. Option (A) 0.9 is the mole fraction of the solvent, which is a common trap. Options (C) and (D) do not match the ratio $3/30$.

Final Answer:

The mole fraction of the solute is 0.1, option (B).

0.1 (B)

Quick Tip: Raoult's law: relative lowering of vapour pressure equals the mole fraction of solute.

73. Calculate the molar mass of nonelectrolyte solute when 6 gram of it is dissolved in 1 dm³ water has osmotic pressure 2.4atm at 300 K ($R = 0.0821\text{atm dm}^3\text{K}^{-1}\text{mol}^{-1}$)

- (A) 45.0g mol⁻¹
- (B) 74.12g mol⁻¹
- (C) 61.58g mol⁻¹
- (D) 90.28g mol⁻¹

Correct Answer: (C) 61.58g mol⁻¹

Solution:**Step 1: Understanding the Concept:**

Osmotic pressure of a dilute solution of a nonelectrolyte follows the van 't Hoff equation, which looks like the ideal gas law.

Step 2: Key Formula or Approach:

$$\pi V = nRT = \frac{w}{M}RT \Rightarrow M = \frac{wRT}{\pi V}$$

Step 3: Detailed Explanation:

Given $w = 6 \text{ g}$, $V = 1 \text{ dm}^3$, $\pi = 2.4 \text{ atm}$, $T = 300 \text{ K}$, $R = 0.0821$.

$$M = \frac{6 \times 0.0821 \times 300}{2.4 \times 1}$$

$$6 \times 0.0821 = 0.4926, \quad 0.4926 \times 300 = 147.78$$

$$M = \frac{147.78}{2.4} = 61.58 \text{ g mol}^{-1}$$

The other options do not result from these values. For example 74.12 would need a pressure near 2.0 atm.

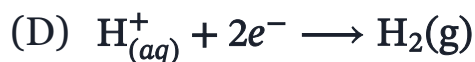
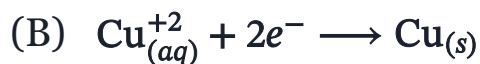
Final Answer:

The molar mass of the solute is 61.58 g/mol, option (C).

$61.58 \text{ g mol}^{-1} \text{ (C)}$

Quick Tip: Use $\pi = (w/M)(RT/V)$ and solve for M.

74. A galvanic cell consist copper electrode and standard hydrogen electrode. If $E^\circ(\text{Cu}_{(aq)}^{+2}|\text{Cu}_{(s)}) = +0.34 \text{ V}$, then identify a reaction taking place at positive electrode during working of cell.



Correct Answer: (B) $\text{Cu}_{(aq)}^{+2} + 2e^{-} \longrightarrow \text{Cu}_{(s)}$

Solution:

Step 1: Understanding the Concept:

In a galvanic cell the electrode with the higher reduction potential is the cathode. It is the positive electrode and reduction takes place there.

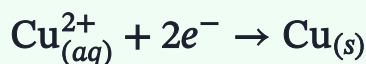
Step 2: Key Formula or Approach:

Compare the potentials. The standard hydrogen electrode (SHE) is defined as 0.00 V. The copper half-cell is +0.34 V.

Step 3: Detailed Explanation:

Since +0.34 V is greater than 0.00 V, copper has the greater tendency to be reduced, so the Cu electrode is the cathode and the positive terminal.

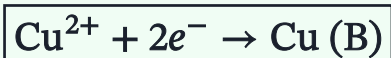
The reaction at the cathode is



The hydrogen electrode is the anode, where $\text{H}_2 \rightarrow 2\text{H}^{+} + 2e^{-}$ occurs (option C), and that is the negative electrode. Option (A) is oxidation of copper and (D) is reduction of H^{+} , neither of which happens at the positive electrode in this cell.

Final Answer:

At the positive (copper) electrode the reaction is reduction of Cu^{2+} , option (B).



Quick Tip: Cu^{2+}/Cu has +0.34 V against SHE at 0 V, so Cu is the cathode (positive) and Cu^{2+} is reduced.



75. If standard reduction potential of Zn, Ni and Fe are -0.76 V , -0.23 V and -0.44 V respectively. For the reaction (Stated below) to be spontaneous find out electrodes X and Y considering above electrode potentials



- (A) X = Ni , Y = Fe
- (B) X = Ni , Y = Zn
- (C) X = Fe , Y = Zn
- (D) X = Zn , Y = Ni

Correct Answer: (D) X = Zn , Y = Ni

Solution:**Step 1: Understanding the Concept:**

In the reaction $\text{X} + \text{Y}^{2+} \rightarrow \text{X}^{2+} + \text{Y}$, X is oxidised (anode) and Y^{2+} is reduced (cathode).

Step 2: Key Formula or Approach:

$$E_{\text{cell}}^{\circ} = E_{\text{cathode}}^{\circ} - E_{\text{anode}}^{\circ} > 0$$

So the reduced species (Y) must have a higher reduction potential than the oxidised species (X).

Step 3: Detailed Explanation:

Reduction potentials: Zn -0.76 V, Fe -0.44 V, Ni -0.23 V.

Check (A) X = Ni, Y = Fe: $E^\circ = -0.44 - (-0.23) = -0.21$ V, not spontaneous.

Check (B) X = Ni, Y = Zn: $E^\circ = -0.76 + 0.23 = -0.53$ V, not spontaneous.

Check (C) X = Fe, Y = Zn: $E^\circ = -0.76 + 0.44 = -0.32$ V, not spontaneous.

Check (D) X = Zn, Y = Ni: $E^\circ = -0.23 - (-0.76) = +0.53$ V, positive, so spontaneous.

Final Answer:

Zinc is oxidised and nickel ion is reduced, so X = Zn and Y = Ni, option (D).

$$X = \text{Zn}, Y = \text{Ni (D)}$$

Quick Tip: A reaction is spontaneous when X (oxidised) has a lower reduction potential than Y (reduced).

76. If standard reduction potential of four electrodes A, B, C, D are $+2.5$ V, $+3.0$ V, -2.0 V and -1.5 V respectively. When is the standard emf of cell maximum?

- (A) 'A' is anode and 'B' is cathode
- (B) 'B' is anode and 'D' is cathode
- (C) 'C' is anode and 'B' is cathode
- (D) 'B' is anode and 'C' is cathode

Correct Answer: (C) 'C' is anode and 'B' is cathode

Solution:

Step 1: Understanding the Concept:

The standard emf of a cell is the difference between the reduction potential of the cathode and that of the anode. In a working cell the cathode has the higher reduction potential.

Step 2: Key Formula or Approach:

$$E^\circ_{\text{cell}} = E^\circ_{\text{cathode}} - E^\circ_{\text{anode}}$$

To make this as large as possible, choose the highest E° as cathode and the lowest as anode.

Step 3: Detailed Explanation:

The potentials are A = +2.5 V, B = +3.0 V, C = -2.0 V, D = -1.5 V.

(A) A anode, B cathode: $3.0 - 2.5 = 0.5$ V.

(B) B anode, D cathode: $-1.5 - 3.0 = -4.5$ V, which is negative, so not a working cell.

(C) C anode, B cathode: $3.0 - (-2.0) = 5.0$ V.

(D) B anode, C cathode: $-2.0 - 3.0 = -5.0$ V, also negative.

The largest positive value is 5.0 V in option (C), which uses the highest and the lowest potentials.

Final Answer:

The cell emf is maximum when C is the anode and B is the cathode, option (C).

C anode, B cathode (C)

Quick Tip: Maximum emf needs the highest reduction potential as cathode and the lowest as anode.



77. Which of the following is an example of fractional order reaction ?

- (A) $2\text{H}_2\text{O}_2(\text{g}) \longrightarrow 2\text{H}_2\text{O}(\text{l}) + \text{O}_2(\text{g})$
- (B) $\text{H}_2(\text{g}) + \text{I}_2(\text{g}) \longrightarrow 2\text{HI}(\text{g})$
- (C) $\text{CH}_3\text{CHO}(\text{g}) \longrightarrow \text{CH}_4(\text{g}) + \text{CO}(\text{g})$
- (D) $2\text{NO}(\text{g}) + 2\text{H}_2(\text{g}) \longrightarrow \text{N}_2(\text{g}) + 2\text{H}_2\text{O}(\text{l})$

Correct Answer: (C) $\text{CH}_3\text{CHO}(\text{g}) \longrightarrow \text{CH}_4(\text{g}) + \text{CO}(\text{g})$

Solution:**Step 1: Understanding the Concept:**

The order of a reaction is the sum of the powers of concentrations in the experimentally determined rate law. It need not be an integer and it cannot be read from the balanced equation.

Step 2: Key Formula or Approach:

Recall the known rate laws:

1. $2\text{H}_2\text{O}_2 \rightarrow 2\text{H}_2\text{O} + \text{O}_2$: rate = $k[\text{H}_2\text{O}_2]$, first order.
2. $\text{H}_2 + \text{I}_2 \rightarrow 2\text{HI}$: rate = $k[\text{H}_2][\text{I}_2]$, second order.
3. $\text{CH}_3\text{CHO} \rightarrow \text{CH}_4 + \text{CO}$: rate = $k[\text{CH}_3\text{CHO}]^{3/2}$.

4. $2\text{NO} + 2\text{H}_2 \rightarrow \text{N}_2 + 2\text{H}_2\text{O}$: rate = $k[\text{NO}]^2[\text{H}_2]$, third order.

Step 3: Detailed Explanation:

Reactions 1, 2 and 4 have whole-number orders 1, 2 and 3. Only the decomposition of acetaldehyde has the order $3/2$, which is a fraction. Its order differs from its molecularity (which is 1 for each elementary step) because the reaction proceeds through a chain mechanism.

Final Answer:

The thermal decomposition of acetaldehyde has order 1.5, so option (C) is the fractional order reaction.



Quick Tip: Acetaldehyde decomposition has an experimental rate law of order $3/2$.

78. What is the time required for 75 % completion of a first order reaction if rate constant is 23.03minute^{-1} ?

- (A) 12.00 s
- (B) 3.6 s
- (C) 36 s
- (D) 6.0 s

Correct Answer: (B) 3.6 s

Solution:

Step 1: Understanding the Concept:

For a first order reaction, the time to reach a given fraction of completion depends only on k . Here 75 % complete means 25 % of the reactant is left.

Step 2: Key Formula or Approach:

$$t = \frac{2.303}{k} \log \frac{[A]_0}{[A]}$$

If 75 % reacts, $[A]_0/[A] = 100/25 = 4$.

Step 3: Detailed Explanation:

$$t = \frac{2.303}{23.03 \text{ min}^{-1}} \log 4 = 0.1 \text{ min} \times 0.6021 = 0.06021 \text{ min}$$

Convert minutes to seconds:

$$t = 0.06021 \times 60 = 3.61 \text{ s} \approx 3.6 \text{ s}$$

Option (C) 36 s comes from forgetting the factor 0.1 in 2.303/23.03.

Options (A) and (D) arise from using the wrong time unit or using 50 percent completion, which would give 1.8 s.

Final Answer:

The time required for 75 percent completion is about 3.6 s, option (B).

3.6 s (B)

Quick Tip: Use $t = (2.303/k) \log(100/25)$ and watch that k is per minute while options are in seconds.



79. For a reaction $A \longrightarrow \text{product}$, $k = 2 \times 10^{-2} \text{s}^{-1}$. If the initial concentration of A is 1.0mol dm^{-3} find the value of $\log \frac{1}{[A]_t}$ after 100 second ?

- (A) 0.135
- (B) 0.270
- (C) 0.430
- (D) 0.868

Correct Answer: (D) 0.868

Solution:

Step 1: Understanding the Concept:

The unit of k is s^{-1} , which tells us the reaction is first order. For a first order reaction the concentration falls exponentially with time.

Step 2: Key Formula or Approach:

$$k = \frac{2.303}{t} \log \frac{[A]_0}{[A]_t} \Rightarrow \log \frac{[A]_0}{[A]_t} = \frac{kt}{2.303}$$

Since $[A]_0 = 1.0$, the left side equals $\log \frac{1}{[A]_t}$.

Step 3: Detailed Explanation:

$$\log \frac{1}{[A]_t} = \frac{2 \times 10^{-2} \times 100}{2.303} = \frac{2}{2.303} = 0.868$$

This means $[A]_t = 10^{-0.868} = 0.135 \text{ mol dm}^{-3}$. Option (A) 0.135 is that concentration and not the logarithm asked for. Option (B) 0.270 is double 0.135, and option (C) 0.430 comes from using $kt = 1$, neither of which answers the question.

Final Answer:

The value of $\log \frac{1}{[A]_t}$ is 0.868, option (D).

0.868 (D)

Quick Tip: A reaction with unit of k in s^{-1} is first order. Use $\log[A]_0/[A] = kt/2.303$.

80. Which of the following is an example of macromolecular colloid?

- (A) Proteins
- (B) Detergents
- (C) Soap
- (D) S_8 sulphur molecules

Correct Answer: (A) Proteins

Solution:

Step 1: Understanding the Concept:

Colloids are classified by the nature of the dispersed particles into multimolecular, macromolecular and associated colloids.

Step 2: Key Formula or Approach:

In a macromolecular colloid, each dispersed particle is a single very large molecule (a macromolecule) of size in the colloidal range.

Step 3: Detailed Explanation:

Proteins are polymers of amino acids with large molar masses, so a single protein molecule is already of colloidal size. Proteins, starch and cellulose are standard examples of macromolecular colloids.

Detergents and soaps are associated colloids (micelles). Their small molecules gather into micelles only above a certain concentration.

Sulphur sol made of S_8 molecules is a multimolecular colloid, where many small molecules clump into a particle.

Final Answer:

Proteins form a macromolecular colloid, option (A).

Proteins (A)

Quick Tip: Macromolecular colloids are made of giant molecules such as proteins, starch or cellulose.

81. Which of the following reactions exhibits the reducing property of ozone ?

- (A) $\text{NO(g)} + \text{O}_3\text{(g)} \longrightarrow \text{NO}_2\text{(g)} + \text{O}_2\text{(g)}$
- (B) $2\text{KI(aq)} + \text{H}_2\text{O(l)} + \text{O}_3\text{(g)} \longrightarrow 2\text{KOH(aq)} + \text{I}_2\text{(g)} + \text{O}_2\text{(g)}$
- (C) $\text{H}_2\text{O}_2\text{(l)} + \text{O}_3\text{(g)} \longrightarrow \text{H}_2\text{O(l)} + 2\text{O}_2\text{(g)}$
- (D) $\text{PbS(s)} + 4\text{O}_3\text{(g)} \longrightarrow \text{PbSO}_4\text{(s)} + 4\text{O}_2\text{(g)}$

Correct Answer: (C) $\text{H}_2\text{O}_2\text{(l)} + \text{O}_3\text{(g)} \longrightarrow \text{H}_2\text{O(l)} + 2\text{O}_2\text{(g)}$

Solution:

Step 1: Understanding the Concept:

Ozone is mostly an oxidising agent because it readily gives up an oxygen atom. Its reducing action is the rare case and is shown only with a few compounds such as H_2O_2 .

Step 2: Key Formula or Approach:

Decide in each reaction whether the other reactant is being oxidised (then ozone is oxidising) or reduced (then ozone is reducing).

Step 3: Check reaction (A):

$\text{NO} + \text{O}_3 \rightarrow \text{NO}_2 + \text{O}_2$: NO is oxidised to NO_2 , so ozone acts as an oxidising agent.

Step 4: Check reactions (B) and (D):

$2\text{KI} + \text{H}_2\text{O} + \text{O}_3 \rightarrow 2\text{KOH} + \text{I}_2 + \text{O}_2$: iodide is oxidised to iodine, so ozone is oxidising.

$\text{PbS} + 4\text{O}_3 \rightarrow \text{PbSO}_4 + 4\text{O}_2$: sulphide is oxidised to sulphate, so ozone is oxidising.

Step 5: Check reaction (C):

$\text{H}_2\text{O}_2 + \text{O}_3 \rightarrow \text{H}_2\text{O} + 2\text{O}_2$: here the peroxide is reduced to water and oxygen is evolved from both reactants. This is the standard textbook example of the reducing action of ozone, in which it reduces hydrogen peroxide.

Final Answer:

The reducing property of ozone is shown by its reaction with hydrogen peroxide, option (C).



Quick Tip: Ozone usually oxidises. Its reducing action is seen with hydrogen peroxide, giving water and oxygen.

82. What is the oxidation state of chlorine in its weakest oxoacid?

- (A) +1
- (B) -1
- (C) +3
- (D) -3

Correct Answer: (A) +1

Solution:

Step 1: Understanding the Concept:

The oxoacids of chlorine are HOCl, HClO₂, HClO₃ and HClO₄. Acid strength rises as the number of oxygen atoms on chlorine increases, because the extra oxygens pull electron density and stabilise the anion.

Step 2: Key Formula or Approach:

Find the weakest acid, then compute the oxidation state of Cl using hydrogen +1 and oxygen -2.

Step 3: Detailed Explanation:

The weakest acid is hypochlorous acid, HOCl, with only one oxygen.

$$(+1) + x + (-2) = 0 \Rightarrow x = +1$$

For comparison, HClO_2 has $x = +3$, HClO_3 has $x = +5$ and HClO_4 has $x = +7$, and these are all stronger acids. The values $+3$ in option (C) belongs to chlorous acid, while -1 and -3 are not oxoacid states at all.

Final Answer:

Chlorine is in the $+1$ state in the weakest oxoacid HOCl , option (A).

+1 (A)

Quick Tip: The weakest oxoacid of chlorine is hypochlorous acid, HOCl , with Cl in $+1$.



83. Which from following pair of elements in their respective oxidation states have same number of unpaired electrons ?

- (A) Fe^{2+} and Mn^{2+}
- (B) Co^{2+} and Ni^{2+}
- (C) Fe^{2+} and Cr^{2+}
- (D) Co^{2+} and Fe^{2+}

Correct Answer: (C) Fe^{2+} and Cr^{2+}

Solution:

Step 1: Understanding the Concept:

The number of unpaired electrons in a transition metal ion comes from its d-electron configuration, filled by Hund's rule (high spin for free ions).

Step 2: Key Formula or Approach:

Atomic configurations: Cr $3d^5 4s^1$, Mn $3d^5 4s^2$, Fe $3d^6 4s^2$, Co $3d^7 4s^2$, Ni $3d^8 4s^2$. For +2 ions the two 4s electrons are lost.

Step 3: Detailed Explanation:

Cr^{2+} : $3d^4$, so 4 unpaired electrons.

Mn^{2+} : $3d^5$, so 5 unpaired electrons.

Fe^{2+} : $3d^6$, one orbital paired and 4 unpaired electrons.

Co^{2+} : $3d^7$, so 3 unpaired electrons.

Ni^{2+} : $3d^8$, so 2 unpaired electrons.

Compare: (A) 4 and 5, (B) 3 and 2, (C) 4 and 4, (D) 3 and 4. Only (C) has equal numbers.

Final Answer:

Fe^{2+} and Cr^{2+} both have 4 unpaired electrons, option (C).

Fe^{2+} and Cr^{2+} (C)

Quick Tip: Write the 3d configuration of each ion and count unpaired electrons: Fe^{2+} and Cr^{2+} both have 4.

84. Which lanthanoid from following exhibits f^{14} configuration in +3 state?

- (A) Tm
- (B) Lu
- (C) Ce
- (D) Dy

Correct Answer: (B) Lu

Solution:

Step 1: Understanding the Concept:

The lanthanoids fill the 4f subshell across the series from cerium ($Z = 58$) to lutetium ($Z = 71$). Lanthanum has none and lutetium completes the subshell. In the +3 state, the ions lose the 6s and 5d or 4f electrons so that only the 4f configuration remains.

Step 2: Key Formula or Approach:

Ln^{3+} has the configuration $[\text{Xe}]4f^n$, where $n = Z - 57$.

Step 3: Detailed Explanation:

Lutetium ($Z = 71$): $n = 71 - 57 = 14$, so $\text{Lu}^{3+} = [\text{Xe}]4f^{14}$.

Thulium ($Z = 69$): $n = 12$, $4f^{12}$.

Dysprosium ($Z = 66$): $n = 9$, $4f^9$.

Cerium ($Z = 58$): $n = 1$, $4f^1$.

Only lutetium reaches the full $4f^{14}$ set.

Final Answer:

Lu in the +3 state has the f^{14} configuration, option (B).

Lu (B)

Quick Tip: Lanthanoid 4f runs from f0 (La) to f14 (Lu). In +3 state, Lu^{3+} has 4f14.

85. What is EAN for metal ion in $[\text{Fe}(\text{CN})_6]^{3-}$?

- (A) 37
- (B) 36
- (C) 35
- (D) 34

Correct Answer: (C) 35

Solution:

Step 1: Understanding the Concept:

Effective atomic number (EAN) is the total number of electrons around the central metal, counting the electrons donated by ligands.

Step 2: Key Formula or Approach:

$$\text{EAN} = Z - (\text{oxidation state}) + 2 \times (\text{number of ligands})$$

Each cyanide ion donates one pair, so 2 electrons.

Step 3: Detailed Explanation:

In $[\text{Fe}(\text{CN})_6]^{3-}$, the oxidation state of Fe is: $x + 6(-1) = -3$, so $x = +3$.
Fe has $Z = 26$.

$$\text{EAN} = 26 - 3 + 2 \times 6 = 23 + 12 = 35$$

Option (B) 36 is the EAN of $[\text{Fe}(\text{CN})_6]^{4-}$ with Fe^{2+} , where $26 - 2 + 12 = 36$, the krypton configuration. For the +3 ion the value is one less.

Final Answer:

The EAN of iron in $[\text{Fe}(\text{CN})_6]^{3-}$ is 35, option (C).

Quick Tip: EAN = atomic number of metal minus oxidation state plus 2 times the number of ligands.

86. Which of the following is a correct increasing order of ligand field strength ?

- (A) $\text{EDTA} < \text{C}_2\text{O}_4^{2-} < \text{H}_2\text{O} < \text{en}$
- (B) $\text{en} < \text{H}_2\text{O} < \text{EDTA} < \text{C}_2\text{O}_4^{2-}$
- (C) $\text{C}_2\text{O}_4^{2-} < \text{H}_2\text{O} < \text{EDTA} < \text{en}$
- (D) $\text{H}_2\text{O} < \text{en} < \text{C}_2\text{O}_4^{2-} < \text{EDTA}$

Correct Answer: (C) $\text{C}_2\text{O}_4^{2-} < \text{H}_2\text{O} < \text{EDTA} < \text{en}$

Solution:

Step 1: Understanding the Concept:

The spectrochemical series arranges ligands by the size of the crystal field splitting they cause. A stronger field ligand gives a larger splitting.

Step 2: Key Formula or Approach:

In the common spectrochemical series the relevant ligands appear in this increasing order:



Oxygen donors such as oxalate and water are weak. EDTA (mixed N and O donors) is stronger. Ethylenediamine (N donors) is stronger

still.

Step 3: Detailed Explanation:

Compare each option with this order.

(A) places EDTA first, which contradicts the series where EDTA is stronger than oxalate and water.

(B) starts with en as the weakest, but en is the strongest of the four.

(D) puts water as the weakest, but oxalate is weaker than water.

(C) lists oxalate, then water, then EDTA, then en, which matches the series.

Final Answer:

The increasing order of ligand field strength is oxalate, water, EDTA, en, option (C).



Quick Tip: Spectrochemical series order here: oxalate < water < EDTA < en.

87. Identify substrate 'S' in the following reaction.



- (A) 3 - chloro - 2 - methylpentane
- (B) 2 - chloro - 3 - methylpentane
- (C) 3 - chloro - 3 - methylpentane
- (D) 2 - chloro - 2 - methylpentane

Correct Answer: (C) 3 - chloro - 3 - methylpentane

Solution:

Step 1: Understanding the Concept:

In the Wurtz reaction, two molecules of an alkyl halide react with sodium in dry ether and the two alkyl groups join:



So the product is made of two identical R groups.

Step 2: Key Formula or Approach:

Break the product at its central C-C bond to get R, then attach X.

Step 3: Detailed Explanation:

The product is 3,4-diethyl-3,4-dimethylhexane. The central bond is between C₃ and C₄. Each of these carbons carries a methyl, an ethyl, and the rest of the hexane chain (which is also an ethyl: C₂-C₁ on one side, C₅-C₆ on the other).

So each half is a carbon bearing one methyl and two ethyl groups: $\text{C}(\text{CH}_3)(\text{C}_2\text{H}_5)_2$.

Its longest chain is five carbons with a methyl on C₃, so the radical is 3-methylpentan-3-yl.

Attaching chlorine gives 3-chloro-3-methylpentane. Options (A) and (B) are secondary chlorides that would give a different coupled product. Option (D) would give 2,3-dimethyl-type chain products.

Final Answer:

The substrate is 3-chloro-3-methylpentane, option (C).

3-chloro-3-methylpentane (C)

Quick Tip: Wurtz joins two alkyl groups. Split the product in half to find the alkyl halide.

88. Select the correct decreasing order of acid strength for the following compounds.

a) Ethanol b) 2-Methyl propan-2-ol c) Phenol d) p - Nitrophenol

(A) $a > b > c > d$

(B) $c > d > b > a$

(C) $d > c > a > b$

(D) $d > b > c > a$

Correct Answer: (C) $d > c > a > b$

Solution:

Step 1: Understanding the Concept:

Acid strength depends on how stable the conjugate base is. Phenoxide ions are stabilised by resonance, so phenols are stronger acids than alcohols. Electron-donating alkyl groups destabilise the alkoxide ion and weaken acidity.

Step 2: Key Formula or Approach:

Compare in three groups: nitrophenol vs phenol, phenol vs alcohols, and primary vs tertiary alcohol.

Step 3: Detailed Explanation:

p-Nitrophenol: the $-\text{NO}_2$ group withdraws electrons by resonance and induction and spreads the negative charge of the phenoxide further, so it is more acidic than phenol.

Phenol: the phenoxide ion is stabilised by resonance with the ring, so it is more acidic than any simple alcohol.

Ethanol: one alkyl group, so the alkoxide is only slightly destabilised.

2-Methylpropan-2-ol: three alkyl groups push electrons towards the oxygen of the alkoxide ion, so it is the weakest acid.

So the order is p-nitrophenol > phenol > ethanol > 2-methylpropan-2-ol, that is $d > c > a > b$.

Final Answer:

The decreasing order of acidity is $d > c > a > b$, option (C).

$$d > c > a > b \text{ (C)}$$

Quick Tip: Phenols are more acidic than alcohols. A nitro group makes phenol more acidic. Tertiary alcohols are the weakest.

89. Which of the following isomers of C_4H_9OH has lowest boiling point ?

- (A) n - Butyl alcohol
- (B) Isobutyl alcohol
- (C) sec-Butyl alcohol
- (D) tert Butyl alcohol

Correct Answer: (D) tert Butyl alcohol

Solution:

Step 1: Understanding the Concept:

All four are isomers of C_4H_9OH , so they have the same molar mass and can all form hydrogen bonds. The difference in boiling point

comes from the shape of the molecule.

Step 2: Key Formula or Approach:

Among isomers, a more branched molecule is more compact. Smaller surface area means weaker van der Waals forces and so a lower boiling point.

Step 3: Detailed Explanation:

n-Butyl alcohol is a straight chain, with the largest surface area, and boils at about 118 degrees C.

Isobutyl alcohol has one branch near the end of the chain and boils at about 108 degrees C.

sec-Butyl alcohol has the OH on a secondary carbon and boils at about 99 degrees C.

tert-Butyl alcohol is the most branched and almost spherical, and boils at about 83 degrees C.

The order of boiling point is primary > secondary > tertiary for isomeric alcohols.

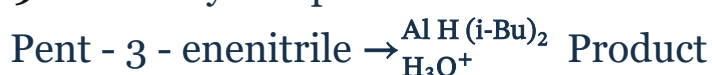
Final Answer:

tert-Butyl alcohol has the lowest boiling point, option (D).

tert-Butyl alcohol (D)

Quick Tip: More branching means a smaller surface area, weaker van der Waals forces and a lower boiling point.

90. Identify the 'product' in the following reaction.



- (A) Pent - 3 - en - 1 - amine
- (B) Pentanal
- (C) Pent - 3 - enal
- (D) Pent - 3 - en - 1 - ol

Correct Answer: (C) Pent - 3 - enal

Solution:

Step 1: Understanding the Concept:

Diisobutylaluminium hydride (DIBAL-H) is a mild, bulky reducing agent. With a nitrile, it adds a single hydride and stops at an imine-aluminium complex. Water then hydrolyses the imine to an aldehyde.

Step 2: Key Formula or Approach:



It does not reduce an isolated C=C bond.

Step 3: Detailed Explanation:

Pent-3-enenitrile is $\text{CH}_3\text{CH}=\text{CH-CH}_2\text{-CN}$. The nitrile carbon becomes the aldehyde carbon, and the chain keeps its double bond between C3 and C4.

Product: $\text{CH}_3\text{CH}=\text{CH-CH}_2\text{-CHO}$, which is pent-3-enal.

Option (A) the amine and option (D) the alcohol need a strong reagent like LiAlH_4 . Option (B) pentanal would require the double bond to be reduced, which DIBAL-H does not do.

Final Answer:

The product is pent-3-enal, option (C).

Pent-3-enal (C)

Quick Tip: DIBAL-H reduces a nitrile to an imine, which hydrolyses to an aldehyde without touching the C=C.

91. Identify the product obtained when isopropyl magnesium chloride in dry ether reacts with dry ice forming a complex, hydrolysed further.

- (A) Propanoic acid
- (B) Propanal
- (C) 2 - Methylpropanoic acid
- (D) 2 - Methylpropanol

Correct Answer: (C) 2 - Methylpropanoic acid

Solution:

Step 1: Understanding the Concept:

A Grignard reagent behaves as a carbanion source. It attacks the carbon of CO₂ (dry ice), giving a magnesium carboxylate complex. Acid hydrolysis then liberates the carboxylic acid.

Step 2: Key Formula or Approach:



The acid has one more carbon than the Grignard's alkyl group.

Step 3: Detailed Explanation:

Isopropyl magnesium chloride is (CH₃)₂CH-MgCl, with R = isopropyl

(3 carbons).

Adding CO_2 gives $(\text{CH}_3)_2\text{CH-COOMgCl}$. Hydrolysis gives $(\text{CH}_3)_2\text{CH-COOH}$.

This is a four-carbon acid with a methyl branch on C_2 , named 2-methylpropanoic acid (isobutyric acid).

Propanoic acid in (A) would come from ethyl magnesium halide. Aldehyde (B) and alcohol (D) are not made from CO_2 .

Final Answer:

The product is 2-methylpropanoic acid, option (C).

2-Methylpropanoic acid (C)

Quick Tip: A Grignard reagent adds to CO_2 to give a carboxylic acid with one more carbon.

92. What is the number of moles of hydrogen atoms needed to obtain one mole ethanamine from acetamide using LiAlH_4 /ether as reducing agent.

- (A) 4
- (B) 3
- (C) 2
- (D) 1

Correct Answer: (A) 4

Solution:

Step 1: Understanding the Concept:

LiAlH_4 reduces the carbonyl of an amide fully to a CH_2 group, so an

amide becomes an amine with the same number of carbons.

Step 2: Key Formula or Approach:



Step 3: Detailed Explanation:

Count hydrogens on both sides.

Acetamide, $\text{C}_2\text{H}_5\text{NO}$, has 5 hydrogens. Ethanamine, $\text{C}_2\text{H}_7\text{N}$, has 7 hydrogens. Water has 2 hydrogens and takes the oxygen.

Hydrogen in products: $7 + 2 = 9$. Hydrogen in acetamide: 5. So the reagent must supply $9 - 5 = 4$ hydrogen atoms.

The $\text{C}=\text{O}$ is converted to CH_2 , using 2 H for the carbon and 2 H for removing the oxygen as water.

Final Answer:

4 moles of hydrogen atoms are needed per mole of ethanamine, option (A).

4 (A)

Quick Tip: LiAlH_4 reduces an amide to an amine: $\text{RCONH}_2 + 4[\text{H}]$ gives $\text{RCH}_2\text{NH}_2 + \text{H}_2\text{O}$.

93. Identify 'B' in the following reaction



- (A) Ethanoic acid
- (B) Ethanamide
- (C) Propanoic acid
- (D) Propanamide

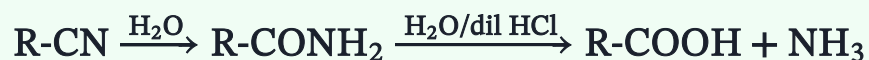
Correct Answer: (C) Propanoic acid

Solution:

Step 1: Understanding the Concept:

Nitriles are hydrolysed in two stages. Partial hydrolysis with water gives an amide, and further hydrolysis with dilute acid gives a carboxylic acid with the loss of ammonia.

Step 2: Key Formula or Approach:



Step 3: Detailed Explanation:

Ethyl cyanide is $\text{CH}_3\text{CH}_2\text{CN}$, propanenitrile.

Step one gives A = $\text{CH}_3\text{CH}_2\text{CONH}_2$, propanamide.

Step two hydrolyses the amide, which releases NH_3 as stated in the equation, and forms B = $\text{CH}_3\text{CH}_2\text{COOH}$, propanoic acid.

Ethanoic acid and ethanamide in (A) and (B) have two carbons, but ethyl cyanide has three carbons including the nitrile carbon.

Propanamide in (D) is the intermediate A, not B.

Final Answer:

B is propanoic acid, option (C).

Propanoic acid (C)

Quick Tip: Nitrile partly hydrolyses to an amide (A). Further hydrolysis gives the acid (B) and ammonia.

94. Which from following amines when heated with ethanolic KOH and chloroform does NOT produce foul smell ?

- (A) Benzenamine
- (B) N , N - Dimethyl aniline
- (C) Propan - 2 - amine
- (D) Phenylmethanamine

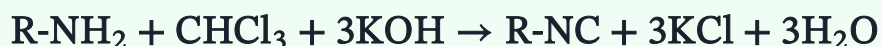
Correct Answer: (B) N , N - Dimethyl aniline

Solution:

Step 1: Understanding the Concept:

In the carbylamine reaction, a primary amine heated with chloroform and ethanolic KOH forms an isocyanide (carbylamine), which has a very foul smell.

Step 2: Key Formula or Approach:



Only compounds with a free -NH_2 group take part.

Step 3: Detailed Explanation:

Benzenamine (aniline) is a primary aromatic amine, so it gives phenyl

isocyanide.

Propan-2-amine is a primary aliphatic amine, so it gives isopropyl isocyanide.

Phenylmethanamine (benzylamine) is a primary amine, so it gives benzyl isocyanide.

N,N-Dimethylaniline is a tertiary amine with no N-H hydrogen. It cannot form an isocyanide, so there is no foul smell.

Final Answer:

N,N-Dimethylaniline does not give the test, option (B).

N,N-Dimethylaniline (B)

Quick Tip: Only primary amines give the carbylamine foul smell. N,N-dimethylaniline is tertiary.

95. Identify glycosidic linkage in lactose.

- (A) $\alpha - 1, 4$
- (B) $\beta - 1, 4$
- (C) $\alpha - 1, 6$
- (D) $\beta - 2, 6$

Correct Answer: (B) $\beta - 1, 4$

Solution:

Step 1: Understanding the Concept:

Disaccharides are two monosaccharides joined by a glycosidic bond between the anomeric carbon of one unit and a hydroxyl carbon of the

other.

Step 2: Key Formula or Approach:

Lactose is milk sugar. Its two units are D-galactose and D-glucose.

Step 3: Detailed Explanation:

The anomeric carbon (C1) of beta-D-galactose is linked to C4 of glucose through oxygen. So the linkage is β -1,4.

Because the glucose unit still has a free anomeric hydroxyl, lactose is a reducing sugar.

For comparison, maltose has an α -1,4 linkage, and sucrose has an α -1, β -2 linkage.

So option (A), α -1,4, is maltose, and options (C) and (D) do not describe lactose.

Final Answer:

The glycosidic linkage in lactose is β -1,4, option (B).

β -1,4 (B)

Quick Tip: Lactose is galactose joined to glucose through a beta-1,4 glycosidic bond.

96. Identify a protein present in nails

- (A) Legumelin
- (B) Keratin
- (C) Myosin
- (D) Insulin

Correct Answer: (B) Keratin

Solution:

Step 1: Understanding the Concept:

Proteins are classified as fibrous (long threads, insoluble in water, structural) and globular (folded, soluble, functional).

Step 2: Key Formula or Approach:

Match each protein with its location and role.

Step 3: Detailed Explanation:

Keratin is a fibrous protein found in hair, nails, horns, wool and the outer skin layer. It is rich in cysteine, whose disulphide bridges make it tough.

Legumelin is a globular protein of the legume (pea) family and is not found in human nails.

Myosin is a fibrous protein of muscle that helps contraction.

Insulin is a hormone, a small globular protein that controls blood sugar.

Final Answer:

Nails contain keratin, option (B).

Keratin (B)

Quick Tip: Nails, hair and horns are made of keratin, a fibrous structural protein.

97. Identify the use of LDP from following.

- (A) Manufacture of buckets
- (B) Preparation of pipes for irrigation
- (C) For making nonstick cook ware
- (D) For making tyre cords

Correct Answer: (B) Preparation of pipes for irrigation

Solution:

Step 1: Understanding the Concept:

Low density polyethene (LDPE) is made by free radical polymerisation of ethene at high pressure. Its chains are branched, so they pack loosely. This gives a soft, flexible, tough plastic.

Step 2: Key Formula or Approach:

Match the property of LDPE (flexible, chemically resistant, easily moulded) with the use.

Step 3: Detailed Explanation:

Flexible tubing and pipes, for example for irrigation, are made from LDPE because they must bend and resist water and chemicals.

Nonstick cookware in option (C) uses PTFE (Teflon), which is a different polymer made from tetrafluoroethene.

Tyre cords in option (D) are made of nylon-6,6 or polyesters, which are strong fibres.

So among the options, the typical use matched with LDPE is the preparation of pipes.

Final Answer:

LDPE is used to prepare pipes for irrigation, option (B).

Pipes for irrigation (B)

Quick Tip: LDPE is soft and flexible, so it is used for flexible pipes such as irrigation pipes.

98. What is a starting material used to obtain viscose rayon ?

- (A) Styrene
- (B) Cellulose
- (C) Acrylamide
- (D) Bisphenol

Correct Answer: (B) Cellulose

Solution:

Step 1: Understanding the Concept:

Rayon is a semi-synthetic fibre. It is made from a natural polymer that is chemically treated and then re-formed into fibres.

Step 2: Key Formula or Approach:

Viscose rayon is also called regenerated cellulose.

Step 3: Detailed Explanation:

Wood pulp (which is mostly cellulose) is treated with NaOH and carbon disulphide to give cellulose xanthate, a viscous solution called viscose. When this is forced through a spinneret into acid, the cellulose is regenerated as fibres.

So the starting material is cellulose.

Styrene is the monomer of polystyrene. Acrylamide gives

polyacrylamide. Bisphenol A is used for polycarbonates and epoxy resins. None of these gives viscose.

Final Answer:

Viscose rayon starts from cellulose, option (B).

Cellulose (B)

Quick Tip: Viscose rayon is regenerated cellulose, made from wood pulp.

99. Which of the following is the use of HDPE ?

- (A) To obtain shopping bags
- (B) To prepare shampoo bottles
- (C) For making carpet
- (D) To obtain food container

Correct Answer: (B) To prepare shampoo bottles

Solution:

Step 1: Understanding the Concept:

High density polyethene (HDPE) is made with Ziegler-Natta catalyst. Its chains are linear with little branching, so they pack closely. This makes it stiff, strong and opaque.

Step 2: Key Formula or Approach:

Match the property (rigid, tough, chemically resistant) with the use.

Step 3: Detailed Explanation:

A shampoo bottle needs a rigid, tough, chemically resistant container that does not crack when squeezed or dropped. HDPE has these properties, so it is used to mould such bottles, along with buckets, drums and pipes.

So option (B) is a standard use of HDPE.

Final Answer:

HDPE is used to prepare shampoo bottles, option (B).

Shampoo bottles (B)

Quick Tip: HDPE is rigid and strong, so it is moulded into bottles such as shampoo bottles.



100. Which from following plants is useful to extract analgesic compound from it ?

- (A) Turmeric
- (B) Cinnamon
- (C) Wintergreen
- (D) Plants of citrus fruits

Correct Answer: (C) Wintergreen

Solution:

Step 1: Understanding the Concept:

Analgesics are drugs that relieve pain without causing unconsciousness. Some come from plants.

Step 2: Key Formula or Approach:

Match each plant with its known active compound.

Step 3: Detailed Explanation:

Wintergreen leaves yield oil of wintergreen, which is methyl salicylate. It is applied on the skin in pain-relief balms and sprays, so it is an analgesic.

Turmeric contains curcumin, known as an antiseptic and anti-inflammatory agent.

Cinnamon gives cinnamaldehyde, used mainly as a flavouring and antimicrobial.

Citrus plants supply vitamin C and citric acid, not an analgesic.

Final Answer:

Wintergreen is the plant from which an analgesic is obtained, option (C).

Wintergreen (C)

Quick Tip: Wintergreen oil is methyl salicylate, a topical analgesic.

Mathematics

Questions 101 to 150 | 2 marks each | 100 marks

101. The difference between the roots of the equation $x^2 + 2x + 4 = 0$ is ...
(where $i = \sqrt{-1}$)

- (A) $2\sqrt{3}$
- (B) $3\sqrt{2}$
- (C) $2i\sqrt{3}$
- (D) $i\sqrt{3}$

Correct Answer: (C) $2i\sqrt{3}$

Solution:

Step 1: Understanding the Concept:

A quadratic with negative discriminant has two complex conjugate roots. The difference of the roots is therefore a purely imaginary number.

Step 2: Key Formula or Approach:

For $ax^2 + bx + c = 0$, the roots are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. The difference of the two roots is $\frac{\sqrt{b^2 - 4ac}}{a}$ in magnitude.

Step 3: Detailed Explanation:

Here $a = 1$, $b = 2$, $c = 4$, so the discriminant is

$$D = 2^2 - 4(1)(4) = 4 - 16 = -12$$

$$\sqrt{D} = \sqrt{-12} = 2\sqrt{3}i$$

The roots are

$$x = \frac{-2 \pm 2\sqrt{3}i}{2} = -1 \pm i\sqrt{3}$$

Difference of the roots:

$$(-1 + i\sqrt{3}) - (-1 - i\sqrt{3}) = 2i\sqrt{3}$$

The real part of each root cancels, so only the imaginary part is left. Options (A) and (B) are real numbers, which cannot be the difference of two conjugate complex roots. Option (D) takes only half of the imaginary difference.

Final Answer:

The difference between the roots is $2i\sqrt{3}$, option (C).

$$\boxed{2i\sqrt{3} \text{ (C)}}$$

Quick Tip: Solve with the quadratic formula to get complex roots, then subtract them.

102. In a test, there are 7 questions of the type 'True or False'. No student got all the answers correct. If the sequence of answers for every student is unique, then the maximum number of students who appeared for the test is ...

- (A) 128
- (B) 126
- (C) 127
- (D) 125

Correct Answer: (C) 127

Solution:

Step 1: Understanding the Concept:

Each of the 7 True/False questions can be answered in exactly 2 ways. Every student has a different sequence of answers, so the number of students cannot exceed the number of possible sequences.

Step 2: Key Formula or Approach:

By the fundamental counting principle, the number of answer sequences is 2^7 . One of them is the all-correct sequence, which no student has.

Step 3: Detailed Explanation:

Total sequences:

$$2^7 = 128$$

Remove the all-correct sequence, since no student got all answers correct:

$$128 - 1 = 127$$

Because every student has a unique sequence, the maximum number of students equals the number of allowed sequences, 127.

Option (A) 128 includes the forbidden all-correct sequence. Options (B) 126 and (D) 125 remove more sequences than the one excluded.

Final Answer:

The maximum number of students is 127, option (C).

127 (C)

Quick Tip: Total sequences 2^7 minus the one all-correct sequence.

103. Let $f_k(x) = \frac{1}{k}(\cos^k x + \sin^k x)$ where $k \in N$, then $f_6(x) - f_4(x) = \dots$

- (A) $\frac{1}{6}$
- (B) $-\frac{1}{12}$
- (C) $-\frac{1}{6}$
- (D) $\frac{1}{12}$

Correct Answer: (B) $-\frac{1}{12}$

Solution:

Step 1: Understanding the Concept:

We need $f_6 - f_4 = \frac{1}{6}(\cos^6 x + \sin^6 x) - \frac{1}{4}(\cos^4 x + \sin^4 x)$. Both sums can be written using $\sin^2 x \cos^2 x$.

Step 2: Key Formula or Approach:

Let $t = \sin^2 x \cos^2 x$. Use $\cos^2 x + \sin^2 x = 1$:

1. $\cos^4 x + \sin^4 x = (\cos^2 x + \sin^2 x)^2 - 2t = 1 - 2t$.

2. $\cos^6 x + \sin^6 x = (\cos^2 x + \sin^2 x)^3 - 3t(\cos^2 x + \sin^2 x) = 1 - 3t$.

Step 3: Detailed Explanation:

$$f_6 = \frac{1-3t}{6}, \quad f_4 = \frac{1-2t}{4}$$

$$f_6 - f_4 = \frac{1}{6} - \frac{t}{2} - \frac{1}{4} + \frac{t}{2}$$

The terms in t cancel exactly:

$$f_6 - f_4 = \frac{1}{6} - \frac{1}{4} = \frac{2-3}{12} = -\frac{1}{12}$$

The result does not depend on x , which is why a single number appears in all four options.

Final Answer:

$$f_6(x) - f_4(x) = -\frac{1}{12}, \text{ option (B).}$$

$$\boxed{-\frac{1}{12} \text{ (B)}}$$

Quick Tip: Write $\cos^6 + \sin^6$ and $\cos^4 + \sin^4$ in terms of $\sin^2$ x $\cos^2$ x.



104. The coordinates of the orthocenter of the triangle whose sides are represented by the lines $4x - 7y + 10 = 0$, $x + y = 5$ and $7x + 4y = 15$ are

...

- (A) (1, 2)
- (B) (1, -2)
- (C) (-1, 2)
- (D) (-1, -2)

Correct Answer: (A) (1, 2)

Solution:

Step 1: Understanding the Concept:

The orthocentre is the point where the three altitudes meet. In a right-angled triangle, it is the vertex at the right angle.

Step 2: Key Formula or Approach:

Two lines are perpendicular if the product of their slopes is -1.

Step 3: Detailed Explanation:

Slope of $4x - 7y + 10 = 0$: $m_1 = \frac{4}{7}$.

Slope of $7x + 4y = 15$: $m_3 = -\frac{7}{4}$.

$$m_1 m_3 = \frac{4}{7} \times \left(-\frac{7}{4}\right) = -1$$

So these two sides are perpendicular and the triangle is right-angled at their intersection. Solve them together:

$$4x - 7y = -10, \quad 7x + 4y = 15$$

Multiply the first by 4 and the second by 7:

$$16x - 28y = -40, \quad 49x + 28y = 105$$

Add the two: $65x = 65$, so $x = 1$. Then $4 - 7y = -10$ gives $y = 2$.
The orthocentre is $(1, 2)$. It also lies on neither of the other options' coordinates, such as $(1, -2)$.

Final Answer:

The orthocentre is $(1, 2)$, option (A).

(1, 2) (A)

Quick Tip: Check whether two sides are perpendicular. If so, the vertex between them is the orthocentre.

105. The combined equation of lines parallel to the coordinate axes and passing through the point of intersection of lines represented by $x^2 - 6xy + 5y^2 + 10x - 14y + 9 = 0$ is ...

- (A) $xy + 2x + y + 2 = 0$
- (B) $xy + 2x - y - 2 = 0$
- (C) $xy - 2x + y - 2 = 0$
- (D) $xy - 2x - y + 2 = 0$

Correct Answer: (D) $xy - 2x - y + 2 = 0$

Solution:

Step 1: Understanding the Concept:

A second degree equation of the form $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$ can represent a pair of straight lines. Their point of intersection is the point we need. The lines through it parallel to the

axes are $x = h$ and $y = k$.

Step 2: Key Formula or Approach:

Factor $x^2 - 6xy + 5y^2 + 10x - 14y + 9$ as $(x - y + a)(x - 5y + b)$, because $x^2 - 6xy + 5y^2 = (x - y)(x - 5y)$.

Step 3: Detailed Explanation:

Expand and compare the coefficients of x , y and the constant term.

Coefficient of x : $a + b = 10$.

Coefficient of y : $-(b + 5a) = -14$, so $5a + b = 14$.

Subtract: $4a = 4$, so $a = 1$ and $b = 9$. The constant term is $ab = 9$, which matches.

The two lines are $x - y + 1 = 0$ and $x - 5y + 9 = 0$.

Subtract them: $4y - 8 = 0$, so $y = 2$ and $x = 1$. The point of intersection is $(1, 2)$.

Lines through $(1, 2)$ parallel to the axes are $x = 1$ and $y = 2$. Their combined equation is

$$(x - 1)(y - 2) = 0 \Rightarrow xy - 2x - y + 2 = 0$$

Final Answer:

The combined equation is $xy - 2x - y + 2 = 0$, option (D).

$xy - 2x - y + 2 = 0 \text{ (D)}$

Quick Tip: Factor the pair of lines, find their meeting point, then write $(x-h)(y-k)=0$.

106. The number of circles passing through the origin and touching the lines $x + y = 1$ and $x - y = 1$ is ...

- (A) 1
- (B) 2
- (C) 3
- (D) 4

Correct Answer: (B) 2

Solution:

Step 1: Understanding the Concept:

A circle touching two lines has its centre at equal distances from both, so its centre lies on one of the angle bisectors. The lines $x + y = 1$ and $x - y = 1$ meet at $(1, 0)$ and are perpendicular. Their bisectors are $y = 0$ and $x = 1$.

Step 2: Key Formula or Approach:

For a circle with centre C passing through the origin, the radius equals $|OC|$. It also equals the distance from C to each line.

Step 3: Case 1: centre on $y = 0$:

Let the centre be $(h, 0)$. Radius = $|h|$.

Distance from $(h, 0)$ to $x + y - 1 = 0$ is $\frac{|h - 1|}{\sqrt{2}}$.

$$\frac{|h - 1|}{\sqrt{2}} = |h| \Rightarrow (h - 1)^2 = 2h^2 \Rightarrow h^2 + 2h - 1 = 0$$

The discriminant is $4 + 4 = 8 > 0$, so there are two real values $h = -1 \pm \sqrt{2}$. That gives 2 circles.

Step 4: Case 2: centre on $x = 1$:

Let the centre be $(1, k)$. Radius squared $= 1 + k^2$. Distance to $x + y = 1$ is $\frac{|k|}{\sqrt{2}}$.

$$\frac{k^2}{2} = 1 + k^2 \Rightarrow k^2 = -2$$

This has no real solution. So no circle exists here.

Final Answer:

Only case 1 gives circles, and it gives 2, so the answer is option (B).

2 (B)

Quick Tip: The centre lies on an angle bisector of the two lines. Check each bisector separately.

107. A parabola has its focus on the positive X-axis and the Y-axis as its directrix. If $P(\alpha, 4)$ is a point on this parabola such that the tangent to the parabola at point P passes through the origin, then the distance of P from origin is

- (A) 4
- (B) $\sqrt{20}$
- (C) 5
- (D) $\sqrt{32}$

Correct Answer: (D) $\sqrt{32}$

Solution:

Step 1: Understanding the Concept:

A parabola with focus $(a, 0)$ on the positive X-axis and directrix the Y-axis (the line $x = 0$) has its vertex midway at $(\frac{a}{2}, 0)$, and its axis is the X-axis.

Step 2: Key Formula or Approach:

A point is on the parabola if its distance from the focus equals its distance from the directrix:

$$\sqrt{(x - a)^2 + y^2} = x \Rightarrow y^2 = 2ax - a^2$$

The tangent at (x_1, y_1) is $yy_1 = a(x + x_1) - a^2$.

Step 3: Detailed Explanation:

The tangent at $P(\alpha, 4)$ passes through the origin. Substitute $(0, 0)$:

$$0 = a(\alpha) - a^2 \Rightarrow \alpha = a \quad (a \neq 0)$$

Now P lies on the parabola:

$$16 = 2a\alpha - a^2 = 2a^2 - a^2 = a^2 \Rightarrow a = 4$$

So $\alpha = 4$ and $P = (4, 4)$.

Distance from the origin:

$$OP = \sqrt{4^2 + 4^2} = \sqrt{32}$$

Option (A) 4 and (C) 5 are not distances to the point $(4, 4)$, and option

(B) $\sqrt{20}$ would correspond to (2, 4).

Final Answer:

The distance of P from the origin is $\sqrt{32}$, option (D).

$$\boxed{\sqrt{32} \text{ (D)}}$$

Quick Tip: Build the parabola with the y-axis as directrix, then use the tangent-through-origin condition.

108. If $\lim_{x \rightarrow k} \frac{x^3 - k^3}{x^2 - k^2} = \lim_{x \rightarrow 0} \frac{1 - \cos(2x)}{x \sin x}$, then the value of k is ...

- (A) $\frac{4}{3}$
- (B) $\frac{3}{4}$
- (C) $\frac{8}{3}$
- (D) $\frac{3}{8}$

Correct Answer: (A) $\frac{4}{3}$

Solution:

Step 1: Understanding the Concept:

We evaluate both limits, equate them, and solve for k .

Step 2: Left limit:

$$\lim_{x \rightarrow k} \frac{x^3 - k^3}{x^2 - k^2} = \lim_{x \rightarrow k} \frac{(x - k)(x^2 + xk + k^2)}{(x - k)(x + k)} = \frac{3k^2}{2k} = \frac{3k}{2}$$

Step 3: Right limit:

Use $1 - \cos 2x = 2 \sin^2 x$:

$$\lim_{x \rightarrow 0} \frac{1 - \cos 2x}{x \sin x} = \lim_{x \rightarrow 0} \frac{2 \sin^2 x}{x \sin x} = \lim_{x \rightarrow 0} \frac{2 \sin x}{x} = 2$$

Step 4: Equate and solve:

$$\frac{3k}{2} = 2 \Rightarrow k = \frac{4}{3}$$

The value $\frac{3}{4}$ in option (B) arises from inverting the equation. Options (C) and (D) come from the wrong value of the right-hand limit.

Final Answer:

The value of k is $\frac{4}{3}$, option (A).

$$\boxed{\frac{4}{3} \text{ (A)}}$$

Quick Tip: Evaluate each limit separately: the left one gives $3k/2$ and the right one gives 2.

109. In a switching circuit, if the combination $(S_1 \wedge S_2)$ is connected in parallel to the combination **Missing open brace for superscript**, then the room is lit only when ...

- (A) S_1 is ON and S_2 is OFF
- (B) S_1 is OFF and S_2 is ON
- (C) S_1 and S_2 both ON or S_1 and S_2 both OFF
- (D) The room is always lit.

Correct Answer: (C) S_1 and S_2 both ON or S_1 and S_2 both OFF

Solution:

Step 1: Understanding the Concept:

Switches in series correspond to $\wedge$ (AND) and switches in parallel correspond to $\vee$ (OR). The room is lit when the circuit has at least one closed path.

Step 2: Key Formula or Approach:

The circuit expression is $(S_1 \wedge S_2) \vee (S'_1 \wedge S'_2)$.

S'_1 is ON exactly when S_1 is OFF.

Step 3: Detailed Explanation:

The first branch conducts when both S_1 and S_2 are ON.

The second branch conducts when S'_1 and S'_2 are both ON, which means both S_1 and S_2 are OFF.

So the lamp is lit when both are ON or both are OFF. This is the biconditional $S_1 \leftrightarrow S_2$.

If S_1 is ON and S_2 is OFF (option A), the first branch is broken at S_2 and the second branch is broken at S'_1 , so no path exists. The same holds for option (B). The room is not always lit, so (D) is wrong.

Final Answer:

The room is lit when both switches are ON or both are OFF, option (C).

Both ON or both OFF (C)

Quick Tip: (S1 and S2) in parallel with (S1' and S2') means S1 and S2 are in the same state.



110. If the truth value of the statement pattern $(\sim p \wedge q) \vee (\sim p \wedge \sim q) \vee (p \wedge \sim q)$ is F , then the truth values of $(p \vee \sim q)$ and $(p \rightarrow q)$ are ... respectively.

- (A) F, F
- (B) F, T
- (C) T, T
- (D) T, F

Correct Answer: (C) T, T

Solution:

Step 1: Understanding the Concept:

We simplify the compound statement and use the fact that it is false to find the truth values of p and q .

Step 2: Key Formula or Approach:

Combine the terms with common factors using the distributive law.

Step 3: Detailed Explanation:

$$(\sim p \wedge q) \vee (\sim p \wedge \sim q) = \sim p \wedge (q \vee \sim q) = \sim p \wedge T = \sim p$$

So the pattern becomes $\sim p \vee (p \wedge \sim q)$.

$$\sim p \vee (p \wedge \sim q) = (\sim p \vee p) \wedge (\sim p \vee \sim q) = T \wedge (\sim p \vee \sim q) = \sim p \vee \sim q = \sim (p \wedge q)$$

The pattern is false only when $p \wedge q$ is true, that is $p = T$ and $q = T$.

Now evaluate:

$$p \vee \sim q = T \vee F = T.$$

$$p \rightarrow q = T \rightarrow T = T.$$

So the two truth values are T and T. Options (A), (B) and (D) each contain an F, which cannot happen here.

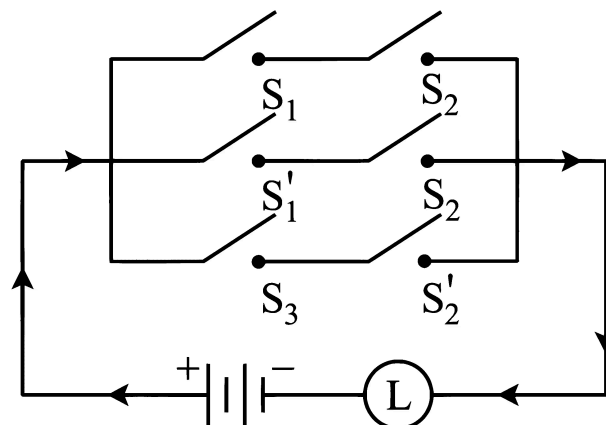
Final Answer:

The truth values are T, T, option (C).

$T, T (C)$

Quick Tip: Simplify the pattern: it equals $\sim(p \text{ and } q)$. It is false only when p and q are both true.

111. The minimum number of switches in the simplified form of the following switching circuit is



- (A) 0
- (B) 1
- (C) 2
- (D) 3

Correct Answer: (C) 2

Solution:

Step 1: Understanding the Concept:

In the figure, three branches are connected in parallel. The top branch has S_1 and S_2 in series. The middle branch has S'_1 and S_2 in series. The bottom branch has S_3 and S'_2 in series.

Step 2: Key Formula or Approach:

Series is AND and parallel is OR, so the circuit is

$$(S_1 \wedge S_2) \vee (S'_1 \wedge S_2) \vee (S_3 \wedge S'_2)$$

Step 3: Detailed Explanation:

Combine the first two branches by taking S_2 as a common factor:

$$(S_1 \wedge S_2) \vee (S'_1 \wedge S_2) = (S_1 \vee S'_1) \wedge S_2 = T \wedge S_2 = S_2$$

The circuit is now $S_2 \vee (S_3 \wedge S'_2)$. By the distributive law:

$$S_2 \vee (S_3 \wedge S'_2) = (S_2 \vee S_3) \wedge (S_2 \vee S'_2) = (S_2 \vee S_3) \wedge T = S_2 \vee S_3$$

So the circuit reduces to $S_2 \vee S_3$, which is two switches in parallel. We cannot remove either one, since the lamp needs to turn on when

either is closed.

Option (A) 0 and (B) 1 would give a circuit that is always on or tied to one switch, which is not what the expression says. Option (D) 3 is the unsimplified form.

Final Answer:

The minimum number of switches is 2, option (C).

2 (C)

Quick Tip: Read the circuit as three parallel branches and simplify with Boolean algebra.

112. In $\triangle ABC$, with usual notations, if Δ denotes the area of triangle ABC then the value of $2s(b + c - a)\tan(\frac{A}{2})$ is equal to ...

- (A) Δ
- (B) 2Δ
- (C) 3Δ
- (D) 4Δ

Correct Answer: (D) 4Δ

Solution:

Step 1: Understanding the Concept:

In a triangle with semi-perimeter $s = \frac{a + b + c}{2}$, the half-angle formula for the tangent connects $\tan \frac{A}{2}$ with the sides and the area Δ .

Step 2: Key Formula or Approach:

$$1. \tan \frac{A}{2} = \frac{\Delta}{s(s-a)}.$$

$$2. b + c - a = 2s - 2a = 2(s - a).$$

Step 3: Detailed Explanation:

Substitute $b + c - a = 2(s - a)$:

$$2s(b + c - a) \tan \frac{A}{2} = 2s \cdot 2(s - a) \cdot \tan \frac{A}{2} = 4s(s - a) \tan \frac{A}{2}$$

Now use the half-angle formula:

$$4s(s - a) \cdot \frac{\Delta}{s(s - a)} = 4\Delta$$

The factors $s(s - a)$ cancel, leaving 4 times the area. Options (A), (B), (C) give Δ , 2Δ and 3Δ , which would arise from missing the factor 2 in $b + c - a$ or in the leading $2s$.

Final Answer:

The value is 4Δ , option (D).

$$\boxed{4\Delta \text{ (D)}}$$

Quick Tip: Use $\tan(A/2) = \Delta / (s(s-a))$ and $b+c-a = 2(s-a)$.

113. If matrix A and its inverse A^{-1} are given by $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & x & 1 \end{bmatrix}$ and A^{-1}

$$= \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} \\ -4 & 3 & y \\ \frac{5}{2} & -\frac{3}{2} & \frac{1}{2} \end{bmatrix}, \text{ then the polar co-ordinates of the points whose}$$

Cartesian co-ordinates are (x, y) are ...

(A) $(2, \frac{7\pi}{4})$

(B) $(\sqrt{2}, \frac{\pi}{4})$

(C) $(\sqrt{2}, \frac{7\pi}{4})$

(D) $(2, \frac{\pi}{4})$

Correct Answer: (C) $(\sqrt{2}, \frac{7\pi}{4})$

Solution:

Step 1: Understanding the Concept:

Since A^{-1} is the inverse of A , their product is the identity matrix.

Comparing particular entries of $AA^{-1} = I$ gives x and y .

Step 2: Key Formula or Approach:

Row i of A times column j of A^{-1} equals 1 if $i = j$ and 0 otherwise.

Step 3: Detailed Explanation:

Row 3 of A is $(3, x, 1)$ and column 1 of A^{-1} is $(\frac{1}{2}, -4, \frac{5}{2})$. Their product must be 0:

$$\frac{3}{2} - 4x + \frac{5}{2} = 0 \Rightarrow 4 - 4x = 0 \Rightarrow x = 1$$

Row 2 of A is $(1, 2, 3)$ and column 3 of A^{-1} is $(\frac{1}{2}, y, \frac{1}{2})$. Their product must be 0:

$$\frac{1}{2} + 2y + \frac{3}{2} = 0 \Rightarrow 2 + 2y = 0 \Rightarrow y = -1$$

The Cartesian point is $(1, -1)$.

Polar form: $r = \sqrt{1^2 + (-1)^2} = \sqrt{2}$. The point lies in the fourth quadrant and $\tan \theta = -1$, so $\theta = \frac{7\pi}{4}$.

Option (B) has the same radius but $\theta = \frac{\pi}{4}$, which is in the first quadrant and would be the point $(1, 1)$.

Final Answer:

The polar coordinates are $(\sqrt{2}, \frac{7\pi}{4})$, option (C).

$$\boxed{(\sqrt{2}, \frac{7\pi}{4}) \text{ (C)}}$$

Quick Tip: Use $A \text{ times } A \text{ inverse equals } I$ to find x and y , then convert (x, y) into polar form.



114. Let $A = \begin{bmatrix} -3 & 2 \\ 1 & 4 \end{bmatrix}$ and if $A^2 - 2A + I = \begin{bmatrix} 18 & p \\ q & 11 \end{bmatrix}$, then ...

- (A) $p = -2, q = -1$
- (B) $p = 2, q = 1$
- (C) $p = -1, q = -2$
- (D) $p = 1, q = 2$

Correct Answer: (A) $p = -2, q = -1$

Solution:

Step 1: Understanding the Concept:

We evaluate the matrix expression $A^2 - 2A + I$ entry by entry and compare it with the given matrix.

Step 2: Detailed Explanation:

$$A^2 = \begin{pmatrix} -3 & 2 \\ 1 & 4 \end{pmatrix} \begin{pmatrix} -3 & 2 \\ 1 & 4 \end{pmatrix} = \begin{pmatrix} 9+2 & -6+8 \\ -3+4 & 2+16 \end{pmatrix} = \begin{pmatrix} 11 & 2 \\ 1 & 18 \end{pmatrix}$$

$$2A = \begin{pmatrix} -6 & 4 \\ 2 & 8 \end{pmatrix}, \quad I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$A^2 - 2A + I = \begin{pmatrix} 11+6+1 & 2-4+0 \\ 1-2+0 & 18-8+1 \end{pmatrix} = \begin{pmatrix} 18 & -2 \\ -1 & 11 \end{pmatrix}$$

Step 3: Compare with the given matrix:

The diagonal entries 18 and 11 match, which confirms the calculation. The off-diagonal entries give $p = -2$ and $q = -1$.

Options (B) and (D) have the wrong signs for both entries. Option (C) swaps the values of p and q .

Final Answer:

$p = -2$ and $q = -1$, option (A).

$$\boxed{p = -2, q = -1 \text{ (A)}}$$

Quick Tip: Compute A squared, subtract $2A$ and add the identity, then compare entries.

115. For $x > 0$ and $(x \log x) < 1$, if $y = \cot^{-1}\left(\frac{x - \log x^{x^2}}{\log e^{x^2} + \log x^x}\right)$, then $\frac{dy}{dx} = \dots$

(A) $\frac{1}{1+x^2} + \frac{2}{x[1+(\log x)^2]}$

(B) $\frac{1}{1+x^2} + \frac{1}{x[1+(\log x)^2]}$

(C) $\frac{-1}{1+x^2} + \frac{1}{x[1+(\log x)^2]}$

(D) $\frac{1}{1+x^2} + \frac{1}{[1+(\log x)^2]}$

Correct Answer: (B) $\frac{1}{1+x^2} + \frac{1}{x[1+(\log x)^2]}$

Solution:

Step 1: Understanding the Concept:

We first simplify the fraction inside $\cot^{-1}$ using logarithm rules, then convert it into a sum of two inverse tangents, which are easy to differentiate.

Step 2: Key Formula or Approach:

1. $\log x^{x^2} = x^2 \log x$, $\log e^{x^2} = x^2$, $\log x^x = x \log x$.

2. $\cot^{-1} u = \tan^{-1} \frac{1}{u}$ and $\tan^{-1} a + \tan^{-1} b = \tan^{-1} \frac{a+b}{1-ab}$ when $ab < 1$.

Step 3: Detailed Explanation:

$$\frac{x - x^2 \log x}{x^2 + x \log x} = \frac{x(1 - x \log x)}{x(x + \log x)} = \frac{1 - x \log x}{x + \log x}$$

So

$$y = \cot^{-1} \frac{1 - x \log x}{x + \log x} = \tan^{-1} \frac{x + \log x}{1 - x \log x}$$

With $a = x$, $b = \log x$, and the condition $x \log x < 1$, this is

$$y = \tan^{-1} x + \tan^{-1}(\log x)$$

Differentiate:

$$\frac{dy}{dx} = \frac{1}{1+x^2} + \frac{1}{1+(\log x)^2} \cdot \frac{1}{x} = \frac{1}{1+x^2} + \frac{1}{x[1+(\log x)^2]}$$

Option (A) has an extra factor 2 on the second term, option (C) has a wrong sign on the first term, and option (D) forgets the $1/x$ from the chain rule.

Final Answer:

$$\frac{dy}{dx} = \frac{1}{1+x^2} + \frac{1}{x[1+(\log x)^2]}, \text{ option (B).}$$

$$\boxed{\frac{1}{1+x^2} + \frac{1}{x[1+(\log x)^2]} \text{ (B)}}$$

Quick Tip: Simplify the logarithms first. The argument becomes $(1 - x \log x)/(x + \log x)$, so $y = \tan^{-1} x + \tan^{-1}(\log x)$.

116. For $x > 0$, if $\sin(\cos^{-1} x + \tan^{-1} x) - \cos(\sin^{-1} x + \tan^{-1} x) = \sin(\cot^{-1} 2)$ then $x =$

- (A) $\frac{1}{\sqrt{2}}$
 (B) $\frac{1}{2}$
 (C) $\frac{\sqrt{3}}{2}$
 (D) 1

Correct Answer: (B) $\frac{1}{2}$

Solution:

Step 1: Understanding the Concept:

Rewrite $\cos^{-1} x = \frac{\pi}{2} - \sin^{-1} x$ so that both terms use $a = \sin^{-1} x$ and $b = \tan^{-1} x$.

Step 2: Key Formula or Approach:

$\cos A - \cos B = -2 \sin \frac{A+B}{2} \sin \frac{A-B}{2}$, or directly $\cos(a - b) - \cos(a + b) = 2 \sin a \sin b$.

Step 3: Detailed Explanation:

$$\sin(\cos^{-1} x + \tan^{-1} x) = \sin\left(\frac{\pi}{2} - a + b\right) = \cos(b - a) = \cos(a - b)$$

So the left side is

$$\cos(a - b) - \cos(a + b) = 2 \sin a \sin b$$

Now $\sin a = x$, and $\sin b = \sin(\tan^{-1} x) = \frac{x}{\sqrt{1+x^2}}$, so

$$2 \sin a \sin b = \frac{2x^2}{\sqrt{1+x^2}}$$

The right side: $\cot^{-1} 2$ is an angle with $\cot = 2$, so $\sin = \frac{1}{\sqrt{5}}$.

Solve:

$$\frac{2x^2}{\sqrt{1+x^2}} = \frac{1}{\sqrt{5}}$$

Test $x = \frac{1}{2}$: the left side is $\frac{2 \cdot \frac{1}{4}}{\sqrt{5/4}} = \frac{1/2}{\sqrt{5}/2} = \frac{1}{\sqrt{5}}$, which matches. The

left side is increasing for $x > 0$, so this is the only positive root.

Options (A), (C) and (D) give larger left-side values (for example $x = 1$ gives $2/\sqrt{2} = \sqrt{2}$).

Final Answer:

$x = \frac{1}{2}$, option (B).

$\frac{1}{2} \text{ (B)}$

Quick Tip: Put $a = \arcsin x$ and $b = \arctan x$. The left side becomes $\cos(a-b) - \cos(a+b) = 2 \sin a \sin b$.

117. If $\tan^{-1} ax + \tan^{-1} 3x = \frac{\pi}{4}$, where $3ax^2 < 1$, then value of a for $x = \frac{1}{6}$ is ...

- (A) 2
- (B) 3
- (C) 4
- (D) 9

Correct Answer: (A) 2

Solution:

Step 1: Understanding the Concept:

The sum of two inverse tangents can be combined into one when the product of the arguments is less than 1, which the condition $3ax^2 < 1$ guarantees.

Step 2: Key Formula or Approach:

$$\tan^{-1} A + \tan^{-1} B = \tan^{-1} \frac{A + B}{1 - AB}, \quad AB < 1$$

Step 3: Detailed Explanation:

Here $A = ax$ and $B = 3x$:

$$\tan^{-1} \frac{ax + 3x}{1 - 3ax^2} = \frac{\pi}{4}$$

Take tangent on both sides, using $\tan \frac{\pi}{4} = 1$:

$$\frac{x(a + 3)}{1 - 3ax^2} = 1$$

Substitute $x = \frac{1}{6}$, so $x^2 = \frac{1}{36}$:

$$\frac{(a + 3)/6}{1 - a/12} = 1 \Rightarrow \frac{a + 3}{6} = 1 - \frac{a}{12}$$

Multiply by 12:

$$2(a + 3) = 12 - a \Rightarrow 3a = 6 \Rightarrow a = 2$$

Check: $\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{2} = \tan^{-1} \frac{5/6}{5/6} = \tan^{-1} 1 = \frac{\pi}{4}$. It holds.

Final Answer:

$a = 2$, option (A).

2 (A)

Quick Tip: Use $\tan^{-1} A + \tan^{-1} B = \tan^{-1}((A+B)/(1-AB))$ and put $x = 1/6$.



118. $\sec^2(\tan^{-1} 3) - \tan^2(\sec^{-1} 3) =$

- (A) 0
- (B) 1
- (C) 2
- (D) 3

Correct Answer: (C) 2

Solution:

Step 1: Understanding the Concept:

Both terms are squares of trigonometric functions of an inverse function. We can use the identity $\sec^2 \theta = 1 + \tan^2 \theta$.

Step 2: Key Formula or Approach:

1. If $\theta = \tan^{-1} 3$, then $\tan \theta = 3$.
2. If $\phi = \sec^{-1} 3$, then $\sec \phi = 3$.

Step 3: Detailed Explanation:

First term:

$$\sec^2(\tan^{-1} 3) = 1 + \tan^2(\tan^{-1} 3) = 1 + 9 = 10$$

Second term:

$$\tan^2(\sec^{-1} 3) = \sec^2(\sec^{-1} 3) - 1 = 9 - 1 = 8$$

Subtract:

$$10 - 8 = 2$$

Options (A), (B) and (D) come from slips such as using $\tan^2 = \sec^2$ or forgetting to square the 3.

Final Answer:

The value of the expression is 2, option (C).

$$\boxed{2 \text{ (C)}}$$

Quick Tip: Use $\sec^2 \theta = 1 + \tan^2 \theta$, so each term can be found directly.

119. The domain of the function $f(x) = \sqrt{\frac{x}{1+x}}$ is ...

- (A) $(-\infty, -1) \cup [0, \infty)$
- (B) $(-\infty, -1] \cup [0, \infty)$
- (C) $(-\infty, -1) \cap [0, \infty)$
- (D) all $\mathbb{R}$

Correct Answer: (A) $(-\infty, -1) \cup [0, \infty)$

Solution:

Step 1: Understanding the Concept:

The square root is defined when its argument is not negative, and the fraction needs a nonzero denominator. So we need $\frac{x}{1+x} \geq 0$ with $x \neq -1$.

Step 2: Key Formula or Approach:

A fraction is non-negative when the numerator and the denominator have the same sign, or the numerator is zero. The critical points are $x = 0$ (zero of numerator) and $x = -1$ (zero of denominator).

Step 3: Detailed Explanation:

Check each interval:

For $x < -1$: numerator negative, denominator negative, so the fraction is positive. Allowed.

For $-1 < x < 0$: numerator negative, denominator positive, so the fraction is negative. Not allowed.

For $x = 0$: the fraction is 0, and $\sqrt{0}$ is defined. Allowed.

For $x > 0$: both positive, so the fraction is positive. Allowed.

At $x = -1$ the function is undefined, so -1 is excluded and 0 is included:

$$(-\infty, -1) \cup [0, \infty)$$

Option (B) wrongly includes -1 . Option (C) uses an intersection, which is empty. Option (D) ignores the restriction.

Final Answer:

The domain is $(-\infty, -1) \cup [0, \infty)$, option (A).

$$\boxed{(-\infty, -1) \cup [0, \infty) \text{ (A)}}$$

Quick Tip: Need $x/(1+x) \geq 0$ and $x \neq -1$. Use the sign chart.

120. If $f(x) = \frac{3^x + 3^{-x} - 2}{\tan x \cdot \log(1+x)}$ for $x \neq 0$, is continuous at $x = 0$, then the value of $f(0)$ is equal to ...

- (A) $2\log 3$
- (B) $(\log 3)^2$
- (C) $\frac{1}{2}\log 3$
- (D) $\log \frac{1}{3}$

Correct Answer: (B) $(\log 3)^2$

Solution:

Step 1: Understanding the Concept:

For f to be continuous at 0, we need $f(0) = \lim_{x \rightarrow 0} f(x)$. So we evaluate the limit.

Step 2: Key Formula or Approach:

1. $3^x + 3^{-x} - 2 = (3^{x/2} - 3^{-x/2})^2$.
2. $\lim_{x \rightarrow 0} \frac{a^x - 1}{x} = \log a$, $\lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = 1$, $\lim_{x \rightarrow 0} \frac{\tan x}{x} = 1$.

Step 3: Detailed Explanation:

Rewrite $3^x + 3^{-x} - 2 = 3^{-x}(3^x - 1)^2$, since

$$3^{-x}(3^{2x} - 2 \cdot 3^x + 1) = 3^x - 2 + 3^{-x}$$

So

$$f(x) = \frac{3^{-x}(3^x - 1)^2}{\tan x \cdot \log(1 + x)} = 3^{-x} \cdot \left(\frac{3^x - 1}{x}\right)^2 \cdot \frac{x}{\tan x} \cdot \frac{x}{\log(1 + x)}$$

Take the limit as $x \rightarrow 0$:

$$3^0 \cdot (\log 3)^2 \cdot 1 \cdot 1 = (\log 3)^2$$

So $f(0) = (\log 3)^2$.

Option (A) $2 \log 3$ and (C) $\frac{1}{2} \log 3$ would appear if the numerator were a first power in x . Option (D) is a logarithm of $\frac{1}{3}$, which is negative and cannot match a square.

Final Answer:

$f(0) = (\log 3)^2$, option (B).

$(\log 3)^2 \text{ (B)}$

Quick Tip: Write the numerator as a perfect square and use the standard limits for $(a^x - 1)/x$ and $\log(1+x)/x$.

121. If **Missing open brace for superscript**, where k is constant of integration, then $f(x) = \dots$

- (A) $2x^3 - \frac{x^2}{2} + x + c$, where c is constant of integration.
(B) $\frac{x^3}{2} + 3x^2 + 4x + c$, where c is constant of integration.
(C) $x^3 + 4x^2 + 6x + c$, where c is constant of integration.
(D) $\frac{2x^3}{3} - x^2 + x + c$, where c is constant of integration.

Correct Answer: (D) $\frac{2x^3}{3} - x^2 + x + c$, where c is constant of integration.

Solution:

Step 1: Understanding the Concept:

If an integral equals a given function plus a constant, differentiating the equation returns the integrand.

Step 2: Key Formula or Approach:

If $\int g(x) dx = G(x) + k$, then $g(x) = G'(x)$. Here $G(x) = (x - 1)e^{x^2}$.

Step 3: Detailed Explanation:

Differentiate G using the product rule:

$$G'(x) = e^{x^2} + (x - 1) \cdot 2x e^{x^2} = e^{x^2} (1 + 2x^2 - 2x)$$

The integrand is $f'(x)e^{x^2}$, so

$$f'(x) = 2x^2 - 2x + 1$$

Integrate to get $f(x)$:

$$f(x) = \frac{2x^3}{3} - x^2 + x + c$$

This is exactly option (D). Option (A) has the wrong coefficients $2x^3$ and $\frac{x^2}{2}$. Options (B) and (C) have positive x^2 terms that do not come from $-2x$.

Final Answer:

$$f(x) = \frac{2x^3}{3} - x^2 + x + c, \text{ option (D).}$$

$$\boxed{\frac{2x^3}{3} - x^2 + x + c \text{ (D)}}$$

Quick Tip: Differentiate both sides to get $f'(x)e^{(x^2)}$, then integrate $f'(x)$.



122. Let $y = \sqrt[p]{x^3y}$. If $\frac{dy}{dx} = \frac{3}{2}$ when $y = 1$, then the value of p is equal to ...

- (A) 1
- (B) 2
- (C) 3
- (D) 4

Correct Answer: (C) 3

Solution:

Step 1: Understanding the Concept:

The relation $y = (x^3y)^{1/p}$ can be cleared of the root by raising to the power p .

Step 2: Key Formula or Approach:

$$y^p = x^3 y \Rightarrow y^{p-1} = x^3 \quad (y \neq 0)$$

Differentiate with respect to x using the chain rule.

Step 3: Detailed Explanation:

$$(p-1)y^{p-2} \frac{dy}{dx} = 3x^2$$

When $y = 1$, the relation $y^{p-1} = x^3$ gives $x^3 = 1$, so $x = 1$.

Substitute $y = 1$, $x = 1$ and $\frac{dy}{dx} = \frac{3}{2}$:

$$(p-1) \cdot 1 \cdot \frac{3}{2} = 3 \Rightarrow p-1 = 2 \Rightarrow p = 3$$

Check with $p = 3$: $y^2 = x^3$, so $2yy' = 3x^2$ and at $(1, 1)$, $y' = \frac{3}{2}$. It holds.

Option (A) $p = 1$ would make the relation $1 = x^3$, which defines no curve. Options (B) and (D) give slopes 3 and 1 at $(1, 1)$.

Final Answer:

$p = 3$, option (C).

3 (C)

Quick Tip: Raise both sides to the p -th power to get $y^{p(p-1)} = x^{3p}$, then differentiate.

123. If Missing open brace for superscript and $y = f\left(\frac{2x-1}{x^2+1}\right)$, then $\frac{dy}{dx}$ at $x = 1$ is

- (A) $\frac{1}{4}\sin(\frac{1}{2})$
- (B) $\frac{1}{4}\sin^2(\frac{1}{2})$
- (C) $\sin^2(\frac{1}{4})$
- (D) $\frac{1}{2}\sin^2(\frac{1}{2})$

Correct Answer: (D) $\frac{1}{2}\sin^2(\frac{1}{2})$

Solution:

Step 1: Understanding the Concept:

Here $y = f(u)$ with $u = \frac{2x-1}{x^2+1}$, so the chain rule applies: $\frac{dy}{dx} = f'(u) \cdot \frac{du}{dx}$.

Step 2: Key Formula or Approach:

$f'(u) = \sin^2 u$. Differentiate u by the quotient rule.

Step 3: Detailed Explanation:

$$\frac{du}{dx} = \frac{2(x^2+1) - (2x-1)(2x)}{(x^2+1)^2} = \frac{-2x^2+2x+2}{(x^2+1)^2}$$

At $x = 1$:

$$u(1) = \frac{1}{2}, \quad \left. \frac{du}{dx} \right|_{x=1} = \frac{-2+2+2}{4} = \frac{1}{2}$$

So

$$\left. \frac{dy}{dx} \right|_{x=1} = \sin^2\left(\frac{1}{2}\right) \cdot \frac{1}{2} = \frac{1}{2}\sin^2\left(\frac{1}{2}\right)$$

Option (B) uses $\frac{1}{4}$, which comes from taking $u'(1) = \frac{1}{4}$. Option (A)

removes the square, and option (C) evaluates the argument at $\frac{1}{4}$, neither being $u(1)$.

Final Answer:

The derivative at $x = 1$ is $\frac{1}{2}\sin^2\left(\frac{1}{2}\right)$, option (D).

$$\boxed{\frac{1}{2}\sin^2\left(\frac{1}{2}\right) \text{ (D)}}$$

Quick Tip: Use the chain rule: $dy/dx = f'(u)$ times u' with $u = (2x-1)/(x^2+1)$.

124. If $f : \mathbb{R} \rightarrow \mathbb{R}$ is an even function then

- (A) Missing open brace for superscript
- (B) Missing open brace for superscript is an even function
- (C) $f(0) = 0$
- (D) Missing open brace for superscript is an odd function

Correct Answer: (D) Missing open brace for superscript is an odd function

Solution:

Step 1: Understanding the Concept:

A function is even when $f(-x) = f(x)$ for all x . Its graph is symmetric about the Y-axis. We find the symmetry of the derivative.

Step 2: Key Formula or Approach:

Differentiate both sides of $f(-x) = f(x)$ using the chain rule.

Step 3: Detailed Explanation:

Differentiate: $-f'(-x) = f'(x)$, so

$$f'(-x) = -f'(x)$$

This is the definition of an odd function, so f' is odd. Example: $f(x) = x^2$ is even and $f'(x) = 2x$ is odd.

Now check the other options.

(A) $f'(0) = 1$ is not necessary. For $f(x) = x^2$, $f'(0) = 0$.

(B) says f' is even, but we showed it is odd.

(C) $f(0) = 0$ is not required: $f(x) = x^2 + 1$ is even and $f(0) = 1$.

Final Answer:

The derivative of an even function is an odd function, option (D).

f' is odd (D)

Quick Tip: Differentiating $f(-x) = f(x)$ shows $f'(-x) = -f'(x)$, so f' is odd.

125. Let $g(x) = f(x) + f(1 - x)$ and $f''(x) < 0, 0 \leq x \leq 1$, then ...

- (A) $g(x)$ increases on $[\frac{1}{2}, 1]$ and $g(x)$ decreases on $[0, \frac{1}{2}]$
- (B) $g(x)$ decreases on $[0, 1]$
- (C) $g(x)$ increases on $[0, 1]$
- (D) $g(x)$ decreases on $[\frac{1}{2}, 1]$ and $g(x)$ increases on $[0, \frac{1}{2}]$

Correct Answer: (D) $g(x)$ decreases on $[\frac{1}{2}, 1]$ and $g(x)$ increases on $[0, \frac{1}{2}]$

Solution:

Step 1: Understanding the Concept:

The sign of $g'(x)$ tells us where g increases or decreases. Since $f'' < 0$, the derivative f' is a decreasing function on $[0, 1]$.

Step 2: Key Formula or Approach:

$$g'(x) = f'(x) - f'(1 - x)$$

Because f' is decreasing, a smaller argument gives a larger value of f' .

Step 3: Detailed Explanation:

Case $0 < x < \frac{1}{2}$: here $x < 1 - x$, so $f'(x) > f'(1 - x)$. Then $g'(x) > 0$ and g increases on $[0, \frac{1}{2}]$.

Case $\frac{1}{2} < x < 1$: here $x > 1 - x$, so $f'(x) < f'(1 - x)$. Then $g'(x) < 0$ and g decreases on $[\frac{1}{2}, 1]$.

At $x = \frac{1}{2}$, $g'(x) = 0$, so g has its maximum there.

This matches option (D). Option (A) has the two behaviours swapped, and options (B) and (C) claim one monotone behaviour on the whole interval, which cannot be since g' changes sign at $\frac{1}{2}$.

Final Answer:

g increases on $[0, \frac{1}{2}]$ and decreases on $[\frac{1}{2}, 1]$, option (D).

(D)

Quick Tip: $g'(x) = f'(x) - f'(1-x)$. With f' negative, f' is decreasing, so compare x with $1-x$.

126. If the function $f(x) = ax^2 + bx + \sin x$ satisfies all the conditions of Rolle's theorem on $[0, \pi]$ and the slope of the tangent to the curve $y = f(x)$ at $x = \frac{\pi}{4}$ is zero, then $a - b =$

- (A) $\frac{\sqrt{2}(1-\pi)}{\pi}$
- (B) $\frac{\sqrt{2}(2+\pi)}{\pi}$
- (C) $\frac{\sqrt{2}(\pi-1)}{\pi}$
- (D) $\frac{\sqrt{2}(\pi+1)}{\pi}$

Correct Answer: (D) $\frac{\sqrt{2}(\pi+1)}{\pi}$

Solution:

Step 1: Understanding the Concept:

For Rolle's theorem on $[0, \pi]$, the function must be continuous, differentiable and satisfy $f(0) = f(\pi)$. The polynomial plus sine is smooth, so only the endpoint condition gives an equation.

Step 2: Key Formula or Approach:

1. $f(0) = f(\pi)$.
2. $f'(x) = 2ax + b + \cos x$, and the slope at $x = \frac{\pi}{4}$ is zero.

Step 3: Detailed Explanation:

$f(0) = 0$ and $f(\pi) = a\pi^2 + b\pi + 0$. Setting them equal:

$$a\pi^2 + b\pi = 0 \Rightarrow b = -a\pi$$

Slope at $\frac{\pi}{4}$:

$$2a \cdot \frac{\pi}{4} + b + \cos \frac{\pi}{4} = 0 \Rightarrow \frac{a\pi}{2} + b + \frac{\sqrt{2}}{2} = 0$$

Substitute $b = -a\pi$:

$$\frac{a\pi}{2} - a\pi + \frac{\sqrt{2}}{2} = 0 \Rightarrow -\frac{a\pi}{2} = -\frac{\sqrt{2}}{2} \Rightarrow a = \frac{\sqrt{2}}{\pi}$$

Then $b = -\sqrt{2}$. So

$$a - b = \frac{\sqrt{2}}{\pi} + \sqrt{2} = \frac{\sqrt{2}(1 + \pi)}{\pi}$$

Option (A) and (C) have the factor $(1 - \pi)$ or $(\pi - 1)$, which come from a sign error in b . Option (B) has $2 + \pi$, which comes from using $\cos \frac{\pi}{4} = \sqrt{2}$ instead of $\frac{\sqrt{2}}{2}$.

Final Answer:

$$a - b = \frac{\sqrt{2}(\pi + 1)}{\pi}, \text{ option (D).}$$

$$\boxed{\frac{\sqrt{2}(\pi + 1)}{\pi} \text{ (D)}}$$

Quick Tip: Rolle's theorem needs $f(0) = f(\pi)$. Combine that with $f'(\pi/4) = 0$ to find a and b .

127. If a particle moves such that the displacement (s) is proportional to the square of the velocity (v), then its acceleration (a) is

- (A) proportional to s^2
- (B) proportional to $1/s$
- (C) proportional to $1/s^2$
- (D) a constant

Correct Answer: (D) a constant

Solution:

Step 1: Understanding the Concept:

Velocity is $v = \frac{ds}{dt}$ and acceleration is $a = \frac{dv}{dt}$. We are told s is proportional to v^2 .

Step 2: Key Formula or Approach:

Write $s = cv^2$ for a constant c , then differentiate both sides with respect to time.

Step 3: Detailed Explanation:

$$\frac{ds}{dt} = 2cv \frac{dv}{dt}$$

Replace $\frac{ds}{dt}$ by v and $\frac{dv}{dt}$ by a :

$$v = 2cva$$

For a moving particle $v \neq 0$, so we may cancel v :

$$a = \frac{1}{2c}$$

This is a constant, since c is a constant. So the acceleration does not

change with time or with displacement.

This matches uniformly accelerated motion, where $v^2 = 2as$. Options (A), (B) and (C) say that a depends on s , which would not be the case here.

Final Answer:

The acceleration is a constant, option (D).

a constant (D)

Quick Tip: Write $s = c v^2$, differentiate with respect to time and use $ds/dt = v$.

128. If $\int e^{x+\tan^{-1}x} \left(\frac{x^2+2}{\sec^2(\tan^{-1}x)} \right) dx = e^{f(x)} + c$, then ...

- (A) $f(x)$ is strictly decreasing on R .
- (B) $f(x)$ is strictly increasing on R^+ and strictly decreasing on R^- .
- (C) $f(x)$ is strictly increasing on R .
- (D) $f(x)$ is strictly decreasing on R^+ and strictly increasing on R^- .

Correct Answer: (C) $f(x)$ is strictly increasing on R .

Solution:

Step 1: Understanding the Concept:

We need the function $f(x)$ in the exponent of the result. If the integrand is the derivative of $e^{f(x)}$, then f is read off directly.

Step 2: Key Formula or Approach:

$$\sec^2(\tan^{-1} x) = 1 + \tan^2(\tan^{-1} x) = 1 + x^2. \text{ Also } \frac{d}{dx} (x + \tan^{-1} x) = 1$$

$$+ \frac{1}{1+x^2} = \frac{x^2+2}{1+x^2}.$$

Step 3: Detailed Explanation:

The integrand is

$$e^{x+\tan^{-1}x} \cdot \frac{x^2+2}{1+x^2} = e^{x+\tan^{-1}x} \cdot \frac{d}{dx} (x + \tan^{-1}x)$$

By the rule $\int e^{g(x)} g'(x) dx = e^{g(x)} + c$, the integral is $e^{x+\tan^{-1}x} + c$. So

$$f(x) = x + \tan^{-1}x$$

Its derivative is $f'(x) = \frac{x^2+2}{1+x^2}$, which is positive for every real x , since both numerator and denominator are positive.

So f is strictly increasing on all of $\mathbb{R}$. Options (A), (B) and (D) all claim decreasing behaviour somewhere, which contradicts $f' > 0$.

Final Answer:

$f(x)$ is strictly increasing on $\mathbb{R}$, option (C).

strictly increasing on $\mathbb{R}$ (C)

Quick Tip: Notice that the integrand is the derivative of $e^{(x + \tan^{-1}x)}$.
So $f(x) = x + \tan^{-1}x$.



129. Let $f(x) = x$, $f_1(x) = f(\log x)$, $f_2(x) = f_1(\log x)$, $f_3(x) = f_2(\log x)$, ... and so on. Then $\int \frac{1}{f(x)f_1(x)f_2(x)\dots f_{2026}(x)} dx = \dots$

- (A) $f_{2025}(x) + c$
(B) $2025f_{2025}(x) + c$
(C) $f_{2027}(x) + c$
(D) $2027f_{2027}(x) + c$

Correct Answer: (C) $f_{2027}(x) + c$

Solution:

Step 1: Understanding the Concept:

The functions are $f = x$, $f_1 = \log x$, $f_2 = \log \log x$, and so on. Each f_k is the logarithm applied k times to x .

Step 2: Key Formula or Approach:

Since $f_k(x) = \log f_{k-1}(x)$, the chain rule gives

$$f'_k(x) = \frac{f'_{k-1}(x)}{f_{k-1}(x)}$$

Step 3: Detailed Explanation:

Start with the first few derivatives:

$$f'_1 = \frac{1}{x} = \frac{1}{f}$$

$$f'_2 = \frac{f'_1}{f_1} = \frac{1}{f f_1}$$

$$f'_3 = \frac{f'_2}{f_2} = \frac{1}{f f_1 f_2}$$

By induction,

$$f'_k(x) = \frac{1}{f f_1 f_2 \cdots f_{k-1}}$$

The integrand has the product up to f_{2026} . That is the derivative of f_{2027} , with $k - 1 = 2026$, so $k = 2027$.

$$\int \frac{dx}{f f_1 \cdots f_{2026}} = f_{2027}(x) + c$$

Options (A) and (B) stop at f_{2025} , which is one step too early, and (D) has an extra multiplier 2027 that the derivative does not produce.

Final Answer:

The integral equals $f_{2027}(x) + c$, option (C).

$$\boxed{f_{2027}(x) + c \text{ (C)}}$$

Quick Tip: Differentiate $f_k(x) = \log \log \dots \log x$ and look for the pattern $1/(f f_1 \dots f_{k-1})$.

130. If $\int_0^2 x(2-x)^b dx = \frac{32}{7}$, where $b \in N$ then $b =$

- (A) 5
- (B) 6
- (C) 7
- (D) 8

Correct Answer: (B) 6

Solution:

Step 1: Understanding the Concept:

The integrand has the awkward factor $(2 - x)^b$. Using the property $\int_0^a f(x) dx = \int_0^a f(a - x) dx$ moves the power to x .

Step 2: Key Formula or Approach:

$$\int_0^2 x(2 - x)^b dx = \int_0^2 (2 - x) x^b dx$$

Step 3: Detailed Explanation:

Expand and integrate:

$$\int_0^2 (2x^b - x^{b+1}) dx = \left[\frac{2x^{b+1}}{b+1} - \frac{x^{b+2}}{b+2} \right]_0^2 = \frac{2^{b+2}}{b+1} - \frac{2^{b+2}}{b+2}$$

$$= 2^{b+2} \cdot \frac{1}{(b+1)(b+2)}$$

Set this equal to $\frac{32}{7}$. Try $b = 6$:

$$\frac{2^8}{7 \cdot 8} = \frac{256}{56} = \frac{32}{7}$$

It matches. For the other options: $b = 5$ gives $\frac{128}{42} = \frac{64}{21}$, $b = 7$ gives $\frac{512}{72} = \frac{64}{9}$, and $b = 8$ gives $\frac{1024}{90}$, none equal to $\frac{32}{7}$.

Final Answer:

$b = 6$, option (B).

6 (B)

Quick Tip: Use the property of replacing x by $2-x$, then integrate and test values of b .

131. $\int_0^2 |4x - 5| dx = \dots$

(A) $\frac{17}{4}$

(B) $\frac{18}{3}$

(C) $\frac{1}{25}$

(D) $\frac{13}{2}$

Correct Answer: (A) $\frac{17}{4}$

Solution:**Step 1: Understanding the Concept:**

The absolute value changes the expression at the point where $4x - 5 = 0$, that is $x = \frac{5}{4}$. This point lies inside $[0, 2]$, so we split the integral there.

Step 2: Key Formula or Approach:

$$|4x - 5| = \begin{cases} 5 - 4x, & x < \frac{5}{4} \\ 4x - 5, & x \geq \frac{5}{4} \end{cases}$$

Step 3: Detailed Explanation:

First part:

$$\int_0^{5/4} (5 - 4x) dx = [5x - 2x^2]_0^{5/4} = \frac{25}{4} - \frac{25}{8} = \frac{25}{8}$$

Second part:

$$\int_{5/4}^2 (4x - 5) dx = [2x^2 - 5x]_{5/4}^2 = (8 - 10) - \left(\frac{25}{8} - \frac{25}{4}\right) = -2 + \frac{25}{8} = \frac{9}{8}$$

Add:

$$\frac{25}{8} + \frac{9}{8} = \frac{34}{8} = \frac{17}{4}$$

Final Answer:

The integral equals $\frac{17}{4}$, option (A).

$$\boxed{\frac{17}{4} \text{ (A)}}$$

Quick Tip: Split the integral at $x = 5/4$, where $4x-5$ changes sign.

132. If the area bounded by $y = x^3 + ax$ (where $a > 0$), the x -axis and the lines $x = -2$ and $x = 1$ is $\frac{37}{4}$ square units, then ...

- (A) $a = 4$
- (B) $a = 2$
- (C) $a = 10$
- (D) $a = 20$

Correct Answer: (B) $a = 2$

Solution:

Step 1: Understanding the Concept:

Since $a > 0$, the function $y = x^3 + ax = x(x^2 + a)$ is negative for $x < 0$ and positive for $x > 0$. It crosses the axis only at $x = 0$. So area must be found in two pieces.

Step 2: Key Formula or Approach:

$$\text{Area} = \int_{-2}^0 |x^3 + ax| dx + \int_0^1 (x^3 + ax) dx$$

Step 3: Detailed Explanation:

Left piece, with the sign reversed:

$$-\int_{-2}^0 (x^3 + ax) dx = -\left[\frac{x^4}{4} + \frac{ax^2}{2} \right]_{-2}^0 = \frac{16}{4} + \frac{4a}{2} = 4 + 2a$$

Right piece:

$$\int_0^1 (x^3 + ax) dx = \frac{1}{4} + \frac{a}{2}$$

Total area:

$$4 + 2a + \frac{1}{4} + \frac{a}{2} = \frac{17}{4} + \frac{5a}{2}$$

Set this to $\frac{37}{4}$:

$$\frac{5a}{2} = \frac{37}{4} - \frac{17}{4} = 5 \Rightarrow a = 2$$

Option (A) $a = 4$ would give area $\frac{17}{4} + 10$, too large. Options (C) and (D) are even larger.

Final Answer:

$a = 2$, option (B).

$$a = 2 \text{ (B)}$$

Quick Tip: The curve is below the axis for $x < 0$ and above for $x > 0$. Split the area at $x = 0$.

133. The rate of disintegration of a radioactive element at any time is proportional to its mass at that time, where k ($k > 0$) is the constant of proportionality. The time during which an original mass of 1.5 gm will disintegrate to a mass of 0.5 gm is ...

- (A) $k \log 3$
- (B) $\frac{1}{k} \log 3$
- (C) $k \log 5$
- (D) $\frac{1}{k} \log 5$

Correct Answer: (B) $\frac{1}{k} \log 3$

Solution:

Step 1: Understanding the Concept:

The rate of disintegration is proportional to the mass present, and the mass decreases with time. So the differential equation has a negative sign.

Step 2: Key Formula or Approach:

$$\frac{dm}{dt} = -km \Rightarrow \frac{dm}{m} = -k dt \Rightarrow \log m = -kt + C$$

Step 3: Detailed Explanation:

At $t = 0$, $m = 1.5$ g, so $C = \log 1.5$. Then

$$\log \frac{m}{1.5} = -kt$$

We want the time when $m = 0.5$ g:

$$\log \frac{0.5}{1.5} = -kt \Rightarrow \log \frac{1}{3} = -kt \Rightarrow -\log 3 = -kt$$

$$t = \frac{1}{k} \log 3$$

Option (A) has k multiplied instead of divided. Options (C) and (D) involve $\log 5$, which would be correct for a drop to one fifth of the mass.

Final Answer:

The time required is $\frac{1}{k} \log 3$, option (B).

$$\boxed{\frac{1}{k} \log 3 \text{ (B)}}$$

Quick Tip: Solve $dm/dt = -km$ to get $m = m_0 e^{(-kt)}$, then set $m = m_0/3$.

134. The differential equation of the family of all parabolas whose axis is the y -axis is ...

- (A) $\frac{d^2y}{dx^2} + x \frac{dy}{dx} = 0$
- (B) $x \frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$
- (C) $\frac{d^2y}{dx^2} - x \frac{dy}{dx} = 0$
- (D) $x \frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$

Correct Answer: (B) $x \frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$

Solution:**Step 1: Understanding the Concept:**

A parabola whose axis is the Y -axis has its vertex on the Y -axis, so its equation is $x^2 = 4a(y - b)$. It has two arbitrary constants, a and b . So the differential equation has order 2.

Step 2: Key Formula or Approach:

Differentiate twice and eliminate a and b .

Step 3: Detailed Explanation:

Differentiate $x^2 = 4a(y - b)$ once:

$$2x = 4a \frac{dy}{dx} \Rightarrow x = 2a \frac{dy}{dx}$$

Differentiate again:

$$1 = 2a \frac{d^2y}{dx^2} \Rightarrow 2a = \frac{1}{y''}$$

Substitute $2a$ in the first result:

$$x = \frac{1}{y''} y' \Rightarrow x y'' = y'$$

$$x \frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$$

This is option (B). Options (A), (C) and (D) have a different relation between x , y' and y'' , and they do not follow from the family equation.

Final Answer:

The differential equation is $x \frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$, option (B).

$$\boxed{x \frac{d^2y}{dx^2} - \frac{dy}{dx} = 0 \text{ (B)}}$$

Quick Tip: Parabolas with the y-axis as axis are $x^2 = 4a(y - b)$. Two constants need two differentiations.

135. The integrating factor of the differential equation $x \frac{dy}{dx} + 2y = x^2 \log x$ is

- (A) x^3
- (B) x^2
- (C) $\log 2x$
- (D) $\log x^2$

Correct Answer: (B) x^2

Solution:

Step 1: Understanding the Concept:

A linear first order equation must be written in the standard form $\frac{dy}{dx} + Py = Q$. The integrating factor is $e^{\int P dx}$.

Step 2: Key Formula or Approach:

Divide the whole equation by x so that the coefficient of $\frac{dy}{dx}$ is 1.

Step 3: Detailed Explanation:

$$\frac{dy}{dx} + \frac{2}{x} y = x \log x$$

Here $P = \frac{2}{x}$. So

$$\int P dx = 2 \log x = \log x^2$$

$$\text{IF} = e^{\log x^2} = x^2$$

Option (A) x^3 would result if the equation was not divided by x and P was taken as 2 instead. Options (C) and (D) are not of the form $e^{\int P dx}$ for this equation.

Final Answer:

The integrating factor is x^2 , option (B).

$$x^2 \text{ (B)}$$

Quick Tip: Divide by x to get standard form $dy/dx + (2/x)y = x \log x$, then $IF = e^{(\text{integral of } 2/x)}$.

136. A spherical raindrop evaporates at a rate proportional to its surface area. The differential equation involving the rate of change of its radius r with time ' t ' is ... (where k is a positive constant)

- (A) $\frac{dr}{dt} + k = 0$
- (B) $\frac{dr}{dt} - k = 0$
- (C) $\frac{dr}{dt} + kr = 0$
- (D) $\frac{dr}{dt} - kr = 0$

Correct Answer: (A) $\frac{dr}{dt} + k = 0$

Solution:

Step 1: Understanding the Concept:

The raindrop is a sphere of radius r . It loses volume at a rate proportional to its surface area, so $\frac{dV}{dt}$ is negative.

Step 2: Key Formula or Approach:

$V = \frac{4}{3}\pi r^3$ and surface area $S = 4\pi r^2$.

Step 3: Detailed Explanation:

Rate of change of volume:

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

Evaporation rate is proportional to surface area, with a minus sign because volume decreases:

$$\frac{dV}{dt} = -k(4\pi r^2)$$

Equate:

$$4\pi r^2 \frac{dr}{dt} = -k 4\pi r^2 \Rightarrow \frac{dr}{dt} = -k$$

$$\frac{dr}{dt} + k = 0$$

The radius shrinks at a constant rate. Options (C) and (D) say that the rate depends on r , which is not the case since the r^2 factors cancel. Option (B) has the wrong sign because the drop gets smaller.

Final Answer:

The differential equation is $\frac{dr}{dt} + k = 0$, option (A).

$$\boxed{\frac{dr}{dt} + k = 0 \text{ (A)}}$$

Quick Tip: Volume loss rate is proportional to surface area. Differentiate $V = \frac{4}{3} \pi r^3$.

137. The order and degree of the differential equation $\sqrt{1 + \frac{1}{(\frac{dy}{dx})^2}} = (\frac{d^2y}{dx^2})^{\frac{3}{2}}$ are ...

- (A) Order 2, Degree 3
- (B) Order 2, Degree 2
- (C) Order 3, Degree 3
- (D) Order 3, Degree 2

Correct Answer: (A) Order 2, Degree 3

Solution:

Step 1: Understanding the Concept:

The order is the highest derivative present. The degree is the power of the highest order derivative after the equation has been made a polynomial in the derivatives.

Step 2: Key Formula or Approach:

Square both sides to remove the outer square root, then clear the fraction.

Step 3: Detailed Explanation:

Given

$$\sqrt{1 + \frac{1}{(y')^2}} = (y'')^{3/2}$$

Square both sides:

$$1 + \frac{1}{(y')^2} = (y'')^3$$

Multiply through by $(y')^2$:

$$(y')^2 + 1 = (y')^2 (y'')^3$$

Now every derivative has an integer power. The highest derivative is y'' , so the order is 2. Its power is 3, so the degree is 3.

Order 2 with degree 2 in option (B) and the order 3 options (C) and (D) do not match: there is no third derivative anywhere.

Final Answer:

The order is 2 and the degree is 3, option (A).

Order 2, Degree 3 (A)

Quick Tip: Remove the radicals and fractions so that all derivatives appear as polynomials, then read off the order and degree.



138. The equation of the plane passing through the points having position vectors $(\vec{a} + \vec{b})$, $(\vec{b} + \vec{c})$ and $(\vec{a} + \vec{c})$ is ...

- (A) $\vec{r} \cdot (\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}) = [\vec{a}\vec{b}\vec{c}]$
- (B) $\vec{r} \cdot (\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}) = 2[\vec{a}\vec{b}\vec{c}]$
- (C) $\vec{r} \cdot (\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{a} \times \vec{c}) = [\vec{a}\vec{b}\vec{c}]$
- (D) $\vec{r} \cdot (\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{a} \times \vec{c}) = 2[\vec{a}\vec{b}\vec{c}]$

Correct Answer: (B) $\vec{r} \cdot (\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}) = 2[\vec{a}\vec{b}\vec{c}]$

Solution:

Step 1: Understanding the Concept:

A plane through three points has a normal vector equal to the cross product of two vectors lying in it. The equation is $\vec{r} \cdot \vec{n} = \vec{p} \cdot \vec{n}$ for any point $\vec{p}$ on the plane.

Step 2: Key Formula or Approach:

Let the points be $P = \vec{a} + \vec{b}$, $Q = \vec{b} + \vec{c}$, $R = \vec{a} + \vec{c}$.

Vectors in the plane:

$$\vec{PQ} = \vec{c} - \vec{a}, \quad \vec{PR} = \vec{c} - \vec{b}$$

Step 3: Detailed Explanation:

Normal:

$$\begin{aligned} \vec{n} &= (\vec{c} - \vec{a}) \times (\vec{c} - \vec{b}) = \vec{c} \times \vec{c} - \vec{c} \times \vec{b} - \vec{a} \times \vec{c} + \vec{a} \times \vec{b} \\ &= \vec{b} \times \vec{c} + \vec{c} \times \vec{a} + \vec{a} \times \vec{b} \end{aligned}$$

Now evaluate $\vec{p} \cdot \vec{n}$ using $\vec{p} = \vec{a} + \vec{b}$. Any scalar triple product with a repeated vector is zero:

$$(\vec{a} + \vec{b}) \cdot (\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}) = \vec{a} \cdot (\vec{b} \times \vec{c}) + \vec{b} \cdot (\vec{c} \times \vec{a})$$

Both terms equal $[\vec{a} \vec{b} \vec{c}]$, so the total is $2[\vec{a} \vec{b} \vec{c}]$.

The plane is

$$\vec{r} \cdot (\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}) = 2[\vec{a} \vec{b} \vec{c}]$$

Option (A) forgets the factor 2. Options (C) and (D) use $\vec{a} \times \vec{c}$ in place of $\vec{c} \times \vec{a}$, which flips the sign of one term and gives a different normal.

Final Answer:

The plane is option (B).

$$\vec{r} \cdot (\vec{a} \times \vec{b} + \vec{b} \times \vec{c} + \vec{c} \times \vec{a}) = 2[\vec{a} \vec{b} \vec{c}] \text{ (B)}$$

Quick Tip: Find a normal using two in-plane vectors, then write $\vec{r} \cdot \vec{n} = (\vec{a} + \vec{b}) \cdot \vec{n}$.



139. If the volume of the tetrahedron whose coterminous edges are given by the vectors $\vec{a} = -2\hat{i} + 3\hat{j} - 3\hat{k}$, $\vec{b} = 4\hat{i} + 5\hat{j} + (\lambda - 10)\hat{k}$, $\vec{c} = 6\hat{i} + 2\hat{j} - 3\hat{k}$ is 11 cubic units, then the sum of the possible values of λ is ...

- (A) 7
- (B) 8
- (C) 1
- (D) 6

Correct Answer: (B) 8

Solution:

Step 1: Understanding the Concept:

The volume of a tetrahedron with coterminous edges $\vec{a}, \vec{b}, \vec{c}$ is

$$\frac{1}{6} |[\vec{a} \vec{b} \vec{c}]|.$$

Step 2: Key Formula or Approach:

$$[\vec{a} \vec{b} \vec{c}] = \begin{vmatrix} -2 & 3 & -3 \\ 4 & 5 & \lambda - 10 \\ 6 & 2 & -3 \end{vmatrix}$$

Step 3: Detailed Explanation:

Expand along the first row:

$$-2 [5(-3) - 2(\lambda - 10)] - 3 [4(-3) - 6(\lambda - 10)] + (-3) [4 \cdot 2 - 5 \cdot 6]$$

$$= -2(5 - 2\lambda) - 3(48 - 6\lambda) - 3(-22)$$

$$= -10 + 4\lambda - 144 + 18\lambda + 66 = 22\lambda - 88$$

The volume is 11, so $|22\lambda - 88| = 66$.

$$22\lambda - 88 = 66 \Rightarrow \lambda = 7$$

$$22\lambda - 88 = -66 \Rightarrow \lambda = 1$$

Sum of the possible values: $7 + 1 = 8$.

Option (A) 7 and (C) 1 are single roots, and (D) 6 does not come from either case.

Final Answer:

The sum of the possible values of λ is 8, option (B).

8 (B)

Quick Tip: Volume = |scalar triple product| / 6. Evaluate the determinant in terms of lambda.



140. The value of $\theta \in (0, \frac{\pi}{2})$ for which vectors $\vec{a} = (\sin\theta)\hat{i} + (\cos\theta)\hat{j}$ and $\vec{b} = \hat{i} - \sqrt{3}\hat{j} + 2\hat{k}$ are perpendicular is

- (A) $\theta = \frac{\pi}{3}$
- (B) $\theta = \frac{\pi}{6}$
- (C) $\theta = \frac{\pi}{4}$
- (D) $\theta = \frac{\pi}{2}$

Correct Answer: (A) $\theta = \frac{\pi}{3}$

Solution:

Step 1: Understanding the Concept:

Two vectors are perpendicular when their dot product is zero.

Step 2: Key Formula or Approach:

$$\vec{a} \cdot \vec{b} = (\sin \theta)(1) + (\cos \theta)(-\sqrt{3}) + (0)(2).$$

Step 3: Detailed Explanation:

$$\sin \theta - \sqrt{3} \cos \theta = 0 \Rightarrow \tan \theta = \sqrt{3}$$

For $\theta \in \left(0, \frac{\pi}{2}\right)$, the angle with tangent $\sqrt{3}$ is $\theta = \frac{\pi}{3}$.

Check: $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$ and $\sqrt{3} \cos \frac{\pi}{3} = \frac{\sqrt{3}}{2}$, which cancel.

Option (B) $\frac{\pi}{6}$ has $\tan = \frac{1}{\sqrt{3}}$, option (C) $\frac{\pi}{4}$ has $\tan = 1$, and option (D) $\frac{\pi}{2}$ is not in the open interval and has no tangent.

Final Answer:

$\theta = \frac{\pi}{3}$, option (A).

$$\theta = \frac{\pi}{3} \text{ (A)}$$

Quick Tip: Perpendicular vectors have zero dot product. Solve $\sin(\theta) - \sqrt{3} \cos(\theta) = 0$.

141. If $|\vec{a}| = |\vec{b}| = 1$, $|\vec{c}| = 2$ and $\vec{a} \times (\vec{a} \times \vec{c}) + \vec{b} = \vec{0}$, then the acute angle between $\vec{a}$ and $\vec{c}$ is ...

- (A) $\frac{\pi}{2}$
- (B) $\frac{\pi}{3}$
- (C) $\frac{\pi}{4}$
- (D) $\frac{\pi}{6}$

Correct Answer: (D) $\frac{\pi}{6}$

Solution:

Step 1: Understanding the Concept:

We use the vector triple product expansion $\vec{a} \times (\vec{a} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{a} - (\vec{a} \cdot \vec{a})\vec{c}$.

Step 2: Key Formula or Approach:

With $|\vec{a}| = 1$, we get $\vec{a} \cdot \vec{a} = 1$, so the triple product equals $(\vec{a} \cdot \vec{c})\vec{a} - \vec{c}$.

Step 3: Detailed Explanation:

The given equation $\vec{a} \times (\vec{a} \times \vec{c}) + \vec{b} = \vec{0}$ becomes

$$\vec{b} = \vec{c} - (\vec{a} \cdot \vec{c})\vec{a}$$

Take the magnitude squared. Let $t = \vec{a} \cdot \vec{c}$:

$$|\vec{b}|^2 = |\vec{c}|^2 - 2t(\vec{a} \cdot \vec{c}) + t^2|\vec{a}|^2 = 4 - 2t^2 + t^2 = 4 - t^2$$

Since $|\vec{b}| = 1$, we get $4 - t^2 = 1$, so $t^2 = 3$.

But $t = |\vec{a}||\vec{c}| \cos \theta = 2 \cos \theta$. So

$$4 \cos^2 \theta = 3 \Rightarrow \cos \theta = \frac{\sqrt{3}}{2}$$

The acute angle is $\theta = \frac{\pi}{6}$.

Option (A) $\frac{\pi}{2}$ would give $t = 0$ and $|\vec{b}| = 2$. Options (B) and (C) give $t^2 = 1$ and $t^2 = 2$, neither of which equals 3.

Final Answer:

The acute angle between $\vec{a}$ and $\vec{c}$ is $\frac{\pi}{6}$, option (D).

$$\boxed{\frac{\pi}{6} \text{ (D)}}$$

Quick Tip: Expand the triple product to $(a \cdot c)a - c$, then take magnitudes of $b = c - (a \cdot c)a$.

142. The value of $|\vec{a} \cdot \vec{b}|^2 + |\vec{a} \times \vec{b}| \cdot |\vec{a} \times \vec{b}|$ is ...

- (A) $-a^2b^2$
- (B) a^2b^2
- (C) $a^2b^2\cos\theta$
- (D) $a^2b^2\sin\theta$

Correct Answer: (B) a^2b^2

Solution:

Step 1: Understanding the Concept:

For two vectors with magnitudes a and b and angle θ between them, the dot product and the magnitude of the cross product are given by simple formulas.

Step 2: Key Formula or Approach:

1. $\vec{a} \cdot \vec{b} = ab \cos \theta$.
2. $|\vec{a} \times \vec{b}| = ab \sin \theta$.

Step 3: Detailed Explanation:

$$|\vec{a} \cdot \vec{b}|^2 = a^2b^2 \cos^2 \theta$$

$$|\vec{a} \times \vec{b}| \cdot |\vec{a} \times \vec{b}| = a^2 b^2 \sin^2 \theta$$

Add:

$$a^2 b^2 (\cos^2 \theta + \sin^2 \theta) = a^2 b^2$$

This is Lagrange's identity. Options (A), (C) and (D) either have the wrong sign or carry a leftover trigonometric factor, which cannot be since $\cos^2 \theta + \sin^2 \theta = 1$.

Final Answer:

The value is $a^2 b^2$, option (B).

$$a^2 b^2 \text{ (B)}$$

Quick Tip: Use $\vec{a} \cdot \vec{b} = ab \cos(\theta)$ and $|\vec{a} \times \vec{b}| = ab \sin(\theta)$, then add.

143. If $\vec{a} \cdot \vec{b} = \beta$ and $\vec{a} \times \vec{b} = \vec{c}$ then $\vec{a} =$

- (A) $\frac{\vec{b} \times \vec{c} - \beta \vec{b}}{|\vec{b}|^2}$
- (B) $\frac{\vec{b} \times \vec{c} - \beta \vec{c}}{|\vec{b}|^2}$
- (C) $\frac{\vec{b} \times \vec{c} + \beta \vec{b}}{|\vec{b}|^2}$
- (D) $\frac{\vec{b} \times \vec{c} + \beta \vec{c}}{|\vec{b}|^2}$

Correct Answer: (C) $\frac{\vec{b} \times \vec{c} + \beta \vec{b}}{|\vec{b}|^2}$

Solution:

Step 1: Understanding the Concept:

We know $\vec{a} \cdot \vec{b} = \beta$ and $\vec{a} \times \vec{b} = \vec{c}$. Crossing $\vec{b}$ with $\vec{c}$ brings out $\vec{a}$ through the triple product rule.

Step 2: Key Formula or Approach:

$$\vec{b} \times (\vec{a} \times \vec{b}) = \vec{a}(\vec{b} \cdot \vec{b}) - \vec{b}(\vec{b} \cdot \vec{a})$$

Step 3: Detailed Explanation:

Substitute $\vec{c} = \vec{a} \times \vec{b}$:

$$\vec{b} \times \vec{c} = \vec{b} \times (\vec{a} \times \vec{b}) = \vec{a}|\vec{b}|^2 - \vec{b}(\vec{a} \cdot \vec{b}) = \vec{a}|\vec{b}|^2 - \beta\vec{b}$$

Solve for $\vec{a}$:

$$\vec{a}|\vec{b}|^2 = \vec{b} \times \vec{c} + \beta\vec{b}$$

$$\vec{a} = \frac{\vec{b} \times \vec{c} + \beta\vec{b}}{|\vec{b}|^2}$$

Options (A) and (B) have a minus sign before $\beta\vec{b}$, which would be the case if the term was moved incorrectly. Option (D) uses $\beta\vec{c}$, but $\vec{c}$ is perpendicular to $\vec{b}$ and cannot be multiplied by a dot product to rebuild the part of $\vec{a}$ along $\vec{b}$.

Final Answer:

$$\vec{a} = \frac{\vec{b} \times \vec{c} + \beta\vec{b}}{|\vec{b}|^2}, \text{ option (C).}$$

$$\frac{\bar{b} \times \bar{c} + \beta \bar{b}}{|\bar{b}|^2} \quad (C)$$

Quick Tip: Compute $\mathbf{b} \times \mathbf{c} = \mathbf{b} \times (\mathbf{a} \times \mathbf{b})$ using the triple product expansion.



144. If the equation of a plane passing through $A(1, p, 2)$, $B(3, 2, 4)$ and parallel to the z axis, is $3x - 2y - q = 0$, then ...

- (A) $p = -1, q = 5$
- (B) $p = 1, q = -5$
- (C) $p = -2, q = -5$
- (D) $p = 2, q = -5$

Correct Answer: (A) $p = -1, q = 5$

Solution:

Step 1: Understanding the Concept:

A plane parallel to the z -axis has a normal perpendicular to the z -axis, so its equation has no z term. That fits the given form $3x - 2y - q = 0$.

Step 2: Key Formula or Approach:

Each point on the plane must satisfy $3x - 2y - q = 0$.

Step 3: Detailed Explanation:

Point $B(3, 2, 4)$:

$$3(3) - 2(2) - q = 0 \Rightarrow 9 - 4 = q \Rightarrow q = 5$$

Point $A(1, p, 2)$:

$$3(1) - 2p - 5 = 0 \Rightarrow -2p = 2 \Rightarrow p = -1$$

So $p = -1$ and $q = 5$. Options (B), (C) and (D) all have $q = -5$, which would give a plane $3x - 2y + 5 = 0$, and B does not lie on it: $9 - 4 + 5 = 10 \neq 0$.

Final Answer:

$p = -1$ and $q = 5$, option (A).

$$p = -1, q = 5 \text{ (A)}$$

Quick Tip: A plane parallel to the z -axis has no z term. Use the two points to find q and p .

145. The angle between the line $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k})$ and the plane $\vec{r} \cdot (2\hat{i} - \hat{j} + \hat{k}) = 8$ is ...

- (A) $\theta = \sin^{-1}(\frac{\sqrt{2}}{3})$
- (B) $\theta = \cos^{-1}(\frac{\sqrt{2}}{3})$
- (C) $\theta = \cos^{-1}(\frac{2}{\sqrt{3}})$
- (D) $\theta = \sin^{-1}(\frac{3}{\sqrt{2}})$

Correct Answer: (A) $\theta = \sin^{-1}(\frac{\sqrt{2}}{3})$

Solution:

Step 1: Understanding the Concept:

The angle between a line and a plane is the complement of the angle between the line and the normal to the plane. So the formula uses sine and not cosine.

Step 2: Key Formula or Approach:

$$\sin \theta = \frac{|\vec{d} \cdot \vec{n}|}{|\vec{d}| |\vec{n}|}$$

where $\vec{d}$ is the direction of the line and $\vec{n}$ is the normal to the plane.

Step 3: Detailed Explanation:

$\vec{d} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{n} = 2\hat{i} - \hat{j} + \hat{k}$.

$$\vec{d} \cdot \vec{n} = 2 - 1 + 1 = 2, \quad |\vec{d}| = \sqrt{3}, \quad |\vec{n}| = \sqrt{6}$$

$$\sin \theta = \frac{2}{\sqrt{3} \cdot \sqrt{6}} = \frac{2}{\sqrt{18}} = \frac{2}{3\sqrt{2}} = \frac{\sqrt{2}}{3}$$

$$\theta = \sin^{-1} \left(\frac{\sqrt{2}}{3} \right)$$

Option (B) uses cosine and so gives the angle with the normal. Option (D) has a ratio larger than 1, which cannot be a sine. Option (C) has a value larger than 1, so it is not a cosine either.

Final Answer:

$$\theta = \sin^{-1} \left(\frac{\sqrt{2}}{3} \right), \text{ option (A).}$$

$$\boxed{\sin^{-1} \left(\frac{\sqrt{2}}{3} \right) \text{ (A)}}$$

Quick Tip: The angle with the plane satisfies $\sin(\theta) = |\mathbf{d} \cdot \mathbf{n}| / (|\mathbf{d}| |\mathbf{n}|)$.

146. The acute angle between the lines $2x = 3y = -z$ and $6x = -y = -4z$ is ...

- (A) $\frac{\pi}{4}$
- (B) $\frac{\pi}{3}$
- (C) $\frac{\pi}{6}$
- (D) $\frac{\pi}{2}$

Correct Answer: (D) $\frac{\pi}{2}$

Solution:

Step 1: Understanding the Concept:

The angle between two lines in space comes from the dot product of their direction ratios. First we need these ratios from the given equations.

Step 2: Key Formula or Approach:

For a line $\frac{x}{l} = \frac{y}{m} = \frac{z}{n}$, the direction ratios are (l, m, n) . Rewrite $2x = 3y = -z = k$ as $x = \frac{k}{2}, y = \frac{k}{3}, z = -k$.

Step 3: Detailed Explanation:

First line: $\left(\frac{1}{2}, \frac{1}{3}, -1\right)$. Multiply by 6 to clear fractions: $(3, 2, -6)$.

Second line: $6x = -y = -4z = k$ gives $x = \frac{k}{6}, y = -k, z = -\frac{k}{4}$.

Direction $\left(\frac{1}{6}, -1, -\frac{1}{4}\right)$. Multiply by 12: $(2, -12, -3)$.

Dot product:

$$3(2) + 2(-12) + (-6)(-3) = 6 - 24 + 18 = 0$$

So $\cos \theta = 0$ and the lines are perpendicular.

$$\theta = \frac{\pi}{2}$$

The other options give $\cos \theta$ values of $\frac{1}{\sqrt{2}}, \frac{1}{2}$ or $\frac{\sqrt{3}}{2}$, none of which matches 0.

Final Answer:

The angle between the lines is $\frac{\pi}{2}$, option (D).

$$\boxed{\frac{\pi}{2} \text{ (D)}}$$

Quick Tip: Convert each line to symmetric form to read its direction ratios, then test the dot product.

147. The maximum value of $z = 4x + y$ subject to the constraints $x + y \leq 5, 2x + y \leq 7, 3x + 2y \leq 11, x \geq 0, y \geq 0$ is ...

- (A) 13
- (B) 8
- (C) 11
- (D) 14

Correct Answer: (D) 14

Solution:

Step 1: Understanding the Concept:

In a linear programming problem the maximum of a linear function occurs at a corner point of the feasible region.

Step 2: Key Formula or Approach:

Constraints: $x + y \leq 5$, $2x + y \leq 7$, $3x + 2y \leq 11$, $x, y \geq 0$. Find the corners and test them in all constraints.

Step 3: Detailed Explanation:

Corner points:

$(0, 0)$: feasible, $z = 0$.

$(3.5, 0)$ from $2x + y = 7$ on the x-axis: check $3(3.5) = 10.5 \leq 11$ and $3.5 \leq 5$. Feasible, $z = 4(3.5) = 14$.

$(0, 5)$ from $x + y = 5$ on the y-axis: check $2(5) = 10 \leq 11$ and $5 \leq 7$. Feasible, $z = 5$.

$(3, 1)$ from $2x + y = 7$ and $3x + 2y = 11$: check $x + y = 4 \leq 5$. Feasible, $z = 13$.

$(1, 4)$ from $x + y = 5$ and $3x + 2y = 11$: check $2x + y = 6 \leq 7$. Feasible, $z = 8$.

The corner $(2, 3)$ from $x + y = 5$ and $2x + y = 7$ fails $3x + 2y = 12 > 11$, so it is not a vertex.

The largest value is 14 at $(3.5, 0)$.

Final Answer:

The maximum value of z is 14, option (D).

14 (D)

Quick Tip: Find the corner points of the feasible region and evaluate z at each.

148. A fair die is thrown 4 times. If getting a prime number on the die is considered as a success, then the probability of getting no success at all is ...

- (A) $\frac{1}{16}$
- (B) $\frac{15}{16}$
- (C) $\frac{1}{81}$
- (D) $\frac{16}{81}$

Correct Answer: (A) $\frac{1}{16}$

Solution:**Step 1: Understanding the Concept:**

Throwing a die 4 times with a fixed chance of success each time is a binomial experiment with $n = 4$.

Step 2: Key Formula or Approach:

$$P(X = r) = {}^nC_r p^r q^{n-r}$$

Step 3: Detailed Explanation:

The primes on a die are 2, 3 and 5, so there are 3 favourable faces out of 6.

$$p = \frac{3}{6} = \frac{1}{2}, \quad q = 1 - p = \frac{1}{2}$$

No success at all means $r = 0$:

$$P(X = 0) = {}^4C_0 \left(\frac{1}{2}\right)^0 \left(\frac{1}{2}\right)^4 = \frac{1}{16}$$

Option (B) $\frac{15}{16}$ is the chance of at least one prime. Options (C) and (D) come from taking the success probability as $\frac{1}{3}$ or $\frac{2}{3}$, which would be right if only two faces counted.

Final Answer:

The probability of no success is $\frac{1}{16}$, option (A).

$\frac{1}{16} \text{ (A)}$

Quick Tip: Primes on a die are 2, 3 and 5, so $p = 1/2$. Use the binomial formula for zero successes.

149. If the p. d. f. of a continuous random variable X is given by $f(x)$
 $= \begin{cases} k(9 + 8x - x^2), & \text{for } -1 \leq x \leq 4 \\ 0, & \text{otherwise} \end{cases}$ then the value of k is ...

- (A) $\frac{1}{125}$
(B) $\frac{3}{250}$
(C) $\frac{3}{125}$
(D) $\frac{1}{250}$

Correct Answer: (B) $\frac{3}{250}$

Solution:

Step 1: Understanding the Concept:

For a continuous random variable, the total area under the pdf equals 1. This fixes the constant k .

Step 2: Key Formula or Approach:

$$\int_{-1}^4 k(9 + 8x - x^2) dx = 1$$

Step 3: Detailed Explanation:

Find the antiderivative:

$$\int (9 + 8x - x^2) dx = 9x + 4x^2 - \frac{x^3}{3}$$

At $x = 4$:

$$36 + 64 - \frac{64}{3} = 100 - \frac{64}{3} = \frac{236}{3}$$

At $x = -1$:

$$-9 + 4 + \frac{1}{3} = -5 + \frac{1}{3} = -\frac{14}{3}$$

Difference:

$$\frac{236}{3} + \frac{14}{3} = \frac{250}{3}$$

So $k \cdot \frac{250}{3} = 1$, which gives

$$k = \frac{3}{250}$$

Option (D) $\frac{1}{250}$ forgets the factor 3 from the cubic term. Options (A) and (C) are based on the number 125, which would arise from a wrong value of the integral.

Final Answer:

$k = \frac{3}{250}$, option (B).

$$\boxed{\frac{3}{250} \text{ (B)}}$$

Quick Tip: Total probability must be 1, so integrate the pdf from -1 to 4 and set it equal to 1.

150. In an entrance test, there are multiple choice questions. There are four possible answers to each question, only one of which is correct. The probability that a student knows the answer to a question is 90%. If he gets the correct answer to a question, then the probability that he was guessing is

- (A) $\frac{37}{40}$
(B) $\frac{1}{37}$
(C) $\frac{36}{37}$
(D) $\frac{1}{9}$

Correct Answer: (B) $\frac{1}{37}$

Solution:

Step 1: Understanding the Concept:

We want the probability that the student guessed, given that the answer is correct. This is a conditional probability that needs Bayes' theorem.

Step 2: Key Formula or Approach:

Let K be the event that the student knows the answer and G the event that the student guesses. Let C be the event of a correct answer.

$$P(K) = 0.9, \quad P(G) = 0.1, \quad P(C|K) = 1, \quad P(C|G) = \frac{1}{4}$$

$$P(G|C) = \frac{P(G)P(C|G)}{P(K)P(C|K) + P(G)P(C|G)}$$

Step 3: Detailed Explanation:

Numerator: $0.1 \times \frac{1}{4} = 0.025$.

Denominator: $0.9 \times 1 + 0.025 = 0.925$.

$$P(G|C) = \frac{0.025}{0.925} = \frac{25}{925} = \frac{1}{37}$$

Option (C) $\frac{36}{37}$ is the probability that the student knew the answer given it is correct. Option (A) and (D) do not match the ratio 25/925.

Final Answer:

The probability that the student was guessing is $\frac{1}{37}$, option (B).

$$\boxed{\frac{1}{37} \text{ (B)}}$$

Quick Tip: Use Bayes' theorem. A guess is correct with probability 1/4, and a known answer is always correct.