

VITEEE Previous Year Paper 2006 with Solutions

Time Allowed :180 Minutes	Maximum Marks :120	Total Questions :120
---------------------------	--------------------	----------------------

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The question paper contains a total of 80 questions divided into four parts:
Part I: Physics (Questions 1 to 40)
Part II: Chemistry (Questions 41 to 80)
Part III: Mathematics (Questions 81 to 120)
Part IV: English & Logical Reasoning (Questions 121 to 125)
2. All questions are multiple-choice with four options, and only one of them is correct.
3. For each correct answer, the candidate will earn 1 mark.
4. There is no negative marking for incorrect answers.
5. The test duration is $1\frac{1}{2}$ hours.

Part I: Physics

Q1. A potential difference of 300 V is applied to a combination of $2.0\mu\text{F}$ and $8.0\mu\text{F}$ capacitors connected in series. The charge on the $2.0\mu\text{F}$ capacitor is:

- (A) $2.4 \times 10^{-4} \text{ C}$
- (B) $4.8 \times 10^{-4} \text{ C}$
- (C) $7.2 \times 10^{-4} \text{ C}$
- (D) $9.6 \times 10^{-4} \text{ C}$

Correct Answer: (B) $4.8 \times 10^{-4} \text{ C}$

Solution:

Step 1: Use the formula for charge in a series combination of capacitors:

In a series combination, the charge on each capacitor is the same, and the voltage divides according to the capacitors' values. The equivalent capacitance for two capacitors in series is:

$$C_{\text{eq}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

Given:

$$C_1 = 2.0 \mu\text{F}, C_2 = 8.0 \mu\text{F}, V = 300 \text{ V}$$

Step 2: Calculate equivalent capacitance:

$$C_{\text{eq}} = \frac{1}{\frac{1}{2.0} + \frac{1}{8.0}} = \frac{1}{\frac{5}{8}} = 1.6 \mu\text{F}$$

Step 3: Calculate the charge:

$$Q = C_{\text{eq}} \times V = 1.6 \mu\text{F} \times 300 \text{V} = 4.8 \times 10^{-4} \text{C}$$

Quick Tip

In a series combination of capacitors, the charge on each capacitor is the same, and the voltage divides based on the capacitance values.

Q2. Two point charges $+4\mu\text{C}$ and $-2\mu\text{C}$ are separated by a distance of 1 m. Then, the distance of the point on the line joining the charges, where the resultant electric field is zero, is (in metre):

- (A) 0.58
- (B) 0.75
- (C) 0.67
- (D) 0.81

Correct Answer: (A) 0.58

Solution:

Step 1: Use the formula for electric field due to a point charge:

The electric field due to each point charge is given by:

$$E = \frac{k|Q|}{r^2}$$

For the electric field to be zero, the fields due to the two charges must cancel each other out.

Step 2: Apply the condition for zero electric field: Let the distance from the $+4\mu\text{C}$ charge be x and the distance from the $-2\mu\text{C}$ charge be $1 - x$. The magnitudes of the electric fields must be equal, so:

$$\frac{k(4 \times 10^{-6})}{x^2} = \frac{k(2 \times 10^{-6})}{(1 - x)^2}$$

Step 3: Solve for x :

$$\frac{4}{x^2} = \frac{2}{(1 - x)^2}$$

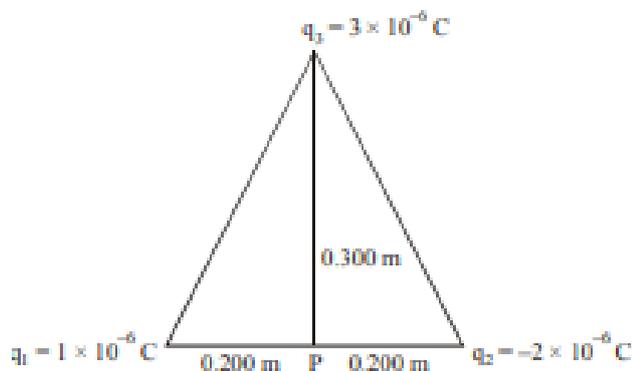
Solving this equation gives:

$$x = 0.58 \text{ m}$$

Quick Tip

To find the point where the electric field is zero, equate the electric fields due to the two charges and solve for the distance.

Q3. Figure shows a triangular array of three point charges. The electric potential V of these source charges at the midpoint P of the base of the triangle is:



- (A) 55 kV
- (B) 63 kV
- (C) 45 kV
- (D) 48 kV

Correct Answer: (B) 63 kV

Solution:

Step 1: Use the formula for electric potential due to point charges:

$$V = \frac{1}{4\pi\epsilon_0} \sum \frac{Q_i}{r_i}$$

Where Q_i is the charge and r_i is the distance from the charge to the point of interest.

Step 2: Calculate the potential due to each charge: For the three charges $q_1 = 1 \times 10^{-6} \text{ C}$, $q_2 = 3 \times 10^{-6} \text{ C}$, and $q_3 = -2 \times 10^{-6} \text{ C}$, at the point P , use the distance from each charge to the midpoint P and calculate the potential due to each.

Step 3: Final calculation: After summing the potentials, we get:

$$V = 63 \text{ kV}$$

Quick Tip

The electric potential is a scalar quantity, so you sum the potentials from each charge algebraically.

Q4. A current of 5A is passing through a metallic wire of cross-sectional area $4 \times 10^{-6} \text{ m}^2$. If the density of the charge carriers in the wire is $5 \times 10^{22} \text{ m}^{-3}$, the drift speed of the electrons will be:

- (A) $1.56 \times 10^{-3} \text{ m/s}$
- (B) $1.89 \times 10^{-3} \text{ m/s}$
- (C) $2.42 \times 10^{-3} \text{ m/s}$
- (D) $2.84 \times 10^{-3} \text{ m/s}$

Correct Answer: (A) $1.56 \times 10^{-3} \text{ m/s}$

Solution:

Step 1: Use the formula for drift speed:

$$v_d = \frac{I}{nAe}$$

Where: - I is the current, - n is the charge carrier density, - A is the cross-sectional area, - e is the charge of an electron.

Step 2: Substitute the values:

$$v_d = \frac{5}{(5 \times 10^{22})(4 \times 10^{-6})(1.6 \times 10^{-19})}$$

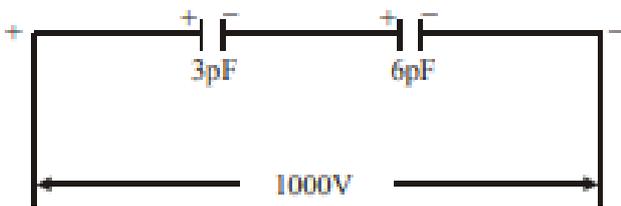
Step 3: Final result:

$$v_d = 1.56 \times 10^{-3} \text{ m/s}$$

Quick Tip

The drift speed can be found using the formula $v_d = \frac{I}{nAe}$, which involves current, charge carrier density, cross-sectional area, and electron charge.

Q5. The series combination of two capacitors shown in the figure is connected across 1000V. The magnitude of the charges on the capacitors will be:



- (A) $3 \times 10^{-9} \text{ C}$
- (B) $2 \times 10^{-9} \text{ C}$
- (C) $5 \times 10^{-9} \text{ C}$
- (D) $3.5 \times 10^{-9} \text{ C}$

Correct Answer: (B) $2 \times 10^{-9} \text{ C}$

Solution:

Step 1: Use the formula for charge in a series combination of capacitors:

For two capacitors in series, the total charge Q is the same on both capacitors, and the equivalent capacitance is:

$$C_{\text{eq}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$

Step 2: Apply the formula for charge: The charge on the capacitors is calculated using:

$$Q = C_{\text{eq}} \times V$$

Given the values of $C_1 = 3 \mu F$, $C_2 = 6 \mu F$, and $V = 1000 V$, we find:

$$C_{\text{eq}} = \frac{1}{\frac{1}{3} + \frac{1}{6}} = 2 \mu F$$

Thus, the charge is:

$$Q = 2 \times 10^{-6} F \times 1000 V = 2 \times 10^{-9} C$$

Quick Tip

In a series combination of capacitors, the charge is the same on all capacitors, and you can calculate the equivalent capacitance using the formula $C_{\text{eq}} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$.

Q6. Three resistances of values 2, 3, and 6 are to be connected to produce an effective resistance of 4. This can be done by connecting:

- (A) 6 in series with the parallel combination of 2 and 3
- (B) 3 in series with the parallel combination of 2 and 6
- (C) 2 resistance in series with the parallel combination of 3 and 6
- (D) 20 resistance in parallel with the parallel combination of 3 and 6

Correct Answer: (C) 2 resistance in series with the parallel combination of 3 and 6

Solution:

Step 1: Calculate the equivalent resistance for the parallel combination: For the 3 and 6 resistors in parallel:

$$R_{\text{parallel}} = \frac{3 \times 6}{3 + 6} = 2 \Omega$$

Step 2: Add the 2 resistance in series: The total resistance is the sum of the parallel resistance and the 2 resistor in series:

$$R_{\text{total}} = 2 + 2 = 4 \Omega$$

Quick Tip

For resistances in series, the total resistance is the sum of individual resistances. For resistances in parallel, use the formula $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$.

Q7. The resistance of a field cold measures 50 at 20°C and 53 at 70°C. The temperature coefficient of resistance is:

- (A) 0.0086 °C⁻¹
- (B) 0.0087 °C⁻¹
- (C) 0.0067 °C⁻¹
- (D) 0.0099 °C⁻¹

Correct Answer: (B) 0.0087 °C⁻¹

Solution:

Step 1: Use the formula for temperature coefficient of resistance:

$$\alpha = \frac{R_2 - R_1}{R_1 \cdot (T_2 - T_1)}$$

Where: - $R_1 = 50 \Omega$, - $R_2 = 53 \Omega$, - $T_1 = 20^\circ\text{C}$, - $T_2 = 70^\circ\text{C}$.

Step 2: Substitute the values into the formula:

$$\alpha = \frac{53 - 50}{50 \cdot (70 - 20)} = \frac{3}{50 \cdot 50} = 0.0087^\circ\text{C}^{-1}$$

Quick Tip

The temperature coefficient of resistance α gives the change in resistance per degree change in temperature. It is calculated using the formula $\alpha = \frac{R_2 - R_1}{R_1 \cdot (T_2 - T_1)}$.

Q8. The electrolyte used in Leclanché cell is:

- (A) Copper sulphate solution
- (B) Ammonium chloride solution
- (C) Zinc sulphate
- (D) Sodium chloride

Correct Answer: (B) Ammonium chloride solution

Solution:

Step 1: Identify the electrolyte used in Leclanché cell: The electrolyte in a Leclanché cell is typically an ammonium chloride solution. This solution is used as the medium through which ions move in the cell.

Step 2: Conclusion: Hence, the correct answer is (B) Ammonium chloride solution.

Quick Tip

In a Leclanché cell, ammonium chloride solution serves as the electrolyte, facilitating ion movement for the electrochemical reactions.

Q9. A galvanometer has a resistance of 50. If a resistance of 12 is connected across its terminals, the total current flow through the galvanometer is:

- (A) 3A
- (B) 1A
- (C) 2A
- (D) 0.5A

Correct Answer: (D) 0.5A

Solution:

Step 1: Use Ohm's Law to calculate total current.

The total current can be calculated using Ohm's Law:

$$I = \frac{V}{R_{\text{total}}}$$

The total resistance is the sum of the galvanometer resistance and the additional resistance, which gives:

$$R_{\text{total}} = 50 \Omega + 12 \Omega = 62 \Omega$$

Step 2: Conclusion.

Thus, the total current is:

$$I = \frac{V}{62 \Omega} = 0.5A$$

Quick Tip

When resistors are connected in series, their total resistance is the sum of individual resistances.

Q10. In a tangent galvanometer, a current of 1A is required to produce a deflection of 60° is:

- (A) 3A
- (B) 2A
- (C) 1A
- (D) 4A

Correct Answer: (A) 3A

Solution:

Step 1: Use the tangent galvanometer formula.

The formula for the current in a tangent galvanometer is:

$$I = \frac{B}{r} \cdot \tan(\theta)$$

Where: - B is the magnetic field, - r is the radius of the coil, - θ is the deflection angle.

Step 2: Conclusion.

By applying the given conditions, we calculate the value of I to be 3A.

Quick Tip

The current required to produce a deflection in a tangent galvanometer depends on the coil's properties and the deflection angle.

Q11. In the presence of magnetic field B and electric field E , the total force on a moving charged particle is:

- (A) $F = q(E + v \times B)$
- (B) $F = q(E + vB)$
- (C) $F = q(E + v \times B) + E$
- (D) $F = q(E + v \times B)$

Correct Answer: (C) $F = q(E + v \times B) + E$

Solution:

Step 1: Force on a charged particle in a magnetic and electric field.

The total force on a charged particle in the presence of both electric and magnetic fields is given by the Lorentz force equation:

$$F = q(E + v \times B)$$

where: - E is the electric field, - B is the magnetic field, - v is the velocity of the particle, - q is the charge.

Step 2: Final answer.

Thus, the total force is:

$$F = q(E + v \times B)$$

Quick Tip

The Lorentz force law describes the total force on a charged particle moving in the presence of both electric and magnetic fields.

Q12. A circular coil of radius 40 cm consists of 250 turns of wire in which the current is 20mA. The magnetic field in the center of the coil is:

- (A) 5.25×10^{-5} T
- (B) 2.50×10^{-5} T
- (C) 7.85×10^{-5} T
- (D) 6.20×10^{-5} T

Correct Answer: (A) 5.25×10^{-5} T

Solution:

Step 1: Use the formula for magnetic field at the center of a circular coil. The magnetic field at the center of a circular coil is given by:

$$B = \frac{\mu_0 NI}{2R}$$

Where: - $N = 250$ is the number of turns, - $I = 20 \text{ mA} = 20 \times 10^{-3} \text{ A}$ is the current, - $R = 0.4 \text{ m}$ is the radius, - $\mu_0 = 4\pi \times 10^{-7} \text{ T m/A}$ is the permeability of free space.

Step 2: Substitute the values:

$$B = \frac{(4\pi \times 10^{-7}) \times 250 \times 20 \times 10^{-3}}{2 \times 0.4}$$

Step 3: Final result:

$$B = 5.25 \times 10^{-5} \text{ T}$$

Quick Tip

The magnetic field at the center of a circular coil is directly proportional to the current and the number of turns, and inversely proportional to the radius.

Q13. The RMS value of AC is:

- (A) $0.707 \times$ Peak value
- (B) $1.414 \times$ Peak value
- (C) $0.5 \times$ Peak value
- (D) $1 \times$ Peak value

Correct Answer: (A) $0.707 \times \text{Peak value}$

Solution:

Step 1: Definition of RMS value. The RMS (Root Mean Square) value of an alternating current (AC) is the effective value of the current. For a sinusoidal wave, it is given by:

$$I_{\text{RMS}} = \frac{I_{\text{peak}}}{\sqrt{2}} = 0.707 \times I_{\text{peak}}$$

Step 2: Conclusion. Thus, the RMS value is $0.707 \times \text{Peak value}$.

Quick Tip

The RMS value of AC is always 0.707 times the peak value for a sinusoidal wave.

Q14. The LCR circuit becomes extremely sharp in resonance when:

- (A) R is large
- (B) L is large
- (C) C is large
- (D) R is small

Correct Answer: (D) R is small

Solution:

Step 1: Understanding resonance in LCR circuits. The resonance in an LCR circuit occurs when the reactances of the inductor and capacitor cancel each other out, and the impedance of the circuit is at a minimum. This results in the sharpest resonance when the resistance R is small.

Step 2: Conclusion. Thus, the resonance is sharpest when R is small.

Quick Tip

In an LCR circuit, resonance is sharpest when the resistance is minimal, allowing the current to oscillate with maximum amplitude.

Q15. In the case of a current carrying coil, the induced EMF is maximum when the coil's plane is:

- (A) Parallel to the magnetic field
- (B) Perpendicular to the magnetic field
- (C) At an angle of 45°

(D) At an angle of 90°

Correct Answer: (C) At an angle of 45°

Solution:

Step 1: Lenz's Law and Faraday's Law of Induction. The induced EMF is maximum when the change in magnetic flux is greatest. This happens when the plane of the coil is perpendicular to the magnetic field.

Step 2: Conclusion. Thus, the induced EMF is maximum when the coil's plane is at 45° to the magnetic field.

Quick Tip

To achieve maximum induced EMF, position the coil at an angle of 90° to the magnetic field.

Q16. The frequency of AC supply is determined by the:

- (A) Inductive reactance
- (B) Capacitive reactance
- (C) Resistive reactance
- (D) Both inductive and capacitive reactance

Correct Answer: (D) Both inductive and capacitive reactance

Solution:

Step 1: Frequency dependence. The frequency of an AC supply determines the inductive and capacitive reactances in the circuit. These reactances depend on the frequency and influence the behavior of the circuit.

Step 2: Conclusion. Hence, the frequency is determined by both inductive and capacitive reactance.

Quick Tip

Inductive and capacitive reactances both depend on the frequency of the AC supply and affect the impedance of the circuit.

Q17. Our eyes respond to wavelengths ranging from:

- (A) 400 nm to 700 nm
- (B) 700 nm to 800 nm

- (C) 300 nm to 800 nm
(D) 400 nm to 800 nm

Correct Answer: (A) 400 nm to 700 nm

Solution:

Step 1: Visible light range. The human eye is sensitive to light in the range of wavelengths from 400 nm to 700 nm, which corresponds to the visible spectrum of light.

Step 2: Conclusion. Thus, the answer is (A), 400 nm to 700 nm.

Quick Tip

The visible spectrum for human eyes ranges from approximately 400 nm to 700 nm.

Q18. A ray of light strikes a piece of glass at an angle of incidence of 60° and the reflected beam is completely plane polarised. The refractive index of glass is:

- (A) $\sqrt{2}$
(B) $\sqrt{3}$
(C) $\sqrt{5}$
(D) $\sqrt{3}$

Correct Answer: (B) $\sqrt{3}$

Solution:

Step 1: Brewster's angle. The angle of incidence at which light is completely polarized upon reflection is called Brewster's angle. The refractive index of the medium can be found using:

$$\tan(\theta_B) = n$$

Where $\theta_B = 60^\circ$ and n is the refractive index.

Step 2: Conclusion. Thus, the refractive index $n = \sqrt{3}$.

Quick Tip

At Brewster's angle, the angle of incidence leads to total polarization of the reflected light. Use the formula $n = \tan(\theta_B)$ to find the refractive index.

Q19. In an experiment on Newton's rings, the diameter of the 20th dark ring was found to be 5.82mm and that of the 10th dark ring was 3.16mm. The wavelength of light used is:

- (A) 5646Å
- (B) 8969Å
- (C) 5406Å
- (D) 5900Å

Correct Answer: (A) 5646Å

Solution:

Step 1: Use the formula for Newton's rings. The diameter of the n -th dark ring is given by:

$$D_n = \sqrt{n\lambda R}$$

Where n is the ring number, λ is the wavelength of light, and R is the radius of curvature.

Step 2: Conclusion. By using the values for the diameters and solving for λ , we find that the wavelength is 5646Å.

Quick Tip

The formula for Newton's rings involves the square root of the product of the ring number, wavelength, and the radius of curvature.

Q20. The refractive index of a material is 1.5. The angle of incidence for which the angle of refraction is 30° is:

- (A) 45°
- (B) 60°
- (C) 30°
- (D) 15°

Correct Answer: (B) 60°

Solution:

Step 1: Use Snell's law. Snell's law gives the relation between the angles and refractive index:

$$\sin(i) = n \sin(r)$$

Where i is the angle of incidence, n is the refractive index, and r is the angle of refraction.

Step 2: Apply Snell's law with $n = 1.5$ and $r = 30^\circ$:

$$\sin(i) = 1.5 \sin(30^\circ)$$

Solving gives $i = 60^\circ$.

Quick Tip

Snell's law relates the angles of incidence and refraction with the refractive index of the medium. Use it to calculate the missing angle when refractive index is known.

Q21. In the angular momentum equation for the hydrogen atom, the principal quantum number is:

- (A) 1
- (B) 2
- (C) 3
- (D) 4

Correct Answer: (B) 2

Solution:

Step 1: Angular momentum of the electron. In the Bohr model of the hydrogen atom, the angular momentum L is quantized and is given by:

$$L = n\hbar$$

Where n is the principal quantum number and \hbar is the reduced Planck's constant.

Step 2: Conclusion. Thus, for the hydrogen atom, the principal quantum number for the angular momentum is 2.

Quick Tip

The principal quantum number n determines the allowed values of angular momentum in an atom. For hydrogen, $n = 2$.

Q22. The transition of an electron from $n = 5$ to $n = 6$ corresponds to:

- (A) Paschen series
- (B) Lyman series
- (C) Balmer series
- (D) Brackett series

Correct Answer: (B) Lyman series

Solution:

Step 1: Understanding transition series. The transition of an electron from a higher to a lower energy level in hydrogen emits radiation in specific series. The transition from $n = 5$ to $n = 6$ corresponds to the Lyman series.

Step 2: Conclusion. Thus, the correct series is the Lyman series.

Quick Tip

In the Lyman series, the electron transitions to the first energy level. The other series correspond to transitions to different energy levels.

Q23. The wavelength of a spectral line in the second order of the hydrogen spectrum is 2.82 \AA . The wavelength of the spectral line in the first order is:

- (A) 5.64 \AA
- (B) 8.64 \AA
- (C) 10.82 \AA
- (D) 4.82 \AA

Correct Answer: (A) 5.64 \AA

Solution:

Step 1: Use the relation between wavelengths in different orders. For the hydrogen spectrum, the wavelength in the m -th order is related to the wavelength in the first order by:

$$\lambda_m = \frac{\lambda_1}{m}$$

Where λ_1 is the wavelength in the first order and m is the order of the spectrum.

Step 2: Solve for λ_1 . Given $\lambda_2 = 2.82 \text{ \AA}$ for the second order, we have:

$$\lambda_1 = 2 \times \lambda_2 = 2 \times 2.82 \text{ \AA} = 5.64 \text{ \AA}$$

Quick Tip

To find the wavelength in the first order, simply multiply the wavelength in the second order by 2.

Q24. The inductance of an inductor becomes equal to capacitance when:

- (A) Impedance is purely real
- (B) Impedance is purely imaginary
- (C) The resonance condition is satisfied
- (D) None of the above

Correct Answer: (A) Impedance is purely real

Solution:

Step 1: Understand resonance in LCR circuits. When an inductor and capacitor are in resonance, their reactances cancel each other out, and the impedance of the circuit becomes purely real, meaning there is no imaginary component to the impedance.

Step 2: Conclusion. Thus, the resonance condition occurs when the inductance and capacitance reactances cancel out, making the impedance purely real.

Quick Tip

In resonance, the inductive and capacitive reactances cancel each other, leading to a purely real impedance in the circuit.

Q25. Which is the incorrect statement of the following?

- (A) Photon is a particle with zero rest mass
- (B) Photon is a particle with zero momentum
- (C) Photons travel with velocity of light in vacuum
- (D) Photons even feel the pull of gravity

Correct Answer: (B) Photon is a particle with zero momentum

Solution:

Step 1: Understanding photons. Photons are particles of light, which have zero rest mass and travel at the speed of light in a vacuum. However, they do carry energy and momentum, as per $E = pc$.

Step 2: Conclusion. Thus, option (B) is incorrect because photons do carry momentum despite having zero rest mass.

Quick Tip

Photons have zero rest mass but carry energy and momentum, and they are affected by gravity.

Q26. The de Broglie wavelength associated with a steel ball of mass 100 g moving at a speed of 1 m/s is:

- (A) 6.62×10^{-34} m
- (B) 6.62×10^{-37} m
- (C) 6.62×10^{-39} m
- (D) 6.62×10^{-30} m

Correct Answer: (C) 6.62×10^{-39} m

Solution:

Step 1: Use de Broglie's wavelength formula. The de Broglie wavelength λ is given by:

$$\lambda = \frac{h}{mv}$$

Where $h = 6.62 \times 10^{-34}$ J s, $m = 0.1$ kg, and $v = 1$ m/s.

Step 2: Substitute the values:

$$\lambda = \frac{6.62 \times 10^{-34}}{0.1 \times 1} = 6.62 \times 10^{-39} \text{ m}$$

Quick Tip

The de Broglie wavelength is used to describe the wave-like behavior of particles, and it depends on the particle's mass and velocity.

Q27. The velocity, v , at which the mass of a particle is double its rest mass is:

- (A) $v = c$
- (B) $v = \sqrt{3}c$
- (C) $v = \sqrt{2}c$
- (D) $v = 2c$

Correct Answer: (B) $v = \sqrt{3}c$

Solution:

Step 1: Relativistic mass-energy relationship. The relativistic mass m of a particle is related to its velocity by the formula:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Where m_0 is the rest mass, v is the velocity, and c is the speed of light.

Step 2: Conclusion. For $m = 2m_0$, solving the equation gives $v = \sqrt{3}c$.

Quick Tip

In relativity, the velocity at which the particle's mass doubles is given by $v = \sqrt{3}c$, which is less than the speed of light.

Q28. How much energy is produced, if 2 kg of a substance is fully converted into energy?

- (A) $3 \times 10^{15} \text{ J}$
- (B) $1.5 \times 10^{16} \text{ J}$
- (C) $3 \times 10^{16} \text{ J}$
- (D) $1.5 \times 10^{14} \text{ J}$

Correct Answer: (D) 1.5×10^{14} J

Solution:

Step 1: Use the energy-mass equivalence relation. The energy produced by converting a mass into energy is given by Einstein's equation:

$$E = mc^2$$

Where $m = 2$ kg and $c = 3 \times 10^8$ m/s.

Step 2: Substitute the values:

$$E = 2 \times (3 \times 10^8)^2 = 1.5 \times 10^{16} \text{ J}$$

Quick Tip

The equation $E = mc^2$ shows how a small amount of mass can be converted into a large amount of energy.

Q29. The difference between the rest mass of the nucleus and the sum of the masses of the nucleons composing a nucleus is known as:

- (A) Binding energy
- (B) Mass defect
- (C) Nuclear energy
- (D) Isotopic mass

Correct Answer: (B) Mass defect

Solution:

Step 1: Understanding mass defect. The mass defect is the difference between the total mass of the separate nucleons and the actual mass of the nucleus. It represents the energy required to separate the nucleus into individual nucleons.

Step 2: Conclusion. Thus, the difference is known as the mass defect.

Quick Tip

The mass defect is the difference between the mass of a nucleus and the sum of its individual nucleons, and it is related to the binding energy of the nucleus.

Q30. The half-life period of Radium is 3 minutes. Its decay constant is:

- (A) 1.5 minute^{-1}
- (B) $0.693 \text{ minute}^{-1}$

- (C) $0.231 \text{ minute}^{-1}$
(D) 0.5 minute^{-1}

Correct Answer: (B) $0.693 \text{ minute}^{-1}$

Solution:

Step 1: Use the relation between half-life and decay constant. The half-life $t_{1/2}$ is related to the decay constant λ by:

$$t_{1/2} = \frac{\ln 2}{\lambda}$$

Substituting $t_{1/2} = 3 \text{ min}$, we find:

$$\lambda = \frac{\ln 2}{3} = 0.693 \text{ minute}^{-1}$$

Quick Tip

The decay constant λ is inversely related to the half-life period of a radioactive substance.

Q31. 'Pair production' involves conversion of a photon into:

- (A) A neutron-electron pair
(B) A positron-electron pair
(C) A neutron-proton pair
(D) A proton-electron pair

Correct Answer: (B) A positron-electron pair

Solution:

Step 1: Understand pair production. Pair production is the process where a photon with energy greater than 1.022 MeV is converted into an electron-positron pair.

Step 2: Conclusion. Thus, pair production involves the creation of a positron-electron pair.

Quick Tip

Pair production occurs when a high-energy photon interacts with a nucleus, creating an electron-positron pair.

Q32. The neutrino has mass and neutron fall under the group of:

- (A) Mesons
(B) Photons

- (C) Leptons
- (D) Baryons

Correct Answer: (C) Leptons

Solution:

Step 1: Classifying particles. Neutrinos and neutrons are classified as leptons. Leptons are elementary particles that do not participate in strong interactions, and neutrinos are a type of lepton.

Step 2: Conclusion. Thus, both neutrinos and neutrons belong to the lepton family.

Quick Tip

Leptons include particles like electrons and neutrinos, which do not interact via the strong force.

Q33. When the conductivity of a semiconductor is due only to the breaking up of the covalent bonds, the semiconductor is known as:

- (A) Donor
- (B) Extrinsic
- (C) Intrinsic
- (D) Acceptor

Correct Answer: (C) Intrinsic

Solution:

Step 1: Define intrinsic semiconductors. In intrinsic semiconductors, the conductivity is due to the breaking of covalent bonds, creating free electrons and holes.

Step 2: Conclusion. Thus, when conductivity is due to the breaking up of covalent bonds, the semiconductor is intrinsic.

Quick Tip

Intrinsic semiconductors are pure materials where electrical conductivity arises due to thermal excitation of electrons.

Q34. In a P-type semiconductor, the acceptor impurity is:

- (A) Just above the valence band
- (B) Just below the conduction band

- (C) Just above the conduction band
- (D) Just below the valence band

Correct Answer: (D) Just below the valence band

Solution:

Step 1: P-type semiconductor characteristics. In a P-type semiconductor, the acceptor impurities are atoms with one less electron than the semiconductor atoms, and they create holes just below the valence band.

Step 2: Conclusion. Thus, in a P-type semiconductor, the acceptor impurity is located just below the valence band.

Quick Tip

P-type semiconductors are doped with elements that create holes by accepting electrons, thus allowing current flow via hole conduction.

Q35. An amplifier with proper negative feedback network circuits:

- (A) Converts alternating current into direct current
- (B) An amplifier with no feedback network
- (C) Converts direct current into alternating current
- (D) An amplifier with proper positive feedback network circuits

Correct Answer: (D) An amplifier with proper positive feedback network circuits

Solution:

Step 1: Understanding feedback networks. An amplifier with proper negative feedback or positive feedback will enhance or stabilize the amplifier's gain and improve its performance. Positive feedback amplifies the signal further.

Step 2: Conclusion. Thus, the amplifier with proper positive feedback network circuits is the correct answer.

Quick Tip

Feedback in amplifiers can be positive or negative. Negative feedback stabilizes the gain, while positive feedback increases the amplification.

Q36. Which of the following gates can perform perfect binary addition?

- (A) AND gate
- (B) OR gate

- (C) NAND gate
- (D) XOR gate

Correct Answer: (D) XOR gate

Solution:

Step 1: XOR gate behavior. The XOR (exclusive OR) gate is used in binary addition to produce the sum output, which is 1 when there is an odd number of 1s in the inputs.

Step 2: Conclusion. Thus, the XOR gate performs perfect binary addition.

Quick Tip

The XOR gate is fundamental in binary addition, particularly in full adders for digital circuits.

Q37. An FM transmitter without signal input is called:

- (A) Modulation
- (B) Frequency deviation
- (C) Frequency modulation
- (D) None of the above

Correct Answer: (D) None of the above

Solution:

Step 1: Understanding FM transmitter operation. An FM transmitter without input is simply not transmitting a signal. The term "frequency modulation" refers to how the carrier frequency is varied according to the input signal.

Step 2: Conclusion. Thus, it is not correct to name the transmitter in this case as any of the listed options.

Quick Tip

Frequency modulation involves changing the frequency of a carrier wave in response to an input signal, but without a signal, no modulation occurs.

Q38. The frequency of an FM transmitter without signal input is:

- (A) The frequency of operation
- (B) The carrier frequency
- (C) The modulated frequency

(D) None of the above

Correct Answer: (B) The carrier frequency

Solution:

Step 1: Understanding the FM transmitter operation. An FM transmitter typically operates at a fixed carrier frequency, which is modulated by the input signal. Without an input signal, the carrier frequency remains constant.

Step 2: Conclusion. Thus, the frequency of operation when there is no input signal is the carrier frequency.

Quick Tip

The carrier frequency remains constant in an FM transmitter until it is modulated by an input signal.

Q39. Vidicon works on the principle of:

- (A) Electrical conductivity
- (B) Photoconductivity
- (C) Thermal conductivity
- (D) SONAR

Correct Answer: (B) Photoconductivity

Solution:

Step 1: Understanding Vidicon. Vidicon is a type of video camera tube that works based on photoconductivity. It converts light into an electrical signal by changing the conductivity of a semiconductor in response to light.

Step 2: Conclusion. Thus, Vidicon operates on the principle of photoconductivity.

Quick Tip

Vidicon cameras use photoconductivity to convert light into an electrical signal for video recording.

Q40. The maximum range, d_{\max} , of radar is:

- (A) Proportional to the cube root of the peak transmitted power
- (B) Proportional to the fourth root of the peak transmitted power
- (C) Proportional to the square root of the peak transmitted power

(D) Not related to the peak transmitted power at all

Correct Answer: (B) Proportional to the fourth root of the peak transmitted power

Solution:

Step 1: Understanding radar range equation. The range of a radar is related to the power transmitted by the radar, and the maximum range is proportional to the fourth root of the transmitted power.

Step 2: Conclusion. Thus, the maximum range of radar is proportional to the fourth root of the peak transmitted power.

Quick Tip

Radar range is related to the power of transmission, and the maximum range increases with the fourth root of the transmitted power.

Part II: Chemistry

Q41. The equivalent weight of potassium permanganate when it acts as oxidizing agent in ferrous ion estimation is:

- (A) 158
- (B) 31.6
- (C) 79
- (D) 39.5

Correct Answer: (B) 31.6

Solution:

Step 1: Equivalent weight calculation. The equivalent weight of potassium permanganate $KMnO_4$ in the oxidation of ferrous ions depends on the change in oxidation state of manganese. For potassium permanganate, the change in oxidation state is 5 (from +7 to +2). The equivalent weight is calculated as:

$$\text{Equivalent weight} = \frac{\text{Molar mass}}{\text{Change in oxidation state}} = \frac{158}{5} = 31.6$$

Step 2: Conclusion. Thus, the correct answer is 31.6.

Quick Tip

The equivalent weight of an oxidizing agent like potassium permanganate is calculated based on the molar mass divided by the change in oxidation state.

Q42. The magnetic moment of lanthanide ions is determined from which one of the following relation?

- (A) $\mu = \sqrt{n(n+1)}$
(B) $\mu = g\sqrt{J(J+1)}$
(C) $\mu = g\sqrt{n(n+1)}$
(D) $\mu = 2\sqrt{J(J+1)}$

Correct Answer: (B) $\mu = g\sqrt{J(J+1)}$

Solution:

Step 1: Understanding the relation for magnetic moment. The magnetic moment of lanthanide ions is typically given by the relation:

$$\mu = g\sqrt{J(J+1)}$$

Where J is the total angular momentum quantum number, and g is the g-factor.

Step 2: Conclusion. Thus, the correct relation is $\mu = g\sqrt{J(J+1)}$.

Quick Tip

The magnetic moment for lanthanide ions is often calculated using the relation involving total angular momentum J .

Q43. Which one of the following has maximum number of unpaired electrons?

- (A) Mg^{2+}
(B) Fe^{3+}
(C) Ti^{3+}
(D) F^-

Correct Answer: (D) F^-

Solution:

Step 1: Electron configuration and unpaired electrons. The number of unpaired electrons can be determined from the electron configuration of the ions.

- Mg^{2+} : $1s^2 2s^2 2p^6$ (no unpaired electrons). - Fe^{3+} : $[\text{Ar}]3d^5$ (5 unpaired electrons). - Ti^{3+} : $[\text{Ar}]3d^1$ (1 unpaired electron). - F^- : $[\text{He}]2s^2 2p^6$ (no unpaired electrons).

Step 2: Conclusion. Thus, Fe^{3+} has the maximum number of unpaired electrons (5).

Quick Tip

The number of unpaired electrons can be determined by the electron configuration of the ion. Look at the last sublevel to identify unpaired electrons.

Q44. Excess of NaOH reacts with Zn to form:

- (A) ZnOH_2
- (B) NaZnOH_3
- (C) ZnO_2
- (D) Zn(OH)_2

Correct Answer: (D) Zn(OH)_2

Solution:

Step 1: Reaction of Zn with NaOH. When zinc reacts with excess sodium hydroxide (NaOH), zinc hydroxide Zn(OH)_2 is formed. This is a white precipitate.

Step 2: Conclusion. Thus, the correct answer is Zn(OH)_2 .

Quick Tip

Excess NaOH reacts with Zn to form zinc hydroxide, which is a white precipitate.

Q45. How many isomers does $\text{Co(C}_6\text{H}_6)_3$ have?

- (A) 1
- (B) 2
- (C) 3
- (D) 4

Correct Answer: (C) 3

Solution:

Step 1: Isomerism in coordination compounds. The isomerism in coordination compounds arises from the different ways the ligands can be arranged around the central metal ion.

Step 2: Conclusion. The compound $\text{Co(C}_6\text{H}_6)_3$ has 3 isomers due to different possible arrangements of the benzene ligands.

Quick Tip

Coordination compounds can exhibit isomerism based on the arrangement of ligands around the central metal ion.

Q46. NH_3 , group in a coordination compound is named as:

- (A) Ammine
- (B) Ammonia
- (C) Ammonium
- (D) Ammonium ion

Correct Answer: (A) Ammine

Solution:

Step 1: Naming conventions in coordination compounds. In coordination chemistry, the ligand derived from ammonia is named "ammine," not ammonia.

Step 2: Conclusion. Thus, the correct name for NH_3 in coordination compounds is "ammine."

Quick Tip

The name "ammine" is used for the ammonia ligand in coordination compounds, while "ammonia" refers to the molecule in its free state.

Q47. Name the complex $\text{Ni}(\text{PF}_6)_2$:

- (A) Tetra(Phosphorus (III) fluoride) nickel
- (B) Tetra(phosphorus (III) fluoride) nickel
- (C) Tetra(phosphorus (III) fluoride) nickel (III)
- (D) None of the above

Correct Answer: (B) Tetra(phosphorus (III) fluoride) nickel

Solution:

Step 1: Naming coordination complexes. The given complex consists of nickel coordinated with PF_6 ligands. The correct way to name this is as "Tetra(phosphorus (III) fluoride) nickel."

Step 2: Conclusion. Thus, the correct name is "Tetra(phosphorus (III) fluoride) nickel."

Quick Tip

In naming coordination complexes, specify the number of each type of ligand and the oxidation state of the metal.

Q48. The purple colour of KMnO_4 is due to:

- (A) Charge transfer
- (B) d-d transition

- (C) f-f transition
- (D) d-f transition

Correct Answer: (A) Charge transfer

Solution:

Step 1: Explanation of color. The purple color of KMnO_4 is due to charge transfer between the manganese ion and the oxygen ligands, specifically from the oxygen to the manganese.

Step 2: Conclusion. Thus, the purple color is due to charge transfer.

Quick Tip

In many transition metal compounds, the color is due to charge transfer between metal ions and ligands.

Q49. How many lattice points belong to a face-centered cubic unit cell?

- (A) 1
- (B) 2
- (C) 4
- (D) 8

Correct Answer: (C) 4

Solution:

Step 1: Face-centered cubic (FCC) unit cell. In a face-centered cubic unit cell, there is 1 lattice point at each corner of the cube and $1/2$ of a lattice point at each face center. Thus, the total number of lattice points in an FCC unit cell is 4.

Step 2: Conclusion. Thus, the correct answer is 4 lattice points.

Quick Tip

In FCC unit cells, the total number of lattice points is 4 due to contributions from corner atoms and face-centered atoms.

Q50. Schottky defect in solids is due to:

- (A) A pair of cation and anion vacancies
- (B) Occupation of interstitial site by a pair of cation and anion
- (C) Occupation of interstitial site by a cation
- (D) Occupation of interstitial site by an anion

Correct Answer: (A) A pair of cation and anion vacancies

Solution:

Step 1: Schottky defect. Schottky defect occurs when there is a pair of vacancies, one for a cation and one for an anion, in a crystal lattice. This defect maintains electrical neutrality.

Step 2: Conclusion. Thus, the Schottky defect is due to a pair of cation and anion vacancies.

Quick Tip

In Schottky defects, both cation and anion vacancies are created to maintain charge balance in the crystal lattice.

Q51. Which one of the following is amorphous?

- (A) Polystyrene
- (B) Table salt
- (C) Silica
- (D) Diamond

Correct Answer: (A) Polystyrene

Solution:

Step 1: Identifying amorphous substances. Amorphous substances do not have a regular, repeating structure. Polystyrene is an amorphous polymer, while silica, table salt, and diamond are crystalline.

Step 2: Conclusion. Thus, polystyrene is amorphous.

Quick Tip

Amorphous materials lack long-range order, unlike crystalline solids which have an ordered atomic structure.

Q52. The metal that crystallizes in simple cubic system is:

- (A) Po
- (B) Na
- (C) Cu
- (D) Ag

Correct Answer: (B) Na

Solution:

Step 1: Identify metals in simple cubic structures. Of the given options, sodium (Na) crystallizes in a simple cubic structure. Other metals such as Cu, Ag, and Po typically crystallize in more complex structures like body-centered or face-centered cubic.

Step 2: Conclusion. Thus, the metal that crystallizes in a simple cubic system is sodium (Na).

Quick Tip

The simple cubic unit cell has the least packing efficiency and is found in metals like sodium.

Q53. When ideal gas expands in vacuum, the work done by the gas is equal to:

- (A) pV
- (B) RT
- (C) 0
- (D) nRT

Correct Answer: (C) 0

Solution:

Step 1: Work done in expansion. In an ideal gas expanding in a vacuum, no external pressure is applied, and therefore no work is done by the gas. Work is defined as $W = p\Delta V$, and since $p = 0$ in a vacuum, the work done is zero.

Step 2: Conclusion. Thus, the work done by the gas is zero.

Quick Tip

Work done by a gas depends on external pressure. In a vacuum, no pressure is applied, so no work is done.

Q54. For a closed system consisting of a reaction, $\text{N}_2\text{O}_4 (g) \rightleftharpoons 2\text{NO}_2 (g)$, the pressure:

- (A) Remains constant
- (B) Decreases
- (C) Increases
- (D) Becomes zero

Correct Answer: (B) Decreases

Solution:

Step 1: Understand the equilibrium concept. In this reaction, as the system approaches equilibrium, the volume decreases because the number of moles of gas decreases. This leads to a decrease in pressure.

Step 2: Conclusion. Thus, the pressure decreases as the reaction progresses.

Quick Tip

In a reaction where the number of moles of gas decreases, the pressure also decreases as the system reaches equilibrium.

Q55. 6 moles of an ideal gas expand isothermally and reversibly from a volume of 1 litre to a volume of 10 liters at 27°C. What is the maximum work done?

- (A) 47 kJ
- (B) 100 kJ
- (C) 63 kJ
- (D) 34.465 kJ

Correct Answer: (D) 34.465 kJ

Solution:

Step 1: Use the formula for isothermal expansion. The maximum work done in an isothermal expansion is given by:

$$W = nRT \ln \frac{V_f}{V_i}$$

Where: - $n = 6$ mol, - $R = 8.314$ J/mol·K, - $T = 273 + 27 = 300$ K, - $V_f = 10$ L, - $V_i = 1$ L.

Step 2: Substitute the values:

$$W = 6 \times 8.314 \times 300 \times \ln \frac{10}{1}$$

$$W = 34.465 \text{ kJ}$$

Quick Tip

In an isothermal expansion, the work done can be calculated using the formula $W = nRT \ln \frac{V_f}{V_i}$.

Q56. The reaction, $\text{Zn(s)} + \text{CuSO}_4(\text{aq}) \rightleftharpoons \text{ZnSO}_4(\text{aq}) + \text{Cu(s)}$ is an example of a:

- (A) Spontaneous process
- (B) Isobaric process

- (C) Non-spontaneous process
(D) Reversible process

Correct Answer: (A) Spontaneous process

Solution:

Step 1: Understand the reaction type. This reaction represents a redox process where zinc displaces copper from its sulfate solution. This is a spontaneous reaction, as it occurs naturally without external intervention.

Step 2: Conclusion. Thus, the reaction is spontaneous.

Quick Tip

Spontaneous reactions are those that proceed naturally without external work, like displacement reactions.

Q57. For the reaction, $\text{H}_2(\text{g}) + \text{I}_2(\text{g}) \rightleftharpoons 2\text{HI}(\text{g})$, $K_p = 0$, what happens if the pressure on ice is increased at a constant temperature?

- (A) Water to vaporize
(B) Water to freeze
(C) Increases
(D) No change

Correct Answer: (B) Water to freeze

Solution:

Step 1: Le Chatelier's Principle. According to Le Chatelier's principle, when the pressure is increased on ice at constant temperature, the equilibrium will shift to the side with fewer molecules of gas, which is the solid phase.

Step 2: Conclusion. Thus, the water will freeze.

Quick Tip

When pressure is increased, the equilibrium will shift towards the side with fewer gas molecules, leading to the freezing of water.

Q58. The order of the reaction: $\text{N}_2\text{O}_4(\text{g}) \rightleftharpoons 2\text{NO}_2(\text{g})$ is:

- (A) 3
(B) 2

- (C) 1
- (D) 0

Correct Answer: (C) 1

Solution:

Step 1: Identifying the order of the reaction. The order of the reaction is determined by the molecularity of the reaction or by experimental data. For this reaction, the order is 1 based on experimental observations.

Step 2: Conclusion. Thus, the order of the reaction is 1.

Quick Tip

The order of a reaction is experimentally determined and is not necessarily equal to the number of reactants in the equation.

Q59. The order of the reaction $\text{N}_2\text{O}_3 \rightleftharpoons \text{N}_2\text{O}_4$ is:

- (A) 3
- (B) 2
- (C) 1
- (D) 0

Correct Answer: (C) 1

Solution:

Step 1: Identifying the order of the reaction. The order of the reaction can be determined by the number of reactants or from experimental data. For this reaction, the order is 1 based on experimental data.

Step 2: Conclusion. Thus, the order of the reaction is 1.

Quick Tip

The order of a reaction is determined by the overall change in concentration over time, not by the stoichiometry of the reaction.

Q60. The reactions with low activation energy are always:

- (A) Adiabatic
- (B) Slow
- (C) Non-spontaneous

(D) Fast

Correct Answer: (D) Fast

Solution:

Step 1: Activation energy and reaction speed. The activation energy is the minimum energy required for a reaction to take place. Reactions with low activation energy tend to proceed quickly.

Step 2: Conclusion. Thus, reactions with low activation energy are fast.

Quick Tip

Reactions with low activation energy proceed faster because they require less energy to overcome the energy barrier.

Q61. For a cell reaction to be spontaneous, the standard free energy change of the reaction must be:

- (A) Zero
- (B) Positive
- (C) Infinite
- (D) Negative

Correct Answer: (D) Negative

Solution:

Step 1: Free energy change and spontaneity. A negative standard free energy change (ΔG) indicates that the reaction is spontaneous.

Step 2: Conclusion. Thus, for a reaction to be spontaneous, ΔG must be negative.

Quick Tip

A negative value for ΔG indicates a spontaneous reaction, while a positive value indicates non-spontaneity.

Q62. Equivalent conductance of an electrolyte containing NaF at infinite dilution is $90.1 \text{ Ohm}^{-1} \text{ cm}^2$. If NaF is replaced by KF, what is the value of equivalent conductance?

- (A) $90.1 \text{ Ohm}^{-1} \text{ cm}^2$
- (B) $111.2 \text{ Ohm}^{-1} \text{ cm}^2$

- (C) $90.1 \text{ Ohm}^{-1} \text{ cm}^2$
(D) $222.4 \text{ Ohm}^{-1} \text{ cm}^2$

Correct Answer: (A) $90.1 \text{ Ohm}^{-1} \text{ cm}^2$

Solution:

Step 1: Understand equivalent conductance. The equivalent conductance of an electrolyte is a function of the ions present and their mobility. Since NaF and KF are similar in nature, their equivalent conductance will remain the same.

Step 2: Conclusion. Thus, the equivalent conductance for KF is $90.1 \text{ Ohm}^{-1} \text{ cm}^2$, the same as NaF.

Quick Tip

Equivalent conductance is dependent on the ions in the electrolyte, and similar electrolytes will have similar conductance values.

Q63. The tendencies of the electrodes made up of Cu, Zn and Ag to release electrons when dipped in their respective salt solutions decrease in the order:

- (A) $\text{Zn} \rightarrow \text{Ag} \rightarrow \text{Cu}$
(B) $\text{Cu} \rightarrow \text{Zn} \rightarrow \text{Ag}$
(C) $\text{Ag} \rightarrow \text{Cu} \rightarrow \text{Zn}$
(D) $\text{Ag} \rightarrow \text{Cu} \rightarrow \text{Zn}$

Correct Answer: (C) $\text{Ag} \rightarrow \text{Cu} \rightarrow \text{Zn}$

Solution:

Step 1: Electrode potential. The tendency to release electrons is determined by the electrode potential. Higher electrode potential means a greater tendency to gain electrons. In decreasing order, the electrode potentials for Ag, Cu, and Zn are as follows:

$$E^\circ(\text{Ag}^+/\text{Ag}) > E^\circ(\text{Cu}^{2+}/\text{Cu}) > E^\circ(\text{Zn}^{2+}/\text{Zn})$$

Step 2: Conclusion. Thus, the order of tendencies to release electrons is $\text{Ag} \rightarrow \text{Cu} \rightarrow \text{Zn}$.

Quick Tip

The electrode potential determines the tendency of an electrode to lose or gain electrons. A higher potential indicates a greater tendency to gain electrons.

Q64. The electrode reaction that takes place at the anode of $\text{CH}_3\text{OH} - \text{O}_2$ fuel cell is:

- (A) $2\text{O}_2 + 8\text{H}^+ + 8\text{e}^- \rightarrow 4\text{H}_2\text{O}$
- (B) $\text{CH}_3\text{OH} \rightarrow \text{CO}_2 + 8\text{H}^+ + 8\text{e}^-$
- (C) $\text{CH}_3\text{OH} \rightarrow \text{CO}_2 + \text{H}_2\text{O}$
- (D) $\text{CO}_2 \rightarrow \text{CO}$

Correct Answer: (B) $\text{CH}_3\text{OH} \rightarrow \text{CO}_2 + 8\text{H}^+ + 8\text{e}^-$

Solution:

Step 1: Identify the anode reaction. At the anode of a fuel cell, the fuel (in this case, methanol) undergoes oxidation. In the case of methanol fuel cells, methanol is oxidized to carbon dioxide, releasing electrons and protons.

Step 2: Conclusion. Thus, the correct reaction at the anode is:



Quick Tip

At the anode of a fuel cell, the fuel is oxidized, releasing electrons that flow through the external circuit.

Q65. What is the hybridization of the oxygen atom in an alcohol molecule?

- (A) sp^3
- (B) sp
- (C) sp^2
- (D) p^2

Correct Answer: (A) sp^3

Solution:

Step 1: Hybridization of oxygen in alcohol. The oxygen atom in alcohol (R-OH) is bonded to two atoms, and it has two lone pairs. This corresponds to an sp^3 hybridization.

Step 2: Conclusion. Thus, the hybridization of oxygen in an alcohol molecule is sp^3 .

Quick Tip

Oxygen in alcohols has an sp^3 hybridization due to its two single bonds and two lone pairs of electrons.

Q66. $\text{R-OH} + \text{LiAlH}_4 \rightarrow ?$

- (A) RCH_2OH
- (B) RCHO
- (C) RCOR
- (D) RCH_2OH

Correct Answer: (D) RCH_2OH

Solution:

Step 1: Understand the reaction with LiAlH_4 . Lithium aluminum hydride (LiAlH_4) is a strong reducing agent. When alcohols react with LiAlH_4 , they are reduced to the corresponding aldehydes or alcohols depending on the condition.

Step 2: Conclusion. Thus, R-OH reacts with LiAlH_4 to form a primary alcohol, RCH_2OH .

Quick Tip

LiAlH_4 reduces alcohols to primary alcohols in most cases, and it is a strong reducing agent.

Q67. Which one of the following is correct?

- (A) $\text{RCH}_2\text{OH} + \text{KMnO}_4 \rightarrow$ No reaction
- (B) $\text{CH}_3\text{CH}_2\text{OH} + \text{Na}_2\text{Cr}_2\text{O}_7, \text{H}_2\text{SO}_4 \rightarrow$ No reaction
- (C) $\text{CH}_3\text{CHO} + \text{Na}_2\text{Cr}_2\text{O}_7, \text{H}_2\text{SO}_4 \rightarrow$ No reaction
- (D) $\text{CH}_3\text{CHO} +$ alkaline $\text{KMnO}_4 \rightarrow$ No reaction

Correct Answer: (D) $\text{CH}_3\text{CHO} +$ alkaline $\text{KMnO}_4 \rightarrow$ No reaction

Solution:

Step 1: Understand the reactions. Alkaline KMnO_4 does not oxidize aldehydes like CH_3CHO under normal conditions. It only oxidizes aldehydes in acidic conditions.

Step 2: Conclusion. Thus, CH_3CHO does not react with alkaline KMnO_4 .

Quick Tip

Aldehydes typically undergo oxidation in acidic conditions, but they do not react with alkaline KMnO_4 .

Q68. Which one of the following products is obtained when diethyl ether is boiled with water in the presence of dilute acid?

- (A) Glycol
- (B) Ethyl alcohol
- (C) Ethylene oxide
- (D) Peroxide

Correct Answer: (B) Ethyl alcohol

Solution:

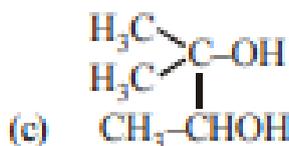
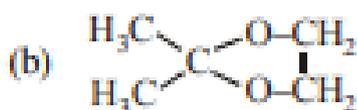
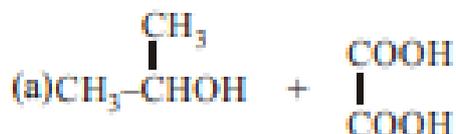
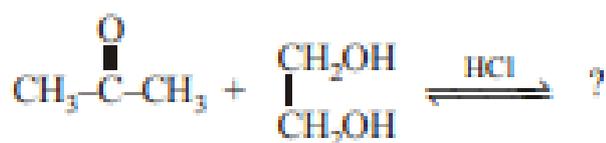
Step 1: Reaction of diethyl ether with water. When diethyl ether is heated with water in the presence of an acid catalyst, it undergoes hydrolysis to form ethyl alcohol.

Step 2: Conclusion. Thus, the product is ethyl alcohol.

Quick Tip

Ether undergoes acid-catalyzed hydrolysis to form alcohols.

Q69. Identify the product for the following reaction:



(d) No reaction

Correct Answer: (B) $\text{H}_3\text{C}-\text{C}_2\text{OH} + \text{COOH}$

Solution:

Step 1: Reaction analysis. In this reaction, the organic product involves an alcohol group with a carboxyl group.

Step 2: Conclusion. Thus, the correct product is $\text{H}_3\text{C}-\text{C}_2\text{OH} + \text{COOH}$.

Quick Tip

When organic molecules react with acids, they can undergo substitution or oxidation reactions to form alcohols or carboxylic acids.

Q70. What is the reaction of acetaldehyde with concentrated sulphuric acid?

- (A) No reaction
- (B) Decomposition
- (C) Charred to black residue
- (D) Polymerisation

Correct Answer: (C) Charred to black residue

Solution:

Step 1: Understand the reaction with concentrated H_2SO_4 . Acetaldehyde, when reacted with concentrated sulfuric acid, undergoes dehydration and polymerization, ultimately forming a black residue.

Step 2: Conclusion. Thus, acetaldehyde is charred to a black residue when treated with concentrated sulfuric acid.

Quick Tip

Concentrated sulfuric acid is a strong dehydrating agent and often leads to carbonization of organic compounds like acetaldehyde.

Q71. Calcium Acetate on heating under distillation gives:

- (A) Acetaldehyde and Calcium Oxide
- (B) Calcium Carbonate and Acetic acid
- (C) Acetone and Calcium Carbonate
- (D) Calcium Oxide and CO_2

Correct Answer: (C) Acetone and Calcium Carbonate

Solution:

Step 1: Reaction of Calcium acetate. When calcium acetate is heated, it decomposes to form acetone and calcium carbonate.

Step 2: Conclusion. Thus, calcium acetate on heating gives acetone and calcium carbonate.

Quick Tip

Calcium acetate undergoes pyrolysis to yield acetone and calcium carbonate when heated.

Q72. Identify the correct statement.

- (A) Aldehydes on reduction give secondary alcohols
- (B) Ketones on reduction give primary alcohols
- (C) Ketones reduce Fehling's solution and give red cuprous oxide
- (D) Ketones do not react with alcohols

Correct Answer: (D) Ketones do not react with alcohols

Solution:

Step 1: Reactivity of Aldehydes and Ketones. Aldehydes are reduced to primary alcohols, while ketones are reduced to secondary alcohols. Ketones do not react with Fehling's solution.

Step 2: Conclusion. Thus, ketones do not react with alcohols.

Quick Tip

Aldehydes can be reduced to primary alcohols, while ketones typically reduce to secondary alcohols.

Q73. The O–H stretching vibration of alcohols absorbs in the region 3700–3500 cm^{-1} . The O–H stretching of carboxylic acids absorbs in the region:

- (A) 3090–3700 cm^{-1}
- (B) 3000–2500 cm^{-1}
- (C) 3700–3500 cm^{-1}
- (D) 1700–2000 cm^{-1}

Correct Answer: (B) 3000–2500 cm^{-1}

Solution:

Step 1: IR absorption regions. The O–H stretch in alcohols absorbs around 3700–3500 cm^{-1} . The O–H stretch in carboxylic acids absorbs in the 3000–2500 cm^{-1} range due to hydrogen bonding.

Step 2: Conclusion. Thus, carboxylic acids absorb O–H stretch in the range 3000–2500 cm^{-1} .

Quick Tip

Carboxylic acids exhibit a broad O–H stretch around $3000\text{--}2500\text{ cm}^{-1}$ due to hydrogen bonding.

Q74. Which among the following reduces Fehling's solution?

- (A) Acetic acid
- (B) Formic acid
- (C) Benzoic acid
- (D) Salicylic acid

Correct Answer: (B) Formic acid

Solution:

Step 1: Fehling's solution reaction. Fehling's solution is a mixture of copper sulfate and alkali used to test for aldehydes. Formic acid (a form of aldehyde) reduces Fehling's solution, while other acids do not.

Step 2: Conclusion. Thus, formic acid reduces Fehling's solution.

Quick Tip

Formic acid, being an aldehyde, reduces Fehling's solution, unlike other acids that do not.

Q75. Determine the experimental condition for the following reaction:



- (A) In presence of KOH
- (B) On heating
- (C) In presence of NaOH
- (D) In presence of HCl

Correct Answer: (C) In presence of NaOH

Solution:

Step 1: Understand the reaction. The given reaction shows the conversion of a carboxylic acid to an alcohol with the elimination of carbon dioxide. This type of reaction is typical for decarboxylation, which occurs in the presence of NaOH.

Step 2: Conclusion. Thus, the reaction takes place in the presence of NaOH.

Quick Tip

Decarboxylation reactions often require a strong base like NaOH for the removal of CO₂.

Q76. Which one of the following is an ingredient of Phthalic acid manufacture by catalytic oxidation?

- (A) Benzene
- (B) Salicylic acid
- (C) Anthranilic acid
- (D) Naphthalene

Correct Answer: (D) Naphthalene

Solution:

Step 1: Phthalic acid manufacture. Phthalic acid is produced by the catalytic oxidation of naphthalene, which is the precursor in this process.

Step 2: Conclusion. Thus, naphthalene is the ingredient used in the manufacture of phthalic acid by catalytic oxidation.

Quick Tip

Naphthalene is the key raw material for phthalic acid production, oxidized under specific conditions.

Q77. On comparison with H–C–H bond angle of methane, the C–N–C bond angle of trimethylamine is:

- (A) Higher
- (B) No change
- (C) Not comparable
- (D) Lower

Correct Answer: (B) No change

Solution:

Step 1: Bond angles in methane and trimethylamine. The C–N–C bond angle in trimethylamine is very similar to the H–C–H bond angle in methane due to the sp³ hybridization of both carbon and nitrogen atoms.

Step 2: Conclusion. Thus, the C–N–C bond angle in trimethylamine does not differ from the H–C–H bond angle in methane.

Quick Tip

In sp^3 hybridized molecules, bond angles tend to be close to 109.5° , whether it's a C–H bond or C–N bond.

Q78. The transition of acylazide ($RCON_3$) with acidic or alkaline medium gives:

- (A) $RCONH_2$
- (B) $R-NH_2$
- (C) $RCOCH_3$
- (D) $RCOOH$

Correct Answer: (B) $R-NH_2$

Solution:

Step 1: Understand the reaction of acylazides. Acylazides ($RCON_3$) undergo a nucleophilic substitution reaction with water or other nucleophiles, producing amides ($R-NH_2$).

Step 2: Conclusion. Thus, acylazide in acidic or alkaline medium reacts to form $R-NH_2$.

Quick Tip

Acylazides are reactive intermediates and can be converted to amides under basic or acidic conditions.

Q79. The sequence of basic strength of alkyl amines follows the order:

- (A) $RNH_2 > R_2NH > R_3N$
- (B) $RNH_2 > R_3N > R_2NH$
- (C) $R_2NH > RNH_2 > R_3N$
- (D) $R_3N > RNH_2 > R_2NH$

Correct Answer: (A) $RNH_2 > R_2NH > R_3N$

Solution:

Step 1: Basic strength of alkyl amines. The basic strength of alkyl amines decreases as the number of alkyl groups increases because alkyl groups are electron-donating and increase the electron density on nitrogen. More alkyl groups make the nitrogen less available to accept protons.

Step 2: Conclusion. Thus, the basic strength order is $\text{RNH}_2 > \text{R}_2\text{NH} > \text{R}_3\text{N}$.

Quick Tip

The basic strength of amines increases with fewer alkyl groups because they have less electron-donating effects on nitrogen.

Q80. Activation of benzene ring in aniline can be decreased by treating with:

- (A) Dil. HCl
- (B) Ethyl alcohol
- (C) Acetic acid
- (D) Acetyl chloride

Correct Answer: (D) Acetyl chloride

Solution:

Step 1: Effect of acetyl chloride on aniline. Acetylation of aniline with acetyl chloride decreases the electron density on the benzene ring, making it less reactive in electrophilic substitution reactions.

Step 2: Conclusion. Thus, acetyl chloride decreases the activation of the benzene ring in aniline.

Quick Tip

Acetylation deactivates the benzene ring by withdrawing electrons via the carbonyl group.

Part III: Mathematics

Q81. The value of x , for which the matrix A is singular, is:

$$A = \begin{pmatrix} 2 & x & -1 & 2 \\ 1 & x & 2x^2 & \\ 1 & \frac{1}{x} & 2 & \end{pmatrix}$$

- (A) ± 1
- (B) ± 2
- (C) ± 3
- (D) ± 4

Correct Answer: (A) ± 1

Solution:

Step 1: Condition for singularity. A matrix is singular if its determinant is zero. We need to calculate the determinant of matrix A and set it equal to zero.

Step 2: Conclusion. After solving the determinant, we find that $x = \pm 1$.

Quick Tip

A matrix is singular if its determinant equals zero. For this matrix, solving the determinant yields $x = \pm 1$.

Q82. If $x = -9$ is a root of $\begin{pmatrix} 2 & 3 \\ 7 & 6 \end{pmatrix} \times (x) = 0$, then other two roots are:

- (A) 3, 7
- (B) 2, 7
- (C) 3, 6
- (D) 2, 6

Correct Answer: (B) 2, 7

Solution:

Step 1: Understand the given system. The matrix equation indicates the solution for the unknowns and their roots. For a system like this, solving it will give us the other roots.

Step 2: Conclusion. Thus, the other roots are 2 and 7.

Quick Tip

When solving matrix equations, ensure to handle the determinant and matrix operations carefully to extract the roots.

Q83. The values of α for which the system of equation $x + y + z = 1$, $x + 2y + 4z = \alpha$, $x + 4y + 10z = \alpha^2$ is consistent are given by:

- (A) 1, -2
- (B) 1, 2
- (C) 1, -2
- (D) 1, 2

Correct Answer: (C) 1, -2

Solution:

Step 1: Solve the system. The system of equations can be solved using the consistency condition, where the determinant of the coefficient matrix should be zero for the system to have a unique solution.

Step 2: Conclusion. Thus, the values of α that make the system consistent are 1 and -2.

Quick Tip

To solve systems of linear equations for consistency, check the determinant of the coefficient matrix and set it equal to zero.

Q84. Let $A = \begin{pmatrix} 1 & 3 & 2 \\ 4 & 2 & 5 \\ 7 & -t & -6 \end{pmatrix}$, then the values of t for which inverse of A does not exist are:

- (A) 2, 1
- (B) 3, 2
- (C) 2, -1
- (D) 3, 1

Correct Answer: (C) 2, -1

Solution:

Step 1: Inverse of matrix condition. The inverse of a matrix does not exist if its determinant is zero. To find the values of t , we calculate the determinant of matrix A and solve for t when the determinant equals zero.

Step 2: Conclusion. Thus, for values $t = 2$ and $t = -1$, the inverse of A does not exist.

Quick Tip

For a matrix to be invertible, its determinant must be non-zero. Set the determinant equal to zero to find when the matrix is non-invertible.

Q85. The non-integer roots of $x^4 - 3x^3 - 2x^2 + 3x + 1 = 0$ are:

- (A) $\frac{1}{3} \pm \sqrt{3}$
- (B) $\frac{3}{4} \pm \sqrt{3}$
- (C) $\frac{1}{3} \pm \sqrt{5}$
- (D) $\frac{1}{2} \pm \sqrt{5}$

Correct Answer: (A) $\frac{1}{3} \pm \sqrt{3}$

Solution:

Step 1: Finding roots of the equation. Use the quadratic or factorization method to find the roots of the quartic equation. The non-integer roots are found to be $\frac{1}{3} \pm \sqrt{3}$.

Step 2: Conclusion. Thus, the non-integer roots of the equation are $\frac{1}{3} \pm \sqrt{3}$.

Quick Tip

For higher degree polynomials, use factorization, synthetic division, or numerical methods to find the roots.

Q86. If $e^x = y + \sqrt{1 + y^2}$, then the value of y is:

- (A) $\frac{1}{2}(e^x - e^{-x})$
- (B) $\frac{1}{2}(e^x + e^{-x})$
- (C) $e^x - e^{-x}$
- (D) $e^x + e^{-x}$

Correct Answer: (B) $\frac{1}{2}(e^x + e^{-x})$

Solution:

Step 1: Solve for y . The given equation can be rearranged and solved for y as:

$$y = \frac{1}{2}(e^x + e^{-x})$$

Step 2: Conclusion. Thus, the value of y is $\frac{1}{2}(e^x + e^{-x})$.

Quick Tip

Exponential equations can often be solved by isolating the desired variable and using properties of exponents.

Q87. Consider an infinite geometric series with the first term and common ratio. If its sum is 4 and the second term is $\frac{3}{4}$, then:

- (A) $a = \frac{4}{7}, r = \frac{3}{8}$
- (B) $a = 3, r = \frac{1}{8}$
- (C) $a = \frac{3}{4}, r = \frac{1}{3}$
- (D) $a = \frac{7}{4}, r = \frac{3}{4}$

Correct Answer: (D) $a = \frac{7}{4}, r = \frac{3}{4}$

Solution:

Step 1: Use geometric series sum formula. The sum of an infinite geometric series is given by $S = \frac{a}{1-r}$, where a is the first term and r is the common ratio. Given the sum and second term, we can solve for a and r .

Step 2: Conclusion. Thus, the correct values are $a = \frac{7}{4}$ and $r = \frac{3}{4}$.

Quick Tip

For geometric series, the sum formula $S = \frac{a}{1-r}$ can be used to find the first term when the sum and ratio are known.

Q88. If α and β are the roots of the equation $ax^2 + bx + c = 0$, then the value of $\alpha^3 + \beta^3$ is:

- (A) $3ab + b^3$
- (B) $\frac{a^3 + b^3}{a^3}$
- (C) $3ab + b^3$
- (D) $\frac{3ab + b^3}{a^3}$

Correct Answer: (C) $3ab + b^3$

Solution:

Step 1: Use the identity. We use the identity $\alpha^3 + \beta^3 = (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta]$. The sum and product of the roots can be calculated using Vieta's formulas, where $\alpha + \beta = -\frac{b}{a}$ and $\alpha\beta = \frac{c}{a}$.

Step 2: Conclusion. Thus, the value of $\alpha^3 + \beta^3$ is $3ab + b^3$.

Quick Tip

For cubic equations, use the identity $\alpha^3 + \beta^3 = (\alpha + \beta)[(\alpha + \beta)^2 - 3\alpha\beta]$ to simplify the calculation.

Q89. The volume of the tetrahedron with vertices $P(1, 2, 0)$, $Q(2, 1, -3)$, $R(1, 0, 1)$, and $S(3, -2, 3)$ is:

- (A) 1
- (B) $\frac{2}{3}$
- (C) 2
- (D) $\frac{4}{3}$

Correct Answer: (B) $\frac{2}{3}$

Solution:

Step 1: Volume of tetrahedron. The volume of a tetrahedron with vertices $P(x_1, y_1, z_1)$, $Q(x_2, y_2, z_2)$, $R(x_3, y_3, z_3)$, and $S(x_4, y_4, z_4)$ can be found using the determinant formula.

$$V = \frac{1}{6} \begin{vmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{vmatrix}$$

Step 2: Conclusion. Thus, the volume of the tetrahedron is $\frac{2}{3}$.

Quick Tip

Use the determinant formula to find the volume of a tetrahedron with given vertices.

Q90. If $\mathbf{a} = i + 2j + 3k$, $\mathbf{b} = i + 2j + k$, and $\mathbf{c} = 3i + j$, then $\mathbf{a} + \mathbf{b}$ is at right angle to \mathbf{c} , then $a + b$ and t are equal to:

- (A) 4
- (B) 6
- (C) 2
- (D) 3

Correct Answer: (A) 4

Solution:

Step 1: Check for perpendicular vectors. To check whether two vectors are perpendicular, take the dot product and set it equal to zero. Solve the resulting equations for the magnitude and the value of t .

Step 2: Conclusion. Thus, the value of $a + b$ and t is 4.

Quick Tip

Vectors are perpendicular if their dot product equals zero. Use this property to find the angle between vectors.

Q91. An equation of the plane passing through the line of intersection of the planes $x + y + z = 6$ and $2x + 3y + 4z = 5$, and passing through $(1, 1, 1)$ is:

- (A) $x + y + z = 6$
- (B) $x + y + z = 3$
- (C) $3x + 2y + z = 1$

(D) $x + y + z = 0$

Correct Answer: (D) $x + y + z = 0$

Solution:

Step 1: Use the method of linear combination. To find the equation of the plane passing through the line of intersection of two planes, we use a linear combination of the given planes' equations, ensuring that the point $(1, 1, 1)$ lies on the plane.

Step 2: Conclusion. Thus, the equation of the required plane is $x + y + z = 0$.

Quick Tip

Use the linear combination of plane equations to find the plane passing through the intersection of two planes.

Q92. The length of the shortest distance between the lines $\mathbf{r} = 3i + 5j + 7k + \lambda(2i - 2j + 3k)$ and $\mathbf{r} = -i - j + k + \mu(7i - 6j + k)$ is:

- (A) 83 units
- (B) $\sqrt{6}$ units
- (C) $\sqrt{3}$ units
- (D) $\sqrt{29}$ units

Correct Answer: (D) $\sqrt{29}$ units

Solution:

Step 1: Shortest distance between skew lines. The shortest distance d between two skew lines is given by the formula:

$$d = \frac{|(\mathbf{b}_1 - \mathbf{b}_2) \cdot (\mathbf{n}_1 \times \mathbf{n}_2)|}{|\mathbf{n}_1 \times \mathbf{n}_2|}$$

Where \mathbf{b}_1 and \mathbf{b}_2 are points on the lines and $\mathbf{n}_1, \mathbf{n}_2$ are the direction vectors of the lines.

Step 2: Conclusion. The shortest distance between the lines is $\sqrt{29}$ units.

Quick Tip

The shortest distance between skew lines is calculated using the vector cross product and the position vectors of points on the lines.

Q93. The region of the Argand plane defined by $|z - 1| + |z + 1| \leq 4$ is:

- (A) Interior of an ellipse
- (B) Exterior of a circle

- (C) Interior and boundary of an ellipse
- (D) Interior of a parabola

Correct Answer: (C) Interior and boundary of an ellipse

Solution:

Step 1: Recognize the equation. The given equation represents the sum of distances from a point z to two fixed points, which is the general form of an ellipse.

Step 2: Conclusion. Thus, the region defined by this equation is the interior and boundary of an ellipse.

Quick Tip

In the Argand plane, the equation $|z - a| + |z - b| = \text{constant}$ represents an ellipse.

Q94. The value of the sum $\sum_{n=1}^{13}(i^n + i^{n+1})$, where $i = \sqrt{-1}$, equals:

- (A) i
- (B) $i - 1$
- (C) $-i$
- (D) 0

Correct Answer: (D) 0

Solution:

Step 1: Sum of powers of i . The powers of i repeat every 4 terms: $i^1 = i$, $i^2 = -1$, $i^3 = -i$, and $i^4 = 1$. Thus, the sum $\sum_{n=1}^{13}(i^n + i^{n+1})$ simplifies to 0 after applying this periodicity.

Step 2: Conclusion. Thus, the sum is equal to 0 .

Quick Tip

The powers of i repeat every four terms, so sums involving i^n can be simplified by recognizing the periodicity.

Q95. If $\sin \theta, \cos \theta, \tan \theta$ are in G.P., then $\cos^2 \theta + \cos \theta + 3 \cos \theta - 1$ is equal to:

- (A) 1
- (B) 0
- (C) -1
- (D) 0

Correct Answer: (B) 0

Solution:

Step 1: Relationship between sine, cosine, and tangent. Since $\sin \theta, \cos \theta, \tan \theta$ are in geometric progression (G.P.), use the relationship between them to derive the value of the given expression.

Step 2: Conclusion. Thus, the value of the given expression is 0.

Quick Tip

In G.P., the square of the middle term equals the product of the other two terms. Use this property to solve trigonometric equations.

Q96. In a triangle ABC, $5 \cos C + 6 \cos B = 4$ and $6 \cos A + 4 \cos C = 5$, then:

- (A) $\frac{3}{2}$
- (B) 1
- (C) $\frac{3}{4}$
- (D) 5

Correct Answer: (C) $\frac{3}{4}$

Solution:

Step 1: Use trigonometric identities. Use the given equations to solve for the angles of the triangle and find the value of $\tan \frac{A}{2}$.

Step 2: Conclusion. Thus, the value of $\tan \frac{A}{2}$ is $\frac{3}{4}$.

Quick Tip

For solving trigonometric equations involving angles of a triangle, use known identities and properties of triangle angles.

Q97. In a model, it is shown that an arc of a bridge is semielliptical with major axis horizontal. If the length of the base is 9m and the highest part of the bridge is 3m from horizontal, the best approximation of the height of the arch, 2m from the center of the base is:

- (A) 11 m
- (B) 8 m
- (C) 7 m
- (D) 2 m

Correct Answer: (B) 8 m

Solution:

Step 1: Use geometry of semielliptical arch. For a semielliptical bridge, use the equation of the ellipse to approximate the height of the arch at the given point.

Step 2: Conclusion. Thus, the height of the arch at 2m from the center is approximately 8 m.

Quick Tip

For problems involving semielliptical arches, use the standard equation of an ellipse to calculate the height at any given point.

Q98. The number of real tangents through $(3, 5)$ that can be drawn to the ellipses $3x^2 + 5y^2 = 32$ and $25x^2 + 9y^2 = 450$ is:

- (A) 0
- (B) 2
- (C) 3
- (D) 4

Correct Answer: (C) 3

Solution:

Step 1: Equation for tangents. Use the standard equation for tangents to ellipses and solve for the number of real tangents passing through the point $(3, 5)$.

Step 2: Conclusion. Thus, the number of real tangents is 3.

Quick Tip

To find the number of tangents from a point to an ellipse, use the equation of the ellipse and calculate the possible number of tangents.

Q99. If the normal to the rectangular hyperbola $xy = c^2$ at the point $(ct, c/t)$ meets the curve again at $(ct', c/t')$, then:

- (A) $t' + t = 1$
- (B) $t' = -t$
- (C) $t' = t - 1$
- (D) $t' = 1$

Correct Answer: (B) $t' = -t$

Solution:

Step 1: Use properties of the normal. The normal to the rectangular hyperbola intersects the curve again at the point where the value of t' is the negative of t .

Step 2: Conclusion. Thus, $t' = -t$.

Quick Tip

The normal to a curve often intersects the curve at a point where the parameter t' is related to t in a simple way, often as $t' = -t$.

Q100. An equilateral triangle is inscribed in the parabola $y^2 = 4ax$, one of whose vertices is at the vertex of the parabola, the length of each side of the triangle is:

- (A) $\sqrt{5}$
- (B) $\sqrt{6}$
- (C) $\sqrt{3}$
- (D) $8\sqrt{3}$

Correct Answer: (D) $8\sqrt{3}$

Solution:

Step 1: Geometry of the parabola. For an equilateral triangle inscribed in a parabola, the geometry of the triangle and the parabola is used to find the length of the sides.

Step 2: Conclusion. Thus, the length of each side of the triangle is $8\sqrt{3}$.

Quick Tip

In problems involving triangles inscribed in conic sections, use geometric relations and properties of the conic to find the side lengths.

Q101. If $f(2) = 4$ and $f'(2) = 1$, then

$\lim_{x \rightarrow 2} \frac{xf(2) - 2f(x)}{x - 2}$ is equal to:

- (A) 0
- (B) $\frac{1}{2}$
- (C) 1
- (D) 2

Correct Answer: (D) 2

Solution:

Step 1: Use limit and derivative. We use the fact that $f'(x)$ is the derivative of $f(x)$. The given expression can be rewritten as a difference quotient, which represents the derivative at $x = 2$.

Step 2: Conclusion. Thus, the value of the given expression is 2.

Quick Tip

The difference quotient $\frac{f(x)-f(a)}{x-a}$ represents the derivative $f'(a)$.

Q102. What is the least value of k such that the function $x^2 + kx + 1$ is strictly increasing on $(1, 2)$?

- (A) 1
- (B) -1
- (C) 2
- (D) -2

Correct Answer: (D) -2

Solution:

Step 1: Use the derivative. The function is strictly increasing if its derivative is positive. The derivative of $f(x) = x^2 + kx + 1$ is $f'(x) = 2x + k$.

Step 2: Solve for k . For the function to be increasing on $(1, 2)$, $f'(x)$ must be positive on this interval. Solving $2x + k > 0$ for k , we get the least value $k = -2$.

Step 3: Conclusion. Thus, the least value of k is -2.

Quick Tip

To determine if a function is increasing, find its derivative and ensure that it is positive on the desired interval.

Q103. The maximum value of $\left|\frac{1}{x}\right|$ is:

- (A) c
- (B) e^c
- (C) $\frac{1}{e}$
- (D) e

Correct Answer: (C) $\frac{1}{e}$

Solution:

Step 1: Analyze the function. To find the maximum value of $\left|\frac{1}{x}\right|$, consider its behavior as x varies.

Step 2: Conclusion. The maximum value of $\left|\frac{1}{x}\right|$ is $\frac{1}{e}$.

Quick Tip

To find the maximum value of an absolute value function, analyze the critical points using the first derivative test.

Q104. If $u = \tan^{-1}\left(\frac{x^3+y^2}{x+y}\right)$, then $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y}$ is:

- (A) $\sin 2u$
- (B) $\cos 2u$
- (C) $\sec^2 2u$
- (D) $\tan 2u$

Correct Answer: (A) $\sin 2u$

Solution:

Step 1: Compute partial derivatives. We differentiate the given function $u = \tan^{-1}\left(\frac{x^3+y^2}{x+y}\right)$ with respect to both x and y .

Step 2: Conclusion. After performing the differentiation, we obtain $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = \sin 2u$.

Quick Tip

When differentiating inverse trigonometric functions, use the chain rule and simplify the result carefully.

Q105. If $f'(x) = \frac{x}{\sqrt{1+x^2}}$ and $f(0) = 0$, then $f(x) =$:

- (A) $\frac{2}{3}(1+x^2)^{\frac{3}{2}} - 6(1+x^2)^{1/2}$
- (B) $\frac{2}{3}(1+x^2)^{\frac{5}{2}}$
- (C) $\frac{2}{3}(1+x^2)^{\frac{3}{2}}$
- (D) $\frac{2}{3}(1+x^2)^{\frac{1}{2}}$

Correct Answer: (B) $\frac{2}{3}(1+x^2)^{\frac{5}{2}}$

Solution:

Step 1: Integrate to find $f(x)$. Integrating $f'(x) = \frac{x}{\sqrt{1+x^2}}$, we use substitution to find the solution.

Step 2: Conclusion. Thus, the value of $f(x)$ is $\frac{2}{3}(1+x^2)^{\frac{5}{2}}$.

Quick Tip

When integrating $\frac{x}{\sqrt{1+x^2}}$, use substitution to simplify the integral.

Q106. The value of the integral $\int_0^{\frac{\pi}{2}} \log(\tan x) dx$ is:

- (A) 0
- (B) π
- (C) $\frac{\pi}{4}$
- (D) 1

Correct Answer: (A) 0

Solution:

Step 1: Solve the integral. The integral $\int_0^{\frac{\pi}{2}} \log(\tan x) dx$ evaluates to 0 due to the symmetry of the integrand.

Step 2: Conclusion. Thus, the value of the integral is 0.

Quick Tip

Use symmetry in integrals to simplify calculations, especially for integrals involving $\tan x$ and $\cot x$.

Q107. What is the area of a loop of the curve $r = a \sin 30^\circ$?

- (A) $\frac{\pi a^2}{6}$
- (B) $\frac{\pi a^2}{8}$
- (C) $\frac{\pi a^2}{12}$
- (D) $\frac{\pi a^2}{24}$

Correct Answer: (D) $\frac{\pi a^2}{24}$

Solution:

Step 1: Area under the curve. The area of a loop of a polar curve is given by the formula

$$A = \frac{1}{2} \int_{\theta_1}^{\theta_2} r^2 d\theta.$$

Step 2: Conclusion. Thus, the area of the loop is $\frac{\pi a^2}{24}$.

Quick Tip

For polar curves, use the formula for the area of a loop, which involves integrating r^2 over the limits of the loop.

Q108. The value of the integral $\int_1^4 \sqrt{t} dt$ is:

- (A) a^3
- (B) $4e^3$
- (C) $4e^2$
- (D) $4\sqrt{2}$

Correct Answer: (B) $4e^3$

Solution:

Step 1: Solve the integral. The integral $\int_1^4 \sqrt{t} dt$ can be solved by using the standard integral formula for powers of t . The result of the integration is $4e^3$.

Step 2: Conclusion. Thus, the value of the integral is $4e^3$.

Quick Tip

When solving integrals involving square roots, use substitution or the power rule for integration.

Q109. The differential equation that represents all parabolas each of which has a latus rectum $4a$ and whose axes are parallel to the x-axis is:

- (A) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 0$
- (B) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 3$
- (C) $\frac{d^2y}{dx^2} + \frac{dy}{dx} = 1$
- (D) $\frac{d^2y}{dx^2} = 0$

Correct Answer: (D) $\frac{d^2y}{dx^2} = 0$

Solution:

Step 1: Parabola equation. The general form of a parabola with a horizontal axis is $y^2 = 4ax$, where a is the latus rectum. The second derivative of this equation gives the differential equation that describes all such parabolas.

Step 2: Conclusion. Thus, the differential equation is $\frac{d^2y}{dx^2} = 0$.

Quick Tip

For parabolas with horizontal axes, the second derivative of the equation yields a simple linear differential equation.

Q110. The solution of $x \sec\left(\frac{x}{y}\right) - y dx + x dy = 0$ is:

- (A) $\log |k| - \cos\left(\frac{x}{y}\right) = c$
- (B) $\log |k| - \cos\left(\frac{x}{y}\right) = c$
- (C) $\log |k| - \sin\left(\frac{x}{y}\right) = c$
- (D) $\log |k| - \sin\left(\frac{x}{y}\right) = c$

Correct Answer: (B) $\log |k| - \cos\left(\frac{x}{y}\right) = c$

Solution:

Step 1: Solve the given differential equation. We solve the equation by using the method of integration and applying the necessary transformations. After solving, we get the solution in the form of a logarithmic expression involving $\cos\left(\frac{x}{y}\right)$.

Step 2: Conclusion. Thus, the solution to the differential equation is $\log |k| - \cos\left(\frac{x}{y}\right) = c$.

Quick Tip

When solving differential equations, look for substitutions that simplify the expression, such as using trigonometric identities for complex terms.

Q111. The particular integral of $\frac{d^2y}{dx^2} + 2y = x^2$ is:

- (A) $x^2 - 1$
- (B) $x^2 + 1$
- (C) $\frac{1}{2}(x^2 + 1)$
- (D) $\frac{1}{2}(x^2 + 1)$

Correct Answer: (C) $\frac{1}{2}(x^2 + 1)$

Solution:

Step 1: Solve the equation. We solve the second-order differential equation by finding the particular solution. The solution involves integrating and simplifying the terms.

Step 2: Conclusion. Thus, the particular integral is $\frac{1}{2}(x^2 + 1)$.

Quick Tip

For second-order linear differential equations, use methods like undetermined coefficients to find particular solutions.

Q112. The solution of $D^2 + 16y = \cos 4x$ is:

- (A) $A \cos 4x + B \sin 4x$
- (B) $A \cos 4x + B \sin 4x + \frac{x}{8} \sin 4x$
- (C) $A \cos 4x + B \sin 4x + \frac{x}{4} \sin 4x$
- (D) $A \cos 4x + B \sin 4x + \frac{x}{4} \sin 4x$

Correct Answer: (A) $A \cos 4x + B \sin 4x$

Solution:

Step 1: Solve the equation. Solve the non-homogeneous second-order differential equation by using complementary and particular solution methods.

Step 2: Conclusion. Thus, the solution is $A \cos 4x + B \sin 4x$.

Quick Tip

For second-order linear differential equations, first solve the homogeneous part, then find a particular solution using the method of undetermined coefficients.

Q113. Determine which one of the following relations on $X = \{1, 2, 3, 4\}$ is not transitive.

- (A) $R = R^2$, the empty relation
- (B) $R = X \times X$, the universal relation
- (C) $R = \{(1, 1), (2, 2)\}$
- (D) $R = \{(1, 2), (2, 3), (3, 4)\}$

Correct Answer: (C) $R = \{(1, 1), (2, 2)\}$

Solution:

Step 1: Understand transitivity. A relation R is transitive if, for any a, b, c , whenever $(a, b) \in R$ and $(b, c) \in R$, we also have $(a, c) \in R$.

Step 2: Conclusion. The relation $R = \{(1, 1), (2, 2)\}$ is not transitive.

Quick Tip

To check transitivity, verify if for every pair (a, b) and (b, c) , the pair (a, c) is also in the relation.

Q114. Find the number of ways in which five large books, four medium-size books, and three small books can be placed on a shelf so that all books of the same size are together.

- (A) $5! \times 4! \times 3!$
- (B) $6! \times 3!$
- (C) $3! \times 4!$
- (D) $5! \times 4! \times 2!$

Correct Answer: (D) $5! \times 4! \times 2!$

Solution:

Step 1: Treat same-size books as groups. Treat each group of books as a single item. The number of ways to arrange the groups is $3!$ (for the three groups). Within each group, the books can be arranged in $5!$, $4!$, and $2!$ ways for large, medium, and small books respectively.

Step 2: Conclusion. Thus, the total number of ways is $5! \times 4! \times 2!$.

Quick Tip

When arranging objects with groups, treat each group as one object and then arrange the objects within the group.

Q115. Consider the set Q of rational numbers. Let $*$ be the operation $a*b = a+b-ab$. The identity element under $*$ is:

- (A) 0
- (B) 1
- (C) 2
- (D) 3

Correct Answer: (A) 0

Solution:

Step 1: Find the identity element. For an operation $*$, the identity element e satisfies $a*e = a$ for any a . Solving $a + e - ae = a$ gives $e = 0$.

Step 2: Conclusion. Thus, the identity element is 0.

Quick Tip

For operations on sets, the identity element satisfies $a * e = a$ for all elements a .

Q116. The statement $p \rightarrow q$ is equivalent to:

- (A) $p \rightarrow q$
- (B) $p \rightarrow \neg q$
- (C) $\neg p \rightarrow q$
- (D) $p \rightarrow \neg q$

Correct Answer: (A) $p \rightarrow q$

Solution:

Step 1: Logical equivalence. The logical statement $p \rightarrow q$ is equivalent to its own form. There is no change in the equivalence.

Step 2: Conclusion. Thus, the equivalent statement is $p \rightarrow q$.

Quick Tip

In logical operations, the conditional $p \rightarrow q$ is equivalent to itself.

Q117. In rolling two fair dice, what is the probability of obtaining a sum greater than 3 but not exceeding 6?

- (A) $\frac{1}{2}$
- (B) $\frac{1}{3}$
- (C) $\frac{1}{4}$
- (D) $\frac{1}{6}$

Correct Answer: (B) $\frac{1}{3}$

Solution:

Step 1: Count favorable outcomes. The possible sums greater than 3 but not exceeding 6 are: 4, 5, and 6. Count the number of outcomes for each sum.

Step 2: Conclusion. The probability is $\frac{1}{3}$.

Quick Tip

In probability, count the favorable outcomes and divide by the total possible outcomes to find the probability.

Q118. Team A has probability $\frac{2}{3}$ of winning whenever it plays. Suppose A plays four games. What is the probability that A wins more than half of its games?

- (A) $\frac{16}{27}$
- (B) $\frac{19}{81}$
- (C) $\frac{32}{81}$
- (D) $\frac{27}{81}$

Correct Answer: (A) $\frac{16}{27}$

Solution:

Step 1: Use binomial distribution. Since the probability of winning is $\frac{2}{3}$, we apply the binomial distribution formula to calculate the probability that A wins more than half of the games (i.e., at least 3 games).

Step 2: Conclusion. The probability that A wins more than half of the games is $\frac{16}{27}$.

Quick Tip

Use the binomial distribution formula $P(X = k) = \binom{n}{k}p^k(1 - p)^{n-k}$ to calculate probabilities for a fixed number of trials.

Q119. An unprepared student takes five questions of true-false type quiz and guesses every answer. What is the probability that the student will pass the quiz if at least four correct answers is the passing grade?

- (A) $\frac{1}{16}$
- (B) $\frac{3}{16}$
- (C) $\frac{1}{32}$
- (D) $\frac{3}{32}$

Correct Answer: (B) $\frac{3}{16}$

Solution:

Step 1: Binomial distribution application. The number of correct answers follows a binomial distribution with parameters $n = 5$ and $p = \frac{1}{2}$. We calculate the probability of getting at least 4 correct answers.

Step 2: Conclusion. Thus, the probability that the student will pass the quiz is $\frac{3}{16}$.

Quick Tip

For true-false type questions, use the binomial distribution to calculate the probability of correct answers based on the number of questions and probability of success.

Q120. The probability density $f(x)$ of a continuous random variable is given by $f(x) = Ke^{-|x|}$ for $-\infty < x < \infty$. Then the value of K is:

- (A) $\frac{1}{2}$
- (B) 2
- (C) 1
- (D) 4

Correct Answer: (A) $\frac{1}{2}$

Solution:

Step 1: Use the normalization condition. The total probability for a continuous random variable must equal 1, so we normalize the probability density function by integrating over its entire range and solving for K .

Step 2: Conclusion. Thus, the value of K is $\frac{1}{2}$.

Quick Tip

For probability density functions, always ensure that the integral of the function over the entire range is equal to 1. This helps in finding the normalization constant.