CBSE Class 10 Mathematics Standard 2026-27

Sample Paper with Solutions

Conducted by the Central Board of Secondary Education (CBSE)

Time Allowed: 3 hours **Maximum Marks: 80**

General Instructions

(i) This Question paper contains **38 questions**. All questions are compulsory.

(ii) This Question paper is divided into **five Sections - A, B, C, D and E**.

(iii) In **Section A**, Question number **1 to 18** are Multiple Choice Questions (MCQs) and Question number **19 and 20** are Assertion - Reason based questions of **1 mark** each.

(iv) In **Section B**, Question number **21 to 25** are Very Short Answer (VSA)-type questions, carrying **2 marks** each.

(v) In **Section C**, Question number **26 to 31** are Short Answer (SA)-type questions carrying **3 marks** each.

(vi) In **Section D**, Question number **32 to 35** are Long Answer (LA)-type questions carrying **5 marks** each.

(vii) In **Section E**, Question number **36 to 38** are case study-based questions carrying **4 marks** each.

(viii) There is no overall choice. However, an internal choice has been provided in **2 questions in Section B, 2 questions in Section C and 2 questions in Section D**. An internal choice has been provided in all the sub parts having **2 marks** of questions in Section E.

(ix) Draw neat and clean figures wherever required. Take $\pi = 22/7$ wherever required, if not stated.

(x) Use of calculators is not allowed.

Section A (20 marks)

This section comprises of 18 multiple choice questions and two questions of assertion and reasoning type of 1 mark each.

1. The greatest number which divides both 134 and 188, leaving remainders 4 and 6 respectively, is: [1 mark]

(A) 13

(B) 26

(C) 39

(D) 65

Correct Answer: (B) 26

Solution:

Concept:

- Let the required number be n. When n divides 134 and leaves remainder 4, it means n divides $134 - 4 = 130$ exactly.
- In the same way, n divides $188 - 6 = 182$ exactly.
- So n is a common divisor of 130 and 182. The greatest such number is their HCF.

Step 1: Find the HCF by prime factorisation $130 = 2 \times 5 \times 13$
$182 = 2 \times 7 \times 13$
The common prime factors are 2 and 13, so $\text{HCF}(130, 182) = 2 \times 13 = 26$.

Step 2: Check the condition on the remainder A remainder must be smaller than the divisor. The remainders are 4 and 6, and $26 > 6$, so 26 is allowed.
Check: $134 = 26 \times 5 + 4$ and $188 = 26 \times 7 + 6$. Both leave the required remainders.

Step 3: Why the other options are wrong Option (A) 13: it divides 130 and 182, but it is only a common factor, not the greatest one. 26 is larger.
Option (C) 39: $130 = 39 \times 3 + 13$, so 39 does not divide 130 exactly.
Option (D) 65: $182 = 65 \times 2 + 52$, so 65 does not divide 182 exactly.

Step 4: Conclusion The greatest number is the HCF of 130 and 182. This is option (B).

Quick Tip to Solve: If a number leaves remainder 4 with 134, it divides $134 - 4$ exactly. Do the same for 188 and take the HCF.

2. If $f(x) = px^2 + qx + r$, $p \neq 0$ and $p + r = q$, then one of the zeroes of $f(x)$ is: [1 mark]

(A) $\frac{q}{p}$

(B) $\frac{r}{p}$

(C) $-\frac{r}{p}$

(D) $-\frac{q}{p}$

Correct Answer: (C) $-\frac{r}{p}$

Solution:

Step 1: Use the given condition We are given $p + r = q$. Replace q in the polynomial by $p + r$.

$$f(x) = px^2 + (p + r)x + r$$

Step 2: Factorise by splitting the middle term

$$f(x) = px^2 + px + rx + r$$

Take px common from the first two terms and r from the last two terms.

$$f(x) = px(x + 1) + r(x + 1) = (x + 1)(px + r)$$

Step 3: Find the zeroes A zero is a value of x that makes $f(x) = 0$. So we put each factor equal to zero.
$x + 1 = 0$ gives $x = -1$.
$px + r = 0$ gives $x = -\frac{r}{p}$, which is allowed because $p \neq 0$.

Step 4: Check the options with an example Take $p = 1$ and $r = 2$, so $q = 3$ and $f(x) = x^2 + 3x + 2 = (x + 1)(x + 2)$. The zeroes are -1 and -2.
Here $\frac{q}{p} = 3$, $\frac{r}{p} = 2$, $-\frac{r}{p} = -2$ and $-\frac{q}{p} = -3$.
Only $-\frac{r}{p}$ is a zero. So options (A), (B) and (D) are wrong.

Step 5: Conclusion One zero of $f(x)$ is -1 and the other is $-\frac{r}{p}$. The option given is (C).

Quick Tip to Solve: Replace q by $p + r$ and factorise the quadratic, or check what $f(-1)$ is.

3. Tarun correctly solved a pair of linear equations in two variables and found their only point of intersection as $(5, -1)$. One of the lines was $x - y = 6$.
Which of the following could have been the other line?
I: $3x - 3y = 18$
II: $2x - 3y = 13$
III: $2x - 3y = 16$ [1 mark]

(A) I only

(B) II only

(C) I and II

(D) II and III

Correct Answer: (B) II only

Solution:

Concept:

- The two lines meet at exactly one point, $(5, -1)$. So the other line must pass through $(5, -1)$, and it must not be the same line as $x - y = 6$.
- If it were the same line, the lines would meet at every point, not at only one point.

Step 1: Check which lines pass through $(5, -1)$ Line I: $3(5) - 3(-1) = 15 + 3 = 18$. It passes through the point.

Line II: $2(5) - 3(-1) = 10 + 3 = 13$. It passes through the point.
Line III: $2(5) - 3(-1) = 10 + 3 = 13$, but the right side is 16. It does not pass through the point.

Step 2: Reject the line that is the same as x - y = 6 Divide Line I by 3: $3x - 3y = 18$ becomes $x - y = 6$. This is the same line as the given one.
So with Line I the system has infinitely many solutions (coincident lines). That is not allowed.

Step 3: Confirm Line II Compare $x - y = 6$ and $2x - 3y = 13$: $\frac{a_1}{a_2} = \frac{1}{2}$ and $\frac{b_1}{b_2} = \frac{-1}{-3} = \frac{1}{3}$.
Since $\frac{1}{2} \neq \frac{1}{3}$, the lines intersect at exactly one point, and we saw that point is $(5, -1)$.

Step 4: Conclusion Only Line II can be the other line. This is option (B).

Quick Tip to Solve: The other line must pass through $(5, -1)$ and must not be the same line as $x - y = 6$.

4. If $(1 - p)$ is a root of the quadratic equation $x^2 + px + 1 - p = 0$, then its roots are: [1 mark]

(A) 0, 1

(B) -1, 1

(C) 0, -1

(D) -1, 2

Correct Answer: (C) 0, -1

Solution:

Step 1: Use the fact that 1 - p is a root A root satisfies the equation. So we put $x = 1 - p$ in $x^2 + px + 1 - p = 0$.

$$(1 - p)^2 + p(1 - p) + 1 - p = 0$$

Step 2: Simplify Expand each part: $(1 - p)^2 = 1 - 2p + p^2$ and $p(1 - p) = p - p^2$.

$$1 - 2p + p^2 + p - p^2 + 1 - p = 0$$

The p^2 terms cancel. The p terms give $-2p + p - p = -2p$. The numbers give $1 + 1 = 2$.

$$2 - 2p = 0 \Rightarrow p = 1$$

Step 3: Write the equation and solve it Put $p = 1$: $x^2 + x + 1 - 1 = 0$, that is $x^2 + x = 0$.

$$x(x + 1) = 0 \Rightarrow x = 0 \text{ or } x = -1$$

Check: $1 - p = 0$ is indeed one of these roots.

Step 4: Why the other options are wrong With $p = 1$, the sum of roots is $-p = -1$ and the product is $1 - p = 0$.
Option (A) 0, 1 has sum 1. Option (B) -1, 1 has sum 0 and product -1. Option (D) -1, 2 has sum 1 and product -2. None matches.

Step 5: Conclusion The roots are 0 and -1. This is option (C).

Quick Tip to Solve: Put $x = 1 - p$ in the equation to find p, then solve the equation.

5. The middle term of the A.P.: 10, 7, 4, ..., -62 is: [1 mark]

(A) -26

(B) -29

(C) -32

(D) -35

Correct Answer: (A) -26

Solution:

Step 1: Note the first term and common difference The A.P. is 10, 7, 4, ..., -62. The first term is $a = 10$.
The common difference is $d = 7 - 10 = -3$. The last term is -62.

Step 2: Find the number of terms Use $a_n = a + (n-1)d$ with $a_n = -62$.

$$-62 = 10 + (n-1)(-3)$$

$$-72 = -3(n-1) \Rightarrow n - 1 = 24 \Rightarrow n = 25$$

Step 3: Locate the middle term There are 25 terms, which is odd, so there is exactly one middle term. It is the $\dfrac{25+1}{2} = 13$th term.

Step 4: Find the 13th term

$$a_{13} = 10 + (13-1)(-3) = 10 - 36 = -26$$

Step 5: Why the other options are wrong The next terms of the A.P. after -26 are -29, -32, -35. They are the 14th, 15th and 16th terms. So options (B), (C) and (D) are not the middle term.

Step 6: Conclusion The middle term is the 13th term, which is -26. This is option (A).

Quick Tip to Solve: First find the number of terms using $a_n = a + (n-1)d$, then pick the middle position.

6. The perimeters of two similar triangles are 56 cm and 70 cm respectively. If one side of the first triangle is 14 cm, then the corresponding side of the second triangle (in cm) is: [1 mark]

(A) 5

(B) 7.5

(C) 10

(D) 17.5

Correct Answer: (D) 17.5

Solution:

Step 1: Recall the rule for similar triangles For similar triangles, the ratio of their perimeters is equal to the ratio of any pair of corresponding sides.

Step 2: Set up the proportion Let the corresponding side of the second triangle be x cm.

$$\frac{56}{70} = \frac{14}{x}$$

Step 3: Solve for x Cross multiply: $56x = 14 \times 70 = 980$.

$$x = \frac{980}{56} = 17.5$$

Step 4: Why the other options are wrong The second triangle has the larger perimeter (70 cm against 56 cm), so its sides are longer than those of the first triangle. The corresponding side must be more than 14 cm.
Options (A) 5, (B) 7.5 and (C) 10 are all less than 14, so they cannot be right.

Step 5: Conclusion The corresponding side is 17.5 cm. This is option (D).

Quick Tip to Solve: In similar triangles the ratio of perimeters equals the ratio of corresponding sides.

7. The point which lies on the perpendicular bisector of the line segment joining the points A $(-3, -4)$ and B $(3, 4)$ is: [1 mark]

(A) $(0, 0)$

(B) $(0, 3)$

(C) $(3, 0)$

(D) $(0, 4)$

Correct Answer: (A) $(0, 0)$

Solution:

Step 1: Recall the property of a perpendicular bisector Every point on the perpendicular bisector of a segment is at the same distance from both end points. The midpoint of the segment also lies on it.

Step 2: Find the midpoint of AB $A(-3, -4)$ and $B(3, 4)$.

$$\left(\frac{-3+3}{2}, \frac{-4+4}{2}\right) = (0, 0)$$

So the point $(0, 0)$ is on the perpendicular bisector.

Step 3: Check the other options with the distance formula Option (B) $(0, 3)$: $PA = \sqrt{9+49} = \sqrt{58}$ and $PB = \sqrt{9+1} = \sqrt{10}$. Not equal.
Option (C) $(3, 0)$: $PA = \sqrt{36+16} = \sqrt{52}$ and $PB = \sqrt{0+16} = 4$. Not equal.
Option (D) $(0, 4)$: $PA = \sqrt{9+64} = \sqrt{73}$ and $PB = \sqrt{9+0} = 3$. Not equal.

Step 4: Conclusion Only $(0, 0)$ lies on the perpendicular bisector. This is option (A).

Quick Tip to Solve: The midpoint of a segment always lies on its perpendicular bisector.

8. If $\tan(A+B)=\sqrt{3}$ and $\tan(A-B)=\frac{1}{\sqrt{3}}$, $0^\circ<A+B<90^\circ$, $A>B$, then the value of A and B respectively are: [1 mark]

(A) $60^\circ, 30^\circ$

(B) $60^\circ, 45^\circ$

(C) $45^\circ, 15^\circ$

(D) $60^\circ, 15^\circ$

Correct Answer: (C) $45^\circ, 15^\circ$

Solution:

Concept:

- We are given the tangent of the sum $A+B$ and of the difference $A-B$.
- Both angles must lie between 0° and 90°.
- We first read off the two angles $A+B$ and $A-B$, and then solve for A and B.

Step 1: Find the angle A + B We know from the table of standard values that $\tan 60^\circ=\sqrt{3}$.
The given range is $0^\circ<A+B<90^\circ$, and tangent takes the value $\sqrt{3}$ only once in this range.
So $A+B=60^\circ$ ... (i)

Step 2: Find the angle A - B We know that $\tan 30^\circ=\frac{1}{\sqrt{3}}$.
Since $A>B$, the difference $A-B$ is positive and smaller than $A+B$, so it lies in the same range.
So $A-B=30^\circ$ ... (ii)

Step 3: Solve the two equations Add (i) and (ii). The B terms cancel:

$$(A+B)+(A-B)=60^\circ+30^\circ$$

$$2A=90^\circ \Rightarrow A=45^\circ$$

Put $A=45^\circ$ in (i): $45^\circ+B=60^\circ$, so $B=15^\circ$.
Check with (ii): $45^\circ-15^\circ=30^\circ$. This matches.

Step 4: Why the other options are wrong Option (A) gives $A+B=90^\circ$, which is outside the allowed range, and $\tan 90^\circ$ is not defined.
Option (B) gives $A+B=105^\circ$, which is outside the range.
Option (D) gives $A+B=75^\circ$ and $A-B=45^\circ$. Then $\tan 75^\circ\neq\sqrt{3}$ and $\tan 45^\circ=1\neq\frac{1}{\sqrt{3}}$.

Step 5: Conclusion A = 45 degrees and B = 15 degrees, which is option (C).

Quick Tip to Solve: Write $A+B$ and $A-B$ as standard angles using $\tan 60^\circ$ and $\tan 30^\circ$, then add and subtract the two equations.

9. T-shirts marked with numbers 4 to 99 are placed in a box. Gunika is fond of numbers. She randomly takes out a T-shirt from this box. The probability that she gets a T-shirt marked with a number that is either a perfect square or a perfect cube is: [1 mark]

(A) $\frac{1}{12}$

(B) $\frac{1}{32}$

(C) $\frac{11}{96}$

(D) $\frac{5}{48}$

Correct Answer: (D) $\frac{5}{48}$

Solution:

Step 1: Count all possible outcomes The T-shirts carry the numbers 4, 5, 6, ..., 99.
The count of numbers from 4 to 99 is $99 - 4 + 1 = 96$.
So total outcomes $= 96$.

Step 2: List the perfect squares from 4 to 99 $2^2, 3^2, \ldots, 9^2$ give 4, 9, 16, 25, 36, 49, 64, 81. That is 8 numbers.
$1^2 = 1$ is left out because 1 is not in the box, and $10^2 = 100$ is too big.

Step 3: List the perfect cubes from 4 to 99 $2^3 = 8$, $3^3 = 27$, $4^3 = 64$. That is 3 numbers.
$1^3 = 1$ is not in the box and $5^3 = 125$ is too big.

Step 4: Remove the number counted twice The number 64 appears in both lists, because $64 = 8^2 = 4^3$.
The numbers that are a square or a cube: $8 + 3 - 1 = 10$.
So favourable outcomes $= 10$.

Step 5: Find the probability

$$P = \frac{\text{favourable outcomes}}{\text{total outcomes}} = \frac{10}{96} = \frac{5}{48}$$

Step 6: Why the other options are wrong Option (A) is $\frac{8}{96}$. It counts only the squares.
Option (B) is $\frac{3}{96}$. It counts only the cubes.
Option (C) is $\frac{11}{96}$. It counts 64 twice, once as a square and once as a cube.

Step 7: Conclusion The probability is 5/48, which is option (D).

Quick Tip to Solve: Count the numbers from 4 to 99, then list the perfect squares and perfect cubes. Do not count 64 twice.

10. **The diameter of a car wheel is 21 cm. The number of complete revolutions it will make in moving 66 km is:** [1 mark]

(A) 10^4

(B) 10^5

(C) 10^6

(D) 10^7

Correct Answer: (B) 10^5

Solution:

Concept:

- In one full turn, a wheel covers a distance equal to its circumference.
- So the number of revolutions is the total distance divided by the circumference.

Step 1: Find the circumference Diameter $= 21$ cm, so radius $r = \frac{21}{2}$ cm.
Circumference $= 2\pi r = \pi \times 21$ cm.
Taking $\pi = \frac{22}{7}$:

$$\text{Circumference} = \frac{22}{7} \times 21 = 66 \text{ cm}$$

Step 2: Change the distance to cm 1 km = 1000 m and 1 m = 100 cm, so 1 km = 100000 cm.

$$66 \text{ km} = 66 \times 100000 \text{ cm} = 6600000 \text{ cm}$$

Step 3: Find the number of revolutions

$$\text{Revolutions} = \frac{6600000}{66} = 100000 = 10^5$$

Step 4: Why the other options are wrong Option (A) 10^4 is 10 times too small. It comes from using 1 km = 10000 cm by mistake.
Options (C) and (D) are 10 and 100 times too large. They come from adding extra zeros during the unit change.

Step 5: Conclusion The wheel makes 100000 complete revolutions, which is option (B).

Quick Tip to Solve: One revolution covers the circumference πd. Divide 66 km (in cm) by this length.

11. **Two cubes each of volume 64 cm^3 are joined end to end to form a solid. The surface area of the resultant cuboid is:** [1 mark]

(A) 192 cm^2

(B) 160 cm^2

(C) 96 cm^2

(D) 80 cm^2

Correct Answer: (B) 160 cm^2

Solution:

Step 1: Find the side of each cube Volume of a cube $= a^3$. So $a^3 = 64$, which gives $a = 4$ cm.

Step 2: Find the dimensions of the cuboid Joining two cubes end to end places them side by side. The length doubles and the other two edges stay the same.
Length $l = 4 + 4 = 8$ cm, breadth $b = 4$ cm, height $h = 4$ cm.

Step 3: Use the surface area formula Total surface area of a cuboid $= 2(lb + bh + hl)$.

Step 4: Put in the values

$$\text{TSA} = 2(8 \times 4 + 4 \times 4 + 4 \times 8)$$

$$= 2(32 + 16 + 32) = 2 \times 80 = 160 \text{ cm}^2$$

Step 5: Why the other options are wrong Option (A) 192 is 2×96. It adds the full surface areas of both cubes, but the two joined faces are hidden and must not be counted.
Option (C) 96 is the surface area of just one cube.
Option (D) 80 is only the sum $lb + bh + hl$. The factor 2 is missing.

Step 6: Conclusion The surface area of the cuboid is 160 square cm, which is option (B).

Quick Tip to Solve: First find the cube's edge from its volume. The joined solid is an $8 \times 4 \times 4$ cuboid.

12. The mean age of a combined group of men and women is 35 years. If the mean ages of the men and women are 38 years and 30 years respectively, then the percentage of women in the group is: [1 mark]

(A) 15

(B) 25.5

(C) 35

(D) 37.5

Correct Answer: (D) 37.5

Solution:

Step 1: Set up the symbols Let the number of men be x and the number of women be y.
The sum of the ages of the men is $38x$, and the sum of the ages of the women is $30y$.

Step 2: Write the mean of the combined group Mean of the whole group = total of all ages ÷ total number of people.

$$\frac{38x + 30y}{x + y} = 35$$

Step 3: Simplify the equation Multiply both sides by $(x + y)$:

$$38x + 30y = 35x + 35y$$

Move the x terms to one side and the y terms to the other:

$$38x - 35x = 35y - 30y \Rightarrow 3x = 5y$$

$$\frac{x}{y} = \frac{5}{3}$$

Step 4: Find the percentage of women Men : Women $= 5 : 3$, so the whole group has $5 + 3 = 8$ parts

and women are 3 of them.

$$\text{Percentage of women} = \frac{3}{8} \times 100 = 37.5$$

Step 5: Why the other options are wrong Option (A) 15 and option (B) 25.5 do not follow from $x : y = 5 : 3$.
Option (C) 35 is just the combined mean age, not a percentage.

Step 6: Conclusion 37.5 percent of the group are women, which is option (D).

Quick Tip to Solve: Write the combined mean as $\frac{38x + 30y}{x + y} = 35$ and find the ratio $x : y$.

13. Two dice are rolled simultaneously. The probability of getting number less than 4 on each die is: [1 mark]

(A) $\frac{1}{4}$

(B) $\frac{1}{9}$

(C) $\frac{1}{36}$

(D) $\frac{1}{6}$

Correct Answer: (A) $\frac{1}{4}$

Solution:

Step 1: Count all outcomes Each die has 6 faces. For two dice the total number of outcomes is $6 \times 6 = 36$.

Step 2: Decide what we want We want a number less than 4 on each die. So each die must show 1, 2 or 3.

Step 3: List the favourable outcomes (1, 1), (1, 2), (1, 3)
(2, 1), (2, 2), (2, 3)
(3, 1), (3, 2), (3, 3)
There are $3 \times 3 = 9$ favourable outcomes.

Step 4: Find the probability

$$P = \frac{9}{36} = \frac{1}{4}$$

Step 5: Why the other options are wrong Option (B) $\frac{1}{9}$ would need only 4 favourable outcomes. We found 9.
Option (C) $\frac{1}{36}$ is the chance of one particular pair such as (1, 1).
Option (D) $\frac{1}{6}$ would need 6 favourable outcomes, so it is also wrong.

Step 6: Conclusion The probability is 1/4, which is option (A).

Quick Tip to Solve: Total outcomes are 36. Each die must show 1, 2 or 3, so count the pairs.

14. PM is a median of Δ PQR with vertices P (5, -6), Q (6, 4) and R (0, 0). The length of PM is: [1 mark]

(A) $2\sqrt{17}$ units

(B) $2\sqrt{15}$ units

(C) $\sqrt{101}$ units

(D) 10 units

Correct Answer: (A) $2\sqrt{17}$ units

Solution:

Concept:

- A median joins a vertex to the mid-point of the opposite side.
- Here PM is the median from P, so M is the mid-point of QR.

Step 1: Find the mid-point M of QR Mid-point formula: $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$.

$$M = (\frac{6+0}{2}, \frac{4+0}{2}) = (3, 2)$$

Step 2: Use the distance formula for PM Distance between (x_1, y_1) and (x_2, y_2) is $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.

$$PM = \sqrt{(5-3)^2+(-6-2)^2}$$

$$= \sqrt{4+64} = \sqrt{68} = \sqrt{4\times 17} = 2\sqrt{17}$$

Step 3: Why the other options are wrong Option (B) $2\sqrt{15} = \sqrt{60}$, but we got $\sqrt{68}$.
Option (C) $\sqrt{101}$ is the length of the side PQ, since $PQ = \sqrt{1^2+10^2}$. It is a side, not the median.
Option (D) 10 would need $PM^2 = 100$, but $PM^2 = 68$.

Step 4: Conclusion The length of the median PM is $2\sqrt{17}$ units, which is option (A).

Quick Tip to Solve: Find the mid-point M of QR first, then use the distance formula between P and M.

15. The arc of a circle is of length 6π cm and the sector it bound has an area of 24π cm^2. The radius of the circle is: [1 mark]

(A) 4 cm

(B) 8 cm

(C) 16 cm

(D) 18 cm

Correct Answer: (B) 8 cm

Solution:

Step 1: Recall the formula for the area of a sector A sector is cut from a circle by two radii and an arc. If the arc has length l and the radius is r, the area of the sector is

$$A = \frac{1}{2} \times l \times r$$

This comes from $l = \frac{\theta}{360^\circ} \times 2\pi r$ and $A = \frac{\theta}{360^\circ} \times \pi r^2$. Dividing the second by the first gives $\frac{A}{l} = \frac{r}{2}$.

Step 2: Put in the given values The arc length is $l = 6\pi$ cm and the area is $A = 24\pi$ cm^2.

$$24\pi = \frac{1}{2} \times 6\pi \times r$$

$$24\pi = 3\pi r$$

Step 3: Solve for the radius Divide both sides by 3π.

$$r = \frac{24\pi}{3\pi} = 8 \text{ cm}$$

Step 4: Why the other options are wrong Put each radius back into $A = \frac{1}{2}lr$ with $l = 6\pi$.

For $r = 4$: $A = \frac{1}{2} \times 6\pi \times 4 = 12\pi$, not 24π. So (A) is wrong.

For $r = 16$: $A = \frac{1}{2} \times 6\pi \times 16 = 48\pi$. So (C) is wrong.

For $r = 18$: $A = \frac{1}{2} \times 6\pi \times 18 = 54\pi$. So (D) is wrong.

Only $r = 8$ gives $\frac{1}{2} \times 6\pi \times 8 = 24\pi$.

Step 5: Conclusion The radius of the circle is 8 cm, which is option (B).

Quick Tip to Solve: Use area of sector $= \frac{1}{2} \times$ arc length $\times$ radius.

16. The mean and median of the data are 35.5 and 32 respectively. The value of mode for this data is: [1 mark]

(A) 23

(B) 26

(C) 25

(D) 30

Correct Answer: (C) 25

Solution:

Step 1: Recall the relation between mean, median and mode For a moderately skewed distribution, the three averages are linked by the empirical relation

$$\text{Mode} = 3 \times \text{Median} - 2 \times \text{Mean}$$

We know the mean and the median, so we can find the mode.

Step 2: Put in the values Mean $= 35.5$ and median $= 32$.

$$\text{Mode} = 3(32) - 2(35.5)$$

$$3(32) = 96 \quad \text{and} \quad 2(35.5) = 71$$

Step 3: Subtract

$$\text{Mode} = 96 - 71 = 25$$

Step 4: Why the other options are wrong If the mode were 23, the relation would give mean $= \frac{3(32) - 23}{2} = 36.5$, not 35.5. So (A) is wrong.

If the mode were 26, mean $= \frac{96 - 26}{2} = 35$, not 35.5. So (B) is wrong.

If the mode were 30, mean $= \frac{96 - 30}{2} = 33$, not 35.5. So (D) is wrong.

Only mode 25 gives mean $= \frac{96 - 25}{2} = 35.5$.

Step 5: Conclusion The mode of the data is 25, which is option (C).

Quick Tip to Solve: Use Mode $= 3$ Median -2 Mean.

17. A flying kite is tied to a point on the ground with the help of a string. The string makes an angle θ with the ground level such that $\tan\theta = \frac{12}{5}$. If the length of the string is 52 m, then the height of the kite above the ground is: [1 mark]

(A) 40 m

(B) 45.5 m

(C) 48 m

(D) 50 m

Correct Answer: (C) 48 m

Solution:

Step 1: Draw the situation as a right triangle The kite is at height h above the ground. The string is the hypotenuse of a right triangle, with length 52 m. The string makes angle θ with the ground, so h is the side opposite to θ.

Step 2: Find sin of theta from tan of theta We have $\tan\theta = \frac{12}{5}$, so take opposite $= 12k$ and adjacent $= 5k$.

Hypotenuse $= \sqrt{(12k)^2 + (5k)^2} = \sqrt{144k^2 + 25k^2} = \sqrt{169k^2} = 13k$.

$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} = \frac{12k}{13k} = \frac{12}{13}$$

Step 3: Use the string length In the triangle, $\sin\theta = \frac{h}{52}$.

$$\frac{h}{52} = \frac{12}{13}$$

$$h = 52 \times \frac{12}{13} = 4 \times 12 = 48 \text{ m}$$

Step 4: Why the other options are wrong The correct ratio is $\frac{h}{52} = \frac{12}{13}$. For 40 m we would get $\frac{40}{52} = \frac{10}{13}$, for 45.5 m we would get $\frac{45.5}{52} = \frac{7}{8}$, and for 50 m we would get $\frac{50}{52} = \frac{25}{26}$.

None of these equals $\frac{12}{13}$, so (A), (B) and (D) are wrong.

Step 5: Conclusion The kite is 48 m above the ground, which is option (C).

Quick Tip to Solve: Find $\sin\theta$ from $\tan\theta = \frac{12}{5}$, then use $h = 52\sin\theta$.

18. 2 cards of diamonds and 4 cards of spades are missing from a pack of 52 cards. A card is drawn at random from this pack. The probability of getting a card of heart is: [1 mark]

(A) $\frac{13}{52}$

(B) $\frac{13}{46}$

(C) $\frac{11}{52}$

(D) $\frac{11}{46}$

Correct Answer: (B) $\frac{13}{46}$

Solution:

Step 1: Find how many cards are left A full pack has 52 cards. Two cards of diamonds and 4 cards of spades are missing.

$$52 - (2 + 4) = 46$$

So the new pack has 46 cards, and any one of them is equally likely to be drawn. The total number of outcomes is 46.

Step 2: Count the favourable cards The missing cards are diamonds and spades, not hearts. So all 13 hearts are still in the pack.
Number of favourable outcomes $= 13$.

Step 3: Apply the probability formula

$$P(\text{heart}) = \frac{\text{favourable outcomes}}{\text{total outcomes}} = \frac{13}{46}$$

Step 4: Why the other options are wrong Option (A) $\frac{13}{52}$ uses the full pack of 52, but 6 cards are missing.
Options (C) $\frac{11}{52}$ and (D) $\frac{11}{46}$ use 11 hearts. That would be right only if 2 hearts were missing, but the missing cards are diamonds and spades, so hearts stay at 13.

Step 5: Conclusion The probability of drawing a heart is $\frac{13}{46}$, option (B).

Quick Tip to Solve: Count the cards left in the pack. Hearts are not among the missing cards.

19. Assertion (A): If three vertices of a parallelogram taken in order are $(-1,-6)$, $(2,-5)$ and $(7,2)$, then its fourth vertex is $(4,1)$.
Reason (R): Diagonals of a parallelogram bisect each other. [1 mark]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(C) Assertion (A) is true but Reason (R) is false.

(D) Assertion (A) is false but Reason (R) is true.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Solution:

Step 1: Check the Assertion using the diagonals Let the vertices in order be $A(-1,-6)$, $B(2,-5)$, $C(7,2)$ and let the fourth vertex be $D(x,y)$.
In parallelogram $ABCD$, the diagonals are AC and BD. They bisect each other, so they have the same mid-point.
Mid-point of $AC = \left(\frac{-1+7}{2}, \frac{-6+2}{2}\right) = (3,-2)$.
Mid-point of $BD = \left(\frac{2+x}{2}, \frac{-5+y}{2}\right)$.

Step 2: Equate the mid-points

$$\frac{2+x}{2} = 3 \Rightarrow x = 4$$

$$\frac{-5+y}{2} = -2 \Rightarrow y = 1$$

So the fourth vertex is $(4,1)$. The Assertion is TRUE.

Step 3: Check the Reason A basic property of a parallelogram is that its diagonals bisect each other. So the Reason is TRUE.

Step 4: Does the Reason explain the Assertion? Yes. We found the fourth vertex by making the mid-points of the two diagonals equal. That step uses exactly the property given in the Reason. So (R) is the correct explanation of (A).

Step 5: Choose the option Both statements are true and (R) explains (A). This is option (A).
Option (B) is wrong because (R) does explain (A). Options (C) and (D) are wrong because both statements are true.

Step 6: Conclusion Option (A) is correct.

Quick Tip to Solve: Diagonals of a parallelogram have the same mid-point. Use it to find the fourth vertex.

20. Assertion (A): If zeroes of the polynomial $(2k-1)x^2+4x-3$ are reciprocal of each other, then $k=-1$.

Reason (R): If $a=c$, then zeroes of the polynomial ax^2+bx+c, $a\neq 0$ are reciprocal of each other. [1 mark]

(A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

(B) Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A).

(C) Assertion (A) is true but Reason (R) is false.

(D) Assertion (A) is false but Reason (R) is true.

Correct Answer: (A) Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A).

Solution:

Step 1: Recall the product of zeros For a quadratic ax^2+bx+c with $a\neq 0$, the product of the zeros is $\frac{c}{a}$.
Two numbers are reciprocals of each other when their product is 1.

Step 2: Check the Assertion For $(2k-1)x^2+4x-3$: $a=2k-1$ and $c=-3$.
Product of zeros $=\frac{-3}{2k-1}$. For reciprocal zeros this must be 1.

$$\frac{-3}{2k-1}=1 \Rightarrow 2k-1=-3 \Rightarrow 2k=-2 \Rightarrow k=-1$$

So the Assertion is TRUE.

Step 3: Check the Reason If $a=c$, the product of zeros is $\frac{c}{a}=1$. So the zeros are reciprocals of each other. The Reason is TRUE.

Step 4: Does the Reason explain the Assertion? Yes. In the Assertion we needed the product to be 1, which means c must equal a. Here that gives $2k-1=-3$, the same as $a=c$. So (R) is the correct explanation of (A).

Step 5: Choose the option Both statements are true and (R) explains (A), so (A) is the answer. Option (B) is wrong because (R) does explain (A). Options (C) and (D) are wrong because neither statement is false.

Step 6: Conclusion Option (A) is correct.

Quick Tip to Solve: Zeroes are reciprocals when their product $\frac{c}{a}$ equals 1.

Section B (10 marks)

This section comprises of 5 Very Short Answer (VSA) type questions of 2 marks each.

21(A). A hall has a length of 9.75 m, breadth of 6.75 m and height of 5.25 m. What is the length of the longest unmarked ruler that can exactly measure the dimensions of the hall? [2 marks]

Solution:

Concept:

- The ruler has no marks, so it can only be used by placing it end to end along a side. To measure all three dimensions exactly, its length must divide the length, the breadth and the height without any remainder.
- The longest such ruler has a length equal to the HCF of the three dimensions.

Step 1: Convert everything to one unit Length $= 9.75$ m $= 975$ cm, breadth $= 6.75$ m $= 675$ cm and height $= 5.25$ m $= 525$ cm.
Whole numbers are easier to factorise than decimals, so we work in centimetres.

Step 2: Prime factorise each number

$$975 = 3 \times 5^2 \times 13$$

$$675 = 3^3 \times 5^2$$

$$525 = 3 \times 5^2 \times 7$$

Step 3: Take the HCF HCF is the product of the common prime factors, each with the smallest power that appears in all three numbers.
The common primes are 3 and 5. The smallest power of 3 is 3^1 and the smallest power of 5 is 5^2.

$$\text{HCF} = 3 \times 5^2 = 3 \times 25 = 75$$

Step 4: Check the answer $975 \div 75 = 13$, $675 \div 75 = 9$ and $525 \div 75 = 7$. All three are whole numbers, so a 75 cm ruler fits exactly. Any longer ruler would not divide all three dimensions.

Step 5: Conclusion The longest unmarked ruler is 75 cm long, which is 0.75 m.

Quick Tip to Solve: The longest ruler that fits every dimension exactly is the HCF of the three lengths. Convert to cm first.

OR

21(B). There are three bells placed at different swings in a park, which toll at intervals of 5, 6 and 8 minutes, respectively. They all toll together when the park opens for the visitors at 10:00 a.m.. How many more times do they all toll together till the park is closed at 7:00 p.m.? [2 marks]

Solution:

Step 1: Find the common tolling interval The bells toll every 5, 6 and 8 minutes. They toll together again after a time that is a common multiple of 5, 6 and 8. The first time they meet again is the LCM.

$$5 = 5, \quad 6 = 2 \times 3, \quad 8 = 2^3$$

$$\text{LCM} = 2^3 \times 3 \times 5 = 120$$

So all three bells toll together every 120 minutes, which is 2 hours.

Step 2: Find how long the park stays open The park opens at 10:00 a.m. and closes at 7:00 p.m. That is 9 hours, or $9 \times 60 = 540$ minutes.

Step 3: List the times they toll together The bells toll together at the opening time. After that they toll together every 2 hours:
10:00 a.m. + 2 h = 12:00 noon, then 2:00 p.m., then 4:00 p.m., then 6:00 p.m.
The next one would be 8:00 p.m., which is after the park closes at 7:00 p.m., so it is not counted.

Step 4: Count The bells toll together at 12:00 noon, 2:00 p.m., 4:00 p.m. and 6:00 p.m. That is 4 more times after the opening toll.

Step 5: Conclusion The bells toll together 4 more times before the park closes.

Quick Tip to Solve: Find the LCM of 5, 6 and 8 to get the gap between joint tolls, then see how many such gaps fit in 9 hours.

22(A). ABCD is a trapezium in which AB is parallel to DC and its diagonals intersect each other at the point O. Show that $\frac{AO}{BO} = \frac{CO}{DO}$. [2 marks]

Solution:

Step 1: Draw and name the figure ABCD is a trapezium with $AB \parallel DC$. The diagonals AC and BD meet at O. We want to show $\frac{AO}{BO} = \frac{CO}{DO}$.
The sides AO and BO lie in triangle AOB, and CO and DO lie in triangle COD. So we compare these two triangles.

Step 2: Find equal angles in triangles AOB and COD Since $AB \parallel DC$ and AC is a transversal, $\angle OAB = \angle OCD$ (alternate interior angles).
Since $AB \parallel DC$ and BD is a transversal, $\angle OBA = \angle ODC$ (alternate interior angles).
Also $\angle AOB = \angle COD$ because they are vertically opposite angles.

Step 3: Prove similarity Two angles of $\triangle AOB$ are equal to two angles of $\triangle COD$. By the AA criterion,

$$\triangle AOB \sim \triangle COD$$

Step 4: Use the ratio of corresponding sides In similar triangles, corresponding sides are in the same

ratio. A corresponds to C, B corresponds to D and O corresponds to O. So

$$\frac{AO}{CO} = \frac{BO}{DO}$$

Interchange the means of this proportion (cross multiply and divide):

$$\frac{AO}{BO} = \frac{CO}{DO}$$

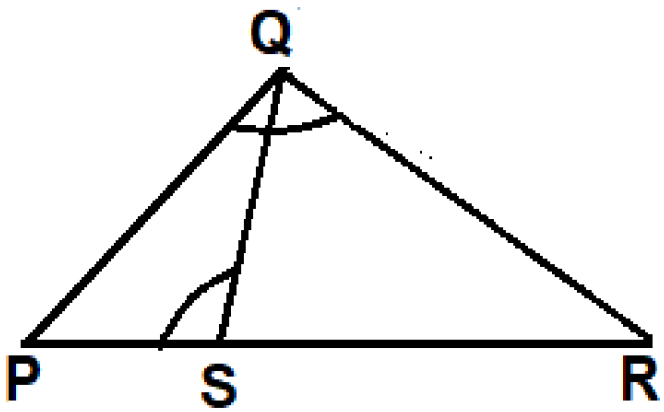
This is what we had to show.

Step 5: Conclusion Using the similarity of triangles AOB and COD, we get the required ratio.

Quick Tip to Solve: Compare triangles AOB and COD. AB parallel to DC gives equal alternate angles, so they are similar by AA.

OR

22(B). In the given figure, if $\angle PQR = \angle QSP$, PQ = 6 cm and PS = 3 cm, then find the length of PR.



[2 marks]

Solution:

Step 1: Pick the two triangles We know $\angle PQR = \angle QSP$. The length PR is a side of $\triangle PRQ$, and PS is a side of $\triangle PQS$. So we compare $\triangle PRQ$ and $\triangle PQS$.

Step 2: Find two equal angles $\angle PQR = \angle QSP$ (given).
$\angle QPR = \angle QPS$ (angle P is common to both triangles, because S lies on PR).
By the AA criterion,

$$\triangle PRQ \sim \triangle PQS$$

Step 3: Write the ratio of corresponding sides In the similarity, P corresponds to P, R corresponds to Q and Q corresponds to S. So the corresponding sides are PR with PQ, and PQ with PS. This gives

$$\frac{PQ}{PS} = \frac{PR}{PQ}$$

Step 4: Put in the values PQ = 6 cm and PS = 3 cm.

$$\frac{6}{3} = \frac{PR}{6}$$

$$PR = \frac{6 \times 6}{3} = 12$$

Step 5: Conclusion The length PR is 12 cm.

Quick Tip to Solve: Show triangle PRQ is similar to triangle PQS by AA (common angle P and the given equal angles), then use the ratio of sides.

23. If $\tan\theta = \frac{1}{\sqrt{5}}$, then find the value of $\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta}$. [2 marks]

Solution:

Step 1: Find cot and sec squared We have $\tan\theta = \frac{1}{\sqrt{5}}$. Since $\cot\theta$ is the reciprocal of $\tan\theta$, $\cot\theta = \sqrt{5}$.

We use the identity $\sec^2\theta = 1 + \tan^2\theta$:

$$\sec^2\theta = 1 + \left(\frac{1}{\sqrt{5}}\right)^2 = 1 + \frac{1}{5} = \frac{6}{5}$$

Step 2: Find cosec squared We use the identity $\text{cosec}^2\theta = 1 + \cot^2\theta$:

$$\text{cosec}^2\theta = 1 + (\sqrt{5})^2 = 1 + 5 = 6$$

Step 3: Substitute in the expression

$$\frac{\text{cosec}^2\theta - \sec^2\theta}{\text{cosec}^2\theta + \sec^2\theta} = \frac{6 - \frac{6}{5}}{6 + \frac{6}{5}}$$

Numerator: $6 - \frac{6}{5} = \frac{30-6}{5} = \frac{24}{5}$. Denominator: $6 + \frac{6}{5} = \frac{30+6}{5} = \frac{36}{5}$.

$$\frac{24/5}{36/5} = \frac{24}{36} = \frac{2}{3}$$

Step 4: Conclusion The value of the expression is 2/3.

Quick Tip to Solve: Use $\sec^2\theta = 1+\tan^2\theta$ and $\text{cosec}^2\theta = 1+\cot^2\theta$ to get both values, then substitute.

24. Find the area of a sector of a circle with diameter 28 cm, If the length of the corresponding arc is 22 cm. [2 marks]

Solution:

Step 1: Find the radius The diameter is 28 cm, so the radius is $r = \frac{28}{2} = 14$ cm.

Step 2: Use the formula for the area of a sector in terms of arc length For a sector with arc length l and radius r, the area is half of the arc length times the radius. This comes from $\frac{\theta}{360^\circ}\pi r^2$ with $l = \frac{\theta}{360^\circ} \times 2\pi r$.

$$\text{Area} = \frac{1}{2} \times l \times r$$

Step 3: Substitute and calculate Here $l = 22$ cm and $r = 14$ cm.

$$\text{Area} = \frac{1}{2} \times 22 \times 14 = 11 \times 14 = 154 \text{ cm}^2$$

Step 4: Conclusion The area of the sector is 154 square centimetres.

Quick Tip to Solve: Area of a sector can be written as half of arc length times radius. Radius is 14 cm.

25. Two dice are thrown simultaneously. Find the probability that the product of the numbers appearing on them is a prime number. [2 marks]

Solution:

Step 1: Count all outcomes Each die has 6 faces, so two dice give $6 \times 6 = 36$ equally likely outcomes. Total outcomes $= 36$.

Step 2: Find when the product is a prime number A prime number has exactly two factors, 1 and itself. A product of two whole numbers a and b is prime only when one of them is 1 and the other is a prime number.
The prime numbers on a die are 2, 3 and 5. So the pairs are:
One die shows 1 and the other shows 2: (1, 2) and (2, 1).
One die shows 1 and the other shows 3: (1, 3) and (3, 1).
One die shows 1 and the other shows 5: (1, 5) and (5, 1).
The pair (1, 1) gives 1, which is not prime. Pairs such as (1, 4) and (1, 6) give 4 and 6, which are not prime. Pairs with both numbers more than 1 give composite products.

Step 3: Apply the probability formula Favourable outcomes $= 6$.

$$P(\text{product is prime}) = \frac{6}{36} = \frac{1}{6}$$

Step 4: Conclusion The probability that the product is a prime number is 1/6.

Quick Tip to Solve: A product is prime only when one die shows 1 and the other shows a prime (2, 3 or 5). Count such pairs out of 36.

Section C (18 marks)

This section comprises of 6 Short Answer (SA) type questions of 3 marks each.

26. Given that $\sqrt{3}$ is irrational number, prove that $(2+3\sqrt{3})$ is an irrational number. [3 marks]

Solution:

Concept:

- We are told that $\sqrt{3}$ is irrational.
- We must prove that $2+3\sqrt{3}$ is also irrational.
- The standard method is proof by contradiction: assume the opposite and show it leads to a false statement.

Step 1: Assume the opposite Suppose $2+3\sqrt{3}$ is a rational number. Then we can write it as a fraction of two integers:

$$2+3\sqrt{3}=\frac{a}{b}$$

Here a and b are integers and $b\neq 0$.

Step 2: Isolate the root Subtract 2 from both sides:

$$3\sqrt{3}=\frac{a}{b}-2=\frac{a-2b}{b}$$

Now divide both sides by 3:

$$\sqrt{3}=\frac{a-2b}{3b}$$

Step 3: Look at the right side Since a and b are integers, $a-2b$ is an integer and $3b$ is a non-zero integer. So $\frac{a-2b}{3b}$ is a ratio of two integers, which means it is a rational number.

Step 4: Find the contradiction The equation says $\sqrt{3}$ equals a rational number. But we are given that $\sqrt{3}$ is irrational. Both cannot be true, so our assumption was wrong.

Step 5: Conclusion Our assumption that $2+3\sqrt{3}$ is rational is false. Hence $2+3\sqrt{3}$ is an irrational number.

Quick Tip to Solve: Assume $2+3\sqrt{3}$ is rational, write it as $\frac{a}{b}$ and make $\sqrt{3}$ the subject.

27. If α and β are the zeroes of the quadratic polynomial $2x^2-8x+5$, then find the value of $(\alpha+\frac{1}{\beta})\times(\beta+\frac{1}{\alpha})$. [3 marks]

Solution:

Concept:

- For a quadratic polynomial ax^2+bx+c with zeroes α and β, the sum and product of the zeroes are given by the coefficients.
- We first find these two values and then rewrite the expression using them.

Step 1: Key Formula

$$\alpha+\beta=-\frac{b}{a},\qquad \alpha\beta=\frac{c}{a}$$

Step 2: Find the sum and the product Here $a = 2$, $b = -8$, $c = 5$.

$$\alpha + \beta = -\frac{-8}{2} = 4$$

$$\alpha\beta = \frac{5}{2}$$

Step 3: Simplify each bracket Take a common denominator in each bracket:

$$\alpha + \frac{1}{\beta} = \frac{\alpha\beta + 1}{\beta}, \qquad \beta + \frac{1}{\alpha} = \frac{\alpha\beta + 1}{\alpha}$$

Multiply the two brackets:

$$\left(\frac{\alpha\beta + 1}{\beta}\right)\left(\frac{\alpha\beta + 1}{\alpha}\right) = \frac{(\alpha\beta + 1)^2}{\alpha\beta}$$

Step 4: Put in the value of the product Use $\alpha\beta = \frac{5}{2}$.

$$\alpha\beta + 1 = \frac{5}{2} + 1 = \frac{7}{2}$$

$$\frac{(7/2)^2}{5/2} = \frac{49}{4} \times \frac{2}{5} = \frac{49}{10}$$

Step 5: Conclusion The value of the expression is $\frac{49}{10}$.

Quick Tip to Solve: Write each bracket over a common denominator, so the product becomes $\frac{(\alpha\beta + 1)^2}{\alpha\beta}$, then use $\alpha\beta = \frac{c}{a}$.

28(A). Ridhi drew a polygon with n sides. The smallest exterior angle is $8°$ and each subsequent exterior angle is $4°$ more than the previous exterior angle. Find the number of sides of the polygon that Ridhi had drawn. [3 marks]

Solution:

Concept:

- The exterior angles are $8°, 12°, 16°, \ldots$, and each one is $4°$ more than the one before.
- So they form an A.P. with n terms, one for each side of the polygon.

Step 1: Key Formula The sum of all exterior angles of any polygon is $360°$. The sum of the first n terms of an A.P. is

$$S_n = \frac{n}{2}\left[2a + (n-1)d\right]$$

Step 2: Write the data First term $a = 8$ and common difference $d = 4$. We need $S_n = 360$.

Step 3: Form the equation

$$\frac{n}{2}\left[2 \times 8 + (n-1) \times 4\right] = 360$$

$$\frac{n}{2}\left[16 + 4n - 4\right] = 360$$

$$n(2n+6) = 360 \implies 2n^2 + 6n - 360 = 0 \implies n^2 + 3n - 180 = 0$$

Step 4: Solve the quadratic We need two numbers with product -180 and sum 3. These are 15 and -12.

$$n^2 + 15n - 12n - 180 = 0$$

$$n(n+15) - 12(n+15) = 0 \implies (n+15)(n-12) = 0$$

So $n = -15$ or $n = 12$.

Step 5: Reject the invalid value The number of sides cannot be negative, so $n = -15$ is rejected. Check: for $n = 12$, the last angle is $8 + 11 \times 4 = 52^\circ$ and the sum is $\frac{12}{2}(8 + 52) = 360^\circ$, which is correct.

Step 6: Conclusion Ridhi drew a polygon with 12 sides.

Quick Tip to Solve: The exterior angles form an A.P. with $a = 8$, $d = 4$. Put $S_n = 360$ and solve for n.

OR

28(B). Find the sum of integers between 1 and 400 that are multiples of 4 as well as of 5. [3 marks]

Solution:

Concept:

- A number that is a multiple of both 4 and 5 must be a multiple of their L.C.M.
- Since 4 and 5 have no common factor, the L.C.M. is 20.
- So we need the sum of all multiples of 20 between 1 and 400.

Step 1: Write the A.P. The multiples of 20 are $20, 40, 60, \ldots$. The question says "between 1 and 400", so 400 is not counted. The largest multiple of 20 below 400 is 380. The A.P. is

$$20, 40, 60, \ldots, 380$$

Here $a = 20$, $d = 20$ and last term $a_n = 380$.

Step 2: Find the number of terms Use $a_n = a + (n-1)d$.

$$20 + (n-1) \times 20 = 380$$

$$(n-1) \times 20 = 360 \implies n - 1 = 18 \implies n = 19$$

Step 3: Find the sum Use $S_n = \frac{n}{2}(a + l)$.

$$S_{19} = \frac{19}{2}(20 + 380) = \frac{19}{2} \times 400 = 19 \times 200 = 3800$$

Step 4: Conclusion The required sum is 3800.

Quick Tip to Solve: Common multiples of 4 and 5 are multiples of 20. Form the A.P. $20, 40, \ldots, 380$ and find its sum.

29. Prove that the parallelogram circumscribing a circle is a rhombus. [3 marks]

Solution:

Concept:

- A parallelogram $ABCD$ circumscribes a circle, which means each of its four sides touches the circle.
- Tangents drawn to a circle from an outside point are equal in length.
- We use this fact at every vertex.

Step 1: Mark the points of contact Let the circle touch the sides AB, BC, CD, DA at P, Q, R, S respectively.

Step 2: Use equal tangents at each vertex

$$AP = AS \quad \text{(i)}$$

$$BP = BQ \quad \text{(ii)}$$

$$CR = CQ \quad \text{(iii)}$$

$$DR = DS \quad \text{(iv)}$$

Each pair is a pair of tangent segments from one vertex to the circle.

Step 3: Add all four equations

$$AP + BP + CR + DR = AS + BQ + CQ + DS$$

Group the terms: $AP + BP = AB$, $CR + DR = CD$, $AS + DS = AD$ and $BQ + CQ = BC$. So

$$AB + CD = AD + BC$$

Step 4: Use the property of a parallelogram Opposite sides of a parallelogram are equal, so $AB = CD$ and $AD = BC$. Put these in:

$$AB + AB = AD + AD \implies 2AB = 2AD \implies AB = AD$$

Step 5: Conclude Two adjacent sides AB and AD are equal. In a parallelogram, opposite sides are already equal, so all four sides are equal. A parallelogram with all sides equal is a rhombus.

Step 6: Conclusion Hence the parallelogram $ABCD$ is a rhombus.

Quick Tip to Solve: Use equal tangents from each vertex to get $AB + CD = AD + BC$, then use $AB = CD$ and $AD = BC$.

30. In what ratio does the x-axis divide the line segment joining the points $(-4, -6)$ and $(-1, 7)$? Also, find the coordinates of the point of division. [3 marks]

Solution:

Concept:

- A point on the x-axis has y-coordinate 0. Suppose the x-axis meets the segment AB at $P(x, 0)$, where $A(-4, -6)$ and $B(-1, 7)$.
- Let P divide AB in the ratio $k : 1$. We use the section formula.

Step 1: Key Formula For a point dividing the segment joining (x_1, y_1) and (x_2, y_2) in the ratio $m : n$,

$$(x, y) = \left(\frac{mx_2 + nx_1}{m+n}, \frac{my_2 + ny_1}{m+n}\right)$$

Step 2: Write the coordinates of P Put $m = k$, $n = 1$:

$$P = \left(\frac{k(-1) + 1(-4)}{k+1}, \frac{k(7) + 1(-6)}{k+1}\right) = \left(\frac{-k-4}{k+1}, \frac{7k-6}{k+1}\right)$$

Step 3: Use that P lies on the x-axis The y-coordinate of P is zero.

$$\frac{7k-6}{k+1} = 0 \implies 7k - 6 = 0 \implies k = \frac{6}{7}$$

So the ratio $k : 1 = \frac{6}{7} : 1 = 6 : 7$.

Step 4: Find the x-coordinate

$$x = \frac{-k-4}{k+1} = \frac{-\frac{6}{7} - 4}{\frac{6}{7} + 1} = \frac{-\frac{34}{7}}{\frac{13}{7}} = -\frac{34}{13}$$

Step 5: Conclusion The x-axis divides the segment in the ratio $6 : 7$, and the point of division is $\left(-\frac{34}{13}, 0\right)$.

Quick Tip to Solve: Let the ratio be $k : 1$ and use the section formula. A point on the x-axis has y-coordinate 0.

31(A). During a math class, Ms. Isha wrote the expression given below on the board and asked the students to simplify it.

\[\frac{\cos\theta}{1 - \sin\theta} + \frac{1 - \sin\theta}{\cos\theta} \]

Jyoti solved it in her note book as follows:

\[\frac{\cos\theta}{1 - \sin\theta} + \frac{1 - \sin\theta}{\cos\theta} \]

\[= \frac{\cos ^ 2\theta + (1 - \sin\theta) ^ 2}{(1 - \sin\theta) \times \cos\theta} \] .... (step 1)

\[= \frac{\cos ^ 2\theta + \cos ^ 2\theta}{(1 - \sin\theta) \times \cos\theta} \] .... (step 2)

\[= \frac{2\cos ^ 2\theta}{(1 - \sin\theta) \times \cos\theta} \] .... (step 3)

\[= \frac{2\cos\theta}{(1 - \sin\theta)} \] .... (step 4)

Identify the step in which Jyoti has made error(s), if any. Rectify the same and find the correct answer. [3 marks]

Solution:

Concept:

- We check each step of Jyoti's work. Step 1 adds the two fractions using the common denominator $(1-\sin\theta)\cos\theta$.
- That is correct. The place to test is the step where $(1-\sin\theta)^2$ is changed.

Step 1: Locate the error In step 2 Jyoti wrote $(1-\sin\theta)^2 = \cos^2\theta$. This is wrong. By the identity $(a-b)^2 = a^2 - 2ab + b^2$,

$$(1-\sin\theta)^2 = 1 - 2\sin\theta + \sin^2\theta$$

This is not equal to $\cos^2\theta$. The error is in **step 2**. Steps 3 and 4 follow from the wrong step 2, so they are also wrong.

Step 2: Redo the work from step 1 Step 1 is correct:

$$\frac{\cos^2\theta + (1-\sin\theta)^2}{(1-\sin\theta)\cos\theta}$$

Expand the square in the numerator:

$$= \frac{\cos^2\theta + 1 + \sin^2\theta - 2\sin\theta}{(1-\sin\theta)\cos\theta}$$

Step 3: Use the identity Since $\sin^2\theta + \cos^2\theta = 1$, the numerator becomes $1 + 1 - 2\sin\theta$.

$$= \frac{2 - 2\sin\theta}{(1-\sin\theta)\cos\theta}$$

Step 4: Cancel the common factor Take 2 common in the numerator: $2 - 2\sin\theta = 2(1-\sin\theta)$. The factor $(1-\sin\theta)$ cancels from top and bottom.

$$= \frac{2(1-\sin\theta)}{(1-\sin\theta)\cos\theta} = \frac{2}{\cos\theta} = 2\sec\theta$$

Step 5: Conclusion Jyoti made the error in step 2. The correct simplified value is $2\sec\theta$.

Quick Tip to Solve: Check whether $(1-\sin\theta)^2$ really equals $\cos^2\theta$. Expand the square and use $\sin^2\theta + \cos^2\theta = 1$.

OR

31(B). **If** $\frac{1}{\sin x - \cos x} = \frac{\operatorname{cosec} x}{\sqrt{2}}$, **then prove that** $\left(\frac{1}{\sin x + \cos x}\right)^2 = \frac{\sec^2 x}{2}$. [3 marks]

Solution:

Concept:

- The given condition connects $\sin x$ and $\cos x$.
- We turn it into a simple equation, square it, and then use that result to simplify the left side of what we must prove.

Step 1: Simplify the given condition Since $\operatorname{cosec} x = \frac{1}{\sin x}$,

$$\frac{1}{\sin x - \cos x} = \frac{1}{\sqrt{2}\sin x}$$

Cross-multiply:

$$\sin x - \cos x = \sqrt{2}\sin x$$

Step 2: Square both sides

$$\sin^2 x + \cos^2 x - 2\sin x \cos x = 2\sin^2 x$$

Use $\sin^2 x + \cos^2 x = 1$:

$$1 - 2\sin x \cos x = 2\sin^2 x \implies 2\sin x \cos x = 1 - 2\sin^2 x \quad \text{(i)}$$

Step 3: Expand the left side of the result to prove

$$\text{LHS} = \left(\frac{1}{\sin x + \cos x}\right)^2 = \frac{1}{\sin^2 x + \cos^2 x + 2\sin x \cos x} = \frac{1}{1 + 2\sin x \cos x}$$

Step 4: Use equation (i) Put $2\sin x \cos x = 1 - 2\sin^2 x$:

$$\text{LHS} = \frac{1}{1 + 1 - 2\sin^2 x} = \frac{1}{2(1 - \sin^2 x)} = \frac{1}{2\cos^2 x} = \frac{\sec^2 x}{2} = \text{RHS}$$

Step 5: Conclusion Hence $\left(\frac{1}{\sin x + \cos x}\right)^2 = \frac{\sec^2 x}{2}$. Proved.

Quick Tip to Solve: From the given condition get $\sin x - \cos x = \sqrt{2}\sin x$, square it, and use the result in $\sin^2 x + \cos^2 x + 2\sin x \cos x$.

Section D (20 marks)

This section comprises of 4 Long Answer (LA) type questions of 5 marks each.

32. In a rectangular park measuring 50 m $\times$ 40 m, the gram panchayat plans to construct a rectangular swimming pool in the middle, surrounded by a uniform-width grass strip throughout the park. Find the dimensions of the swimming pool, if the area of the grassy strip is 1184 m^2. [5 marks]

Solution:

Concept:

- The park is a rectangle of 50 m by 40 m. The pool sits in the middle and the grass strip goes all around it with the same width on every side.
- Let the width of the grass strip be x m.
- The strip is present on both sides of the length and on both sides of the breadth, so it cuts $2x$ m from each dimension.

Step 1: Write the pool dimensions Length of the pool = $(50 - 2x)$ m.
Breadth of the pool = $(40 - 2x)$ m.

Step 2: Find the area of the pool Area of the park = $50 \times 40 = 2000$ m^2.
The park is made of the pool and the grass strip. So

$$\text{Area of pool} = 2000 - 1184 = 816 \text{ m}^2$$

Step 3: Form the equation Area of pool = length $\times$ breadth, so

$$(50 - 2x)(40 - 2x) = 816$$

$$2000 - 100x - 80x + 4x^2 = 816$$

$$4x^2 - 180x + 1184 = 0$$

Divide every term by 4:

$$x^2 - 45x + 296 = 0$$

Step 4: Solve the quadratic equation We look for two numbers whose product is 296 and whose sum is 45. These are 8 and 37.

$$x^2 - 37x - 8x + 296 = 0$$

$$x(x - 37) - 8(x - 37) = 0$$

$$(x - 8)(x - 37) = 0$$

So $x = 8$ or $x = 37$.

Step 5: Reject the value that cannot work If $x = 37$, then $50 - 2x = -24$, which is negative. A length cannot be negative, so $x = 37$ is rejected.
So the strip width is $x = 8$ m.

Step 6: Find the pool dimensions Length of the pool = $50 - 2(8) = 50 - 16 = 34$ m.

Breadth of the pool = $40 - 2(8) = 40 - 16 = 24$ m.
Check: $34 \times 24 = 816$ m^2, and $2000 - 816 = 1184$ m^2, which matches the grass strip.

Step 7: Conclusion The swimming pool is 34 m long and 24 m wide.

Quick Tip to Solve: Let the strip width be x m. The pool is $(50 - 2x)$ by $(40 - 2x)$, and its area is $2000 - 1184$.

33. Prove that a line drawn parallel to one side of a triangle to intersect the other two sides in distinct points, divides the other two sides in the same ratio.
Using the above theorem solve the following:
PQRS is a trapezium with PQ || SR. X and Y are points on non-parallel sides PS and QR respectively such that XY || PQ. Show that $\frac{PX}{XS} = \frac{QY}{YR}$. [5 marks]

Solution:

Step 1: Given, to prove and construction Given: In $\triangle ABC$, a line DE is parallel to BC and meets AB at D and AC at E.
To prove: $\frac{AD}{DB} = \frac{AE}{EC}$.
Construction: Join B to E and C to D. Draw $DM \perp AC$ and $EN \perp AB$.

Step 2: Write the areas of the triangles We use the formula area = $\frac{1}{2} \times$ base $\times$ height.

$$ar(\triangle ADE) = \tfrac{1}{2} \times AD \times EN, \qquad ar(\triangle BDE) = \tfrac{1}{2} \times DB \times EN$$

Dividing, the common part $\frac{1}{2}EN$ cancels:

$$\frac{ar(\triangle ADE)}{ar(\triangle BDE)} = \frac{AD}{DB} \qquad \cdots (1)$$

In the same way, using DM as the common height:

$$\frac{ar(\triangle ADE)}{ar(\triangle CDE)} = \frac{\frac{1}{2} \times AE \times DM}{\frac{1}{2} \times EC \times DM} = \frac{AE}{EC} \qquad \cdots (2)$$

Step 3: Show that two triangles have equal area $\triangle BDE$ and $\triangle CDE$ stand on the same base DE and lie between the same parallel lines DE and BC. Triangles on the same base and between the same parallels have equal area.

$$ar(\triangle BDE) = ar(\triangle CDE) \qquad \cdots (3)$$

Step 4: Join the results to finish the theorem From (1), (2) and (3), the right-hand sides of (1) and (2) have the same denominator area, so

$$\frac{AD}{DB} = \frac{AE}{EC}$$

This proves the theorem (Basic Proportionality Theorem).

Step 5: Apply the theorem to the trapezium: draw the diagonal In trapezium $PQRS$, $PQ \parallel SR$ and $XY \parallel PQ$. So $XY \parallel SR$ as well.
Join the diagonal PR. Let it meet XY at M.

Step 6: Use the theorem in triangle PSR In $\triangle PSR$, $XM \parallel SR$ (because $XY \parallel SR$). By the theorem,

$$\frac{PX}{XS} = \frac{PM}{MR} \qquad \cdots (i)$$

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Step 7: Use the theorem in triangle PQR In $\triangle PQR$, $MY \parallel PQ$ (because $XY \parallel PQ$). The line MY meets PR at M and QR at Y. By the theorem,

$$\frac{QY}{YR} = \frac{PM}{MR} \qquad \cdots (ii)$$

Step 8: Compare (i) and (ii) The right-hand sides of (i) and (ii) are the same, so the left-hand sides are equal.

Step 9: Conclusion Hence the theorem is proved and $\frac{PX}{XS} = \frac{QY}{YR}$.

Quick Tip to Solve: Prove the theorem with areas of triangles on the same base between the same parallels. For the trapezium, join the diagonal PR and use the theorem twice.

34(A). A tent is in the shape of a cylinder surmounted by a conical top. If the height and radius of the cylindrical part are 3 m and 14 m respectively, and the total height of the tent is 13.5 m, then find the area of the canvas required for making the tent, keeping a provision of 26 m^2 of canvas for stitching and wastage. Also, find the cost of the canvas to be purchased at the rate of Rs 250 per m^2.

[5 marks]

Solution:

Concept:

- The tent has two parts: a cylinder at the bottom and a cone on top. The canvas covers only the curved surfaces. The cylinder is open at the top because the cone sits on it, and the tent is pitched on the ground, so we need the curved surface of the cylinder and the curved surface of the cone.
- Given: radius $r = 14$ m, cylinder height $h = 3$ m, total height = 13.5 m.

Step 1: Find the height of the cone

$$\text{Height of cone} = 13.5 - 3 = 10.5 \text{ m}$$

Step 2: Find the slant height of the cone The radius, the height and the slant height l of a cone form a right triangle, so

$$l = \sqrt{r^2 + H^2} = \sqrt{14^2 + 10.5^2} = \sqrt{196 + 110.25} = \sqrt{306.25} = 17.5 \text{ m}$$

Step 3: Curved surface area of the cone

$$\text{CSA}_{cone} = \pi r l = \frac{22}{7} \times 14 \times 17.5 = 22 \times 2 \times 17.5 = 770 \text{ m}^2$$

Step 4: Curved surface area of the cylinder

$$\text{CSA}_{cyl} = 2\pi r h = 2 \times \frac{22}{7} \times 14 \times 3 = 2 \times 22 \times 2 \times 3 = 264 \text{ m}^2$$

Step 5: Total canvas needed Area of the tent = $770 + 264 = 1034$ m^2.
Add the 26 m^2 for stitching and wastage:

$$1034 + 26 = 1060 \text{ m}^2$$

Step 6: Cost of the canvas Cost = area × rate = $1060 \times 250 = 265000$.
So the canvas costs Rs 2,65,000.

Step 7: Conclusion Area of canvas required is 1060 m^2 and the cost is Rs 2,65,000.

Quick Tip to Solve: Cone height = total height - cylinder height. Find the slant height, then add the curved surface areas of cone and cylinder plus 26 m^2.

OR

34(B). A solid wooden toy is in the form of a hemisphere surmounted by a cone of the same radius. The radius of hemisphere is 3.5 cm and the total wood used in making the toy is $166\frac{5}{6}$ cm^3. Find the height of the conical part. Also, find the cost of painting the hemispherical part of the toy at the rate of Rs 15 per cm^2. [5 marks]

Solution:

Concept:

- The toy is a hemisphere with a cone fixed on its flat face. Both have radius $r = 3.5$ cm. Let the cone height be h cm.
- The wood used is the volume of the whole toy.

Step 1: Write the volume equation

$$\text{Volume of toy} = \text{Volume of cone} + \text{Volume of hemisphere}$$

$$\frac{1001}{6} = \frac{1}{3}\pi r^2 h + \frac{2}{3}\pi r^3$$

Step 2: Find the volume of the hemisphere

$$\frac{2}{3} \times \frac{22}{7} \times (3.5)^3 = \frac{2}{3} \times \frac{22}{7} \times \frac{343}{8} = \frac{539}{6}\ \mathrm{cm}^3$$

Step 3: Find the volume of the cone

$$\text{Volume of cone} = \frac{1001}{6} - \frac{539}{6} = \frac{462}{6} = 77\ \mathrm{cm}^3$$

Step 4: Find the height of the cone

$$\frac{1}{3} \times \frac{22}{7} \times (3.5)^2 \times h = 77$$

$$\frac{1}{3} \times \frac{22}{7} \times 12.25 \times h = 77 \Rightarrow \frac{38.5}{3}h = 77$$

$$h = \frac{77 \times 3}{38.5} = 6\ \mathrm{cm}$$

Step 5: Curved surface area of the hemisphere Only the hemispherical part is painted, so we need its curved surface:

$$2\pi r^2 = 2 \times \frac{22}{7} \times (3.5)^2 = 2 \times \frac{22}{7} \times 12.25 = 77\ \mathrm{cm}^2$$

Step 6: Cost of painting Cost $= 77 \times 15 = 1155$. So the painting costs Rs 1155.

Step 7: Conclusion The height of the conical part is 6 cm and the cost of painting the hemispherical part is Rs 1155.

Quick Tip to Solve: Total volume = cone volume + hemisphere volume. Find the hemisphere volume first, then solve for the cone height.

35(A). Manish is standing on level ground and observes a kite flying 200 m away from him at an angle of elevation of 30°. Mahesh, standing on the roof of a 50 m high building on the opposite side of the kite, observes the same kite at an angle of elevation of 45°. Find the distance between the kite and Mahesh. [5 marks]

Solution:

Step 1: Draw the situation in words Let Manish stand at A on the ground. Let the kite be at B, so $AB = 200$ m and $\angle BAE = 30^\circ$, where E is the point on the ground directly below the kite.
Let Mahesh be at D on the roof of the building, with $DC = 50$ m and DC vertical. Draw DR parallel to the ground meeting BE at R. Then $RE = DC = 50$ m and $\angle BDR = 45^\circ$.

Step 2: Find the height of the kite In right triangle AEB, $\sin 30^\circ = \dfrac{BE}{AB}$.

$$\frac{BE}{200} = \frac{1}{2} \Rightarrow BE = 100 \text{ m}$$

Step 3: Find how much higher the kite is than Mahesh

$$BR = BE - RE = 100 - 50 = 50 \text{ m}$$

Step 4: Find the distance BD In right triangle BRD, $\sin 45^\circ = \dfrac{BR}{BD}$.

$$\frac{50}{BD} = \frac{1}{\sqrt{2}} \Rightarrow BD = 50\sqrt{2} \text{ m}$$

This is about 70.7 m.

Step 5: Conclusion The kite is $50\sqrt{2}$ m away from Mahesh.

Quick Tip to Solve: Use $\sin 30^\circ$ to find the kite's height, subtract 50 m, then use $\sin 45^\circ$ in the small right triangle.

OR

35(B). The angles of depression of two ships from the top of a lighthouse and on the same side of it are found to be 45° and 30°. If the ships are 200 m apart and one ship is exactly behind the other then find the height of lighthouse. [5 marks]

Solution:

Step 1: Set up the figure Let the lighthouse be AB with $AB = h$ m, with B at the foot. Let the nearer ship be at C (angle of depression 45°) and the farther ship at D (angle of depression 30°). The ships are on a line with B, so $CD = 200$ m.
The angle of depression equals the angle of elevation of A from the ship, so $\angle ACB = 45^\circ$ and $\angle ADB = 30^\circ$.

Step 2: Use the nearer ship In right triangle ABC, $\tan 45^\circ = \dfrac{AB}{BC}$.

$$\frac{h}{BC} = 1 \Rightarrow BC = h \qquad \cdots (i)$$

Step 3: Use the farther ship The farther ship is $BD = BC + CD = h + 200$ from the foot. In right triangle ABD, $\tan 30^\circ = \dfrac{AB}{BD}$.

$$\frac{h}{h + 200} = \frac{1}{\sqrt{3}}$$

Cross-multiply:

$$\sqrt{3}\, h = h + 200$$

$$h(\sqrt{3} - 1) = 200$$

Step 4: Solve for h

$$h = \frac{200}{\sqrt{3} - 1}$$

Rationalise by multiplying the top and bottom by $\sqrt{3} + 1$:

$$h = \frac{200(\sqrt{3} + 1)}{(\sqrt{3})^2 - 1^2} = \frac{200(\sqrt{3} + 1)}{2} = 100(\sqrt{3} + 1)$$

This is about 273.2 m.

Step 5: Conclusion The height of the lighthouse is $100(\sqrt{3} + 1)$ m.

Quick Tip to Solve: Nearer ship is at distance h from the foot, farther ship at $h + 200$. Use $\tan 45^\circ$ and $\tan 30^\circ$.

Section E (12 marks)

This section comprises of 3 case-study-based questions of 4 marks each with sub parts. Each case study question has three sub parts (i), (ii), (iii) of marks 1, 1, 2 respectively.

36(i). A student entrepreneur started a company that manufactures sanitizers in two sizes - small and large. The cost of a small bottle of sanitizer is Rs 10 and that of a large bottle is Rs 15. In June, the company sold 1000 bottles and recorded a total sale of Rs 12,750. Seeing the increased demand, the company decided to increase the price of both sanitizer bottles by Rs 2 each. In the next month, the company sold 2500 bottles and recorded a total sale of Rs 34,250.
Based on the above information, answer the following questions:
Form a linear equation in two variables representing the sale for June. [1 mark]

Solution:

Step 1: Choose the variables Let the number of small bottles sold in June be x and the number of large bottles sold in June be y. We need an equation for the money collected, so we use the prices.

Step 2: Find the sale from each size A small bottle costs Rs 10, so x small bottles bring in $10x$ rupees. A large bottle costs Rs 15, so y large bottles bring in $15y$ rupees.

Step 3: Add the two amounts The total sale in June was Rs 12,750. So the two amounts must add up to 12750.

$$10x + 15y = 12750$$

Dividing every term by 5 gives the simpler form $2x + 3y = 2550$.

Step 4: Conclusion The linear equation for the June sale is

Quick Tip to Solve: Multiply the price of each size by the number of bottles of that size and add. The total is the June sale.

36(ii). Form a linear equation in two variables representing the sale for July. [1 mark]

Solution:

Step 1: Find the new prices Both prices go up by Rs 2.
New price of a small bottle $= 10 + 2 = 12$ rupees.
New price of a large bottle $= 15 + 2 = 17$ rupees.

Step 2: Choose the variables for July Let x be the number of small bottles and y the number of large bottles sold in July. These are new unknowns, because the sale numbers changed from June.

Step 3: Write the sale equation Small bottles bring in $12x$ rupees and large bottles bring in $17y$ rupees. The recorded total for July is Rs 34,250.

$$12x + 17y = 34250$$

Step 4: Conclusion The linear equation for the July sale is

Quick Tip to Solve: Add Rs 2 to both prices first, then multiply each new price by its number of bottles.

36(iii)(A). (A) How many sanitizer bottles of each type were actually sold in June? [2 marks]

Solution:

Step 1: Write both equations Let x be the small bottles and y the large bottles sold in June.
Total bottles sold in June is 1000, so $x + y = 1000$ ... (1)
Total sale is Rs 12,750, so $10x + 15y = 12750$ ... (2)

Step 2: Use substitution From (1), $x = 1000 - y$. Put this in (2).

$$10(1000 - y) + 15y = 12750$$

$$10000 - 10y + 15y = 12750$$

$$5y = 12750 - 10000 = 2750$$

$$y = \frac{2750}{5} = 550$$

Step 3: Find the other variable $x = 1000 - 550 = 450$.

Step 4: Check the answer Bottles: $450 + 550 = 1000$.
Sale: $10 \times 450 + 15 \times 550 = 4500 + 8250 = 12750$. Both conditions are met.

Step 5: Conclusion In June, the company sold 450 small bottles and 550 large bottles.

Quick Tip to Solve: Use x + y = 1000 along with 10x + 15y = 12750 and solve the pair by substitution or elimination.

OR

36(iii)(B). (B) How many sanitizer bottles of each type were sold in July? [2 marks]

Solution:

Step 1: Write both equations Let x be the small bottles and y the large bottles sold in July.
Total bottles is 2500, so $x + y = 2500$ ... (1)
Total sale is Rs 34,250 at the new prices, so $12x + 17y = 34250$ ... (2)

Step 2: Use elimination Multiply (1) by 12 so that the x coefficients match.

$$12x + 12y = 30000$$

... (3)
Subtract (3) from (2).

$$5y = 34250 - 30000 = 4250$$

$$y = \frac{4250}{5} = 850$$

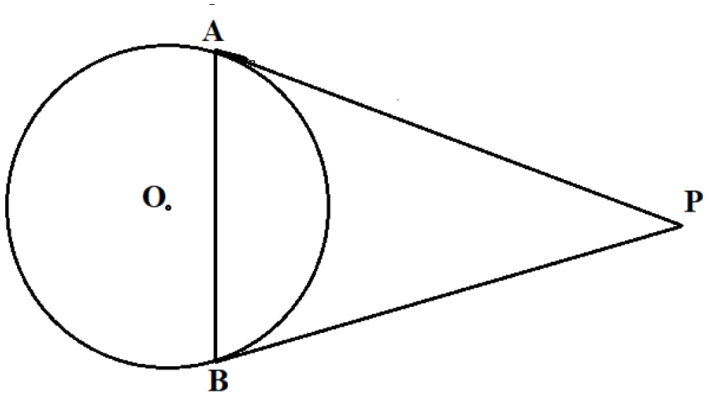
Step 3: Find x $x = 2500 - 850 = 1650$.

Step 4: Check the answer Bottles: $1650 + 850 = 2500$.
Sale: $12 \times 1650 + 17 \times 850 = 19800 + 14450 = 34250$. Both conditions are met.

Step 5: Conclusion In July, the company sold 1650 small bottles and 850 large bottles.

Quick Tip to Solve: Use x + y = 2500 along with 12x + 17y = 34250 and solve the pair.

37(i). **In a technology park in Hyderabad, a company installed a circular meditation garden for its employees as shown in the figure.**



Arjun, a designer, stood at a point P outside the garden and drew two tangents PA and PB to the circular boundary. The radius of the garden is 9 m, and the distance of point P from the centre O is 18 m. For preparing safety guidelines, Arjun needed the lengths of the tangents. During a demonstration to interns, he asked them to compute the angle formed between the two tangents using the properties of circle.
Based on the above information, answer the following questions:
What is the measure of $\angle OAP$? [1 mark]

Solution:

Step 1: Recall the tangent property A tangent to a circle is perpendicular to the radius drawn to the point of contact. Here PA touches the circle at A, and OA is the radius to A.

Step 2: Apply it So OA is perpendicular to PA.

$$\angle OAP = 90^\circ$$

Step 3: Check with the given lengths In $\triangle OAP$, $OA = 9$ and $OP = 18$. If the angle at A is 90 degrees, then $AP^2 = 18^2 - 9^2 = 324 - 81 = 243$, which is a valid positive length. So the figure is consistent.

Step 4: Conclusion The radius is perpendicular to the tangent at the point of contact, so

Quick Tip to Solve: Recall the angle between a tangent and the radius at the point of contact.

37(ii). **Calculate the length of the tangent PA.** [1 mark]

Solution:

Step 1: Use the right angle at A From part (i), $\angle OAP = 90^\circ$. So $\triangle OAP$ is right angled at A, with hypotenuse $OP = 18$ m and one side $OA = 9$ m.

Step 2: Apply Pythagoras theorem

$$OP^2 = OA^2 + AP^2$$

$$18^2 = 9^2 + AP^2$$

$$324 = 81 + AP^2$$

$$AP^2 = 243$$

Step 3: Take the square root $243 = 81 \times 3$, so

$$AP = \sqrt{243} = 9\sqrt{3} \text{ m}$$

This is about 15.59 m.

Step 4: Conclusion The length of the tangent PA is

Quick Tip to Solve: Triangle OAP has a right angle at A. Use Pythagoras theorem with OP = 18 and OA = 9.

37(iii)(A). (A) Calculate the measure of $\angle AOB$. [2 marks]

Solution:

Step 1: Draw the triangle and name the angle Join OA, OB and OP. Let $\angle AOP = \theta$. In right $\triangle OAP$, the angle at A is 90 degrees, $OA = 9$ and $OP = 18$.

Step 2: Use cosine

$$\cos\theta = \frac{OA}{OP} = \frac{9}{18} = \frac{1}{2} = \cos 60^\circ$$

So $\theta = 60^\circ$.

Step 3: Show OP bisects angle AOB In $\triangle OAP$ and $\triangle OBP$: $OA = OB$ (radii), $AP = BP$ (tangents from an external point are equal), and OP is common. So the triangles are congruent, and $\angle AOP = \angle BOP = 60^\circ$.

Step 4: Add the two angles

$$\angle AOB = 2\theta = 2 \times 60^\circ = 120^\circ$$

Step 5: Conclusion The angle subtended at the centre by the points of contact is

Quick Tip to Solve: In right triangle OAP, find angle AOP using cos = OA/OP. Then double it, since OP bisects angle AOB.

OR

37(iii)(B). (B) In the above design, if PA and PB were inclined at 60°, then what would be their length? [2 marks]

Solution:

Step 1: Note what is given The two tangents PA and PB are inclined at 60°, so $\angle APB = 60^\circ$. The radius is still 9 m, as in the design.

Step 2: Find angle OPA The line OP bisects $\angle APB$ because $\triangle OAP \cong \triangle OBP$.

$$\angle OPA = \frac{1}{2} \times 60^\circ = 30^\circ$$

Step 3: Use tangent ratio in right triangle OAP The angle at A is 90 degrees.

$$\tan 30^\circ = \frac{OA}{AP} \quad \Rightarrow \quad \frac{1}{\sqrt{3}} = \frac{9}{AP}$$

Step 4: Solve

$$AP = 9\sqrt{3} \text{ m}$$

Since $PB = PA$, each tangent is $9\sqrt{3}$ m, about 15.59 m long.

Step 5: Conclusion Each tangent has length

Quick Tip to Solve: OP bisects the angle between the tangents, so angle OPA = 30 degrees. Then use tan 30 = OA/AP.

38(i). **A school conducted a weekly test for Class X students, before the commencement of the pre-board examination and recorded their scores (out of 50). To analyse performance patterns, the academic coordinator grouped the marks into intervals. The grouped frequency distribution is as below:**

Marks Obtained	0 - 10	10 - 20	20 - 30	30 - 40	40 - 50
Number of students	3	6	12	15	14

The academic coordinator computed the central tendencies to judge overall learning level, the most common performance range and to understand consistency across the batch for planning the remedial sessions.
Based on the above information, answer the following questions:
Identify the class with the most common performance range. [1 mark]

Solution:

Concept:

- The most common performance range is the class that has the highest frequency.
- This class is called the modal class.

Step 1: Read the frequencies 0 - 10: 3 students
10 - 20: 6 students
20 - 30: 12 students
30 - 40: 15 students
40 - 50: 14 students

Step 2: Pick the largest frequency The largest frequency is 15, and it belongs to the class 30 - 40. The next largest is 14 (40 - 50), which is smaller.

Step 3: Conclusion The class with the most common performance range is

Quick Tip to Solve: The most common range is the class with the highest frequency (modal class).

38(ii). Find the class interval containing the median. [1 mark]

Solution:

Step 1: Build the cumulative frequency column Add the frequencies one after the other.
0 - 10: cf = 3
10 - 20: cf = 3 + 6 = 9
20 - 30: cf = 9 + 12 = 21
30 - 40: cf = 21 + 15 = 36
40 - 50: cf = 36 + 14 = 50

Step 2: Find N by 2 The total number of students is $N = 50$, so

$$\frac{N}{2} = 25$$

Step 3: Locate the median class The median class is the first class whose cumulative frequency is greater than or equal to 25. The cf of 20 - 30 is 21, which is less than 25. The cf of 30 - 40 is 36, which is greater than 25. So the median class is 30 - 40.

Step 4: Conclusion The class interval containing the median is

Quick Tip to Solve: Make a cumulative frequency column and find the first class whose cf is at least N/2 = 25.

38(iii)(A). (A) What is the average performance of the students? [2 marks]

Solution:

Step 1: Choose the method The average performance is the mean. For grouped data we use the direct method: Mean $= \frac{\sum f_i x_i}{\sum f_i}$, where x_i is the class mark (midpoint) of each class.

Step 2: Find the class marks 0 - 10: $x = 5$
10 - 20: $x = 15$
20 - 30: $x = 25$
30 - 40: $x = 35$
40 - 50: $x = 45$

Step 3: Find each f times x $3 \times 5 = 15$
$6 \times 15 = 90$
$12 \times 25 = 300$
$15 \times 35 = 525$
$14 \times 45 = 630$

Step 4: Add up $\sum f_i = 3 + 6 + 12 + 15 + 14 = 50$.
$\sum f_i x_i = 15 + 90 + 300 + 525 + 630 = 1560$.

Step 5: Divide

$$\text{Mean} = \frac{1560}{50} = 31.2$$

Step 6: Conclusion The average (mean) performance of the students is

Quick Tip to Solve: Average means mean. Use class marks and the formula sum(f x) / sum(f).

OR

38(iii)(B). (B) Find the mode of the data. [2 marks]

Solution:

Step 1: Identify the modal class The highest frequency is 15, so the modal class is 30 - 40. Its lower limit is $l = 30$ and the class width is $h = 10$.

Step 2: Write the frequencies Frequency of the modal class: $f_1 = 15$.
Frequency of the class before it: $f_0 = 12$.
Frequency of the class after it: $f_2 = 14$.

Step 3: Recall the mode formula

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

Step 4: Substitute

$$\text{Mode} = 30 + \left(\frac{15 - 12}{2 \times 15 - 12 - 14}\right) \times 10$$

$$= 30 + \frac{3}{30 - 26} \times 10 = 30 + \frac{3}{4} \times 10$$

$$= 30 + 7.5 = 37.5$$

Step 5: Conclusion The mode of the data is

Quick Tip to Solve: The modal class is 30 - 40. Use Mode = l + [(f1 - f0) / (2f1 - f0 - f2)] x h.