

MHT CET 2026 PCM Question Paper April 19 Shift 1 with Solutions



General Instructions

- (i) This booklet contains 150 questions, each provided with a complete, step-by-step solution.
- (ii) It comprises 150 single-correct multiple-choice questions.
- (iii) Attempt each question on your own before reviewing the given solution.

Physics

Questions 1 to 50 | 1 mark each | 50 marks

1. For two vectors $\vec{P}$ and $\vec{Q}$, $\vec{P} \cdot \vec{Q} = |\vec{P} \times \vec{Q}|$

The magnitude of $\vec{R} = \vec{P} + \vec{Q}$ is ($\cos 45^\circ = \frac{1}{\sqrt{2}}$) ?

- (A) $\sqrt{P^2 + Q^2}$
- (B) $\frac{\sqrt{P^2 + Q^2}}{\sqrt{2}}$
- (C) $\sqrt{P^2 + Q^2 + \frac{PQ}{\sqrt{2}}}$
- (D) $\sqrt{P^2 + Q^2 + \sqrt{2}PQ}$

Correct Answer: (D) $\sqrt{P^2 + Q^2 + \sqrt{2}PQ}$

Solution:

Step 1: Find the Angle:

$\vec{P} \cdot \vec{Q} = PQ \cos \theta$ and $|\vec{P} \times \vec{Q}| = PQ \sin \theta$. Setting them equal gives $\tan \theta = 1$, so $\theta = 45^\circ$.

Step 2: Resultant Formula:

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos \theta}$$

Step 3: Substitute:

With $\cos 45^\circ = \frac{1}{\sqrt{2}}$, $2PQ \cos 45^\circ = \sqrt{2} PQ$. So

$$R = \sqrt{P^2 + Q^2 + \sqrt{2} PQ}$$

Step 4: Check the Other Options:

Option (A) corresponds to $\theta = 90^\circ$. Option (C) has $\frac{PQ}{\sqrt{2}}$, which would come from forgetting the factor 2 in $2PQ \cos \theta$. Option (B) is not a valid form. So (D) is correct.

Final Answer:

The magnitude is $\sqrt{P^2 + Q^2 + \sqrt{2} PQ}$, option (D).

$(D) \sqrt{P^2 + Q^2 + \sqrt{2} PQ}$

Quick Tip: Equal dot and cross magnitudes mean the angle between the vectors is 45 degrees.

2. Two stones of masses m and $3m$ are whirled in horizontal circles, the heavier one in a radius $\frac{r}{3}$ and the lighter one in radius r . When both the stones experience same centripetal forces, the tangential speed of lighter stone is ' n ' times that of the value of heavier stone. The value of ' n ' is

- (A) 1
- (B) 2
- (C) 3
- (D) 4

Correct Answer: (C) 3

Solution:

Step 1: Centripetal Force:

For a mass m moving at speed v in a circle of radius r , $F = \frac{mv^2}{r}$.

Step 2: Write for Both Stones:

Lighter stone: mass m , radius r , speed v_1 : $F = \frac{mv_1^2}{r}$.

Heavier stone: mass $3m$, radius $\frac{r}{3}$, speed v_2 : $F = \frac{3m v_2^2}{r/3} = \frac{9m v_2^2}{r}$.

Step 3: Equate:

$$\frac{mv_1^2}{r} = \frac{9mv_2^2}{r} \Rightarrow v_1^2 = 9v_2^2 \Rightarrow v_1 = 3v_2.$$

Step 4: Result:

The lighter stone's speed is 3 times that of the heavier stone, so $n = 3$.

Options 1, 2 and 4 would give $v_1^2 = v_2^2$, $4v_2^2$, $16v_2^2$, none equal to $9v_2^2$.

Final Answer:

The value of n is 3, option (C).

Quick Tip: Equate $m v^2 / r$ for both stones and solve for the speed ratio.

3. Two particles having mass 'M' and 'm' are moving in a circular path with radius 'R' and 'r' respectively. The time period for both the particles is same. The ratio of angular velocity of the first particle to that of the second particle will be

- (A) 1 : 1
- (B) 1 : 2
- (C) 2 : 3
- (D) 3 : 4

Correct Answer: (A) 1 : 1

Solution:

Step 1: Key Relation:

For circular motion, $\omega = \frac{2\pi}{T}$.

Step 2: Apply:

Both particles have the same period T , so $\omega_1 = \frac{2\pi}{T} = \omega_2$.

Step 3: Ratio:

$\omega_1 : \omega_2 = 1 : 1$. The masses M, m and radii R, r do not appear in the formula for angular velocity, so they do not change the ratio.

Step 4: Check the Other Options:

The ratios 1 : 2, 2 : 3 and 3 : 4 would need different periods. Only equal periods give 1 : 1, so (A) is correct.

Final Answer:

The ratio is 1 : 1, option (A).

(A) 1 : 1

Quick Tip: Angular velocity depends only on the time period, not on mass or radius.

4. Which of the following person is in an inertial frame of reference?

- (A) A pilot in an aeroplane which is taking off.
- (B) A driver driving a bus which is moving with constant velocity.
- (C) A child revolving in a merry - go - round.
- (D) A man in a train which is slowing down to stop.

Correct Answer: (B) A driver driving a bus which is moving with constant velocity.

Solution:

Step 1: Understanding the Concept:

An inertial frame is one in which Newton's first law holds. Any frame moving with constant velocity, with no acceleration, is inertial. A frame that speeds up, slows down or turns is non-inertial.

Step 2: Check (A):

A plane taking off speeds up, so it accelerates. Its frame is non-

inertial.

Step 3: Check (B):

A bus moving with constant velocity has zero acceleration. The driver is in an inertial frame.

Step 4: Check (C) and (D):

A merry-go-round moves in a circle, so it has centripetal acceleration. A train slowing down has deceleration. Both frames are non-inertial.

Final Answer:

Only the bus with constant velocity is an inertial frame, option (B).

(B)

Quick Tip: An inertial frame moves with constant velocity; accelerating frames are non-inertial.

5. The spring is initially in unstretched condition. It is first stretched by a length ' x ' and the work done is W_1 . Then again it is stretched by further length ' x ' when the work done is W_2 . The value of W_2 is given by

- (A) W_1
- (B) $2W_1$
- (C) $3W_1$
- (D) $4W_1$

Correct Answer: (C) $3W_1$

Solution:

Step 1: Work in Stretching a Spring:

The work to stretch a spring from natural length by s is $\frac{1}{2}ks^2$.

Step 2: First Stretch:

$$W_1 = \frac{1}{2}kx^2.$$

Step 3: Second Stretch:

Total work to reach extension $2x$: $\frac{1}{2}k(2x)^2 = 2kx^2$. So

$$W_2 = 2kx^2 - \frac{1}{2}kx^2 = \frac{3}{2}kx^2 = 3W_1$$

Step 4: Check the Options:

Option (A) W_1 would be true for a constant force, but a spring force grows with extension. Option (B) $2W_1$ and (D) $4W_1$ match neither $\frac{3}{2}kx^2$ nor the difference found above.

Final Answer:

$W_2 = 3W_1$, option (C).

(C) $3W_1$

Quick Tip: Work to stretch from 0 to x is half kx^2 ; from x to $2x$ subtract it from the work to $2x$.

6. A thin uniform rod AB of mass ' m ' and length ' l ' is hinged at one end A to the ground level. Initially the rod stands vertically and is allowed to fall freely to the ground in the vertical plane. The angular velocity of the rod when its B end strikes the ground is
(g = acceleration due to gravity)

(A) $\sqrt{\frac{2g}{l}}$

(B) $\sqrt{\frac{3g}{l}}$

(C) $\sqrt{\frac{mg}{l}}$

(D) $\sqrt{\frac{mg}{3l}}$

Correct Answer: (B) $\sqrt{\frac{3g}{l}}$

Solution:

Step 1: Energy Conservation:

The rod rotates about the hinge A. Its centre of mass is at $l/2$ and falls through $l/2$ in going from vertical to horizontal. The loss of potential energy becomes rotational kinetic energy.

Step 2: Write the Equation:

$$mg\frac{l}{2} = \frac{1}{2}I\omega^2, \quad I = \frac{ml^2}{3}$$

Step 3: Solve:

$$mg\frac{l}{2} = \frac{1}{2} \cdot \frac{ml^2}{3}\omega^2 \Rightarrow \omega^2 = \frac{3g}{l}$$

$$\omega = \sqrt{\frac{3g}{l}}$$

Step 4: Check the Options:

Option (A) $\sqrt{2g/l}$ comes from using $I = \frac{ml^2}{2}$ or point-mass reasoning. Options (C) and (D) contain m in the answer, but ω cannot depend on mass because m cancels. So (B) is correct.

Final Answer:

The angular velocity is $\sqrt{3g/l}$, option (B).

(B) $\sqrt{\frac{3g}{l}}$

Quick Tip: Use energy conservation with $I = ml^2 / 3$ about the hinge.

7. The radius of gyration of a solid sphere of radius 'R' and mass 'M' about its diameter is K_d and that about a tangent of a solid sphere is K_t . The ratio of K_d to K_t is

- (A) $(\frac{7}{5})^{1/2}$
- (B) $(\frac{7}{2})^{1/2}$
- (C) $(\frac{2}{7})^{1/2}$
- (D) $(\frac{2}{5})^{1/2}$

Correct Answer: (C) $(\frac{2}{7})^{1/2}$

Solution:

Step 1: Diameter:

$$I_d = \frac{2}{5}MR^2. \text{ Since } I = MK^2, K_d^2 = \frac{2}{5}R^2.$$

Step 2: Tangent (Parallel Axis Theorem):

$$I_t = I_d + MR^2 = \frac{2}{5}MR^2 + MR^2 = \frac{7}{5}MR^2. \text{ So } K_t^2 = \frac{7}{5}R^2.$$

Step 3: Ratio:

$$\frac{K_d}{K_t} = \sqrt{\frac{2/5}{7/5}} = \sqrt{\frac{2}{7}}$$

Step 4: Check the Options:

Options (A) and (B) give a ratio greater than 1, but $K_t > K_d$ because the tangent is farther from the centre. Option (D) $\sqrt{2/5}$ is just K_d/R . So (C) is correct.

Final Answer:

The ratio is $(2/7)^{1/2}$, option (C).

$$\boxed{(C) \left(\frac{2}{7}\right)^{1/2}}$$

Quick Tip: Use $I = (2/5)MR^2$ about a diameter and the parallel axis theorem for the tangent.

8. A geostationary satellite is orbiting the earth at a height of $4R$ above the surface of the earth, where R is the radius of the earth. Another satellite is orbiting the earth at a height $1.5R$ from the surface of the earth with periodic time 'T' in hour. The value of 'T' in hour is

- (A) $\frac{6}{\sqrt{2}}$
- (B) $6\sqrt{2}$
- (C) 4
- (D) 8

Correct Answer: (B) $6\sqrt{2}$

Solution:

Step 1: Radii From the Earth Centre:

Geostationary satellite: $r_1 = R + 4R = 5R$, period $T_1 = 24$ h.

Second satellite: $r_2 = R + 1.5R = 2.5R$.

Step 2: Kepler's Third Law:

$$\frac{T_2^2}{T_1^2} = \left(\frac{r_2}{r_1}\right)^3 = \left(\frac{2.5}{5}\right)^3 = \frac{1}{8}$$

Step 3: Solve:

$$T_2 = 24 \times \frac{1}{2\sqrt{2}} = \frac{12}{\sqrt{2}} = 6\sqrt{2} \text{ h}$$

Step 4: Check the Other Options:

$6/\sqrt{2} \approx 4.2$ h would need a radius ratio of about 0.31, and 4 h and 8 h

correspond to $T^2/T_1^2 = 1/36$ and $1/9$, which do not match $1/8$. So (B) is correct.

Final Answer:

The period is $6\sqrt{2}$ hours, option (B).

(B) $6\sqrt{2}$ h

Quick Tip: Use Kepler's third law T^2 proportional to r^3 with r measured from the centre of the earth.

9. One large soap bubble of diameter 'D' breaks into 64 bubbles having surface tension 'T'. The change in surface energy is

- (A) $2\pi TD^2$
- (B) $4\pi TD^2$
- (C) $6\pi TD^2$
- (D) $8\pi TD^2$

Correct Answer: (C) $6\pi TD^2$

Solution:

Step 1: Radius of the Small Bubbles:

Volume is conserved: $64 \cdot \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$, so $r = \frac{R}{4}$, where $R = \frac{D}{2}$.

Step 2: Surface Area and Energy:

A soap bubble has two free surfaces (inside and outside), so the surface energy of a bubble of radius ρ is $2 \times 4\pi\rho^2 T = 8\pi\rho^2 T$.

Initial: $8\pi R^2 T = 8\pi \frac{D^2}{4} T = 2\pi TD^2$.

Step 3: Final Energy:

$$\text{Final: } 64 \times 8\pi r^2 T = 64 \times 8\pi \frac{R^2}{16} T = 32\pi R^2 T = 8\pi TD^2.$$

Step 4: Change:

$$\Delta E = 8\pi TD^2 - 2\pi TD^2 = 6\pi TD^2$$

This is option (C).

Final Answer:

The change in surface energy is $6\pi TD^2$, option (C).

$$\boxed{(C) \ 6\pi TD^2}$$

Quick Tip: A soap bubble has two surfaces; find the new radius from volume conservation.

10. The pressures inside two soap bubbles A and B are 1.02 atmosphere and 1.04 atmosphere respectively. The ratio of volume of bubble A to that of bubble B is (outside pressure = 1 atmosphere)

- (A) 1 : 8
- (B) 8 : 1
- (C) 4 : 1
- (D) 1 : 4

Correct Answer: (B) 8 : 1

Solution:

Step 1: Excess Pressure:

For a soap bubble, $\Delta P = \frac{4T}{r}$, so $r \propto \frac{1}{\Delta P}$.

Step 2: Compute:

$\Delta P_A = 1.02 - 1 = 0.02$ atm and $\Delta P_B = 1.04 - 1 = 0.04$ atm. So $\frac{r_A}{r_B} = \frac{0.04}{0.02} = 2$.

Step 3: Volume Ratio:

$$\frac{V_A}{V_B} = \left(\frac{r_A}{r_B}\right)^3 = 2^3 = 8$$

Step 4: Check the Options:

Option (A) 1 : 8 is the inverse. Options (C) and (D) correspond to ratios of 4 and 1/4, which would need a radius ratio of $4^{1/3}$, not 2. So (B) is correct.

Final Answer:

The volume ratio is 8 : 1, option (B).

(B) 8 : 1

Quick Tip: Excess pressure in a soap bubble is $4T/r$, so volume goes as $1/(\text{excess pressure})^3$.

11. An in-compressible fluid flows steadily through a horizontal cylindrical pipe. The pipe has radius ' $3R$ ' at point A and ' $1.5R$ ' at point B, further along the flow direction at the same level. If the velocity at point A is ' v ', then that at point B is

- (A) v
- (B) $2v$
- (C) $3v$
- (D) $4v$

Correct Answer: (D) $4v$

Solution:

Step 1: Continuity Equation:

For an incompressible fluid in steady flow, $A_1v_1 = A_2v_2$.

Step 2: Areas:

$$A_A = \pi(3R)^2 = 9\pi R^2 \text{ and } A_B = \pi(1.5R)^2 = 2.25\pi R^2.$$

Step 3: Solve:

$$v_B = \frac{A_A}{A_B}v = \frac{9}{2.25}v = 4v$$

Step 4: Check the Options:

The radius ratio is 2, but speed depends on the area ratio, which is $2^2 = 4$. Option (B) $2v$ comes from using the radius ratio directly. So (D) is correct.

Final Answer:

The speed at B is $4v$, option (D).

$$(D) 4v$$

Quick Tip: Use the equation of continuity $A_1 v_1 = A_2 v_2$.

12. When 170 J of energy is incident on a surface of a body, 17 J of energy is reflected by it. If the coefficient of absorption is 0.7, then the amount of energy transmitted will be

- (A) 3.4 J
- (B) 11.9 J
- (C) 34.0 J
- (D) 119.0 J

Correct Answer: (C) 34.0 J

Solution:

Step 1: Reflection Coefficient:

$$r = \frac{17}{170} = 0.1.$$

Step 2: Sum Rule:

$$r + a + t = 1. \text{ With } a = 0.7: t = 1 - 0.1 - 0.7 = 0.2.$$

Step 3: Transmitted Energy:

$$0.2 \times 170 = 34.0 \text{ J.}$$

Step 4: Check the Options:

3.4 J would be 0.02, 11.9 J is the absorbed energy 0.07×170 with a decimal slip, and 119 J is the absorbed energy 0.7×170 . Only 34.0 J

matches the transmission coefficient.

Final Answer:

The transmitted energy is 34.0 J, option (C).

(C) 34.0 J

Quick Tip: Reflection, absorption and transmission coefficients add up to 1.

13. A black body is at temperature 827°C . The rate at which it emits energy is proportional to

- (A) $(827)^4$
- (B) $(827)^2$
- (C) $(1100)^4$
- (D) $(1100)^2$

Correct Answer: (C) $(1100)^4$

Solution:

Step 1: Stefan's Law:

The energy emitted per second by a black body is proportional to the fourth power of its absolute temperature: $E \propto T^4$.

Step 2: Convert the Temperature:

$$T = 827 + 273 = 1100 \text{ K.}$$

Step 3: Result:

So $E \propto (1100)^4$.

Step 4: Check the Other Options:

Options (A) and (B) use the Celsius value 827, which is not an absolute scale. Option (D) uses the correct temperature but the wrong power, 2. So (C) is correct.

Final Answer:

The rate is proportional to $(1100)^4$, option (C).

(C) $(1100)^4$

Quick Tip: Stefan's law needs the absolute temperature, so convert Celsius to Kelvin.

14. If the two temperatures on Fahrenheit scale differ by 54°F , then the difference in temperature on the Celsius scale is

- (A) 30°C
- (B) 35°C
- (C) 40°C
- (D) 45°C

Correct Answer: (A) 30°C

Solution:

Step 1: Scale Relation:

$\frac{C}{100} = \frac{F - 32}{180}$. For a difference, the constant 32 cancels: $\frac{\Delta C}{100} = \frac{\Delta F}{180}$.

Step 2: Substitute:

$$\Delta C = \frac{100}{180} \times 54 = \frac{5}{9} \times 54 = 30.$$

Step 3: Check the Options:

35, 40 and 45 would need $\Delta F = 63, 72, 81$. The given difference is 54, so only 30 fits.

Final Answer:

The difference is 30°C , option (A).

(A) 30°C

Quick Tip: The size of one Celsius degree is $9/5$ of a Fahrenheit degree.



15. A gas at normal temperature is suddenly compressed to one-fourth of its original volume. If $\gamma = 1.5$, then the increase in the temperature of the gas in Kelvin is (γ is the ratio of specific heats)

- (A) 273
- (B) 373
- (C) 473
- (D) 573

Correct Answer: (A) 273

Solution:

Step 1: Adiabatic Relation:

For a sudden (adiabatic) compression, $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$.

Step 2: Substitute:

Normal temperature is taken as $T_1 = 273$ K. With $V_2 = V_1/4$ and $\gamma = 1.5$:

$$T_2 = T_1 \left(\frac{V_1}{V_2} \right)^{\gamma-1} = 273 \times 4^{0.5} = 273 \times 2 = 546 \text{ K}$$

Step 3: Increase:

$$T_2 - T_1 = 546 - 273 = 273 \text{ K.}$$

Step 4: Check the Other Options:

373, 473 and 573 K would need final temperatures of 646, 746 and 846 K, i.e. factors of 2.37, 2.73 and 3.10, which do not equal $4^{0.5} = 2$. So (A) is correct.

Final Answer:

The temperature rises by 273 K, option (A).

(A) 273

Quick Tip: Use $T V^{\gamma-1} = \text{constant}$ with normal temperature taken as 273 K.

16. The thermodynamic process in which no work is done by the gas or on the gas is

- (A) isothermal process
- (B) adiabatic process
- (C) isobaric process
- (D) isochoric process

Correct Answer: (D) isochoric process

Solution:

Step 1: Work Formula:

Work done by a gas is $W = \int P dV$. It is zero when $dV = 0$.

Step 2: Check Each Process:

Isothermal: volume changes at constant temperature, so work is done.

Adiabatic: volume changes without heat flow, so work is done at the cost of internal energy.

Isobaric: volume changes at constant pressure, so $W = P\Delta V \neq 0$.

Isochoric: volume is constant, so $W = 0$.

Step 3: Conclusion:

Only the isochoric process has no work done.

Final Answer:

The isochoric process does no work, option (D).

(D) isochoric process

Quick Tip: Work is P times the volume change, so a constant-volume process does no work.

17. The kinetic energy of an ideal gas is E_0 at 27°C . When the temperature is increased to 177°C , then kinetic energy will be

- (A) $E_0/2$
- (B) $2E_0/3$
- (C) $3E_0/2$
- (D) $2E_0$

Correct Answer: (C) $3E_0/2$

Solution:

Step 1: Key Fact:

The mean kinetic energy of an ideal gas is proportional to its absolute temperature: $E \propto T$.

Step 2: Convert:

$$T_1 = 27 + 273 = 300 \text{ K and } T_2 = 177 + 273 = 450 \text{ K.}$$

Step 3: Ratio:

$$\frac{E}{E_0} = \frac{450}{300} = \frac{3}{2} \Rightarrow E = \frac{3}{2}E_0$$

Step 4: Check the Others:

Using Celsius values would give $177/27 \approx 6.6$, which is not an option. $2E_0/3$ is the inverse ratio, and $2E_0$ would need the temperature to double to 600 K.

Final Answer:

The kinetic energy becomes $\frac{3}{2}E_0$, option (C).

$$(C) \frac{3E_0}{2}$$

Quick Tip: Kinetic energy of an ideal gas is proportional to the absolute temperature.



18. A particle executes a simple harmonic motion with a periodic time 8 second. At time $t = 0$, it is at a mean position. The ratio of the distance traveled by a particle in the 2nd and that in the 1st second of its motion is ($\sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}$, $\sin 90^\circ = \cos 0^\circ = 1$)

- (A) $\frac{1}{\sqrt{2}}$
- (B) $\sqrt{2}$
- (C) $\sqrt{2} - 1$
- (D) $\frac{1}{\sqrt{2}-1}$

Correct Answer: (C) $\sqrt{2} - 1$

Solution:

Step 1: Equation of Motion:

Starting at the mean position, $x = A \sin \omega t$ with $\omega = \frac{2\pi}{T} = \frac{2\pi}{8}$
 $= \frac{\pi}{4}$ rad/s.

Step 2: Position at Each Second:

$t = 0$: $x = 0$. $t = 1$: $x = A \sin 45^\circ = \frac{A}{\sqrt{2}}$. $t = 2$: $x = A \sin 90^\circ = A$.

Step 3: Distances:

Distance in the 1st second = $\frac{A}{\sqrt{2}}$. Distance in the 2nd second = $A - \frac{A}{\sqrt{2}}$. The particle moves in one direction in both seconds (before the extreme position at $t = 2$), so distance equals displacement.

Step 4: Ratio:

$$\frac{A\left(1 - \frac{1}{\sqrt{2}}\right)}{A/\sqrt{2}} = \sqrt{2}\left(1 - \frac{1}{\sqrt{2}}\right) = \sqrt{2} - 1$$

Option (D) $1/(\sqrt{2} - 1)$ is the inverse, and (A), (B) are different numbers.

Final Answer:

The ratio is $\sqrt{2} - 1$, option (C).

$$\boxed{(C) \sqrt{2} - 1}$$

Quick Tip: Write $x = A \sin(2\pi t / T)$ and find displacement at each second.

19. The third overtone of a closed pipe of length ' L_c ' has the same frequency as the third overtone of the open pipe of length ' L_o '. Both the pipes have same diameters. The ratio $L_c : L_o$ is equal to

- (A) 8 : 7
- (B) 7 : 8
- (C) 5 : 3
- (D) 3 : 2

Correct Answer: (B) 7 : 8

Solution:

Step 1: Closed Pipe:

A closed pipe has only odd harmonics: $f = \frac{(2k+1)v}{4L_c}$. The first overtone is the 3rd harmonic, the second is the 5th and the third overtone is the 7th harmonic: $f_c = \frac{7v}{4L_c}$.

Step 2: Open Pipe:

An open pipe has all harmonics: $f = \frac{nv}{2L_0}$. The third overtone is the 4th harmonic: $f_0 = \frac{4v}{2L_0} = \frac{2v}{L_0}$.

Step 3: Equate:

$$\frac{7v}{4L_c} = \frac{2v}{L_0} \Rightarrow \frac{L_c}{L_0} = \frac{7}{8}$$

Step 4: Check the Other Options:

8 : 7 is the inverse. 5 : 3 and 3 : 2 would come from using the wrong harmonic numbers, such as 5th for the closed pipe and 3rd for the open pipe. So (B) is correct.

Final Answer:

The ratio is 7 : 8, option (B).

(B) 7 : 8

Quick Tip: Third overtone is the 7th harmonic for a closed pipe and the 4th harmonic for an open pipe.

20. A tuning fork gives 5 beats per second with a 33 cm length of sonometer wire. If the length of the wire is shortened by 1 cm, the number of beats is still the same. The frequency of the fork is

- (A) 320 Hz
- (B) 325 Hz
- (C) 330 Hz
- (D) 340 Hz

Correct Answer: (B) 325 Hz

Solution:

Step 1: Set Up:

For a sonometer wire, $f \propto \frac{1}{L}$. Shortening the wire from 33 cm to 32 cm raises its frequency. Let the fork frequency be n .

Step 2: Use the Beat Condition:

With 33 cm the wire frequency is f_1 and with 32 cm it is $f_2 > f_1$. Both give 5 beats, so the fork lies between them: $n - f_1 = 5$ and $f_2 - n = 5$. So $f_1 = n - 5$ and $f_2 = n + 5$.

Step 3: Ratio of Frequencies:

$$\frac{f_2}{f_1} = \frac{33}{32} \Rightarrow \frac{n+5}{n-5} = \frac{33}{32}$$

$$32n + 160 = 33n - 165 \Rightarrow n = 325 \text{ Hz}$$

Step 4: Check:

$f_1 = 320 \text{ Hz}$ and $f_2 = 330 \text{ Hz}$, and $330/320 = 33/32$. Both give 5 beats with the 325 Hz fork. So (B) is correct.

Final Answer:

The frequency of the fork is 325 Hz, option (B).

(B) 325 Hz

Quick Tip: Wire frequency is inversely proportional to length; equal beats on both sides put the fork frequency between the two wire frequencies.

21. A set of 14 tuning forks is arranged in a series of increasing frequencies. Each fork produces 'x' beats per second with the preceding fork and the last fork is an octave of the first fork. If the seventh fork has the frequency of 114 Hz, the value of x is

- (A) 4
- (B) 5
- (C) 6
- (D) 7

Correct Answer: (C) 6

Solution:

Step 1: Arithmetic Progression:

Let the first fork have frequency f_1 . Each next fork is x Hz higher, so $f_n = f_1 + (n - 1)x$.

Step 2: Use the Octave Condition:

The 14th fork is an octave of the first: $f_{14} = 2f_1$, so $f_1 + 13x = 2f_1$
 $\Rightarrow f_1 = 13x$.

Step 3: Use the 7th Fork:

$$f_7 = f_1 + 6x = 13x + 6x = 19x = 114 \text{ Hz.}$$

Step 4: Solve:

$$x = \frac{114}{19} = 6 \text{ Hz}$$

Check: $f_1 = 78$ Hz and $f_{14} = 78 + 78 = 156$ Hz, which is double. So (C) is correct.

Final Answer:

The value of x is 6, option (C).

(C) 6

Quick Tip: Frequencies form an arithmetic progression; the last is twice the first.

22. Which one of the following statements regarding pitch of sound is 'WRONG' ?

- (A) Pitch refers to sharpness of sound.
- (B) Tone refers to the single frequency of that wave.
- (C) High pitch sound need not be louder.
- (D) In general male sound is sharper than that of a female.

(A) D

(B) A

(C) C

(D) B

Correct Answer: (A) D

Solution:

Step 1: Statement (A):

"Pitch refers to sharpness of sound" is correct. Pitch is the sensation that tells us whether a sound is sharp (high) or flat (low), and it depends on frequency.

Step 2: Statement (B):

"Tone refers to the single frequency of that wave" is correct. A tone is a sound of a single frequency.

Step 3: Statement (C):

"High pitch sound need not be louder" is correct, because loudness depends on amplitude and pitch depends on frequency.

Step 4: Statement (D):

"In general male sound is sharper than that of a female" is wrong. A female voice has a higher frequency, so it is sharper (higher pitched)

than a male voice. So statement (D) is the WRONG one.

Step 5: Match the Choice:

The answer options are labelled differently: option A of the question corresponds to statement (D).

Final Answer:

Statement (D) is wrong, which is option A of the question.

(A) statement (D)

Quick Tip: Pitch is the sharpness of a sound and is set by frequency, not loudness; a female voice is sharper than a male voice.



23. ' n ' small spherical drops of the same size which are charged to ' V ' volt each coalesce to form a single big drop. The potential of a big drop is

- (A) nV
- (B) $\frac{V}{n}$
- (C) $(n^{\frac{1}{3}})V$
- (D) $(n^{\frac{2}{3}})V$

Correct Answer: (D) $(n^{\frac{2}{3}})V$

Solution:

Step 1: Charge and Radius:

Each small drop has charge q and radius r , with $V = \frac{kq}{r}$. The big drop has charge $Q = nq$. Volume is conserved: $\frac{4}{3}\pi R^3 = n \cdot \frac{4}{3}\pi r^3$, so $R = n^{1/3}r$.

Step 2: Potential of the Big Drop:

$$V' = \frac{kQ}{R} = \frac{k nq}{n^{1/3}r} = n^{2/3} \frac{kq}{r} = n^{2/3}V$$

Step 3: Check the Options:

nV would hold if the radius did not change. V/n is a decrease and $n^{1/3}V$ forgets that charge also adds up. So (D) is correct.

Final Answer:

The potential of the big drop is $n^{2/3}V$, option (D).

(D) $n^{2/3}V$

Quick Tip: Charge adds up and the radius scales as $n^{1/3}$.

24. Two electric dipoles of moment P and $8P$ are placed in opposite directions on a line at a distance of 24 cm. The electric field will be zero at the point between the dipoles whose distance from the dipole of moment P will be x_1 . When $8P$ is replaced by $27P$ keeping all other quantities the same, the distance x_1 now becomes x_2 . The difference $|x_2 - x_1|$ is (All the distances are measured from the centers of the dipole.)

- (A) 2 cm
- (B) 4 cm
- (C) 6 cm
- (D) 8 cm

Correct Answer: (A) 2 cm

Solution:

Step 1: Field of a Dipole on Its Axis:

The magnitude is $E = \frac{2kp}{x^3}$, proportional to $\frac{p}{x^3}$. With the dipoles placed in opposite directions, the two fields point opposite between them, so they can cancel.

Step 2: First Case:

Let x_1 be the distance from the dipole P , so the distance from $8P$ is $24 - x_1$.

$$\frac{P}{x_1^3} = \frac{8P}{(24 - x_1)^3} \Rightarrow \frac{24 - x_1}{x_1} = 2 \Rightarrow x_1 = 8 \text{ cm}$$

Step 3: Second Case:

With $27P$: $\frac{24 - x_2}{x_2} = \sqrt[3]{27} = 3 \Rightarrow x_2 = 6 \text{ cm}$.

Step 4: Difference:

$|x_2 - x_1| = |6 - 8| = 2 \text{ cm}$. Option (A) is correct. Options 4, 6 and 8 cm would need $x_2 = 4, 2, 0$ or similar values that do not satisfy the cube-root relation.

Final Answer:

The difference is 2 cm, option (A).

(A) 2 cm

Quick Tip: On the axis a dipole field goes as p/x^3 ; the fields cancel where the two magnitudes are equal.



25. A parallel plate capacitor of capacitance ' C ' is connected to a battery and charged to a potential ' V '. Another capacitor of capacitance ' $3C$ ' is charged to a potential ' $3V$ '. The charging battery is then disconnected and both the capacitors are connected in parallel to each other such that positive terminal of one is connected to negative terminal of the other. The final energy of the configuration is

- (A) $1.5 CV^2$
- (B) $6.5 CV^2$
- (C) $8.0 CV^2$
- (D) $18.0 CV^2$

Correct Answer: (C) $8.0 CV^2$

Solution:

Step 1: Initial Charges:

First capacitor: $q_1 = CV$. Second capacitor: $q_2 = (3C)(3V) = 9CV$.

Step 2: Connect Positive to Negative:

When the positive plate of one is joined to the negative plate of the other, the charges partly cancel. The net charge is

$$q = 9CV - CV = 8CV$$

The equivalent capacitance in parallel is $C + 3C = 4C$.

Step 3: Common Voltage:

$$V' = \frac{q}{C_{eq}} = \frac{8CV}{4C} = 2V$$

Step 4: Final Energy:

$$U = \frac{1}{2}C_{eq}V'^2 = \frac{1}{2}(4C)(2V)^2 = 8CV^2$$

The values $1.5CV^2$, $6.5CV^2$ and $18CV^2$ do not follow from a net charge of $8CV$ on a total capacitance of $4C$. So (C) is correct.

Final Answer:

The final energy is $8.0 CV^2$, option (C).

(C) $8.0 CV^2$

Quick Tip: Opposite polarity means the charges subtract; then find the common voltage and the stored energy.

◆

26. A slab of material of dielectric constant K has the same area as the plates of a parallel plate capacitor but has a thickness $(4/5)d$, where d is the separation of the plates. The capacitance in the presence and absence of dielectric are C and C_0 respectively. The ratio (C/C_0) is

- (A) $\frac{K+4}{5K}$
 (B) $\frac{4K}{K+5}$
 (C) $\frac{5K}{K+4}$
 (D) $\frac{K+5}{4K}$

Correct Answer: (C) $\frac{5K}{K+4}$

Solution:

Step 1: Formula With a Partial Slab:

For a slab of thickness t and dielectric constant K in a gap d :

$$C = \frac{\epsilon_0 A}{d - t + \frac{t}{K}}$$

Step 2: Substitute:

With $t = \frac{4d}{5}$: $d - t = \frac{d}{5}$, and $\frac{t}{K} = \frac{4d}{5K}$. So

$$C = \frac{\epsilon_0 A}{\frac{d}{5} + \frac{4d}{5K}} = \frac{\epsilon_0 A}{d} \cdot \frac{5K}{K+4}$$

Step 3: Ratio:

Without the dielectric $C_0 = \frac{\epsilon_0 A}{d}$. So

$$\frac{C}{C_0} = \frac{5K}{K+4}$$

Step 4: Check the Options:

For $K = 1$ (no dielectric) the ratio must be 1. Option (C) gives $5/5 = 1$. Option (A) gives $5/5 = 1$ too, but for large K the capacitance should approach $5 C_0$ because only the $d/5$ air gap remains. Option (C) tends to 5, while (A) tends to $1/5$. So (C) is correct.

Final Answer:

The ratio is $\frac{5K}{K+4}$, option (C).

$$\boxed{(C) \frac{5K}{K+4}}$$

Quick Tip: The gap not filled by the slab and the slab itself act like two capacitors in series.



27. In a meter bridge experiment, the balance point is obtained at length ' l_1 ' cm from left hand when resistances in the left gap and right gap are 15Ω and $R\Omega$ respectively. When the resistance R is shunted with equal resistance the new balance point is at $(1.6 l_1)$. The resistance R in ohm is

- (A) 25
- (B) 30
- (C) 45
- (D) 60

Correct Answer: (C) 45

Solution:

Step 1: Balance Condition:

For a meter bridge, $\frac{P}{Q} = \frac{l}{100-l}$.

First case: $\frac{15}{R} = \frac{l_1}{100 - l_1}$.

Step 2: After Shunting:

Shunting R with an equal resistance gives $\frac{R}{2}$. Now $\frac{15}{R/2} = \frac{30}{R}$
 $= \frac{1.6l_1}{100 - 1.6l_1}$.

Step 3: Divide the Two Equations:

$$\frac{30/R}{15/R} = 2 = \frac{1.6l_1(100 - l_1)}{l_1(100 - 1.6l_1)} \Rightarrow 2(100 - 1.6l_1) = 1.6(100 - l_1)$$

$$200 - 3.2l_1 = 160 - 1.6l_1 \Rightarrow 1.6l_1 = 40 \Rightarrow l_1 = 25 \text{ cm}$$

Step 4: Find R:

$$\frac{15}{R} = \frac{25}{75} = \frac{1}{3} \Rightarrow R = 45 \Omega. \text{ Check: new balance } 1.6 \times 25 = 40 \text{ cm, and}$$

$$\frac{30}{45} = \frac{40}{60}. \text{ True. So (C) is correct.}$$

Final Answer:

The resistance is 45 ohm, option (C).

(C) 45 Ω

Quick Tip: Use the balance condition for both situations and eliminate l_1 .

28. In a potentiometer circuit when two cells of e.m.f. 1.5 volt and 1.2 volt are connected to assist each other, balancing length is 270 cm. What will be the balancing length in cm when these two cells are connected in opposition?

- (A) 90
- (B) 81
- (C) 60
- (D) 30

Correct Answer: (D) 30

Solution:

Step 1: Principle:

In a potentiometer the balancing length is proportional to the e.m.f.:
 $E \propto l$.

Step 2: Cells Assisting:

Net e.m.f. = $1.5 + 1.2 = 2.7$ V and $l = 270$ cm. So the potential gradient is $\frac{2.7}{270} = 0.01$ V/cm.

Step 3: Cells in Opposition:

Net e.m.f. = $1.5 - 1.2 = 0.3$ V. Balancing length = $\frac{0.3}{0.01} = 30$ cm.

Step 4: Check the Other Options:

90 cm, 81 cm and 60 cm correspond to e.m.f.s of 0.9 V, 0.81 V and 0.6 V, none of which equals 0.3 V.

Final Answer:

The balancing length is 30 cm, option (D).

(D) 30 cm

Quick Tip: Balancing length is proportional to the net e.m.f.

29. The voltage across a lamp is (6.0 ± 0.3) volt and the current passing through it is (4.0 ± 0.1) ampere. The power consumed in watt will be (using percentage error)

- (A) (24.0 ± 1.8)
- (B) (24.0 ± 0.75)
- (C) (22.0 ± 0.4)
- (D) (18.0 ± 0.4)

Correct Answer: (A) (24.0 ± 1.8)

Solution:

Step 1: Power:

$$P = VI = 6.0 \times 4.0 = 24.0 \text{ W.}$$

Step 2: Percentage Errors:

$$\frac{\Delta V}{V} \times 100 = \frac{0.3}{6.0} \times 100 = 5\%. \quad \frac{\Delta I}{I} \times 100 = \frac{0.1}{4.0} \times 100 = 2.5\%.$$

Step 3: Add for a Product:

$$\text{For a product, } \frac{\Delta P}{P} = \frac{\Delta V}{V} + \frac{\Delta I}{I} = 5\% + 2.5\% = 7.5\%.$$

Step 4: Absolute Error:

$$\Delta P = 0.075 \times 24.0 = 1.8 \text{ W. So } P = (24.0 \pm 1.8) \text{ W, which is option (A).}$$

Final Answer:

The power is (24.0 ± 1.8) W, option (A).

$(A) (24.0 \pm 1.8)$

Quick Tip: Multiply V and I; percentage errors add for a product.

30. A diamagnetic liquid is filled in a U-tube. One arm of a U-tube is placed in an external magnetic field with the meniscus in line with the field. The level of the liquid in that arm will

- (A) rise.
- (B) fall.
- (C) oscillate slowly.
- (D) remains as it is.

Correct Answer: (B) fall.

Solution:**Step 1: Property of Diamagnetic Substances:**

A diamagnetic substance has a negative susceptibility. It is weakly repelled by a magnet and tends to move from a strong field region to a weak field region.

Step 2: Apply:

The meniscus of the liquid sits in the strong magnetic field. The liquid moves away from the stronger field, so the level of the liquid in that arm goes down.

Step 3: Compare With Paramagnetic:

A paramagnetic liquid would be attracted to the strong field, and the level would rise. The diamagnetic liquid behaves in the opposite way, so the level falls.

Final Answer:

The liquid level in that arm falls, option (B).

(B) fall

Quick Tip: Diamagnetic materials are pushed toward weaker field regions.

31. The frequency of oscillation of a small magnet in a magnetic field of induction 'B' is ' n '. If the frequency of oscillations of the same magnet in a field of induction 'X' falls to ($\frac{n}{3}$), the value of 'X' is

- (A) $3B$
- (B) $9B$
- (C) $\frac{B}{3}$
- (D) $\frac{B}{9}$

Correct Answer: (D) $\frac{B}{9}$

Solution:**Step 1: Formula:**

For a small magnet in a field, $T = 2\pi\sqrt{\frac{I}{MB}}$, so $f = \frac{1}{2\pi}\sqrt{\frac{MB}{I}}$. Thus $f \propto \sqrt{B}$.

Step 2: Ratio:

$$\frac{f_X}{f_B} = \sqrt{\frac{X}{B}} = \frac{n/3}{n} = \frac{1}{3}$$

Step 3: Solve:

$$\frac{X}{B} = \frac{1}{9} \Rightarrow X = \frac{B}{9}.$$

Step 4: Check the Options:

$3B$ and $9B$ would raise the frequency. $B/3$ forgets the square root. Only $B/9$ makes the frequency fall to $n/3$.

Final Answer:

The field is $B/9$, option (D).

(D) $\frac{B}{9}$

Quick Tip: Frequency of a magnet oscillating in a field varies as the square root of B .

32. Two circular coils X (smaller) and Y (bigger), each having a single turn, carry equal currents in the same direction and subtend the same angle at point 'O' along the axis of the coil. The distances of the centers of coil to point 'O' are d and $(\frac{d}{2})$ for coil Y and X respectively. The radii of coils Y and X are $(2r)$ and (r) respectively. The magnetic induction due to the bigger coil at point 'O' is B_y and that due to smaller coil X at point 'O' is B_x . ($d \gg r$) The relation between B_x and B_y is

- (A) $B_y = B_x$
- (B) $B_y = 2B_x$
- (C) $B_x = 2B_y$
- (D) $B_x = 4B_y$

Correct Answer: (C) $B_x = 2B_y$

Solution:

Step 1: Axial Field of a Circular Coil:

$$B = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}$$

For $x \gg R$ this becomes $B \approx \frac{\mu_0 I R^2}{2x^3}$. The condition $d \gg r$ lets us use this form for both coils.

Step 2: Coil Y:

Radius $2r$, distance d : $B_y = \frac{\mu_0 I (2r)^2}{2d^3} = \frac{4\mu_0 I r^2}{2d^3}$.

Step 3: Coil X:

Radius r , distance $d/2$: $B_x = \frac{\mu_0 I r^2}{2(d/2)^3} = \frac{8\mu_0 I r^2}{2d^3}$.

Step 4: Compare:

$$\frac{B_x}{B_y} = \frac{8}{4} = 2 \Rightarrow B_x = 2B_y$$

This is option (C). The two coils subtend the same angle at O because $\frac{2r}{d} = \frac{r}{d/2}$, but that does not make the fields equal, since the field also

depends on the distance.

Final Answer:

$B_x = 2B_y$, option (C).

$$(C) B_x = 2B_y$$

Quick Tip: Use the axial field formula and the condition d much larger than r .

33. Two thin long parallel wires, W_1 and W_2 separated by distance ' a ', carry currents i and $3i$ respectively in the same direction. The magnitude of the force per unit length exerted by wire W_1 on wire W_2 is

- (A) $\frac{\mu_0 i^2}{2\pi a}$
- (B) $\frac{\mu_0 i^2}{\pi a}$
- (C) $\frac{3\mu_0 i^2}{2\pi a}$
- (D) $\frac{2\mu_0 i^2}{3\pi a}$

Correct Answer: (C) $\frac{3\mu_0 i^2}{2\pi a}$

Solution:

Step 1: Formula:

Force per unit length between two long parallel wires carrying currents i_1 and i_2 , at separation a , is

$$\frac{F}{l} = \frac{\mu_0 i_1 i_2}{2\pi a}$$

Step 2: Substitute:

With $i_1 = i$ and $i_2 = 3i$:

$$\frac{F}{l} = \frac{\mu_0(i)(3i)}{2\pi a} = \frac{3\mu_0 i^2}{2\pi a}$$

Step 3: Direction:

The currents are in the same direction, so the force is attractive. Its magnitude is the same on both wires (Newton's third law).

Step 4: Check the Options:

Option (A) uses $i \cdot i$, forgetting the 3. Option (B) has πa instead of $2\pi a$ with the 1 factor. Option (D) has no basis in the formula.

Final Answer:

The force per unit length is $\frac{3\mu_0 i^2}{2\pi a}$, option (C).

(C) $\frac{3\mu_0 i^2}{2\pi a}$

Quick Tip: The force per length between parallel wires is $\mu_0 i_1 i_2 / (2\pi a)$.

34. Two different coils have self-inductance $3L$ and L . The current in both the coils is increased at the same constant rate. At certain instant of time, the power given to the two coils is same. At that time there was current and voltage induced in the two coils. At the same instant, the ratio of energy stored in the first coil to that in the second coil is

- (A) 1 : 9
- (B) 1 : 3
- (C) 3 : 1
- (D) 9 : 1

Correct Answer: (B) 1 : 3

Solution:

Step 1: Power Delivered to an Inductor:

The induced emf is $\varepsilon = L \frac{di}{dt}$, and the power given to the coil is $P = \varepsilon i = L \frac{di}{dt} i$.

Step 2: Equal Power, Equal Rate:

Both currents rise at the same rate, so $\frac{di}{dt}$ is the same. Equal power gives

$$(3L)i_1 = (L)i_2 \Rightarrow i_2 = 3i_1$$

Step 3: Energy Stored:

$$U = \frac{1}{2} Li^2.$$

$$\frac{U_1}{U_2} = \frac{\frac{1}{2}(3L)i_1^2}{\frac{1}{2}L(3i_1)^2} = \frac{3}{9} = \frac{1}{3}$$

Step 4: Check the Options:

1 : 9 would be the ratio if we compared i^2 alone, and 3 : 1 and 9 : 1 are inverses or wrong. So (B) is correct.

Final Answer:

The energy ratio is 1 : 3, option (B).

(B) 1 : 3

Quick Tip: Power is $L i (di/dt)$; equal power and equal di/dt give the ratio of currents.



35. Consider two coils in which a current in one coil carrying 6 A causes the change in the flux in the second coil 12×10^{-4} weber/turn. The second coil has 2000 turns. The mutual inductance between the coils is

- (A) 0.2 H
- (B) 0.3 H
- (C) 0.4 H
- (D) 2.4 H

Correct Answer: (C) 0.4 H

Solution:**Step 1: Formula:**

Flux linked with the second coil is $N_2\phi_2$. Mutual inductance is M
 $= \frac{N_2\phi_2}{I_1}$.

Step 2: Substitute:

$$M = \frac{2000 \times 12 \times 10^{-4}}{6} = \frac{2.4}{6} = 0.4 \text{ H}$$

Step 3: Check the Other Options:

0.2 H and 0.3 H come from using fewer turns, and 2.4 H is the total flux linkage without dividing by the current of 6 A.

Final Answer:

The mutual inductance is 0.4 H, option (C).

(C) 0.4 H

Quick Tip: Mutual inductance is N times the flux per turn divided by the current.



36. Magnetic flux linked with the coil in weber is given by the equation $\Phi = 5t^2 + 6t + 11$. The e.m.f. induced in the coil in the 5th second will be

- (A) 10 V
- (B) 51 V
- (C) 102 V
- (D) 166 V

Correct Answer: (B) 51 V

Solution:

Step 1: Faraday's Law:

The induced e.m.f. is the rate of change of flux. For the 5th second, which runs from $t = 4$ s to $t = 5$ s, the e.m.f. is the average rate over this second:

$$\varepsilon = \left| \frac{\Phi(5) - \Phi(4)}{5 - 4} \right|$$

Step 2: Compute the Flux Values:

$$\Phi(5) = 5(25) + 6(5) + 11 = 125 + 30 + 11 = 166 \text{ Wb.}$$

$$\Phi(4) = 5(16) + 6(4) + 11 = 80 + 24 + 11 = 115 \text{ Wb.}$$

Step 3: Average e.m.f.:

$$\varepsilon = \frac{166 - 115}{1} = 51 \text{ V}$$

Step 4: Check the Other Options:

10 V is $d\Phi/dt$ at $t = 1$ s minus 6, which is not meaningful. 102 V and 166 V do not match any rate of change of this flux over the 5th second.

Final Answer:

The e.m.f. induced in the 5th second is 51 V, option (B).

(B) 51 V

Quick Tip: The e.m.f. in the 5th second is the average rate of change of flux between $t = 4$ s and $t = 5$ s.

37. For a series LCR circuit inductive reactance X_L is equal to resistance R and also equal to twice the capacitive reactance X_C . The impedance of the circuit and the phase difference between voltage V and current i are respectively

(A) $\sqrt{5}R, \tan^{-1}(\frac{1}{2})$

(B) $\sqrt{5}R, \tan^{-1}(2)$

(C) $\frac{\sqrt{5}R}{2}, \tan^{-1}(\frac{1}{2})$

(D) $\frac{\sqrt{5}R}{2}, \tan^{-1}(2)$

Correct Answer: (C) $\frac{\sqrt{5}R}{2}, \tan^{-1}(\frac{1}{2})$

Solution:

Step 1: Reactances:

Let $R = X_L$ and $X_C = \frac{X_L}{2} = \frac{R}{2}$. The net reactance is

$$X = X_L - X_C = R - \frac{R}{2} = \frac{R}{2}$$

Step 2: Impedance:

$$Z = \sqrt{R^2 + X^2} = \sqrt{R^2 + \frac{R^2}{4}} = \frac{\sqrt{5}R}{2}$$

Step 3: Phase Difference:

$$\tan \phi = \frac{X_L - X_C}{R} = \frac{R/2}{R} = \frac{1}{2} \Rightarrow \phi = \tan^{-1}\left(\frac{1}{2}\right)$$

Step 4: Check the Other Options:

Options (A) and (B) have $Z = \sqrt{5}R$, which would be right if $X = 2R$.

Option (D) has the right impedance but the phase $\tan^{-1} 2$, which would need $X = 2R$.

Final Answer:

$Z = \frac{\sqrt{5}R}{2}$ and the phase is $\tan^{-1}(1/2)$, option (C).

$$\boxed{(C) \frac{\sqrt{5}R}{2}, \tan^{-1}\left(\frac{1}{2}\right)}$$

Quick Tip: Net reactance is X_L minus X_C ; impedance is the root of R squared plus X squared.



38. An alternating voltage $e = 150\sqrt{2}\sin 100t$ volt is applied to a capacitor of capacity $2\mu\text{F}$. The root mean square value of current in the circuit is

- (A) 300 mA
- (B) 150 mA
- (C) 30 mA
- (D) 15 mA

Correct Answer: (C) 30 mA

Solution:**Step 1: Read the Voltage:**

$e = 150\sqrt{2}\sin 100t$ gives peak voltage $150\sqrt{2}$ V and $\omega = 100$ rad/s. The r.m.s. voltage is $\frac{150\sqrt{2}}{\sqrt{2}} = 150$ V.

Step 2: Capacitive Reactance:

$$X_C = \frac{1}{\omega C} = \frac{1}{100 \times 2 \times 10^{-6}} = 5000 \, \Omega$$

Step 3: R.M.S. Current:

$$I_{rms} = \frac{V_{rms}}{X_C} = \frac{150}{5000} = 0.03 \, \text{A} = 30 \, \text{mA}$$

Step 4: Check the Other Options:

300 mA and 150 mA would need $X_C = 500 \, \Omega$ and $1000 \, \Omega$. 15 mA would need $10,000 \, \Omega$. Only 5000 ohm gives 30 mA.

Final Answer:

The r.m.s. current is 30 mA, option (C).

(C) 30 mA

Quick Tip: Find the capacitive reactance from omega and C, then divide the r.m.s. voltage by it.

39. In an LCR circuit at resonance, the a.c. source current is

- A) maximum in a series LCR circuit only.
- B) maximum in a parallel LCR circuit only.
- C) maximum in both series and parallel LCR circuits.
- D) minimum in both series and parallel LCR circuits.

The correct answer is

- (A) B
- (B) D
- (C) C
- (D) A

Correct Answer: (D) A

Solution:

Step 1: Series Resonance:

At resonance $X_L = X_C$. In a series circuit the impedance is $Z = R$, the minimum possible value, so the current drawn from the source is maximum.

Step 2: Parallel Resonance:

In a parallel LCR circuit, the inductor and capacitor branch currents are nearly equal and opposite at resonance. The current drawn from the source is minimum, because the impedance is maximum.

Step 3: Statements:

- (A) Maximum in a series LCR circuit only: true.
- (B) Maximum in a parallel circuit only: not true.
- (C) Maximum in both: not true, since the parallel circuit draws minimum current.
- (D) Minimum in both: not true, since the series circuit draws

maximum current.

Step 4: Match the Choice:

Statement (A) is correct. In the answer list, the option that gives statement (A) is option D.

Final Answer:

Statement (A) is correct, which is option D.

(D) statement (A)

Quick Tip: At resonance the series circuit has minimum impedance (maximum current) while the parallel circuit has maximum impedance (minimum current).



40. A glass slab of thickness 6.0 cm is placed on the piece of paper on which an inkdot is marked. By how much distance would an inkdot appear to be raised? The velocity of light in glass is $2 \times 10^8 \text{ ms}^{-1}$ and that in air is $3 \times 10^8 \text{ ms}^{-1}$.

- (A) 2.0 cm
- (B) 3.0 cm
- (C) 4.0 cm
- (D) 5.0 cm

Correct Answer: (A) 2.0 cm

Solution:

Step 1: Refractive Index:

$$\mu = \frac{c}{v} = \frac{3 \times 10^8}{2 \times 10^8} = 1.5$$

Step 2: Normal Shift:

For a slab of thickness t , viewed from above, the apparent shift is

$$\text{shift} = t \left(1 - \frac{1}{\mu} \right) = 6 \left(1 - \frac{1}{1.5} \right) = 6 \times \frac{1}{3} = 2.0 \text{ cm}$$

Step 3: Check the Other Options:

3.0 cm would need $\mu = 2$, 4.0 cm would need $\mu = 3$, and 5.0 cm would need $\mu = 6$. With $\mu = 1.5$, only 2.0 cm fits.

Final Answer:

The ink dot appears raised by 2.0 cm, option (A).

(A) 2.0 cm

Quick Tip: Apparent shift is $t(1 - 1/\mu)$, with $\mu = c/v$.

41. In Young's double slit experiment, the wavelength of light used is λ . The intensity on the screen at a point for path difference ' λ ' is 'X'. The intensity at the point for path difference $(\frac{\lambda}{6})$ is $(\cos 180^\circ = -1, \cos 30^\circ = \frac{\sqrt{3}}{2})$

- (A) $\frac{X}{6}$
- (B) $\frac{X}{2}$
- (C) $\frac{3X}{4}$
- (D) $\frac{4X}{3}$

Correct Answer: (C) $\frac{3X}{4}$

Solution:

Step 1: Intensity Formula:

$$I = 4I_0 \cos^2 \frac{\phi}{2}, \text{ where } \phi = \frac{2\pi}{\lambda} \Delta.$$

Step 2: Path Difference lambda:

$\phi = 2\pi$, so $I = 4I_0 \cos^2 \pi = 4I_0$. This is given as X , so $4I_0 = X$.

Step 3: Path Difference lambda/6:

$$\phi = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{6} = \frac{\pi}{3}. \text{ Then}$$

$$I = 4I_0 \cos^2 \frac{\pi}{6} = 4I_0 \times \frac{3}{4} = 3I_0$$

Step 4: Express in X:

$3I_0 = \frac{3}{4}(4I_0) = \frac{3X}{4}$. So (C) is correct. Option (A) $X/6$ and (B) $X/2$ correspond to $\cos^2$ values of $1/6$ and $1/2$, and (D) $4X/3$ exceeds the maximum intensity X .

Final Answer:

The intensity is $\frac{3X}{4}$, option (C).

$$(C) \frac{3X}{4}$$

Quick Tip: Intensity is $I = 4 I_0 \cos^2(\phi/2)$, with $\phi = 2\pi / \lambda$ times the path difference.



42. In a single slit diffraction pattern, the distance between the plane of the slit and the screen is 1.4 m. The width of the slit is 0.66 mm. The second maximum is formed at the distance of 2.8 mm from the center of the screen. The wavelength of light used is

- (A) 6500Å
- (B) 5600Å
- (C) 5280Å
- (D) 4600Å

Correct Answer: (C) 5280Å

Solution:

Step 1: Condition for Secondary Maxima:

In a single slit pattern, the n th secondary maximum is at $a \sin \theta = (n + \frac{1}{2})\lambda$. For the second maximum, $n = 2$: $a \sin \theta = \frac{5}{2}\lambda$.

Step 2: Small Angle:

$\sin \theta \approx \frac{y}{D}$, with $y = 2.8$ mm and $D = 1.4$ m. So $\frac{ay}{D} = \frac{5}{2}\lambda$.

Step 3: Solve for the Wavelength:

$$\lambda = \frac{2ay}{5D} = \frac{2 \times 0.66 \times 10^{-3} \times 2.8 \times 10^{-3}}{5 \times 1.4}$$

$$\lambda = \frac{3.696 \times 10^{-6}}{7} = 5.28 \times 10^{-7} \text{ m} = 5280 \text{ \AA}$$

Step 4: Check the Other Options:

Using 2λ or $3\lambda/2$ instead of $5\lambda/2$ would give 6600 Å or 8800 Å, which do not match the options. So (C) is correct.

Final Answer:

The wavelength is 5280 Å, option (C).

(C) 5280 Å

Quick Tip: Secondary maxima occur at a $\sin \theta = (n + 1/2) \lambda$, so the second maximum has $n = 2$.

43. In Young's double slit experiment, for the n^{th} dark fringe ($n = 1, 2, 3, \dots$) the phase difference of the interfering waves in radian will be

- (A) $n \cdot \frac{\pi}{2}$
- (B) $(2n + 1)\pi$
- (C) $(2n - 1)\pi$
- (D) $(2n - 1)\frac{\pi}{2}$

Correct Answer: (C) $(2n - 1)\pi$

Solution:

Step 1: Path Difference:

For the n th dark fringe ($n = 1, 2, 3, \dots$), the path difference is $\Delta = (2n - 1)\frac{\lambda}{2}$.

Step 2: Phase Difference:

$$\phi = \frac{2\pi}{\lambda} \Delta = \frac{2\pi}{\lambda} (2n - 1) \frac{\lambda}{2} = (2n - 1)\pi$$

Step 3: Check the Options:

For $n = 1$ the first dark fringe has phase π . Option (C) gives π . Option (B) $(2n + 1)\pi$ gives 3π for $n = 1$, which is the second dark fringe. Option (D) $(2n - 1)\pi/2$ gives $\pi/2$ for $n = 1$, which is not destructive. Option (A) $n\pi/2$ gives $\pi/2$ too.

Final Answer:

The phase difference is $(2n - 1)\pi$, option (C).

$(C) (2n - 1)\pi$

Quick Tip: Dark fringes occur at path difference $(2n - 1)\lambda/2$, and phase difference is $2\pi/\lambda$ times path difference.

44. A photoelectric surface is illuminated successively by monochromatic light of wavelength λ and $(\lambda/3)$. If the maximum kinetic energy of the emitted photo electrons in the second case is 4 times that in the first case, the work function of the surface of the material is (h = Planck's constant, c = speed of light)

- (A) $\frac{3hc}{\lambda}$
- (B) $\frac{hc}{3\lambda}$
- (C) $\frac{hc}{2\lambda}$
- (D) $\frac{hc}{\lambda}$

Correct Answer: (B) $\frac{hc}{3\lambda}$

Solution:

Step 1: Einstein's Equation:

$$K = \frac{hc}{\lambda} - \phi.$$

Step 2: Two Cases:

First case: $K_1 = \frac{hc}{\lambda} - \phi$. Second case, wavelength $\frac{\lambda}{3}$: $K_2 = \frac{3hc}{\lambda} - \phi$.

Step 3: Use $K_2 = 4 K_1$:

$$\frac{3hc}{\lambda} - \phi = 4 \left(\frac{hc}{\lambda} - \phi \right) \Rightarrow 3\phi = \frac{hc}{\lambda} \Rightarrow \phi = \frac{hc}{3\lambda}$$

Step 4: Check the Options:

For $\phi = \frac{hc}{3\lambda}$: $K_1 = \frac{2hc}{3\lambda}$ and $K_2 = \frac{8hc}{3\lambda}$, a ratio of 4. This is option (B).

Other options give a different ratio.

Final Answer:

The work function is $\frac{hc}{3\lambda}$, option (B).

$$\boxed{(B) \frac{hc}{3\lambda}}$$

Quick Tip: Apply Einstein's equation twice and use the factor of 4 between the kinetic energies.

45. According to Einstein's photoelectric equation, the graph of the kinetic energy of the emitted photoelectrons versus the frequency of incident radiation gives a straight line whose slope

- (A) depends on the intensity of incident radiation.
- (B) depends on the nature of the metal used.
- (C) is the same for all metals and independent of the intensity of radiation.
- (D) depends on both the intensity of incident radiation and the nature of metal used.

Correct Answer: (C) is the same for all metals and independent of the intensity of radiation.

Solution:**Step 1: Einstein's Equation:**

$K_{max} = h\nu - \phi$. Plotting K_{max} on the y-axis against ν on the x-axis gives a straight line $y = mx + c$.

Step 2: Slope and Intercept:

The slope is h , Planck's constant. The intercept on the y-axis is $-\phi$, which depends on the metal. The intercept on the frequency axis is the threshold frequency.

Step 3: Check the Statements:

The slope h is a universal constant, so it is the same for all metals. Intensity changes only the number of photoelectrons, not the K_{max} -versus-frequency line. So (C) is correct and (A), (B), (D) are not.

Final Answer:

The slope is the same for all metals and independent of intensity, option (C).

(C)

Quick Tip: From $K = h\nu - \phi$, the slope is Planck's constant.

46. The shortest wavelength in the Balmer series of hydrogen atom is equal to the shortest wavelength in the Brackett series of a hydrogen like atom of atomic number 'Z'. The value of 'Z' is

- (A) 6
- (B) 4
- (C) 3
- (D) 2

Correct Answer: (D) 2

Solution:

Step 1: Series Limit Formula:

For a hydrogen-like atom, $\frac{1}{\lambda} = RZ^2 \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$. The shortest wavelength has $n_2 = \infty$, so $\frac{1}{\lambda_{min}} = \frac{RZ^2}{n_1^2}$.

Step 2: Hydrogen Balmer Limit:

$$n_1 = 2, Z = 1: \frac{1}{\lambda} = \frac{R}{4}.$$

Step 3: Brackett Limit for Atom Z:

$$n_1 = 4: \frac{1}{\lambda} = \frac{RZ^2}{16}.$$

Step 4: Equate:

$$\frac{R}{4} = \frac{RZ^2}{16} \Rightarrow Z^2 = 4 \Rightarrow Z = 2$$

Option (D) is correct. For $Z = 3, 4, 6$, the Brackett limit would be at $\frac{9R}{16}, R, \frac{36R}{16}$, none of which equals $R/4$.

Final Answer:

The atomic number is 2, option (D).

(D) 2

Quick Tip: The shortest wavelength of a series corresponds to the transition from infinity.

47. A radioactive sample has half-life of 5 years. The percentage of fraction decayed in 10 years will be

- (A) 25%
- (B) 50%
- (C) 75%
- (D) 90%

Correct Answer: (C) 75%

Solution:

Step 1: Number of Half-Lives:

$$n = \frac{10}{5} = 2.$$

Step 2: Remaining Fraction:

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^2 = \frac{1}{4} = 25\%.$$

Step 3: Decayed Fraction:

$$\text{Decayed} = 100\% - 25\% = 75\%.$$

Step 4: Check the Other Options:

25% is the remaining amount, not the decayed one. 50% is the decay after one half-life. 90% would need more than three half-lives.

Final Answer:

75% of the sample has decayed, option (C).

(C) 75%

Quick Tip: After n half-lives the remaining fraction is $(1/2)^n$.

48. The material used for solar cell should have band gap

- (A) equal to zero.
- (B) equal to 0.08 eV.
- (C) between 1.0 eV to 1.8 eV.
- (D) equal to 2 eV.

- (A) D
- (B) A
- (C) C
- (D) B

Correct Answer: (C) C

Solution:

Step 1: Requirement:

A solar cell must absorb photons from sunlight, whose energy is mostly between about 1 eV and 3 eV. The band gap must be small enough for photons to excite electrons across, but large enough to give a useful voltage.

Step 2: Check the Statements:

- (A) Zero band gap: this is a conductor, with no useful junction voltage.
- (B) 0.08 eV: too small, so thermal energy alone would excite carriers.
- (C) 1.0 to 1.8 eV: matches the visible and near infrared photon energies, as in silicon (1.1 eV) and GaAs (1.4 eV).
- (D) 2 eV: too large, so much of the sunlight is not absorbed.

Step 3: Match the Choice:

Statement (C) is correct, which is option C of the question.

Final Answer:

The band gap should be between 1.0 eV and 1.8 eV, option C.

(C) between 1.0 eV and 1.8 eV

Quick Tip: Solar cell materials have band gaps close to the energy of visible light photons, between 1.0 and 1.8 eV.

49. For a two input logic gate, when the inputs are 'zero' and 'zero' the output is 'one'. When the inputs are 'one' and 'zero' the output is 'zero'. The type of logic gate is

- (A) NAND
- (B) OR
- (C) NOR
- (D) XOR

Correct Answer: (C) NOR

Solution:

Step 1: Given Rows:

Inputs (0, 0) give output 1. Inputs (1, 0) give output 0.

Step 2: Check Each Gate:

NAND: (0, 0) $\rightarrow$ 1 fits, but (1, 0) $\rightarrow$ 1, which does not match.

OR: $(0,0) \rightarrow 0$, which does not match.

NOR: $(0,0) \rightarrow 1$ fits, and $(1,0) \rightarrow 0$ fits.

XOR: $(0,0) \rightarrow 0$, which does not match.

Step 3: Conclusion:

Only the NOR gate satisfies both rows. A NOR gate gives 1 only when both inputs are 0.

Final Answer:

The gate is a NOR gate, option (C).

(C) NOR

Quick Tip: Check the given rows against the truth tables of NAND, OR, NOR and XOR.

50. In n-type semiconductor

- A) pentavalent impurity are dopants and holes are majority carriers.
- B) trivalent impurity are dopants and electrons are minority carriers.
- C) pentavalent impurity are dopants and electrons are majority carriers.
- D) trivalent impurity are dopants and holes are majority carriers.

- (A) A
- (B) C
- (C) B
- (D) D

Correct Answer: (B) C

Solution:

Step 1: Doping:

An n-type semiconductor is made by adding a pentavalent impurity (such as phosphorus or arsenic) to silicon or germanium. Each impurity atom has five valence electrons, of which four form bonds and one is left free.

Step 2: Carriers:

The free electrons are the majority carriers. Holes are the minority carriers.

Step 3: Check the Statements:

(A) Says holes are majority carriers, which describes p-type material, so it does not match.

(B) Trivalent impurity with electrons minority: this describes p-type material.

(C) Pentavalent impurity, electrons majority: this describes n-type material.

(D) Trivalent impurity with holes majority: this describes p-type material.

Step 4: Match the Choice:

Statement (C) is the correct one. In the answer list, the option that gives statement (C) is option B.

Final Answer:

Statement (C) is correct, which is option B.

(B) statement (C)

Quick Tip: n-type material is doped with pentavalent atoms and has electrons as majority carriers.

Chemistry

Questions 51 to 100 | 1 mark each | 50 marks

51. What is the percentage atom economy when formula weight of product is 65 u and the sum of formula weight of reactant is 130 u?

- (A) 65%
- (B) 70%
- (C) 40%
- (D) 50%

Correct Answer: (D) 50%

Solution:

Step 1: Understanding the Concept:

Atom economy tells us what fraction of the mass of all reactants ends up in the desired product.

A higher value means less waste, which is the idea behind green chemistry.

Step 2: Key Formula:

$$\text{Atom economy} = \frac{\text{formula weight of product}}{\text{sum of formula weights of reactants}} \times 100$$

Step 3: Substitute the Values:

Formula weight of product = 65 u.

Sum of formula weights of reactants = 130 u.

$$\text{Atom economy} = \frac{65}{130} \times 100 = 0.5 \times 100 = 50\%$$

Step 4: Check the Other Options:

65% would need the ratio 0.65, which is not what 65/130 gives. 70% and 40% correspond to ratios 0.70 and 0.40, which would need product weights of 91 u and 52 u. Only the ratio 0.5 matches the data, so (D) is correct.

Final Answer:

The atom economy is 50%, which is option (D).

(D) 50%

Quick Tip: Atom economy is the product formula weight divided by the total reactant formula weight, times 100.

52. What quantity of H₂O in gram is present in 0.25 mol of it?

- (A) 4.5 g
- (B) 0.25 g
- (C) 18 g
- (D) 5.4 g

Correct Answer: (A) 4.5 g

Solution:

Step 1: Understanding the Concept:

One mole of a substance has a mass in grams equal to its molar mass.
So the mass of any amount is moles multiplied by molar mass.

Step 2: Find the Molar Mass of Water:

H_2O : $2 \times 1 + 16 = 18$ g per mol.

Step 3: Calculate the Mass:

$$m = n \times M = 0.25 \times 18 = 4.5 \text{ g}$$

Step 4: Check the Other Options:

0.25 g is just the number of moles written as grams, a unit mix-up. 18 g is the mass of one full mole, four times too large. 5.4 g would come from using a wrong molar mass of 21.6 g per mol. So (A) is correct.

Final Answer:

0.25 mol of water has a mass of 4.5 g, which is option (A).

(A) 4.5 g

Quick Tip: Mass equals moles times molar mass, and the molar mass of water is 18 g per mol.

53. What is the wave number of lowest transition associated with Paschen series?

(A) $\bar{\nu} = R_H(\frac{5}{36})\text{cm}^{-1}$

(B) $\bar{\nu} = R_H(\frac{36}{5})\text{cm}^{-1}$

(C) $\bar{\nu} = R_H(\frac{144}{7})\text{cm}^{-1}$

(D) $R_H(\frac{7}{144})\text{cm}^{-1}$

Correct Answer: (D) $R_H(\frac{7}{144})\text{cm}^{-1}$

Solution:

Step 1: Understanding the Concept:

The Paschen series is the set of lines emitted when an electron falls to the level $n_1 = 3$ from higher levels $n_2 = 4, 5, 6, \dots$

The "lowest transition" means the line with the smallest energy, hence the smallest wave number.

Step 2: Key Formula:

$$\bar{\nu} = R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

The difference is smallest when n_2 is closest to n_1 , so take $n_2 = 4$.

Step 3: Calculation:

$$\bar{\nu} = R_H \left(\frac{1}{3^2} - \frac{1}{4^2} \right) = R_H \left(\frac{1}{9} - \frac{1}{16} \right) = R_H \left(\frac{16 - 9}{144} \right) = R_H \frac{7}{144}$$

Step 4: Check the Other Options:

$5/36$ equals $\frac{1}{4} - \frac{1}{9}$, which is the first line of the Balmer series (3 to 2). $36/5$ and $144/7$ are reciprocals of these values, so they are not wave numbers at all. So (D) is correct.

Final Answer:

The lowest-energy Paschen line has wave number $\frac{7}{144} R_H \text{ cm}^{-1}$, option (D).

$$\boxed{(D) R_H \left(\frac{7}{144} \right) \text{ cm}^{-1}}$$

Quick Tip: Paschen series ends at $n = 3$, and its lowest-energy line is the transition from $n = 4$ to $n = 3$.

54. Which among the following molecule exhibits paramagnetism?

- (A) O_2
- (B) O_3
- (C) N_2
- (D) F_2

Correct Answer: (A) O_2

Solution:**Step 1: Understanding the Concept:**

Paramagnetism comes from unpaired electrons. Molecular orbital (MO) theory shows the electron count in each orbital, so we can see which molecule has an unpaired electron.

Step 2: Fill the MOs for O_2 :

O_2 has 16 electrons. The configuration is $\sigma 1s^2 \sigma^* 1s^2 \sigma 2s^2 \sigma^* 2s^2 \sigma 2p_z^2 \pi 2p_x^2 = \pi 2p_y^2 \pi^* 2p_x^1 = \pi^* 2p_y^1$.

The two electrons in the degenerate π^* orbitals stay unpaired (Hund's

rule), so O_2 is paramagnetic.

Step 3: Check the Other Options:

N_2 (14 electrons) fills every orbital up to $\sigma 2p_z^2$ in pairs, so it is diamagnetic. F_2 (18 electrons) has all the π^* orbitals completely filled, so it is diamagnetic. O_3 has all electrons paired in its resonance structures, so it is diamagnetic.

Step 4: Conclusion:

Only O_2 has unpaired electrons.

Final Answer:

Oxygen is paramagnetic because of two unpaired electrons in π^* orbitals, option (A).

(A) O_2

Quick Tip: A molecule is paramagnetic if it has an unpaired electron in its molecular orbitals; draw the MO configuration.

55. Identify correct order for repulsion between electron pair present in valence shell of central atom of molecule?

- (A) $Bp-Bp > Lp-Bp > Lp-Lp$
- (B) $Lp-Lp > Lp-Bp > Bp-Bp$
- (C) $Lp-Bp > Bp-Bp > Lp-Lp$
- (D) $Lp-Lp > Bp-Bp > Lp-Bp$

Correct Answer: (B) $Lp-Lp > Lp-Bp > Bp-Bp$

Solution:

Step 1: Understanding the Concept:

VSEPR theory says electron pairs around the central atom arrange themselves to minimise repulsion. The strength of repulsion depends on how much space each pair occupies.

Step 2: Compare the Pairs:

A lone pair (Lp) is held by only one nucleus. It spreads out more and takes more space around the central atom.

A bond pair (Bp) is shared by two nuclei, so it is pulled along the bond axis and is more compact.

Step 3: Rank the Repulsions:

Lp-Lp repulsion is strongest because both clouds are large.

Lp-Bp repulsion is in the middle.

Bp-Bp repulsion is weakest because both clouds are compact.

$$\text{Lp-Lp} > \text{Lp-Bp} > \text{Bp-Bp}$$

Step 4: Check the Other Options:

Option (A) is exactly reversed. Options (C) and (D) place Bp-Bp above or below Lp-Bp in a way that breaks the pattern; they do not follow the lone pair spreading argument. This order is also why bond angles shrink in NH_3 and H_2O .

Final Answer:

The correct order is $\text{Lp-Lp} > \text{Lp-Bp} > \text{Bp-Bp}$, option (B).

$$(B) \text{ Lp-Lp} > \text{Lp-Bp} > \text{Bp-Bp}$$

Quick Tip: Lone pairs sit closer to the nucleus and spread out more, so they repel most.



56. What is the number of sp^3 hybrid carbon atoms in $\text{HO}(\text{CH}_2)_2\text{CH}(\text{CH}_3)_2$?

- (A) 2
- (B) 3
- (C) 4
- (D) 5

Correct Answer: (D) 5

Solution:

Step 1: Understanding the Concept:

A carbon atom forming only single bonds uses four sigma bonds and is sp^3 hybridised. A double bond makes it sp^2 and a triple bond makes it sp .

Step 2: Expand the Structure:

The compound is $\text{HO} - \text{CH}_2 - \text{CH}_2 - \text{CH}(\text{CH}_3) - \text{CH}_3$, that is 3-methylbutan-1-ol.

Step 3: Count the Carbons:

CH_2 attached to OH: 4 single bonds.

Second CH_2 : 4 single bonds.

CH (branch carbon): 4 single bonds.

Two CH_3 groups: 4 single bonds each.

Total carbons = 5, and all five are sp^3 .

Step 4: Check the Other Options:

2, 3 or 4 would suggest some carbons have multiple bonds, but the molecule has no double or triple bond. So (D) is correct.

Final Answer:

All 5 carbon atoms are sp^3 hybridised, option (D).

(D) 5

Quick Tip: A carbon with four single bonds is sp^3 ; count every carbon in the chain.



57. Which of the following is not an illustration of viscosity?

- (A) Gradation of lubricant oils
- (B) Indication of Cardiovascular disease
- (C) Cleansing action of soap
- (D) Thickening of glass panes of old buildings

Correct Answer: (C) Cleansing action of soap

Solution:

Step 1: Understanding the Concept:

Viscosity is the internal resistance of a liquid to flow. We should look for the option that is not related to flow resistance.

Step 2: Check Option (A):

Lubricant oils are graded by viscosity (for example SAE grades). So (A) illustrates viscosity.

Step 3: Check Option (B):

Blood viscosity changes in cardiovascular disease, and doctors use it as an indicator. So (B) illustrates viscosity.

Step 4: Check Option (C):

Soap cleans by forming micelles that trap grease and make an emulsion. This is a surface effect, not a flow resistance effect. So (C) is NOT an illustration of viscosity.

Step 5: Check Option (D):

Old window glass is thicker at the bottom because glass is a supercooled liquid and has flowed very slowly under gravity. This is a slow-flow, high-viscosity effect, so (D) illustrates viscosity.

Final Answer:

The cleansing action of soap does not illustrate viscosity, so the answer is option (C).

(C) Cleansing action of soap

Quick Tip: Cleansing by soap works through emulsification and micelles, not resistance to flow.

58. Identify from following, the relation between standard free energy change in a reaction and equilibrium constant K_c .

(A) $\Delta G^\circ = RT \ln K_c$

(B) $-\Delta G^\circ = \frac{RT}{\ln K_c}$

(C) $\Delta G^\circ = \frac{\ln K_c}{RT}$

(D) $-\Delta G^\circ = RT \ln K_c$

Correct Answer: (D) $-\Delta G^\circ = RT \ln K_c$

Solution:

Step 1: Understanding the Concept:

At equilibrium the reaction quotient equals the equilibrium constant. Under standard conditions the free energy change is tied to K_c by a logarithmic relation.

Step 2: Key Formula:

From $\Delta G = \Delta G^\circ + RT \ln Q$ and $\Delta G = 0$, $Q = K_c$ at equilibrium:

$$0 = \Delta G^\circ + RT \ln K_c$$

$$-\Delta G^\circ = RT \ln K_c$$

Step 3: Interpret:

If $K_c > 1$, then $\ln K_c > 0$ and ΔG° is negative, so the reaction favours products. This matches common sense.

Step 4: Check the Other Options:

(A) lacks the minus sign, which would give the wrong direction. (B) and (C) put $\ln K_c$ or RT in the denominator, but the relation is a product and not a ratio. So (D) is correct.

Final Answer:

The relation is $-\Delta G^\circ = RT \ln K_c$, option (D).

$$(D) \quad -\Delta G^\circ = RT \ln K_c$$

Quick Tip: Use the standard relation $\Delta G = -RT \ln K$.

59. A gas absorbs 120 J of heat and expands by $300 \times 10^{-6} \text{ m}^3$ against a constant external pressure of $2 \times 10^5 \text{ N m}^{-2}$. What will be the change in internal energy of the system ?

- (A) 180 J
- (B) 240 J
- (C) 120 J
- (D) 60 J

Correct Answer: (D) 60 J

Solution:**Step 1: Understanding the Concept:**

By the first law, $\Delta U = q + w$, where w is work done on the system. When the gas expands, work is done by the gas and w is negative.

Step 2: Work Done by the Gas:

$$w_{\text{by}} = P \Delta V = (2 \times 10^5 \text{ N m}^{-2})(300 \times 10^{-6} \text{ m}^3) = 60 \text{ J}$$

Step 3: Apply the First Law:

Heat absorbed $q = +120 \text{ J}$ and $w = -60 \text{ J}$.

$$\Delta U = 120 - 60 = 60 \text{ J}$$

Step 4: Check the Other Options:

180 J comes from adding the work instead of subtracting it. 120 J ignores the work done. 240 J doubles the heat. None of these respects the sign of the work, so (D) is correct.

Final Answer:

The internal energy of the gas increases by 60 J, option (D).

(D) 60 J

Quick Tip: Work done by the gas is P times the volume change; then use $\Delta U = q - w$.

60. An ideal gas expands from initial volume 5dm^3 to 15dm^3 against a constant external pressure of 2 atm. What is the work done by the gas ?

- (A) -1202.6 J
- (B) -2026.4 J
- (C) -1101.32 J
- (D) -1013.2 J

Correct Answer: (B) -2026.4 J

Solution:

Step 1: Understanding the Concept:

Expansion against a constant external pressure gives $w = -P_{ext}\Delta V$.
The negative sign means the system loses energy as work.

Step 2: Find the Volume Change:

$$\Delta V = 15 - 5 = 10 \text{ dm}^3 = 10 \text{ L.}$$

Step 3: Calculate the Work:

$$w = -2 \text{ atm} \times 10 \text{ L} = -20 \text{ L atm}$$

Use $1 \text{ L atm} = 101.32 \text{ J}$.

$$w = -20 \times 101.32 = -2026.4 \text{ J}$$

Step 4: Check the Other Options:

-1013.2 J corresponds to only 10 L atm , as if the pressure were 1 atm .
 -1202.6 J and -1101.32 J do not match any sensible product of P , ΔV and 101.32 . So (B) is correct.

Final Answer:

The work done is -2026.4 J , option (B).

(B) -2026.4 J

Quick Tip: For a constant external pressure, $w = \text{minus } P \text{ times the change in volume}$; convert L atm to joules.

61. A weak base is 1.3% dissociated in its aqueous solution. If K_b for weak base is 1.69×10^{-5} at 298 K. Find the concentration of aqueous solution of weak base.

- (A) 1 M
- (B) 0.1 M
- (C) 0.01 M
- (D) 0.001 M

Correct Answer: (B) 0.1 M

Solution:

Step 1: Understanding the Concept:

For a weak base with small degree of dissociation α , the base constant is $K_b = C\alpha^2$, where C is the initial concentration.

Step 2: Convert the Data:

$\alpha = 1.3\% = 0.013$, so $\alpha^2 = 1.69 \times 10^{-4}$.

Step 3: Solve for C:

$$C = \frac{K_b}{\alpha^2} = \frac{1.69 \times 10^{-5}}{1.69 \times 10^{-4}} = 0.1 \text{ M}$$

Step 4: Check the Other Options:

If C were 1 M, then $\alpha = \sqrt{1.69 \times 10^{-5}} = 0.0041$, which is 0.41%. For 0.01 M, α would be 4.1%. For 0.001 M it would be about 13%. Only 0.1 M gives 1.3%. So (B) is correct.

Final Answer:

The concentration of the base solution is 0.1 M, option (B).

(B) 0.1 M

Quick Tip: Use $K_b = C \alpha^2$ for a weak base with small dissociation.

62. What is the pH of solution containing $5 \times 10^{-4} \text{MH}^+$ ion?

- (A) 4.699
- (B) 3.301
- (C) 5.401
- (D) 3.725

Correct Answer: (B) 3.301

Solution:**Step 1: Understanding the Concept:**

pH is defined as $\text{pH} = -\log[\text{H}^+]$.

Step 2: Calculation:

$$\text{pH} = -\log(5 \times 10^{-4}) = -(\log 5 + \log 10^{-4})$$

$$= -(0.699 - 4) = 4 - 0.699 = 3.301$$

Step 3: Check the Other Options:

4.699 is $4 + 0.699$, a sign mistake. 5.401 and 3.725 do not come from any correct handling of the logarithm. As an estimate, $[\text{H}^+] = 5 \times 10^{-4}$ lies between 10^{-3} (pH 3) and 10^{-4} (pH 4), so the pH must be between 3 and 4, which only 3.301 satisfies. Option (B) is correct.

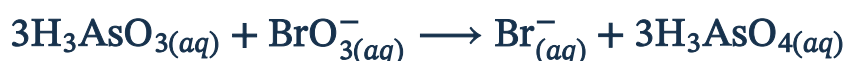
Final Answer:

The pH is 3.301, option (B).

(B) 3.301

Quick Tip: pH is the negative log of the hydrogen ion concentration; $\log 5$ is 0.699.

63. Identify change in oxidation state of oxidising agent in following redox reaction.



- (A) -1 to $+5$
- (B) $+5$ to -1
- (C) $+3$ to $+5$
- (D) $+5$ to $+3$

Correct Answer: (B) $+5$ to -1

Solution:**Step 1: Understanding the Concept:**

An oxidising agent gains electrons, so its oxidation number goes down. We find which species is reduced.

Step 2: Oxidation Numbers:

In H_3AsO_3 : $3 + x - 6 = 0$, so As is +3. In H_3AsO_4 : $3 + x - 8 = 0$, so As is +5. Arsenic is oxidised.

In BrO_3^- : $x - 6 = -1$, so Br is +5. In Br^- , Br is -1. Bromine is reduced.

Step 3: Identify the Oxidising Agent:

Bromate, BrO_3^- , is the oxidising agent because it takes electrons from arsenious acid. Its oxidation state changes from +5 to -1.

Step 4: Check the Other Options:

(C) +3 to +5 is the change for arsenic, the reducing agent. (A) and (D) have the wrong direction or wrong values. So (B) is correct.

Final Answer:

The oxidising agent is bromate and Br goes from +5 to -1, option (B).

(B) + 5 to - 1

Quick Tip: The oxidising agent is reduced; find the oxidation number of bromine before and after.

64. Which element from following forms superoxide with oxygen?

- (A) Sodium
- (B) Potassium
- (C) Lithium
- (D) Magnesium

Correct Answer: (B) Potassium

Solution:

Step 1: Understanding the Concept:

On burning in excess oxygen, alkali metals give different oxides depending on the size of the cation. Lithium gives the normal oxide, sodium gives mainly the peroxide, and K, Rb and Cs give superoxides (MO_2).

Step 2: Why Size Matters:

The superoxide ion O_2^- is large. A large, weakly polarising cation such as K^+ stabilises it through high lattice energy.

Step 3: Check Each Option:

Sodium: forms Na_2O_2 (peroxide) mainly.

Potassium: forms KO_2 (superoxide).

Lithium: forms Li_2O (normal oxide).

Magnesium: forms MgO (normal oxide).

Step 4: Conclusion:

Potassium is the only element in the list that forms a superoxide.

Final Answer:

Potassium forms the superoxide KO_2 , option (B).

(B) Potassium

Quick Tip: Larger alkali metal cations (K, Rb, Cs) stabilise the large superoxide ion.

65. What is the number of chiral carbon atoms in 2-chlorobutane?

- (A) 4
- (B) 3
- (C) 2
- (D) 1

Correct Answer: (D) 1

Solution:

Step 1: Understanding the Concept:

A carbon is chiral if it is bonded to four different atoms or groups.

Step 2: Write the Structure:

2-chlorobutane is $\text{CH}_3 - \text{CH}(\text{Cl}) - \text{CH}_2 - \text{CH}_3$.

Step 3: Test Each Carbon:

C1 (CH_3): has three H atoms, so it is not chiral.

C2: has H, Cl, CH_3 and C_2H_5 , four different groups, so it is chiral.

C3 (CH_2): has two H atoms, so it is not chiral.

C4 (CH_3): three H atoms, so it is not chiral.

Step 4: Conclusion:

Only C2 is a stereocentre, so the molecule has 1 chiral carbon and exists as a pair of enantiomers.

Final Answer:

There is exactly one chiral carbon, option (D).

(D) 1

Quick Tip: A chiral carbon has four different groups attached; check each carbon of $\text{CH}_3\text{CHClCH}_2\text{CH}_3$.



66. Identify the aldehyde from following so that the aldehydic group is NOT attached with any sp^3 hybrid 'c' atom.

- (A) Methanal
- (B) Ethanal
- (C) 2-Methyl Propanal
- (D) Propanal

Correct Answer: (A) Methanal

Solution:

Step 1: Understanding the Concept:

In an aldehyde the $-\text{CHO}$ group is attached to either an alkyl group or to hydrogen. We want the one where it is not attached to any sp^3 carbon.

Step 2: Check the Options:

Methanal, HCHO : CHO is attached to two hydrogens and no carbon at all. This matches.

Ethanal, CH_3CHO : CHO attached to a CH_3 carbon, which is sp^3 .

2-Methylpropanal: CHO attached to the sp^3 CH carbon.

Propanal, $\text{CH}_3\text{CH}_2\text{CHO}$: CHO attached to an sp^3 CH_2 carbon.

Step 3: Conclusion:

Only methanal is not bonded to an sp^3 carbon, so (A) is correct.

Final Answer:

Methanal is the only aldehyde with no alkyl carbon next to CHO, option (A).

(A) Methanal

Quick Tip: Only formaldehyde has the CHO group attached to hydrogen instead of a carbon.

67. Which from following is a primary benzylic alcohol?

- (A) $\text{C}_6\text{H}_5\text{CH}_2\text{OH}$
- (B) $\text{C}_6\text{H}_5 - \text{CH} \begin{smallmatrix} \text{CH}_3 \\ \text{OH} \end{smallmatrix}$
- (C) $\text{H}_2\text{C} = \text{CH} - \text{CH} \begin{smallmatrix} \text{CH}_3 \\ \text{OH} \end{smallmatrix}$
- (D) $\text{H}_2\text{C} = \text{CH} - \text{CH}_2\text{OH}$

Correct Answer: (A) $\text{C}_6\text{H}_5\text{CH}_2\text{OH}$

Solution:**Step 1: Understanding the Concept:**

In a benzylic alcohol the $-\text{OH}$ is on an sp^3 carbon directly attached to an aromatic ring. It is primary if that carbon is bonded to only one other carbon (the ring).

Step 2: Check (A):

$\text{C}_6\text{H}_5\text{CH}_2\text{OH}$ is benzyl alcohol. The carbinol carbon is attached to the ring, two H and OH, so it is primary and benzylic.

Step 3: Check (B):

$\text{C}_6\text{H}_5\text{CH}(\text{OH})\text{CH}_3$ is benzylic but the carbinol carbon bonds to two carbons, so it is secondary.

Step 4: Check (C) and (D):

(C) is a secondary allylic alcohol. (D) is allyl alcohol, a primary allylic alcohol. Neither has a benzene ring, so they are not benzylic.

Final Answer:

Benzyl alcohol is the primary benzylic alcohol, option (A).



Quick Tip: A benzylic alcohol has OH on a carbon attached to a benzene ring; primary means that carbon has only one other carbon.



68. Which from the following aldehydes contains two $-\text{CHO}$ groups in a molecule?

- (A) Acrolein
- (B) O-Tolualdehyde
- (C) Phthaldehyde
- (D) Butyraldehyde

Correct Answer: (C) Phthaldehyde

Solution:

Step 1: Understanding the Concept:

We need the common-name aldehyde whose structure carries two $-\text{CHO}$ groups.

Step 2: Check Each Option:

Acrolein (propenal): $\text{CH}_2 = \text{CH} - \text{CHO}$, one CHO group.

o-Tolualdehyde: 2-methylbenzaldehyde, one CHO group on the ring.

Phthaldehyde: benzene-1,2-dicarbaldehyde, $\text{C}_6\text{H}_4(\text{CHO})_2$, two CHO groups.

Butyraldehyde (butanal): $\text{CH}_3\text{CH}_2\text{CH}_2\text{CHO}$, one CHO group.

Step 3: Conclusion:

Only phthaldehyde contains two aldehyde groups.

Final Answer:

Phthaldehyde has two CHO groups, option (C).

(C) Phthaldehyde

Quick Tip: Phthaldehyde is benzene-1,2-dicarbaldehyde, so it has two CHO groups.

69. Gold crystallizes as fcc unit cell, the edge length of unit cell is 408 pm. What is the radius of gold atom?

- (A) 86.6 pm
- (B) 115.4 pm
- (C) 144.2 pm
- (D) 175.2 pm

Correct Answer: (C) 144.2 pm

Solution:

Step 1: Understanding the Concept:

In a face-centred cubic cell, the atoms touch along the face diagonal. Its length is $\sqrt{2}a$, and it equals four atomic radii.

Step 2: Key Formula:

$$\sqrt{2}a = 4r \Rightarrow r = \frac{\sqrt{2}a}{4} = \frac{a}{2\sqrt{2}}$$

Step 3: Calculation:

$$r = \frac{1.414 \times 408}{4} = \frac{576.9}{4} = 144.2 \text{ pm}$$

Step 4: Check the Other Options:

The other values do not come from the fcc relation. Using the bcc relation $r = \sqrt{3}a/4$ would give 176.6 pm, which is close to (D) but belongs to a different lattice. (C) is correct.

Final Answer:

The radius of the gold atom is 144.2 pm, option (C).

(C) 144.2 pm

Quick Tip: In an fcc cell atoms touch along the face diagonal, so $a = 2\sqrt{2}r$.

70. What is the number of tetrahedral voids present in 0.5 mole of compound forming hcp structure?

- (A) 1.223×10^{23}
- (B) 3.628×10^{23}
- (C) 4.487×10^{23}
- (D) 6.022×10^{23}

Correct Answer: (D) 6.022×10^{23}

Solution:

Step 1: Understanding the Concept:

In close packing (hcp or ccp) with N atoms, there are N octahedral voids and $2N$ tetrahedral voids.

Step 2: Number of Atoms:

0.5 mol of atoms = $0.5 \times 6.022 \times 10^{23} = 3.011 \times 10^{23}$ atoms.

Step 3: Number of Tetrahedral Voids:

$$2N = 2 \times 3.011 \times 10^{23} = 6.022 \times 10^{23}$$

Step 4: Check the Other Options:

3.011×10^{23} would be the number of atoms or octahedral voids. The other values do not match any simple multiple of N . So (D) is correct.

Final Answer:

The number of tetrahedral voids is 6.022×10^{23} , option (D).

$$(D) 6.022 \times 10^{23}$$

Quick Tip: An hcp lattice has 2 tetrahedral voids per atom, so find the number of atoms first.

71. Which among the following pairs of solutions is isotonic in nature?

- (A) 0.1 M urea and 0.1 M sucrose
- (B) 0.1 M urea and 0.1 M KCl
- (C) 0.1 M sucrose and 0.1 M KCl
- (D) 0.1 M KCl and 0.1 M CaCl_2

Correct Answer: (A) 0.1 M urea and 0.1 M sucrose

Solution:

Step 1: Understanding the Concept:

Isotonic solutions have the same osmotic pressure at the same temperature. Osmotic pressure depends on the effective number of particles: $\pi = iCRT$.

Step 2: Find the Particle Concentration:

0.1 M urea: non-electrolyte, $i = 1$, effective 0.1 M.

0.1 M sucrose: non-electrolyte, $i = 1$, effective 0.1 M.

0.1 M KCl: $i \approx 2$, effective about 0.2 M.

0.1 M CaCl_2 : $i \approx 3$, effective about 0.3 M.

Step 3: Match the Pairs:

Urea and sucrose both give 0.1 M of particles, so their osmotic pressures are equal. Every other pair has different effective

concentrations: urea and KCl (0.1 and 0.2), sucrose and KCl (0.1 and 0.2), KCl and CaCl_2 (0.2 and 0.3).

Final Answer:

Urea and sucrose at 0.1 M each are isotonic, option (A).

(A) 0.1 M urea and 0.1 M sucrose

Quick Tip: Isotonic solutions have equal osmotic pressure, which depends on the total particle concentration.

72. A solution of non volatile solute has boiling point elevation 0.70 K if K_b for the solvent is 2.44K kg mol^{-1} . What is the molality of the solution?

- (A) 3.48 m
- (B) 2.86 m
- (C) 0.286 m
- (D) 0.0186 m

Correct Answer: (C) 0.286 m

Solution:

Step 1: Understanding the Concept:

The elevation in boiling point of a dilute solution of a non-volatile solute is proportional to its molality: $\Delta T_b = K_b m$.

Step 2: Rearrange:

$$m = \frac{\Delta T_b}{K_b}$$

Step 3: Substitute:

$$m = \frac{0.70}{2.44} = 0.2869 \approx 0.286 \text{ m}$$

Step 4: Check the Other Options:

3.48 m is $2.44/0.70$, the ratio inverted. 2.86 m is ten times too large, a decimal slip. 0.0186 m does not match the formula. Option (C) is correct.

Final Answer:

The molality is 0.286 m, option (C).

(C) 0.286 m

Quick Tip: Use $\Delta T_b = K_b \text{ times molality}$.

73. van't Hoff factor for BaCl_2 is 2.47, calculate the percentage dissociation of in its aqueous solution.

- (A) 25.6%
- (B) 35.7%
- (C) 73.5%
- (D) 13.6%

Correct Answer: (C) 73.5%

Solution:

Step 1: Understanding the Concept:

The van't Hoff factor relates to degree of dissociation. For $\text{BaCl}_2 \rightarrow \text{Ba}^{2+} + 2\text{Cl}^-$, one formula unit gives $n = 3$ particles.

Step 2: Key Formula:

$$i = 1 + (n - 1)\alpha = 1 + 2\alpha$$

Step 3: Calculation:

$$2.47 = 1 + 2\alpha \Rightarrow \alpha = \frac{1.47}{2} = 0.735$$

Percentage dissociation = $0.735 \times 100 = 73.5\%$.

Step 4: Check the Other Options:

25.6%, 35.7% and 13.6% would give i of 1.51, 1.71 and 1.27. None of these equals 2.47. So (C) is correct.

Final Answer:

BaCl_2 is 73.5% dissociated, option (C).

(C) 73.5%

Quick Tip: For BaCl_2 , $i = 1 + 2\alpha$ since each formula unit gives three ions.

74. What is number of moles of electron passed when a current of 2 Amp flows through solution of electrolyte for 10 minutes?

- (A) 1.784×10^{-2}
- (B) 1.243×10^{-2}
- (C) 2.022×10^{-2}
- (D) 8.041×10^{-2}

Correct Answer: (B) 1.243×10^{-2}

Solution:

Step 1: Understanding the Concept:

One mole of electrons carries a charge of 1 faraday, $F = 96500$ C.

Step 2: Find the Charge:

$$t = 10 \times 60 = 600 \text{ s.}$$

$$Q = It = 2 \times 600 = 1200 \text{ C}$$

Step 3: Moles of Electrons:

$$n = \frac{Q}{F} = \frac{1200}{96500} = 1.243 \times 10^{-2} \text{ mol}$$

Step 4: Check the Other Options:

1.784×10^{-2} would need 1722 C, 2.022×10^{-2} would need 1951 C, 8.041×10^{-2} would need 7760 C. None matches 1200 C. So (B) is correct.

Final Answer:

About 1.243×10^{-2} mol of electrons pass, option (B).

$$(B) 1.243 \times 10^{-2}$$

Quick Tip: Charge = current times time; divide by the Faraday constant.

75. What is the conductivity of 0.02M AgNO_3 solution having cell constant 1.2cm^{-1} and resistance 95.0ohms ?

- (A) $1.140 \times 10^{-2} \Omega^{-1}\text{cm}^{-1}$
- (B) $1.263 \times 10^{-2} \Omega^{-1}\text{cm}^{-1}$
- (C) $1.340 \times 10^{-2} \Omega^{-1}\text{cm}^{-1}$
- (D) $1.463 \times 10^{-2} \Omega^{-1}\text{cm}^{-1}$

Correct Answer: (B) $1.263 \times 10^{-2} \Omega^{-1}\text{cm}^{-1}$

Solution:

Step 1: Understanding the Concept:

The conductivity κ of a solution is the cell constant G^* times the conductance, and conductance is $1/R$.

Step 2: Key Formula:

$$\kappa = \frac{G^*}{R}$$

Step 3: Calculation:

$$\kappa = \frac{1.2 \text{ cm}^{-1}}{95.0 \Omega} = 1.263 \times 10^{-2} \Omega^{-1}\text{cm}^{-1}$$

Step 4: Check the Other Options:

Multiplying instead of dividing would give 114, a huge number. The other options (1.140, 1.340, 1.463 times 10^{-2}) correspond to R values of 105, 89.6 and 82, none of them 95.0 ohm. So (B) is correct.

Final Answer:

The conductivity is $1.263 \times 10^{-2} \Omega^{-1}\text{cm}^{-1}$, option (B).

$(B) 1.263 \times 10^{-2} \Omega^{-1}\text{cm}^{-1}$

Quick Tip: Conductivity is the cell constant divided by resistance.

76. During the electrolysis of aqueous NaCl the product formed at cathode is?

- (A) $\text{Cl}_{2(g)}$
- (B) $\text{O}_{2(g)}$
- (C) $\text{Na}_{(s)}$
- (D) $\text{H}_{2(g)}$

Correct Answer: (D) $\text{H}_{2(g)}$

Solution:**Step 1: Understanding the Concept:**

In electrolysis of an aqueous salt, water competes with the ions. At the cathode, the species that is reduced most easily (higher reduction potential) wins.

Step 2: Possible Cathode Reactions:

$\text{Na}^+ + e^- \rightarrow \text{Na}$ has $E^\circ = -2.71 \text{ V}$.

$2\text{H}_2\text{O} + 2e^- \rightarrow \text{H}_2 + 2\text{OH}^-$ has $E^\circ = -0.83 \text{ V}$.

The water reaction has the less negative potential, so it occurs.

Step 3: Product at the Cathode:

Hydrogen gas bubbles out of the cathode, and the solution near it becomes basic because of OH^- (this is the basis of the chlor-alkali process).

Step 4: Check the Other Options:

Cl_2 forms at the anode, not the cathode. O_2 is an anode product of water oxidation, and it is not formed at the cathode. Sodium metal is not deposited because Na^+ is harder to reduce than water. So (D) is correct.

Final Answer:

Hydrogen gas is evolved at the cathode, option (D).

(D) $\text{H}_{2(g)}$

Quick Tip: Water is reduced more easily than Na^+ , so hydrogen gas forms at the cathode.

77. Which of the following is an elementary reaction ?

- (A) $\text{O}_{3(g)} \rightarrow \text{O}_{2(g)} + \text{O}_{(g)}$
(B) $2\text{NO}_2\text{Cl}_{(g)} \rightarrow 2\text{NO}_{2(g)} + \text{Cl}_{2(g)}$
(C) $2\text{NO}_{2(g)} + \text{F}_{2(g)} \rightarrow 2\text{NO}_2\text{F}_{(g)}$
(D) $2\text{NO}_{(g)} + \text{Cl}_{2(g)} \rightarrow 2\text{NOCl}_{(g)}$

Correct Answer: (A) $\text{O}_{3(g)} \rightarrow \text{O}_{2(g)} + \text{O}_{(g)}$

Solution:

Step 1: Understanding the Concept:

An elementary reaction occurs in one step, with the collision of at most three species. Complex reactions have several steps.

Step 2: Check Option (A):

$\text{O}_3 \rightarrow \text{O}_2 + \text{O}$ involves a single molecule breaking into two pieces in one step. This is a unimolecular elementary reaction.

Step 3: Check the Other Options:

$2\text{NO}_2\text{Cl} \rightarrow 2\text{NO}_2 + \text{Cl}_2$ is known to occur in two steps (slow step produces $\text{NO}_2 + \text{Cl}$), so it is complex.

$2\text{NO}_2 + \text{F}_2 \rightarrow 2\text{NO}_2\text{F}$ is known to occur in two steps through NO_2F and F atoms.

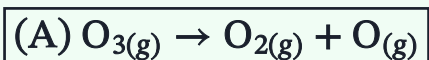
$2\text{NO} + \text{Cl}_2 \rightarrow 2\text{NOCl}$ has three reacting molecules, and its mechanism involves an intermediate NOCl_2 , so it is not a single step.

Step 4: Conclusion:

The decomposition of ozone as written is the elementary step.

Final Answer:

Option (A) is a single-step unimolecular process.



Quick Tip: An elementary reaction occurs in a single step; its molecularity is small and equals the number of reacting species.



78. A first order reaction take 30 minutes for 75% decomposition, calculate its rate constant ?

- (A) 0.0238minute^{-1}
- (B) 0.0463minute^{-1}
- (C) 0.0715minute^{-1}
- (D) 0.0957minute^{-1}

Correct Answer: (B) 0.0463minute^{-1}

Solution:

Step 1: Understanding the Concept:

For a first-order reaction, $k = \frac{2.303}{t} \log \frac{[A]_0}{[A]}$.

Step 2: Set Up the Data:

75% has decomposed, so 25% remains. If $[A]_0 = 100$, then $[A] = 25$ and the ratio is 4.

$t = 30 \text{ min.}$

Step 3: Calculation:

$$k = \frac{2.303}{30} \log 4 = \frac{2.303 \times 0.6021}{30} = \frac{1.3866}{30} = 0.0462 \text{ min}^{-1} \approx 0.0463 \text{ min}^{-1}$$

Step 4: Check the Other Options:

0.0238 and 0.0715 and 0.0957 min^{-1} do not satisfy $kt = \ln 4 = 1.386$ for $t = 30 \text{ min}$. For 0.0463, $kt = 1.389$, which matches. So (B) is correct.

Final Answer:

The rate constant is 0.0463 min^{-1} , option (B).

(B) 0.0463 min^{-1}

Quick Tip: Use $k = (2.303/t) \log (a/(a - x))$; 75% decomposition leaves 25 percent.



79. Rate constant of decomposition of hydrogen peroxide is $0.0204 \text{ minutes}^{-1}$, calculate half life of reaction.

- (A) 17.56 minutes
- (B) 33.97 minutes
- (C) 51.50 minutes
- (D) 68.70 minutes

Correct Answer: (B) 33.97 minutes

Solution:

Step 1: Understanding the Concept:

The decomposition of hydrogen peroxide is first order, and the half-life of a first-order reaction does not depend on concentration.

Step 2: Key Formula:

$$t_{1/2} = \frac{0.693}{k}$$

Step 3: Calculation:

$$t_{1/2} = \frac{0.693}{0.0204 \text{ min}^{-1}} = 33.97 \text{ min}$$

Step 4: Check the Other Options:

17.56 min would need $k = 0.0395$, 51.50 min would need $k = 0.01346$, and 68.70 min would need $k = 0.0101$. Since the given k is 0.0204, only 33.97 min is consistent. Option (B) is correct.

Final Answer:

The half-life is 33.97 min, option (B).

(B) 33.97 min

Quick Tip: For a first-order reaction $t_{1/2} = 0.693$ divided by k .

80. Which of the following properties of colloid is used to measure the rate of migration of sol particles?

- (A) Tyndall effect
- (B) Brownian motion
- (C) Electrophoresis
- (D) Electroosmosis

Correct Answer: (C) Electrophoresis

Solution:

Step 1: Understanding the Concept:

Sol particles carry an electric charge. In an electric field they move toward the oppositely charged electrode, and the speed of this movement can be measured.

Step 2: Check Each Property:

Tyndall effect: scattering of light by colloidal particles, used to detect them.

Brownian motion: random zig-zag motion caused by solvent molecules hitting the particles; it keeps the sol stable.

Electrophoresis: movement of sol particles under an applied electric field, whose rate can be measured.

Electroosmosis: movement of the dispersion medium, not the particles, under the field.

Step 3: Conclusion:

The rate of migration of the sol particles is measured through electrophoresis.

Final Answer:

Electrophoresis measures the migration of charged sol particles, option (C).

(C) Electrophoresis

Quick Tip: Electrophoresis is the movement of charged sol particles in an electric field.

81. Which among the following oxoacids of chlorine exhibits +3 oxidation state of chlorine?

- (A) HClO
- (B) HClO_2
- (C) HClO_3
- (D) HClO_4

Correct Answer: (B) HClO_2

Solution:

Step 1: Understanding the Concept:

In oxoacids, H is +1 and O is -2. The sum of oxidation numbers is zero, so we can solve for Cl.

Step 2: Calculate for Each Acid:

HClO : $1 + x - 2 = 0$, so $x = +1$.

HClO_2 : $1 + x - 4 = 0$, so $x = +3$.

HClO_3 : $1 + x - 6 = 0$, so $x = +5$.

HClO_4 : $1 + x - 8 = 0$, so $x = +7$.

Step 3: Conclusion:

Chlorous acid, HClO_2 , has chlorine in the +3 state. So (B) is correct, and the others are hypochlorous (+1), chloric (+5) and perchloric (+7)

acids.

Final Answer:

HClO₂ shows the +3 state, option (B).

(B) HClO₂

Quick Tip: Calculate the oxidation number of Cl in HClO, HClO₂, HClO₃ and HClO₄.



82. Identify general electronic configuration exhibited by 2nd series of transition elements.

- (A) [Ar]4d¹⁻¹⁰4s²
- (B) [Kr]4d¹⁻¹⁰5s⁰⁻²
- (C) [Xe]4d¹⁻¹⁰4s²
- (D) [Rn]4d¹⁻¹⁰6s²

Correct Answer: (B) [Kr]4d¹⁻¹⁰5s⁰⁻²

Solution:

Step 1: Understanding the Concept:

The second transition series (Y to Cd) fills the 4d subshell. The core before it is krypton, [Kr].

Step 2: Write the Configuration:

The 5s orbital holds 0 to 2 electrons, because some elements such as Pd (4d¹⁰5s⁰) and Ag (4d¹⁰5s¹) are exceptions.

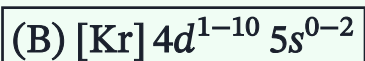
[Kr] 4d¹⁻¹⁰ 5s⁰⁻²

Step 3: Check the Other Options:

Options (A) and (C) place $4s^2$ after the 4d, which is wrong because the 4s subshell is already filled in the core. (C) also uses [Xe] as the core, which belongs to the third series, and (D) with [Rn] and 6s belongs to the 6th period. Only (B) is the right pattern.

Final Answer:

The 4d series has the configuration [Kr] $4d^{1-10}5s^{0-2}$, option (B).



Quick Tip: The second series fills 4d with 5s outer electrons, after krypton.

83. Identify metals present in brass.

- (A) Ni and Cu
- (B) Fe and Zn
- (C) Cr and Sn
- (D) Cu and Zn

Correct Answer: (D) Cu and Zn

Solution:**Step 1: Understanding the Concept:**

An alloy is a mixture of a metal with other elements. Common alloys are tested often in the d-block chapter.

Step 2: Composition of Brass:

Brass is about 60-80% copper and 20-40% zinc.

Step 3: Compare the Options:

Ni and Cu: this pairing is found in the alloy called cupronickel (coins).

Fe and Zn: not a standard alloy pair.

Cr and Sn: not a standard alloy pair either.

Cu and Zn: brass.

Step 4: Conclusion:

The metals in brass are copper and zinc, option (D).

Final Answer:

Brass contains Cu and Zn, option (D).

(D) Cu and Zn

Quick Tip: Brass is an alloy of copper and zinc.

84. What is the highest possible oxidation state exhibited by manganese ?

- (A) +8
- (B) +5
- (C) +6
- (D) +7

Correct Answer: (D) +7

Solution:

Step 1: Understanding the Concept:

The highest oxidation state of a 3d element is usually the sum of its 4s and 3d electrons.

Step 2: Manganese:

Mn has the configuration $[\text{Ar}]3d^54s^2$, which has 7 valence electrons. Removing all of them gives +7, as in KMnO_4 and Mn_2O_7 .

Step 3: Check the Other Options:

+8 is not possible because there are only 7 electrons available. +5 and +6 are real states (manganate(V) and manganate(VI) ions), but they are not the highest. So (D) is correct.

Final Answer:

The highest oxidation state of Mn is +7, option (D).

(D) + 7

Quick Tip: Manganese uses all 7 valence electrons ($3d^5 4s^2$) to show +7.

85. What type of hybridization is found in $[\text{Ni}(\text{Cl})_4]^{2-}$?

- (A) sp^3
- (B) sp^3d^2
- (C) d^2sp^3
- (D) dsp^2

Correct Answer: (A) sp^3

Solution:

Step 1: Understanding the Concept:

The hybridisation of a complex depends on the metal electron configuration and the strength of the ligand field.

Step 2: Find the Oxidation State:

In $[\text{NiCl}_4]^{2-}$, $x - 4 = -2$, so Ni is +2 and has the configuration $3d^8$.

Step 3: Ligand Field:

Chloride is a weak field ligand. It does not force the d electrons to pair, so the 3d orbitals stay occupied and the 4s and 4p orbitals are used.

Step 4: Hybridisation:

One 4s and three 4p orbitals mix to give four sp^3 orbitals, so the shape is tetrahedral and the complex is paramagnetic with two unpaired electrons.

A dsp^2 square planar shape would need strong field ligands such as CN^- . The sp^3d^2 and d^2sp^3 types give six-coordinate octahedral shapes, which need six ligands.

Final Answer:

The complex is tetrahedral with sp^3 hybridisation, option (A).

(A) sp^3

Quick Tip: Chloride is a weak field ligand, so Ni(II) with Cl forms a tetrahedral sp^3 complex.

86. What is the oxidation state of cobalt in $[\text{Co}(\text{NH}_3)_6]^{3+}$?

- (A) +5
- (B) +4
- (C) +3
- (D) +2

Correct Answer: (C) +3

Solution:

Step 1: Understanding the Concept:

The oxidation state of the central metal equals the charge of the complex minus the total charge of the ligands.

Step 2: Calculation:

Let the oxidation state of Co be x . Ammonia is a neutral ligand (charge 0).

$$x + 6(0) = +3 \Rightarrow x = +3$$

Step 3: Check the Other Options:

+5 and +4 would give a complex charge of +5 or +4, and +2 would give 2+. Only +3 matches the given 3+. So (C) is correct.

Final Answer:

Cobalt is in the +3 state, option (C).

(C) + 3

Quick Tip: Ammonia is neutral, so the metal carries the full charge of the complex.

87. Identify reagent B in following reaction.



Correct Answer: (A) $\text{Ag} - \text{O} - \text{N} = \text{O}$

Solution:

Step 1: Understanding the Concept:

The nitrite ion, NO_2^- , is an ambident nucleophile: it can attack through N or through O.

Step 2: Silver Nitrite:

AgNO_2 is largely covalent, so the nitrogen lone pair is free for attack. The product is a nitroalkane, R-NO_2 .



Step 3: Check the Other Options:

KNO_2 is ionic and attacks through oxygen, which gives alkyl nitrite R-O-N=O . NaOR gives an ether and alcoholic KCN gives a nitrile.

Option (A) is the reagent that gives the nitro compound. Option (A),

Ag-O-N=O, is the written form of silver nitrite.

Final Answer:

Silver nitrite gives the nitroalkane, option (A).

(A) AgNO_2

Quick Tip: Silver nitrite (AgNO_2) is covalent and attacks through nitrogen to give nitroalkane.

88. Which of the following n mole compound is obtained when $2n$ moles of $\text{C}_6\text{H}_5\text{Cl}$ treated of with $2n$ mole sodium atom in dry ether?

- (A) Biphenyl
- (B) Toluene
- (C) Phenol
- (D) Benzene

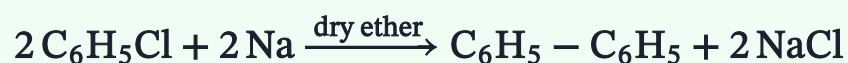
Correct Answer: (A) Biphenyl

Solution:

Step 1: Understanding the Concept:

When an aryl halide is heated with sodium metal in dry ether, two aryl groups join. This is the Fittig reaction.

Step 2: Balanced Reaction:



So $2n$ mol of chlorobenzene and $2n$ mol of sodium give n mol of biphenyl.

Step 3: Check the Other Options:

Benzene would form by reduction with a reagent such as Zn and acid. Phenol needs hydrolysis (aqueous NaOH at high temperature and pressure). Toluene needs methyl halide in the Wurtz-Fittig reaction, which is absent here. So (A) is correct.

Final Answer:

The product is biphenyl, option (A).

(A) Biphenyl

Quick Tip: Two aryl halides couple with sodium in dry ether to give biphenyl (Fittig reaction).



89. Which from the following compounds is not obtained when methyl bromide is treated with ethyl bromide in presence of sodium metal in dry ether?

- (A) n-Butane
- (B) Ethane
- (C) Propane
- (D) 2-methyl propane

Correct Answer: (D) 2-methyl propane

Solution:

Step 1: Understanding the Concept:

In the Wurtz reaction two alkyl groups join. With two different halides, three couplings are possible.

Step 2: Possible Couplings:

$\text{CH}_3 + \text{CH}_3$ gives ethane.

$\text{CH}_3 + \text{C}_2\text{H}_5$ gives propane.

$\text{C}_2\text{H}_5 + \text{C}_2\text{H}_5$ gives n-butane.

Step 3: Check Option (D):

2-methylpropane, $\text{CH}_3\text{CH}(\text{CH}_3)_2$, has a branched carbon skeleton. Joining methyl and ethyl groups can never create a branch, so it cannot be formed.

Step 4: Conclusion:

Ethane, propane and n-butane are all products. 2-methylpropane is not, so (D) is the answer.

Final Answer:

2-methylpropane cannot form, option (D).

(D) 2-methylpropane

Quick Tip: Cross Wurtz coupling gives three alkanes: propane, ethane, n-butane.

90. Which of the following pair of reagents is used for the conversion of carboxylic acid to alcohol ?

- (A) $\text{LiAlH}_4/\text{H}_3\text{O}^+$
- (B) H_2/Ni - heat
- (C) $\text{B}_2\text{H}_6/\text{H}_2\text{O}_2, \text{OH}^-$
- (D) H_2/Pd

Correct Answer: (A) $\text{LiAlH}_4/\text{H}_3\text{O}^+$

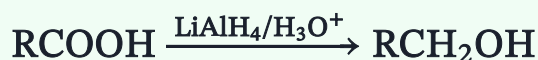
Solution:

Step 1: Understanding the Concept:

A carboxylic acid is reduced to a primary alcohol, $\text{RCOOH} \rightarrow \text{RCH}_2\text{OH}$. This needs a strong hydride donor.

Step 2: Standard Reagent:

Lithium aluminium hydride in dry ether, followed by hydrolysis with dilute acid, performs this reduction.



Step 3: Check the Other Options:

H_2/Ni and H_2/Pd are catalytic hydrogenation reagents. They reduce $\text{C}=\text{C}$ and $\text{C}\equiv\text{C}$ triple bonds easily but leave the carboxyl group untouched under ordinary conditions. The pair in (C), $\text{B}_2\text{H}_6/\text{H}_2\text{O}_2, \text{OH}^-$, is the hydroboration-oxidation sequence normally written for converting alkenes into alcohols, and the textbook reagent for the acid-to-alcohol conversion is the hydride in (A).

Final Answer:

LiAlH_4 followed by acid work-up is the standard reagent, option (A).



Quick Tip: Lithium aluminium hydride followed by acidic work-up reduces RCOOH to a primary alcohol.

91. Which from the following hydroxy compounds contain only one $-\text{OH}$ group in a molecule?

- (A) Crotonyl alcohol
- (B) Ethylene glycol
- (C) Pyrogallol
- (D) Hydroquinone

Correct Answer: (A) Crotonyl alcohol

Solution:

Step 1: Understanding the Concept:

Alcohols and phenols are classified as mono-, di- or trihydric by the number of $-\text{OH}$ groups.

Step 2: Count the OH Groups:

Crotonyl (crotyl) alcohol: $\text{CH}_3\text{CH}=\text{CHCH}_2\text{OH}$, one OH.

Ethylene glycol: $\text{HOCH}_2\text{CH}_2\text{OH}$, two OH.

Pyrogallol: benzene-1,2,3-triol, three OH.

Hydroquinone: benzene-1,4-diol, two OH.

Step 3: Conclusion:

Only crotonyl alcohol is monohydric, so (A) is correct.

Final Answer:

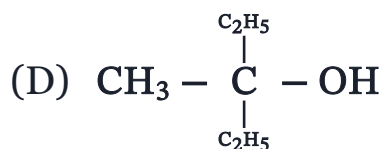
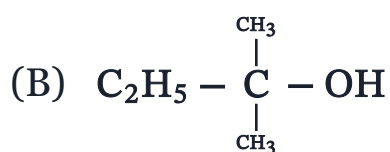
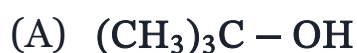
Crotonyl alcohol has a single OH group, option (A).

(A) Crotonyl alcohol

Quick Tip: Count the OH groups: crotyl alcohol has one; glycol has two; pyrogallol has three; hydroquinone has two.



92. Identify the compound obtained when ethyl methyl ketone is treated with CH_3MgBr and hydrolyzed.

**Solution:****Step 1: Understanding the Concept:**

Grignard reagents add to the carbonyl carbon. Aldehydes (except HCHO) give secondary alcohols, and ketones give tertiary alcohols after hydrolysis.

Step 2: Reaction:

Ethyl methyl ketone is $\text{CH}_3\text{COC}_2\text{H}_5$. The CH_3^- group of CH_3MgBr adds to the carbonyl carbon:



The product is 2-methylbutan-2-ol.

Step 3: Check the Other Options:

(A) $(\text{CH}_3)_3\text{COH}$ comes from acetone. (C) $(\text{C}_2\text{H}_5)_3\text{COH}$ and (D) with two ethyl groups would need ethyl Grignard reagents. Only (B) has one ethyl and two methyl groups on the carbinol carbon.

Final Answer:

The product is 2-methylbutan-2-ol, option (B).



Quick Tip: A ketone with a Grignard reagent gives a tertiary alcohol after hydrolysis.

93. Which from the following is NOT a monocarboxylic acid?

- (A) Toluic acid
- (B) Acrylic acid
- (C) Phthalic acid
- (D) Isobutyric acid

Correct Answer: (C) Phthalic acid

Solution:

Step 1: Understanding the Concept:

A monocarboxylic acid has exactly one -COOH group.

Step 2: Check the Options:

Toluic acid: methylbenzoic acid, one COOH .

Acrylic acid: $\text{CH}_2 = \text{CH-COOH}$, one COOH .

Phthalic acid: benzene-1,2-dicarboxylic acid, two COOH .

Isobutyric acid: $(\text{CH}_3)_2\text{CH-COOH}$, one COOH .

Step 3: Conclusion:

Phthalic acid has two carboxyl groups, so it is a dicarboxylic acid and not a monocarboxylic acid.

Final Answer:

Phthalic acid is dicarboxylic, option (C).

(C) Phthalic acid

Quick Tip: Phthalic acid is benzene-1,2-dicarboxylic acid, with two COOH groups.



94. Which from following compounds is obtained when propionamide is treated with Br_2 and concentrated aqueous KOH solution ?

- (A) $\text{CH}_3\text{CH}_2\text{CH}_2\text{COOH}$
(B) CH_3COCH_3
(C) $\text{CH}_3\text{CH}_2\text{NH}_2$
(D) $\text{CH}_3\text{CH}_2\text{Br}$

Correct Answer: (C) $\text{CH}_3\text{CH}_2\text{NH}_2$

Solution:

Step 1: Understanding the Concept:

In Hofmann bromamide degradation, an amide reacts with Br_2 and aqueous KOH to give a primary amine with one carbon atom fewer.

Step 2: Reaction:



Propionamide has three carbons, so the amine has two: ethylamine.

Step 3: Check the Other Options:

(A) has four carbons, so it has more carbon than the amide. (B) is acetone, which has no nitrogen. (D) is ethyl bromide, which is not formed since nitrogen is retained. So (C) is correct.

Final Answer:

The product is ethylamine, option (C).

(C) $\text{CH}_3\text{CH}_2\text{NH}_2$

Quick Tip: Hofmann degradation removes one carbon atom as carbonate and gives a primary amine.

95. Which from following amines is obtained by Gabriel phthalimide synthesis?

- (A) $\text{C}_6\text{H}_5\text{NH}_2$
- (B) $\text{C}_2\text{H}_5\text{NH}_2$
- (C) $(\text{CH}_3)_2\text{NH}$
- (D) $\text{C}_2\text{H}_5\text{N}(\text{CH}_3)_2$

Correct Answer: (B) $\text{C}_2\text{H}_5\text{NH}_2$

Solution:

Step 1: Understanding the Concept:

Gabriel phthalimide synthesis converts an alkyl halide into a primary amine through N-alkylphthalimide, followed by hydrolysis.

Step 2: Limits of the Method:

Aryl halides do not undergo nucleophilic substitution by phthalimide anion, so aromatic primary amines like aniline cannot be made this way. Also, only primary aliphatic amines can result, not secondary or tertiary.

Step 3: Check the Options:

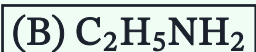
- (A) Aniline: aromatic amine, not possible.
- (B) Ethylamine: primary aliphatic amine, possible from ethyl halide.
- (C) Dimethylamine: secondary amine, not possible.
- (D) $\text{C}_2\text{H}_5\text{N}(\text{CH}_3)_2$: tertiary amine, not possible.

Step 4: Conclusion:

Only ethylamine can be prepared, so (B) is correct.

Final Answer:

Gabriel synthesis gives ethylamine, option (B).



Quick Tip: Gabriel synthesis gives only primary aliphatic amines.

96. Which from following compounds is classified as oligosaccharide?

- (A) Ribose
- (B) Sucrose
- (C) Fructose
- (D) Glucose

Correct Answer: (B) Sucrose

Solution:**Step 1: Understanding the Concept:**

Carbohydrates are classified by the number of simple sugar units they give on hydrolysis. Monosaccharides give none, and oligosaccharides give 2 to 10 units.

Step 2: Check the Options:

Ribose: a monosaccharide (aldopentose).

Sucrose: a disaccharide that gives glucose and fructose on hydrolysis, so it is an oligosaccharide.

Fructose: a monosaccharide (ketohexose).

Glucose: a monosaccharide (aldohexose).

Step 3: Conclusion:

Sucrose is the only oligosaccharide listed.

Final Answer:

Sucrose is an oligosaccharide, option (B).

(B) Sucrose

Quick Tip: Sucrose gives two monosaccharides on hydrolysis, so it is an oligosaccharide (disaccharide).



97. Which from following, sugar is an aldohexose?

- (A) Ribose
- (B) Fructose
- (C) Lactose
- (D) Glucose

Correct Answer: (D) Glucose

Solution:**Step 1: Understanding the Concept:**

The name "aldohexose" means a sugar with an aldehyde group and six carbon atoms.

Step 2: Check the Options:

Ribose: aldehyde, five carbons, so aldopentose.

Fructose: ketone, six carbons, so ketohexose.

Lactose: a disaccharide, not a single sugar.

Glucose: aldehyde, six carbons, so aldohexose.

Step 3: Conclusion:

Glucose is the aldohexose.

Final Answer:

Glucose is an aldohexose, option (D).

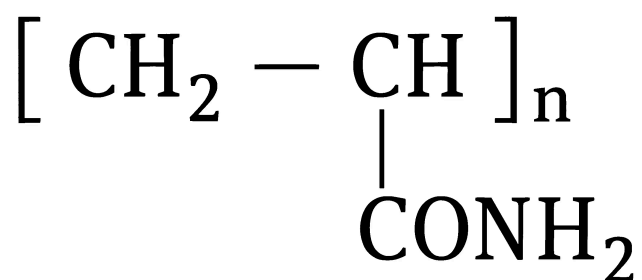
(D) Glucose

Quick Tip: An aldohexose has six carbons and an aldehyde group; glucose fits.

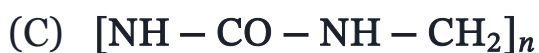
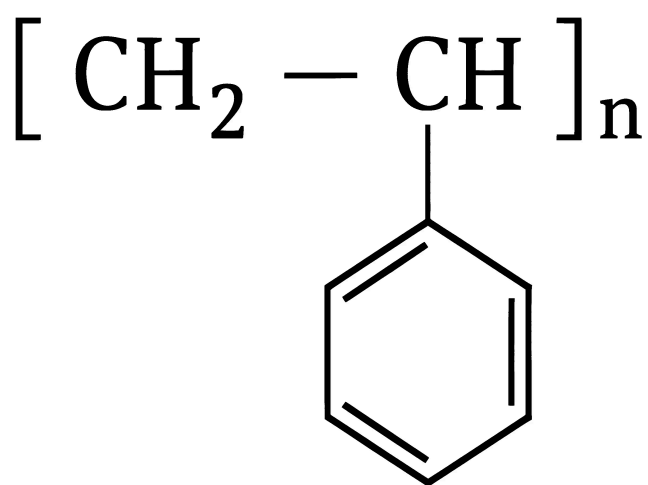


98. Identify ureaformaldehyde resin from following polymers?

(A)



(B)



Correct Answer: (C) $[\text{NH} - \text{CO} - \text{NH} - \text{CH}_2]_n$

Solution:

Step 1: Understanding the Concept:

Urea-formaldehyde resin is a condensation polymer formed from urea (NH_2CONH_2) and formaldehyde (HCHO). The repeating unit contains $-\text{NH}-\text{CO}-\text{NH}-\text{CH}_2-$.

Step 2: Check the Options:

Option (A) shows $-\text{[CH}_2 - \text{CH}(\text{CONH}_2)]_n-$, which is polyacrylamide, an addition polymer of acrylamide.

Option (B) shows a chain with a phenyl group on every second carbon, which is polystyrene.

Option (C) is $[\text{NH-CO-NH-CH}_2]_n$, the urea-formaldehyde repeating unit.

Option (D) is $[\text{NH-(CH}_2)_5\text{-CO}]_n$, which is nylon-6.

Step 3: Conclusion:

Only option (C) matches the urea-formaldehyde resin.

Final Answer:

The repeat unit with urea and CH_2 is option (C).



Quick Tip: Urea-formaldehyde resin repeats the unit NH-CO-NH-CH_2 .

99. Identify thermosetting polymer from following.

- (A) Ureaformaldehyde resin
- (B) Polystyrene
- (C) Buna-S
- (D) Terylene

Correct Answer: (A) Ureaformaldehyde resin

Solution:

Step 1: Understanding the Concept:

Thermosetting polymers are heavily cross-linked and become hard and infusible once heated and moulded. They cannot be remoulded.

Step 2: Check the Options:

Urea-formaldehyde resin: forms a cross-linked network, so it is thermosetting.

Polystyrene: linear chain, thermoplastic.

Buna-S: a synthetic rubber, an elastomer.

Terylene: a linear polyester, which is a fibre (thermoplastic).

Step 3: Conclusion:

Urea-formaldehyde resin is the thermosetting polymer.

Final Answer:

Urea-formaldehyde resin is thermosetting, option (A).

(A) Urea-formaldehyde resin

Quick Tip: Urea-formaldehyde resin is cross-linked and sets permanently on heating.

100. Identify antimicrobial compound from following.

- (A) Curcumin
- (B) Methylsalicylate
- (C) Cinnamaldehyde
- (D) Ascorbic acid

Correct Answer: (C) Cinnamaldehyde

Solution:

Step 1: Understanding the Concept:

Antimicrobials are substances that kill or stop the growth of microorganisms such as bacteria and fungi.

Step 2: Check Cinnamaldehyde:

Cinnamaldehyde is the main compound of cinnamon oil. It shows antimicrobial activity and is used as a natural preservative and flavouring, so it is the antimicrobial compound in the list.

Step 3: Other Options:

Curcumin is the yellow pigment of turmeric. Methyl salicylate is oil of wintergreen, used as a pain-relieving liniment (analgesic). Ascorbic acid is vitamin C, used as an antioxidant. These compounds are associated with other roles, so the item that matches the question is cinnamaldehyde.

Final Answer:

Cinnamaldehyde is the antimicrobial compound, option (C).

(C) Cinnamaldehyde

Quick Tip: Cinnamaldehyde from cinnamon oil has antimicrobial action.

Mathematics

Questions 101 to 150 | 2 marks each | 100 marks

101. Let a be an integer selected at random from the set $\{0, 1, 2, 3, \dots, 9\}$. The probability that the equation $ax^2 - ax + 1 = 0$ has real roots is ...

- (A) $\frac{3}{5}$
- (B) $\frac{1}{2}$
- (C) $\frac{2}{5}$
- (D) $\frac{5}{9}$

Correct Answer: (A) $\frac{3}{5}$

Solution:

Step 1: Understanding the Concept:

The equation $ax^2 - ax + 1 = 0$ is a quadratic only when $a \neq 0$. For $a = 0$ it becomes $1 = 0$, which has no solution at all.

Real roots need a non-negative discriminant.

Step 2: Discriminant Condition:

$$D = (-a)^2 - 4(a)(1) = a^2 - 4a = a(a - 4) \geq 0$$

With $a > 0$ this gives $a - 4 \geq 0$, so $a \geq 4$.

Step 3: Count Favourable Values:

From $\{0, 1, 2, \dots, 9\}$ the values $a = 4, 5, 6, 7, 8, 9$ qualify. That is 6 values. $a = 1, 2, 3$ give a negative discriminant, and $a = 0$ gives no equation.

Step 4: Probability:

Total outcomes = 10, so

$$P = \frac{6}{10} = \frac{3}{5}$$

Options (B), (C), (D) correspond to 5, 4 and 5.55 favourable values out of 10, which do not match the count.

Final Answer:

The probability is $\frac{3}{5}$, option (A).

(A) $\frac{3}{5}$

Quick Tip: A quadratic needs a nonzero a and discriminant at least zero; count the integers from 0 to 9 that qualify.



102. The smallest positive integer n for which $\frac{(1+i)^n}{(1-i)^{n-2}}$ is a real number, is

...

- (A) 1
- (B) 2
- (C) 3
- (D) 4

Correct Answer: (A) 1

Solution:

Step 1: Understanding the Concept:

A complex number is real when its argument is a multiple of π . Polar form makes powers easy.

Step 2: Polar Forms:

$$1 + i = \sqrt{2}e^{i\pi/4} \text{ and } 1 - i = \sqrt{2}e^{-i\pi/4}.$$

Step 3: Simplify the Ratio:

$$\frac{(1 + i)^n}{(1 - i)^{n-2}} = \frac{(\sqrt{2})^n e^{in\pi/4}}{(\sqrt{2})^{n-2} e^{-i(n-2)\pi/4}} = 2e^{i\pi(2n-2)/4} = 2e^{i\pi(n-1)/2}$$

Step 4: Reality Condition:

The number is real when $\frac{\pi(n-1)}{2}$ is a multiple of π , that is when $n - 1$ is even, so n is odd.

The smallest positive odd integer is $n = 1$. Check: for $n = 1$ the value is $\frac{1+i}{(1-i)^{-1}} = (1+i)(1-i) = 2$, which is real.

Final Answer:

The smallest such n is 1, option (A).

(A) 1

Quick Tip: Write $1+i$ and $1-i$ in polar form and compare the arguments.

103. Four men and three women are to be arranged at a round table. The number of arrangements in which no two women are together is

- (A) $7!$
- (B) $3! \times 4!$
- (C) $3! \times 3!$
- (D) $6!$

Correct Answer: (B) $3! \times 4!$

Solution:

Step 1: Understanding the Concept:

In a circular arrangement of n distinct people, positions are relative, giving $(n - 1)!$ arrangements.

Step 2: Seat the Men:

Four men can be seated around the round table in $(4 - 1)! = 3! = 6$ ways.

Step 3: Place the Women:

Four men create exactly 4 gaps between neighbours. To keep women apart, each woman goes into a different gap.

Choose 3 of the 4 gaps and arrange the 3 women there: ${}^4P_3 = 4!/1! = 24 = 4!$ ways.

Step 4: Multiply:

$$3! \times 4! = 6 \times 24 = 144$$

Option (A), $7! = 5040$, counts all linear arrangements, and (D), $6! = 720$, counts all circular arrangements with no condition. Option (C) $3! \times 3! = 36$ undercounts the gaps choice.

Final Answer:

The number of arrangements is $3! \times 4!$, option (B).

$$(B) 3! \times 4!$$

Quick Tip: Seat the men first around the table, then place the women in the gaps.

104. The value of $\cos(\frac{\pi}{5})$ is ...

- (A) $\frac{-1+\sqrt{5}}{4}$
(B) $\frac{-1+\sqrt{5}}{2}$
(C) $\frac{1+\sqrt{5}}{4}$
(D) $\frac{1-\sqrt{5}}{4}$

Correct Answer: (C) $\frac{1+\sqrt{5}}{4}$

Solution:**Step 1: Understanding the Concept:**

We want $\cos 36^\circ$. Let $A = \cos 36^\circ$ and $B = \cos 72^\circ$. Two simple facts about A and B will fix A .

Step 2: Find the Product:

Multiply and divide by $\sin 36^\circ$ and use the double-angle formula twice:

$$AB = \frac{\sin 36^\circ \cos 36^\circ \cos 72^\circ}{\sin 36^\circ} = \frac{\frac{1}{2} \sin 72^\circ \cos 72^\circ}{\sin 36^\circ} = \frac{\frac{1}{4} \sin 144^\circ}{\sin 36^\circ} = \frac{1}{4}$$

because $\sin 144^\circ = \sin 36^\circ$.

Step 3: Find the Difference:

$A - B = \cos 36^\circ - \cos 72^\circ = 2 \sin 54^\circ \sin 18^\circ$. Now $\sin 54^\circ = \cos 36^\circ = A$ and $\sin 18^\circ = \cos 72^\circ = B$. So

$$A - B = 2AB = \frac{1}{2}$$

Step 4: Solve for A:

Put $B = A - \frac{1}{2}$ in $AB = \frac{1}{4}$:

$$A \left(A - \frac{1}{2} \right) = \frac{1}{4} \Rightarrow 4A^2 - 2A - 1 = 0 \Rightarrow A = \frac{2 \pm \sqrt{20}}{8} = \frac{1 \pm \sqrt{5}}{4}$$

$\cos 36^\circ$ is positive, so $A = \frac{1 + \sqrt{5}}{4} \approx 0.809$.

Step 5: Check the Other Options:

Option (A) $\frac{-1+\sqrt{5}}{4} \approx 0.309$ is $\cos 72^\circ$. Option (D) is its negative. Option (B) is about 0.618, which is too small for $\cos 36^\circ$.

Final Answer:

$\cos(\pi/5) = \frac{1 + \sqrt{5}}{4}$, option (C).

$(C) \frac{1 + \sqrt{5}}{4}$

Quick Tip: Let $A = \cos 36$ and $B = \cos 72$; their product is $1/4$ and their difference is $1/2$.



105. Let $x \in [0, 6\pi]$ satisfy the equation $\cos x - \sin x = -1$. If $x = k(\frac{\pi}{3})$ where $k \in N$, then find the number of possible values of k is

- (A) 3
- (B) 4
- (C) 6
- (D) 8

Correct Answer: (A) 3

Solution:

Step 1: Reduce the Equation:

Write $\cos x - \sin x = \sqrt{2} \cos\left(x + \frac{\pi}{4}\right)$. So

$$\cos\left(x + \frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$$

Step 2: General Solution:

$x + \frac{\pi}{4} = 2n\pi \pm \frac{3\pi}{4}$. This gives $x = 2n\pi + \frac{\pi}{2}$ or $x = 2n\pi + \pi$.

Step 3: List Values in $[0, 6\pi]$:

From $x = 2n\pi + \pi/2$: $\frac{\pi}{2}, \frac{5\pi}{2}, \frac{9\pi}{2}$.

From $x = 2n\pi + \pi$: $\pi, 3\pi, 5\pi$.

Step 4: Keep Those of the Form $k(\pi/3)$:

Write each as $k\pi/3$: $\pi/2$ gives $k = 3/2$, $5\pi/2$ gives $15/2$, $9\pi/2$ gives $27/2$.

None is a natural number.

π gives $k = 3$, 3π gives $k = 9$, 5π gives $k = 15$.

So k takes 3 values: 3, 9, 15.

Final Answer:

There are 3 possible values of k , option (A).

(A) 3

Quick Tip: Solve for x in $[0, 6\pi]$, then keep only the values that are whole multiples of $\pi/3$.

106. A line making equal intercepts on coordinate axes and is tangent to the circle $x^2 + y^2 = 4$. The length of each intercept made by line on the coordinate axes is ...

- (A) $\sqrt{2}$
- (B) 2
- (C) $2\sqrt{2}$
- (D) 4

Correct Answer: (C) $2\sqrt{2}$

Solution:

Step 1: Understanding the Concept:

A line with equal intercepts c on both axes has the equation $x + y = c$. It is tangent to a circle when its distance from the centre equals the radius.

Step 2: Apply the Condition:

The circle $x^2 + y^2 = 4$ has centre $(0, 0)$ and radius 2. Distance from the origin to $x + y - c = 0$:

$$\frac{|c|}{\sqrt{1^2 + 1^2}} = \frac{|c|}{\sqrt{2}} = 2 \Rightarrow |c| = 2\sqrt{2}$$

Step 3: Read the Intercept:

The length of each intercept is $|c| = 2\sqrt{2}$.

Step 4: Check the Options:

$\sqrt{2}$, 2 and 4 would give distances 1, $\sqrt{2}$ and $2\sqrt{2}$ from the origin, none equal to the radius 2. Only $2\sqrt{2}$ works.

Final Answer:

Each intercept has length $2\sqrt{2}$, option (C).

$(C) 2\sqrt{2}$

Quick Tip: A line with equal intercepts is $x + y = c$; its distance from the origin must equal the radius.

107. A straight line L passes through the point of intersection of the lines $x - y + 1 = 0$ and $2x + y - 7 = 0$. If L intersects the positive x -axis at $A(a, 0)$ and the positive y -axis at $B(0, b)$, then the minimum area of the triangle OAB (where O is the origin) is

- (A) 6 square units.
- (B) 12 square units.
- (C) 24 square units.
- (D) 48 square units.

Correct Answer: (B) 12 square units.

Solution:

Step 1: Find the Fixed Point:

Adding $x - y + 1 = 0$ and $2x + y - 7 = 0$ gives $3x - 6 = 0$, so $x = 2$ and then $y = 3$. The line L passes through $(2, 3)$.

Step 2: Write the Line:

With intercepts a and b on the positive axes, $\frac{x}{a} + \frac{y}{b} = 1$. Passing through $(2, 3)$:

$$\frac{2}{a} + \frac{3}{b} = 1$$

Step 3: Minimise the Area:

Area = $\frac{1}{2}ab$. By AM-GM,

$$1 = \frac{2}{a} + \frac{3}{b} \geq 2\sqrt{\frac{6}{ab}} \Rightarrow \sqrt{ab} \geq 2\sqrt{6} \Rightarrow ab \geq 24$$

So the minimum area is $\frac{1}{2}(24) = 12$ square units. Equality needs $2/a = 3/b = 1/2$, i.e. $a = 4$, $b = 6$.

Step 4: Check:

For $a = 4$, $b = 6$: $2/4 + 3/6 = 1$, valid, and area = 12. Options 6 is

impossible because $ab \geq 24$ forces area at least 12.

Final Answer:

The minimum area is 12 square units, option (B).

(B) 12 square units

Quick Tip: Find the point of intersection, write the intercept form, and minimise ab with AM-GM.



108. If the combined equation of angle bisectors of the lines $x^2 - 2pxy - y^2 = 0$ is $x^2 - 2qxy - y^2 = 0$, then which of the following is true?

- (A) $2p + q = 0$
- (B) $2p + 3q = 0$
- (C) $pq = 1$
- (D) $pq + 1 = 0$

Correct Answer: (D) $pq + 1 = 0$

Solution:

Step 1: Understanding the Concept:

For the pair $ax^2 + 2hxy + by^2 = 0$, the combined equation of the angle bisectors is

$$\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$$

Step 2: Identify the Coefficients:

For $x^2 - 2pxy - y^2 = 0$: $a = 1$, $b = -1$, $2h = -2p$, so $h = -p$.

Step 3: Form the Bisector Equation:

$$\frac{x^2 - y^2}{2} = \frac{xy}{-p} \Rightarrow x^2 - y^2 = -\frac{2}{p}xy \Rightarrow x^2 + \frac{2}{p}xy - y^2 = 0$$

Step 4: Compare With the Given Form:

The bisector pair is given as $x^2 - 2qxy - y^2 = 0$. Matching the xy coefficients: $-2q = \frac{2}{p}$, so $pq = -1$, or $pq + 1 = 0$.

This matches option (D). Options (A) and (B) are linear relations in p and q , while the actual relation is a product, and option (C) has the wrong sign.

Final Answer:

The relation is $pq + 1 = 0$, option (D).

(D) $pq + 1 = 0$

Quick Tip: Use the bisector equation $(x^2 - y^2)/(a - b) = xy/h$ for the pair $ax^2 + 2hxy + by^2 = 0$.



109. A tangent having slope $-\frac{1}{2}$ to the ellipse $3x^2 + 4y^2 = 12$ intersects the X-axis and Y-axis at the points A and B respectively. if O is the origin, then the area of $\triangle AOB$ is ...

- (A) 4 sq. units
- (B) 8 sq. units
- (C) 12 sq. units
- (D) 16 sq. units

Correct Answer: (A) 4 sq. units

Solution:

Step 1: Standard Form:

Divide $3x^2 + 4y^2 = 12$ by 12: $\frac{x^2}{4} + \frac{y^2}{3} = 1$, so $a^2 = 4$ and $b^2 = 3$.

Step 2: Tangent With Slope m :

A tangent with slope m is $y = mx \pm \sqrt{a^2m^2 + b^2}$. For $m = -\frac{1}{2}$:

$$\sqrt{4 \cdot \frac{1}{4} + 3} = \sqrt{4} = 2 \Rightarrow y = -\frac{x}{2} \pm 2$$

Step 3: Intercepts:

Take $y = -\frac{x}{2} + 2$. It meets the y-axis at $B(0, 2)$ and the x-axis (set $y = 0$) at $A(4, 0)$.

Step 4: Area:

$$\text{Area} = \frac{1}{2} \times OA \times OB = \frac{1}{2} \times 4 \times 2 = 4 \text{ sq. units}$$

The other tangent $y = -\frac{x}{2} - 2$ gives the same area by symmetry.

Options 8, 12 and 16 would need intercept products of 16, 24 and 32.

Final Answer:

The area of the triangle is 4 square units, option (A).

(A) 4 sq. units

Quick Tip: Write the ellipse in standard form and use the tangent $y = mx$ plus or minus $\sqrt{a^2 m^2 + b^2}$.



110. If the function $f(x) = \frac{2\sqrt{2} - (\cos x + \sin x)^3}{1 - \sin 2x}$ is continuous at $x = \frac{\pi}{4}$, then the value of $f(\frac{\pi}{4})$ is ...

- (A) $\frac{3\sqrt{2}}{2}$
- (B) $\frac{5\sqrt{2}}{2}$
- (C) 0
- (D) $\sqrt{2}$

Correct Answer: (A) $\frac{3\sqrt{2}}{2}$

Solution:**Step 1: Understanding the Concept:**

Continuity at $x = \pi/4$ means $f(\pi/4) = \lim_{x \rightarrow \pi/4} f(x)$. At $x = \pi/4$ both numerator and denominator are zero, so we simplify first.

Step 2: Substitution:

Let $u = \cos x + \sin x$. Then $u^2 = 1 + \sin 2x$, so $1 - \sin 2x = 2 - u^2$. Also $2\sqrt{2} = (\sqrt{2})^3$. As $x \rightarrow \pi/4$, $u \rightarrow \sqrt{2}$.

Step 3: Simplify:

$$f(x) = \frac{(\sqrt{2})^3 - u^3}{2 - u^2} = \frac{(\sqrt{2} - u)(2 + \sqrt{2}u + u^2)}{(\sqrt{2} - u)(\sqrt{2} + u)} = \frac{2 + \sqrt{2}u + u^2}{\sqrt{2} + u}$$

Step 4: Take the Limit:

$$\lim_{u \rightarrow \sqrt{2}} \frac{2 + \sqrt{2}u + u^2}{\sqrt{2} + u} = \frac{2 + 2 + 2}{2\sqrt{2}} = \frac{6}{2\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$$

So $f(\pi/4) = \frac{3\sqrt{2}}{2}$.

Final Answer:

The value is $\frac{3\sqrt{2}}{2}$, option (A).

(A) $\frac{3\sqrt{2}}{2}$

Quick Tip: For continuity, $f(\pi/4)$ equals the limit; factor the numerator as a difference of cubes.

111. The value of $\lim_{x \rightarrow 0} \left(\frac{8}{x^8} \right) [1 - \cos \frac{x^2}{2} - \cos \frac{x^2}{4} + \cos \frac{x^2}{2} \cdot \cos \frac{x^2}{4}]$ is equal to ...

- (A) $\frac{1}{8}$
- (B) $\frac{1}{32}$
- (C) $\frac{1}{16}$
- (D) 0

Correct Answer: (B) $\frac{1}{32}$

Solution:

Step 1: Factorise the Bracket:

Let $A = \frac{x^2}{2}$ and $B = \frac{x^2}{4}$. The bracket is $1 - \cos A - \cos B + \cos A \cos B$
 $= (1 - \cos A)(1 - \cos B)$.

Step 2: Use the Small-Angle Estimate:

For small θ , $1 - \cos \theta \approx \frac{\theta^2}{2}$. So

$$1 - \cos A \approx \frac{x^4}{8}, \quad 1 - \cos B \approx \frac{x^4}{32}$$

Step 3: Multiply:

The bracket is about $\frac{x^4}{8} \cdot \frac{x^4}{32} = \frac{x^8}{256}$.

Step 4: Take the Limit:

$$\lim_{x \rightarrow 0} \frac{8}{x^8} \cdot \frac{x^8}{256} = \frac{8}{256} = \frac{1}{32}$$

Option (A) $\frac{1}{8}$ would need the bracket to be $x^8/64$, and option (D) would need a higher power of x . So (B) is correct.

Final Answer:

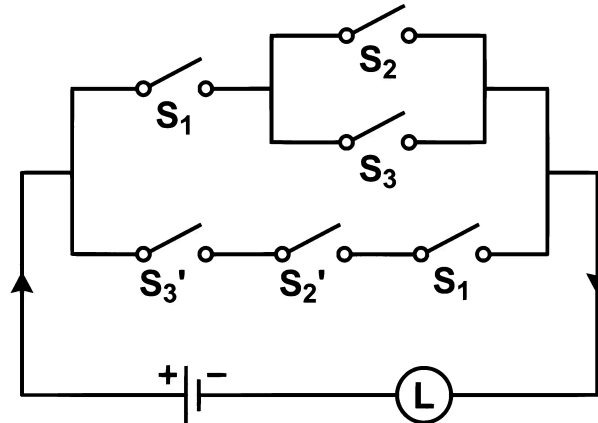
The limit is $\frac{1}{32}$, option (B).

$(B) \frac{1}{32}$

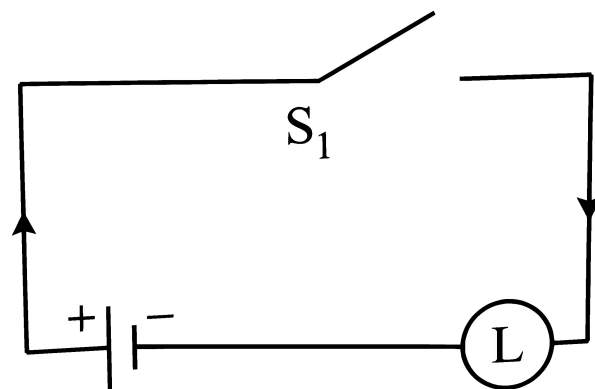
Quick Tip: The bracket factorises as $(1 - \cos A)(1 - \cos B)$.



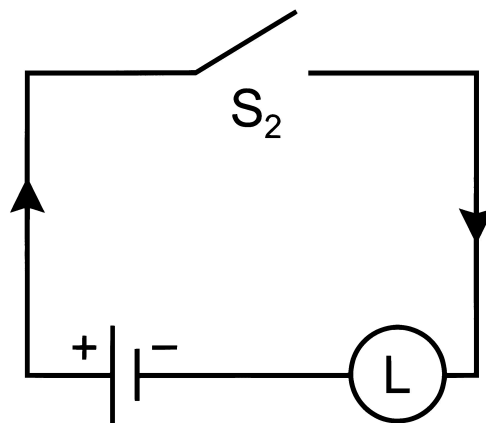
112. The simplified switching circuit for the following circuit is



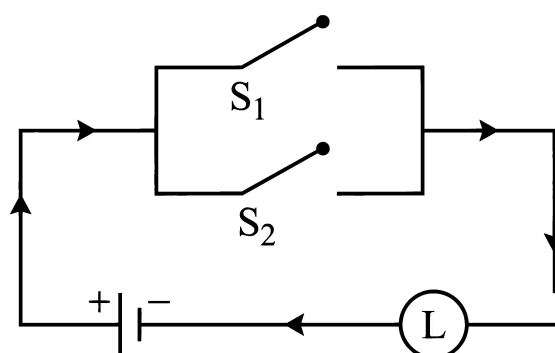
(A)



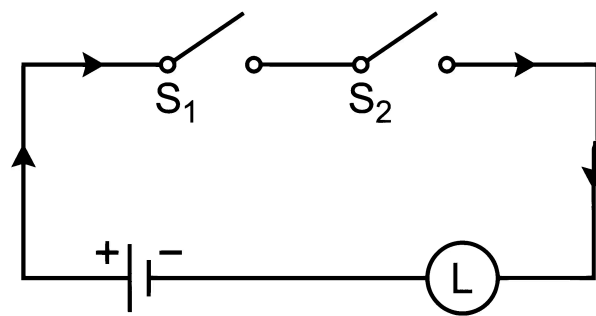
(B)



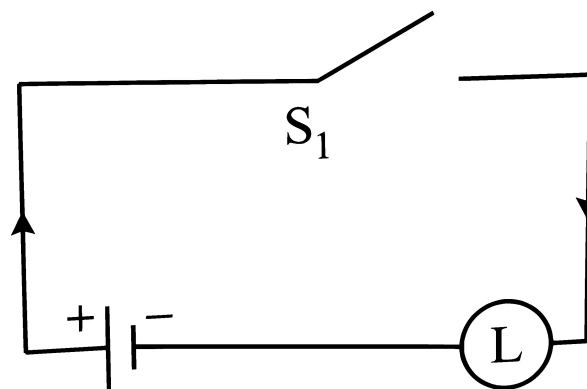
(C)



(D)



Correct Answer: (A)



Solution:

Step 1: Read the Circuit:

The figure has two parallel branches between the battery and the lamp.

Upper branch: S_1 in series with a parallel pair S_2 and S_3 . Expression: $S_1(S_2 + S_3)$.

Lower branch: S'_3 , S'_2 and S_1 in series. Expression: $S_1 S'_2 S'_3$.

Step 2: Combine the Branches:

Parallel branches mean OR. So the circuit is

$$S_1(S_2 + S_3) + S_1 S'_2 S'_3 = S_1 [S_2 + S_3 + S'_2 S'_3]$$

Step 3: Simplify:

By De Morgan, $S_2' S_3' = (S_2 + S_3)'$. So $S_2 + S_3 + (S_2 + S_3)' = 1$ (a variable plus its complement is 1).

$$S_1 \cdot 1 = S_1$$

Step 4: Choose the Option:

The simplified circuit is just a single switch S_1 in series with the lamp. Option (A) shows exactly that. Option (B) shows only S_2 , (C) shows S_1 and S_2 in parallel, and (D) shows them in series, none of which equals S_1 alone.

Final Answer:

The circuit reduces to the single switch S_1 , option (A).

(A) Single switch S_1

Quick Tip: Write the Boolean expression of the circuit and simplify it.

113. The logical statement $(p \vee q) \wedge [(\sim p \wedge q) \vee (p \wedge \sim q)] \wedge \sim q$ is logically equivalent to ...

- (A) $p \wedge \sim q$
- (B) $\sim p \wedge q$
- (C) $p \wedge q$
- (D) $\sim p \vee \sim q$

Correct Answer: (A) $p \wedge \sim q$

Solution:

Step 1: Understanding the Concept:

The middle bracket $(\sim p \wedge q) \vee (p \wedge \sim q)$ is true exactly when p and q have different truth values (exclusive OR).

Step 2: Combine With the Last Factor:

If $\sim q$ is true then q is false. For the exclusive OR to be true with q false, p must be true. So the middle bracket together with $\sim q$ gives $p \wedge \sim q$.

Step 3: Algebra Check:

$$[(\sim p \wedge q) \vee (p \wedge \sim q)] \wedge \sim q = (\sim p \wedge q \wedge \sim q) \vee (p \wedge \sim q \wedge \sim q) = F \vee (p \wedge \sim q) = p \wedge \sim q$$

Step 4: Include the First Factor:

The factor $p \vee q$ is already true whenever $p \wedge \sim q$ is true (since p is true). So it does not change the result.

$$(p \vee q) \wedge (p \wedge \sim q) = p \wedge \sim q$$

This is option (A).

Final Answer:

The statement is equivalent to $p \wedge \sim q$, option (A).

$(A) \ p \wedge \sim q$

Quick Tip: Use the exclusive-or shape of the middle bracket and the final $\sim q$.

114. Which of the following logical statements is a tautology?

- (A) $[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$
- (B) $(p \rightarrow q) \wedge (p \wedge \sim q)$
- (C) $[(p \vee q) \wedge \sim p] \wedge \sim q$
- (D) $[(p \vee \sim q) \vee (\sim p \wedge q)] \wedge r$

Correct Answer: (A) $[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$

Solution:

Step 1: Understanding the Concept:

A tautology is true in every row of its truth table.

Step 2: Check Option (A):

$[(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$ is the law of contrapositive reasoning (modus tollens). If $p \rightarrow q$ holds and q is false, then p must be false. The only way for the whole implication to fail would need the antecedent true and $\sim p$ false, i.e. p true, q false, and $p \rightarrow q$ true, which is impossible. So (A) is a tautology.

Step 3: Check Option (B):

$(p \rightarrow q) \wedge (p \wedge \sim q)$: the second part makes $p \rightarrow q$ false, so the conjunction is always false. It is a contradiction.

Step 4: Check Options (C) and (D):

(C) $[(p \vee q) \wedge \sim p] \wedge \sim q$: needs p false, q false, and $p \vee q$ true,

impossible. It is a contradiction.

(D) $[(p \vee \sim q) \vee (\sim p \wedge q)] \wedge r$: false whenever r is false, so it is not a tautology.

Final Answer:

Only option (A) is a tautology.

$$(A) [(p \rightarrow q) \wedge \sim q] \rightarrow \sim p$$

Quick Tip: A tautology is true for every combination of truth values; test modus tollens.

115. In $\triangle ABC$, with usual notations, if the sides a , b and c are in the ratio $18 : 17 : 7$, then $\cot \frac{A}{2} : \cot \frac{B}{2} : \cot \frac{C}{2} =$

(A) $4 : 3 : 14$

(B) $14 : 4 : 3$

(C) $3 : 4 : 14$

(D) $4 : 14 : 3$

Correct Answer: (C) $3 : 4 : 14$

Solution:

Step 1: Setup:

Let $a = 18k$, $b = 17k$, $c = 7k$. The semi-perimeter is s
 $= \frac{18k + 17k + 7k}{2} = 21k$.

Step 2: Key Formula:

In a triangle, $\cot \frac{A}{2} = \frac{s-a}{r}$, $\cot \frac{B}{2} = \frac{s-b}{r}$, $\cot \frac{C}{2} = \frac{s-c}{r}$, where r is

the inradius.

Step 3: Compute:

$$s - a = 3k, s - b = 4k, s - c = 14k.$$

Step 4: Ratio:

$$\cot \frac{A}{2} : \cot \frac{B}{2} : \cot \frac{C}{2} = (s - a) : (s - b) : (s - c) = 3 : 4 : 14$$

Option (B) $14 : 4 : 3$ and the others reorder the same numbers, so the correct assignment matters: the largest side a goes with the smallest $s - a$, giving $3 : 4 : 14$.

Final Answer:

The ratio is $3 : 4 : 14$, option (C).

(C) $3 : 4 : 14$

Quick Tip: Use $\cot(A/2) = (s - a)/r$ and find $s - a$, $s - b$, $s - c$.

116. If $A = [a_{ij}]_{3 \times 3}$, where $a_{ij} = \begin{cases} 1, & \text{if } i + j \text{ is even} \\ 0, & \text{if } i + j \text{ is odd} \end{cases}$, then $\text{adj}(A) = \dots$

$$(A) \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$(B) \begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$(C) \begin{bmatrix} 0 & 1 & 0 \\ 1 & 1 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$$(D) \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Correct Answer: (B) $\begin{bmatrix} 1 & 0 & -1 \\ 0 & 0 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

Solution:

Step 1: Build the Matrix:

Entry is 1 when $i + j$ is even and 0 when odd. So row 1 is $(1, 0, 1)$, row 2 is $(0, 1, 0)$ and row 3 is $(1, 0, 1)$.

Step 2: Cofactors:

$$C_{11} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1, C_{12} = -\begin{vmatrix} 0 & 0 \\ 1 & 1 \end{vmatrix} = 0, C_{13} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1.$$

$$C_{21} = -\begin{vmatrix} 0 & 1 \\ 0 & 1 \end{vmatrix} = 0, C_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0, C_{23} = -\begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix} = 0.$$

$$C_{31} = \begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = -1, C_{32} = -\begin{vmatrix} 1 & 1 \\ 0 & 0 \end{vmatrix} = 0, C_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1.$$

Step 3: Adjoint:

The adjoint is the transpose of the cofactor matrix. The cofactor matrix has rows $(1, 0, -1)$, $(0, 0, 0)$, $(-1, 0, 1)$. It is symmetric, so the adjoint is the same matrix.

Step 4: Check the Options:

Option (B) has exactly these rows. Options (A), (C) and (D) have only 0 and 1 as entries, but the cofactors C_{13} and C_{31} are -1 , so they cannot be the adjoint.

Final Answer:

$\text{adj}(A)$ has rows $(1, 0, -1), (0, 0, 0), (-1, 0, 1)$, option (B).

(B)

Quick Tip: Build the matrix from the rule, then find the cofactors.



117. The inverse of matrix $\begin{bmatrix} 1 + pq & p & 0 \\ q & 1 + pq & p \\ 0 & q & 1 \end{bmatrix}$ is ...

- (A) $\begin{bmatrix} 1 + pq & p & 0 \\ q & 1 + pq & p \\ 0 & q & 1 \end{bmatrix}$
- (B) $\begin{bmatrix} 1 & p & p^2 \\ q & 1 + pq & p + p^2q \\ q^2 & q + pq^2 & 1 + pq + p^2q^2 \end{bmatrix}$
- (C) $\begin{bmatrix} 1 & -p & p^2 \\ -q & 1 + pq & -(p + p^2q) \\ q^2 & -(q + pq^2) & 1 + pq + p^2q^2 \end{bmatrix}$
- (D) $\begin{bmatrix} 1 & -p & p^2 \\ -q & 1 + pq & p + p^2q \\ q^2 & q + pq^2 & 1 + pq + p^2q^2 \end{bmatrix}$

Correct Answer: (C)
$$\begin{bmatrix} 1 & -p & p^2 \\ -q & 1 + pq & -(p + p^2q) \\ q^2 & -(q + pq^2) & 1 + pq + p^2q^2 \end{bmatrix}$$

Solution:

Step 1: Determinant:

For M with rows $(1 + pq, p, 0)$, $(q, 1 + pq, p)$, $(0, q, 1)$, expand along the first row:

$$\det M = (1 + pq) [(1 + pq)(1) - pq] - p [q(1) - 0] = (1 + pq) - pq = 1$$

Since $\det M = 1$, $M^{-1} = \text{adj } M$.

Step 2: Cofactors:

$$C_{11} = (1 + pq) - pq = 1, C_{12} = -(q \cdot 1 - 0) = -q, C_{13} = q^2.$$

$$C_{21} = -(p \cdot 1 - 0) = -p, C_{22} = 1 + pq, C_{23} = -(q(1 + pq) - 0) = -(q + pq^2).$$

$$C_{31} = p^2, C_{32} = -((1 + pq)p - 0) = -(p + p^2q), C_{33} = (1 + pq)^2 - pq = 1 + pq + p^2q^2.$$

Step 3: Adjoint:

The inverse is the transpose of the cofactor matrix. Its rows are:

$$(1, -p, p^2), (-q, 1 + pq, -(p + p^2q)), (q^2, -(q + pq^2), 1 + pq + p^2q^2).$$

Step 4: Match the Option:

This is exactly option (C). Option (B) has first row $(1, p, p^2)$ with no minus signs, so it differs. Option (D) has the right signs in the first row and column but $+(p + p^2q)$ and $+(q + pq^2)$ in the lower right, where we found minus signs. Option (A) is M itself, which is not equal to its own inverse because $M^2 \neq I$.

Final Answer:

The inverse is option (C).

(C)

Quick Tip: Find the determinant first, then the cofactors; here the determinant is 1.

118. $\sum_{k=1}^{2026} \sin^{-1}(\cos \frac{k\pi}{4}) =$

(A) $-\frac{\pi}{4}$

(B) 0

(C) $\frac{\pi}{4}$

(D) $\frac{\pi}{2}$

Correct Answer: (C) $\frac{\pi}{4}$

Solution:**Step 1: Understanding the Concept:**

$\sin^{-1}(\cos \theta)$ returns a value in $[-\pi/2, \pi/2]$. We evaluate it for $\theta = k\pi/4$, which repeats every 8 values of k .

Step 2: One Full Period:

$k = 1: \cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$, so the term is $\frac{\pi}{4}$.

$k = 2: \cos \frac{\pi}{2} = 0$, term 0.

$k = 3: \cos \frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$, term $-\frac{\pi}{4}$.

$k = 4: \cos \pi = -1$, term $-\frac{\pi}{2}$.

$k = 5: \cos \frac{5\pi}{4} = -\frac{\sqrt{2}}{2}$, term $-\frac{\pi}{4}$.

$k = 6: \cos \frac{3\pi}{2} = 0$, term 0.

$$k = 7: \cos \frac{7\pi}{4} = \frac{\sqrt{2}}{2}, \text{ term } \frac{\pi}{4}.$$

$$k = 8: \cos 2\pi = 1, \text{ term } \frac{\pi}{2}.$$

Step 3: Sum of a Period:

$$\frac{\pi}{4} + 0 - \frac{\pi}{4} - \frac{\pi}{2} - \frac{\pi}{4} + 0 + \frac{\pi}{4} + \frac{\pi}{2} = 0$$

Step 4: Use the Period:

$2026 = 8 \times 253 + 2$. The first 2024 terms add up to 0. The last two terms are those of $k \equiv 1, 2$: $\frac{\pi}{4} + 0 = \frac{\pi}{4}$.

Options (A) and (B) would need those leftover terms to cancel, and (D) would need an extra $\pi/4$.

Final Answer:

The sum is $\frac{\pi}{4}$, option (C).

$$\boxed{(C) \frac{\pi}{4}}$$

Quick Tip: The terms repeat with period 8 and each block of 8 sums to zero.

119. If $\sin^{-1}(\tan \frac{\pi}{4}) - \sin^{-1}(\sqrt{\frac{3}{x}}) = \frac{\pi}{6}$ then x is a root of the equation

(A) $x^2 - x - 6 = 0$

(B) $x^2 - x - 12 = 0$

(C) $x^2 + x - 12 = 0$

(D) $x^2 + x - 6 = 0$

Correct Answer: (B) $x^2 - x - 12 = 0$

Solution:

Step 1: Simplify the First Term:

$$\tan \frac{\pi}{4} = 1, \text{ so } \sin^{-1}(1) = \frac{\pi}{2}.$$

Step 2: Solve for the Second Term:

$$\frac{\pi}{2} - \sin^{-1} \sqrt{\frac{3}{x}} = \frac{\pi}{6} \Rightarrow \sin^{-1} \sqrt{\frac{3}{x}} = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$$

Step 3: Find x:

$$\sqrt{\frac{3}{x}} = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2} \Rightarrow \frac{3}{x} = \frac{3}{4} \Rightarrow x = 4$$

Step 4: Test the Equations:

$$x^2 - x - 6 \text{ at } 4: 16 - 4 - 6 = 6 \neq 0.$$

$$x^2 - x - 12 \text{ at } 4: 16 - 4 - 12 = 0. \text{ True.}$$

$$x^2 + x - 12 \text{ at } 4: 16 + 4 - 12 = 8 \neq 0.$$

$$x^2 + x - 6 \text{ at } 4: 16 + 4 - 6 = 14 \neq 0.$$

Only (B) has 4 as a root.

Final Answer:

$x = 4$ is a root of $x^2 - x - 12 = 0$, option (B).

$$\boxed{\text{(B) } x^2 - x - 12 = 0}$$

Quick Tip: Evaluate $\sin^{-1}(\tan(\pi/4)) = \pi/2$, then solve for the second term.



120. If $f(x) = x^2$ and $g(x) = [x^2]$ where $[\cdot]$ represents the greatest integer function then, $(f \circ g)(\frac{3}{2}) + (g \circ f)(\frac{3}{2})$ is equal to ...

- (A) 2
- (B) 5
- (C) 9
- (D) 10

Correct Answer: (C) 9

Solution:

Step 1: Understanding the Concept:

$f \circ g$ means $f(g(x))$ and $g \circ f$ means $g(f(x))$. The inner function is applied first.

Step 2: Compute $(f \circ g)(3/2)$:

$g(3/2) = [(3/2)^2] = [2.25] = 2$. Then $f(2) = 2^2 = 4$.

Step 3: Compute $(g \circ f)(3/2)$:

$f(3/2) = \frac{9}{4} = 2.25$. Then $g(2.25) = [(2.25)^2] = [5.0625] = 5$.

Step 4: Add:

$$4 + 5 = 9$$

Option (A) 2 and (D) 10 come from using the wrong order of

composition or from dropping the greatest integer. The correct sum is 9.

Final Answer:

The sum is 9, option (C).

(C) 9

Quick Tip: Compute each composite carefully, applying the inner function first.

121. If $y = \sin(2\sin^{-1}x)$ then $\frac{dy}{dx} = ..$

- (A) $\frac{2-4x^2}{\sqrt{1-x^2}}$
- (B) $\frac{2+4x^2}{\sqrt{1-x^2}}$
- (C) $\frac{2-4x^2}{\sqrt{1+x^2}}$
- (D) $\frac{2+4x^2}{\sqrt{1+x^2}}$

Correct Answer: (A) $\frac{2-4x^2}{\sqrt{1-x^2}}$

Solution:

Step 1: Substitute:

Let $\theta = \sin^{-1} x$, so $x = \sin \theta$. Then $y = \sin 2\theta = 2 \sin \theta \cos \theta$
 $= 2x\sqrt{1-x^2}$.

Step 2: Differentiate With the Product Rule:

$$\frac{dy}{dx} = 2\sqrt{1-x^2} + 2x \cdot \frac{-x}{\sqrt{1-x^2}}$$

Step 3: Combine:

$$\frac{dy}{dx} = \frac{2(1 - x^2) - 2x^2}{\sqrt{1 - x^2}} = \frac{2 - 4x^2}{\sqrt{1 - x^2}}$$

Step 4: Check the Options:

Options (C) and (D) have $\sqrt{1 + x^2}$ in the denominator, which cannot arise from derivatives of $\sin^{-1}$. Option (B) has $+4x^2$, but the second term is subtracted. So (A) is correct.

Final Answer:

$$\frac{dy}{dx} = \frac{2 - 4x^2}{\sqrt{1 - x^2}}, \text{ option (A).}$$

$(A) \frac{2 - 4x^2}{\sqrt{1 - x^2}}$

Quick Tip: Put $x = \sin t$, so $y = \sin 2t$, and differentiate with the chain rule.

122. Let $x = at^2 - 1$, where $a > 0$ and $y = t^3 + 1$. If at $t = 1$, $\frac{d^2y}{dx^2} = \frac{3}{16}$, then the value of a is...

- (A) 3
- (B) -2
- (C) 1
- (D) 2

Correct Answer: (D) 2

Solution:

Step 1: First Derivative:

$$x = at^2 - 1 \Rightarrow \frac{dx}{dt} = 2at. \quad y = t^3 + 1 \Rightarrow \frac{dy}{dt} = 3t^2. \text{ So}$$

$$\frac{dy}{dx} = \frac{3t^2}{2at} = \frac{3t}{2a}$$

Step 2: Second Derivative:

$$\frac{d^2y}{dx^2} = \frac{d}{dt} \left(\frac{3t}{2a} \right) \bigg/ \frac{dx}{dt} = \frac{3/(2a)}{2at} = \frac{3}{4a^2t}$$

Step 3: Use $t = 1$:

$$\text{At } t = 1: \frac{3}{4a^2} = \frac{3}{16} \Rightarrow a^2 = 4. \text{ Since } a > 0, a = 2.$$

Step 4: Check the Options:

$a = -2$ is excluded by $a > 0$. For $a = 3$ and $a = 1$ the value would be $1/12$ and $3/4$. Only $a = 2$ works.

Final Answer:

The value is $a = 2$, option (D).

(D) 2

Quick Tip: Differentiate in t twice, using $dy/dx = (dy/dt)/(dx/dt)$.

123. If $y = \cot^{-1}\left(\frac{1+\sin 5x}{\cos 5x}\right)$, then the value of $\frac{dy}{dx}$ is

- (A) -5
- (B) 5
- (C) $-\frac{2}{5}$
- (D) $-\frac{5}{2}$

Correct Answer: (D) $-\frac{5}{2}$

Solution:

Step 1: Simplify the Argument:

For any angle t , $\frac{1 + \sin t}{\cos t} = \tan\left(\frac{\pi}{4} + \frac{t}{2}\right)$. With $t = 5x$:

$$\frac{1 + \sin 5x}{\cos 5x} = \tan\left(\frac{\pi}{4} + \frac{5x}{2}\right)$$

Step 2: Use the Inverse Identity:

$\cot^{-1}(\tan \varphi) = \frac{\pi}{2} - \varphi$. So

$$y = \frac{\pi}{2} - \frac{\pi}{4} - \frac{5x}{2} = \frac{\pi}{4} - \frac{5x}{2}$$

Step 3: Differentiate:

$$\frac{dy}{dx} = -\frac{5}{2}$$

Step 4: Check the Options:

(A) -5 and (B) 5 forget the half-angle factor. (C) $-2/5$ inverts the

fraction. Only $-5/2$ fits, option (D).

Final Answer:

The derivative is $-\frac{5}{2}$, option (D).

$$\boxed{(D) -\frac{5}{2}}$$

Quick Tip: Rewrite $(1 + \sin t)/\cos t$ as $\tan(\pi/4 + t/2)$.



124. The derivative of $\log_{10}x$ with respect to $\log_x 10$ is

- (A) $-\log_{10}x^2$
- (B) $-(\log_{10}x)^2$
- (C) $(\log_x 10)^2$
- (D) $\frac{x^2}{100}$

Correct Answer: (B) $-(\log_{10}x)^2$

Solution:

Step 1: Write in Natural Logs:

$$\text{Let } u = \log_{10} x = \frac{\ln x}{\ln 10} \text{ and } v = \log_x 10 = \frac{\ln 10}{\ln x}.$$

Step 2: Differentiate Both:

$$\frac{du}{dx} = \frac{1}{x \ln 10} \cdot \frac{dv}{dx} = -\frac{\ln 10}{x(\ln x)^2}.$$

Step 3: Divide:

$$\frac{du}{dv} = \frac{1/(x \ln 10)}{-\ln 10/(x(\ln x)^2)} = -\frac{(\ln x)^2}{(\ln 10)^2} = -(\log_{10} x)^2$$

Step 4: Check the Options:

Option (C) has the wrong sign. Options (A) and (D) do not match the form $(\ln x / \ln 10)^2$. So (B) is correct.

Final Answer:

The derivative is $-(\log_{10} x)^2$, option (B).

$$\boxed{(B) - (\log_{10} x)^2}$$

Quick Tip: Write both functions in natural logarithms and divide the derivatives.

125. A tank with a rectangular base and rectangular sides, open at the top is made. Depth of the tank is 4 m and its volume is 36 cubic meters. For making a tank cost of base material used is Rs. 100 per sq. meter and that of sides is Rs. 50 per sq. meter. Then minimum cost of tank is

- (A) Rs. 1100
- (B) Rs. 2200
- (C) Rs. 3300
- (D) Rs. 4400

Correct Answer: (C) Rs. 3300

Solution:

Step 1: Set Up:

Depth = 4 m and volume = 36 m^3 , so the base area is $lb = 36/4 = 9 \text{ m}^2$.

Step 2: Cost Function:

Base cost: $100 \times 9 = 900$ rupees. Side area: $2(l + b) \times 4 = 8(l + b)$, at 50 per sq. m, giving $400(l + b)$.

$$C = 900 + 400(l + b)$$

Step 3: Minimise:

With $lb = 9$, AM-GM gives $l + b \geq 2\sqrt{lb} = 6$, with equality at $l = b = 3$.

Step 4: Minimum Cost:

$$C_{\min} = 900 + 400 \times 6 = 900 + 2400 = 3300$$

So the minimum cost is Rs. 3300. Option (A) 1100 and (B) 2200 are below the side cost alone.

Final Answer:

The minimum cost is Rs. 3300, option (C).

(C) Rs. 3300

Quick Tip: Fix the base area from the volume, then minimise the cost of the sides using AM-GM.

126. A particle is fired straight up from the ground. Its height in feet after t second is given by $s(t) = 128t - 16t^2$. The velocity of the particle when it hits the ground is...

- (A) -128 ft/sec
- (B) 128 ft/sec
- (C) 0 ft/sec
- (D) 256 ft/sec

Correct Answer: (A) -128 ft/sec

Solution:

Step 1: Find the Landing Time:

The particle is on the ground when $s = 0$: $128t - 16t^2 = 16t(8 - t) = 0$.
So $t = 0$ (launch) or $t = 8$ s (landing).

Step 2: Velocity Function:

$$v = \frac{ds}{dt} = 128 - 32t.$$

Step 3: Evaluate at Landing:

$$v(8) = 128 - 32(8) = 128 - 256 = -128 \text{ ft/s.}$$

Step 4: Meaning:

The negative sign shows the particle moves downward at 128 ft/s when it hits the ground. The speed equals the launch speed, so the magnitude is 128 . Option (B) has the wrong sign, (C) is the velocity at the peak, and (D) is unrelated.

Final Answer:

The velocity is -128 ft/s, option (A).

$$(A) - 128 \text{ ft/s}$$

Quick Tip: Find the time when $s = 0$ again, then differentiate $s(t)$.

127. The value of c satisfied by the Rolle's theorem for the function $f(x) = x^2(1 - x)^2$, $x \in [0, 1]$ is...

- (A) 0
- (B) 1
- (C) $\frac{1}{2}$
- (D) -1

Correct Answer: (C) $\frac{1}{2}$

Solution:

Step 1: Check the Conditions:

$f(x) = x^2(1 - x)^2$ is a polynomial, so it is continuous and differentiable. Also $f(0) = 0 = f(1)$. Rolle's theorem applies.

Step 2: Differentiate:

$$f'(x) = 2x(1 - x)^2 - 2x^2(1 - x) = 2x(1 - x)[(1 - x) - x] = 2x(1 - x)(1 - 2x)$$

Step 3: Solve $f'(c) = 0$:

The roots are $x = 0$, $x = 1$, $x = \frac{1}{2}$. The point c must lie in the open interval $(0, 1)$, so $c = \frac{1}{2}$.

Step 4: Check the Options:

Options 0 and 1 are the endpoints, which are not in the open interval.
Option -1 lies outside $[0, 1]$. So (C) is correct.

Final Answer:

$$c = \frac{1}{2}, \text{ option (C).}$$

$$\boxed{\text{(C) } \frac{1}{2}}$$

Quick Tip: Rolle's theorem gives a point c where $f'(c) = 0$.

128. The value of integral $\int \frac{dx}{\sin^2 x + \tan^2 x}$ is...

- (A) $\frac{-1}{2\tan x} + \frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right) + c$
(B) $\frac{-1}{2\tan x} - \frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right) + c$
(C) $\frac{1}{2\tan x} - \frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right) + c$
(D) $\frac{1}{2\tan x} + \frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right) + c$

Correct Answer: (B) $\frac{-1}{2\tan x} - \frac{1}{2\sqrt{2}} \tan^{-1}\left(\frac{\tan x}{\sqrt{2}}\right) + c$

Solution:**Step 1: Rewrite in Terms of $\tan x$:**

Let $t = \tan x$, so $dx = \frac{dt}{1+t^2}$, $\sin^2 x = \frac{t^2}{1+t^2}$. Then

$$\sin^2 x + \tan^2 x = \frac{t^2}{1+t^2} + t^2 = \frac{t^2(2+t^2)}{1+t^2}$$

Step 2: Form the Integral:

$$I = \int \frac{1+t^2}{t^2(2+t^2)} \cdot \frac{dt}{1+t^2} = \int \frac{dt}{t^2(t^2+2)}$$

Step 3: Partial Fractions:

$$\frac{1}{t^2(t^2+2)} = \frac{1}{2} \left(\frac{1}{t^2} - \frac{1}{t^2+2} \right). \text{ So}$$

$$I = \frac{1}{2} \left[-\frac{1}{t} - \frac{1}{\sqrt{2}} \tan^{-1} \frac{t}{\sqrt{2}} \right] + c$$

Step 4: Back-Substitute:

$$I = -\frac{1}{2 \tan x} - \frac{1}{2\sqrt{2}} \tan^{-1} \left(\frac{\tan x}{\sqrt{2}} \right) + c$$

Option (B) matches. Options (A) and (D) have a plus sign in front of the arctan term, and (C) has the wrong sign for $1/(2 \tan x)$.

Final Answer:

The integral is option (B).

(B)

Quick Tip: Convert to $\tan x$ using $t = \tan x$, then use partial fractions.

129. The value of integral $\int \frac{\sqrt{x^2+1} [\log(x^2+1)-2\log x]}{x^4} dx$ is equal to...

- (A) $(1 + \frac{1}{x^2})^{3/2} [\frac{-1}{3} \log(1 + \frac{1}{x^2}) + \frac{2}{9}] + c$
(B) $(1 + \frac{1}{x^2})^{3/2} [\frac{-1}{3} \log(1 + \frac{1}{x^2}) - \frac{2}{9}] + c$
(C) $(1 + \frac{1}{x^2})^{3/2} [\frac{-1}{3} \log(1 + \frac{1}{x^2}) + \frac{2}{3}] + c$
(D) $(1 + \frac{1}{x^2})^{3/2} [\frac{-1}{3} \log(1 + \frac{1}{x^2}) - \frac{2}{3}] + c$

Correct Answer: (A) $(1 + \frac{1}{x^2})^{3/2} [\frac{-1}{3} \log(1 + \frac{1}{x^2}) + \frac{2}{9}] + c$

Solution:

Step 1: Simplify the Log Part:

$$\log(x^2 + 1) - 2 \log x = \log \frac{x^2 + 1}{x^2} = \log \left(1 + \frac{1}{x^2} \right).$$

Step 2: Substitute:

$$\text{Let } t = 1 + \frac{1}{x^2}. \text{ Then } dt = -\frac{2}{x^3} dx. \text{ Also } \frac{\sqrt{x^2 + 1}}{x^4} dx = \frac{\sqrt{x^2 + 1}}{x} \cdot \frac{dx}{x^3} \\ = \sqrt{t} \cdot \left(-\frac{dt}{2} \right).$$

$$I = -\frac{1}{2} \int \sqrt{t} \log t \, dt$$

Step 3: Integrate by Parts:

Take $u = \log t$ and $dv = t^{1/2} dt$, so $v = \frac{2}{3} t^{3/2}$.

$$\int t^{1/2} \log t \, dt = \frac{2}{3} t^{3/2} \log t - \frac{2}{3} \int t^{1/2} dt = \frac{2}{3} t^{3/2} \log t - \frac{4}{9} t^{3/2}$$

Step 4: Final Form:

$$I = -\frac{1}{2} \left[\frac{2}{3} t^{3/2} \log t - \frac{4}{9} t^{3/2} \right] + c = t^{3/2} \left[-\frac{1}{3} \log t + \frac{2}{9} \right] + c$$

This is option (A). Options (C) and (D) have the constant $2/3$, which would need $\frac{4}{3}$ in the bracket before halving, and (B) has the wrong sign on the constant.

Final Answer:

The integral is option (A).

(A)

Quick Tip: Rewrite the bracket as $\log(1 + 1/x^2)$ and substitute $t = 1 + 1/x^2$.

130. If $f(x) = \frac{1}{\log x}$ and $g(x) = \frac{1}{(\log x)^2}$, then the value of $\int [f(x) - g(x)] dx$ is...

- (A) $(\log x)^2 + c$
- (B) $x \log x + c$
- (C) $\frac{x}{\log x} + c$
- (D) $\frac{1}{\log x} + c$

Correct Answer: (C) $\frac{x}{\log x} + c$

Solution:

Step 1: Spot the Pattern:

We need $\int \left[\frac{1}{\log x} - \frac{1}{(\log x)^2} \right] dx$. Try the derivative of $\frac{x}{\log x}$.

Step 2: Differentiate:

$$\frac{d}{dx} \left(\frac{x}{\log x} \right) = \frac{\log x \cdot 1 - x \cdot \frac{1}{x}}{(\log x)^2} = \frac{\log x - 1}{(\log x)^2} = \frac{1}{\log x} - \frac{1}{(\log x)^2}$$

Step 3: Integrate:

Since the integrand is the derivative of $\frac{x}{\log x}$,

$$\int [f(x) - g(x)]dx = \frac{x}{\log x} + c$$

Step 4: Check the Options:

Option (B) $x \log x$ has derivative $\log x + 1$, and (A) and (D) do not differentiate back to the integrand. So (C) is correct.

Final Answer:

The integral is $\frac{x}{\log x} + c$, option (C).

$(C) \frac{x}{\log x} + c$

Quick Tip: The expression is the derivative of $x/\log x$.

131. If $\int \frac{dx}{x^{7/2}(x^4+1)^{3/8}} = m\left(\frac{x^4+1}{x^4}\right)^n + c$, where c is a constant of integration, then the value of $\frac{n}{m}$ is...

- (A) $-\frac{1}{16}$
 (B) $-\frac{25}{16}$
 (C) $\frac{25}{4}$
 (D) $-\frac{25}{4}$

Correct Answer: (B) $-\frac{25}{16}$

Solution:

Step 1: Rewrite the Integrand:

$(x^4 + 1)^{3/8} = x^{3/2}(1 + x^{-4})^{3/8}$. So the denominator becomes $x^{7/2}$
 $\cdot x^{3/2}(1 + x^{-4})^{3/8} = x^5(1 + x^{-4})^{3/8}$.

$$I = \int x^{-5}(1 + x^{-4})^{-3/8} dx$$

Step 2: Substitute:

Let $t = 1 + x^{-4}$. Then $dt = -4x^{-5}dx$, so $x^{-5}dx = -\frac{1}{4}dt$.

Step 3: Integrate:

$$I = -\frac{1}{4} \int t^{-3/8} dt = -\frac{1}{4} \cdot \frac{t^{5/8}}{5/8} = -\frac{2}{5} t^{5/8} + c$$

Step 4: Compare With the Given Form:

$\frac{x^4 + 1}{x^4} = t$, so $I = m t^n + c$ with $m = -\frac{2}{5}$ and $n = \frac{5}{8}$.

$$\frac{n}{m} = \frac{5/8}{-2/5} = -\frac{25}{16}$$

Final Answer:

The ratio is $-\frac{25}{16}$, option (B).

$$(B) - \frac{25}{16}$$

Quick Tip: Factor x^4 out of the bracket and substitute $t = 1 + x^{-4}$.

132. If $[x]$ is the greatest integer function not greater than x , then the value of $\int_0^2 x[x^2] dx$ is...

- (A) $\frac{3}{2}$
- (B) $\frac{5}{2}$
- (C) 3
- (D) 5

Correct Answer: (C) 3

Solution:**Step 1: Find the Breakpoints:**

$[x^2]$ changes at $x^2 = 1, 2, 3$, i.e. $x = 1, \sqrt{2}, \sqrt{3}$. So on $[0, 1)$ it is 0, on $[1, \sqrt{2})$ it is 1, on $[\sqrt{2}, \sqrt{3})$ it is 2, and on $[\sqrt{3}, 2)$ it is 3.

Step 2: Split the Integral:

$$I = 0 + \int_1^{\sqrt{2}} x dx + 2 \int_{\sqrt{2}}^{\sqrt{3}} x dx + 3 \int_{\sqrt{3}}^2 x dx$$

Step 3: Evaluate:

$$\int x \, dx = \frac{x^2}{2}. \text{ So}$$

$$I = \frac{2-1}{2} + 2 \cdot \frac{3-2}{2} + 3 \cdot \frac{4-3}{2} = \frac{1}{2} + 1 + \frac{3}{2} = 3$$

Step 4: Check the Options:

(A) $\frac{3}{2}$ and (B) $\frac{5}{2}$ miss one of the three pieces, and (D) 5 overcounts. The sum of the pieces is 3.

Final Answer:

The value is 3, option (C).

(C) 3

Quick Tip: Split $[0, 2]$ where the greatest integer $[x^2]$ changes value.

133. The value of $\int_2^4 (\{x\} + [x]) \, dx = \dots$ (where $\{x\}$ and $[x]$ are the fractional part function and the greatest integer function, respectively)

- (A) 2
- (B) 4
- (C) 6
- (D) 8

Correct Answer: (C) 6

Solution:

Step 1: Key Identity:

Every real x satisfies $x = [x] + \{x\}$. So $\{x\} + [x] = x$.

Step 2: Integrate:

$$\int_2^4 x \, dx = \left[\frac{x^2}{2} \right]_2^4 = \frac{16}{2} - \frac{4}{2} = 8 - 2 = 6$$

Step 3: Check:

Direct split: on $[2, 3)$ the integrand is x , on $[3, 4)$ it is also x . The integral over both is the same 6. Option (A) 2 is just the interval length, (B) 4 and (D) 8 do not equal $\int x \, dx$ on this interval.

Final Answer:

The value is 6, option (C).

(C) 6

Quick Tip: Since $\{x\} + [x] = x$, the integrand is simply x .

134. If the area of the region bounded by the parabola $y^2 = 4kx$ and the line $x = k$, (where $k > 0$) is $\frac{128}{3}$ sq. units, then the value of $\sin^{-1}(\frac{2}{k})$ is equal to...

- (A) $\frac{\pi}{4}$
- (B) $\frac{\pi}{6}$
- (C) $\frac{\pi}{3}$
- (D) $\frac{\pi}{2}$

Correct Answer: (B) $\frac{\pi}{6}$

Solution:

Step 1: Area Setup:

The parabola $y^2 = 4kx$ and the line $x = k$ enclose a region symmetric about the x-axis. Its area is twice the area of the upper half.

Step 2: Compute:

Upper half: $y = 2\sqrt{kx}$.

$$A = 2 \int_0^k 2\sqrt{kx} \, dx = 4\sqrt{k} \cdot \frac{2}{3} k^{3/2} = \frac{8k^2}{3}$$

Step 3: Solve for k:

$$\frac{8k^2}{3} = \frac{128}{3} \Rightarrow k^2 = 16 \Rightarrow k = 4 \text{ since } k > 0.$$

Step 4: Evaluate:

$$\sin^{-1}\left(\frac{2}{4}\right) = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$$

Option (A) $\pi/4$ would need $k = 2\sqrt{2}$, and (C) $\pi/3$ would need $k = 4/\sqrt{3}$.

Final Answer:

The value is $\frac{\pi}{6}$, option (B).

$$\boxed{(B) \frac{\pi}{6}}$$

Quick Tip: Compute the area symmetric about the x-axis and solve for k.

135. The area (in sq. units) of the region enclosed by $\{(x, y) | y \leq x^2, xy \leq 8, y \geq 1\}$ is...

- (A) $8\log 2 - \frac{14}{3}$
- (B) $8\log 2 + \frac{7}{3}$
- (C) $16\log 2 + \frac{7}{3}$
- (D) $16\log 2 - \frac{14}{3}$

Correct Answer: (D) $16\log 2 - \frac{14}{3}$

Solution:**Step 1: Intersections:**

The curves $y = x^2$ and $y = 8/x$ meet where $x^3 = 8$, at $(2, 4)$. The line $y = 1$ meets $y = x^2$ at $x = 1$ and $y = 8/x$ at $x = 8$.

Step 2: Describe the Region:

Consider the region in the first quadrant, which is the standard region intended here. It lies above $y = 1$ and below the lower of the two curves. For $1 \leq x \leq 2$ the lower curve is $y = x^2$, and for $2 \leq x \leq 8$ it is $y = 8/x$.

Step 3: Integrate:

$$A = \int_1^2 (x^2 - 1) dx + \int_2^8 \left(\frac{8}{x} - 1 \right) dx$$

First part: $\left[\frac{x^3}{3} - x \right]_1^2 = \left(\frac{8}{3} - 2 \right) - \left(\frac{1}{3} - 1 \right) = \frac{2}{3} + \frac{2}{3} = \frac{4}{3}.$

Second part: $[8 \ln x - x]_2^8 = 8 \ln 4 - 6 = 16 \ln 2 - 6.$

Step 4: Total:

$$A = \frac{4}{3} + 16 \ln 2 - 6 = 16 \ln 2 - \frac{14}{3}$$

This matches option (D).

Final Answer:

The area is $16 \ln 2 - \frac{14}{3}$, option (D).

(D) $16 \log 2 - \frac{14}{3}$

Quick Tip: Find the intersection points and integrate between the curves in the first quadrant.

136. The solution of the differential equation $\frac{dy}{dx} = \frac{x-y}{x+y}$, when $x = 0$ and $y = 0$ represents

- (A) Circle
- (B) Ellipse
- (C) Hyperbola
- (D) Pair of straight Lines

Correct Answer: (D) Pair of straight Lines

Solution:

Step 1: Substitute $y = vx$:

$$\frac{dy}{dx} = v + x \frac{dv}{dx} = \frac{1-v}{1+v}. \text{ So}$$

$$x \frac{dv}{dx} = \frac{1-v}{1+v} - v = \frac{1-2v-v^2}{1+v}$$

Step 2: Separate the Variables:

$$\int \frac{1+v}{1-2v-v^2} dv = \int \frac{dx}{x}$$

The numerator is $-\frac{1}{2}$ times the derivative of the denominator, so
 $-\frac{1}{2} \ln |1-2v-v^2| = \ln |x| + c_1$.

Step 3: General Solution:

$$x^2(1-2v-v^2) = C, \text{ and with } v = y/x: x^2 - 2xy - y^2 = C.$$

Step 4: Use the Condition:

The curve passes through $(0,0)$, so $C = 0$ and the solution is $x^2 - 2xy - y^2 = 0$. This factorises into two straight lines through the origin (discriminant $h^2 - ab = 1 + 1 = 2 > 0$).

So the solution is a pair of straight lines, option (D). A circle, ellipse or hyperbola would have a constant term $C \neq 0$.

Final Answer:

The solution represents a pair of straight lines, option (D).

(D) Pair of straight lines

Quick Tip: This is a homogeneous equation; put $y = vx$ and integrate.

137. The solution of the differential equation $\frac{dy}{dx} = \frac{a+bx}{c+dy}$ represents a family of circles centered at the origin if...

- (A) $a = c = 0, b + d = 0$
- (B) $a = c = 0, b = d$
- (C) $b = d = 0, a + c = 0$
- (D) $b = d = 0, a = c$

Correct Answer: (A) $a = c = 0, b + d = 0$

Solution:

Step 1: Separate the Variables:

$$(c + dy) dy = (a + bx) dx.$$

Step 2: Integrate:

$$cy + \frac{d}{2}y^2 = ax + \frac{b}{2}x^2 + k \Rightarrow \frac{b}{2}x^2 - \frac{d}{2}y^2 + ax - cy + k = 0$$

Step 3: Conditions for a Circle at the Origin:

For a circle with centre at the origin there must be no linear terms: $a = 0$ and $c = 0$. Also the coefficients of x^2 and y^2 must be equal: $\frac{b}{2} = -\frac{d}{2}$, i.e. $b + d = 0$.

Step 4: Check the Options:

Option (A) $a = c = 0$, $b + d = 0$ satisfies both. Option (B) has $b = d$, which would give a hyperbola. Options (C) and (D) put $b = d = 0$, which removes the quadratic terms and leaves a straight line.

Final Answer:

The condition is $a = c = 0$, $b + d = 0$, option (A).

$$\boxed{(A) \ a = c = 0, \ b + d = 0}$$

Quick Tip: Separate variables, integrate, and demand no linear terms and equal x^2, y^2 coefficients.

138. If $A(\vec{a})$, $B(\vec{b})$ and $C(\vec{c})$ are vertices of $\triangle ABC$. Point D divides segment BC internally in the ratio 2 : 1. Point E divides segment AD internally in the ratio 1 : 2, then the position vector of E is _____

- (A) $\frac{3\vec{a}+4\vec{b}+2\vec{c}}{9}$
(B) $\frac{6\vec{a}+2\vec{b}+\vec{c}}{9}$
(C) $\frac{3\vec{a}+2\vec{b}+4\vec{c}}{9}$
(D) $\frac{6\vec{a}+\vec{b}+2\vec{c}}{9}$

Correct Answer: (D) $\frac{6\vec{a}+\vec{b}+2\vec{c}}{9}$

Solution:**Step 1: Find D:**

D divides BC internally in the ratio 2:1, so

$$\vec{d} = \frac{2\vec{c} + 1 \cdot \vec{b}}{2 + 1} = \frac{\vec{b} + 2\vec{c}}{3}$$

Step 2: Find E:

E divides AD internally in the ratio 1:2, so E is one third of the way from A to D:

$$\vec{e} = \frac{2\vec{a} + 1 \cdot \vec{d}}{1 + 2} = \frac{2\vec{a} + \vec{d}}{3}$$

Step 3: Substitute:

$$\vec{e} = \frac{2\vec{a} + \frac{\vec{b} + 2\vec{c}}{3}}{3} = \frac{6\vec{a} + \vec{b} + 2\vec{c}}{9}$$

Step 4: Check the Options:

The coefficients $6 + 1 + 2 = 9$, which equals the denominator, as required for a point inside the triangle. This is option (D). Option (B) swaps the weights of B and C, and (A) and (C) do not have 6 as the largest coefficient.

Final Answer:

The position vector of E is $\frac{6\vec{a} + \vec{b} + 2\vec{c}}{9}$, option (D).

(D)

Quick Tip: Use the section formula twice: first for D on BC, then for E on AD.

139. If $|\vec{a}| = 3$, $|\vec{b}| = 4$, $|\vec{c}| = 5$ such that each vector is perpendicular to the sum of the other two, then $|\vec{a} + \vec{b} + \vec{c}|$ is equal to

- (A) $5\sqrt{3}$
- (B) $10\sqrt{2}$
- (C) $5\sqrt{2}$
- (D) $4\sqrt{3}$

Correct Answer: (C) $5\sqrt{2}$

Solution:

Step 1: Translate the Conditions:

$\vec{a} \cdot (\vec{b} + \vec{c}) = 0$, $\vec{b} \cdot (\vec{a} + \vec{c}) = 0$, $\vec{c} \cdot (\vec{a} + \vec{b}) = 0$. Write $x = \vec{a} \cdot \vec{b}$, $y = \vec{b} \cdot \vec{c}$, $z = \vec{c} \cdot \vec{a}$. Then $x + z = 0$, $x + y = 0$, $y + z = 0$.

Step 2: Solve:

Adding all three gives $x + y + z = 0$, so $x = y = z = 0$. All pairs are perpendicular.

Step 3: Magnitude:

$$|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(x + y + z) = 9 + 16 + 25 + 0 = 50$$

Step 4: Result:

$|\vec{a} + \vec{b} + \vec{c}| = \sqrt{50} = 5\sqrt{2}$. Option (A) $5\sqrt{3}$ would need a squared length of 75, and (B) $10\sqrt{2}$ of 200.

Final Answer:

The magnitude is $5\sqrt{2}$, option (C).

$$(C) 5\sqrt{2}$$

Quick Tip: The conditions force every pair of vectors to be perpendicular.

140. The value of x so that the volume of the parallelopiped formed by the vectors $\hat{i} + x\hat{j} + \hat{k}$, $\hat{j} + x\hat{k}$ and $x\hat{i} + \hat{k}$ is minimum, is

- (A) -3
- (B) 3
- (C) $\frac{1}{\sqrt{3}}$
- (D) $\sqrt{3}$

Correct Answer: (C) $\frac{1}{\sqrt{3}}$

Solution:

Step 1: Volume Expression:

Volume = absolute value of $\begin{vmatrix} 1 & x & 1 \\ 0 & 1 & x \\ x & 0 & 1 \end{vmatrix}$. Expanding along the first row:

$$1(1 - 0) - x(0 - x^2) + 1(0 - x) = 1 + x^3 - x$$

So $V = |1 + x^3 - x|$.

Step 2: Find the Stationary Points of the Expression:

Let $g(x) = 1 + x^3 - x$. Then $g'(x) = 3x^2 - 1 = 0 \Rightarrow x = \pm \frac{1}{\sqrt{3}}$.

Step 3: Classify:

$g''(x) = 6x$. At $x = \frac{1}{\sqrt{3}}$, $g'' > 0$, so g has a local minimum there. At $x = -\frac{1}{\sqrt{3}}$, $g'' < 0$, a local maximum.

Step 4: Compare With the Options:

At $x = \frac{1}{\sqrt{3}}$: $g = 1 + \frac{1}{3\sqrt{3}} - \frac{1}{\sqrt{3}} = 1 - \frac{2}{3\sqrt{3}} \approx 0.615$.

For the other options: $g(-3) = -23$, so $V = 23$; $g(3) = 25$; $g(\sqrt{3}) = 1 + 3\sqrt{3} - \sqrt{3} \approx 4.46$. Among the given values, $x = \frac{1}{\sqrt{3}}$ gives the least volume, 0.615.

Final Answer:

The local minimum occurs at $x = \frac{1}{\sqrt{3}}$, option (C).

(C) $\frac{1}{\sqrt{3}}$

Quick Tip: The volume is the absolute scalar triple product; minimise the cubic $1 + x^3 - x$.

141. The sum of all real values of λ for which the vectors $\vec{a} = \lambda\hat{i} + \hat{j} + \hat{k}$, $\vec{b} = \hat{i} + \lambda\hat{j} + 2\hat{k}$, $\vec{c} = 2\hat{i} + 3\hat{j} + \lambda\hat{k}$ are coplanar is...

- (A) 9
- (B) 7
- (C) 0
- (D) cant determine

Correct Answer: (C) 0

Solution:

Step 1: Condition:

Three vectors are coplanar when $\begin{vmatrix} \lambda & 1 & 1 \\ 1 & \lambda & 2 \\ 2 & 3 & \lambda \end{vmatrix} = 0$.

Step 2: Expand:

$$\lambda(\lambda^2 - 6) - 1(\lambda - 4) + 1(3 - 2\lambda) = \lambda^3 - 6\lambda - \lambda + 4 + 3 - 2\lambda = \lambda^3 - 9\lambda + 7$$

Step 3: Roots:

We need the sum of real roots of $\lambda^3 - 9\lambda + 7 = 0$. The cubic has values $f(-4) = -21 < 0$, $f(0) = 7 > 0$, $f(2) = -3 < 0$, $f(3) = 7 > 0$, so it changes sign three times and has three real roots.

Step 4: Sum:

For $\lambda^3 + 0\lambda^2 - 9\lambda + 7 = 0$, the sum of roots is $-(\text{coefficient of } \lambda^2) = 0$.
Option (C).

Final Answer:

The sum of the values of λ is 0, option (C).

(C) 0

Quick Tip: Coplanar vectors have a zero scalar triple product; sum the real roots of the cubic.

142. If ABC is a right-angled triangle in which BC is the longest side and the position vector of B and C are respectively $3\hat{i} - 2\hat{j} + \hat{k}$ and $5\hat{i} + \hat{j} - 3\hat{k}$, then the value of $\overline{AB} \cdot \overline{AC} + \overline{BA} \cdot \overline{BC} + \overline{CA} \cdot \overline{CB}$ is

- (A) 25
- (B) 27
- (C) 29
- (D) 31

Correct Answer: (C) 29

Solution:

Step 1: Locate the Right Angle:

In a right-angled triangle the hypotenuse is the longest side. Since BC is the longest, the right angle is at A . So $\overline{AB} \cdot \overline{AC} = 0$.

Step 2: Reduce the Other Terms:

$$\overline{BA} \cdot \overline{BC} = \overline{BA} \cdot (\overline{BA} + \overline{AC}) = |\overline{BA}|^2 + 0 = |\overline{AB}|^2.$$

$$\overline{CA} \cdot \overline{CB} = \overline{CA} \cdot (\overline{CA} + \overline{AB}) = |\overline{CA}|^2 + 0 = |\overline{AC}|^2.$$

Step 3: Add:

The sum is $0 + |\overline{AB}|^2 + |\overline{AC}|^2 = |\overline{BC}|^2$ by Pythagoras.

Step 4: Compute BC:

$$\overline{BC} = (5 - 3)\hat{i} + (1 + 2)\hat{j} + (-3 - 1)\hat{k} = 2\hat{i} + 3\hat{j} - 4\hat{k}.$$

$$|\overline{BC}|^2 = 4 + 9 + 16 = 29$$

So the value is 29. Options 25, 27 and 31 do not equal this sum of squares.

Final Answer:

The value is 29, option (C).

(C) 29

Quick Tip: Since BC is the longest side, angle A is 90 degrees; the sum reduces to BC squared.

143. The plane $\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 1$ cuts the axes at the points A, B, C then the area of triangle ABC is

- (A) $\sqrt{29}$
- (B) $\sqrt{41}$
- (C) $\sqrt{61}$
- (D) $\sqrt{51}$

Correct Answer: (C) $\sqrt{61}$

Solution:**Step 1: Intercept Points:**

The plane meets the axes at $A(2, 0, 0)$, $B(0, 3, 0)$ and $C(0, 0, 4)$.

Step 2: Edge Vectors:

$$\overrightarrow{AB} = (-2, 3, 0) \text{ and } \overrightarrow{AC} = (-2, 0, 4).$$

Step 3: Cross Product:

$$\overrightarrow{AB} \times \overrightarrow{AC} = (3 \cdot 4 - 0 \cdot 0, 0 \cdot (-2) - (-2)(4), (-2)(0) - 3(-2)) = (12, 8, 6)$$

Step 4: Area:

$$\text{Area} = \frac{1}{2} \sqrt{144 + 64 + 36} = \frac{1}{2} \sqrt{244} = \frac{1}{2} \cdot 2\sqrt{61} = \sqrt{61}$$

Option (C) is correct.

Final Answer:

The area is $\sqrt{61}$, option (C).

$$\boxed{(C) \sqrt{61}}$$

Quick Tip: Find the three axis intercept points and use half the magnitude of the cross product.

144. From a point P (a, b, c), perpendiculars PA and PB are drawn to XY plane and ZX plane respectively. If O is the origin, then the equation of plane OAB is

- (A) $\frac{x}{a} - \frac{y}{b} - \frac{z}{c} = 0$
- (B) $\frac{x}{a} - \frac{y}{b} + \frac{z}{c} = 0$
- (C) $\frac{x}{a} + \frac{y}{b} - \frac{z}{c} = 0$
- (D) $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 0$

Correct Answer: (A) $\frac{x}{a} - \frac{y}{b} - \frac{z}{c} = 0$

Solution:

Step 1: Find A and B:

The foot of the perpendicular from $P(a, b, c)$ to the XY plane is $A(a, b, 0)$. The foot on the ZX plane is $B(a, 0, c)$.

Step 2: Normal Vector:

The plane passes through O , so its normal is $\overrightarrow{OA} \times \overrightarrow{OB}$:

$$(a, b, 0) \times (a, 0, c) = (bc - 0, 0 \cdot a - ac, 0 - ab) = (bc, -ac, -ab)$$

Step 3: Equation:

$bcx - acy - abz = 0$. Divide by abc :

$$\frac{x}{a} - \frac{y}{b} - \frac{z}{c} = 0$$

Step 4: Check:

Substitute $A(a, b, 0)$: $1 - 1 - 0 = 0$. Substitute $B(a, 0, c)$: $1 - 0 - 1 = 0$.

Both lie on the plane, so option (A) is correct. The other options fail one of these checks.

Final Answer:

The plane is $\frac{x}{a} - \frac{y}{b} - \frac{z}{c} = 0$, option (A).

$(A) \frac{x}{a} - \frac{y}{b} - \frac{z}{c} = 0$

Quick Tip: Find the coordinates of A and B, then use the cross product as the plane normal.

145. If p is the shortest distance between the lines $\frac{x+1}{7} = \frac{y+1}{-6} = z+1$ and $\vec{r} = (3\hat{i} + 5\hat{j} + 7\hat{k}) + \mu(\hat{i} - 2\hat{j} + \hat{k})$ then $[p]$ is... , (where $[.]$ denotes the greatest integer function.)

- (A) 5
- (B) 20
- (C) 10
- (D) 8

Correct Answer: (C) 10

Solution:

Step 1: Write the Lines:

Line 1: $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$, point $P_1(-1, -1, -1)$, direction $\vec{d}_1 = (7, -6, 1)$.

Line 2: point $P_2(3, 5, 7)$, direction $\vec{d}_2 = (1, -2, 1)$.

Step 2: Cross Product:

$$\vec{d}_1 \times \vec{d}_2 = ((-6)(1) - (1)(-2), (1)(1) - (7)(1), (7)(-2) - (-6)(1)) = (-4, -6, -8)$$

$$|\vec{d}_1 \times \vec{d}_2| = \sqrt{16 + 36 + 64} = \sqrt{116}.$$

Step 3: Distance:

$\vec{P_1P_2} = (4, 6, 8)$. Then $(4, 6, 8) \cdot (-4, -6, -8) = -116$.

$$p = \frac{116}{\sqrt{116}} = \sqrt{116} \approx 10.77$$

Step 4: Greatest Integer:

Since $10^2 = 100 < 116 < 121 = 11^2$, $10 < \sqrt{116} < 11$, so $[p] = 10$.

Option (C).

Final Answer:

$[p] = 10$, option (C).

(C) 10

Quick Tip: Use the formula $|(a_2 - a_1) \cdot (d_1 \times d_2)| / |d_1 \times d_2|$.

146. The symmetric form of the equation of the line $x = ay + b$, $z = cy + d$ is

- (A) $\frac{x-a}{b} = y = \frac{z-c}{d}$
 (B) $\frac{x-b}{a} = \frac{y-d}{c} = z$
 (C) $x = \frac{y-a}{b} = \frac{z-c}{d}$
 (D) $\frac{x-b}{a} = y = \frac{z-d}{c}$

Correct Answer: (D) $\frac{x-b}{a} = y = \frac{z-d}{c}$

Solution:**Step 1: Solve for the Common Variable:**

From $x = ay + b$: $y = \frac{x-b}{a}$. From $z = cy + d$: $y = \frac{z-d}{c}$.

Step 2: Equate:

Both expressions equal the same y , and $y = \frac{y-0}{1}$. So

$$\frac{x-b}{a} = \frac{y-0}{1} = \frac{z-d}{c}$$

Step 3: Read the Result:

The line passes through $(b, 0, d)$ with direction ratios $(a, 1, c)$, and its symmetric form is $\frac{x - b}{a} = y = \frac{z - d}{c}$. This is option (D).

Step 4: Check the Others:

Option (A) uses $x - a$ over b , swapping constant and slope. Option (B) pairs $y - d$ with c , which is wrong because y is the free variable here. Option (C) has the same swapping error as (A).

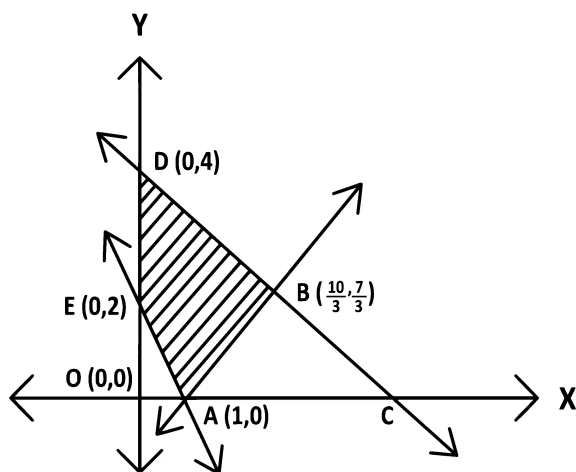
Final Answer:

The symmetric form is $\frac{x - b}{a} = y = \frac{z - d}{c}$, option (D).

$$(D) \frac{x - b}{a} = y = \frac{z - d}{c}$$

Quick Tip: Solve each equation for y and equate.

147. The shaded region in the provided graph represents the solution set for which of the following systems of linear inequalities?



- (A) $2x + y \geq 2, x - y \geq 1, x + 2y \leq 8, x \geq 0, y \geq 0$
 (B) $x + 2y \geq 2, x - y \geq 1, x + 2y \leq 8, x \geq 0, y \geq 0$
 (C) $2x + y \geq 2, x - y \leq 1, x + 2y \leq 8, x \geq 0, y \geq 0$
 (D) $2x + y \geq 2, x - y \leq 1, 2x + y \leq 8, x \geq 0, y \geq 0$

Correct Answer: (C) $2x + y \geq 2, x - y \leq 1, x + 2y \leq 8, x \geq 0, y \geq 0$

Solution:

Step 1: Boundary Lines From the Figure:

Line through $E(0, 2)$ and $A(1, 0)$: $2x + y = 2$.

Line through $A(1, 0)$ and $B(\frac{10}{3}, \frac{7}{3})$: slope 1, so $x - y = 1$.

Line through $D(0, 4)$ and $B(\frac{10}{3}, \frac{7}{3})$: $x + 2y = 8$, since $\frac{10}{3} + \frac{14}{3} = 8$.

Step 2: Pick a Test Point:

The shaded region lies inside the polygon $EABD$, for example near $(1, 2)$.

$2x + y = 4 \geq 2$, so $2x + y \geq 2$ holds.

$x - y = -1 \leq 1$, so $x - y \leq 1$ holds.

$x + 2y = 5 \leq 8$, so $x + 2y \leq 8$ holds.

Step 3: Axis Conditions:

The region is in the first quadrant, so $x \geq 0$, $y \geq 0$.

Step 4: Match:

The system is $2x + y \geq 2$, $x - y \leq 1$, $x + 2y \leq 8$, $x \geq 0$, $y \geq 0$, which is option (C). Option (A) has $x - y \geq 1$, which would pick the region below the line AB . Option (B) uses $x + 2y \geq 2$, a line that is not in the figure. Option (D) uses $2x + y \leq 8$, also not in the figure.

Final Answer:

The system is option (C).

(C)

Quick Tip: Find each boundary line from the labelled points and test a point in the shaded region.



148. If in 6 trials, X is a binomial random variable which follows the relation $9P(x = 4) = P(x = 2)$, then the probability of failure is...

- (A) 0.125
- (B) 0.25
- (C) 0.375
- (D) 0.75

Correct Answer: (D) 0.75

Solution:

Step 1: Binomial Formula:

$$P(X = r) = \binom{6}{r} p^r q^{6-r} \text{ with } q = 1 - p.$$

Step 2: Form the Equation:

$$9\binom{6}{4} p^4 q^2 = \binom{6}{2} p^2 q^4$$

Both binomials equal 15, so they cancel: $9p^4 q^2 = p^2 q^4$.

Step 3: Simplify:

Divide by $p^2 q^2$: $9p^2 = q^2$, so $q = 3p$ (probabilities are positive).

Step 4: Solve:

$p + q = 1 \Rightarrow p + 3p = 1 \Rightarrow p = \frac{1}{4}$. The probability of failure is $q = \frac{3}{4} = 0.75$. Option (D).

Final Answer:

The probability of failure is 0.75, option (D).

(D) 0.75

Quick Tip: Write both probabilities with the binomial formula and solve for p/q.



149. Given the probability density function (p.d.f.) of the random variable X , $f(x) = \frac{1}{2a}$, $0 < x < 2a$, $a > 0$
 $= 0$, otherwise, then which of the following is correct ?

- (A) $P(X < \frac{a}{2}) = P(X > \frac{a}{2})$
 (B) $P(X < \frac{a}{2}) < P(X > \frac{3a}{2})$
 (C) $P(X < \frac{a}{2}) > P(X > \frac{3a}{2})$
 (D) $P(X < \frac{a}{2}) = P(X > \frac{3a}{2})$

Correct Answer: (D) $P(X < \frac{a}{2}) = P(X > \frac{3a}{2})$

Solution:

Step 1: Density:

$f(x) = \frac{1}{2a}$ on $(0, 2a)$, so probability over an interval equals its length divided by $2a$.

Step 2: Compute the Probabilities:

$$P\left(X < \frac{a}{2}\right) = \frac{a/2}{2a} = \frac{1}{4}.$$

$$P\left(X > \frac{a}{2}\right) = \frac{2a - a/2}{2a} = \frac{3}{4}.$$

$$P\left(X > \frac{3a}{2}\right) = \frac{2a - 3a/2}{2a} = \frac{1}{4}.$$

Step 3: Compare:

$P(X < a/2) = P(X > 3a/2) = \frac{1}{4}$. So (D) is true.

Step 4: Check the Others:

(A) compares $\frac{1}{4}$ and $\frac{3}{4}$, which are not equal. (B) says $\frac{1}{4} < \frac{1}{4}$, false. (C) says $\frac{1}{4} > \frac{1}{4}$, false.

Final Answer:

The correct statement is option (D).

(D)

Quick Tip: Compute each probability as area under the constant density.

150. Consider a game of tossing a six sided fair die. If the face that comes up is 6, the player wins Rs. 36 and he loses Rs. k^2 , where k is the face that comes up $k = \{1, 2, 3, 4, 5\}$, then the expected winning amount in this game in Rs. is...

- (A) $\frac{19}{6}$
- (B) $-\frac{19}{6}$
- (C) $\frac{3}{2}$
- (D) $-\frac{3}{2}$

Correct Answer: (B) $-\frac{19}{6}$

Solution:

Step 1: Setup:

Each face has probability $\frac{1}{6}$. If the face is 6 the player wins Rs. 36. For $k = 1, 2, 3, 4, 5$, he loses Rs. k^2 , so his winning is $-k^2$.

Step 2: Expected Value:

$$E = \frac{1}{6} [36 - (1 + 4 + 9 + 16 + 25)] = \frac{1}{6} (36 - 55)$$

Step 3: Compute:

$$E = \frac{-19}{6}$$

Step 4: Check the Options:

A positive value like $\frac{19}{6}$ comes from reversing the sign of the loss. $\frac{3}{2}$ and $-\frac{3}{2}$ do not come from this sum (they would need totals of +9 and -9). Only $-\frac{19}{6}$ fits, option (B).

Final Answer:

The expected winning is $-\frac{19}{6}$ rupees, option (B).

$$\boxed{(B) - \frac{19}{6}}$$

Quick Tip: Find the expected value as the sum of each outcome times its probability.