

MHT CET 2026 PCM Question Paper April 19 Shift 2 with Solutions



General Instructions

- (i) This booklet contains 150 questions, each provided with a complete, step-by-step solution.
- (ii) It comprises 150 single-correct multiple-choice questions.
- (iii) Attempt each question on your own before reviewing the given solution.

Physics

Questions 1 to 50 | 1 mark each | 50 marks

1. A cuboid has volume $V = l \times l \times \sqrt{l}$, where l is the length of one side. When the length l is measured by a meter scale of least count 1 mm, the relative percentage error in the measurement of its volume is 6.25%. Then the relative percentage error in measurement of its length and the value of the length l is

- (A) 2% and 2 cm
- (B) 2.5% and 4 cm
- (C) 1.8% and 3.6 cm
- (D) 1% and 3 cm

Correct Answer: (B) 2.5% and 4 cm

Solution:

Step 1: Understand the concept

For a quantity $V = l^a$, the relative error in V is a times the relative

error in l . Here $V = l \times l \times \sqrt{l} = l^{5/2}$.

Step 2: Apply the rule

$$\frac{\Delta V}{V} = \frac{5}{2} \frac{\Delta l}{l}$$

Given $\frac{\Delta V}{V} \times 100 = 6.25\%$, we get

$$\frac{\Delta l}{l} \times 100 = \frac{6.25}{2.5} = 2.5\%$$

Step 3: Find the length

The least count of the scale is 1 mm, so $\Delta l = 1 \text{ mm} = 0.1 \text{ cm}$. Then

$$l = \frac{\Delta l}{0.025} = \frac{0.1}{0.025} = 4 \text{ cm}$$

Step 4: Check the options

Only option (B) gives both 2.5% and 4 cm. Check option (A): 2% of 2 cm is 0.04 cm, not the 0.1 cm least count. Option (D): 1% of 3 cm is 0.03 cm, again not 0.1 cm.

Final Answer:

The error is 2.5% and the length is 4 cm. This is option (B).

(B) 2.5% and 4 cm

Quick Tip: Powers multiply the relative error: V goes as l to the power 2.5.

2. A particle is performing U.C.M. along a circle of radius R . In half the period of revolution, its displacement and distance covered are respectively

(A) $\sqrt{2}R, 2\pi R$

(B) $2R, \pi R$

(C) $2R, 2\pi R$

(D) $R, \pi R$

Correct Answer: (B) $2R, \pi R$

Solution:

Step 1: Understand the concept

Displacement is the shortest straight line from the starting point to the end point. Distance is the actual length of the path travelled.

Step 2: Half a period

In half the period the particle completes half of the circle, ending at the point diametrically opposite to its start.

Step 3: Displacement

The straight line joining two opposite points of a circle is its diameter, so the displacement is $2R$.

Step 4: Distance

The path is a half circle, so the distance is half the circumference: $\frac{1}{2} \times 2\pi R = \pi R$. So the answer is displacement $2R$ and distance πR , option (B). The value $2\pi R$ is the distance for a full revolution, where the displacement would be zero.

Final Answer:

Displacement is $2R$ and distance is πR . This is option (B).

$$(B) \ 2R, \ \pi R$$

Quick Tip: Displacement is the straight line from start to end, and distance is the arc length travelled.

3. The equation of the trajectory of a ball projected at an angle θ with the horizontal, is given as $y = x - \frac{gx^2}{2}$

The initial velocity of the ball is

[Given : $\tan 45^\circ = 1$, $\cos 45^\circ = \frac{1}{\sqrt{2}}$]

(A) $2\sqrt{2} \text{ m/s}$

(B) 2 m/s

(C) $\sqrt{2} \text{ m/s}$

(D) $\frac{1}{\sqrt{2}} \text{ m/s}$

Correct Answer: (C) $\sqrt{2} \text{ m/s}$

Solution:**Step 1: Understand the concept**

A projectile launched with speed u at angle θ follows

$$y = x \tan \theta - \frac{g x^2}{2u^2 \cos^2 \theta}$$

Step 2: Compare with the given equation

The given equation is $y = x - \frac{gx^2}{2}$. Matching the coefficient of x : $\tan \theta = 1$, so $\theta = 45^\circ$.

Step 3: Match the x^2 term

$$\frac{g}{2u^2 \cos^2 \theta} = \frac{g}{2} \Rightarrow u^2 \cos^2 \theta = 1$$

With $\cos^2 45^\circ = \frac{1}{2}$, we get $u^2 = 2$.

Step 4: Result

$u = \sqrt{2}$ m/s, option (C). Option (A) would give $u^2 = 8$ and (B) would give $u^2 = 4$, which do not satisfy $u^2 \cos^2 \theta = 1$.

Final Answer:

The initial speed is $\sqrt{2}$ m/s. This is option (C).

$(C) \sqrt{2} \text{ m/s}$

Quick Tip: Compare the given trajectory with $y = x \tan(\theta) - \frac{gx^2}{2u^2 \cos^2(\theta)}$.

4. A stone of mass 40 g is tied to a light inextensible string of length 50 cm and whirled in a vertical circle at the rate of 30 revolutions per minute. The tension in the string when it is at the lowest point of the circle is (Take gravitational acceleration, $g = 10 \text{ m/s}^2$ and $\pi^2 = 10$)

- (A) 0.4 N
- (B) 0.5 N
- (C) 0.6 N
- (D) 0.7 N

Correct Answer: (C) 0.6 N

Solution:

Step 1: Understand the concept

At the lowest point both the weight (down) and the tension (up) act. The net upward force provides the centripetal force: $T - mg = m\omega^2 r$.

Step 2: Convert the data

$m = 40 \text{ g} = 0.04 \text{ kg}$, $r = 50 \text{ cm} = 0.5 \text{ m}$. The frequency is 30 rev/min = 0.5 rev/s, so $\omega = 2\pi f = \pi \text{ rad/s}$.

Step 3: Compute

$$m\omega^2 r = 0.04 \times \pi^2 \times 0.5 = 0.04 \times 10 \times 0.5 = 0.2 \text{ N}$$

$$mg = 0.04 \times 10 = 0.4 \text{ N}$$

Step 4: Result

$T = 0.4 + 0.2 = 0.6 \text{ N}$, option (C). The value 0.4 N is the weight alone and 0.2 N is the centripetal term alone.

Final Answer:

The tension is 0.6 N. This is option (C).

(C) 0.6 N

Quick Tip: At the lowest point the tension must supply the weight plus the centripetal force.

5. A body of mass 'm' attached at the end of a string is just completing the loop in a vertical circle. The apparent weight of the body at the lowest point in its path is

(g = gravitational acceleration)

(A) $6mg$

(B) $3mg$

(C) $1mg$

(D) zero

Correct Answer: (A) $6mg$

Solution:

Step 1: Understand the concept

A body "just completing" a vertical loop has zero tension at the top, so gravity alone provides the centripetal force there: $mg = \frac{mv_t^2}{R}$, so $v_t^2 = gR$.

Step 2: Speed at the bottom

Use energy conservation between the top and bottom (height difference $2R$):

$$\frac{1}{2}mv_b^2 = \frac{1}{2}mv_t^2 + mg(2R) \Rightarrow v_b^2 = gR + 4gR = 5gR$$

Step 3: Force at the bottom

At the lowest point, $T - mg = \frac{mv_b^2}{R} = 5mg$, so $T = 6mg$.

Step 4: Apparent weight

The apparent weight is the force the body exerts on the string, equal to the tension, so it is $6mg$, option (A). The values $3mg$ and mg do not fit $v_b^2 = 5gR$.

Final Answer:

The apparent weight at the lowest point is 6 mg. This is option (A).

(A) $6mg$

Quick Tip: Just completing the loop means the speed at the top is $\sqrt{gR}$, which makes the speed at the bottom $\sqrt{5gR}$.

6. Two particles of masses 2 g and 4 g are situated at the opposite ends, A and B of a wooden bar respectively. Let $l(AB) = 9$ cm. The center of mass of the system will be

- (A) 6 cm from B.
- (B) 3 cm from A.
- (C) 2 cm from B.
- (D) 6 cm from A.

Correct Answer: (D) 6 cm from A.

Solution:

Step 1: Understand the concept

The centre of mass of two particles lies on the line joining them, closer to the heavier particle. Its position is the mass-weighted average of positions.

Step 2: Set up

Put A at $x = 0$ with mass 2 g and B at $x = 9$ cm with mass 4 g.

Step 3: Compute

$$x_{cm} = \frac{2(0) + 4(9)}{2 + 4} = \frac{36}{6} = 6 \text{ cm}$$

Step 4: Interpret

The centre of mass is 6 cm from A, which is 3 cm from B. This is option (D). Options (A) and (B) swap which end is heavier, and (C) uses a wrong distance.

Final Answer:

The centre of mass is 6 cm from A. This is option (D).

(D) 6 cm from A

Quick Tip: Place A at the origin and use the weighted average of positions.

7. A force of $3\hat{i} + 2\hat{j} - \hat{k}$ N acts on a particle with position vector $\hat{i} + \hat{j} - \hat{k}$ m. The magnitude of torque of given force is

- (A) $\sqrt{5} \text{ N m}$
- (B) $\sqrt{8} \text{ N m}$
- (C) $\sqrt{6} \text{ N m}$
- (D) $\sqrt{10} \text{ N m}$

Correct Answer: (C) $\sqrt{6} \text{ N m}$

Solution:

Step 1: Understand the concept

The torque about the origin is $\vec{\tau} = \vec{r} \times \vec{F}$. Its magnitude is then found from the components.

Step 2: Write the vectors

$\vec{r} = (1, 1, -1) \text{ m}$ and $\vec{F} = (3, 2, -1) \text{ N}$.

Step 3: Compute the cross product

$$\vec{\tau} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -1 \\ 3 & 2 & -1 \end{vmatrix} = \hat{i}(-1 + 2) - \hat{j}(-1 + 3) + \hat{k}(2 - 3) = \hat{i} - 2\hat{j} - \hat{k}$$

Step 4: Magnitude

$$|\vec{\tau}| = \sqrt{1 + 4 + 1} = \sqrt{6} \text{ N m}$$

Option (C).

Final Answer:

The torque magnitude is $\sqrt{6} \text{ N m}$. This is option (C).

$$(C) \sqrt{6} \text{ N m}$$

Quick Tip: Torque is the cross product of position vector and force.

8. A satellite is revolving around a planet in a circular orbit close to its surface. Let ρ be mean density and R be the radius of the planet; then the period of the satellite is

(G = Universal constant of gravitation).

(A) $\sqrt{\frac{4\pi}{\rho G}}$

(B) $\sqrt{\frac{2\pi}{\rho G}}$

(C) $\sqrt{\frac{\pi}{\rho G}}$

(D) $\sqrt{\frac{3\pi}{\rho G}}$

Correct Answer: (D) $\sqrt{\frac{3\pi}{\rho G}}$

Solution:

Step 1: Understand the concept

A satellite orbiting close to the surface has orbital radius R . Gravity provides the centripetal force: $\frac{GMm}{R^2} = m \frac{4\pi^2}{T^2} R$.

Step 2: Solve for T

$$T^2 = \frac{4\pi^2 R^3}{GM}$$

Step 3: Use the density

The mass is $M = \rho \cdot \frac{4}{3}\pi R^3$. Substituting:

$$T^2 = \frac{4\pi^2 R^3}{G\rho \cdot \frac{4}{3}\pi R^3} = \frac{3\pi}{G\rho}$$

Step 4: Result

$T = \sqrt{\frac{3\pi}{\rho G}}$, option (D). The radius cancels, so the period depends only on density. Options (A), (B) and (C) have the wrong numerical factors of 4π , 2π and π .

Final Answer:

The period is $\sqrt{3\pi / (\rho G)}$. This is option (D).

$(D) \sqrt{\frac{3\pi}{\rho G}}$

Quick Tip: For a satellite close to the surface, the orbital radius equals R and gravity supplies the centripetal force.



9. A metal wire of length 'L' and density 'd' floats horizontally on the free surface of water. The maximum radius of the wire does not sink in water is
[T = surface tension of water , g = gravitational acceleration]

- (A) $\sqrt{\frac{2dg}{\pi T}}$
 (B) $\sqrt{\frac{2T}{\pi dg}}$
 (C) $\sqrt{\frac{\pi dg}{2T}}$
 (D) $\sqrt{\frac{T}{\pi dg}}$

Correct Answer: (B) $\sqrt{\frac{2T}{\pi dg}}$

Solution:

Step 1: Understand the concept

A thin wire floats because surface tension pulls on the water surface along its length. The surface acts on both sides of the wire, so the upward force is $2TL$ (taking the force as vertical in the limiting case).

Step 2: Weight of the wire

The volume is $\pi r^2 L$ and the density is d , so the weight is $\pi r^2 L d g$.

Step 3: Balance

$$\pi r^2 L d g = 2TL$$

Step 4: Solve

$$r^2 = \frac{2T}{\pi dg} \Rightarrow r = \sqrt{\frac{2T}{\pi dg}}$$

Option (B). The length L cancels, so the maximum radius does not depend on it. Option (D) would result if only one side of the wire were counted.

Final Answer:

The maximum radius is $\sqrt{2T/(\pi d g)}$. This is option (B).

$$(B) \sqrt{\frac{2T}{\pi d g}}$$

Quick Tip: The wire floats when the upward surface tension force equals its weight; the force acts along both sides of the wire.

10. The energy needed for breaking a liquid drop of radius 'R' into 'n' droplets each of radius 'r' is

[T = surface tension of the liquid]

(A) $4\pi TR^2[\frac{R}{r} - 1]$

(B) $4\pi TR[\frac{R}{r} - 1]$

(C) $4\pi T[\frac{R^2}{r^2} - 1]$

(D) $4\pi T[1 + \frac{R^3}{r}]$

Correct Answer: (A) $4\pi TR^2[\frac{R}{r} - 1]$

Solution:**Step 1: Understand the concept**

Breaking a drop into smaller droplets increases the total surface area.

The work done equals the surface tension times the increase in area:

$$W = T \Delta A.$$

Step 2: Find the number of droplets

The volume is conserved: $\frac{4}{3}\pi R^3 = n \cdot \frac{4}{3}\pi r^3$, so $n = \frac{R^3}{r^3}$.

Step 3: Change in area

$$\Delta A = n \cdot 4\pi r^2 - 4\pi R^2 = 4\pi \left(\frac{R^3}{r} - R^2 \right) = 4\pi R^2 \left(\frac{R}{r} - 1 \right)$$

Step 4: Result

$$W = 4\pi TR^2 \left[\frac{R}{r} - 1 \right]$$

Option (A). The other options have wrong powers of R or r and do not reduce to zero when $r = R$ (where no energy should be needed).

Final Answer:

The energy is $4\pi TR^2 (R/r - 1)$. This is option (A).

$$\boxed{(A) \ 4\pi TR^2 \left[\frac{R}{r} - 1 \right]}$$

Quick Tip: The energy needed equals surface tension times the increase in surface area.

11. In most liquids, with rise in temperature surface tension of liquid

- (A) decreases
- (B) increases
- (C) remains unchanged
- (D) first decreases and then increases

Correct Answer: (A) decreases

Solution:

Step 1: Understand the concept

Surface tension arises from the cohesive forces between molecules at the surface of a liquid. It is a measure of the energy needed to increase the surface area.

Step 2: Effect of temperature

On heating, molecules move faster and move further apart, so the average cohesive force between them weakens.

Step 3: Result

With weaker cohesion, less energy is needed to create surface, so the surface tension decreases as the temperature rises. It becomes zero at the critical temperature, where the boundary between liquid and vapour disappears.

Step 4: Options

The surface tension does not increase (B), stay constant (C), or rise again after falling (D) for most liquids. The answer is option (A).

Final Answer:

Surface tension decreases as temperature rises. This is option (A).

(A) Decreases

Quick Tip: Higher temperature weakens the cohesive forces between liquid molecules.

12. Out of the following, which statement is NOT true about black body radiation ?

- (A) Intensity is more for shorter wavelengths (λ).
- (B) Intensity is same for all wavelengths (λ).
- (C) Intensity is less for longer wavelengths (λ).
- (D) A black body emits all wavelengths (λ).

Correct Answer: (B) Intensity is same for all wavelengths (λ).

Solution:

Step 1: Understand the concept

A black body is an ideal absorber and emitter. Its emitted radiation is spread over all wavelengths, but the intensity varies with wavelength in a smooth curve described by Planck's law.

Step 2: Look at the curve

At a given temperature the intensity rises from zero at very short wavelengths, reaches a peak at λ_m (given by Wien's law, $\lambda_m T = \text{constant}$), and then falls for long wavelengths.

Step 3: Check statement (B)

The statement "intensity is the same for all wavelengths" would need a flat curve. The actual curve is not flat, because the intensity changes strongly with λ . So (B) is NOT true.

Step 4: Other statements

Statements (A), (C) and (D) are the usual textbook ways of describing the black body curve: radiation is emitted at all wavelengths, and the intensity depends on wavelength, in contrast to the flat behaviour in

(B). The statement that is not true is (B).

Final Answer:

Black body intensity is not the same for all wavelengths. This is option (B).

(B) Intensity is same for all wavelengths

Quick Tip: The spectrum of a black body is a curve with a peak: intensity depends on wavelength and is not flat.

13. 15 g of ice at 0°C is added to a vessel containing water at 40°C . The mass of water and water equivalent of the vessel is 60 g. Assuming that negligible heat is taken from the surroundings, the final temperature of the mixture will be

[$L_{\text{ice}} = 80 \text{ cal/g}$, $S_{\text{water}} = 1 \text{ cal/g}$]

- (A) 30°C
- (B) 22°C
- (C) 16°C
- (D) 10°C

Correct Answer: (C) 16°C

Solution:

Step 1: Understand the concept

Heat lost by the warm water and vessel goes into melting the ice and then warming the melted ice from 0°C to the final temperature T .

Step 2: Check whether the ice melts fully

Heat available as the water cools from 40°C to 0°C : $60 \times 1 \times 40 = 2400$ cal. Heat needed to melt 15 g: $15 \times 80 = 1200$ cal. Since $2400 > 1200$, all the ice melts.

Step 3: Write the heat balance

$$60 \times 1 \times (40 - T) = 15 \times 80 + 15 \times 1 \times (T - 0)$$

$$2400 - 60T = 1200 + 15T$$

Step 4: Solve

$75T = 1200$, so $T = 16^{\circ}\text{C}$, option (C). The value 30°C would result if the latent heat were ignored, and 10°C is lower than the heat balance allows.

Final Answer:

The final temperature is 16 degrees C. This is option (C).

(C) 16°C

Quick Tip: First check whether all ice melts, then apply heat lost = heat gained.



14. An ideal gas having pressure P , volume V and temperature T undergoes a thermodynamic process in which $dW = 0$ and $dQ < 0$. Then, for the gas

- (A) T will increase.
- (B) V will increase.
- (C) T will decrease.
- (D) P may increase or decrease.

Correct Answer: (C) T will decrease.

Solution:

Step 1: Understand the concept

The first law of thermodynamics says $dQ = dU + dW$. For an ideal gas, the internal energy depends only on temperature: $dU = nC_v dT$.

Step 2: Apply the conditions

Since $dW = 0$, we have $dU = dQ$. Since $dQ < 0$, we get $dU < 0$, so $nC_v dT < 0$.

Step 3: Conclusion on temperature

$dT < 0$, so the temperature T decreases, option (C).

Step 4: Check the other options

No work is done, so the volume does not change (an isochoric process), which rules out (B). With constant V , $P \propto T$, so the pressure must fall along with T , and it cannot rise, which rules out (D). Option (A) has the wrong sign for temperature change.

Final Answer:

Temperature decreases. This is option (C).

(C) T will decrease

Quick Tip: With no work done, the first law gives $dU = dQ$, and dU depends on temperature change.

15. An ideal gas at pressure 'P' and temperature 'T' is enclosed in a vessel of volume 'V'. Some gas leaks through a hole from the vessel and the pressure of the enclosed gas falls to P^1 . Assuming that the temperature of the gas remains constant during the leakage, the number of moles of the gas that have leaked is

- (A) $\frac{P^1 V}{RT}$
- (B) $\frac{V}{RT}(P - P^1)$
- (C) $\frac{2V}{RT}(P + P^1)$
- (D) $\frac{V}{RT}(P + P^1)$

Correct Answer: (B) $\frac{V}{RT}(P - P^1)$

Solution:

Step 1: Understand the concept

The vessel volume and the temperature stay constant. The ideal gas law $PV = nRT$ tells us the moles in the vessel at any pressure.

Step 2: Initial and final moles

Initially $n_1 = \frac{PV}{RT}$. After leakage, $n_2 = \frac{P^1 V}{RT}$.

Step 3: Moles that leaked

$$\Delta n = n_1 - n_2 = \frac{V}{RT}(P - P^1)$$

Step 4: Check the options

Option (A) is the moles remaining, not the moles leaked. Options (C) and (D) add the pressures, which is not a difference. So the answer is option (B).

Final Answer:

Moles leaked equal $V(P - P_1)/(RT)$. This is option (B).

$$(B) \frac{V}{RT} (P - P^1)$$

Quick Tip: Moles leaked equal initial moles minus final moles.



16. Assuming the expression for the pressure exerted by the gas, it can be shown that pressure is

- (A) $(\frac{2}{3})^{rd}$ of kinetic energy per unit volume of a gas.
- (B) $(\frac{3}{4})^{th}$ of kinetic energy per unit volume of a gas.
- (C) $(\frac{1}{3})^{rd}$ of kinetic energy per unit volume of a gas.
- (D) $(\frac{3}{2})^{nd}$ times kinetic energy per unit volume of a gas.

Correct Answer: (A) $(\frac{2}{3})^{rd}$ of kinetic energy per unit volume of a gas.

Solution:**Step 1: Recall the formula**

From kinetic theory, $P = \frac{1}{3} \rho \overline{v^2}$ where ρ is the density of the gas and $\overline{v^2}$ is the mean square speed.

Step 2: Rewrite using energy

The translational kinetic energy per unit volume is $E = \frac{1}{2}\rho \overline{v^2}$, so $\rho \overline{v^2} = 2E$.

Step 3: Substitute

$$P = \frac{1}{3} \times 2E = \frac{2}{3}E$$

Step 4: Result

The pressure is two thirds of the kinetic energy per unit volume, option (A). The fractions $\frac{1}{3}$ and $\frac{3}{4}$ and $\frac{3}{2}$ do not result from this substitution.

Final Answer:

Pressure is two thirds of kinetic energy per unit volume. This is option (A).

$$\boxed{(A) \frac{2}{3}}$$

Quick Tip: The pressure formula $P = (1/3) \rho v^2$ can be rewritten using kinetic energy per unit volume.

17. An ideal gas ($\gamma = 1.5$) is expanded adiabatically. To reduce the root mean square velocity of molecules two times, the gas should be expanded

- (A) 20 times
- (B) 16 times
- (C) 12 times
- (D) 8 times

Correct Answer: (B) 16 times

Solution:

Step 1: Understand the concept

The rms speed is $v_{rms} = \sqrt{\frac{3RT}{M}}$, so $v_{rms} \propto \sqrt{T}$. Halving the speed means T falls to one fourth.

Step 2: Temperature ratio

$$\frac{v_2}{v_1} = \frac{1}{2} \text{ gives } \frac{T_2}{T_1} = \frac{1}{4}.$$

Step 3: Adiabatic relation

For an adiabatic process $TV^{\gamma-1} = \text{constant}$, so

$$\frac{V_2}{V_1} = \left(\frac{T_1}{T_2} \right)^{\frac{1}{\gamma-1}} = 4^{\frac{1}{0.5}} = 4^2$$

Step 4: Result

$V_2 = 16V_1$, so the gas must be expanded 16 times, option (B). Options (A), (C) and (D) are not powers of 4.

Final Answer:

The volume must increase 16 times. This is option (B).

(B) 16 times

Quick Tip: The rms speed varies as the square root of T , and for an adiabatic process $T V^{(\gamma - 1)}$ is constant.

18. A spring force constant 180 N/m is loaded with a mass 0.2 kg . The amplitude of oscillations is 4 cm . When mass comes to equilibrium position, its velocity is

- (A) 0.012 m/s
- (B) 0.12 m/s
- (C) 1.2 m/s
- (D) 12 m/s

Correct Answer: (C) 1.2 m/s

Solution:

Step 1: Understand the concept

In simple harmonic motion the speed is greatest at the mean (equilibrium) position and equals $v_{\max} = A\omega$.

Step 2: Find ω

For a spring-mass system $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{180}{0.2}} = \sqrt{900} = 30 \text{ rad/s}$.

Step 3: Compute

The amplitude is $A = 4 \text{ cm} = 0.04 \text{ m}$, so

$$v_{\max} = A\omega = 0.04 \times 30 = 1.2 \text{ m/s}$$

Step 4: Check

The result is option (C). The amplitude must be in metres. Using $A = 4$ with $\omega = 30$ would give 120, and a slip of a factor of 10 in the amplitude would give 0.12 or 12, which are the other listed values.

Final Answer:

The velocity is 1.2 m/s. This is option (C).

(C) 1.2 m/s

Quick Tip: The speed at the equilibrium position is the maximum speed, A times omega.

19. A particle executing linear S.H.M. has velocities V_1 and V_2 at distance x_1 and x_2 respectively, from the mean position, its angular velocity is

(A) $\sqrt{\frac{V_2^2 - V_1^2}{x_1^2 - x_2^2}}$

(B) $\sqrt{\frac{V_2^2 - V_1^2}{x_2^2 - x_1^2}}$

(C) $\sqrt{\frac{V_2^2 - V_1^2}{x_1 x_2}}$

(D) $\sqrt{\frac{V_1 V_2}{x_1 + x_2}}$

Correct Answer: (A) $\sqrt{\frac{V_2^2 - V_1^2}{x_1^2 - x_2^2}}$

Solution:

Step 1: Understand the concept

For linear SHM, the speed at displacement x is $v^2 = \omega^2(A^2 - x^2)$.

Step 2: Write for the two positions

$$V_1^2 = \omega^2(A^2 - x_1^2) \text{ and } V_2^2 = \omega^2(A^2 - x_2^2).$$

Step 3: Subtract

$$V_2^2 - V_1^2 = \omega^2(x_1^2 - x_2^2)$$

Step 4: Solve

$$\omega = \sqrt{\frac{V_2^2 - V_1^2}{x_1^2 - x_2^2}}$$

Option (A). Option (B) has the denominator in the opposite order and would give a negative value under the root whenever $V_2 > V_1$ and $x_1 > x_2$.

Final Answer:

Angular velocity is $\sqrt{(V_2^2 - V_1^2)/(x_1^2 - x_2^2)}$. This is option (A).

$$(A) \sqrt{\frac{V_2^2 - V_1^2}{x_1^2 - x_2^2}}$$

Quick Tip: Use $v^2 = \omega^2(A^2 - x^2)$ at the two positions and subtract.

20. In a resonance tube open at one end, the end correction is 1.1 cm. If the shortest length of resonating air column with a tuning fork is 18 cm, the next resonating length will be

- (A) 45.9 cm
- (B) 49.6 cm
- (C) 51.3 cm
- (D) 56.2 cm

Correct Answer: (D) 56.2 cm

Solution:

Step 1: Understand the concept

A resonance tube open at one end behaves as a closed pipe.

Resonance occurs when $L + e = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \dots$ where e is the end correction.

Step 2: First resonance

$L_1 + e = \frac{\lambda}{4}$ gives $18 + 1.1 = 19.1$ cm, so $\lambda = 76.4$ cm.

Step 3: Next resonance

$$L_2 + e = \frac{3\lambda}{4} = 3 \times 19.1 = 57.3 \text{ cm}$$

Step 4: Result

$L_2 = 57.3 - 1.1 = 56.2$ cm, option (D). The value 51.3 cm would come from adding 33.3 cm, and 45.9 and 49.6 cm are not of the form $3 \times 19.1 - 1.1$.

Final Answer:

The next resonating length is 56.2 cm. This is option (D).

(D) 56.2 cm

Quick Tip: In a tube closed at one end, resonances occur at lengths with end correction equal to $\lambda/4$, $3\lambda/4$ and so on.

21. For stationary wave, $y = 12\cos\left(\frac{\pi x}{10}\right)\sin(36\pi t)$ cm, the distance between a node and the successive antinode is

- (A) 20 cm
- (B) 12 cm
- (C) 10 cm
- (D) 5 cm

Correct Answer: (D) 5 cm

Solution:**Step 1: Understand the concept**

In a stationary wave $y = 2A \cos(kx) \sin(\omega t)$, the quantity $k = \frac{2\pi}{\lambda}$.

Nodes and antinodes alternate, and the distance between a node and the next antinode is $\frac{\lambda}{4}$.

Step 2: Find k

Comparing with $\cos\left(\frac{\pi x}{10}\right)$ gives $k = \frac{\pi}{10} \text{ cm}^{-1}$.

Step 3: Find the wavelength

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{\pi/10} = 20 \text{ cm}$$

Step 4: Result

The node to antinode distance is $\frac{20}{4} = 5 \text{ cm}$, option (D). The value 10 cm is the distance between adjacent nodes, and 20 cm is the full wavelength.

Final Answer:

The distance is 5 cm. This is option (D).

(D) 5 cm

Quick Tip: Read the wave number k from the cosine term; the node-to-antinode distance is a quarter of the wavelength.

22. Two waves of same frequency (n) approaching each other with same velocity of 18 m/s interfere. The distance between two consecutive antinodes is

- (A) $\frac{18}{n}$
- (B) $\frac{10}{n}$
- (C) $\frac{9}{n}$
- (D) $\frac{n}{18}$

Correct Answer: (C) $\frac{9}{n}$

Solution:

Step 1: Understand the concept

Two identical waves travelling in opposite directions form a stationary wave. The distance between two consecutive antinodes is $\frac{\lambda}{2}$.

Step 2: Find the wavelength

The wave speed is $v = 18$ m/s and the frequency is n , so $\lambda = \frac{v}{n} = \frac{18}{n}$ m.

Step 3: Compute

$$\text{distance} = \frac{\lambda}{2} = \frac{9}{n} \text{ m}$$

Step 4: Result

Option (C). The value $\frac{18}{n}$ is the full wavelength, which is the distance between alternate antinodes, not consecutive ones.

Final Answer:

Consecutive antinodes are $9/n$ metres apart. This is option (C).

$(C) \frac{9}{n}$

Quick Tip: The wavelength is velocity over frequency, and consecutive antinodes are half a wavelength apart.

23. The equation of a progressive wave is given by $y = 6\cos(100t - 4x)$ where y is in μm , x in metre and t in second. The ratio of the maximum particle velocity to the velocity of wave is

- (A) 1.5×10^{-5}
- (B) 2.0×10^{-6}
- (C) 2.4×10^{-5}
- (D) 2.8×10^{-6}

Correct Answer: (C) 2.4×10^{-5}

Solution:

Step 1: Understand the concept

For $y = A \cos(\omega t - kx)$, the maximum particle velocity is $v_{p,max} = A\omega$, and the wave speed is $v = \frac{\omega}{k}$.

Step 2: Read the data

$A = 6 \mu\text{m} = 6 \times 10^{-6} \text{ m}$, $\omega = 100 \text{ rad/s}$ and $k = 4 \text{ m}^{-1}$.

Step 3: Compute

$$v_{p,max} = 6 \times 10^{-6} \times 100 = 6 \times 10^{-4} \text{ m/s. } v = \frac{100}{4} = 25 \text{ m/s.}$$

Step 4: Ratio

$$\frac{v_{p,max}}{v} = \frac{6 \times 10^{-4}}{25} = 2.4 \times 10^{-5}$$

Option (C).

Final Answer:

The ratio is 2.4×10^{-5} . This is option (C).

$$(C) 2.4 \times 10^{-5}$$

Quick Tip: Maximum particle velocity is A times ω and the wave velocity is ω over k .



24. A body sends a wave 150 mm long through medium P and 0.30 m long in medium Q. If velocity of the wave in medium P is 80 cm s^{-1} , the velocity of wave in medium Q is

- (A) 4.0 m s^{-1}
- (B) 3.2 m s^{-1}
- (C) 2.0 m s^{-1}
- (D) 1.6 m s^{-1}

Correct Answer: (D) 1.6 m s^{-1}

Solution:

Step 1: Understand the concept

When a wave passes from one medium into another, its frequency is fixed by the source and does not change. So $v = f\lambda$ gives $v \propto \lambda$.

Step 2: Convert units

$\lambda_P = 150 \text{ mm} = 0.15 \text{ m}$, $\lambda_Q = 0.30 \text{ m}$, and $v_P = 80 \text{ cm/s} = 0.8 \text{ m/s}$.

Step 3: Set up the ratio

$$\frac{v_Q}{v_P} = \frac{\lambda_Q}{\lambda_P} = \frac{0.30}{0.15} = 2$$

Step 4: Result

$v_Q = 2 \times 0.8 = 1.6 \text{ m/s}$, option (D). The frequency is $\frac{0.8}{0.15} = 5.33 \text{ Hz}$ in both media.

Final Answer:

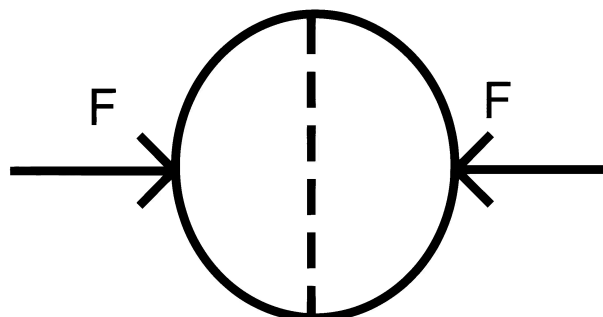
The velocity in medium Q is 1.6 m/s . This is option (D).

(D) 1.6 m s^{-1}

Quick Tip: The frequency stays the same in both media, so the speed is proportional to the wavelength.



- 25.** An uniformly charged thin spherical shell of radius 'R' has uniform surface charge density ' σ '. It is made of two hemispherical identical shells held together by pressing them with force 'F' as shown. F is proportional to
[ϵ_0 = permittivity of free space]



- (A) $\frac{1}{\epsilon_0} \frac{\sigma^2}{R^2}$
 (B) $\frac{1}{\epsilon_0} \frac{\sigma^2}{R}$
 (C) $\frac{1}{\epsilon_0} \sigma^2 R^2$
 (D) $\frac{1}{\epsilon_0} \sigma^2 R$

Correct Answer: (C) $\frac{1}{\epsilon_0} \sigma^2 R^2$

Solution:

Step 1: Understand the figure

The figure shows a uniformly charged spherical shell split along a plane into two identical hemispheres, held together by equal and opposite forces F pressing them toward each other.

Step 2: Electrostatic pressure

Each small patch of the surface is repelled by the rest of the charge. The outward force per unit area (electrostatic pressure) on a surface charge σ is $\frac{\sigma^2}{2\epsilon_0}$.

Step 3: Net force on a hemisphere

The outward forces on the hemisphere add up to the pressure times the area of the flat circular cross-section, πR^2 :

$$F = \frac{\sigma^2}{2\epsilon_0} \times \pi R^2$$

Step 4: Result

So $F \propto \frac{1}{\epsilon_0} \sigma^2 R^2$, option (C). The pressing force must balance this repulsion. Options with R^2 in the denominator, or with a single power of R , do not follow from pressure times area.

Final Answer:

F is proportional to $\sigma^2 R^2 / \epsilon_0$. This is option (C).

$$(C) \frac{1}{\epsilon_0} \sigma^2 R^2$$

Quick Tip: The outward electrostatic pressure on a charged surface is $\sigma^2 / 2\epsilon_0$; multiply by the cross-section area of a hemisphere.

26. Two equal positive charges each of value 'q' are placed at points A and B, where $AB = 3x$. A third charge $-3q$ is placed at point C at a distance x from A on AB. The potential energy of the system is nearly ($\epsilon_0 =$ permittivity of free space)

- (A) $\frac{q^2}{4\pi\epsilon_0 x}$
- (B) $\frac{-2q^2}{4\pi\epsilon_0 x}$
- (C) $\frac{3q^2}{4\pi\epsilon_0 x}$
- (D) $\frac{-4q^2}{4\pi\epsilon_0 x}$

Correct Answer: (D) $\frac{-4q^2}{4\pi\epsilon_0 x}$

Solution:**Step 1: Understand the concept**

The potential energy of a system of point charges is the sum of $\frac{q_i q_j}{4\pi\epsilon_0 r_{ij}}$ over every pair of charges.

Step 2: Find the distances

$AB = 3x$ and $AC = x$, so $CB = 3x - x = 2x$. The charges are q at A, q at B and $-3q$ at C.

Step 3: Add the three pair energies

$$U = \frac{1}{4\pi\epsilon_0} \left[\frac{q \cdot q}{3x} + \frac{q(-3q)}{x} + \frac{q(-3q)}{2x} \right] = \frac{q^2}{4\pi\epsilon_0 x} \left[\frac{1}{3} - 3 - \frac{3}{2} \right]$$

Step 4: Evaluate

The bracket is $\frac{2 - 18 - 9}{6} = -\frac{25}{6} \approx -4.17$. So $U \approx \frac{-4q^2}{4\pi\epsilon_0 x}$, which is nearly option (D). The question says "nearly", so rounding -4.17 to -4 is intended.

Final Answer:

The potential energy is nearly $-4 q^2 / (4 \pi \epsilon_0 x)$. This is option (D).

(D) $\frac{-4q^2}{4\pi\epsilon_0 x}$

Quick Tip: Add the pair energies kq_1q_2/r for all three pairs.



27. A particle of mass ' m ' and charge ' q ', initially at rest, is accelerated by a uniform electric field ' E ' through a distance ' D ' and is then allowed to approach a fixed static charge ' Q ' of the same sign. The distance of the closest approach of the charge q is

[ϵ_0 = permittivity of free space]

- (A) $Q/4\pi\epsilon_0 D$
- (B) $Q/4\pi\epsilon_0 ED$
- (C) $Q/2\pi\epsilon_0 D^2$
- (D) $Q/4\pi\epsilon_0 E$

Correct Answer: (B) $Q/4\pi\epsilon_0 ED$

Solution:

Step 1: Understand the concept

The charge gains kinetic energy equal to the work done by the field. Later, at the closest approach, all that kinetic energy has become electrostatic potential energy.

Step 2: Energy gained

The force is qE over a distance D , so the kinetic energy gained is $K = qED$.

Step 3: Energy at closest approach

At the closest distance r the speed is zero, so $K = \frac{qQ}{4\pi\epsilon_0 r}$ (the starting potential energy at large distance is zero).

Step 4: Solve

$$qED = \frac{qQ}{4\pi\epsilon_0 r} \Rightarrow r = \frac{Q}{4\pi\epsilon_0 ED}$$

Option (B). The charge q and the mass m cancel out.

Final Answer:

The closest approach is $Q/(4\pi\epsilon_0 ED)$. This is option (B).

$$(B) \frac{Q}{4\pi\epsilon_0 ED}$$

Quick Tip: Work done by the field becomes kinetic energy, which is then converted to potential energy at the closest approach.

28. In an oscillating LC circuit the maximum charge on the capacitor is Q . When the energy is stored equally between the electric and magnetic fields, the charge on the capacitor (q) is

- (A) Q
- (B) $\frac{Q}{2}$
- (C) $\frac{Q}{\sqrt{2}}$
- (D) $\frac{Q}{\sqrt{3}}$

Correct Answer: (C) $\frac{Q}{\sqrt{2}}$

Solution:

Step 1: Understand the concept

In an LC circuit the total energy is constant and shifts between the capacitor (electric field) and the inductor (magnetic field). The total is $\frac{Q^2}{2C}$, the energy when the capacitor holds its maximum charge Q .

Step 2: Equal sharing

If the energy is shared equally, the electric part is half the total:

$$\frac{q^2}{2C} = \frac{1}{2} \cdot \frac{Q^2}{2C}$$

Step 3: Solve

$$q^2 = \frac{Q^2}{2} \Rightarrow q = \frac{Q}{\sqrt{2}}$$

Step 4: Check the options

$q = Q$ means all the energy is electric, $q = \frac{Q}{2}$ gives only a quarter of the energy in the capacitor, and $q = \frac{Q}{\sqrt{3}}$ gives one third. The result is option (C).

Final Answer:

The charge is $Q/\sqrt{2}$. This is option (C).

(C) $\frac{Q}{\sqrt{2}}$

Quick Tip: Total energy is Q^2 over $2C$; set the electric energy equal to half of it.



29. A parallel plate air capacitor has capacity 'C' farad, potential 'V' volt and energy 'E' joule. When the gap between the plates is completely filled with dielectric

- (A) V increases, E decreases.
- (B) V decreases, E increases.
- (C) both V and E increase.
- (D) both V and E decrease.

Correct Answer: (D) both V and E decrease.

Solution:

Step 1: Understand the concept

The capacitor is charged and then isolated, so the charge Q on its plates cannot change when the dielectric is inserted. Filling the gap with a dielectric of constant $K > 1$ raises the capacitance to $C' = KC$.

Step 2: Effect on potential

$$V' = \frac{Q}{C'} = \frac{Q}{KC} = \frac{V}{K}$$

So the potential decreases.

Step 3: Effect on energy

$$E' = \frac{Q^2}{2C'} = \frac{E}{K}$$

The stored energy also decreases, as the plates must do work on the dielectric as it is pulled in.

Step 4: Result

Both V and E decrease, option (D). Options (A), (B) and (C) each have at least one of the two quantities increasing, which would need C' to be less than C .

Final Answer:

Both V and E decrease. This is option (D).

(D) Both V and E decrease

Quick Tip: For an isolated charged capacitor the charge stays constant, so a larger capacitance lowers both V and E.



30. When a galvanometer is shunted by a resistance 'S', its current capacity increases 'n' times. If the same galvanometer is shunted by another resistance ' S^1 ', its current capacity will increase to ' n^1 '. The value of n in terms of n^1 , S and S^1 is

- (A) $\frac{n^1 + S}{S^1}$
- (B) $\frac{S(n^1 - 1) - S^1}{S}$
- (C) $\frac{(n^1 + 1)S^1}{S}$
- (D) $\frac{S + S^1(n^1 - 1)}{S}$

Correct Answer: (D) $\frac{S + S^1(n^1 - 1)}{S}$

Solution:**Step 1: Understand the concept**

With a shunt S across a galvanometer of resistance G and full scale current I_g , the total current I satisfies $I_g G = (I - I_g)S$.

Step 2: Use the factor n

Current capacity becomes nI_g , so $I_g G = (nI_g - I_g)S$, which gives $G = (n - 1)S$.

Step 3: Second shunt

Similarly, with shunt S^1 we get $G = (n^1 - 1)S^1$.

Step 4: Equate and solve

$$(n - 1)S = (n^1 - 1)S^1 \Rightarrow n = 1 + \frac{S^1(n^1 - 1)}{S} = \frac{S + S^1(n^1 - 1)}{S}$$

Option (D).

Final Answer:

n equals $(S + S^1 (n^1 - 1))/S$. This is option (D).

(D) $\frac{S + S^1(n^1 - 1)}{S}$

Quick Tip: Use $I_g G = (I - I_g) S$ for each shunt and equate the galvanometer resistance.

31. The resistances in the two gaps of a balanced meter bridge are ' X ' Ω and ' $3X$ ' Ω respectively. If the resistances are interchanged the balance point shifts by

- (A) 25 cm
- (B) 33.3 cm
- (C) 50 cm
- (D) 75 cm

Correct Answer: (C) 50 cm

Solution:

Step 1: Understand the concept

For a meter bridge, balance gives $\frac{X}{3X} = \frac{l}{100 - l}$, with l measured from the end where X is connected.

Step 2: First balance point

$\frac{1}{3} = \frac{l}{100 - l}$ gives $100 - l = 3l$, so $l = 25$ cm.

Step 3: After interchange

The ratio becomes $\frac{3X}{X} = 3 = \frac{l'}{100 - l'}$, so $l' = 300 - 3l'$ and $l' = 75$ cm.

Step 4: Shift

The shift is $75 - 25 = 50$ cm, option (C).

Final Answer:

The balance point shifts by 50 cm. This is option (C).

(C) 50 cm

Quick Tip: At balance, the lengths are in the same ratio as the resistances; interchanging the resistances swaps the lengths.

32. A current carrying circular coil of radius 'R' produces magnetic field B_1 at an axial point P at a distance 'x' from its centre and B_2 at point Q placed at its centre respectively . If $B_2 = 8B_1$, the value of x is

- (A) $3R$
 (B) $\sqrt{3} R$
 (C) $\frac{R}{2\sqrt{3}}$
 (D) $\frac{2R}{\sqrt{3}}$

Correct Answer: (B) $\sqrt{3} R$

Solution:

Step 1: Understand the concept

On the axis of a circular coil, the field at distance x from the centre is

$$B_1 = \frac{\mu_0 I R^2}{2(R^2 + x^2)^{3/2}}. \text{ At the centre } (x = 0) \text{ it is } B_2 = \frac{\mu_0 I}{2R}.$$

Step 2: Take the ratio

$$\frac{B_2}{B_1} = \frac{(R^2 + x^2)^{3/2}}{R^3} = 8$$

Step 3: Solve

$$(R^2 + x^2)^{3/2} = 8R^3, \text{ so } R^2 + x^2 = (8)^{2/3} R^2 = 4R^2.$$

Step 4: Result

$$x^2 = 3R^2, \text{ so } x = \sqrt{3} R, \text{ option (B).}$$

Final Answer:

The distance is $\sqrt{3} R$. This is option (B).

(B) $\sqrt{3} R$

Quick Tip: Compare the axial field formula with the centre value and solve for x .



33. The magnetic induction produced inside an ideal solenoid, depends on which of the following quantities ?

- (a) number of turns per unit length.
- (b) radius of the wire.
- (c) current flowing through it.
- (d) permeability of the medium.

- (A) (a), (b) and (d)
- (B) (a), (b) and (c)
- (C) (b), (c) and (d)
- (D) (a), (c) and (d)

Correct Answer: (D) (a), (c) and (d)

Solution:

Step 1: Understand the concept

Inside a long ideal solenoid, the field is uniform and equals $B = \mu n I$, where n is the number of turns per unit length, I is the current and μ is the permeability of the medium inside.

Step 2: Match the quantities

(a) the number of turns per unit length, n , appears directly. (c) the current I appears directly. (d) the permeability μ of the medium appears directly.

Step 3: Radius of the wire

(b) The radius of the wire does not appear in the formula. The field depends on the current and the turns per unit length, not on the thickness of the wire.

Step 4: Result

The field depends on (a), (c) and (d), option (D).

Final Answer:

The field depends on turns per unit length, current and permeability. This is option (D).

(D) (a), (c) and (d)

Quick Tip: Inside an ideal solenoid $B = \mu n I$, so it depends on n , I and the medium.

34. Two coils P and S have a mutual inductance of (π) mH. The secondary coil S has resistance $4\ \Omega$ and self inductance $(60/\pi)$ mH. If the current in the primary is $I_p = 12\sin(50\pi t)$, then the maximum value of the current induced in coil S is [Take $\pi^2 = 10$]

- (A) 2 A
- (B) 1.8 A
- (C) 1.5 A
- (D) 1.2 A

Correct Answer: (D) 1.2 A

Solution:

Step 1: Understand the concept

The changing current in P induces an emf in S: $\varepsilon_S = M \frac{dI_p}{dt}$. The secondary has resistance and self inductance, so its current is limited by its impedance.

Step 2: Maximum emf

$I_p = 12 \sin(50\pi t)$, so $\frac{dI_p}{dt} = 600\pi \cos(50\pi t)$ with peak 600π A/s. With $M = \pi \times 10^{-3}$ H:

$$\varepsilon_{max} = \pi \times 10^{-3} \times 600\pi = 0.6\pi^2 = 6 \text{ V}$$

Step 3: Impedance of S

Reactance $X_L = \omega L = 50\pi \times \frac{60}{\pi} \times 10^{-3} = 3 \Omega$. Resistance is 4Ω , so $Z = \sqrt{4^2 + 3^2} = 5 \Omega$.

Step 4: Maximum current

$$I_{max} = \frac{6}{5} = 1.2 \text{ A}$$

Option (D). If the self inductance were ignored, we would get $\frac{6}{4} = 1.5$ A, which is option (C) and is a trap.

Final Answer:

The maximum induced current is 1.2 A. This is option (D).

(D) 1.2 A

Quick Tip: The induced current is limited by the impedance of the secondary, which includes its inductive reactance.



35. A solenoid is connected to a battery so that a steady current flows through it. If an iron core is inserted into the solenoid, then the current in the coil

- (A) will not change
- (B) will increase
- (C) will decrease
- (D) may increase or decrease depending upon the direction of the current

Correct Answer: (C) will decrease

Solution:

Step 1: Understand the concept

The battery drives a steady current through the solenoid. Inserting an iron core increases the inductance, so the magnetic flux linked with the coil increases while the core moves in.

Step 2: Apply Lenz's law

A rising flux induces an emf that opposes the change, that is, an emf that acts against the battery emf. This opposing induced emf is $-\frac{d\Phi}{dt}$ in size.

Step 3: Effect on the current

The net emf in the circuit is the battery emf minus the induced emf, so the current in the coil decreases while the core is being inserted.

Step 4: Result

The current decreases, option (C). The direction of the current does not matter, because the induced emf always opposes the change in flux, so (D) is not needed, and (A) and (B) ignore the induced emf.

Final Answer:

The current decreases during insertion because the induced emf opposes the battery. This is option (C).

(C) Decreases

Quick Tip: While the core is being pushed in, the flux grows, and the induced emf opposes the battery.



36. The magnetic energy stored in an inductor of inductance 4 H carrying a current of 1.5 A is

- (A) 4.5 mJ
- (B) 3 μ J
- (C) 45 μ J
- (D) 4500 mJ

Correct Answer: (D) 4500 mJ

Solution:**Step 1: Understand the concept**

The energy stored in the magnetic field of an inductor carrying a current I is $U = \frac{1}{2}LI^2$.

Step 2: Substitute

$$L = 4 \text{ H and } I = 1.5 \text{ A.}$$

Step 3: Compute

$$U = \frac{1}{2} \times 4 \times (1.5)^2 = 2 \times 2.25 = 4.5 \text{ J}$$

Step 4: Convert units

4.5 J = 4500 mJ, option (D). The value 4.5 mJ is off by a factor of 1000 because it treats 4.5 as if it were already in millijoules.

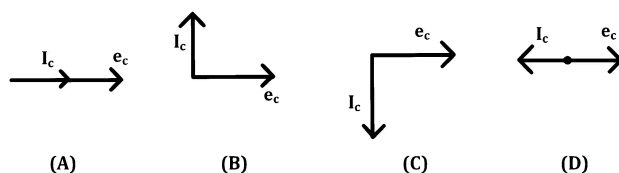
Final Answer:

The energy is 4.5 J, which is 4500 mJ. This is option (D).

(D) 4500 mJ

Quick Tip: Magnetic energy in an inductor is one half $L I^2$.

37. When a.c. source is connected across a pure capacitor, the correct phase relation between current and emf is shown in figure.



- (A) (C)
- (B) (A)
- (C) (B)
- (D) (D)

Correct Answer: (C) (B)

Solution:

Step 1: Understand the concept

In an AC circuit with a pure capacitor, the current leads the emf (voltage) by a phase angle of 90° , or $\frac{\pi}{2}$. Phasors rotate anticlockwise, so the leading phasor is 90° ahead in the anticlockwise direction.

Step 2: Read the figure

Diagram (A) shows both phasors along the same direction, meaning they are in phase (a pure resistor). Diagram (B) shows e_c along the horizontal direction and I_c pointing up, 90° ahead anticlockwise of e_c . Diagram (C) shows I_c pointing down, 90° behind e_c . Diagram (D) shows the two in opposite directions, a phase difference of 180° .

Step 3: Match

Only diagram (B) shows the current ahead of the emf by 90° , so it represents the capacitor.

Step 4: Result

The figure labels its four drawings (A) to (D), and the question options point to them in a different order. The drawing labelled (B) is the third option of the question, which is option (C). The drawing labelled (C) shows the current lagging, as in an inductor.

Final Answer:

Diagram (B) shows the current leading the emf by 90 degrees. This is option (C).

(C) Diagram (B)

Quick Tip: For a pure capacitor the current leads the emf by 90 degrees.



38. In series LCR circuit $R = 10\ \Omega$ and impedance is $30\ \Omega$. An r.m.s. voltage 210 V, is applied across the circuit. The true power consumed in AC circuit is

- (A) 360 W
- (B) 420 W
- (C) 490 W
- (D) 600 W

Correct Answer: (C) 490 W

Solution:**Step 1: Understand the concept**

The true (average) power in an AC circuit is $P = V_{rms}I_{rms} \cos \phi$, where the power factor is $\cos \phi = \frac{R}{Z}$.

Step 2: Find the current

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{210}{30} = 7\text{ A.}$$

Step 3: Find the power factor

$$\cos \phi = \frac{10}{30} = \frac{1}{3}.$$

Step 4: Compute

$$P = 210 \times 7 \times \frac{1}{3} = 490 \text{ W}$$

This equals $I_{rms}^2 R = 49 \times 10 = 490 \text{ W}$. Option (C). The value 1470 W would be the apparent power VI , which is not asked.

Final Answer:

The true power is 490 W. This is option (C).

(C) 490 W

Quick Tip: True power is V_{rms} squared times R over Z squared.

39. Light travels a distance 'x' in time ' t_0 ' in air and ' $4x$ ' in time ' t_1 ' in another denser medium. The critical angle for this medium is

- (A) $\sin^{-1}(\frac{t_1}{t_0})$
- (B) $\sin^{-1}(\frac{4t_0}{t_1})$
- (C) $\sin^{-1}(\frac{4t_1}{t_0})$
- (D) $\sin^{-1}(\frac{t_0}{4t_1})$

Correct Answer: (B) $\sin^{-1}(\frac{4t_0}{t_1})$

Solution:

Step 1: Understand the concept

The refractive index of the medium is $\mu = \frac{c}{v}$, where c is the speed in air and v is the speed in the medium. The critical angle satisfies $\sin C = \frac{1}{\mu}$.

Step 2: Find the speeds

In air: $c = \frac{x}{t_0}$. In the medium: $v = \frac{4x}{t_1}$.

Step 3: Find μ

$$\mu = \frac{c}{v} = \frac{x/t_0}{4x/t_1} = \frac{t_1}{4t_0}$$

Step 4: Critical angle

$$\sin C = \frac{1}{\mu} = \frac{4t_0}{t_1} \Rightarrow C = \sin^{-1} \left(\frac{4t_0}{t_1} \right)$$

Option (B). The medium is denser, so $t_1 > 4t_0$ and the ratio is less than 1, so the inverse sine is defined.

Final Answer:

The critical angle is $\arcsin(4t_0 / t_1)$. This is option (B).

$$\boxed{(B) \sin^{-1} \left(\frac{4t_0}{t_1} \right)}$$

Quick Tip: Refractive index is the ratio of speeds, and the critical angle satisfies $\sin C = 1/\mu$.

40. In a biprism experiment, a steady interference pattern is observed on the screen kept at a distance of 100 cm using a light of wavelength 5000 Å. Without changing the distance between the virtual images of the slit, the source of light is replaced by a source of wavelength 6400 Å. Now, to reduce the fringe width by 20% of its initial value, the screen should be moved

- (A) towards the source by 37.5 cm
- (B) towards the source by 62.5 cm
- (C) away from the source by 62.5 cm
- (D) away from the source by 37.5 cm

Correct Answer: (A) towards the source by 37.5 cm

Solution:

Step 1: Understand the concept

In a biprism experiment, the fringe width is $\beta = \frac{\lambda D}{d}$. Here d , the separation between the virtual images, is unchanged.

Step 2: Set up the condition

Initially $\beta_1 = \frac{5000 \times 100}{d}$ (taking D in cm and λ in Angstrom, the units cancel in the ratio). After changing the source and the screen, we want $\beta_2 = 0.8 \beta_1$:

$$6400 D' = 0.8 \times 5000 \times 100$$

Step 3: Solve for the new distance

$$D' = \frac{400000}{6400} = 62.5 \text{ cm}$$

Step 4: Result

The screen goes from 100 cm to 62.5 cm, so it is moved by 37.5 cm towards the source (the biprism), option (A). The other options: 62.5 cm is the new distance, not the movement, and moving away would increase the fringe width.

Final Answer:

The screen moves 37.5 cm towards the source. This is option (A).

(A) towards the source by 37.5 cm

Quick Tip: Fringe width is proportional to λD , so set the new product to 80 percent of the old one.

41. A ray of light is incident at polarising angle θ on air-glass interface. If λ_a and λ_g are the wavelengths of light in air and glass respectively then

- (A) $\lambda_a = \lambda_g \cot \theta$
- (B) $\lambda_g = \lambda_a \cot \theta$
- (C) $\lambda_a = \lambda_g \tan^2 \theta$
- (D) $\lambda_g = \lambda_a \tan^2 \theta$

Correct Answer: (B) $\lambda_g = \lambda_a \cot \theta$

Solution:

Step 1: Understand the concept

Brewster's law says that at the polarising angle θ , $\tan \theta = \mu$, where μ is the refractive index of glass relative to air.

Step 2: Link μ with wavelength

The frequency does not change on entering glass, so $\mu = \frac{c}{v} = \frac{\lambda_a}{\lambda_g}$.

Step 3: Combine

$$\tan \theta = \frac{\lambda_a}{\lambda_g} \Rightarrow \lambda_g = \frac{\lambda_a}{\tan \theta} = \lambda_a \cot \theta$$

Step 4: Result

Option (B). Option (A) has λ_a and λ_g interchanged, and the $\tan^2 \theta$ options have a wrong power of the tangent.

Final Answer:

$\lambda_g = \lambda_a \cot \theta$. This is option (B).

$(B) \lambda_g = \lambda_a \cot \theta$

Quick Tip: At the polarising angle $\tan(\theta)$ equals the refractive index, which is also the ratio of wavelengths in air and glass.

42. In biprism experiment, the maximum intensity is I_0 . If the path difference between the two interfering waves is $\frac{\lambda}{3}$, then intensity at the point on the screen is

$$[\sin 30^\circ = \cos 60^\circ = 0.5, \sin 60^\circ = \cos 30^\circ = \sqrt{3}/2]$$

- (A) $\frac{I_0}{4}$
- (B) $\frac{I_0}{3}$
- (C) $\frac{I_0}{2}$
- (D) I_0

Correct Answer: (A) $\frac{I_0}{4}$

Solution:

Step 1: Understand the concept

A path difference Δx corresponds to a phase difference $\phi = \frac{2\pi}{\lambda} \Delta x$.

For two equal sources, the intensity is $I = I_0 \cos^2 \frac{\phi}{2}$, where I_0 is the maximum intensity.

Step 2: Find the phase difference

$$\phi = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{3} = \frac{2\pi}{3}$$

Step 3: Compute

$$I = I_0 \cos^2 \frac{\pi}{3} = I_0 \cos^2 60^\circ = I_0 \left(\frac{1}{2}\right)^2 = \frac{I_0}{4}$$

Step 4: Result

The intensity is $\frac{I_0}{4}$, option (A). The value $I_0/2$ would be at a path difference of $\lambda/4$, and I_0 occurs at zero path difference.

Final Answer:

The intensity is $I_0/4$. This is option (A).

$$(A) \frac{I_0}{4}$$

Quick Tip: Convert path difference to phase difference and use $I = I_0 \cos^2$ of half the phase.



43. When light of wavelength ' λ ' is incident on a photosensitive surface, the stopping potential is ' V '. When a light of wavelength 1.5λ is incident on the same surface, the stopping potential is ' $\frac{V}{4}$ '. Threshold wavelength for the surface is

- (A) $\frac{6}{5}\lambda$
- (B) $\frac{7.5}{4}\lambda$
- (C) $\frac{7.5}{9}\lambda$
- (D) $\frac{9}{5}\lambda$

Correct Answer: (D) $\frac{9}{5}\lambda$

Solution:

Step 1: Understand the concept

Einstein's equation gives $eV_0 = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$, where λ_0 is the threshold wavelength.

Step 2: Write the two cases

Case 1: $eV = \frac{hc}{\lambda} - \phi$. Case 2: $\frac{eV}{4} = \frac{hc}{1.5\lambda} - \phi$, where $\phi = \frac{hc}{\lambda_0}$.

Step 3: Eliminate ϕ

Subtract: $\frac{3eV}{4} = \frac{hc}{\lambda} \left(1 - \frac{2}{3}\right) = \frac{hc}{3\lambda}$, so $eV = \frac{4hc}{9\lambda}$.

Step 4: Find the threshold

$$\phi = \frac{hc}{\lambda} - eV = \frac{hc}{\lambda} \left(1 - \frac{4}{9}\right) = \frac{5hc}{9\lambda}. \text{ So}$$

$$\lambda_0 = \frac{hc}{\phi} = \frac{9}{5}\lambda$$

Option (D).

Final Answer:

The threshold wavelength is $9\lambda / 5$. This is option (D).

$$\boxed{(D) \frac{9}{5}\lambda}$$

Quick Tip: Write Einstein equations for both wavelengths and eliminate the work function to find the threshold wavelength.

44. If the ionisation energy for the hydrogen atom is 13.6 eV, then the energy required to excite it from the ground state to the next higher state is nearly

- (A) -10.2 eV
- (B) -3.4 eV
- (C) 10.2 eV
- (D) 13.6 eV

Correct Answer: (C) 10.2 eV

Solution:

Step 1: Understand the concept

The energy of the hydrogen atom in the n th level is $E_n = -\frac{13.6}{n^2}$ eV.

The ionisation energy of 13.6 eV is the energy needed to take the electron from $n = 1$ to infinity.

Step 2: Energies of the two levels

$$E_1 = -13.6 \text{ eV and } E_2 = -\frac{13.6}{4} = -3.4 \text{ eV.}$$

Step 3: Excitation energy

$$\Delta E = E_2 - E_1 = -3.4 - (-13.6) = 10.2 \text{ eV}$$

Step 4: Result

The energy required is +10.2 eV, option (C). The value -3.4 eV is the energy of the first excited state, not the energy needed, and -10.2 eV has the wrong sign because the atom absorbs this energy.

Final Answer:

The required energy is 10.2 eV. This is option (C).

(C) 10.2 eV

Quick Tip: Energy levels are $-13.6/n^2$ eV, so the first excitation goes from $n = 1$ to $n = 2$.



45. The de-Broglie wavelength of an electron moving in the n^{th} Bohr orbit of radius 'r' is

(A) $n\pi r$

(B) $\frac{nr}{\pi}$

(C) $\frac{2\pi r}{n}$

(D) $\frac{nr}{2\pi}$

Correct Answer: (C) $\frac{2\pi r}{n}$

Solution:

Step 1: Understand the concept

De Broglie explained Bohr's quantisation by saying that the electron's standing wave must fit the orbit: the circumference of the orbit equals a whole number of wavelengths.

Step 2: Write the condition

$$2\pi r = n\lambda$$

Step 3: Solve

$$\lambda = \frac{2\pi r}{n}$$

Step 4: Result

Option (C). This is equivalent to Bohr's angular momentum condition

$$mvr = \frac{nh}{2\pi} \text{ with } \lambda = \frac{h}{mv}.$$

Final Answer:

The wavelength is $2\pi r / n$. This is option (C).

$$(C) \frac{2\pi r}{n}$$

Quick Tip: In the n th Bohr orbit the circumference holds n de Broglie wavelengths.

46. The magnetic moment of electron due to orbital motion is proportional to
(n = principal quantum number)

- (A) n
- (B) n^2
- (C) $\frac{1}{n}$
- (D) $\frac{1}{n^2}$

Correct Answer: (A) n

Solution:

Step 1: Understand the concept

An electron orbiting in a circle is a current loop. Its magnetic moment is $\mu = \frac{e}{2m}L$, where L is the orbital angular momentum.

Step 2: Use Bohr's quantisation

In the n th Bohr orbit, $L = \frac{nh}{2\pi}$.

Step 3: Substitute

$$\mu = \frac{e}{2m} \cdot \frac{nh}{2\pi} = n \frac{eh}{4\pi m}$$

Step 4: Result

The magnetic moment is proportional to n , option (A). The factor $\frac{eh}{4\pi m}$ is the Bohr magneton, which is the same for every orbit.

Final Answer:

The magnetic moment is proportional to n . This is option (A).

(A) n

Quick Tip: Orbital magnetic moment equals $e L / 2m$, and L is $n h$ over 2π .



47. A radioactive element has rate of disintegration 16,000 disintegrations per minute at a particular instant. After four minutes, it becomes 2000 disintegrations per minute. The decay constant per minute is

- (A) $0.25\log_e 2$
- (B) $0.50\log_e 3$
- (C) $0.75\log_e 2$
- (D) $0.8\log_e 3$

Correct Answer: (C) $0.75\log_e 2$

Solution:**Step 1: Understand the concept**

The rate of disintegration (activity) of a radioactive sample falls as $A = A_0 e^{-\lambda t}$.

Step 2: Substitute

$A_0 = 16000$, $A = 2000$ and $t = 4$ min:

$$\frac{2000}{16000} = \frac{1}{8} = e^{-4\lambda}$$

Step 3: Solve for λ

$$4\lambda = \ln 8 = 3 \ln 2 \Rightarrow \lambda = 0.75 \ln 2 \text{ per minute}$$

Step 4: Check

Activity falls to $\frac{1}{8}$ in three half-lives, so the half-life is $\frac{4}{3}$ min and $\lambda = \frac{\ln 2}{4/3} = 0.75 \ln 2$. Option (C).

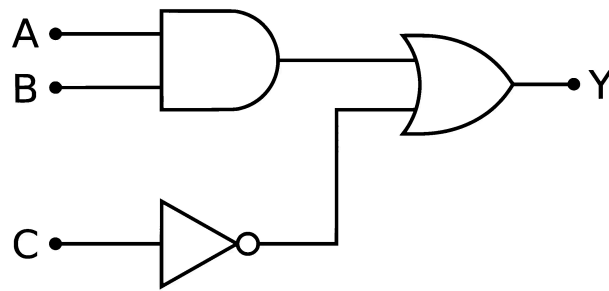
Final Answer:

The decay constant is $0.75 \log_e 2$ per minute. This is option (C).

$(C) 0.75 \log_e 2$

Quick Tip: Activity falls exponentially: $A = A_0 e^{(-\lambda t)}$.

48. The Boolean expression for the given combination of logic gates is



(A) $Y = (A + B) \cdot \bar{C}$

(B) $Y = (A \cdot B) \cdot \bar{C}$

(C) $Y = (A + B) + C$

(D) $Y = (A \cdot B) + \bar{C}$

Correct Answer: (D) $Y = (A \cdot B) + \bar{C}$

Solution:

Step 1: Read the circuit

The figure shows inputs A and B going into an AND gate. Input C goes into a NOT gate (triangle with a bubble). The outputs of the AND gate and the NOT gate then feed an OR gate, whose output is Y.

Step 2: Write each gate

AND output: $A \cdot B$. NOT output: $\bar{C}$.

Step 3: Combine in the OR gate

$$Y = (A \cdot B) + \bar{C}$$

Step 4: Check the options

Option (A) and (B) have $\bar{C}$ multiplied, which would be an AND, while

the figure shows an OR. Option (C) lacks the NOT on C and has OR instead of AND on A and B. So the answer is option (D).

Final Answer:

The output is (A AND B) OR (NOT C). This is option (D).

$$(D) Y = (A \cdot B) + \bar{C}$$

Quick Tip: Trace the signal through each gate: AND on A and B, NOT on C, then OR.

49. In a transistor, comparing the doping of emitter, base and collector, the part which is heavily doped and that which is lightly doped are respectively

- (A) collector and emitter
- (B) emitter and base
- (C) collector and base
- (D) emitter and collector

Correct Answer: (B) emitter and base

Solution:

Step 1: Understand the concept

A transistor has three regions: the emitter, the base and the collector. Each has a different doping level to suit its role.

Step 2: Emitter

The emitter emits (injects) a large number of charge carriers into the

base, so it is heavily doped.

Step 3: Base

The base is thin and lightly doped, so that only a few of the carriers combine in it and most pass on to the collector.

Step 4: Result

The collector is moderately doped and larger in size, to collect the carriers and dissipate heat. The heavily doped part is the emitter and the lightly doped part is the base, option (B).

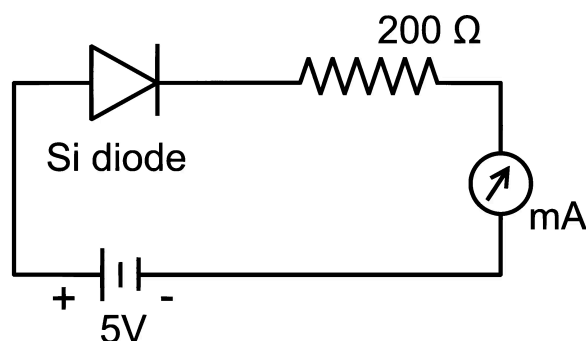
Final Answer:

The emitter is heavily doped and the base is lightly doped. This is option (B).

(B) Emitter and base

Quick Tip: The emitter must supply many carriers, so it is heavily doped, and the base is thin and lightly doped.

50. In the following electrical circuit, the reading in the milliammeter is
(Take knee voltage of silicon diode = 0.7 V)



- (A) 25.5 mA
- (B) 21.5 mA
- (C) 18.5 mA
- (D) 15.8 mA

Correct Answer: (B) 21.5 mA

Solution:

Step 1: Read the circuit

The figure shows a 5 V battery in series with a silicon diode, a 200 Ω resistor and a milliammeter. The diode symbol points from the positive terminal side toward the resistor, so the diode is forward biased.

Step 2: Account for the diode

A forward-biased silicon diode conducts once the voltage across it reaches the knee voltage of 0.7 V, and then it drops about 0.7 V. So the voltage across the resistor is $5 - 0.7 = 4.3$ V.

Step 3: Apply Ohm's law

$$I = \frac{4.3}{200} = 0.0215 \text{ A} = 21.5 \text{ mA}$$

Step 4: Check

Option (B). The value 25 mA would be $5/200$, which ignores the diode drop.

Final Answer:

The milliammeter reads 21.5 mA. This is option (B).

(B) 21.5 mA

Quick Tip: The diode is forward biased, so subtract the knee voltage from the battery voltage before applying Ohm's law.

Chemistry

Questions 51 to 100 | 1 mark each | 50 marks

51. Chlorine has two isotopes Cl^{37} and Cl^{35} with percentage abundance of 25% and 75% respectively. Find average atomic mass of chlorine ?

- (A) 35
- (B) 37
- (C) 35.5
- (D) 36

Correct Answer: (C) 35.5

Solution:

Step 1: Understand the concept

The average atomic mass of an element is the weighted mean of the masses of its isotopes. Each isotope contributes in proportion to its percentage abundance in nature.

Step 2: Write the formula

$$\text{Average mass} = \frac{(m_1 \times a_1) + (m_2 \times a_2)}{100}$$

Here m is the mass number of an isotope and a is its percentage abundance.

Step 3: Substitute the data

Cl-37 has abundance 25% and Cl-35 has abundance 75%.

$$\text{Average mass} = \frac{37 \times 25 + 35 \times 75}{100} = \frac{925 + 2625}{100} = \frac{3550}{100} = 35.5$$

Step 4: Check the other options

The value must lie between 35 and 37, and it must sit closer to 35 because Cl-35 is three times more abundant. So 35 and 37 are plain isotope masses, not averages, and 36 is the simple mean that ignores abundance. Only 35.5 fits the weighted result.

Final Answer:

The weighted average works out to 35.5 u. This is option (C).

(C) 35.5

Quick Tip: Average atomic mass is the abundance-weighted mean of the isotopic masses.

52. Calculate the shortest wavelength in hydrogen spectrum emission of Lyman series ($R_H = 109677 \text{ cm}^{-1}$)

- (A) $9.117 \times 10^{-6} \text{ cm}$
- (B) $9.241 \times 10^{-6} \text{ cm}$
- (C) $9.360 \times 10^{-6} \text{ cm}$
- (D) $9.482 \times 10^{-6} \text{ cm}$

Correct Answer: (A) $9.117 \times 10^{-6} \text{ cm}$

Solution:

Step 1: Understand the question

The shortest wavelength of a spectral series is the series limit. It comes from a jump from $n_2 = \infty$ down to the lowest level of that series. For the Lyman series the lowest level is $n_1 = 1$.

Step 2: Write the Rydberg formula

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

Step 3: Substitute the limits

With $n_1 = 1$ and $n_2 = \infty$, the second term vanishes.

$$\frac{1}{\lambda} = 109677 \times (1 - 0) = 109677 \text{ cm}^{-1}$$

Step 4: Find the wavelength

$$\lambda = \frac{1}{109677} = 9.117 \times 10^{-6} \text{ cm}$$

Step 5: Check the other options

The other values are slightly longer than the true limit. A transition from a finite level such as $n_2 = 6$ gives a longer wavelength than the limit, and no transition can give a wavelength below $1/R_H$. So only $9.117 \times 10^{-6} \text{ cm}$ is the series limit.

Final Answer:

The Lyman series limit is $9.117 \times 10^{-6} \text{ cm}$. This is option (A).

$$(A) 9.117 \times 10^{-6} \text{ cm}$$

Quick Tip: For the series limit of Lyman, put $n_1 = 1$ and $n_2 = \text{infinity}$ in the Rydberg formula.



53. What is the number of bond pair of electrons and lone pair of electrons present in the valence shell of chalcogens in their hydride ?

- (A) 2 - Bond pair and 1- lone pair of electrons
- (B) 2 - Bond pair and 2- lone pair of electrons
- (C) 3 - Bond pair and 1- lone pair of electrons
- (D) 3 - Bond pair and 2- lone pair of electrons

Correct Answer: (B) 2 - Bond pair and 2- lone pair of electrons

Solution:

Step 1: Understand the question

Chalcogens are the group 16 elements (O, S, Se, Te). Each has 6 electrons in its valence shell. Their hydrides have the general formula H_2X , such as H_2O and H_2S .

Step 2: Count bond pairs

Each hydrogen shares one electron with the central atom through one single bond. Two hydrogens give 2 bond pairs.

Step 3: Count lone pairs

Of the 6 valence electrons, 2 are used in bonding. The remaining 4 electrons form $4/2 = 2$ lone pairs.

Step 4: Conclusion with check

Total electron pairs around X is 4 (2 bond + 2 lone), which gives a bent shape, as seen in water. Options with 3 bond pairs would need three hydrogens (like NH_3), and an option with only 1 lone pair would leave the valence shell short of the 6 electrons that group 16 has.

Final Answer:

The hydride has 2 bond pairs and 2 lone pairs. This is option (B).

(B) 2 bond pairs and 2 lone pairs

Quick Tip: Chalcogens have 6 valence electrons and form H_2X , so two electrons are shared and two pairs stay unshared.



54. Which of the following is favourable condition for the formation of ionic bond ?

- (A) Low ionization enthalpy of metal and low negative value of electron gain enthalpy of non metal.
- (B) Low ionization enthalpy of metal and high negative value of electron gain enthalpy of non metal.
- (C) High ionization enthalpy of metal and high negative value of electron gain enthalpy of non metal.
- (D) High ionization enthalpy of metal and low negative value of electron gain enthalpy of non metal.

Correct Answer: (B) Low ionization enthalpy of metal and high negative value of electron gain enthalpy of non metal.

Solution:

Step 1: Understand the concept

An ionic bond forms by complete transfer of electrons from a metal to a non-metal. The metal becomes a cation and the non-metal becomes an anion. These ions then attract each other.

Step 2: Condition for the metal

Forming the cation needs energy equal to the ionization enthalpy. A low ionization enthalpy makes it cheap for the metal to lose its electron.

Step 3: Condition for the non-metal

Forming the anion releases energy equal to the electron gain enthalpy. A highly negative value means more energy is released, so the anion forms readily.

Step 4: Check the options

Option (A) has a poorly negative electron gain enthalpy, so the non-metal is unwilling to accept electrons. Option (C) makes the metal reluctant to lose electrons. Option (D) has both factors unfavourable. Only option (B) has both factors favourable, and a large lattice energy then stabilises the solid.

Final Answer:

Low ionization enthalpy of the metal with high negative electron gain enthalpy of the non-metal favours ionic bonding. This is option (B).

(B) Low IE of metal, high negative $\Delta_{eg}H$ of non-metal

Quick Tip: An ionic bond forms easily when the metal loses electrons easily and the non-metal gains them with a large release of energy.

55. Which is the correct decreasing order of boiling points for different compounds from following ?

- (A) $\text{CH}_3\text{Cl} > \text{CH}_3\text{Br} > \text{CH}_2\text{Br}_2 > \text{CHBr}_3$
- (B) $\text{CH}_3\text{Br} > \text{CH}_2\text{Br}_2 > \text{CHBr}_3 > \text{CH}_3\text{Cl}$
- (C) $\text{CH}_2\text{Br}_2 > \text{CHBr}_3 > \text{CH}_3\text{Br} > \text{CH}_3\text{Cl}$
- (D) $\text{CHBr}_3 > \text{CH}_2\text{Br}_2 > \text{CH}_3\text{Br} > \text{CH}_3\text{Cl}$

Correct Answer: (D) $\text{CHBr}_3 > \text{CH}_2\text{Br}_2 > \text{CH}_3\text{Br} > \text{CH}_3\text{Cl}$

Solution:

Step 1: Understand the concept

All four compounds are polar molecules. Their boiling points depend mainly on the strength of van der Waals forces. These forces grow with molecular mass, size of the electron cloud and surface area.

Step 2: Compare the molecules

Molar masses: $\text{CH}_3\text{Cl} = 50.5$, $\text{CH}_3\text{Br} = 95$, $\text{CH}_2\text{Br}_2 = 174$, $\text{CHBr}_3 = 253$ g/mol. Bromine is larger and more polarisable than chlorine, so each added Br raises the dispersion forces strongly.

Step 3: Known boiling points

CH_3Cl about 249 K, CH_3Br about 277 K, CH_2Br_2 about 370 K and CHBr_3 about 422 K. The trend rises steadily with mass.

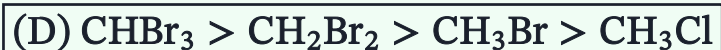
Step 4: Why the other orders fail

Option (A) puts the lightest compounds at the top and the heaviest at the bottom, which is exactly reversed. Options (B) and (C) place CH_3Cl or CH_3Br above heavier bromides, which breaks the mass trend. Only option (D) lists the compounds from heaviest to lightest.

Final Answer:

Boiling point falls from CHBr_3 to CH_3Cl as molar mass decreases.

This is option (D).



Quick Tip: Boiling point of similar haloalkanes rises with molecular mass and surface area, so more bromine atoms means a higher boiling point.



56. Which from following gases is least soluble in water under same conditions of temperature and pressure ?

- (A) HCl
- (B) CO_2
- (C) NH_3
- (D) O_2

Correct Answer: (D) O_2

Solution:**Step 1: Understand the concept**

Solubility of a gas in water depends on how well the gas interacts with water molecules. Polar gases, and gases that react with water, dissolve far better than non-polar gases.

Step 2: Check HCl

HCl is polar and reacts with water to give H_3O^+ and Cl^- . It is extremely soluble, about 450 volumes in one volume of water.

Step 3: Check NH_3 and CO_2

NH_3 forms hydrogen bonds with water and gives NH_4^+ and OH^- , so it is highly soluble (about 700 volumes). CO_2 reacts partly to give carbonic acid and dissolves about one volume per volume of water.

Step 4: Check O_2

O_2 is non-polar, forms no hydrogen bonds and does not react. Only about 0.03 volume dissolves per volume of water. This is the smallest value, so it is the least soluble.

Final Answer:

Oxygen is non-polar and unreactive toward water, so it dissolves least. This is option (D).

(D) O_2

Quick Tip: Polar gases or gases that react with water dissolve well; a non-polar gas like oxygen dissolves very little.

57. Calculate standard Gibbs energy for any gaseous reaction if values of K_p and R respectively are 3.5×10^{17} ; $8.314 \text{ JK}^{-1} \text{ mol}^{-1}$

- (A) $-100.1 \text{ kJ mol}^{-1}$
- (B) $-79.5 \text{ kJ mol}^{-1}$
- (C) $-71.4 \text{ kJ mol}^{-1}$
- (D) $-89.5 \text{ kJ mol}^{-1}$

Correct Answer: (A) $-100.1 \text{ kJ mol}^{-1}$

Solution:

Step 1: Understand the concept

The standard Gibbs energy change is linked to the equilibrium constant by $\Delta G^\circ = -RT \ln K_p = -2.303 RT \log K_p$. The temperature is taken as 298 K, the standard value.

Step 2: Find $\log K_p$

$$\log(3.5 \times 10^{17}) = \log 3.5 + 17 = 0.544 + 17 = 17.544$$

Step 3: Substitute

$$\Delta G^\circ = -2.303 \times 8.314 \times 298 \times 17.544 \text{ J mol}^{-1}$$

First $2.303 \times 8.314 \times 298 = 5705.8 \text{ J/mol}$. Then $5705.8 \times 17.544 \approx 1.001 \times 10^5 \text{ J/mol}$.

Step 4: Result and check

$$\Delta G^\circ \approx -100.1 \text{ kJ mol}^{-1}$$

The sign is negative because K_p is very large, which means the reaction is strongly spontaneous. The other options (-79.5, -71.4, -89.5) would need a smaller $\log K_p$, about 13.9, 12.5 or 15.7.

Final Answer:

Delta G is about -100.1 kJ per mole. This is option (A).

$$(A) - 100.1 \text{ kJ mol}^{-1}$$

Quick Tip: Use the standard relation $\Delta G = -2.303 RT \log K_p$ at 298 K.



58. Calculate the work done for the following reaction at 27 °C



(A) 2494.2 J

(B) 124.71 J

(C) 3741.3 J

(D) 187.07 J

Correct Answer: (A) 2494.2 J

Solution:

Step 1: Understand the concept

For a gaseous reaction at constant pressure, the work done is $w = -P\Delta V = -\Delta n_g RT$, where Δn_g is the change in moles of gas.

Step 2: Find Δn_g

Reactants: 1 mol C_2H_4 + 1 mol H_2 = 2 mol gas. Product: 1 mol C_2H_6 .

So $\Delta n_g = 1 - 2 = -1$.

Step 3: Substitute

Temperature: 27°C = 300 K.

$$w = -(-1)(8.314)(300) = +2494.2 \text{ J}$$

Step 4: Interpret

The sign is positive because the gas volume shrinks and the surroundings do work on the system. The magnitude is 2494.2 J, matching option (A). The other values (124.71, 3741.3, 187.07) do not equal RT for this temperature and are wrong multiples.

Final Answer:

Work done on the system is 2494.2 J. This is option (A).

(A) 2494.2 J

Quick Tip: Find the change in moles of gas, then use $w = -(\Delta n) RT$ for work done on the system.



59. Which from following is NOT a state function ?

- (A) Volume
- (B) Pressure
- (C) Work
- (D) Temperature

Correct Answer: (C) Work

Solution:**Step 1: Understand the concept**

A state function has a value fixed by the present state of the system. Its change depends only on the initial and final states and never on

the route.

Step 2: Check the state variables

Volume, pressure and temperature are properties that describe the state of a system. Each has a definite value for a given state, so each is a state function.

Step 3: Check work

Work depends on how the change is carried out. For example, expanding a gas from V_1 to V_2 reversibly gives more work than expanding it against a smaller constant pressure. The same end states give different work, so work is a path function.

Step 4: Conclusion

Only work is not a state function. Heat is also a path function, but it is not listed.

Final Answer:

Work depends on the path, so it is not a state function. This is option (C).

(C) Work

Quick Tip: A state function depends only on the initial and final states, not on the path taken.

60. Calculate the $[\text{H}_3\text{O}^+]$ if pH of the solution is 4.25

- (A) $6.123 \times 10^{-5} \text{ M}$
- (B) $5.623 \times 10^{-5} \text{ M}$
- (C) $4.508 \times 10^{-5} \text{ M}$
- (D) $4.085 \times 10^{-5} \text{ M}$

Correct Answer: (B) $5.623 \times 10^{-5} \text{ M}$

Solution:

Step 1: Understand the concept

pH is defined as $\text{pH} = -\log[\text{H}_3\text{O}^+]$. Reversing this gives $[\text{H}_3\text{O}^+] = 10^{-\text{pH}}$.

Step 2: Substitute

$$[\text{H}_3\text{O}^+] = 10^{-4.25} = 10^{-5} \times 10^{0.75}$$

Step 3: Evaluate

$10^{0.75} = 5.623$ (since $\log 5.623 = 0.75$). So

$$[\text{H}_3\text{O}^+] = 5.623 \times 10^{-5} \text{ M}$$

Step 4: Check the other options

Option (A) 6.123×10^{-5} has $\text{pH} = 4.21$, option (C) 4.508×10^{-5} has $\text{pH} = 4.35$ and option (D) 4.085×10^{-5} has $\text{pH} = 4.39$. None of them equals 4.25. Only 5.623×10^{-5} gives $-\log(5.623 \times 10^{-5}) = 4.25$.

Final Answer:

The hydronium ion concentration is $5.623 \times 10^{-5} \text{ M}$. This is option (B).

$$(B) 5.623 \times 10^{-5} \text{ M}$$

Quick Tip: Use $[\text{H}_3\text{O}^+] = 10^{(-\text{pH})}$.

61. If $[\text{H}_3\text{O}^+]$ of a solution is 1×10^{-4} . What is the value of pOH at 298 K ?

- (A) 4
- (B) 10
- (C) 12
- (D) 14

Correct Answer: (B) 10

Solution:

Step 1: Understand the concept

At 298 K, the ion product of water gives $\text{pH} + \text{pOH} = 14$. So we need the pH first.

Step 2: Find pH

$$\text{pH} = -\log(1 \times 10^{-4}) = 4$$

Step 3: Find pOH

$$\text{pOH} = 14 - 4 = 10$$

Step 4: Check the options

The value 4 is the pH, not the pOH, so (A) is a mix-up. The value 12 would mean pH 2, and 14 would mean pH 0. Neither fits a hydronium concentration of 10^{-4} M. The solution is acidic, so the pOH must be larger than 7, and 10 satisfies this.

Final Answer:

The pOH is 10. This is option (B).

(B) 10

Quick Tip: First get pH from the hydronium concentration, then use $\text{pH} + \text{pOH} = 14$.

62. Calculate the molar concentration of weak monobasic acid if $K_a = 1.8 \times 10^{-5}$ and degree of dissociation is 0.01 ?

- (A) 1.8×10^{-1}
- (B) 1.8×10^{-2}
- (C) 5.55×10^{-3}
- (D) 5.55×10^{-4}

Correct Answer: (A) 1.8×10^{-1}

Solution:**Step 1: Understand the concept**

For a weak monobasic acid HA of concentration C , the Ostwald

dilution law says $K_a = \frac{C\alpha^2}{1 - \alpha}$. When α is very small, $1 - \alpha \approx 1$ and the law becomes $K_a = C\alpha^2$.

Step 2: Rearrange

$$C = \frac{K_a}{\alpha^2}$$

Step 3: Substitute

Given $K_a = 1.8 \times 10^{-5}$ and $\alpha = 0.01$:

$$C = \frac{1.8 \times 10^{-5}}{(0.01)^2} = \frac{1.8 \times 10^{-5}}{10^{-4}} = 1.8 \times 10^{-1} \text{ M}$$

Step 4: Check the options

The value 1.8×10^{-2} would need α^2 to be 1×10^{-3} , and 5.55×10^{-3} and 5.55×10^{-4} come from inverting the ratio or mishandling the powers of ten. Only 0.18 M matches the correct formula.

Final Answer:

The molar concentration is 0.18 M . This is option (A).

(A) $1.8 \times 10^{-1} \text{ M}$

Quick Tip: Use Ostwald dilution law, $K_a = C\alpha^2$, for a weak acid.

63. Which of the following equimolar solutions is the best conductor of electricity at same temperature ?

- (A) CH_3COOH
- (B) NH_4OH
- (C) HCl
- (D) $\text{C}_6\text{H}_{12}\text{O}_6$

Correct Answer: (C) HCl

Solution:

Step 1: Understand the concept

The conductivity of an electrolyte solution depends on the number of free ions and their mobility. At equal molarity, the substance that ionises most gives the highest conductivity.

Step 2: Check HCl

HCl is a strong acid and ionises completely into H^+ and Cl^- . The H^+ ion also has an exceptionally high mobility, so HCl conducts best.

Step 3: Check the others

CH_3COOH is a weak acid and ionises only slightly. NH_4OH is a weak base and also ionises slightly. $\text{C}_6\text{H}_{12}\text{O}_6$ (glucose) is a non-electrolyte and gives no ions at all.

Step 4: Conclusion

Complete ionisation puts HCl far ahead of the weak electrolytes and the non-electrolyte.

Final Answer:

HCl is a strong electrolyte, so it conducts best. This is option (C).

(C) HCl

Quick Tip: A strong electrolyte that ionises fully gives the most ions and conducts best.

64. Which of the following statements is false regarding hydrogen ?

- (A) It has two isotopes.
- (B) In periodic table hydrogen is placed separately above group 1.
- (C) When one electron is lost it forms proton.
- (D) It does not exist freely.

Correct Answer: (A) It has two isotopes.

Solution:

Step 1: Understand the question

We need the statement that is false. Check each against the known facts about hydrogen.

Step 2: Check statement (A)

Hydrogen has three isotopes: protium (^1H), deuterium (^2H) and tritium (^3H). Saying it has only two is wrong, so (A) is FALSE.

Step 3: Check statement (B)

Hydrogen has one valence electron like alkali metals but also needs one electron to complete its shell like halogens. So it is placed separately at the top, above group 1. (B) is TRUE.

Step 4: Check statements (C) and (D)

Losing its single electron leaves a bare proton, $\text{H} \rightarrow \text{H}^+ + e^-$, so (C) is TRUE. Hydrogen is very reactive and light, so it is found in compounds like water and hydrocarbons, not freely in nature. (D) is TRUE.

Final Answer:

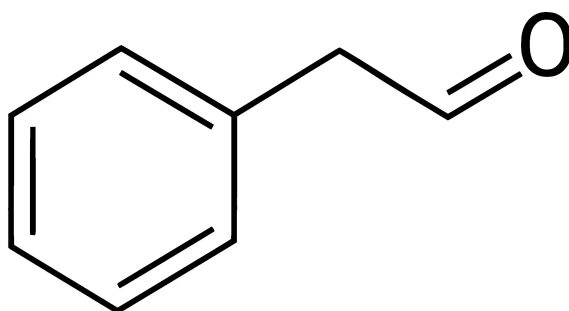
Only the statement about two isotopes is false. This is option (A).

(A) It has two isotopes

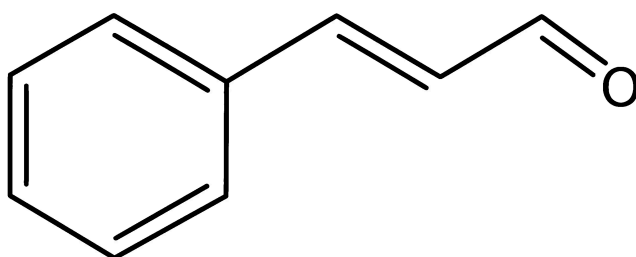
Quick Tip: Hydrogen has three isotopes, protium, deuterium and tritium, so the statement about two isotopes is the false one.

65. Which among the following is an aromatic aldehyde ?

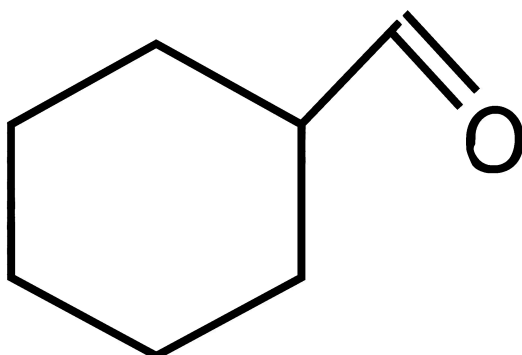
(A)



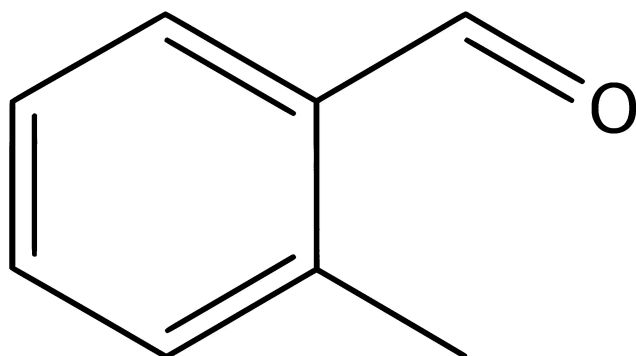
(B)



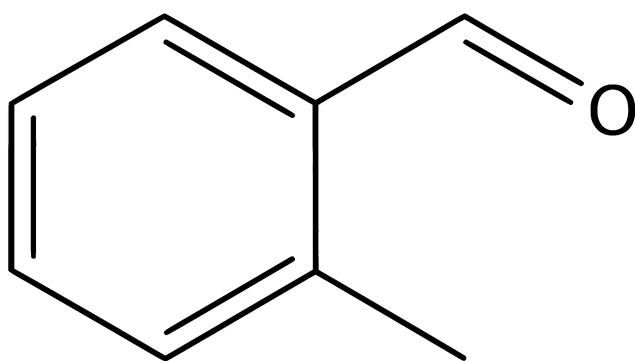
(C)



(D)



Correct Answer: (D)



Solution:

Step 1: Understand the concept

An aromatic aldehyde has the formyl group (-CHO) attached directly to a carbon of the benzene ring. The simplest example is benzaldehyde, $\text{C}_6\text{H}_5\text{CHO}$.

Step 2: Look at option (A)

The structure is a benzene ring joined to a CH_2 group, which then carries the CHO . This is phenylacetaldehyde. The CHO is on the side chain, not on the ring, so it does not fit the definition.

Step 3: Look at options (B) and (C)

Option (B) is a benzene ring joined to a CH=CH-CHO chain (cinnamaldehyde), so the CHO is again on the side chain. Option (C) is a saturated cyclohexane ring with a CHO group, an alicyclic aldehyde with no aromatic ring at all.

Step 4: Look at option (D)

Option (D) is a benzene ring with a CHO group on one ring carbon and a methyl group on the neighbouring carbon. This is 2-methylbenzaldehyde, with the CHO directly on the ring. So it is the aromatic aldehyde.

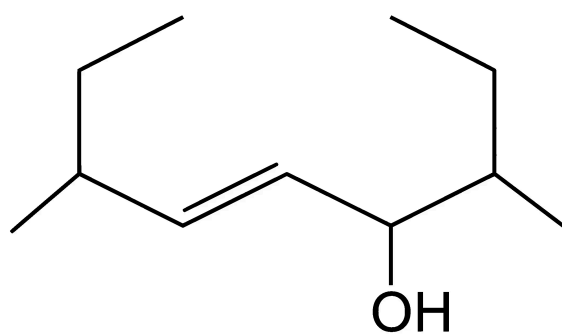
Final Answer:

The CHO group is attached directly to the ring in option (D). This is option (D).

(D) 2-Methylbenzaldehyde

Quick Tip: In an aromatic aldehyde the CHO group is bonded directly to the benzene ring.

66. What is IUPAC name of the following compound ?



- (A) 2,6-diethylhept-4-en-3-ol
- (B) 2-ethyl-6-methyloct-3-en-5-ol
- (C) 3,7-Dimethylnon-5-en-4-ol
- (D) 7-ethyl-3-methyl-oct-5-en-4-ol

Correct Answer: (C) 3,7-Dimethylnon-5-en-4-ol

Solution:**Step 1: Read the structure**

The skeleton has an OH on one carbon, and a C=C bond next to it. On the left of the double bond is a CH carrying a methyl group and an

ethyl group. On the right of the OH carbon is a CH carrying a methyl group and an ethyl group.

Step 2: Find the parent chain

The longest chain that contains both the OH carbon and the double bond runs through both ethyl groups: $\text{CH}_3\text{CH}_2 - \text{CH}(\text{CH}_3) - \text{CH}=\text{CH} - \text{CH}(\text{OH}) - \text{CH}(\text{CH}_3) - \text{CH}_2\text{CH}_3$. It has 9 carbons, so the parent is nonene.

Step 3: Number the chain

The -OH group has the first priority for numbering. Numbering from the right end gives $\text{C}_1 = \text{CH}_3$, $\text{C}_2 = \text{CH}_2$, $\text{C}_3 = \text{CH}(\text{CH}_3)$, $\text{C}_4 = \text{CH}(\text{OH})$, $\text{C}_5 = \text{CH}=\text{CH}$, $\text{C}_6 = \text{CH}$, $\text{C}_7 = \text{CH}(\text{CH}_3)$, C_8 , C_9 . Numbering from the left would put OH on C_6 , which is higher.

Step 4: Assemble the name

OH at C_4 , double bond at C_5 , methyl groups at C_3 and C_7 . The name is 3,7-dimethylnon-5-en-4-ol. The other names choose a shorter chain or give wrong locants.

Final Answer:

The correct name is 3,7-dimethylnon-5-en-4-ol. This is option (C).

(C) 3,7-Dimethylnon-5-en-4-ol

Quick Tip: Choose the longest chain containing both OH and the double bond, then number from the end that gives OH the lowest locant.

67. Which among the following is allylic halide ?

- (A) $\text{CH}_3 - \text{CH}_2 - \text{CH}_2 - \text{X}$
(B) $\text{C}_6\text{H}_5\text{CH}_2 - \text{X}$
(C) $\text{CH}_2 = \text{CH} - \text{CH}_2 - \text{X}$
(D) $\text{CH}_3 - \text{CH} = \text{CH} - \text{X}$

Correct Answer: (C) $\text{CH}_2 = \text{CH} - \text{CH}_2 - \text{X}$

Solution:

Step 1: Understand the definition

In an allylic halide the halogen is bonded to an sp^3 hybridised carbon that is directly attached to a carbon-carbon double bond. The group is $\text{C}=\text{C}-\text{C}-\text{X}$.

Step 2: Check (C)

$\text{CH}_2 = \text{CH} - \text{CH}_2 - \text{X}$: the carbon holding X is sp^3 and is next to the $\text{CH}=\text{CH}_2$ double bond. So it is an allylic halide.

Step 3: Check the others

(A) $\text{CH}_3\text{CH}_2\text{CH}_2\text{X}$ has no double bond, so it is a primary alkyl halide.
(B) $\text{C}_6\text{H}_5\text{CH}_2\text{X}$ has X on a carbon next to a benzene ring, so it is a benzylic halide. (D) $\text{CH}_3\text{CH}=\text{CH}-\text{X}$ has X directly on a double bond carbon, so it is a vinylic halide.

Step 4: Conclusion

Only (C) fits the allylic definition.

Final Answer:

The halogen sits on the carbon next to the double bond only in option (C). This is option (C).



Quick Tip: An allylic halide has the halogen on a carbon next to a C=C double bond, that is, on an sp^3 carbon attached to the double bond.

68. IUPAC name of glycerol is -

- (A) Propane - 1, 2-diol.
- (B) Propane - 1, 3-diol.
- (C) Propane - 1,2,3-triol.
- (D) Propene - 1,2,3-triol.

Correct Answer: (C) Propane - 1,2,3-triol.

Solution:

Step 1: Know the structure

Glycerol is $\text{HOCH}_2 - \text{CH}(\text{OH}) - \text{CH}_2\text{OH}$. It has a chain of 3 carbons and one OH on each carbon.

Step 2: Choose the parent chain

Three carbons with only single bonds give propane. The suffix for an alcohol is "-ol".

Step 3: Count the OH groups

There are three OH groups, so we use the prefix "tri" and the suffix "-triol". Their positions are on C1, C2 and C3.

Step 4: Name and check

The name is propane-1,2,3-triol. Option (A) has only 2 OH groups,

option (B) has 2 OH groups at C1 and C3, and option (D) uses "propene", which would need a double bond that glycerol does not have.

Final Answer:

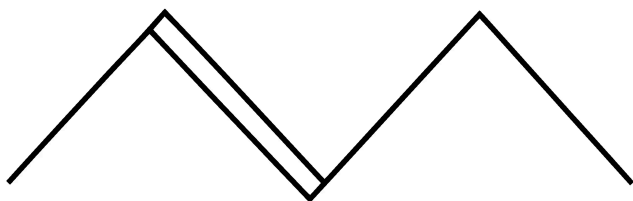
Glycerol is propane-1,2,3-triol. This is option (C).

(C) Propane-1,2,3-triol

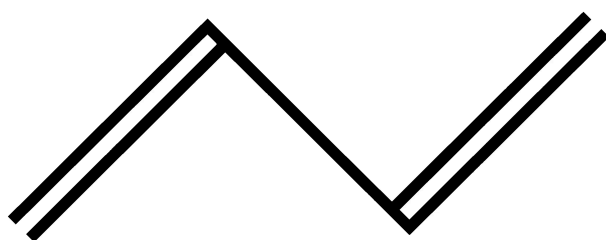
Quick Tip: Glycerol has three OH groups on a three-carbon chain.

69. Which from following structures represents 1,3 butadiene ?

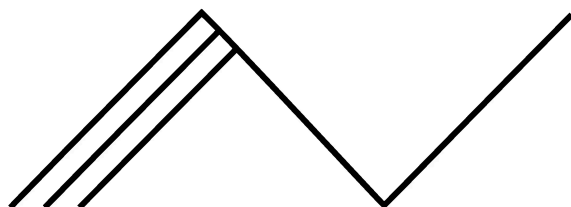
(A)



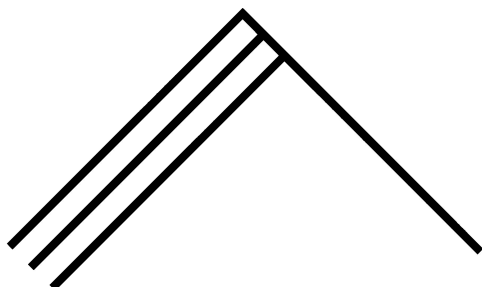
(B)



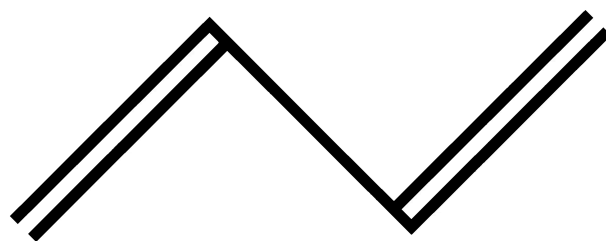
(C)



(D)



Correct Answer: (B)



Solution:

Step 1: Understand the name

1,3-Butadiene has a four carbon chain ("but"), with two double bonds ("diene"). The locants 1 and 3 show that one double bond starts at C1 and the other at C3, giving $\text{CH}_2 = \text{CH} - \text{CH} = \text{CH}_2$.

Step 2: Read the skeletal drawings

In a skeletal formula each bend or end is a carbon. A double bond is drawn as two parallel lines.

Step 3: Check option (A)

The drawing is a zigzag of five carbons with one double bond in the first part. It is a pentene, not a butadiene.

Step 4: Check options (B), (C) and (D)

Option (B) shows four carbons with a double bond at each end joined by a single bond in the middle, which is $\text{CH}_2 = \text{CH} - \text{CH} = \text{CH}_2$.

Option (C) has a triple bond and (D) also has a triple bond, so neither is a diene. Only (B) is 1,3-butadiene.

Final Answer:

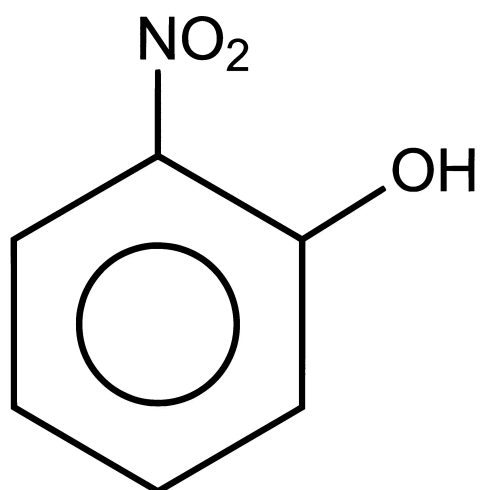
Option (B) shows the two double bonds at C1 and C3 of a four-carbon chain. This is option (B).



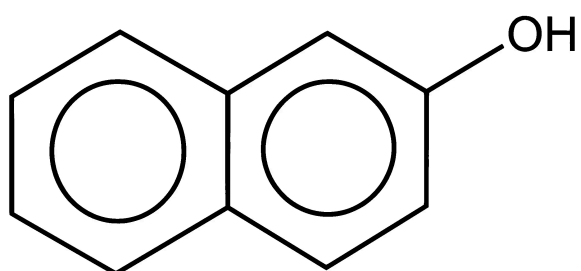
Quick Tip: 1,3-Butadiene has four carbons with double bonds starting at C1 and C3.

70. Which of the following is NOT a phenol ?

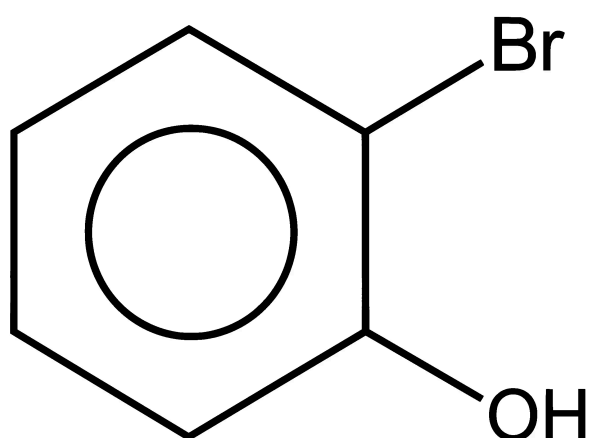
(A)



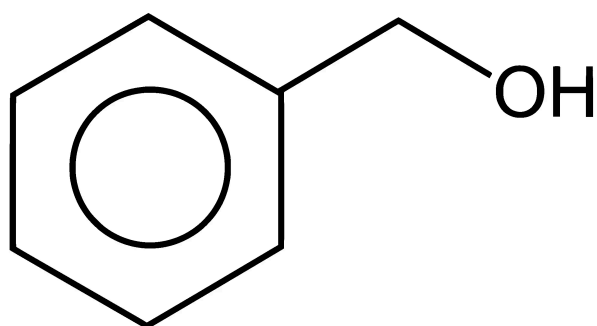
(B)



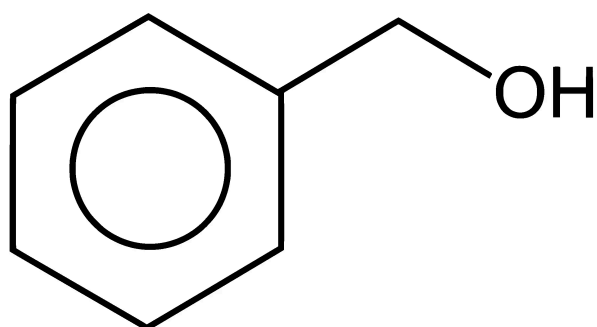
(C)



(D)



Correct Answer: (D)



Solution:

Step 1: Understand the concept

A phenol is a compound in which a hydroxyl group is attached directly to a carbon of an aromatic ring. Substituents on the ring do not change this.

Step 2: Check options (A), (B) and (C)

Option (A) is a benzene ring with NO₂ and OH on neighbouring carbons (2-nitrophenol), so OH is on the ring. Option (B) is naphthalene with OH on a ring carbon (2-naphthol), which is also a phenol. Option (C) is a benzene ring with Br and OH on adjacent carbons (2-bromophenol).

Step 3: Check option (D)

Option (D) is a benzene ring joined to a CH_2 group that carries the OH. The OH sits on a side-chain sp^3 carbon, not on the ring. This is benzyl alcohol, an alcohol.

Step 4: Conclusion

Benzyl alcohol lacks a ring-bonded OH group, so it is not a phenol.

Final Answer:

In benzyl alcohol the OH is on a side-chain carbon, not on the ring. This is option (D).

(D) Benzyl alcohol

Quick Tip: A phenol has an OH group bonded directly to a carbon of an aromatic ring.

71. Find the number of structural isomers possible for hexane ?

- (A) Two
- (B) Four
- (C) Three
- (D) Five

Correct Answer: (D) Five

Solution:

Step 1: Understand the question

Hexane has the molecular formula C_6H_{14} . We count every different carbon skeleton that fits this formula, that is, the chain isomers.

Step 2: Six-carbon and five-carbon chains

Chain of 6 carbons: n-hexane. Chain of 5 carbons with one methyl: the methyl can be on C2 (2-methylpentane) or on C3 (3-methylpentane). A methyl on C1 would just extend the chain, so it is not new.

Step 3: Four-carbon chains

Chain of 4 carbons with two methyl groups: both on C2 give 2,2-dimethylbutane, and one on C2 and one on C3 give 2,3-dimethylbutane. Other placements repeat these or lengthen the chain.

Step 4: Count

n-Hexane, 2-methylpentane, 3-methylpentane, 2,2-dimethylbutane and 2,3-dimethylbutane make 5. A three-carbon chain cannot hold six carbons without repeating the earlier structures. So the answer is five.

Final Answer:

Hexane has five structural isomers, counting itself. This is option (D).

(D) 5

Quick Tip: Draw the longest chain first, then shorten it by one carbon and place a methyl at different spots, and so on.

72. Which of the following hydrocarbons has stronger London dispersion forces ?

- (A) Methane
- (B) Ethane
- (C) n-pentane
- (D) neo-pentane

Correct Answer: (C) n-pentane

Solution:

Step 1: Understand the concept

London dispersion forces arise from temporary dipoles in non-polar molecules. Their strength increases with the number of electrons and with the surface area available for contact.

Step 2: Compare the sizes

Methane (CH_4) has 10 electrons, ethane (C_2H_6) has 18, and both pentane isomers (C_5H_{12}) have 42. So the pentanes are far ahead of methane and ethane.

Step 3: Compare the pentane isomers

n-Pentane is a long, straight chain that lies along its neighbours and has a large surface of contact. Neopentane is a compact, nearly spherical molecule with little surface in contact.

Step 4: Conclusion

So n-pentane has stronger London forces, which is why it boils at 309 K while neopentane boils at 283 K.

Final Answer:

The long straight chain of n-pentane makes more contact than neopentane. This is option (C).

(C) n-Pentane

Quick Tip: London dispersion forces grow with the surface area and number of electrons, so a long unbranched chain beats a compact one.

73. Identify the major product of following reaction.



- (A) 1 - Bromopropane
- (B) 2 - Bromopropane
- (C) 3 - Bromopropene
- (D) 2 - Bromopropan-1-ol

Correct Answer: (A) 1 - Bromopropane

Solution:

Step 1: Understand the reaction

Propene, $\text{CH}_3\text{CH}=\text{CH}_2$, reacts with HBr in the presence of a peroxide. This is the peroxide (Kharasch) effect, which changes the usual orientation of addition.

Step 2: Mechanism

The peroxide produces a bromine radical $\text{Br}\cdot$ from HBr. The radical adds to the terminal carbon CH_2 of the double bond so that the more stable secondary radical $\text{CH}_3\dot{\text{C}}\text{H} - \text{CH}_2\text{Br}$ forms.

Step 3: Complete the step

The secondary radical removes H from another HBr molecule, giving $\text{CH}_3\text{CH}_2\text{CH}_2\text{Br}$ and a new $\text{Br}\cdot$, so the chain continues.

Step 4: Check the options

The product is 1-bromopropane (A). 2-Bromopropane (B) is the Markovnikov product without peroxide. 3-Bromopropene (C) needs substitution at the allylic position, and (D) needs an OH group, which is not available in this reaction.

Final Answer:

Anti-Markovnikov addition gives 1-bromopropane. This is option (A).

(A) 1-Bromopropane

Quick Tip: With peroxide, HBr adds to propene by a free radical path against Markovnikov, giving the 1-bromo product.

74. Calculate the total volume of simple cubic unit cell in cm^3 if void volume is $2.0 \times 10^{-23} \text{ cm}^3$

- (A) 4.2×10^{-23}
- (B) 3.8×10^{-23}
- (C) 3.4×10^{-23}
- (D) 4.6×10^{-23}

Correct Answer: (A) 4.2×10^{-23}

Solution:**Step 1: Understand the concept**

In a simple cubic unit cell the atoms touch along the edge, so $a = 2r$. The occupied fraction (packing efficiency) is the volume of the one

atom divided by the volume of the cell.

Step 2: Find the packing fraction

$$\text{Packing efficiency} = \frac{\frac{4}{3}\pi r^3}{(2r)^3} = \frac{\pi}{6} = 0.524$$

So the void fraction is $1 - 0.524 = 0.476$.

Step 3: Use the void volume

$$V_{\text{void}} = 0.476 \times V_{\text{total}} \Rightarrow V_{\text{total}} = \frac{2.0 \times 10^{-23}}{0.476}$$

Step 4: Evaluate

$$V_{\text{total}} = 4.2 \times 10^{-23} \text{ cm}^3$$

The other options correspond to void fractions of 0.53, 0.59 and 0.43, which do not match the simple cubic cell.

Final Answer:

The total volume is $4.2 \times 10^{-23} \text{ cm}^3$. This is option (A).

$$\boxed{(A) 4.2 \times 10^{-23} \text{ cm}^3}$$

Quick Tip: The packing efficiency of a simple cubic cell is 52.4 percent, so void is 47.6 percent of the total volume.

75. Which of the following statements is true about schottky defect ?

- (A) The density of a substance decreases.
- (B) The electrical neutrality of the substance changes.
- (C) It is found in ionic compounds with larger difference between sizes of cation and anion.
- (D) It develops when smaller cations occupy interstitial sites from their normal sites.

Correct Answer: (A) The density of a substance decreases.

Solution:

Step 1: Understand the defect

A Schottky defect occurs when equal numbers of cations and anions leave their lattice sites, creating vacancies. It is found in ionic solids where the cation and anion are similar in size, such as NaCl and KCl.

Step 2: Check statement (A)

Because ions are missing from the crystal but the volume does not change, mass per unit volume decreases. So the density decreases. (A) is TRUE.

Step 3: Check statement (B)

Equal numbers of positive and negative ions go missing, so the crystal remains electrically neutral. (B) is FALSE.

Step 4: Check statements (C) and (D)

The Schottky defect appears when the cation and anion sizes are nearly equal, not when they differ widely. So (C) is FALSE. Statement (D), a smaller cation moving to an interstitial site, describes a Frenkel defect, so it is FALSE for Schottky.

Final Answer:

Only the statement about decreased density is true. This is option (A).

(A) The density of a substance decreases

Quick Tip: In Schottky defect equal numbers of cations and anions are missing, so mass drops while volume stays the same.



76. In ionic solid, anions are arranged in hcp array and cations occupy $\frac{1}{2}$ tetrahedral voids. What is the formula of ionic compound ?

[Consider A = Cation ; B = anion]

- (A) AB_2
- (B) AB
- (C) A_2B
- (D) AB_3

Correct Answer: (B) AB

Solution:**Step 1: Understand the concept**

In a hexagonal close packed (hcp) array, if there are N atoms (here anions B), there are N octahedral voids and $2N$ tetrahedral voids.

Step 2: Count the anions and voids

Take N anions of B. The number of tetrahedral voids is $2N$.

Step 3: Count the cations

Cations A occupy one half of the tetrahedral voids, so the number of A is $\frac{1}{2} \times 2N = N$.

Step 4: Write the formula

The ratio is $A : B = N : N = 1 : 1$, so the formula is AB. Option (A) AB_2 would need all tetrahedral voids to be half empty with fewer cations, option (C) A_2B needs all tetrahedral voids filled, and (D) does not fit a void count of $2N$.

Final Answer:

The cation to anion ratio is $1 : 1$. This is option (B).

(B) AB

Quick Tip: In hcp, N anions give $2N$ tetrahedral voids; half of them occupied means N cations.



77. Calculate the molality of aqueous solution of electrolyte that freezes at -0.93°C if K_f for water and van't Hoff factor respectively are $1.86\text{ K kg mol}^{-1}$ and 1.25 . (Freezing point of water = 0°C)

- (A) 0.2 m
- (B) 0.5 m
- (C) 0.3 m
- (D) 0.4 m

Correct Answer: (D) 0.4 m

Solution:

Step 1: Understand the concept

For an electrolyte, the freezing point depression is $\Delta T_f = i K_f m$, where i is the van't Hoff factor and m is the molality.

Step 2: Find the depression

Water freezes at 0°C and the solution freezes at -0.93°C , so $\Delta T_f = 0.93\text{ K}$.

Step 3: Substitute

$$m = \frac{\Delta T_f}{i K_f} = \frac{0.93}{1.25 \times 1.86} = \frac{0.93}{2.325} = 0.4\text{ m}$$

Step 4: Check the options

If we had ignored i , we would get $0.93/1.86 = 0.5\text{ m}$, which is option (B), a trap. The correct use of $i = 1.25$ gives 0.4 m .

Final Answer:

The molality is 0.4 m . This is option (D).

(D) 0.4 m

Quick Tip: Use depression in freezing point = $i \times K_f \times m$.

78. Calculate the solubility of certain gas in solvent with pressure 3 atm at 25°C (Henry's law constant is $3.0 \times 10^{-2}\text{ mol dm}^{-3}\text{ atm}^{-1}$)

- (A) 0.07 M
- (B) 0.08 M
- (C) 0.09 M
- (D) 0.1 M

Correct Answer: (C) 0.09 M

Solution:

Step 1: Understand the law

Henry's law says the solubility of a gas in a liquid is directly proportional to its partial pressure: $S = K_H \times P$.

Step 2: Substitute the data

$K_H = 3.0 \times 10^{-2} \text{ mol dm}^{-3} \text{ atm}^{-1}$ and $P = 3 \text{ atm}$.

Step 3: Calculate

$$S = 3.0 \times 10^{-2} \times 3 = 9.0 \times 10^{-2} = 0.09 \text{ mol dm}^{-3}$$

Step 4: Check the options

The values 0.07, 0.08 and 0.1 M would need pressures of 2.33, 2.67 and 3.33 atm. Only 0.09 M corresponds to exactly 3 atm.

Final Answer:

The gas solubility is 0.09 M. This is option (C).

(C) 0.09 M

Quick Tip: Henry law: solubility = $K_H \times \text{pressure}$.

79. What is molar conductivity of CH_3COOH at zero concentration if the molar conductivities of H_2SO_4 , K_2SO_4 and CH_3COOK at zero concentrations are respectively x , y and $z \text{ } \Omega^{-1}\text{cm}^2\text{mol}^{-1}$?

- (A) $\frac{(x-y)}{2} + z$
(B) $(x - y + 2z)$
(C) $(x + y - z)$
(D) $\frac{(x-y)}{2} + 2z$

Correct Answer: (A) $\frac{(x-y)}{2} + z$

Solution:

Step 1: Understand the concept

Kohlrausch's law says the limiting molar conductivity of an electrolyte is the sum of the contributions of its ions: $\Lambda_m^\circ = \lambda_+^\circ + \lambda_-^\circ$ (with stoichiometric numbers).

Step 2: Write the three given values

$$\Lambda^\circ(\text{H}_2\text{SO}_4) = 2\lambda_{\text{H}^+} + \lambda_{\text{SO}_4} = x. \quad \Lambda^\circ(\text{K}_2\text{SO}_4) = 2\lambda_{\text{K}^+} + \lambda_{\text{SO}_4} = y.$$

$$\Lambda^\circ(\text{CH}_3\text{COOK}) = \lambda_{\text{CH}_3\text{COO}^-} + \lambda_{\text{K}^+} = z.$$

Step 3: Combine

$$\text{Subtract: } x - y = 2\lambda_{\text{H}^+} - 2\lambda_{\text{K}^+}, \text{ so } \lambda_{\text{H}^+} - \lambda_{\text{K}^+} = (x - y)/2.$$

Step 4: Find acetic acid

$$\Lambda^\circ(\text{CH}_3\text{COOH}) = \lambda_{\text{H}^+} + \lambda_{\text{CH}_3\text{COO}^-} = (\lambda_{\text{H}^+} - \lambda_{\text{K}^+}) + (\lambda_{\text{CH}_3\text{COO}^-} + \lambda_{\text{K}^+}) = \frac{x - y}{2} + z$$

Final Answer:

The limiting molar conductivity is $(x-y)/2 + z$. This is option (A).

$$(A) \frac{x-y}{2} + z$$

Quick Tip: Apply Kohlrausch law: write acetic acid as a combination of the three given electrolytes.

80. What must be the molarity of BaCl_2 solution to have molar conductivity $240 \Omega^{-1}\text{cm}^2\text{mol}^{-1}$ and conductivity $0.012 \Omega^{-1}\text{cm}^{-1}$ at 25°C ?

- (A) 0.01 M
- (B) 0.02 M
- (C) 0.05 M
- (D) 0.1 M

Correct Answer: (C) 0.05 M

Solution:**Step 1: Understand the relation**

Molar conductivity and conductivity are linked by $\Lambda_m = \frac{\kappa \times 1000}{C}$, where C is the molarity in mol per litre and κ is in $\Omega^{-1}\text{cm}^{-1}$.

Step 2: Rearrange

$$C = \frac{\kappa \times 1000}{\Lambda_m}$$

Step 3: Substitute

$$C = \frac{0.012 \times 1000}{240} = \frac{12}{240} = 0.05 \text{ M}$$

Step 4: Check the options

If the 1000 factor is forgotten we get 5×10^{-5} , not listed. The values 0.01, 0.02 and 0.1 M would give $\kappa = 0.0024$, 0.0048 and 0.024, so they do not match.

Final Answer:

The molarity is 0.05 M. This is option (C).

(C) 0.05 M

Quick Tip: Use molar conductivity = 1000 x conductivity / molarity.

81. Half life of a first order reaction is 900 second. If initial concentration of reactant is 0.08 mol dm^{-3} find concentration that remains after 35 minute ?

- (A) $0.159 \text{ mol dm}^{-3}$
- (B) $0.0159 \text{ mol dm}^{-3}$
- (C) 1.05 mol dm^{-3}
- (D) $0.759 \text{ mol dm}^{-3}$

Correct Answer: (B) $0.0159 \text{ mol dm}^{-3}$

Solution:

Step 1: Understand the concept

For a first order reaction, the concentration halves every half life.

After n half lives, $[A] = [A]_0/2^n$ with $n = t/t_{1/2}$.

Step 2: Convert time

$t = 35 \text{ min} = 35 \times 60 = 2100 \text{ s}$. Half life is 900 s , so $n = 2100/900 = 2.333$.

Step 3: Calculate the rate constant route

$$k = 0.693/900 = 7.7 \times 10^{-4} \text{ s}^{-1}. \text{ Then } \log \frac{[A]_0}{[A]} = \frac{kt}{2.303} \\ = \frac{7.7 \times 10^{-4} \times 2100}{2.303} = 0.7021, \text{ so } [A]_0/[A] = 5.04.$$

Step 4: Result

$$[A] = \frac{0.08}{5.04} = 0.0159 \text{ mol dm}^{-3}$$

The other options are larger than the initial concentration or are the wrong order of magnitude, which cannot be so since concentration only falls.

Final Answer:

The remaining concentration is $0.0159 \text{ mol per dm}^3$. This is option (B).

$(B) 0.0159 \text{ mol dm}^{-3}$

Quick Tip: Convert 35 minutes to seconds, count the half lives, then divide the initial concentration by 2 to that power.

82. Consider the reaction,

Double subscripts: use braces to clarify

If the rate of formation of SO_4^{2-} at a particular time is $2.2 \times 10^{-2} \text{ mol dm}^{-3} \text{ s}^{-1}$. Calculate the rate of consumption of I^- .

- (A) $1.1 \times 10^{-2} \text{ mol dm}^{-3} \text{ s}^{-1}$
- (B) $2.2 \times 10^{-2} \text{ mol dm}^{-3} \text{ s}^{-1}$
- (C) $3.3 \times 10^{-2} \text{ mol dm}^{-3} \text{ s}^{-1}$
- (D) $4.4 \times 10^{-2} \text{ mol dm}^{-3} \text{ s}^{-1}$

Correct Answer: (C) $3.3 \times 10^{-2} \text{ mol dm}^{-3} \text{ s}^{-1}$

Solution:

Step 1: Understand the concept

For $a\text{A} \rightarrow b\text{B}$ the rate of reaction is $-\frac{1}{a} \frac{d[\text{A}]}{dt} = \frac{1}{b} \frac{d[\text{B}]}{dt}$. Each species rate is divided by its coefficient.

Step 2: Write the rate expression

For $3\text{I}^- + \text{S}_2\text{O}_8^{2-} \rightarrow \text{I}_3^- + 2\text{SO}_4^{2-}$:

$$-\frac{1}{3} \frac{d[\text{I}^-]}{dt} = \frac{1}{2} \frac{d[\text{SO}_4^{2-}]}{dt}$$

Step 3: Substitute

$$-\frac{d[\text{I}^-]}{dt} = \frac{3}{2} \times 2.2 \times 10^{-2} = 3.3 \times 10^{-2} \text{ mol dm}^{-3} \text{ s}^{-1}$$

Step 4: Check options

2.2×10^{-2} would be right only if iodide had the same coefficient as sulphate, and 1.1×10^{-2} or 4.4×10^{-2} come from mistaken ratios of $1/2$ or 2 . The true ratio is $3 : 2$.

Final Answer:

Iodide is used at $3.3 \times 10^{-2} \text{ mol dm}^{-3} \text{ s}^{-1}$. This is option (C).

$(C) 3.3 \times 10^{-2} \text{ mol dm}^{-3} \text{ s}^{-1}$

Quick Tip: Divide each rate by its stoichiometric coefficient to compare rates.



83. For a first order reaction, half life is 5 hour. What time is required to reduce 10 g of reactant to 2.5 g ?

- (A) 3 hour
- (B) 4 hour
- (C) 5 hour
- (D) 10 hour

Correct Answer: (D) 10 hour

Solution:**Step 1: Understand the concept**

In a first order reaction the amount of reactant halves in every half life, regardless of the starting amount.

Step 2: Count the half lives

10 g $\xrightarrow{t_{1/2}}$ 5 g $\xrightarrow{t_{1/2}}$ 2.5 g. So two half lives are needed.

Step 3: Find the time

$$t = 2 \times t_{1/2} = 2 \times 5 = 10 \text{ h}$$

Step 4: Check the options

3 h and 4 h are shorter than one half life of 5 h, so less than half would have decayed. 5 h gives only 5 g. Only 10 h gives 2.5 g.

Final Answer:

Two half lives take 10 hours. This is option (D).

(D) 10 h

Quick Tip: Going from 10 g to 2.5 g takes two half lives.

84. Which of following forces is responsible for physisorption ?

- (A) Coulombic force
- (B) Electrostatic force
- (C) London force of dispersion
- (D) Hydrogen bonding

Correct Answer: (C) London force of dispersion

Solution:

Step 1: Understand the concept

Adsorption can be physical (physisorption) or chemical (chemisorption). They differ in the type of force between the adsorbate and the surface.

Step 2: Physisorption forces

Physisorption is caused by weak van der Waals forces, mainly London dispersion forces. These forces act between any molecules, so the process is non-specific and reversible, with a low enthalpy of about 20-40 kJ/mol.

Step 3: Compare with chemisorption

Chemisorption involves chemical bonds, which are strong, specific and need activation energy, with an enthalpy of 80-240 kJ/mol.

Step 4: Why not the others

Coulombic and electrostatic forces act between ions and are typical of ionic bonds, and hydrogen bonding needs special groups (N-H, O-H, F-H). None of them is the general force that causes physisorption.

Final Answer:

Physisorption arises from London dispersion forces. This is option (C).

(C) London force of dispersion

Quick Tip: Physisorption involves weak van der Waals forces, namely London dispersion forces.

85. Which of the following ranges of particle size defines nano scale ?

- (A) 10-100 nm
- (B) 1000-2000 nm
- (C) 1-100 nm
- (D) 100-1000 nm

Correct Answer: (C) 1-100 nm

Solution:

Step 1: Understand the concept

Nanoscience deals with materials whose size is measured in nanometres. A nanometre is 10^{-9} m.

Step 2: State the range

By the usual definition, the nanoscale lies between 1 nm and 100 nm in at least one dimension.

Step 3: Compare with the options

Option (A) 10-100 nm leaves out the 1-10 nm range, which still counts as nano. Options (B) 1000-2000 nm and (D) 100-1000 nm correspond to particles of micrometre size.

Step 4: Conclusion

Hence 1-100 nm is the right range.

Final Answer:

Nano scale means 1-100 nm. This is option (C).

(C) 1-100 nm

Quick Tip: Nanoscale is 1 to 100 nanometres.

86. Which of the following is the correct decreasing order of bond dissociation enthalpy of halogens ?

- (A) $F_2 > Cl_2 > Br_2 > I_2$
- (B) $I_2 > Br_2 > Cl_2 > F_2$
- (C) $Cl_2 > Br_2 > F_2 > I_2$
- (D) $Br_2 > I_2 > F_2 > Cl_2$

Correct Answer: (C) $Cl_2 > Br_2 > F_2 > I_2$

Solution:

Step 1: Recall the data

Bond dissociation enthalpies (kJ/mol): $Cl_2 = 242$, $Br_2 = 193$, $F_2 = 158$, $I_2 = 151$.

Step 2: Understand the trend

From chlorine to iodine the bond lengthens as the atoms grow, so the bond weakens: $Cl_2 > Br_2 > I_2$.

Step 3: Why fluorine is unusual

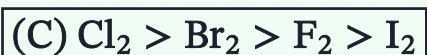
The F-F bond is very short, so the lone pairs on the two small F atoms repel each other strongly. This weakens the bond, and its enthalpy falls below that of chlorine and bromine.

Step 4: Final order

Combining these gives $\text{Cl}_2 > \text{Br}_2 > \text{F}_2 > \text{I}_2$. Option (A) assumes a smooth decrease down the group, and (B) the reverse, while (D) does not fit the data.

Final Answer:

Chlorine has the highest and iodine the lowest bond enthalpy. This is option (C).



Quick Tip: Fluorine is an exception: its small size causes strong lone pair repulsion that weakens the F-F bond.

87. Which from following is NOT an inner transition element ?

- (A) Rf
- (B) Ac
- (C) Lu
- (D) Ho

Correct Answer: (A) Rf

Solution:

Step 1: Understand the concept

Inner transition elements are the f-block elements: the lanthanoids (Ce to Lu, 4f series, with La as the parent) and the actinoids (Th to Lr, 5f series, with Ac as the parent).

Step 2: Check Ac, Lu and Ho

Ac ($Z = 89$) heads the actinoid series. Lu ($Z = 71$) is the last member of the lanthanoid series. Ho ($Z = 67$) is a lanthanoid. All three belong to the f-block series.

Step 3: Check Rf

Rutherfordium ($Z = 104$) comes after the actinoids. It is a 6d element placed in group 4 below Hf, so it is a d-block (transition) element.

Step 4: Conclusion

Rf is the one that is not an inner transition element.

Final Answer:

Rf is a 6d block element, not an f-block element. This is option (A).

(A) Rf

Quick Tip: Inner transition elements are the lanthanoids (4f) and actinoids (5f); Rf is a 6d element.

88. Identify species having metal atom in +6 oxidation state from following

- (A) MnO_4^-
- (B) $[\text{Cr}(\text{CN})_6]^{3-}$
- (C) Cr_2O_3
- (D) CrO_2Cl_2

Correct Answer: (D) CrO_2Cl_2

Solution:

Step 1: Method

Set the sum of oxidation numbers of all atoms equal to the charge on the species.

Step 2: Check (A) and (B)

In MnO_4^- : $x + 4(-2) = -1$, so $x = +7$. In $[\text{Cr}(\text{CN})_6]^{3-}$: $x + 6(-1) = -3$, so $x = +3$.

Step 3: Check (C)

In Cr_2O_3 : $2x + 3(-2) = 0$, so $x = +3$.

Step 4: Check (D)

In CrO_2Cl_2 : $x + 2(-2) + 2(-1) = 0$, so $x = +6$. This is chromyl chloride, with chromium in the +6 state.

Final Answer:

Chromium is +6 in chromyl chloride. This is option (D).

(D) CrO_2Cl_2

Quick Tip: Assign oxidation states using charge balance on each species.

89. Which element from following is NOT regarded as transition element ?

- (A) Cu
- (B) Sc
- (C) Cd
- (D) Au

Correct Answer: (C) Cd

Solution:

Step 1: Understand the concept

A transition element has a partly filled d subshell either in the atom or in at least one of its common oxidation states.

Step 2: Check Cu, Au and Sc

Cu ($3d^{10}4s^1$) forms Cu^{2+} with $3d^9$. Au forms Au^{3+} with $5d^8$. Sc has $3d^14s^2$ in the atom. Each has an incomplete d subshell in the atom or an ion, so each is regarded as a transition element.

Step 3: Check Cd

Cd has $4d^{10}5s^2$ and its only common ion Cd^{2+} is $4d^{10}$. The d subshell is full in both the atom and the ion.

Step 4: Conclusion

Cd is the element not regarded as a transition element.

Final Answer:

Cd has a completely filled d subshell in atom and ion. This is option (C).

(C) Cd

Quick Tip: A transition element has an incomplete d subshell in the atom or in its common ions; Cd has a full d10 in both.

90. Which from following coordinate complexes contains neutral ligand ?

- (A) Pentacarbonyliron (0)
- (B) Hexacyanoferrate (II) ion
- (C) Trioxalatocobaltate (III) ion
- (D) Sodium hexanitrocobaltate (III)

Correct Answer: (A) Pentacarbonyliron (0)

Solution:

Step 1: Understand the concept

A ligand is neutral if it has no net charge. Carbonyl (CO), ammine (NH_3) and aqua (H_2O) are common neutral ligands.

Step 2: Check (A)

Pentacarbonyliron(0), $[\text{Fe}(\text{CO})_5]$, has five CO ligands, which are neutral. The oxidation state of iron is 0 and the complex is neutral.

Step 3: Check (B), (C) and (D)

Hexacyanoferrate(II) has six CN^- ligands, which are anionic.

Trioxalatocobaltate(III) has three $\text{C}_2\text{O}_4^{2-}$ ligands, also anionic.

Sodium hexanitrocobaltate(III) has six NO_2^- ligands, again anionic.

Step 4: Conclusion

Only (A) contains a neutral ligand.

Final Answer:

Carbonyl is a neutral ligand. This is option (A).

(A) Pentacarbonyliron(0)

Quick Tip: Look for ligands such as CO, NH_3 or H_2O that carry no charge.

91. Which from following complexes is an example of MA_4BC type distereoisomers ?

- (A) $[\text{Pt}(\text{NH}_3)(\text{H}_2\text{O})\text{Cl}_2]$
- (B) $[\text{Pt}(\text{NH}_3)_4\text{ClBr}]^{2+}$
- (C) $\text{Pt}(\text{NH}_3)_2\text{Cl}_2$
- (D) $[\text{Co}(\text{NH}_3)_4\text{Cl}_2]^+$

Correct Answer: (B) $[\text{Pt}(\text{NH}_3)_4\text{ClBr}]^{2+}$

Solution:

Step 1: Understand the notation

In MA_4BC , M is the metal, A is a ligand present 4 times, and B and C are two different ligands, each once. The coordination number is 6.

Step 2: Check (B)

$[\text{Pt}(\text{NH}_3)_4\text{ClBr}]^{2+}$ has four NH_3 (A), one Cl (B) and one Br (C). The coordination number is 6, so it is MA_4BC . It can have Cl and Br either adjacent (cis) or opposite (trans), giving geometrical isomers.

Step 3: Check the others

(A) $[\text{Pt}(\text{NH}_3)(\text{H}_2\text{O})\text{Cl}_2]$ has four ligands of three kinds, so it is MABC_2 type. (C) $\text{Pt}(\text{NH}_3)_2\text{Cl}_2$ is MA_2B_2 . (D) $[\text{Co}(\text{NH}_3)_4\text{Cl}_2]^+$ is MA_4B_2 .

Step 4: Conclusion

Only (B) matches MA_4BC .

Final Answer:

Four identical ligands plus two different ones fit option (B). This is option (B).



Quick Tip: MA₄BC means four identical ligands and two different ligands B and C in an octahedral complex.

92. Identify the product B in following reaction.



- (A) Ethane
- (B) Ethanamine
- (C) Ethanol
- (D) Methanol

Correct Answer: (B) Ethanamine

Solution:

Step 1: First step

$CH_3Br + KCN \rightarrow CH_3CN + KBr$. The cyanide ion replaces bromide by S_N2 , so A is methyl cyanide (acetonitrile).

Step 2: Second step

Sodium in ethanol supplies nascent hydrogen, which reduces the nitrile group (Mendius reduction): $CH_3CN + 4[H] \rightarrow CH_3CH_2NH_2$.

Step 3: Identify B

B is $CH_3CH_2NH_2$, ethanamine, with two carbons.

Step 4: Why not others

Ethane would need loss of the CN group, and ethanol and methanol

have no nitrogen, so the amine nitrogen could not have been lost during a reduction with Na and ethanol.

Final Answer:

Reduction of the nitrile gives ethanamine. This is option (B).

(B) Ethanamine

Quick Tip: KCN gives the nitrile and Na with ethanol reduces it to a primary amine with one more carbon.

93. What is the product formed on Rosenmund reduction of benzoyl chloride ?

- (A) Benzene
- (B) Benzyl alcohol
- (C) Benzaldehyde
- (D) Chlorobenzene

Correct Answer: (C) Benzaldehyde

Solution:

Step 1: Understand the reaction

Rosenmund reduction is the catalytic hydrogenation of an acyl chloride to an aldehyde. The catalyst is palladium on barium sulphate, partly poisoned with sulphur or quinoline.

Step 2: Write the equation



Step 3: Why the reaction stops

The poisoned catalyst is not active enough to reduce the aldehyde further to the alcohol, so the reaction stops at the aldehyde stage.

Step 4: Check the options

Benzyl alcohol would form only with a stronger reducing agent, benzene needs removal of the whole acyl group and chlorobenzene needs a different reaction. So benzaldehyde is right.

Final Answer:

Benzoyl chloride gives benzaldehyde. This is option (C).

(C) Benzaldehyde

Quick Tip: Rosenmund reduction converts an acyl chloride to an aldehyde using H_2 over Pd on $BaSO_4$.

94. Identify the reagent used in following reaction.

Benzoic acid $\xrightarrow[\Delta]{\text{Reagent}}$ Benzoyl chloride + phosphorous oxychloride
+ Hydrogen chloride

- (A) PCl_3
- (B) HCl
- (C) PCl_5
- (D) $SOCl_2$

Correct Answer: (C) PCl_5

Solution:

Step 1: Understand the reaction

A carboxylic acid is converted to an acyl chloride when the OH group is replaced by Cl. Different reagents leave different by-products.

Step 2: Write with PCl_5



Step 3: Match by-products

The question lists benzoyl chloride, phosphorus oxychloride (POCl_3) and HCl. This exactly matches the PCl_5 reaction.

Step 4: Check others

PCl_3 gives H_3PO_3 as the by-product, SOCl_2 gives SO_2 and HCl, and HCl alone cannot replace the acid OH group.

Final Answer:

PCl_5 gives POCl_3 and HCl as by-products. This is option (C).

(C) PCl_5

Quick Tip: The by-products POCl_3 and HCl point to PCl_5 .

95. Which from following reactions occurs by removal of -CO- group from amide ?

- (A) Gabriel phthalimide synthesis
- (B) Ammonolysis of halides
- (C) Mendius reduction
- (D) Hofmann degradation

Correct Answer: (D) Hofmann degradation

Solution:

Step 1: Understand the concept

We need the reaction in which an amide loses its -CO- group. This gives an amine with one carbon less.

Step 2: Hofmann degradation

$\text{RCONH}_2 + \text{Br}_2 + 4\text{NaOH} \rightarrow \text{RNH}_2 + 2\text{NaBr} + \text{Na}_2\text{CO}_3 + 2\text{H}_2\text{O}$. The carbonyl carbon leaves as carbonate, so the amine has one carbon fewer than the amide.

Step 3: Check others

Gabriel synthesis uses a phthalimide anion and an alkyl halide to make a primary amine. Ammonolysis uses ammonia and an alkyl halide. Mendius reduction reduces a nitrile with Na and alcohol. None of these removes a CO group from an amide.

Step 4: Conclusion

Hofmann degradation is the answer.

Final Answer:

Hofmann degradation drops the carbonyl carbon as carbonate. This is option (D).

(D) Hofmann degradation

Quick Tip: In Hofmann bromamide degradation the amide loses its carbonyl carbon as carbonate.



96. Which from following pairs of carbohydrates contains galactose in both of them as one of the constituent ?

- (A) Sucrose and maltose
- (B) Maltose and lactose
- (C) Lactose and Raffinose
- (D) Sucrose and lactose

Correct Answer: (C) Lactose and Raffinose

Solution:

Step 1: Recall the units

Sucrose = glucose + fructose. Maltose = glucose + glucose. Lactose = glucose + galactose. Raffinose = galactose + glucose + fructose (a trisaccharide).

Step 2: Check (A) and (B)

Sucrose and maltose contain no galactose. In (B), maltose has none, even though lactose has.

Step 3: Check (D)

Sucrose has no galactose, even though lactose has.

Step 4: Check (C)

Lactose has galactose and raffinose has galactose, so both contain it.

Final Answer:

Lactose and raffinose both have galactose. This is option (C).

(C) Lactose and Raffinose

Quick Tip: Lactose is glucose plus galactose, and raffinose has galactose, glucose and fructose.



97. Which from following amino acids has a unique structure so that the side chain connects to the back bone of amino acid at two points ?

- (A) Histidine
- (B) Proline
- (C) Tryptophan
- (D) Tyrosine

Correct Answer: (B) Proline

Solution:

Step 1: Understand the concept

In most amino acids the side chain is attached to the alpha carbon only. Proline is different because its side chain loops back to the nitrogen of the backbone.

Step 2: Structure of proline

Proline has a five membered pyrrolidine ring. The side chain is a three-carbon chain joined to the alpha carbon at one end and to the

nitrogen at the other end, so it connects at two points.

Step 3: Check the others

Histidine has an imidazole side chain, tryptophan has an indole side chain and tyrosine has a phenol side chain. Each ring is attached only at one point, through a CH_2 to the alpha carbon.

Step 4: Conclusion

Only proline fits.

Final Answer:

Proline has a cyclic structure with the side chain closing onto the nitrogen. This is option (B).

(B) Proline

Quick Tip: Proline is a cyclic amino acid with a pyrrolidine ring.

98. Which from following contains mainly saturated fats ?

- (A) Margarine
- (B) Olive oil
- (C) Safflower oil
- (D) Fish oil

Correct Answer: (A) Margarine

Solution:

Step 1: Understand the concept

Saturated fats have no $C=C$ bonds in their fatty acid chains and are solid at room temperature. Unsaturated fats have $C=C$ bonds and are usually liquid oils.

Step 2: Check margarine

Margarine is produced by hydrogenating vegetable oils, which removes most double bonds. It therefore holds mainly saturated fats and is semi-solid.

Step 3: Check the oils

Olive oil is rich in oleic acid, which is monounsaturated. Safflower oil is rich in linoleic acid, which is polyunsaturated. Fish oil is rich in omega-3 polyunsaturated fatty acids.

Step 4: Conclusion

Margarine is the one with mainly saturated fats.

Final Answer:

Margarine is hydrogenated, so it is rich in saturated fat. This is option (A).

(A) Margarine

Quick Tip: Margarine is made by hydrogenating vegetable oils, so it is rich in saturated fat.

99. Which from following polymers is classified as branched chain polymer?

- (A) High density polythene
- (B) Low density polythene
- (C) PVC
- (D) Melamine

Correct Answer: (B) Low density polythene

Solution:

Step 1: Understand the concept

Polymers can be linear, branched or cross-linked in structure, depending on how the chains are connected.

Step 2: Branched example

Low density polythene (LDPE) is prepared at high pressure and high temperature by free radical polymerisation. Chain transfer creates side branches, so the chains pack loosely and the density is low.

Step 3: Check the others

High density polythene is made with Ziegler-Natta catalyst and has linear chains. PVC has linear chains. Melamine polymer has a cross-linked three dimensional network.

Step 4: Conclusion

LDPE is classified as a branched chain polymer.

Final Answer:

Low density polythene has branched chains. This is option (B).

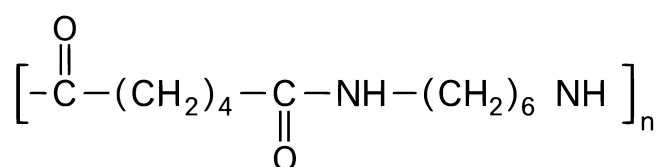
(B) Low density polythene

Quick Tip: LDPE is made by high pressure free radical polymerisation, which gives branched chains.

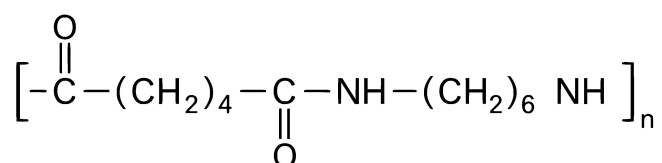


100. Which from following polymers is obtained by condensation polymerisation method ?

- (A) $[\text{CF}_2 - \text{CF}_2]_n$
- (B) $[\text{CH}_2 - \text{CH}_2]_n$
- (C) $[\text{NH} - (\text{CH}_2)_5 - \text{CO}]_n$
- (D)



Correct Answer: (D)



Solution:

Step 1: Understand the concept

Condensation polymerisation joins bifunctional monomers with the loss of a small molecule such as water or HCl. Addition polymerisation joins unsaturated monomers without any loss.

Step 2: Check (A) and (B)

$[\text{CF}_2 - \text{CF}_2]_n$ (Teflon) comes from tetrafluoroethene, and $[\text{CH}_2 - \text{CH}_2]_n$ (polythene) comes from ethene. Both are made by addition of a double bond monomer.

Step 3: Check (D)

The structure in (D) is $[\text{CO} - (\text{CH}_2)_4 - \text{CO} - \text{NH} - (\text{CH}_2)_6 - \text{NH}]_n$, which is nylon-6,6. It is made from adipic acid and hexamethylenediamine with loss of water, a typical condensation polymerisation.

Step 4: Option (C)

Option (C), nylon-6, is made from caprolactam by ring opening and is the other polyamide in the list. Nylon-6,6 in option (D) is the standard example of condensation polymerisation, so (D) is the answer.

Final Answer:

Nylon-6,6 is the textbook condensation polymer. This is option (D).

(D) Nylon-6,6

Quick Tip: Condensation polymerisation joins two different monomers with loss of small molecules like water.

Mathematics

Questions 101 to 150 | 2 marks each | 100 marks

101. If $z = \sum_{n=0}^{2026} i^n$, where $i = \sqrt{-1}$, then one of the values of $\sqrt{z}$ is...

- (A) $e^{i\frac{\pi}{4}}$
- (B) $\frac{1}{\sqrt{2}}e^{i\frac{\pi}{4}}$
- (C) $e^{i\frac{\pi}{2}}$
- (D) $\frac{1}{\sqrt{2}}e^{i\frac{\pi}{2}}$

Correct Answer: (A) $e^{i\frac{\pi}{4}}$

Solution:

Step 1: Understand the concept

The powers of i cycle with period 4: $i^0 = 1, i^1 = i, i^2 = -1, i^3 = -i$. Any four consecutive powers add up to $1 + i - 1 - i = 0$.

Step 2: Count the terms

The sum runs from $n = 0$ to $n = 2026$, which is 2027 terms. Since $2027 = 4 \times 506 + 3$, there are 506 full blocks, each summing to zero, and 3 leftover terms.

Step 3: Sum the leftover terms

The leftover terms are $i^{2024}, i^{2025}, i^{2026}$. Because 2024 is a multiple of 4, these equal 1, i , -1 . So

$$z = 1 + i - 1 = i$$

Step 4: Take the square root

Write $z = i = e^{i\pi/2}$. Then $\sqrt{z} = e^{i\pi/4}$ (the other root is $e^{i5\pi/4}$). This is option (A). The options with factor $\frac{1}{\sqrt{2}}$ have modulus less than 1, but $|z| = 1$ gives $|\sqrt{z}| = 1$. Option (C) $e^{i\pi/2}$ squares to -1 , not i .

Final Answer:

z equals i , so one square root is $e^{i\pi/4}$. This is option (A).

(A) $e^{i\pi/4}$

Quick Tip: Powers of i repeat every four terms and each block of four sums to zero.

102. Point A(5, 12) rotated about the origin O in the XY-plane through an angle of 30° in the anticlockwise direction to a new position B. The ordinate of point B is...

- (A) $6\sqrt{3} + \frac{5}{2}$
- (B) $\frac{5\sqrt{3}}{2} - 6$
- (C) $\frac{5\sqrt{3}}{2} + 6$
- (D) $6\sqrt{3} - \frac{5}{2}$

Correct Answer: (A) $6\sqrt{3} + \frac{5}{2}$

Solution:**Step 1: Understand the concept**

When a point (x, y) is rotated about the origin anticlockwise by an angle θ , its new coordinates are $(x \cos \theta - y \sin \theta, x \sin \theta + y \cos \theta)$.

Step 2: Substitute the data

Here $x = 5$, $y = 12$ and $\theta = 30^\circ$, with $\sin 30^\circ = \frac{1}{2}$ and $\cos 30^\circ = \frac{\sqrt{3}}{2}$.

Step 3: Compute the ordinate

$$y_B = 5 \sin 30^\circ + 12 \cos 30^\circ = \frac{5}{2} + 6\sqrt{3}$$

Step 4: Check the other options

The abscissa is $5 \cos 30^\circ - 12 \sin 30^\circ = \frac{5\sqrt{3}}{2} - 6$, which is option (B), so that option gives the abscissa and not the ordinate. Options (C) and (D) mix the signs. The ordinate is $6\sqrt{3} + \frac{5}{2}$.

Final Answer:

The ordinate of B is $6\sqrt{3} + \frac{5}{2}$. This is option (A).

$(A) \ 6\sqrt{3} + \frac{5}{2}$

Quick Tip: Rotating (x, y) by an angle t gives y-coordinate $x \sin t + y \cos t$.

103. If ${}^nC_4$, ${}^nC_5$ and ${}^nC_6$ are in arithmetic progression (A.P.), then the value of n is...

- (A) 5 or 11
- (B) 7 or 14
- (C) 8 or 15
- (D) 6 or 13

Correct Answer: (B) 7 or 14

Solution:

Step 1: Write the AP condition

Three terms are in A.P. when the middle term is the mean: $2 {}^nC_5 = {}^nC_4 + {}^nC_6$.

Step 2: Divide by ${}^nC_5$

Use ${}^nC_4/{}^nC_5 = \frac{5}{n-4}$ and ${}^nC_6/{}^nC_5 = \frac{n-5}{6}$. Then

$$2 = \frac{5}{n-4} + \frac{n-5}{6}$$

Step 3: Clear the fractions

Multiply by $6(n-4)$: $12(n-4) = 30 + (n-5)(n-4)$. So $12n - 48 = n^2 - 9n + 50$, which gives

$$n^2 - 21n + 98 = 0$$

Step 4: Solve

$(n-7)(n-14) = 0$, so $n = 7$ or $n = 14$. Check $n = 7$: ${}^7C_4 = 35$, ${}^7C_5 = 21$, ${}^7C_6 = 7$, and $35 + 7 = 42 = 2 \times 21$. Option (B) is correct.

Final Answer:

The values of n are 7 and 14. This is option (B).

(B) 7 or 14

Quick Tip: Use the AP condition $2 {}^nC_5 = {}^nC_4 + {}^nC_6$ and divide by ${}^nC_5$.

104. The family of straight lines $4ax + 3by + c = 0$ such that $a + b + c = 0$ (where a, b, c are real constants) are concurrent at the point...

- (A) $(4, 3)$
- (B) $(\frac{1}{2}, \frac{1}{3})$
- (C) $(\frac{1}{4}, \frac{1}{3})$
- (D) $(\frac{1}{3}, \frac{1}{2})$

Correct Answer: (C) $(\frac{1}{4}, \frac{1}{3})$

Solution:

Step 1: Understand the concept

A family of lines that all pass through one fixed point has an equation of the form $(\text{line 1}) + \lambda (\text{line 2}) = 0$. The fixed point is where both lines vanish.

Step 2: Remove c

Since $a + b + c = 0$, we have $c = -a - b$. Substitute into $4ax + 3by + c = 0$:

$$4ax + 3by - a - b = 0$$

Step 3: Group the parameters

$$a(4x - 1) + b(3y - 1) = 0$$

This holds for every a and b only when $4x - 1 = 0$ and $3y - 1 = 0$.

Step 4: Read the point

$x = \frac{1}{4}$ and $y = \frac{1}{3}$, so the lines are concurrent at $(\frac{1}{4}, \frac{1}{3})$. Check: $4a \cdot \frac{1}{4} + 3b \cdot \frac{1}{3} + c = a + b + c = 0$. The other points do not satisfy this check.

Final Answer:

All lines pass through $(1/4, 1/3)$. This is option (C).

$$(C) \left(\frac{1}{4}, \frac{1}{3} \right)$$

Quick Tip: Rewrite the family as $a(4x - 1) + b(3y - 1) = 0$ using $c = -a - b$.

105. If two lines represented by $x^2 - (1 + \sqrt{3})xy + \sqrt{3}y^2 = 0$ make angles α and β with the X-axis, then $\tan(\alpha + \beta)$ is...

- (A) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$
- (B) $\frac{1+\sqrt{3}}{1-\sqrt{3}}$
- (C) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$
- (D) $\frac{\sqrt{3}+1}{2}$

Correct Answer: (C) $\frac{\sqrt{3}+1}{\sqrt{3}-1}$

Solution:

Step 1: Understand the concept

A homogeneous equation in x and y represents two lines through the origin. If a line makes angle θ with the X-axis, then $y = mx$ with m

$= \tan \theta$.

Step 2: Form the quadratic in m

Put $y = mx$ in $x^2 - (1 + \sqrt{3})xy + \sqrt{3}y^2 = 0$ and divide by x^2 :

$$\sqrt{3}m^2 - (1 + \sqrt{3})m + 1 = 0$$

Its roots are $\tan \alpha$ and $\tan \beta$.

Step 3: Sum and product

$$\tan \alpha + \tan \beta = \frac{1 + \sqrt{3}}{\sqrt{3}}, \quad \tan \alpha \tan \beta = \frac{1}{\sqrt{3}}$$

Step 4: Apply the formula

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta} = \frac{(1 + \sqrt{3})/\sqrt{3}}{(\sqrt{3} - 1)/\sqrt{3}} = \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$

This is option (C). Option (B) has the denominator $1 - \sqrt{3}$, which has the wrong sign.

Final Answer:

$\tan(\alpha + \beta)$ is $(\sqrt{3} + 1)/(\sqrt{3} - 1)$. This is option (C).

$$\boxed{(C) \frac{\sqrt{3} + 1}{\sqrt{3} - 1}}$$

Quick Tip: Treat the pair as a quadratic in the slope $m = y/x$ and use sum and product of roots.



106. Let PA and PB be the tangent segments drawn from point $P(6, 8)$ to the circle with the centre at origin O . The radius of circle for which the area of quadrilateral $PAOB$ is maximum, is...

- (A) 5
- (B) $5\sqrt{2}$
- (C) $\frac{5}{\sqrt{2}}$
- (D) $\frac{5}{2}$

Correct Answer: (B) $5\sqrt{2}$

Solution:

Step 1: Understand the figure

$P = (6, 8)$ lies outside the circle with centre O at the origin. The tangents PA and PB are equal and each is perpendicular to the radius at its point of contact.

Step 2: Express the area

$OP = \sqrt{36 + 64} = 10$. The tangent length is $PA = \sqrt{OP^2 - r^2} = \sqrt{100 - r^2}$. The quadrilateral is two right triangles OAP and OBP , so

$$\text{Area} = 2 \times \frac{1}{2} r \sqrt{100 - r^2} = r \sqrt{100 - r^2}$$

Step 3: Maximise

The area is largest when its square $r^2(100 - r^2)$ is largest. Let $u = r^2$.

Then $u(100 - u)$ is maximum at $u = 50$.

Step 4: Result

$r^2 = 50$, so $r = 5\sqrt{2}$. Then the area is $5\sqrt{2} \cdot \sqrt{50} = 50$. Option (A) $r = 5$ gives only $5\sqrt{75} = 43.3$, and $r = \frac{5}{\sqrt{2}}$ gives $\frac{5}{\sqrt{2}}\sqrt{87.5} = 33.1$. So $5\sqrt{2}$ is the maximum.

Final Answer:

The area is greatest for radius $5\sqrt{2}$. This is option (B).

(B) $5\sqrt{2}$

Quick Tip: Area of PAOB equals r times the tangent length, so maximise r times $\sqrt{100 - r^2}$.

107. A circle passes through the point $(0, 1)$ and touches the parabola $y = x^2$ at the point $(1, 1)$. The centre of the circle is...

- (A) $(-\frac{1}{2}, -\frac{5}{2})$
- (B) $(\frac{1}{2}, -\frac{5}{2})$
- (C) $(\frac{1}{2}, \frac{5}{4})$
- (D) $(-\frac{1}{2}, \frac{5}{4})$

Correct Answer: (C) $(\frac{1}{2}, \frac{5}{4})$

Solution:

Step 1: Understand the setup

The circle passes through $(0, 1)$ and $(1, 1)$, and it touches $y = x^2$ at $(1, 1)$. So the radius to $(1, 1)$ is perpendicular to the tangent of the

parabola there.

Step 2: Normal to the parabola

$\frac{dy}{dx} = 2x = 2$ at $x = 1$, so the tangent slope is 2 and the normal slope is $-\frac{1}{2}$. The centre (h, k) lies on $y - 1 = -\frac{1}{2}(x - 1)$, so $k - 1 = -\frac{1}{2}(h - 1)$.

Step 3: Use the two points

The centre is equidistant from $(0, 1)$ and $(1, 1)$: $h^2 + (k - 1)^2 = (h - 1)^2 + (k - 1)^2$. So $h^2 = h^2 - 2h + 1$, giving $h = \frac{1}{2}$.

Step 4: Find k

$k - 1 = -\frac{1}{2}(\frac{1}{2} - 1) = \frac{1}{4}$, so $k = \frac{5}{4}$. The centre is $(\frac{1}{2}, \frac{5}{4})$, option (C).

The options with $-\frac{5}{2}$ place the centre far below both points and would not give equal distances to both.

Final Answer:

The centre is $(1/2, 5/4)$. This is option (C).

$$\boxed{(C) \left(\frac{1}{2}, \frac{5}{4}\right)}$$

Quick Tip: The centre lies on the normal to the parabola at $(1,1)$ and on the perpendicular bisector of the chord to $(0,1)$.

108. The eccentricity of the ellipse represented by the equation $7x^2 + 16y^2 - 14x + 64y - 377 = 0$ is...

- (A) $\frac{3}{4}$
(B) $\frac{\sqrt{7}}{4}$
(C) $\frac{1}{2}$
(D) $\frac{3}{8}$

Correct Answer: (A) $\frac{3}{4}$

Solution:

Step 1: Complete the square

Group the terms: $7(x^2 - 2x) + 16(y^2 + 4y) = 377$. Add the squares:
 $7(x - 1)^2 - 7 + 16(y + 2)^2 - 64 = 377$, so

$$7(x - 1)^2 + 16(y + 2)^2 = 448$$

Step 2: Standard form

Divide by 448:

$$\frac{(x - 1)^2}{64} + \frac{(y + 2)^2}{28} = 1$$

Here $a^2 = 64$ and $b^2 = 28$, with the major axis along the x-direction because 64 is bigger.

Step 3: Find e

$$e^2 = 1 - \frac{b^2}{a^2} = 1 - \frac{28}{64} = \frac{36}{64} \Rightarrow e = \frac{6}{8} = \frac{3}{4}$$

Step 4: Check the options

$\frac{\sqrt{7}}{4}$ equals $\sqrt{1 - \frac{9}{16}}$, which is the value of b/a for this ellipse, a common mix-up. Options (C) and (D) do not come from the numbers 64 and 28.

Final Answer:

The eccentricity is $3/4$. This is option (A).

$$(A) \frac{3}{4}$$

Quick Tip: Complete the squares to reach standard form, then use $e^2 = 1 - b^2/a^2$.

109. If $\lim_{x \rightarrow 1} \frac{x^3 + ax^2 + bx + c}{x^2 - 2x + 1} = 2026$ then the value of $a - c$ is...

- (A) 2
- (B) 1
- (C) -1
- (D) -2

Correct Answer: (D) -2

Solution:

Step 1: Understand the concept

The denominator $x^2 - 2x + 1 = (x - 1)^2$ is zero at $x = 1$. For the limit to be finite, the numerator must also vanish at $x = 1$ to the second order, so $(x - 1)^2$ is a factor of it.

Step 2: Factorise the numerator

The cubic is monic, so $x^3 + ax^2 + bx + c = (x - 1)^2(x + k) = x^3 + (k - 2)x^2 + (1 - 2k)x + k$. Therefore $a = k - 2$, $b = 1 - 2k$ and $c = k$.

Step 3: Use the limit value

Cancelling $(x - 1)^2$ gives

$$\lim_{x \rightarrow 1} (x + k) = 1 + k = 2026 \Rightarrow k = 2025$$

Step 4: Find a - c

$a = 2023$ and $c = 2025$, so $a - c = (k - 2) - k = -2$. Option (D). This does not depend on the value 2026.

Final Answer:

$a - c$ equals -2. This is option (D).

(D) - 2

Quick Tip: For a finite limit the numerator needs a double root at $x = 1$, so write it as $(x-1)^2 (x + k)$.

110. If the truth value of $[(p \vee q) \wedge (q \rightarrow r) \wedge (\sim r)] \rightarrow (p \wedge q)$ is false, then which of the following is **NOT** true?

- (A) Truth value of $p \rightarrow q$ is False
- (B) Truth value of $p \rightarrow r$ is False.
- (C) Truth value of $(\sim q) \rightarrow p$ is False.
- (D) The truth value of $(\sim p) \wedge r$ is False.

Correct Answer: (C) Truth value of $(\sim q) \rightarrow p$ is False.

Solution:

Step 1: Understand the concept

$A \rightarrow B$ is false only when A is true and B is false. The given statement is false, so $(p \vee q) \wedge (q \rightarrow r) \wedge (\sim r)$ is true while $p \wedge q$ is false.

Step 2: Find the truth values

$\sim r$ true gives $r = F$. Then $q \rightarrow r$ true needs $q = F$. Then $p \vee q$ true needs $p = T$. Check: $p \wedge q = F$, as needed. So $(p, q, r) = (T, F, F)$.

Step 3: Evaluate each option

(A) $p \rightarrow q = T \rightarrow F = \text{False}$, so "is False" is true. (B) $p \rightarrow r = T \rightarrow F = \text{False}$, so "is False" is true. (D) $(\sim p) \wedge r = F \wedge F = \text{False}$, so "is False" is true.

Step 4: Option (C)

$(\sim q) \rightarrow p = T \rightarrow T = \text{True}$. So the claim that its truth value is False is not true. Option (C) is the answer.

Final Answer:

Only option (C) makes a wrong claim. This is option (C).

(C) $(\sim q) \rightarrow p$ is not False

Quick Tip: A conditional is false only when the antecedent is true and the consequent is false.

111. If p, q, r are simple propositions with truth values T, F, T respectively, then which of the following is not a true statement?

- (A) $[q \wedge (p \rightarrow q)] \rightarrow p$
- (B) $(p \wedge q) \rightarrow (q \vee \sim p)$
- (C) $[(\sim p \vee q) \wedge \sim r] \leftrightarrow p$
- (D) $(p \wedge q) \vee (\sim q \vee r)$

Correct Answer: (C) $[(\sim p \vee q) \wedge \sim r] \leftrightarrow p$

Solution:

Step 1: Understand the task

We have $p = T, q = F, r = T$. A statement is "not true" when its truth value is False.

Step 2: Option (A)

$q \wedge (p \rightarrow q) = F \wedge (T \rightarrow F) = F \wedge F = F$. Then $F \rightarrow p$ is True. So (A) is true.

Step 3: Options (B) and (D)

(B): $p \wedge q = F$, so $F \rightarrow (\dots)$ is True. (D): $(p \wedge q) \vee (\sim q \vee r) = F \vee (T \vee T) = T$. Both are true.

Step 4: Option (C)

$\sim p \vee q = F \vee F = F$. Then $F \wedge \sim r = F \wedge F = F$. The biconditional $F \leftrightarrow p$ is $F \leftrightarrow T = F$. So (C) is false, which means it is not a true statement.

Final Answer:

Option (C) evaluates to False. This is option (C).

$$(C) [(\sim p \vee q) \wedge \sim r] \leftrightarrow p$$

Quick Tip: Substitute $p = T$, $q = F$, $r = T$ into each compound statement.

112. If $\sim p \rightarrow q$ is false and $q \leftrightarrow r$ is false, then the truth value of (p, q, r) is...

- (A) (T, F, T)
- (B) (F, T, F)
- (C) (F, F, T)
- (D) (F, T, T)

Correct Answer: (C) (F, F, T)

Solution:

Step 1: Use the first statement

$\sim p \rightarrow q$ is false only if $\sim p$ is true and q is false. So $p = F$ and $q = F$.

Step 2: Use the second statement

$q \leftrightarrow r$ is false when q and r have different truth values. Since $q = F$, we need $r = T$.

Step 3: Combine

$(p, q, r) = (F, F, T)$.

Step 4: Check the options

(A) (T, F, T) has $p = T$, so $\sim p \rightarrow q$ would be true. (B) (F, T, F) has $q = T$, making $\sim p \rightarrow q$ true. (D) (F, T, T) also has $q = T$. Only (C) satisfies both conditions.

Final Answer:

The values are (F, F, T). This is option (C).

(C) (F, F, T)

Quick Tip: A false conditional means the antecedent is true and the consequent is false.

113. In $\triangle ABC$, with usual notations, $(a + b + c)(b + c - a)(c + a - b)(a + b - c) = 3b^2c^2$, then $\angle A =$

- (A) 60° or 120°
- (B) 30° or 150°
- (C) 45° or 135°
- (D) 30° or 90°

Correct Answer: (A) 60° or 120°

Solution:**Step 1: Understand the concept**

Rewrite two pairs using $(b + c)^2 - a^2$ and $a^2 - (b - c)^2$: $(a + b + c)(b + c - a) = (b + c)^2 - a^2$ and $(c + a - b)(a + b - c) = a^2 - (b - c)^2$.

Step 2: Use the cosine rule

$(b + c)^2 - a^2 = 2bc(1 + \cos A)$ and $a^2 - (b - c)^2 = 2bc(1 - \cos A)$, using $a^2 = b^2 + c^2 - 2bc \cos A$.

Step 3: Multiply

The product is $4b^2c^2(1 - \cos^2 A) = 4b^2c^2 \sin^2 A$. Setting this equal to $3b^2c^2$:

$$\sin^2 A = \frac{3}{4} \Rightarrow \sin A = \frac{\sqrt{3}}{2}$$

Step 4: Find A

Since A is an angle of a triangle, $A = 60^\circ$ or 120° . Both are possible, so option (A). The other options have $\sin A = \frac{1}{2}$, $\frac{1}{\sqrt{2}}$ or $\frac{1}{2}$ and 1, which do not satisfy $\sin A = \frac{\sqrt{3}}{2}$.

Final Answer:

Angle A is 60 degrees or 120 degrees. This is option (A).

(A) 60° or 120°

Quick Tip: The product of the four factors is 16 times area squared, which equals 4 b squared c squared sin squared A.

114. If ABC is a triangle of area Δ with $a = 2, b = \frac{7}{2}, c = \frac{5}{2}$, where a, b, c are the lengths of the sides of the triangle opposite to angles A, B and C respectively, then $\frac{2\sin A - \sin 2A}{2\sin A + \sin 2A}$ is equal to...

- (A) $\frac{3}{4\Delta}$
- (B) $(\frac{3}{4\Delta})^2$
- (C) $\frac{45}{4\Delta}$
- (D) $(\frac{45}{4\Delta})^2$

Correct Answer: (B) $(\frac{3}{4\Delta})^2$

Solution:

Step 1: Simplify the expression

$$\frac{2 \sin A - \sin 2A}{2 \sin A + \sin 2A} = \frac{2 \sin A(1 - \cos A)}{2 \sin A(1 + \cos A)} = \frac{1 - \cos A}{1 + \cos A} = \tan^2 \frac{A}{2}$$

Step 2: Half angle formula

In a triangle, $\tan \frac{A}{2} = \frac{(s-b)(s-c)}{\Delta}$, where $s = \frac{a+b+c}{2}$.

Step 3: Compute s

$a = 2, b = \frac{7}{2}, c = \frac{5}{2}$, so $s = \frac{8}{2} = 4$. Then $s - b = \frac{1}{2}$ and $s - c = \frac{3}{2}$, so $(s - b)(s - c) = \frac{3}{4}$.

Step 4: Result

$\tan \frac{A}{2} = \frac{3}{4\Delta}$, so the expression equals $\left(\frac{3}{4\Delta}\right)^2$. Option (B). Option (A) is $\tan \frac{A}{2}$ without the square, and the options with 45 come from using $s(s - a)$ wrongly.

Final Answer:

The expression is tan squared of A/2, equal to $(3/(4 \Delta))^2$ squared. This is option (B).

$$\boxed{(B) \left(\frac{3}{4\Delta}\right)^2}$$

Quick Tip: The expression equals tan squared of A/2, and $\tan(A/2) = (s-b)(s-c)/\text{area}$.

115. If matrix $A = \begin{bmatrix} -1 & 2025 & 2026 \\ 0 & 2 & 2027 \\ 0 & 0 & -1 \end{bmatrix}$, then the sum of all elements in $\text{adj}(A^{-1})$ is equal to...

- (A) 1013
- (B) 2026
- (C) 3039
- (D) 6078

Correct Answer: (C) 3039

Solution:

Step 1: Understand the concept

For an invertible matrix M we have $\text{adj}(M) = |M| M^{-1}$. Taking $M = A^{-1}$, we get $\text{adj}(A^{-1}) = |A^{-1}| (A^{-1})^{-1} = \frac{A}{|A|}$.

Step 2: Find the determinant

A is upper triangular, so $|A|$ is the product of the diagonal: $(-1)(2)(-1) = 2$.

Step 3: Sum the elements of A

$$-1 + 2025 + 2026 + 0 + 2 + 2027 + 0 + 0 + (-1) = 6078.$$

Step 4: Result

The sum of all elements of $\text{adj}(A^{-1}) = \frac{6078}{2} = 3039$. Option (C). The value 6078 is the sum of A itself, without dividing by the determinant.

Final Answer:

The sum is 3039. This is option (C).

Quick Tip: $\text{adj}(A \text{ inverse}) = \det(A \text{ inverse})$ times A , which equals A divided by $\det A$.

116. The inverse of the matrix $A = \begin{bmatrix} 2 & -1 & 4 \\ 4 & -3 & 1 \\ 1 & 2 & 1 \end{bmatrix}$ is B

$$= \frac{1}{37} \begin{bmatrix} -5 & 9 & 11 \\ -3 & -2 & 14 \\ 11 & -5 & k \end{bmatrix}, \text{ then the value of } k \text{ is...}$$

- (A) 1
- (B) -1
- (C) 2
- (D) -2

Correct Answer: (D) -2

Solution:

Step 1: Understand the concept

The inverse is $A^{-1} = \frac{1}{|A|} \text{adj}(A)$, and the adjoint is the transpose of the cofactor matrix. So the entry in position (3,3) is $C_{33}/|A|$.

Step 2: Find the determinant

$$|A| = 2(-3 - 2) - (-1)(4 - 1) + 4(8 + 3) = -10 + 3 + 44 = 37$$

This matches the factor $\frac{1}{37}$ in B .

Step 3: Find the cofactor C_{33}

$$C_{33} = (+1) \begin{vmatrix} 2 & -1 \\ 4 & -3 \end{vmatrix} = 2(-3) - (-1)(4) = -6 + 4 = -2.$$

Step 4: Result

$k = -2$, option (D). Check using the first row of A and the third column of B : $2 \cdot 11 + (-1) \cdot 14 + 4k = 0$, so $22 - 14 + 4k = 0$ and $k = -2$.

Final Answer:

The value of k is -2 . This is option (D).

(D) -2

Quick Tip: The (3,3) entry of the inverse is the cofactor C_{33} divided by the determinant.

117. The value of $\sin^{-1}(\sin \frac{5\pi}{6}) + \cos^{-1}(\cos \frac{7\pi}{6}) + \tan^{-1}(\tan \frac{2\pi}{3})$ is equal to...

- (A) $\frac{2\pi}{3}$
- (B) $\frac{4\pi}{3}$
- (C) $\frac{5\pi}{3}$
- (D) π

Correct Answer: (A) $\frac{2\pi}{3}$

Solution:

Step 1: Understand the concept

The principal ranges are $\sin^{-1}$: $[-\frac{\pi}{2}, \frac{\pi}{2}]$, $\cos^{-1}$: $[0, \pi]$ and $\tan^{-1}$:

$(-\frac{\pi}{2}, \frac{\pi}{2})$. We must shift each angle into the right range.

Step 2: First term

$$\sin \frac{5\pi}{6} = \sin \left(\pi - \frac{\pi}{6} \right) = \sin \frac{\pi}{6}, \text{ so } \sin^{-1}(\dots) = \frac{\pi}{6}.$$

Step 3: Second and third terms

$$\cos \frac{7\pi}{6} = \cos \left(2\pi - \frac{5\pi}{6} \right) = \cos \frac{5\pi}{6}, \text{ and } \frac{5\pi}{6} \text{ is in } [0, \pi], \text{ so it gives } \frac{5\pi}{6}.$$

$$\tan \frac{2\pi}{3} = \tan \left(\frac{2\pi}{3} - \pi \right) = \tan \left(-\frac{\pi}{3} \right), \text{ giving } -\frac{\pi}{3}.$$

Step 4: Add

$$\frac{\pi}{6} + \frac{5\pi}{6} - \frac{\pi}{3} = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$$

Naively cancelling the inverse and the function would give $\frac{5\pi}{6} + \frac{7\pi}{6} + \frac{2\pi}{3} = \frac{8\pi}{3}$, which is wrong because the angles are outside the principal ranges.

Final Answer:

The sum is $2\pi / 3$. This is option (A).

$(A) \frac{2\pi}{3}$

Quick Tip: Bring each angle into the principal range before applying the inverse function.

118. Let $f(x) = 1 - \frac{1}{x}$, $g_2(x) = f(f(x))$, $g_3(x) = f(f(f(x)))$ and so on. If $\int x \cdot g_{2026}(x) dx = \int g_{2025}(x) dx + h(x) + c$, then $h(x) = \dots$

- (A) x
 (B) $-x$
 (C) $\log x$
 (D) $-\log x$

Correct Answer: (B) $-x$

Solution:

Step 1: Compute the first few functions

$$g_1(x) = f(x) = 1 - \frac{1}{x}. \text{ Then } g_2(x) = f(f(x)) = 1 - \frac{1}{1 - 1/x} = 1 - \frac{x}{x-1} = \frac{-1}{x-1} = \frac{1}{1-x}. \text{ And } g_3(x) = f\left(\frac{1}{1-x}\right) = 1 - (1-x) = x.$$

Step 2: Use the period

Because $g_3(x) = x$, the functions repeat with period 3. Now $2026 = 3 \times 675 + 1$, so $g_{2026} = g_1 = 1 - \frac{1}{x}$. And $2025 = 3 \times 675$, so $g_{2025} = g_3 = x$.

Step 3: Integrate

$$\int x \left(1 - \frac{1}{x}\right) dx = \int (x - 1) dx = \frac{x^2}{2} - x + C_1$$

$$\int g_{2025} dx = \int x dx = \frac{x^2}{2}$$

Step 4: Compare

$\frac{x^2}{2} - x = \frac{x^2}{2} + h(x) + c$, so $h(x) = -x$ (the constants merge into c).

Option (B). The options with $\log x$ would come from wrongly integrating $\frac{1}{x}$, which does not appear after multiplying by x .

Final Answer:

$h(x)$ equals $-x$. This is option (B).

(B) $-x$

Quick Tip: Compute g_1 , g_2 , g_3 and notice that $g_3(x) = x$, so the sequence repeats every 3 steps.

119. If $g(x) = 1 - \sqrt{x}$ and $f(g(x)) = 5 + 4\sqrt{x} + x$, then the value of $f(6)$ is...

- (A) 5
- (B) 10
- (C) 15
- (D) 20

Correct Answer: (B) 10

Solution:**Step 1: Understand the concept**

We know $f(g(x))$ but want f itself. Let $t = g(x) = 1 - \sqrt{x}$, so that $\sqrt{x} = 1 - t$ and $x = (1 - t)^2$.

Step 2: Express f in terms of t

$$f(t) = 5 + 4(1 - t) + (1 - t)^2$$

Expand: $5 + 4 - 4t + 1 - 2t + t^2 = t^2 - 6t + 10$.

Step 3: Evaluate at 6

$$f(6) = 36 - 36 + 10 = 10$$

Step 4: Check

The function $f(t) = t^2 - 6t + 10$ is fixed for every value t that g can take, and we use the same polynomial at $t = 6$. Note that $f(0) = 10$ as well, since $t = 0$ corresponds to $x = 1$, giving $5 + 4 + 1 = 10$. This matches option (B).

Final Answer:

$f(6)$ equals 10. This is option (B).

(B) 10

Quick Tip: Put $t = 1 - \sqrt{x}$, write $\sqrt{x} = 1 - t$, and express the right side in t .

120. If the function $f(x) = \left(\frac{5x-8}{8-3x}\right)^{\frac{3}{2x-4}}$, for $x \neq 2$ is continuous at $x = 2$, then the value of $f(2)$ is...

- (A) e^{12}
- (B) e^6
- (C) e^3
- (D) $e^{\frac{3}{2}}$

Correct Answer: (B) e^6

Solution:

Step 1: Understand the concept

For continuity at $x = 2$ we need $f(2) = \lim_{x \rightarrow 2} f(x)$. At $x = 2$ the base is $\frac{5 \cdot 2 - 8}{8 - 6} = \frac{2}{2} = 1$ and the exponent $\frac{3}{2x - 4}$ tends to infinity. This is the form 1^∞ .

Step 2: Use the standard result

For 1^∞ : $\lim [u(x)]^{v(x)} = e^{\lim v(x)(u(x)-1)}$.

Step 3: Compute the exponent limit

$$u - 1 = \frac{5x - 8 - 8 + 3x}{8 - 3x} = \frac{8(x - 2)}{8 - 3x}. \text{ Multiply by } \frac{3}{2(x - 2)}:$$

$$\frac{8(x - 2)}{8 - 3x} \cdot \frac{3}{2(x - 2)} = \frac{12}{8 - 3x} \rightarrow \frac{12}{2} = 6$$

Step 4: Result

$f(2) = e^6$, option (B). The value e^{12} would come from forgetting the factor $\frac{1}{2}$ in $2x - 4 = 2(x - 2)$.

Final Answer:

$f(2)$ equals e^6 . This is option (B).

(B) e^6

Quick Tip: This is a 1 to the power infinity form, so use e raised to the limit of (base - 1) times the exponent.

121. If $x^m + y^m = k$, ($m \neq 1$) and $y'' = \frac{ax^b}{y^c}$ such that $a + b + c = 0$, then the value of k is...

- (A) 1
- (B) 2
- (C) 3
- (D) m

Correct Answer: (C) 3

Solution:

Step 1: First derivative

From $x^m + y^m = k$: $mx^{m-1} + my^{m-1}y' = 0$, so $y' = -\left(\frac{x}{y}\right)^{m-1}$.

Step 2: Second derivative

Differentiating: $y'' = -(m-1)\left(\frac{x}{y}\right)^{m-2} \cdot \frac{y - xy'}{y^2}$. Now $y - xy' = y + \frac{x^m}{y^{m-1}} = \frac{y^m + x^m}{y^{m-1}} = \frac{k}{y^{m-1}}$.

Step 3: Simplify

$$y'' = -(m-1) \frac{x^{m-2}}{y^{m-2}} \cdot \frac{k}{y^{m-1} y^2} = \frac{-(m-1)k x^{m-2}}{y^{2m-1}}$$

Step 4: Compare and solve

So $a = -(m-1)k$, $b = m-2$, $c = 2m-1$. Then $a + b + c = -(m-1)k + 3m - 3 = (m-1)(3-k) = 0$. Since $m \neq 1$, $k = 3$. Option (C).

Final Answer:

k equals 3. This is option (C).

(C) 3

Quick Tip: Differentiate twice and compare with ax^b / y^c , then use $a + b + c = 0$.



122. If $y = \sin^{-1}\left(\frac{25-x^2}{25+x^2}\right)$, then **Missing open brace for superscript** is...

- (A) $-\frac{5}{13}$
- (B) $\frac{5}{13}$
- (C) $\frac{13}{5}$
- (D) $\frac{2}{7}$

Correct Answer: (A) $-\frac{5}{13}$

Solution:

Step 1: Understand the concept

The expression $\frac{25-x^2}{25+x^2}$ looks like $\frac{1-\tan^2 \theta}{1+\tan^2 \theta} = \cos 2\theta$ with $x = 5 \tan \theta$.

Step 2: Simplify y

Then $y = \sin^{-1}(\cos 2\theta) = \sin^{-1}\left[\sin\left(\frac{\pi}{2} - 2\theta\right)\right] = \frac{\pi}{2} - 2\theta$. This holds for $0 \leq \theta \leq \frac{\pi}{2}$, which includes $x = 1$. So

$$y = \frac{\pi}{2} - 2 \tan^{-1} \frac{x}{5}$$

Step 3: Differentiate

$$y' = -2 \cdot \frac{1}{1+x^2/25} \cdot \frac{1}{5} = -\frac{10}{25+x^2}$$

Step 4: Evaluate at $x = 1$

$y'(1) = -\frac{10}{26} = -\frac{5}{13}$, option (A). Option (B) has the right size but the wrong sign.

Final Answer:

y' at 1 is $-5/13$. This is option (A).

$$\boxed{(A) - \frac{5}{13}}$$

Quick Tip: Substitute $x = 5 \tan(\theta)$ to turn the expression into $\cos 2\theta$.



123. If $f(x)$ and $g(x)$ are inverse functions of each other and $f(x) = x + e^x$, then **Missing open brace for superscript...**

- (A) $\frac{1}{1+e^{g(x)}}$
- (B) $\frac{e^{g(x)}}{1+e^{g(x)}}$
- (C) $\frac{1}{1+g(x)}$
- (D) $e^{g(x)}$

Correct Answer: (A) $\frac{1}{1+e^{g(x)}}$

Solution:

Step 1: Understand the concept

If g is the inverse of f , then $f(g(x)) = x$. Differentiating gives $f'(g(x)) \cdot g'(x) = 1$, so $g'(x) = \frac{1}{f'(g(x))}$.

Step 2: Find f'

$f(x) = x + e^x$, so $f'(x) = 1 + e^x$.

Step 3: Substitute

Evaluate f' at $g(x)$: $f'(g(x)) = 1 + e^{g(x)}$.

Step 4: Result

$$g'(x) = \frac{1}{1 + e^{g(x)}}$$

Option (B) has an extra $e^{g(x)}$ in the numerator, option (C) uses $g(x)$ instead of $e^{g(x)}$, and option (D) drops the denominator. Only (A) matches.

Final Answer:

$g'(x)$ equals $1/(1 + e^{g(x)})$. This is option (A).

$(A) \frac{1}{1 + e^{g(x)}}$

Quick Tip: The derivative of an inverse function is the reciprocal of the derivative of the original at the matching point.

124. If $3y^2 - 2xy - x = 0$, then the value of $\frac{dy}{dx}$ at $y = 2$ is...

- (A) $\frac{5}{36}$
- (B) $\frac{35}{36}$
- (C) $\frac{25}{36}$
- (D) $\frac{36}{25}$

Correct Answer: (C) $\frac{25}{36}$

Solution:

Step 1: Express x in terms of y

From $3y^2 - 2xy - x = 0$ we get $x(2y + 1) = 3y^2$, so $x = \frac{3y^2}{2y + 1}$.

Step 2: Differentiate with respect to y

$$\frac{dx}{dy} = \frac{6y(2y + 1) - 3y^2 \cdot 2}{(2y + 1)^2} = \frac{6y^2 + 6y}{(2y + 1)^2}$$

Step 3: Evaluate at $y = 2$

$$\frac{dx}{dy} = \frac{24 + 12}{25} = \frac{36}{25}$$

Step 4: Invert

$\frac{dy}{dx} = \frac{25}{36}$, option (C). Option (D) is $\frac{dx}{dy}$ itself and was not inverted.

Final Answer:

dy/dx at $y = 2$ is $25/36$. This is option (C).

$(C) \frac{25}{36}$

Quick Tip: Solve for x as a function of y , differentiate, and invert the derivative.

125. Rolle's theorem holds for monic quadratic polynomial $f(x)$ on the interval $[\alpha, \alpha + 3]$ where $f(\alpha) = 0$. Similarly, $g(x) = f(x) + 2$ also follows Rolle's theorem in the interval $[\beta, 3]$ where $g(3) = 0$, such that the value of c is the same for both $f(x)$ and $g(x)$. Then the value of $(f \circ g)(\alpha)$ is...

- (A) -4
- (B) 4
- (C) -2
- (D) 2

Correct Answer: (C) -2

Solution:

Step 1: Rolle for f

f is monic quadratic with $f(\alpha) = 0$ and Rolle on $[\alpha, \alpha + 3]$ needs $f(\alpha + 3) = 0$. So $f(x) = (x - \alpha)(x - \alpha - 3)$ and $c = \alpha + \frac{3}{2}$ (the vertex).

Step 2: Rolle for g

$g(x) = f(x) + 2$ has the same vertex, so c is the same. Write $g(x) = (x - c)^2 - \frac{1}{4}$, since $f(x) = (x - c)^2 - \frac{9}{4}$.

Step 3: Use $g(3) = 0$

$(3 - c)^2 = \frac{1}{4}$ so $c = \frac{5}{2}$ or $c = \frac{7}{2}$. Also $g(\beta) = 0$ gives $\beta = 2c - 3$, and the interval $[\beta, 3]$ needs $\beta < 3$, so $c = \frac{5}{2}$, $\beta = 2$, and $\alpha = c - \frac{3}{2} = 1$.

Step 4: Compute

$g(\alpha) = g(1) = (1 - \frac{5}{2})^2 - \frac{1}{4} = \frac{9}{4} - \frac{1}{4} = 2$. Then $f(2) = (2 - 1)(2 - 4) = -2$. Option (C).

Final Answer:

$(f \circ g)(\alpha)$ equals -2 . This is option (C).

Quick Tip: Rolle needs equal end values, so a monic quadratic has its vertex at the point c .



126. If the rate of increase of surface area of a spherical balloon is $5 \text{ cm}^2/\text{sec}$ and rate of increase of volume of a spherical balloon is $10 \text{ cm}^3/\text{sec}$, then the radius of the balloon at that time is...

- (A) 3 cm
- (B) 5 cm
- (C) 6 cm
- (D) 4 cm

Correct Answer: (D) 4 cm

Solution:

Step 1: Understand the concept

For a sphere of radius r , the surface area is $S = 4\pi r^2$ and the volume is $V = \frac{4}{3}\pi r^3$. All quantities change with time, so we differentiate with respect to t .

Step 2: Differentiate

$$\frac{dS}{dt} = 8\pi r \frac{dr}{dt} = 5, \quad \frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt} = 10$$

Step 3: Divide the equations

Dividing the second by the first removes $\frac{dr}{dt}$ and the factor $4\pi r$:

$$\frac{4\pi r^2}{8\pi r} = \frac{10}{5} \Rightarrow \frac{r}{2} = 2$$

Step 4: Result

$r = 4$ cm, option (D). Check: $\frac{dr}{dt} = \frac{5}{8\pi \cdot 4} = \frac{5}{32\pi}$, and then $\frac{dV}{dt} = 4\pi \cdot 16 \cdot \frac{5}{32\pi} = 10$, which matches the given rate.

Final Answer:

The radius is 4 cm. This is option (D).

(D) 4 cm

Quick Tip: Divide dV/dt by dS/dt so that dr/dt cancels, leaving r .

127. The value of integral $\int x^3 \cos x \, dx$ is...

- (A) $x^3 \sin x + x^2 \cos x - 6x \sin x + 6 \cos x + c$
- (B) $x^3 \sin x + 3x^2 \sin x - 6x \sin x - 6 \cos x + c$
- (C) $x^3 \sin x + 3x^2 \cos x - 6x \sin x - 6 \cos x + c$
- (D) $x^3 \sin x + 3x^2 \cos x - 6x \sin x + 6 \cos x + c$

Correct Answer: (C) $x^3 \sin x + 3x^2 \cos x - 6x \sin x - 6 \cos x + c$

Solution:**Step 1: Understand the concept**

We use $\int u \, dv = uv - \int v \, du$ and take $u = x^n$ each time, so the power

of x falls by one at every step.

Step 2: First two steps

$$\int x^3 \cos x \, dx = x^3 \sin x - 3 \int x^2 \sin x \, dx$$

$$\int x^2 \sin x \, dx = -x^2 \cos x + 2 \int x \cos x \, dx$$

Step 3: Next step

$\int x \cos x \, dx = x \sin x + \cos x$. Substitute back: $\int x^2 \sin x \, dx = -x^2 \cos x + 2x \sin x + 2 \cos x$.

Step 4: Combine

$$x^3 \sin x - 3(-x^2 \cos x + 2x \sin x + 2 \cos x) = x^3 \sin x + 3x^2 \cos x - 6x \sin x - 6 \cos x + c$$

This is option (C). Differentiating it gives back $x^3 \cos x$, since the lower terms cancel in pairs.

Final Answer:

The integral is $x^3 \sin x + 3x^2 \cos x - 6x \sin x - 6 \cos x + c$. This is option (C).

$(C) \, x^3 \sin x + 3x^2 \cos x - 6x \sin x - 6 \cos x + c$

Quick Tip: Apply integration by parts repeatedly, lowering the power of x

each time.



128. If u and v are functions of x , then $\int \frac{1}{v^3} \left(uv \frac{du}{dx} - u^2 \frac{dv}{dx} \right) dx =$

(A) $\log uv + c$

(B) $\log \frac{u}{v} + c$

(C) $\frac{v^2}{2u^2} + c$

(D) $\frac{u^2}{2v^2} + c$

Correct Answer: (D) $\frac{u^2}{2v^2} + c$

Solution:

Step 1: Understand the concept

We look for a function whose derivative is the integrand. The form $\frac{u u'}{v^2} - \frac{u^2 v'}{v^3}$ suggests a quotient with u^2 and v^2 .

Step 2: Expand the integrand

$$\frac{1}{v^3} \left(uv \frac{du}{dx} - u^2 \frac{dv}{dx} \right) = \frac{u}{v^2} \frac{du}{dx} - \frac{u^2}{v^3} \frac{dv}{dx}$$

Step 3: Compare with a derivative

$$\frac{d}{dx} \left(\frac{u^2}{2v^2} \right) = \frac{2u u'}{2v^2} - \frac{2u^2 v'}{2v^3} = \frac{u u'}{v^2} - \frac{u^2 v'}{v^3}$$

Step 4: Result

The integrand is exactly this derivative, so the integral is $\frac{u^2}{2v^2} + c$,

option (D). Option (B) $\log \frac{u}{v}$ has derivative $\frac{u'}{u} - \frac{v'}{v}$, which is different.

Final Answer:

The integral is $u^2/(2v^2) + c$. This is option (D).

$$(D) \frac{u^2}{2v^2} + c$$

Quick Tip: Spot that the integrand is the derivative of u squared over $2v$ squared.

129. If $f(x) = |5x - 3|$ is defined on interval $[0, 1]$, then the value of $\int_0^1 f(x) dx$ is...

- (A) $\frac{13}{10}$
- (B) $\frac{9}{10}$
- (C) $\frac{3}{10}$
- (D) $\frac{17}{10}$

Correct Answer: (A) $\frac{13}{10}$

Solution:

Step 1: Understand the concept

The absolute value changes its formula where $5x - 3 = 0$, that is, at $x = \frac{3}{5}$. For $x < \frac{3}{5}$ it equals $3 - 5x$, and for $x > \frac{3}{5}$ it equals $5x - 3$.

Step 2: First part

$$\int_0^{3/5} (3 - 5x) dx = \left[3x - \frac{5x^2}{2} \right]_0^{3/5} = \frac{9}{5} - \frac{9}{10} = \frac{9}{10}$$

Step 3: Second part

$$\int_{3/5}^1 (5x - 3) dx = \left[\frac{5x^2}{2} - 3x \right]_{3/5}^1 = \left(\frac{5}{2} - 3 \right) - \left(\frac{9}{10} - \frac{9}{5} \right) = -\frac{1}{2} + \frac{9}{10} = \frac{2}{5}$$

Step 4: Add

$\frac{9}{10} + \frac{4}{10} = \frac{13}{10}$, option (A). The value $\frac{9}{10}$ is only the first part, and $\frac{17}{10}$ would come from a sign error in the second part.

Final Answer:

The integral is $13/10$. This is option (A).

(A) $\frac{13}{10}$

Quick Tip: Split the integral at the point where $5x - 3$ changes sign, $x = 3/5$.

130. The value of integral $\int_0^\infty \frac{1}{1+e^x} dx$ is...

- (A) $\log 2$
- (B) $\log\left(\frac{2}{e}\right)$
- (C) $-\log e$
- (D) $\log\left(\frac{4}{e}\right)$

Correct Answer: (A) $\log 2$

Solution:

Step 1: Understand the concept

The upper limit is infinity, so this is an improper integral. We first find an antiderivative and then take the limit.

Step 2: Rewrite the integrand

Multiply the numerator and denominator by e^{-x} :

$$\frac{1}{1 + e^x} = \frac{e^{-x}}{e^{-x} + 1}$$

The numerator is the negative of the derivative of the denominator.

Step 3: Antiderivative

$$\int \frac{e^{-x}}{1 + e^{-x}} dx = -\ln(1 + e^{-x})$$

Step 4: Apply the limits

At ∞ , $e^{-x} \rightarrow 0$ so the value is $-\ln 1 = 0$. At 0, the value is $-\ln 2$. So the integral is $0 - (-\ln 2) = \ln 2 = \log 2$, option (A).

Final Answer:

The integral converges to $\log 2$. This is option (A).

(A) $\log 2$

Quick Tip: Multiply top and bottom by e^{-x} to get an integrand of the form $-f'/f$.

131. If $f(x)$ is an antiderivative of $\frac{x+1}{\sqrt{x-1}}$ with respect to x , then value of $f(5) - f(2)$ is...

- (A) 4
- (B) $\frac{14}{3}$
- (C) 7
- (D) $\frac{26}{3}$

Correct Answer: (D) $\frac{26}{3}$

Solution:

Step 1: Understand the concept

$f(5) - f(2)$ equals the definite integral of the integrand from 2 to 5, since f is an antiderivative.

Step 2: Substitute

Let $u = x - 1$, so $x + 1 = u + 2$. When $x = 2$, $u = 1$, and when $x = 5$, $u = 4$.

$$\int_1^4 \frac{u+2}{\sqrt{u}} du = \int_1^4 (u^{1/2} + 2u^{-1/2}) du$$

Step 3: Integrate

$$\left[\frac{2}{3}u^{3/2} + 4u^{1/2} \right]_1^4$$

Step 4: Evaluate

At $u = 4$: $\frac{2}{3} \cdot 8 + 4 \cdot 2 = \frac{16}{3} + 8 = \frac{40}{3}$. At $u = 1$: $\frac{2}{3} + 4 = \frac{14}{3}$. The

difference is $\frac{26}{3}$, option (D).

Final Answer:

The value is $26/3$. This is option (D).

$$\boxed{(D) \frac{26}{3}}$$

Quick Tip: Substitute $u = x - 1$ so that the root becomes a simple power.

132. The area (in sq. units) of the region bounded by the curve $y = 2x - x^2$ and the X-axis is...

- (A) $\frac{4}{3}$
- (B) $\frac{8}{3}$
- (C) $\frac{20}{3}$
- (D) $\frac{2}{3}$

Correct Answer: (A) $\frac{4}{3}$

Solution:

Step 1: Understand the concept

The curve $y = 2x - x^2 = x(2 - x)$ is a downward parabola cutting the X-axis at $x = 0$ and $x = 2$. Between these points $y \geq 0$, so the area is the integral of y .

Step 2: Set up

$$A = \int_0^2 (2x - x^2) dx = \left[x^2 - \frac{x^3}{3} \right]_0^2$$

Step 3: Evaluate

$$A = 4 - \frac{8}{3} = \frac{4}{3}$$

Step 4: Check the options

The value $\frac{8}{3}$ would be twice the area and appears if a factor is doubled, and $\frac{2}{3}$ is half. The area is $\frac{4}{3}$, option (A).

Final Answer:

The area is $4/3$ square units. This is option (A).

(A) $\frac{4}{3}$

Quick Tip: Find where the parabola meets the X axis and integrate between the roots.

133. The area (in square units) of the region bounded by the circle $x^2 + y^2 = 9$ and the parabola $y^2 \leq 8x$ is...

- (A) $\frac{8\sqrt{2}}{3} + \frac{9\pi}{2} - 2\sqrt{2} - 9\sin^{-1}\frac{1}{3}$
- (B) $\frac{8\sqrt{2}}{3} + \frac{9\pi}{2} + 2\sqrt{2} + 9\sin^{-1}\frac{1}{3}$
- (C) $\frac{4\sqrt{2}}{3} + \frac{9\pi}{4} - \sqrt{2} - \frac{9}{2}\sin^{-1}\frac{1}{3}$
- (D) $\frac{4\sqrt{2}}{3} + \frac{9\pi}{4} + \sqrt{2} + \frac{9}{2}\sin^{-1}\frac{1}{3}$

Correct Answer: (A) $\frac{8\sqrt{2}}{3} + \frac{9\pi}{2} - 2\sqrt{2} - 9\sin^{-1}\frac{1}{3}$

Solution:

Step 1: Understand the concept

The region is inside the circle and inside the parabola $y^2 \leq 8x$. Both curves are symmetric about the X-axis, so we compute the upper half and double it.

Step 2: Find the intersection

Substitute $y^2 = 8x$ in $x^2 + y^2 = 9$: $x^2 + 8x - 9 = 0$, so $x = 1$ (since $x > 0$). Then $y = \pm 2\sqrt{2}$.

Step 3: Area from $x = 0$ to 1 (parabola)

$$2 \int_0^1 \sqrt{8x} \, dx = 2 \cdot 2\sqrt{2} \cdot \frac{2}{3} = \frac{8\sqrt{2}}{3}$$

Step 4: Area from $x = 1$ to 3 (circle)

$$2 \int_1^3 \sqrt{9 - x^2} \, dx = 2 \left[\frac{x}{2} \sqrt{9 - x^2} + \frac{9}{2} \sin^{-1} \frac{x}{3} \right]_1^3 = \frac{9\pi}{2} - 2\sqrt{2} - 9 \sin^{-1} \frac{1}{3}$$

The total is $\frac{8\sqrt{2}}{3} + \frac{9\pi}{2} - 2\sqrt{2} - 9 \sin^{-1} \frac{1}{3}$, option (A).

Final Answer:

The area is $8 \sqrt{2}/3 + 9 \pi/2 - 2 \sqrt{2} - 9 \arcsin(1/3)$. This is option (A).

$$(A) \frac{8\sqrt{2}}{3} + \frac{9\pi}{2} - 2\sqrt{2} - 9 \sin^{-1} \frac{1}{3}$$

Quick Tip: Find where the circle and parabola cross, then add the parabola part and the circle part, each doubled for symmetry.



134. The differential equation of all lines where the length of the normal from the origin is p and the inclination of the normal is α is... (where p and α are arbitrary constants)

- (A) $\frac{d^2y}{dx^2} = 0$
- (B) $\frac{dy}{dx} = 0$
- (C) $\frac{dy}{dx} = -\cot\alpha$
- (D) $\frac{d^2y}{dx^2} = \csc^2\alpha$

Correct Answer: (A) $\frac{d^2y}{dx^2} = 0$

Solution:

Step 1: Understand the concept

A line whose normal from the origin has length p and inclination α has the equation $x \cos \alpha + y \sin \alpha = p$. Here both p and α are arbitrary constants, so the family is all straight lines (two parameters).

Step 2: Solve for y

$$y = -x \cot \alpha + p \csc \alpha$$

This is of the form $y = mx + c$ with two arbitrary constants $m = -\cot \alpha$ and $c = p \csc \alpha$.

Step 3: Eliminate the constants

Differentiate once: $\frac{dy}{dx} = -\cot \alpha$. Differentiate again: $\frac{d^2y}{dx^2} = 0$. Since

two constants must be eliminated, we need the second order equation.

Step 4: Check the options

Option (B) $\frac{dy}{dx} = 0$ is only the horizontal lines, and option (C) fixes the slope, which is not arbitrary. Option (D) is not zero for a straight line. So the answer is $\frac{d^2y}{dx^2} = 0$, option (A).

Final Answer:

The differential equation is $y'' = 0$. This is option (A).

$$(A) \frac{d^2y}{dx^2} = 0$$

Quick Tip: The family has two arbitrary constants, so the equation is of second order.

135. If the solution of the differential equation $(1 + x^3)\frac{dy}{dx} + 6x^2y = 1 + x^2$ is $y = \frac{1}{(1+x^3)^s} [x + \frac{x^p}{p} + \frac{x^q}{q} + \frac{x^r}{r} + c]$, then the LCM of p, q, r and s is...

- (A) 1
- (B) 6
- (C) 4
- (D) 12

Correct Answer: (D) 12

Solution:

Step 1: Convert to standard form

Divide by $(1 + x^3)$: $\frac{dy}{dx} + \frac{6x^2}{1 + x^3}y = \frac{1 + x^2}{1 + x^3}$.

Step 2: Integrating factor

$$\text{IF} = e^{\int \frac{6x^2}{1+x^3} dx} = e^{2\ln(1+x^3)} = (1 + x^3)^2$$

Step 3: Solve

$$y(1 + x^3)^2 = \int \frac{1 + x^2}{1 + x^3} (1 + x^3)^2 dx = \int (1 + x^2)(1 + x^3) dx = \int (1 + x^2 + x^3 + x^5) dx$$

This is $x + \frac{x^3}{3} + \frac{x^4}{4} + \frac{x^6}{6} + c$.

Step 4: Compare

Hence $s = 2$ and $\{p, q, r\} = \{3, 4, 6\}$. The LCM of 3, 4, 6, 2 is 12, option (D).

Final Answer:

The LCM is 12. This is option (D).

(D) 12

Quick Tip: The integrating factor is $(1 + x^3)^2$, since the coefficient of y over the coefficient of y -prime is $6x^2/(1 + x^3)$.

136. The solution of the differential equation $\frac{dy}{dx} = \cos(x + y)$ is...

- (A) $\cot(\frac{x+y}{2}) = x + c$
(B) $\tan(\frac{x+y}{2}) = x + c$
(C) $-\sin(x + y) = x + c$
(D) $\sec(x - y) + x = c$

Correct Answer: (B) $\tan(\frac{x+y}{2}) = x + c$

Solution:

Step 1: Substitute

Let $v = x + y$. Then $\frac{dv}{dx} = 1 + \frac{dy}{dx} = 1 + \cos v$.

Step 2: Use a half angle identity

$1 + \cos v = 2 \cos^2 \frac{v}{2}$. So $\frac{dv}{dx} = 2 \cos^2 \frac{v}{2}$.

Step 3: Separate and integrate

$$\int \frac{dv}{2 \cos^2(v/2)} = \int dx \Rightarrow \int \frac{1}{2} \sec^2 \frac{v}{2} dv = x + c$$

$$\tan \frac{v}{2} = x + c$$

Step 4: Result

Resubstitute $v = x + y$: $\tan \frac{x + y}{2} = x + c$, option (B). Option (A) uses $\cot$, whose derivative has the wrong sign, and the others do not reduce to this form.

Final Answer:

The solution is $\tan((x + y)/2) = x + c$. This is option (B).

$$(B) \tan \frac{x+y}{2} = x+c$$

Quick Tip: Put $v = x + y$ so that the variables separate.

137. If $\vec{a}, \vec{b}, \vec{c}$ are three vectors such that $\vec{a} \perp (\vec{b} + \vec{c})$, $\vec{b} \perp (\vec{c} + \vec{a})$, and $\vec{c} \perp (\vec{a} + \vec{b})$ and $|\vec{a}| = 1, |\vec{b}| = 2, |\vec{c}| = 3$, then $|\vec{a} + \vec{b} + \vec{c}|$ is...

- (A) $\sqrt{8}$
- (B) 8
- (C) 14
- (D) $\sqrt{14}$

Correct Answer: (D) $\sqrt{14}$

Solution:

Step 1: Translate the conditions

Perpendicular vectors have zero dot product: $\vec{a} \cdot (\vec{b} + \vec{c}) = 0$, $\vec{b} \cdot (\vec{c} + \vec{a}) = 0$, $\vec{c} \cdot (\vec{a} + \vec{b}) = 0$. Writing $p = \vec{a} \cdot \vec{b}$, $q = \vec{b} \cdot \vec{c}$, $r = \vec{c} \cdot \vec{a}$, we have $p + r = 0$, $q + p = 0$, $r + q = 0$.

Step 2: Solve

Adding all three gives $2(p + q + r) = 0$. Subtracting the pairs gives $p = q = r = 0$. So the vectors are mutually perpendicular.

Step 3: Use the square of the magnitude

$$|\vec{a} + \vec{b} + \vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2(p + q + r)$$

Step 4: Result

$= 1 + 4 + 9 + 0 = 14$, so the magnitude is $\sqrt{14}$, option (D). The value 14 is the square of the magnitude, not the magnitude.

Final Answer:

The magnitude is $\sqrt{14}$. This is option (D).

(D) $\sqrt{14}$

Quick Tip: Adding the three perpendicularity conditions shows that every pairwise dot product is zero.

138. Two adjacent sides of a parallelogram ABCD are given by $\overrightarrow{AB} = 2\hat{i} + 10\hat{j} + 11\hat{k}$ and $\overrightarrow{AD} = -\hat{i} + 2\hat{j} + 2\hat{k}$. The side AD is rotated by an acute angle α in the plane of the parallelogram so that AD becomes **Missing open brace for superscript**. If **Missing open brace for superscript** makes a right angle with the side AB, then the cosine of the angle α is...

- (A) $\frac{8}{9}$
- (B) $\frac{\sqrt{17}}{9}$
- (C) $\frac{1}{9}$
- (D) $\frac{4\sqrt{5}}{9}$

Correct Answer: (B) $\frac{\sqrt{17}}{9}$

Solution:

Step 1: Understand the setup

The side AD is rotated in the plane of the parallelogram until it is perpendicular to AB . If θ is the angle between $\overrightarrow{AD}$ and $\overrightarrow{AB}$, the acute rotation needed is $\alpha = 90^\circ - \theta$.

Step 2: Find cos of theta

$$\overrightarrow{AB} \cdot \overrightarrow{AD} = -2 + 20 + 22 = 40. |\overrightarrow{AB}| = \sqrt{4 + 100 + 121} = 15 \text{ and } |\overrightarrow{AD}| = \sqrt{1 + 4 + 4} = 3. \text{ So } \cos \theta = \frac{40}{45} = \frac{8}{9}.$$

Step 3: Find sin of theta

$$\sin \theta = \sqrt{1 - \frac{64}{81}} = \frac{\sqrt{17}}{9}.$$

Step 4: Result

$\cos \alpha = \cos(90^\circ - \theta) = \sin \theta = \frac{\sqrt{17}}{9}$, option (B). The value $\frac{8}{9}$ is $\cos \theta$, which is a trap.

Final Answer:

The cosine of alpha is $\sqrt{17}/9$. This is option (B).

$$\boxed{(B) \frac{\sqrt{17}}{9}}$$

Quick Tip: Rotating AD to be perpendicular to AB in the plane means α is the complement of the angle between AD and AB .

139. The maximum volume of a parallelepiped (in cubic units) with vectors $(2a\hat{i} + \hat{k})$, $(a\hat{j} - a\hat{k})$, and $(3\hat{i} + a\hat{j})$, where $a \in [0, 1]$, as its coterminal edges is...

- (A) $\frac{1}{\sqrt{2}}$
- (B) $\sqrt{2}$
- (C) 2
- (D) $2\sqrt{2}$

Correct Answer: (B) $\sqrt{2}$

Solution:

Step 1: Understand the concept

The volume with coterminal edges $\vec{p}, \vec{q}, \vec{r}$ is $||[\vec{p} \ \vec{q} \ \vec{r}]||$, the modulus of the determinant of their components.

Step 2: Build the determinant

$$\begin{vmatrix} 2a & 0 & 1 \\ 0 & a & -a \\ 3 & a & 0 \end{vmatrix} = 2a(0 + a^2) - 0 + 1(0 - 3a) = 2a^3 - 3a$$

Step 3: Maximise the modulus

Let $f(a) = 2a^3 - 3a$. Then $f'(a) = 6a^2 - 3 = 0$ at $a = \frac{1}{\sqrt{2}}$ (inside $[0, 1]$).

$$f\left(\frac{1}{\sqrt{2}}\right) = \frac{2}{2\sqrt{2}} - \frac{3}{\sqrt{2}} = -\sqrt{2}.$$

Step 4: Compare end points

$f(0) = 0$ and $f(1) = -1$. The largest modulus is $\sqrt{2}$, so the maximum volume is $\sqrt{2}$, option (B).

Final Answer:

The maximum volume is $\sqrt{2}$ cubic units. This is option (B).

$$\boxed{(B) \sqrt{2}}$$

Quick Tip: The volume is the modulus of the scalar triple product; maximise its modulus on $[0, 1]$.

140. Let $O(0, 0)$, $A(-1, 2)$ and $B(1, 3)$ be the vertices of $\triangle OAB$. The bisector of angle O intersects side AB at point D . The value of $\vec{OD} \cdot \vec{AB}$ is equal to...

- (A) $-5(\sqrt{2} + 1)$
- (B) $5(1 - \sqrt{2})$
- (C) $5(\sqrt{2} + 1)$
- (D) $5(\sqrt{2} - 1)$

Correct Answer: (D) $5(\sqrt{2} - 1)$

Solution:

Step 1: Understand the concept

The bisector of angle O meets AB at D so that $\frac{AD}{DB} = \frac{OA}{OB}$ (angle bisector theorem).

Step 2: Find the lengths

$OA = \sqrt{1 + 4} = \sqrt{5}$ and $OB = \sqrt{1 + 9} = \sqrt{10}$. So $AD : DB = \sqrt{5} : \sqrt{10} = 1 : \sqrt{2}$.

Step 3: Find D

By the section formula, $\vec{OD} = \frac{\sqrt{2}\vec{OA} + 1 \cdot \vec{OB}}{\sqrt{2} + 1} = \frac{(-\sqrt{2} + 1, 2\sqrt{2} + 3)}{\sqrt{2} + 1}$.

Step 4: Dot product

$\vec{AB} = (2, 1)$. Then $\vec{OD} \cdot \vec{AB} = \frac{2(1 - \sqrt{2}) + 2\sqrt{2} + 3}{\sqrt{2} + 1} = \frac{5}{\sqrt{2} + 1} = 5(\sqrt{2} - 1)$, option (D).

Final Answer:

The dot product is $5(\sqrt{2} - 1)$. This is option (D).

(D) $5(\sqrt{2} - 1)$

Quick Tip: The angle bisector divides the opposite side in the ratio of the adjacent sides.



141. The perpendicular distance from the origin to the plane containing the points $(1, -2, 1)$, $(2, -1, -3)$ and $(0, 1, 5)$ is...(in units)

- (A) $\frac{1}{\sqrt{17}}$
- (B) $\frac{3}{\sqrt{26}}$
- (C) $\frac{5}{\sqrt{17}}$
- (D) $\frac{7}{\sqrt{26}}$

Correct Answer: (C) $\frac{5}{\sqrt{17}}$

Solution:**Step 1: Find two vectors in the plane**

Take $P = (1, -2, 1)$. Then $\vec{PQ} = (1, 1, -4)$ and $\vec{PR} = (-1, 3, 4)$.

Step 2: Normal vector

$$\vec{PQ} \times \vec{PR} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & -4 \\ -1 & 3 & 4 \end{vmatrix} = (16, 0, 4)$$

So the normal direction is (4, 0, 1).

Step 3: Plane equation

$4x + 0y + z = d$. Using P : $4 + 1 = 5$, so the plane is $4x + z = 5$. Check Q : $8 - 3 = 5$. Check R : $0 + 5 = 5$.

Step 4: Distance

$$\text{distance} = \frac{|5|}{\sqrt{16 + 0 + 1}} = \frac{5}{\sqrt{17}}$$

Option (C).

Final Answer:

The distance is $5/\sqrt{17}$. This is option (C).

$$\boxed{(C) \frac{5}{\sqrt{17}}}$$

Quick Tip: Find the normal as the cross product of two vectors in the plane, then use the distance formula.

142. If α, β, γ are the direction angles of the line $x = 4z + 3$ and $y = 2 - 3z$, then the value of $\cos\alpha + \cos\beta + \cos\gamma$ is...

- (A) $\frac{8}{\sqrt{26}}$
- (B) $\frac{6}{\sqrt{26}}$
- (C) $\frac{4}{\sqrt{26}}$
- (D) $\frac{2}{\sqrt{26}}$

Correct Answer: (D) $\frac{2}{\sqrt{26}}$

Solution:

Step 1: Write in symmetric form

From $x = 4z + 3$ we get $\frac{x-3}{4} = z$. From $y = 2 - 3z$ we get $\frac{y-2}{-3} = z$.

So

$$\frac{x-3}{4} = \frac{y-2}{-3} = \frac{z}{1}$$

Step 2: Direction ratios

The direction ratios are $(4, -3, 1)$. The magnitude is $\sqrt{16 + 9 + 1} = \sqrt{26}$.

Step 3: Direction cosines

$$\cos \alpha = \frac{4}{\sqrt{26}}, \cos \beta = \frac{-3}{\sqrt{26}}, \cos \gamma = \frac{1}{\sqrt{26}}.$$

Step 4: Add

$$\cos \alpha + \cos \beta + \cos \gamma = \frac{4 - 3 + 1}{\sqrt{26}} = \frac{2}{\sqrt{26}}$$

Option (D). Options (A) to (C) come from adding the magnitudes of the ratios or leaving out a term.

Final Answer:

The sum of direction cosines is $2/\sqrt{26}$. This is option (D).

$$(D) \frac{2}{\sqrt{26}}$$

Quick Tip: Write the line in symmetric form to read off direction ratios, then divide by the magnitude.



143. The shortest distance between the lines $\vec{r} = (4\hat{i} - \hat{j}) + \lambda(\hat{i} + 2\hat{j} - 3\hat{k})$ and $\vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \mu(\hat{i} + 4\hat{j} - 5\hat{k})$ is...

- (A) $\frac{1}{\sqrt{2}}$
- (B) $\frac{1}{2}$
- (C) $\frac{1}{\sqrt{3}}$
- (D) $\frac{1}{3}$

Correct Answer: (C) $\frac{1}{\sqrt{3}}$

Solution:

Step 1: Understand the concept

For lines $\vec{r} = \vec{a}_1 + \lambda\vec{d}_1$ and $\vec{r} = \vec{a}_2 + \mu\vec{d}_2$, the shortest distance is

$$\frac{|(\vec{a}_2 - \vec{a}_1) \cdot (\vec{d}_1 \times \vec{d}_2)|}{|\vec{d}_1 \times \vec{d}_2|}.$$

Step 2: Cross product

$$\vec{d}_1 = (1, 2, -3), \vec{d}_2 = (1, 4, -5).$$

$$\vec{d}_1 \times \vec{d}_2 = (2(-5) - (-3)(4), (-3)(1) - (1)(-5), 1 \cdot 4 - 2 \cdot 1) = (2, 2, 2)$$

Step 3: Dot product

$$\vec{a}_2 - \vec{a}_1 = (-3, 0, 2). \text{ So } (\vec{a}_2 - \vec{a}_1) \cdot (2, 2, 2) = -6 + 0 + 4 = -2.$$

Step 4: Result

$$|\vec{d}_1 \times \vec{d}_2| = 2\sqrt{3}, \text{ so the distance is } \frac{2}{2\sqrt{3}} = \frac{1}{\sqrt{3}}, \text{ option (C).}$$

Final Answer:

The shortest distance is $1/\sqrt{3}$. This is option (C).

$(C) \frac{1}{\sqrt{3}}$

Quick Tip: The shortest distance between skew lines is $|(a_2 - a_1) \cdot (d_1 \times d_2)| / |d_1 \times d_2|$.

144. Let $\vec{r} \cdot (3\hat{i} - 2\hat{j} + 7\hat{k}) = 32$ is the equation of a plane and the line having direction ratios $(5, b, 3)$ is parallel to the plane, then the value of b is...

- (A) $b = 13$
- (B) $b = 15$
- (C) $b = 16$
- (D) $b = 18$

Correct Answer: (D) $b = 18$

Solution:

Step 1: Understand the concept

A line with direction ratios (l, m, n) is parallel to the plane $\vec{r} \cdot \vec{N} = d$ when $\vec{N}$ is perpendicular to the direction, that is, $\vec{N} \cdot (l, m, n) = 0$.

Step 2: Identify the vectors

The normal is $\vec{N} = (3, -2, 7)$ and the direction is $(5, b, 3)$.

Step 3: Set the dot product to zero

$$3(5) + (-2)(b) + 7(3) = 0 \Rightarrow 15 - 2b + 21 = 0$$

Step 4: Solve

$2b = 36$, so $b = 18$, option (D). Check: $15 - 36 + 21 = 0$.

Final Answer:

The value of b is 18. This is option (D).

(D) $b = 18$

Quick Tip: A line is parallel to a plane when its direction is perpendicular to the plane normal.

145. A line passing through the points $(1, -1, 2)$ and $(2, 0, 1)$ meets the XY plane and the YZ plane at points A and B respectively. The distance AB is equal to...

- (A) $2\sqrt{2}$
- (B) $2\sqrt{3}$
- (C) $3\sqrt{2}$
- (D) $3\sqrt{3}$

Correct Answer: (D) $3\sqrt{3}$

Solution:

Step 1: Find the line

The points are $(1, -1, 2)$ and $(2, 0, 1)$, so the direction is $(1, 1, -1)$. A point is $(1 + t, -1 + t, 2 - t)$.

Step 2: Meet the XY plane

On the XY plane $z = 0$, so $2 - t = 0$ and $t = 2$. This gives $A = (3, 1, 0)$.

Step 3: Meet the YZ plane

On the YZ plane $x = 0$, so $1 + t = 0$ and $t = -1$. This gives $B = (0, -2, 3)$.

Step 4: Distance

$$AB = \sqrt{(3 - 0)^2 + (1 + 2)^2 + (0 - 3)^2} = \sqrt{27} = 3\sqrt{3}$$

Option (D).

Final Answer:

The distance AB is $3\sqrt{3}$. This is option (D).

$$(D) 3\sqrt{3}$$

Quick Tip: Write the line in parametric form and find the parameter values at which $x = 0$ and $z = 0$.

146. An airplane can carry a maximum of 250 passengers. A profit of Rs 1500 is made on each executive class ticket and a profit of Rs 900 is made on each economy class ticket. The airline reserves at least 30 seats for executive class. However at least 4 times as many passengers prefer to travel by economy class than by executive class. Let x_1 be the number of passengers of executive class and x_2 be the number of passengers of economy class. Formulate the LPP in order to maximize the profit for the airline...

- (A) Maximize $z = 1500x_1 + 900x_2$ subject to $x_1 + x_2 \leq 250$, $x_1 \leq 30$, $x_2 \leq 4x_1$, $x_1 \geq 0$, $x_2 \geq 0$.
- (B) Minimize $z = 150x_1 + 90x_2$ subject to $x_1 + x_2 \leq 250$, $x_1 \geq 30$, $x_2 \geq 4x_1$, $x_1 \geq 0$, $x_2 \geq 0$.
- (C) Minimize $z = 1500x_1 + 900x_2$ subject to $x_1 + x_2 \leq 250$, $x_1 \geq 30$, $x_2 \geq 4x_1$, $x_1 \geq 0$, $x_2 \geq 0$.
- (D) Maximize $z = 1500x_1 + 900x_2$ subject to $x_1 + x_2 \leq 250$, $x_1 \geq 30$, $x_2 \geq 4x_1$, $x_1 \geq 0$, $x_2 \geq 0$.

Correct Answer: (D) Maximize $z = 1500x_1 + 900x_2$ subject to $x_1 + x_2 \leq 250$, $x_1 \geq 30$, $x_2 \geq 4x_1$, $x_1 \geq 0$, $x_2 \geq 0$.

Solution:

Step 1: Define the variables

Let x_1 be the number of executive class passengers and x_2 the number of economy class passengers.

Step 2: Objective

Profit is Rs 1500 per executive ticket and Rs 900 per economy ticket. The airline wants the largest profit, so we maximise $z = 1500x_1 + 900x_2$.

Step 3: Constraints

Capacity: $x_1 + x_2 \leq 250$. At least 30 executive seats: $x_1 \geq 30$. At least 4 times as many prefer economy as executive: $x_2 \geq 4x_1$. Non-negativity: $x_1, x_2 \geq 0$.

Step 4: Match the options

This is option (D). Option (A) has the inequalities reversed for x_1 and x_2 . Options (B) and (C) say "Minimize", which does not fit a profit problem.

Final Answer:

Option (D) states the correct formulation. This is option (D).

$$(D) \text{ Maximize } z = 1500x_1 + 900x_2$$

Quick Tip: Maximise profit, with at least 30 executive seats, at least four times as many economy as executive passengers, and at most 250 passengers.

147. The probability that a bomb will hit the target is 0.8. Out of 6 bombs dropped, probability that at least 1 will miss the target is...

- (A) $(\frac{1}{5})^6$
- (B) $1 - (\frac{1}{5})^6$
- (C) $(\frac{4}{5})^6$
- (D) $1 - (\frac{4}{5})^6$

Correct Answer: (D) $1 - (\frac{4}{5})^6$

Solution:

Step 1: Understand the concept

The bombs are independent. "At least 1 will miss" is the complement of "all 6 hit the target".

Step 2: Probability of all hits

$P(\text{hit}) = 0.8 = \frac{4}{5}$, so $P(\text{all 6 hit}) = (\frac{4}{5})^6$.

Step 3: Use the complement

$$P(\text{at least 1 miss}) = 1 - \left(\frac{4}{5}\right)^6$$

Step 4: Check the options

$(\frac{1}{5})^6$ is the probability that all six miss, which is different from at least one missing. $(\frac{4}{5})^6$ is the probability of all hits. The correct choice is option (D).

Final Answer:

The probability is $1 - (4/5)^6$. This is option (D).

$$(D) 1 - \left(\frac{4}{5}\right)^6$$

Quick Tip: At least one miss is the complement of all six bombs hitting.

148. Two cards are drawn at random from a box which contains 5 cards numbered 1, 1, 2, 2 & 3. If X denotes the sum of the numbers, then the expected value of X is...

- (A) 2.6
- (B) 3.8
- (C) 3.2
- (D) 3.6

Correct Answer: (D) 3.6

Solution:

Step 1: Count the outcomes

Two cards from five can be chosen in ${}^5C_2 = 10$ ways. The cards are $1_a, 1_b, 2_a, 2_b, 3$.

Step 2: Distribution of X

Sum 2: the two 1s, 1 way. Sum 3: one 1 and one 2, $2 \times 2 = 4$ ways. Sum 4: (1, 3) in 2 ways and (2, 2) in 1 way, total 3 ways. Sum 5: (2, 3) in 2 ways. Check: $1 + 4 + 3 + 2 = 10$.

Step 3: Expected value

$$E(X) = \frac{2(1) + 3(4) + 4(3) + 5(2)}{10} = \frac{2 + 12 + 12 + 10}{10} = \frac{36}{10} = 3.6$$

Step 4: Check the options

3.8 would come from counting the sum of 4 as 4 ways. 2.6 and 3.2 are too low for these sums. The value is 3.6, option (D).

Final Answer:

The expected value is 3.6. This is option (D).

(D) 3.6

Quick Tip: List the possible sums with their probabilities from the 10 equally likely pairs.

149. Bag A contains 3 white and 5 black balls while bag B contains 4 white and 3 black balls. A ball is selected at random from bag A and put in bag B. If a ball is now selected at random from bag B then the probability that this ball is white ball is...

- (A) $\frac{20}{64}$
- (B) $\frac{31}{64}$
- (C) $\frac{35}{64}$
- (D) $\frac{32}{64}$

Correct Answer: (C) $\frac{35}{64}$

Solution:

Step 1: Understand the concept

The ball moved from A to B is either white or black. Bag B then has 8

balls, and we use total probability over the two cases.

Step 2: Case 1: a white ball moved

$P = \frac{3}{8}$. Bag B now has 5 white and 3 black, so $P(\text{white from B}) = \frac{5}{8}$.

Contribution: $\frac{3}{8} \cdot \frac{5}{8} = \frac{15}{64}$.

Step 3: Case 2: a black ball moved

$P = \frac{5}{8}$. Bag B now has 4 white and 4 black, so $P(\text{white from B}) = \frac{4}{8}$.

Contribution: $\frac{5}{8} \cdot \frac{4}{8} = \frac{20}{64}$.

Step 4: Add

$$P = \frac{15}{64} + \frac{20}{64} = \frac{35}{64}$$

Option (C).

Final Answer:

The probability is $35/64$. This is option (C).

(C) $\frac{35}{64}$

Quick Tip: Use the total probability theorem over the colour of the ball transferred from bag A.

150. The lengths of the shadows of a tree of height p , thrown by sun's rays at three different moments are p , $2p$ and $3p$. The sum of the angles of elevation of the sun's rays at these three moments is equal to...

- (A) $\frac{2\pi}{3}$
- (B) $\frac{3\pi}{2}$
- (C) $\frac{\pi}{4}$
- (D) $\frac{\pi}{2}$

Correct Answer: (D) $\frac{\pi}{2}$

Solution:

Step 1: Understand the concept

If the tree has height p and the shadow has length s , the angle of elevation θ of the sun satisfies $\tan \theta = \frac{p}{s}$.

Step 2: Find the three tangents

Shadows $p, 2p, 3p$ give $\tan \theta_1 = 1, \tan \theta_2 = \frac{1}{2}, \tan \theta_3 = \frac{1}{3}$. So $\theta_1 = \frac{\pi}{4}$, $\theta_2 = \tan^{-1} \frac{1}{2}$ and $\theta_3 = \tan^{-1} \frac{1}{3}$.

Step 3: Add the last two

$$\tan^{-1} \frac{1}{2} + \tan^{-1} \frac{1}{3} = \tan^{-1} \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{6}} = \tan^{-1} 1 = \frac{\pi}{4}$$

Step 4: Total

$$\theta_1 + \theta_2 + \theta_3 = \frac{\pi}{4} + \frac{\pi}{4} = \frac{\pi}{2}, \text{ option (D).}$$

Final Answer:

The sum of the three angles is $\pi/2$. This is option (D).

(D) $\frac{\pi}{2}$

Quick Tip: The tangent of each elevation angle is tree height divided by shadow length.