MHT CET 2026 PCM Question Paper April 20 Shift 1 with Solutions

General Instructions

(i) This booklet contains 150 questions, each provided with a complete, step-by-step solution.

(ii) It comprises 150 single-correct multiple-choice questions.

(iii) Attempt each question on your own before reviewing the given solution.

Physics

Questions 1 to 50 | 1 mark each | 50 marks

1. The area of parallelogram formed by vectors $\vec{P} = 2\hat{i} - \hat{j} + 5\hat{k}$ and $\vec{Q} = 3\hat{i} - 2\hat{j} + 4\hat{k}$ is

(A) $\sqrt{72}$

(B) $\sqrt{86}$

(C) $\sqrt{104}$

(D) $\sqrt{240}$

Correct Answer: (B) $\sqrt{86}$

Solution:

Step 1: Understanding the Concept:

The area of a parallelogram with adjacent sides $\vec{P}$ and $\vec{Q}$ equals the magnitude of their cross product, $|\vec{P} \times \vec{Q}|$.

Step 2: Compute the cross product:

$$\vec{P} \times \vec{Q} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 5 \\ 3 & -2 & 4 \end{vmatrix}$$

$\hat{i}$ component: $(-1)(4) - (5)(-2) = -4 + 10 = 6$.
$\hat{j}$ component: $-[(2)(4) - (5)(3)] = -[8 - 15] = 7$.
$\hat{k}$ component: $(2)(-2) - (-1)(3) = -4 + 3 = -1$.
So $\vec{P} \times \vec{Q} = 6\hat{i} + 7\hat{j} - \hat{k}$.

Step 3: Magnitude:

$$|\vec{P} \times \vec{Q}| = \sqrt{36 + 49 + 1} = \sqrt{86}$$

Step 4: Why the other options are wrong.
$\sqrt{72}$, $\sqrt{104}$ and $\sqrt{240}$ come from sign mistakes in the components, for example taking the $\hat{j}$ component with the wrong sign.

Final Answer:
The area is $\sqrt{86}$ square units, option (B).

$$\boxed{\sqrt{86}}$$

Quick Tip: Area equals the magnitude of the cross product P x Q.

2. The percentage error in the measurement of mass of a body and its speed are 0.73% and 1.84% respectively. The percentage error in the measurement of its momentum and kinetic energy are respectively

(A) 2.57% , 2.75%

(B) 2.57% , 4.41%

(C) 1.46% , 1.84%

(D) 1.84% , 3.70%

Correct Answer: (B) 2.57% , 4.41%

Solution:

Step 1: Understanding the Concept:

For a product of quantities raised to powers, the percentage errors add, each multiplied by its power.

Step 2: Momentum:

$p = mv$, so $\frac{\Delta p}{p} \times 100 = 0.73\% + 1.84\% = 2.57\%$.

Step 3: Kinetic energy:

$K = \frac{1}{2}mv^2$. The factor $\frac{1}{2}$ is a constant with no error, and v appears squared:

$$\frac{\Delta K}{K} \times 100 = 0.73\% + 2(1.84\%) = 0.73 + 3.68 = 4.41\%$$

Step 4: Why the other options are wrong.

Option (A) takes 2.75% for kinetic energy, which uses $0.73 \times 2 + 1.84 \neq$ the correct combination. Option (C) and (D) use averages or half values, but errors in a product always add.

Final Answer:

The errors are 2.57% and 4.41%, option (B).

$\boxed{2.57\%,\ 4.41\%}$

Quick Tip: p = mv adds the percentage errors; KE = half m v squared doubles the error in v.

3. A boat crosses a river from one bank A to another bank B which is opposite. The distance between them is D. The speed of water is V_W and that of boat relative to water is V_B. If $V_B = 2V_W$, the time taken by the boat to cross the river directly along AB is $(sin30° = \frac{1}{2})$, $(cos30° = \frac{\sqrt{3}}{2})$

(A) $\frac{D}{V_B\sqrt{2}}$

(B) $\frac{D\sqrt{2}}{V_B}$

(C) $\frac{2D}{V_B\sqrt{3}}$

(D) $\frac{\sqrt{3}D}{2V_B}$

Correct Answer: (C) $\frac{2D}{V_B\sqrt{3}}$

Solution:

Step 1: Understanding the Concept:

To reach the point directly opposite, the boat is steered upstream at an angle θ to the line AB so that the sideways velocity cancels the water flow.

Step 2: Condition for direct crossing:

$V_B \sin\theta = V_W$. With $V_B = 2V_W$, $\sin\theta = \frac{1}{2}$, so $\theta = 30°$.

Step 3: Speed across the river:

The component perpendicular to the banks is $V_B \cos 30° = V_B \cdot \frac{\sqrt{3}}{2}$.

Step 4: Time:

$$t = \frac{D}{V_B \cos 30^\circ} = \frac{2D}{\sqrt{3}\, V_B}$$

Step 5: Why the other options are wrong.

Options (A) and (B) have $\sqrt{2}$, which would arise if the angle were 45°.

Option (D) has $\frac{\sqrt{3}D}{2V_B}$, the reciprocal of the correct factor.

Final Answer:

The time is $\frac{2D}{V_B\sqrt{3}}$, option (C).

$$\boxed{\frac{2D}{V_B\sqrt{3}}}$$

Quick Tip: To go straight across, the upstream component of boat velocity must cancel the river current.

4. The weight of man in lift moving in upward direction with an acceleration 'a' is 660 N. When the lift moves in the downward direction with the same acceleration, his weight is found to be 380 N. The real weight of the man when the lift is at rest is

(A) 400 N

(B) 460 N

(C) 520 N

(D) 640 N

Correct Answer: (C) 520 N

Solution:

Step 1: Understanding the Concept:

The apparent weight is the normal force from the floor. When the lift accelerates upward with a, the apparent weight is $m(g + a)$. When it accelerates downward with the same a, it is $m(g - a)$.

Step 2: Write the equations:

$m(g + a) = 660$ N and $m(g - a) = 380$ N.

Step 3: Add them:

$$2mg = 660 + 380 = 1040 \Rightarrow mg = 520 \text{ N}$$

The real weight is mg, the weight at rest.

Step 4: Check:

Subtracting gives $2ma = 280$, so $ma = 140$ N, which is consistent: $520 + 140 = 660$ and $520 - 140 = 380$.

Step 5: Why the other options are wrong.

400 N, 460 N and 640 N do not lie midway between 380 N and 660 N, and the real weight must be their average.

Final Answer:

The real weight is 520 N, option (C).

$$\boxed{520 \text{ N}}$$

Quick Tip: Upward acceleration gives m(g+a), downward gives m(g-a); add the two.

5. A solid sphere rolls down from the top of an inclined plane. On reaching the bottom of the plane, its velocity is 'V_1'. When the same sphere slides down from the top of the same plane of same height, its velocity on reaching the bottom is 'V_2'. The ratio $V_1 : V_2$ is (neglect friction)

(A) $\sqrt{7} : \sqrt{5}$

(B) $\sqrt{7} : \sqrt{3}$

(C) $\sqrt{3} : \sqrt{5}$

(D) $\sqrt{5} : \sqrt{7}$

Correct Answer: (D) $\sqrt{5} : \sqrt{7}$

Solution:

Step 1: Understanding the Concept:

For a solid sphere of mass m, radius R, $I = \frac{2}{5}mR^2$. Using energy conservation from the top of the incline at height h.

Step 2: Rolling case:

$mgh = \frac{1}{2}mV_1^2 + \frac{1}{2}I\omega^2$ with $\omega = V_1/R$:

$$mgh = \frac{1}{2}mV_1^2\left(1 + \frac{2}{5}\right) = \frac{1}{2}mV_1^2 \cdot \frac{7}{5} \Rightarrow V_1^2 = \frac{10}{7}gh$$

Step 3: Sliding case:

No friction, so no rotation: $mgh = \frac{1}{2}mV_2^2$, giving $V_2^2 = 2gh$.

Step 4: Ratio:

$$\frac{V_1^2}{V_2^2} = \frac{10/7}{2} = \frac{5}{7} \Rightarrow V_1 : V_2 = \sqrt{5} : \sqrt{7}$$

Step 5: Why the other options are wrong.

Options (A) and (B) put the larger number first, which would make the rolling sphere faster. Rolling is slower because some energy goes into rotation. Option (C) uses $\sqrt{3}$, which does not match the moment of inertia of a sphere.

Final Answer:

The ratio is $\sqrt{5} : \sqrt{7}$, option (D).

$$\boxed{\sqrt{5} : \sqrt{7}}$$

Quick Tip: Rolling converts part of the energy into rotation; sliding without friction does not.

6. Two identical rings A and B of same mass and radius are revolving, ring A arounds its own diameter and ring B about tangential axis in its own plane. Both the rings A and B have same rotational kinetic energy. The ratio of the angular velocity of ring B to that of ring A is

(A) $1:3$

(B) $1:\sqrt{3}$

(C) $3:2$

(D) $\sqrt{3}:2$

Correct Answer: (B) $1:\sqrt{3}$

Solution:

Step 1: Understanding the Concept:

Rotational KE is $\frac{1}{2}I\omega^2$. With equal KE, $I_A\omega_A^2 = I_B\omega_B^2$, so $\frac{\omega_B}{\omega_A} = \sqrt{\frac{I_A}{I_B}}$.

Step 2: Moments of inertia:

Ring A about its diameter: $I_A = \frac{1}{2}MR^2$.

Ring B about a tangent in its plane: use the parallel axis theorem, $I_B = \frac{1}{2}MR^2 + MR^2 = \frac{3}{2}MR^2$.

Step 3: Ratio:

$$\frac{\omega_B}{\omega_A} = \sqrt{\frac{\frac{1}{2}MR^2}{\frac{3}{2}MR^2}} = \sqrt{\frac{1}{3}} = \frac{1}{\sqrt{3}}$$

So $\omega_B : \omega_A = 1:\sqrt{3}$.

Step 4: Why the other options are wrong.

The ratio 1:3 comes from forgetting the square root. Options (C) and (D) involve 2, as if a different moment of inertia such as MR^2 was used for one of the rings.

Final Answer:

The ratio is $1:\sqrt{3}$, option (B).

$$\boxed{1:\sqrt{3}}$$

Quick Tip: Find the moment of inertia of a ring about a diameter and about an in-plane tangent.

7. A body is rotating about its own axis. Its rotational kinetic energy is 'x' and its angular momentum is 'y'. Hence its moment of inertia about its own axis is

(A) $\frac{x}{2y}$

(B) $\frac{y}{2x}$

(C) $\frac{x^2}{2y}$

(D) $\frac{y^2}{2x}$

Correct Answer: (D) $\frac{y^2}{2x}$

Solution:

Step 1: Understanding the Concept:

Rotational kinetic energy is $x = \frac{1}{2}I\omega^2$ and angular momentum is $y = I\omega$.

Step 2: Eliminate omega:

From $y = I\omega$, $\omega = \frac{y}{I}$. Substitute into the energy:

$$x = \frac{1}{2}I \cdot \frac{y^2}{I^2} = \frac{y^2}{2I}$$

Step 3: Solve for I:

$$I = \frac{y^2}{2x}$$

Step 4: Why the other options are wrong.

$\frac{x}{2y}$ and $\frac{y}{2x}$ fail dimensional checks: I has units of kg m^2, while $\frac{y}{x}$ has units of seconds. $\frac{x^2}{2y}$ also has wrong units.

Final Answer:

The moment of inertia is $\frac{y^2}{2x}$, option (D).

$$\boxed{\frac{y^2}{2x}}$$

Quick Tip: Combine K = (1/2) I w^2 and L = I w to eliminate w.

8. The percentage decrease in the weight of a body when taken to a height of 48 km above the surface of the earth is (Radius of the earth is 6400 km)

(A) 1%

(B) 1.5%

(C) 2%

(D) 2.5%

Correct Answer: (B) 1.5%

Solution:

Step 1: Understanding the Concept:

Weight is mg, so the fractional change in weight equals the fractional

change in g. At a height h much smaller than R:

$$g_h \approx g\left(1 - \frac{2h}{R}\right)$$

Step 2: Compute:

$$\frac{\Delta g}{g} = \frac{2h}{R} = \frac{2 \times 48}{6400} = \frac{96}{6400} = 0.015$$

Step 3: Percentage:

$0.015 \times 100 = 1.5\%$. The weight decreases by 1.5%.

Step 4: Why the other options are wrong.

1% would result from using h/R without the 2 and a different height.

2% and 2.5% are too large for $h = 48$ km.

Final Answer:

The weight decreases by **1.5%**, option (B).

$$\boxed{1.5\%}$$

Quick Tip: For small h, g falls by 2h/R.

9. If 450 erg of work is done in blowing a soap bubble of radius 'r', the additional work required to be done to blow it to a radius equal to $3r$ is

(A) 2400 erg

(B) 3000 erg

(C) 3600 erg

(D) 4000 erg

Correct Answer: (C) 3600 erg

Solution:

Step 1: Understanding the Concept:

Work done in blowing a bubble equals surface tension times the increase in surface area. A soap bubble has two surfaces, so $W = T \times 2 \times 4\pi r^2 = 8\pi r^2 T$.

Step 2: Scaling:

$W \propto r^2$. When the radius becomes $3r$, the work becomes 9 times larger (starting from zero radius):

$$W_2 = 9 \times 450 = 4050 \text{ erg}$$

Step 3: Additional work:

$$W_2 - W_1 = 4050 - 450 = 3600 \text{ erg}$$

Step 4: Why the other options are wrong.

2400 erg would result from an area ratio of $\frac{16}{3}$-type slip. 3000 erg and 4000 erg do not match a factor of 8 times the first work (3600 = 8 x 450).

Final Answer:

The additional work is 3600 erg, option (C).

$$\boxed{3600 \text{ erg}}$$

Quick Tip: A soap bubble has two surfaces, so work is T times 2 times 4 pi r squared; work scales as r squared.

10. The excess pressure inside a soap bubble of radius 2.5 cm is 48 dyne/cm^2. The surface tension of soap solution in dyne /cm is

(A) 25

(B) 30

(C) 35

(D) 40

Correct Answer: (B) 30

Solution:

Step 1: Understanding the Concept:

A soap bubble has two free surfaces (inside and outside), so its excess pressure is $\Delta P = \frac{4T}{r}$.

Step 2: Solve for T:

$$T = \frac{\Delta P\, r}{4} = \frac{48 \times 2.5}{4} = \frac{120}{4} = 30 \text{ dyne/cm}$$

Step 3: Why the other options are wrong.

Using $\Delta P = \frac{2T}{r}$, which is for a single surface (a liquid drop), would give 60 dyne/cm. Values 25, 35 and 40 do not follow from the formula.

Final Answer:

The surface tension is 30 dyne/cm, option (B).

$$\boxed{30 \text{ dyne/cm}}$$

Quick Tip: Excess pressure in a soap bubble is 4T/r.

11. In case of liquid, if Reynold's number 'R_n' is 1450, then the flow of liquid will

(A) be streamline.

(B) be turbulent.

(C) change from streamline to turbulent flow.

(D) be sometimes turbulent and sometimes streamline.

Correct Answer: (C) change from streamline to turbulent flow.

Solution:

Step 1: Understanding the Concept:

The Reynolds number R_n tells whether the flow of a liquid is smooth or turbulent.

Step 2: Ranges used in the Maharashtra Board text:

For $R_n < 1000$ the flow is streamline. For $1000 < R_n < 2000$ the flow is unsteady and changes from streamline to turbulent. For $R_n > 2000$

the flow is turbulent.

Step 3: Apply:
$R_n = 1450$ lies between 1000 and 2000, so the flow is in the transition range, where it changes from streamline to turbulent.

Step 4: Why the other options are wrong.
Option (A) would need R_n below 1000. Option (B) would need R_n above 2000. Option (D) does not describe a standard classification.

Final Answer:
The flow changes from streamline to turbulent, option (C).

Changes from streamline to turbulent (C)

Quick Tip: Between about 1000 and 2000 the flow is unstable and changes from streamline to turbulent.

12. A sphere is at temperature 600 K. In an external environment of 200 K, its cooling rate is R. When the temperature of the sphere falls to 400 K then cooling rate R' will become

(A) $\frac{16}{3}R$

(B) $\frac{16}{9}R$

(C) $\frac{9}{16}R$

(D) $\frac{3}{16}R$

Correct Answer: (D) $\frac{3}{16}R$

Solution:

Step 1: Understanding the Concept:

A hot sphere in a cooler surrounding loses heat by radiation. By Stefan-Boltzmann law the net rate of heat loss is proportional to $T^4 - T_0^4$, where T_0 is the temperature of the surroundings.

Step 2: Set up the ratio:

Initial: $R \propto 600^4 - 200^4$. Later: $R' \propto 400^4 - 200^4$.

Take 200 as a common unit: $600 = 3, 400 = 2, 200 = 1$.

Step 3: Compute:

$$\frac{R'}{R} = \frac{2^4 - 1^4}{3^4 - 1^4} = \frac{16 - 1}{81 - 1} = \frac{15}{80} = \frac{3}{16}$$

Step 4: Why the other options are wrong.

$\frac{16}{3}R$ and $\frac{16}{9}R$ are larger than R, but cooling slows as the sphere gets closer to the surrounding temperature. $\frac{9}{16}R$ comes from taking $\frac{3^2}{4^2}$, which is not the Stefan law.

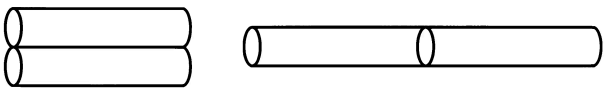
Final Answer:

The new cooling rate is $\frac{3}{16}R$, option (D).

$$\boxed{\frac{3}{16}R}$$

Quick Tip: Rate of cooling by radiation is proportional to T^4 minus To^4.

13. Two rods of same length, radius and material transfer a given amount of heat in 't' second when they are joined as shown in fig (1). But when they are joined as shown in fig (2) then they will transfer same heat in same condition in time



(fig 1) (fig 2)

(A) $2t$

(B) $3t$

(C) $4t$

(D) $6t$

Correct Answer: (C) $4t$

Solution:

Step 1: Understanding the Figures:

Figure 1 shows two identical rods side by side, so heat flows through both at once (parallel). Figure 2 shows the same rods joined end to end (series), so heat must flow through twice the length.

Step 2: Heat flow formula:

Heat conducted in time t is $Q = \frac{KA(\Delta T)}{L}t$. Let one rod have area A and length L.

Step 3: Compare:

Parallel: effective area $2A$, length L, so $Q = \frac{K(2A)\Delta T}{L}t$.

Series: area A, length $2L$, so $Q = \frac{KA\Delta T}{2L}t'$.

Step 4: Equate the same heat Q:

$$\frac{2KA\Delta T}{L}t = \frac{KA\Delta T}{2L}t' \Rightarrow t' = 4t$$

Step 5: Why the other options are wrong.

$2t$ results from changing only the length or only the area. $3t$ and $6t$ do not follow from the factor of 2 in both area and length.

Final Answer:

The time is $4t$, option (C).

$$\boxed{4t}$$

Quick Tip: Parallel doubles the conducting area; series doubles the length. Heat rate ratio is 4.

14. A rigid diatomic gas ($\gamma = \frac{7}{5}$) is compressed adiabatically to volume ($\frac{V_i}{32}$), where V_i is the initial volume. The initial temperature of the gas is 'T_i' K and the final temperature is '$x\,T_i$' K. The value of x is

(A) 2

(B) 3

(C) 4

(D) 5

Correct Answer: (C) 4

Solution:

Step 1: Understanding the Concept:

In an adiabatic process of an ideal gas, $TV^{\gamma-1} = \text{constant}$. Here $\gamma - 1 = \frac{7}{5} - 1 = \frac{2}{5}$.

Step 2: Apply:

$$T_i V_i^{2/5} = T_f \left(\frac{V_i}{32}\right)^{2/5} \Rightarrow T_f = T_i \cdot 32^{2/5}$$

Step 3: Evaluate:

$32 = 2^5$, so $32^{2/5} = 2^{5\times\frac{2}{5}} = 2^2 = 4$. Thus $T_f = 4T_i$, so $x = 4$.

Step 4: Why the other options are wrong.

$x = 2$ would need a volume ratio of $2^{5/2}$, and $x = 3, 5$ are not powers of 2, so they cannot come from $32^{2/5}$.

Final Answer:

The value of x is 4, option (C).

$$\boxed{4}$$

Quick Tip: Use T V^(gamma-1) = constant with gamma = 7/5.

15. An ideal rigid diatomic gas ($\gamma = \frac{7}{5}$) undergoes an adiabatic change . If the relation between temperature and volume is $TV^x = \text{constant}$ then the value of x is

(A) 0.25

(B) 0.30

(C) 0.40

(D) 0.60

Correct Answer: (C) 0.40

Solution:

Step 1: Understanding the Concept:

For an ideal gas undergoing an adiabatic change, $PV^{\gamma} = \text{constant}$.

Using $P = \frac{nRT}{V}$ we get $TV^{\gamma-1} = \text{constant}$.

Step 2: Compute:

For a rigid diatomic gas, $\gamma = \frac{7}{5}$. So

$$x = \gamma - 1 = \frac{7}{5} - 1 = \frac{2}{5} = 0.4$$

Step 3: Why the other options are wrong.

The value 0.25 would result from using $\gamma = 1.25$. Values 0.30 and 0.60 do not match $\gamma - 1$ for $\gamma = 1.4$.

Final Answer:

The value of x is 0.40, option (C).

$$\boxed{0.40}$$

Quick Tip: x equals gamma minus 1.

16. Gas at pressure P, temperature T, and volume V is filled in jar A. Another jar B is filled with gas having parameters $2P$, $\frac{V}{4}$, $2T$. The ratio of number of molecules of jar B to those of jar A is

(A) 1 : 2

(B) 1 : 4

(C) 2 : 1

(D) 4 : 1

Correct Answer: (B) 1 : 4

Solution:

Step 1: Understanding the Concept:

From $PV = NkT$, the number of molecules is $N = \frac{PV}{kT}$, so $N \propto \frac{PV}{T}$.

Step 2: Jar A:

$N_A \propto \frac{PV}{T}$.

Step 3: Jar B:

$N_B \propto \frac{(2P)(V/4)}{2T} = \frac{PV}{4T}$.

Step 4: Ratio:

$$\frac{N_B}{N_A} = \frac{PV/4T}{PV/T} = \frac{1}{4}$$

So $N_B : N_A = 1 : 4$.

Step 5: Why the other options are wrong.

1:2 results from forgetting the extra factor of $\frac{1}{2}$ from the doubled temperature. 2:1 and 4:1 are the inverted ratios.

Final Answer:

The ratio is 1:4, option (B).

$$\boxed{1:4}$$

Quick Tip: Number of molecules is proportional to PV/T.

17. A simple pendulum of length l_1 has periodic time 2.4 s. Another simple pendulum of length $l_2 < l_1$, has periodic time 1.8 s. The periodic time of simple pendulum of length $(l_1 - l_2)$ is nearly

(A) 1.2 s

(B) 1.4 s

(C) 1.6 s

(D) 1.8 s

Correct Answer: (C) 1.6 s

Solution:

Step 1: Understanding the Concept:

For a simple pendulum $T = 2\pi\sqrt{\frac{l}{g}}$, so $l = \frac{gT^2}{4\pi^2}$, meaning $l \propto T^2$.

Step 2: Subtract lengths:

$l_1 - l_2 \propto T_1^2 - T_2^2$. So the period T of the pendulum with length $l_1 - l_2$ satisfies

$$T^2 = T_1^2 - T_2^2 = (2.4)^2 - (1.8)^2 = 5.76 - 3.24 = 2.52$$

Step 3: Take the root:

$T = \sqrt{2.52} \approx 1.59$ s, which is nearly 1.6 s.

Step 4: Why the other options are wrong.

1.2 s is $2.4 - 1.8$ taken directly as a difference in periods, which is wrong because T is not linear in length. 1.4 s and 1.8 s do not equal $\sqrt{2.52}$.

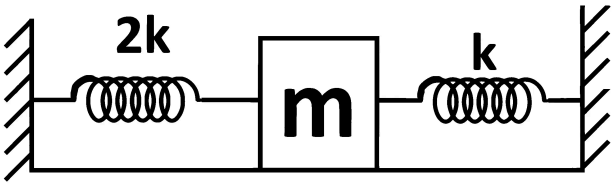
Final Answer:

The period is nearly 1.6 s, option (C).

$$\boxed{1.6 \text{ s}}$$

Quick Tip: T squared is proportional to length, so lengths subtract in T squared.

18. Two springs of force constants '$2k$' and 'k' are connected to a mass 'm' as shown. Mass is displaced slightly to one side and released. The frequency of oscillation of the two springs-mass system is



(A) $\frac{1}{2\pi}\sqrt{\frac{m}{k}}$

(B) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$

(C) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

(D) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$

Correct Answer: (D) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$

Solution:

Step 1: Understanding the Figure:

The mass is attached to a spring of constant $2k$ on the left wall and a spring of constant k on the right wall. When the mass moves, one spring stretches while the other compresses, and both give a restoring force in the same direction.

Step 2: Effective constant:

The two springs act in parallel, so $k_{\text{eff}} = 2k + k = 3k$.

Step 3: Frequency:

$$f = \frac{1}{2\pi}\sqrt{\frac{k_{\text{eff}}}{m}} = \frac{1}{2\pi}\sqrt{\frac{3k}{m}}$$

Step 4: Why the other options are wrong.

$\frac{1}{2\pi}\sqrt{\frac{m}{k}}$ has m and k inverted. Options $\sqrt{\frac{k}{m}}$ and $\sqrt{\frac{2k}{m}}$ use only one spring or a wrong sum.

Final Answer:

The frequency is $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$, option (D).

$$\boxed{\frac{1}{2\pi}\sqrt{\frac{3k}{m}}}$$

Quick Tip: Springs on both sides of a mass act in parallel, so their constants add.

19. Two oscillating simple pendulums with time periods T and $\frac{4T}{3}$ are in phase at a given time. They will be again in phase after an elapse of time

(A) $5T$

(B) $4T$

(C) $3T$

(D) $2T$

Correct Answer: (B) $4T$

Solution:

Step 1: Understanding the Concept:

Two oscillators are in phase when each has completed a whole number of oscillations at the same instant. The first time this happens after the start is the least common multiple of their periods.

Step 2: Set up:

Periods are T and $\frac{4T}{3}$. We need $nT = m \cdot \frac{4T}{3}$ for the smallest whole numbers n, m.

Step 3: Solve:

$\frac{n}{m} = \frac{4}{3}$, so $n = 4, m = 3$. The time elapsed is $4T$.

Step 4: Check:

In time $4T$ the first pendulum completes 4 oscillations. The second has period $\frac{4T}{3}$, so it completes $\frac{4T}{4T/3} = 3$ oscillations. Both are whole numbers.

Step 5: Why the other options are wrong.

At $2T$ and $3T$, the second pendulum has completed 1.5 and 2.25

oscillations. At $5T$ it has completed 3.75.

Final Answer:

They are in phase again after $4T$, option (B).

$$\boxed{4T}$$

Quick Tip: They are in phase again when the shorter period has completed one more oscillation than the longer.

20. A wire of length 'L', diameter 'd', density of material 'ρ' is under tension 'T' has fundamental frequency of vibration 'n_A'. Another wire of length '$2L$', diameter '$3d$', density of material '2ρ' is vibrated under tension '$2T$', the fundamental frequency of vibration becomes 'n_B'. The ratio $n_B : n_A$ is

(A) 1 : 2

(B) 1 : 4

(C) 1 : 6

(D) 1 : 8

Correct Answer: (C) 1 : 6

Solution:

Step 1: Understanding the Concept:

The fundamental frequency of a stretched wire is $n = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$, where $\mu = \rho \cdot \frac{\pi d^2}{4}$ is the mass per unit length.

Step 2: Write the dependence:

$$n \propto \frac{1}{L}\sqrt{\frac{T}{\rho d^2}}$$

Step 3: Ratio:

Wire A: L, T, ρ, d. Wire B: $2L, 2T, 2\rho, 3d$.

$$\frac{n_B}{n_A} = \frac{L}{2L}\sqrt{\frac{2T}{T}\cdot\frac{\rho}{2\rho}\cdot\frac{d^2}{9d^2}} = \frac{1}{2}\sqrt{2\cdot\frac{1}{2}\cdot\frac{1}{9}} = \frac{1}{2}\cdot\frac{1}{3} = \frac{1}{6}$$

Step 4: Why the other options are wrong.

1:2 comes from using only the length ratio. 1:4 and 1:8 do not come out of the ratios of tension, density and diameter together.

Final Answer:

The ratio $n_B : n_A$ is 1:6, option (C).

$$\boxed{1:6}$$

Quick Tip: n = (1/2L) sqrt(T/mu) with mass per unit length mu = rho pi d squared over 4.

21. The 'Loudness' and 'Pitch' of sound respectively are the human perception to

(A) intensity of sound and frequency of sound.

(B) intensity of sound and velocity of sound.

(C) frequency of sound and velocity of sound.

(D) frequency of sound and intensity of sound.

Correct Answer: (A) intensity of sound and frequency of sound.

Solution:

Step 1: Understanding the Concept:

Loudness and pitch are the subjective sensations produced in the ear by physical properties of a sound wave.

Step 2: Detailed Explanation:

Loudness is how strong a sound feels, and it depends on the intensity (amplitude) of the sound wave. Pitch is how high or low a sound feels, and it depends on the frequency of the wave: higher frequency gives a higher pitch.

Step 3: Why the other options are wrong.

The speed of sound depends on the medium (its elasticity and density) and not on how we perceive pitch or loudness. So any option that links loudness or pitch to velocity of sound is incorrect.

Final Answer:

Loudness relates to intensity and pitch to frequency, option (A).

Intensity and frequency (A)

Quick Tip: Loudness relates to intensity; pitch relates to frequency.

22. A tuning fork of frequency 'n' is held near the open end of a tube which is dipped in water and length of the tube is adjusted until resonance occurs. If the two shortest lengths that produce resonance are l_1 and l_2, the speed of sound in air is (neglect end correction)

(A) $2n(l_2 - l_1)$

(B) $n(l_2 - l_1)$

(C) $\frac{n}{2}(l_2 - l_1)$

(D) $\frac{2n}{(l_2 - l_1)}$

Correct Answer: (A) $2n(l_2 - l_1)$

Solution:

Step 1: Understanding the Concept:

A tube closed at one end (by water) resonates when its air column length is an odd multiple of $\frac{\lambda}{4}$. The first two shortest resonating lengths are $l_1 = \frac{\lambda}{4}$ and $l_2 = \frac{3\lambda}{4}$ (neglecting end correction).

Step 2: Subtract:

$$l_2 - l_1 = \frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{\lambda}{2} \Rightarrow \lambda = 2(l_2 - l_1)$$

Step 3: Speed:

$$v = n\lambda = 2n(l_2 - l_1)$$

Step 4: Why the other options are wrong.

$n(l_2 - l_1)$ would be the speed if $l_2 - l_1$ were a full wavelength. $\frac{n}{2}(l_2$

$-l_1$) and $\frac{2n}{l_2-l_1}$ are not valid: the last has wrong units.

Final Answer:

The speed of sound is $2n(l_2 - l_1)$, option (A).

$$\boxed{2n(l_2 - l_1)}$$

Quick Tip: Consecutive resonances in a closed tube differ by half a wavelength.

23. The ratio of the speed of sound in helium gas to that in nitrogen gas at 295 K is (Ratio of specific heats γ for helium is $\frac{5}{3}$ and that for nitrogen is $\frac{7}{5}$. Molecular weight for helium and nitrogen are 4 and 28 respectively.)

(A) $\sqrt{\frac{2}{7}}$

(B) $\sqrt{\frac{3}{5}}$

(C) $\frac{\sqrt{3}}{5}$

(D) $\frac{5}{\sqrt{3}}$

Correct Answer: (D) $\frac{5}{\sqrt{3}}$

Solution:

Step 1: Understanding the Concept:

The speed of sound in an ideal gas is $v = \sqrt{\frac{\gamma RT}{M}}$. At the same temperature, $v \propto \sqrt{\frac{\gamma}{M}}$.

Step 2: Compute the ratio:

$$\frac{v_{He}}{v_{N_2}} = \sqrt{\frac{\gamma_{He}}{\gamma_{N_2}} \cdot \frac{M_{N_2}}{M_{He}}} = \sqrt{\frac{5/3}{7/5} \cdot \frac{28}{4}}$$

$\frac{5/3}{7/5} = \frac{25}{21}$, and multiplying by 7 gives $\frac{25\times7}{21} = \frac{25}{3}$.

Step 3: Take the root:

$$\frac{v_{He}}{v_{N_2}} = \sqrt{\frac{25}{3}} = \frac{5}{\sqrt{3}}$$

Step 4: Why the other options are wrong.

$\sqrt{\frac{2}{7}}$ and $\sqrt{\frac{3}{5}}$ are less than 1, but helium, being much lighter, carries sound faster. $\frac{\sqrt{3}}{5}$ is the inverted ratio.

Final Answer:

The ratio is $\frac{5}{\sqrt{3}}$, option (D).

$$\boxed{\frac{5}{\sqrt{3}}}$$

Quick Tip: Speed of sound in a gas is sqrt(gamma R T / M).

24. A train is moving towards a stationary observer with speed 34 m/s. A train sounds a whistle of frequency 450 Hz. If the speed of sound is 340 m/s, the frequency heard by the observer in Hz is

(A) 440

(B) 480

(C) 500

(D) 540

Correct Answer: (C) 500

Solution:

Step 1: Understanding the Concept:

When a source moves toward a stationary observer, the observed frequency increases because the wavefronts are compressed.

Step 2: Formula:

$$f' = f \cdot \frac{v}{v - v_s}$$

Step 3: Compute:

$$f' = 450 \times \frac{340}{340 - 34} = 450 \times \frac{340}{306} = 450 \times \frac{10}{9} = 500 \text{ Hz}$$

Step 4: Why the other options are wrong.

440 Hz is lower than the source frequency, which would be heard if the train were moving away. 480 Hz and 540 Hz do not match $\frac{340}{306}$, and 540 Hz would be 450×1.2.

Final Answer:

The frequency heard is 500 Hz, option (C).

500 Hz

Quick Tip: Source moving toward a stationary observer: f = fo v/(v - vs).

25. Four electric charges $+q$, $+q$, $-q$, and $-q$ are placed in order at the corners of a square of side '$2r$'. The electric potential at a point midway between the two negative charges is

(A) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r}[\frac{1}{\sqrt{5}}-1]$

(B) $\frac{1}{4\pi\varepsilon_0}\frac{q}{r}[\frac{1}{\sqrt{5}}+1]$

(C) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r}[1-\sqrt{5}]$

(D) $\frac{1}{4\pi\varepsilon_0}\frac{q}{r}[1+\sqrt{5}]$

Correct Answer: (A) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r}[\frac{1}{\sqrt{5}}-1]$

Solution:

Step 1: Understanding the Figure:

The charges $+q, +q, -q, -q$ are placed in order at the corners of a square of side $2r$. The two negative charges are at adjacent corners, so the midpoint P of the side joining them is at distance r from each.

Step 2: Distances to the positive charges:

Each positive charge sits at the corner adjacent to one negative charge, across the side. From P, the distance to such a corner is $\sqrt{(2r)^2+r^2}=r\sqrt{5}$.

Step 3: Add potentials:

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{-q}{r} + \frac{-q}{r} + \frac{q}{r\sqrt{5}} + \frac{q}{r\sqrt{5}}\right] = \frac{1}{4\pi\varepsilon_0} \cdot \frac{2q}{r}\left[\frac{1}{\sqrt{5}} - 1\right]$$

Step 4: Why the other options are wrong.

Options (B) and (D) have $\frac{q}{r}$ with a plus sign inside the bracket, which would make the potential positive, but the nearer charges are negative. Option (C) has $1 - \sqrt{5}$, which comes from taking the distances the wrong way round.

Final Answer:

The potential is $\frac{1}{4\pi\varepsilon_0}\frac{2q}{r}\left[\frac{1}{\sqrt{5}} - 1\right]$, option (A).

$$\boxed{\frac{1}{4\pi\varepsilon_0}\frac{2q}{r}\left[\frac{1}{\sqrt{5}} - 1\right]}$$

Quick Tip: Add the potentials of the four charges at the midpoint of one side; distances are r and r root 5.

26. Two point charges q_1 and q_2 are 'l' distance apart. If one of the charges is doubled and the distance between them is halved. The magnitude of the force becomes 'n' times, where 'n' is

(A) 2

(B) 4

(C) 8

(D) 16

Correct Answer: (C) 8

Solution:

Step 1: Understanding the Concept:

Coulomb's law gives $F = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{l^2}$. The force is proportional to the product of the charges and inversely proportional to the square of the distance.

Step 2: Apply the changes:

One charge doubles, so the product $q_1 q_2$ becomes $2q_1 q_2$. The distance becomes $\frac{l}{2}$, so l^2 becomes $\frac{l^2}{4}$.

$$F' = \frac{1}{4\pi\varepsilon_0}\frac{2q_1 q_2}{l^2/4} = 8F$$

Step 3: Result:

The force becomes 8 times, so $n = 8$.

Step 4: Why the other options are wrong.

$n = 2$ counts only the doubled charge. $n = 4$ counts only the halved distance. $n = 16$ would need both charges doubled with the distance halved.

Final Answer:

The force becomes 8 times, option (C).

$$\boxed{8}$$

Quick Tip: Coulomb's force varies directly with q1 q2 and inversely with r

squared.

27. In a parallel plate air capacitor of plate separation 'd', a dielectric slab of thickness 't' is introduced between the plates. The capacitance becomes one-third of the original value. The dielectric constant of the slab will be

(A) $\frac{t}{d+t}$

(B) $\frac{t}{2d+t}$

(C) $\frac{t}{d-2t}$

(D) $\frac{2t}{2d-t}$

Correct Answer: (B) $\frac{t}{2d+t}$

Solution:

Step 1: Understanding the Concept:

With a dielectric slab of thickness t and constant K between plates a distance d apart, the capacitance is

$$C' = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$$

Without the slab, $C = \frac{\varepsilon_0 A}{d}$.

Step 2: Use the given ratio:

$C' = \frac{C}{3}$ means $d - t + \frac{t}{K} = 3d$.

Step 3: Solve for K:

$$\frac{t}{K} = 3d - d + t = 2d + t \Rightarrow K = \frac{t}{2d + t}$$

Step 4: Why the other options are wrong.

$\frac{t}{d+t}$ would correspond to a capacitance ratio of $\frac{1}{2}$. The other two forms do not satisfy $d - t + \frac{t}{K} = 3d$.

Final Answer:

The dielectric constant is $\frac{t}{2d+t}$, option (B).

$$\boxed{\frac{t}{2d+t}}$$

Quick Tip: With a slab, C = eo A / (d - t + t/K).

28. A parallel plate air capacitor having area of each plate 'A' and the distance between the plates 'd' has uniform electric field E in the space between the plates. The energy stored in the capacitor is (ε_0 = permittivity of free space.)

(A) $\frac{1}{2}\varepsilon_0 E^2$

(B) $\frac{1}{2}A\varepsilon_0 E$

(C) $\frac{1}{2}A\varepsilon_0 E^2 d$

(D) $\frac{1}{2}A\varepsilon_0^2 E \cdot d$

Correct Answer: (C) $\frac{1}{2}A\varepsilon_0 E^2 d$

Solution:

Step 1: Understanding the Concept:

Energy stored in a capacitor is $U = \frac{1}{2}CV^2$. For a parallel plate capacitor $C = \frac{\varepsilon_0 A}{d}$ and $V = Ed$.

Step 2: Substitute:

$$U = \frac{1}{2} \cdot \frac{\varepsilon_0 A}{d} \cdot (Ed)^2 = \frac{1}{2}\varepsilon_0 A E^2 d$$

Step 3: Interpretation:

This equals the energy density $\frac{1}{2}\varepsilon_0 E^2$ multiplied by the volume Ad between the plates.

Step 4: Why the other options are wrong.

$\frac{1}{2}\varepsilon_0 E^2$ is only the energy per unit volume. $\frac{1}{2}A\varepsilon_0 E$ and $\frac{1}{2}A\varepsilon_0^2 Ed$ have wrong powers of E and ε_0.

Final Answer:

The stored energy is $\frac{1}{2}A\varepsilon_0 E^2 d$, option (C).

$$\boxed{\frac{1}{2}A\varepsilon_0 E^2 d}$$

Quick Tip: Energy density is (1/2) eo E squared; multiply by volume A d.

29. In meter bridge experiment, two resistances X and Y in the two gaps give a null point dividing the wire in the ratio $2 : 3$. When each resistance is increased by $30\,\Omega$, the null point divides the wire in the ratio $5 : 6$. The resistance X and Y are respectively

(A) $20\,\Omega$, $30\,\Omega$

(B) $22\,\Omega$, $33\,\Omega$

(C) $32\,\Omega$, $48\,\Omega$

(D) $40\,\Omega$, $60\,\Omega$

Correct Answer: (A) $20\,\Omega$, $30\,\Omega$

Solution:

Step 1: Understanding the Concept:

In a meter bridge the null point divides the wire in the ratio of the resistances in the two gaps. Here the null point divides the wire in the ratio $2:3$ in the first case, and $5:6$ when each resistance is increased by $30\,\Omega$.

Step 2: First balance:

$\frac{X}{Y} = \frac{2}{3}$, so let $X = 2k$ and $Y = 3k$.

Step 3: Second balance:

$\frac{X+30}{Y+30} = \frac{5}{6}$:

$$6(2k + 30) = 5(3k + 30) \Rightarrow 12k + 180 = 15k + 150 \Rightarrow k = 10$$

Step 4: Resistances:

$X = 20\,\Omega$ and $Y = 30\,\Omega$.

Step 5: Why the other options are wrong.

The pairs 22 and 33, 32 and 48, and 40 and 60 do keep the ratio 2:3, but adding 30 gives ratios $\frac{52}{63}$, $\frac{62}{78}$ and $\frac{70}{90}$, none equal to $\frac{5}{6}$.

Final Answer:

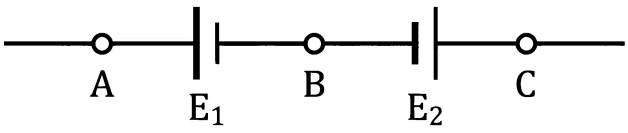
X = 20 ohm and Y = 30 ohm, option (A).

$$\boxed{X = 20\,\Omega,\ Y = 30\,\Omega}$$

Quick Tip: Balance condition: X/Y = l/(100 - l), equivalent to the ratio of the wire parts.

30.

Two cells of e.m.f. E_1 and E_2 ($E_1 > E_2$) are connected as shown in figure.



When a potentiometer is connected between points A and B the balancing length of potentiometer wire is 412 cm. When same potentiometer wire is connected between points A and C the balancing length is 103 cm. The ratio $E_1 : E_2$ is

(A) 6 : 1

(B) 4 : 1

(C) 4 : 3

(D) 3 : 4

Correct Answer: (C) 4 : 3

Solution:

Step 1: Understanding the Figure:

The circuit is a chain A, cell E_1, point B, cell E_2, point C. In the

symbols, the longer line is the positive terminal. E_1 has its positive side toward A, and E_2 has its positive side toward C, so the two cells are connected in opposition at B.

Step 2: Potential differences:

Across AB only E_1 acts, so the balancing length is 412 cm for E_1. Across AC the net emf is $E_1 - E_2$, balanced at 103 cm.

Step 3: Use proportionality to length:

$\frac{E_1}{E_1 - E_2} = \frac{412}{103} = 4$. So $E_1 = 4E_1 - 4E_2$, giving $4E_2 = 3E_1$.

Step 4: Ratio:

$$E_1 : E_2 = 4 : 3$$

Step 5: Why the other options are wrong.

6:1 and 4:1 would result from assuming $E_1 + E_2$ or E_2 alone is balanced. 3:4 is the inverse, which would violate $E_1 > E_2$.

Final Answer:

The ratio $E_1 : E_2$ is 4:3, option (C).

$$\boxed{4 : 3}$$

Quick Tip: The cells oppose each other, so A to C measures E1 minus E2.

31. The periodic time of a magnet in vibration magnetometer is T_0. If a magnet is replaced by another magnet whose moment of inertia is three times and magnetic moment is one-third of the original magnet, then time period of new magnet will be

(A) $\frac{T_0}{3}$

(B) $\frac{T_0}{\sqrt{3}}$

(C) $3\,T_0$

(D) $\sqrt{3}\,T_0$

Correct Answer: (C) $3\,T_0$

Solution:

Step 1: Understanding the Concept:

A magnet oscillating in a vibration magnetometer has time period $T = 2\pi\sqrt{\frac{I}{MB_H}}$, where I is the moment of inertia, M the magnetic moment and B_H the horizontal field.

Step 2: Apply the changes:

New $I' = 3I$ and $M' = \frac{M}{3}$:

$$T' = 2\pi\sqrt{\frac{3I}{(M/3)B_H}} = 2\pi\sqrt{\frac{9I}{MB_H}} = 3T_0$$

Step 3: Why the other options are wrong.

$\frac{T_0}{3}$ and $\frac{T_0}{\sqrt{3}}$ invert the effect. $\sqrt{3}\,T_0$ comes from changing only I or only M, but both changes together multiply $\frac{I}{M}$ by 9.

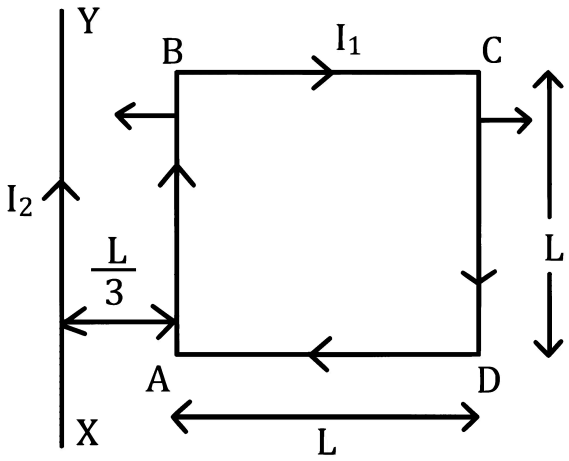
Final Answer:

The new time period is $3T_0$, option (C).

$\boxed{3T_0}$

Quick Tip: T = 2 pi sqrt(I / (M B)).

32. A square loop ABCD of side L carrying a current I_1 is placed at a distance $(L/3)$ from a conductor coplaner with a straight conductor XY carrying current I_2 as shown in figure. The net force on the loop will be (μ_0 = magnetic permeability)



(A) $\frac{\mu_0 I_1 I_2}{3\pi}$

(B) $\frac{3\mu_0 I_1 I_2}{8\pi}$

(C) $\frac{9\mu_0 I_1 I_2}{8\pi}$

(D) $\frac{3\mu_0 I_1 I_2}{4\pi}$

Correct Answer: (C) $\frac{9\mu_0 I_1 I_2}{8\pi}$

Solution:

Step 1: Understanding the Figure:

The wire XY carries I_2 upward. In the loop, side AB (nearest the wire) carries I_1 upward, so its current is parallel to I_2 and is attracted

toward the wire. Side CD (far side) carries I_1 downward, antiparallel to I_2, so it is repelled away from the wire. The forces on the sides BC and DA are equal and opposite and cancel.

Step 2: Force on AB (distance $\frac{L}{3}$):

$$F_{AB} = \frac{\mu_0 I_1 I_2}{2\pi (L/3)} \cdot L = \frac{3\mu_0 I_1 I_2}{2\pi}$$

directed toward the wire.

Step 3: Force on CD (distance $\frac{L}{3} + L = \frac{4L}{3}$):

$$F_{CD} = \frac{\mu_0 I_1 I_2}{2\pi (4L/3)} \cdot L = \frac{3\mu_0 I_1 I_2}{8\pi}$$

directed away from the wire.

Step 4: Net force:

The two act in opposite directions, so

$$F = \frac{3\mu_0 I_1 I_2}{2\pi} - \frac{3\mu_0 I_1 I_2}{8\pi} = \frac{12-3}{8\pi}\mu_0 I_1 I_2 = \frac{9\mu_0 I_1 I_2}{8\pi}$$

This is directed toward the wire.

Step 5: Why the other options are wrong.

$\frac{3\mu_0 I_1 I_2}{4\pi}$ and $\frac{3\mu_0 I_1 I_2}{8\pi}$ are the sum or only the far-side force. $\frac{\mu_0 I_1 I_2}{3\pi}$ does not match either side.

Final Answer:

The net force is $\frac{9\mu_0 I_1 I_2}{8\pi}$, option (C).

$$\boxed{\frac{9\mu_0 I_1 I_2}{8\pi}}$$

Quick Tip: Parallel currents attract and opposite currents repel; the near side wins.

33. A long solenoid carrying a current produces a magnetic field 'B' along its axis. If the currunt is tripled and the number of turns per cm is halved then new value of magnetic field will be

(A) $\frac{B}{2}$

(B) B

(C) $\frac{3}{2}B$

(D) $3B$

Correct Answer: (C) $\frac{3}{2}B$

Solution:

Step 1: Understanding the Concept:

The magnetic field along the axis inside a long solenoid is $B = \mu_0 nI$, where n is the number of turns per unit length.

Step 2: Apply the changes:

The current becomes $3I$ and the turns per unit length become $\frac{n}{2}$:

$$B' = \mu_0 \cdot \frac{n}{2} \cdot 3I = \frac{3}{2}\mu_0 nI = \frac{3}{2}B$$

Step 3: Why the other options are wrong.

$\frac{B}{2}$ accounts only for the halved turns. $3B$ accounts only for the tripled current. B would need the two changes to cancel, which is not the case since $3 \times \frac{1}{2} = \frac{3}{2}$.

Final Answer:

The new field is $\frac{3}{2}B$, option (C).

$$\boxed{\frac{3}{2}B}$$

Quick Tip: B = muo n I inside a long solenoid.

34. The self inductance of an air-core inductor (solenoid) is 0.03 mH. By introducing an iron core into inductor the self-inductance increases to 30 mH. The relative permeability of the core used is

(A) 10^{-3}

(B) 10^{-2}

(C) 10^{2}

(D) 10^{3}

Correct Answer: (D) 10^{3}

Solution:

Step 1: Understanding the Concept:

The self inductance of a solenoid is $L = \mu n^2 Al$. Replacing air (μ_0) by an iron core ($\mu = \mu_r \mu_0$) multiplies L by μ_r, when the geometry stays the same.

Step 2: Compute:

$$\mu_r = \frac{L_{\text{iron}}}{L_{\text{air}}} = \frac{30\ \text{mH}}{0.03\ \text{mH}} = 1000 = 10^3$$

Step 3: Why the other options are wrong.
10^{-3} and 10^{-2} would make the inductance decrease. 10^2 would give only 3 mH.

Final Answer:
The relative permeability is 10^3, option (D).

$$\boxed{10^3}$$

Quick Tip: Inductance with a core is mu_r times the air-core value.

35. A coil of effective area 3 m^2 is placed at right angles to a magnetic field of induction 0.05 Wb/m^2. If the field is decreased to 20% of its original value in 10 second, the e.m.f. induced in the coil will be

(A) 15 mV

(B) 12 mV

(C) 10 mV

(D) 5 mV

Correct Answer: (B) 12 mV

Solution:

Step 1: Understanding the Concept:
By Faraday's law, the induced emf is $|\varepsilon| = \frac{\Delta\Phi}{\Delta t}$. With the coil

perpendicular to the field, $\Phi = BA$.

Step 2: Compute the change:
Initial $B = 0.05$ Wb/m^2. Final $B = 0.2 \times 0.05 = 0.01$ Wb/m^2. So $\Delta B = 0.04$ Wb/m^2.

Step 3: Find the emf:

$$\varepsilon = \frac{A\,\Delta B}{\Delta t} = \frac{3 \times 0.04}{10} = 0.012\ \text{V} = 12\ \text{mV}$$

Step 4: Why the other options are wrong.
15 mV would come from $\Delta B = 0.05$ (field dropping to zero). 10 mV and 5 mV do not match the area and field change.

Final Answer:
The induced emf is 12 mV, option (B).

$$\boxed{12\ \text{mV}}$$

Quick Tip: emf = A times change in B divided by time.

36. In a series LCR circuit, the voltage across R is 100 V, $R = 1\,\text{K}\Omega$ and $C = 2\,\mu\text{F}$. The angular frequency ω is 200 rad s^{-1}. At resonance, the voltage across 'L' is

(A) 150 V

(B) 200 V

(C) 250 V

(D) 300 V

Correct Answer: (C) 250 V

Solution:

Step 1: Understanding the Concept:

At resonance of a series LCR circuit, the inductive reactance equals the capacitive reactance, $X_L = X_C$. The impedance is just R, and the current is $I = \frac{V_R}{R}$.

Step 2: Find the current:

$I = \frac{100}{1000} = 0.1$ A.

Step 3: Find the reactance:

$$X_C = \frac{1}{\omega C} = \frac{1}{200 \times 2 \times 10^{-6}} = \frac{1}{4 \times 10^{-4}} = 2500\ \Omega$$

At resonance $X_L = X_C = 2500\ \Omega$.

Step 4: Voltage across L:

$$V_L = IX_L = 0.1 \times 2500 = 250\ \text{V}$$

Step 5: Why the other options are wrong.

150 V, 200 V and 300 V would require X_L of 1500, 2000 or 3000 Ω, which do not match $\frac{1}{\omega C} = 2500\ \Omega$.

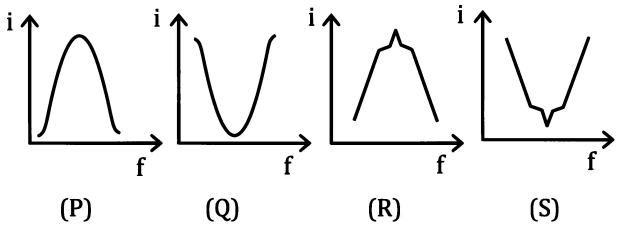
Final Answer:

The voltage across L is 250 V, option (C).

$$\boxed{250\ \text{V}}$$

Quick Tip: At resonance X_L = X_C, so V_L = I X_C with I = V_R / R.

37. Which graph shows the correct variation of r.m.s. current 'i' with frequency 'f' of a.c. in case of series resonant circuit ?



(A) (R)

(B) (P)

(C) (S)

(D) (Q)

Correct Answer: (B) (P)

Solution:

Step 1: Understanding the Concept:

In a series LCR circuit the impedance is $Z = \sqrt{R^2 + (X_L - X_C)^2}$. It is smallest, equal to R, at the resonant frequency where $X_L = X_C$.

Step 2: Effect on current:

$i_{rms} = \frac{V}{Z}$. At low frequency X_C is large, so Z is large and the current is small. At resonance Z is minimum, so the current is maximum. At high frequency X_L is large and the current falls again.

Step 3: Read the graphs:
Graph (P) shows a smooth bell-shaped curve with one maximum in the middle, rising from low values and falling again. This is the required shape.

Step 4: Why the other graphs are wrong.
Graphs (Q) and (S) are valleys, showing a minimum of current, which is the behaviour of a parallel resonant circuit. Graph (R) has a sharp spike on top of straight slopes, which is not the smooth variation of $\frac{V}{Z}$.

Final Answer:
Graph (P) is correct, option (B).

$$\boxed{\text{(P)}}$$

Quick Tip: Current rises to a maximum at resonance and falls on both sides, so the graph is a smooth peak.

38. When an inductance 'L' and resistor 'R' are connected in series to 25V, 50 Hz supply, a current of 0.5 A flows in the circuit. The current lags behind in phase from applied voltage by $(\frac{\pi}{3})$ radian. The value of R is $(cos60° = \frac{1}{2})$

(A) $20\,\Omega$

(B) $25\,\Omega$

(C) $40\,\Omega$

(D) $50\,\Omega$

Correct Answer: (B) $25\,\Omega$

Solution:

Step 1: Understanding the Concept:

For a series LR circuit, the impedance is $Z = \frac{V_{rms}}{I_{rms}}$, and the resistance is $R = Z\cos\phi$, where ϕ is the phase angle between voltage and current.

Step 2: Find the impedance:

$Z = \frac{25}{0.5} = 50\,\Omega$.

Step 3: Find R:

$$R = Z\cos\frac{\pi}{3} = 50 \times \frac{1}{2} = 25\,\Omega$$

Step 4: Why the other options are wrong.

$50\,\Omega$ is the full impedance (the value if ϕ were zero). $20\,\Omega$ and $40\,\Omega$ do not match $Z\cos 60°$.

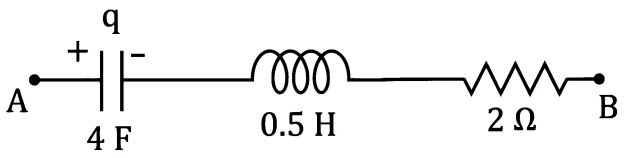
Final Answer:

R is $25\,\Omega$, option (B).

$$\boxed{25\,\Omega}$$

Quick Tip: Z = V/I and R = Z cos(phi).

39. In the circuit shown charge q varies with time t as $q = t^2 - 5$, where q is in coulomb and t is in second. At time $t = 3$ second, voltage V_{AB} in volt will be



(A) 8

(B) 12

(C) 14

(D) 18

Correct Answer: (C) 14

Solution:

Step 1: Understanding the Figure:

From A to B the circuit has a 4 F capacitor with its positive plate toward A, then a 0.5 H inductor, then a 2 Ω resistor. The charge on the positive plate is $q = t^2 - 5$ coulomb.

Step 2: Current:

The charge on the plate connected to A is increasing, so the current flows from A into the capacitor and on through L and R toward B. Its value is

$$I = \frac{dq}{dt} = 2t$$

At $t = 3$ s, $I = 6$ A, and $\frac{dI}{dt} = 2$ A/s.

Step 3: Voltage drops from A to B:
Capacitor: $\frac{q}{C} = \frac{3^2-5}{4} = \frac{4}{4} = 1$ V.
Inductor: $L\frac{dI}{dt} = 0.5 \times 2 = 1$ V.
Resistor: $IR = 6 \times 2 = 12$ V.

Step 4: Add them:

$$V_{AB} = 1 + 1 + 12 = 14 \text{ V}$$

Step 5: Why the other options are wrong.
8 V, 12 V and 18 V come from leaving out one drop or using $q = t^2 - 5$ at the wrong time. For example 12 V is the resistor drop alone.

Final Answer:
$V_{AB} = 14$ V, option (C).

$$\boxed{14 \text{ V}}$$

Quick Tip: The current is dq/dt; add the drops across C, L and R along the current direction.

40. The prism has refracting angle A. The second refracting surface of the prism is silvered. Light ray falling on first refracting surface with angle of incidence $2A$, reaches the second surface and returns back through same path due to reflection at the silvered surface. The refractive index of the material of prism is

(A) $2sinA$

(B) $2cosA$

(C) $\frac{1}{2}sinA$

(D) $\frac{1}{2}cosA$

Correct Answer: (B) $2cosA$

Solution:

Step 1: Understanding the Concept:

The ray returns along the same path when it strikes the silvered second surface at normal incidence, so that it is reflected back on itself.

Step 2: Angle conditions:

For a prism, $r_1 + r_2 = A$. If the ray falls normally on the second surface, $r_2 = 0$, so $r_1 = A$.

Step 3: Apply Snell's law at the first surface:

$$\mu = \frac{\sin i}{\sin r_1} = \frac{\sin 2A}{\sin A} = \frac{2\sin A\cos A}{\sin A} = 2\cos A$$

Step 4: Why the other options are wrong.

$2\sin A$ would result from $\sin 2A$ divided by $\cos A$. Options with a factor $\frac{1}{2}$ cannot be greater than 1 for typical prism angles, whereas the

refractive index must exceed 1.

Final Answer:

The refractive index is $2\cos A$, option (B).

$$\boxed{2\cos A}$$

Quick Tip: The ray retraces its path only if it hits the silvered face normally, so r1 = A.

41. Light of wavelength 580 nm is incident normally on a slit of width 'a'. The distance between slit and screen is 2.5 m and the distance of the second order maximum from the centre of the screen is 14.5 mm in a diffraction pattern. The value of 'a' is

(A) 0.12×10^{-3} m

(B) 0.25×10^{-3} m

(C) 0.36×10^{-3} m

(D) 0.50×10^{-3} m

Correct Answer: (B) 0.25×10^{-3} m

Solution:

Step 1: Understanding the Concept:

In single slit diffraction, secondary maxima occur at $a\sin\theta = \left(n + \frac{1}{2}\right)\lambda$, where $n = 1, 2, \ldots$. For the second order maximum, $n = 2$, so $a\sin\theta = \frac{5}{2}\lambda$.

Step 2: Small angles:

$\sin\theta \approx \frac{y}{D}$. So

$$a \cdot \frac{y}{D} = \frac{5\lambda}{2} \Rightarrow a = \frac{5\lambda D}{2y}$$

Step 3: Substitute:

$\lambda = 580 \times 10^{-9}$ m, $D = 2.5$ m, $y = 14.5 \times 10^{-3}$ m.

$$a = \frac{5 \times 580 \times 10^{-9} \times 2.5}{2 \times 14.5 \times 10^{-3}} = \frac{7.25 \times 10^{-6}}{29 \times 10^{-3}} = 2.5 \times 10^{-4} \text{ m}$$

Step 4: Result:

$a = 0.25 \times 10^{-3}$ m.

Step 5: Why the other options are wrong.

0.12×10^{-3} m would follow from using $a\sin\theta = 5\lambda$-type slips, and 0.5×10^{-3} m from using 2λ instead of $\frac{5}{2}\lambda$.

Final Answer:

The slit width is 0.25×10^{-3} m, option (B).

$$\boxed{0.25 \times 10^{-3} \text{ m}}$$

Quick Tip: Secondary maxima of a single slit lie at a sin(theta) = (n + 1/2) lambda; the second one has n = 2.

42. In an interference experiment, the m^{th} bright fringe for light of wavelength λ_1 coincides with the n^{th} dark fringe for light of wavelength λ_2 . The ratio $\frac{\lambda_2}{\lambda_1}$ is

(A) $\frac{m}{n-1}$

(B) $\frac{m}{(2n-1)}$

(C) $\frac{2m}{(2n-1)}$

(D) $\frac{2m}{(2n+1)}$

Correct Answer: (C) $\frac{2m}{(2n-1)}$

Solution:

Step 1: Understanding the Concept:

In Young's experiment, the position of the m^{th} bright fringe is $y = \frac{m\lambda D}{d}$. The position of the n^{th} dark fringe is $y = \frac{(2n-1)\lambda D}{2d}$.

Step 2: Equate the positions:

$$\frac{m\lambda_1 D}{d} = \frac{(2n-1)\lambda_2 D}{2d}$$

Step 3: Solve:

$$\frac{\lambda_2}{\lambda_1} = \frac{2m}{2n-1}$$

Step 4: Why the other options are wrong.

$\frac{m}{n-1}$ and $\frac{m}{2n-1}$ lack the factor 2 from the dark fringe formula. $\frac{2m}{2n+1}$ uses $2n+1$, which is the formula for the dark fringe counted from $n = 0$, not from $n = 1$.

Final Answer:

The ratio is $\frac{2m}{2n-1}$, option (C).

$$\boxed{\frac{2m}{2n-1}}$$

Quick Tip: Bright fringe: y = m lambda D/d; dark fringe: y = (2n - 1) lambda D/(2d).

43. In a biprism experiment, the distance between 4^{th} and 13^{th} bright band on the same side is 'y' when light of wavelength 6000 Å is used. For a light of wavelength 'λ' the distance between 6^{th} and 16^{th} bright band on the same side is again 'y'. The value of 'λ' in Å units is

(A) 6500

(B) 6300

(C) 5400

(D) 4800

Correct Answer: (C) 5400

Solution:

Step 1: Understanding the Concept:

In a biprism experiment the fringe width is $\beta = \frac{\lambda D}{d}$, proportional to λ when D and d are fixed.

Step 2: Count the fringe widths:

The distance between the 4th and 13th bright bands on the same side is $13 - 4 = 9$ fringe widths, so $y = 9\beta_1$.

The distance between the 6th and 16th bright bands is $16 - 6 = 10$ fringe widths, so $y = 10\beta_2$.

Step 3: Equate:

$9\lambda_1 = 10\lambda$, so

$$\lambda = \frac{9 \times 6000}{10} = 5400 \text{ Å}$$

Step 4: Why the other options are wrong.

6500 and 6300 Å are larger than 6000 Å, but more fringes within the same distance mean narrower fringes, so λ must be smaller. 4800 Å would correspond to a ratio $\frac{9}{10}$ mistaken for $\frac{4}{5}$.

Final Answer:

The wavelength is 5400 angstrom, option (C).

$$\boxed{5400 \text{ Å}}$$

Quick Tip: Fringe width is proportional to wavelength; count the number of fringe widths.

44. The proton and α - particle are accelerated through same potential difference. Then the ratio of the de-Broglie wavelength of proton and α - particle is (mass of α-particle is 4 times mass of proton, charge of α-particle is 2 times charge of proton)

(A) $3\sqrt{3}$

(B) $3\sqrt{2}$

(C) $2\sqrt{3}$

(D) $2\sqrt{2}$

Correct Answer: (D) $2\sqrt{2}$

Solution:

Step 1: Understanding the Concept:

A charge q accelerated through potential difference V gains kinetic energy qV, so its momentum is $p = \sqrt{2mqV}$. The de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqV}}$$

Step 2: Ratio:

Same V for both, so $\lambda \propto \frac{1}{\sqrt{mq}}$:

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} = \sqrt{4 \times 2} = \sqrt{8} = 2\sqrt{2}$$

Step 3: Why the other options are wrong.

$3\sqrt{3}$ and $3\sqrt{2}$ involve a factor of 3, which does not appear in the mass or charge ratios. $2\sqrt{3}$ would need mq ratio 12.

Final Answer:

The ratio is $2\sqrt{2}$, option (D).

$$\boxed{2\sqrt{2}}$$

Quick Tip: lambda = h / sqrt(2 m q V) for a particle accelerated through V.

45. Two identical photocathodes receive light of frequencies n_1 and n_2. If the velocities of the emitted photoelectrons of mass m are V_1 and V_2 respectively, then (h = Planck's constant)

(A) $V_1 + V_2 = [\frac{2h}{m}(n_1 + n_2)]^{\frac{1}{2}}$

(B) $V_1 - V_2 = [\frac{2h}{m}(n_1 - n_2)]^{\frac{1}{2}}$

(C) $V_1^2 + V_2^2 = \frac{2h}{m}(n_1 + n_2)$

(D) $V_1^2 - V_2^2 = \frac{2h}{m}(n_1 - n_2)$

Correct Answer: (D) $V_1^2 - V_2^2 = \frac{2h}{m}(n_1 - n_2)$

Solution:

Step 1: Understanding the Concept:

Einstein's photoelectric equation is $h\nu = \phi + \frac{1}{2}mV^2$. Both cathodes are identical, so the work function ϕ is the same.

Step 2: Write both equations:

$hn_1 = \phi + \frac{1}{2}mV_1^2$ and $hn_2 = \phi + \frac{1}{2}mV_2^2$.

Step 3: Subtract:

$$h(n_1 - n_2) = \frac{1}{2}m(V_1^2 - V_2^2) \Rightarrow V_1^2 - V_2^2 = \frac{2h}{m}(n_1 - n_2)$$

Step 4: Why the other options are wrong.
Options with $V_1 + V_2$ or $V_1 - V_2$ have the speeds themselves rather than their squares. Option (C) adds the equations, which would leave 2ϕ in the result.

Final Answer:
$V_1^2 - V_2^2 = \frac{2h}{m}(n_1 - n_2)$, option (D).

$$\boxed{V_1^2 - V_2^2 = \frac{2h}{m}(n_1 - n_2)}$$

Quick Tip: Use Einstein's equation for the two identical cathodes and subtract.

46. An electron in the hydrogen atom jumps from n^{th} energy state to the ground state. The wavelength so emitted illuminates a photosensitive material having work function **2.65** eV. If the maximum kinetic energy of the emitted photoelectrons is **10.1** eV, then the value of 'n' is

(A) 2

(B) 3

(C) 4

(D) 5

Correct Answer: (C) 4

Solution:

Step 1: Understanding the Concept:
Photon energy absorbed by the metal equals the work function plus the maximum kinetic energy of the photoelectron: $h\nu = \phi + K_{max}$.

Step 2: Photon energy:

$h\nu = 2.65 + 10.1 = 12.75$ eV.

Step 3: Hydrogen transition:

The electron jumps from level n to the ground state, so

$$h\nu = 13.6\left(1 - \frac{1}{n^2}\right) = 12.75$$

$$1 - \frac{1}{n^2} = \frac{12.75}{13.6} = 0.9375 \Rightarrow \frac{1}{n^2} = 0.0625 = \frac{1}{16}$$

So $n = 4$.

Step 4: Why the other options are wrong.

For $n = 2$, the photon is 10.2 eV; for $n = 3$, 12.09 eV; for $n = 5$, 13.06 eV. Only $n = 4$ gives 12.75 eV.

Final Answer:

The value of n is 4, option (C).

$$\boxed{4}$$

Quick Tip: Photon energy = work function + max kinetic energy = E_n - E_1.

47. The angular momentum of an electron in Bohr's hydrogen atom having energy (-0.544) eV is
(h = Planck's constant)

(A) $\frac{h}{\pi}$

(B) $\frac{3h}{\pi}$

(C) $\frac{5h}{2\pi}$

(D) $\frac{7h}{2\pi}$

Correct Answer: (C) $\frac{5h}{2\pi}$

Solution:

Step 1: Understanding the Concept:
In Bohr's hydrogen atom, the energy of level n is $E_n = -\frac{13.6}{n^2}$ eV, and the angular momentum is $L = \frac{nh}{2\pi}$.

Step 2: Find n:

$$\frac{13.6}{n^2} = 0.544 \Rightarrow n^2 = \frac{13.6}{0.544} = 25 \Rightarrow n = 5$$

Step 3: Angular momentum:

$$L = \frac{nh}{2\pi} = \frac{5h}{2\pi}$$

Step 4: Why the other options are wrong.
$\frac{h}{\pi}$ corresponds to $n = 2$, $\frac{3h}{\pi}$ to $n = 6$, and $\frac{7h}{2\pi}$ to $n = 7$, each with a different energy from -0.544 eV.

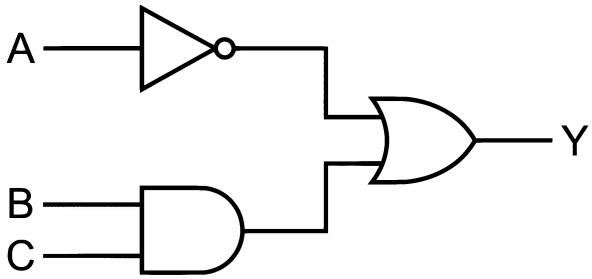
Final Answer:

The angular momentum is $\frac{5h}{2\pi}$, option (C).

$$\boxed{\frac{5h}{2\pi}}$$

Quick Tip: Find n from E = -13.6 / n squared, then L = n h / 2 pi.

48. The output Y of the following digital logic circuit will be '1' (one) for the inputs



(A) $A = 1, B = 0, C = 0$

(B) $A = 1, B = 0, C = 1$

(C) $A = 1, B = 1, C = 0$

(D) $A = 1, B = 1, C = 1$

Correct Answer: (D) $A = 1, B = 1, C = 1$

Solution:

Step 1: Understanding the Figure:

Input A passes through a NOT gate. Inputs B and C go into an AND gate. The outputs of the NOT gate and the AND gate feed an OR gate, which gives Y.

Step 2: Boolean expression:

$$Y = \overline{A} + B \cdot C$$

Step 3: Check each option:

(A) $A = 1, B = 0, C = 0$: $\overline{A} = 0$, $BC = 0$, so $Y = 0$.

(B) $A = 1, B = 0, C = 1$: $\overline{A} = 0$, $BC = 0$, so $Y = 0$.

(C) $A = 1, B = 1, C = 0$: $\overline{A} = 0$, $BC = 0$, so $Y = 0$.

(D) $A = 1, B = 1, C = 1$: $\overline{A} = 0$, $BC = 1$, so $Y = 0 + 1 = 1$.

Step 4: Conclusion:

With $A = 1$, the NOT gate gives 0, so Y is 1 only when both B and C are 1.

Final Answer:

Y is 1 for $A = B = C = 1$, option (D).

$$\boxed{A = 1,\ B = 1,\ C = 1}$$

Quick Tip: Y = (NOT A) OR (B AND C).

49. In a common emitter transistor amplifier, AC current gain is **65**, the load resistance is **5400 Ω** and input resistance of the transistor is **450 Ω**. The voltage gain is

(A) 460

(B) 540

(C) 780

(D) 7800

Correct Answer: (C) 780

Solution:

Step 1: Understanding the Concept:

In a common emitter amplifier, the voltage gain is $A_V = \beta_{ac} \times \frac{R_L}{R_{in}}$, where β_{ac} is the ac current gain, R_L the load resistance and R_{in} the input resistance.

Step 2: Compute:

$$A_V = 65 \times \frac{5400}{450} = 65 \times 12 = 780$$

Step 3: Why the other options are wrong.

460 and 540 do not match the product of 65 and 12. 7800 results from a slip of a factor of 10, such as using $R_L = 54000\ \Omega$.

Final Answer:

The voltage gain is 780, option (C).

$$\boxed{780}$$

Quick Tip: Voltage gain = current gain times (R_load / R_input).

50. In the band structure of n-type semiconductor , the free electrons donated by impurity atoms occupy energy levels in

(A) the conduction band.

(B) the valence band.

(C) the band gap and are close to conduction band.

(D) the band gap and are close to valence band.

Correct Answer: (C) the band gap and are close to conduction band.

Solution:

Step 1: Understanding the Concept:
An n-type semiconductor is made by adding pentavalent impurity atoms (donors) such as phosphorus to silicon. Each donor atom has one extra electron that is not needed for bonding.

Step 2: Energy band picture:
The extra electrons occupy a discrete donor energy level in the forbidden gap, very close to the bottom of the conduction band, only about 0.01 to 0.05 eV below it.

Step 3: Detailed Explanation:
Because this level is so close to the conduction band, a very small amount of thermal energy at room temperature lifts these electrons into the conduction band, where they become free electrons for conduction. The donor levels themselves lie in the band gap close to the conduction band.

Step 4: Why the other options are wrong.
The valence band (B) holds the bonding electrons, not the donor electrons. Option (D) describes acceptor levels of a p-type

semiconductor, which lie in the gap close to the valence band.

Final Answer:

Donor levels lie in the band gap close to the conduction band, option (C).

Option (C)

Quick Tip: Donor levels lie in the band gap just below the conduction band.

Chemistry

Questions 51 to 100 | 1 mark each | 50 marks

51. What is average atomic mass of chlorine if its two isotopes Cl^{35} and Cl^{37} exists in relative abundance of 75% and 25% respectively ?

(A) 35.0

(B) 35.5

(C) 37.0

(D) 37.5

Correct Answer: (B) 35.5

Solution:

Step 1: Understanding the Concept:

The atomic mass listed in the periodic table is not the mass of one isotope. It is the weighted average of all natural isotopes, where each isotope counts in proportion to its relative abundance.

Chlorine has two isotopes here, Cl^{35} at 75% and Cl^{37} at 25%.

Step 2: Key Formula or Approach:

$$\text{Average mass} = \frac{(m_1 \times a_1) + (m_2 \times a_2)}{100}$$

where m is the isotopic mass and a is the percentage abundance.

Step 3: Detailed Explanation:

Substitute the given values:

$$\text{Average mass} = \frac{(35 \times 75) + (37 \times 25)}{100} = \frac{2625 + 925}{100} = \frac{3550}{100} = 35.5$$

The answer must lie between 35 and 37 and sit closer to 35 because the lighter isotope is three times as abundant. This matches **35.5**.

Step 4: Why the other options are wrong.
Option (A) **35.0** ignores the heavier isotope completely. Option (C) **37.0** ignores the lighter isotope. Option (D) **37.5** lies above both isotopic masses, which is impossible for a weighted average.

Final Answer:
The average atomic mass of chlorine is **35.5** u, option (B).

$$\boxed{35.5}$$

Quick Tip: Average atomic mass is the abundance-weighted mean of the isotopic masses.

52. What is the SI unit of luminous intensity ?

(A) Ampere

(B) Coulomb

(C) Candela

(D) Ohm

Correct Answer: (C) Candela

Solution:

Step 1: Understanding the Question:
The SI system has seven base quantities: length, mass, time, electric current, temperature, amount of substance and luminous intensity. Each has one base unit.

Step 2: Key Fact:
The base unit of luminous intensity is the candela, symbol cd. It measures how strongly a source shines in a given direction as seen by the human eye.

Step 3: Detailed Explanation:
Match the base units with their quantities:
metre for length, kilogram for mass, second for time, ampere for electric current, kelvin for temperature, mole for amount of substance, candela for luminous intensity.
So the unit asked for in the question is the candela.

Step 4: Why the other options are wrong.
Option (A) Ampere is the unit of electric current. Option (B) Coulomb is the unit of electric charge, a derived unit equal to one ampere-second. Option (D) Ohm is the unit of electrical resistance, also a derived unit. None of these measures brightness of a source.

Final Answer:
The SI unit of luminous intensity is the candela, option (C).

Candela (C)

Quick Tip: Luminous intensity is one of the seven SI base quantities, measured in candela.

53. Which element from following has positive electron gain enthalpy?

(A) Xenon

(B) Bromine

(C) Iodine

(D) Chlorine

Correct Answer: (A) Xenon

Solution:

Step 1: Understanding the Concept:
Electron gain enthalpy is the energy change when a neutral gaseous atom takes up an electron. Most nonmetals release energy, so the value is negative. A positive value means energy must be supplied.

Step 2: Key Approach:
An atom with a completely filled valence shell has no vacant stable place for the new electron. The incoming electron must go into the next higher shell, which costs energy.

Step 3: Detailed Explanation:
Xenon is a noble gas with the configuration $[Kr]4d^{10}5s^2 5p^6$, a complete octet in its outermost shell. An extra electron would have to enter the $6s$ level. This is unfavourable, so energy is absorbed and the electron gain enthalpy of Xenon is positive.
Halogens such as Cl, Br and I are one electron short of an octet. They gain an electron easily and release energy.

Step 4: Why the other options are wrong.
Bromine (B), iodine (C) and chlorine (D) all have large negative

electron gain enthalpies. Chlorine has the most negative value in the periodic table, about -349 kJ/mol.

Final Answer:
Only the noble gas Xenon has a positive electron gain enthalpy, option (A).

Xenon (A)

Quick Tip: Noble gases have a full octet, so adding an electron needs energy.

54. Identify linear molecule from following.

(A) SO_2

(B) BCl_3

(C) CO_2

(D) NH_3

Correct Answer: (C) CO_2

Solution:

Step 1: Understanding the Concept:
The shape of a molecule follows from VSEPR theory. A molecule is linear when the central atom has two electron regions and no lone pair, so the bond angle is $180°$.

Step 2: Detailed Explanation of the keyed option:
In CO_2 carbon forms two double bonds, O=C=O, and has no lone

pair. Two regions of electron density stay as far apart as possible, so the O-C-O angle is 180° and the molecule is linear (sp hybridised carbon).

Step 3: Why the other options are wrong.
SO_2 has one lone pair on sulphur, which pushes the two S=O bonds into a bent shape of about 119°. BCl_3 has three bond pairs on boron and no lone pair, giving a trigonal planar shape with 120° angles. NH_3 has three bond pairs and one lone pair on nitrogen, giving a pyramidal shape.

Final Answer:
Only carbon dioxide is linear, option (C).

$$\boxed{CO_2 \text{ (C)}}$$

Quick Tip: Look for a central atom with two bond pairs and no lone pair.

55. Find the number of moles of CO_2 present in a sample occuping $4 \times 10^{-3}\ m^3$ at $1.104 \times 10^5\ Nm^{-2}$ pressure ($R \times T = 2208 J\ mol^{-1}$)

(A) 0.1 mole

(B) 0.2 mole

(C) 0.3 mole

(D) 0.4 mole

Correct Answer: (B) 0.2 mole

Solution:

Step 1: Understanding the Concept:

The ideal gas equation connects pressure, volume, amount of gas and temperature. Since the question gives the product RT directly, no separate temperature is needed.

Step 2: Key Formula:

$$PV = nRT \Rightarrow n = \frac{PV}{RT}$$

Step 3: Detailed Explanation:

Given $P = 1.104 \times 10^5$ N m^{-2}, $V = 4 \times 10^{-3}$ m^3 and $RT = 2208$ J mol^{-1}.

$$PV = 1.104 \times 10^5 \times 4 \times 10^{-3} = 441.6 \text{ J}$$

$$n = \frac{441.6}{2208} = 0.2 \text{ mol}$$

Units check: N m^{-2} times m^3 gives N m = J, and J divided by J mol^{-1} gives mol.

Step 4: Why the other options are wrong.

Values 0.1, 0.3 and 0.4 mol would need PV of 220.8, 662.4 and 883.2 J, which do not match the product of the given pressure and volume.

Final Answer:

The sample contains 0.2 mole of CO_2, option (B).

$\boxed{0.2 \text{ mol}}$

Quick Tip: Use PV = nRT directly with the given values of P, V and RT.

56. If 2 mole of an ideal gas expand isothermally and reversibly at 27°C from 1 dm^3 to $1m^3$ calculate work done? $[R = 8.314 J\ K^{-1}mol^{-1}]$

(A) -49.95 kJ

(B) -99.90 kJ

(C) -34.46 kJ

(D) -68.92 kJ

Correct Answer: (C) -34.46 kJ

Solution:

Step 1: Understanding the Concept:

In a reversible isothermal expansion of an ideal gas the system does work on the surroundings, so the work done by the gas appears with a negative sign in the convention used for MHT CET.

Step 2: Key Formula:

$$W = -2.303\, nRT \log_{10} \frac{V_2}{V_1}$$

Step 3: Detailed Explanation:

Convert volumes to the same unit: $1\ m^3 = 1000\ dm^3$, so V_2/V_1

$= 1000/1 = 1000$ and $\log_{10} 1000 = 3$.

Temperature $T = 27 + 273 = 300$ K and $n = 2$.

$$W = -2.303 \times 2 \times 8.314 \times 300 \times 3$$

$$2.303 \times 2 \times 8.314 = 38.29, \quad 38.29 \times 300 \times 3 = 34461 \text{ J}$$

$$W \approx -34.46 \text{ kJ}$$

Step 4: Why the other options are wrong.

Option (A) -49.95 kJ and (B) -99.90 kJ come from using a wrong volume ratio or a wrong log value. Option (D) -68.92 kJ is exactly double the correct value, so it results from using $n = 4$ or an extra factor of 2.

Final Answer:

The work done is about -34.46 kJ, option (C).

$$\boxed{-34.46 \text{ kJ}}$$

Quick Tip: Use W = -2.303 nRT log(V2/V1) with both volumes in the same unit.

57. Identify the law for the statement "Overall the enthalpy change for a reaction is equal to sum of enthalpy changes of individual steps in the reaction"

(A) First law of thermodynamics

(B) Hess's law

(C) Avogadro's law

(D) Second law of thermodynamics

Correct Answer: (B) Hess's law

Solution:

Step 1: Understanding the Question:
The statement says the total enthalpy change for a reaction equals the sum of the enthalpy changes of its individual steps. We must name the law that says this.

Step 2: Key Concept:
Enthalpy is a state function, so its change depends only on the initial and final states, not on the path. This is the content of Hess's law of constant heat summation.

Step 3: Detailed Explanation:
If a reaction can be done in one step or as a series of steps, the overall ΔH is the same in both cases:

$$\Delta H_{\text{overall}} = \Delta H_1 + \Delta H_2 + \Delta H_3 + \ldots$$

This lets us calculate enthalpies that cannot be measured directly, for example the formation of CO from carbon and oxygen.

Step 4: Why the other options are wrong.

The first law of thermodynamics (A) deals with conservation of energy, $\Delta U = q + W$. Avogadro's law (C) says equal volumes of gases at the same temperature and pressure hold equal numbers of molecules. The second law (D) concerns entropy and the direction of spontaneous change.

Final Answer:

The statement is Hess's law, option (B).

Hess's law (B)

Quick Tip: The statement describes the path independence of enthalpy change, which is Hess law.

58. Calculate $\triangle G^\circ$ for reaction,

$CH_{4(g)} + H_{2(g)} \rightarrow C_2H_{6(g)}$ ($K_p = 2 \times 10^{17}$, $R = 8.314 JK^{-1}mol^{-1}$)

(A) -64.695 kJ mol^{-1}

(B) -98.716 kJ mol^{-1}

(C) -44.08 kJ mol^{-1}

(D) -58.78 kJ mol^{-1}

Correct Answer: (B) -98.716 kJ mol^{-1}

Solution:

Step 1: Understanding the Concept:

The standard Gibbs energy change of a reaction is linked to its equilibrium constant. A large K_p means a strongly negative ΔG°.

Step 2: Key Formula:

$$\Delta G^\circ = -2.303\, RT \log_{10} K_p$$

The temperature is not stated, so the standard temperature of 298 K is used.

Step 3: Detailed Explanation:

$\log(2 \times 10^{17}) = \log 2 + 17 = 0.301 + 17 = 17.301.$

$$\Delta G^\circ = -2.303 \times 8.314 \times 298 \times 17.301$$

$$2.303 \times 8.314 = 19.147, \quad 19.147 \times 298 = 5705.8$$

$$\Delta G^\circ = -5705.8 \times 17.301 \approx -98717 \text{ J mol}^{-1} \approx -98.7 \text{ kJ mol}^{-1}$$

Step 4: Why the other options are wrong.

The other values (-64.695, -44.08, -58.78 kJ) correspond to smaller $\log K$ values, such as K near 10^{11} to 10^8. They do not match $K_p = 2 \times 10^{17}$.

Final Answer:

$\Delta G^\circ \approx -98.716$ kJ mol^{-1}, option (B).

$$\boxed{-98.716 \text{ kJ mol}^{-1}}$$

Quick Tip: Use delta G = -2.303 RT log Kp at 298 K.

59. What is the normal pH range of human blood?

(A) 6.0 to 6.4

(B) 7.36 to 7.42

(C) 6.4 to 6.95

(D) 7.8 to 8.0

Correct Answer: (B) 7.36 to 7.42

Solution:

Step 1: Understanding the Question:
Blood is a buffered fluid. Its pH is held in a very narrow window because even small changes disturb enzymes and oxygen transport.

Step 2: Key Fact:
The normal pH range of human blood is 7.36 to 7.42, so blood is slightly alkaline. The bicarbonate-carbonic acid buffer, HCO_3^-/H_2CO_3, together with haemoglobin and phosphates, maintains this value.

Step 3: Detailed Explanation:
A pH above 7 means the solution is basic, and 7.36-7.42 is just above neutral. If blood pH falls below about 7.35 the condition is called acidosis, and if it rises above about 7.45 it is called alkalosis. Both are harmful.

Step 4: Why the other options are wrong.
Ranges 6.0-6.4 (A) and 6.4-6.95 (C) are acidic and would be fatal for

blood. The range 7.8-8.0 (D) is much too alkaline for normal blood. Only 7.36-7.42 is the accepted value.

Final Answer:

The normal blood pH range is **7.36** to **7.42**, option (B).

$$\boxed{7.36 \text{ to } 7.42}$$

Quick Tip: Blood is slightly alkaline and kept in a very narrow window by buffers.

60. Calculate the pOH of **0.01** M monobasic acid that is completely dissociated, at **298** K.

(A) 02

(B) 12

(C) 03

(D) 11

Correct Answer: (B) 12

Solution:

Step 1: Understanding the Concept:

A monobasic acid gives one H^+ per molecule. If it dissociates completely, the hydrogen ion concentration equals the acid concentration.

Step 2: Key Formulas:

$$pH = -\log[H^+], \quad pH + pOH = 14 \text{ at } 298 \text{ K}$$

Step 3: Detailed Explanation:
$[H^+] = 0.01 = 10^{-2}$ M, so $pH = 2$.
Then $pOH = 14 - 2 = 12$.
Check: $[OH^-] = K_w/[H^+] = 10^{-14}/10^{-2} = 10^{-12}$ M, so $pOH = 12$.

Step 4: Why the other options are wrong.
The value 02 (A) is the pH, not the pOH, a common slip. Value 03 (C) and 11 (D) would need acid concentrations of 0.001 M or 0.0001 M, which is not the case here.

Final Answer:
The pOH is 12, option (B).

$$\boxed{pOH = 12}$$

Quick Tip: Complete dissociation gives [H+] = 0.01 M, so pH = 2 and pOH = 14 - pH.

61. A weak monobasic acid is 4% dissociated in 0.05 M solution. What is percent dissociation in 0.1 M solution ?

(A) 5.715%

(B) 8.516%

(C) 2.828%

(D) 4.620%

Correct Answer: (C) 2.828%

Solution:

Step 1: Understanding the Concept:

For a weak monobasic acid the degree of dissociation α is small, so Ostwald's dilution law applies: $K_a = c\alpha^2$.

Step 2: Key Approach:

K_a is constant at a given temperature, so $c_1\alpha_1^2 = c_2\alpha_2^2$, which gives $\alpha_2 = \alpha_1\sqrt{c_1/c_2}$.

Step 3: Detailed Explanation:

Given $\alpha_1 = 4\%$ at $c_1 = 0.05$ M, and the new concentration is $c_2 = 0.1$ M.

$$\alpha_2 = 4 \times \sqrt{\frac{0.05}{0.1}} = 4 \times \sqrt{0.5} = 4 \times 0.7071$$

$$\alpha_2 = 2.828\%$$

The solution became more concentrated, so the percent dissociation falls.

Step 4: Why the other options are wrong.

Values 5.715%, 8.516% and 4.620% are all larger than or close to 4%. Dissociation of a weak acid decreases when concentration increases, so a higher value like 5.715% or 8.516% cannot be right.

Final Answer:

The percent dissociation becomes 2.828%, option (C).

2.828%

Quick Tip: Ostwald dilution law: degree of dissociation varies as the square root of 1/c.

62. What is the difference in oxidation number of nitrogen when nitric acid is converted into nitrous oxide ?

(A) +1

(B) +2

(C) +4

(D) +3

Correct Answer: (C) +4

Solution:

Step 1: Understanding the Concept:
Oxidation number of an element in a compound is found by assuming the sum of all oxidation numbers equals the net charge. Hydrogen is +1 and oxygen is -2.

Step 2: Nitrogen in nitric acid:
In HNO_3: $+1 + x + 3(-2) = 0$, so $x = +5$.

Step 3: Nitrogen in nitrous oxide:
In N_2O: $2x + (-2) = 0$, so $x = +1$.

Step 4: Detailed Explanation:
The change in oxidation number is $+5 \rightarrow +1$, a decrease of $5 - 1 = 4$.

The magnitude of the difference is 4, meaning each nitrogen gains 4 electrons (it is reduced).

Step 5: Why the other options are wrong.
The value +1 (A) is the oxidation number of N in N_2O itself, not the difference. A difference of +2 (B) or +3 (D) would need a final oxidation number of +3 or +2, as in N_2O_3 or NO, not N_2O.

Final Answer:
The difference is 4, option (C).

$\boxed{+4}$

Quick Tip: Find the oxidation number of nitrogen in each compound, then subtract.

63. Which from following oxides is amphoteric in nature?

(A) BaO

(B) N_2O_5

(C) CO_2

(D) Al_2O_3

Correct Answer: (D) Al_2O_3

Solution:

Step 1: Understanding the Concept:
An amphoteric oxide shows both acidic and basic behaviour, reacting with acids to give salts and with bases to give salts too.

Step 2: Detailed Explanation of the keyed option:
Aluminium oxide, Al_2O_3, reacts with acid:

$$Al_2O_3 + 6HCl \rightarrow 2AlCl_3 + 3H_2O$$

and with alkali:

$$Al_2O_3 + 2NaOH + 3H_2O \rightarrow 2Na[Al(OH)_4]$$

So it is amphoteric.

Step 3: Why the other options are wrong.
BaO (A) is a strongly basic oxide of an alkaline earth metal. N_2O_5 (B) is an acidic oxide of a nonmetal that forms nitric acid with water. CO_2 (C) is also acidic and gives carbonic acid with water.

Final Answer:
Aluminium oxide is amphoteric, option (D).

$$\boxed{Al_2O_3 \text{ (D)}}$$

Quick Tip: An amphoteric oxide reacts with both acids and bases.

64. Identify the catalyst used to transform carbon monoxide from water gas into carbon dioxide

(A) Potassium dichromate

(B) Sodium chromate

(C) Iron chromate

(D) Nickel

Correct Answer: (C) Iron chromate

Solution:

Step 1: Understanding the Process:

Water gas is a mixture of CO and H_2. To get more hydrogen, CO in it is converted to CO_2 by steam. This is called the water gas shift reaction.

Step 2: Key Reaction:

$$CO + H_2O \xrightarrow{\text{catalyst, 673 K}} CO_2 + H_2$$

Step 3: Detailed Explanation:

The catalyst used in this step is iron chromate (the iron-chromium catalyst), at about 673 K. The CO_2 formed is then removed by scrubbing with sodium arsenite solution, leaving nearly pure hydrogen. This is how dihydrogen is made on the industrial scale.

Step 4: Why the other options are wrong.

Potassium dichromate (A) and sodium chromate (B) are oxidising agents used in solution, not catalysts for this gas phase process. Nickel (D) is used for steam reforming of methane and for hydrogenation, not for the shift step.

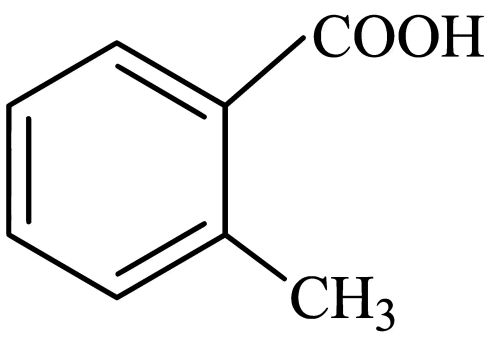
Final Answer:

The catalyst is iron chromate, option (C).

Iron chromate (C)

Quick Tip: Iron chromate, a Fe-Cr oxide mixture, catalyses the shift of CO to CO2.

65. What is IUPAC name of following compound?



(A) Benzoic acid

(B) 2-Methylbenzoic acid

(C) o-Toluic acid

(D) 2-Methylcyclohexylcarboxylic acid

Correct Answer: (B) 2-Methylbenzoic acid

Solution:

Step 1: Understanding the Figure:

The figure shows a benzene ring with a $-COOH$ group on one carbon and a $-CH_3$ group on the neighbouring (adjacent) ring carbon.

Step 2: Key Rule:

In IUPAC naming, the carboxylic acid is the principal functional group, so the parent is benzoic acid. The ring carbon bearing –COOH is numbered 1, and the substituent gets the lowest possible locant.

Step 3: Detailed Explanation:

The methyl group is on the carbon next to C-1, so it takes locant 2. The name is therefore 2-methylbenzoic acid.

Step 4: Why the other options are wrong.

Benzoic acid (A) leaves out the methyl group. 2-Methylcyclohexylcarboxylic acid (D) would have a saturated ring with no double bonds, but the figure shows an aromatic ring. Option (C), o-toluic acid, is the common (trivial) name of the same compound, whereas the question asks for the systematic IUPAC name, which is 2-methylbenzoic acid.

Final Answer:

The IUPAC name is 2-methylbenzoic acid, option (B).

2-Methylbenzoic acid (B)

Quick Tip: Name the benzene ring bearing COOH as benzoic acid and number the carbon with COOH as 1.

66. Which from following is a correct priority order for selection of principal functional group for nomenclature of polyfunctional compound according to IUPAC system ?

(A) $-CHO > -CONH_2 > -OH > -C \equiv C-$

(B) $-CONH_2 > -CHO > -OH > -C \equiv C-$

(C) $-C \equiv C- > -OH > -CONH_2 > -CHO$

(D) $-C \equiv C- > -CHO > -CONH_2 > -OH$

Correct Answer: (B) $-CONH_2 > -CHO > -OH > -C \equiv C-$

Solution:

Step 1: Understanding the Concept:

When a molecule has more than one functional group, IUPAC picks one as the principal group using a fixed priority list. It becomes the suffix of the name.

Step 2: Key Order:

The general priority is: carboxylic acid > sulphonic acid > anhydride > ester > acid halide > amide > nitrile > aldehyde > ketone > alcohol > amine > alkene > alkyne.

Step 3: Detailed Explanation:

From the list, the four groups in the question rank as amide $-CONH_2$ first, aldehyde $-CHO$ second, alcohol $-OH$ third, and alkyne $-C \equiv C-$ last:

$$-CONH_2 > -CHO > -OH > -C \equiv C-$$

This matches option (B).

Step 4: Why the other options are wrong.

Option (A) puts the aldehyde above the amide, which is the wrong way round. Options (C) and (D) place the alkyne first, but alkynes are

among the lowest priority groups, below alcohols and aldehydes.

Final Answer:

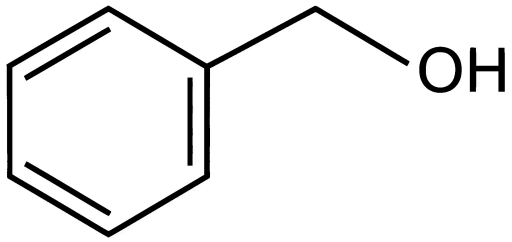
The correct order is amide, aldehyde, alcohol, alkyne, option (B).

$$\boxed{-CONH_2 > -CHO > -OH > -C \equiv C-}$$

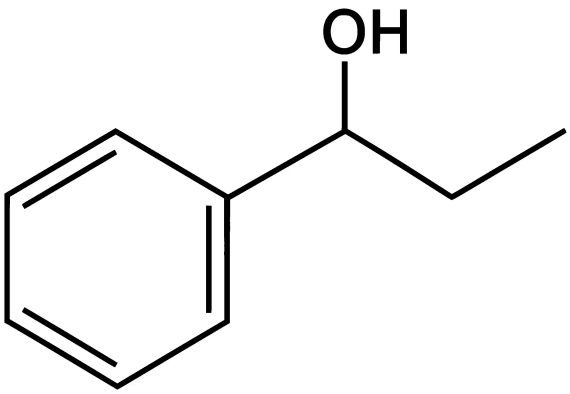
Quick Tip: Amide ranks above aldehyde, then alcohol, then alkyne.

67. Which of the following is tertiary benzylic alcohol?

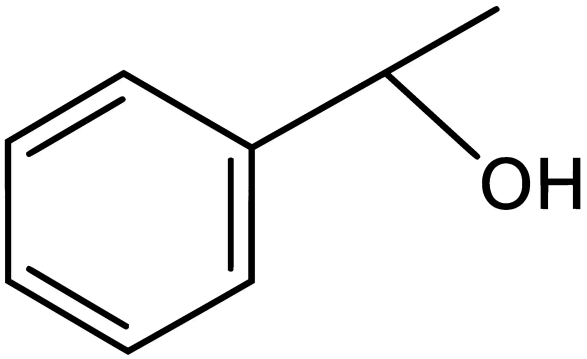
(A)



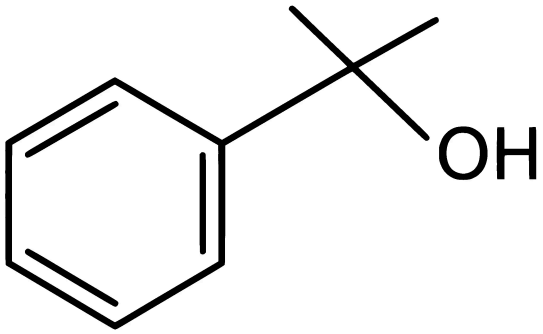
(B)



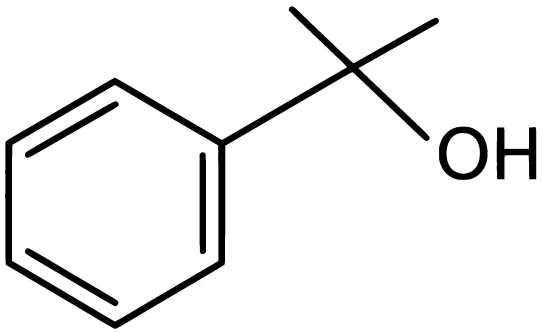
(C)



(D)



Correct Answer: (D)



Solution:

Step 1: Understanding the Concept:

A benzylic alcohol has its $-OH$ on a carbon directly attached to the benzene ring. It is tertiary if that carbon is bonded to three other carbon atoms (one of them the ring carbon) and carries no hydrogen.

Step 2: Check each structure:

(A) is benzyl alcohol, $C_6H_5CH_2OH$: the carbinol carbon bears two hydrogens, so it is primary.

(B) is 1-phenylpropan-1-ol: the carbinol carbon bears one hydrogen and is attached to the ring and an ethyl group, so it is secondary.

(C) is 1-phenylethanol: again the carbinol carbon bears one hydrogen

with a methyl and the ring, so it is secondary.

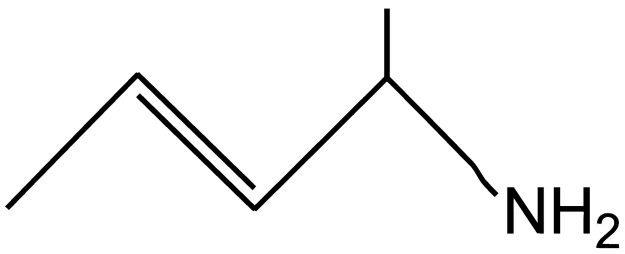
Step 3: The keyed structure:
(D) is 2-phenylpropan-2-ol, $C_6H_5C(CH_3)_2OH$. The carbinol carbon is attached to the ring and to two methyl groups, has no hydrogen, and carries the -OH group. So it is a tertiary benzylic alcohol.

Final Answer:
Structure (D), 2-phenylpropan-2-ol, is the tertiary benzylic alcohol.

(D)

Quick Tip: Benzylic means the OH carbon touches the ring; tertiary means that carbon has three carbon neighbours.

68. What is IUPAC name of the following compound ?



(A) Pent-2-en-4-amine

(B) Pent-3-en-2-amine

(C) 1-Methylbut-2-en-1-amine

(D) 4-Methylbut-2-en-4-amine

Correct Answer: (B) Pent-3-en-2-amine

Solution:

Step 1: Understanding the Structure:

The figure shows a five-carbon chain with a double bond and an $-NH_2$ group. Written out: $CH_3 - CH = CH - CH(CH_3) - NH_2$, where the carbon bearing NH_2 also carries a methyl that is part of the longest chain.

Step 2: Key Rule:

The longest chain containing the principal group (amine) and the double bond is chosen. The amine, being the principal group, gets the lowest locant, then the double bond.

Step 3: Detailed Explanation:

Chain: CH_3 (C1) - $CH(NH_2)$ (C2) - $CH =$ (C3) $=$ CH (C4) - CH_3 (C5). Numbering from the end nearest the amine gives $-NH_2$ at C2 and the double bond at C3. Name: pent-3-en-2-amine.

Step 4: Why the other options are wrong.

Pent-2-en-4-amine (A) numbers from the wrong end, giving the amine a higher locant (4) than necessary. Options (C) and (D) choose a four-carbon chain, but the longest chain has five carbons.

Final Answer:

The name is pent-3-en-2-amine, option (B).

Pent-3-en-2-amine (B)

Quick Tip: Number the chain so the amine group gets the lowest locant.

69. Which of the following alkenes on oxidation by $KMnO_4$ in dil H_2SO_4 forms adipic acid ?

(A) Hex-1-ene

(B) Hex-2-ene

(C) Hex-3-ene

(D) Cyclohexene

Correct Answer: (D) Cyclohexene

Solution:

Step 1: Understanding the Concept:
Hot acidic $KMnO_4$ cleaves a $C=C$ bond completely. Each carbon of the double bond becomes a carboxylic acid group (or a ketone if it was fully substituted).

Step 2: Adipic acid:
Adipic acid is hexanedioic acid, $HOOC\text{-}(CH_2)_4\text{-}COOH$, with six carbons and two $-COOH$ groups at the two ends.

Step 3: Detailed Explanation:
Cyclohexene is a six-carbon ring with one double bond. Cleaving the double bond opens the ring and turns both former alkene carbons into $-COOH$, leaving four CH_2 groups in between:

$$\text{Cyclohexene} \xrightarrow{KMnO_4/\text{dil } H_2SO_4} HOOC\text{-}(CH_2)_4\text{-}COOH$$

Step 4: Why the other options are wrong.
Hex-1-ene (A) breaks into pentanoic acid and CO_2. Hex-2-ene (B) gives ethanoic acid and butanoic acid. Hex-3-ene (C) gives two

molecules of propanoic acid. None of these gives a dicarboxylic acid because the chains are open.

Final Answer:
Cyclohexene gives adipic acid, option (D).

Cyclohexene (D)

Quick Tip: Oxidative cleavage of cyclohexene opens the ring to a six-carbon dicarboxylic acid.

70. Calculate the radius of atom of metal forming fcc unit cell having edge length 360 pm

(A) 180.04 pm

(B) 127.26 pm

(C) 155.91 pm

(D) 254.67 pm

Correct Answer: (B) 127.26 pm

Solution:

Step 1: Understanding the Concept:
In a face-centred cubic (fcc) unit cell, atoms touch each other along the face diagonal, not along the cube edge.

Step 2: Key Formula:
Face diagonal $= \sqrt{2}\,a = 4r$, so

$$r = \frac{a}{2\sqrt{2}}$$

Step 3: Detailed Explanation:

With $a = 360$ pm:

$$r = \frac{360}{2 \times 1.414} = \frac{360}{2.828} = 127.3 \text{ pm}$$

This is the value 127.26 pm in the options.

Step 4: Why the other options are wrong.

180 pm is $a/2$, which is the radius for a simple cubic cell where atoms touch along the edge. 155.91 pm is $\sqrt{3}a/4$, which belongs to the bcc cell. 254.67 pm is roughly $2r$, the diameter, not the radius.

Final Answer:

The radius is about 127.26 pm, option (B).

$$\boxed{127.26 \text{ pm}}$$

Quick Tip: For fcc, face diagonal equals 4r so r = a/(2 root 2).

71. What is the coordination number of particle in simple cubic structure ?

(A) 2

(B) 4

(C) 6

(D) 12

Correct Answer: (C) 6

Solution:

Step 1: Understanding the Concept:
Coordination number is the number of nearest neighbour particles that touch a given particle in a crystal.

Step 2: Detailed Explanation:
In a simple cubic (primitive) structure, particles sit at the corners of a cube. A given particle has one neighbour in each of the six directions: left and right, front and back, up and down. So the number of nearest neighbours is 6.

Step 3: Comparison:
For comparison, bcc has coordination number 8, and fcc and hcp (both closest packed) have 12.

Step 4: Why the other options are wrong.
The value 2 (A) is characteristic of a linear chain, and 4 (B) is found in a square arrangement or in tetrahedral coordination. The value 12 (D) belongs to closest packed structures, and simple cubic is not close packed.

Final Answer:
The coordination number is 6, option (C).

6

Quick Tip: Each particle in a simple cubic lattice touches six neighbours.

72. Calculate the density of an element having molar mass 225g mol^{-1} forming bcc structure $[a^3 \times N_A = 75\text{cm}^3\text{mol}^{-1}]$

(A) $6.0\ \text{g cm}^{-3}$

(B) $2.81\ \text{g cm}^{-3}$

(C) $9.24\ \text{g cm}^{-3}$

(D) $11.36\ \text{g cm}^{-3}$

Correct Answer: (A) $6.0\ \text{g cm}^{-3}$

Solution:

Step 1: Understanding the Concept:

Density of a crystal equals the mass of atoms in one unit cell divided by the volume of the cell.

Step 2: Key Formula:

$$\rho = \frac{Z \times M}{a^3 \times N_A}$$

For bcc, $Z = 2$ atoms per unit cell (8 corners at 1/8 each plus 1 body centre).

Step 3: Detailed Explanation:

Given $M = 225\ \text{g mol}^{-1}$ and $a^3 N_A = 75\ \text{cm}^3\ \text{mol}^{-1}$.

$$\rho = \frac{2 \times 225}{75} = \frac{450}{75} = 6.0 \text{ g cm}^{-3}$$

Step 4: Why the other options are wrong.
Using $Z = 1$ (simple cubic) would give 3.0, and $Z = 4$ (fcc) would give 12. The other options 2.81, 9.24 and 11.36 do not follow from any valid value of Z with these data.

Final Answer:
The density is 6.0 g cm^{-3}, option (A).

$$\boxed{6.0 \text{ g cm}^{-3}}$$

Quick Tip: Use rho = Z M / (a^3 N_A) with Z = 2 for bcc.

73. Calculate ΔT_b of a 0.45 m solution of a nonvolatile solute in solvent if molal elevation constant of solvent is 3.0 K kg mol^{-1}

(A) 0.94 K

(B) 2.69 K

(C) 3.21 K

(D) 1.35 K

Correct Answer: (D) 1.35 K

Solution:

Step 1: Understanding the Concept:
Adding a nonvolatile solute raises the boiling point of the solvent.

This is a colligative property, depending only on the number of solute particles.

Step 2: Key Formula:

$$\Delta T_b = K_b \times m$$

Step 3: Detailed Explanation:

With $K_b = 3.0$ K kg mol^{-1} and $m = 0.45$ mol kg^{-1}:

$$\Delta T_b = 3.0 \times 0.45 = 1.35 \text{ K}$$

No van't Hoff factor is needed because the solute is not stated to ionise.

Step 4: Why the other options are wrong.

The values 0.94, 2.69 and 3.21 K do not equal the product of 3.0 and 0.45. For example 3.21 K would need a molality of 1.07.

Final Answer:

The elevation in boiling point is 1.35 K, option (D).

$$\boxed{1.35 \text{ K}}$$

Quick Tip: Delta Tb = Kb times m for a nonvolatile solute.

74. Calculate the mass of nonvolatile solute dissolved in 0.3 dm^3 water having osmotic pressure 0.1 atm at 300K.
[Molar mass of solute = 328 g mol^{-1}, R = 0.082 dm^3atm $K^{-1}mol^{-1}$]

(A) 0.4 g

(B) 0.6 g

(C) 0.8 g

(D) 1.0 g

Correct Answer: (A) 0.4 g

Solution:

Step 1: Understanding the Concept:
Osmotic pressure is given by the van't Hoff equation, similar in form to the ideal gas law.

Step 2: Key Formula:

$$\pi = CRT = \frac{n}{V}RT \quad \Rightarrow \quad n = \frac{\pi V}{RT}$$

Step 3: Detailed Explanation:

$$n = \frac{0.1 \times 0.3}{0.082 \times 300} = \frac{0.03}{24.6} = 1.22 \times 10^{-3} \text{ mol}$$

Mass $= n \times M = 1.22 \times 10^{-3} \times 328 = 0.4$ g.

Step 4: Why the other options are wrong.
0.6 g, 0.8 g and 1.0 g would need moles of 1.83×10^{-3}, 2.44×10^{-3} and 3.05×10^{-3}, requiring osmotic pressures of 0.15, 0.2 and 0.25 atm for

this volume.

Final Answer:

The mass of solute is 0.4 g, option (A).

$$\boxed{0.4\ \text{g}}$$

Quick Tip: Use pi = CRT with C in mol per dm3, then multiply moles by molar mass.

75. Which from following solutions in water on complete dissociation exhibits minimum freezing point depression?

(A) 0.2 m Potassium chloride

(B) 0.1 m Sodium chloride

(C) 0.05 m Aluminium phosphate

(D) 0.15 m Magnesium sulphate

Correct Answer: (C) 0.05 m Aluminium phosphate

Solution:

Step 1: Understanding the Concept:

Freezing point depression is $\Delta T_f = iK_f m$. With the same solvent (water), the depression depends only on $i \times m$, the effective molality of particles after complete dissociation.

Step 2: Detailed Explanation:

Find i from the number of ions each formula unit gives:

KCl gives 2 ions, so $i \times m = 2 \times 0.2 = 0.40$.

NaCl gives 2 ions, so $2 \times 0.1 = 0.20$.
$AlPO_4$ gives 2 ions (Al^{3+} and PO_4^{3-}), so $2 \times 0.05 = 0.10$.
$MgSO_4$ gives 2 ions, so $2 \times 0.15 = 0.30$.

Step 3: Compare:
The smallest effective molality is 0.10 for aluminium phosphate, so its freezing point depression is the least.

Step 4: Why the other options are wrong.
KCl (A), NaCl (B) and MgSO4 (D) have effective molalities of 0.40, 0.20 and 0.30, all higher than 0.10, so each lowers the freezing point more.

Final Answer:
Aluminium phosphate gives the smallest depression, option (C).

0.05 m aluminium phosphate (C)

Quick Tip: Compare i times m for each solution; the smallest value gives the smallest depression.

76. Which of the following reactions occurs at cathode during recharging of lead accumulator ?

(A) Double subscripts: use braces to clarify

(B) $PbO_{2(s)} + 4H^+ 2e^- \rightarrow Pb^{2+} + 2H_2O_{(\ell)}$

(C) Double subscripts: use braces to clarify

(D) Double subscripts: use braces to clarify

Correct Answer: (D) Double subscripts: use braces to clarify

Solution:

Step 1: Understanding the Concept:

A lead accumulator has a lead anode and a lead dioxide cathode while discharging. When it is recharged by an external source, the roles of the electrodes reverse: the electrode that was the anode during discharge now acts as the cathode.

Step 2: Key Idea:

The cathode is always the site of reduction. During discharge the lead electrode forms $PbSO_4$ on its surface. On recharge, that $PbSO_4$ must be turned back into lead.

Step 3: Detailed Explanation:

The reaction at the cathode on recharging is:

$$PbSO_{4(s)} + 2e^- \rightarrow Pb_{(s)} + SO_4^{2-}{}_{(aq)}$$

Lead(II) gains two electrons, so this is a reduction, as expected for a cathode.

Step 4: Why the other options are wrong.

Option (A) shows PbO_2 being reduced, which is the cathode reaction during discharge, not recharge. Option (B) is not a proper balanced reaction of the cell. Option (C) is an oxidation (electrons on the product side), the anode reaction during discharge.

Final Answer:

The recharging cathode reaction is $PbSO_4 + 2e^- \rightarrow Pb + SO_4^{2-}$, option (D).

$$\boxed{PbSO_4 + 2e^- \rightarrow Pb + SO_4^{2-}}$$

Quick Tip: On recharging, PbSO4 at the cathode is reduced back to Pb.

77. What is the quantity of electricity required to produce 8 g of Mg (molar mass = $24 g\ mol^{-1}$) from $MgCl_2$ solution ?

(A) 0.222 F

(B) 0.444 F

(C) 0.666 F

(D) 0.888 F

Correct Answer: (C) 0.666 F

Solution:

Step 1: Understanding the Concept:

Faraday's first law says the mass deposited is proportional to the charge passed. One mole of electrons carries 1 faraday (96500 C).

Step 2: Key Reaction:

$$Mg^{2+} + 2e^- \rightarrow Mg$$

So 2 mol of electrons, that is 2 F, are needed to deposit 1 mol of Mg.

Step 3: Detailed Explanation:

Moles of Mg $= 8/24 = 0.333$ mol.
Charge needed $= 0.333 \times 2\ \text{F} = 0.666\ \text{F}$.

Step 4: Why the other options are wrong.
0.222 F is one third of the answer, from using the 1:1 ratio with a wrong molar mass. 0.444 F and 0.888 F do not match 8 g with a charge of 2 F per mole.

Final Answer:
The quantity of electricity needed is 0.666 F, option (C).

$$\boxed{0.666\ \text{F}}$$

Quick Tip: Mg^{2+} needs 2 faradays per mole; find moles of Mg first.

78. What is the molar conductivity of 0.02 M KI solution if its conductivity is $4.37 \times 10^{-4}\ \Omega^{-1}\ \text{cm}^{-1}$?

(A) $8.74\Omega^{-1}\ \text{cm}^2\text{mol}^{-1}$

(B) $21.85\Omega^{-1}\ \text{cm}^2\text{mol}^{-1}$

(C) $43.70\Omega^{-1}\ \text{cm}^2\text{mol}^{-1}$

(D) $13.65\Omega^{-1}\ \text{cm}^2\text{mol}^{-1}$

Correct Answer: (B) $21.85\Omega^{-1}\ \text{cm}^2\text{mol}^{-1}$

Solution:

Step 1: Understanding the Concept:
Molar conductivity Λ_m is the conductivity of all the ions produced by one mole of electrolyte in a solution volume that contains that one

mole.

Step 2: Key Formula:

$$\Lambda_m = \frac{\kappa \times 1000}{M}$$

where κ is in $\Omega^{-1}\text{cm}^{-1}$ and M is in mol L^{-1}; the factor 1000 converts litres to cm^3.

Step 3: Detailed Explanation:

$$\Lambda_m = \frac{4.37 \times 10^{-4} \times 1000}{0.02} = \frac{0.437}{0.02} = 21.85\ \Omega^{-1}\text{cm}^2\text{mol}^{-1}$$

Step 4: Why the other options are wrong.
8.74 is obtained by using 0.05 M instead of 0.02 M. 43.70 comes from dividing by 0.01 M, a factor-of-two slip. 13.65 does not follow from the given data.

Final Answer:
The molar conductivity is $21.85\ \Omega^{-1}\text{cm}^2\text{mol}^{-1}$, option (B).

$$\boxed{21.85\ \Omega^{-1}\text{cm}^2\text{mol}^{-1}}$$

Quick Tip: Molar conductivity = kappa x 1000 / molarity.

79. Identify correct statement regarding order of reaction from following.

(A) Rate of zero order reaction depends on initial concentration of reactant.

(B) Decomposition of acetaldehyde is a first order reaction.

(C) Half life of first order reaction is independent of initial concentration of reactant.

(D) Half life of zero order reaction is independent of initial concentration of reactant

Correct Answer: (C) Half life of first order reaction is independent of initial concentration of reactant.

Solution:

Step 1: Understanding the Concept:
The half-life of a reaction depends on its order. For a first-order reaction it is a constant, while for other orders it changes with the starting concentration.

Step 2: Key Formulas:
First order: $t_{1/2} = \frac{0.693}{k}$, independent of $[A]_0$.
Zero order: $t_{1/2} = \frac{[A]_0}{2k}$, proportional to $[A]_0$.

Step 3: Detailed Explanation:
Statement (C) says the half-life of a first-order reaction does not depend on the initial concentration. This agrees with $t_{1/2} = 0.693/k$, which contains only the rate constant. So (C) is correct.

Step 4: Why the other options are wrong.
(A) is false: for a zero order reaction the rate equals k and does not depend on concentration of reactant. (B) is false: decomposition of acetaldehyde follows a fractional (3/2) order, not first. (D) is false:

zero order half-life is $[A]_0/2k$, directly proportional to the initial concentration.

Final Answer:
Statement (C) is correct.

$$\boxed{\text{(C)}}$$

Quick Tip: Half-life of a first-order reaction does not depend on initial concentration.

80. Calculate rate constant of a first order reaction having half life 1 minute 40 second?

(A) $1.76 \times 10^{-3} s^{-1}$

(B) $2.31 \times 10^{-3} s^{-1}$

(C) $6.93 \times 10^{-3} s^{-1}$

(D) $4.61 \times 10^{-3} s^{-1}$

Correct Answer: (C) $6.93 \times 10^{-3} s^{-1}$

Solution:

Step 1: Understanding the Concept:
For a first-order reaction the rate constant and the half-life are linked by a fixed relation.

Step 2: Key Formula:

$$k = \frac{0.693}{t_{1/2}}$$

Step 3: Detailed Explanation:

Convert the half-life into seconds first: 1 min 40 s = 60 + 40 = 100 s.

$$k = \frac{0.693}{100\ \text{s}} = 6.93 \times 10^{-3}\ \text{s}^{-1}$$

Step 4: Why the other options are wrong.

2.31×10^{-3} would come from $t_{1/2} = 300$ s, and 1.76×10^{-3} and 4.61×10^{-3} from other wrong conversions, such as using 1.40 minutes or 150 s.

Final Answer:

The rate constant is $6.93 \times 10^{-3}\ \text{s}^{-1}$, option (C).

$$\boxed{6.93 \times 10^{-3}\ \text{s}^{-1}}$$

Quick Tip: Convert the half-life to seconds, then k = 0.693 / t half.

81. In a reaction,

$2N_2O_{5(g)} \rightarrow 4\,NO_{2(g)} + O_{2(g)}$

N_2O_5 disappears at a rate of 0.06 mol $dm^{-3}\ s^{-1}$

Calculate rate of formation of $O_{2(g)}$?

(A) 0.02 mol dm^{-3} s^{-1}

(B) 0.03 mol dm^{-3} s^{-1}

(C) 0.04 mol dm^{-3} s^{-1}

(D) 0.06 mol dm^{-3} s^{-1}

Correct Answer: (B) 0.03 mol dm^{-3} s^{-1}

Solution:

Step 1: Understanding the Concept:

For a reaction $aA \rightarrow bB + cC$, the rate of reaction is $-\frac{1}{a}\frac{d[A]}{dt} = \frac{1}{b}\frac{d[B]}{dt} = \frac{1}{c}\frac{d[C]}{dt}$.

Step 2: Setting up:

For $2N_2O_5 \rightarrow 4NO_2 + O_2$:

$$-\frac{1}{2}\frac{d[N_2O_5]}{dt} = \frac{1}{4}\frac{d[NO_2]}{dt} = \frac{d[O_2]}{dt}$$

Step 3: Detailed Explanation:

N_2O_5 disappears at 0.06 mol dm^{-3} s^{-1}, so

$$\frac{d[O_2]}{dt} = \frac{1}{2} \times 0.06 = 0.03 \text{ mol } dm^{-3}s^{-1}$$

Step 4: Why the other options are wrong.

0.06 (D) would hold only if the coefficients of N_2O_5 and O_2 were equal. 0.02 and 0.04 do not match the 2:1 ratio of N_2O_5 used to O_2 formed.

Final Answer:

Rate of formation of O_2 is 0.03 mol dm^{-3} s^{-1}, option (B).

$$\boxed{0.03 \text{ mol dm}^{-3}\text{s}^{-1}}$$

Quick Tip: Divide each rate by its stoichiometric coefficient to get the same reaction rate.

82. Identify the term used to describe movement of colloidal particles under applied electric potential without using semipermeable membrane.

(A) Dialysis

(B) Electrophoresis

(C) Electroosmosis

(D) Brownian motion

Correct Answer: (B) Electrophoresis

Solution:

Step 1: Understanding the Concept:

Colloidal particles carry an electric charge. In an applied electric field they migrate toward the electrode of opposite charge.

Step 2: Detailed Explanation:

The migration of the dispersed colloidal particles themselves under an electric potential, without any semipermeable membrane, is called electrophoresis. Positively charged sols such as ferric hydroxide move to the cathode, and negatively charged sols such as arsenious sulphide move to the anode.

Step 3: Why the other options are wrong.
Dialysis (A) separates colloidal particles from dissolved ions using a semipermeable membrane. Electroosmosis (C) is the movement of the dispersion medium, not the particles, under an electric field when the particles are held fixed. Brownian motion (D) is the random zigzag motion of colloidal particles due to molecular collisions and needs no electric field.

Final Answer:
The term is electrophoresis, option (B).

Electrophoresis (B)

Quick Tip: Movement of particles under an electric field with no membrane is electrophoresis.

83. Which element from following has highest melting point?

(A) Cr

(B) V

(C) Mn

(D) Fe

Correct Answer: (A) Cr

Solution:

Step 1: Understanding the Concept:
In the 3d series the melting point rises as the number of unpaired d

electrons rises, because more electrons take part in metallic bonding. It peaks near the middle of the series and then falls.

Step 2: Detailed Explanation:
Chromium has the configuration $3d^5 4s^1$, which gives six unpaired electrons, the maximum for the series. Strong metallic bonding results, and the melting point is very high, about 2180 K. Vanadium ($3d^3 4s^2$) is also very high.

Step 3: Comparison with the others:
Manganese has the configuration $3d^5 4s^2$ with extra stability from a half-filled d set, but its electrons are held back from bonding, so its melting point is much lower, about 1519 K. Iron has four unpaired electrons and melts at about 1811 K.

Step 4: Why the keyed option fits:
Among the four given metals, chromium has a melting point far above that of manganese and iron, and it is the metal in the question with the strongest metallic bonding.

Final Answer:
Chromium has the highest melting point of the listed metals, option (A).

Cr (A)

Quick Tip: Chromium has six unpaired electrons forming strong metallic bonds.

84. Identify the catalyst used in synthesis of gasoline by Fischer Tropsch process.

(A) Platinized asbestos

(B) Fe-Cr

(C) Co-Th

(D) MnO_2

Correct Answer: (C) Co-Th

Solution:

Step 1: Understanding the Process:

Synthesis gas, a mixture of CO and H_2, can be turned into liquid hydrocarbons such as gasoline by the Fischer-Tropsch process.

Step 2: Key Reaction:

$$nCO + (2n + 1)H_2 \xrightarrow{\text{Co-Th, 470-670 K}} C_nH_{2n+2} + nH_2O$$

Step 3: Detailed Explanation:

The catalyst used is cobalt with thoria (ThO_2) as a promoter, called Co-Th. The process runs at about 470 K to 670 K under pressure.

Step 4: Why the other options are wrong.

Platinised asbestos (A) is the catalyst of the contact process for SO_2 oxidation. Fe-Cr (B) is used in the water gas shift reaction. MnO_2 (D) catalyses decomposition of potassium chlorate and hydrogen peroxide.

Final Answer:

The catalyst is Co-Th, option (C).

Co-Th (C)

Quick Tip: Fischer-Tropsch uses a cobalt-thoria catalyst on CO and H2.

85. Identify the formula of Bis(ethylenediamine)dithiocyanatoplatinum(IV)ion from following.

(A) $[Pt(en)_2(NCS)]^{2+}$

(B) $Pt[(en)_2(SCN)_2]^{4+}$

(C) $[Pt(en)_2(SCN)_2]^{2+}$

(D) $Pt[(en)_3(SCN)_2]^{4+}$

Correct Answer: (C) $[Pt(en)_2(SCN)_2]^{2+}$

Solution:

Step 1: Understanding the Concept:

To write the formula from the name, put the metal first, then ligands in alphabetical order inside square brackets, and finally the overall charge.

Step 2: Decode the name:

"Bis(ethylenediamine)" means two neutral en ligands.

"Dithiocyanato" means two SCN^- ligands, each with charge -1.

"Platinum(IV)" means the metal has charge +4.

Step 3: Detailed Explanation:

Charge on the complex ion $= +4 + 2(0) + 2(-1) = +2$.
The formula is $[Pt(en)_2(SCN)_2]^{2+}$.

Step 4: Why the other options are wrong.
(A) has only one thiocyanate ligand, so it has the wrong ligand count. (B) and (D) write the metal outside the bracket and give a charge of 4+, which is not the net charge. Option (D) also has three en ligands instead of two.

Final Answer:
The formula is $[Pt(en)_2(SCN)_2]^{2+}$, option (C).

$$\boxed{[Pt(en)_2(SCN)_2]^{2+}}$$

Quick Tip: Add the charges: Pt(IV) plus neutral en plus two SCN- gives +2.

86. What is the coordination number of central metal ion in $[Co(en)_3]^{3+}$ complex ?

(A) 2

(B) 4

(C) 6

(D) 8

Correct Answer: (C) 6

Solution:

Step 1: Understanding the Concept:
Coordination number is the number of donor atoms directly bonded

to the central metal ion, not the number of ligand molecules.

Step 2: Key Point:
Ethylenediamine (en), $NH_2CH_2CH_2NH_2$, is a bidentate ligand. It binds the metal through two nitrogen atoms.

Step 3: Detailed Explanation:
In $[Co(en)_3]^{3+}$ there are 3 en ligands, each attached through 2 donor atoms:

$$\text{Coordination number} = 3 \times 2 = 6$$

The geometry is octahedral.

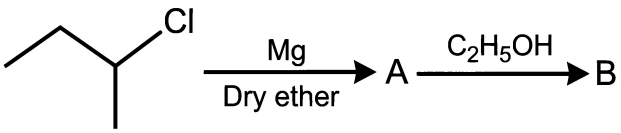
Step 4: Why the other options are wrong.
The value 2 or 4 would apply to a complex with fewer donor atoms, such as $[Ag(NH_3)_2]^+$ or $[Ni(CN)_4]^{2-}$. The value 8 is not found here since only six donor atoms are present.

Final Answer:
The coordination number is 6, option (C).

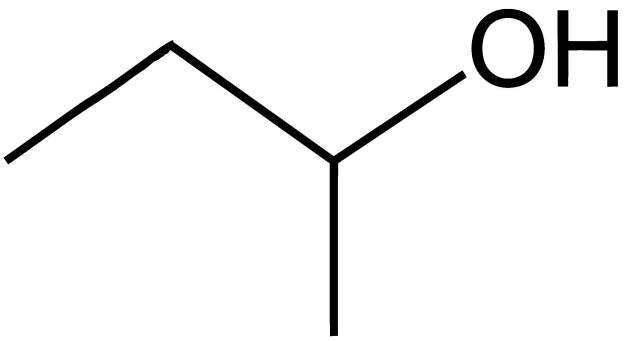
$$\boxed{6}$$

Quick Tip: Each en is bidentate and donates two pairs, so 3 x 2 = 6.

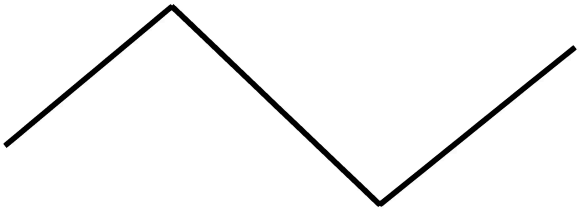
87. Identify the product 'B' in the following reaction sequence.



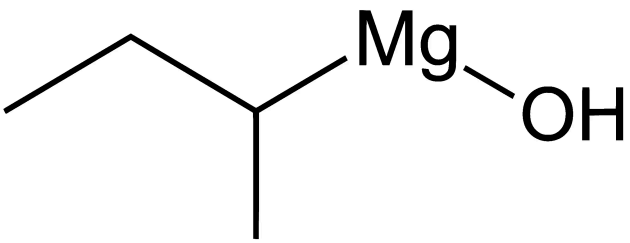
(A)



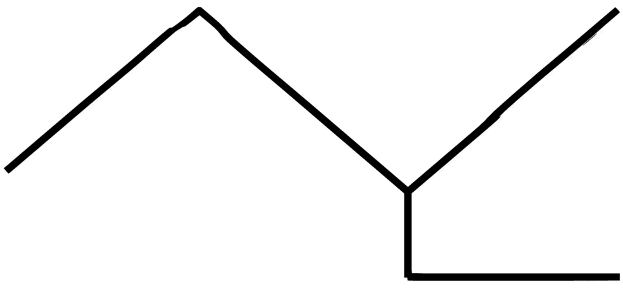
(B)



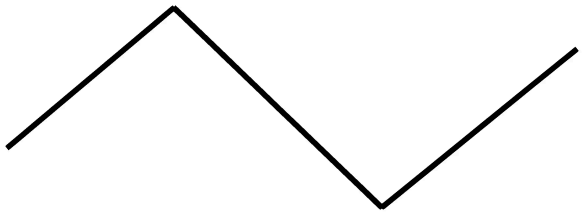
(C)



(D)



Correct Answer: (B)



Solution:

Step 1: Understanding the Reaction:

The figure shows 2-chlorobutane (sec-butyl chloride) reacting with magnesium in dry ether, then with ethanol.

Step 2: Step A:

Magnesium inserts into the C-Cl bond to form the Grignard reagent, sec-butylmagnesium chloride, $CH_3CH_2CH(CH_3)MgCl$.

Step 3: Step B:

A Grignard reagent is a strong base; the carbon attached to Mg is strongly nucleophilic and carbanion-like. It takes a proton from ethanol, which has an acidic O-H hydrogen:

$$CH_3CH_2CH(CH_3)MgCl + C_2H_5OH \rightarrow CH_3CH_2CH_2CH_3 + Mg(OC_2H_5)Cl$$

So B is butane, the four-carbon zigzag chain.

Step 4: Why the other options are wrong.

The 2-butanol structure (A) needs oxidation with oxygen, not an alcohol. The Mg-OH compound (C) does not form from ethanol. The branched six-carbon alkane (D) would need coupling of two alkyl groups, which does not happen here.

Final Answer:
Product B is butane, option (B).

Butane (B)

Quick Tip: A Grignard reagent reacts with any compound having an acidic hydrogen, such as an alcohol, to give an alkane.

88. Which among the following is NOT a feature of SN^2 mechanism ?

(A) Single step mechanism

(B) Backside attack of nucleophile

(C) Formation of planar carbocation intermediate

(D) Involves simultaneous bond breaking and bond forming

Correct Answer: (C) Formation of planar carbocation intermediate

Solution:

Step 1: Understanding the Concept:
In the S_N2 (bimolecular nucleophilic substitution) mechanism the nucleophile attacks the carbon at the same moment that the leaving group departs.

Step 2: Features of SN2:
It is a single-step mechanism. The nucleophile attacks from the side opposite to the leaving group (backside attack). Bond formation and bond breaking occur simultaneously through a pentacoordinate transition state, and the configuration is inverted.

Step 3: Detailed Explanation:
Formation of a planar carbocation intermediate belongs to the S_N1 mechanism, which is a two-step process. In S_N2 no carbocation is formed, so option (C) is not a feature of it.

Step 4: Why the other options are wrong.
(A), (B) and (D) are all genuine features of S_N2: single step, backside attack and simultaneous bond making and breaking.

Final Answer:
Planar carbocation formation is not an SN2 feature, option (C).

$\boxed{(C)}$

Quick Tip: SN2 is a one-step concerted process with no carbocation intermediate.

89. Which reagent is used to convert Alkyl halide into Alkyl nitrite ?

(A) Silver nitrite

(B) Silver cyanide

(C) Potassium nitrite

(D) Silver carboxylate

Correct Answer: (C) Potassium nitrite

Solution:

Step 1: Understanding the Concept:
The nitrite ion, NO_2^-, is an ambident nucleophile. It can attack through oxygen to give an alkyl nitrite or through nitrogen to give a nitroalkane.

Step 2: Key Rule:
Ionic salts such as KNO2 and NaNO2 are largely ionic, so the more electronegative oxygen, which carries the free negative charge, attacks the carbon. Covalent salts such as $AgNO_2$ attack through nitrogen, because the nitrogen lone pair is available.

Step 3: Detailed Explanation:

$$R\text{-}X + KNO_2 \rightarrow R\text{-}O\text{-}N{=}O + KX$$

The product is an alkyl nitrite. With $AgNO_2$ the product would be a nitroalkane, $R\text{-}NO_2$.

Step 4: Why the other options are wrong.
Silver nitrite (A) gives the nitro compound. Silver cyanide (B) gives isocyanides (via N attack). Silver carboxylate (D) gives esters, RCOOR.

Final Answer:
Potassium nitrite gives the alkyl nitrite, option (C).

Potassium nitrite (C)

Quick Tip: Ionic KNO2 attacks through oxygen, giving R-O-N=O (alkyl nitrite).

90. Which of the following on reaction with Grignard's reagent followed by hydrolysis forms tertiary alcohol ?

(A) HCHO

(B) CH_3CHO

(C) CH_3-CH_2-CHO

(D) $CH_3 - CO - CH_3$

Correct Answer: (D) $CH_3 - CO - CH_3$

Solution:

Step 1: Understanding the Concept:
A Grignard reagent adds to the carbonyl carbon. After hydrolysis, the type of alcohol depends on the carbonyl compound used.

Step 2: Key Rules:
Formaldehyde (HCHO) gives a primary alcohol. Any other aldehyde gives a secondary alcohol. A ketone gives a tertiary alcohol.

Step 3: Detailed Explanation:
Acetone, CH_3COCH_3, reacts with, for instance, CH_3MgBr:

$$CH_3COCH_3 + CH_3MgBr \rightarrow (CH_3)_3C\text{-}OMgBr \xrightarrow{H_2O} (CH_3)_3C\text{-}OH$$

The product is tert-butyl alcohol, a tertiary alcohol, because the carbinol carbon has three alkyl groups.

Step 4: Why the other options are wrong.
HCHO (A) gives a primary alcohol. CH_3CHO (B) and CH_3CH_2CHO (C) are aldehydes and give secondary alcohols.

Final Answer:

Acetone gives a tertiary alcohol, option (D).

$$\boxed{CH_3COCH_3 \text{ (D)}}$$

Quick Tip: Only a ketone gives a tertiary alcohol; HCHO gives primary and other aldehydes give secondary.

91. Identify product 'B' in following series of reactions

2-methylpropan-2-ol $\xrightarrow[363K]{20\%H_2SO_4}$ A $\xrightarrow[\text{Conc. } H_2SO_4]{H_2O\Delta}$ B

(A) n-Butylalcohol

(B) sec-Butylalcohol

(C) Iso-butylalcohol

(D) tert-Butylalcohol

Correct Answer: (D) tert-Butylalcohol

Solution:

Step 1: Understanding the Reaction Sequence:

2-Methylpropan-2-ol is tert-butyl alcohol. Step 1 uses 20% H_2SO_4 at 363 K, which dehydrates tertiary alcohols by a mild route. Step 2 hydrates the product.

Step 2: Step A:

Dehydration gives the alkene 2-methylpropene (isobutene):

$$(CH_3)_3C\text{-}OH \xrightarrow{20\%\ H_2SO_4,\ 363\ K} (CH_3)_2C{=}CH_2 + H_2O$$

Step 3: Step B:
Acid-catalysed hydration follows Markovnikov addition. The proton adds to the CH_2 carbon, giving the stable tertiary carbocation, then water adds to it:

$$(CH_3)_2C{=}CH_2 + H_2O \xrightarrow{H^+} (CH_3)_3C\text{-}OH$$

So B is tert-butyl alcohol.

Step 4: Why the other options are wrong.
Anti-Markovnikov hydration would be needed for n-butyl alcohol, isobutyl alcohol or sec-butyl alcohol, and acid hydration does not do that. A tertiary carbocation is the most stable, so only tert-butyl alcohol forms.

Final Answer:
B is tert-butyl alcohol, option (D).

tert-Butyl alcohol (D)

Quick Tip: Dehydration gives 2-methylpropene; hydration follows Markovnikov rule to the tertiary alcohol.

92. Which among the following compounds has highest solubility in water?

(A) Phenol

(B) p-resol

(C) o-Nitrophenol

(D) p-Nitrophenol

Correct Answer: (A) Phenol

Solution:

Step 1: Understanding the Concept:
Solubility of a phenol in water depends on how well it forms hydrogen bonds with water, and on the size of the non-polar part of the molecule.

Step 2: Detailed Explanation:
Phenol has one polar -OH group and a plain benzene ring. Its -OH can form hydrogen bonds with water freely, so it dissolves best of the four. Adding a methyl group (p-cresol) increases the non-polar part and lowers solubility.

Step 3: Why the other options are wrong.
p-Cresol (B) has an extra hydrophobic methyl group. o-Nitrophenol (C) forms an intramolecular hydrogen bond between OH and NO_2, so its OH is not free to bond with water and it is much less soluble. p-Nitrophenol (D) has a large non-polar nitro-substituted ring and strong intermolecular attraction between its own molecules, so it dissolves less than phenol.

Final Answer:
Phenol has the highest solubility in water, option (A).

Phenol (A)

Quick Tip: Smaller hydrophobic groups and free OH hydrogen bonding with water favour solubility.

93. Which of the following reaction represents Rosenmund reduction ?

(A) $R-CO-Cl \xrightarrow[Pd-BaSO_4]{H_2} R-CHO + HCl$

(B) $R-CN \xrightarrow[H_3O^+]{SnCl_2,HCl} R-CHO + NH_4Cl$

(C) $R-CHO \xrightarrow[\Delta]{Zn-Hg\ conc.HCl} R-CH_3 + H_2O$

(D) $R-CO-R \xrightarrow[ii)\ KOH,\ HO-CH_2-CH_2-OH,\Delta]{i)\ H_2N-NH_2} R-CH_2-R$

Correct Answer: (A) $R-CO-Cl \xrightarrow[Pd-BaSO_4]{H_2} R-CHO + HCl$

Solution:

Step 1: Understanding the Concept:

Rosenmund reduction is the partial hydrogenation of an acyl chloride (acid chloride) to an aldehyde.

Step 2: Key Details:

The catalyst is palladium on barium sulphate, and it is partially poisoned (with quinoline or sulphur) so the aldehyde is not reduced further to an alcohol.

Step 3: Detailed Explanation:

$$R\text{-}CO\text{-}Cl + H_2 \xrightarrow{Pd\text{-}BaSO_4} R\text{-}CHO + HCl$$

This is option (A).

Step 4: Why the other options are wrong.
Option (B) is the Stephen reduction (nitrile to aldehyde with $SnCl_2/HCl$). Option (C) is Clemmensen reduction (Zn-Hg with conc. HCl). Option (D) is Wolff-Kishner reduction (hydrazine then KOH in ethylene glycol).

Final Answer:
Option (A) represents the Rosenmund reduction.

$$\boxed{(A)}$$

Quick Tip: Rosenmund reduction converts an acyl chloride to an aldehyde using H2 over poisoned Pd-BaSO4.

94. Identify the substrate 'A' in the following reaction.

$$A \xrightarrow[ii)\Delta]{i)\ \text{Moist } Ag_2O} CH_3CH_2N(CH_3)_2 + CH_2 = CH_2$$

(A) Diethyldimethylammonium halide

(B) Ethyltrimethylammonium halide

(C) Diethyldimethylammonium hydroxide

(D) Ethyltrimethylammonium hydroxide

Correct Answer: (A) Diethyldimethylammonium halide

Solution:

Step 1: Understanding the Reaction:
This is the Hofmann elimination. A quaternary ammonium halide is converted to the hydroxide using moist silver oxide, and on heating

the hydroxide loses an alkene and a tertiary amine.

Step 2: Work backwards:
The products are $CH_3CH_2N(CH_3)_2$ (ethyldimethylamine) and ethene. Together these have the atoms of a cation with the nitrogen carrying two methyl groups and two ethyl groups: $[(C_2H_5)_2N(CH_3)_2]^+$.

Step 3: Detailed Explanation:
Starting substrate A is the halide, diethyldimethylammonium halide:

$$(C_2H_5)_2N^+(CH_3)_2X^- \xrightarrow{Ag_2O/H_2O} (C_2H_5)_2N^+(CH_3)_2OH^- \xrightarrow{\Delta} CH_3CH_2N(CH_3)_2 + CH_2{=}CH_2 + H_2O$$

Step 4: Why the other options are wrong.
Ethyltrimethylammonium (B, D) has only one ethyl group and could give ethene plus trimethylamine, not ethyldimethylamine. The hydroxide (C) is the intermediate formed after the treatment with Ag_2O, whereas the starting substrate A has to be the halide.

Final Answer:
The substrate is diethyldimethylammonium halide, option (A).

Diethyldimethylammonium halide (A)

Quick Tip: Moist Ag2O converts the quaternary ammonium halide into its hydroxide, which on heating eliminates an alkene.

95. Which from following amino acids is an essential amino acid ?

(A) Phenylalanine

(B) Tyrosine

(C) Glutamine

(D) Alanine

Correct Answer: (A) Phenylalanine

Solution:

Step 1: Understanding the Concept:
Essential amino acids are those the human body cannot synthesise and must get from the diet.

Step 2: Detailed Explanation:
Phenylalanine is one of the ten essential amino acids, along with valine, leucine, isoleucine, lysine, methionine, threonine, tryptophan, histidine and arginine. It must be taken in through the diet.

Step 3: Why the other options are wrong.
Tyrosine (B) is made in the body from phenylalanine, so it is non-essential. Glutamine (C) and alanine (D) are also made by the body from other compounds and are non-essential.

Final Answer:
Phenylalanine is the essential amino acid, option (A).

Phenylalanine (A)

Quick Tip: Phenylalanine cannot be made by the body and must come from food.

96. Identify a natural source of ascorbic acid from following.

(A) Clove

(B) Wintergreen

(C) Citrus fruits

(D) Cinnamon

Correct Answer: (C) Citrus fruits

Solution:

Step 1: Understanding the Concept:
Ascorbic acid is the chemical name for vitamin C, a water-soluble vitamin.

Step 2: Detailed Explanation:
Vitamin C is found abundantly in citrus fruits such as oranges, lemons and amla, and also in guava and tomato. Deficiency leads to scurvy, with bleeding gums and slow wound healing.

Step 3: Why the other options are wrong.
Clove (A) and cinnamon (D) are spices known for essential oils, such as eugenol and cinnamaldehyde, not for vitamin C. Wintergreen (B) is the source of methyl salicylate, used as a pain reliever.

Final Answer:
Citrus fruits are a natural source of ascorbic acid, option (C).

Citrus fruits (C)

Quick Tip: Ascorbic acid is vitamin C, abundant in citrus fruits.

97. What type of glycosidic bond is present in maltose?

(A) β-1, 4

(B) α-1, 4

(C) α-1, 6

(D) β-2, 4

Correct Answer: (B) α-1, 4

Solution:

Step 1: Understanding the Concept:
Maltose is a reducing disaccharide made of two glucose units. The glycosidic bond connects the anomeric carbon of one unit to a hydroxyl of the other.

Step 2: Detailed Explanation:
In maltose the C-1 of one α-D-glucose is joined to the C-4 of a second glucose unit through an oxygen bridge. This is an α-1,4-glycosidic bond. The second glucose keeps a free anomeric carbon, which is why maltose is a reducing sugar.

Step 3: Why the other options are wrong.
The β-1,4 linkage (A) is found in cellobiose and cellulose. The α-1,6 linkage (C) is the branching point in amylopectin and glycogen. A β -2,4 linkage (D) is not a standard linkage in common disaccharides.

Final Answer:

Maltose has an α-1,4 glycosidic bond, option (B).

$$\boxed{\alpha\text{-1,4 (B)}}$$

Quick Tip: Maltose is two alpha-D-glucose units joined by an alpha-1,4 linkage.

98. Which from following polymers is obtained by condensation polymerization method?

(A) Polythene

(B) Polyacrylonitrile

(C) Teflon

(D) Dacron

Correct Answer: (D) Dacron

Solution:

Step 1: Understanding the Concept:

In condensation polymerisation, monomers with two functional groups join with elimination of small molecules such as water or methanol. In addition polymerisation, unsaturated monomers add to each other with no by-product.

Step 2: Detailed Explanation:

Dacron (terylene, polyethylene terephthalate) is made from ethylene glycol and terephthalic acid, or its dimethyl ester, with removal of water or methanol:

$$n\,HOCH_2CH_2OH + n\,HOOC\text{-}C_6H_4\text{-}COOH \rightarrow [\text{-}OCH_2CH_2O\text{-}CO\text{-}C_6H_4\text{-}CO\text{-}]_n + 2n\,H_2O$$

Step 3: Why the other options are wrong.
Polythene (A) comes from ethene, polyacrylonitrile (B) from acrylonitrile and Teflon (C) from tetrafluoroethene. All three are made by addition polymerisation of a C=C monomer.

Final Answer:
Dacron is a condensation polymer, option (D).

Dacron (D)

Quick Tip: Dacron forms from terephthalic acid and ethylene glycol with loss of water or methanol.

99. Which from following polymers is used to obtain shampoo bottles?

(A) HDPE

(B) LDPE

(C) PP

(D) PS

Correct Answer: (A) HDPE

Solution:

Step 1: Understanding the Concept:
Polythene comes in two main forms. Low density polythene (LDPE) is flexible and has branched chains. High density polythene (HDPE) has

linear chains packed closely, so it is rigid and strong.

Step 2: Detailed Explanation:
Shampoo bottles need a firm, chemical-resistant container that keeps its shape, which suits HDPE. HDPE is also used for buckets, pipes, toys and milk bottles.

Step 3: Why the other options are wrong.
LDPE (B) is soft and used for squeeze bags, films and flexible containers. PP (C), polypropylene, is used for ropes, bottle caps and crates. PS (D), polystyrene, is used for disposable cups and insulation, and is brittle.

Final Answer:
HDPE is used for shampoo bottles, option (A).

HDPE (A)

Quick Tip: HDPE is rigid and tough, so it is used for shampoo bottles and milk jugs.

100. Identify the monomers used for preparation of glyptal?

(A) Styrene and phosgene

(B) Ethylene glycol and phthalic acid

(C) Butadiene and ethylene glycol

(D) Acrylamide and urea

Correct Answer: (B) Ethylene glycol and phthalic acid

Solution:

Step 1: Understanding the Concept:

Glyptal (also called alkyd resin) is a condensation polymer of the polyester family.

Step 2: Detailed Explanation:

It is formed by the condensation of ethylene glycol and phthalic acid (or phthalic anhydride):

$$n\,HOCH_2CH_2OH + n\,C_6H_4(COOH)_2 \rightarrow \text{glyptal} + n\,H_2O$$

Glyptal is used in paints and lacquers.

Step 3: Why the other options are wrong.

Styrene and phosgene (A) are not monomers for any common polymer pair. Butadiene and ethylene glycol (C) do not form glyptal. Acrylamide and urea (D) are not used for glyptal; urea combines with formaldehyde to give urea-formaldehyde resin.

Final Answer:

The monomers are ethylene glycol and phthalic acid, option (B).

$$\boxed{\text{Ethylene glycol and phthalic acid (B)}}$$

Quick Tip: Glyptal is a polyester from ethylene glycol and phthalic acid.

Mathematics

Questions 101 to 150 | 2 marks each | 100 marks

101. If w is a complex cube root of unity , then the value of $w^{10} - w^7 + w^5 - w^2 + 1$ is.

(A) 0

(B) -1

(C) 1

(D) w

Correct Answer: (C) 1

Solution:

Step 1: Understanding the Concept:

A complex cube root of unity w satisfies $w^3 = 1$ and $1 + w + w^2 = 0$. Any power of w can be reduced by removing multiples of 3 from the exponent.

Step 2: Reduce each power:

$w^{10} = w^9 \cdot w = w$.

$w^7 = w^6 \cdot w = w$.

$w^5 = w^3 \cdot w^2 = w^2$.

w^2 stays as it is.

Step 3: Substitute:

$$w^{10} - w^7 + w^5 - w^2 + 1 = w - w + w^2 - w^2 + 1 = 1$$

Step 4: Why the other options are wrong.

The value 0 would result if the constant 1 were missing. The value -1 and w come from sign or exponent slips, such as treating w^{10} as w^2 or forgetting that the terms cancel in pairs.

Final Answer:

The value of the expression is 1, option (C).

$$\boxed{1}$$

Quick Tip: Use w^3 = 1 to reduce every power of w to w or w squared.

102. If $a_1, a_2, a_3, \ldots, a_n$ are in arithmetic progression with common difference d, then $tan[tan^{-1}(\frac{d}{1+a_1a_2}) + tan^{-1}(\frac{d}{1+a_2a_3}) + \ldots + tan^{-1}(\frac{d}{1+a_{n-1}a_n})]$ = _____

(A) $\frac{a_1-a_n}{1+a_1a_n}$

(B) $\frac{a_n-a_1}{1-a_1a_n}$

(C) $\frac{a_n-a_1}{1+a_1a_n}$

(D) $\frac{a_1+a_n}{1+a_1a_n}$

Correct Answer: (C) $\frac{a_n-a_1}{1+a_1a_n}$

Solution:

Step 1: Understanding the Concept:

For an AP with common difference d, each term of the form $\tan^{-1}\frac{d}{1+a_ka_{k+1}}$ can be written as a difference of two inverse tangents.

Step 2: Key Formula:

$$\tan^{-1} x - \tan^{-1} y = \tan^{-1} \frac{x-y}{1+xy}$$

Put $x = a_{k+1}$ and $y = a_k$. Since $a_{k+1} - a_k = d$, we get

$$\tan^{-1} \frac{d}{1 + a_k a_{k+1}} = \tan^{-1} a_{k+1} - \tan^{-1} a_k$$

Step 3: Detailed Explanation:

The sum telescopes:

$$(\tan^{-1} a_2 - \tan^{-1} a_1) + (\tan^{-1} a_3 - \tan^{-1} a_2) + \cdots + (\tan^{-1} a_n - \tan^{-1} a_{n-1}) = \tan^{-1} a_n - \tan^{-1} a_1$$

Taking tangent of both sides:

$$\tan(\tan^{-1} a_n - \tan^{-1} a_1) = \frac{a_n - a_1}{1 + a_1 a_n}$$

Step 4: Why the other options are wrong.

Options (A) and (D) have the wrong numerator ($a_1 - a_n$ has the wrong sign, and $a_1 + a_n$ is not a difference). Option (B) has $1 - a_1 a_n$ in the denominator, which belongs to the tangent of a sum, not of a difference.

Final Answer:

The value is $\frac{a_n - a_1}{1 + a_1 a_n}$, option (C).

$$\boxed{\frac{a_n - a_1}{1 + a_1 a_n}}$$

Quick Tip: Write each term as tan inverse of a_{k+1} minus tan inverse of a_k.

103. In a right-angled $\triangle ABC$, the measures of the angles are in an Arithmetic Progression (A.P.) If its smallest side is 4 units, then the area of $\triangle ABC$ is.....

(A) 16 sq. units

(B) $8\sqrt{3}$ sq. units

(C) $16\sqrt{3}$ sq. units

(D) 32 sq. units

Correct Answer: (B) $8\sqrt{3}$ sq. units

Solution:

Step 1: Understanding the Concept:

The three angles sum to $180°$ and are in AP. Let them be $a - d,\ a,\ a + d$. Then $3a = 180°$, so the middle angle is $60°$.

Step 2: Find the angles:

One angle is $90°$, so $60 + d = 90$, $d = 30°$. The angles are $30°,\ 60°,\ 90°$.

Step 3: Find the sides:

Sides are in the ratio $1 : \sqrt{3} : 2$. The smallest side (opposite $30°$) is 4, so the other leg is $4\sqrt{3}$ and the hypotenuse is 8.

Step 4: Area:

$$\text{Area} = \frac{1}{2} \times 4 \times 4\sqrt{3} = 8\sqrt{3} \text{ sq. units}$$

Step 5: Why the other options are wrong.
16 would be the area of a right isosceles triangle with legs $4\sqrt{2}$ or a different ratio. $16\sqrt{3}$ results from taking the hypotenuse as 8 and a leg of 4 and multiplying without the half. 32 does not match this triangle.

Final Answer:
The area is $8\sqrt{3}$ sq. units, option (B).

$$\boxed{8\sqrt{3} \text{ sq. units}}$$

Quick Tip: Angles in AP with a right angle must be 30, 60, 90.

104. If $sec^{-1}(\frac{x^2+y^2}{x^2-y^2}) = 2a$ such that $y\frac{dy}{dx} = x \cdot f(a)$ then the value of $f(\frac{2\pi}{3})$ is

(A) -3

(B) $\sqrt{3}$

(C) 3

(D) $\frac{1}{2}$

Correct Answer: (C) 3

Solution:

Step 1: Understanding the Concept:
Here a acts as a constant parameter. The relation $\sec^{-1}\frac{x^2+y^2}{x^2-y^2} = 2a$ defines y as a function of x, and we need $y\frac{dy}{dx}$ in terms of x and a function of a.

Step 2: Simplify the relation:

Taking secant, $\frac{x^2+y^2}{x^2-y^2} = \sec 2a$, so $\frac{x^2-y^2}{x^2+y^2} = \cos 2a$.

By componendo and dividendo, $\frac{y^2}{x^2} = \frac{1-\cos 2a}{1+\cos 2a} = \tan^2 a$.

So $y = x \tan a$.

Step 3: Differentiate:

$\frac{dy}{dx} = \tan a$. Therefore

$$y\frac{dy}{dx} = x \tan a \cdot \tan a = x \tan^2 a$$

So $f(a) = \tan^2 a$.

Step 4: Evaluate:

$f\left(\frac{2\pi}{3}\right) = \tan^2 \frac{2\pi}{3} = (-\sqrt{3})^2 = 3$.

Step 5: Why the other options are wrong.

Option (A) -3 cannot occur because $\tan^2$ is never negative. Option (B) $\sqrt{3}$ is $|\tan 60°|$ without squaring. Option (D) $1/2$ has no connection to this function.

Final Answer:

$f(2\pi/3) = 3$, option (C).

$\boxed{3}$

Quick Tip: Convert the secant relation to tan a = y/x and differentiate with a constant.

105. The value of $cos15° - sin15°$ is .....

(A) $\sqrt{2}$

(B) $-\sqrt{2}$

(C) $\frac{1}{\sqrt{2}}$

(D) $-\frac{1}{\sqrt{2}}$

Correct Answer: (C) $\frac{1}{\sqrt{2}}$

Solution:

Step 1: Understanding the Concept:

An expression $\cos x - \sin x$ can be written as a single cosine using the amplitude-phase form.

Step 2: Key Identity:

$$\cos x - \sin x = \sqrt{2}\cos(x + 45°)$$

Step 3: Detailed Explanation:

Take $x = 15°$:

$$\cos 15° - \sin 15° = \sqrt{2}\cos 60° = \sqrt{2} \times \frac{1}{2} = \frac{1}{\sqrt{2}}$$

Check numerically: $\cos 15° = 0.9659$ and $\sin 15° = 0.2588$, so the difference is $0.7071 = 1/\sqrt{2}$.

Step 4: Why the other options are wrong.

$\sqrt{2} = 1.414$ is too large because both values are below 1. The negative options are impossible since $\cos 15° > \sin 15°$, so the difference is

positive.

Final Answer:

The value is $\frac{1}{\sqrt{2}}$, option (C).

$$\boxed{\frac{1}{\sqrt{2}}}$$

Quick Tip: Write cos x minus sin x as root 2 cos(x + 45 degrees).

106. The general solution of $cosx + sinx = cos2x + sin2x$ is $x = np\pi$ or $x = \frac{nq\pi}{3} + \frac{\pi}{6}$ for $n \in Z$ then $p : q =$

(A) 1 : 1

(B) 1 : 2

(C) 2 : 3

(D) 2 : 1

Correct Answer: (A) 1 : 1

Solution:

Step 1: Understanding the Concept:

Use the sum-to-product formulas to factorise the equation and solve each factor.

Step 2: Rearrange:

$\cos x - \cos 2x = \sin 2x - \sin x$.

Left side: $\cos x - \cos 2x = 2 \sin \frac{3x}{2} \sin \frac{x}{2}$.

Right side: $\sin 2x - \sin x = 2 \cos \frac{3x}{2} \sin \frac{x}{2}$.

Step 3: Solve:

$$2\sin\frac{x}{2}\left(\sin\frac{3x}{2}-\cos\frac{3x}{2}\right)=0$$

Case 1: $\sin\frac{x}{2}=0$ gives $\frac{x}{2}=n\pi$, so $x=2n\pi$.
Case 2: $\tan\frac{3x}{2}=1$ gives $\frac{3x}{2}=n\pi+\frac{\pi}{4}$, so $x=\frac{2n\pi}{3}+\frac{\pi}{6}$.

Step 4: Compare with the given form:
$x=np\pi$ means $p=2$. $x=\frac{nq\pi}{3}+\frac{\pi}{6}$ means $q=2$. So $p:q=2:2=1:1$.

Step 5: Why the other options are wrong.
Ratios 1:2, 2:3 and 2:1 would need q or p to take values other than 2, which does not match the solution set above.

Final Answer:
The ratio is 1 : 1, option (A).

$$\boxed{1:1}$$

Quick Tip: Group the equation as cos x - cos 2x = sin 2x - sin x and factor.

107. The number of common solutions of the pair of equations $2sin^2\theta-2cos^2\theta+1=0$ and $2sin^2\theta+3sin\theta-2=0$ in the interval $[0,2\pi]$, is

(A) 1

(B) 2

(C) 3

(D) 4

Correct Answer: (B) 2

Solution:

Step 1: Equation 1:

$2\sin^2\theta - 2\cos^2\theta + 1 = 0$ means $-2\cos 2\theta + 1 = 0$, so $\cos 2\theta = \frac{1}{2}$.

Then $2\theta = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}$ in $[0, 4\pi]$, so $\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$.

Step 2: Equation 2:

$2\sin^2\theta + 3\sin\theta - 2 = 0$ factors as $(2\sin\theta - 1)(\sin\theta + 2) = 0$.

$\sin\theta = -2$ is impossible, so $\sin\theta = \frac{1}{2}$, giving $\theta = \frac{\pi}{6}, \frac{5\pi}{6}$ in $[0, 2\pi]$.

Step 3: Common solutions:

The angles in both lists are $\frac{\pi}{6}$ and $\frac{5\pi}{6}$. That is 2 common solutions.

Step 4: Why the other options are wrong.

Option 1 misses one of the two angles. Options 3 and 4 may come from counting $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$, which satisfy the first equation but have $\sin\theta = -\frac{1}{2}$, so they fail the second.

Final Answer:

There are 2 common solutions, option (B).

$\boxed{2}$

Quick Tip: Solve each equation on [0, 2 pi] and count the angles that satisfy both.

108. The area of a triangle formed by a line with the coordinate axes is 49 sq. units. If the perpendicular drawn from the origin to this line makes an angle of 45° with the positive X-axis, then the equation of line is....

(A) $x + y = 7$

(B) $x + y = 7\sqrt{2}$

(C) $x + y = \sqrt{2}$

(D) $x + y = 2$

Correct Answer: (B) $x + y = 7\sqrt{2}$

Solution:

Step 1: Understanding the Concept:

If the perpendicular from the origin to a line has length p and makes angle α with the positive x-axis, the line is $x\cos\alpha + y\sin\alpha = p$.

Step 2: Set up:

With $\alpha = 45°$: $\frac{x}{\sqrt{2}} + \frac{y}{\sqrt{2}} = p$, that is $x + y = p\sqrt{2}$.

Both intercepts equal $p\sqrt{2}$.

Step 3: Use the area:

$$\text{Area} = \frac{1}{2}(p\sqrt{2})(p\sqrt{2}) = p^2 = 49$$

So $p = 7$ and the line is $x + y = 7\sqrt{2}$.

Step 4: Why the other options are wrong.

$x + y = 7$ has intercepts 7 and area 24.5. $x + y = \sqrt{2}$ and $x + y = 2$ have areas 1 and 2, far from 49.

Final Answer:

The line is $x + y = 7\sqrt{2}$, option (B).

$$\boxed{x + y = 7\sqrt{2}}$$

Quick Tip: Use the normal form x cos a + y sin a = p with a = 45 degrees.

109. If the equation $16x^2 - 24xy + 9y^2 - 8x + 6y - 35 = 0$ represents a pair of straight lines, then the equation of the locus of points equidistant from these two lines is.....

(A) $4x - 3y - 1 = 0$

(B) $4x - 3y + 1 = 0$

(C) $8x - 6y - 1 = 0$

(D) $8x - 6y + 1 = 0$

Correct Answer: (A) $4x - 3y - 1 = 0$

Solution:

Step 1: Understanding the Concept:

The given equation represents a pair of lines. Look for a perfect square in the second-degree terms: $16x^2 - 24xy + 9y^2 = (4x - 3y)^2$. This tells us that the lines are parallel.

Step 2: Factorise:

Let $u = 4x - 3y$. The equation is $u^2 - 2u - 35 = 0$, because $-8x + 6y = -2(4x - 3y)$.

$$(u - 7)(u + 5) = 0$$

So the lines are $4x - 3y = 7$ and $4x - 3y = -5$.

Step 3: Locus:
Points equidistant from two parallel lines lie on the parallel line midway between them: $4x - 3y = \frac{7+(-5)}{2} = 1$.
So the locus is $4x - 3y - 1 = 0$.

Step 4: Why the other options are wrong.
$4x - 3y + 1 = 0$ corresponds to $u = -1$, not the midpoint. Options (C) and (D) have coefficients $8x - 6y$, which gives $u = \frac{1}{2}$ or $-\frac{1}{2}$, also not 1.

Final Answer:
The locus is $4x - 3y - 1 = 0$, option (A).

$$\boxed{4x - 3y - 1 = 0}$$

Quick Tip: Factor the second-degree expression into two parallel lines and take their midline.

110. The equation of the circle which passes through the points $(2, 3)$ and $(4, 5)$ and whose centre lies on a straight line $4x - y - 3 = 0$, is

(A) $(x-1)^2+(y-6)^2=10$

(B) $(x-3)^2+(y-4)^2=2$

(C) $x^2+(y-7)^2=20$

(D) $(x-2)^2+(y-5)^2=4$

Correct Answer: (D) $(x-2)^2+(y-5)^2=4$

Solution:

Step 1: Understanding the Concept:

The centre (h,k) of the circle must satisfy the line $4h-k-3=0$ and must be equidistant from $(2,3)$ and $(4,5)$, that is, on the perpendicular bisector of the chord joining them.

Step 2: Perpendicular bisector:

Midpoint of the two points is $(3,4)$. The chord has slope $\frac{5-3}{4-2}=1$, so the bisector has slope -1: $y-4=-(x-3)$, that is, $x+y=7$.

Step 3: Find the centre:

Solve $4x-y=3$ and $x+y=7$. Adding gives $5x=10$, so $x=2$, $y=5$.

Centre is $(2,5)$.

Radius squared $=(2-2)^2+(3-5)^2=4$.

Step 4: Result:

The circle is $(x-2)^2+(y-5)^2=4$.

Step 5: Why the other options are wrong.

Option (A) has centre $(1,6)$: $4-6-3\neq 0$. Option (B) has centre $(3,4)$: $12-4-3=5\neq 0$. Option (C) has centre $(0,7)$: $0-7-3\neq 0$.

Final Answer:

The circle is $(x-2)^2+(y-5)^2=4$, option (D).

$$\boxed{(x-2)^2+(y-5)^2=4}$$

Quick Tip: Test each option: the centre must lie on 4x - y - 3 = 0 and be equidistant from both points.

111. If θ is the eccentric angle of a point on the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ such that the distance of the point from the center is 5, then θ =.....

(A) 0

(B) $\frac{\pi}{6}$

(C) $\frac{\pi}{3}$

(D) $\frac{\pi}{2}$

Correct Answer: (A) 0

Solution:

Step 1: Understanding the Concept:

A point on the ellipse $\frac{x^2}{25}+\frac{y^2}{9}=1$ with eccentric angle θ is $(5\cos\theta,\ 3\sin\theta)$.

Step 2: Use the distance condition:

$$OP^2=25\cos^2\theta+9\sin^2\theta=25$$

$$25\cos^2\theta+9(1-\cos^2\theta)=25\Rightarrow 16\cos^2\theta=16$$

So $\cos^2\theta = 1$, which gives $\theta = 0$ (or π).

Step 3: Interpretation:
The distance from the centre is 5 only at the end of the major axis, $(\pm 5, 0)$, where the semi-major axis $a = 5$.

Step 4: Why the other options are wrong.
At $\theta = \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}$ the distance OP is less than 5, for example 3 at $\theta = \frac{\pi}{2}$.

Final Answer:
The eccentric angle is 0, option (A).

$$\boxed{0}$$

Quick Tip: Use P = (a cos t, b sin t) and set OP squared equal to 25.

112. $\lim_{x\to 0}\left[\frac{x\cdot\log(1+4x)}{(e^{4x}-1)^2}\right] = \cdots$

(A) $\frac{1}{4}$

(B) $\frac{1}{16}$

(C) $\frac{1}{3}$

(D) $\frac{1}{9}$

Correct Answer: (A) $\frac{1}{4}$

Solution:

Step 1: Understanding the Concept:
We need the standard limits $\lim_{x\to 0}\frac{\log(1+ax)}{x} = a$ and $\lim_{x\to 0}\frac{e^{ax}-1}{x} = a$.

Step 2: Rewrite the expression:

$$\frac{x\log(1+4x)}{(e^{4x}-1)^2} = \frac{\log(1+4x)}{x}\cdot\frac{x^2}{(e^{4x}-1)^2} = \frac{\log(1+4x)}{x}\cdot\left(\frac{x}{e^{4x}-1}\right)^2$$

Step 3: Apply limits:

$$\lim = 4\times\left(\frac{1}{4}\right)^2 = 4\times\frac{1}{16} = \frac{1}{4}$$

Step 4: Why the other options are wrong.

$\frac{1}{16}$ comes from forgetting the factor 4 from the logarithm. $\frac{1}{3}$ and $\frac{1}{9}$ do not follow from these standard limits.

Final Answer:

The limit is $\frac{1}{4}$, option (A).

$$\boxed{\frac{1}{4}}$$

Quick Tip: Use log(1+4x) ~ 4x and (e^{4x}-1) ~ 4x as x tends to 0.

113. Consider the following statements.

p: If $3^4 > 4^3$, then $3^3 > 4^4$

q: The roots of the equation $x^2 - 2x + 2 = 0$ are real if and only if Mumbai is in Maharashtra.

r: Statement p is true or statement q is false.

Which of the following has truth value T (true)?

(A) $(p \vee q) \wedge r$

(B) $p \vee (q \wedge r)$

(C) $p \wedge (q \vee r)$

(D) $(p \wedge q) \vee r$

Correct Answer: (D) $(p \wedge q) \vee r$

Solution:

Step 1: Truth value of p:

$3^4 = 81 > 4^3 = 64$ is true. $3^3 = 27 > 4^4 = 256$ is false. A true hypothesis with a false conclusion makes "if...then" false. So p is F.

Step 2: Truth value of q:

The roots of $x^2 - 2x + 2 = 0$ have discriminant $4 - 8 = -4 < 0$, so the roots are not real: statement F. "Mumbai is in Maharashtra" is T. A biconditional between F and T is F. So q is F.

Step 3: Truth value of r:

r is "p is true or q is false" which is $p \vee \sim q = F \vee T = T$. So r is T.

Step 4: Evaluate the options:

(A) $(p \vee q) \wedge r = (F \vee F) \wedge T = F$.

(B) $p \vee (q \wedge r) = F \vee (F \wedge T) = F$.

(C) $p \wedge (q \vee r) = F \wedge T = F$.

(D) $(p \wedge q) \vee r = F \vee T = T$.

Final Answer:

Only option (D) is true.

$$\boxed{(p \wedge q) \vee r}$$

Quick Tip: Find the truth values of p, q and r first, then evaluate each option.

114. Which of the following is/are not true ?

I) If 1 is not a prime number, then 2 is not a prime number.

II) e is a vowel and $12 \times 3 = 36$

III) It is not true that 14 is a composite number and 3 is even number.

IV) $\sqrt{5}$ is an irrational number, but $3 + \sqrt{5}$ is a complex number.

(A) Only I and IV

(B) Only I

(C) Only II and III

(D) Only I, II and IV

Correct Answer: (B) Only I

Solution:

Step 1: Statement I:

"If 1 is not a prime number, then 2 is not a prime number." The hypothesis (1 is not prime) is true. The conclusion (2 is not prime) is false, since 2 is prime. True implies false is false, so I is not true.

Step 2: Statement II:

"e is a vowel and $12 \times 3 = 36$". Both parts are true, so II is true.

Step 3: Statement III:

"It is not true that 14 is a composite number and 3 is even." The inside is T and F, so the conjunction is F, and its negation is T. So III is true.

Step 4: Statement IV:

"$\sqrt{5}$ is irrational, but $3 + \sqrt{5}$ is a complex number." Both parts are true, since every real number is also a complex number. So IV is true.

Step 5: Conclusion:

Only statement I is not true.

Final Answer:

Only statement I is not true, option (B).

Only I

Quick Tip: Check each statement using truth tables for if-then, and, and not.

115. The negation of the inverse of the statement $\sim p \vee \sim q$ is...

(A) $p \wedge q$

(B) $\sim p \wedge q$

(C) $\sim p \wedge \sim q$

(D) $p \vee q$

Correct Answer: (C) $\sim p \wedge \sim q$

Solution:

Step 1: Understanding the Concept:

The inverse of a conditional $p \to q$ is $\sim p \to \sim q$. The statement $\sim p \vee \sim q$ can be read as the conditional $p \to \sim q$, because $a \to b \equiv \sim a \vee b$.

Step 2: Take the inverse:

For $p \to \sim q$ the inverse is $\sim p \to \sim (\sim q) = \sim p \to q$.

Writing this as a disjunction, $\sim p \to q \equiv p \vee q$.

Step 3: Take the negation:

$$\sim (p \vee q) \equiv \sim p \wedge \sim q$$

by De Morgan's law.

Step 4: Why the other options are wrong.

$p \wedge q$ and $p \vee q$ have the wrong connective or lack the negations. $\sim p \wedge q$ keeps q un-negated, but De Morgan's law negates both parts.

Final Answer:

The negation is $\sim p \wedge \sim q$, option (C).

$$\boxed{\sim p \wedge \sim q}$$

Quick Tip: Read the statement as p implies not q, take its inverse, then negate.

116. If $A = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}, B = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ such that $XA = B^T$ and $A^{-1}Y = B$, then $XY =$

(A) $[-1]$

(B) $[1]$

(C) $[-2]$

(D) $[2]$

Correct Answer: (D) $[2]$

Solution:

Step 1: Understanding the Concept:

From $XA = B^T$, multiply on the right by A^{-1}: $X = B^T A^{-1}$. From $A^{-1}Y = B$, multiply on the left by A: $Y = AB$.

Step 2: Multiply:

$$XY = B^T A^{-1} AB = B^T (A^{-1}A)B = B^T B$$

The matrix A cancels out, so we do not need to compute A^{-1} at all.

Step 3: Compute:

$B = [1, 0, 1]^T$, so $B^T B = 1 \cdot 1 + 0 \cdot 0 + 1 \cdot 1 = 2$, the 1×1 matrix $[2]$.

Step 4: Why the other options are wrong.

$[-1]$, $[1]$ and $[-2]$ would need a different sum of squares. $B^T B$ is a sum of squares, so it can never be negative.

Final Answer:

$XY = [2]$, option (D).

$$\boxed{[2]}$$

Quick Tip: Express X and Y using the inverse of A and watch A and its inverse cancel.

117. Let $A = \begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$, where a and b are the roots of the equation $x^2 - 4x + 2 = 0$. If $A + A^{-1} = kI_2$, then the value of k is _____

(A) 2

(B) $2\sqrt{2}$

(C) 4

(D) 1

Correct Answer: (C) 4

Solution:

Step 1: Understanding the Concept:

For a 2×2 matrix $\begin{pmatrix} a & 1 \\ 1 & b \end{pmatrix}$, the inverse is $\frac{1}{ab-1}\begin{pmatrix} b & -1 \\ -1 & a \end{pmatrix}$.

Step 2: Use the roots:

For $x^2 - 4x + 2 = 0$, the sum of roots is $a + b = 4$ and the product is $ab = 2$. So $ab - 1 = 1$ and

$$A^{-1} = \begin{pmatrix} b & -1 \\ -1 & a \end{pmatrix}$$

Step 3: Add:

$$A + A^{-1} = \begin{pmatrix} a+b & 0 \\ 0 & a+b \end{pmatrix} = 4I$$

So $k = 4$.

Step 4: Why the other options are wrong.
Options 2 and 1 and $2\sqrt{2}$ come from using the product (2) or its root instead of the sum of the roots, which is the number that appears on the diagonal.

Final Answer:
$k = 4$, option (C).

$$\boxed{4}$$

Quick Tip: Use sum and product of roots a + b = 4 and ab = 2 to find the inverse.

118. The domain of the function $f(x) = \sqrt{x-1} + \sqrt{6-x}$ is...

(A) $[1, \infty)$

(B) $[1, 6]$

(C) $(-\infty, 1)$

(D) $(6, \infty)$

Correct Answer: (B) $[1, 6]$

Solution:

Step 1: Understanding the Concept:

A square root is defined only for non-negative arguments. A sum is defined where each term is defined.

Step 2: Conditions:

$x - 1 \geq 0 \Rightarrow x \geq 1$.

$6 - x \geq 0 \Rightarrow x \leq 6$.

Step 3: Combine:

Both must hold, so $1 \leq x \leq 6$. The domain is $[1, 6]$.

Step 4: Why the other options are wrong.

$[1, \infty)$ forgets the second root. $(-\infty, 1)$ and $(6, \infty)$ each make one of the radicands negative.

Final Answer:

The domain is $[1, 6]$, option (B).

$$\boxed{[1, 6]}$$

Quick Tip: Both expressions under the roots must be non-negative.

119. The domain of the function $f(x) = {}^{(10-x)}C_{(x-6)}$ is...

(A) $[6, 10]$

(B) $\{6, 7, 8, 9, 10\}$

(C) $[6, 8]$

(D) $\{6, 7, 8\}$

Correct Answer: (D) $\{6, 7, 8\}$

Solution:

Step 1: Understanding the Concept:

The symbol ${}^{n}C_{r}$ needs n and r to be non-negative integers with $0 \leq r \leq n$. Here $n = 10 - x$ and $r = x - 6$.

Step 2: Conditions:

$x - 6 \geq 0 \Rightarrow x \geq 6$.

$x - 6 \leq 10 - x \Rightarrow 2x \leq 16 \Rightarrow x \leq 8$.

Also x must be an integer so that n and r are integers.

Step 3: Domain:

$x \in \{6, 7, 8\}$.

Step 4: Why the other options are wrong.

Intervals such as $[6, 10]$ or $[6, 8]$ include non-integers, which are not allowed. The set $\{6, 7, 8, 9, 10\}$ includes 9 and 10, where $r > n$ (for $x = 9$, $n = 1$ and $r = 3$).

Final Answer:

The domain is $\{6, 7, 8\}$, option (D).

$$\boxed{\{6, 7, 8\}}$$

Quick Tip: For nCr we need r a non-negative integer and r not greater than n.

120. Let $[x]$ denotes the greatest integer less than or equal to x and $f(x) = [tan^2 x]$, then which of the following is true ?

(A) $\lim_{x \to 0} f(x)$ does not exist

(B) $f(x)$ is continuous at $x = 0$

(C) $f(x)$ is not differentiable at $x = 0$

(D) Missing open brace for superscript

Correct Answer: (B) $f(x)$ is continuous at $x = 0$

Solution:

Step 1: Understanding the Concept:

For $f(x) = [\tan^2 x]$, find the values of $\tan^2 x$ near $x = 0$.

Step 2: Detailed Explanation:

When $|x| < \frac{\pi}{4}$, we have $0 \leq \tan^2 x < 1$, so $[\tan^2 x] = 0$. At $x = 0$, $f(0) = 0$. So $f(x) = 0$ in a whole neighbourhood of 0.

Step 3: Conclusions:

$\lim_{x \to 0} f(x) = 0 = f(0)$, so f is continuous at 0. Since f is constant near 0, it is also differentiable there with $f'(0) = 0$.

Step 4: Check the options:

(A) is false because the limit exists and equals 0. (B) is true. (C) is false because f is differentiable at 0. (D) is false because $f'(0) = 0$, not 1.

Final Answer:

The function is continuous at 0, option (B).

$$\boxed{\text{(B)}}$$

Quick Tip: Near 0, tan squared x lies between 0 and 1, so the floor is 0.

121. If $\sqrt{y+x}+\sqrt{y-x}=c$ and $\frac{dy}{dx}=K-\sqrt{\frac{y^2}{x^2}-1}$, then the value of K is

(A) $\frac{-x}{y}$

(B) $\frac{-y}{x}$

(C) $\frac{x}{y}$

(D) $\frac{y}{x}$

Correct Answer: (D) $\frac{y}{x}$

Solution:

Step 1: Understanding the Concept:

Differentiate $\sqrt{y+x}+\sqrt{y-x}=c$ with respect to x, then solve for $\frac{dy}{dx}$.

Step 2: Differentiate:

$$\frac{y'+1}{2\sqrt{y+x}}+\frac{y'-1}{2\sqrt{y-x}}=0$$

Multiply through: $(y'+1)\sqrt{y-x}+(y'-1)\sqrt{y+x}=0$.

So $y'(\sqrt{y-x}+\sqrt{y+x})=\sqrt{y+x}-\sqrt{y-x}$.

Step 3: Simplify:

$$y' = \frac{\sqrt{y+x} - \sqrt{y-x}}{\sqrt{y+x} + \sqrt{y-x}}$$

Multiply numerator and denominator by $\sqrt{y+x} - \sqrt{y-x}$:

$$y' = \frac{(y+x) + (y-x) - 2\sqrt{y^2 - x^2}}{2x} = \frac{y - \sqrt{y^2 - x^2}}{x} = \frac{y}{x} - \sqrt{\frac{y^2}{x^2} - 1}$$

Step 4: Compare:

Matching with $\frac{dy}{dx} = K - \sqrt{\frac{y^2}{x^2} - 1}$ gives $K = \frac{y}{x}$.

Final Answer:

$K = \frac{y}{x}$, option (D).

$$\boxed{\frac{y}{x}}$$

Quick Tip: Square the relation and differentiate implicitly, then simplify.

122. If $y = (x^2 + 1)^{sinx}$ for $x > 0$ such that $\frac{dy}{dx} = y[\frac{2xsinx}{g(x)} + cosx \cdot log[g(x)]]$, then the function $\frac{1}{g(x)}$ is...

(A) increasing

(B) strictly increasing.

(C) decreasing

(D) strictly decreasing

Correct Answer: (D) strictly decreasing

Solution:

Step 1: Understanding the Concept:

Take logarithms: $\log y = \sin x \cdot \log(x^2+1)$, then differentiate to compare with the given form.

Step 2: Differentiate:

$$\frac{1}{y}\frac{dy}{dx} = \cos x \log(x^2+1) + \sin x \cdot \frac{2x}{x^2+1}$$

So $\frac{dy}{dx} = y\left[\frac{2x\sin x}{x^2+1} + \cos x \log(x^2+1)\right]$.

Comparing with the given form, $g(x) = x^2+1$.

Step 3: Monotonicity of 1/g:

$\frac{1}{g(x)} = \frac{1}{x^2+1}$. Its derivative is $-\frac{2x}{(x^2+1)^2}$, which is negative for $x > 0$.

So $\frac{1}{g(x)}$ is strictly decreasing for $x > 0$.

Step 4: Options:

Option (D), strictly decreasing, is the precise description that holds for $x > 0$. The increasing options are false because the derivative is negative.

Final Answer:

$1/g(x)$ is strictly decreasing, option (D).

Strictly decreasing (D)

Quick Tip: Differentiate y = (x squared + 1)^{sin x} logarithmically to identify g(x).

123. The derivative of $f(sinx)$ with respect to $g(secx)$ at $x = \frac{\pi}{4}$, given that Missing open brace for superscript and Missing open brace for superscript is.....

(A) $\frac{\sqrt{3}}{2}$

(B) $\frac{3}{2}$

(C) $\frac{1}{3}$

(D) $\frac{-1}{3}$

Correct Answer: (B) $\frac{3}{2}$

Solution:

Step 1: Understanding the Concept:

To differentiate one function of x with respect to another, divide their derivatives with respect to x: $\frac{du}{dv} = \frac{du/dx}{dv/dx}$.

Step 2: Compute each:

$\frac{d}{dx} f(\sin x) = f'(\sin x)\cos x.$

$\frac{d}{dx} g(\sec x) = g'(\sec x)\sec x \tan x.$

Step 3: Evaluate at $x = \frac{\pi}{4}$:

$\sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}, \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}, \sec \frac{\pi}{4} = \sqrt{2}, \tan \frac{\pi}{4} = 1.$

Numerator: $3 \times \frac{1}{\sqrt{2}}$. Denominator: $1 \times \sqrt{2} \times 1 = \sqrt{2}.$

$$\frac{3/\sqrt{2}}{\sqrt{2}} = \frac{3}{2}$$

Step 4: Why the other options are wrong.

$\frac{\sqrt{3}}{2}$ and $\frac{1}{3}$ do not follow from the given values. $-\frac{1}{3}$ has the wrong sign, since all factors here are positive.

Final Answer:

The derivative is $\frac{3}{2}$, option (B).

$$\boxed{\frac{3}{2}}$$

Quick Tip: Use the chain rule on both functions and divide: (df/dx)/(dg/dx).

124. A spherical iron ball 10 cm in radius is coated with a layer of ice of uniform thickness that melts at a rate of $50\text{cm}^3/\text{min}$. When the thickness of ice is 5 cm, the rate at which the thickness of ice decreases is...

(A) $\frac{1}{18\pi}$ cm/min

(B) $\frac{1}{36\pi}$ cm/min

(C) $\frac{5}{6\pi}$ cm/min

(D) $\frac{1}{54\pi}$ cm/min

Correct Answer: (A) $\frac{1}{18\pi}$ cm/min

Solution:

Step 1: Understanding the Concept:

The ball has radius 10 cm and ice thickness t. Outer radius is $R = 10 + t$. The volume of ice is the volume of a spherical shell: $V = \frac{4}{3}\pi(R^3 - 10^3)$.

Step 2: Differentiate:

$$\frac{dV}{dt} = 4\pi R^2 \frac{dR}{dt}$$

Because the ice melts at a rate of $50\ \text{cm}^3/\text{min}$, $\frac{dV}{dt} = -50$.

Step 3: Evaluate:

When thickness is 5 cm, $R = 15$ cm, so $R^2 = 225$.

$$\frac{dR}{dt} = \frac{-50}{4\pi \times 225} = -\frac{50}{900\pi} = -\frac{1}{18\pi}$$

The thickness $t = R - 10$ changes at the same rate as R, so it decreases at $\frac{1}{18\pi}$ cm/min.

Step 4: Why the other options are wrong.

$\frac{1}{36\pi}$ results from using $\frac{4}{3}\pi R^2$ or R = 15 with the wrong formula, $\frac{1}{54\pi}$ from a different slip, and $\frac{5}{6\pi}$ from using $R = 5$.

Final Answer:

The thickness decreases at $\frac{1}{18\pi}$ cm/min, option (A).

$$\boxed{\frac{1}{18\pi}\ \text{cm/min}}$$

Quick Tip: The volume of ice is a spherical shell; differentiate volume with respect to time.

125. If the line $ax + by + 5 = 0$ is a normal to the curve $xy = 1$ then .....

(A) $a > 0, b < 0$

(B) $a < 0, b < 0$

(C) $a > 0, b = 0$

(D) $a > 0, b > 0$

Correct Answer: (A) $a > 0, b < 0$

Solution:

Step 1: Understanding the Concept:

For the curve $xy = 1$, $y = \frac{1}{x}$, so $\frac{dy}{dx} = -\frac{1}{x^2}$. The slope of the tangent is always negative.

Step 2: Slope of the normal:

The normal slope is the negative reciprocal: $m = x^2 > 0$. So any normal has a positive slope (and cannot be vertical or horizontal).

Step 3: Compare with the line:

The line $ax + by + 5 = 0$ has slope $-\frac{a}{b}$, which must be positive. This needs a and b to have opposite signs, with $b \neq 0$.

Step 4: Check the options:

(A) $a > 0, b < 0$: opposite signs, so the slope is positive. This is possible.

(B) and (D) have same signs, giving a negative slope, which is

impossible.
(C) $b = 0$ gives a vertical line, which cannot be a normal here.

Final Answer:
Option (A) is correct.

$$\boxed{a > 0,\ b < 0}$$

Quick Tip: Slope of the normal to xy = 1 is x squared, which is always positive.

126. On the interval $[0, 1]$, the function $f(x) = x^{25}(1 - x)^{75}$ attains its maximum value at the point $x =$....

(A) 0

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) $\frac{1}{3}$

Correct Answer: (B) $\frac{1}{4}$

Solution:

Step 1: Understanding the Concept:
The function $f(x) = x^{25}(1 - x)^{75}$ is zero at both ends of $[0, 1]$ and positive inside. So its maximum is at an interior critical point.

Step 2: Differentiate:

$$f'(x) = 25x^{24}(1 - x)^{75} - 75x^{25}(1 - x)^{74} = 25x^{24}(1 - x)^{74}\,[(1 - x) - 3x] = 25x^{24}(1 - x)^{74}(1 - 4x)$$

Step 3: Critical point:

On $(0,1)$, $f'(x) = 0$ only when $1 - 4x = 0$, that is $x = \frac{1}{4}$. For $x < \frac{1}{4}$ the derivative is positive and for $x > \frac{1}{4}$ it is negative, so $x = \frac{1}{4}$ is a maximum.

Step 4: Why the other options are wrong.

$x = 0$ gives $f = 0$, the minimum value. $x = \frac{1}{2}$ and $x = \frac{1}{3}$ lie in the region where f is already decreasing.

Final Answer:

The maximum is at $x = \frac{1}{4}$, option (B).

$$\boxed{x = \frac{1}{4}}$$

Quick Tip: Differentiate, set to zero, and use the sign change to confirm a maximum.

127. If $f(x) = \int \frac{x}{(x^2+4)(x^2+9)}\,dx$ and $f(0) = \frac{1}{5}\log(\frac{2}{3})$, then $f(1) =$

(A) $\frac{1}{5}\log(2)$

(B) $\frac{1}{10}\log(2)$

(C) $-\frac{1}{5}\log(2)$

(D) $-\frac{1}{10}\log(2)$

Correct Answer: (D) $-\frac{1}{10}\log(2)$

Solution:

Step 1: Understanding the Concept:

Let $u = x^2$, so $x\,dx = \frac{du}{2}$. The integral becomes $\frac{1}{2}\int \frac{du}{(u+4)(u+9)}$.

Step 2: Partial fractions:

$$\frac{1}{(u+4)(u+9)} = \frac{1}{5}\left(\frac{1}{u+4} - \frac{1}{u+9}\right)$$

So $f(x) = \frac{1}{10}\log\frac{x^2+4}{x^2+9} + C$.

Step 3: Find C:

$f(0) = \frac{1}{10}\log\frac{4}{9} + C = \frac{1}{10}\cdot 2\log\frac{2}{3} + C = \frac{1}{5}\log\frac{2}{3} + C$. This equals the given $\frac{1}{5}\log\frac{2}{3}$, so $C = 0$.

Step 4: Evaluate:

$$f(1) = \frac{1}{10}\log\frac{5}{10} = \frac{1}{10}\log\frac{1}{2} = -\frac{1}{10}\log 2$$

Step 5: Why the other options are wrong.

Options with $\frac{1}{5}$ come from forgetting the factor $\frac{1}{2}$ from $x\,dx = du/2$. The positive options have the wrong sign because $\log\frac{1}{2} < 0$.

Final Answer:

$f(1) = -\frac{1}{10}\log 2$, option (D).

$$\boxed{-\frac{1}{10}\log 2}$$

Quick Tip: Substitute u = x squared and split into partial fractions.

128. $\int \frac{x^4(x^{10}-1)}{x^{20}+3x^{10}+1}\,dx = \cdots$

(A) $tan^{-1}(x^5 + \frac{1}{x^5}) + c$

(B) $\frac{1}{5}tan^{-1}(x^5 + \frac{1}{x^5}) + c$

(C) $tan^{-1}(x^{10} + \frac{1}{x^{10}}) + c$

(D) $\frac{1}{10}tan^{-1}(x^{10} + \frac{1}{x^{10}}) + c$

Correct Answer: (B) $\frac{1}{5}tan^{-1}(x^5 + \frac{1}{x^5}) + c$

Solution:

Step 1: Understanding the Concept:

Divide the numerator and denominator by x^{10}:

$$\frac{x^4(x^{10}-1)}{x^{20}+3x^{10}+1} = \frac{x^4 - x^{-6}}{x^{10}+3+x^{-10}}$$

Step 2: Substitute:

Let $t = x^5 + x^{-5}$. Then $dt = 5(x^4 - x^{-6})\,dx$, so $(x^4 - x^{-6})dx = \frac{dt}{5}$.

Also $t^2 = x^{10} + 2 + x^{-10}$, so $x^{10} + x^{-10} + 3 = t^2 + 1$.

Step 3: Integrate:

$$\int \frac{dt/5}{t^2+1} = \frac{1}{5}\tan^{-1} t + c = \frac{1}{5}\tan^{-1}\left(x^5 + \frac{1}{x^5}\right) + c$$

Step 4: Why the other options are wrong.

Options (A) and (C) miss the factor $\frac{1}{5}$ from $dt = 5(\ldots)dx$. Option (D) uses x^{10} in the inverse tangent and the factor $\frac{1}{10}$, which does not match the substitution.

Final Answer:

The integral is $\frac{1}{5}\tan^{-1}(x^5 + x^{-5}) + c$, option (B).

$$\boxed{\frac{1}{5}\tan^{-1}\left(x^5 + \frac{1}{x^5}\right) + c}$$

Quick Tip: Divide top and bottom by x^10 and substitute t = x^5 + 1/x^5.

129. $\int (e^{\log(\sin x)} + \cos x)x\,dx =$

(A) $x(\sin x + \cos x) + (\sin x - \cos x) + c$

(B) $x(\sin x - \cos x) + (\sin x - \cos x) + c$

(C) $x(\sin x + \cos x) + (\sin x + \cos x) + c$

(D) $x(\sin x - \cos x) + (\sin x + \cos x) + c$

Correct Answer: (D) $x(\sin x - \cos x) + (\sin x + \cos x) + c$

Solution:

Step 1: Simplify:

$e^{\log(\sin x)} = \sin x$, so the integral is $\int x(\sin x + \cos x)\,dx$.

Step 2: Integration by parts:

Take $u = x$ and $dv = (\sin x + \cos x)dx$. Then $v = -\cos x + \sin x$.

$$\int u\,dv = uv - \int v\,du = x(\sin x - \cos x) - \int (\sin x - \cos x)dx$$

Step 3: Finish:

$\int(\sin x - \cos x)dx = -\cos x - \sin x$. So

$$x(\sin x - \cos x) + \cos x + \sin x + c = x(\sin x - \cos x) + (\sin x + \cos x) + c$$

Step 4: Why the other options are wrong.

Options (A) and (C) have $x(\sin x + \cos x)$ as the first term, which uses the integrand itself instead of its antiderivative. Option (B) has the wrong sign on the last bracket because the integral of $\sin x - \cos x$ was not subtracted correctly.

Final Answer:

The integral is $x(\sin x - \cos x) + (\sin x + \cos x) + c$, option (D).

$$\boxed{x(\sin x - \cos x) + (\sin x + \cos x) + c}$$

Quick Tip: Simplify e^{log sin x} to sin x and use integration by parts.

130. The value of $\int sin4xcos3x\,dx$ is

(A) $-\frac{1}{14}cos7x - \frac{1}{2}cosx + c$

(B) $-\frac{1}{14}cos7x + \frac{1}{2}cosx + c$

(C) $\frac{1}{14}cos7x - \frac{1}{2}cosx + c$

(D) $\frac{1}{14}cos7x + \frac{1}{2}cosx + c$

Correct Answer: (A) $-\frac{1}{14}cos7x - \frac{1}{2}cosx + c$

Solution:

Step 1: Understanding the Concept:

Use the product-to-sum identity $2\sin A\cos B = \sin(A+B) + \sin(A - B)$.

Step 2: Apply:

$$\sin 4x \cos 3x = \frac{1}{2}[\sin 7x + \sin x]$$

Step 3: Integrate:

$$\int \frac{1}{2}(\sin 7x + \sin x)\,dx = \frac{1}{2}\left(-\frac{\cos 7x}{7} - \cos x\right) + c = -\frac{1}{14}\cos 7x - \frac{1}{2}\cos x + c$$

Step 4: Why the other options are wrong.

Options with $+\frac{1}{14}\cos 7x$ or $+\frac{1}{2}\cos x$ have sign errors, because the integral of $\sin$ is $-\cos$.

Final Answer:

The integral is $-\frac{1}{14}\cos 7x - \frac{1}{2}\cos x + c$, option (A).

$$\boxed{-\frac{1}{14}\cos 7x - \frac{1}{2}\cos x + c}$$

Quick Tip: Convert the product to a sum using 2 sin A cos B.

131. If $[x]$ denotes the greatest integer less than or equal to x, then the value of the integral $\int_0^2 x^2[x]\,dx$ is equal to

(A) $\frac{8}{3}$

(B) $\frac{3}{8}$

(C) $\frac{7}{3}$

(D) 0

Correct Answer: (C) $\frac{7}{3}$

Solution:

Step 1: Understanding the Concept:

The greatest integer function $[x]$ is 0 for $0 \le x < 1$ and 1 for $1 \le x < 2$. Split the integral at $x = 1$.

Step 2: Split:

$$\int_0^2 x^2[x]\,dx = \int_0^1 x^2 \cdot 0\,dx + \int_1^2 x^2 \cdot 1\,dx$$

Step 3: Evaluate:

$$\int_1^2 x^2\,dx = \left[\frac{x^3}{3}\right]_1^2 = \frac{8}{3} - \frac{1}{3} = \frac{7}{3}$$

Step 4: Why the other options are wrong.

$\frac{8}{3}$ takes $\int_0^2 x^2$ without the floor. $\frac{3}{8}$ is its reciprocal. 0 would hold only if $[x]$ were 0 on the whole interval.

Final Answer:

The integral equals $\frac{7}{3}$, option (C).

$$\boxed{\frac{7}{3}}$$

Quick Tip: Split the interval at 1: the floor is 0 on [0,1) and 1 on [1,2).

132. If $\int_0^1 \frac{dx}{\sqrt{x+1}-\sqrt{x}} = \sqrt{2}k$, then the value of k is...

(A) $\frac{4}{\sqrt{3}}$

(B) $\frac{4}{3}$

(C) $\frac{2}{\sqrt{3}}$

(D) $\frac{2}{3}$

Correct Answer: (B) $\frac{4}{3}$

Solution:

Step 1: Rationalise:

$$\frac{1}{\sqrt{x+1}-\sqrt{x}} \cdot \frac{\sqrt{x+1}+\sqrt{x}}{\sqrt{x+1}+\sqrt{x}} = \frac{\sqrt{x+1}+\sqrt{x}}{(x+1)-x} = \sqrt{x+1}+\sqrt{x}$$

Step 2: Integrate:

$$\int_0^1 (\sqrt{x+1}+\sqrt{x})dx = \frac{2}{3}\left[(x+1)^{3/2}+x^{3/2}\right]_0^1$$

Step 3: Evaluate:

At $x = 1$: $2^{3/2}+1 = 2\sqrt{2}+1$. At $x = 0$: $1+0 = 1$.

$$\frac{2}{3}\left[(2\sqrt{2}+1)-1\right]=\frac{2}{3}\cdot 2\sqrt{2}=\frac{4\sqrt{2}}{3}$$

Comparing with $\sqrt{2}\,k$ gives $k=\frac{4}{3}$.

Step 4: Why the other options are wrong.

$\frac{4}{\sqrt{3}}$ and $\frac{2}{\sqrt{3}}$ contain a root of 3, which never appears here. $\frac{2}{3}$ forgets the factor 2 in $2\sqrt{2}$.

Final Answer:

$k=\frac{4}{3}$, option (B).

$$\boxed{\frac{4}{3}}$$

Quick Tip: Multiply by the conjugate: 1/(sqrt(x+1) - sqrt x) = sqrt(x+1) + sqrt x.

133. The area of the region lying in the first quadrant and bounded by the curve $y=4x^2$, the Y-axis, and the lines $y=2$, $y=4$ is...

(A) $\frac{1}{3}[8+2\sqrt{2}]$ sq. units

(B) $\frac{1}{3}[8-2\sqrt{2}]$ sq. units

(C) $\frac{1}{2}[8-2\sqrt{2}]$ sq. units

(D) $\frac{1}{2}[8+2\sqrt{2}]$ sq. units

Correct Answer: (B) $\frac{1}{3}[8-2\sqrt{2}]$ sq. units

Solution:

Step 1: Understanding the Concept:

The region is bounded by the Y-axis, the curve and two horizontal lines, so we integrate along the Y-axis: $A = \int_{y_1}^{y_2} x\,dy$.

Step 2: Express x in terms of y:

From $y = 4x^2$, in the first quadrant $x = \frac{\sqrt{y}}{2}$.

Step 3: Integrate:

$$A = \int_2^4 \frac{\sqrt{y}}{2}\,dy = \frac{1}{2} \cdot \frac{2}{3}\left[y^{3/2}\right]_2^4 = \frac{1}{3}\left(8 - 2\sqrt{2}\right)$$

because $4^{3/2} = 8$ and $2^{3/2} = 2\sqrt{2}$.

Step 4: Why the other options are wrong.

Option (A) has $8 + 2\sqrt{2}$, a sign slip when subtracting the lower limit. Options (C) and (D) have the factor $\frac{1}{2}$, which comes from forgetting the $\frac{2}{3}$ in the antiderivative.

Final Answer:

The area is $\frac{1}{3}[8 - 2\sqrt{2}]$ sq. units, option (B).

$$\boxed{\frac{1}{3}\left[8 - 2\sqrt{2}\right] \text{ sq. units}}$$

Quick Tip: Integrate x as a function of y between y = 2 and y = 4.

134. The area bounded by the curves $y = |x| - 1$ and $y = -|x| + 1$ is

(A) 1 sq.unit

(B) 2 sq. units

(C) $2\sqrt{2}$ sq.units

(D) 4 sq.units

Correct Answer: (B) 2 sq. units

Solution:

Step 1: Understanding the Concept:

$y = 1 - |x|$ is an upward-opening-down tent with peak at $(0, 1)$. $y = |x| - 1$ is a V with its vertex at $(0, -1)$. They meet where $|x| - 1 = 1 - |x|$, that is $|x| = 1$.

Step 2: Set up the integral:

The region lies between $x = -1$ and $x = 1$, with the tent above the V. By symmetry:

$$A = 2\int_0^1 [(1 - x) - (x - 1)]\,dx = 2\int_0^1 (2 - 2x)\,dx$$

Step 3: Evaluate:

$$2\left[2x - x^2\right]_0^1 = 2(2 - 1) = 2$$

Step 4: Check by geometry:

The four vertices are $(0, 1), (1, 0), (0, -1), (-1, 0)$, a square with diagonals of length 2. Its area is $\frac{1}{2} \times 2 \times 2 = 2$.

Step 5: Why the other options are wrong.

Area 1 is half of the square, 4 is the area of a square with diagonal 2 $\sqrt{2}$, and $2\sqrt{2}$ would be its perimeter-style value, not the area.

Final Answer:

The area is 2 sq. units, option (B).

$$\boxed{\text{2 sq. units}}$$

Quick Tip: The two V-shaped graphs form a square; integrate the gap between them.

135. If $y = f(x)$ is a monotonically increasing function such that $(\frac{dy}{dx})^2 = 6 - \frac{dy}{dx}$ and $y(0) = 5$, then $y(3) = \cdots$

(A) 23

(B) 14

(C) 13

(D) 11

Correct Answer: (D) 11

Solution:

Step 1: Understanding the Concept:

Let $p = \frac{dy}{dx}$. The equation $p^2 = 6 - p$ is a quadratic in p.

Step 2: Solve:

$p^2 + p - 6 = 0$, so $(p + 3)(p - 2) = 0$, giving $p = -3$ or $p = 2$.

Step 3: Use monotonicity:

Since y is monotonically increasing, $\frac{dy}{dx} \geq 0$, so $p = 2$. Then $y = 2x + c$. With $y(0) = 5$, $c = 5$, so $y = 2x + 5$.

Step 4: Evaluate:

$y(3) = 6 + 5 = 11$.

Step 5: Why the other options are wrong.

The value 23 would come from $p = 6$, 14 from $p = 3$, and 13 from $p = \frac{8}{3}$. Using $p = -3$ would give $y(3) = -4$, which is not an option and is excluded by monotonicity.

Final Answer:

$y(3) = 11$, option (D).

$\boxed{11}$

Quick Tip: Solve the quadratic for dy/dx and pick the positive root for an increasing function.

136. If $tanx$ is an integrating factor of the differential equation $\frac{dy}{dx} + Py = Q$, then P can be

(A) $2sec2x$

(B) $tan2x$

(C) $sin2x$

(D) $2csc2x$

Correct Answer: (D) $2csc2x$

Solution:

Step 1: Understanding the Concept:

For $\frac{dy}{dx} + Py = Q$ the integrating factor is I.F. $= e^{\int P\,dx}$.

Step 2: Set up:

We need $e^{\int P\,dx} = \tan x$, so $\int P\,dx = \log(\tan x)$. Differentiate:

$$P = \frac{d}{dx}\log\tan x = \frac{\sec^2 x}{\tan x} = \frac{1}{\sin x \cos x}$$

Step 3: Simplify:

$$P = \frac{2}{2\sin x \cos x} = \frac{2}{\sin 2x} = 2\csc 2x$$

Step 4: Why the other options are wrong.

$2\sec 2x$, $\tan 2x$ and $\sin 2x$ do not equal $\frac{2}{\sin 2x}$. For example at $x = \frac{\pi}{4}$, the correct P is 2, while $\tan 2x$ is undefined and $\sin 2x = 1$.

Final Answer:

$P = 2\csc 2x$, option (D).

$$\boxed{2\csc 2x}$$

Quick Tip: The integrating factor is e^{integral of P}, so P is the log derivative of tan x.

137. The order and degree of the differential equation $\left(\frac{d^3y}{dx^3}\right)^{\frac{2}{3}} - 3\frac{d^2y}{dx^2} + 5\frac{dy}{dx} + 4 = 0$ are respectively

(A) 2, 3

(B) 3, 2

(C) 3, not defined

(D) not defined, 3

Correct Answer: (B) 3, 2

Solution:

Step 1: Understanding the Concept:

Order is the highest derivative present. Degree is the power of the highest-order derivative after the equation is made free of fractional powers of derivatives.

Step 2: Order:

The highest derivative is $\frac{d^3y}{dx^3}$, so the order is 3.

Step 3: Degree:

Isolate the fractional term: $\left(\frac{d^3y}{dx^3}\right)^{2/3} = 3\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 4.$

Cube both sides to remove the power $\frac{2}{3}$: $\left(\frac{d^3y}{dx^3}\right)^2 = (3y'' - 5y' - 4)^3.$

The highest derivative now appears with power 2, so the degree is 2.

Step 4: Why the other options are wrong.

Degree 3 (A) comes from reading the 3 of the cube instead of the power of y'''. "Not defined" does not apply because the equation can be made polynomial in derivatives.

Final Answer:

Order 3 and degree 2, option (B).

$$\boxed{3,\ 2}$$

Quick Tip: Clear the fractional power first, then read off the degree.

138. Let $\bar{a} = \hat{i} + \hat{j} + \hat{k}$, $\bar{b} = \hat{i} - 3\hat{j} + 2\hat{k}$ and $\bar{c} = 3\hat{i} - 2\hat{k}$. If a vector $\bar{p}$ satisfies the conditions $\bar{p} \cdot \bar{c} = 0$ and $\bar{p} \times \bar{a} = \bar{b} \times \bar{a}$, then the value of $|\bar{p}|$ =...

(A) $\sqrt{13}$

(B) $\sqrt{14}$

(C) $\sqrt{17}$

(D) $\sqrt{19}$

Correct Answer: (C) $\sqrt{17}$

Solution:

Step 1: Understanding the Concept:

The condition $\bar{p} \times \bar{a} = \bar{b} \times \bar{a}$ gives $(\bar{p} - \bar{b}) \times \bar{a} = 0$. Two vectors with zero cross product are parallel, so $\bar{p} - \bar{b} = \lambda\bar{a}$.

Step 2: Write p:

$\bar{p} = \bar{b} + \lambda\bar{a} = (1 + \lambda)\hat{i} + (-3 + \lambda)\hat{j} + (2 + \lambda)\hat{k}$.

Step 3: Use the dot condition:

$\bar{p} \cdot \bar{c} = 3(1 + \lambda) + 0 - 2(2 + \lambda) = 3 + 3\lambda - 4 - 2\lambda = \lambda - 1 = 0$, so $\lambda = 1$.

Step 4: Magnitude:

$\bar{p} = 2\hat{i} - 2\hat{j} + 3\hat{k}$, so $|\bar{p}| = \sqrt{4 + 4 + 9} = \sqrt{17}$.

Step 5: Why the other options are wrong.

$\sqrt{13}$, $\sqrt{14}$ and $\sqrt{19}$ arise from arithmetic slips in the components,

such as using $\lambda = 0$ or $\lambda = 2$. With $\lambda = 0$, $\bar{p} = \bar{b}$ has magnitude $\sqrt{14}$, but then $\bar{p} \cdot \bar{c} = 3 - 4 \neq 0$.

Final Answer:

$|\bar{p}| = \sqrt{17}$, option (C).

$$\boxed{\sqrt{17}}$$

Quick Tip: p x a = b x a means (p - b) is parallel to a, so p = b + lambda a.

139. The volume of the tetrahedron whose vertices are A$(-1, 2, 3)$, B $(3, -2, 1)$, C$(p, 1, 3)$, D$(-1, -2, 4)$ is $\frac{16}{3}$ cubic units then the value of p is

(A) $\frac{-10}{3}$

(B) 5

(C) 8

(D) 10

Correct Answer: (A) $\frac{-10}{3}$

Solution:

Step 1: Understanding the Concept:

The volume of a tetrahedron with vertices A, B, C, D is $V = \frac{1}{6}\left|[\vec{AB}\,\vec{AC}\,\vec{AD}]\right|$.

Step 2: Find the edge vectors:

$\vec{AB} = (4, -4, -2)$, $\vec{AC} = (p + 1, -1, 0)$, $\vec{AD} = (0, -4, 1)$.

Step 3: Triple product:

$$\begin{vmatrix} 4 & -4 & -2 \\ p+1 & -1 & 0 \\ 0 & -4 & 1 \end{vmatrix} = 4(-1) + 4(p+1) - 2(-4(p+1)) = 12(p+1) - 4 = 12p + 8$$

Step 4: Use the volume:

$\frac{1}{6}|12p + 8| = \frac{16}{3}$, so $|12p + 8| = 32$.

Either $12p + 8 = 32$, giving $p = 2$, or $12p + 8 = -32$, giving $p = -\frac{10}{3}$.

Only $p = -\frac{10}{3}$ appears among the options.

Step 5: Why the other options are wrong.

Values 5, 8 and 10 give $12p + 8 = 68, 104, 128$, none equal to ± 32.

Final Answer:

$p = -\frac{10}{3}$, option (A).

$$\boxed{-\frac{10}{3}}$$

Quick Tip: Volume = one sixth of the absolute scalar triple product of AB, AC, AD.

140. Let $\overline{OD} = \hat{i} + 2\hat{j} + 6\hat{k}$, $\overline{CB} = -3\hat{i} - 2\hat{k}$ be the diagonals of the parallelogram OBDC and $\overline{OA} = \hat{i} + 2\hat{j} + 3\hat{k}$ be another vector. Then the volume of a parallelopiped determined by vectors $\overline{OA}$, $\overline{OB}$, and $\overline{OC}$ (in cubic units), is

(A) 3

(B) 6

(C) 9

(D) 12

Correct Answer: (C) 9

Solution:

Step 1: Understanding the Concept:

In parallelogram OBDC the vertex D is opposite O, and C and B are opposite each other. So $\overline{OD} = \overline{OB} + \overline{OC}$ and $\overline{CB} = \overline{OB} - \overline{OC}$.

Step 2: Find OB and OC:

$\overline{OB} = \frac{1}{2}(\overline{OD} + \overline{CB}) = \frac{1}{2}(-2\hat{i} + 2\hat{j} + 4\hat{k}) = -\hat{i} + \hat{j} + 2\hat{k}.$

$\overline{OC} = \frac{1}{2}(\overline{OD} - \overline{CB}) = \frac{1}{2}(4\hat{i} + 2\hat{j} + 8\hat{k}) = 2\hat{i} + \hat{j} + 4\hat{k}.$

Step 3: Triple product:

$$\begin{vmatrix} 1 & 2 & 3 \\ -1 & 1 & 2 \\ 2 & 1 & 4 \end{vmatrix} = 1(4-2) - 2(-4-4) + 3(-1-2) = 2 + 16 - 9 = 9$$

Step 4: Why the other options are wrong.

Values 3, 6 and 12 arise from sign or arithmetic slips, for example dropping the $-2(-8)$ term gives $2 - 9 = -7$.

Final Answer:

The volume is 9 cubic units, option (C).

9 cubic units

Quick Tip: Diagonals of parallelogram OBDC give OB and OC by sum and difference.

141. If $\bar{a}$ and $\bar{b}$ are unit vectors perpendicular to each other, then $[\bar{a} + (\bar{a} \times \bar{b})\,\bar{b} + (\bar{a} \times \bar{b})\,(\bar{a} \times \bar{b})] = \cdots$

(A) -1

(B) 1

(C) 2

(D) 3

Correct Answer: (B) 1

Solution:

Step 1: Understanding the Concept:

Let $\bar{c} = \bar{a} \times \bar{b}$. The scalar triple product $[\bar{a} + \bar{c},\ \bar{b} + \bar{c},\ \bar{c}]$ is a determinant, and it does not change when we subtract a multiple of one row from another.

Step 2: Simplify:

Subtract the third row $\bar{c}$ from each of the first two rows:

$$[\bar{a} + \bar{c},\ \bar{b} + \bar{c},\ \bar{c}] = [\bar{a},\ \bar{b},\ \bar{c}]$$

Step 3: Evaluate:

$$[\bar{a}\ \bar{b}\ \bar{c}] = \bar{a} \cdot (\bar{b} \times \bar{c}) = (\bar{a} \times \bar{b}) \cdot \bar{c} = |\bar{a} \times \bar{b}|^2$$

For perpendicular unit vectors, $|\bar{a} \times \bar{b}| = |\bar{a}||\bar{b}| \sin 90° = 1$, so the value is 1.

Step 4: Why the other options are wrong.
The value -1 would need a negative square. Values 2 and 3 come from adding extra terms, such as $\bar{c} \cdot \bar{c}$ more than once, which cancel out in the determinant.

Final Answer:
The value is 1, option (B).

$$\boxed{1}$$

Quick Tip: Subtract the third vector from the first two rows; the triple product is unchanged.

142. If $\bar{a}$, $\bar{b}$ and $\bar{c}$ are three vectors such that $|\bar{a} + \bar{b} + \bar{c}| = 1$, $\bar{c} = \lambda(\bar{a} \times \bar{b})$ and $|\bar{a}| = \frac{1}{\sqrt{3}}$, $|\bar{b}| = \frac{1}{\sqrt{2}}$, $|\bar{c}| = \frac{1}{\sqrt{6}}$, then the angle between $\bar{a}$ and $\bar{b}$ is

(A) $\frac{\pi^c}{6}$

(B) $\frac{\pi^c}{4}$

(C) $\frac{\pi^c}{3}$

(D) $\frac{\pi^c}{2}$

Correct Answer: (D) $\frac{\pi^c}{2}$

Solution:

Step 1: Understanding the Concept:

Because $\bar{c} = \lambda(\bar{a} \times \bar{b})$, the vector $\bar{c}$ is perpendicular to both $\bar{a}$ and $\bar{b}$, hence perpendicular to $\bar{a} + \bar{b}$.

Step 2: Use Pythagoras:

$$|\bar{a} + \bar{b} + \bar{c}|^2 = |\bar{a} + \bar{b}|^2 + |\bar{c}|^2$$

So $1 = |\bar{a} + \bar{b}|^2 + \frac{1}{6}$, giving $|\bar{a} + \bar{b}|^2 = \frac{5}{6}$.

Step 3: Expand:

$$|\bar{a} + \bar{b}|^2 = |\bar{a}|^2 + |\bar{b}|^2 + 2\bar{a} \cdot \bar{b} = \frac{1}{3} + \frac{1}{2} + 2\bar{a} \cdot \bar{b} = \frac{5}{6} + 2\bar{a} \cdot \bar{b}$$

Equating with $\frac{5}{6}$ gives $\bar{a} \cdot \bar{b} = 0$.

Step 4: Conclusion:

The dot product is zero, so the angle is $\frac{\pi}{2}$.

Step 5: Why the other options are wrong.

Angles $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}$ would give a non-zero dot product $|\bar{a}||\bar{b}| \cos\theta$, which would make $|\bar{a} + \bar{b}|^2$ differ from $\frac{5}{6}$.

Final Answer:

The angle is $\frac{\pi}{2}$, option (D).

$$\boxed{\frac{\pi}{2}}$$

Quick Tip: c is perpendicular to a and b, so |a+b+c| squared splits as |a+b| squared plus |c| squared.

143. The acute angle between the line $\vec{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} + \hat{j} + \hat{k})$ and the plane $\vec{r} \cdot (2\hat{i} + p\hat{j} + \hat{k}) = 8$ is $sin^{-1}(\frac{\sqrt{2}}{3})$, then the value of p are...

(A) $p = 1$ or $p = 17$

(B) $p = -1$ or $p = -17$

(C) $p = 6$ or $p = 3$

(D) $p = -6$ or $p = -3$

Correct Answer: (B) $p = -1$ or $p = -17$

Solution:

Step 1: Understanding the Concept:

The angle ϕ between a line with direction $\bar{d}$ and a plane with normal $\bar{n}$ satisfies $\sin\phi = \frac{|\bar{d}\cdot\bar{n}|}{|\bar{d}||\bar{n}|}$.

Step 2: Set up:

$\bar{d} = (1, 1, 1)$ and $\bar{n} = (2, p, 1)$. Then $\bar{d} \cdot \bar{n} = 3 + p$, $|\bar{d}| = \sqrt{3}$, $|\bar{n}| = \sqrt{5 + p^2}$.

$$\frac{|3 + p|}{\sqrt{3}\sqrt{5 + p^2}} = \frac{\sqrt{2}}{3}$$

Step 3: Solve:

Square both sides: $\frac{(3+p)^2}{3(5+p^2)} = \frac{2}{9}$, so $9(9 + 6p + p^2) = 6(5 + p^2)$.

$81 + 54p + 9p^2 = 30 + 6p^2$, so $3p^2 + 54p + 51 = 0$, that is $p^2 + 18p + 17$

$= 0$.

$(p+1)(p+17) = 0$, so $p = -1$ or $p = -17$.

Step 4: Why the other options are wrong.

Option (A) has the positive values 1 and 17, which come from the sign of p being flipped. With $p = 1$: $\sin\phi = \frac{4}{\sqrt{3}\sqrt{6}} = \frac{4}{3\sqrt{2}}$, which is not $\frac{\sqrt{2}}{3}$.

Final Answer:

$p = -1$ or $p = -17$, option (B).

$$\boxed{p = -1 \text{ or } p = -17}$$

Quick Tip: sin of the angle equals |d . n| / (|d||n|) with direction d and normal n.

144. The shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$ is

(A) $5\sqrt{30}$

(B) $3\sqrt{30}$

(C) $2\sqrt{30}$

(D) $\sqrt{30}$

Correct Answer: (B) $3\sqrt{30}$

Solution:

Step 1: Understanding the Concept:

For skew lines through points P_1, P_2 with directions $\bar{d}_1$, $\bar{d}_2$, the shortest distance is $\frac{|(\overrightarrow{P_1P_2})\cdot(\bar{d}_1\times\bar{d}_2)|}{|\bar{d}_1\times\bar{d}_2|}$.

Step 2: Collect data:

$P_1 = (3, 8, 3)$, $\bar{d}_1 = (3, -1, 1)$; $P_2 = (-3, -7, 6)$, $\bar{d}_2 = (-3, 2, 4)$.

$\overrightarrow{P_1P_2} = (-6, -15, 3)$.

Step 3: Cross product:

$$\bar{d}_1 \times \bar{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{vmatrix} = (-4-2)\hat{i} - (12+3)\hat{j} + (6-3)\hat{k} = (-6, -15, 3)$$

$$|\bar{d}_1 \times \bar{d}_2| = \sqrt{36 + 225 + 9} = \sqrt{270} = 3\sqrt{30}.$$

Step 4: Distance:

Dot product: $(-6)(-6) + (-15)(-15) + (3)(3) = 36 + 225 + 9 = 270$.

$$d = \frac{270}{3\sqrt{30}} = \frac{90}{\sqrt{30}} = 3\sqrt{30}$$

Step 5: Why the other options are wrong.

$5\sqrt{30}$, $2\sqrt{30}$ and $\sqrt{30}$ are other multiples of $\sqrt{30}$ that do not equal $\frac{90}{\sqrt{30}}$.

Final Answer:

The shortest distance is $3\sqrt{30}$, option (B).

$$\boxed{3\sqrt{30}}$$

Quick Tip: Use d = |(P2 - P1) . (d1 x d2)| / |d1 x d2|.

145. A plane meets the co-ordinate axes in A, B, C such that the centroid of the triangle ABC is the point $(1, r, r^2)$, then the equation of the plane is,

(A) $x + ry + r^2z = 3r^2$

(B) $r^2x + ry + z = 3r^2$

(C) $x + ry + r^2z = 3$

(D) $r^2x + ry + z = 3$

Correct Answer: (B) $r^2x + ry + z = 3r^2$

Solution:

Step 1: Understanding the Concept:

Let the plane meet the axes at $A(a, 0, 0)$, $B(0, b, 0)$, $C(0, 0, c)$. The centroid of $\triangle ABC$ is $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$.

Step 2: Match the centroid:

$\frac{a}{3} = 1$, $\frac{b}{3} = r$, $\frac{c}{3} = r^2$, so $a = 3$, $b = 3r$, $c = 3r^2$.

Step 3: Plane equation:

$$\frac{x}{3} + \frac{y}{3r} + \frac{z}{3r^2} = 1$$

Multiply by $3r^2$:

$$r^2x + ry + z = 3r^2$$

Step 4: Why the other options are wrong.

Options (A) and (C) have x with coefficient 1 and z with r^2, which is

the reverse assignment of intercepts. Option (D) has the right coefficients but the wrong right-hand side 3 instead of $3r^2$.

Final Answer:

The plane is $r^2x + ry + z = 3r^2$, option (B).

$$\boxed{r^2x + ry + z = 3r^2}$$

Quick Tip: The centroid of the intercept triangle gives the three intercepts directly.

146. If M denotes the midpoint of the line joining A$(4, 5, -10)$ and B $(-1, 2, 1)$, then the equation of the plane through M and perpendicular to AB is:

(A) $\bar{r} \cdot (-5\hat{i} - 3\hat{j} + 11\hat{k}) + \frac{135}{2} = 0$

(B) $\bar{r} \cdot (\frac{3}{2}\hat{i} + \frac{7}{2}\hat{j} - \frac{9}{2}\hat{k}) + \frac{135}{2} = 0$

(C) $\bar{r} \cdot (4\hat{i} + 5\hat{j} - 10\hat{k}) + 4 = 0$

(D) $\bar{r} \cdot (-\hat{i} + 2\hat{j} + \hat{k}) + 4 = 0$

Correct Answer: (A) $\bar{r} \cdot (-5\hat{i} - 3\hat{j} + 11\hat{k}) + \frac{135}{2} = 0$

Solution:

Step 1: Understanding the Concept:

A plane perpendicular to AB through M has normal $\overrightarrow{AB}$ and passes through M.

Step 2: Find M and the normal:

$M = (\frac{4-1}{2}, \frac{5+2}{2}, \frac{-10+1}{2}) = (\frac{3}{2}, \frac{7}{2}, -\frac{9}{2})$.

Normal $\overrightarrow{AB} = B - A = (-5, -3, 11)$.

Step 3: Equation:

$\bar{r} \cdot \bar{n} = \bar{m} \cdot \bar{n}$ with $\bar{m} \cdot \bar{n} = \frac{3}{2}(-5) + \frac{7}{2}(-3) + \left(-\frac{9}{2}\right)(11) = -\frac{15}{2} - \frac{21}{2} - \frac{99}{2}$ $= -\frac{135}{2}$.

$$\bar{r} \cdot (-5\hat{i} - 3\hat{j} + 11\hat{k}) + \frac{135}{2} = 0$$

Step 4: Why the other options are wrong.

Option (B) uses the midpoint as the normal. Option (C) uses $\overrightarrow{OA}$ as the normal. Option (D) uses the vector $-\hat{i} + 2\hat{j} + \hat{k}$, which is not parallel to AB.

Final Answer:

Option (A) is the plane.

$$\boxed{\bar{r} \cdot (-5\hat{i} - 3\hat{j} + 11\hat{k}) + \frac{135}{2} = 0}$$

Quick Tip: AB gives the normal vector; plug in the midpoint to find the constant.

147. The difference between the maximum value and the minimum value of the objective function $z = 3x + y$ subject to the constraints $2x + 3y \leq 6$, $x + y \geq 1$, $x \geq 0$, $y \geq 0$ is....

(A) 7

(B) 3

(C) 8

(D) 1

Correct Answer: (C) 8

Solution:

Step 1: Understanding the Concept:

The optimum of a linear objective function over a bounded feasible region occurs at a corner point.

Step 2: Feasible region:

The constraints are $2x + 3y \leq 6$, $x + y \geq 1$, $x \geq 0$, $y \geq 0$. Corner points: $(1, 0)$ from $x + y = 1$ and $y = 0$; $(3, 0)$ from $2x + 3y = 6$ and $y = 0$; $(0, 2)$ from $2x + 3y = 6$ and $x = 0$; $(0, 1)$ from $x + y = 1$ and $x = 0$.

Step 3: Evaluate z = 3x + y:

At $(1, 0)$: 3. At $(3, 0)$: 9. At $(0, 2)$: 2. At $(0, 1)$: 1.

Step 4: Result:

Maximum is 9 and minimum is 1, so the difference is $9 - 1 = 8$.

Step 5: Why the other options are wrong.

Option 7 would be $9 - 2$, taking the wrong minimum. Option 3 is $3 - 0$ -type slips, and 1 is the minimum itself.

Final Answer:

The difference is 8, option (C).

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Quick Tip: Find the corner points of the feasible region and evaluate z at each.

148. A random variable $X \sim B(n, p)$ follows a binomial distribution with $n = 6$. If $9P(X = 4) = P(X = 2)$, then the probability of success p is..

(A) 0.125

(B) 0.75

(C) 0.25

(D) 0.375

Correct Answer: (C) 0.25

Solution:

Step 1: Understanding the Concept:

For $X \sim B(6, p)$, $P(X = r) = \binom{6}{r} p^r q^{6-r}$ with $q = 1 - p$.

Step 2: Write the condition:

$9P(X = 4) = P(X = 2)$ gives

$$9\binom{6}{4} p^4 q^2 = \binom{6}{2} p^2 q^4$$

Since $\binom{6}{4} = \binom{6}{2} = 15$, they cancel: $9p^4q^2 = p^2q^4$.

Step 3: Simplify:

Divide by p^2q^2: $9p^2 = q^2$, so $3p = q = 1 - p$ (taking positive roots).

Then $4p = 1$, so $p = \frac{1}{4} = 0.25$.

Step 4: Why the other options are wrong.
$p = 0.125$ gives $q = 0.875 \neq 3p$. $p = 0.75$ gives $q = 0.25$, so $q = \frac{p}{3}$, the reverse ratio. $p = 0.375$ gives $q = 0.625 \neq 1.125$.

Final Answer:
The probability of success is 0.25, option (C).

$$\boxed{0.25}$$

Quick Tip: Write both probabilities with C(6,r) p^r q^(6-r) and compare.

149. Let X be a continuous random variable with the probability density function(p.d.f.) given by

$$f(x) = \begin{cases} kx, & 0 \leq x < 1 \\ k, & 1 \leq x < 2 \\ -kx + 3k, & 2 \leq x < 3 \\ 0, & \text{otherwise} \end{cases}$$

$P(2 < X \leq 3) = \cdots$

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{1}{4}$

(D) $\frac{1}{5}$

Correct Answer: (C) $\frac{1}{4}$

Solution:

Step 1: Understanding the Concept:

For a probability density function, the total area under the graph is 1. This fixes the constant k.

Step 2: Find k:

$$\int_0^1 kx\,dx + \int_1^2 k\,dx + \int_2^3 (-kx + 3k)\,dx = 1$$

First piece: $\frac{k}{2}$. Second piece: k. Third piece: $k\left[-\frac{x^2}{2} + 3x\right]_2^3$
$= k\left[\left(-\frac{9}{2} + 9\right) - (-2 + 6)\right] = k\left(\frac{9}{2} - 4\right) = \frac{k}{2}$.
Total: $\frac{k}{2} + k + \frac{k}{2} = 2k = 1$, so $k = \frac{1}{2}$.

Step 3: Required probability:

$$P(2 < X \leq 3) = \int_2^3 (-kx + 3k)\,dx = \frac{k}{2} = \frac{1}{4}$$

Step 4: Why the other options are wrong.

$\frac{1}{2}$ is the value of k, not the probability. $\frac{1}{3}$ and $\frac{1}{5}$ do not follow from the areas $\frac{1}{4}, \frac{1}{2}, \frac{1}{4}$.

Final Answer:

The probability is $\frac{1}{4}$, option (C).

$$\boxed{\frac{1}{4}}$$

Quick Tip: Total probability is 1, which fixes k; then integrate the last piece.

150. In a certain city, the ratio of men to women is 5 : 4. It is found that 80% of men and 90% of women are literate. If a person selected at random is found to be illiterate, then the probability that the person is a man is....

(A) $\frac{2}{3}$

(B) $\frac{5}{7}$

(C) $\frac{1}{3}$

(D) $\frac{2}{7}$

Correct Answer: (B) $\frac{5}{7}$

Solution:

Step 1: Understanding the Concept:

Let M be "man" and W be "woman", with $P(M) = \frac{5}{9}$ and $P(W) = \frac{4}{9}$.

Let I be "illiterate".

Step 2: Find illiteracy rates:

$P(I|M) = 1 - 0.80 = 0.20$ and $P(I|W) = 1 - 0.90 = 0.10$.

Step 3: Bayes theorem:

$$P(M|I) = \frac{P(M)P(I|M)}{P(M)P(I|M) + P(W)P(I|W)} = \frac{\frac{5}{9}(0.2)}{\frac{5}{9}(0.2) + \frac{4}{9}(0.1)} = \frac{1}{1+0.4} = \frac{1}{1.4} = \frac{5}{7}$$

Step 4: Why the other options are wrong.

$\frac{2}{3}$ and $\frac{1}{3}$ ignore the different illiteracy rates. $\frac{2}{7}$ is the probability that the illiterate person is a woman, $1 - \frac{5}{7}$.

Final Answer:

The probability is $\frac{5}{7}$, option (B).

$$\boxed{\frac{5}{7}}$$

Quick Tip: Apply Bayes theorem with ratio 5:4 and illiteracy rates 20% and 10%.