

MHT CET 2026 PCM Question Paper April 20 Shift 2 with Solutions



General Instructions

- (i) This booklet contains 150 questions, each provided with a complete, step-by-step solution.
- (ii) It comprises 150 single-correct multiple-choice questions.
- (iii) Attempt each question on your own before reviewing the given solution.

Physics

Questions 1 to 50 | 1 mark each | 50 marks

1. A physical quantity x is related as $x = \sqrt{ab^2c^3d^{-4}}$. Relative errors in the quantities a , b , c and d are 2%, 1%, 3% and 4% respectively. Relative error in x will be

- (A) 30%
- (B) 28%
- (C) 24%
- (D) 18%

Correct Answer: (B) 28%

Solution:

Step 1: Understanding the Concept

When quantities are multiplied or divided, their relative errors add. A power n on a quantity multiplies its relative error by n .

Step 2: Key Formula or Approach

For $x = a^{1/2}b^2c^3d^{-4}$:

$$\frac{\Delta x}{x} = \frac{1}{2} \frac{\Delta a}{a} + 2 \frac{\Delta b}{b} + 3 \frac{\Delta c}{c} + 4 \frac{\Delta d}{d}$$

The negative power on d only changes the sign inside the formula, but errors are added in the worst case, so we use $|-4| = 4$.

Step 3: Substitute

$$\frac{\Delta x}{x} = \frac{1}{2}(2) + 2(1) + 3(3) + 4(4) \%$$

$$= 1 + 2 + 9 + 16 = 28 \%$$

Step 4: Check the options

A value of 30% would come from forgetting the half on a and using 2 instead of 1. The value 24% would come from losing the 4 in the d term. The result is 28%, option (B).

Final Answer:

The maximum relative error is $1 + 2 + 9 + 16 = 28$ percent, option (B).

28%

Quick Tip: Powers multiply the relative errors; add all the contributions.

2. If two vectors $\vec{A} = 4\hat{i} + n\hat{j} + 2\hat{k}$ and $\vec{B} = 2\hat{i} + 2\hat{j} - \hat{k}$ are mutually perpendicular to each other then value of 'n' is

- (A) +3
- (B) +1
- (C) -3
- (D) -1

Correct Answer: (C) -3

Solution:

Step 1: Understanding the Concept

Two vectors are perpendicular when the angle between them is 90 degrees, so $\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos 90^\circ = 0$.

Step 2: Key Formula or Approach

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

Step 3: Substitute

$$(4)(2) + (n)(2) + (2)(-1) = 0$$

$$8 + 2n - 2 = 0 \Rightarrow 2n = -6$$

$$n = -3$$

Step 4: Check the options

The option +3 comes from a sign slip when moving 6 across. The values ± 1 do not make the sum zero: for $n = 1$ the sum is 8, and for $n = -1$ it is 4. So only $n = -3$ works, option (C).

Final Answer:

The dot product $6 + 2n$ must be zero, so $n = -3$, option (C).

$$\boxed{-3}$$

Quick Tip: Perpendicular vectors have zero dot product.



3. A particle completes 2 revolutions in a circular path of radius 3 cm. The angular displacement of the particle will be (in radian)

- (A) π
- (B) 2π
- (C) 3π
- (D) 4π

Correct Answer: (D) 4π

Solution:**Step 1: Understanding the Concept**

Angular displacement is the angle swept by the radius line. One full revolution sweeps an angle of 2π radians, whatever the radius.

Step 2: Calculation

$$\theta = 2 \text{ revolutions} \times 2\pi \frac{\text{rad}}{\text{rev}} = 4\pi \text{ rad}$$

Step 3: Why the radius is not used

The radius of 3 cm would matter only for the distance travelled, which is $s = r\theta = 3 \times 4\pi = 12\pi$ cm. The question asks for the angle, which depends only on the number of turns.

Step 4: Check the options

π is half a turn, 2π is one turn and 3π is one and a half turns. Two full turns give 4π , option (D).

Final Answer:

Two revolutions sweep 4 pi radians, option (D).

$$\boxed{4\pi}$$

Quick Tip: One revolution is 2 pi radians; the radius does not matter.

4. A car is travelling at 40 m/s on a circular path of radius 40 m. It is increasing its speed at the rate of 2m/s^2 . Its net acceleration is (in m/s^2) nearly

- (A) 4m/s^2
- (B) 8m/s^2
- (C) 16m/s^2
- (D) 40m/s^2

Correct Answer: (D) 40m/s^2

Solution:

Step 1: Understanding the Concept

A car moving on a circle while speeding up has two perpendicular accelerations: the centripetal one pointing to the centre and the tangential one along the motion.

Step 2: Centripetal acceleration

$$a_c = \frac{v^2}{r} = \frac{40^2}{40} = 40 \text{ m/s}^2$$

Step 3: Tangential acceleration

$$a_t = 2 \text{ m/s}^2$$

Step 4: Net acceleration

The two are at right angles, so

$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{1600 + 4} = \sqrt{1604} \approx 40.05 \text{ m/s}^2$$

Step 5: Conclusion

The value is nearly 40 m/s^2 , option (D). The tangential part is tiny compared with the centripetal part, so it changes the answer only in the second decimal place.

Final Answer:

The centripetal part is 40 and the tangential part is 2, so the net is about 40 m/s squared, option (D).

$$\approx 40 \text{ m/s}^2$$

Quick Tip: Combine the centripetal and tangential accelerations at right angles.

5. Two spheres are projected at angles 30° and 45° with the horizontal. The maximum height reached by both is same. The ratio of their initial velocities is, ($\sin 45^\circ = \frac{1}{\sqrt{2}}$, $\sin 30^\circ = 0.5$)

- (A) 2 : 3
- (B) $\sqrt{2} : 1$
- (C) 3 : 1
- (D) $\sqrt{2} : \sqrt{3}$

Correct Answer: (B) $\sqrt{2} : 1$

Solution:

Step 1: Understanding the Concept

The maximum height of a projectile is $H = \frac{u^2 \sin^2 \theta}{2g}$. Equal heights mean equal values of $u \sin \theta$.

Step 2: Set equal

$$u_1 \sin 30^\circ = u_2 \sin 45^\circ$$

Step 3: Solve for the ratio

$$\frac{u_1}{u_2} = \frac{\sin 45^\circ}{\sin 30^\circ} = \frac{1/\sqrt{2}}{1/2} = \frac{2}{\sqrt{2}} = \sqrt{2}$$

So $u_1 : u_2 = \sqrt{2} : 1$.

Step 4: Check the options

The ratio 2:3 and 3:1 do not come from the sine values. The ratio $\sqrt{2} : \sqrt{3}$ would come from using $\tan$ or mixing in $\cos$. The correct one is option (B).

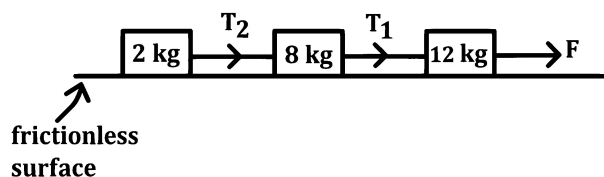
Final Answer:

Equal heights need equal vertical speeds, so $u_1 : u_2 = \sin 45^\circ : \sin 30^\circ = \sqrt{2} : 1$, option (B).

$$\boxed{\sqrt{2} : 1}$$

Quick Tip: Maximum height depends on the vertical part of the speed, $u \sin(\theta)$.

6. Three blocks of different masses connected with inextensible string are pulled by a force F on a frictionless surface as shown in the figure. The ratio of tensions T_1 to T_2 is



- (A) 1 : 5
- (B) 5 : 1
- (C) 10 : 1
- (D) 1 : 10

Correct Answer: (B) 5 : 1

Solution:

Step 1: Understanding the Concept

The figure shows three blocks of 2 kg, 8 kg and 12 kg on a frictionless surface, joined by strings and pulled by a force F on the 12 kg block. The string T_2 joins 2 kg and 8 kg, and T_1 joins 8 kg and 12 kg.

Step 2: Common acceleration

All blocks move together:

$$a = \frac{F}{2 + 8 + 12} = \frac{F}{22}$$

Step 3: Tension T_2

It pulls only the 2 kg block:

$$T_2 = 2a = \frac{2F}{22}$$

Step 4: Tension T_1

It pulls the 2 kg and 8 kg blocks together (10 kg):

$$T_1 = 10a = \frac{10F}{22}$$

Step 5: Ratio

$$\frac{T_1}{T_2} = \frac{10}{2} = 5$$

So $T_1 : T_2 = 5 : 1$, option (B).

Final Answer:

The tension nearer the force carries 10 kg and the other carries 2 kg, so the ratio is 5 to 1, option (B).

$$\boxed{5 : 1}$$

Quick Tip: Find the common acceleration, then apply Newton second law to the blocks behind each string.

7. A solid sphere of mass 5 kg and a disc of mass 4 kg have the same radius. The ratio of moment of inertia of the sphere about its tangent to the moment of inertia of the disc about a tangent in its plane will be $x : y$. The value of x and y respectively is

- (A) 5, 8
- (B) 6, 7
- (C) 7, 5
- (D) 4, 3

Correct Answer: (C) 7, 5

Solution:

Step 1: Understanding the Concept

Moment of inertia about a tangent is found with the parallel axis theorem: $I = I_{cm} + Mr^2$, because the tangent is at distance r from the centre.

Step 2: Solid sphere (mass 5 kg)

$$I_{cm} = \frac{2}{5}Mr^2, \quad I_{tan} = \frac{2}{5}Mr^2 + Mr^2 = \frac{7}{5}Mr^2 = \frac{7}{5}(5)r^2 = 7r^2$$

Step 3: Disc (mass 4 kg), tangent in its plane

The axis through the centre along a diameter has $I = \frac{1}{2}Mr^2$. So

$$I_{tan} = \frac{1}{2}Mr^2 + Mr^2 = \frac{3}{2}Mr^2 = \frac{3}{2}(4)r^2 = 6r^2$$

Step 4: Ratio

$$x : y = 7r^2 : 5r^2 = 7 : 5$$

So $x = 7$ and $y = 5$, option (C).

Final Answer:

The sphere gives $7r^2$ and the disc gives $5r^2$, so $x = 7$ and $y = 5$, option (C).

7, 5

Quick Tip: Use the parallel axis theorem for each body about its tangent.

8. Select the correct statement out of the following

- (A) Acceleration due to gravity decreases as we go up or down from earth's surface.
- (B) Acceleration due to gravity is same at a height ' h ' and depth ' d ' from earth's surface, if $h = d$.
- (C) Acceleration due to gravity at 640 km above the earth surface decreases by 10% of its value at earth's surface (earth's radius is 6400 km)
- (D) There is reduction in acceleration due to gravity at poles due to rotation of earth as the poles are lying on the axis of rotation and do not revolve.

Correct Answer: (A) Acceleration due to gravity decreases as we go up or down from earth's surface.

Solution:**Step 1: Understanding the Concept**

Acceleration due to gravity is largest at the surface of the earth. It falls both going up and going down.

Step 2: Statement (A)

At height h : $g_h = g(1 - \frac{2h}{R})$ approximately. At depth d : $g_d = g(1 - \frac{d}{R})$. Both are less than g . So (A) is true.

Step 3: Statement (B)

For equal values, $2h = d$, not $h = d$. So the claim $h = d$ is wrong.

Step 4: Statement (C)

At $h = 640$ km, $g' = g(\frac{R}{R+h})^2 = g(\frac{1}{1.1})^2 \approx 0.83g$. That is a decrease of about 17%, not 10%.

Step 5: Statement (D)

Rotation reduces the effective g at the equator, and at the poles there is no such reduction. The statement is the wrong way round.

Step 6: Conclusion

Only (A) is correct.

Final Answer:

Gravity is greatest at the surface and falls above and below it, option (A).

(A)

Quick Tip: The value of g falls both above the surface and below it.



9. If we dip capillary tubes of different radii r_n in water and the water rises to different heights h_n in them, then ($n = 1, 2, 3, \dots$)

- (A) $\frac{h_n}{r_n^2} = \text{constant}$
- (B) $\frac{h_n}{r_n} = \text{constant}$
- (C) $h_n r_n^2 = \text{constant}$
- (D) $h_n r_n = \text{constant}$

Correct Answer: (D) $h_n r_n = \text{constant}$

Solution:

Step 1: Understanding the Concept

In a capillary tube, the height of the liquid column is given by Jurin's law.

Step 2: Key Formula or Approach

$$h = \frac{2T \cos \theta}{r \rho g}$$

For the same liquid (water) in the same place, T , θ , ρ and g are constant.

Step 3: Reduce

$$h \propto \frac{1}{r} \Rightarrow h r = \frac{2T \cos \theta}{\rho g} = \text{constant}$$

Step 4: Check the options

h/r and h/r^2 would mean a rise that grows with radius, which is opposite to what is seen. hr^2 is wrong because the radius enters only once. So $h_n r_n = \text{constant}$, option (D).

Final Answer:

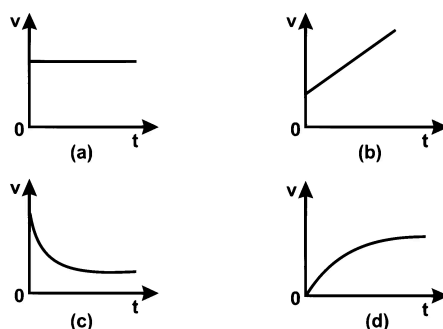
The product of rise and radius is constant, option (D).

$$h_n r_n = \text{constant}$$

Quick Tip: Jurin law: $h = 2T \cos(\theta) / (r \rho g)$, so h is inversely proportional to r .



10. A small metal sphere is falling through a viscous liquid. The variation of velocity (V) with time (t) is shown correctly in graph



- (A) b
- (B) a
- (C) d
- (D) c

Correct Answer: (C) d

Solution:**Step 1: Understanding the Concept**

A sphere released in a viscous liquid has three forces: weight down, buoyancy up and viscous drag up. The drag grows with speed.

Step 2: Early motion

At the start the speed is zero, so the drag is zero. The net force is large, so the acceleration is large. The curve starts from the origin and rises steeply.

Step 3: Later motion

As the speed grows, the drag rises and the net force falls. The acceleration falls, so the curve bends over.

Step 4: Terminal velocity

Finally drag plus buoyancy equals weight, the net force is zero, and the speed stays constant at the terminal velocity. The graph becomes flat.

Step 5: Match the graphs

Graph (a) is constant speed from the start. Graph (b) is a straight line with a nonzero start. Graph (c) falls with time. Graph (d) rises from zero and flattens, which is what we found. So the answer is graph (d), option (C).

Final Answer:

The speed grows from zero and levels off at terminal velocity, which is graph (d), option (C).

graph (d)

Quick Tip: The sphere speeds up from rest and its speed levels off at terminal velocity.

11. On opposite sides of a wide vertical vessel filled with water (density ρ), two identical holes are drilled, each having cross-section area A . The height difference between holes is x . The resultant force of reaction of water flowing out of vessel is (g = acceleration due to gravity)

- (A) ρAgx
- (B) $2\rho Agx$
- (C) $3\rho Agx$
- (D) $4\rho Agx$

Correct Answer: (B) $2\rho Agx$

Solution:

Step 1: Understanding the Concept

Water leaving a hole carries momentum. By Newton's third law, the water pushes the vessel in the opposite direction with a force equal to the rate of change of momentum.

Step 2: Force from one hole

Speed of outflow (Torricelli): $v = \sqrt{2gh}$. Mass leaving per second: ρAv . So

$$F = \rho Av \cdot v = \rho Av^2 = 2\rho Ag h$$

Step 3: Two holes on opposite sides

The two reaction forces point in opposite directions, so they subtract. The holes are at depths h_1 and h_2 with $h_2 - h_1 = x$:

$$F_{net} = 2\rho Ag(h_2 - h_1) = 2\rho Agx$$

Step 4: Conclusion

The resultant force is $2\rho Agx$, option (B).

Final Answer:

Each jet gives $2\rho Agx$ and the opposite jets subtract, so the net is $2\rho Agx$, option (B).

$$2\rho Agx$$

Quick Tip: Each jet pushes the vessel back with force ρAv^2 ; the jets on opposite sides partly cancel.

12. Two stars A and B radiate maximum energy at wavelength 3.9×10^{-7} m and 5.2×10^{-7} m respectively. The ratio of the temperature of star A to that of star B will be

- (A) 2 : 3
- (B) 3 : 2
- (C) 3 : 4
- (D) 4 : 3

Correct Answer: (D) 4 : 3

Solution:**Step 1: Understanding the Concept**

Wien's displacement law says $\lambda_m T = b$, a constant. A hotter star radiates its peak energy at a shorter wavelength.

Step 2: Write for both stars

$$\lambda_A T_A = \lambda_B T_B \Rightarrow \frac{T_A}{T_B} = \frac{\lambda_B}{\lambda_A}$$

Step 3: Substitute

$$\frac{T_A}{T_B} = \frac{5.2 \times 10^{-7}}{3.9 \times 10^{-7}} = \frac{52}{39} = \frac{4}{3}$$

Step 4: Check the options

Star A has the shorter wavelength, so it is the hotter one and the ratio must be greater than 1. That rules out 2:3 and 3:4. The value 3:2 would need a wavelength ratio of 1.5, which does not match 5.2/3.9. So the answer is 4:3, option (D).

Final Answer:

The temperature ratio equals the inverse wavelength ratio, 5.2 over 3.9, which is 4 to 3, option (D).

4 : 3

Quick Tip: Wien displacement law: wavelength of peak emission times temperature is constant.

13. A piece of metal of 850 K is dropped into 1 kg of water at 300 K. If equilibrium temperature of water is 350 K then the heat capacity of the metal expressed in $\frac{J}{K}$ is (Sp. heat of water = $1 \frac{\text{cal}}{\text{g}^\circ\text{C}}$)

- (A) 420
- (B) 240
- (C) 100
- (D) 24

Correct Answer: (A) 420

Solution:

Step 1: Understanding the Concept

In an isolated mixing, heat lost by the hot metal equals heat gained by the water.

Step 2: Heat gained by water

Mass 1 kg = 1000 g, specific heat 1 cal/g C = 4.2 J/g K, temperature rise 350 - 300 = 50 K:

$$Q = 1000 \times 4.2 \times 50 = 210000 \text{ J}$$

Step 3: Heat lost by the metal

Let the heat capacity be C . The metal cools from 850 K to 350 K, a fall of 500 K:

$$C \times 500 = 210000$$

$$C = 420 \text{ J/K}$$

Step 4: Check the options

The value 240 comes from using the wrong fall of temperature. The values 100 and 24 are far off. The answer is 420 J/K, option (A).

Final Answer:

The water gains 210000 J while the metal falls 500 K, so its heat capacity is 420 J per K, option (A).

420 J/K

Quick Tip: Heat lost by the metal equals heat gained by the water.



14. A sphere having temperature 600 K is losing heat due to radiation. At this temperature its rate of cooling is R . The rate of cooling of this sphere at 400 K is $\frac{x}{243}$. The value of x is (Temperature of surrounding is 300 K)

- (A) $50R$
- (B) $45R$
- (C) $40R$
- (D) $35R$

Correct Answer: (D) $35R$

Solution:**Step 1: Understanding the Concept**

By Stefan's law, the net radiation loss is proportional to $T^4 - T_0^4$, with surroundings at $T_0 = 300 \text{ K}$. The rate of cooling is proportional to this

loss.

Step 2: At 600 K

$$600^4 - 300^4 = (6^4 - 3^4) \times 10^8 = (1296 - 81) \times 10^8 = 1215 \times 10^8$$

Step 3: At 400 K

$$400^4 - 300^4 = (4^4 - 3^4) \times 10^8 = (256 - 81) \times 10^8 = 175 \times 10^8$$

Step 4: Ratio

$$\frac{R'}{R} = \frac{175}{1215} = \frac{35}{243}$$

$$\text{So } R' = \frac{35R}{243}.$$

Step 5: Compare with the form given

The rate is written as $\frac{x}{243}$, so $x = 35R$, option (D).

Final Answer:

The ratio of the radiation terms is 175 to 1215, which is 35/243, so $x = 35R$, option (D).

$$\boxed{35R}$$

Quick Tip: Rate of cooling is proportional to T to the fourth minus T_0 to the fourth for radiation.

15. A black body is at temperature of 5780 K. The energy of radiation emitted by the body at wavelength 300 nm is U_1 , at wavelength 500 nm is U_2 and that at 900 nm is U_3 respectively. Wien's constant $b = 2.89 \times 10^6 \text{ nmK}$. This shows that

- (A) $U_1 < U_2 < U_3$
- (B) $U_1 > U_2 > U_3$
- (C) $U_1 < U_2 > U_3$
- (D) $U_1 > U_2 < U_3$

Correct Answer: (C) $U_1 < U_2 > U_3$

Solution:

Step 1: Understanding the Concept

A black body radiates the most energy at the wavelength $\lambda_m = \frac{b}{T}$. The spectrum rises to a peak and then falls on either side.

Step 2: Peak wavelength

$$\lambda_m = \frac{2.89 \times 10^6 \text{ nm K}}{5780 \text{ K}} = 500 \text{ nm}$$

Step 3: Compare

The wavelength 500 nm is exactly at the peak, so U_2 is the largest. The wavelengths 300 nm and 900 nm are on opposite sides of the peak, so both energies are smaller than U_2 .

Step 4: Conclusion

$$U_1 < U_2 > U_3$$

This is option (C). We cannot say that U_1 is more or less than U_3 from the given data, and the answer does not need it.

Final Answer:

The peak lies at 500 nm, so the energy at 500 nm is greater than at 300 nm and at 900 nm, option (C).

$$U_1 < U_2 > U_3$$

Quick Tip: Find the wavelength of peak emission with Wien law and compare each given wavelength with it.



16. The efficiency of Carnot engine which operates between the two temperatures $T_1 = 500$ K and $T_2 = 300$ K is

- (A) 75%
- (B) 50%
- (C) 25%
- (D) 40%

Correct Answer: (D) 40%

Solution:

Step 1: Understanding the Concept

The efficiency of a Carnot engine depends only on the temperatures of the hot source and the cold sink, both in kelvin.

Step 2: Key Formula or Approach

$$\eta = 1 - \frac{T_2}{T_1}$$

Step 3: Substitute

$$\eta = 1 - \frac{300}{500} = 1 - 0.6 = 0.4$$

$$\eta = 40\%$$

Step 4: Check the options

75% would need the sink at 125 K. 50% needs a sink at 250 K. 25% needs a sink at 375 K. Only 40% matches 300 K, option (D).

Final Answer:

The efficiency is $1 - 300/500 = 40$ percent, option (D).

40%

Quick Tip: Carnot efficiency is 1 minus the ratio of the sink temperature to the source temperature.

17. An ideal gas having pressure P , volume V and temperature T is expanded isothermally, to a volume $3V$ and final pressure P_I . The same gas is expanded adiabatically to a volume $3V$, the final pressure being P_A . The ratio $\frac{P_A}{P_I}$ is ($\frac{C_P}{C_V} = \gamma$)

- (A) 3γ
- (B) 3^γ
- (C) $3^{(1-\gamma)}$
- (D) $3^{(\gamma-1)}$

Correct Answer: (C) $3^{(1-\gamma)}$

Solution:

Step 1: Understanding the Concept

In an isothermal process $PV = \text{constant}$. In an adiabatic process $PV^\gamma = \text{constant}$.

Step 2: Isothermal expansion

$$PV = P_I(3V) \Rightarrow P_I = \frac{P}{3}$$

Step 3: Adiabatic expansion

$$PV^\gamma = P_A(3V)^\gamma \Rightarrow P_A = \frac{P}{3^\gamma}$$

Step 4: Ratio

$$\frac{P_A}{P_I} = \frac{P/3^\gamma}{P/3} = 3^{1-\gamma}$$

Step 5: Check

Since $\gamma > 1$, the exponent is negative and the ratio is below 1. This agrees with the fact that the adiabatic pressure falls more, because the gas also cools. The answer is $3^{(1-\gamma)}$, option (C).

Final Answer:

The adiabatic pressure is P over 3 to the γ and the isothermal one is P over 3 , giving $3^{(1-\gamma)}$, option (C).

$$3^{(1-\gamma)}$$

Quick Tip: Isothermal: PV constant. Adiabatic: PV to the γ constant.

18. The displacement of a particle varies with time according to the relation

$$x = a \sin \omega t + b \cos \omega t$$

- (A) The motion is simple harmonic motion with an amplitude $\sqrt[3]{a^2 + b^2}$
- (B) The motion is simple harmonic motion with an amplitude $\sqrt{a^2 + b^2}$
- (C) The motion is periodic but not simple harmonic motion
- (D) The motion is simple harmonic with an amplitude $(a^2 + b^2)$

Correct Answer: (B) The motion is simple harmonic motion with an amplitude $\sqrt{a^2 + b^2}$

Solution:

Step 1: Understanding the Concept

A sum of a sine and a cosine of the same angular frequency is a single sinusoid with the same frequency.

Step 2: Rewrite

Let $a = A \cos \phi$ and $b = A \sin \phi$. Then

$$x = A \cos \phi \sin \omega t + A \sin \phi \cos \omega t = A \sin(\omega t + \phi)$$

Step 3: Find A

$$a^2 + b^2 = A^2 \Rightarrow A = \sqrt{a^2 + b^2}$$

Step 4: Nature of motion

$x = A \sin(\omega t + \phi)$ is the standard form of simple harmonic motion with amplitude A .

Step 5: Check the options

The cube root and the plain sum $a^2 + b^2$ are not amplitudes. The motion is simple harmonic, not merely periodic. So the answer is (B).

Final Answer:

The expression equals a single sine with amplitude root of a squared plus b squared, so it is SHM, option (B).

$\text{SHM, amplitude } \sqrt{a^2 + b^2}$

Quick Tip: Combine the sine and cosine into a single sine with an amplitude and phase.

19. For a particle executing S.H.M., the potential energy is n times the kinetic energy when its displacement from mean position is $(\frac{2\sqrt{2}}{3})A$, where A is the amplitude of S.H.M. The value of n is

- (A) 2
- (B) 4
- (C) 6
- (D) 8

Correct Answer: (D) 8

Solution:

Step 1: Understanding the Concept

In SHM, potential energy at displacement x is $\frac{1}{2}kx^2$, and kinetic energy is $\frac{1}{2}k(A^2 - x^2)$.

Step 2: Ratio

$$n = \frac{PE}{KE} = \frac{x^2}{A^2 - x^2}$$

Step 3: Substitute $x = \frac{2\sqrt{2}}{3}A$

$$x^2 = \frac{8}{9}A^2, \quad A^2 - x^2 = \frac{1}{9}A^2$$

$$n = \frac{8/9}{1/9} = 8$$

Step 4: Check the options

The value 2 or 4 would put the particle much nearer the mean position. At 8 the particle is close to the extreme, where PE dominates. So option (D).

Final Answer:

At that position PE is 8/9 of the energy and KE is 1/9, so $n = 8$, option (D).

8

Quick Tip: PE is half kx^2 and KE is half $k(A^2 - x^2)$.



20. A particle is performing S.H.M. about $x = 0$, with an amplitude a and time period T . The speed of the particle at $x = \frac{a}{3}$ will be

- (A) $\frac{2\pi a}{T}$
- (B) $\frac{4\pi a}{3T}$
- (C) $\frac{\sqrt{3}\pi^2 a}{2T}$
- (D) $\frac{4\sqrt{2}\pi a}{3T}$

Correct Answer: (D) $\frac{4\sqrt{2}\pi a}{3T}$

Solution:

Step 1: Understanding the Concept

In SHM the speed at displacement x is $v = \omega\sqrt{a^2 - x^2}$, and $\omega = \frac{2\pi}{T}$.

Step 2: Substitute

$$v = \frac{2\pi}{T} \sqrt{a^2 - \frac{a^2}{9}} = \frac{2\pi}{T} \cdot a \sqrt{\frac{8}{9}}$$

$$\sqrt{\frac{8}{9}} = \frac{2\sqrt{2}}{3}$$

Step 3: Result

$$v = \frac{2\pi}{T} \cdot \frac{2\sqrt{2}a}{3} = \frac{4\sqrt{2}\pi a}{3T}$$

Step 4: Check the options

The option $\frac{2\pi a}{T}$ is the maximum speed at the mean position. The others have a different numerical factor. The answer is option (D).

Final Answer:

The speed is omega times root of (a squared - a squared/9), which is 4 root 2 pi a over 3T, option (D).

$$\boxed{\frac{4\sqrt{2}\pi a}{3T}}$$

Quick Tip: Speed in SHM is omega times root of (a squared minus x squared).



21. Two sound waves each of wavelength λ and same amplitude A interfere at point Q . If the path difference is $\frac{\lambda}{4}$, the amplitude of the resultant wave at point Q is $[\sin \frac{\pi}{2} = 1, \cos \frac{\pi}{2} = 0]$

- (A) A
- (B) $\sqrt{2}A$
- (C) $3A$
- (D) $\sqrt{3}A$

Correct Answer: (B) $\sqrt{2}A$

Solution:

Step 1: Understanding the Concept

The phase difference is $\phi = \frac{2\pi}{\lambda} \times \text{path difference}$.

Step 2: Phase difference

$$\phi = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$$

Step 3: Resultant amplitude

For two waves of equal amplitude A :

$$A_R = 2A \cos \frac{\phi}{2} = 2A \cos \frac{\pi}{4} = 2A \cdot \frac{1}{\sqrt{2}} = \sqrt{2}A$$

Step 4: Check the options

Amplitude A would need a phase difference of 120° . Amplitude $3A$ is impossible because the maximum is $2A$. So option (B).

Final Answer:

The phase difference is $\pi/2$, so the amplitude is root 2 times A , option (B).

$$\boxed{\sqrt{2}A}$$

Quick Tip: A path difference of $\lambda/4$ means a phase difference of $\pi/2$.

22. Two monoatomic ideal gases '1' and '2' of molecular masses m_1 and m_2 respectively are enclosed in separate containers kept at the same temperature. The ratio of the speed of sound in gas '1' to that in gas '2' is

- (A) $\sqrt{\frac{m_1}{m_2}}$
- (B) $\sqrt{\frac{m_2}{m_1}}$
- (C) $\frac{m_1}{m_2}$
- (D) $\frac{m_2}{m_1}$

Correct Answer: (B) $\sqrt{\frac{m_2}{m_1}}$

Solution:

Step 1: Understanding the Concept

The speed of sound in an ideal gas is $v = \sqrt{\frac{\gamma RT}{M}}$. Both gases are monatomic, so $\gamma = \frac{5}{3}$ for each. The temperature is the same.

Step 2: Ratio

$$\frac{v_1}{v_2} = \sqrt{\frac{M_2}{M_1}} = \sqrt{\frac{m_2}{m_1}}$$

Step 3: Meaning

A heavier gas has a smaller speed of sound. Gas 1 with larger molecular mass m_1 is slower, so the ratio has m_2 on top.

Step 4: Check the options

Option (A) is the inverse. Options (C) and (D) lack the square root. So the answer is (B).

Final Answer:

At equal temperature the speed varies as one over root of molecular mass, so the ratio is root of m_2 over m_1 , option (B).

$$\sqrt{\frac{m_2}{m_1}}$$

Quick Tip: Speed of sound in a gas is proportional to root of T over molar mass.

23. Two waves are represented as

$$y_1 = a_1 \sin\left(\omega t - \frac{2\pi x}{\lambda}\right) \text{ and}$$

$$y_2 = a_2 \cos\left(\omega t - \frac{2\pi x}{\lambda} + \frac{\pi}{6}\right)$$

The path difference between two waves is

(A) $\frac{\lambda}{5}$

(B) $\frac{\lambda}{4}$

(C) $\frac{\lambda}{3}$

(D) $\frac{\lambda}{2}$

Correct Answer: (C) $\frac{\lambda}{3}$

Solution:

Step 1: Understanding the Concept

Convert the cosine wave into a sine wave so that the phases can be compared.

Step 2: Convert

$$\cos \theta = \sin\left(\theta + \frac{\pi}{2}\right)$$

$$y_2 = a_2 \sin\left(\omega t - \frac{2\pi x}{\lambda} + \frac{\pi}{6} + \frac{\pi}{2}\right) = a_2 \sin\left(\omega t - \frac{2\pi x}{\lambda} + \frac{2\pi}{3}\right)$$

Step 3: Phase difference

Compared with y_1 , the extra phase is $\phi = \frac{2\pi}{3}$.

Step 4: Path difference

$$\Delta x = \frac{\lambda}{2\pi} \phi = \frac{\lambda}{2\pi} \cdot \frac{2\pi}{3} = \frac{\lambda}{3}$$

This is option (C).

Final Answer:

The phase difference is 2π over 3, which corresponds to a path difference of λ over 3, option (C).

$$\boxed{\frac{\lambda}{3}}$$

Quick Tip: Write both waves with a sine, find the phase difference, then convert to path difference.

24. The equation of a wave on a string of linear mass density 0.02kg m^{-1} is $Y = 0.01\sin[2\pi(\frac{t}{0.02} - \frac{x}{0.50})]$ m. The tension in the string is

- (A) 12.50 N
- (B) 6.25 N
- (C) 25 N
- (D) 50 N

Correct Answer: (A) 12.50 N

Solution:

Step 1: Understanding the Concept

Compare the given wave with $y = A \sin(\omega t - kx)$ to find the speed of the wave. Then use $v = \sqrt{T/\mu}$.

Step 2: Wave speed

The wave is $y = 0.01 \sin \left[2\pi \left(\frac{t}{0.02} - \frac{x}{0.50} \right) \right]$. So the period is 0.02 s and the wavelength is 0.50 m.

$$v = \frac{\lambda}{T} = \frac{0.50}{0.02} = 25 \text{ m/s}$$

Step 3: Tension

$$T = \mu v^2 = 0.02 \times 625 = 12.5 \text{ N}$$

Step 4: Check the options

6.25 N would come from using $v = 12.5$. 25 N and 50 N come from dropping the square. The tension is 12.50 N, option (A).

Final Answer:

The wave speed is 25 m/s, so the tension is 0.02 times 625 = 12.5 N, option (A).

12.50 N

Quick Tip: Read omega and k from the wave equation, find the speed, then use $v = \text{root of } T \text{ over } \mu$.

◆

25. If charge $+q$ is taken from one point to another over an equipotential surface, then

- (A) work done on the charge continuously increases.
- (B) work is done by the charge.
- (C) work done on the charge is constant.
- (D) no work is done.

Correct Answer: (D) no work is done.

Solution:

Step 1: Understanding the Concept

The work done in moving a charge between two points is $W = q(V_B - V_A)$, where V is the electric potential.

Step 2: Equipotential surface

All points on an equipotential surface are at the same potential. So $V_B - V_A = 0$ for any two points on it.

Step 3: Calculation

$$W = q \times 0 = 0$$

Step 4: Check the options

(A) and (C) say work is done, which is not so. (B) says the charge does work, which is also not so. The electric force is always perpendicular to the surface, so it has no component along the displacement. No work is done, option (D).

Final Answer:

The potential difference is zero on an equipotential surface, so no work is done, option (D).

No work is done

Quick Tip: Work done equals charge times potential difference, which is zero on an equipotential surface.

26. A charge Q is placed at each corner of a cube of side r . The potential at the centre of the cube is (ϵ_0 = permittivity of free space)

- (A) $\frac{4Q}{\pi\epsilon_0 r\sqrt{3}}$
(B) $\frac{8Q}{\pi\epsilon_0 r\sqrt{3}}$
(C) $\frac{16Q}{3\pi\epsilon_0 r}$
(D) $\frac{32Q}{\pi\epsilon_0 r\sqrt{3}}$

Correct Answer: (A) $\frac{4Q}{\pi\epsilon_0 r\sqrt{3}}$

Solution:

Step 1: Understanding the Concept

Potential is a scalar, so the potentials of the eight charges simply add. Every corner is at the same distance from the centre.

Step 2: Distance

The body diagonal of a cube of side r is $r\sqrt{3}$. The centre is halfway along it:

$$d = \frac{r\sqrt{3}}{2}$$

Step 3: Potential of one charge

$$V_1 = \frac{Q}{4\pi\epsilon_0 d} = \frac{Q}{4\pi\epsilon_0} \cdot \frac{2}{r\sqrt{3}}$$

Step 4: Total

$$V = 8V_1 = \frac{8Q}{4\pi\epsilon_0} \cdot \frac{2}{r\sqrt{3}} = \frac{4Q}{\pi\epsilon_0 r\sqrt{3}}$$

Step 5: Conclusion

This is option (A). Option (B) doubles the result, which happens if the distance is taken as $r\sqrt{3}/4$.

Final Answer:

Eight equal potentials at distance $r\sqrt{3}/2$ give $4Q$ over $\pi\epsilon_0 r\sqrt{3}$, option (A).

$$\boxed{\frac{4Q}{\pi\epsilon_0 r\sqrt{3}}}$$

Quick Tip: All eight charges are at the same distance from the centre, so add their potentials.

27. Select the 'WRONG' statement about polar molecules.

- (A) A molecule in which centre of mass of positive charges does not coincide with the centre of mass of negative charges is a polar molecule.
- (B) They have permanent dipole moment.
- (C) They act as tiny electric dipoles.
- (D) They have symmetrical shape.

Correct Answer: (D) They have symmetrical shape.

Solution:

Step 1: Understanding the Concept

A polar molecule has a permanent separation between its positive and negative charge centres. This gives it a permanent dipole moment.

Step 2: Statement (A)

Positive and negative charge centres not coinciding is the definition of a polar molecule. True.

Step 3: Statement (B)

Because of this separation, it has a permanent dipole moment. True.

Step 4: Statement (C)

Each polar molecule behaves like a tiny electric dipole. True.

Step 5: Statement (D)

Symmetrical shape is a feature of non-polar molecules, whose charge centres coincide, as in CO_2 or CH_4 . Polar molecules like H_2O are bent and unsymmetrical. So (D) is the wrong statement.

Final Answer:

Polar molecules have an unsymmetrical charge arrangement, so the symmetrical-shape statement is wrong, option (D).

(D)

Quick Tip: A polar molecule has a lopsided charge distribution, so its shape is not symmetrical.



28. Two point charges placed in air separated by distance R exert a force F on each other. Now the space between those charges is filled with dielectric constant K . At distance R_1 between the charges, the same force is exerted as F . Then

(A) $R_1 = \sqrt{KR}$

(B) $R_1 = R$

(C) $R_1 = \frac{R}{K}$

(D) $R_1 = \frac{R}{\sqrt{K}}$

Correct Answer: (D) $R_1 = \frac{R}{\sqrt{K}}$

Solution:

Step 1: Understanding the Concept

In air, $F = \frac{kq_1q_2}{R^2}$. In a medium of dielectric constant K , the force is reduced by the factor K .

Step 2: Force in the medium

$$F' = \frac{kq_1q_2}{KR_1^2}$$

Step 3: Set equal to the force in air

$$\frac{kq_1q_2}{KR_1^2} = \frac{kq_1q_2}{R^2} \Rightarrow KR_1^2 = R^2$$

$$R_1 = \frac{R}{\sqrt{K}}$$

Step 4: Check the options

Since $K > 1$, the charges must be brought nearer. That rules out $\sqrt{K}R$ and R . Option (C) would be right only if the force were proportional to $1/R$. The answer is option (D).

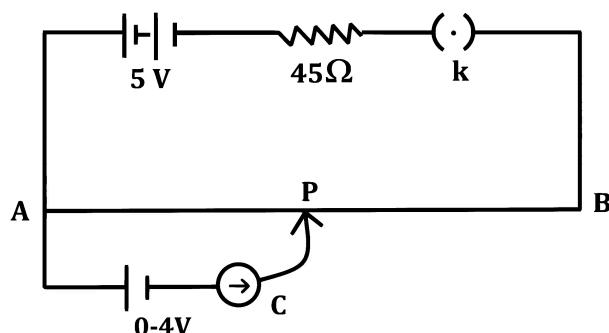
Final Answer:

The force in the dielectric is kq_1q_2 over $K R_1$ squared, so R_1 must equal R over root K , option (D).

$$R_1 = \frac{R}{\sqrt{K}}$$

Quick Tip: A dielectric reduces the force by K , so the distance must shrink by root K to restore the force.

29. Figure shows a potentiometer wire AB having resistance of 5Ω and length 10 m. An e.m.f. is 0.4 V of battery, the balancing length AP is (internal resistance is negligible)



- (A) 8 m
- (B) 10 m
- (C) 6 m
- (D) 4 m

Correct Answer: (A) 8 m

Solution:

Step 1: Understanding the Concept

The figure shows a driver battery of 5 V in series with a 45 ohm resistor, a key and the potentiometer wire AB of 5 ohm and length 10 m. A cell of emf 0.4 V is connected through a galvanometer to the sliding point P.

Step 2: Current in the wire

$$I = \frac{5}{45 + 5} = 0.1 \text{ A}$$

Step 3: Potential across the wire

$$V_{AB} = I \times 5 = 0.1 \times 5 = 0.5 \text{ V}$$

Step 4: Potential gradient

$$k = \frac{0.5}{10} = 0.05 \text{ V/m}$$

Step 5: Balancing length

At balance, the cell emf equals the potential drop along AP:

$$0.4 = k l \Rightarrow l = \frac{0.4}{0.05} = 8 \text{ m}$$

This is option (A).

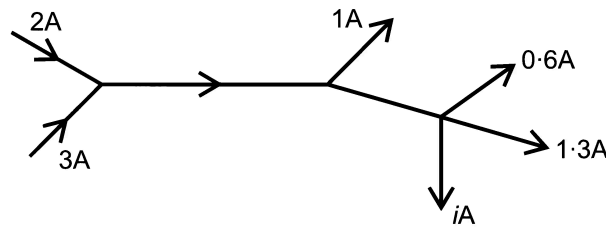
Final Answer:

The wire carries 0.1 A, so it drops 0.5 V over 10 m, and 0.4 V is balanced at 8 m, option (A).

8 m

Quick Tip: Find the potential gradient of the wire from the driver circuit, then divide the cell emf by it.

30. The currents in different parts of the electric circuit are shown in following figure. The value of current i is



- (A) 0.7 A
- (B) 1.4 A
- (C) 2.1 A
- (D) 2.8 A

Correct Answer: (C) 2.1 A

Solution:

Step 1: Understanding the Concept

Kirchhoff's current law says that the total current flowing into a junction equals the total current flowing out of it.

Step 2: Current entering

Two wires bring 2 A and 3 A into the first junction, so a total of 5 A flows along the main wire.

Step 3: First split

A branch of 1 A leaves, so the remaining current in the main wire is $5 - 1 = 4$ A.

Step 4: Second junction

This 4 A divides into 0.6 A, 1.3 A and the unknown i :

$$4 = 0.6 + 1.3 + i$$

$$i = 4 - 1.9 = 2.1 \text{ A}$$

Step 5: Conclusion

The current is 2.1 A, option (C).

Final Answer:

The currents in total are 5 A and the known outflows are 2.9 A, so $i = 2.1 \text{ A}$, option (C).

2.1 A

Quick Tip: Current entering a junction equals current leaving it.

31. An electron is moving with initial velocity $\vec{v} = V_0 \hat{j}$ in a magnetic field $\vec{B} = B_0 \hat{i}$. Its de-Broglie wavelength will

- (A) decrease with time.
- (B) increase and decrease periodically.
- (C) remains constant.
- (D) increase with time.

Correct Answer: (C) remains constant.

Solution:

Step 1: Understanding the Concept

The de Broglie wavelength is $\lambda = \frac{h}{p} = \frac{h}{mv}$. It changes only if the speed changes.

Step 2: Effect of the magnetic field

The magnetic force on a moving charge is $\vec{F} = q\vec{v} \times \vec{B}$. It is always perpendicular to the velocity, so it does no work. The kinetic energy and the speed stay constant. Only the direction of motion changes, and the electron moves along a circle or a helix.

Step 3: Result

Since $|v|$ is constant, p is constant and λ is constant.

Step 4: Check the options

Decrease, increase, or periodic change would all need a changing speed. So the wavelength remains constant, option (C).

Final Answer:

A magnetic force does no work, so the speed and de Broglie wavelength stay constant, option (C).

Remains constant

Quick Tip: A magnetic force does no work, so the speed and hence the wavelength stay the same.

32. A thin circular wire carrying current I has magnetic moment M . The shape of the wire is changed to a square and it carries the same current. It will have magnetic moment M_1 . The ratio M to M_1 is

- (A) $\frac{2}{\pi}$
- (B) $\frac{4}{\pi}$
- (C) $\frac{\pi}{4}$
- (D) $\frac{\pi}{2}$

Correct Answer: (B) $\frac{4}{\pi}$

Solution:

Step 1: Understanding the Concept

The magnetic moment of a current loop is $M = IA$. The same wire is reshaped, so its length L is the same and the current is the same. Only the area changes.

Step 2: Circle

$$L = 2\pi r \Rightarrow r = \frac{L}{2\pi}, \quad A_c = \pi r^2 = \frac{L^2}{4\pi}$$

Step 3: Square

$$L = 4s \Rightarrow s = \frac{L}{4}, \quad A_s = \frac{L^2}{16}$$

Step 4: Ratio

$$\frac{M}{M_1} = \frac{A_c}{A_s} = \frac{L^2/4\pi}{L^2/16} = \frac{4}{\pi}$$

This is option (B). The circle encloses the larger area for a given perimeter, so the ratio is greater than 1.

Final Answer:

For the same wire length the circle has area $L^2 / 4\pi$ and the square $L^2 / 16$, giving $4/\pi$, option (B).

$$\boxed{\frac{4}{\pi}}$$

Quick Tip: Same wire length, so compare the areas of a circle and a square.

33. A circular arc of wire of radius of curvature r subtends an angle of $\frac{\pi}{5}$ radian at its centre. If current i is flowing in it then the magnetic induction at its centre is (μ_0 = permeability of free space)

- (A) $\frac{\mu_0 i}{4r}$
- (B) $\frac{\mu_0 i}{8r}$
- (C) $\frac{\mu_0 i}{16r}$
- (D) $\frac{\mu_0 i}{20r}$

Correct Answer: (D) $\frac{\mu_0 i}{20r}$

Solution:

Step 1: Understanding the Concept

A full circle of radius r gives $B = \frac{\mu_0 i}{2r}$ at the centre. For an arc, the field is in proportion to the angle subtended.

Step 2: Formula for an arc

$$B = \frac{\mu_0 i}{4\pi r} \theta$$

Step 3: Substitute $\theta = \frac{\pi}{5}$

$$B = \frac{\mu_0 i}{4\pi r} \cdot \frac{\pi}{5} = \frac{\mu_0 i}{20r}$$

Step 4: Check with the fraction of a circle

The angle $\frac{\pi}{5}$ is $\frac{1}{10}$ of 2π . So the field is $\frac{1}{10}$ of $\frac{\mu_0 i}{2r}$, which is $\frac{\mu_0 i}{20r}$. Both ways give option (D).

Final Answer:

The arc is one tenth of a circle, so the field is one tenth of $\mu_0 i$ over $2r$, which is $\mu_0 i$ over $20r$, option (D).

$$\boxed{\frac{\mu_0 i}{20r}}$$

Quick Tip: The field at the centre of an arc is $\mu_0 i$ over $4\pi r$ times the angle.

34. A solenoid of 1000 turns is wound uniformly on a glass tube 4 m long and 0.3 m in diameter. The magnetic intensity at the centre of the solenoid when a current of 4 A flows through it is

- (A) $2 \times 10^3 \text{ A/m}$
- (B) $16 \times 10^3 \text{ A/m}$
- (C) $4 \times 10^3 \text{ A/m}$
- (D) 10^3 A/m

Correct Answer: (D) 10^3 A/m

Solution:

Step 1: Understanding the Concept

Inside a long solenoid, the magnetic intensity is $H = nI$, where n is the number of turns per unit length.

Step 2: Turns per metre

$$n = \frac{1000}{4} = 250 \text{ turns/m}$$

Step 3: Calculation

$$H = nI = 250 \times 4 = 1000 \text{ A/m} = 10^3 \text{ A/m}$$

Step 4: Check the options

The diameter of 0.3 m is not needed because the solenoid is long compared with its width (4 m against 0.3 m). Using 1000 turns without dividing by length would give 4000 A/m, which is option (C), a common slip. The answer is option (D).

Final Answer:

With 250 turns per metre and 4 A, $H = nI = 1000 \text{ A/m}$, option (D).

$$10^3 \text{ A/m}$$

Quick Tip: Magnetic intensity inside a long solenoid is $H = nI$.

35. A circular disc of radius 20 cm is placed in uniform magnetic field of induction $\frac{7}{22} \text{ Wb/m}^2$ in such a way that its axis makes an angle of 60° with $\vec{B}$. The magnetic flux linked with the disc is ($\cos 60^\circ = 0.5$)

- (A) 0.01 Wb
- (B) 0.02 Wb
- (C) 0.06 Wb
- (D) 0.08 Wb

Correct Answer: (B) 0.02 Wb

Solution:

Step 1: Understanding the Concept

Magnetic flux is $\Phi = BA \cos \theta$, where θ is the angle between $\vec{B}$ and the normal to the surface. The axis of the disc is its normal.

Step 2: Area

$$A = \pi r^2 = \frac{22}{7} \times (0.2)^2 = \frac{22}{7} \times 0.04$$

Step 3: Flux

$$\Phi = \frac{7}{22} \times \frac{22}{7} \times 0.04 \times \cos 60^\circ = 0.04 \times 0.5 = 0.02 \text{ Wb}$$

Step 4: Check the options

The value 0.04 Wb would result if the angle were zero. The value 0.01 Wb would come from also multiplying by 0.5 twice. Our result is 0.02 Wb, option (B).

Final Answer:

The flux is B times area times $\cos 60$, which is 0.04 times $0.5 = 0.02$ Wb, option (B).

0.02 Wb

Quick Tip: Flux is $B A \cos(\theta)$ with θ measured from the normal (axis).

36. An aeroplane having a wing span of 30 m flies due north with the speed of 170 m/s. Magnetic field $B = 3.6 \times 10^{-5}$ T. The potential difference between the tips of the wings will be

- (A) 136.4 mV
- (B) 183.6 mV
- (C) 272.8 mV
- (D) 367.2 mV

Correct Answer: (B) 183.6 mV

Solution:

Step 1: Understanding the Concept

A conductor of length l moving with velocity v perpendicular to a magnetic field B develops a motional emf $\varepsilon = Blv$. The wings act as such a rod.

Step 2: Substitute

$$\begin{aligned}\varepsilon &= 3.6 \times 10^{-5} \times 30 \times 170 \\ &= 3.6 \times 10^{-5} \times 5100 = 0.1836 \text{ V} \\ &= 183.6 \text{ mV}\end{aligned}$$

Step 3: Check the options

The value 136.4 mV and 272.8 mV come from a different product. 367.2 mV is twice the correct value and would come from using a wing span of 60 m. The answer is 183.6 mV, option (B).

Final Answer:

The emf is $B l v = 3.6 \times 10^{-5} \times 30 \times 170 = 0.1836 \text{ V}$, option (B).

183.6 mV

Quick Tip: The wing tips behave as the ends of a rod moving across the field: $\text{emf} = B l v$.

37. In an LCR series circuit an alternating voltage source of frequency F is connected. The current leads the voltage by 45° . The value of L is

- (A) $\frac{1+2\pi FCR}{2\pi FC}$
- (B) $\frac{1+2\pi FCR}{4\pi^2 F^2 C}$
- (C) $\frac{1-2\pi FCR}{4\pi^2 F^2 C}$
- (D) $\frac{1+2\pi FR}{4\pi^2 F^2 C}$

Correct Answer: (C) $\frac{1-2\pi FCR}{4\pi^2 F^2 C}$

Solution:

Step 1: Understanding the Concept

In a series LCR circuit the phase angle ϕ satisfies $\tan \phi = \frac{X_C - X_L}{R}$ when the current leads. The current leads by 45° , so $\tan 45^\circ = 1$.

Step 2: Equation

$$X_C - X_L = R$$

$$\frac{1}{2\pi FC} - 2\pi FL = R$$

Step 3: Solve for L

$$2\pi FL = \frac{1}{2\pi FC} - R = \frac{1 - 2\pi FCR}{2\pi FC}$$

$$L = \frac{1 - 2\pi FCR}{4\pi^2 F^2 C}$$

Step 4: Check the options

Options (B) and (D) have a plus sign, which would belong to a voltage-leading situation. Option (A) has the wrong dimension for an inductance. The answer is option (C).

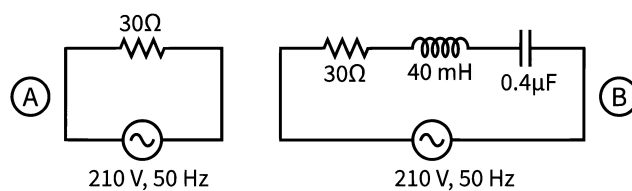
Final Answer:

Equal reactance difference and resistance give $L = (1 - 2\pi F C R) \text{ over } 4\pi^2 \text{ squared } F \text{ squared } C$, option (C).

$$\frac{1 - 2\pi F C R}{4\pi^2 F^2 C}$$

Quick Tip: Current leads the voltage when the circuit is capacitive; tan of the phase angle is $(X_C - X_L) \text{ over } R$.

38. Given below are the two circuits, choose the correct option.



- (A) At resonance, current in (B) is less than that in (A)
- (B) The r.m.s. current in (B) can be larger than that in (A)
- (C) The r.m.s. current in (A) is always equal to that in (B)
- (D) The r.m.s. current in (B) can never be larger than that in (A)

Correct Answer: (D) The r.m.s. current in (B) can never be larger than that in (A)

Solution:

Step 1: Understanding the Concept

Circuit A has only a 30 ohm resistor on a 210 V, 50 Hz supply. Circuit B has the same resistor in series with a 40 mH inductor and a 0.4 microfarad capacitor on the same supply.

Step 2: Current in A

$$I_A = \frac{210}{30} = 7 \text{ A}$$

Step 3: Impedance of B

$$Z_B = \sqrt{R^2 + (X_L - X_C)^2} \geq R = 30 \Omega$$

So

$$I_B = \frac{210}{Z_B} \leq \frac{210}{30} = 7 \text{ A}$$

The reactances make Z_B larger than R , or at best equal to it when $X_L = X_C$.

Step 4: Numbers here

$X_L = 2\pi(50)(0.04) \approx 12.6 \Omega$ and $X_C = \frac{1}{2\pi(50)(0.4 \times 10^{-6})} \approx 7960 \Omega$. So Z_B is large and I_B is very small.

Step 5: Check the options

At resonance the currents would be equal, not less, so (A) is wrong.

Since $I_B \leq I_A$ always, (B) is wrong. (C) is wrong because they are equal

only at resonance. So (D): the current in B can never exceed that in A.

Final Answer:

The impedance of B is never smaller than $30\ \Omega$, so its current never exceeds the $7\ \text{A}$ in A, option (D).

(D)

Quick Tip: The impedance of circuit B is at least R , so its current cannot exceed V over R , the current of circuit A.



39. In a L-R circuit of $4\ \text{mH}$ inductance and $3\ \Omega$ resistance, e.m.f. $E = \cos(1000t)\ \text{V}$ is applied. The amplitude of current is

- (A) $1.6\ \text{A}$
- (B) $1\ \text{A}$
- (C) $0.2\ \text{A}$
- (D) $0.4\ \text{A}$

Correct Answer: (C) $0.2\ \text{A}$

Solution:

Step 1: Understanding the Concept

The applied emf is $E = \cos(1000t)\ \text{V}$. So the peak emf is $E_0 = 1\ \text{V}$ and $\omega = 1000\ \text{rad/s}$.

Step 2: Inductive reactance

$$X_L = \omega L = 1000 \times 4 \times 10^{-3} = 4\ \Omega$$

Step 3: Impedance

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{3^2 + 4^2} = 5 \, \Omega$$

Step 4: Amplitude of current

$$I_0 = \frac{E_0}{Z} = \frac{1}{5} = 0.2 \, \text{A}$$

Step 5: Check the options

The value 1 A is for a resistance of 1 ohm, and 0.4 A or 1.6 A would need an impedance of 2.5 or 0.625 ohm. The answer is 0.2 A, option (C).

Final Answer:

The reactance is 4 ohm and the impedance is 5 ohm, so the current amplitude is 0.2 A, option (C).

0.2 A

Quick Tip: The impedance of an LR circuit is the root of R squared plus X_L squared.

40. A current of 4 A is flowing at 230 V in the primary coil of a transformer. If the voltage produced in the secondary coil is 2300 V and 40 % of power is lost, then the current in the secondary will be

- (A) 0.24 A
- (B) 0.2 A
- (C) 0.12 A
- (D) 0.4 A

Correct Answer: (A) 0.24 A

Solution:

Step 1: Understanding the Concept

Power lost means the output power is smaller than the input power. Here 40% is lost, so 60% is delivered.

Step 2: Input power

$$P_{in} = V_p I_p = 230 \times 4 = 920 \text{ W}$$

Step 3: Output power

$$P_{out} = 0.60 \times 920 = 552 \text{ W}$$

Step 4: Secondary current

$$I_s = \frac{P_{out}}{V_s} = \frac{552}{2300} = 0.24 \text{ A}$$

Step 5: Check the options

An ideal transformer would give $920/2300 = 0.4$ A, which is option (D). With the 40% loss it is smaller. So 0.24 A, option (A).

Final Answer:

The output power is 552 W at 2300 V, so the secondary current is 0.24 A, option (A).

0.24 A

Quick Tip: Output power is 60 percent of input power, then divide by the secondary voltage.



41. A thin glass prism has refractive index 1.5. The correct relation between the angle of minimum deviation (δm) and angle of refraction (r) is

- (A) $\delta m = \frac{r}{2}$
- (B) $\delta m = 2r$
- (C) $\delta m = r$
- (D) $\delta m = \frac{3r}{2}$

Correct Answer: (C) $\delta m = r$

Solution:**Step 1: Understanding the Concept**

For a thin prism, the deviation is $\delta = (\mu - 1)A$. At minimum deviation

the ray passes symmetrically, so the angle of refraction inside each face is $r = \frac{A}{2}$.

Step 2: Compute

$$\delta_m = (1.5 - 1)A = 0.5A = \frac{A}{2}$$

$$r = \frac{A}{2}$$

Step 3: Compare

$$\delta_m = r$$

Step 4: Check the options

$\delta_m = \frac{r}{2}$ would need $\mu = 1.25$. $\delta_m = 2r$ would need $\mu = 2$. $\frac{3r}{2}$ would need $\mu = 1.75$. Only $\delta_m = r$ matches $\mu = 1.5$, option (C).

Final Answer:

With index 1.5 the deviation is A over 2, which equals the angle of refraction r , option (C).

$$\boxed{\delta_m = r}$$

Quick Tip: For a thin prism the deviation is $(\mu - 1) A$, and at minimum deviation r equals A over 2.

42. In diffraction experiment from single slit, the angular width of the central maxima does NOT depend upon

- (A) width of the slit
- (B) wavelength of light used.
- (C) ratio of wavelength and slit width.
- (D) distance of the slit from the screen.

Correct Answer: (D) distance of the slit from the screen.

Solution:

Step 1: Understanding the Concept

For a single slit, the angular half-width of the central maximum is $\theta \approx \frac{\lambda}{a}$. So the total angular width is $\frac{2\lambda}{a}$.

Step 2: Dependence

The angular width depends on the wavelength λ , on the slit width a , and so on the ratio λ/a .

Step 3: Distance to the screen

The screen distance D does not appear in the formula for the angle. It appears only in the linear width on the screen, $2\lambda D/a$.

Step 4: Conclusion

The angular width does NOT depend on the distance of the slit from the screen, option (D).

Final Answer:

The angle is $2\lambda/a$, which contains no screen distance, option (D).

distance of the slit from the screen

Quick Tip: Angular width of the central maximum is $2\lambda/a$; the screen distance only sets the linear width.



43. To improve the resolving power of a compound microscope we should

- (A) decrease the diameter of the objective lens.
- (B) decrease the focal length of the eye-piece.
- (C) increase the refractive index of the medium between the object and objective lens.
- (D) increase the wavelength of light.

Correct Answer: (C) increase the refractive index of the medium between the object and objective lens.

Solution:

Step 1: Understanding the Concept

The resolving power of a microscope is

$$R = \frac{2\mu \sin \theta}{1.22\lambda}$$

where μ is the refractive index of the medium between the object and the objective and θ is the half-angle of the cone of light.

Step 2: Check the options

- (A) Decreasing the objective diameter lowers $\sin \theta$ and so lowers R .
- (B) The eyepiece focal length affects magnification only, not resolving

power.

(C) Increasing μ , for example by using oil, increases R . This is the correct way.

(D) A longer wavelength makes R smaller. Violet light is better than red.

Step 3: Conclusion

Option (C).

Final Answer:

The resolving power is proportional to the refractive index of the medium at the objective, option (C).

Increase the refractive index of the medium

Quick Tip: Resolving power of a microscope is $\frac{2 \mu \sin(\theta)}{1.22 \lambda}$.

44. In the diffraction pattern, the first maximum is at 30° , when a monochromatic light of wavelength λ is incident on a slit of width a . For the same wavelength, if the slit width is changed, so that first maximum is at 45° . The slit width is changed by ($\sin 30^\circ = \frac{1}{2}$, $\sin 45^\circ = \frac{1}{\sqrt{2}}$)

(A) $3\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)\lambda$

(B) $\frac{3}{\sqrt{2}}\lambda$

(C) $3\sqrt{2}\lambda$

(D) $3\left(\frac{\sqrt{2}+1}{\sqrt{2}}\right)\lambda$

Correct Answer: (A) $3\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)\lambda$

Solution:

Step 1: Understanding the Concept

For a single slit, the first secondary maximum (the first maximum beside the centre) occurs at

$$a \sin \theta = \frac{3\lambda}{2}$$

Step 2: Slit width in each case

$$a_1 = \frac{3\lambda}{2 \sin 30^\circ} = 3\lambda, \quad a_2 = \frac{3\lambda}{2 \sin 45^\circ} = \frac{3\lambda}{\sqrt{2}}$$

Step 3: Change

$$a_1 - a_2 = 3\lambda - \frac{3\lambda}{\sqrt{2}} = 3\lambda \left(\frac{\sqrt{2}-1}{\sqrt{2}} \right)$$

So the slit width is reduced by this amount. This is option (A).

Final Answer:

The widths are 3λ and $\frac{3\lambda}{\sqrt{2}}$, so the change is $3(\sqrt{2}-1)/\sqrt{2}$ times λ , option (A).

$$\boxed{3\left(\frac{\sqrt{2}-1}{\sqrt{2}}\right)\lambda}$$

Quick Tip: The first secondary maximum satisfies a $\sin(\theta) = \frac{3\lambda}{2}$ over 2.



45. The ratio of de-Broglie wavelength of an α -particle and proton accelerated from rest by the same potential is $\frac{1}{\sqrt{m}}$. The value of m is

- (A) 2
- (B) 8
- (C) 4
- (D) 5

Correct Answer: (B) 8

Solution:

Step 1: Understanding the Concept

A particle of mass m and charge q accelerated from rest through a potential V has momentum $p = \sqrt{2mqV}$. So $\lambda = \frac{h}{\sqrt{2mqV}}$.

Step 2: Ratio

Same V for both:

$$\frac{\lambda_{\alpha}}{\lambda_p} = \sqrt{\frac{m_p q_p}{m_{\alpha} q_{\alpha}}}$$

Step 3: Insert values

An alpha particle has mass about 4 times that of a proton and charge 2 times:

$$\frac{\lambda_{\alpha}}{\lambda_p} = \sqrt{\frac{1 \times 1}{4 \times 2}} = \frac{1}{\sqrt{8}}$$

Step 4: Compare

The given ratio is $\frac{1}{\sqrt{m}}$, so $m = 8$, option (B).

Final Answer:

The alpha has 4 times the mass and twice the charge, so the wavelength ratio is 1 over root 8 and $m = 8$, option (B).

8

Quick Tip: De Broglie wavelength after acceleration through V is h over root of $2 m q V$.

46. The kinetic energy of the electron in an orbit of radius r in hydrogen atom is proportional to (e = electronic charge)

- (A) $\frac{e^2}{2r^2}$
- (B) $\frac{e^2}{r^2}$
- (C) $\frac{e^2}{2r}$
- (D) $\frac{e^2}{4r}$

Correct Answer: (C) $\frac{e^2}{2r}$

Solution:

Step 1: Understanding the Concept

In a Bohr orbit the electrostatic force provides the centripetal force on the electron.

Step 2: Force balance

In Gaussian-style units (the form in which the options are written):

$$\frac{mv^2}{r} = \frac{e^2}{r^2} \Rightarrow mv^2 = \frac{e^2}{r}$$

Step 3: Kinetic energy

$$KE = \frac{1}{2}mv^2 = \frac{e^2}{2r}$$

In SI units this reads $\frac{e^2}{8\pi\epsilon_0 r}$, which has the same dependence on r .

Step 4: Conclusion

The kinetic energy is proportional to $\frac{e^2}{r}$. Among the options, $\frac{e^2}{2r}$ has exactly this form, so the answer is option (C).

Final Answer:

The kinetic energy equals e^2 over $2r$, which varies as one over r , option (C).

$$\boxed{\frac{e^2}{2r}}$$

Quick Tip: The Coulomb force supplies the centripetal force, which gives kinetic energy proportional to $1/r$.

47. Which of the following is an integral multiple of $\frac{h}{2\pi}$ in Bohr's model of an hydrogen atom?

- (A) Radius of an atom
- (B) Kinetic energy
- (C) Potential energy
- (D) Angular momentum

Correct Answer: (D) Angular momentum

Solution:

Step 1: Understanding the Concept

Bohr's postulate says that the electron moves only in orbits where its angular momentum is quantised:

$$L = mvr = n\frac{h}{2\pi}, \quad n = 1, 2, 3, \dots$$

Step 2: Check the options

- (A) The radius varies as n^2 , so it is not an integral multiple of $h/2\pi$.
- (B) Kinetic energy varies as $1/n^2$.
- (C) Potential energy varies as $-1/n^2$.
- (D) Angular momentum is exactly $n\frac{h}{2\pi}$.

Step 3: Conclusion

Only the angular momentum is an integral multiple of $\frac{h}{2\pi}$, option (D).

Final Answer:

Bohr's rule fixes the angular momentum at $n h$ over 2π , option (D).

Angular momentum

Quick Tip: Bohr quantisation: angular momentum is $n h$ over 2π .

48. Activity of a radioactive sample decreases to $\left(\frac{1}{4}\right)^{th}$ of its original value in 4 days. Then in 16 days its activity will become x times the original value. The value of x is

- (A) $\frac{1}{16}$
- (B) $\frac{1}{32}$
- (C) $\frac{1}{256}$
- (D) $\frac{1}{128}$

Correct Answer: (C) $\frac{1}{256}$

Solution:

Step 1: Understanding the Concept

Activity halves every half-life. A fall to $\left(\frac{1}{4}\right)$ of the original value takes two half-lives.

Step 2: Half-life

$$2T_{1/2} = 4 \text{ days} \Rightarrow T_{1/2} = 2 \text{ days}$$

Step 3: Number of half-lives in 16 days

$$n = \frac{16}{2} = 8$$

Step 4: Activity

$$\frac{A}{A_0} = \left(\frac{1}{2}\right)^8 = \frac{1}{256}$$

Step 5: Check the options

$\frac{1}{16}$ would be after 4 half-lives (8 days). The value $\frac{1}{256}$ is option (C).
Another way: after 4 days it is $\frac{1}{4}$, after 8 days $\frac{1}{16}$, after 12 days $\frac{1}{64}$,
after 16 days $\frac{1}{256}$.

Final Answer:

Activity quarters every 4 days, so in 16 days it becomes $1/256$ of the original, option (C).

$$\boxed{\frac{1}{256}}$$

Quick Tip: Activity falling to a quarter takes two half-lives.

49. In Boolean expression $A + B = Y$ implies that

- (A) Y exists when A or B exists but not when both A and B exist.
- (B) Y exists only when A and B both exist.
- (C) Y exists when both A and B do not exist.
- (D) Y exists when A exists or B exists or both A and B exist

Correct Answer: (D) Y exists when A exists or B exists or both A and B exist

Solution:

Step 1: Understanding the Concept

In Boolean algebra the plus sign means logical OR. The expression $Y = A + B$ describes an OR gate.

Step 2: Truth table

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

Step 3: Reading the table

Y is 1 whenever A is 1, or B is 1, or both are 1. It is 0 only when both inputs are 0.

Step 4: Check the options

(A) describes XOR, which excludes the case of both. (B) describes AND. (C) describes NOR. (D) describes OR, which matches.

Final Answer:

Y is 1 when A, or B, or both are 1, so it is an OR gate, option (D).

Y exists when A or B or both exist

Quick Tip: A plus sign in Boolean algebra is the OR operation.

50. The depletion region of p-n junction

- (A) increases if reverse biased.
- (B) increases if forward biased.
- (C) decreases if reverse biased.
- (D) remains same in reverse and forward biasing.

Correct Answer: (A) increases if reverse biased.

Solution:**Step 1: Understanding the Concept**

The depletion region is the layer near the p-n junction that has no free charge carriers, only fixed ions.

Step 2: Reverse bias

The positive terminal of the battery goes to the n side and the negative terminal to the p side. Free electrons and holes are pulled away from the junction, so more fixed ions are exposed and the depletion region increases. This is option (A).

Step 3: Forward bias

The carriers are pushed towards the junction and fill in part of the

depletion layer. So the region shrinks, which is why (B) and (D) do not hold.

Step 4: Check

Statement (C) says it decreases when reverse biased, which is the reverse of what happens.

Final Answer:

Reverse bias pulls carriers away from the junction and widens the depletion region, option (A).

Increases if reverse biased

Quick Tip: Reverse bias pulls charge carriers away from the junction, so the depletion region widens.

Chemistry

Questions 51 to 100 | 1 mark each | 50 marks

51. What is the number of molecules in 2.125 g of ammonia?

- (A) 6.022×10^{23}
- (B) 3.011×10^{23}
- (C) 1.505×10^{22}
- (D) 7.527×10^{22}

Correct Answer: (D) 7.527×10^{22}

Solution:

Step 1: Understanding the Concept

The number of molecules in a sample is found by first converting its mass into moles. One mole of any substance holds 6.022×10^{23} particles (Avogadro number, N_A).

Step 2: Key Formula or Approach

$$n = \frac{\text{given mass}}{\text{molar mass}}, \quad \text{Number of molecules} = n \times N_A$$

Step 3: Molar mass of ammonia

Ammonia is NH_3 . Atomic masses are $\text{N} = 14$ and $\text{H} = 1$, so the molar mass is $14 + 3 \times 1 = 17 \text{ g mol}^{-1}$.

Step 4: Moles present

$$n = \frac{2.125}{17} = 0.125 \text{ mol}$$

Step 5: Number of molecules

$$N = 0.125 \times 6.022 \times 10^{23} = 7.5275 \times 10^{22} \approx 7.527 \times 10^{22}$$

Step 6: Check the other options

Option (A) is the value for a full 1 mol (17 g), and (B) is for 0.5 mol (8.5 g). Option (C) is far too small because it would need only about 0.025 mol. Only (D) matches 0.125 mol.

Final Answer:

The sample is 0.125 mol of NH_3 , which holds 7.527×10^{22} molecules, option (D).

$$\boxed{7.527 \times 10^{22}}$$

Quick Tip: Convert mass to moles using the molar mass, then multiply by Avogadro number.

52. Identify pair of orbitals having similar $(n + l)$ values from the following.

- (A) 3s and 3p
- (B) 2p and 4s
- (C) 3d and 4p
- (D) 2p and 3p

Correct Answer: (C) 3d and 4p

Solution:

Step 1: Understanding the Concept

The $(n + l)$ rule says that the energy of an orbital rises with the sum of its principal quantum number n and azimuthal quantum number l . Orbitals with the same sum have similar energy.

Step 2: Key Formula or Approach

For s, p, d, f orbitals the value of l is 0, 1, 2, 3 in that order. Compute $n + l$ for every orbital in each pair and compare.

Step 3: Check option (A): 3s and 3p

3s: $3 + 0 = 3$. 3p: $3 + 1 = 4$. The values differ, so (A) is not the pair.

Step 4: Check option (B): 2p and 4s

2p: $2 + 1 = 3$. 4s: $4 + 0 = 4$. Different again.

Step 5: Check option (C): 3d and 4p

3d: $3 + 2 = 5$. 4p: $4 + 1 = 5$. Both equal 5, so this pair has similar $(n + l)$ values.

Step 6: Check option (D): 2p and 3p

2p: $2 + 1 = 3$. 3p: $3 + 1 = 4$. Different.

Final Answer:

3d and 4p both have $n + l = 5$, so the answer is option (C).

3d and 4p

Quick Tip: Add n and l for each orbital in the pair and look for equal sums.



53. What is the proportionality between (I) Bond length and size of atom
(II) Bond length and multiplicity of bond respectively?

- (A) (I)- direct (II)- inverse
- (B) (I)- direct (II)- direct
- (C) (I)-inverse (II)-direct
- (D) (I)-inverse (II)-inverse

Correct Answer: (A) (I)- direct (II)- inverse

Solution:**Step 1: Understanding the Concept**

Bond length is the average distance between the nuclei of two bonded atoms. Two things control it: how big the atoms are and how many electron pairs are shared between them.

Step 2: Part I: bond length and size of atom

Bond length is roughly the sum of the covalent radii of the two atoms. If the atoms become bigger, the sum becomes bigger. So bond length is directly proportional to atomic size. For example, the H-I bond is longer than the H-F bond because iodine is much larger than fluorine.

Step 3: Part II: bond length and multiplicity

Multiplicity means single, double or triple bond. More shared electron pairs pull the nuclei closer together. So bond length falls as multiplicity rises: C-C is 154 pm, C=C is 134 pm and the triple bond is 120 pm. This is an inverse relationship.

Step 4: Compare with options

Option (A) gives I direct and II inverse, which matches both facts. Options (B), (C) and (D) each reverse at least one relationship, so they are wrong.

Final Answer:

Bond length is directly related to atomic size and inversely related to bond multiplicity, option (A).

(I) direct, (II) inverse

Quick Tip: Larger atoms form longer bonds; more bonds between two atoms pull them closer.

54. A mixture of 0.5 mol N_2 gas, 1.0 mol O_2 gas and 1.5 mol H_2 gas exerts a total pressure of 18 bar. Find the partial pressure of $\text{O}_{2(g)}$.

- (A) 4.0 bar
- (B) 6.0 bar
- (C) 8.0 bar
- (D) 10.0 bar

Correct Answer: (B) 6.0 bar

Solution:

Step 1: Understanding the Concept

Dalton's law says that each gas in a mixture exerts a partial pressure equal to its mole fraction multiplied by the total pressure.

Step 2: Key Formula or Approach

$$p_i = x_i P_{\text{total}}, \quad x_i = \frac{n_i}{n_{\text{total}}}$$

Step 3: Total moles

$$n_{\text{total}} = 0.5 + 1.0 + 1.5 = 3.0 \text{ mol}$$

Step 4: Mole fraction of oxygen

$$x_{\text{O}_2} = \frac{1.0}{3.0} = \frac{1}{3}$$

Step 5: Partial pressure of oxygen

$$p_{\text{O}_2} = \frac{1}{3} \times 18 = 6.0 \text{ bar}$$

Step 6: Check the other options

The partial pressure of N_2 would be $\frac{0.5}{3} \times 18 = 3 \text{ bar}$ and that of H_2 would be 9 bar. The three values add up to 18 bar, as Dalton's law demands. The values 4, 8 and 10 bar do not match any gas in the

mixture.

Final Answer:

The partial pressure of oxygen is 6.0 bar, option (B).

6.0 bar

Quick Tip: Partial pressure equals mole fraction times total pressure.



55. The enthalpy of combustion of C, H₂, and CH₄ are -390 , -285 and -890 kJ/mol. Find the enthalpy of formation of methane.

- (A) -70 kJ
- (B) -111 kJ
- (C) -170 kJ
- (D) -85 kJ

Correct Answer: (A) -70 kJ

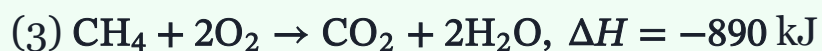
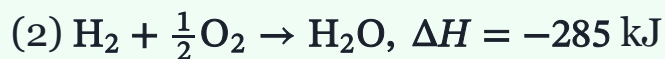
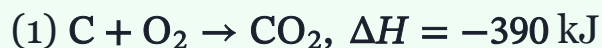
Solution:

Step 1: Understanding the Concept

Enthalpy is a state function, so we can combine known reactions (Hess's law) to get the enthalpy of the target reaction.

Step 2: Target reaction



Step 3: Given combustion data**Step 4: Combine them**

Take (1) as it is, take (2) twice, and reverse (3):

$$\Delta_f H = \Delta H_1 + 2\Delta H_2 - \Delta H_3$$

$$\Delta_f H = -390 + 2(-285) - (-890) = -390 - 570 + 890$$

$$\Delta_f H = -70 \text{ kJ mol}^{-1}$$

Step 5: Check

Adding the three equations cancels CO_2 , H_2O and O_2 and leaves exactly the target reaction, so the answer is -70 kJ , option (A).

Final Answer:

The enthalpy of formation of methane is -70 kJ per mole, option (A).

-70 kJ

Quick Tip: Write the formation reaction of methane and combine the three combustion equations with Hess law.

56. Calculate the change in internal energy if an ideal gas expands from 0.5dm^3 to 3.5dm^3 at constant pressure 2 bar by absorbing 1800 J heat energy.

- (A) 1200 J.
- (B) -1200 J.
- (C) 2400 J.
- (D) -2400 J.

Correct Answer: (A) 1200 J.

Solution:

Step 1: Understanding the Concept

The first law of thermodynamics is $\Delta U = q + w$, where w is work done on the system. For expansion against constant pressure, work is done by the gas, so w is negative.

Step 2: Key Formula or Approach

$$w = -P_{\text{ext}}\Delta V, \quad \Delta U = q + w$$

Step 3: Work done

$$\Delta V = 3.5 - 0.5 = 3.0 \text{ dm}^3$$

$$w = -2 \text{ bar} \times 3.0 \text{ dm}^3 = -6 \text{ bar dm}^3$$

Since $1 \text{ bar dm}^3 = 100 \text{ J}$, this gives $w = -600 \text{ J}$.

Step 4: Apply the first law

The system absorbs heat, so $q = +1800 \text{ J}$.

$$\Delta U = 1800 + (-600) = 1200 \text{ J}$$

Step 5: Why the others are wrong

Option (B) has the wrong sign. Options (C) and (D) come from adding the 600 J incorrectly or from doubling the answer. Only +1200 J follows from the first law.

Final Answer:

The change in internal energy is +1200 J, option (A).

1200 J

Quick Tip: Find the work from $-P$ times the volume change, then use the first law.

57. Which of the following processes exhibits the decrease in internal energy of a system?

- (A) Isothermal expansion of an ideal gas.
- (B) Isothrmal compression of an ideal gas.
- (C) Adiabatic expansion of an ideal gas.
- (D) Adiabatic compression of an ideal gas

Correct Answer: (C) Adiabatic expansion of an ideal gas.

Solution:

Step 1: Understanding the Concept

The first law gives $\Delta U = q + w$. For an ideal gas the internal energy depends only on temperature, so ΔU is negative only if the temperature falls.

Step 2: Check option (A): isothermal expansion

Temperature stays constant, so $\Delta U = 0$ for an ideal gas. There is no decrease.

Step 3: Check option (B): isothermal compression

Again the temperature is constant, so $\Delta U = 0$.

Step 4: Check option (C): adiabatic expansion

In an adiabatic process $q = 0$, so $\Delta U = w$. The gas expands and does work on the surroundings, which makes w negative. So $\Delta U < 0$, and the gas cools. This is a decrease in internal energy.

Step 5: Check option (D): adiabatic compression

Here the surroundings do work on the gas, so w is positive and ΔU is positive. The gas warms up.

Final Answer:

Only the adiabatic expansion of an ideal gas lowers its internal energy, option (C).

Adiabatic expansion

Quick Tip: In an adiabatic expansion no heat enters, so the gas does work

at the cost of its own energy.



58. There are two different solutions, A and B. The pH of solution A is 4. Find the pH of solution B having $[H^+]$ three times that of solution A.

- (A) 3.5229
- (B) 4.229
- (C) 3.229
- (D) 3.5223

Correct Answer: (A) 3.5229

Solution:

Step 1: Understanding the Concept

The pH is defined as $pH = -\log[H^+]$. A higher hydrogen ion concentration gives a lower pH.

Step 2: Solution A

$$pH_A = 4 \Rightarrow [H^+]_A = 10^{-4} \text{ M}$$

Step 3: Solution B

$$[H^+]_B = 3 \times 10^{-4} \text{ M}$$

$$pH_B = -\log(3 \times 10^{-4}) = 4 - \log 3$$

Using $\log 3 = 0.4771$:

$$\text{pH}_B = 4 - 0.4771 = 3.5229$$

Step 4: Check the options

The pH must be less than 4 because B is more acidic. That removes (B) 4.229. Option (C) 3.229 would need a larger rise in acidity than threefold. Option (D) 3.5223 has the wrong last digits. The correct value is 3.5229, option (A).

Final Answer:

The pH of solution B is 3.5229, option (A).

3.5229

Quick Tip: Use $\text{pH} = -\log[\text{H}^+]$ and note that tripling $[\text{H}^+]$ subtracts $\log 3$ from the pH.

59. Calculate the pH of buffer solution containing 0.1M CH_3COOH and 0.1 M CH_3COONa [$K_a = 1.8 \times 10^{-5}$]

- (A) 4.745
- (B) 5.741
- (C) 2.876
- (D) 7.001

Correct Answer: (A) 4.745

Solution:

Step 1: Understanding the Concept

A mixture of a weak acid and its salt is an acidic buffer. Its pH follows the Henderson-Hasselbalch equation.

Step 2: Key Formula or Approach

$$\text{pH} = \text{p}K_a + \log \frac{[\text{salt}]}{[\text{acid}]}$$

Step 3: Find pKa

$$\text{p}K_a = -\log(1.8 \times 10^{-5}) = 5 - \log 1.8 = 5 - 0.2553 = 4.745$$

Step 4: Concentration term

The salt and acid are both 0.1 M, so the ratio is 1 and $\log 1 = 0$.

Step 5: Result

$$\text{pH} = 4.745 + 0 = 4.745$$

Step 6: Why the other options fail

The pH of a buffer made from a weak acid must be close to its pKa, which is near 4.7. Option (D) 7.001 is neutral. Option (C) 2.876 is the pH of a plain 0.1 M acetic acid solution without salt. Option (B) 5.741 would need a salt to acid ratio of 10.

Final Answer:

The buffer has equal acid and salt, so pH equals pKa, which is 4.745, option (A).

4.745

Quick Tip: Use the Henderson-Hasselbalch equation; equal acid and salt concentrations give $\text{pH} = \text{pKa}$.

60. What is the pH of 1 molar HCl solution? (assume complete dissociation)

- (A) 1
- (B) 2
- (C) Zero
- (D) 7

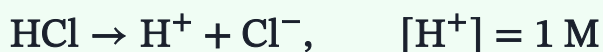
Correct Answer: (C) Zero

Solution:**Step 1: Understanding the Concept**

HCl is a strong acid. The question tells us to assume complete dissociation, so each mole of HCl gives one mole of H^+ .

Step 2: Key Formula or Approach

$$\text{pH} = -\log[\text{H}^+]$$

Step 3: Hydrogen ion concentration**Step 4: Calculation**

$$\text{pH} = -\log(1) = 0$$

Step 5: Why the other options are wrong

A pH of 1 needs 0.1 M acid and a pH of 2 needs 0.01 M acid. A pH of 7 belongs to pure water. Because 1 M is the unit concentration, the logarithm is zero. A pH of zero is perfectly allowed, since the pH scale is not limited to values between 0 and 14 in strong solutions.

Final Answer:

The pH of 1 M HCl is 0, option (C).

0

Quick Tip: Strong acid gives $[\text{H}^+]$ equal to its molarity; take the negative log.

61. What is the oxidation state of Xe in XeOF_4 ?

- (A) +4
- (B) -4
- (C) -6
- (D) +6

Correct Answer: (D) +6

Solution:

Step 1: Understanding the Concept

The oxidation numbers in a neutral molecule add to zero. Fluorine is always -1 in compounds, and oxygen is -2 here.

Step 2: Set up the equation

Let the oxidation state of Xe be x . The molecule XeOF_4 has one oxygen and four fluorine atoms.

$$x + (-2) + 4(-1) = 0$$

Step 3: Solve

$$x - 2 - 4 = 0 \Rightarrow x = +6$$

Step 4: Check the options

A value of +4 belongs to XeF_4 , which has no oxygen. The negative values -4 and -6 do not work because the sum of the oxidation numbers would then be far from zero. So Xe is in the +6 state, option (D).

Final Answer:

Xe is in the +6 oxidation state in XeOF_4 , option (D).

+6

Quick Tip: Fix O at -2 and F at -1, then make the molecule neutral.

62. What is the oxidation number of Cl in HOClO_2 ?

- (A) +7
- (B) +5
- (C) +3
- (D) +1

Correct Answer: (B) +5

Solution:**Step 1: Understanding the Concept**

HOClO_2 is the structural way of writing chloric acid, HClO_3 . Its atoms are one H, one Cl and three O.

Step 2: Key Formula or Approach

Sum of oxidation numbers = 0 for a neutral molecule. H is +1 and O is -2.

Step 3: Calculation

Let Cl be x :

$$(+1) + x + 3(-2) = 0$$

$$x = +5$$

Step 4: Check the other options

The value +7 belongs to perchloric acid HClO_4 . The value +3 belongs to chlorous acid HClO_2 , and +1 belongs to hypochlorous acid, HOCl . Our molecule has three oxygen atoms, so chlorine is +5, option (B).

Final Answer:

Chlorine in HOClO_2 has oxidation number +5, option (B).

+5

Quick Tip: Rewrite HOClO_2 as HClO_3 and balance the charges.

63. What is the number of electrons gained by Cl atom when n mole of ClO_4^- is transformed into n mole of ClO_2^- ion?

- (A) $3n$
- (B) $4n$
- (C) n
- (D) $2n$

Correct Answer: (B) $4n$

Solution:

Step 1: Understanding the Concept

The number of electrons gained by an atom equals the fall in its oxidation number. We calculate the oxidation number of chlorine in both ions.

Step 2: Chlorine in ClO_4^-

$$x + 4(-2) = -1 \Rightarrow x = +7$$

Step 3: Chlorine in ClO_2^-

$$y + 2(-2) = -1 \Rightarrow y = +3$$

Step 4: Electrons per Cl atom

+7 $\rightarrow$ +3 means 4 electrons gained per atom

Step 5: For n moles

One mole of Cl atoms gains 4 moles of electrons. So n mol of ClO_4^- gain $4n$ mol of electrons.

Step 6: Check the options

The other options $3n$, n and $2n$ come from wrong oxidation numbers such as +6 or +5 for the product. The correct answer is option (B).

Final Answer:

Each Cl atom goes from +7 to +3, gaining 4 electrons, so n mol gain $4n$ electrons, option (B).

$$4n$$

Quick Tip: Find the oxidation number of Cl in each ion and take the difference.

64. Choose a false statement regarding the properties of sodium hydroxide.

- (A) It is highly water soluble.
- (B) It absorbs atmospheric CO_2 to form Na_2CO_3 .
- (C) the solution of NaOH turns blue litmus red.
- (D) NaOH is a white deliquescent solid.

Correct Answer: (C) the solution of NaOH turns blue litmus red.

Solution:

Step 1: Understanding the Concept

We must find the statement that is not true. Sodium hydroxide is a strong base and a white solid, so we check each statement against that.

Step 2: Check statement (A)

NaOH dissolves very easily in water and gives out heat while dissolving. So (A) is true.

Step 3: Check statement (B)

NaOH absorbs CO₂ from air and forms sodium carbonate:



So (B) is true.

Step 4: Check statement (C)

A basic solution turns red litmus blue. Blue litmus turns red only in an acidic solution. NaOH solution is strongly basic, so it cannot turn blue litmus red. So (C) is FALSE.

Step 5: Check statement (D)

NaOH is a white, translucent, deliquescent solid that absorbs moisture from air. So (D) is true.

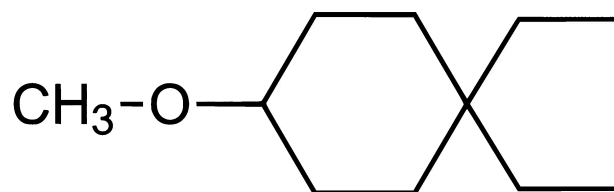
Final Answer:

Statement (C) is false because a basic NaOH solution does not turn blue litmus red.

(C)

Quick Tip: NaOH is a strong base, so its solution turns red litmus blue, not blue litmus red.

65. The IUPAC name of the following compound is,



- (A) 1, 1 - diethyl - 4 - methoxycyclohexane
- (B) 1, 1 - dimethyl - 4 - methoxycyclohexane
- (C) 4, 4 - dimethyl - 1 - methoxycyclohexane
- (D) 4, 4 - Diethyl - 1 - methoxycyclohexane

Correct Answer: (A) 1, 1 - diethyl - 4 - methoxycyclohexane

Solution:

Step 1: Understanding the Concept

The figure shows a six-membered saturated ring (cyclohexane). One ring carbon carries a CH_3O group (methoxy). The carbon on the opposite side of the ring carries two ethyl groups, drawn as two two-carbon chains.

Step 2: Key Formula or Approach

Ethers are named as alkoxy-alkanes. The ring is the parent. Number the ring so that the substituents get the lowest locants, then write the names alphabetically.

Step 3: Locants

The two ethyl groups sit on one carbon and the methoxy is on the carbon directly across the ring (position 4 from it). Numbering from the diethyl carbon gives locants 1,1,4. Numbering from the methoxy

carbon gives 1,4,4. The set 1,1,4 is lower at the first point of difference.

Step 4: Alphabetical order

"Diethyl" is alphabetised under e and "methoxy" under m, so ethyl comes first.

1,1-diethyl-4-methoxycyclohexane

Step 5: Eliminate others

Options (B) and (C) say dimethyl, but the groups drawn have two carbons each. Option (D) numbers from the wrong carbon.

Final Answer:

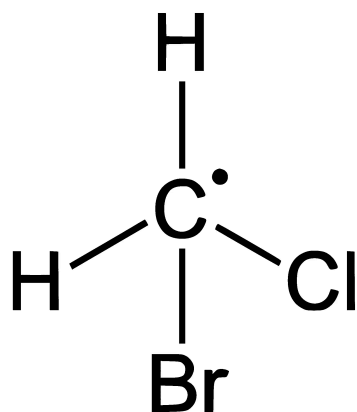
The compound is 1,1-diethyl-4-methoxycyclohexane, option (A).

1,1-diethyl-4-methoxycyclohexane

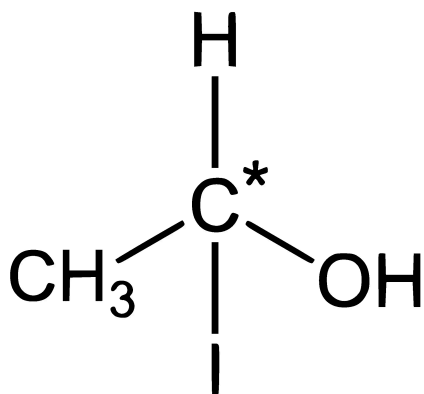
Quick Tip: Pick the ring as parent, number from the carbon bearing the ethyl groups, then list substituents alphabetically.

66. Which of the following molecule has Chiral Carbon?

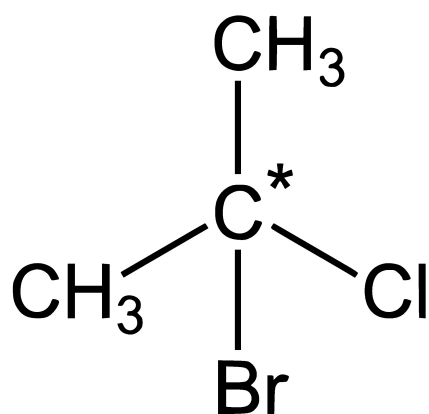
(A)



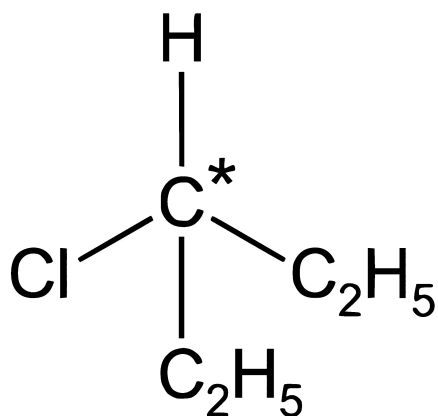
(B)



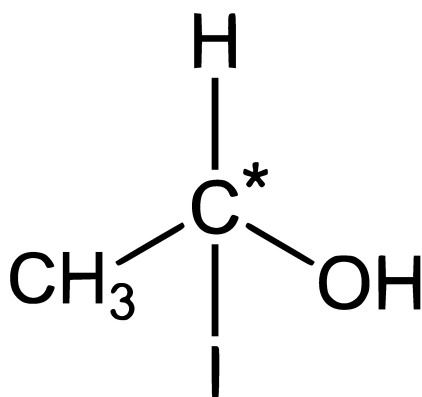
(C)



(D)



Correct Answer: (B)



Solution:

Step 1: Understanding the Concept

A carbon atom is chiral when it is attached to four different atoms or groups. If any two of its four groups are the same, it is not chiral.

Step 2: Check option (A)

The starred carbon in the first figure carries H, H, Cl and Br and also an unpaired electron dot. It has two identical hydrogens, so it is not a chiral centre.

Step 3: Check option (B)

The starred carbon holds H, CH₃, OH and I. All four are different, so it is a chiral carbon.

Step 4: Check option (C)

The starred carbon holds two CH₃ groups, plus Cl and Br. Two identical groups, so it is not chiral.

Step 5: Check option (D)

The starred carbon holds H, Cl and two C₂H₅ groups. Two identical ethyl groups, so it is not chiral.

Final Answer:

Only the carbon with H, methyl, OH and iodine has four different groups, option (B).

(B)

Quick Tip: A chiral carbon needs four different groups; check each drawn

carbon for repeated groups.



67. Which one of the following series contains *only* electrophiles?

- (A) R-OH , Cl^+ , NH_3
- (B) H_2O , R_2N^- , H_3O^+
- (C) BF_3 , SO_3 , NO^+
- (D) AlCl_3 , R-NH_2 , PCl_3

Correct Answer: (C) BF_3 , SO_3 , NO^+

Solution:

Step 1: Understanding the Concept

An electrophile is an electron-loving species. It is either positively charged or has an incomplete octet, so it accepts a pair of electrons.

Step 2: Check option (A)

R-OH and NH_3 have lone pairs and act as nucleophiles. Only Cl^+ is an electrophile. So the series is mixed.

Step 3: Check option (B)

H_2O and R_2N^- are nucleophiles because they have lone pairs. Only H_3O^+ is an electrophile.

Step 4: Check option (C)

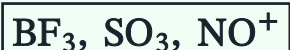
BF_3 has an incomplete octet on boron. SO_3 has an electron-poor sulfur. NO^+ is a cation. All three accept electron pairs, so all are electrophiles.

Step 5: Check option (D)

AlCl_3 is an electrophile but R-NH_2 is a nucleophile through the nitrogen lone pair. PCl_3 also has a lone pair on phosphorus.

Final Answer:

BF_3 , SO_3 and NO^+ are all electron-deficient, so the answer is option (C).



Quick Tip: Electrophiles are electron-deficient species that accept an electron pair.

68. Which of the following reaction steps is known as propagation step

- (A) $\text{Cl-Cl} \rightarrow \text{Cl}^\bullet + \text{Cl}^\bullet$
- (B) $\text{CH}_3 + \text{Cl}_2 \rightarrow \text{CH}_3\text{-Cl} + \text{Cl}^\bullet$
- (C) $\text{CH}_3^\bullet + \text{Cl}^\bullet \rightarrow \text{CH}_3\text{-Cl}$
- (D) $\text{CH}_3^\bullet + \text{CH}_3^\bullet \rightarrow \text{CH}_3\text{-CH}_3$

Correct Answer: (B) $\text{CH}_3 + \text{Cl}_2 \rightarrow \text{CH}_3\text{-Cl} + \text{Cl}^\bullet$

Solution:

Step 1: Understanding the Concept

A free radical chain reaction has three stages: initiation, propagation and termination. In propagation a radical reacts with a molecule and forms a new radical, so the chain keeps going.

Step 2: Check option (A)

$\text{Cl-Cl} \rightarrow 2\text{Cl}^\bullet$ makes radicals from a molecule using light. This is the

initiation step.

Step 3: Check option (B)

A methyl radical reacts with a chlorine molecule to give CH_3Cl and a new chlorine radical. One radical is used up and another is made. This is a propagation step.

Step 4: Check option (C)

$\text{CH}_3\cdot + \text{Cl}\cdot \rightarrow \text{CH}_3\text{Cl}$ joins two radicals into a stable molecule. No radical is left, so this is termination.

Step 5: Check option (D)

Two methyl radicals form ethane. Again no radical remains, so it is termination.

Final Answer:

Step (B) uses one radical and creates another, so it is the propagation step.

(B)

Quick Tip: A propagation step uses one radical and makes a new radical so the chain continues.

69. A metal crystallizes in a bcc unit cell having unit cell volume $2.5 \times 10^{-23} \text{cm}^3$. Find the number of unit cells in 18 g metal if the density of metal is 7.2g cm^{-3} .

- (A) 1.0×10^{23}
- (B) 2.0×10^{23}
- (C) 0.5×10^{23}
- (D) 1.5×10^{23}

Correct Answer: (A) 1.0×10^{23}

Solution:

Step 1: Understanding the Concept

The volume of the whole sample is mass divided by density. Dividing this by the volume of a single unit cell gives the number of unit cells. The bcc detail is not needed for this count.

Step 2: Key Formula or Approach

$$V = \frac{m}{\rho}, \quad N_{\text{cells}} = \frac{V}{V_{\text{cell}}}$$

Step 3: Volume of the sample

$$V = \frac{18}{7.2} = 2.5 \text{ cm}^3$$

Step 4: Number of unit cells

$$N = \frac{2.5}{2.5 \times 10^{-23}} = 1.0 \times 10^{23}$$

Step 5: Why this works

Unit cells fill space with no gaps, so the sample volume is the number of cells times the volume of one cell. The bcc fact only matters if we want atoms, which would be twice this number, 2.0×10^{23} , option (B).

Final Answer:

The sample contains 1.0×10^{23} unit cells, option (A).

$$1.0 \times 10^{23}$$

Quick Tip: Find the volume of the sample from the density, then divide by the volume of one unit cell.



70. A metal has a fcc structure if the edge length of unit cell is 200 pm, calculate the volume occupied by particles in a unit cell.

- (A) $2.08 \times 10^{-24} \text{cm}^3$
- (B) $5.92 \times 10^{-24} \text{cm}^3$
- (C) $8 \times 10^{-24} \text{cm}^3$
- (D) $3.14 \times 10^{-24} \text{cm}^3$

Correct Answer: (B) $5.92 \times 10^{-24} \text{cm}^3$

Solution:

Step 1: Understanding the Concept

In a face-centred cubic (fcc) cell the atoms touch along the face diagonal. There are 4 atoms per unit cell. The volume occupied equals 4 times the volume of one sphere.

Step 2: Key Formula or Approach

$$\sqrt{2}a = 4r \Rightarrow r = \frac{\sqrt{2}a}{4}$$

Step 3: Radius

$$r = \frac{1.414 \times 200}{4} = 70.7 \text{ pm} = 7.07 \times 10^{-9} \text{ cm}$$

Step 4: Volume of one atom

$$V_1 = \frac{4}{3}\pi r^3 = \frac{4}{3}(3.1416)(7.07 \times 10^{-9})^3 = 1.48 \times 10^{-24} \text{ cm}^3$$

Step 5: Volume of four atoms

$$V = 4 \times 1.48 \times 10^{-24} = 5.92 \times 10^{-24} \text{ cm}^3$$

Step 6: Check

The cell volume is $a^3 = (2 \times 10^{-8})^3 = 8 \times 10^{-24} \text{ cm}^3$, which is option (C). The fraction occupied is $5.92/8 = 0.74$, the known packing efficiency of fcc, so the result is consistent.

Final Answer:

The four atoms in the fcc cell fill $5.92 \times 10^{-24} \text{ cm}^3$, option (B).

$5.92 \times 10^{-24} \text{ cm}^3$

Quick Tip: Use $r = \frac{a\sqrt{2}}{4}$ for fcc, then multiply the volume of one sphere by 4 atoms per cell.



71. Which of the following defects is observed in steel?

- (A) Substitutional impurity defect
- (B) Interstitial impurity defect
- (C) Vacancy defect
- (D) Schottky defect

Correct Answer: (B) Interstitial impurity defect

Solution:

Step 1: Understanding the Concept

Steel is iron with a small amount of carbon. Carbon atoms are much smaller than iron atoms, so they can fit into the empty spaces (interstices) of the iron lattice.

Step 2: What kind of defect is this

When a foreign atom sits in a void of the crystal instead of replacing a host atom, the defect is called an interstitial impurity defect.

Step 3: Check the options

- (A) Substitutional impurity defect: the foreign atom takes the place of a host atom, as in brass. Not the case for carbon in steel.
- (B) Interstitial impurity defect: the small carbon atoms occupy gaps in the iron lattice. This is correct.
- (C) Vacancy defect: a lattice site is empty. This is a different defect.
- (D) Schottky defect: equal numbers of cations and anions are missing,

seen in ionic solids like NaCl.

Final Answer:

Carbon atoms occupy gaps in the iron lattice, an interstitial impurity defect, option (B).

Interstitial impurity defect

Quick Tip: Carbon atoms sit in the gaps between iron atoms in steel.



72. When the bottle of soft drink is opened, what would take place from the following?

- (A) Solubility of dissolved gas decreases.
- (B) bottle released internal pressure.
- (C) effervescences evolved from the bottle.
- (D) increase in external pressure.

Identify the correct choice.

- (A) (A), (B) and (C) only.
- (B) (A), (B) and (D) only.
- (C) (A), (B), (C) and (D).
- (D) (B), (C) and (D) only.

Correct Answer: (A) (A), (B) and (C) only.

Solution:

Step 1: Understanding the Concept

A soft drink is made by dissolving CO_2 in water under high pressure. By Henry's law the solubility of a gas rises with its pressure.

Step 2: Check statement (A)

When the cap is opened, the pressure above the liquid falls to atmospheric. So the solubility of the dissolved gas decreases. True.

Step 3: Check statement (B)

The gas trapped above the liquid rushes out, so the internal pressure is released. True.

Step 4: Check statement (C)

Because the gas is no longer soluble at the lower pressure, it comes out as bubbles. This is effervescence. True.

Step 5: Check statement (D)

Opening the cap does not change the outside atmospheric pressure. It is the internal pressure that falls. So (D) does not happen.

Step 6: Conclusion

The correct choice is (A), (B) and (C) only, option (A).

Final Answer:

Statements (A), (B) and (C) occur, while (D) does not, so the answer is option (A).

(A), (B) and (C) only

Quick Tip: Henry law: opening the bottle drops the pressure above the liquid, and the gas leaves the solution.

73. Find the osmotic pressure of 0.01 M solution at 25 °C ($R = 0.0821 \text{ L atm mol}^{-1} \text{ K}^{-1}$)

- (A) 0.201 atm
- (B) 0.245 atm
- (C) 0.082 atm
- (D) 0.41 atm

Correct Answer: (B) 0.245 atm

Solution:

Step 1: Understanding the Concept

Osmotic pressure of a dilute solution follows the van 't Hoff relation, which looks like the ideal gas equation.

Step 2: Key Formula or Approach

$$\pi = CRT$$

where C is molarity, $R = 0.0821 \text{ L atm mol}^{-1} \text{ K}^{-1}$ and T is in kelvin.

Step 3: Convert temperature

$$T = 25 + 273 = 298 \text{ K}$$

Step 4: Calculation

$$\pi = 0.01 \times 0.0821 \times 298 = 0.2447 \text{ atm} \approx 0.245 \text{ atm}$$

Step 5: Check the options

Option (C) 0.082 comes from forgetting T. Option (D) 0.41 and (A) 0.201 do not follow from any correct use of the formula. The right value is 0.245 atm, option (B).

Final Answer:

The osmotic pressure is about 0.245 atm, option (B).

0.245 atm

Quick Tip: Apply $\pi = CRT$ with T converted to kelvin.

74. Identify the colligative property from the followings?

- (A) Vapour pressure of solution.
- (B) Osmotic pressure of solution.
- (C) Boiling point of solution.
- (D) Freezing point of solution.

Correct Answer: (B) Osmotic pressure of solution.

Solution:**Step 1: Understanding the Concept**

Colligative properties depend only on the number of solute particles dissolved, not on their nature. The four standard colligative properties are relative lowering of vapour pressure, elevation of boiling point, depression of freezing point and osmotic pressure.

Step 2: Why osmotic pressure fits

Osmotic pressure is given by $\pi = CRT$. It depends on the concentration of particles and the temperature, and not on what the solute is. It is a colligative property in its own right.

Step 3: The other options

Vapour pressure of a solution, its boiling point and its freezing point are properties of the solution that change when solute is added. The colligative quantities are the relative lowering of vapour pressure, the elevation of boiling point and the depression of freezing point, which are changes caused by the solute.

Step 4: Conclusion

Among the options as stated, osmotic pressure is the colligative property, option (B).

Final Answer:

Osmotic pressure depends only on the number of dissolved particles, option (B).

Osmotic pressure

Quick Tip: Osmotic pressure is itself a colligative property; the others change as lowering, elevation or depression.

75. Determine the electrode potential of $\text{Sn}^{2+}(0.01\text{M})|\text{Sn(s)}$ at 25°C if E°_{Sn} is -0.136 V .

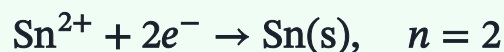
- (A) 0.186 V
- (B) -0.186 V
- (C) -0.195 V
- (D) 0.195 V

Correct Answer: (C) -0.195 V

Solution:

Step 1: Understanding the Concept

The electrode potential changes with ion concentration according to the Nernst equation. The electrode reaction is a reduction:



Step 2: Key Formula or Approach

$$E = E^{\circ} - \frac{0.0591}{n} \log \frac{1}{[\text{Sn}^{2+}]}$$

Step 3: Substitute

$$E = -0.136 - \frac{0.0591}{2} \log \frac{1}{0.01}$$

$$\log \frac{1}{0.01} = \log 100 = 2$$

$$E = -0.136 - 0.0591 = -0.1951 \text{ V}$$

Step 4: Check the options

The dilute solution makes the reduction less favourable, so the potential is lower than the standard value of -0.136 V . Options (A) and (D) are positive and so cannot follow. Option (B) -0.186 V is not given by the equation. The answer is -0.195 V , option (C).

Final Answer:

The electrode potential is -0.195 V , option (C).

$$\boxed{-0.195\text{ V}}$$

Quick Tip: Use the Nernst equation for a metal in its own ion; the reaction is $\text{Sn}^{2+} + 2\text{e}^-$ going to Sn .



76. If molar conductance at infinite dilution for CH_3COO^- and H^+ ions are $50\text{ S cm}^2\text{mol}^{-1}$ and $350\text{ S cm}^2\text{mol}^{-1}$ respectively, molar conductivity of $5 \times 10^{-2}\text{ M}$ CH_3COOH is $20\text{ S cm}^2\text{mol}^{-1}$. What is the hydrogen ion concentration in mol/dm^3 of CH_3COOH ?

- (A) 5×10^{-2}
- (B) 5×10^{-3}
- (C) 2.5×10^{-3}
- (D) 2.5×10^{-4}

Correct Answer: (C) 2.5×10^{-3}

Solution:

Step 1: Understanding the Concept

For a weak electrolyte the degree of dissociation is the ratio of molar conductivity at the given concentration to the molar conductivity at infinite dilution.

Step 2: Key Formula or Approach

$$\alpha = \frac{\Lambda_m}{\Lambda_m^\circ}, \quad [\text{H}^+] = c\alpha$$

Step 3: Limiting molar conductivity

By Kohlrausch's law:

$$\Lambda_m^\circ(\text{CH}_3\text{COOH}) = \lambda^\circ(\text{CH}_3\text{COO}^-) + \lambda^\circ(\text{H}^+) = 50 + 350 = 400 \text{ S cm}^2\text{mol}^{-1}$$

Step 4: Degree of dissociation

$$\alpha = \frac{20}{400} = 0.05$$

Step 5: Hydrogen ion concentration

$$[\text{H}^+] = 5 \times 10^{-2} \times 0.05 = 2.5 \times 10^{-3} \text{ mol dm}^{-3}$$

Step 6: Check the options

The value 5×10^{-2} is the acid concentration, which would mean 100 percent dissociation. The option 5×10^{-3} would need $\alpha = 0.1$. The

answer is option (C).

Final Answer:

Only 5 percent of the acid ionises, giving $[H^+] = 2.5 \times 10^{-3} \text{ mol/dm}^3$, option (C).

$$2.5 \times 10^{-3}$$

Quick Tip: Find the limiting molar conductivity first, then the degree of dissociation, then $[H^+] = c \text{ times } \alpha$.



77. Which of the following expressions indicates the correct relationship between molar conductivity of strong electrolyte and its concentration?

- (A) $\Lambda = \Lambda_0 + \sqrt{c}$
- (B) $\Lambda = \Lambda_0 - a\sqrt{c}$
- (C) $\Lambda = \Lambda_0 + a\sqrt{c}$
- (D) $\Lambda = \Lambda_0 - \sqrt{c}$

Correct Answer: (B) $\Lambda = \Lambda_0 - a\sqrt{c}$

Solution:

Step 1: Understanding the Concept

For a strong electrolyte, the molar conductivity increases slightly as the solution is diluted, because ion-ion attractions weaken.

Step 2: Key Formula or Approach

Debye-Huckel-Onsager (Kohlrausch) equation:

$$\Lambda = \Lambda^\circ - a\sqrt{c}$$

Here Λ° is the limiting molar conductivity, c is the concentration and a is a constant that depends on the solvent, temperature and type of electrolyte.

Step 3: Check the options

Options (A) and (C) have a plus sign, which would make conductivity rise with concentration. This is the opposite of what is seen. Option (D) is missing the constant a , which carries the dependence on the type of electrolyte. Option (B) has the minus sign and the constant, so it is correct.

Step 4: Graph meaning

A plot of Λ against $\sqrt{c}$ is a straight line with intercept Λ° and slope $-a$.

Final Answer:

The correct relation is $\Lambda = \Lambda_0 - a\sqrt{c}$, option (B).

$$\Lambda = \Lambda_0 - a\sqrt{c}$$

Quick Tip: Debye-Huckel-Onsager equation: molar conductivity falls linearly with the square root of concentration.

78. What is the role of catalyst in reaction?

- (A) To increase activation energy of a reaction
- (B) To decrease the equilibrium constant of the reaction
- (C) To supply energy to the reactants
- (D) To decrease activation energy of a reaction

Correct Answer: (D) To decrease activation energy of a reaction

Solution:

Step 1: Understanding the Concept

A catalyst speeds up a reaction without being used up. It does this by providing a different reaction path with a lower activation energy.

Step 2: Check option (A)

Raising the activation energy would slow the reaction. A catalyst does the opposite. Wrong.

Step 3: Check option (B)

A catalyst does not change the equilibrium constant. It speeds up the forward and backward reactions equally. Wrong.

Step 4: Check option (C)

A catalyst does not supply energy to the reactants. It just makes it easier to cross the energy barrier. Wrong.

Step 5: Check option (D)

Lowering the activation energy lets a larger fraction of molecules have enough energy to react. By the Arrhenius equation $k = Ae^{-E_a/RT}$, the rate constant then rises. Correct.

Final Answer:

A catalyst lowers the activation energy, option (D).

Decreases the activation energy

Quick Tip: A catalyst opens an alternative path with a lower energy barrier.

79. Calculate the rate constant of first order reaction $A \rightarrow B$ having rate $5.4 \times 10^{-6} \text{ mol dm}^{-3} \text{ s}^{-1}$ and $[A] = 0.3 \text{ M}$.

(A) $1.52 \times 10^{-5} \text{ s}^{-1}$

(B) $1.68 \times 10^{-5} \text{ s}^{-1}$

(C) $1.72 \times 10^{-5} \text{ s}^{-1}$

(D) $1.80 \times 10^{-5} \text{ s}^{-1}$

Correct Answer: (D) $1.80 \times 10^{-5} \text{ s}^{-1}$

Solution:

Step 1: Understanding the Concept

For a first-order reaction the rate is proportional to the concentration of the reactant to the first power.

Step 2: Key Formula or Approach

$$\text{rate} = k[A]$$

Step 3: Substitute

$$k = \frac{\text{rate}}{[A]} = \frac{5.4 \times 10^{-6} \text{ mol dm}^{-3} \text{ s}^{-1}}{0.3 \text{ mol dm}^{-3}}$$

Step 4: Calculate

$$k = 1.8 \times 10^{-5} \text{ s}^{-1}$$

Step 5: Units check

The unit of k for a first-order reaction is s^{-1} , matching the options. The other values, 1.52, 1.68 and 1.72 (all times 10^{-5}), would come from a different rate or concentration.

Final Answer:

The rate constant is $1.8 \times 10^{-5} \text{ s}^{-1}$, option (D).

$$1.80 \times 10^{-5} \text{ s}^{-1}$$

Quick Tip: For a first-order reaction, $\text{rate} = k[\text{A}]$.

80. What is the order and molecularity of the following elementary reaction?



- (A) The reaction is second order and bimolecular.
- (B) The reaction is first order and bimolecular.
- (C) The reaction is second order and unimolecular.
- (D) The reaction is zero order and unimolecular.

Correct Answer: (A) The reaction is second order and bimolecular.

Solution:**Step 1: Understanding the Concept**

Order is the sum of the powers of concentrations in the rate law. Molecularity is the number of reacting molecules colliding in an

elementary step.

Step 2: Order

The rate is $k[\text{NO}_2]^2$, so the order is 2 (second order).

Step 3: Molecularity

The reaction is stated to be elementary. Two NO_2 molecules collide, so the molecularity is 2. A reaction with molecularity two is called bimolecular (the option spells it "biomolecular").

Step 4: Check the options

(B) says first order, which is wrong because the power is 2. (C) says unimolecular, which is wrong because two molecules take part. (D) gives zero order, which is wrong. Only (A) has second order and bimolecular.

Final Answer:

The reaction is second order and bimolecular, option (A).

Second order, bimolecular

Quick Tip: For an elementary step, molecularity is the number of molecules colliding; order is the sum of powers in the rate law.

81. Which of the following nanomaterials has one dimension < 100 nm?

- (A) Quantum dots
- (B) Nanorings
- (C) Thin films
- (D) Nanowires

Correct Answer: (C) Thin films

Solution:

Step 1: Understanding the Concept

Nanomaterials are classified by how many of their dimensions fall in the nanoscale (below 100 nm). Zero-, one-, two- and three-dimensional confinement give different shapes.

Step 2: Classification

- Quantum dots are confined in all three dimensions, so all three are below 100 nm (zero-dimensional objects).
- Nanowires and nanorods have two dimensions (the cross-section) below 100 nm while the length is large.
- Nanorings are small closed loops with all dimensions in the nanometre range.
- Thin films have only the thickness below 100 nm, while length and width are large.

Step 3: Match the options

(A) Quantum dots: three nano dimensions. (B) Nanorings: all nano.
(D) Nanowires: two nano dimensions. (C) Thin films: exactly one nano dimension, the thickness.

Step 4: Conclusion

The material with only one dimension below 100 nm is the thin film,

option (C).

Final Answer:

A thin film is nanoscale only in its thickness, so the answer is option (C).

Thin films

Quick Tip: Count how many dimensions are confined below 100 nm: thin films are confined in one only.

82. In Haber process of production of ammonia Al_2O_3 is used as

- (A) Adsorbate
- (B) Inhibitor
- (C) Catalyst
- (D) Promoter

Correct Answer: (D) Promoter

Solution:

Step 1: Understanding the Concept

In the Haber process, nitrogen and hydrogen combine to give ammonia over finely divided iron. Small amounts of other substances are added to make the iron catalyst work better.

Step 2: Key idea

A substance that is not a catalyst by itself but raises the activity of the catalyst is called a promoter.

Step 3: Check the options

- (A) Adsorbate: this is the substance that sticks to the surface, such as the reacting gases. Al_2O_3 is not a reactant.
- (B) Inhibitor: this slows a reaction. Al_2O_3 does not slow the process.
- (C) Catalyst: the catalyst is iron.
- (D) Promoter: Al_2O_3 (with K_2O) increases the efficiency of the iron catalyst. Correct.

Final Answer:

Aluminium oxide improves the iron catalyst, so it acts as a promoter, option (D).

Promoter

Quick Tip: The Haber catalyst is iron; Al_2O_3 and K_2O improve its activity.

83. Which of the following pairs of elements does NOT include transition elements?

- (A) Mn and Ag
- (B) Zr and Au
- (C) Mo and Pt
- (D) Sn and Pm

Correct Answer: (D) Sn and Pm

Solution:

Step 1: Understanding the Concept

Transition elements (d-block) have partly filled d orbitals in their atoms or in their common ions. They lie in groups 3 to 12 of the periodic table.

Step 2: Check the pairs

(A) Mn (group 7) and Ag (group 11) are both d-block elements.

(B) Zr (group 4) and Au (group 11) are both d-block elements.

(C) Mo (group 6) and Pt (group 10) are both d-block elements.

(D) Sn (group 14) is a p-block element. Pm (promethium) is a lanthanoid, an f-block element. Neither belongs to the d-block.

Step 3: Conclusion

The pair with no d-block element is Sn and Pm, option (D).

Final Answer:

Tin is a p-block metal and promethium is an f-block lanthanoid, so option (D) has no transition element.

Sn and Pm

Quick Tip: A transition element has a partly filled d subshell; tin is a p-block metal and promethium is an f-block element.



84. Which of the following prediction is correct when the transition metal ion is colourless?

- (A) it contains 0 unpaired electrons
- (B) it contains 1 unpaired electrons
- (C) it contains 2 unpaired electrons
- (D) it contains 3 unpaired electrons

Correct Answer: (A) it contains 0 unpaired electrons

Solution:

Step 1: Understanding the Concept

Transition metal ions are coloured because an electron can jump between split d orbitals by absorbing visible light (a d-d transition). For this jump an orbital in the d set must be able to take a paired or unpaired electron from a lower one.

Step 2: Link to unpaired electrons

If the d subshell is empty (d^0) or completely full (d^{10}), no d-d transition can occur. Such ions have zero unpaired electrons and appear colourless, for example Sc^{3+} (d^0) and Zn^{2+} (d^{10}).

Step 3: Check the options

(A) 0 unpaired electrons: consistent with a colourless ion. (B), (C) and (D) all have unpaired electrons, which usually leads to colour.

Step 4: Conclusion

The correct prediction is zero unpaired electrons, option (A).

Final Answer:

A colourless transition metal ion has no unpaired d electrons, option (A).

0 unpaired electrons

Quick Tip: Colour in transition ions comes from d-d transitions, which need unpaired d electrons.

85. Which of the following complex is neutral?

- (A) $\text{K}_3[\text{Fe}(\text{CN})_5\text{NO}]$
- (B) $[\text{Zn}(\text{NH}_3)_4]^{2+}$
- (C) $[\text{Co}(\text{NH}_3)_5\text{Cl}]^{2+}$
- (D) $[\text{Ni}(\text{CO})_4]$

Correct Answer: (D) $[\text{Ni}(\text{CO})_4]$

Solution:

Step 1: Understanding the Concept

A complex is neutral when the sum of the charge on the metal and the charges on the ligands equals zero, and no counter-ion is needed.

Step 2: Check option (A)

$\text{K}_3[\text{Fe}(\text{CN})_5\text{NO}]$ is a salt. The complex ion inside the bracket carries a charge of $3-$, balanced by three K^+ . So the complex ion is not neutral.

Step 3: Check option (B)

$[\text{Zn}(\text{NH}_3)_4]^{2+}$ has NH_3 neutral and Zn^{2+} , so the overall charge is $+2$.

Step 4: Check option (C)

$[\text{Co}(\text{NH}_3)_5\text{Cl}]^{2+}$: Co^{3+} plus Cl^- gives $+2$.

Step 5: Check option (D)

$[\text{Ni}(\text{CO})_4]$: carbon monoxide is neutral and nickel is in the zero oxidation state, so the complex has no charge. It is neutral.

Final Answer:

$[\text{Ni}(\text{CO})_4]$ has a neutral ligand and nickel in the zero state, so it is neutral, option (D).



Quick Tip: A complex is neutral when the oxidation state of the metal cancels the charge of the ligands.

86. Which of the following coordinate complexes is heteroleptic?

- (A) Pentacarbonyliron(0)
- (B) Triamminetrinitrocobalt(III)
- (C) Tetraamminecopper(II) ion
- (D) Tetracyanonickelate(II) ion

Correct Answer: (B) Triamminetrinitrocobalt(III)

Solution:

Step 1: Understanding the Concept

In a homoleptic complex the metal is bonded to only one kind of ligand. In a heteroleptic complex the metal is bonded to more than one kind of ligand.

Step 2: Check option (A)

Pentacarbonyliron(o): only CO ligands. Homoleptic.

Step 3: Check option (B)

Triamminetrinitrocobalt(III): three NH_3 ligands and three NO_2^- ligands. Two different ligands, so it is heteroleptic. Formula: $[\text{Co}(\text{NH}_3)_3(\text{NO}_2)_3]$.

Step 4: Check option (C)

Tetraamminecopper(II) ion: only ammine ligands. Homoleptic.

Step 5: Check option (D)

Tetracyanonickelate(II) ion: only cyanide ligands. Homoleptic.

Final Answer:

Triamminetrinitrocobalt(III) has two types of ligand, so it is heteroleptic, option (B).

Triamminetrinitrocobalt(III)

Quick Tip: A heteroleptic complex has more than one kind of ligand.

87. What is the major product formed when 2-bromopentane reacts with alcoholic KOH?

- (A) Pent-1-ene
- (B) Pent-2-ene
- (C) Pentane
- (D) Pentan-2-ol

Correct Answer: (B) Pent-2-ene

Solution:

Step 1: Understanding the Concept

Alcoholic KOH removes HBr from an alkyl halide (dehydrohalogenation, E2 elimination) and gives an alkene. Aqueous KOH would give substitution instead.

Step 2: Structure

2-Bromopentane is $\text{CH}_3\text{-CHBr-CH}_2\text{-CH}_2\text{-CH}_3$. The hydrogen can be lost from C1 or from C3.

Step 3: Two possible alkenes

Loss of H from C1 gives pent-1-ene, a terminal alkene with one alkyl group on the double bond. Loss of H from C3 gives pent-2-ene, an internal alkene with an alkyl group on each double-bond carbon.

Step 4: Saytzeff rule

The more substituted, more stable alkene is the major product. So pent-2-ene (mostly trans) is major.

Step 5: Other options

Pentane would need reduction, and pentan-2-ol needs aqueous KOH.

Final Answer:

By the Saytzeff rule the more substituted alkene, pent-2-ene, is the major product, option (B).

Pent-2-ene

Quick Tip: Alcoholic KOH causes dehydrohalogenation and the more substituted alkene forms (Saytzeff rule).

88. In Finkelstein reaction, which one of the following reagents is used?

- (A) HI in dry acetone
- (B) NaI in dry acetone
- (C) NaOI in dry acetone
- (D) NaI in dry ether

Correct Answer: (B) NaI in dry acetone

Solution:

Step 1: Understanding the Concept

The Finkelstein reaction prepares alkyl iodides from alkyl chlorides or bromides by halogen exchange.

Step 2: Reaction



Step 3: Why dry acetone

Sodium iodide dissolves in acetone but sodium chloride and sodium bromide do not. The salt that forms precipitates and leaves the solution, which pushes the equilibrium towards the alkyl iodide.

Step 4: Check the options

(A) HI is an acid and is not used here. (C) NaOI is a hypo-iodite and is

not a halide source. (D) NaI in dry ether is not the standard medium, since NaI is poorly soluble there. The correct reagent is NaI in dry acetone, option (B).

Final Answer:

The Finkelstein reaction uses NaI in dry acetone, option (B).

NaI in dry acetone

Quick Tip: The Finkelstein reaction swaps a halogen for iodine using NaI in dry acetone.

89. Which of the following alcohols contains the hydroxy group, at the side chain of the aromatic ring?

- (A) allylic alcohol
- (B) vinylic alcohol
- (C) aromatic alcohols
- (D) aliphatic alcohol

Correct Answer: (C) aromatic alcohols

Solution:

Step 1: Understanding the Concept

Alcohols are classified by the kind of carbon atom that holds the hydroxyl group. Aromatic alcohols have the OH on a side chain attached to the aromatic ring, for example benzyl alcohol, $\text{C}_6\text{H}_5\text{CH}_2\text{OH}$.

Step 2: Why option (C) fits

In benzyl alcohol the ring carries a CH_2 side chain and the OH is on that side chain carbon, not on the ring itself. So it is an aromatic alcohol.

Step 3: What the other types are

(A) Allylic alcohols have OH on a carbon next to a carbon-carbon double bond, as in $\text{CH}_2=\text{CH}-\text{CH}_2\text{OH}$.

(B) Vinylic alcohols have OH directly on a double-bonded carbon, as in $\text{CH}_2=\text{CH}-\text{OH}$.

(D) Aliphatic alcohols have OH on a saturated open-chain carbon, as in ethanol.

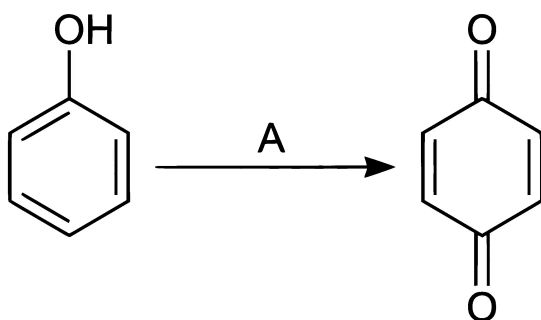
Final Answer:

An OH on the side chain of an aromatic ring defines an aromatic alcohol, option (C).

Aromatic alcohols

Quick Tip: Alcohols are grouped by the carbon that carries OH; benzylic OH on a side chain is classed as aromatic.

90. In the following reaction, condition 'A' is -



- (A) Reduction with Zinc dust
- (B) Dehydration with H_2SO_4 / 443 K
- (C) Treatment with CS_2 / low temperature
- (D) Oxidation with CrO_3

Correct Answer: (D) Oxidation with CrO_3

Solution:

Step 1: Understanding the Concept

The figure shows phenol, a benzene ring with one OH, turning into a ring with two C=O groups on opposite carbons. This product is p-benzoquinone, so the change is an oxidation.

Step 2: Check the options

- (A) Reduction with zinc dust removes the OH and gives benzene. It does not add oxygen.
- (B) Dehydration with H_2SO_4 at 443 K is used to turn alcohols into alkenes, not for phenols.
- (C) Treatment with CS_2 at low temperature is used for bromination in a non-polar solvent to give mono-bromophenols.
- (D) Oxidation with CrO_3 (chromic acid) converts phenol into p-benzoquinone, a conjugated cyclic diketone. Correct.

Final Answer:

Phenol is oxidised by chromic acid to p-benzoquinone, option (D).

Oxidation with CrO_3

Quick Tip: Phenol is oxidised to p-benzoquinone by chromic acid.

91. Which of the following compounds exhibits Schiff test?

- (A) Ketones
- (B) Carboxylic acids
- (C) Aldehydes
- (D) Ethers

Correct Answer: (C) Aldehydes

Solution:

Step 1: Understanding the Concept

Schiff's reagent is a decolourised solution of rosaniline hydrochloride (magenta) treated with sulfurous acid. It is used to detect aldehydes.

Step 2: The test

When an aldehyde is added, the colourless reagent turns pink or magenta within a short time.

Step 3: Check the options

- (A) Ketones do not restore the colour under the same conditions, so the test is negative.
- (B) Carboxylic acids do not react with Schiff's reagent.
- (C) Aldehydes give the pink colour. Correct.
- (D) Ethers are unreactive and give no colour.

Final Answer:

Only aldehydes restore the pink colour of Schiff's reagent, option (C).

Aldehydes

Quick Tip: Schiff reagent turns pink with aldehydes.

92. Which of the following reactions is preferentially used to prepare the corresponding paraffins from alkanone?

- (A) Clemmensen reduction
- (B) Etard reaction
- (C) Rosenmund reduction
- (D) Haloform reaction

Correct Answer: (A) Clemmensen reduction

Solution:

Step 1: Understanding the Concept

An alkanone is a ketone. Paraffins are alkanes. So we want to reduce the C=O group to CH₂.

Step 2: Clemmensen reduction



Step 3: Check the options

(A) Clemmensen reduction does exactly this, giving the alkane.

Correct.

(B) Etard reaction oxidises toluene to benzaldehyde with chromyl chloride.

(C) Rosenmund reduction converts an acyl chloride into an aldehyde.

(D) Haloform reaction converts methyl ketones into a carboxylate and

a haloform.

Final Answer:

Zinc amalgam with concentrated HCl reduces the ketone to an alkane, option (A).

Clemmensen reduction

Quick Tip: Clemmensen reduction uses Zn-Hg and conc. HCl to turn C=O into CH₂.

93. Identify the reason for the highest boiling point of R-COOH as compared to alkanes, ethers, alcohols and aldehydes.

- (A) Formation of carboxylate ion
- (B) Van der waals force of attraction
- (C) Formation of intramolecular hydrogen bonding
- (D) Formation of intermolecular hydrogen bonding

Correct Answer: (D) Formation of intermolecular hydrogen bonding

Solution:

Step 1: Understanding the Concept

The boiling point depends on how strongly molecules attract each other. Alkanes have only weak van der Waals forces, ethers and aldehydes have dipole attraction, and alcohols have hydrogen bonding.

Step 2: Why acids are highest

Carboxylic acids have both an O-H and a C=O group. Two molecules join through two hydrogen bonds to make a dimer. The hydrogen bonds are between separate molecules, so they are intermolecular. This strong association needs a lot of heat to break.

Step 3: Check the options

(A) Carboxylate ion formation is an ionisation and not the reason. (B) Van der Waals forces are present in all, so they cannot explain the highest value. (C) Intramolecular hydrogen bonding is within one molecule and would lower the boiling point. (D) Intermolecular hydrogen bonding is correct.

Final Answer:

Intermolecular hydrogen bonding, which forms dimers, gives carboxylic acids the highest boiling points, option (D).

Intermolecular hydrogen bonding

Quick Tip: Carboxylic acid molecules join in pairs through two hydrogen bonds between molecules.

94. Aniline on treatment with NaNO_2 / HCl at about 0°C gives a compound A, which on further treatment with CuCN / KCN gives a compound B, which when reacts with H_2 / Ni gives a compound C. The compound C on reaction with HNO_2 gives a compound D. The structure of D is:

- (A) $\text{C}_6\text{H}_5\text{-CH}_2\text{-OH}$
- (B) $\text{C}_6\text{H}_5\text{-NH-CH}_2\text{-CH}_3$
- (C) $\text{C}_6\text{H}_5\text{-NH-OH}$
- (D) $\text{C}_6\text{H}_5\text{-CH}_2\text{-NH}_2$

Correct Answer: (A) $\text{C}_6\text{H}_5\text{-CH}_2\text{-OH}$

Solution:

Step 1: Understanding the Concept

We follow the sequence one step at a time.

Step 2: Step to A

Aniline with NaNO_2/HCl at 0°C gives benzenediazonium chloride, $\text{C}_6\text{H}_5\text{N}_2^+\text{Cl}^-$.

Step 3: Step to B

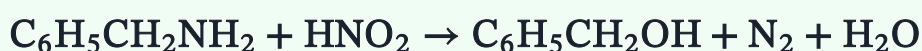
CuCN/KCN replaces the diazonium group by CN (Sandmeyer reaction) and gives benzonitrile, $\text{C}_6\text{H}_5\text{CN}$.

Step 4: Step to C

H_2/Ni reduces the nitrile to a primary amine, benzylamine, $\text{C}_6\text{H}_5\text{CH}_2\text{NH}_2$.

Step 5: Step to D

A primary aliphatic amine reacts with nitrous acid to form an alcohol with the release of nitrogen:



Step 6: Conclusion

D is benzyl alcohol, option (A).

Final Answer:

The sequence ends in benzyl alcohol, option (A).



Quick Tip: Follow the chain: diazonium salt, cyanide, amine by reduction, alcohol with nitrous acid.

95. Identify the reagent used to distinguish primary and secondary amines by Hinsberg test.

- (A) NaNO_2 and HCl
- (B) FeCl_3 (alc)
- (C) $\text{C}_6\text{H}_5\text{SO}_2\text{Cl}$
- (D) CHCl_3 and KOH (alc)

Correct Answer: (C) $\text{C}_6\text{H}_5\text{SO}_2\text{Cl}$

Solution:**Step 1: Understanding the Concept**

The Hinsberg test separates primary, secondary and tertiary amines using benzenesulfonyl chloride, $\text{C}_6\text{H}_5\text{SO}_2\text{Cl}$.

Step 2: What happens

A primary amine gives a sulfonamide that has an acidic N-H, so it dissolves in alkali. A secondary amine gives a sulfonamide with no

acidic hydrogen, which is insoluble in alkali. A tertiary amine does not react in the same way.

Step 3: Check the options

(A) NaNO_2/HCl is used for the nitrous acid test. (B) Alcoholic FeCl_3 is a test for phenols. (D) CHCl_3 and alcoholic KOH is the carbylamine test for primary amines only. (C) Benzenesulfonyl chloride is the Hinsberg reagent.

Final Answer:

Hinsberg's reagent is benzenesulfonyl chloride, option (C).



Quick Tip: Hinsberg reagent is benzenesulfonyl chloride.

96. Identify the product when glucose reacts with bromine water.

- (A) Saccharic acid
- (B) Cyanohydrin
- (C) Gluconic acid
- (D) Nucleic acid

Correct Answer: (C) Gluconic acid

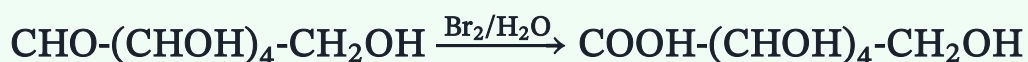
Solution:

Step 1: Understanding the Concept

Glucose has an aldehyde group at C1 and a primary alcohol at C6. Bromine water is a mild oxidising agent.

Step 2: Reaction

Bromine water oxidises only the aldehyde group to a carboxylic acid group, leaving the rest of the chain unchanged:



The product is gluconic acid.

Step 3: Compare the others

Saccharic acid forms with strong oxidation by nitric acid, where both end groups become COOH. Cyanohydrin forms with HCN. Nucleic acids are polymers and have nothing to do with this reaction.

Final Answer:

Bromine water turns the CHO group of glucose into COOH, giving gluconic acid, option (C).

Gluconic acid

Quick Tip: Bromine water oxidises only the aldehyde group of glucose to a carboxylic acid.

97. What type of compound the Legumelin is?

- (A) Carbohydrate
- (B) Globular protein
- (C) Fibrous protein
- (D) Enzyme

Correct Answer: (B) Globular protein

Solution:

Step 1: Understanding the Concept

Proteins are grouped by shape. Fibrous proteins form long threads and are insoluble in water. Globular proteins fold into a compact spherical form and dissolve in water.

Step 2: About Legumelin

Legumelin is an albumin-type protein present in legumes such as peas and beans. Albumins are water-soluble globular proteins.

Step 3: Check the options

(A) It is not a carbohydrate. (C) Fibrous proteins include keratin and myosin, which are not water-soluble. (D) Although many enzymes are globular proteins, Legumelin is classed as a storage protein and not as an enzyme. So (B) globular protein is the best description.

Final Answer:

Legumelin is a water-soluble albumin, so it is a globular protein, option (B).

Globular protein

Quick Tip: Legumelin is a simple globular protein found in legumes.

98. Which of the following is fully Fluorinated polymer?

- (A) PVC
- (B) Teflon
- (C) neoprene
- (D) Buna-S

Correct Answer: (B) Teflon

Solution:

Step 1: Understanding the Concept

We need a polymer in which all the hydrogen atoms of the monomer have been replaced by fluorine.

Step 2: Teflon

Teflon is made by polymerising tetrafluoroethylene, $\text{CF}_2=\text{CF}_2$:



There is no hydrogen in the chain, so it is fully fluorinated.

Step 3: Others

PVC is polyvinyl chloride and has chlorine. Neoprene is based on chloroprene and has chlorine. Buna-S is a copolymer of butadiene and styrene with no fluorine.

Final Answer:

Teflon is built from $\text{CF}_2=\text{CF}_2$ and is fully fluorinated, option (B).

Teflon

Quick Tip: Teflon is polytetrafluoroethylene, with every hydrogen replaced by fluorine.



99. Which of the following is a cross-linked, thermosetting polymer?

- (A) Bakelite
- (B) PVC
- (C) Polystyrene
- (D) PTFE (Teflon)

Correct Answer: (A) Bakelite

Solution:

Step 1: Understanding the Concept

A thermosetting polymer sets hard on heating and cannot be softened again. It has strong cross-links that join the chains in a three-dimensional network.

Step 2: Bakelite

Bakelite is formed by the condensation of phenol with formaldehyde. The product is heavily cross-linked, so it is thermosetting. It is used in electrical switches and handles.

Step 3: Others

PVC, polystyrene and PTFE are linear or slightly branched chains with no cross-links. They soften on heating and are thermoplastics.

Final Answer:

Bakelite is a cross-linked thermosetting polymer, option (A).

Bakelite

Quick Tip: Bakelite has a three-dimensional network from phenol-formaldehyde condensation.

100. What is the chemical name of Aspirin?

- (A) salicylic acid
- (B) acetyl salicylic acid
- (C) chloroxylenol
- (D) thymol

Correct Answer: (B) acetyl salicylic acid

Solution:

Step 1: Understanding the Concept

Aspirin is a well-known analgesic and antipyretic. It is made by acetylating the phenolic OH group of salicylic acid with acetic anhydride.

Step 2: Reaction



Step 3: Check the options

(A) Salicylic acid is the starting material, not aspirin. (C) Chloroxylenol is an antiseptic (Dettol). (D) Thymol is also an antiseptic. (B) Acetyl salicylic acid is aspirin.

Final Answer:

Aspirin is acetyl salicylic acid, option (B).

Acetyl salicylic acid

Quick Tip: Aspirin is the acetyl ester of salicylic acid.

Mathematics

Questions 101 to 150 | 2 marks each | 100 marks

101. The statement having truth value 'T' from the following is

- (A) $\sin(x)$ is an even function.
- (B) Every square matrix is non-singular.
- (C) The product of complex number and its conjugate is purely imaginary.
- (D) A square of any real number is non-negative.

Correct Answer: (D) A square of any real number is non-negative.

Solution:

Step 1: Understanding the Concept

A statement has truth value T if it is always true. A single counter-example makes it false.

Step 2: Check statement (A)

$\sin(-x) = -\sin x$, so $\sin x$ is an odd function. For example, $\sin(-30^\circ) = -\frac{1}{2}$ but $\sin 30^\circ = \frac{1}{2}$. So "even" is false.

Step 3: Check statement (B)

A square matrix is non-singular only when its determinant is not zero.

The matrix $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ has determinant 0, so it is singular. So (B) is false.

Step 4: Check statement (C)

For $z = a + ib$, $z\bar{z} = a^2 + b^2$, which is a real number. For $z = 1$ it is 1, which is not purely imaginary. So (C) is false.

Step 5: Check statement (D)

For any real x , $x^2 \geq 0$. So (D) is true.

Final Answer:

Only the statement that the square of any real number is non-negative is always true, option (D).

(D)

Quick Tip: A statement is true only if it holds for every case; test each one with a quick example.



102. If $i = \sqrt{-1}$ then $[i^{18} + (\frac{1}{i})^{25}]^3 =$

- (A) $4 + 4i$
- (B) $2 + 2i$
- (C) $4 - 4i$
- (D) $2 - 2i$

Correct Answer: (D) $2 - 2i$

Solution:**Step 1: Understanding the Concept**

Powers of i repeat in a cycle of four: $i^1 = i$, $i^2 = -1$, $i^3 = -i$, $i^4 = 1$.

Step 2: First term

$18 = 4 \times 4 + 2$, so $i^{18} = i^2 = -1$.

Step 3: Second term

$\frac{1}{i} = -i$. So $\left(\frac{1}{i}\right)^{25} = (-i)^{25} = -i^{25}$. Since $25 = 4 \times 6 + 1$, $i^{25} = i$. So the term is $-i$.

Step 4: Bracket

$$i^{18} + \left(\frac{1}{i}\right)^{25} = -1 - i = -(1 + i)$$

Step 5: Cube

$$(1 + i)^2 = 2i, \quad (1 + i)^3 = 2i(1 + i) = 2i - 2$$

$$[-(1 + i)]^3 = -(2i - 2) = 2 - 2i$$

This is option (D).

Final Answer:

The bracket equals $-1 - i$, and its cube is $2 - 2i$, option (D).

$$\boxed{2 - 2i}$$

Quick Tip: Reduce the powers of i using the cycle of four, then cube.

103. The number of permutations of the letters of the word INSTITUTION are

- (A) 6652800
- (B) 221760
- (C) 1108800
- (D) 554400

Correct Answer: (D) 554400

Solution:

Step 1: Understanding the Concept

When some objects repeat, the number of distinct arrangements is the total factorial divided by the factorial of each repeat count.

Step 2: Count the letters

INSTITUTION has 11 letters: I appears 2 times, N appears 3 times, T appears 3 times, and S, U, O appear once each.

Step 3: Formula

$$\frac{11!}{2! 3! 3!}$$

Step 4: Calculate

$$11! = 39916800, \quad 2! 3! 3! = 2 \times 6 \times 6 = 72$$

$$\frac{39916800}{72} = 554400$$

Step 5: Check the options

6652800 is $11!/6$, which corrects only one repeat. 221760 and 1108800 do not match any correct handling of the repeats. The answer is 554400, option (D).

Final Answer:

The word has 11 letters with I twice, N three times and T three times, giving 554400 arrangements, option (D).

554400

Quick Tip: Divide n factorial by the factorials of the repeated letters.

104. If $x + y = \frac{\pi}{4}$, then $(1 + \tan x)(1 + \tan y) =$

- (A) 1
- (B) 2
- (C) 3
- (D) 0

Correct Answer: (B) 2

Solution:

Step 1: Understanding the Concept

We use the sum formula for tangent and the fact that $\tan \frac{\pi}{4} = 1$.

Step 2: Key Formula or Approach

$$\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$$

Step 3: Use the given condition

$$\frac{\tan x + \tan y}{1 - \tan x \tan y} = \tan \frac{\pi}{4} = 1$$

$$\tan x + \tan y = 1 - \tan x \tan y$$

Step 4: Expand the product

$$(1 + \tan x)(1 + \tan y) = 1 + \tan x + \tan y + \tan x \tan y$$

Replace $\tan x + \tan y$ by $1 - \tan x \tan y$:

$$= 1 + (1 - \tan x \tan y) + \tan x \tan y = 2$$

So the value is 2, option (B).

Final Answer:

The product is always 2 when $x + y = \pi/4$, option (B).

2

Quick Tip: Expand $\tan(x+y) = 1$ and substitute into the product.

105. A straight line makes equal negative intercepts on the coordinate axes. If the perpendicular distance from the origin to the line is 4 units, then the equation of the line is

- (A) $x + y + 4 = 0$
- (B) $x + y + 4\sqrt{2} = 0$
- (C) $x - y + 4 = 0$
- (D) $x - y + 4\sqrt{2} = 0$

Correct Answer: (B) $x + y + 4\sqrt{2} = 0$

Solution:

Step 1: Understanding the Concept

A line with equal intercepts a on both axes has the form $\frac{x}{a} + \frac{y}{a} = 1$, that is $x + y = a$. Negative intercepts mean $a < 0$.

Step 2: Write the line

$$x + y - a = 0, \quad a < 0$$

Let $a = -k$ with $k > 0$. The line is $x + y + k = 0$.

Step 3: Distance from origin

$$\frac{|k|}{\sqrt{1^2 + 1^2}} = \frac{k}{\sqrt{2}} = 4 \Rightarrow k = 4\sqrt{2}$$

Step 4: Equation

$$x + y + 4\sqrt{2} = 0$$

Step 5: Check the options

Option (A) $x + y + 4 = 0$ is at distance $4/\sqrt{2} = 2\sqrt{2}$, not 4. Options (C) and (D) have unequal-sign intercepts, so they do not match. The answer is (B).

Final Answer:

The line is $x + y + 4\sqrt{2} = 0$, option (B).

$$x + y + 4\sqrt{2} = 0$$

Quick Tip: Equal intercepts give slope -1; use the distance-from-origin formula to find the constant.

106. If the angle made by the lines represented by the equation $ax^2 + 2hxy + by^2 = 0$ with X-axis are α and β , then $\tan(\alpha + \beta)$ is

- (A) $\frac{h}{a+b}$
- (B) $\frac{2h}{a-b}$
- (C) $\frac{2h}{a+b}$
- (D) $\frac{h}{a-b}$

Correct Answer: (B) $\frac{2h}{a-b}$

Solution:**Step 1: Understanding the Concept**

The lines given by $ax^2 + 2hxy + by^2 = 0$ pass through the origin. If they make angles α and β with the X-axis, their slopes are $m_1 = \tan \alpha$

and $m_2 = \tan \beta$.

Step 2: Slope equation

Put $y = mx$ in the equation:

$$a + 2hm + bm^2 = 0$$

$$m_1 + m_2 = -\frac{2h}{b}, \quad m_1 m_2 = \frac{a}{b}$$

Step 3: Tangent of the sum

$$\tan(\alpha + \beta) = \frac{m_1 + m_2}{1 - m_1 m_2} = \frac{-2h/b}{1 - a/b}$$

$$= \frac{-2h}{b - a} = \frac{2h}{a - b}$$

Step 4: Check

Option (B) is exactly this. Options (A) and (D) miss the factor 2 and option (C) has a plus sign in the denominator.

Final Answer:

$$\tan(\alpha + \beta) = \frac{2h}{a - b}, \text{ option (B).}$$

$$\boxed{\frac{2h}{a - b}}$$

Quick Tip: The slopes $\tan(\alpha)$ and $\tan(\beta)$ are roots of $bm^2 + 2hm + a = 0$.



107. The equation of a circle whose center lies on $x + 2y = 0$ and touching the lines $3x - 4y + 8 = 0$ and $3x - 4y - 28 = 0$ is

- (A) $(x - 2)^2 + (y + 1)^2 = 324$
- (B) $(x - 2)^2 + (y - 1)^2 = 324$
- (C) $5(x - 2)^2 + 5(y + 1)^2 = 324$
- (D) $25(x - 2)^2 + 25(y + 1)^2 = 324$

Correct Answer: (D) $25(x - 2)^2 + 25(y + 1)^2 = 324$

Solution:

Step 1: Understanding the Concept

The two lines $3x - 4y + 8 = 0$ and $3x - 4y - 28 = 0$ are parallel. A circle touching both has its diameter equal to the distance between them and its centre on the line midway between them.

Step 2: Radius

$$d = \frac{|8 - (-28)|}{\sqrt{3^2 + 4^2}} = \frac{36}{5}, \quad r = \frac{18}{5}, \quad r^2 = \frac{324}{25}$$

Step 3: Locus of the centre

The centre is equidistant from both lines, so it lies on the mid line:

$$3x - 4y + \frac{8 - 28}{2} = 0 \Rightarrow 3x - 4y - 10 = 0$$

Step 4: Solve with the given line

$x + 2y = 0 \Rightarrow x = -2y$. Substitute:

$$-6y - 4y = 10 \Rightarrow y = -1, \quad x = 2$$

Step 5: Equation

$$(x - 2)^2 + (y + 1)^2 = \frac{324}{25} \Rightarrow 25(x - 2)^2 + 25(y + 1)^2 = 324$$

This is option (D). Options (A) and (B) have the radius squared as 324, which is 25 times too large.

Final Answer:

The centre is (2, -1) and the radius is $18/5$, giving $25(x - 2)^2 + 25(y + 1)^2 = 324$, option (D).

$$25(x - 2)^2 + 25(y + 1)^2 = 324$$

Quick Tip: The lines are parallel, so the diameter equals the distance between them and the centre is on the midline.

108. If P be any point on the ellipse $16x^2 + 25y^2 = 400$ with foci S and S' and area of $\triangle PS S'$ is 9 square units, then the abscissa of point P is.....

- (A) $\frac{7\sqrt{5}}{4}$
 (B) $\frac{4\sqrt{7}}{5}$
 (C) $\frac{5\sqrt{7}}{4}$
 (D) $\frac{10}{7}$

Correct Answer: (C) $\frac{5\sqrt{7}}{4}$

Solution:

Step 1: Understanding the Concept

Write the ellipse in standard form and find the foci. The base of the triangle is the distance SS' and its height is the y -coordinate of P .

Step 2: Standard form

$$16x^2 + 25y^2 = 400 \Rightarrow \frac{x^2}{25} + \frac{y^2}{16} = 1$$

So $a = 5$, $b = 4$, and $c^2 = a^2 - b^2 = 9$, giving $c = 3$. The foci are $(\pm 3, 0)$ and $SS' = 6$.

Step 3: Area

$$\text{Area} = \frac{1}{2} \times 6 \times |y| = 3|y| = 9 \Rightarrow |y| = 3$$

Step 4: Find x

$$\frac{x^2}{25} = 1 - \frac{9}{16} = \frac{7}{16} \Rightarrow x^2 = \frac{175}{16}$$

$$x = \pm \frac{5\sqrt{7}}{4}$$

Step 5: Check the options

The positive value $\frac{5\sqrt{7}}{4}$ is option (C). Option (A) has the 5 and 7 in the wrong places, and (B) comes from using $x^2/16$.

Final Answer:

The ordinate of P is 3, so its abscissa is $\frac{5\sqrt{7}}{4}$, option (C).

$$\boxed{\frac{5\sqrt{7}}{4}}$$

Quick Tip: Area of triangle PSS-dash is half base $2ae$ times the height $|y|$.

109. The value of $\lim_{n \rightarrow \infty} \left[\frac{1}{1-n^2} + \frac{2}{1-n^2} + \dots + \frac{n}{1-n^2} \right]^3$ is

- (A) 8
- (B) -8
- (C) $\frac{1}{8}$
- (D) $\frac{-1}{8}$

Correct Answer: (D) $\frac{-1}{8}$

Solution:

Step 1: Understanding the Concept

All the fractions share the denominator $1 - n^2$, so the numerators can

be added first.

Step 2: Add the numerators

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}$$

The bracket becomes

$$\frac{n(n+1)}{2(1-n^2)}$$

Step 3: Take the limit

Divide the top and bottom by n^2 :

$$\frac{1 + \frac{1}{n}}{2\left(\frac{1}{n^2} - 1\right)} \rightarrow \frac{1}{2(0 - 1)} = -\frac{1}{2}$$

Step 4: Cube

$$\left(-\frac{1}{2}\right)^3 = -\frac{1}{8}$$

This is option (D). The sign matters: because the denominator $1 - n^2$ is negative for large n , the limit is negative, so options (A) and (C) are ruled out.

Final Answer:

The bracket tends to $-1/2$, so the limit of its cube is $-1/8$, option (D).

$$\boxed{-\frac{1}{8}}$$

Quick Tip: Add the numerators with $n(n+1)/2$, then divide top and bottom by n squared.



110. If $\sim p \vee q$ is false then which of the following is correct?

- (A) $p \leftrightarrow q$ is T
- (B) $p \rightarrow q$ is T
- (C) $q \rightarrow p$ is T
- (D) $q \rightarrow p$ is F

Correct Answer: (C) $q \rightarrow p$ is T

Solution:

Step 1: Understanding the Concept

An OR statement is false only when both components are false.

Step 2: Find the truth values

$\sim p \vee q$ is false, so $\sim p$ is F and q is F . So p is T and q is F .

Step 3: Check each option

(A) $p \leftrightarrow q$ is T only when both have equal values. Here T and F differ, so it is F .

(B) $p \rightarrow q$ with $p = T$ and $q = F$ is F .

(C) $q \rightarrow p$ with $q = F$ is T , because a false antecedent makes the implication true. So (C) is correct.

(D) $q \rightarrow p$ is F is the opposite of (C), so it is wrong.

Final Answer:

With p true and q false, the implication q to p is true, option (C).

$$q \rightarrow p \text{ is T}$$

Quick Tip: A disjunction is false only when both parts are false.

111. The contrapositive of $\sim q \rightarrow p$ is equivalent to

- (A) $p \rightarrow q$
- (B) $p \wedge q$
- (C) $p \vee q$
- (D) $\sim p \rightarrow \sim q$

Correct Answer: (C) $p \vee q$

Solution:**Step 1: Understanding the Concept**

The contrapositive of $A \rightarrow B$ is $\sim B \rightarrow \sim A$, and it is logically equivalent to the original.

Step 2: Apply

Here $A = \sim q$ and $B = p$. So the contrapositive is

$$\sim p \rightarrow \sim (\sim q) = \sim p \rightarrow q$$

Step 3: Convert to OR

An implication $X \rightarrow Y$ is equivalent to $\sim X \vee Y$. So

$$\sim p \rightarrow q \equiv \sim (\sim p) \vee q = p \vee q$$

Step 4: Check the options

(A) $p \rightarrow q$ is $\sim p \vee q$, which differs. (B) $p \wedge q$ is stronger. (D) $\sim p \rightarrow \sim q$ is not equivalent. The match is (C).

Final Answer:

The contrapositive is equivalent to $p \vee q$, option (C).

$$\boxed{p \vee q}$$

Quick Tip: The contrapositive of A implies B is not-B implies not-A.

112. The angles of $\triangle ABC$ are in A.P. and $b : c = \sqrt{3} : \sqrt{2}$ then $\angle A =$

- (A) 30°
- (B) 90°
- (C) 105°
- (D) 75°

Correct Answer: (D) 75°

Solution:**Step 1: Understanding the Concept**

If the three angles are in A.P., the middle angle is the mean of the

other two. Their sum is 180° , so $3B = 180^\circ$ and $B = 60^\circ$.

Step 2: Use the sine rule

$$\frac{b}{c} = \frac{\sin B}{\sin C} \Rightarrow \frac{\sqrt{3}}{\sqrt{2}} = \frac{\sin 60^\circ}{\sin C}$$

$$\sin C = \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2}}{2}$$

Step 3: Find C

$C = 45^\circ$ or 135° . The value 135° cannot hold because $B + C = 195^\circ$ exceeds 180° . So $C = 45^\circ$.

Step 4: Find A

$$A = 180^\circ - 60^\circ - 45^\circ = 75^\circ$$

Also the A.P. check: 45, 60, 75 is an A.P. The answer is option (D).

Final Answer:

The angles are 45, 60 and 75 degrees, so angle A is 75 degrees, option (D).

$$\boxed{75^\circ}$$

Quick Tip: Angles in A.P. force $B = 60$ degrees; the sine rule then gives C and A.

113. With the usual notations, if the lengths of the sides of the triangle are 3 units, 5 units and 7 units, then the largest angle of the triangle is

- (A) $\frac{\pi}{2}$
- (B) $\frac{\pi}{3}$
- (C) $\frac{2\pi}{3}$
- (D) $\frac{\pi}{4}$

Correct Answer: (C) $\frac{2\pi}{3}$

Solution:

Step 1: Understanding the Concept

The largest angle lies opposite the longest side, which is 7. Use the cosine rule.

Step 2: Key Formula or Approach

$$\cos \theta = \frac{a^2 + b^2 - c^2}{2ab}$$

with $a = 3, b = 5, c = 7$.

Step 3: Calculation

$$\cos \theta = \frac{9 + 25 - 49}{2 \times 3 \times 5} = \frac{-15}{30} = -\frac{1}{2}$$

$$\theta = \frac{2\pi}{3}$$

Step 4: Check the options

$\pi/2$ would need $c^2 = 34$. $\pi/3$ and $\pi/4$ are acute, which cannot be the largest angle here because $3^2 + 5^2 < 7^2$ shows an obtuse angle. The answer is (C).

Final Answer:

The largest angle has cosine $-1/2$, so it is $\frac{2\pi}{3}$, option (C).

$$\boxed{\frac{2\pi}{3}}$$

Quick Tip: The largest angle is opposite the longest side; use the cosine rule.

114. If $A = \begin{bmatrix} 1 & -5 \\ -2 & 4 \end{bmatrix}$, then $A^{-1} =$

(A) $-\frac{1}{6} \begin{bmatrix} -4 & 5 \\ 2 & -1 \end{bmatrix}$

(B) $\frac{1}{14} \begin{bmatrix} -1 & 5 \\ 2 & -4 \end{bmatrix}$

(C) $\frac{1}{14} \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$

(D) $-\frac{1}{6} \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$

Correct Answer: (D) $-\frac{1}{6} \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$

Solution:

Step 1: Understanding the Concept

For $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, the inverse is $\frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Step 2: Determinant

$$|A| = (1)(4) - (-5)(-2) = 4 - 10 = -6$$

Step 3: Adjoint

Swap the diagonal entries and change the sign of the other two:

$$\text{adj } A = \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$$

Step 4: Inverse

$$A^{-1} = -\frac{1}{6} \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$$

Step 5: Check

Multiply A by this: first row first column is $(4 - 10)/(-6) = 1$. The off-diagonal $(5 - 5)/(-6) = 0$. So it is correct, option (D).

Final Answer:

The inverse is $-\frac{1}{6} \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}$, option (D).

$$\boxed{-\frac{1}{6} \begin{bmatrix} 4 & 5 \\ 2 & 1 \end{bmatrix}}$$

Quick Tip: Inverse of a 2 by 2 matrix: swap the diagonal, change signs of the others, divide by the determinant.

115. Let $A = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 2 & 0 \\ 1 & 1 & 4 \end{bmatrix}$ and $pC_{11} + 4C_{21} - 5C_{32} = -2$, where C_{ij} denotes

the cofactor of an element a_{ij} of matrix A , then the value of p is :

- (A) -2
- (B) 2
- (C) 4
- (D) 3

Correct Answer: (B) 2

Solution:

Step 1: Understanding the Concept

The cofactor of a_{ij} is $C_{ij} = (-1)^{i+j}M_{ij}$, where M_{ij} is the determinant left after deleting row i and column j .

Step 2: Find the three cofactors

$$C_{11} = + \begin{vmatrix} 2 & 0 \\ 1 & 4 \end{vmatrix} = 8.$$

$$C_{21} = - \begin{vmatrix} 1 & 2 \\ 1 & 4 \end{vmatrix} = -(4 - 2) = -2.$$

$$C_{32} = - \begin{vmatrix} 3 & 2 \\ 1 & 0 \end{vmatrix} = -(0 - 2) = 2.$$

Step 3: Substitute

$$p(8) + 4(-2) - 5(2) = -2$$

$$8p - 8 - 10 = -2 \Rightarrow 8p = 16 \Rightarrow p = 2$$

Step 4: Conclusion

The value is $p = 2$, option (B).

Final Answer:

The cofactors are 8, -2 and 2, which give $p = 2$, option (B).

2

Quick Tip: Cofactor C_{ij} is $(-1)^{(i+j)}$ times the minor M_{ij} .



116. If $3\sin^{-1}\left(\frac{2x}{1+x^2}\right) - 4\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right) + 2\tan^{-1}\left(\frac{2x}{1-x^2}\right) = \frac{\pi}{3}$ then $x = ?$

(A) $\sqrt{3}$

(B) 1

(C) $\frac{1}{\sqrt{3}}$

(D) -1

Correct Answer: (C) $\frac{1}{\sqrt{3}}$

Solution:

Step 1: Understanding the Concept

Three standard identities hold for $0 \leq x \leq 1$:

$$\sin^{-1} \frac{2x}{1+x^2} = 2 \tan^{-1} x, \quad \cos^{-1} \frac{1-x^2}{1+x^2} = 2 \tan^{-1} x, \quad \tan^{-1} \frac{2x}{1-x^2} = 2 \tan^{-1} x$$

Step 2: Substitute

$$3(2 \tan^{-1} x) - 4(2 \tan^{-1} x) + 2(2 \tan^{-1} x) = \frac{\pi}{3}$$

Step 3: Combine

$$(6 - 8 + 4) \tan^{-1} x = 2 \tan^{-1} x = \frac{\pi}{3}$$

$$\tan^{-1} x = \frac{\pi}{6} \Rightarrow x = \tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$$

Step 4: Check

$x = 1/\sqrt{3}$ lies in the allowed range $0 \leq x < 1$. The value $x = \sqrt{3}$ lies outside the range for these identities, and $x = -1$ gives a negative left-hand side. Answer: (C).

Final Answer:

The equation reduces to $2 \tan^{-1} x = \pi/3$, so $x = 1/\sqrt{3}$, option (C).

$$\boxed{\frac{1}{\sqrt{3}}}$$

Quick Tip: Convert each inverse function to $2 \tan^{-1} x$, valid for x between 0 and 1.

117. If $2\sin^{-1}x - 3\cos^{-1}x = 4$, then $2\sin^{-1}x + 3\cos^{-1}x =$

(A) $\frac{6\pi-4}{5}$

(B) $\frac{6\pi+4}{5}$

(C) $\frac{5\pi-4}{6}$

(D) $\frac{5\pi+4}{6}$

Correct Answer: (A) $\frac{6\pi-4}{5}$

Solution:

Step 1: Understanding the Concept

For $x \in [-1, 1]$: $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$. Let $s = \sin^{-1}x$, so $\cos^{-1}x = \frac{\pi}{2} - s$.

Step 2: Use the given equation

$$2s - 3\left(\frac{\pi}{2} - s\right) = 4 \Rightarrow 5s = 4 + \frac{3\pi}{2}$$

$$s = \frac{8 + 3\pi}{10}$$

Step 3: Required expression

$$2s + 3\left(\frac{\pi}{2} - s\right) = \frac{3\pi}{2} - s = \frac{3\pi}{2} - \frac{8 + 3\pi}{10}$$

$$= \frac{15\pi - 8 - 3\pi}{10} = \frac{12\pi - 8}{10} = \frac{6\pi - 4}{5}$$

This is option (A).

Final Answer:

Using the identity for sin inverse plus cos inverse, the value is $\frac{6\pi - 4}{5}$, option (A).

$$\boxed{\frac{6\pi - 4}{5}}$$

Quick Tip: Use $\sin^{-1} x + \cos^{-1} x = \pi/2$ to get both unknowns.



118. If $f(x) = \frac{2x-1}{x+5}$, $x \neq -5$ then $f^{-1}(x)$ is equal to

- (A) $\frac{x+5}{2x-1}$, $x \neq \frac{1}{2}$
- (B) $\frac{5x+1}{2-x}$, $x \neq 2$
- (C) $\frac{5x-1}{2-x}$, $x \neq 2$
- (D) $\frac{x-5}{2x+1}$, $x \neq -\frac{1}{2}$

Correct Answer: (B) $\frac{5x+1}{2-x}$, $x \neq 2$

Solution:

Step 1: Understanding the Concept

To find an inverse function, write $y = f(x)$, solve for x in terms of y , and then rename y as x .

Step 2: Solve

$$y = \frac{2x-1}{x+5} \Rightarrow y(x+5) = 2x-1$$

$$xy + 5y = 2x - 1 \Rightarrow x(y - 2) = -1 - 5y$$

$$x = \frac{5y + 1}{2 - y}$$

Step 3: Rename

$$f^{-1}(x) = \frac{5x + 1}{2 - x}, \quad x \neq 2$$

Step 4: Why $x \neq 2$

The denominator $2 - x$ cannot be zero, and the value 2 is the horizontal asymptote of f , which is never reached. So the domain of the inverse excludes 2. Option (B).

Final Answer:

The inverse function is $\frac{5x + 1}{2 - x}$ with $x \neq 2$, option (B).

$$\boxed{\frac{5x + 1}{2 - x}, x \neq 2}$$

Quick Tip: Put $y = f(x)$, solve for x , then swap the letters.

119. If the function $f(x)$ defined by $f(x) = \begin{cases} ax + 1 & \text{if } x \leq 3 \\ bx + 3 & \text{if } x > 3 \end{cases}$ is continuous at $x = 3$, then $(a - b) = \dots\dots\dots$

- (A) $\frac{2}{3}$
- (B) $\frac{3}{2}$
- (C) 2
- (D) 3

Correct Answer: (A) $\frac{2}{3}$

Solution:

Step 1: Understanding the Concept

A function is continuous at $x = 3$ if the left-hand limit, the right-hand limit and the value at 3 are all equal.

Step 2: Evaluate the pieces

Left side and value: $f(3) = 3a + 1$.

Right-hand limit: $\lim_{x \rightarrow 3^+} (bx + 3) = 3b + 3$.

Step 3: Equate

$$3a + 1 = 3b + 3$$

$$3a - 3b = 2 \Rightarrow a - b = \frac{2}{3}$$

Step 4: Check the options

The result is $\frac{2}{3}$, option (A). Options (C) and (D) come from forgetting to divide by 3.

Final Answer:

Equating the two pieces at $x = 3$ gives $a - b = 2/3$, option (A).

$$\boxed{\frac{2}{3}}$$

Quick Tip: Continuity at $x = 3$ means the left and right values are equal.

120. If $f(x), g(x)$ be twice differentiable functions, satisfying

Missing open brace for superscript and $f(2) = 3g(2) = 9$ then $f(x) - g(x)$ at $x = 4$ is equal to

- (A) 0
- (B) 10
- (C) 8
- (D) 2

Correct Answer: (B) 10

Solution:**Step 1: Understanding the Concept**

Let $h(x) = f(x) - g(x)$. Then $h''(x) = f''(x) - g''(x) = 0$, so h' is constant and h is a linear function.

Step 2: Find the constants

$f'(1) = 4$ and $2g'(1) = 4 \Rightarrow g'(1) = 2$. So

$$h'(1) = 4 - 2 = 2$$

Since h' is constant, $h'(x) = 2$ for all x . So $h(x) = 2x + c$.

Step 3: Use the second condition

$f(2) = 9$ and $3g(2) = 9 \Rightarrow g(2) = 3$. So $h(2) = 9 - 3 = 6$.

$$4 + c = 6 \Rightarrow c = 2$$

Step 4: Evaluate at 4

$$h(4) = 2(4) + 2 = 10$$

This is option (B).

Final Answer:

The difference $f - g$ equals $2x + 2$, so at $x = 4$ it is 10, option (B).

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Quick Tip: Equal second derivatives mean the difference of the two functions is a linear function.

121. If $y = \cos^2[\cot^{-1}(\sqrt{\frac{1-x}{1+x}})]$ then $\frac{dy}{dx} = \dots\dots\dots$

- (A) $\frac{-1}{2}$
(B) $\frac{1}{2}$
(C) 1
(D) -1

Correct Answer: (A) $\frac{-1}{2}$

Solution:

Step 1: Understanding the Concept

The expression inside has $\sqrt{\frac{1-x}{1+x}}$, which simplifies with the substitution $x = \cos 2\theta$.

Step 2: Substitute

$$\frac{1-x}{1+x} = \frac{1-\cos 2\theta}{1+\cos 2\theta} = \tan^2 \theta \Rightarrow \sqrt{\frac{1-x}{1+x}} = \tan \theta$$

Step 3: Simplify

$$\cot^{-1}(\tan \theta) = \frac{\pi}{2} - \theta$$

$$y = \cos^2\left(\frac{\pi}{2} - \theta\right) = \sin^2 \theta = \frac{1 - \cos 2\theta}{2} = \frac{1-x}{2}$$

Step 4: Differentiate

$$\frac{dy}{dx} = -\frac{1}{2}$$

This is option (A).

Final Answer:

The expression simplifies to $(1 - x)/2$, so the derivative is $-1/2$, option (A).

$$\boxed{-\frac{1}{2}}$$

Quick Tip: Substitute $x = \cos 2\theta$ to simplify the inner expression.

122. If $\sqrt{\frac{x}{y}} + \sqrt{\frac{y}{x}} = 6$, then $\frac{dy}{dx} =$

- (A) $\frac{x+17y}{17x-y}$
- (B) $\frac{x-17y}{17x-y}$
- (C) $\frac{x-17y}{17x+y}$
- (D) $\frac{x+17y}{17x+y}$

Correct Answer: (B) $\frac{x-17y}{17x-y}$

Solution:

Step 1: Understanding the Concept

Put $t = \sqrt{x/y}$. The equation becomes $t + \frac{1}{t} = 6$.

Step 2: Remove the roots

Square both sides:

$$\frac{x}{y} + \frac{y}{x} + 2 = 36 \Rightarrow \frac{x^2 + y^2}{xy} = 34$$

$$x^2 + y^2 = 34xy$$

Step 3: Differentiate

$$2x + 2y \frac{dy}{dx} = 34y + 34x \frac{dy}{dx}$$

$$\frac{dy}{dx} (2y - 34x) = 34y - 2x$$

$$\frac{dy}{dx} = \frac{17y - x}{y - 17x} = \frac{x - 17y}{17x - y}$$

This is option (B).

Final Answer:

After squaring, $x^2 + y^2 = 34xy$, and differentiating gives $(x - 17y)/(17x - y)$, option (B).

$$\boxed{\frac{x - 17y}{17x - y}}$$

Quick Tip: Let $t = \sqrt{x/y}$ and turn the equation into a relation between x and y without roots.

123. If $x = 4t^3 + 3$, $y = 3t^4 + 4$ and $\frac{\frac{d^2x}{dy^2}}{(\frac{dx}{dy})^n}$ is constant then the value of n is

- (A) 1
- (B) 2
- (C) 6
- (D) 5

Correct Answer: (D) 5

Solution:

Step 1: Understanding the Concept

Both x and y depend on t , so we find $\frac{dx}{dy}$ first and then differentiate it with respect to y using the chain rule.

Step 2: First derivative

$$\frac{dx}{dt} = 12t^2, \quad \frac{dy}{dt} = 12t^3 \Rightarrow \frac{dx}{dy} = \frac{1}{t}$$

Step 3: Second derivative

$$\frac{d^2x}{dy^2} = \frac{d}{dt} \left(\frac{1}{t} \right) \cdot \frac{dt}{dy} = -\frac{1}{t^2} \cdot \frac{1}{12t^3} = -\frac{1}{12t^5}$$

Step 4: Form the ratio

$$\frac{d^2x/dy^2}{(dx/dy)^n} = \frac{-1/(12t^5)}{t^{-n}} = -\frac{t^{n-5}}{12}$$

Step 5: Constant condition

The ratio does not depend on t only when $n - 5 = 0$, so $n = 5$. Option (D).

Final Answer:

The ratio equals $-t^{(n-5)}/12$, which is constant only for $n = 5$, option (D).

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Quick Tip: Find dx/dy as a function of t , then differentiate again with respect to y .

124. If $x = a\sin^3 t$ and $y = a\cos^3 t$, then the value of $\frac{d^2y}{dx^2}$ at $t = \frac{\pi}{3}$ is equal to

- (A) $\frac{-32}{27a}$
- (B) $\frac{-32}{3\sqrt{3}a}$
- (C) $\frac{32}{27a}$
- (D) $\frac{-32}{9a}$

Correct Answer: (C) $\frac{32}{27a}$

Solution:**Step 1: Understanding the Concept**

For parametric curves, $\frac{d^2y}{dx^2} = \frac{\frac{d}{dt} \left(\frac{dy}{dx} \right)}{dx/dt}$.

Step 2: First derivative

$$\frac{dx}{dt} = 3a \sin^2 t \cos t, \quad \frac{dy}{dt} = -3a \cos^2 t \sin t$$

$$\frac{dy}{dx} = -\frac{\cos t}{\sin t} = -\cot t$$

Step 3: Second derivative

$$\frac{d}{dt}(-\cot t) = \csc^2 t$$

$$\frac{d^2y}{dx^2} = \frac{\csc^2 t}{3a \sin^2 t \cos t} = \frac{1}{3a \sin^4 t \cos t}$$

Step 4: At $t = \pi/3$

$$\sin^4 t = \left(\frac{\sqrt{3}}{2}\right)^4 = \frac{9}{16}, \quad \cos t = \frac{1}{2}.$$

$$\frac{1}{3a \cdot \frac{9}{16} \cdot \frac{1}{2}} = \frac{32}{27a}$$

This is option (C).

Final Answer:

At $t = \pi/3$ the second derivative equals $32/(27a)$, option (C).

$$\frac{32}{27a}$$

Quick Tip: Find $dy/dx = -\cot t$, then differentiate with respect to t and divide by dx/dt .

125. An aeroplane at an altitude of 1 km is flying horizontally at 600 km / hr, passes directly over an observer. Then the rate at which it is approaching the observer when it is 1250 meters away from him is.....

- (A) 360km/hr
- (B) 430km/hr
- (C) 600km/hr
- (D) 250km/hr

Correct Answer: (A) 360km/hr

Solution:

Step 1: Understanding the Concept

Let the plane be at horizontal distance x from the point above the observer and at height $h = 1$ km. The distance from the observer is s , so $s^2 = x^2 + 1$.

Step 2: Differentiate

$$2s \frac{ds}{dt} = 2x \frac{dx}{dt} \Rightarrow \frac{ds}{dt} = \frac{x}{s} \frac{dx}{dt}$$

Step 3: Values at the given moment

$s = 1250 \text{ m} = 1.25 \text{ km}$. Then $x = \sqrt{1.25^2 - 1} = \sqrt{0.5625} = 0.75 \text{ km}$. The speed is $\frac{dx}{dt} = -600 \text{ km/h}$ (distance decreasing).

Step 4: Calculate

$$\frac{ds}{dt} = \frac{0.75}{1.25} \times (-600) = -360 \text{ km/h}$$

The plane is approaching the observer at 360 km/h, option (A).

Final Answer:

The distance to the observer shrinks at 360 km/h, option (A).

360 km/hr

Quick Tip: Join the plane, the observer and the point below the plane in a right triangle and differentiate the Pythagoras relation.



126. The line $x + y = 0$ touches the curve $y^2 = ax^3 + b$ at $(1, -1)$ then values of a and b respectively are

- (A) $\frac{1}{2}, \frac{2}{5}$
- (B) $\frac{1}{3}, \frac{2}{3}$
- (C) $\frac{2}{3}, \frac{1}{3}$
- (D) $\frac{2}{5}, \frac{1}{2}$

Correct Answer: (C) $\frac{2}{3}, \frac{1}{3}$

Solution:

Step 1: Understanding the Concept

If a line touches a curve at a point, two things hold: the point lies on the curve, and the slope of the curve at that point equals the slope of the line.

Step 2: Use the point

$(1, -1)$ is on $y^2 = ax^3 + b$:

$$1 = a + b$$

Step 3: Use the slope

The line $x + y = 0$ has slope -1 . Differentiate the curve:

$$2y \frac{dy}{dx} = 3ax^2 \Rightarrow \frac{dy}{dx} = \frac{3ax^2}{2y}$$

At $(1, -1)$:

$$\frac{3a}{-2} = -1 \Rightarrow a = \frac{2}{3}$$

Step 4: Find b

$$b = 1 - a = 1 - \frac{2}{3} = \frac{1}{3}$$

So $a = \frac{2}{3}$ and $b = \frac{1}{3}$, option (C). Option (B) has the values swapped.

Final Answer:

The curve passes through (1, -1) and has slope -1 there, giving $a = 2/3$ and $b = 1/3$, option (C).

$$a = \frac{2}{3}, b = \frac{1}{3}$$

Quick Tip: The point lies on the curve and the slope of the curve at that point equals the slope of the line.



127. The function $f(x) = x(x + 3)e^{-(\frac{1}{2})x}$ satisfies all the conditions of Rolle's theorem in $[-3, 0]$, then $c =$

- (A) -3
- (B) -2
- (C) -1
- (D) 0

Correct Answer: (B) -2

Solution:

Step 1: Understanding the Concept

Rolle's theorem says that if $f(a) = f(b)$, then some c in (a, b) has $f'(c) = 0$. Here $f(-3) = 0$ and $f(0) = 0$.

Step 2: Differentiate

$$f(x) = (x^2 + 3x)e^{-x/2}$$

$$f'(x) = e^{-x/2} \left[(2x + 3) - \frac{x^2 + 3x}{2} \right]$$

Step 3: Set to zero

The exponential is never zero, so

$$4x + 6 - x^2 - 3x = 0 \Rightarrow x^2 - x - 6 = 0$$

$$(x - 3)(x + 2) = 0 \Rightarrow x = 3 \text{ or } x = -2$$

Step 4: Choose c

Only $x = -2$ lies inside $(-3, 0)$. The root $x = 3$ is outside the interval.
So $c = -2$, option (B).

Final Answer:

Setting $f'(x) = 0$ gives $x = -2$ or 3 , and only -2 lies in the interval, option (B).

$$\boxed{-2}$$

Quick Tip: In Rolle theorem the point c is where the derivative is zero inside the interval.

128. If $\int f(x)dx = g(x) + c$ then $\int f^{-1}(x)dx =$

- (A) $xf^{-1}(x) + c$
- (B) $f(g^{-1}(x)) + c$
- (C) $xf^{-1}(x) - g(f^{-1}(x)) + c$
- (D) $g^{-1}(x) + c$

Correct Answer: (C) $xf^{-1}(x) - g(f^{-1}(x)) + c$

Solution:

Step 1: Understanding the Concept

Let $y = f^{-1}(x)$, so $x = f(y)$ and $dx = f'(y) dy$.

Step 2: Substitute

$$\int f^{-1}(x) dx = \int y f'(y) dy$$

Step 3: Integrate by parts

Take $u = y$ and $dv = f'(y) dy$:

$$\int y f'(y) dy = y f(y) - \int f(y) dy$$

Step 4: Use the given information

$\int f(y) dy = g(y) + c$. Also $f(y) = x$ and $y = f^{-1}(x)$. So

$$\int f^{-1}(x) dx = xf^{-1}(x) - g(f^{-1}(x)) + c$$

This is option (C). Option (A) forgets the subtraction of g .

Final Answer:

The integral of the inverse function is $xf^{-1}(x) - g(f^{-1}(x)) + c$, option (C).

$$xf^{-1}(x) - g(f^{-1}(x)) + c$$

Quick Tip: Substitute $x = f(y)$ so that the inverse function becomes y , then integrate by parts.

129. $\int \frac{\log x}{(1+\log x)^2} dx =$

- (A) $\frac{1}{1+\log x} + c$
(B) $-\frac{1}{1+\log x} + c$
(C) $\frac{x}{1+\log x} + c$
(D) $-\frac{x}{1+\log x} + c$

Correct Answer: (C) $\frac{x}{1+\log x} + c$

Solution:**Step 1: Understanding the Concept**

Write the numerator as $(1 + \log x) - 1$ so that the integrand splits into two simpler parts.

Step 2: Split

$$\frac{\log x}{(1 + \log x)^2} = \frac{1}{1 + \log x} - \frac{1}{(1 + \log x)^2}$$

Step 3: Integrate the first term by parts

$$\int \frac{1}{1 + \log x} dx = \frac{x}{1 + \log x} + \int \frac{x \cdot \frac{1}{x}}{(1 + \log x)^2} dx = \frac{x}{1 + \log x} + \int \frac{1}{(1 + \log x)^2} dx$$

Step 4: Combine

The last integral cancels the second term in the split:

$$\int \frac{\log x}{(1 + \log x)^2} dx = \frac{x}{1 + \log x} + c$$

Step 5: Check

Differentiating $\frac{x}{1 + \log x}$ gives $\frac{(1 + \log x) - 1}{(1 + \log x)^2} = \frac{\log x}{(1 + \log x)^2}$. So option (C).

Final Answer:

The antiderivative is x divided by $(1 + \log x)$, option (C).

$$\boxed{\frac{x}{1 + \log x} + c}$$

Quick Tip: Write $\log x$ over $(1 + \log x)$ squared as a derivative of x over $(1 + \log x)$.

130. $\int \frac{1}{\sqrt{2x - x^2}} dx =$

- (A) $\sin^{-1}(x - 1) + c$
(B) $\cos^{-1}(x - 1) + c$
(C) $\tan^{-1}(x - 1) + c$
(D) $\sin^{-1}x + c$

Correct Answer: (A) $\sin^{-1}(x - 1) + c$

Solution:

Step 1: Understanding the Concept

Complete the square so that the root has the form $\sqrt{a^2 - u^2}$.

Step 2: Complete the square

$$2x - x^2 = 1 - (x^2 - 2x + 1) = 1 - (x - 1)^2$$

Step 3: Integrate

Let $u = x - 1$, $du = dx$:

$$\int \frac{du}{\sqrt{1 - u^2}} = \sin^{-1} u + c = \sin^{-1}(x - 1) + c$$

Step 4: Check the options

Differentiating $\cos^{-1}(x - 1)$ gives a negative sign, so (B) is not an antiderivative. $\tan^{-1}(x - 1)$ has a different form, and $\sin^{-1} x$ has the wrong argument. The answer is (A).

Final Answer:

Completing the square gives sin inverse of $(x - 1)$, option (A).

$$\sin^{-1}(x - 1) + c$$

Quick Tip: Complete the square under the root to reach the sin inverse standard form.

131. $\int (x^{21} + x^6 + x^3)(2x^{18} + 7x^3 + 14)^{\frac{1}{3}} dx =$

(A) $\frac{1}{56}(2x^{18} + 7x^3 + 14)^{\frac{4}{3}} + c$

(B) $(2x^{18} + 7x^3 + 14)^{\frac{4}{3}} + c$

(C) $(2x^{21} + 7x^6 + 14x^3)^{\frac{4}{3}} + c$

(D) $\frac{1}{56}(2x^{21} + 7x^6 + 14x^3)^{\frac{4}{3}} + c$

Correct Answer: (D) $\frac{1}{56}(2x^{21} + 7x^6 + 14x^3)^{\frac{4}{3}} + c$

Solution:

Step 1: Understanding the Concept

Try the substitution $u = 2x^{21} + 7x^6 + 14x^3$, because its derivative looks like the bracket in the integrand.

Step 2: Differentiate u

$$du = (42x^{20} + 42x^5 + 42x^2) dx = 42x^2(x^{18} + x^3 + 1) dx$$

Step 3: Rewrite the integrand

$$x^{21} + x^6 + x^3 = x^3(x^{18} + x^3 + 1)$$

$$2x^{18} + 7x^3 + 14 = \frac{u}{x^3}$$

So the integrand is

$$x^3(x^{18} + x^3 + 1) \cdot \frac{u^{1/3}}{x} = x^2(x^{18} + x^3 + 1)u^{1/3}$$

Step 4: Integrate

$$\int x^2(x^{18} + x^3 + 1)u^{1/3}dx = \frac{1}{42} \int u^{1/3}du = \frac{1}{42} \cdot \frac{3}{4}u^{4/3} = \frac{u^{4/3}}{56}$$

The result is option (D).

Final Answer:

With $u = 2x^{21} + 7x^6 + 14x^3$ the integral is $u^{4/3}/56$, option (D).

$$\boxed{\frac{1}{56}(2x^{21} + 7x^6 + 14x^3)^{4/3} + c}$$

Quick Tip: Factor x cubed from the bracket to spot that the integrand is a constant times u to the one third times du .

132. The value of the integral $\int_{1/e}^e \frac{|\log x|}{x^2} dx$ is

- (A) $\frac{e^2-1}{2e}$
(B) $\frac{2}{e}$
(C) $2 - \frac{2}{e}$
(D) $1 - \frac{1}{e}$

Correct Answer: (C) $2 - \frac{2}{e}$

Solution:

Step 1: Understanding the Concept

$\log x < 0$ for $x < 1$ and $\log x \geq 0$ for $x \geq 1$. So we split the integral at $x = 1$.

Step 2: Antiderivative

By parts:

$$\int \frac{\log x}{x^2} dx = -\frac{\log x}{x} - \frac{1}{x} = -\frac{\log x + 1}{x}$$

Step 3: Part from $1/e$ to 1

Here $|\log x| = -\log x$, so the integral is $\left[\frac{\log x + 1}{x} \right]_{1/e}^1 = \frac{1}{1} - \frac{0}{1/e} = 1$.

Step 4: Part from 1 to e

$$\left[-\frac{\log x + 1}{x} \right]_1^e = -\frac{2}{e} + 1$$

Step 5: Total

$$1 + 1 - \frac{2}{e} = 2 - \frac{2}{e}$$

This is option (C).

Final Answer:

The two halves give 1 and $1 - 2/e$, so the integral is $2 - 2/e$, option (C).

$$2 - \frac{2}{e}$$

Quick Tip: Split the interval at $x = 1$ where $\log x$ changes sign, and integrate $\log x$ over x squared by parts.



133. If $\int_{-\pi/2}^{\pi/2} (\sin^2 x + \sin^3 x) dx = k$, then the value of k

(A) 0

(B) 1

(C) π

(D) $\frac{\pi}{2}$

Correct Answer: (D) $\frac{\pi}{2}$

Solution:

Step 1: Understanding the Concept

For an interval symmetric about zero, an odd function integrates to zero and an even function integrates to twice the integral over half the interval.

Step 2: Split

$$k = \int_{-\pi/2}^{\pi/2} \sin^2 x \, dx + \int_{-\pi/2}^{\pi/2} \sin^3 x \, dx$$

Step 3: The sin cubed part

$\sin^3(-x) = -\sin^3 x$, so it is odd and its integral is 0.

Step 4: The sin squared part

$\sin^2 x$ is even:

$$\int_{-\pi/2}^{\pi/2} \sin^2 x \, dx = 2 \int_0^{\pi/2} \frac{1 - \cos 2x}{2} dx = \left[x - \frac{\sin 2x}{2} \right]_0^{\pi/2} = \frac{\pi}{2}$$

Step 5: Total

$$k = \frac{\pi}{2} + 0 = \frac{\pi}{2}$$

This is option (D).

Final Answer:

The sin cubed term vanishes by symmetry and the sin squared term gives $\pi/2$, option (D).

$$\boxed{\frac{\pi}{2}}$$

Quick Tip: The integral of an odd function over a symmetric interval is zero.

134. If $I_n = \int_0^{\pi/4} \tan^n x dx$, $n \in N$ then $I_{n+2} + I_n$ is equal to

- (A) $\frac{1}{n}$
- (B) $\frac{1}{n+1}$
- (C) $\frac{1}{n-1}$
- (D) $\frac{1}{n-2}$

Correct Answer: (B) $\frac{1}{n+1}$

Solution:

Step 1: Understanding the Concept

Combine the two integrals under one integral sign.

Step 2: Add

$$I_n + I_{n+2} = \int_0^{\pi/4} \tan^n x (1 + \tan^2 x) dx = \int_0^{\pi/4} \tan^n x \sec^2 x dx$$

Step 3: Substitute

Let $t = \tan x$, $dt = \sec^2 x dx$. When $x = 0$, $t = 0$. When $x = \pi/4$, $t = 1$.

$$\int_0^1 t^n dt = \frac{1}{n+1}$$

Step 4: Conclusion

$I_{n+2} + I_n = \frac{1}{n+1}$, option (B). The other options have different denominators that do not come from $\int t^n dt$.

Final Answer:

Adding the integrals gives the integral of t^n from 0 to 1, which is $1/(n+1)$, option (B).

$$\boxed{\frac{1}{n+1}}$$

Quick Tip: Add the two integrals and use sec squared as the derivative of tan.



135. The area in square units of the region bounded by the curve $y = \sqrt{16 - x^2}$ and lines $x = 0, x = 4$ above the X-axis is

- (A) 16π
- (B) 12π
- (C) 8π
- (D) 4π

Correct Answer: (D) 4π

Solution:

Step 1: Understanding the Concept

$y = \sqrt{16 - x^2}$ is the upper half of the circle $x^2 + y^2 = 16$ of radius 4. The lines $x = 0$ and $x = 4$ cut out the first quadrant part.

Step 2: Area by integration

$$A = \int_0^4 \sqrt{16 - x^2} dx = \left[\frac{x}{2} \sqrt{16 - x^2} + 8 \sin^{-1} \frac{x}{4} \right]_0^4$$

$$= 0 + 8 \cdot \frac{\pi}{2} - 0 = 4\pi$$

Step 3: Check by geometry

The area of the full circle is $\pi r^2 = 16\pi$. One quarter is 4π . Both methods agree, so the answer is option (D). The options 16π , 12π and 8π correspond to the whole circle, three quarters and a half.

Final Answer:

The region is a quarter of a circle of radius 4, so its area is 4 pi, option (D).

$$\boxed{4\pi}$$

Quick Tip: The curve $y = \text{root of } (16 - x^2)$ is the upper half of a circle of radius 4.

136. The general solution of the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2 + 5$ is

- (A) $\frac{x^4}{4} + \frac{5x^2}{2} - xy = c$
- (B) $\frac{x^4}{4} - \frac{5x^2}{2} - xy = c$
- (C) $\frac{x^4}{4} - \frac{5x^2}{2} + xy = c$
- (D) $\frac{x^4}{4} + \frac{5x^2}{2} + xy = c$

Correct Answer: (A) $\frac{x^4}{4} + \frac{5x^2}{2} - xy = c$

Solution:

Step 1: Understanding the Concept

The equation is linear: $\frac{dy}{dx} + P y = Q$ with $P = \frac{1}{x}$ and $Q = x^2 + 5$.

Step 2: Integrating factor

$$\text{I.F.} = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$$

Step 3: Multiply through

$$x \frac{dy}{dx} + y = x^3 + 5x \Rightarrow \frac{d}{dx}(xy) = x^3 + 5x$$

Step 4: Integrate

$$xy = \frac{x^4}{4} + \frac{5x^2}{2} + c'$$

Step 5: Rearrange

$$\frac{x^4}{4} + \frac{5x^2}{2} - xy = c$$

This is option (A). Options with a minus sign on $5x^2/2$ have the wrong sign from the integral of $5x$.

Final Answer:

The integrating factor is x and the solution is $x^4/4 + 5x^2/2 - xy = c$, option (A).

$$\frac{x^4}{4} + \frac{5x^2}{2} - xy = c$$

Quick Tip: Multiply by the integrating factor x to make the left side a perfect derivative.

137. The order and degree of the differential equation $\sqrt{1 + \frac{1}{(\frac{dy}{dx})^2}} = (\frac{d^2y}{dx^2})^{\frac{3}{2}}$, respectively are

- (A) 2, 1
- (B) 2, 3
- (C) 1, 2
- (D) 3, 2

Correct Answer: (B) 2, 3

Solution:

Step 1: Understanding the Concept

Order is the order of the highest derivative. Degree is the power of that highest derivative after the equation is made a polynomial in the derivatives (free of fractional powers and roots).

Step 2: Remove the roots

$$\sqrt{1 + \frac{1}{(y')^2}} = (y'')^{3/2}$$

Square both sides:

$$1 + \frac{1}{(y')^2} = (y'')^3$$

Multiply by $(y')^2$:

$$(y')^2 + 1 = (y')^2(y'')^3$$

Step 3: Read off

The highest derivative is y'' , so the order is 2. Its power is 3, so the degree is 3.

Step 4: Conclusion

The order and degree are 2 and 3, option (B).

Final Answer:

After clearing the roots the highest derivative is of order 2 and has power 3, option (B).

2, 3

Quick Tip: Remove the radicals by squaring, then read the highest derivative and its power.



138. Let $\vec{a} = 2\hat{i} + \hat{k}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$, and $\vec{c} = 4\hat{i} - 3\hat{j} + 7\hat{k}$. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r} \cdot \vec{a} = 0$, Then $\vec{r} \cdot \vec{c} =$

- (A) -14
- (B) 34
- (C) -7
- (D) 20

Correct Answer: (B) 34

Solution:

Step 1: Understanding the Concept

$\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ means $(\vec{r} - \vec{c}) \times \vec{b} = \vec{0}$. So $\vec{r} - \vec{c}$ is parallel to $\vec{b}$.

Step 2: Write r

$$\vec{r} = \vec{c} + \lambda \vec{b}$$

Step 3: Use the dot condition

$$\vec{r} \cdot \vec{a} = 0 \Rightarrow \vec{c} \cdot \vec{a} + \lambda \vec{b} \cdot \vec{a} = 0$$

$\vec{c} \cdot \vec{a} = 8 + 7 = 15$ and $\vec{b} \cdot \vec{a} = 2 + 1 = 3$. So $15 + 3\lambda = 0$, $\lambda = -5$.

Step 4: Find r

$$\vec{r} = (4, -3, 7) - 5(1, 1, 1) = (-1, -8, 2)$$

Step 5: Required dot product

$$\vec{r} \cdot \vec{c} = (-1)(4) + (-8)(-3) + (2)(7) = -4 + 24 + 14 = 34$$

This is option (B).

Final Answer:

The vector $\mathbf{r}$ equals $\mathbf{c} - 5\mathbf{b}$, and its dot product with $\mathbf{c}$ is 34, option (B).

34

Quick Tip: From $\mathbf{r} \times \mathbf{b} = \mathbf{c} \times \mathbf{b}$ we get $\mathbf{r} - \mathbf{c}$ parallel to $\mathbf{b}$.

139. If $\vec{u} = \hat{i} + 2\hat{j} - 2\hat{k}$, $\vec{v} = 2\hat{i} + \hat{k}$ and $\vec{w}$ is unit vector then the maximum value of scalar triple product $[\vec{u}\vec{v}\vec{w}]$ is

- (A) $-3\sqrt{5}$
- (B) 0
- (C) $3\sqrt{5}$
- (D) $\sqrt{54}$

Correct Answer: (C) $3\sqrt{5}$

Solution:

Step 1: Understanding the Concept

$[\vec{u} \vec{v} \vec{w}] = (\vec{u} \times \vec{v}) \cdot \vec{w}$. For a unit vector $\vec{w}$, this is largest when $\vec{w}$ points along $\vec{u} \times \vec{v}$, and the maximum equals $|\vec{u} \times \vec{v}|$.

Step 2: Cross product

$$\vec{u} = (1, 2, -2), \vec{v} = (2, 0, 1).$$

$$\vec{u} \times \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & -2 \\ 2 & 0 & 1 \end{vmatrix} = (2, -5, -4)$$

Step 3: Magnitude

$$|\vec{u} \times \vec{v}| = \sqrt{4 + 25 + 16} = \sqrt{45} = 3\sqrt{5}$$

Step 4: Conclusion

The maximum value is $3\sqrt{5}$, option (C). The value $-3\sqrt{5}$ is the minimum, and $\sqrt{54}$ does not match the cross product magnitude.

Final Answer:

The maximum equals the length of $\vec{u} \times \vec{v}$, which is $3\sqrt{5}$, option (C).

$$\boxed{3\sqrt{5}}$$

Quick Tip: The scalar triple product is largest when $\vec{w}$ is parallel to the cross product of $\vec{u}$ and $\vec{v}$.

140. If $\vec{a}, \vec{b}, \vec{c}$ are three non-coplanar vectors and $\vec{p}, \vec{q}, \vec{r}$ are defined as $\vec{p} = \frac{\vec{b} \times \vec{c}}{[\vec{a}\vec{b}\vec{c}]}$, $\vec{q} = \frac{\vec{c} \times \vec{a}}{[\vec{a}\vec{b}\vec{c}]}$, $\vec{r} = \frac{\vec{a} \times \vec{b}}{[\vec{a}\vec{b}\vec{c}]}$, then $[(\vec{a} + \vec{b}) \cdot \vec{p} + (\vec{b} + \vec{c}) \cdot \vec{q} + (\vec{c} + \vec{a}) \cdot \vec{r}]$ is equal to

- (A) 0
(B) 1
(C) 2
(D) 3

Correct Answer: (D) 3

Solution:

Step 1: Understanding the Concept

The vectors $\vec{p}, \vec{q}, \vec{r}$ are the reciprocal system to $\vec{a}, \vec{b}, \vec{c}$. They satisfy

$$\vec{a} \cdot \vec{p} = \vec{b} \cdot \vec{q} = \vec{c} \cdot \vec{r} = 1, \quad \vec{b} \cdot \vec{p} = \vec{c} \cdot \vec{p} = \vec{a} \cdot \vec{q} = \vec{c} \cdot \vec{q} = \vec{a} \cdot \vec{r} = \vec{b} \cdot \vec{r} = 0$$

The zeros arise because $\vec{b} \times \vec{c}$ is perpendicular to $\vec{b}$ and $\vec{c}$.

Step 2: Evaluate each term

$$(\vec{a} + \vec{b}) \cdot \vec{p} = 1 + 0 = 1.$$

$$(\vec{b} + \vec{c}) \cdot \vec{q} = 1 + 0 = 1.$$

$$(\vec{c} + \vec{a}) \cdot \vec{r} = 1 + 0 = 1.$$

Step 3: Sum

$$1 + 1 + 1 = 3$$

This is option (D).

Final Answer:

Each bracket equals 1 for the reciprocal system, so the sum is 3, option (D).

Quick Tip: The vectors $\mathbf{p}$, $\mathbf{q}$, $\mathbf{r}$ form the reciprocal system, so $\mathbf{a} \cdot \mathbf{p}$ equals 1 and $\mathbf{b} \cdot \mathbf{p}$ equals 0.

141. The volume of the parallelepiped whose coterminous edges are $2\hat{i} + \hat{j} - \hat{k}$, $3\hat{i} - \hat{j} - \hat{k}$, $\hat{j} + 3\hat{k}$ is

- (A) 16 cu. units
- (B) 6 cu. units
- (C) 2 cu. units
- (D) 12 cu. units

Correct Answer: (A) 16 cu. units

Solution:

Step 1: Understanding the Concept

The volume of a parallelepiped with coterminous edges $\vec{a}, \vec{b}, \vec{c}$ is $||[\vec{a} \ \vec{b} \ \vec{c}]||$, the absolute value of the determinant of their components.

Step 2: Determinant

$$\begin{vmatrix} 2 & 1 & -1 \\ 3 & -1 & -1 \\ 0 & 1 & 3 \end{vmatrix}$$

Expand along the first row:

$$2((-1)(3) - (-1)(1)) - 1(3 \cdot 3 - (-1) \cdot 0) + (-1)(3 \cdot 1 - (-1) \cdot 0)$$

$$= 2(-3 + 1) - 9 - 3 = -4 - 9 - 3 = -16$$

Step 3: Volume

$$V = |-16| = 16 \text{ cubic units}$$

This is option (A).

Final Answer:

The determinant of the three edges is -16, so the volume is 16 cubic units, option (A).

16 cu. units

Quick Tip: The volume of a parallelepiped equals the absolute value of the determinant of its edge vectors.

142. Given the following expression

- A) $(\vec{a} \times \vec{b}) \cdot \vec{c}$
- B) $\vec{a} \times (\vec{b} \cdot \vec{c})$
- C) $\vec{a} \cdot (\vec{b} \cdot \vec{c})$
- D) $|\vec{a}|(\vec{b} \cdot \vec{c})$
- E) $(\vec{a} \cdot \vec{b}) \times (\vec{b} \cdot \vec{c})$

Then which of the following is not correct

- (A) B and E are meaningful
- (B) A and D are meaningful
- (C) B, C and E are meaningless
- (D) A is meaningful but B is meaningless

Correct Answer: (A) B and E are meaningful

Solution:

Step 1: Understanding the Concept

A dot product of two vectors is a scalar. A cross product is defined only between two vectors. So a cross product of a vector with a scalar, or of two scalars, has no meaning.

Step 2: Check each expression

A) $(\vec{a} \times \vec{b}) \cdot \vec{c}$: cross first gives a vector, then dot with $\vec{c}$ gives a scalar. Meaningful.

B) $\vec{a} \times (\vec{b} \cdot \vec{c})$: $\vec{b} \cdot \vec{c}$ is a scalar, and a vector cannot be crossed with a scalar. Meaningless.

C) $\vec{a} \cdot (\vec{b} \cdot \vec{c})$: a vector dotted with a scalar. Meaningless.

D) $|\vec{a}|(\vec{b} \cdot \vec{c})$: scalar times scalar. Meaningful.

E) $(\vec{a} \cdot \vec{b}) \times (\vec{b} \cdot \vec{c})$: cross product of two scalars. Meaningless.

Step 3: Compare with the options

The statement "B and E are meaningful" is not correct, because both are meaningless. That is option (A).

Final Answer:

Expressions B and E are meaningless, so the claim that they are meaningful is the incorrect one, option (A).

(A)

Quick Tip: A cross product needs two vectors and a dot product gives a scalar; check that each operation is defined.

143. If the perpendicular distance of the plane passing through the point $Q(1, 0, -1)$ and containing the line $\vec{r} = (\hat{i} - 3\hat{j} + \hat{k}) + \lambda(2\hat{i} - 2\hat{j} + \hat{k})$ from origin is $\frac{p}{\sqrt{53}}$ then $p = \dots$

- (A) 4
- (B) 1
- (C) 5
- (D) 9

Correct Answer: (C) 5

Solution:

Step 1: Understanding the Concept

The plane contains the line with point $P(1, -3, 1)$ and direction $\vec{d} = (2, -2, 1)$, and also passes through $Q(1, 0, -1)$.

Step 2: Normal vector

$$\overrightarrow{PQ} = (0, 3, -2)$$

$$\vec{n} = \vec{d} \times \overrightarrow{PQ} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -2 & 1 \\ 0 & 3 & -2 \end{vmatrix} = (1, 4, 6)$$

Step 3: Equation of the plane

Through $Q(1, 0, -1)$:

$$1(x - 1) + 4y + 6(z + 1) = 0 \Rightarrow x + 4y + 6z + 5 = 0$$

Step 4: Distance from the origin

$$\frac{|5|}{\sqrt{1 + 16 + 36}} = \frac{5}{\sqrt{53}}$$

Comparing with $\frac{p}{\sqrt{53}}$ gives $p = 5$, option (C).

Final Answer:

The plane is $x + 4y + 6z + 5 = 0$, which is 5 over root 53 from the origin, so $p = 5$, option (C).

5

Quick Tip: Find the normal as the cross product of the line direction and a vector joining a point on the line to Q.

144. If the line joining points $(2, 1, 4)$ and $(a - 1, 4, -1)$ is parallel to the line joining points $(0, 2, b - 1)$ and $(5, 3, -2)$ then the values of b and a are respectively

- (A) $18, \frac{2}{3}$
- (B) $\frac{3}{2}, 18$
- (C) $\frac{2}{3}, 18$
- (D) $-\frac{2}{3}, 18$

Correct Answer: (C) $\frac{2}{3}, 18$

Solution:

Step 1: Understanding the Concept

Two lines are parallel when their direction ratios are proportional.

Step 2: Direction ratios

Line 1 through $(2, 1, 4)$ and $(a - 1, 4, -1)$: $(a - 3, 3, -5)$.

Line 2 through $(0, 2, b - 1)$ and $(5, 3, -2)$: $(5, 1, -1 - b)$.

Step 3: Proportion

$$\frac{a - 3}{5} = \frac{3}{1} = \frac{-5}{-1 - b}$$

Step 4: Solve for a

$$a - 3 = 15 \Rightarrow a = 18$$

Step 5: Solve for b

$$\frac{-5}{-1 - b} = 3 \Rightarrow -1 - b = -\frac{5}{3} \Rightarrow b = \frac{2}{3}$$

So $b = \frac{2}{3}$ and $a = 18$, option (C). The order in the options is b first, then a.

Final Answer:

Proportional direction ratios give $a = 18$ and $b = 2/3$, option (C).

$$b = \frac{2}{3}, a = 18$$

Quick Tip: Parallel lines have proportional direction ratios.

145. Lines $\vec{r} = \vec{a} + \lambda\vec{b}$ and $\vec{r} = \vec{b} + \mu\vec{a}$ intersect at point $(2, 4, -4)$. If $|\vec{a} - \vec{b}| = 4$, then $\vec{a} \cdot \vec{b} =$

- (A) 5
- (B) 10
- (C) -5
- (D) -10

Correct Answer: (A) 5

Solution:

Step 1: Understanding the Concept

At the common point, $\vec{a} + \lambda\vec{b} = \vec{b} + \mu\vec{a}$. Rearranged, $(1 - \mu)\vec{a} = (1 - \lambda)\vec{b}$.

Step 2: Independence

The vectors $\vec{a}$ and $\vec{b}$ are not parallel (otherwise the lines would not be

distinct lines in general). So both coefficients must vanish: $\mu = 1$ and $\lambda = 1$.

Step 3: Point of intersection

$$\vec{a} + \vec{b} = (2, 4, -4) \Rightarrow |\vec{a} + \vec{b}|^2 = 4 + 16 + 16 = 36$$

Step 4: Use the given length

$$|\vec{a} - \vec{b}|^2 = 16$$

Step 5: Subtract

$$|\vec{a} + \vec{b}|^2 - |\vec{a} - \vec{b}|^2 = 4 \vec{a} \cdot \vec{b}$$

$$36 - 16 = 20 = 4 \vec{a} \cdot \vec{b} \Rightarrow \vec{a} \cdot \vec{b} = 5$$

This is option (A).

Final Answer:

The intersection point is $\vec{a} + \vec{b}$, so the squares of the sum and difference give $\vec{a} \cdot \vec{b} = 5$, option (A).

5

Quick Tip: At the intersection $\vec{a} + \lambda \vec{b} = \mu \vec{b} + \vec{a}$, which forces

lambda and mu to be 1 for non-parallel vectors.



146. The Cartesian equations of the line passing through $A(0, 1, 1)$ and parallel to the X-axis are

- (A) $y = 1, z = 1$
- (B) $x = 0$
- (C) $x + y + z = 2$
- (D) $y = z$

Correct Answer: (A) $y = 1, z = 1$

Solution:

Step 1: Understanding the Concept

A line parallel to the X-axis has direction ratios $(1, 0, 0)$. Along such a line only x changes.

Step 2: Write the equations

The line passes through $A(0, 1, 1)$, so

$$\frac{x - 0}{1} = \frac{y - 1}{0} = \frac{z - 1}{0}$$

This means $y - 1 = 0$ and $z - 1 = 0$, that is $y = 1, z = 1$.

Step 3: Check the options

(B) $x = 0$ is a plane, not a line. (C) $x + y + z = 2$ is a plane. (D) $y = z$ is also a plane. Only (A) gives two plane equations that together define the line.

Final Answer:

A line parallel to the X-axis through (0, 1, 1) is $y = 1$ and $z = 1$, option (A).

$$y = 1, z = 1$$

Quick Tip: A line parallel to the x-axis keeps y and z fixed.

147. The LPP maximize $z = 2x + 5y$ subject to $x + 3y \leq 6$, $2x + 6y \leq 18$, $x \geq 0$, $y \geq 0$ has

- (A) Unique solution
- (B) Infinite solutions
- (C) No solution
- (D) Unbounded feasible region

Correct Answer: (A) Unique solution

Solution:**Step 1: Understanding the Concept**

The maximum of a linear objective over a bounded feasible region occurs at a corner point.

Step 2: Simplify the constraints

$2x + 6y \leq 18$ is the same as $x + 3y \leq 9$. Since $x + 3y \leq 6$ is stronger, the second constraint is redundant.

Step 3: Corner points

With $x, y \geq 0$ the region is the triangle with vertices (0, 0), (6, 0) and

(0, 2).

Step 4: Evaluate $z = 2x + 5y$

At (0, 0): 0. At (6, 0): 12. At (0, 2): 10.

Step 5: Conclusion

The maximum is 12 at the single point (6, 0), so the problem has a unique solution, option (A). The region is bounded, so (D) is wrong, and the objective line is not parallel to an edge, so (B) is wrong.

Final Answer:

The maximum value 12 occurs at only one corner, (6, 0), so the solution is unique, option (A).

Unique solution

Quick Tip: Draw the feasible region, find its corners and compare z at each corner.

148. Four cards are drawn successively with replacement from well shuffled deck of 52 cards, then the probability that only two cards are club cards is

- (A) $\frac{26}{128}$
- (B) $\frac{24}{128}$
- (C) $\frac{27}{128}$
- (D) $\frac{28}{128}$

Correct Answer: (C) $\frac{27}{128}$

Solution:

Step 1: Understanding the Concept

Drawing with replacement makes the four draws independent. The chance of a club in one draw is $\frac{13}{52} = \frac{1}{4}$, and of a non-club is $\frac{3}{4}$.

Step 2: Binomial formula

$$P(X = 2) = {}^4C_2 \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^2$$

Step 3: Calculate

$${}^4C_2 = 6, \quad \left(\frac{1}{4}\right)^2 \left(\frac{3}{4}\right)^2 = \frac{9}{256}$$

$$P = 6 \times \frac{9}{256} = \frac{54}{256} = \frac{27}{128}$$

Step 4: Check the options

The value $\frac{27}{128}$ is option (C). The others have different numerators and do not follow from this formula.

Final Answer:

The probability is 6 times $(1/4)^2$ times $(3/4)^2$, which is $27/128$, option (C).

$$\boxed{\frac{27}{128}}$$

Quick Tip: Each draw is a club with probability $1/4$; use the binomial formula for exactly two successes in four trials.

149. For the following probability distribution of a random variable X , the Expected value and Variance of X are respectively

$X = x$	1	2	3
$P(X = x)$	$1/5$	$2/5$	$2/5$

- (A) $\frac{27}{5}, \frac{27}{25}$
(B) $\frac{11}{5}, \frac{14}{25}$
(C) $\frac{4}{5}, \frac{14}{25}$
(D) $\frac{7}{5}, \frac{11}{25}$

Correct Answer: (B) $\frac{11}{5}, \frac{14}{25}$

Solution:

Step 1: Understanding the Concept

For a discrete random variable the mean is $E(X) = \sum xP(x)$, and the variance is $E(X^2) - [E(X)]^2$.

Step 2: Mean

$$E(X) = 1 \cdot \frac{1}{5} + 2 \cdot \frac{2}{5} + 3 \cdot \frac{2}{5} = \frac{1 + 4 + 6}{5} = \frac{11}{5}$$

Step 3: Second moment

$$E(X^2) = 1 \cdot \frac{1}{5} + 4 \cdot \frac{2}{5} + 9 \cdot \frac{2}{5} = \frac{1 + 8 + 18}{5} = \frac{27}{5}$$

Step 4: Variance

$$\text{Var}(X) = \frac{27}{5} - \left(\frac{11}{5}\right)^2 = \frac{135}{25} - \frac{121}{25} = \frac{14}{25}$$

Step 5: Check

The pair $\left(\frac{11}{5}, \frac{14}{25}\right)$ is option (B). Option (A) gives the second moment as the mean.

Final Answer:

The mean is $11/5$ and the variance is $14/25$, option (B).

$\frac{11}{5}, \frac{14}{25}$

Quick Tip: Use $E(X) = \sum x p$ and $\text{Var}(X) = E(X^2) - E(X)^2$.

150. A fair die is rolled indefinitely. Player A wins if two consecutive rolls show 3 or 5, and player B wins if two consecutive rolls show 1 or 2 or 4 or 6. The probability that player A wins in the long run is

- (A) $\frac{2}{3}$
- (B) $\frac{5}{21}$
- (C) $\frac{1}{7}$
- (D) $\frac{2}{21}$

Correct Answer: (B) $\frac{5}{21}$

Solution:

Step 1: Understanding the Concept

Call a roll of 3 or 5 an A-roll (probability $\frac{1}{3}$) and any other roll a B-roll (probability $\frac{2}{3}$). Player A wins on two A-rolls in a row. Player B wins on two B-rolls in a row.

Step 2: States

Let a be the probability that A eventually wins when the last roll was an A-roll, and b when the last roll was a B-roll.

$$a = \frac{1}{3} \cdot 1 + \frac{2}{3} b, \quad b = \frac{2}{3} \cdot 0 + \frac{1}{3} a$$

Step 3: Solve

$$b = \frac{a}{3} \Rightarrow a = \frac{1}{3} + \frac{2a}{9} \Rightarrow \frac{7a}{9} = \frac{1}{3} \Rightarrow a = \frac{3}{7}, \quad b = \frac{1}{7}$$

Step 4: First roll

$$P(\text{A wins}) = \frac{1}{3}a + \frac{2}{3}b = \frac{1}{7} + \frac{2}{21} = \frac{5}{21}$$

This is option (B).

Final Answer:

Solving the two recursion equations gives $5/21$ for the chance that A wins, option (B).

$$\frac{5}{21}$$

Quick Tip: Set up two equations for A winning given the type of the last roll.