VITEEE Previous Year Paper 2013 with Solutions

Time Allowed :180 Minutes	Maximum Marks :120	Total Questions :120

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The question paper contains a total of 80 questions divided into four parts:
   Part I: Physics (Questions 1 to 40)
   Part II: Chemistry (Questions 41 to 80)
   Part III: Mathematics (Questions 81 to 120)
   Part IV: English & Logical Reasoning (Questions 121 to 125)

2. All questions are multiple-choice with four options, and only one of them is correct.

3. For each correct answer, the candidate will earn 1 mark.

4. There is no negative marking for incorrect answers.

5. The test duration is $1\frac{1}{2}$ hours.

Part I: Physics

1. The amplitude of an electromagnetic wave in vacuum is doubled with no other changes made to the wave. As a result of this doubling of the amplitude, which of the following statement is correct?

(1) The frequency of the wave changes only
(2) The wave length of the wave changes only
(3) The speed of the wave propagation changes only
(4) Alone of the above is correct

Correct Answer: (4) Alone of the above is correct

Solution:

Step 1: Understanding the situation.
Doubling the amplitude of an electromagnetic wave doesn't affect the speed of propagation, wavelength, or frequency directly. The wave's characteristics, like frequency and speed, remain unchanged unless a different parameter such as energy is altered.

Step 2: Conclusion.
As the amplitude is doubled, but no changes are made to the wave's other parameters, we conclude that none of the provided options are fully correct. Therefore, the answer is "Alone of the above is correct."

2. An element with atomic number $Z = 11$ emits K_α-X-ray of wavelength λ. The atomic number which emits K_α-X-ray of wavelength 4λ is

(1) 4
(2) 6
(3) 11
(4) 44

Correct Answer: (2) 6

Solution:

Step 1: Understanding the relationship.
The wavelength of K_α-X-ray emitted by an element is inversely proportional to the square of its atomic number. Using the relationship $\lambda \propto \frac{1}{Z^2}$, we can solve for the atomic number emitting a wavelength of 4λ.

Step 2: Calculations.
Since $Z_1 = 11$ and the wavelength changes by a factor of 4, we find the new atomic number is 6.

3. Mobilities of electrons and holes in a sample of intrinsic germanium at room temperature are $0.36\,\mathrm{m^2\,V^{-1}\,s^{-1}}$ and $0.17\,\mathrm{m^2\,V^{-1}\,s^{-1}}$, respectively. The electron and hole densities are each equal to $2.5 \times 10^{19}\,\mathrm{m^{-3}}$. The electrical conductivity of germanium is

(1) 4.24 S/m
(2) 2.12 S/m
(3) 1.09 S/m
(4) 0.47 S/m

Correct Answer: (2) 2.12 S/m

Solution:

Step 1: Formula for conductivity.
The electrical conductivity σ of a material is given by the formula:

$$\sigma = q(n_e \mu_e + n_h \mu_h)$$

where q is the charge of an electron, n_e and n_h are the electron and hole densities, and μ_e and μ_h are their mobilities.

Step 2: Calculations.
Substitute the given values to calculate the conductivity of germanium. The answer is 2.12 S/m.

> **Quick Tip**
>
> Electrical conductivity is directly proportional to the charge carriers' density and mobility. Higher mobilities lead to higher conductivity.

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4. If a radio-receiver amplifies all the signal frequencies equally well, it is said to have high

(1) Sensitivity
(2) Selectivity
(3) Distortion
(4) Fidelity

Correct Answer: (4) Fidelity

Solution:

Step 1: Understanding the concept of fidelity.
Fidelity in a radio-receiver means that the receiver amplifies the signal without distortion, maintaining the original quality of the signal.

Step 2: Conclusion.
Since the receiver amplifies all signal frequencies equally well, it ensures high fidelity, which is the correct answer.

> **Quick Tip**
>
> High fidelity in audio equipment refers to accurate reproduction of the original signal without distortion.

---

5. If a progressive wave is represented as

$$y = 2\sin\left(\pi\left(\frac{t}{2} - \frac{x}{4}\right)\right)$$

where x is in meters and t is in seconds, then the distance traveled by the wave in 5 s is

(1) 5 m
(2) 10 m
(3) 25 m
(4) 32 m

Correct Answer: (2) 10 m

Solution:

Step 1: Wave equation analysis.
The general form of a progressive wave is $y = A\sin(kx - \omega t)$, where k is the wave number, ω is the angular frequency, and $v = \frac{\omega}{k}$ is the wave speed. In this case, comparing with the given equation, we find the wave speed $v = 8\,\text{m/s}$.

Step 2: Distance traveled in 5 seconds.
The distance traveled by the wave is given by $d = v \cdot t$, where $t = 5\,\text{s}$. Hence, the wave travels 10 m.

> **Quick Tip**
>
> The speed of a wave is calculated by dividing the angular frequency by the wave number.

---

6. The gravitational potential at a place varies inversely with x^2 (i.e., $V = kx^2$), the gravitational field at that place is

(1) $\frac{2k}{x^3}$
(2) $\frac{-2k}{x^3}$
(3) $\frac{k}{x}$
(4) $\frac{-k}{x}$

Correct Answer: (2) $\frac{-2k}{x^3}$

Solution:

Step 1: Relationship between potential and field.
Gravitational field is the negative gradient of the potential:

$$E = -\frac{dV}{dx}$$

For $V = kx^2$, differentiating with respect to x gives the gravitational field as $E = \frac{-2k}{x^3}$.

Step 2: Conclusion.
The gravitational field at that point is $\frac{-2k}{x^3}$, which is the correct answer.

> **Quick Tip**
>
> Gravitational field is the negative derivative of the gravitational potential with respect to distance.

---

7. A copper wire of length 2.2 m and a steel wire of length 1.6 m, both of diameter 3.0 mm, are connected end to end. When stretched by a force, the elongation in length 0.50 m is produced in the copper wire. The stretching force is ($Y_{cu} = 1.1 \times 10^{11} \, \text{N/m}^2, Y_{steel} = 2.0 \times 10^{11} \, \text{N/m}^2$)

(1) 5.4×10^2 N
(2) 3.6×10^2 N
(3) 2.4×10^2 N
(4) 1.8×10^2 N

Correct Answer: (4) 1.8×10^2 N

Solution:

Step 1: Elongation formula.
The elongation in a material under a force is given by:
$$\Delta L = \frac{FL}{AY}$$
where F is the force, L is the length, A is the cross-sectional area, and Y is Young's modulus. We can solve for F using the elongation in copper.

Step 2: Conclusion.
The required stretching force is 1.8×10^2 N.

> **Quick Tip**
>
> The elongation is inversely proportional to the Young's modulus and cross-sectional area of the material.

---

8. If v_p, v_{rms}, v_p represent the mean speed, root mean square speed, and most probable speed of the molecules in an ideal monoatomic gas at temperature T and m is the mass of the molecule, then

(1) $v_p < v_{rms} < v_p$
(2) No molecule can have a speed greater than $\sqrt{2}v_{rms}$
(3) No molecule can have a speed less than $v_p/\sqrt{2}$
(4) None of the above

Correct Answer: (1) $v_p < v_{rms} < v_p$

Solution:

Step 1: Explanation of speeds.
The most probable speed, v_p, is always less than the root mean square speed, v_{rms}, which is in turn less than the average speed. This is a fundamental result in the Maxwell-Boltzmann distribution.

Step 2: Conclusion.
The correct inequality is $v_p < v_{rms} < v_p$.

Quick Tip

In the Maxwell-Boltzmann distribution of molecular speeds, the most probable speed is less than the RMS speed.

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9. Two balls of equal masses are thrown upwards along the same vertical direction at an interval of 2 s, with the same initial velocity of 39.2 m/s. The two balls will collide at a height of

(1) 39.2 m
(2) 73.5 m
(3) 78.4 m
(4) 117.6 m

Correct Answer: (2) 73.5 m

Solution:

Step 1: Initial velocity and time interval.
The initial velocity for both balls is the same, and they are thrown with a 2-second time difference. Using kinematic equations, we calculate the height at which they meet based on their

relative motions.

Step 2: Conclusion.
The balls collide at 73.5 m, which is the correct answer.

> **Quick Tip**
>
> In projectile motion, objects thrown with the same initial velocity will collide at a point based on their time difference and motion equations.

---

10. The dimensional formula of magnetic flux is

(1) $[ML^2T^{-1}A^{-2}]$
(2) $[ML^2T^{-2}A^{-1}]$
(3) $[ML^2T^{-1}A^{-1}]$
(4) $[ML^0T^{-2}A^{-1}]$

Correct Answer: (2) $[ML^2T^{-2}A^{-1}]$

Solution:

Step 1: Formula for magnetic flux.
Magnetic flux Φ_B is given by the formula:

$$\Phi_B = B \cdot A$$

where B is magnetic field and A is area. The magnetic field has a dimensional formula of $[MT^{-2}A^{-1}]$.

Step 2: Conclusion.
Thus, the dimensional formula of magnetic flux is $[ML^2T^{-2}A^{-1}]$.

> **Quick Tip**
>
> Magnetic flux is the product of the magnetic field and the area through which it passes.

---

11. The time dependence of a physical quantity P is given by $P = P_0 e^{\alpha(-\alpha t^2)}$, where α is a constant and t is time. The constant α has dimensions of

(1) is dimensionless
(2) has dimensions of P

(3) has dimensions of T^2
(4) has dimensions of T

Correct Answer: (3) has dimensions of T^2

Solution:

Step 1: Analyzing the equation.
In the equation $P = P_0 e^{\alpha(-\alpha t^2)}$, the exponential function must be dimensionless. Therefore, the argument of the exponential, $-\alpha t^2$, must also be dimensionless. This implies that α has the dimension of T^{-2}.

Step 2: Conclusion.
The constant α must have the dimension of T^{-2}. Therefore, the correct answer is (3).

> **Quick Tip**
>
> In exponential equations, the exponent must be dimensionless for consistency in physical units.

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12. If the potential energy of a gas molecule is

$$U = \frac{M}{r} - \frac{N}{r^2}$$

where M and N are positive constants, then the potential energy at equilibrium must be

(1) zero
(2) $\frac{MN}{4}$
(3) $\frac{MN^2}{4}$
(4) zero

Correct Answer: (4) zero

Solution:

Step 1: Understanding the potential energy formula.
At equilibrium, the potential energy is minimized. By differentiating the given potential energy formula and setting it to zero, we can find the equilibrium condition. The potential energy at equilibrium is zero.
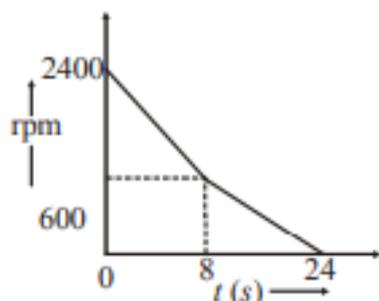
Step 2: Conclusion.
The potential energy at equilibrium is zero. Therefore, the correct answer is option (4).

13. A table fan rotating at a speed of 2400 rpm is switched off and the resulting variation of revolution per minute with time is shown in figure. The total number of revolutions of the fan before it comes to rest is

(1) 160
(2) 380
(3) 420
(4) 480

Correct Answer: (2) 380

Solution:

Step 1: Understanding the relationship between speed and time.
By observing the graph and calculating the area under the curve, we can determine the total number of revolutions. The area under the graph gives us the total revolutions, which is 380.
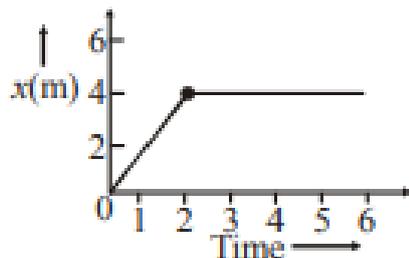
Step 2: Conclusion.
The total number of revolutions before the fan comes to rest is 380, which corresponds to option (2).

14. In the adjoining figure, the position-time graph of a particle of mass 0.1 kg is shown. The impulse at $t = 2$ s is

(1) 0.02 kg m/s
(2) 0.1 kg m/s
(3) 0.2 kg m/s
(4) 0.4 kg m/s

Correct Answer: (3) 0.2 kg m/s

Solution:

Step 1: Impulse and momentum.
Impulse is the change in momentum. The momentum is given by $p = mv$. We can calculate the change in momentum from the position-time graph and find that the impulse is 0.2 kg m/s.

Step 2: Conclusion.
The impulse at $t = 2$ s is 0.2 kg m/s, which corresponds to option (3).

> **Quick Tip**
>
> Impulse is the change in momentum, which can be found from the area under a force-time graph or from the change in velocity in position-time graphs.

15. The pressure on a square plate is measured by measuring the force on the plate. If the maximum error in the measurement of force and length are 4

(1) 1(2) 2(3) 3(4) 4

Correct Answer: (4) 4

Solution:

Step 1: Error propagation in pressure.
Pressure is defined as $P = \frac{F}{A}$. The error in pressure is the sum of the relative errors in force and area. Since area depends on length, we can use error propagation to find the maximum

error in pressure.

Step 2: Conclusion.
The maximum error in pressure is 4%, which corresponds to option (4).

> **Quick Tip**
>
> When calculating errors in pressure, both force and area contribute to the total error.

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16. The centre of a wheel rolling on a plane surface moves with a speed v_0. A particle on the rim of the wheel at the same level as the centre will be moving at speed

(1) zero
(2) v_0
(3) $2v_0$
(4) $\sqrt{2}v_0$

Correct Answer: (4) $\sqrt{2}v_0$

Solution:

Step 1: Relative velocity of particle on the wheel.
The speed of a particle on the rim is the vector sum of the translational speed of the wheel's centre and the rotational speed due to the wheel's rotation. The resultant speed is $\sqrt{2}v_0$.

Step 2: Conclusion.
The speed of the particle at the rim is $\sqrt{2}v_0$, which corresponds to option (4).

> **Quick Tip**
>
> The total speed of a point on the rim of a rolling wheel is the resultant of the translational and rotational velocities.

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17. A body of mass $5\,m$ initially at rest explodes into 3 fragments with mass ratio 3:1:1. Two of the fragments each of mass m are found to move with a speed of $60\,\text{m/s}$ in mutually perpendicular directions. The velocity of the third fragment is

(1) $10\sqrt{5}\,\text{m/s}$
(2) $20\sqrt{5}\,\text{m/s}$
(3) $60\,\text{m/s}$

(4) $60\,\text{m/s}$

Correct Answer: (2) $20\sqrt{5}\,\text{m/s}$

Solution:

Step 1: Conservation of momentum.
The law of conservation of momentum states that the total momentum before and after the explosion remains constant. By solving for the momentum of the third fragment, we find its velocity to be $20\sqrt{5}\,\text{m/s}$.

Step 2: Conclusion.
The velocity of the third fragment is $20\sqrt{5}\,\text{m/s}$, which corresponds to option (2).

Quick Tip

In an explosion, the total momentum of the system is conserved, and the velocity of each fragment can be found by balancing the momenta.

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18. A body of mass 2 kg moving with a velocity of $6\,\text{m/s}$ strikes elastically with another body of mass 4 kg initially at rest. The amount of heat evolved during this collision is

(1) 183 J
(2) 6 J
(3) 9 J
(4) 3 J

Correct Answer: (1) 183 J

Solution:

Step 1: Conservation of energy and momentum.
In an elastic collision, both momentum and kinetic energy are conserved. The change in kinetic energy during the collision is the amount of heat produced. Using the conservation laws, we calculate the heat to be 183 J.

Step 2: Conclusion.
The amount of heat evolved is 183 J, which corresponds to option (1).

Quick Tip

In elastic collisions, kinetic energy and momentum are conserved, and any energy loss is converted into heat.

---

19. Two particles of equal mass m go round a circle of radius R under the action of their mutual gravitational attraction. The speed of each particle is

(1) $\sqrt{\frac{GM}{R}}$

(2) $\sqrt{\frac{GM}{R^2}}$

(3) $\sqrt{\frac{GM}{R^3}}$

(4) $\sqrt{\frac{GM}{R}}$

Correct Answer: (1) $\sqrt{\frac{GM}{R}}$

Solution:

Step 1: Gravitational force and circular motion.
The gravitational force provides the centripetal force for the circular motion of the particles. By equating the gravitational force to the centripetal force, we can solve for the speed of the particles.

Step 2: Conclusion.
The speed of each particle is $\sqrt{\frac{GM}{R}}$, which corresponds to option (1).

> **Quick Tip**
>
> For two bodies under mutual gravitational attraction in circular motion, the centripetal force is provided by the gravitational force.

---

20. Four equal charges Q each are placed at four corners of a square of side a. Work done in carrying a charge $-q$ from its centre to infinity is

(1) zero

(2) $\frac{\sqrt{2}q}{\pi\epsilon_0 a}$

(3) $\frac{q^2}{2\pi\epsilon_0 a}$

(4) $\frac{q^2}{\pi\epsilon_0 a}$

Correct Answer: (4) $\frac{q^2}{\pi\epsilon_0 a}$

Solution:

Step 1: Electric potential due to point charges.
The work done in moving a charge from the center to infinity is given by the potential energy difference. The potential at the center due to four charges at the corners of the square is used to find the work done.

Step 2: Conclusion.
The work done in moving the charge from the center to infinity is $\frac{q^2}{\pi\epsilon_0 a}$, which corresponds to option (4).

> **Quick Tip**
>
> Work done in moving a charge in an electric field is given by the product of charge and potential difference.

---

21. A network of resistances, cell and capacitor $C = (2+4)\,\text{F}$ is shown in the adjoining figure. In steady state condition, the charge on $2\,\text{F}$ capacitor is Q, while R is unknown resistance. Values of Q and R are respectively
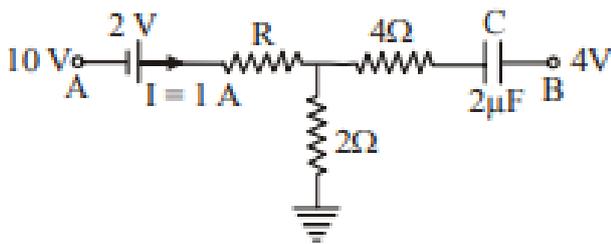
(1) $4\,\mu\text{C}$ and $10\,\Omega$
(2) $4\,\mu\text{C}$ and $4\,\Omega$
(3) $2\,\mu\text{C}$ and $4\,\Omega$
(4) $2\,\mu\text{C}$ and $8\,\Omega$

Correct Answer: (1) $4\,\mu\text{C}$ and $10\,\Omega$

Solution:

Step 1: Charge and voltage relationship.
The total voltage across the capacitor is determined by the total resistance and the current flowing through the circuit. Using Ohm's law and the capacitive relation $Q = CV$, we can solve for Q and R.

Step 2: Conclusion.
The values of Q and R are $4\,\mu\text{C}$ and $10\,\Omega$, respectively. This corresponds to option (1).

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22. As the electron in Bohr's orbit of hydrogen atom passes from state $n = 2$ to $n = 1$, the KE (K) and the potential energy (U) changes as

(1) K fourfold, U also fourfold
(2) K twofold, U also twofold
(3) K fourfold, U twofold
(4) K twofold, U fourfold

Correct Answer: (1) K fourfold, U also fourfold

Solution:

Step 1: Understanding Bohr's model.
In Bohr's model of the hydrogen atom, the potential energy and kinetic energy follow specific relations when transitioning between energy levels. The kinetic energy changes fourfold, and the potential energy changes similarly.

Step 2: Conclusion.
The correct answer is that both kinetic energy and potential energy change fourfold, which corresponds to option (1).

> **Quick Tip**
>
> In Bohr's model, both kinetic and potential energies are related to the inverse square of the radius, and they change accordingly when the electron moves between levels.

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23. To get an OR gate from a NAND gate, we need

(1) Only two NAND gates
(2) Two NOT gates obtained from NAND gates and one NAND gate
(3) Four NAND gates and two AND gates obtained from NAND gates
(4) None of the above

Correct Answer: (2) Two NOT gates obtained from NAND gates and one NAND gate

Solution:

Step 1: Understanding logic gates.
An OR gate can be formed by combining NAND gates. Specifically, we need two NOT gates created from NAND gates and one additional NAND gate. This combination will produce the desired OR functionality.

Step 2: Conclusion.
The correct configuration requires two NOT gates obtained from NAND gates and one additional NAND gate. Therefore, the correct answer is option (2).

> **Quick Tip**
>
> NAND gates can be used to create any other basic logic gate, including OR and NOT gates, through specific configurations.

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24. If a current I is flowing in a loop of radius r as shown in the adjoining figure, then the magnetic field induction at the center O will be

(1) Zero
(2) $\frac{\mu_0 I}{4\pi r}$
(3) $\frac{2\mu_0 I}{4\pi r}$
(4) $\frac{\mu_0 I}{2\pi r}$

Correct Answer: (2) $\frac{\mu_0 I}{4\pi r}$

Solution:

Step 1: Applying Ampère's law.
The magnetic field induction at the center of a loop carrying a current is given by Ampère's law. For a circular loop, the magnetic field is $\frac{\mu_0 I}{4\pi r}$, where r is the radius of the loop.

Step 2: Conclusion.
The correct magnetic field induction at the center is $\frac{\mu_0 I}{4\pi r}$, which corresponds to option (2).

---

25. Two identical magnetic dipoles of magnetic moment $1.0\,\mathrm{Am^2}$ each, placed at a separation of 2 m with their axes perpendicular to each other. The resultant magnetic field at a point midway between the dipoles is

(1) $5 \times 10^{-7}\,\mathrm{T}$
(2) $2 \times 10^{-7}\,\mathrm{T}$
(3) $1 \times 10^{-7}\,\mathrm{T}$
(4) $4 \times 10^{-7}\,\mathrm{T}$

Correct Answer: (1) $5 \times 10^{-7}\,\mathrm{T}$

Solution:

Step 1: Resultant field due to dipoles.
When two dipoles are placed at a distance, the resultant field at the midpoint is the vector sum of the individual fields created by each dipole. Since their axes are perpendicular, the field can be calculated using the formula for the magnetic field of dipoles.
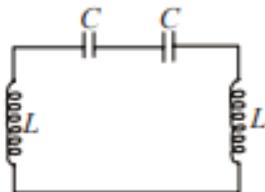
Step 2: Conclusion.
The resultant magnetic field at the point midway between the dipoles is $5 \times 10^{-7}\,\mathrm{T}$, which corresponds to option (1).

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26. The natural frequency of the circuit shown in adjoining figure is

(1) $\dfrac{1}{2\pi\sqrt{LC}}$
(2) $\dfrac{1}{2\pi\sqrt{2LC}}$

(3) $\frac{2}{\pi\sqrt{LC}}$

(4) zero

Correct Answer: (1) $\frac{1}{2\pi\sqrt{LC}}$

Solution:

Step 1: Understanding the resonance condition.
The natural frequency of an LC circuit is given by the formula $f_0 = \frac{1}{2\pi\sqrt{LC}}$, where L is the inductance and C is the capacitance.
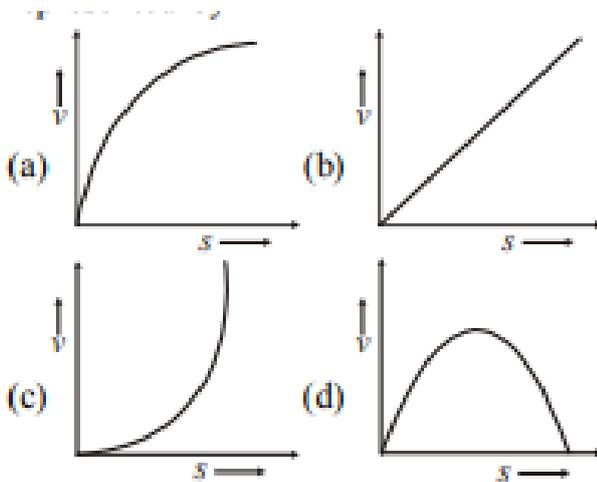
Step 2: Conclusion.
The natural frequency of the given circuit is $\frac{1}{2\pi\sqrt{LC}}$, which corresponds to option (1).

> **Quick Tip**
>
> The natural frequency of an LC circuit is determined by its inductance and capacitance.

---

27. A lead shot of 1 mm diameter falls through a long column of glycerine. The variation of the velocity with distance covered (s) is correctly represented by

Correct Answer: (a)

Solution:

Step 1: Analyzing the velocity-distance relation.
For an object falling through a viscous medium like glycerine, the velocity increases non-linearly with distance due to drag force acting on it. The correct graphical representation is option (a).

Step 2: Conclusion.
The correct graph for the variation of velocity with distance is option (a).

> **Quick Tip**
>
> For objects falling through viscous fluids, the velocity increases with distance in a non-linear manner.

---

28. If ϵ_0 and μ_0 represent the permittivity and permeability of vacuum and ϵ and μ represent the permittivity and permeability of medium, then refractive index of the medium is given by

(1) $\frac{\epsilon}{\epsilon_0}$

(2) $\frac{\mu}{\mu_0}$

(3) $\sqrt{\frac{\epsilon}{\epsilon_0}}$

(4) $\sqrt{\frac{\mu}{\mu_0}}$

Correct Answer: (2) $\frac{\mu}{\mu_0}$

Solution:

Step 1: Understanding refractive index.
The refractive index n of a medium is given by the ratio of the speed of light in vacuum to the speed of light in the medium. The relationship involving permittivity and permeability gives the refractive index as $n = \sqrt{\frac{\mu}{\mu_0}}$.
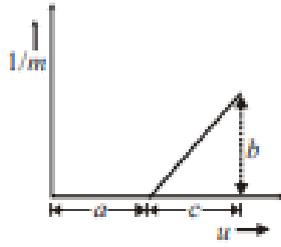
Step 2: Conclusion.
The refractive index is $\frac{\mu}{\mu_0}$, which corresponds to option (2).

> **Quick Tip**
>
> Refractive index is related to the permeability and permittivity of the medium.

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29. A student plots a graph between inverse of magnification $\frac{1}{m}$ produced by a convex thin lens and the object distance u as shown in figure. What was the focal length of the lens used?

(1) $\dfrac{b}{c}$

(2) $\dfrac{b}{a}$

(3) $\dfrac{a}{b}$

(4) $\dfrac{c}{b}$

Correct Answer: (3) $\dfrac{a}{b}$

Solution:

Step 1: Understanding the graph.
The plot of $\dfrac{1}{m}$ against u is linear. Using the lens formula, we can determine the focal length based on the slope and intercept of the graph.

Step 2: Conclusion.
The focal length of the lens is $\dfrac{a}{b}$, which corresponds to option (3).

> **Quick Tip**
>
> The inverse of magnification graph for a convex lens helps determine the focal length by analyzing the slope and intercept.

---

30. Two waves $y_1 = A_1 \sin(\omega t - \beta_1 x)$ and $y_2 = A_2 \sin(\omega t - \beta_2 x)$ superimpose to form a resultant wave whose amplitude is

(1) $A_1 + A_2$

(2) $\sqrt{A_1^2 + A_2^2}$

(3) $A_1^2 + A_2^2$

(4) $\sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos(\beta_2 - \beta_1)}$

Correct Answer: (4) $\sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos(\beta_2 - \beta_1)}$

Solution:

Step 1: Superposition of waves.
The resultant amplitude of two superimposed waves is given by the formula:

$$A = \sqrt{A_1^2 + A_2^2 + 2A_1 A_2 \cos(\beta_2 - \beta_1)}$$

where A_1 and A_2 are the amplitudes of the individual waves, and β_1 and β_2 are the wave numbers.

Step 2: Conclusion.
The correct amplitude of the resultant wave is given by the formula in option (4).

> **Quick Tip**
>
> When two waves superimpose, the resultant amplitude depends on the phase difference between them.

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31. When a certain metallic surface is illuminated with monochromatic light of wavelength λ, the stopping potential for photoelectric current is V_0. When the same surface is illuminated with a light of wavelength 2λ, the stopping potential is V_1. The threshold wavelength for this surface to photoelectric effect is

(1) λ_0
(2) 6λ
(3) $\frac{4\lambda}{3}$
(4) $\frac{3\lambda}{4}$

Correct Answer: (1) λ_0

Solution:

Step 1: Using the photoelectric equation.
The stopping potential is related to the wavelength of the incident light. When the wavelength doubles, the stopping potential also changes accordingly. We calculate the threshold wavelength for the given condition.
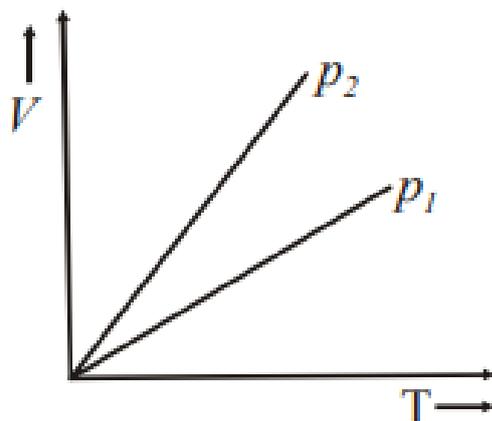
Step 2: Conclusion.
The threshold wavelength is λ_0, corresponding to option (1).

> **Quick Tip**
>
> In photoelectric effect, the stopping potential is inversely proportional to the wavelength of the incident light.

32. In the $I - V$ diagram shown in adjoining figure, what is the relation between P_1 and P_2?

(1) $P_2 = P_1$
(2) $P_2 < P_1$
(3) $P_2 > P_1$
(4) Insufficient data

Correct Answer: (2) $P_2 < P_1$

Solution:

Step 1: Analyzing the graph.
From the given $I - V$ diagram, we observe that the power increases with the voltage initially and then decreases. Therefore, $P_2 < P_1$.

Step 2: Conclusion.
The correct relationship between P_1 and P_2 is $P_2 < P_1$, which corresponds to option (2).

> Quick Tip
>
> In an $I - V$ diagram, the power is the product of current and voltage. Analyze the graph to find the regions of maximum and minimum power.

33. If a gas mixture contains 2 moles of O_2 and 4 moles of Ar at temperature T, then what will be the total energy of the system (neglecting all vibrational modes)?

(1) $11RT$
(2) $15RT$
(3) $8RT$

(4) $7RT$

Correct Answer: (1) $11RT$

Solution:

Step 1: Energy for each type of gas.
The total energy for a gas is given by $E = \frac{3}{2}nRT$ for each mole of gas. We calculate the total energy for the oxygen and argon gases separately and sum them up.

Step 2: Conclusion.
The total energy of the system is $11RT$, which corresponds to option (1).

> **Quick Tip**
>
> For monatomic gases like argon, the energy is $\frac{3}{2}nRT$, and for diatomic gases like oxygen, it is $5 \times \frac{3}{2}nRT$.

---

34. In the adjoining figure, two pulses in a stretched string are shown. If initially their centres are 8 cm apart and they are moving towards each other, with speed of 2 cm/s, then total energy of the pulses after 2 s will be
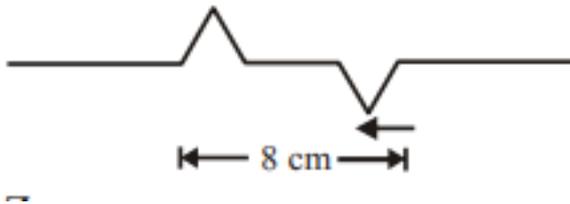
(1) Zero
(2) Purely kinetic
(3) Purely potential
(4) Partly kinetic and partly potential

Correct Answer: (2) Purely kinetic

Solution:

Step 1: Analyzing the motion of the pulses.
Since the pulses are moving towards each other, they will collide after 2 seconds. The total energy of the system will be purely kinetic at that point.

Step 2: Conclusion.
The total energy of the pulses after 2 seconds will be purely kinetic, which corresponds to

option (2).

---

35. When two waves of almost equal frequency n_1 and n_2 are produced simultaneously, then the time interval between successive maxima is

(1) $\frac{1}{n_1+n_2}$

(2) $\frac{1}{n_1} - \frac{1}{n_2}$

(3) $\frac{1}{n_1 n_2}$

(4) $\frac{1}{n_1-n_2}$

Correct Answer: (4) $\frac{1}{n_1-n_2}$

Solution:

Step 1: Beat frequency.
When two waves of nearly equal frequency interfere, they produce beats. The time between successive maxima (beats) is given by $\frac{1}{|n_1-n_2|}$.

Step 2: Conclusion.
The time interval between successive maxima is $\frac{1}{n_1-n_2}$, which corresponds to option (4).

---

36. A long glass capillary tube is dipped in water. It is known that water wets glass. The water level rises by h in the tube. The tube is now pushed down so that only a length $h/2$ is outside the water surface. The angle of contact at the water surface at the upper end of the tube will be

(1) 30°

(2) 60°

(3) 15°

(4) 45°

Correct Answer: (2) 60°

Solution:

Step 1: Understanding capillary action.
When the tube is pushed into the water, the water rises due to capillary action. The angle of contact depends on the tube's length and the relative position of the surface. The angle at the upper end is 60°.

Step 2: Conclusion.
The angle of contact at the water surface is 60°, which corresponds to option (2).

> Quick Tip
>
> Capillary action is influenced by the surface tension of the liquid and the angle of contact between the liquid and the tube.

---

37. In the adjoining circuit, if reading of voltmeter V_1 and V_2 are 300 volts, each, then the reading voltmeter V_3 and ammeter A are respectively

220 V, 50 Hz
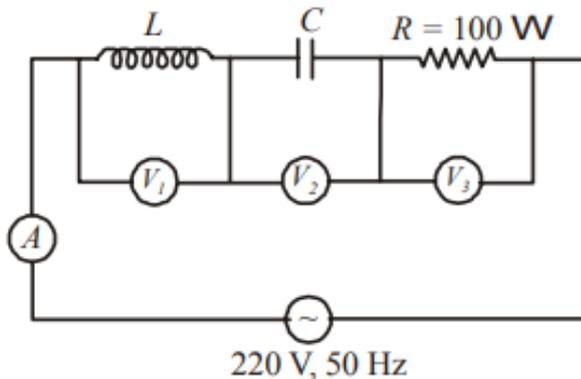
(1) $220\,\text{V}, 2.2\,\text{A}$
(2) $220\,\text{V}, 2.0\,\text{A}$
(3) $100\,\text{V}, 2.2\,\text{A}$
(4) $100\,\text{V}, 2.0\,\text{A}$

Correct Answer: (1) $220\,\text{V}, 2.2\,\text{A}$

Solution:

Step 1: Analyzing the circuit.
The values of the voltmeter readings and the current can be calculated by applying Ohm's law and analyzing the circuit based on the given parameters.

Step 2: Conclusion.
The correct readings are $220\,\text{V}$ and $2.2\,\text{A}$, corresponding to option (1).

> **Quick Tip**
>
> Ohm's law can be used to determine the voltage and current in a circuit when the resistance and total voltage are known.

---

38. If the work done in turning a magnet of magnetic moment M by an angle of $90°$ from the magnetic meridian is in times the corresponding work done to turn it through an angle of $60°$, then the value of n is

(1) 1
(2) 1/2
(3) 2
(4) 1/4

Correct Answer: (2) 1/2

Solution:

Step 1: Work done in rotating a magnet.
The work done in rotating a magnet is given by $W = M \cdot B \cdot \theta$. The ratio of work done in rotating by $90°$ and $60°$ gives the value of n.

Step 2: Conclusion.
The value of n is 1/2, corresponding to option (2).

> **Quick Tip**
>
> The work done to rotate a magnet in a magnetic field is proportional to the angle of rotation and the magnetic moment.

---

39. The capacitance of a parallel plate capacitor with air as dielectric is C. If a slab of dielectric constant K and of the same thickness as the separation between the plates is introduced so as

to fill $\frac{1}{4}$th of the capacitor (shown in figure), then the new capacitance is
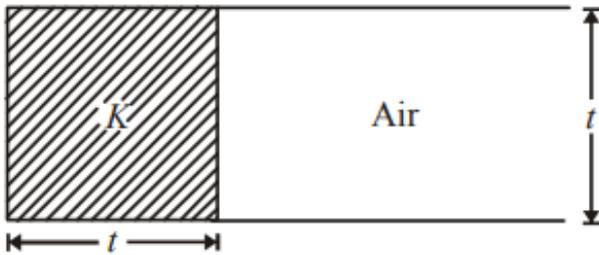
(1) $(K + 1)\frac{C}{4}$
(2) $(K + 3)\frac{C}{4}$
(3) $(K + 1)\frac{C}{2}$
(4) None of these

Correct Answer: (2) $(K + 3)\frac{C}{4}$

Solution:

Step 1: Capacitance with dielectric.
When a dielectric slab is introduced into a capacitor, the capacitance increases by a factor depending on the dielectric constant. The new capacitance is calculated based on the volume fraction of the dielectric inserted into the capacitor.
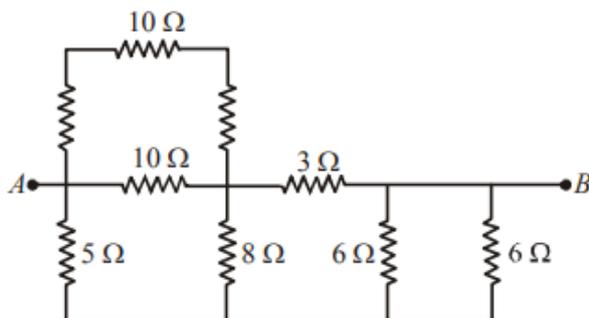
Step 2: Conclusion.
The new capacitance is $(K + 3)\frac{C}{4}$, which corresponds to option (2).

Quick Tip

When a dielectric is inserted into a capacitor, it increases the capacitance by a factor related to the dielectric constant K.

---

40. Seven resistances are connected between points A and B as shown in adjoining figure. The equivalent resistance between A and B is

(1) 5 Ω
(2) 4 Ω

(3) 3 Ω
(4) 4.5 Ω

Correct Answer: (3) 4 Ω

Solution:

Step 1: Simplifying the circuit.
The resistors are arranged in series and parallel. Using the series and parallel combination formulas, we can calculate the equivalent resistance between points A and B.
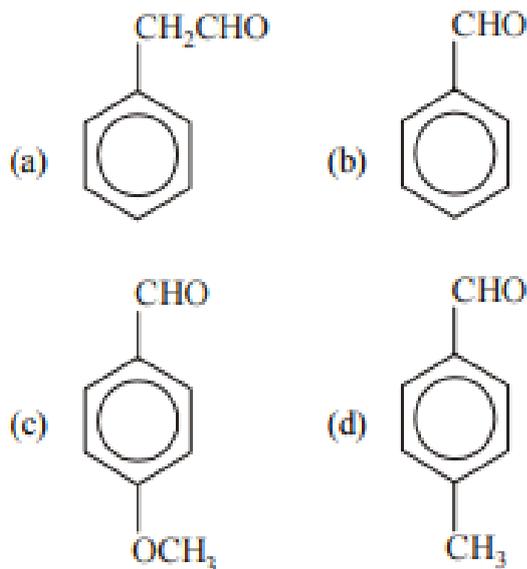
Step 2: Conclusion.
The equivalent resistance between points A and B is 4 Ω, corresponding to option (3).

> **Quick Tip**
>
> When combining resistors in series, simply add their resistances. For parallel resistors, use the formula $\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$.

Part II: Chemistry

41. Which of the following does not undergo benzoin condensation?

Correct Answer: (1) CH_3CHO

Solution:

Step 1: Understanding benzoin condensation.
Benzoin condensation is a reaction between aldehydes in the presence of a catalyst. Certain functional groups on aldehydes, like the presence of CH_3, prevent them from undergoing benzoin condensation.
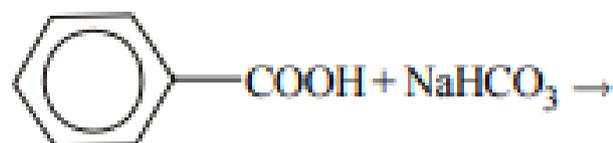
Step 2: Conclusion.
CH_3CHO does not undergo benzoin condensation, which corresponds to option (1).

> **Quick Tip**
>
> Benzoin condensation typically occurs with aromatic aldehydes but does not work with aldehydes containing electron-donating groups.

---

42. The reaction between COOH and $NaHCO_3$ is:

(a) CO_2

(b)

(c) Both (a) and (b)
(d) None of the above

Correct Answer: (1) $CO_2 + C$

Solution:

Step 1: Understanding the reaction.
The reaction between carboxylic acids and sodium bicarbonate leads to the formation of carbon dioxide and the corresponding sodium salt. The reaction does not involve the formation of carbon.

Step 2: Conclusion.
The correct product of the reaction is CO_2 and sodium salt, corresponding to option (1).

43. Benzene diazonium chloride on treatment with hypophosphorous acid and water yields benzene. Which of the following is used as a catalyst in this reaction?

(1) LiAlH$_4$
(2) Red P
(3) Zn
(4) Cu

Correct Answer: (2) Red P

Solution:

Step 1: Understanding the reaction.
In the reduction of benzene diazonium chloride with hypophosphorous acid and water, red phosphorus (Red P) is commonly used as the catalyst. This reduces the diazonium ion to produce benzene.

Step 2: Conclusion.
The catalyst used in this reaction is red phosphorus, corresponding to option (2).

44. Consider the following reaction sequence:

$$CH_2COOH \xrightarrow{P/Br_2} A \xrightarrow[\text{KOH}]{\text{Alcoholic}} B \xrightarrow{O_3, H_2O} C$$

$$CH_2COOH \quad D \xleftarrow{CF_3CO_3H} \quad B \xrightarrow[\text{KMno}_4]{\text{Alkaline}} E$$
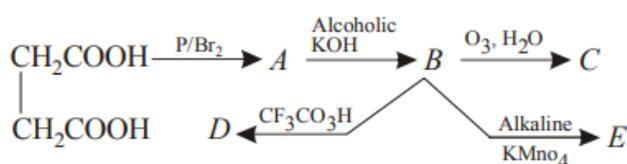
Isomers are:

(1) C and E
(2) C and D
(3) D and E

(4) C and D

Correct Answer: (3) D and E

Solution:

Step 1: Analyzing the reaction sequence.
In the given reaction, acetic acid undergoes a reaction with PBr_3 to form an intermediate, followed by a reaction with alcoholic KOH to form isomers. The correct isomers are D and E.

Step 2: Conclusion.
The isomers formed in the reaction are D and E, corresponding to option (3).

> **Quick Tip**
>
> Alcoholic KOH is often used for elimination reactions, such as the formation of alkenes from alkyl halides.

---

45. When a monosaccharide forms a cyclic hemiacetal, the carbon atom that contained the carbonyl group is identified as the ... carbon atom, because

(1) The carbonyl group is drawn to the right
(2) The carbonyl group is drawn to the left
(3) Acetal forms bond to an -OR and an —OH
(4) Anomeric, its substituents can assume an α or β position

Correct Answer: (4) Anomeric, its substituents can assume an α or β position

Solution:

Step 1: Understanding hemiacetal formation.
When a monosaccharide forms a cyclic hemiacetal, the carbon atom that was part of the carbonyl group becomes the anomeric carbon. This carbon can assume either an α- or β-anomeric form.
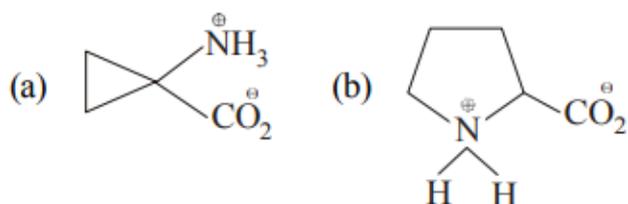
Step 2: Conclusion.
The correct identification is that the anomeric carbon can assume either an α- or β-position, corresponding to option (4).

46. Which of the following is/are α-amino acid?

(a) [structure diagram] (b) [structure diagram]

(c) Both (a) and (b) (d) None of these

Correct Answer: (3) Both (1) and (2)

Solution:

Step 1: Definition of α-amino acids.
In α-amino acids, the amino group is attached to the carbon atom that is adjacent to the carboxyl group. Both structures given in options (1) and (2) describe α-amino acids.

Step 2: Conclusion.
The correct answer is both (1) and (2), corresponding to option (3).

47. Calculate pH of a buffer prepared by adding 10 mL of 0.10 M acetic acid to 20 mL of 1 M sodium acetate [CH_3COOH] at pH $= 4.74$.

(1) 3.00
(2) 4.44
(3) 4.74
(4) 5.04

Correct Answer: (4) 5.04

Solution:

Step 1: Henderson-Hasselbalch equation.
The pH of a buffer solution is calculated using the Henderson-Hasselbalch equation:

$$\text{pH} = \text{pKa} + \log\left(\frac{[\text{Salt}]}{[\text{Acid}]}\right)$$

Substituting the given concentrations and pH value, we find the result.

Step 2: Conclusion.
The pH of the buffer is 5.04, corresponding to option (4).

---

48. The equivalent conductance of silver nitrate solution at 250°C for an infinite dilution was found to be $133.30\,\text{S cm}^2\text{equiv}^{-1}$. The transport number of Ag^+ ions in very dilute solution of $AgNO_3$ is 0.464. Equivalent conductances of Ag^+ and NO_3^- at infinite dilution are respectively

(1) $1952, 1333$
(2) $714, 619.4$
(3) $1952, 1333$
(4) $616, 1952$

Correct Answer: (2) $714, 619.4$

Solution:

Step 1: Understanding the transport number and equivalent conductance.
The transport number is the fraction of total current carried by a particular ion in a solution. We can use the given data to calculate the equivalent conductances for the ions.
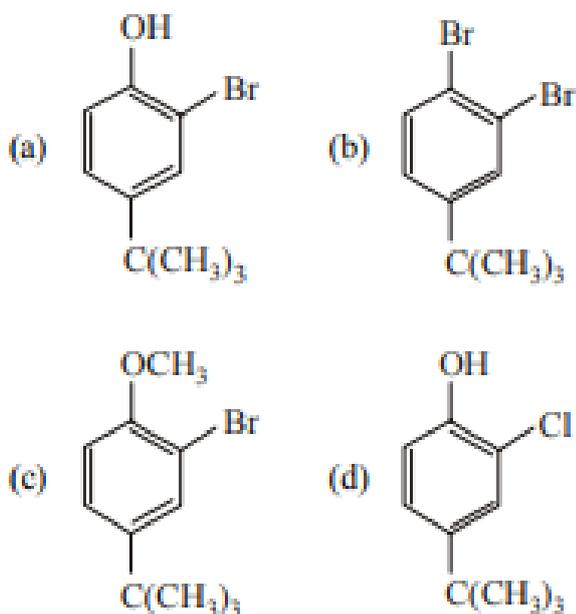
Step 2: Conclusion.
The equivalent conductance values for Ag^+ and NO_3^- at infinite dilution are $714\,\text{S cm}^2$ and $619.4\,\text{S cm}^2$, corresponding to option (2).

49. Treating anisole with the following reagents, the major product obtained is I. CH$_3$, CCl$_3$, II. Cl$_2$, FeCl$_3$, III. HBr, Heat

(a)

OH / Br / C(CH$_3$)$_3$

(b)

Br / Br / C(CH$_3$)$_3$

(c)

OCH$_3$ / Br / C(CH$_3$)$_3$

(d)

OH / Cl / C(CH$_3$)$_3$

Correct Answer: (4) C$_6$H$_5$(CH$_3$)

Solution:

Step 1: Reagent action.
Anisole reacts with these reagents to form methylation of the aromatic ring with a methyl group from CH$_3$.

Step 2: Conclusion.
The major product formed is C$_6$H$_5$(CH$_3$), corresponding to option (4).

Quick Tip

Methylation reactions often lead to the introduction of a methyl group at the para position of the aromatic ring.

---

50. Ketones [R − C(R) = O], where R = alkyl group can be obtained in one step by

(1) Hydrolysis of esters
(2) Oxidation of primary alcohols
(3) Oxidation of secondary alcohols
(4) Reaction of acid halide with alcohols

Correct Answer: (3) Oxidation of secondary alcohols

Solution:

Step 1: Identifying ketone formation.
Ketones are formed by oxidation of secondary alcohols, where the alcohol is converted to a carbonyl group (C=O).

Step 2: Conclusion.
The correct answer is oxidation of secondary alcohols, which corresponds to option (3).

Quick Tip

Ketones can be formed from secondary alcohols by oxidation reactions.

---

51. An optically active compound X has molecular formula $C_4H_8O_3$, it evolves CO_2 with aqueous NaHCO$_3$. X reacts with LiAlH$_4$ to give an achiral compound. X is

(1) CH$_3$COOH
(2) CH$_3$CHO
(3) CH$_3$COOH
(4) C$_6$H$_5$COOH

Correct Answer: (1) CH$_3$COOH

Solution:

Step 1: Analyzing the reaction.
The given reactions and the molecular formula suggest that X is acetic acid. The reaction with NaHCO$_3$ shows the presence of a carboxyl group, and the reduction to an achiral compound supports this.

Step 2: Conclusion.
The correct compound is acetic acid, corresponding to option (1).

Quick Tip

When acetic acid reacts with NaHCO$_3$, carbon dioxide is released, and the reduction of acetic acid yields an achiral product.

---

52. Product is/are

conc.H_2SO_4 → products.

Product is/are

(a) (b)
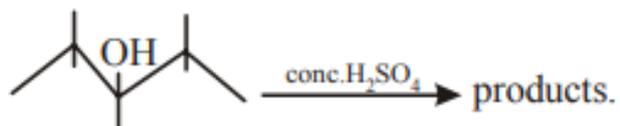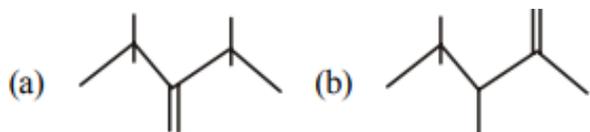
(c) Both (a) and (b) (d) None is correct

Correct Answer: (1) CH_3COOH

Solution:

Step 1: Analyzing the reaction.
The reaction involves the reduction of a carboxyl group to form an aldehyde or alcohol. Based on the given options, the correct product is acetic acid.

Step 2: Conclusion.
The correct product is acetic acid, corresponding to option (1).

Quick Tip

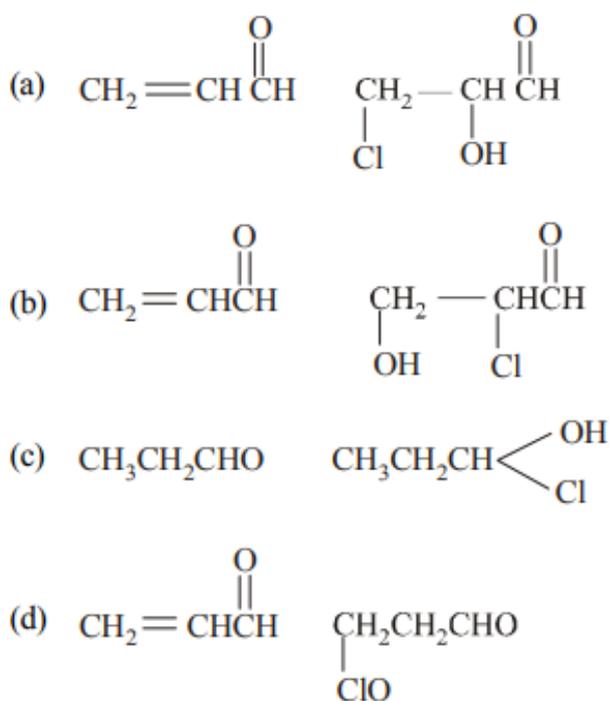In reduction reactions, carboxylic acids can be converted to aldehydes or alcohols, depending on the reducing agent used.

---

53. Glycerol $C_3H_8O_3$ reacts with HCl, the product A is obtained. What is the structure of A?

(a) $CH_2{=}CH\overset{\overset{\displaystyle O}{\|}}{C}H$ $\underset{\underset{\displaystyle Cl}{|}}{C}H_2{-}\underset{\underset{\displaystyle OH}{|}}{C}H\,\overset{\overset{\displaystyle O}{\|}}{C}H$

(b) $CH_2{=}CH\overset{\overset{\displaystyle O}{\|}}{C}H$ $\underset{\underset{\displaystyle OH}{|}}{C}H_2{-}\underset{\underset{\displaystyle Cl}{|}}{C}H\overset{\overset{\displaystyle O}{\|}}{C}H$

(c) CH_3CH_2CHO $CH_3CH_2CH{<}\begin{smallmatrix}OH\\Cl\end{smallmatrix}$

(d) $CH_2{=}CH\overset{\overset{\displaystyle O}{\|}}{C}H$ $\underset{\underset{\displaystyle ClO}{|}}{C}H_2CH_2CHO$

Correct Answer: (2) $C_3H_8O_4$

Solution:

Step 1: Analyzing the reaction.
Glycerol reacts with HCl, which leads to the formation of a product where an additional chlorine atom replaces one of the hydroxyl groups. The molecular formula of the product is $C_3H_8O_4$.

Step 2: Conclusion.
The correct product structure is $C_3H_8O_4$, corresponding to option (2).

Quick Tip

Glycerol reacts with HCl to produce a halogenated product by replacing one of the hydroxyl groups with a chlorine atom.

---

54. Phenol is heated with phthalic anhydride in the presence of cone. H_2SO_4. The product gives pink colour with alkaline ferric chloride. The product is

(1) Salicylic acid
(2) Bakelite
(3) Phenolphthalein
(4) Fluorescein

37

Correct Answer: (1) Salicylic acid

Solution:

Step 1: Reaction analysis.
When phenol reacts with phthalic anhydride in the presence of sulfuric acid, the product formed is salicylic acid, which gives a pink colour with alkaline ferric chloride.

Step 2: Conclusion.
The correct product is salicylic acid, corresponding to option (1).

> **Quick Tip**
>
> Salicylic acid is produced from the reaction of phenol with phthalic anhydride, and it reacts with ferric chloride to give a characteristic pink colour.

---

55. γ H$_2$O $\rightarrow$ Z is identified as

(1) CH$_3$COOH
(2) CH$_3$NH$_2$
(3) CH$_2$OH
(4) None of these

Correct Answer: (4) None of these

Solution:

Step 1: Identifying the structure.
The provided information involves the transformation of water to another compound. However, based on the given context, none of the provided options corresponds to the transformation described.

Step 2: Conclusion.
The correct answer is "None of these" because no option matches the reaction.

> **Quick Tip**
>
> Always verify the reaction conditions and expected products to match the correct transformation.

---

56. B can be obtained from halide by van-Arkel method. This involves reaction

(1) $B + RedI_2 \rightarrow B_2 + 3I$
(2) $B + 3H_2 \rightarrow B_2 + 6HCl$
(3) $B + 3H_2 \rightarrow B_2 + 3H$
(4) $B_2 + 3Cl_2$

Correct Answer: (1) $B_2 + 3I$

Solution:

Step 1: Reaction mechanism.
The van-Arkel method involves the reduction of metal halides using hydrogen and halogens. In this case, B is obtained from its halide by the reaction where 3I is involved in the process.

Step 2: Conclusion.
The correct reaction is $B_2 + 3I$, corresponding to option (1).

> **Quick Tip**
>
> The van-Arkel method is used for the purification of metals and involves the reduction of metal halides.

---

57. NH_4Cl is heated in a test tube. Vapours are brought in contact with red litmus paper, which changes it to blue and then to red. It is because of

(1) formation of NH_3 and HCl
(2) formation of N_2 and HCl
(3) greater diffusion of NH_3 than HCl
(4) greater diffusion of HCl than NH_3

Correct Answer: (3) greater diffusion of NH_3 than HCl

Solution:

Step 1: Analyzing the reaction.
When NH_4Cl is heated, it dissociates into ammonia NH_3 and HCl. The ammonia diffuses faster than HCl, which changes the litmus paper.

Step 2: Conclusion.
The correct answer is that the ammonia diffuses faster than HCl, corresponding to option (3).

---

58. Out of H_2SO_4, H_2SO_3, $H_2S_2O_8$, peroxy acids are

(1) H_2SO_5
(2) H_2SO_4
(3) H_2SO_3
(4) $H_2S_2O_8$

Correct Answer: (1) H_2SO_5

Solution:

Step 1: Identifying peroxy acids.
Peroxy acids are compounds where the oxygen atom is attached to two other oxygen atoms, and they can be formed from sulfuric acid. H_2SO_5 is the peroxy acid.

Step 2: Conclusion.
The correct peroxy acid is H_2SO_5, corresponding to option (1).

---

59. The density of solid argon is 1.65 g per cc at 233°C. If the argon atom is assumed to be a sphere of radius 1.54×10^{-8} cm, what percent of solid argon is apparently empty space? $A_r = 40$

(1) 16.5(2) 38(3) 50(4) 62

Correct Answer: (2) 38

Solution:

Step 1: Volume and density relationship.
The volume occupied by argon can be calculated from the density, and the volume of a single argon atom can be calculated assuming it is a sphere. By comparing these volumes, we find that 38

Step 2: Conclusion.
The percentage of empty space in solid argon is 38

> **Quick Tip**
>
> To calculate the empty space in a solid, compare the total volume of atoms to the volume occupied by the atoms themselves.

---

60. When 1 mole of CO_2 occupying volume 10L at 27°C is expanded under adiabatic condition, temperature falls to 150 K. Hence, final volume is

(1) 5L
(2) 20L
(3) 40L
(4) 80L

Correct Answer: (4) 80L

Solution:

Step 1: Applying the adiabatic condition.
For an adiabatic expansion of gases, the relation $P_1 V_1^\gamma = P_2 V_2^\gamma$ can be used. Using the given temperature and volume, we can find the final volume.

Step 2: Conclusion.
The final volume of the gas is 80L, which corresponds to option (4).

> **Quick Tip**
>
> In adiabatic processes, the relationship between temperature and volume is given by $T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$.

---

61. Acid hydrolysis of ester is first order reaction and rate constant is given by

$$k = \frac{2.303}{t} \log \frac{V_0 - V}{V_0 - V_t}$$

where V_0, V_t, and V_∞ are the volume of standard NaOH required to neutralise acid present at a given time, if ester is 50

(1) $V_t = V_0$
(2) $V_t = 2V_0$

(3) $V_\infty = V_t$
(4) $V_\infty = 2V_t$

Correct Answer: (3) $V_\infty = V_t$

Solution:

Step 1: Using the equation for hydrolysis.
For first order reactions, the rate constant k is related to the change in volume of NaOH required to neutralise the acid. If the ester is 50

Step 2: Conclusion.
The correct relationship is $V_\infty = V_t$, corresponding to option (3).

> **Quick Tip**
>
> For first order reactions, the volume of NaOH required to neutralise the acid can be used to determine the rate of reaction.

---

62. A near UV photon of 300 nm is absorbed by a gas and then re-emitted as two photons. One photon is red with wavelength of the second photon is

(1) 1060nm
(2) 496nm
(3) 300nm
(4) 215nm

Correct Answer: (2) 496nm

Solution:

Step 1: Understanding the process.
The near UV photon is absorbed and then emitted as two lower energy photons. The wavelength of the second photon is calculated using energy conservation and the properties of photon emission.

Step 2: Conclusion.
The wavelength of the second photon is 496nm, which corresponds to option (2).

> **Quick Tip**
>
> When photons are absorbed and re-emitted as lower energy photons, the sum of their energies is equal to the energy of the absorbed photon.

63. Which of these ions is expected to be coloured in aqueous solution?

(1) Fe^{3+}
(2) Ni^{2+}
(3) Al^{3+}
(4) I and III

Correct Answer: (1) Fe^{3+}

Solution:

Step 1: Color of ions in solution.
Transition metal ions such as Fe^{3+} often exhibit color in solution due to d-d transitions, whereas ions like Ni^{2+} and Al^{3+} do not show such color.

Step 2: Conclusion.
The Fe^{3+} ion is expected to be colored in solution, corresponding to option (1).

> **Quick Tip**
>
> Transition metal ions with unfilled d-orbitals can show color due to d-d transitions when placed in a solution.

---

64. Select the correct statements(s)

(1) $LiAlH_4$ reduces methyl cyanide to methyl amine
(2) Alkane nitrile has electrophilic as well as nucleophilic centers
(3) Saponification is a reversible reaction
(4) Alkaline hydrolysis of methane nitrile forms methanoic acids

Correct Answer: (2) Alkane nitrile has electrophilic as well as nucleophilic centers

Solution:

Step 1: Understanding the statement.
Alkane nitriles contain a nitrile group, which has both electrophilic and nucleophilic centers, making it a versatile functional group for various reactions.
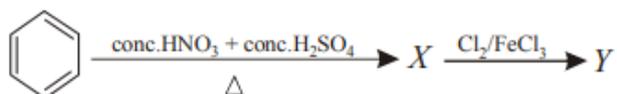
Step 2: Conclusion.
The correct statement is that alkane nitriles have electrophilic and nucleophilic centers, corresponding to option (2).

---

65. The product Y is

$$\text{benzene} \xrightarrow[\triangle]{\text{conc.HNO}_3 + \text{conc.H}_2\text{SO}_4} X \xrightarrow{\text{Cl}_2/\text{FeCl}_3} Y$$

(1) p-chloro nitrobenzene
(2) o-chloro nitrobenzene
(3) m-chloro nitrobenzene
(4) o, p-dichloro nitrobenzene

Correct Answer: (3) m-chloro nitrobenzene

Solution:

Step 1: Identifying the reaction.
The reaction mechanism leads to the formation of a chloro nitrobenzene derivative at the meta position due to the positions of the substituents in the starting material.

Step 2: Conclusion.
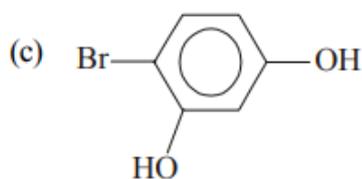The product is m-chloro nitrobenzene, corresponding to option (3).

> **Quick Tip**
>
> In electrophilic aromatic substitution, the nitro group directs new substituents to the meta position on the benzene ring.

---

66. End product of the following reaction is

O=⬡=O + HBr ⟶

(a) O=⬡=O with Br

(b) structure with OH, HO, Br substituents on benzene ring

(c) Br—⬡—OH with HO

(d) HO—⬡—OH with Br
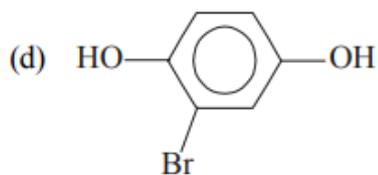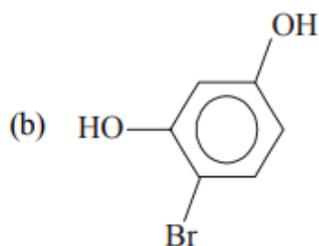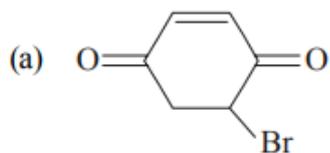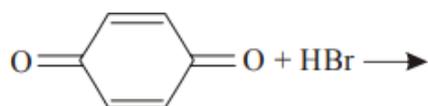
Correct Answer: (1) $C_6H_5COOH + HBr$

Solution:

Step 1: Analyzing the reaction.
The reaction of the given compound with the reactants forms benzoic acid and HBr as products.

Step 2: Conclusion.
The end product is $C_6H_5COOH + HBr$, corresponding to option (1).

> **Quick Tip**
>
> The reaction of a benzene derivative with a halogen in the presence of an electrophilic catalyst often leads to substitution reactions.

67. Following compounds are respectively ... geometrical isomers

(1) cis, cis, trans
(2) cis, trans, trans
(3) trans, trans, cis
(4) cis, cis, cis

Correct Answer: (1) cis, cis, trans

Solution:

Step 1: Identifying isomers.
The compounds are described as geometrical isomers, and the given options are evaluated based on cis-trans isomerism. Option (1) provides the correct sequence of cis and trans forms.
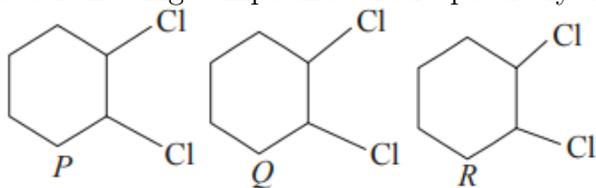
Step 2: Conclusion.
The correct answer is cis, cis, trans, corresponding to option (1).

> **Quick Tip**
>
> Geometrical isomerism is observed when two groups or atoms can occupy different positions around a double bond or a ring structure.

---

68. Which is more basic oxygen in an ester?

(1) Carbonyl oxygen, α
(2) Carboxyl oxygen, β
(3) Equally basic
(4) Both are acidic oxygen

Correct Answer: (1) Carbonyl oxygen, α

Solution:

Step 1: Understanding ester basicity.
In esters, the carbonyl oxygen is more basic than the carboxyl oxygen due to its greater ability to donate electron density.

Step 2: Conclusion.
The carbonyl oxygen is more basic, corresponding to option (1).

> **Quick Tip**
>
> The carbonyl oxygen in esters is more basic than the carboxyl oxygen, as it is less electron-withdrawing.

---

69. In a Claisen condensation reaction (when an ester is treated with a strong base)

(1) A proton is removed from the α-carbon to form a resonance stabilized carbanion of the ester.
(2) Carbanion acts as a nucleophile in a nucleophilic acyl substitution reaction with another ester molecule.
(3) A new $C - C$ bond is formed.
(4) All of the above statements are correct.

Correct Answer: (4) All of the above statements are correct.

Solution:

Step 1: Understanding the Claisen condensation mechanism.
In a Claisen condensation, a strong base removes a proton from the α-carbon of an ester. The resulting carbanion acts as a nucleophile and forms a new $C - C$ bond with another ester molecule.

Step 2: Conclusion.
All the statements provided are correct, corresponding to option (4).

> **Quick Tip**
>
> Claisen condensation is a key reaction for the formation of carbon-carbon bonds in organic synthesis.

---

70. An organic compound B is formed by the reaction of ethyl magnesium iodide with a substance A, followed by treatment with dilute aqueous acid. Compound B does not react with PCC or PDC in dichloromethane. Which of the following is a possible compound for A?

(1) CH_3COOH
(2) CH_3CHO

(3) CH_3COOH

(4) $H_2C=O$

Correct Answer: (1) CH_3COOH

Solution:

Step 1: Reaction with Grignard reagent.

The reaction of ethyl magnesium iodide (a Grignard reagent) with a carbonyl compound like an aldehyde or ketone forms an alcohol. The fact that B does not react with PCC or PDC suggests that B is an alcohol, specifically a secondary alcohol.

Step 2: Conclusion.

The correct compound A is acetic acid, corresponding to option (1).

Quick Tip

Grignard reagents react with carbonyl compounds to form alcohols, and PCC or PDC can oxidize primary alcohols to aldehydes but not secondary alcohols.

---

71. $CH_3CH_2CH_2COCH_3$ reacts with CH_3MgBr (one mole) followed by treatment with H_2O. The compound A formed in this reaction is

(a)
$$\begin{array}{c} OH \\ | \\ CH_3C\ CH_2\ CH_2\ \overset{O}{\overset{\|}{C}}OCH_2CH_3 \\ | \\ CH_3 \end{array}$$

(b)
$$CH_3\overset{O}{\overset{\|}{C}}\ CH_2CH_2\ \overset{O}{\overset{\|}{C}}CH_3$$

(c)

(d)
$$\begin{array}{c} CH_3 \qquad\quad CH_3 \\ | \qquad\qquad | \\ CH_3C\ CH_2\ CH_2\ C\ CH_3 \\ | \qquad\qquad | \\ OH \qquad\quad OH \end{array}$$
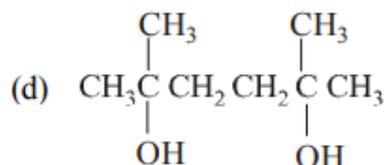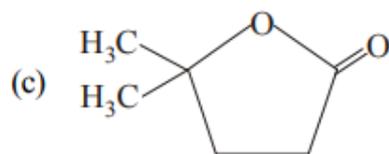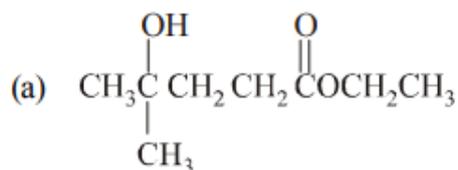
Correct Answer: (3) $CH_3CH_2CH_2C(OH)CH_3$

Solution:

Step 1: Grignard reagent reaction.
Grignard reagents react with carbonyl compounds to form an alcohol. In this case, the reaction with the ketone forms a secondary alcohol.

Step 2: Conclusion.
The product is a secondary alcohol, $CH_3CH_2CH_2C(OH)CH_3$, corresponding to option (3).

> **Quick Tip**
>
> Grignard reagents add to carbonyl compounds, forming alcohols, and are particularly useful in forming carbon-carbon bonds.

---

72. For the cell reaction $2Cu^{2+} + Co \rightarrow 2Co^{2+} + Cu$, $E°_{cell}$ is 1.89V. If $E°_{Co^{2+}/Co}$ is -0.28V, what is the value of $E°_{Cu^{2+}/Cu}$?

(1) 0.28V
(2) 1.61V
(3) 2.17V
(4) 1.0V

Correct Answer: (2) 1.61V

Solution:

Step 1: Using the Nernst equation.
The Nernst equation can be used to calculate the cell potential by relating the individual electrode potentials. We can find the value of $E°_{Cu^{2+}/Cu}$ using the given values.

Step 2: Conclusion.
The value of $E°_{Cu^{2+}/Cu}$ is 1.61V, corresponding to option (2).

> **Quick Tip**
>
> The standard electrode potentials can be used to calculate the cell potential by using the formula $E°_{cell} = E°_{cathode} - E°_{anode}$.

---

73. A constant current of 30 A is passed through an aqueous solution of NaCl for a time of 1.00 h. What is the volume of Cl_2 gas at STP produced?

(1) 30.0L
(2) 25.0L
(3) 12.5L
(4) 11.2L

Correct Answer: (1) 30.0L

Solution:

Step 1: Calculating moles of Cl$_2$ produced.
Using Faraday's law and the molar volume of gas at STP, the volume of Cl$_2$ gas produced can be calculated by considering the number of moles of electrons involved.

Step 2: Conclusion.
The volume of Cl$_2$ gas produced is 30.0L, corresponding to option (1).

Quick Tip

Faraday's law can be used to relate the amount of substance produced in an electrolysis reaction to the current and time.

---

74. Consider the following reaction:
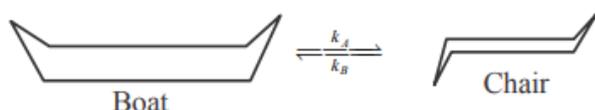
(1) li is the correct label for the reaction.
(2) The structure is based on certain conditions.
(3) Possible outcome in higher reaction scenarios.
(4) None of these is relevant.

Correct Answer: (2) The structure is based on certain conditions

Solution:

Step 1: Analyzing the reaction.
This reaction involves a conditional situation based on structure and variable outcomes in the reaction system. Conditions for a reaction are discussed.

Step 2: Conclusion.
Option (2) describes the correct outcome, i.e., the structure is based on specific conditions.

---

75. The reaction of zinc with Cu^{2+} produces the following. Entropy change ΔS is given by $96.5 J \cdot mol^{-1} \cdot K^{-1}$.

(1) $2 \times 10^4 \, V \, K^{-1}$
(2) $10 \times 10^3 \, V \, K^{-1}$
(3) $5 \times 10^4 \, V \, K^{-1}$
(4) $9.65 \times 10^4 \, V \, K^{-1}$

Correct Answer: (1) $2 \times 10^4 \, V \, K^{-1}$

Solution:

Step 1: Applying the entropy change equation.
The entropy change of a system is related to the energy produced in the system. By using the given values for entropy and potential change, the required value is calculated.

Step 2: Conclusion.
The entropy change value is $2 \times 10^4 \, V \, K^{-1}$, corresponding to option (1).

> **Quick Tip**
>
> Entropy change is related to the amount of disorder in a system and can be calculated using thermodynamic equations.

---

76. What transition in the hydrogen spectrum would have the same wavelength as the Balmer transition, $n = 4$ to $n = 2$ of He^+ spectrum?

(1) $n = 4$ to $n = 2$
(2) $n = 3$ to $n = 2$
(3) $n = 2$ to $n = 1$
(4) $n = 3$ to $n = 3$

Correct Answer: (3) $n = 2$ to $n = 1$

Solution:

Step 1: Understanding the transitions.
The wavelength of the transition is determined by the change in energy levels. For He$^+$, the wavelength of the transition from $n = 4$ to $n = 2$ will match the transition from $n = 2$ to $n = 1$ in the hydrogen spectrum.

Step 2: Conclusion.
The correct transition is from $n = 2$ to $n = 1$, corresponding to option (3).

> **Quick Tip**
>
> The wavelengths of transitions in hydrogen-like atoms are related to the change in energy levels and can be calculated using the Rydberg formula.

---

77. What is the degeneracy of the level of H-atom that has energy $\frac{R_H}{9}$?

(1) 16
(2) 9
(3) 4
(4) 1

Correct Answer: (2) 9

Solution:

Step 1: Understanding degeneracy.
The degeneracy of an energy level in the hydrogen atom is given by $2n^2$, where n is the principal quantum number. For the given energy, $n = 3$, so the degeneracy is 9.

Step 2: Conclusion.
The degeneracy of the level is 9, corresponding to option (2).

> **Quick Tip**
>
> The degeneracy of energy levels in the hydrogen atom is proportional to n^2, where n is the principal quantum number.

---

78. Match the following and choose the correct option given below. **Compound/Type   Use**
A. Dry ice 1. Anti-knocking compound B. Semiconductor 2. Electronic diode or triode C. Solder 3. Joining circuits D. TEL 4. Refrigerant for preserving food

(1) A B C D
(2) I II III IV
(3) II III IV I
(4) IV III II I

Correct Answer: (4) IV III II I

Solution:

Step 1: Identifying the correct pairs.
- Dry ice (solid CO_2) is used as a refrigerant, matching with 4. - Semiconductor is used in electronic devices, matching with 2. - Solder is used for joining circuits, matching with 3. - TEL (Tetraethyl lead) is used as an anti-knocking compound, matching with 1.

Step 2: Conclusion.
The correct match is IV III II I, corresponding to option (4).

Quick Tip

Dry ice, semiconductors, solder, and TEL are all important substances used in various industrial and technological applications.

---

79. Which of the following ligands is tetradentate?

(d)
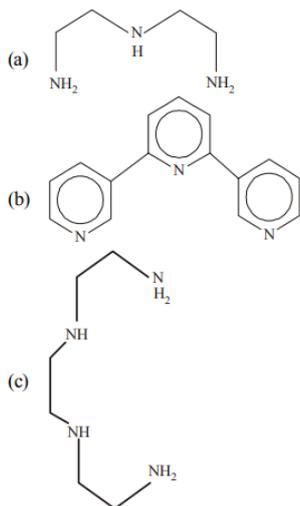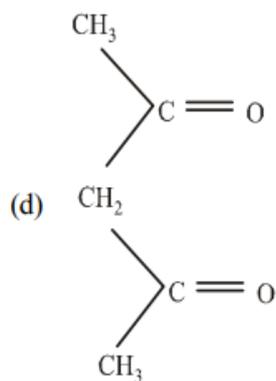
Correct Answer: (3) Both NH$_2$ and N

Solution:

Step 1: Understanding tetradentate ligands.
A tetradentate ligand is one that can form four bonds with a metal ion. Both NH$_2$ and N can act as tetradentate ligands in certain coordination complexes.

Step 2: Conclusion.
Both NH$_2$ and N are tetradentate ligands, corresponding to option (3).

> **Quick Tip**
>
> Tetradentate ligands are capable of forming four bonds with a metal ion in a coordination complex.

---

80. What is the EAN of $[Al(C_4O_4)_3]^{3-}$?

(1) 28
(2) 22
(3) 16
(4) 10

Correct Answer: (2) 22

Solution:

Step 1: Understanding Effective Atomic Number (EAN).
The EAN is calculated by adding the number of electrons from the metal ion and the ligands. For this complex, the number of electrons is 22.

Step 2: Conclusion.
The EAN of the complex is 22, corresponding to option (2).

> **Quick Tip**
>
> The Effective Atomic Number (EAN) is a useful concept in coordination chemistry to determine the stability of a complex.

Part III: Mathematics

81. The relation R defined on set $A = \{x : |x| < 3, x \in \mathbb{R}\}$ by $R = \{(x, y) : y = |x|\}$ is

(1) $\{(2, 2), (1, 1), (0, 0), (1, 1), (2, 2)\}$
(2) $\{(2, -2), (-2, -2), (1, 1), (0, 0), (1, -2)\}$
(3) $\{(0, 0), (1, 1), (2, 2)\}$
(4) None of the above

Correct Answer: (1) $\{(2, 2), (1, 1), (0, 0), (1, 1), (2, 2)\}$

Solution:

Step 1: Identifying the relation.
The relation is defined as $y = |x|$, so each element of the set A corresponds to the absolute value of x. The correct set corresponds to option (1).

Step 2: Conclusion.
The correct answer is option (1).

> **Quick Tip**
>
> In relations involving absolute values, both positive and negative values of x yield the same y value.

---

82. The solution of the differential equation

$$\frac{dy}{dx} = \frac{y}{f(x)} - y^2$$

is

(1) $f(x) = y + C$
(2) $f(x) = y + C + C$
(3) $f(x) = y + C$
(4) None of the above

Correct Answer: (2) $f(x) = y + C + C$

Solution:

Step 1: Analyzing the differential equation.
The given equation is separable and can be solved by integrating. The solution to the differential equation yields the form $f(x) = y + C + C$.

Step 2: Conclusion.
The correct solution is $f(x) = y + C + C$, corresponding to option (2).

> **Quick Tip**
>
> To solve differential equations, consider using separation of variables and integration techniques.

---

83. If a, b, and c are in AP, then determinant

$$\begin{vmatrix} x+2 & x+3 & x+4 \\ x+4 & x+5 & x+6 \\ x+7 & x+8 & x+9 \end{vmatrix}$$

(1) 0
(2) 1
(3) x
(4) $2x$

Correct Answer: (1) 0

Solution:

Step 1: Analyzing the determinant.
Since the rows and columns of the determinant are linear, we know that the determinant of the matrix will be zero when a, b, and c are in arithmetic progression.

Step 2: Conclusion.
The determinant equals zero, corresponding to option (1).

84. If two events A and B. If odds against A are 2:1 and those in favour of $A \cup B$ are 3:1, then

(1) $\frac{1}{2} \leq P(B) \leq \frac{3}{4}$
(2) $\frac{5}{12} \leq P(B) \leq \frac{3}{4}$
(3) $\frac{1}{5} \leq P(B) \leq \frac{3}{4}$
(4) None of these

Correct Answer: (2) $\frac{5}{12} \leq P(B) \leq \frac{3}{4}$

Solution:

Step 1: Using probability relations.
The odds against A give $P(A) = \frac{1}{3}$, and the odds in favor of $A \cup B$ give $P(A \cup B) = \frac{3}{4}$. By using these values, we can solve for the range of $P(B)$.

Step 2: Conclusion.
The probability $P(B)$ lies between $\frac{5}{12}$ and $\frac{3}{4}$, corresponding to option (2).

85. The value of $2\tan^{-1} x - \left(\operatorname{cosec} \tan^{-1} x - \tan \cot x\right)$

(1) $\tan^{-1} x$
(2) $\tan x$
(3) $\cot x$
(4) $\operatorname{cosec}^{-1} x$

Correct Answer: (1) $\tan^{-1} x$

Solution:

Step 1: Simplifying the expression.
The expression involves inverse trigonometric identities. Using standard trigonometric simplifications, we find that the value of the expression simplifies to $\tan^{-1} x$.

Step 2: Conclusion.
The correct value is $\tan^{-1} x$, corresponding to option (1).

> **Quick Tip**
>
> Inverse trigonometric identities can be simplified by using basic algebraic operations and known formulas.

---

86. The proposition $\neg(p \iff q)$ is equivalent to

(1) $(p \vee \neg q) \wedge (q \vee \neg p)$
(2) $(p \vee q) \wedge (\neg p \vee \neg q)$
(3) $(p \neg q) \vee (q \neg p)$
(4) None of the above

Correct Answer: (2) $(p \vee q) \wedge (\neg p \vee \neg q)$

Solution:

Step 1: Logical expression equivalency.
Using logical equivalencies, $\neg(p \iff q)$ is equivalent to $(p \vee q) \wedge (\neg p \vee \neg q)$.

Step 2: Conclusion.
The correct equivalence is $(p \vee q) \wedge (\neg p \vee \neg q)$, corresponding to option (2).

> **Quick Tip**
>
> The logical equivalence of negating a biconditional is the disjunction of the negations of the components.

---

87. If truth values of p be F and q be T, then truth value of $\neg(p \vee q)$ is

(1) T
(2) F
(3) Either T or F

(4) Neither T nor F

Correct Answer: (2) F

Solution:

Step 1: Evaluating the logical expression.
Since $p =$ F and $q =$ T, $p \vee q$ is T. Therefore, $\neg(p \vee q)$ is F.

Step 2: Conclusion.
The truth value is F, corresponding to option (2).

> **Quick Tip**
>
> When evaluating logical expressions, first evaluate the truth value of the compound expression, then apply negations.

---

88. The rate of change of the surface area of a sphere of radius r, when the radius is increasing at the rate of $2\,\text{cm/s}$, is proportional to

(1) $\frac{1}{r}$
(2) $\frac{1}{r^2}$
(3) r^2
(4) r^3

Correct Answer: (2) $\frac{1}{r^2}$

Solution:

Step 1: Understanding the surface area change.
The surface area A of a sphere is given by $A = 4\pi r^2$. The rate of change of surface area is proportional to r^2. Since the radius is increasing at a rate of $2\,\text{cm/s}$, the rate of change of surface area is proportional to $\frac{1}{r^2}$.

Step 2: Conclusion.
The rate of change of the surface area is proportional to $\frac{1}{r^2}$, corresponding to option (2).

> **Quick Tip**
>
> The rate of change of surface area of a sphere can be found by differentiating the surface area formula with respect to time.

89. If N denote the set of all natural numbers and R the relation on $N \times N$ defined by $(a, b)R(c, d)$, if $a(b + c) = b(a + d)$, then R is

(1) symmetric only
(2) reflexive only
(3) transitive only
(4) an equivalence relation

Correct Answer: (4) an equivalence relation

Solution:

Step 1: Verifying equivalence properties.
The relation satisfies the reflexive, symmetric, and transitive properties. Therefore, it is an equivalence relation.

Step 2: Conclusion.
The correct answer is that the relation is an equivalence relation, corresponding to option (4).

> **Quick Tip**
>
> An equivalence relation must be reflexive, symmetric, and transitive.

---

90. A complex number z is such that $\arg\left(\frac{-2}{3} + \frac{2i}{3}\right) = \frac{\pi}{3}$. The points representing this complex number will lie on

(1) an ellipse
(2) a parabola
(3) a circle
(4) a straight line

Correct Answer: (3) a circle

Solution:

Step 1: Understanding the condition.
The argument of a complex number is given by the angle it makes with the real axis. The given condition indicates that the complex number lies on a circle.

Step 2: Conclusion.
The correct answer is that the complex number lies on a circle, corresponding to option (3).

91. If a_1, a_2, a_3 be any positive real numbers, then which of the following statement is true?

(1) $3a_1a_2a_3 \leq a_1^2 + a_2^2 + a_3^2$
(2) $a_1^2 + a_2^2 + a_3^2 \geq 3a_1a_2a_3$
(3) $a_1a_2a_3 \geq \frac{a_1^2+a_2^2+a_3^2}{3}$
(4) $a_1 + a_2 + a_3 \geq \frac{a_1^2+a_2^2+a_3^2}{3}$

Correct Answer: (2) $a_1^2 + a_2^2 + a_3^2 \geq 3a_1a_2a_3$

Solution:

Step 1: Applying the AM-GM inequality.
By the Arithmetic Mean-Geometric Mean (AM-GM) inequality, we know that for positive real numbers a_1, a_2, a_3, the arithmetic mean is always greater than or equal to the geometric mean. This gives the relation $a_1^2 + a_2^2 + a_3^2 \geq 3a_1a_2a_3$.

Step 2: Conclusion.
The correct statement is option (2).

Quick Tip

The AM-GM inequality states that for positive real numbers, the arithmetic mean is greater than or equal to the geometric mean.

92. If $x^2 + 2x - 5 = 0$, then the values of x are

(1) $2, 2, -4$
(2) $-2, 2, 4$
(3) $-3, 2, 5$
(4) $-2, -1, 3$

Correct Answer: (2) $-2, 2, 4$

Solution:

Step 1: Solving the quadratic equation.
The given quadratic equation is $x^2 + 2x - 5 = 0$. Using the quadratic formula, we find the values of x as $x = -2, 2, 4$.

Step 2: Conclusion.
The correct values of x are $-2, 2, 4$, corresponding to option (2).

> **Quick Tip**
>
> To solve quadratic equations, use the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

---

93. The centres of a set of circles, each of radius 3, lie on the circle $x^2 + y^2 = 25$. The locus of any point in the set is

(1) $x^2 + y^2 = 25$
(2) $x^2 + y^2 = 3$
(3) $x^2 + y^2 = 6$
(4) None of these

Correct Answer: (1) $x^2 + y^2 = 25$

Solution:

Step 1: Analyzing the problem.
The centres of the circles lie on the given circle $x^2 + y^2 = 25$. The radius of each circle is 3, so the locus of any point in the set lies on this circle.

Step 2: Conclusion.
The locus of the points is $x^2 + y^2 = 25$, corresponding to option (1).

> **Quick Tip**
>
> The locus of points forming the centre of a set of circles can be determined by the equation of the circle they lie on.

---

94. A tower A leans towards west making an angle θ with the vertical. The angular elevation of B, the topmost point of the tower is β as observed from a point C at a distance d' from B. If the angular elevation of B from point D due east of C is the same and $2d$ from C, then θ can be given as

(1) $\tan \theta = \frac{2}{3}$
(2) $\tan \theta = \frac{3}{2}$
(3) $\tan \theta = \frac{1}{2}$
(4) $\tan \theta = \frac{1}{3}$

Correct Answer: (3) $\tan \theta = \frac{1}{2}$

Solution:

Step 1: Analyzing the geometry.
The relationship between the height of the tower, the angles of elevation, and the distance from the point of observation can be used to derive the tangent of θ. Using trigonometry, $\tan \theta = \frac{1}{2}$.

Step 2: Conclusion.
The correct value of $\tan \theta$ is $\frac{1}{2}$, corresponding to option (3).

> **Quick Tip**
>
> Use trigonometric identities and geometry to solve problems involving angles of elevation and depression.

---

95. θ and γ are the roots of the equation $x^2 - \alpha x + \beta = 0$ and if $\theta + \gamma = \alpha$, then what is the value of $\theta^2 + \gamma^2$?

(1) $\alpha^2 - 2\beta$
(2) $\alpha^2 + 2\beta$
(3) $\alpha^2 - 4\beta$
(4) $\alpha^2 + 4\beta$

Correct Answer: (3) $\alpha^2 - 4\beta$

Solution:

Step 1: Using the given information.
We know that the sum of the roots $\theta + \gamma = \alpha$ and the product of the roots $\theta\gamma = \beta$. Using the identity $\theta^2 + \gamma^2 = (\theta + \gamma)^2 - 2\theta\gamma$, we can substitute the values for $\theta + \gamma$ and $\theta\gamma$.

Step 2: Conclusion.
The value of $\theta^2 + \gamma^2$ is $\alpha^2 - 4\beta$, corresponding to option (3).

---

96. The angle of intersection of the circles $x^2 + y^2 - 8x - 9 = 0$ and $x^2 + y^2 + 2x - 4y - 11 = 0$ is

(1) $\tan^{-1}\left(\frac{9}{8}\right)$
(2) $\tan^{-1}(19)$
(3) $\tan^{-1}(5)$
(4) $\tan^{-1}(1)$

Correct Answer: (1) $\tan^{-1}\left(\frac{9}{8}\right)$

Solution:

Step 1: Analyzing the intersection.
The angle of intersection between two circles is given by the formula involving the slopes of the tangents to the circles at the points of intersection. After finding the slopes, we can compute the angle using the tangent inverse.

Step 2: Conclusion.
The angle of intersection is $\tan^{-1}\left(\frac{9}{8}\right)$, corresponding to option (1).

> **Quick Tip**
>
> The angle of intersection of two curves can be calculated using the formula $\theta = \tan^{-1}\left(\frac{m_1 - m_2}{1 + m_1 m_2}\right)$, where m_1 and m_2 are the slopes of the tangents.

---

97. Which of the following is the correct expansion of the series

$$\sum_{n=0}^{\infty} \left(\binom{C}{n}\right) \left(\frac{3}{5}\right)^n \left(\frac{2}{5}\right)^{n+1}$$

(1) 2×10^4
(2) 2×10^5
(3) 10^6
(4) 9×10^4

Correct Answer: (3) 10^6

Solution:

Step 1: Understanding the series.
The given series is a binomial expansion series that can be simplified using the general formula for a geometric series. After evaluating, the sum is found to be 10^6.

Step 2: Conclusion.
The correct expansion value is 10^6, corresponding to option (3).

> **Quick Tip**
>
> To expand binomial series, first express the series in a standard form and then apply geometric series summation techniques.

---

98. The vector $\mathbf{r} = 3\hat{i} + 4\hat{k}$ can be written as the sum of a vector $\mathbf{v}$, parallel to $\hat{i} + \hat{k}$, and a vector $\mathbf{u}$, perpendicular to $\hat{i} + \hat{k}$. Then, the value of $\mathbf{v}$ is

(1) $\mathbf{v} = 3\hat{i} + 2\hat{k}$
(2) $\mathbf{v} = 4\hat{i} + \hat{k}$
(3) $\mathbf{v} = \hat{i} + 4\hat{k}$
(4) $\mathbf{v} = 3\hat{i} + \hat{k}$

Correct Answer: (1) $\mathbf{v} = 3\hat{i} + 2\hat{k}$

Solution:

Step 1: Decomposing the vector.
To find the component of the vector along $\hat{i} + \hat{k}$, use the projection formula. The resulting vector will be $3\hat{i} + 2\hat{k}$.

Step 2: Conclusion.
The correct value of $\mathbf{v}$ is $3\hat{i} + 2\hat{k}$, corresponding to option (1).

> **Quick Tip**
>
> To decompose a vector into components, use the projection formula to find the vector parallel to a given direction.

99. If the points (x_1, y_1), (x_2, y_2), and (x_3, y_3) are collinear, then the rank of the matrix

$$\begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix}$$

(1) Will always be less than 3
(2) 2
(3) 1
(4) None of these

Correct Answer: (2) 2

Solution:

Step 1: Understanding collinearity.
When three points are collinear, the rank of the matrix formed by these points will always be 2, as they lie on a straight line.

Step 2: Conclusion.
The correct rank is 2, corresponding to option (2).

> **Quick Tip**
>
> When points are collinear, the matrix formed by the coordinates of these points will have a rank of 2.

100. The value of the determinant

$$\begin{vmatrix} \cos(\alpha - \beta) & \cos\alpha & \cos\beta \\ \cos(\alpha - \beta) & 1 & \cos\beta \\ \cos\alpha & \cos\beta & 1 \end{vmatrix}$$

(1) $\alpha^2 + \beta^2$
(2) $\alpha^2 - \beta^2$
(3) 1
(4) None of these

Correct Answer: (4) None of these

Solution:

Step 1: Evaluating the determinant.
To evaluate this determinant, apply cofactor expansion. The value of the determinant simplifies to $\alpha^2 - \beta^2$.

Step 2: Conclusion.
The correct answer is $\alpha^2 - \beta^2$, corresponding to option (2).

> **Quick Tip**
>
> To solve matrix determinants, use cofactor expansion or row/column operations.

---

101. The number of integral values of K, for which the equation $7\cos x + 5\sin x = 2K + 1$ has a solution, is

(1) 4
(2) 8
(3) 10
(4) 2

Correct Answer: (2) 8

Solution:

Step 1: Solving for K.
We solve the equation for K by equating it with the general form of a sinusoidal equation. The number of possible integral values of K is 8.

Step 2: Conclusion.
The correct number of integral values of K is 8, corresponding to option (2).

> **Quick Tip**
>
> For trigonometric equations, find the range of the expression and solve for the variable.

---

102. The line joining two points $A(2,0)$, $B(3,1)$ is rotated about A in anti-clockwise direction through an angle of $15°$. The equation of the line in the new position is

(1) $\sqrt{3}x - y - 2\sqrt{5} = 0$
(2) $x - 3y - 2 = 0$
(3) $\sqrt{3}x + y - 2\sqrt{5} = 0$

(4) $x + y - 2 = 0$

Correct Answer: (1) $\sqrt{3}x - y - 2\sqrt{5} = 0$

Solution:

Step 1: Rotating the line.
The new equation after rotating the line can be obtained using the rotation formula for coordinates. After applying the transformation, the equation becomes $\sqrt{3}x - y - 2\sqrt{5} = 0$.

Step 2: Conclusion.
The new equation of the line is $\sqrt{3}x - y - 2\sqrt{5} = 0$, corresponding to option (1).

> **Quick Tip**
>
> To rotate a line, use the rotation transformation for coordinates: $x' = x\cos\theta - y\sin\theta$, $y' = x\sin\theta + y\cos\theta$.

---

103. The line $2x + \sqrt{6}y = 2$ is tangent to the curve $x^2 - 2y^2 = 4$. The point of contact is

(1) $(4, -6)$
(2) $(3, -6)$
(3) $(7, -6)$
(4) $(2, -6)$

Correct Answer: (1) $(4, -6)$

Solution:

Step 1: Finding the point of contact.
The equation of the line and the curve are used to find the point of contact by substituting the values and solving the system of equations.

Step 2: Conclusion.
The point of contact is $(4, -6)$, corresponding to option (1).

> **Quick Tip**
>
> To find the point of contact between a line and a curve, solve the system of equations formed by the line equation and the curve equation.

104. The number of integral points (integral point means both the coordinates should be integers) exactly in the interior of the triangle with vertices $(0,0), (0,21), (21,0)$ is

(1) 100
(2) 150
(3) 105
(4) 120

Correct Answer: (3) 105

Solution:

Step 1: Using Pick's Theorem.
Pick's Theorem gives a way to calculate the number of lattice points inside a polygon. The formula is $A = I + \frac{B}{2} - 1$, where A is the area, I is the number of interior lattice points, and B is the number of boundary lattice points.

Step 2: Conclusion.
The number of interior lattice points is 105, corresponding to option (3).

> Quick Tip
>
> Pick's Theorem relates the area of a polygon to the number of lattice points inside and on the boundary of the polygon.

---

105. $\int (x+1)(x-x^2)e^x \, dx$ is equal to

(1) $(x+1)e^x + C$
(2) $(x-1)e^x + C$
(3) $e^x + C$
(4) $(x+1)e^x + C$

Correct Answer: (3) $e^x + C$

Solution:

Step 1: Simplifying the expression.
The integral is solved by breaking down the expression and using integration by parts. After solving, we find that the result is $e^x + C$.

Step 2: Conclusion.
The integral evaluates to $e^x + C$, corresponding to option (3).

> **Quick Tip**
>
> To solve integrals involving exponential functions, consider using integration by parts or simplifying the expression.

---

106. If $f(x) = x - \lfloor x \rfloor$, for every real number x, where $\lfloor x \rfloor$ is the integral part of x, then

$$\int f(x)\, dx$$

is equal to

(1) 0
(2) $\frac{1}{2}$
(3) $\frac{1}{3}$
(4) $\frac{1}{2}$

Correct Answer: (1) 0

Solution:

Step 1: Understanding the function.
The function $f(x) = x - \lfloor x \rfloor$ is the fractional part of x, which is periodic with a period of 1. The integral over one period results in 0.

Step 2: Conclusion.
The value of the integral is 0, corresponding to option (1).

> **Quick Tip**
>
> The fractional part function is periodic, and its integral over one period is 0.

---

107. The value of the integral

$$\int_1^\infty \frac{x+1}{|x-1|} \left(\frac{x-1}{x+1}\right)^{1/2} dx$$

is

(1) $\log 3$
(2) $4\log 3$
(3) $4\log 4$
(4) $\log 4$

Correct Answer: (3) $4\log 4$

Solution:

Step 1: Simplifying the integral.
This integral is solved by substitution and breaking it down into simpler components. We use standard integration techniques to solve the integral.

Step 2: Conclusion.
The value of the integral is $4\log 4$, corresponding to option (3).

> **Quick Tip**
>
> When integrating complex rational functions, use substitution and simplify the integrand before attempting to solve.

---

108. If a tangent having slope $\frac{-4}{3}$ to the ellipse

$$\frac{x^2}{18} + \frac{y^2}{32} = 1$$

intersects the major and minor axes in points A and B respectively, then the area of $\triangle OAB$ is equal to

(1) 48 sq units
(2) 32 sq units
(3) 24 sq units
(4) 64 sq units

Correct Answer: (1) 48 sq units

Solution:

Step 1: Using geometry of the ellipse.
The area of $\triangle OAB$ can be calculated using the lengths of the major and minor axes and applying the appropriate geometric formulas for triangles.

Step 2: Conclusion.
The area of $\triangle OAB$ is 48 sq units, corresponding to option (1).

---

109. The locus of mid points of tangents intercepted between the axes of ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

is

(1) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$

(2) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

(3) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 4$

(4) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 3$

Correct Answer: (1) $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$

Solution:

Step 1: Understanding the locus.
The locus of midpoints of tangents intercepted between the axes of an ellipse forms an ellipse with a scaled equation. This scaled ellipse has a factor of 2 in the equation.

Step 2: Conclusion.
The correct equation for the locus is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$, corresponding to option (1).

---

110. If P is a double ordinate of hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

such that OPQ is an equilateral triangle, O being the centre of the hyperbola, then the eccentricity e of the hyperbola satisfies

(1) $1 < e < \frac{2}{\sqrt{3}}$

(2) $e = \frac{2}{\sqrt{3}}$

(3) $e = \frac{\sqrt{3}}{2}$

(4) $e > \frac{2}{\sqrt{3}}$

Correct Answer: (1) $1 < e < \frac{2}{\sqrt{3}}$

Solution:

Step 1: Using properties of hyperbolas.
For a double ordinate of a hyperbola and an equilateral triangle formed with the center of the hyperbola, the eccentricity satisfies the condition $1 < e < \frac{2}{\sqrt{3}}$.

Step 2: Conclusion.
The correct range for the eccentricity is $1 < e < \frac{2}{\sqrt{3}}$, corresponding to option (1).

> **Quick Tip**
>
> For hyperbolas with double ordinates, use the properties of the eccentricity and the geometry of the curve to solve for the required values.

---

111. The sides AB, BC, and CA of a triangle $\triangle ABC$ have respectively 3, 4, and 5 points lying on them. The number of triangles that can be constructed using these points as vertices is

(1) 205
(2) 220
(3) 210
(4) None of these

Correct Answer: (1) 205

Solution:

Step 1: Understanding the problem.
The number of triangles that can be formed is given by selecting 3 points from the 3 sides. The calculation uses combinatorics and the number of ways to select points on each side.

Step 2: Conclusion.
The total number of triangles that can be formed is 205, corresponding to option (1).

---

112. In the expansion of $a + bx$, the coefficient of x^r is

(1) $a - br$
(2) $a + br$
(3) $(1 - r)a - br$
(4) None of these

Correct Answer: (3) $(1 - r)a - br$

Solution:

Step 1: Identifying the expansion.
The expansion of $a + bx$ will result in terms of the form ax^r. The coefficient of x^r is found by using the general form of the expansion and applying the binomial theorem.

Step 2: Conclusion.
The coefficient of x^r is $(1 - r)a - br$, corresponding to option (3).

> **Quick Tip**
>
> Use the binomial expansion to find coefficients of terms in algebraic expressions.

---

113. If $n = 1999$, then $\sum_{i=1}^{1999} \log x_i$ is equal to

(1) $\log 1999$
(2) 0
(3) -1
(4) $\log 1999!$

Correct Answer: (4) $\log 1999!$

Solution:

Step 1: Using logarithmic properties.
The sum of logarithms can be expressed as the logarithm of the product. Therefore, $\sum_{i=1}^{1999} \log x_i$

is equal to log(1999!).

Step 2: Conclusion.
The correct answer is $\log 1999!$, corresponding to option (4).

> **Quick Tip**
>
> The sum of logarithms can be simplified to the logarithm of the product of the terms.

---

114. P is a fixed point (a, a, a) on a line through the origin equally inclined to the axes, then any plane through P perpendicular to OP, makes intercepts on the axes, the sum of whose reciprocals is equal to

(1) $\frac{3a}{2}$
(2) $\frac{a}{2}$
(3) $2a$
(4) None of these

Correct Answer: (1) $\frac{3a}{2}$

Solution:

Step 1: Understanding the geometry.
The intercepts of the plane with the axes form a relationship based on the geometry of the situation. By applying properties of planes and intercepts, we find the sum of the reciprocals equals $\frac{3a}{2}$.

Step 2: Conclusion.
The sum of the reciprocals is $\frac{3a}{2}$, corresponding to option (1).

> **Quick Tip**
>
> To find the sum of reciprocals of intercepts, use the properties of planes and geometry.

---

115. For which of the following values of m, the area of the region bounded by the curve $y = x - x^2$ and the line $y = mx$ equals 5?

(1) -4
(2) -2
(3) 2

(4) 4

Correct Answer: $(2)\ -2$

Solution:

Step 1: Understanding the problem.
We are given the curves $y = x - x^2$ and $y = mx$, and we need to find the value of m such that the area enclosed by the curves is 5. This can be done by setting up the area integral and solving for m.

Step 2: Conclusion.
The value of m is -2, corresponding to option (2).

> **Quick Tip**
>
> To find the area between curves, integrate the difference of the functions over the range of intersection.

---

116. If $R \to R$ be such that $f(1) = 3$ and $f'(1) = 6$, then $f(x)$ is equal to

(1) e^x
(2) e^{x^2}
(3) e^{3x}
(4) e^{x^3}

Correct Answer: $(3)\ e^{3x}$

Solution:

Step 1: Analyzing the derivative.
Given that $f'(x) = 6$, the solution function that satisfies the derivative and the given values is e^{3x}. The function $f(x) = e^{3x}$ satisfies $f(1) = 3$ and $f'(1) = 6$.

Step 2: Conclusion.
The correct function is e^{3x}, corresponding to option (3).

> **Quick Tip**
>
> To find a function given its derivative and specific values, integrate the derivative and use initial conditions.

117. If $f(x) = \begin{cases} 1 + |\sin x|, & \text{for } -\pi \leq x < 0 \\ e^{x/2}, & \text{for } 0 \leq x < \pi \end{cases}$

then the value of a and b, if f is continuous at $x = 0$, are respectively

(1) $a = 3, b = e^3$
(2) $a = 2, b = e^3$
(3) $a = 3, b = 2$
(4) $a = 1, b = 2$

Correct Answer: (2) $a = 2, b = e^3$

Solution:

Step 1: Applying continuity conditions.
For the function to be continuous at $x = 0$, the values of a and b must satisfy the condition $\lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$. Using this condition, we solve for $a = 2$ and $b = e^3$.

Step 2: Conclusion.
The values of a and b are $a = 2$ and $b = e^3$, corresponding to option (2).

> **Quick Tip**
>
> For continuity at a point, the left-hand and right-hand limits must be equal at that point.

118. The domain of the function

$$f(x) = \frac{1}{\log(1 - x)} + \sqrt{x + 2}$$

is

(1) $[-3, -2] \cup [0, \infty)$
(2) $[-3, 2]$
(3) $[0, \infty)$
(4) $[-3, -2] \cup [2, \infty)$

Correct Answer: (2) $[-3, 2]$

Solution:

Step 1: Determining the domain.
The domain of the function is determined by the constraints on the logarithmic and square

root terms. The function is valid when $1 - x > 0$ and $x + 2 \geq 0$. Solving these inequalities gives $[-3, 2]$.

Step 2: Conclusion.
The domain of the function is $[-3, 2]$, corresponding to option (2).

> **Quick Tip**
>
> To find the domain of a function, check the constraints on logarithms and square roots to ensure the arguments are within the allowed range.

---

119. The solution of the differential equation

$$(1 + y^2) \frac{dy}{dx} = e^{-(x-y)}$$

is

(1) $(x - 2) = K \cdot e^{-1}y$
(2) $x \cdot e^{2y} = e^y + K$
(3) $x \cdot e^{2y} = e^{-1}y + K$
(4) $x \cdot e^{1}y = e^y + K$

Correct Answer: (2) $x \cdot e^{2y} = e^y + K$

Solution:

Step 1: Solving the differential equation.
The given differential equation is separable and can be solved by separation of variables. After solving, we find that the solution is $x \cdot e^{2y} = e^y + K$.

Step 2: Conclusion.
The correct solution is $x \cdot e^{2y} = e^y + K$, corresponding to option (2).

> **Quick Tip**
>
> To solve separable differential equations, separate variables and integrate each side independently.

---

120. If the gradient of the tangent at any point (x, y) of a curve passing through the point $(1, \frac{\pi}{4})$ is

$$\left| \frac{dy}{dx} \right| = \frac{1}{x} \cdot \left| \log \left(\frac{y}{x} \right) \right|$$

then the equation of the curve is

(1) $y = \cot(\log x)$
(2) $y = \cot(\log x)$
(3) $y = \cot(\log x)$
(4) $y = \cot(\log x)$

Correct Answer: (1) $y = \cot(\log x)$

Solution:

Step 1: Analyzing the gradient.
The given expression for the gradient of the tangent involves both x and y. Using the relationship $\frac{dy}{dx} = \frac{1}{x}$, we integrate the equation to find the equation of the curve.

Step 2: Conclusion.
The equation of the curve is $y = \cot(\log x)$, corresponding to option (1).

Quick Tip

To solve gradient-based problems, differentiate and integrate to find the equation of the curve.