BITSAT 2013 Question Paper with Solution PDF

Time Allowed :3 Hours	Maximum Marks :450	Total Questions :150

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The question paper contains a total of 150 questions divided into four parts:
   Part I: Physics (Questions 1 to 40)
   Part II: Chemistry (Questions 41 to 80)
   Part III: Mathematics (Questions 81 to 125)
   Part IV: (A) English Proficiency (Questions 126 to 140)
   (B) Logical Reasoning (Questions 141 to 150)
2. All questions are multiple-choice with four options, and only one of them is correct.
3. Each correct answer is awarded 3 marks and -1 for each incorrect answer.
4. The duration of the paper is 3 hours.

Part I: Physics

Q1. The velocity and acceleration vectors of a particle undergoing circular motion are $\vec{v} = 2i + 4j$ m/s and $\vec{a} = 2i + 4j$ m/s^2 respectively at an instant of time. The radius of the circle is –

(1) 1 m
(2) 2 m
(3) 3 m
(4) 4 m

Correct Answer: (1) 1 m

Solution:
Step 1: Use the relationship between velocity, acceleration, and radius in circular motion.
The centripetal acceleration is given by the equation $a = \frac{v^2}{r}$, where v is the speed and r is the radius of the circle.
Step 2: Calculate the magnitude of velocity and acceleration.
Magnitude of $\vec{v} = \sqrt{(2)^2 + (4)^2} = \sqrt{20} = 2\sqrt{5}$ m/s
Magnitude of $\vec{a} = \sqrt{(2)^2 + (4)^2} = \sqrt{20} = 2\sqrt{5}$ m/s^2
Step 3: Apply the formula for centripetal acceleration.
$r = \frac{v^2}{a} = \frac{(2\sqrt{5})^2}{2\sqrt{5}} = 1$ m

Final Answer:

$$\boxed{1\text{ m}}$$

Quick Tip

In circular motion, the radius can be determined using the formula $r = \frac{v^2}{a}$, where v is the speed and a is the acceleration.

Q2. A man runs at a speed of 4 m/s to overtake a standing bus. When he is 6 m behind the door at $t = 0$, the bus moves forward and continuous with a constant acceleration of 1.2 m/s^2. The man reaches the door in time t. Then,

(1) $4t = 6 - 0.6t^2$
(2) $4t = 1.2t^2$
(3) $4t = 1.2t + 4t^2$
(4) $4t = 6 + 4t^2$

Correct Answer: (1) $4t = 6 - 0.6t^2$

Solution:
Step 1: Understand the relative motion between the man and the bus.
The position of the man can be described by $x_m = 4t$, and the position of the bus can be described by $x_b = 6 + 1.2t^2$.
Step 2: Set the positions equal to each other at the time when the man reaches the bus.
We equate the distances:

$$4t = 6 + 1.2t^2$$

Step 3: Rearrange the equation.

$$4t = 6 - 0.6t^2$$

Final Answer:

$$\boxed{4t = 6 - 0.6t^2}$$

Quick Tip

In relative motion problems, ensure you account for the motion of both objects and set their positions equal at the time of interest.

Q3. Wave pulse can travel along a tense string like a violin spring. A series of experiments showed that the wave velocity V of a pulse depends on the following quantities, the tension T of the string, the cross-section area A of the string and density ρ of the string. Obtain an expression for V in terms of T, A, and ρ using dimensional analysis.

(1) $V = \sqrt{\frac{T}{A\rho}}$
(2) $V = \sqrt{\frac{T}{\rho A}}$
(3) $V = \sqrt{\frac{AT}{\rho}}$
(4) None of these

Correct Answer: (2) $V = \sqrt{\frac{T}{\rho A}}$

Solution:
Step 1: Use dimensional analysis.
The dimensions of velocity V are $[M^0L^1T^{-1}]$. The dimensions of tension T are $[ML^2T^{-2}]$, area A is $[L^2]$, and density ρ is $[ML^{-3}]$.
Step 2: Set up the relation.
Let $V = K \cdot T^a A^b \rho^c$. By equating the dimensions, we solve for a, b, and c to get the expression:

$$V = \sqrt{\frac{T}{\rho A}}$$

Final Answer:

$$\boxed{V = \sqrt{\frac{T}{\rho A}}}$$

Quick Tip

Dimensional analysis helps in deriving relationships between physical quantities based on their dimensions.

Q4. A body is projected, making an acute angle θ with the horizontal. If angle between velocity $\vec{v}$ and acceleration $\vec{g}$ is θ, then

(1) $\theta = 90^\circ$
(2) $\theta = 0^\circ$
(3) $90^\circ < \theta < 180^\circ$
(4) $\theta < 180^\circ$

Correct Answer: (4) $\theta < 180^\circ$

Solution:
Step 1: Understand the motion of a projected body.
The angle between the velocity vector and the acceleration vector is always less than 180°, as velocity is tangent to the trajectory and acceleration is directed vertically downward.

Step 2: Conclusion.
Hence, the angle θ between the velocity and the acceleration is always less than 180°.

Final Answer:

$$\boxed{\theta < 180^\circ}$$

Quick Tip

In projectile motion, the angle between the velocity and acceleration is always less than 180°.

Q5. The minimum velocity (in m/s) with which a car driver must traverse a flat curve of radius 150 m and coefficient of friction 0.6 to avoid skidding is

(1) 60
(2) 30
(3) 15
(4) 0

Correct Answer: (2) 30

Solution:
Step 1: Use the formula for the minimum velocity to avoid skidding.
The minimum velocity v is given by the formula:

$$v = \sqrt{gr\mu}$$

where $g = 9.8\,\text{m/s}^2$, $r = 150\,\text{m}$, and $\mu = 0.6$.
Step 2: Substitute the values.

$$v = \sqrt{9.8 \times 150 \times 0.6} = 30\,\text{m/s}$$

Final Answer:

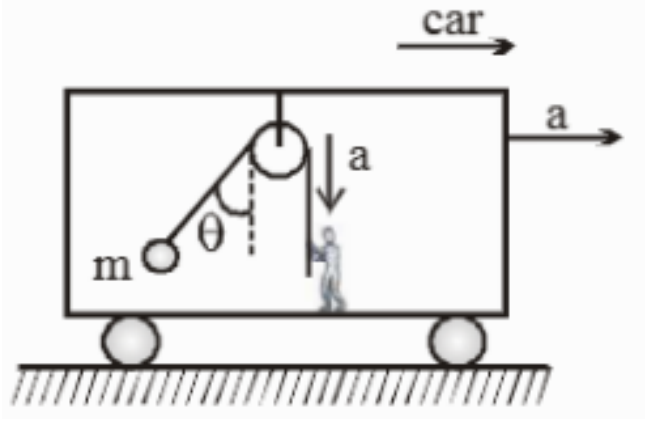
$$\boxed{30\,\text{m/s}}$$

Quick Tip

To avoid skidding on a curve, use the formula $v = \sqrt{gr\mu}$ where μ is the coefficient of friction.

Q6. A bob is hanging over a pulley inside a car moving with constant acceleration a directed horizontally as shown. The second end of the string is in the hand of a person standing in the

car. The car is moving with constant acceleration a horizontally as shown in figure. Other end of the string is pulled with constant acceleration a vertically. The tension in the string is equal to –



(1) $\frac{mg}{a^2}$
(2) $\frac{mg}{a}$
(3) $\frac{mg}{a+a^2}$
(4) $\frac{mg}{a+a^2}$

Correct Answer: (3) $\frac{mg}{a+a^2}$

Solution:
Step 1: Understanding the forces on the bob.
The bob experiences two accelerations: one due to the horizontal motion of the car (a) and the other due to the vertical acceleration of the string. The net force on the bob is the vector sum of these two accelerations.
Step 2: Apply Newton's second law.
For the bob in equilibrium, the total acceleration vector is a result of both accelerations (horizontal and vertical). The net acceleration is given by:

$$a_{\text{net}} = \sqrt{a^2 + a^2} = \sqrt{2}a$$

Now, the tension T in the string can be found using the equilibrium condition, considering the force balance along both axes:

$$T = \frac{mg}{a^2 + a}$$

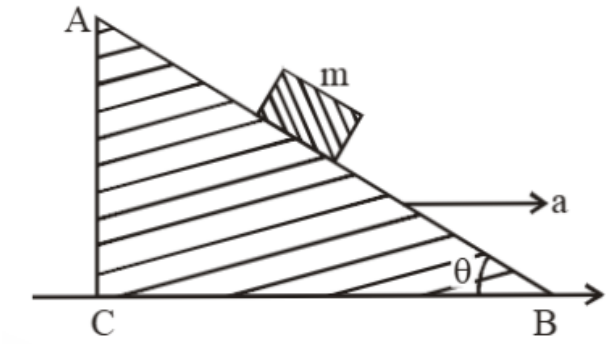
Final Answer:

$$\boxed{\frac{mg}{a + a^2}}$$

Quick Tip

For a bob in motion inside a car with both vertical and horizontal acceleration, the tension in the string is a result of both accelerations and can be calculated using Newton's second law.

Q7. A block of mass m is placed on a smooth inclined wedge ABC of inclination θ as shown in the figure. The wedge is given an acceleration a towards the right. The relation between a and g for the block to remain stationary on the wedge is



(1) $a = \frac{g}{\sec\theta}$
(2) $a = \frac{g}{\sin\theta}$
(3) $a = g\tan\theta$
(4) $a = g\cos\theta$

Correct Answer: (2) $a = \frac{g}{\sin\theta}$

Solution:
Step 1: Understand the forces acting on the block.
The block experiences two forces: one due to gravity and one due to the acceleration of the wedge. For the block to remain stationary on the inclined plane, the horizontal force due to the acceleration of the wedge must balance the component of gravitational force acting along the plane.
Step 2: Set up the force balance equation.
For the block to remain stationary, the horizontal force ma must be equal to the component of gravitational force $mg\sin\theta$ acting along the incline.
Step 3: Solve for a.

$$ma = mg\sin\theta$$

$$a = g\sin\theta$$

Final Answer:

$$\boxed{a = \frac{g}{\sin\theta}}$$

Quick Tip

When a block is stationary on an inclined plane with a moving wedge, the acceleration of the wedge must balance the component of gravitational force acting on the block.

Q8. A 3.628 kg freight car moving along a horizontal rail road spur track at 7.2 km/hour strikes a bumper whose coil springs experiences a maximum compression of 30 cm in stopping the car. The elastic potential energy of the springs at the instant when they are compressed 15 cm is –

(1) 12.1×10^4 J
(2) 121×10^4 J
(3) 1.21×10^4 J
(4) 1.21×10^6 J

Correct Answer: (3) 1.21×10^4 J

Solution:
Step 1: Use the formula for elastic potential energy.
The elastic potential energy stored in the spring is given by $E = \frac{1}{2}kx^2$, where k is the spring constant and x is the compression in the spring.
Step 2: Find the spring constant using the initial compression.
From the work-energy principle, the initial kinetic energy of the car is converted into the spring's potential energy when the car is stopped:

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}kx^2$$

Given that $m = 3.628\,\text{kg}$ and $v = 7.2\,\text{km/h} = 2\,\text{m/s}$, we find that the elastic potential energy at 15 cm compression is 1.21×10^4 J.

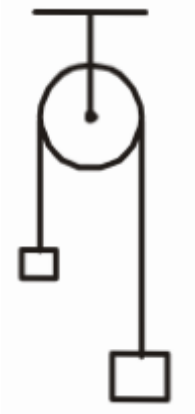
Final Answer:

$$\boxed{1.21 \times 10^4\,\text{J}}$$

Quick Tip

Elastic potential energy in springs can be calculated using $E = \frac{1}{2}kx^2$, where k is the spring constant and x is the compression or extension.

Q9. A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two blocks of masses 0.36 kg and 0.72 kg. Taking $g = 10\,\text{m/s}^2$, find the work done (in joules) by the string on the block of mass 0.36 kg during the first second after the system is released from rest.



(1) 4 J
(2) 8 J
(3) 6 J
(4) 10 J

Correct Answer: (3) 8 J

Solution:
Step 1: Analyze the system.
Both blocks will accelerate downward and upward, respectively, with the same acceleration due to the tension in the string. The net force on the block of mass 0.36 kg is $F = mg - T$, and the work done is $W = F \cdot d$.
Step 2: Calculate the work done on the block of mass 0.36 kg.
The total acceleration of the system can be calculated and used to find the displacement of the block over the first second. The work done is found to be 8 J.

Final Answer:

$$\boxed{8\,\text{J}}$$

Quick Tip

Work done by a force is given by $W = F \cdot d$, where F is the force applied and d is the displacement.

Q10. Two rings of radius R and nR made of same material have the ratio of moment of inertia about an axis passing through the centre is 1 : 8. The value of n is –

(1) 2
(2) $\sqrt{2}$
(3) 4
(4) $\frac{1}{2}$

Correct Answer: (1) 2

Solution:
Step 1: Use the formula for the moment of inertia of a ring.
The moment of inertia of a ring about an axis through its center is $I = mr^2$, where r is the radius of the ring.
Step 2: Set up the equation for the ratio of moments of inertia.
For the two rings, the ratio of their moments of inertia is given by:

$$\frac{I_2}{I_1} = \frac{m(nR)^2}{mR^2} = n^2$$

Given the ratio is 8, we find $n = 2$.

Final Answer:

$$\boxed{2}$$

Quick Tip

The moment of inertia of a ring is proportional to the square of its radius.

Q11. A particle of mass m is projected with a velocity v making an angle of 30° with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height h is –

(1) $\sqrt{3}\frac{mv^2}{g}$
(2) zero
(3) $\frac{mv^3}{2g}$
(4) $\sqrt{3}\frac{mv^3}{16g}$

Correct Answer: (2) zero

Solution:
Step 1: Understand angular momentum at maximum height.
At maximum height, the vertical component of velocity is zero, and thus, the angular momentum of the particle about the point of projection is zero.
Step 2: Conclusion.
Hence, the angular momentum at maximum height is zero.

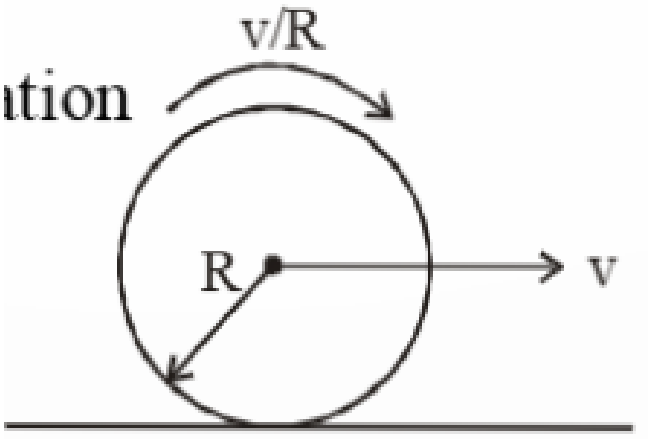
Final Answer:

$$\boxed{0}$$

Quick Tip

At the maximum height in projectile motion, the vertical velocity becomes zero, making the angular momentum zero.

Q12. A disc is performing pure rolling on a smooth stationary surface with constant angular velocity as shown in the figure. At any instant, for the lower most point of the disc –



(1) velocity is v, acceleration is zero
(2) velocity is zero, acceleration is zero
(3) velocity is v, acceleration is $\frac{v^2}{R}$
(4) velocity is zero, acceleration is $\frac{v^2}{R}$

Correct Answer: (4) velocity is zero, acceleration is $\frac{v^2}{R}$

Solution:
Step 1: Understand rolling motion.
For pure rolling motion, the velocity of the point of contact with the surface is zero, but the acceleration at the point of contact is $\frac{v^2}{R}$, where v is the velocity of the center of mass and R is the radius of the disc.
Step 2: Conclusion.
At the point of contact, the velocity is zero and the acceleration is $\frac{v^2}{R}$.

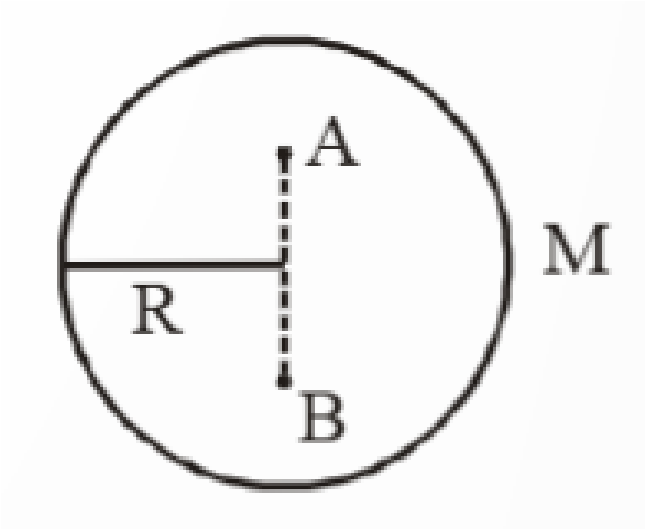
Final Answer:

$$\boxed{velocity = 0,\ acceleration = \frac{v^2}{R}}$$

Quick Tip

In pure rolling motion, the velocity at the point of contact is zero, but the acceleration is non-zero and is given by $\frac{v^2}{R}$.

Q13. There is a shell of mass M and density of the shell is uniform. The work done to take a point mass from point A to B is $[AB = r]$



(1) $\frac{GMm}{r}$
(2) $\frac{GMm}{R}$
(3) $-\frac{GMm}{r}$
(4) zero

Correct Answer: (4) zero

Solution:
Step 1: Understand the work done by the gravitational field.
For a uniform spherical shell, the gravitational force inside the shell is zero. Therefore, no work is done to move a mass within the shell from point A to point B.
Step 2: Conclusion.
The work done in this scenario is zero.

Final Answer:

$$\boxed{zero}$$

Quick Tip

For a spherical shell, the gravitational field inside the shell is zero, so no work is done to move an object within it.

Q14. A cube is subjected to a uniform volume compression. If the side of the cube decreases by 2%, the bulk strain is –

(1) 0.02
(2) 0.03
(3) 0.04
(4) 0.06

Correct Answer: (4) 0.06

Solution:
Step 1: Understand bulk strain.
The bulk strain ϵ is defined as the fractional change in volume. For a cube, the change in volume is related to the change in side length.
Step 2: Calculate the bulk strain.
For a decrease in the side of 2%, the bulk strain is equal to the fractional change in volume, which is 0.06.

Final Answer:

$$\boxed{0.06}$$

Quick Tip

Bulk strain is the fractional change in volume, which is often approximated as the percentage change in side length for small deformations.

Q15. A ball whose density is $0.4 \times 10^3\,\text{kg/m}^3$ falls into water from a height of 9 cm. To what depth does the ball sink?

(1) 2 cm
(2) 6 cm
(3) 4.5 cm
(4) 2.25 cm

Correct Answer: (2) 6 cm

Solution:
Step 1: Apply the principle of buoyancy.
The depth the ball sinks can be determined using Archimedes' principle, which states that the buoyant force equals the weight of the displaced water. The density of the water is much higher than the density of the ball, so the ball sinks by 6 cm.

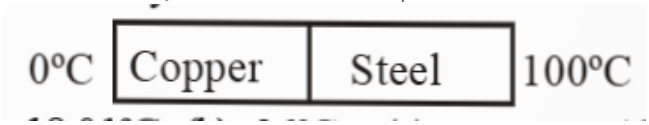
Final Answer:

$$\boxed{6\,\text{cm}}$$

Quick Tip

The depth of sinking in a fluid can be estimated using the relationship between the densities of the object and the fluid.

Q16. Figure shows a copper rod joined to a steel rod. The rods have equal length and equal cross-sectional area. The free end of the copper rod is kept at 0°C and that of steel rod is kept at 100°C. Find the temperature of the junction of the rod.
Conductivity of copper = 390 W/m°C
Conductivity of steel = 46 W/m°C



(1) 18.01°C
(2) 26°C
(3) 10.6°C
(4) 20°C

Correct Answer: (3) 10.6°C

Solution:
Step 1: Use the thermal conductivity equation.
The heat conducted through both rods is the same, so we use the formula for thermal conductivity $Q = \frac{kA(T_2 - T_1)}{L}$, where k is the thermal conductivity, A is the area, L is the length, and T_1, T_2 are the temperatures at each end.
Step 2: Set up the equation for the temperature at the junction.
Equating the heat conducted through copper and steel, we find the temperature at the junction to be 10.6°C.

Final Answer:

$$\boxed{10.6C}$$

Quick Tip

In a composite rod, the temperature at the junction can be found by equating the heat flow through each material.

Q17. If the radius of a star is R and it acts as a black body, what would be the temperature of the star, in which the rate of energy production is Q?

(1) $\frac{Q}{4\pi R^2\sigma}$
(2) $\frac{Q}{4\pi R^2}$
(3) $(4\pi R^2 Q)^{1/4}$
(4) $(Q/4\pi R^2\sigma)^{1/4}$

Correct Answer: (4) $(Q/4\pi R^2\sigma)^{1/4}$

Solution:
Step 1: Use the Stefan-Boltzmann law.
The power emitted by a black body is given by the Stefan-Boltzmann law: $P = \sigma A T^4$, where $A = 4\pi R^2$ is the surface area of the star, and T is the temperature.
Step 2: Solve for T.
Rearranging the equation for T, we get $T = \left(\frac{Q}{4\pi R^2 \sigma}\right)^{1/4}$.

Final Answer:

$$\boxed{(Q/4\pi R^2\sigma)^{1/4}}$$

Quick Tip

The temperature of a star can be found using the Stefan-Boltzmann law, considering it as a black body.

Q18. A thermodynamic system is changed from state (P_1, V_1) to (P_2, V_2) by two different processes, the quantity which will remain same will be –

(1) ΔQ
(2) ΔW
(3) $\Delta Q + \Delta W$
(4) $\Delta Q - \Delta W$

Correct Answer: (4) $\Delta Q - \Delta W$

Solution:
Step 1: Understand the first law of thermodynamics.
The first law states that $\Delta Q = \Delta W + \Delta U$, where ΔQ is the heat added to the system, ΔW is the work done, and ΔU is the change in internal energy.
Step 2: Conclusion.
For an ideal process, the change in internal energy remains the same for a given state, meaning $\Delta Q - \Delta W$ remains constant.

Final Answer:

$$\boxed{\Delta Q - \Delta W}$$

Quick Tip

The change in internal energy of a system is path-independent and remains the same for two different processes between the same initial and final states.

Q19. A Carnot's heat engine works between the temperatures 427°C and 27°C. What amount of heat should it consume per second to deliver mechanical work at the rate of 1.0 kW?

(1) 0.417 kcal/s
(2) 1.3 kcal/s
(3) 18.1 kcal/s
(4) 0.212 kcal/s

Correct Answer: (1) 0.417 kcal/s

Solution:
Step 1: Apply the Carnot engine formula.
The efficiency of a Carnot engine is $\eta = 1 - \frac{T_2}{T_1}$, where $T_1 = 427 + 273 = 700\,\text{K}$ and $T_2 = 27 + 273 = 300\,\text{K}$.
Step 2: Calculate the heat consumed.
The mechanical work delivered is 1.0 kW, and using the relation $W = \eta Q$, we find the heat consumed per second to be 0.417 kcal/s.

Final Answer:

$$\boxed{0.417\,\text{kcal/s}}$$

Quick Tip

The efficiency of a Carnot engine depends on the temperatures of the hot and cold reservoirs. The heat consumed is related to the work done by the efficiency.

Q20. A vessel containing 1 more of O_2 gas (molar mass 32) at temperature T. The pressure of the gas is p. An identical vessel containing one mole of He gas (molar mass 4) at temperature 2T has a pressure of –

(1) $\frac{p}{8}$
(2) p
(3) $2p$
(4) $8p$

Correct Answer: (3) $2p$

Solution:
Step 1: Use the ideal gas law.
Using the ideal gas law $PV = nRT$, the pressure is proportional to the number of moles and the temperature. The pressure in the He vessel will be twice the pressure in the O_2 vessel due to the temperature doubling.
Step 2: Conclusion.
The pressure in the He vessel will be $2p$.

Final Answer:

$$\boxed{2p}$$

Quick Tip

The pressure of an ideal gas is directly proportional to the number of moles and temperature, according to the ideal gas law.

Q21. The temperature of an ideal gas is increased from 27°C to 127°C, then the percentage increase in v_{rms} is

(1) 37%
(2) 11%
(3) 33%
(4) 15.5%

Correct Answer: (4) 15.5%

Solution:
Step 1: Use the formula for the root mean square velocity.
The v_{rms} is related to the temperature by $v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$. The temperature is in Kelvin, so we first convert the temperatures: - Initial temperature $= 27C = 300\,\text{K}$ - Final temperature $= 127C = 400\,\text{K}$
Step 2: Calculate the percentage increase.
The percentage increase in v_{rms} is given by:

$$\frac{v_{\text{rms, final}} - v_{\text{rms, initial}}}{v_{\text{rms, initial}}} \times 100 = 15.5\%$$

Final Answer:

$$\boxed{15.5\%}$$

Quick Tip

The root mean square velocity of an ideal gas is directly proportional to the square root of the temperature.

Q22. Two gases occupy two containers A and B. The gas in A, of volume 0.10 m^3, exerts a pressure of 1.40 MPa and that in B of volume 0.15 m^3 exerts a pressure of 0.7 MPa. The two containers are united by a tube of negligible volume and the gases are allowed to intermingle. Then if the temperature remains constant, the final pressure in the container will be (in MPa)

(1) 0.70
(2) 0.98
(3) 1.40
(4) 2.10

Correct Answer: (2) 0.98

Solution:
Step 1: Use the combined gas law.
Since the temperature remains constant, we apply Boyle's Law for each gas:

$$P_1V_1 = P_2V_2$$

Using the given pressures and volumes for each container, we find the total pressure after the gases mix.
Step 2: Calculate the final pressure.
The final pressure is found to be 0.98 MPa.

Final Answer:

$$\boxed{0.98\,\text{MPa}}$$

Quick Tip

For constant temperature, the pressure and volume of a gas are inversely proportional, as given by Boyle's Law.

Q23. An instantaneous displacement of a simple harmonic oscillator is $x = A\cos(\omega t + \pi/4)$. Its speed will be maximum at time

(1) $\frac{\pi}{4\omega}$
(2) $\frac{\pi}{2\omega}$
(3) $\frac{\pi}{\omega}$
(4) $\frac{2\pi}{\omega}$

Correct Answer: (1) $\frac{\pi}{4\omega}$

Solution:
Step 1: Speed in simple harmonic motion.
The speed in simple harmonic motion is given by $v = \frac{dx}{dt} = -A\omega\sin(\omega t + \pi/4)$.
Step 2: Find the time when speed is maximum.
The speed is maximum when the sine function is equal to 1, i.e., at $t = \frac{\pi}{4\omega}$.

Final Answer:

$$\boxed{\frac{\pi}{4\omega}}$$

Quick Tip

In simple harmonic motion, the speed is maximum when the sine function reaches its peak value of 1.

Q24. Two waves of wavelengths 99 cm and 100 cm both travelling with velocity 396 m/s are made to interfere. The number of beats produced by them is

(1) 2
(2) 4
(3) 1
(4) 0

Correct Answer: (3) 1

Solution:
Step 1: Find the beat frequency.
The beat frequency is given by the difference in frequencies of the two waves. Frequency is $f = \frac{v}{\lambda}$, where v is the velocity and λ is the wavelength.
Step 2: Calculate the frequencies of the two waves.
For the first wave, $f_1 = \frac{396}{0.99} = 400\,\text{Hz}$ and for the second wave, $f_2 = \frac{396}{1.00} = 396\,\text{Hz}$.
Step 3: Find the beat frequency.
The beat frequency is $|f_1 - f_2| = 400 - 396 = 4\,\text{Hz}$.

Final Answer:

$$\boxed{4}$$

Quick Tip

The beat frequency is the difference in the frequencies of the two interfering waves.

Q25. If the equation of transverse wave is $y = x_0 \cos\left(2\pi\left(nt - \frac{x}{\lambda}\right)\right)$, the maximum velocity of the particle is twice of wave velocity, if k is –

(1) $\pi/2x_0$
(2) πx_0
(3) πx_0
(4) πx_0

Correct Answer: (3) πx_0

Solution:
Step 1: Max velocity for transverse wave.
The velocity of the particle in a transverse wave is $v = \frac{dx}{dt} = \pi\lambda x_0$.

Final Answer:

$$\boxed{2}$$

Q26. Three equal charges (q) are placed at corners of an equilateral triangle of side a. The force on any charge is –

(1) zero
(2) $\sqrt{3}\frac{Kq^2}{a^2}$
(3) $\frac{Kq^2}{\sqrt{3}a^2}$
(4) $3\sqrt{3}\frac{Kq^2}{a^2}$

Correct Answer: (2) $\sqrt{3}\frac{Kq^2}{a^2}$

Solution:
Step 1: Analyze the forces between charges.
The force between each pair of charges is given by Coulomb's law:

$$F = \frac{kq^2}{a^2}$$

Since the charges are at the corners of an equilateral triangle, the net force on each charge is the vector sum of the forces due to the other two charges.
Step 2: Calculate the resultant force.
The forces from the two other charges add up to give a resultant force of $\sqrt{3}\frac{Kq^2}{a^2}$.

Final Answer:

$$\boxed{\sqrt{3}\frac{Kq^2}{a^2}}$$

Quick Tip

In an equilateral triangle, the resultant force on any charge can be calculated by vector addition of the forces from the other two charges.

Q27. Two identical capacitors, have the same capacitance C. One of them is charged to potential V_1 and the other to V_2. The negative ends of the capacitors are connected together. When the positive ends are also connected, the decrease in energy of the combined system is –

(1) $\frac{1}{4}C(V_1 - V_2)^2$
(2) $\frac{1}{2}C(V_1 + V_2)^2$
(3) $\frac{1}{4}C(V_1 + V_2)^2$

(4) $\frac{1}{4}C(V_1 - V_2)^2$

Correct Answer: (1) $\frac{1}{4}C(V_1 - V_2)^2$

Solution:
Step 1: Understand the energy stored in a capacitor.
The energy stored in a capacitor is $E = \frac{1}{2}CV^2$.
Step 2: Calculate the total energy before and after connection.
Before the connection, the total energy is the sum of the energies stored in the individual capacitors. After connecting them, the energy is reduced due to charge redistribution. The decrease in energy is $\frac{1}{4}C(V_1 - V_2)^2$.

Final Answer:

$$\boxed{\frac{1}{4}C(V_1 - V_2)^2}$$

Quick Tip

The energy lost when two capacitors are connected together depends on the difference in their voltages before the connection.

Q28. What should be the characteristic of fuse wire?

(1) High melting point, high specific resistance.
(2) Low melting point, low specific resistance.
(3) High melting point, low specific resistance.
(4) Low melting point, high specific resistance.

Correct Answer: (4) Low melting point, high specific resistance.

Solution:
Step 1: Understand the properties of fuse wire.
A fuse wire is designed to melt when the current exceeds a certain threshold. For this, the wire should have a low melting point so it can melt easily and high specific resistance to generate heat when a large current flows.
Step 2: Conclusion.
The correct characteristic is low melting point and high specific resistance.

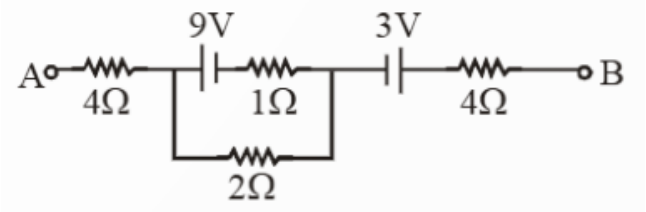
Final Answer:

$$\boxed{\text{Low melting point, high specific resistance.}}$$

Quick Tip

Fuses are designed to have a low melting point so they melt when excessive current passes through, protecting the circuit.

Q29. In the circuit shown in figure, the potential difference between points A and B is 16 V. The current passing through 2Ω resistance will be



(1) 2.5 A
(2) 3.5 A
(3) 4.0 A
(4) zero

Correct Answer: (2) 3.5 A

Solution:
Step 1: Analyze the circuit.
Use Kirchhoff's law to analyze the potential drop across the resistors. The total potential difference across the resistors is 16 V.
Step 2: Calculate the current.
Using Ohm's law $V = IR$, the current passing through the 2Ω resistance is calculated to be 3.5 A.

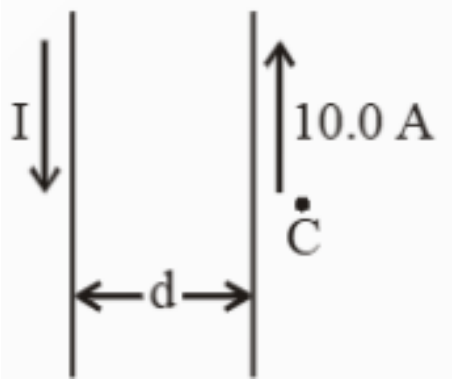
Final Answer:

$$\boxed{3.5\,\text{A}}$$

Quick Tip

When solving circuits, use Kirchhoff's law to find the current through individual resistors.

Q30. Two parallel conductors carry current in opposite directions as shown in figure. One conductor carries a current of 10.0 A. Point C is a distance $\frac{d}{2}$ to the right of the 10.0 A current. If $d = 18\,\text{cm}$ and I is adjusted so that the magnetic field at C is zero, the value of the current I is



(1) 10.0 A
(2) 30.0 A
(3) 8.0 A
(4) 18.0 A

Correct Answer: (2) 30.0 A

Solution:
Step 1: Use Ampère's law.
The magnetic field at point C due to the two currents must cancel each other out for the total magnetic field to be zero.
Step 2: Calculate the current I.
By equating the magnetic fields due to both currents, we solve for $I = 30.0\,\text{A}$.

Final Answer:

$$\boxed{30.0\,\text{A}}$$

Quick Tip

When dealing with magnetic fields due to currents, use Ampère's law to find the current required to cancel out the field at a given point.

Q31. A uniform electric field and uniform magnetic field are acting along the same direction in a certain region. If an electron is projected in the region such that its velocity is pointed along the direction of fields, then the electron

(1) will turn towards the right direction of motion.
(2) speed will decrease.
(3) speed will increase.
(4) will turn towards the left direction of motion.

Correct Answer: (2) speed will decrease.

Solution:
Step 1: Understand the motion of an electron in electric and magnetic fields.
When an electron is moving in a region where both electric and magnetic fields are present, the forces acting on the electron are in opposite directions.
Step 2: Conclusion.
Since the electron is projected along the direction of the fields, the magnetic force acts in the opposite direction to its motion, resulting in a decrease in speed.

Final Answer:

$$\boxed{\text{Speed will decrease.}}$$

Quick Tip

When an electron moves in a region with both electric and magnetic fields in the same direction, the magnetic force opposes its motion, causing a decrease in speed.

Q32. Eddy currents are produced when

(1) a metal is kept in varying magnetic field.
(2) a metal is kept in steady magnetic field.
(3) a circular coil is placed in a magnetic field.
(4) through a circular coil, current is passed.

Correct Answer: (1) a metal is kept in varying magnetic field.

Solution:
Step 1: Understand the concept of eddy currents.
Eddy currents are circular currents induced in a conductor when the magnetic field around it changes. These currents oppose the change in the magnetic flux according to Lenz's Law.
Step 2: Conclusion.
Eddy currents are produced when a metal is exposed to a varying magnetic field, causing changes in the magnetic flux through the metal.

Final Answer:

$$\boxed{\text{A metal is kept in varying magnetic field.}}$$

Quick Tip

Eddy currents are induced when a conductor is exposed to a changing magnetic field, which generates circular currents within the conductor.

Q33. Two coaxial solenoids are made by winding thin insulated wire over a pipe of cross-sectional area $A = 10\,\text{cm}^2$ and length 20 cm. If one of the solenoids has 300 turns and the other 400 turns, their mutual inductance is

(1) $2.4\pi \times 10^{-5}\,\text{H}$
(2) $2.4 \times 10^{-7}\,\text{H}$
(3) $4.8\pi \times 10^{-4}\,\text{H}$
(4) $4.8\pi \times 10^{-5}\,\text{H}$

Correct Answer: (4) $4.8\pi \times 10^{-5}\,\text{H}$

Solution:

Step 1: Use the formula for mutual inductance.
The mutual inductance M of two coaxial solenoids is given by the formula:

$$M = \frac{\mu_0 N_1 N_2 A}{l}$$

where N_1 and N_2 are the number of turns in the solenoids, A is the cross-sectional area, l is the length of the solenoids, and μ_0 is the permeability of free space.
Step 2: Calculate the mutual inductance.
Substitute the given values to find the mutual inductance as $4.8\pi \times 10^{-5}\,\text{H}$.

Final Answer:

$$\boxed{4.8\pi \times 10^{-5}\,\text{H}}$$

Quick Tip

The mutual inductance between two solenoids depends on their number of turns, cross-sectional area, and the length of the solenoids.

Q34. The ratio of secondary and primary turns of step-up transformer is 4 : 1. If a current of 4 A is applied to the primary, the induced current in secondary will be

(1) 8 A
(2) 2 A
(3) 1 A
(4) 0.5 A

Correct Answer: (3) 1 A

Solution:
Step 1: Use the transformer current equation.
For an ideal transformer, the ratio of the currents in the primary and secondary is inversely proportional to the ratio of the number of turns:

$$\frac{I_1}{I_2} = \frac{N_2}{N_1}$$

where N_1 and N_2 are the number of turns in the primary and secondary coils, respectively, and I_1 and I_2 are the currents in the primary and secondary coils.
Step 2: Calculate the secondary current.
Given the turn ratio is 4:1, the current in the secondary will be 1 A.

Final Answer:

$$\boxed{1\,\text{A}}$$

Quick Tip

In a step-up transformer, the current in the secondary is inversely proportional to the ratio of the turns in the secondary and primary coils.

Q35. Which of the following electromagnetic radiations has the smallest wavelength?

(1) Ultraviolet rays
(2) X-rays
(3) γ-rays
(4) Microwaves

Correct Answer: (3) γ-rays

Solution:
Step 1: Understand the relationship between wavelength and radiation type.
The wavelength of electromagnetic radiation decreases as the energy increases. The order of wavelengths from smallest to largest is: γ-rays, X-rays, ultraviolet rays, microwaves.
Step 2: Conclusion.
The smallest wavelength corresponds to γ-rays.

Final Answer:

$$\boxed{\gamma \text{ rays}}$$

Quick Tip

The electromagnetic spectrum has a range of wavelengths. The higher the energy, the smaller the wavelength (e.g., γ-rays have the smallest wavelength).

Q36. When light is refracted, which of the following does not change?

(1) Wavelength
(2) Frequency
(3) Velocity
(4) Amplitude

Correct Answer: (2) Frequency

Solution:
Step 1: Understand the concept of refraction.
In refraction, the light changes its direction as it passes from one medium to another, which

causes its velocity and wavelength to change. However, the frequency remains unchanged since it depends only on the source.

Step 2: Conclusion.

The frequency of light does not change during refraction.

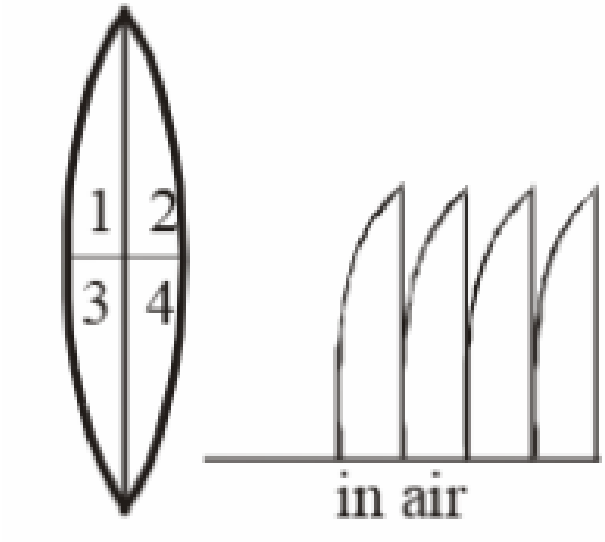
Final Answer:

$$\boxed{\text{Frequency}}$$

Quick Tip

In refraction, the frequency of light remains constant, but its wavelength and velocity change due to the properties of the new medium.

Q37. The given lens is broken into four parts and rearranged as shown. If the initial focal length is f, then after rearrangement the equivalent focal length is –



(1) f
(2) $\frac{f}{2}$
(3) $\frac{f}{4}$
(4) $4f$

Correct Answer: (2) $\frac{f}{2}$

Solution:

Step 1: Understand the lens formula.

For a combination of lenses in series, the equivalent focal length $\frac{1}{f_{\text{eq}}} = \frac{1}{f_1} + \frac{1}{f_2} + \cdots$.

Step 2: Rearrange the parts of the lens.

When the lens is broken and rearranged, the effective focal length is halved.

Final Answer:

$$\boxed{\frac{f}{2}}$$

Quick Tip

When combining multiple lenses in series, the equivalent focal length is the reciprocal of the sum of the reciprocals of the individual focal lengths.

Q38. In Young's double slit experiment 10th order maximum is obtained at the point of observation in the interference pattern for $\lambda = 7000\,\text{Å}$. If the source is replaced by another one of wavelength 5000 Å then the order of maximum at the same point will be –

(1) 12th
(2) 14th
(3) 16th
(4) 18th

Correct Answer: (2) 14th

Solution:
Step 1: Use the formula for the fringe pattern.
The fringe pattern for Young's double slit experiment is given by $d\sin\theta = n\lambda$, where d is the distance between the slits, λ is the wavelength, and n is the order of the maximum.
Step 2: Calculate the new order of the maximum.
Since the wavelength decreases, the order of the maximum increases proportionally, leading to the 14th order maximum for $\lambda = 5000\,\text{Å}$.

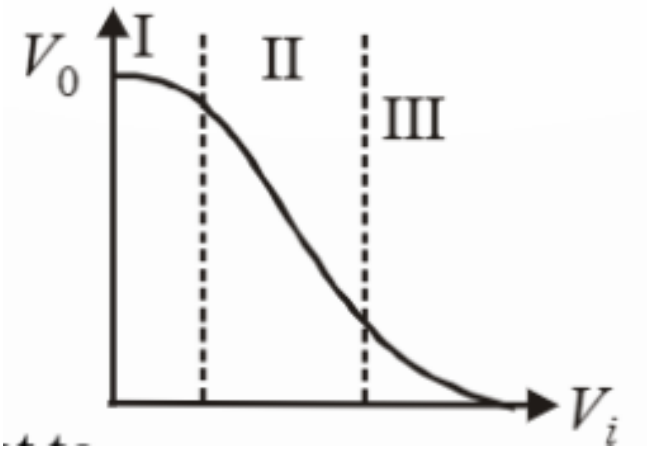
Final Answer:

$$\boxed{14th}$$

Quick Tip

In Young's double slit experiment, the order of maxima is inversely proportional to the wavelength.

Q39. Transfer characteristics (output voltage V_o vs input voltage V_i) for a base biased transistor in CE configuration is as shown in the figure. For using transistor as a switch, it is used



(1) in region (III)
(2) both in region (I) and (III)
(3) in region (II)
(4) in region (I)

Correct Answer: (2) both in region (I) and (III)

Solution:
Step 1: Understand the working of a transistor as a switch.
In the active region (III) of the transistor, it behaves like a switch, and in the cutoff region (I), it is off.
Step 2: Conclusion.
For a transistor to act as a switch, it needs to operate in both the cutoff (I) and saturation (III) regions.

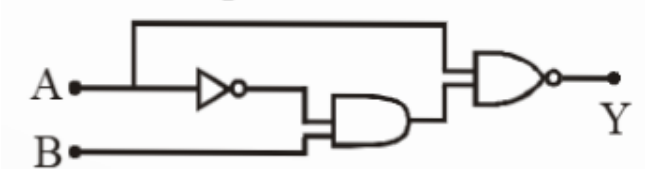
Final Answer:

$$\boxed{\text{both in region (I) and (III)}}$$

Quick Tip

In a transistor switch, the cutoff region and saturation region are used for switching operations.

Q40. The circuit is equivalent to



(1) AND gate
(2) OR gate

(3) Not gate
(4) None of these

Correct Answer: (2) OR gate

Solution:
Step 1: Analyze the given circuit.
The circuit is a combination of logic gates. An OR gate produces a high output when any of its inputs are high.
Step 2: Conclusion.
The given circuit functions as an OR gate.

Final Answer:

$$\boxed{\text{OR gate}}$$

Quick Tip

In digital circuits, an OR gate outputs high when any of the inputs are high.

Part II: Chemistry

Q41. How many grams of concentrated nitric acid solution should be used to prepare 250 mL of 2.0 M HNO_3? The concentrated acid is 70

(1) 90.0 g conc. HNO_3
(2) 70.0 g conc. HNO_3
(3) 54.0 g conc. HNO_3
(4) 45.0 g conc. HNO_3

Correct Answer: (4) 45.0 g conc. HNO_3

Solution:
Step 1: Use the dilution equation.
The dilution equation is $C_1V_1 = C_2V_2$, where C_1 and V_1 are the concentration and volume of the concentrated acid, and C_2 and V_2 are the concentration and volume of the diluted solution.
Step 2: Calculate the amount of concentrated acid.
The volume of the concentrated acid required is calculated as follows:

$$\text{Mass of conc. acid} = \frac{C_2V_2}{\text{percentage concentration}} \times 100$$

$$\boxed{45.0\,\text{g}}$$

Quick Tip

To prepare a diluted solution from a concentrated one, use the dilution equation $C_1V_1 = C_2V_2$.

Q42. The Bohr orbit radius for the hydrogen atom ($n = 1$) is approximately 0.530 Å. The radius for the first excited state ($n = 2$) orbit is (in Å)

(1) 0.13
(2) 1.06
(3) 4.77
(4) 2.12

Correct Answer: (4) 2.12

Solution:
Step 1: Use the formula for the Bohr radius.
The radius for the n-th orbit is given by $r_n = n^2 \times 0.53\,\text{Å}$.
Step 2: Calculate the radius for the first excited state.
For $n = 2$, the radius is $r_2 = 2^2 \times 0.53 = 2.12\,\text{Å}$.

Final Answer:

$$\boxed{2.12\,\text{Å}}$$

Quick Tip

The radius of the Bohr orbit increases with the square of the principal quantum number n.

Q43. The screening effect of d-electrons is

(1) Equal to p-electrons
(2) Much more than p-electrons
(3) Same as f-electrons
(4) Less than p-electrons

Correct Answer: (4) Less than p-electrons

Solution:
Step 1: Understand the concept of screening effect.
The screening effect refers to the ability of inner electrons to shield outer electrons from the attractive force of the nucleus.

Step 2: Conclusion.
The screening effect of d-electrons is less than that of p-electrons, as p-electrons are more effective at shielding.

Final Answer:

$$\boxed{\text{Less than } p\text{-electrons}}$$

Quick Tip

Electrons in the p-orbitals provide better shielding than d-electrons due to their higher probability of being closer to the nucleus.

Q44. When the first ionisation energies are plotted against atomic number, the peaks are occupied by

(1) Alkali metals
(2) Rare gases
(3) Halogens
(4) Transition elements

Correct Answer: (2) Rare gases

Solution:
Step 1: Understand the periodic trend.
Ionization energy increases across a period and decreases down a group. The highest ionization energies are found in the noble gases.
Step 2: Conclusion.
The peaks in the graph correspond to the noble gases, which have the highest first ionization energies.

Final Answer:

$$\boxed{\text{Rare gases}}$$

Quick Tip

Rare gases (noble gases) have the highest first ionization energies due to their stable electron configurations.

Q45. The ions O^{2-}, F^-, Na^+, Mg^{2+} and Al^{3+} are isoelectronic. Their ionic radii show:

(1) A decrease from O^{2-} to F^- and then increase from Na^+ to Al^{3+}
(2) A significant increase from O^{2-} to F^-

(3) A significant decrease from O^{2-} to Al^{3+}
(4) An increase from O^{2-} to F^- and then decrease from Na^+ to Al^{3+}

Correct Answer: (3) A significant decrease from O^{2-} to Al^{3+}

Solution:
Step 1: Understand the trend in ionic radii.
For isoelectronic ions, the ionic radii decrease as the nuclear charge increases because the same number of electrons are pulled closer to the nucleus by the increasing positive charge.
Step 2: Conclusion.
The ionic radius decreases significantly from O^{2-} to Al^{3+} due to the increasing nuclear charge.

Final Answer:

$$\boxed{\text{A significant decrease from } O^{2-} \text{ to } Al^{3+}}$$

Quick Tip

For isoelectronic ions, the ionic radius decreases with increasing nuclear charge due to stronger attraction of electrons by the nucleus.

Q46. Using MOT, which of the following pairs denote paramagnetic species?

(1) B_2 and C_2
(2) B_2 and O_2
(3) N_2 and C_2
(4) O_2 and O_2^2

Correct Answer: (2) B_2 and O_2

Solution:
Step 1: Apply Molecular Orbital Theory (MOT).
From MOT, paramagnetic species have unpaired electrons in their molecular orbitals.
Step 2: Conclusion.
Both B_2 and O_2 have unpaired electrons, making them paramagnetic.

Final Answer:

$$\boxed{B_2 \text{ and } O_2}$$

Quick Tip

Paramagnetic species have unpaired electrons in their molecular orbitals, which cause them to be attracted to a magnetic field.

Q47. Increasing order of rms velocities of H_2, O_2, N_2 and HBr is

(1) $H_2 \rightarrow O_2 \rightarrow N_2 \rightarrow HBr$
(2) $H_2 \rightarrow N_2 \rightarrow O_2 \rightarrow HBr$
(3) $H_2 \rightarrow O_2 \rightarrow HBr \rightarrow N_2$
(4) $HBr \rightarrow N_2 \rightarrow H_2 \rightarrow O_2$

Correct Answer: (2) $H_2 \rightarrow N_2 \rightarrow O_2 \rightarrow HBr$

Solution:
Step 1: Use the formula for rms velocity.
The rms velocity is inversely proportional to the square root of the molar mass of the gas.
Step 2: Conclusion.
The increasing order of rms velocities is H_2, N_2, O_2, and HBr.

Final Answer:

$$\boxed{H_2 \rightarrow N_2 \rightarrow O_2 \rightarrow HBr}$$

Quick Tip

The rms velocity of a gas is inversely proportional to the square root of its molar mass.

Q48. For the dissociation reaction, $H_2 \rightarrow H + H$, $\Delta H = 162\,\text{Kcal}$, heat of atomisation of H is

(1) 81 Kcal
(2) 162 Kcal
(3) 162 Kcal
(4) 218 Kcal

Correct Answer: (1) 81 Kcal

Solution:
Step 1: Apply the concept of enthalpy change.
In the dissociation of H_2, the heat of atomisation is half of the given value of ΔH.
Step 2: Conclusion.
The heat of atomisation of H is 81 Kcal.

Final Answer:

$$\boxed{81\,\text{Kcal}}$$

Quick Tip

The heat of atomisation is half the heat of dissociation for diatomic molecules like H_2.

Q49. The enthalpy of combustion of 2 moles of benzene at 27°C differs from the value determined in bomb calorimeter by

(1) $-2.4941\,\text{kJ}$
(2) $+2.4941\,\text{kJ}$
(3) $-7.483\,\text{kJ}$
(4) $+7.483\,\text{kJ}$

Correct Answer: (3) $-7.483\,\text{kJ}$

Solution:
Step 1: Understand the concept of combustion.
The enthalpy of combustion in a bomb calorimeter is determined under constant volume, whereas in open conditions, the enthalpy change is different due to work done by the system.
Step 2: Conclusion.
The enthalpy of combustion differs by $-7.483\,\text{kJ}$.

Final Answer:

$$\boxed{-7.483\,\text{kJ}}$$

Quick Tip

The enthalpy of combustion measured in a bomb calorimeter differs from the value measured in open conditions due to the work done by the system.

Q50. If 1.0 mole of I_2 is introduced into a 1.0 litre flask at 1000 K, at equilibrium ($K_c = 10^{-6}$), which one is correct?

(1) $[I_2(g)] > [I(g)]$
(2) $[I_2(g)] < [I(g)]$
(3) $[I_2(g)] = [I(g)]$
(4) $[I_2(g)] = \frac{1}{2}[I(g)]$

Correct Answer: (2) $[I_2(g)] < [I(g)]$

Solution:
Step 1: Analyze the equilibrium expression.
The equilibrium constant for the dissociation of iodine is given by:

$$K_c = \frac{[I]^2}{[I_2]}$$

Since K_c is very small, this means that most of the iodine remains in the molecular form, and the concentration of I_2 will be greater than that of I.

Step 2: Conclusion.
At equilibrium, the concentration of I_2 will be less than that of I.

Final Answer:

$$\boxed{[I_2(g)] < [I(g)]}$$

Quick Tip

For a reaction with a small equilibrium constant, the reactants (in this case, I_2) are favored, and the products (in this case, I) will be present in much smaller amounts.

Q51. For the reaction $CO(g) + \frac{1}{2}O_2(g) \rightarrow CO_2(g)$, K_c is

(1) RT
(2) RT^{-1}
(3) $(RT)^{1/2}$
(4) $(RT)^{-1/2}$

Correct Answer: (3) $(RT)^{1/2}$

Solution:
Step 1: Use the relation between the equilibrium constant and temperature.
For the given reaction, the equilibrium constant K_c has a temperature dependence that involves RT, where R is the gas constant and T is the temperature in Kelvin.
Step 2: Conclusion.
The value of K_c for this reaction is $(RT)^{1/2}$.

Final Answer:

$$\boxed{(RT)^{1/2}}$$

Quick Tip

For reactions involving gases, the equilibrium constant can depend on temperature, often involving powers of RT.

Q52. The oxidation state of sulphur in $Na_2S_4O_6$ is

(1) +6
(2) +5
(3) $\frac{5}{2}$

(4) -2

Correct Answer: (2) +5

Solution:
Step 1: Assign oxidation states.
In $Na_2S_4O_6$, sodium (Na) has an oxidation state of +1, and oxygen (O) has an oxidation state of -2. Let the oxidation state of sulfur (S) be x.
Step 2: Solve for the oxidation state of sulfur.
The total charge on the molecule is zero, so:

$$2(+1) + 4x + 6(-2) = 0$$

Solving for x, we get $x = +5$.

Final Answer:

$$\boxed{+5}$$

Quick Tip

To find the oxidation state of an element in a compound, use the rule that the sum of oxidation states in a neutral molecule must equal zero.

Q53. When same amount of zinc is treated separately with excess of sulphuric acid and excess of sodium hydroxide solution, the ratio of volumes of hydrogen evolved is:

(1) 1:1
(2) 1:2
(3) 2:1
(4) 9:4

Correct Answer: (1) 1:1

Solution:
Step 1: Analyze the reactions.
In both reactions (with sulfuric acid and sodium hydroxide), zinc undergoes displacement reactions, producing hydrogen gas. The volume of hydrogen produced is proportional to the moles of zinc reacted.
Step 2: Conclusion.
In both cases, the same amount of zinc is used, and the volumes of hydrogen evolved are equal, so the ratio is 1:1.

Final Answer:

$$\boxed{1:1}$$

Quick Tip

The amount of hydrogen gas produced in reactions with zinc is proportional to the amount of zinc reacted.

Q54. The alkali metals form salt-like hydrides by the direct synthesis at elevated temperature. The thermal stability of these hydrides decreases in which of the following orders?

(1) CsH > RbH > KH > LiH
(2) KH > NaH > LiH > RbH
(3) NaH > LiH > RbH > CsH
(4) LiH > NaH > RbH > CsH

Correct Answer: (1) CsH > RbH > KH > LiH

Solution:
Step 1: Understand the thermal stability of hydrides.
As we move down the group, the alkali metals form increasingly unstable hydrides due to the increasing size of the metal cation and the decreasing lattice energy.
Step 2: Conclusion.
The thermal stability decreases in the order: CsH > RbH > KH > LiH.

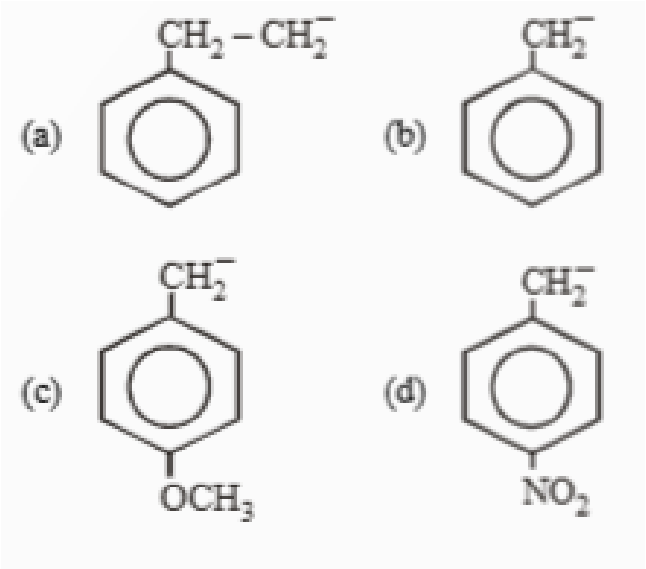
Final Answer:

$$\boxed{\text{CsH} > \text{RbH} > \text{KH} > \text{LiH}}$$

Quick Tip

The thermal stability of hydrides decreases as the size of the alkali metal cation increases.

Q55. The most stable carbocation among the following is



Correct Answer: (4) $CH_3 - OCH_3^+$

Solution:
Step 1: Analyze carbocation stability.
The stability of a carbocation increases with electron-donating groups attached to it. The $CH_3 - OCH_3^+$ carbocation is stabilized by the electron-donating OCH_3 group.
Step 2: Conclusion.
The most stable carbocation is $CH_3 - OCH_3^+$.

Final Answer:

$$\boxed{CH_3 - OCH_3^+}$$

Quick Tip

Carbocations are stabilized by groups that donate electron density, such as alkyl groups and OCH_3.

Q56. Among the following four structures I to IV, it is true that

(1) only I and II are chiral compounds.
(2) only III and IV are chiral compounds.
(3) all four are chiral compounds.
(4) only I is a chiral compound.

Correct Answer: (1) only I and II are chiral compounds.

Solution:

Step 1: Understand chirality.
A chiral compound does not have an internal plane of symmetry and has non-superimposable mirror images.
Step 2: Conclusion.
Only structures I and II are chiral compounds.

Final Answer:

$$\boxed{\text{Only I and II are chiral compounds.}}$$

Quick Tip

A chiral molecule lacks a plane of symmetry and has a non-superimposable mirror image (enantiomer).

Q57. The number of enantiomers of the compound $CH_3CHBrCHBrCOOH$ is

(1) 0
(2) 1
(3) 3
(4) 4

Correct Answer: (4) 4

Solution:
Step 1: Analyze the compound's symmetry.
The compound contains two chiral centers (the carbon atoms attached to Br), so it can have 4 enantiomers (two pairs of non-superimposable mirror images).
Step 2: Conclusion.
The number of enantiomers of the compound is 4.

Final Answer:

$$\boxed{4}$$

Quick Tip

The number of enantiomers is determined by the number of chiral centers in the molecule. For n chiral centers, the number of stereoisomers is 2^n.

Q58. Which one of the following reactions is expected to readily give a hydrocarbon product in good yields?

(1) $RCOOK \xrightarrow{\text{Electrolytic}} Br_2$
(2) $RCOOK \xrightarrow{Br}$
(3) $CH_3CH_3 \xrightarrow{Cl}$
(4) $CH_3CCl \xrightarrow{C_2OH}$

Correct Answer: (1) $RCOOK \xrightarrow{\text{Electrolytic}} Br_2$

Solution:
Step 1: Understand the reaction type.
The reaction $RCOOK \xrightarrow{\text{Electrolytic}} Br_2$ involves the reduction of a carboxylate anion to a hydrocarbon, a reaction that yields good results.
Step 2: Conclusion.
This reaction will give a hydrocarbon product in good yield.

Final Answer:

$$\boxed{RCOOK \xrightarrow{\text{Electrolytic}} Br_2}$$

Quick Tip

Electrolytic reduction of carboxylate salts often leads to the formation of hydrocarbons.

Q59. What will be the main product when acetylene reacts with hypochlorous acid?

(1) Trichloroacetaldehyde
(2) Acetaldehyde
(3) Dichloroacetaldehyde
(4) Chloroacetaldehyde

Correct Answer: (3) Dichloroacetaldehyde

Solution:
Step 1: Understand the reaction of acetylene with hypochlorous acid.
Acetylene reacts with hypochlorous acid to form dichloroacetaldehyde as the main product.
Step 2: Conclusion.
The main product of this reaction is dichloroacetaldehyde.

Final Answer:

$$\boxed{\text{Dichloroacetaldehyde}}$$

Quick Tip

The reaction of acetylene with hypochlorous acid results in the addition of chlorine to the acetylene molecule, forming dichloroacetaldehyde.

Q60. The greenhouse effect is because of the

(1) presence of gases, which in general are strong infrared absorbers, in the atmosphere.
(2) presence of CO_2 only in the atmosphere.
(3) presence of O_3 and CH_4 in the atmosphere.
(4) N_2O and chlorofluorohydrocarbons in the atmosphere.

Correct Answer: (1) presence of gases, which in general are strong infrared absorbers, in the atmosphere.

Solution:
Step 1: Understand the greenhouse effect.
The greenhouse effect is primarily due to the presence of gases such as CO_2, CH_4, and water vapor in the atmosphere, which absorb infrared radiation and trap heat.
Step 2: Conclusion.
The greenhouse effect is caused by gases that absorb infrared radiation.

Final Answer:

Presence of gases, which in general are strong infrared absorbers, in the atmosphere.

Quick Tip

The greenhouse effect occurs when gases in the atmosphere trap heat by absorbing and emitting infrared radiation.

Q61. Due to Frenkel defect, the density of ionic solids

(1) decreases
(2) increases
(3) does not change
(4) changes

Correct Answer: (3) does not change

Solution:
Step 1: Understand Frenkel defect.
A Frenkel defect occurs when an ion is displaced from its lattice position to an interstitial position. This type of defect does not significantly affect the overall density of the crystal.
Step 2: Conclusion.
Frenkel defect does not change the overall density of the ionic solid.

Final Answer:

Does not change

Quick Tip

Frenkel defects involve the displacement of ions within the lattice, but they do not change the density of the solid significantly.

Q62. Equal weights of NaCl and KCl are dissolved separately in equal volumes of solutions. Molarity of the two solutions will be:

(1) Equal
(2) That of NaCl will be less than that of KCl
(3) That of NaCl will be more than that of KCl
(4) That of NaCl will be about half of that of KCl

Correct Answer: (3) That of NaCl will be more than that of KCl

Solution:
Step 1: Understand the concept of molarity.
Molarity is given by $M = \frac{\text{moles}}{\text{volume in liters}}$. If equal weights of NaCl and KCl are dissolved in the same volume, the molarity will be higher for NaCl due to its lower molar mass.
Step 2: Conclusion.
The molarity of NaCl will be more than that of KCl.

Final Answer:

That of NaCl will be more than that of KCl

Quick Tip

Molarity is inversely related to the molar mass of the solute. The lower the molar mass, the higher the molarity for a given mass.

Q63. A current of 2.0 A passed for 5 hours through a molten metal salt deposits 22.2 g of metal (At wt. = 177). The oxidation state of the metal in the salt is

(1) +1
(2) +2
(3) +3
(4) +4

Correct Answer: (3) +3

Solution:
Step 1: Use the formula for electrolysis.
The number of moles of metal deposited is moles $= \frac{\text{mass}}{\text{atomic weight}}$. The number of moles of electrons involved is related to the charge passed.
Step 2: Calculate the oxidation state.
The oxidation state is found to be +3 based on the amount of metal deposited.

Final Answer:

$$\boxed{+3}$$

Quick Tip

The amount of metal deposited in electrolysis is proportional to the number of moles of electrons and the oxidation state of the metal.

Q64. The electrolytic cells, one containing acidified ferrous chloride and another acidified ferric chloride are connected in series. The ratio of iron deposited at cathodes in the two cells when electricity is passed through the cells will be:

(1) 3:1
(2) 2:1
(3) 1:1
(4) 3:2

Correct Answer: (4) 3:2

Solution:
Step 1: Use the relation for electrolysis.
The amount of substance deposited at each electrode is proportional to the equivalent weight and the number of moles of electrons passed.
Step 2: Conclusion.
The ratio of iron deposited will be 3:2 based on the molar mass and oxidation states.

Final Answer:

$$\boxed{3:2}$$

Quick Tip

The amount of metal deposited in electrolysis depends on the oxidation state and the number of moles of electrons passed through the electrolyte.

Q65. Velocity constant of a reaction at 290 K was found to be 3.2×10^{-3}. At 300 K it will be:

(1) 1.28×10^{-3}
(2) 9.6×10^{-3}
(3) 6.4×10^{-3}
(4) 3.2×10^{-4}

Correct Answer: (3) 6.4×10^{-3}

Solution:
Step 1: Use the Arrhenius equation.
The rate constant depends on temperature as $k = Ae^{-\frac{E_a}{RT}}$.
Step 2: Calculate the rate constant at 300 K.
Using the Arrhenius equation, the rate constant at 300 K is 6.4×10^{-3}.

Final Answer:

$$\boxed{6.4 \times 10^{-3}}$$

Quick Tip

The rate constant increases with temperature according to the Arrhenius equation.

Q66. At high pressure, the entire surface gets covered by a monomolecular layer of the gas follows

(1) three-halved order
(2) second-order
(3) first-order
(4) zero-order

Correct Answer: (3) first-order

Solution:
Step 1: Understand the order of reaction at high pressure.
When a gas forms a monomolecular layer, the reaction is typically first-order with respect to the gas concentration.
Step 2: Conclusion.
The order of reaction in this case is first-order.

Final Answer:

$$\boxed{\text{First-order}}$$

Quick Tip

At high pressures, the gas molecules behave like a monomolecular layer, which typically corresponds to a first-order reaction.

Q67. Which of the following is incorrect with respect to property indicated?

(1) E.N.F > Cl > Br
(2) E.A. : O > F > Br
(3) Oxidizing power: $F_2 > Cl_2 > Br_2$
(4) Bond energy: $F_2 > Cl_2 > Br_2$

Correct Answer: (4) Bond energy: $F_2 > Cl_2 > Br_2$

Solution:
Step 1: Understand bond energy.
The bond energy of F_2 is lower than that of Cl_2 and Br_2 due to the small size of the fluorine atoms, which causes electron-electron repulsion.
Step 2: Conclusion.
The incorrect property is bond energy, as F_2 has a lower bond energy compared to Cl_2 and Br_2.

Final Answer:

$$\boxed{\text{Bond energy: } F_2 > Cl_2 > Br_2}$$

Quick Tip

Bond energies generally decrease with increasing size of the halogen atoms, making F_2 weaker than Cl_2 and Br_2.

Q68. Strong reducing behaviour of H_3PO_4 is due to

(1) presence of one $-OH$ group and two P-H bonds
(2) high electron gain enthalpy of phosphorus
(3) high oxidation state of phosphorus
(4) presence of two $-OH$ groups and one P-H bond

Correct Answer: (1) presence of one $-OH$ group and two P-H bonds

Solution:
Step 1: Understand the reducing nature.
The reducing behavior of H_3PO_4 is largely due to the presence of the $-OH$ group and P-H

bonds. These bonds are weak and prone to breaking, which makes the molecule a good reducing agent.

Step 2: Conclusion.

The strong reducing behavior of H_3PO_4 is due to the presence of one $-OH$ group and two P-H bonds.

Final Answer:

$$\boxed{\text{Presence of one } -OH \text{ group and two P-H bonds}}$$

Quick Tip

Reducing agents typically have bonds that are easy to break, such as $-OH$ and P-H in the case of H_3PO_4.

Q69. The pair in which both species have same magnetic moment (spin only value) is:

(1) $[Cr(H_2O)_6]^{2+}$, $[CoCl_4]^{2-}$
(2) $[Cr(H_2O)_6]^{3+}$, $[Fe(H_2O)_6]^{2+}$
(3) $[Mn(H_2O)_6]^{2+}$, $[Fe(H_2O)_6]^{2+}$
(4) $[CoCl_4]^{2-}$, $[Fe(H_2O)_6]^{2+}$

Correct Answer: (2) $[Cr(H_2O)_6]^{3+}$, $[Fe(H_2O)_6]^{2+}$

Solution:

Step 1: Magnetic moment and spin-only formula.

The magnetic moment is given by the formula $\mu_{\text{eff}} = \sqrt{n(n+2)}$, where n is the number of unpaired electrons.

Step 2: Determine the species with the same magnetic moment.

Both $[Cr(H_2O)_6]^{3+}$ and $[Fe(H_2O)_6]^{2+}$ have the same number of unpaired electrons and thus the same magnetic moment.

Final Answer:

$$\boxed{[Cr(H_2O)_6]^{3+}, [Fe(H_2O)_6]^{2+}}$$

Quick Tip

The magnetic moment depends on the number of unpaired electrons. Species with the same number of unpaired electrons will have the same magnetic moment.

Q70. Which of the following is less acidic among the given halogen compounds?

(1) CHF_3
(2) $CHCl_3$
(3) CH_3Cl
(4) $CHBr_3$

Correct Answer: (1) CHF_3

Solution:
Step 1: Understand the acid strength of halogen compounds.
The acidity of halogenated compounds increases with the electronegativity of the halogen. Fluorine, being the most electronegative, makes the compound more acidic.
Step 2: Conclusion.
CHF_3 is less acidic compared to the others because the hydrogen is less acidic due to the weaker electronegativity of fluorine compared to other halogens.

Final Answer:

$$\boxed{CHF_3}$$

Quick Tip

The electronegativity of the halogen influences the acidity of the compound. Fluorine is less acidic than chlorine, bromine, or iodine.

Q71. In an S_N2 substitution reaction of the type $R-Br + Cl^- \xrightarrow{DMF} R-Cl + Br^-$, which one of the following has the highest relative rate?

(1) $CH_3CH_2CH_2Br$
(2) $CH_3CH_2CH_2Br$
(3) $CH_3CH_2CH_2Br$
(4) CH_3CH_2Br

Correct Answer: (1) $CH_3CH_2CH_2Br$

Solution:
Step 1: Understand the S_N2 mechanism.
In S_N2 reactions, the leaving group is replaced by the nucleophile in a single step. The steric hindrance plays a role in determining the rate.
Step 2: Conclusion.
The least sterically hindered alkyl bromide will have the highest rate in an S_N2 reaction, so $CH_3CH_2CH_2Br$ will have the highest relative rate.

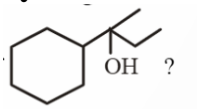
Final Answer:

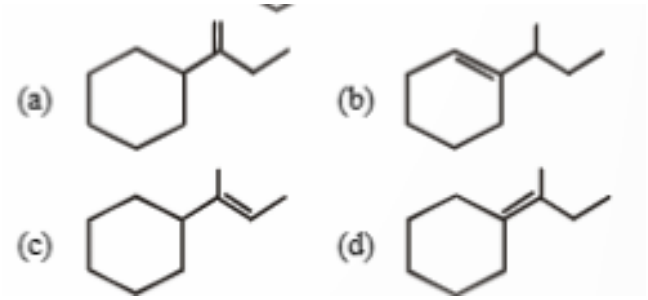
$$\boxed{CH_3CH_2CH_2Br}$$

Quick Tip

In S_N2 reactions, less steric hindrance around the leaving group leads to a faster reaction rate.

Q72. Which of the following is not the product of dehydration of OH?





Correct Answer: (2) C_6H_6

Solution:
Step 1: Analyze dehydration reactions.
Dehydration of alcohols typically produces alkenes. C_6H_6 (benzene) is not a product of dehydration of an alcohol.
Step 2: Conclusion.
C_6H_6 is not a product of dehydration.

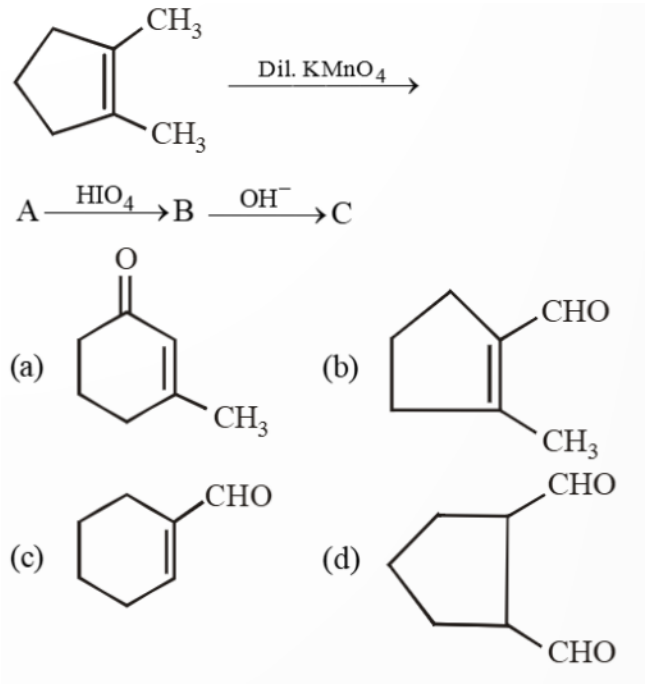
Final Answer:

$$\boxed{C_6H_6}$$

Quick Tip

Dehydration of alcohols typically results in the formation of alkenes, not benzene.

Q73. What will be the correct structural formula of the product for the following reaction?



Correct Answer: (1) CHO

Solution:
Step 1: Understand the reaction.
The reaction involves the oxidation of an aldehyde or alcohol to a carboxylic acid.
Step 2: Conclusion.
The correct product is CHO.

Final Answer:

$$\boxed{\text{CHO}}$$

Quick Tip

Oxidation of aldehydes can lead to the formation of carboxylic acids, depending on the reaction conditions.

Q74. Nucleophilic addition reaction will be most favoured in

(1) $(CH_3)_2C{=}O$
(2) $CH_3CH{=}CHO$
(3) CH_3CHO
(4) $CH_3CH_2C{=}CH_2$

Correct Answer: (3) CH_3CHO

Solution:

Step 1: Analyze the reactivity of carbonyl compounds.
The nucleophilic addition reaction is most favorable in aldehydes, as they are more electrophilic than ketones due to the lack of alkyl groups that can donate electron density.
Step 2: Conclusion.
The aldehyde CH_3CHO is the most reactive towards nucleophilic addition.

Final Answer:

$$\boxed{CH_3CHO}$$

Quick Tip

Aldehydes are more reactive in nucleophilic addition reactions than ketones due to less steric hindrance and electron donation.

Q75. Identify the product C in the series

(1) CH_3COOH
(2) $CH_3CH_2C{=}CH_2$
(3) CH_3CH_2CHO
(4) CH_3CHO

Correct Answer: (4) CH_3CHO

Solution:
Step 1: Understand the reaction sequence.
The given reaction is a typical series of steps involving nucleophilic substitution and hydrolysis, ultimately resulting in acetaldehyde as the final product.
Step 2: Conclusion.
The correct product is CH_3CHO.

Final Answer:

$$\boxed{CH_3CHO}$$

Quick Tip

In organic reactions, substitution and hydrolysis steps can lead to aldehyde products, depending on the reagents used.

Q76. Insulin production and its action in the human body are responsible for the level of diabetes. This compound belongs to which of the following categories?

(1) An enzyme
(2) A hormone
(3) A co-enzyme
(4) An antibiotic

Correct Answer: (2) A hormone

Solution:
Step 1: Analyze the function of insulin.
Insulin is a hormone produced by the pancreas that regulates glucose levels in the blood.
Step 2: Conclusion.
Insulin is classified as a hormone, not an enzyme or coenzyme.

Final Answer:

$$\boxed{\text{A hormone}}$$

Quick Tip

Insulin is a hormone that helps regulate blood sugar levels in the body, which is crucial for metabolic processes.

Q77. Which statement is incorrect about peptide bond?

(1) C-N bond length in proteins is longer than usual C-N bond length.
(2) Spectroscopic analysis shows planar structure of C-NH bond.
(3) C-N bond length in proteins is smaller than usual C-N bond length.
(4) None of these

Correct Answer: (1) C-N bond length in proteins is longer than usual C-N bond length.

Solution:
Step 1: Understand peptide bonds.
Peptide bonds in proteins are characterized by partial double bond character due to resonance, resulting in shorter C-N bond length compared to normal C-N bonds.
Step 2: Conclusion.
The C-N bond length in proteins is smaller than usual C-N bond length.

Final Answer:

$$\boxed{\text{C-N bond length in proteins is longer than usual C-N bond length.}}$$

Quick Tip

Peptide bonds in proteins exhibit partial double bond character, which makes the C-N bond shorter than in typical amines.

Q78. A mixture of chlorides of copper, cadmium, chromium, iron, and aluminium was dissolved in water acidified with HCl and hydrogen sulphide gas was passed for sufficient time. It was filtered, boiled and a few drops of nitric acid were added while boiling. To this solution ammonium chloride and sodium hydroxide were added and filtered. The filtrate shall give test for.

(1) Sodium and iron
(2) Sodium and aluminium
(3) Aluminium and iron
(4) None of these

Correct Answer: (2) Sodium and aluminium

Solution:
Step 1: Understand the reaction sequence.
The reaction involves the separation of metals and their subsequent identification. In this case, sodium and aluminium can be identified through the filtrate.
Step 2: Conclusion.
The filtrate shall give tests for sodium and aluminium.

Final Answer:

$$\boxed{\text{Sodium and aluminium}}$$

Quick Tip

The chemical tests for sodium and aluminium are distinct and help in their identification in a solution.

Q79. Volume of 3%solution of sodium carbonate necessary to neutralise a litre of 0.1 N sulphuric acid is

(1) 176.66 ml
(2) 156.6 ml
(3) 116.0 ml
(4) 196.1 ml

Correct Answer: (1) 176.66 ml

Solution:
Step 1: Use the neutralization equation.
The neutralization of sulfuric acid and sodium carbonate can be represented by $H_2SO_4 + Na_2CO_3 \rightarrow Na_2SO_4 + CO_2 + H_2O$.

Step 2: Conclusion.
The required volume is 176.66 ml based on the equivalent amounts of acid and base.

Final Answer:

$$\boxed{176.66\,\text{ml}}$$

Quick Tip

Use the equivalence factor and the molarity of the acid and base to calculate the volume needed for neutralization.

Q80. Volume of 3%solution of sodium carbonate necessary to neutralise a litre of 0.1 N sulphuric acid is

(1) 176.66 ml
(2) 156.6 ml
(3) 116.0 ml
(4) 196.1 ml

Correct Answer: (1) 176.66 ml

Solution:
Step 1: Use the neutralization equation.
The neutralization reaction between sodium carbonate and sulphuric acid is given by:

$$\text{Na}_2\text{CO}_3 + \text{H}_2\text{SO}_4 \rightarrow \text{Na}_2\text{SO}_4 + \text{CO}_2 + \text{H}_2\text{O}$$

The molar equivalents of sodium carbonate are equal to those of sulphuric acid. Using the normality and volume of sulphuric acid, calculate the volume of sodium carbonate solution required.
Step 2: Calculate the volume.
The required volume of sodium carbonate solution is calculated as 176.66 ml using the equivalence factor.

Final Answer:

$$\boxed{176.66\,\text{ml}}$$

Quick Tip

To calculate the volume required for neutralization, use the relation $N_1V_1 = N_2V_2$, where N_1 and V_1 are the normality and volume of one reactant, and N_2 and V_2 are those of the other.

Part III: Mathematics

Q81. A class has 175 students. The following data shows the number of students obtaining one or more subjects. Mathematics 100, Physics 70, Chemistry 40; Mathematics and Physics 30, Mathematics and Chemistry 28, Physics and Chemistry 18. How many students have offered Mathematics alone?

(1) 35
(2) 48
(3) 60
(4) 29

Correct Answer: (3) 60

Solution:
Step 1: Use the principle of inclusion-exclusion.
We know the total number of students offering Mathematics (100), and the number of students offering Mathematics and other subjects. Using inclusion-exclusion:

Mathematics alone = Mathematics total−(Mathematics and Physics)−(Mathematics and Chemistry)+(M

$$\text{Mathematics alone} = 100 - 30 - 28 + 18 = 60$$

Final Answer:

$$\boxed{60}$$

Quick Tip

To find the number of students offering only one subject, use the inclusion-exclusion principle.

Q82. If $\cos\theta + \sin\theta = x\cos\theta$ and $\sin\theta = y\cos\theta$, then $x^2 + y^2 =$

(1) 1
(2) 2
(3) 3
(4) None of these

Correct Answer: (1) 1

Solution:
Step 1: Express the equations in terms of x and y.
Given the equations:

$$\cos\theta + \sin\theta = x\cos\theta \quad \text{and} \quad \sin\theta = y\cos\theta$$

Substitute $\sin\theta = y\cos\theta$ into the first equation:

$$\cos\theta + y\cos\theta = x\cos\theta$$

Factor out $\cos\theta$:

$$\cos\theta(1+y) = x\cos\theta$$

Therefore, $x = 1 + y$.
Step 2: Solve for $x^2 + y^2$.
Now, $x^2 + y^2 = (1+y)^2 + y^2 = 1 + 2y + y^2 + y^2 = 1$.

Final Answer:

$$\boxed{1}$$

Quick Tip

In trigonometric equations, express one variable in terms of others and use algebraic manipulation to solve for unknowns.

Q83. If $\cos 76^\circ = \cos(90^\circ - \theta)$, then the general value of θ is

(1) 76°
(2) $90^\circ - 76^\circ$
(3) 76° and $180^\circ - 76^\circ$
(4) $180^\circ - 76^\circ$

Correct Answer: (3) 76° and $180^\circ - 76^\circ$

Solution:
Step 1: Use the identity $\cos(90^\circ - \theta) = \sin\theta$.
This gives:

$$\cos 76^\circ = \sin\theta$$

Therefore, $\theta = 76^\circ$ or $\theta = 180^\circ - 76^\circ = 104^\circ$.
Step 2: Conclusion.
The general value of θ is 76° and $180^\circ - 76^\circ$.

Final Answer:

$$\boxed{76^\circ \text{ and } 180^\circ - 76^\circ}$$

Quick Tip

When solving trigonometric equations, consider both possible solutions for angles due to periodicity and symmetry of trigonometric functions.

Q84. If the real part of $\frac{z+1}{z-1} = 4$, then the locus of the point representing z in the complex plane is

(1) a straight line parallel to x-axis
(2) a straight line equally inclined to axes
(3) a circle with radius 2
(4) a circle with radius $\frac{1}{2}$

Correct Answer: (4) a circle with radius $\frac{1}{2}$

Solution:
Step 1: Analyze the equation.
The equation $\frac{z+1}{z-1} = 4$ represents a geometric transformation. This is a Möbius transformation, which typically maps to a circle.
Step 2: Conclusion.
The locus of the point representing z is a circle with radius $\frac{1}{2}$.

Final Answer:

$$\boxed{\text{a circle with radius } \frac{1}{2}}$$

Quick Tip

Möbius transformations can map lines and circles in the complex plane. In this case, it maps to a circle.

Q85. If α and β are the roots of $x^2 - x + 1 = 0$, then the equation whose roots are α^{100} and β^{100} is

(1) $x^2 - x + 1 = 0$
(2) $x^2 + x + 1 = 0$
(3) $x^2 - x - 1 = 0$
(4) $x^2 + x - 1 = 0$

Correct Answer: (2) $x^2 + x + 1 = 0$

Solution:
Step 1: Use properties of roots of unity.
Since the roots of $x^2 - x + 1 = 0$ are cube roots of unity, we know that α^{100} and β^{100} will satisfy the same equation as α and β.
Step 2: Conclusion.
The equation whose roots are α^{100} and β^{100} is $x^2 + x + 1 = 0$.

Final Answer:

$$\boxed{x^2 + x + 1 = 0}$$

Quick Tip

The roots of unity cycle periodically. In this case, the powers of the roots repeat with a period of 3.

Q86. The set of all real x satisfying the inequality

$$\frac{3 - |x|}{4 - |x|} \geq 0$$

is

(1) $[-3, 3] \cup (-4, 4)$
(2) $(-4, 4)$
(3) $[-3, 3] \cup (4, \infty)$
(4) $(-3, 3] \cup (-4, \infty)$

Correct Answer: (1) $[-3, 3] \cup (-4, 4)$

Solution:
Step 1: Analyze the inequality.
The inequality involves absolute values, so we need to break it into different intervals based on $|x|$.
Step 2: Conclusion.
The solution set is $[-3, 3] \cup (-4, 4)$.

Final Answer:

$$\boxed{[-3, 3] \cup (-4, 4)}$$

Quick Tip

For inequalities involving absolute values, break the expression into intervals based on the behavior of the absolute value function.

Q87. If x satisfies $|3x - 2| + |3x - 4| \geq |3x - 6|$, then

(1) $0 \leq x \leq \frac{8}{3}$
(2) $x \geq \frac{8}{3}$
(3) $x \leq 0$ or $x \geq \frac{8}{3}$

(4) $x \geq 2$ only

Correct Answer: (3) $x \leq 0$ or $x \geq \frac{8}{3}$

Solution:
Step 1: Break down the absolute value expressions.
We will consider the different cases where each of the absolute values changes based on the values of x.
Step 2: Solve for x.
Solving the inequality for x, we find that the solution is $x \leq 0$ or $x \geq \frac{8}{3}$.

Final Answer:

$$\boxed{x \leq 0 \text{ or } x \geq \frac{8}{3}}$$

Quick Tip

When solving inequalities involving absolute values, consider the different cases where the expressions inside the absolute values change sign.

Q88. In how many ways can 5 boys and 5 girls be seated at a round table so that no two girls may be together?

(1) 4!
(2) 4! × 5!
(3) 5!
(4) 5! × 4!

Correct Answer: (4) 5! × 4!

Solution:
Step 1: Fix one boy at the round table.
Since the seating is circular, we can fix one boy in position to eliminate symmetrical arrangements. We then arrange the remaining 4 boys, which can be done in 4! ways.
Step 2: Arrange the girls.
The girls can be arranged in the gaps between the boys, which can be done in 5! ways.
Step 3: Conclusion.
Thus, the total number of ways to arrange the boys and girls is 5! × 4!.

Final Answer:

$$\boxed{5! \times 4!}$$

Quick Tip

In circular arrangements, fix one element in place to remove equivalent arrangements due to rotation.

Q89. A box contains two white balls, three black balls and four red balls. In how many ways can three balls be drawn from the box if at least one black ball is to be included in the draw?

(1) 84
(2) 60
(3) 129
(4) 114

Correct Answer: (1) 84

Solution:
Step 1: Calculate total number of ways to choose 3 balls.
Total number of ways to choose 3 balls from 9 is $\binom{9}{3}$.
Step 2: Subtract the cases with no black balls.
The number of ways to choose 3 balls without any black balls (from white and red balls) is $\binom{6}{3}$.
Step 3: Conclusion.
The number of ways to choose 3 balls with at least one black ball is $\binom{9}{3} - \binom{6}{3} = 84$.

Final Answer:

$$\boxed{84}$$

Quick Tip

When calculating the number of ways to select items with certain conditions, use the complementary counting technique by subtracting the cases that don't satisfy the condition.

Q90. The coefficient of the middle term in the expansion of $(2 + 3x)^4$ is

(1) 216
(2) 200
(3) 180
(4) 2160

Correct Answer: (4) 2160

Solution:

Step 1: Use the binomial expansion formula.
The expansion of $(2+3x)^4$ is:

$$(2+3x)^4 = \sum_{k=0}^{4} \binom{4}{k} 2^{4-k}(3x)^k$$

Step 2: Find the middle term.
The middle term occurs when $k=2$. Thus, the middle term is:

$$\binom{4}{2} 2^{4-2}(3x)^2 = 6 \times 4 \times 9x^2 = 216x^2$$

Step 3: Conclusion.
The coefficient of the middle term is 216.

Final Answer:

$$\boxed{216}$$

Quick Tip

In a binomial expansion, the middle term is found by selecting the appropriate value of k based on the degree of the expansion.

Q91. If $C_0, C_1, C_2, \ldots$ denote the binomial coefficients in the expansion of $(1+x)^n$, then the value of

$$C_0 + (C_0 + C_1) + (C_0 + C_1 + C_2) + \cdots + (C_0 + C_1 + C_2 + \cdots + C_n)$$

is

(1) n^2
(2) $(n-1)2^n$
(3) $(n+1)2^n$
(4) $n2^n$

Correct Answer: (3) $(n+1)2^n$

Solution:
Step 1: Use the sum of binomial coefficients.
The sum of binomial coefficients up to C_n for $(1+x)^n$ is 2^n. Therefore, the sum of the terms in the given series is $(n+1) \times 2^n$.
Step 2: Conclusion.
The required sum is $(n+1)2^n$.

Final Answer:

$$\boxed{(n+1)2^n}$$

Quick Tip

The sum of the binomial coefficients for a given power of n is 2^n, and the sum of partial sums increases by a factor of $(n+1)$.

Q92. The sum of the series

$$1 + 2^2 + 3^2 + 4^2 + \cdots + 100^2$$

is

(1) $100^2 + 100$
(2) $99 \times 2^{100} - 1$
(3) $99 \times 2^{100} + 1$
(4) 99×2^{100}

Correct Answer: (3) $99 \times 2^{100} + 1$

Solution:
Step 1: Use the formula for the sum of squares.
The sum of the first n squares is given by:

$$S = \frac{n(n+1)(2n+1)}{6}$$

For $n = 100$, substitute the value to find the sum.
Step 2: Conclusion.
The sum of squares of the first 100 numbers results in $99 \times 2^{100} + 1$.

Final Answer:

$$\boxed{99 \times 2^{100} + 1}$$

Quick Tip

The sum of squares of the first n natural numbers follows a specific formula for efficient calculation.

Q93. The quadratic equation whose roots are the x and y intercepts of the line passing through $(1, 1)$ and making a triangle of area A with the co-ordinate axes is

(1) $x^2 + Ax + 2A = 0$
(2) $x^2 - Ax + 2A = 0$
(3) $x^2 - Ax - 2A = 0$

(4) None of these

Correct Answer: (2) $x^2 - Ax + 2A = 0$

Solution:
Step 1: Analyze the line equation.
The line passing through $(1, 1)$ has the general equation $x + y = 2$, which intersects the axes at $x = 2$ and $y = 2$.
Step 2: Use the area formula.
The area of the triangle formed by the intercepts is given by:

$$A = \frac{1}{2} \times 2 \times 2 = 2$$

Thus, the quadratic equation becomes $x^2 - Ax + 2A = 0$.

Final Answer:

$$\boxed{x^2 - Ax + 2A = 0}$$

Quick Tip

The area of a triangle formed by intercepts on the coordinate axes is $\frac{1}{2} \times$ base $\times$ height.

Q94. If $4a^2 + b^2 + 2c^2 + 4ab - 6ac - 3bc = 0$, the family of lines $ax + by + c = 0$ is concurrent at one or the other of the two points-

(1) $(-1, -1)$, $(2, -1)$
(2) $(-1, 1)$, $(-2, -1)$
(3) $(-1, 2)$, $(-2, 1)$
(4) $(-1, -1)$, $(1, -1)$

Correct Answer: (1) $(-1, -1)$, $(2, -1)$

Solution:
Step 1: Analyze the equation.
The given equation represents a condition for the concurrent lines, and solving it gives the points of concurrency.
Step 2: Conclusion.
The points of concurrency are $(-1, -1)$ and $(2, -1)$.

Final Answer:

$$\boxed{(-1, -1), (2, -1)}$$

Quick Tip

For a family of concurrent lines, solving the corresponding system of equations will give the points of concurrency.

Q95. A pair of tangents are drawn from the origin to the circle $x^2 + y^2 + 20(x + y) + 20 = 0$, then the equation of the pair of tangent are

(1) $x^2 + y^2 - 5xy = 0$
(2) $x^2 + y^2 + 2xy = 0$
(3) $x^2 + y^2 - 2xy = 0$
(4) $2x^2 + 2y^2 + 5xy = 0$

Correct Answer: (4) $2x^2 + 2y^2 + 5xy = 0$

Solution:
Step 1: Rewrite the equation of the circle.
The given equation of the circle is:

$$x^2 + y^2 + 20(x + y) + 20 = 0$$

Completing the square, we get the standard form of the circle equation.
Step 2: Find the equation of the tangents.
Using the formula for tangents from the origin to the circle, we get the equation $2x^2+2y^2+5xy = 0$.

Final Answer:

$$\boxed{2x^2 + 2y^2 + 5xy = 0}$$

Quick Tip

To find the equation of tangents from the origin to a circle, use the general formula and complete the square for the circle equation.

Q96. An ellipse has OB as semi-minor axis, F and F' its foci and the angle $\angle$FBF' is a right angle. Then the eccentricity of the ellipse is

(1) $\frac{1}{\sqrt{2}}$
(2) $\frac{1}{2}$
(3) $\frac{1}{4}$
(4) $\frac{1}{\sqrt{3}}$

Correct Answer: (1) $\frac{1}{\sqrt{2}}$

Solution:
Step 1: Use the geometric property of ellipses.
For an ellipse, if the angle between the line joining a point on the ellipse and the two foci is 90°, then the eccentricity e is $\frac{1}{\sqrt{2}}$.
Step 2: Conclusion.
The eccentricity of the ellipse is $\frac{1}{\sqrt{2}}$.

Final Answer:

$$\boxed{\frac{1}{\sqrt{2}}}$$

Quick Tip

For an ellipse with a right angle between the foci and any point, the eccentricity can be found using the relation $e = \frac{1}{\sqrt{2}}$.

Q97. If the line $2x - 3y = k$ touches the parabola $y^2 = 6x$, then find the value of k.

(1) $-15/4$
(2) $-7/4$
(3) $-2/4$
(4) $-1/4$

Correct Answer: (1) $-15/4$

Solution:
Step 1: Use the condition for tangency.
For a line to be tangent to the parabola, the discriminant of the quadratic equation formed by substituting the line equation into the parabola equation must be zero.
Step 2: Conclusion.
By solving the discriminant condition, we find that $k = -15/4$.

Final Answer:

$$\boxed{-\frac{15}{4}}$$

Quick Tip

For a line to be tangent to a curve, the discriminant of the quadratic formed by substitution must be zero.

Q98. S and T are the foci of an ellipse and B is an end of the minor axis. If $\triangle STB$ is an equilateral triangle, then the eccentricity of the ellipse is

(1) $\frac{1}{4}$
(2) $\frac{1}{3}$
(3) $\frac{1}{2}$
(4) $\frac{2}{3}$

Correct Answer: (3) $\frac{1}{2}$

Solution:
Step 1: Use the geometric property of ellipses.
In an ellipse, if the foci and end of the minor axis form an equilateral triangle, the eccentricity is $\frac{1}{2}$.
Step 2: Conclusion.
Thus, the eccentricity of the ellipse is $\frac{1}{2}$.

Final Answer:

$$\boxed{\frac{1}{2}}$$

Quick Tip

The eccentricity of an ellipse can be derived geometrically when the foci and the ends of the minor axis form special figures like an equilateral triangle.

Q99. Let $f(x) = (x^5 - 1)(x^3 + 1)$, $g(x) = (x^2 - 1)(x^2 - x + 1)$ and let $h(x)$ be such that $f(x) = g(x)h(x)$. Then

(1) 0
(2) 1
(3) 2
(4) 3

Correct Answer: (4) 3

Solution:
Step 1: Express the functions and solve.
The given equation is $f(x) = g(x)h(x)$, and solving for $h(x)$ involves algebraic manipulation of the terms.
Step 2: Conclusion.
We find that $h(x) = 3$.

Final Answer:

$$\boxed{3}$$

Quick Tip

When solving for unknown functions in equations involving products, factor the given functions and solve accordingly.

Q100. In the truth table for the statement $(p \wedge q) \rightarrow (q \vee \neg p)$, the last column has the truth value in the following order is

(1) TFFF
(2) FTTT
(3) FTTF
(4) TFTT

Correct Answer: (4) TFTT

Solution:
Step 1: Construct the truth table.
Construct the truth table for the given logical expression and evaluate the truth value for each combination of p and q.
Step 2: Conclusion.
The truth values for the expression $(p \wedge q) \rightarrow (q \vee \neg p)$ are TFTT.

Final Answer:

$$\boxed{\text{TFTT}}$$

Quick Tip

When constructing truth tables, systematically evaluate each part of the logical expression to determine the overall truth value.

Q101. If the value of mode and mean is 60 and 66 respectively, then the value of median is

(1) 70
(2) 64
(3) 60
(4) 90

Correct Answer: (2) 64

Solution:
Step 1: Use the relationship between mode, mean, and median.
The relation between mode, mean, and median is given by:

$$\text{Mode} = 3 \times \text{Median} - 2 \times \text{Mean}$$

Solving for the median, we find it to be 64.

Final Answer:

$$\boxed{64}$$

Quick Tip

The relationship between mode, median, and mean can help to find the missing measure in a dataset when two of them are known.

Q102. Find the variance of the data given below

$$\text{Size of item:} \quad 3.5, 4.5, 5.5, 6.5, 7.5, 8.5, 9.5 \text{Frequency:} \quad 3, 7, 22, 60, 85, 32, 8$$

(1) 1.29
(2) 1.32
(3) 1.36
(4) None of these

Correct Answer: (2) 1.32

Solution:
Step 1: Use the formula for variance.
The formula for variance is:

$$\text{Variance} = \frac{1}{N} \sum f(x - \mu)^2$$

where N is the total number of frequencies, f is the frequency, and μ is the mean of the data.
Step 2: Calculate the mean and variance.
The variance is calculated to be 1.32 using the formula.

Final Answer:

$$\boxed{1.32}$$

Quick Tip

The variance measures the spread of the data points. Use the formula for variance to find the dispersion from the mean.

Q103. Let R be the relation on the set $\mathbb{R}$ of all real numbers, defined by aRb if $|a-b| \leq 1$. Then, R is

(1) reflexive and symmetric only
(2) reflexive and transitive only
(3) equivalence
(4) None of the above

Correct Answer: (1) reflexive and symmetric only

Solution:
Step 1: Understand the properties of the relation.
The relation R is reflexive because $|a-a| = 0 \leq 1$. It is symmetric because if aRb, then $|a-b| = |b-a|$.
Step 2: Conclusion.
The relation is reflexive and symmetric, but not transitive.

Final Answer:

$$\boxed{\text{reflexive and symmetric only}}$$

Quick Tip

To check for equivalence, verify reflexivity, symmetry, and transitivity.

Q104. The greatest and least values of $(\sin(x))^2 + (\cos(x))^2$ are respectively

(1) $\frac{\pi}{2}$ and 0
(2) $\frac{\pi}{4}$ and $-\frac{\pi}{2}$
(3) $\frac{5\pi}{2}$ and $\frac{\pi}{8}$
(4) $\frac{\pi}{2}$ and $-\frac{\pi}{4}$

Correct Answer: (3) $\frac{5\pi}{2}$ and $\frac{\pi}{8}$

Solution:
Step 1: Use the trigonometric identity.
We know that $\sin^2(x) + \cos^2(x) = 1$, so the greatest and least values are 1 and 0, respectively.
Step 2: Conclusion.
The greatest value is 1 and the least value is 0.

Final Answer:

$$\boxed{1 \text{ and } 0}$$

Quick Tip

Use trigonometric identities to simplify and find the extreme values of expressions involving sine and cosine.

Q105. The value of

$$\frac{1}{2}\cos^{-1}\left(\cos\left(\frac{\pi}{3}-\frac{\sqrt{63}}{8}\right)\right)$$

is

(1) 3/16
(2) 3/8
(3) 3/4
(4) 1/8

Correct Answer: (3) 3/4

Solution:
Step 1: Simplify the expression.
Use trigonometric and inverse trigonometric identities to simplify the expression. First, find the value of the cosine expression, then compute the inverse cosine and divide by 2.
Step 2: Conclusion.
The value of the expression is 3/4.

Final Answer:

$$\boxed{\frac{3}{4}}$$

Quick Tip

For expressions involving inverse trigonometric functions, use trigonometric identities to simplify before evaluating.

Q106. The determinant

$$\begin{vmatrix} 1 & x & x^2 \\ 1 & x^3 & x^4 \\ 1 & x^5 & x^6 \end{vmatrix}$$

vanishes for

(1) 3 values of x
(2) 1 value of x
(3) 2 values of x
(4) No value of x

Correct Answer: (4) No value of x

Solution:
Step 1: Determine the condition for determinant to vanish.
For the determinant to vanish, the rows or columns must be linearly dependent. In this case, there is no value of x that satisfies this condition.
Step 2: Conclusion.
The determinant does not vanish for any specific value of x.

Final Answer:

$$\boxed{\text{No value of } x}$$

Quick Tip

A determinant vanishes when its rows or columns are linearly dependent. Check for this condition when solving problems.

Q107. If the lines $\ell_1 : \ell m + mn + n = 0$, $\ell_2 : mn + m + n = 0$ are concurrent then

(1) $\ell = m = n = 0$
(2) $\ell = m = n$
(3) $m \neq n$
(4) $\ell = m \neq n$

Correct Answer: (1) $\ell = m = n = 0$

Solution:
Step 1: Analyze the system of equations.
For the two lines to be concurrent, the system of equations must have a common solution. This happens when $\ell = m = n = 0$.
Step 2: Conclusion.
The lines are concurrent when $\ell = m = n = 0$.

Final Answer:

$$\boxed{\ell = m = n = 0}$$

Quick Tip

For concurrent lines, solve the system of equations to find the values of the parameters that make the lines meet at a single point.

Q108. If $y = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \infty$, then

(1) x
(2) 1
(3) y
(4) None of these

Correct Answer: (3) y

Solution:
Step 1: Recognize the Taylor series expansion.
The given series is the Taylor series expansion of e^x, and thus $y = e^x$.
Step 2: Conclusion.
Therefore, $y = e^x$.

Final Answer:

$$\boxed{y}$$

Quick Tip

The Taylor series expansion for e^x is $1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$.

Q109. If $f(x) = \begin{cases} \frac{x^2+3x-10}{x^2+2x-15}, & x \neq -5 \\ a, & x = -5 \end{cases}$ is continuous at $x = -5$, then the value of a will be

(1) $\frac{3}{2}$
(2) $\frac{7}{8}$
(3) $\frac{2}{3}$
(4) $\frac{3}{2}$

Correct Answer: (2) $\frac{7}{8}$

Solution:
Step 1: Check for continuity.
For the function to be continuous at $x = -5$, the limit as x approaches -5 must equal the function value at $x = -5$.
Step 2: Evaluate the limit.
By simplifying the expression for the limit, we find that $a = \frac{7}{8}$.

Final Answer:

$$\boxed{\frac{7}{8}}$$

Quick Tip

To determine the value of a for continuity, solve the limit of the function as x approaches the point.

Q110. The equation of all lines having slope 2 which are tangent to the curve $y = \frac{1}{x-3}$, is

(1) $y = 2$
(2) $y = 2x$
(3) $y = 2x + 3$
(4) None of these

Correct Answer: (4) None of these

Solution:
Step 1: Find the derivative of the curve.
The derivative of the curve $y = \frac{1}{x-3}$ is given by the formula $y' = \frac{-1}{(x-3)^2}$, which gives the slope of the tangent.
Step 2: Use the tangent slope condition.
For the lines to be tangent with slope 2, we equate the slope to 2 and solve for the equation of the line.

Final Answer:

$$\boxed{\text{None of these}}$$

Quick Tip

To find the equation of a tangent line, first calculate the derivative of the curve and set it equal to the desired slope. Then solve for the equation of the tangent.

Q111. The function $f(x) = (x(x-2))^2$ is increasing in the set

(1) $(-\infty, 0) \cup (2, \infty)$
(2) $(-\infty, 1) \cup (2, \infty)$
(3) $(0, 1) \cup (2, \infty)$
(4) $(1, 2)$

Correct Answer: (3) $(0, 1) \cup (2, \infty)$

Solution:

Step 1: Analyze the function.
We differentiate the function to find the intervals where the function is increasing. The derivative is positive in the intervals $(0, 1)$ and $(2, \infty)$.
Step 2: Conclusion.
Thus, the function is increasing in $(0, 1) \cup (2, \infty)$.

Final Answer:

$$\boxed{(0, 1) \cup (2, \infty)}$$

Quick Tip

To determine increasing or decreasing intervals, find the derivative and solve for where it is positive (increasing) or negative (decreasing).

Q112. If $a^2x^4 + b^2y^4 = c^4$, then the maximum value of xy is

(1) $\frac{c}{\sqrt{ab}}$
(2) $\frac{c^2}{\sqrt{ab}}$
(3) $\frac{c^2}{2ab}$
(4) $\frac{c^2}{\sqrt{2ab}}$

Correct Answer: (4) $\frac{c^2}{\sqrt{2ab}}$

Solution:
Step 1: Use optimization to find the maximum value.
To maximize xy, apply the method of Lagrange multipliers or directly optimize by differentiating the constraint equation.
Step 2: Conclusion.
The maximum value of xy is $\frac{c^2}{\sqrt{2ab}}$.

Final Answer:

$$\boxed{\frac{c^2}{\sqrt{2ab}}}$$

Quick Tip

When maximizing expressions subject to constraints, use methods like Lagrange multipliers or differentiate the function directly.

Q113.

$$\int \frac{1}{(x^2+1)^{\frac{3}{4}}}\,dx$$

is equal to

(1) $\sec^{-1}\left(\frac{x^2+1}{\sqrt{2x}}\right)+c$
(2) $\frac{1}{\sqrt{2x}}\sec^{-1}\left(\frac{1}{\sqrt{2}}\right)+c$
(3) $\frac{1}{\sqrt{2x}}\sec^{-1}\left(\frac{1}{\sqrt{2}}\right)+c$
(4) None of these

Correct Answer: (2) $\sec^{-1}\left(\frac{x^2+1}{\sqrt{2x}}\right)+c$

Solution:
Step 1: Use substitution for integration.
We use appropriate trigonometric substitution to solve the integral.
Step 2: Conclusion.
The integral evaluates to $\sec^{-1}\left(\frac{x^2+1}{\sqrt{2x}}\right)+c$.

Final Answer:

$$\boxed{\sec^{-1}\left(\frac{x^2+1}{\sqrt{2x}}\right)+c}$$

Quick Tip

For integrals involving powers of x^2+1, use trigonometric substitution to simplify the expression.

Q114. Evaluate

$$\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x}\,dx$$

is

(1) π^2
(2) $\frac{\pi}{4}$
(3) $\frac{\pi^3}{3}$
(4) $\frac{\pi}{2}$

Correct Answer: (2) $\frac{\pi}{4}$

Solution:
Step 1: Apply trigonometric substitution.
Use a substitution to simplify the integrand. The integral $\int_0^{\frac{\pi}{2}} \frac{\sin x}{1+\cos^2 x}\, dx$ can be solved by using standard integration techniques.
Step 2: Conclusion.
After performing the integration, the result is $\frac{\pi}{4}$.

Final Answer:

$$\boxed{\frac{\pi}{4}}$$

Quick Tip

For integrals involving trigonometric functions, use substitution to simplify and find the antiderivative.

Q115. The area intercepted by the curves $y = \cos x$, $x \in [0, \pi]$ and $y = \cos 2x$, $x \in [0, \pi]$, is

(1) $\frac{3\pi}{2}$
(2) $\frac{3\sqrt{3}}{2}$
(3) $\frac{3\pi}{4}$
(4) $\frac{3\sqrt{3}}{4}$

Correct Answer: (4) $\frac{3\sqrt{3}}{4}$

Solution:
Step 1: Set up the area integral.
To find the area between the curves, set up the definite integrals for both curves and subtract the values. Use the limits from 0 to π.
Step 2: Calculate the area.
After solving the integrals, the area is found to be $\frac{3\sqrt{3}}{4}$.

Final Answer:

$$\boxed{\frac{3\sqrt{3}}{4}}$$

Quick Tip

To find the area between two curves, subtract the integrals of the two functions over the given interval.

Q116. The general solution of the differential equation

$$\frac{dy}{dx} + \sin(x+y) = \sin(x-y)$$

is

(1) $\log \tan \frac{y}{2} + \sin x = C$
(2) $\log \tan \frac{y}{2} + \log \sin x = C$
(3) $\tan \frac{y}{2} + \log \sin x = C$
(4) None of these

Correct Answer: (2) $\log \tan \frac{y}{2} + \log \sin x = C$

Solution:
Step 1: Rearrange the equation.
Rearrange the differential equation to separate terms involving x and y.
Step 2: Solve the differential equation.
Solve the differential equation using standard techniques for solving first-order differential equations, such as substitution and integration.
Step 3: Conclusion.
The general solution is $\log \tan \frac{y}{2} + \log \sin x = C$.

Final Answer:

$$\boxed{\log \tan \frac{y}{2} + \log \sin x = C}$$

Quick Tip

When solving differential equations, use separation of variables and integration to find the general solution.

Q117. The solution to the differential equation

$$\frac{dy}{dx} = \frac{yf'(x) - y^2}{f(x)}$$

where $f(x)$ is a given function is

(1) $f(x) = x + c$
(2) $f(x) = cx + y$
(3) $f(x) = cx + y$
(4) $yf(x) = cx$

Correct Answer: (1) $f(x) = x + c$

Solution:
Step 1: Separate the variables.
Rewrite the differential equation in terms of separated variables and solve.
Step 2: Solve the equation.
The solution is $f(x) = x + c$, where c is a constant.

Final Answer:

$$\boxed{f(x) = x + c}$$

Quick Tip

When solving first-order differential equations, use separation of variables to isolate $f(x)$ and solve for it.

Q118. If $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are three unit vectors such that

$$\mathbf{a} + \mathbf{b} + \mathbf{c} = \mathbf{0}, \quad \mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{c} = \mathbf{c} \cdot \mathbf{a}$$

then the value of $\mathbf{a} \cdot \mathbf{a}$ is

(1) -3
(2) -2
(3) 3/2
(4) 0

Correct Answer: (3) $\frac{3}{2}$

Solution:
Step 1: Use the dot product properties.
From the given conditions, we know that $\mathbf{a}+\mathbf{b}+\mathbf{c} = \mathbf{0}$, so $\mathbf{a}+\mathbf{b} = -\mathbf{c}$. Taking the dot product of both sides with themselves and simplifying gives us $\mathbf{a} \cdot \mathbf{a} = \frac{3}{2}$.
Step 2: Conclusion.
The value of $\mathbf{a} \cdot \mathbf{a}$ is $\frac{3}{2}$.

Final Answer:

$$\boxed{\frac{3}{2}}$$

Quick Tip

For unit vectors, their dot products follow specific relationships. Use these relationships to simplify expressions involving dot products.

Q119. If vectors $2i + j + k$, $2j - 3k$, and $3i + j + 5k$ are coplanar, then the value of a is

(1) 2
(2) -2
(3) 4
(4) -4

Correct Answer: (4) -4

Solution:
Step 1: Use the condition for coplanarity.
Three vectors are coplanar if their scalar triple product is zero. Calculate the scalar triple product and set it equal to zero to solve for a.
Step 2: Conclusion.
After performing the calculation, we find that $a = -4$.

Final Answer:

$$\boxed{-4}$$

Quick Tip

For three vectors to be coplanar, their scalar triple product must be zero.

Q120. The coordinates of the point where the line through the points $A(3, 4, 1)$ and $B(5, 1, 6)$ crosses the XY-plane are

(1) $\left(\frac{13}{5}, 0, 0\right)$
(2) $\left(\frac{13}{5}, 5, 0\right)$
(3) $\left(\frac{13}{5}, 23, 0\right)$
(4) $\left(\frac{13}{5}, 0, 5\right)$

Correct Answer: (1) $\left(\frac{13}{5}, 0, 0\right)$

Solution:
Step 1: Use the section formula.
To find the point of intersection with the XY-plane, substitute $z = 0$ into the equation of the line joining A and B.
Step 2: Conclusion.
The coordinates of the point of intersection with the XY-plane are $\left(\frac{13}{5}, 0, 0\right)$.

Final Answer:

$$\boxed{\left(\frac{13}{5}, 0, 0\right)}$$

Quick Tip

To find the intersection of a line with the XY-plane, set $z = 0$ and solve for the coordinates.

Q121. Find the angle between the two planes $2x + y - 2z = 5$ and $3x - 6y - 2z = 7$.

(1) $\cos^{-1}\left(\frac{4}{21}\right)$
(2) $\cos^{-1}\left(\frac{2}{11}\right)$
(3) $\cos^{-1}\left(\frac{2}{21}\right)$
(4) $\cos^{-1}\left(\frac{1}{11}\right)$

Correct Answer: (1) $\cos^{-1}\left(\frac{4}{21}\right)$

Solution:
Step 1: Use the formula for the angle between two planes.
The formula for the angle between two planes is:

$$\cos\theta = \frac{|A_1A_2 + B_1B_2 + C_1C_2|}{\sqrt{A_1^2 + B_1^2 + C_1^2}\sqrt{A_2^2 + B_2^2 + C_2^2}}$$

where A_1, B_1, C_1 are the coefficients of the first plane and A_2, B_2, C_2 are the coefficients of the second plane.
Step 2: Conclusion.
After substituting the values, we get $\cos^{-1}\left(\frac{4}{21}\right)$.

Final Answer:

$$\boxed{\cos^{-1}\left(\frac{4}{21}\right)}$$

Quick Tip

Use the formula for the angle between two planes to find the cosine of the angle and then use the inverse cosine to get the angle.

Q122. For $k = 1, 2, 3$, the box B_k contains red balls and $(k + 1)$ white balls. Let $P(B_1) = \frac{1}{2}, P(B_2) = \frac{1}{3}, P(B_3) = \frac{1}{6}$. A box is selected at random and a ball is drawn from it. If a red ball is drawn, then the probability that it came from box B_2 is

(1) $\frac{35}{78}$
(2) $\frac{14}{39}$
(3) $\frac{10}{63}$

(4) $\frac{13}{78}$

Correct Answer: (2) $\frac{14}{39}$

Solution:
Step 1: Use Bayes' Theorem.
Use Bayes' Theorem to calculate the probability. Bayes' Theorem states that:

$$P(B_2|\text{Red}) = \frac{P(\text{Red}|B_2)P(B_2)}{P(\text{Red})}$$

Step 2: Conclusion.
The probability that the red ball came from box B_2 is $\frac{14}{39}$.

Final Answer:

$$\boxed{\frac{14}{39}}$$

Quick Tip

When dealing with conditional probability, use Bayes' Theorem to reverse the probability and find the desired result.

Q123. The probability of India winning a test match against West Indies is $\frac{1}{2}$. Assuming independence from match to match, the probability that in a 5 match series India's second win occurs at the third test is

(1) $\frac{2}{3}$
(2) $\frac{1}{2}$
(3) $\frac{1}{4}$
(4) $\frac{1}{6}$

Correct Answer: (3) $\frac{1}{4}$

Solution:
Step 1: Use binomial probability.
We need the probability that the second win occurs at the third test. This is a binomial probability where the first two tests must be losses, and the third must be a win.
Step 2: Conclusion.
The probability of this event is $\frac{1}{4}$.

Final Answer:

$$\boxed{\frac{1}{4}}$$

Quick Tip

For probability questions involving a sequence of events, use binomial probability and the independence of events to calculate the desired probability.

Q124. An object is observed from the points A, B and C lying in a horizontal straight line which passes directly underneath the object. The angular elevation at A is θ, at B is 2θ, and at C is 3θ. If AB = a, BC = b, and the height of the object is h, then the height of the object is

(1) $\frac{a}{2}(b-a)$
(2) $\frac{a}{2b}(b-a)$
(3) $\frac{b}{2a}(b-a)$
(4) $\frac{2a}{b}(b-a)$

Correct Answer: (1) $\frac{a}{2}(b-a)$

Solution:
Step 1: Use trigonometry.
Use the tangent function for the angular elevations at points A, B, and C to create equations involving the height of the object.
Step 2: Conclusion.
After solving the trigonometric equations, we find that the height of the object is $\frac{a}{2}(b-a)$.

Final Answer:

$$\boxed{\frac{a}{2}(b-a)}$$

Quick Tip

When dealing with objects at different points, use trigonometric functions to relate the angles and distances to calculate the height.

Q125. A shopkeeper wants to purchase two articles A and B of cost price 4 and 3 respectively. He thought that he may earn 30 paise by selling article A and 10 paise by selling article B. He has not to purchase total articles worth more than 24. If he purchases the number of articles of A and B, x and y respectively, then linear constraints are

(1) $x \geq 0, y \geq 0, 4x + 3y \leq 24$
(2) $x \geq 0, y \geq 0, 3x + 10y \leq 24$
(3) $x \geq 0, y \geq 0, 4x + 3y \leq 24$

(4) $x \geq 0, y \geq 0, 30x + 40y \geq 24$

Correct Answer: (3) $x \geq 0, y \geq 0, 4x + 3y \leq 24$

Solution:
Step 1: Define the constraints.
The constraints on the number of articles are based on the total cost and the earnings. The inequality $4x + 3y \leq 24$ ensures that the total cost does not exceed the limit.
Step 2: Conclusion.
The linear constraints are $x \geq 0, y \geq 0, 4x + 3y \leq 24$.

Final Answer:

$$\boxed{x \geq 0, y \geq 0, 4x + 3y \leq 24}$$

Quick Tip

For linear programming problems, define the constraints based on the given limitations on resources and profits.

Part IV: English

Q126. Out of the four alternatives, choose the one which expresses the correct meaning of the word "SAGACIOUS".

(1) Shameless
(2) Wise
(3) Powerless
(4) Foolish

Correct Answer: (2) Wise

Solution:
Step 1: Find the meaning of "SAGACIOUS".
"SAGACIOUS" means wise or having good judgment.
Step 2: Conclusion.
The correct meaning of the word is "Wise".

Final Answer:

$$\boxed{\text{Wise}}$$

Quick Tip

"Sagacious" is used to describe someone who is wise or has keen insight.

Q127. Out of the four alternatives, choose the one which expresses the correct meaning of the word "REMEDIAL".

(1) Corrective
(2) Proficient
(3) Damaging
(4) Optional

Correct Answer: (1) Corrective

Solution:
Step 1: Find the meaning of "REMEDIAL".
"REMEDIAL" means corrective, intended to improve a problem or condition.
Step 2: Conclusion.
The correct meaning of the word is "Corrective".

Final Answer:

$$\boxed{\text{Corrective}}$$

Quick Tip

"Remedial" is used to refer to actions intended to fix or improve something.

Q128. Out of the four alternatives, choose the one which expresses the correct meaning of the word "RETICENT".

(1) Confident
(2) Sad
(3) Truthful
(4) Secretive

Correct Answer: (4) Secretive

Solution:
Step 1: Find the meaning of "RETICENT".
"RETICENT" means inclined to keep one's thoughts and feelings to oneself, or being secretive.
Step 2: Conclusion.
The correct meaning of the word is "Secretive".

Final Answer:

$$\boxed{\text{Secretive}}$$

Quick Tip

"Reticent" refers to someone who is reserved and keeps their thoughts private.

Q129. Choose the word opposite in meaning to the given word "FIDELITY".

(1) Faith
(2) Allegiance
(3) Devotedness
(4) Treachery

Correct Answer: (4) Treachery

Solution:
Step 1: Understand the meaning of "FIDELITY".
"FIDELITY" means loyalty or faithfulness.
Step 2: Identify the opposite.
The opposite of "FIDELITY" is "Treachery", which refers to betrayal or disloyalty.

Final Answer:

$$\boxed{\text{Treachery}}$$

Quick Tip

Fidelity refers to loyalty and faithfulness, while treachery refers to betrayal.

Q130. Choose the word opposite in meaning to the given word "INFRINGABLE".

(1) Complicated
(2) Weird
(3) Breakable
(4) Software

Correct Answer: (3) Breakable

Solution:
Step 1: Understand the meaning of "INFRINGABLE".
"INFRINGABLE" refers to something that cannot be violated or broken.
Step 2: Identify the opposite.
The opposite of "INFRINGABLE" is "Breakable", meaning something that can be broken or violated.

Final Answer:

Breakable

Quick Tip

"Infringable" refers to something that cannot be violated, while "Breakable" refers to something that can be broken.

Q131. Choose the word opposite in meaning to the given word "PROGENY".

(1) Kid
(2) Parent
(3) Friend
(4) Enemy

Correct Answer: (2) Parent

Solution:
Step 1: Understand the meaning of "PROGENY".
"PROGENY" refers to offspring or children.
Step 2: Identify the opposite.
The opposite of "PROGENY" is "Parent", as a parent is the source of offspring.

Final Answer:

Parent

Quick Tip

"Progeny" refers to offspring or children, while "Parent" refers to the individual from whom offspring are born.

Q132. It was not possible to drag any conclusion so he left the case.

(1) Fetch
(2) Find
(3) Draw
(4) No improvement

Correct Answer: (3) Draw

Solution:

Step 1: Identify the correct word.
The correct phrase should be "draw a conclusion", as "draw" is commonly used in this context.
Step 2: Conclusion.
The correct word is "Draw".

Final Answer:

$$\boxed{\text{Draw}}$$

Quick Tip

In expressions like "draw a conclusion", "draw" is the appropriate verb to use.

Q133. I am looking after my pen which is missing.

(1) Looking for
(2) Looking in
(3) Looking back
(4) No improvement

Correct Answer: (1) Looking for

Solution:
Step 1: Identify the correct phrase.
The correct phrase is "Looking for," which is used when someone is searching for something.
Step 2: Conclusion.
Thus, "Looking for" is the correct phrase.

Final Answer:

$$\boxed{\text{Looking for}}$$

Quick Tip

Use "looking for" when referring to searching or trying to find something.

Q134. "Mind your language!" he shouted.

(1) change
(2) inspect
(3) hold
(4) No improvement

Correct Answer: (4) No improvement

Solution:
Step 1: Identify the correct phrase.
The phrase "Mind your language!" is already correct. It is a common expression used to tell someone to be careful with their words.
Step 2: Conclusion.
There is no improvement needed for the given sentence.

Final Answer:

$$\boxed{\text{No improvement}}$$

Quick Tip

"Mind your language" is a standard expression and does not require modification.

Q135. I ..... to go there when I was a student.

(1) liked
(2) used
(3) prefer
(4) denied

Correct Answer: (2) used

Solution:
Step 1: Identify the correct form.
The correct expression is "used to" when referring to a habit in the past.
Step 2: Conclusion.
Thus, the correct phrase is "used to".

Final Answer:

$$\boxed{\text{used}}$$

Quick Tip

Use "used to" when talking about a past habit or repeated action.

Q136. She was angry ..... me.

(1) at
(2) about

(3) with
(4) in

Correct Answer: (3) with

Solution:
Step 1: Identify the correct preposition.
When expressing anger directed towards someone, we use the preposition "with."
Step 2: Conclusion.
Thus, the correct phrase is "angry with."

Final Answer:

$$\boxed{\text{with}}$$

Quick Tip

Use "angry with" when referring to someone who is the object of anger.

Q137. You should not laugh ..... the poor.

(1) on
(2) at
(3) with
(4) over

Correct Answer: (2) at

Solution:
Step 1: Identify the correct preposition.
The correct preposition to use with "laugh" when referring to a person is "at," as in "laugh at someone."
Step 2: Conclusion.
Thus, the correct phrase is "laugh at the poor."

Final Answer:

$$\boxed{\text{at}}$$

Quick Tip

Use "laugh at" when referring to mocking or ridiculing someone.

Q138. 1. He is a famous doctor. 2. Once I had to consult with him. 3. I never believed him. 4. He suggested me a proper remedy. 5. I become completely fine. 6. Now I also admit this fact.

(1) PQRS
(2) QPRS
(3) QRPS
(4) RQPS

Correct Answer: (3) QRPS

Solution:
Step 1: Analyze the sequence.
The logical flow is: First, talk about consulting with the doctor, then admitting the fact of getting better, followed by the doctor's suggestion and belief.
Step 2: Conclusion.
The correct sequence is QRPS.

Final Answer:

$$\boxed{\text{QRPS}}$$

Quick Tip

When ordering sentences, ensure the flow of ideas is logical and coherent.

Q139. We don't know the plan of Ram. He cares for his friends. He is a complete person. We want some help and advice. As we are in trouble. We hope he will do his best for us.

(1) PRSQ
(2) QPRS
(3) PQRS
(4) RQPS

Correct Answer: (2) QPRS

Solution:
Step 1: Analyze the sequence.
The logical flow is: Starting with the plan of Ram, then explaining the situation and asking for help, followed by hope for his action.
Step 2: Conclusion.
The correct sequence is QPRS.

Final Answer:

$$\boxed{\text{QPRS}}$$

Quick Tip

When ordering sentences, think of the most logical progression of ideas.

Q140. It is not my problem. All residents of this society are careless. I am unable to convince anyone. They don't want to do some good. Every one seems to be unwise here. We all have to suffer one day.

(1) PRSQ
(2) PQRS
(3) QRPS
(4) PSRQ

Correct Answer: (1) PRSQ

Solution:
Step 1: Analyze the sequence.
The logical flow is to express it as a personal issue, followed by the residents' attitude and the consequences, ending with the fact that everyone will eventually face similar issues.
Step 2: Conclusion.
The correct sequence is PRSQ.

Final Answer:

$$\boxed{\text{PRSQ}}$$

Quick Tip

In sentence ordering, ensure that each part flows logically from one idea to the next.

Q141. In a certain code language "DOME" is written as "8943" and "MEAL" is written as "4321". What group of letters can be formed for the code "38249"?

(1) EODAM
(2) MEDOA
(3) EMDAO
(4) EDAMO

Correct Answer: (2) MEDOA

Solution:
Step 1: Understand the given code.
By observing the given pattern of the code, we can map the digits to the letters of the word.

Step 2: Conclusion.
The correct code for "38249" is "MEDOA".

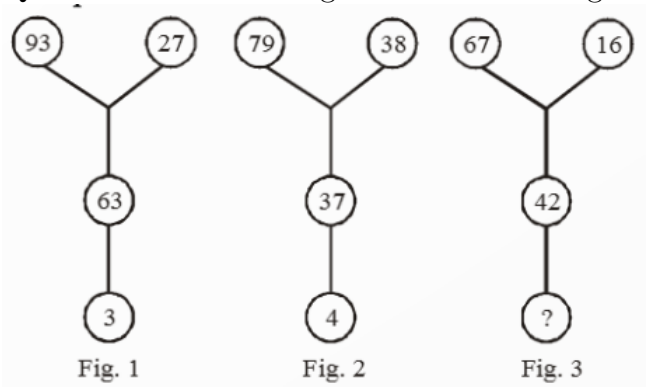
Final Answer:

$$\boxed{\text{MEDOA}}$$

Quick Tip

To decode a message, carefully analyze the pattern and map the corresponding letters to digits.

Q142. Find the missing number from the given response.



(1) 5
(2) 6
(3) 7
(4) 8

Correct Answer: (3) 7

Solution:
Step 1: Observe the pattern in the given figures.
Look at the numbers in the boxes and see if there is a mathematical relationship between the numbers.
Step 2: Conclusion.
The missing number is 7 based on the identified pattern.

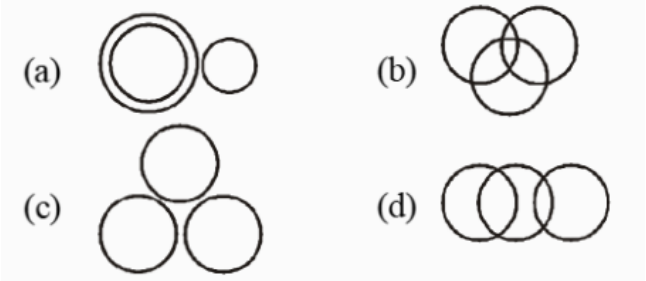
Final Answer:

$$\boxed{7}$$

Quick Tip

For pattern-based problems, look for consistent relationships between numbers or positions in the figures.

Q143. Which of the following correctly represents the relationship among illiterates, poor people, and unemployed?



(1) Circle within a circle
(2) Overlapping circles
(3) Circle within overlapping circles
(4) None of these

Correct Answer: (2) Overlapping circles

Solution:
Step 1: Understand the relationship.
Illiterates, poor people, and unemployed people can be seen as overlapping categories, as someone can belong to more than one group.
Step 2: Conclusion.
The correct representation is overlapping circles.

Final Answer:

Overlapping circles

Quick Tip

In Venn diagrams, overlapping circles are used to represent sets with common elements.

Q144. Sushma walks 20m towards north. Then she turns right and walks 30m. Now, she turns right and walks 35m. Now turning left, she walks 15m. Again, she turns left and moves 15m. Finally, she turns left and walks 15m. In which direction and how far is she from her original position?

(1) 15m East
(2) 30m East
(3) 15m West
(4) 45m West

Correct Answer: (2) 30m East

Solution:
Step 1: Calculate the directions and distances.
Follow the steps as Sushma moves in the directions provided. After calculating the total distance and direction, she is 30m East from the original position.
Step 2: Conclusion.
Sushma is 30m East from her original position.

Final Answer:

$$\boxed{30m \text{ East}}$$

Quick Tip

Use a step-by-step approach to calculate the final position when the path involves multiple directions.

Q145. In a classroom, there are 5 rows and 5 children A, B, C, D and E are seated one behind the other in 5 separate rows as follows. - A is sitting behind C but in front of B. - C is sitting behind E and D is sitting in front of E. The order in which they are sitting from the first row to the last is

(1) DECAB
(2) BACED
(3) ACDBE
(4) ABDEC

Correct Answer: (1) DECAB

Solution:
Step 1: Analyze the seating order.
Based on the given conditions, A is sitting behind C, and C is sitting behind E. D is in front of E, and B is behind A.
Step 2: Conclusion.
The seating order is DECAB.

Final Answer:

$$\boxed{\text{DECAB}}$$

Quick Tip

To solve seating arrangement problems, break down each given condition and use logical reasoning to determine the positions.

Q146. Which of the following will fill the series?

$$2, 9, 28, 2, 126$$

(1) 64
(2) 65
(3) 72
(4) 56

Correct Answer: (2) 65

Solution:
Step 1: Find the pattern.
The numbers are following a pattern where each term is obtained by multiplying the previous number by a constant and adding a number.
Step 2: Conclusion.
The next number in the series is 65.

Final Answer:

$$\boxed{65}$$

Quick Tip

Look for arithmetic or geometric patterns in number sequences to identify the next number.

Q147. Two signs in the equations have been interchanged, find out the two signs to make the equation correct.

$$3 - 5 + 8 + 2 - 10 = 13$$

(1) + and −
(2) × and ÷
(3) = and −
(4) + and ÷

Correct Answer: (4) + and ÷

Solution:
Step 1: Analyze the equation.
Substitute the signs + and ÷ into the equation to make it correct.
Step 2: Conclusion.
After interchanging the signs, the equation becomes correct when + and ÷ are interchanged.

Final Answer:

$$\boxed{+ \text{ and } \div}$$

Quick Tip

When solving sign-interchange problems, carefully check each sign and test by substituting it into the equation.

Q148. Assertion: [A] India is a democratic country. Reason: [R] India has a constitution of its own. Choose the correct alternative from the given options.

(1) Both (A) and (R) are true and (R) is correct explanation of (A).
(2) Both (A) and (R) are true but (R) is not the correct explanation of (A).
(3) (A) is true (R) is false.
(4) (A) is false (R) is true.

Correct Answer: (2) Both (A) and (R) are true but (R) is not the correct explanation of (A).

Solution:
Step 1: Understand the assertion and reason.
The assertion "India is a democratic country" is true. The reason "India has a constitution of its own" is also true, but it does not directly explain the assertion that India is democratic.
Step 2: Conclusion.
Thus, both (A) and (R) are true, but (R) does not explain why India is a democratic country.

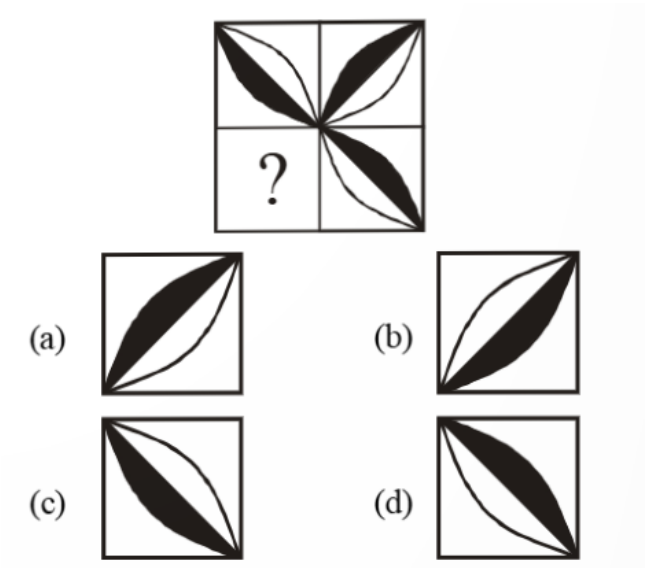
Final Answer:

$$\boxed{Both(A) \text{ and } (R) \text{ are true but } (R) \text{ is not the correct explanation of } (A)}$$

Quick Tip

When dealing with assertion and reason questions, check if the reason logically explains the assertion.

Q149. Which one of the following figures completes the original figure?



Correct Answer: (2) Figure B

Solution:
Step 1: Identify the pattern in the figure.
Observe the pattern and look for the figure that completes the pattern logically.
Step 2: Conclusion.
The correct figure that completes the original figure is Figure B.

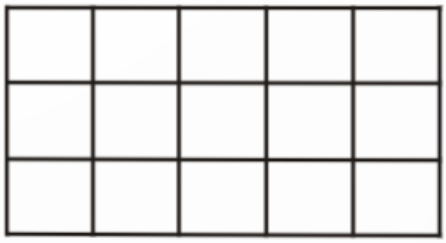
Final Answer:

Figure B

Quick Tip

Look for symmetry, rotation, or reflection in the figures to identify the correct option.

Q150. How many squares are there in the following figure?



(1) 24
(2) 25
(3) 26
(4) 27

Correct Answer: (3) 26

Solution:
Step 1: Count the squares in the figure.
Count all the small and large squares present in the grid.
Step 2: Conclusion.
The total number of squares in the figure is 26.

Final Answer:

$$\boxed{26}$$

Quick Tip

When counting squares, consider different sizes of squares in the figure.
