

VITEEE Previous Year Paper 2015 with Solutions

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| Time Allowed :180 Minutes | Maximum Marks :120 | Total Questions :120 |
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General Instructions

Read the following instructions very carefully and strictly follow them:

1. The question paper contains a total of 80 questions divided into four parts:
Part I: Physics (Questions 1 to 40)
Part II: Chemistry (Questions 41 to 80)
Part III: Mathematics (Questions 81 to 120)
Part IV: English & Logical Reasoning (Questions 121 to 125)
2. All questions are multiple-choice with four options, and only one of them is correct.
3. For each correct answer, the candidate will earn 1 mark.
4. There is no negative marking for incorrect answers.
5. The test duration is $1\frac{1}{2}$ hours.

Part I: Physics

1. When a hydrogen atom is raised from ground energy level to excited energy level, then (1) potential energy increases and kinetic energy decreases
(2) kinetic energy increases and potential energy decreases
(3) Both KE and PE increase
(4) Both KE and PE decrease

Correct Answer: (2) kinetic energy increases and potential energy decreases

Solution:

In this case, when a hydrogen atom is raised to an excited state, it gains potential energy, but the kinetic energy decreases because the atom is less tightly bound in the higher energy state. The potential energy increases as the electron moves farther from the nucleus.

Quick Tip

In atomic physics, when an electron moves to a higher energy level, the kinetic energy decreases and potential energy increases.

2. The half-life for α -decay of uranium U^{228} is 4.47×10^8 yr. If a rock contains 60% of the original U^{228} atoms, then its age is

- (A) 1.2×10^7 yr
- (B) 3.3×10^8 yr
- (C) 4.2×10^9 yr
- (D) 6.5×10^9 yr

Correct Answer: (C) 4.2×10^9 yr

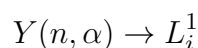
Solution:

Using the formula for half-life, we can calculate the age of the rock. The fraction of original U^{228} left is 60%. This corresponds to the decay equation, and we can calculate the time by taking the logarithm of the fraction and dividing by the decay constant.

Quick Tip

The age of a rock can be determined by using the half-life formula based on the fraction of original isotopes remaining.

3. A nuclear transformation is given by



. The nucleus of element Y is (A) Be^{11}

- (B) B^{10}
- (C) B^9
- (D) C^{12}

Correct Answer: (B) B^{10}

Solution:

In this reaction, the nucleus Y undergoes a nuclear transformation, and by analyzing the atomic mass and number of the particles involved, we deduce that the resultant nucleus is B^{10} .

Quick Tip

To determine the resulting nucleus in a nuclear transformation, balance the atomic number and mass number of the reactants and products.

4. The angular momentum of an electron in Bohr's hydrogen atom whose energy is -3.4 eV is
(A) $\frac{h}{2\pi}$

(B) $\frac{h}{\pi}$

(C) $\frac{h}{2}$

(D) $\frac{h}{\pi}$

Correct Answer: (A) $\frac{h}{2\pi}$

Solution:

For the electron in Bohr's model, the angular momentum is quantized and given by $mvr = \frac{nh}{2\pi}$, where $n = 1$ for the ground state. The angular momentum for a -3.4 eV energy corresponds to $\frac{h}{2\pi}$.

Quick Tip

In Bohr's model, the angular momentum of an electron is quantized and is integral multiples of $\frac{h}{2\pi}$.

5. When the momentum of a photon is changed by an amount p' , then the corresponding change in the de-Broglie wavelength is found to be 0.20%. Then, the original momentum of the photon was (A) 300 p

(B) 500 p

(C) 400 p

(D) 100 p

Correct Answer: (C) 400 p

Solution:

The change in de-Broglie wavelength is related to the change in momentum of the photon. By applying the appropriate relationship between momentum and wavelength, we can find the original momentum of the photon to be 400 p.

Quick Tip

For photons, the de-Broglie wavelength is inversely proportional to momentum, so a small change in momentum results in a proportional change in wavelength.

6. Suppose a beam of electrons with each electron having energy E_0 incident on a metal surface kept in an evacuated chamber. Then, (A) electrons can be emitted with any energy less than

E_0

(B) electrons can be emitted with any energy up to E_0

(C) electrons can be emitted with any energy with a maximum of E_0

(D) electrons can be emitted with any energy with a maximum of $E_0 - \phi$, where ϕ being work function

Correct Answer: (D) electrons can be emitted with any energy with a maximum of $E_0 - \phi$, where ϕ being work function

Solution:

When electrons strike a metal surface, they can be emitted with a maximum energy equal to the incident energy minus the work function ϕ , which is the minimum energy required to release the electron from the metal.

Quick Tip

The maximum kinetic energy of emitted electrons is determined by the incident energy minus the work function of the material.

7. An n-type semiconductor is (A) neutral

(B) positively charged

(C) negatively charged

(D) negatively or positively charged depending on the amount of impurity added

Correct Answer: (D) negatively or positively charged depending on the amount of impurity added

Solution:

An n-type semiconductor is formed by doping a semiconductor with elements that donate electrons, making it negatively charged. However, its charge may change based on the impurity added.

Quick Tip

In n-type semiconductors, the majority charge carriers are electrons, which come from the donor impurities.

8. The de-Broglie wavelength of an electron moving with a velocity of 10^6 m/s is: (1)

- 1.23×10^{-10} m
(2) 1.23×10^{-9} m
(3) 1.23×10^{-8} m
(4) 1.23×10^{-7} m

Correct Answer: (1) 1.23×10^{-10} m

Solution:

Using the de-Broglie wavelength formula $\lambda = \frac{h}{mv}$, where $h = 6.63 \times 10^{-34}$ J·s, m is the mass of the electron (9.11×10^{-31} kg), and v is the velocity of the electron, we calculate the wavelength as:

$$\lambda = \frac{6.63 \times 10^{-34}}{(9.11 \times 10^{-31})(10^6)} \approx 1.23 \times 10^{-10} \text{ m.}$$

Quick Tip

The de-Broglie wavelength of an electron is inversely proportional to its momentum ($p = mv$), so a higher velocity results in a smaller wavelength.

9. The ionization energy of hydrogen in the ground state is (1) 13.6 eV

- (2) 27.2 eV
(3) 5.0 eV
(4) 18.0 eV

Correct Answer: (1) 13.6 eV

Solution:

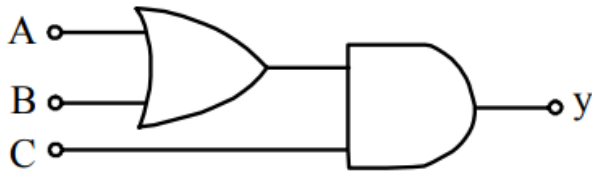
The ionization energy of hydrogen in the ground state is the energy required to remove the electron from the $n = 1$ level to the $n = \infty$ level. From Bohr's model, the ionization energy for hydrogen is given by:

$$E = \frac{13.6}{n^2} \text{ eV, for } n = 1 \text{ (ground state)} \Rightarrow E = 13.6 \text{ eV.}$$

Quick Tip

The ionization energy of hydrogen in its ground state is always 13.6 eV, which corresponds to the energy required to completely remove the electron from the atom.

10. The electric field at the center of a uniformly charged spherical shell is



- (1) Zero
- (2) $k \cdot \frac{Q}{R^2}$
- (3) $k \cdot \frac{Q}{R}$
- (4) $k \cdot \frac{Q}{2R^2}$

Correct Answer: (1) Zero

Solution:

According to Gauss's law, the electric field inside a uniformly charged spherical shell is zero at all points inside the shell, including the center. This result is due to the symmetry of the shell. The net flux through a Gaussian surface inside the shell is zero, implying that the electric field is zero.

Quick Tip

The electric field inside a uniformly charged spherical shell is always zero, regardless of the point inside the shell.

11. Equal charges q each are placed at the vertices of an equilateral triangle of side r . The magnitude of electric field intensity at any vertex is (1) $\frac{2q}{4\pi\epsilon_0 r^2}$

- (2) $\frac{q}{4\pi\epsilon_0 r^2}$
- (3) $\frac{\sqrt{3}q}{4\pi\epsilon_0 r^2}$
- (4) $\frac{\sqrt{2}q}{4\pi\epsilon_0 r^2}$

Correct Answer: (3) $\frac{\sqrt{3}q}{4\pi\epsilon_0 r^2}$

Solution:

For an equilateral triangle with charges at the vertices, the resultant electric field at any vertex is the vector sum of the electric fields due to the other two charges. This results in a field of magnitude $\frac{\sqrt{3}q}{4\pi\epsilon_0 r^2}$.

Quick Tip

The electric field at the vertex of an equilateral triangle formed by charges is the result of the vector sum of the fields from the other two charges.

12. Two point masses, m each carrying charges $-q$ and $+q$ are attached to the ends of a massless rigid non-conducting wire of length L . When this arrangement is placed in a uniform electric field, then it deflects through an angle θ . The minimum time needed by rod to align itself along the field is (1) $\frac{2\pi mL}{qE}$

(2) $\frac{\pi mL}{2qE}$

(3) $\frac{\pi mL}{qE}$

(4) $\frac{2\pi qE}{mL}$

Correct Answer: (3) $\frac{\pi mL}{qE}$

Solution:

The time for the rod to align itself with the electric field depends on the torque applied due to the electric field. Using the relation $\tau = I\alpha$ and the moment of inertia for a rod, the time is given by $\frac{\pi mL}{qE}$.

Quick Tip

When dealing with torque in rotational motion, the time for alignment can be calculated using the equation involving the moment of inertia and torque.

13. A condenser of capacitance C is fully charged by a 200V supply. It is then discharged through a small coil of resistance r embedded in thermally insulated block of specific heat 250 J/K-g and mass 100 g. If the temperature of the block rises by 0.4 K, then the value of C is (1) 300 μF

(2) 200 μF

(3) 400 μF

(4) 500 μF

Correct Answer: (3) 400 μF

Solution:

Using the energy dissipated in the resistor and equating it to the thermal energy absorbed by

the block, we can solve for C . The energy dissipated by the capacitor is given by $E = \frac{1}{2}CV^2$, and the thermal energy is $Q = mc\Delta T$.

Quick Tip

Energy dissipated by the capacitor equals the heat absorbed by the block, allowing us to find the capacitance.

14. The masses of three copper wires are in the ratio 3 : 2 : 5 and their lengths are in the ratio 3 : 2 : 5. Then, the ratio of their electrical resistances is (1) 1: 5: 15

- (2) 3: 2: 5
- (3) 3: 4: 5
- (4) 5: 3: 2

Correct Answer: (1) 1: 5: 15

Solution:

The resistance R of a wire is given by $R = \rho \frac{L}{A}$, where ρ is the resistivity, L is the length, and A is the cross-sectional area. Using the given ratios for mass and length, the resistance ratio is 1:5:15.

Quick Tip

The resistance ratio depends on both the length and cross-sectional area, with mass and length related through the density and volume of the wire.

15. A 30V-90W lamp is operated on a 120V DC line. A resistor is connected in series with the lamp in order to glow it properly. The value of resistance

- (1) 10 Ω
- (2) 30 Ω
- (3) 20 Ω
- (4) 40 Ω

Correct Answer: (3) 20 Ω

Solution:

Using the power formula $P = \frac{V^2}{R}$, we calculate the required resistance for the lamp to operate at its specified power. The total resistance needed is found to be 20Ω .

Quick Tip

Use the formula $P = \frac{V^2}{R}$ to determine the resistance needed for devices working at a specific power.

16. The magnetic field at the centroid of the triangle is (1) $1.66 \times 10^{-5} \text{ T}$

(2) $1.22 \times 10^{-5} \text{ T}$

(3) $1.33 \times 10^{-5} \text{ T}$

(4) $1.44 \times 10^{-4} \text{ T}$

Correct Answer: (1) $1.66 \times 10^{-5} \text{ T}$

Solution:

The magnetic field at the centroid is determined by applying Ampere's Law and calculating the field produced by currents in the sides of the triangle.

Quick Tip

For a current-carrying triangle, the magnetic field at the centroid can be calculated using the Biot-Savart law.

17. In a potentiometer experiment, the balancing length of a cell is 560 cm. When an external resistance of 10Ω is connected in parallel to the cell, the balancing length changes by 60 cm. The internal resistance of a cell is (1) 14Ω

(2) 16Ω

(3) 1.2Ω

(4) 1.2Ω

Correct Answer: (1) 14Ω

Solution:

By using the potentiometer relation $V = IR$ and the formula for balancing length in a potentiometer experiment, we can calculate the internal resistance of the cell as 14Ω . This is based on the change in the balancing length when an external resistance is connected.

Quick Tip

In a potentiometer experiment, the change in the balancing length due to the addition of external resistance helps in calculating the internal resistance of the cell.

18. Two sources of equal emf are connected to a resistance R . The internal resistance of the sources are r_1 and r_2 , ($r_1 > r_2$). If the potential difference across the source having internal resistance r_2 is zero, then (1) $R = \frac{r_2}{r_1 - r_2}$

(2) $R = \frac{r_1 + r_2}{r_1}$

(3) $R = \frac{r_1 r_2}{r_1 + r_2}$

(4) $R = r_2 - r_1$

Correct Answer: (3) $R = \frac{r_1 r_2}{r_1 + r_2}$

Solution:

In this case, when the potential difference across the second source becomes zero, it implies that the two internal resistances are balanced. The effective resistance is given by the formula $R = \frac{r_1 r_2}{r_1 + r_2}$.

Quick Tip

In problems involving internal resistance of sources, the total resistance in the circuit can be determined using the parallel resistance formula.

19. An electron of mass 9.0×10^{-31} kg under the action of a magnetic field moves in a circle of radius 2 cm at a speed of 3×10^6 m/s. A proton of mass 1.8×10^{-27} kg moves in a circle of same radius in the same magnetic field, then its speed will become (1) 1.5×10^3 m/s

(2) 3×10^6 m/s

(3) 6×10^4 m/s

(4) 1.0×10^5 m/s

Correct Answer: (3) 6×10^4 m/s

Solution:

Using the formula for the motion of charged particles in a magnetic field $r = \frac{mv}{qB}$, we can equate the radius of the electron and proton's motion to find the proton's velocity. The speed of the proton turns out to be 6×10^4 m/s.

Quick Tip

The velocity of a charged particle moving in a magnetic field is inversely proportional to its mass for a given radius and charge.

20. A horizontal rod of mass 0.01 kg and length 10 cm is placed on a frictionless plane inclined at an angle 60° with the horizontal and with the length of the rod parallel to the edge of the inclined plane. A uniform magnetic field is applied 'Vertically downwards'. The current through the rod is 1.7 A, then the value of magnetic field induction B for which the rod remains stationary in the inclined plane is (1) 1 T

- (2) 3 T
- (3) 2 T
- (4) 4 T

Correct Answer: (3) 2 T

Solution:

Using the relation for the force on a current-carrying wire in a magnetic field, $F = BIL \sin(\theta)$, and equating it with the gravitational force, we can calculate the required magnetic field as 2 T.

Quick Tip

The force on a current-carrying conductor in a magnetic field depends on the length of the conductor, the current, and the angle between the magnetic field and the conductor.

21. A current of 2 A is flowing in the sides of an equilateral triangle of side 9 cm. The magnetic field at the centroid of the triangle is (1) 1.66×10^{-5} T

- (2) 1.22×10^{-5} T
- (3) 1.33×10^{-5} T
- (4) 1.44×10^{-4} T

Correct Answer: (1) 1.66×10^{-5} T

Solution:

The magnetic field at the centroid of a current-carrying triangle is determined using Ampere's law and Biot-Savart law. The current in the sides of the triangle produces a magnetic field at the centroid, which is 1.66×10^{-5} T.

Quick Tip

To find the magnetic field at the centroid of a current-carrying triangle, use the Biot-Savart law or Ampere's circuital law.

22. The direction of magnetic field dB due to current element dl at a distance r is the direction of (1) $r \times dl$

(2) $dl \times r$

(3) $(r\hat{d})$

(4) dl

Correct Answer: (1) $r \times dl$

Solution:

The magnetic field produced by a current element is given by the Biot-Savart law, where the direction of the magnetic field is perpendicular to both the current element dl and the position vector r , and follows the right-hand rule.

Quick Tip

The direction of the magnetic field due to a current element is given by the right-hand rule, where the field is perpendicular to both the current and position vector.

23. A galvanometer with a scale divided into 100 equal divisions has a current sensitivity of 10 divisions per milliamper and a voltage sensitivity of 2 divisions per millivolt. The galvanometer resistance will be (1) 4Ω

(2) 5Ω

(3) 3Ω

(4) 7Ω

Correct Answer: (2) 5Ω

Solution:

The resistance of the galvanometer can be found by using the relation $R = \frac{V}{I}$. The voltage sensitivity and current sensitivity provide the necessary values for calculating the resistance of the galvanometer.

Quick Tip

The resistance of a galvanometer can be calculated using its voltage and current sensitivity.

24. The earth is considered as a short magnet with its centre coinciding with the geometric centre of earth. The angle of dip δ related to the magnetic latitude λ is (1) $\tan \delta = \frac{1}{2 \tan \lambda}$

(2) $\tan \delta = 2 \tan \lambda$

(3) $\tan \delta = \tan \lambda$

(4) $\tan \delta = 2 \tan \lambda$

Correct Answer: (2) $\tan \delta = 2 \tan \lambda$

Solution:

The angle of dip is related to the latitude using the formula $\tan \delta = 2 \tan \lambda$, which describes the relationship between the dip angle and the magnetic latitude of the location.

Quick Tip

The dip angle and magnetic latitude are related through a tangent function in the Earth's magnetic field.

25. Which of the following statement related to hysteresis loop is incorrect? (1) The curve of

B against H for a ferromagnetic material is called hysteresis loop

(2) The area of $B - H$ curve is a measure of power dissipated per cycle per unit area of the specimen

(3) Coercivity is a measure of the magnetic field required to destroy the residual magnetism of ferromagnetic material

(4) The retentivity of a specimen is the measure of magnetic field remaining in the specimen when the magnetising field is removed

Correct Answer: (4) The retentivity of a specimen is the measure of magnetic field remaining in the specimen when the magnetising field is removed

Solution:

The correct statement is that the retentivity of a specimen is the amount of residual magnetism that remains in the specimen even after the external magnetic field is removed. The other statements are correct as they describe various characteristics of the hysteresis loop.

Quick Tip

The hysteresis loop describes the behavior of a magnetic material under changing magnetic fields, and retentivity refers to the remaining magnetism after the external field is removed.

26. A magnetic needle lying parallel to the magnetic field requires W units of work to turn it through an angle 45° . The torque required to maintain the needle in this position will be (1)

- (1) \sqrt{W}
- (2) 1
- (3) $\sqrt{3}W$
- (4) $\frac{W}{\sqrt{2}}$

Correct Answer: (4) $\frac{W}{\sqrt{2}}$

Solution:

The torque required to maintain the needle in the magnetic field is calculated by using the work-energy theorem and the angle of deflection. The work done by the magnetic field is related to the torque by $\tau = \frac{W}{\theta}$.

Quick Tip

For a magnetic needle, the torque is related to the work done when the needle moves through an angle.

27. An induced emf has (1) a direction same as field direction

- (2) a direction opposite to the field direction
- (3) no direction of its own
- (4) None of the above

Correct Answer: (2) a direction opposite to the field direction

Solution:

According to Lenz's law, the induced emf always opposes the change in the magnetic field that caused it. This means the induced emf has a direction opposite to the applied field direction.

Quick Tip

Lenz's law states that the direction of induced emf always opposes the change in magnetic flux that produces it.

28. A coil of area 5 cm^2 having 20 turns is placed in a uniform magnetic field of 10^3 gauss. The normal to the plane of coil makes an angle 30° with the magnetic field. The flux through the coil is (1) $6.67 \times 10^{-4} \text{ wb}$

- (2) $3.2 \times 10^{-5} \text{ wb}$
(3) $5.9 \times 10^{-4} \text{ wb}$
(4) $8.65 \times 10^{-4} \text{ wb}$

Correct Answer: (1) $6.67 \times 10^{-4} \text{ wb}$

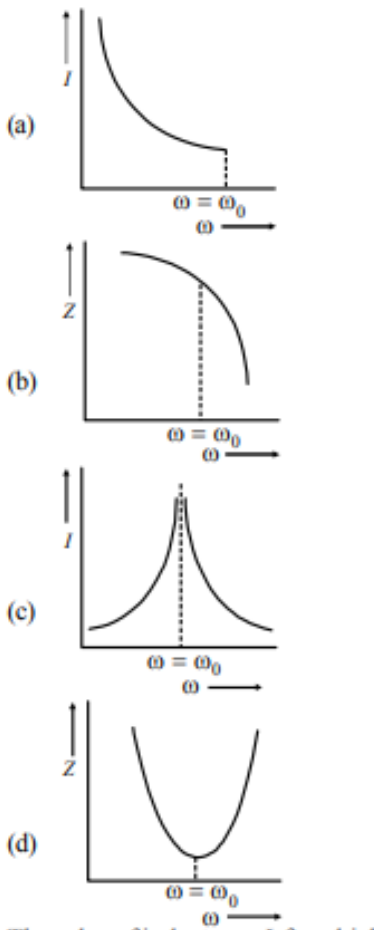
Solution:

Using the formula for magnetic flux $\Phi = BA \cos \theta$, where $B = 10^3 \text{ gauss} = 10^{-1} \text{ T}$, $A = 5 \text{ cm}^2 = 5 \times 10^{-4} \text{ m}^2$, and $\theta = 30^\circ$, we calculate the flux as $6.67 \times 10^{-4} \text{ wb}$.

Quick Tip

The magnetic flux is given by the product of magnetic field strength, area, and the cosine of the angle between the normal to the coil and the magnetic field.

29. The current graph for resonance in an LC circuit is



Correct Answer: (2) Z vs ω

Solution:

In an LC circuit, resonance occurs when the impedance Z is minimized, and the current I is maximized at a specific angular frequency ω_0 . The resonance condition is often represented by the relationship between impedance and angular frequency.

Quick Tip

At resonance in an LC circuit, the impedance is purely resistive, and the current reaches its maximum value.

30. The value of inductance L for which the current is maximum in series LCR circuit with $C = 10 \mu F$ and $R = 1000 \Omega$ is (1) 10 mH

- (2) 50 mH
- (3) 200 mH
- (4) 100 mH

Correct Answer: (3) 200 mH

Solution:

The resonance condition for a series LCR circuit is given by $\omega_0 = \frac{1}{\sqrt{LC}}$, where ω_0 is the resonant angular frequency. Using the given values for C and R , we can calculate the inductance $L = 200$ mH.

Quick Tip

For maximum current in a series LCR circuit, the inductance must satisfy the resonance condition $\omega_0 = \frac{1}{\sqrt{LC}}$.

31. A ray of light is incident on a plane mirror at an angle of 30° . At what angle with the horizontal must a plane mirror be placed so that the reflected ray becomes vertically upwards?

- (1) 40°
- (2) 20°
- (3) 30°
- (4) 60°

Correct Answer: (2) 20°

Solution:

The angle of reflection must be equal to the angle of incidence, and for the reflected ray to be vertical, the angle of the mirror with the horizontal should be 20° . This ensures the total angle of reflection is 90° .

Quick Tip

For a ray to be reflected vertically, the angle of the mirror with the horizontal should be half of the angle of incidence.

32. A compound microscope having magnifying power 35 with its eye-piece of focal length 10 cm. Assume that the final image is at least distance of distinct vision then the magnification produced by the objective is (1) 4

- (2) 5
- (3) 10
- (4) -10

Correct Answer: (3) 10

Solution:

The total magnification M of a compound microscope is given by the product of the magnifications produced by the objective and eyepiece. Using the formula for the total magnification and the given values, we find that the magnification produced by the objective is 10.

Quick Tip

The total magnification of a microscope is the product of the magnification produced by the objective and the eyepiece.

33. The refractive index for a prism is given as $\mu = \cot \frac{A}{2}$. Then, angle of minimum deviation in terms of angle of prism is (1) $90^\circ A$

- (2) $2A$
- (3) $180^\circ - 2A$
- (4) $180^\circ - 2A$

Correct Answer: (2) $2A$

Solution:

The refractive index μ is related to the angle of minimum deviation using the equation $\mu = \cot \frac{A}{2}$, where A is the angle of the prism and the angle of minimum deviation is given by $2A$.

Quick Tip

For a prism, the refractive index can be related to the angle of deviation and the angle of the prism using trigonometric formulas.

34. Two convex lenses of power 2D and 5D are separated by a distance $\frac{1}{3}$ m. The power of optical system formed is (1) +2D

- (2) -2D
- (3) +3D
- (4) -3D

Correct Answer: (3) +3D

Solution:

The total power of the system is the sum of the individual powers of the lenses, but since the lenses are separated by a distance, the effective power is given by $P = P_1 + P_2 - \frac{d}{f_1 f_2}$.

Quick Tip

When combining lenses with a separation distance, use the formula for the effective power of the system.

35. Two light rays having the same wavelength in vacuum are in phase initially. Then, the first ray travels a path L_1 through a medium of refractive index μ_1 , while the second ray travels a path L_2 through a medium of refractive index μ_2 . The two waves are then combined to observe interference. The phase difference between the two waves is (1) $\frac{2\pi}{\lambda}(\mu_1 L_1 - \mu_2 L_2)$

- (2) $\frac{2\pi}{\lambda}(L_1 - L_2)$
 (3) $\frac{2\pi}{\lambda}(\mu_1 L_1 - \mu_2 L_2)$
 (4) $\frac{2\pi}{\lambda}(\mu_1 L_1 + \mu_2 L_2)$

Correct Answer: (1) $\frac{2\pi}{\lambda}(\mu_1 L_1 - \mu_2 L_2)$

Solution:

The phase difference due to different paths in different media is given by $\Delta\phi = \frac{2\pi}{\lambda}(\mu_1 L_1 - \mu_2 L_2)$.

Quick Tip

For interference, the phase difference is proportional to the path length and refractive index of the medium.

36. Two polaroids are kept crossed to each other. If one of them is rotated an angle 60° , the percentage of incident light now transmitted through the system is (1) 10(2) 12.5(3) 20(4) 25

Correct Answer: (2) 12.5

Solution:

The transmission of light through two polaroids is governed by Malus' law, and the percentage of light transmitted when the angle between the polaroids is 60° is 12.5

Quick Tip

Malus' law states that the intensity of light passing through a polaroid is proportional to the square of the cosine of the angle between the light's polarization and the polaroid axis.

37. An electromagnetic wave propagating along north has its electric field vertically upward. The magnetic field vector points towards (1) downward

- (2) east
- (3) north
- (4) south

Correct Answer: (4) south

Solution:

In an electromagnetic wave, the electric field, magnetic field, and the direction of wave propagation are mutually perpendicular. Since the wave is propagating north and the electric field is vertical, the magnetic field must point south.

Quick Tip

In an electromagnetic wave, the electric and magnetic fields are perpendicular to each other and to the direction of propagation.

38. Pick out the wrong statement. (1) Gauss's law of magnetism is given by $\oint \mathbf{B} \cdot d\mathbf{s} = 0$

- (2) An EM wave is a wave radiated by a charge at rest and propagates through electric field only
- (3) A time varying electric field is a source of changing magnetic field
- (4) Faraday's law of EM induction is $\int \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_B}{dt}$

Correct Answer: (2) An EM wave is a wave radiated by a charge in motion, not at rest, and propagates through both electric and magnetic fields.

Solution:

An electromagnetic wave is created by a charge in motion, not at rest, and it propagates through both electric and magnetic fields. The statement about it propagating through only the electric field is incorrect.

Quick Tip

Electromagnetic waves require both electric and magnetic fields for propagation, and are generated by accelerating charges.

39. When sunlight is scattered by atmospheric atoms and molecules the amount of scattering of light of wavelength 880 nm is A . Then, the amount of scattering of light of wavelength 330 nm is approximately (1) $10A$

- (2) $20A$
- (3) $40A$
- (4) $50.5A$

Correct Answer: (3) $40A$

Solution:

The scattering of light follows a λ^{-4} dependence, where λ is the wavelength of the light. Thus, the ratio of the scattering intensities for the two wavelengths is $\left(\frac{880}{330}\right)^4$, giving approximately 40 times the scattering for the shorter wavelength.

Quick Tip

The scattering of light by particles is inversely proportional to the fourth power of the wavelength.

40. The ratio of volume occupied by an atom to the volume of the nucleus is (1) 10^{15}

- (2) 10^{12}
- (3) 10^{10}
- (4) $1 : 10^{15}$

Correct Answer: (4) $1 : 10^{15}$

Solution:

The volume occupied by an atom is much larger than the volume of the nucleus. The ratio of the two volumes is $1 : 10^{15}$, meaning the atomic volume is approximately 15 orders of magnitude greater than the nuclear volume.

Quick Tip

The volume occupied by an atom is much larger than that of the nucleus due to the vast amount of empty space in atoms.

Part II: Chemistry

41. When copper is treated with a certain concentration of nitric acid, nitric oxide and nitrogen dioxide are liberated in equal volumes according to the equation,



The coefficients of x and y are respectively (1) 2 and 3

- (2) 3 and 5
- (3) 2 and 6
- (4) 1 and 3

Correct Answer: (1) 2 and 3

Solution:

By balancing the reaction for the liberation of nitric oxide and nitrogen dioxide, the stoichiometric coefficients for copper and nitric acid are found to be 2 and 3, respectively.

Quick Tip

In redox reactions, balance the number of atoms and charges on both sides of the equation to determine the stoichiometric coefficients.

42. A saturated solution of H_2S in 0.1M HCl at 25°C contains S^{2-} ion concentration of $10^{-23} \text{ mol L}^{-1}$. The solubility product of some sulfides are $CuS = 10^{-44}$, $FeS = 10^{-14}$, and $MnS = 10^{-25}$. If 0.01M solution of these salts in 1M HCl are saturated with H_2S , which of these will be precipitated? (1) All

- (2) All except MnS
- (3) All except MnS and FeS
- (4) Only CuS

Correct Answer: (3) All except MnS and FeS

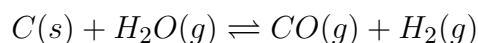
Solution:

The solubility product indicates the solubility of the salts in solution. By comparing the ion product of each sulfide with its solubility product, we determine that CuS will precipitate because its ion product exceeds the solubility product, while MnS and FeS will not.

Quick Tip

The solubility product can be used to predict whether a salt will precipitate when the concentration of its ions exceeds the product of its solubility constant.

43. Consider the water gas equilibrium reaction,



Which of the following statements is true at equilibrium? (1) If the amount of $C(s)$ is increased,

less water would be formed

(2) If the amount of $C(s)$ is increased, more CO and H_2 would be formed

(3) If the pressure on the system is increased by halving the volume, more water would be formed

(4) If the pressure on the system is increased by halving the volume, more CO and H_2 would be formed

Correct Answer: (2) If the amount of $C(s)$ is increased, more CO and H_2 would be formed

Solution:

Increasing the amount of carbon shifts the equilibrium towards the products, increasing the concentrations of CO and H_2 according to Le Chatelier's principle.

Quick Tip

Le Chatelier's principle states that a system at equilibrium will adjust to counteract changes in concentration, pressure, or temperature.

44. The chemical composition of slag formed during the smelting process in the extraction of copper is (1) $Cu_2O + FeS$

(2) $CuFeS_2$

(3) $CuS + FeO$

(4) $FeSiO_3$

Correct Answer: (4) $FeSiO_3$

Solution:

During copper extraction, the slag typically forms as a by-product, consisting mainly of iron silicate $FeSiO_3$.

Quick Tip

In metallurgical processes, slag is a waste product, often formed from the impurities in ores, such as iron silicate in copper extraction.

45. XCl_2 (excess) + $YCl_2 \rightarrow XCl_4 + YI$ Ore of Y formed is (1) siderite

- (2) cinnabar
- (3) hornsilver
- (4) chalcopyrite

Correct Answer: (4) chalcopyrite

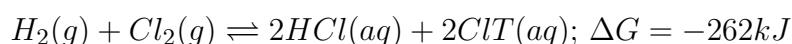
Solution:

Chalcopyrite is a copper iron sulfide mineral and is typically involved in processes that produce Y , such as copper extraction.

Quick Tip

Chalcopyrite is a significant copper ore and plays an essential role in copper extraction reactions.

46. For the given reaction,



The value of free energy of formation (ΔG_f) for the ion Cl^- is (1) $-131.2 \text{ kJ mol}^{-1}$

- (2) $+131.2 \text{ kJ mol}^{-1}$
- (3) $-262.4 \text{ kJ mol}^{-1}$
- (4) $+262.4 \text{ kJ mol}^{-1}$

Correct Answer: (2) $+131.2 \text{ kJ mol}^{-1}$

Solution:

The free energy of formation is half of ΔG for a molecule involving two ions, so ΔG_f for Cl^- is $+131.2 \text{ kJ mol}^{-1}$.

Quick Tip

The free energy of formation for a substance is calculated by dividing the total free energy change by the number of molecules involved.

47. The molarity of NO_3^- in the solution after 2L of 3M $AgNO_3$ is mixed with 3L of 1M $BaCl_2$ is (1) 1.2M
(2) 0.5M
(3) 1.8M
(4) 0.4M

Correct Answer: (2) 0.5M

Solution:

By applying the dilution formula $C_1V_1 = C_2V_2$, we find that the molarity of NO_3^- after mixing the two solutions is 0.5M.

Quick Tip

Use the dilution formula to calculate the final concentration after mixing solutions of different concentrations.

-
48. Amongst NO_3^- , SO_3^{2-} , CO_3^{2-} , SO_4^{2-} the non-planar species are (1) CO_3^{2-} , SO_3^{2-}
(2) SO_3^{2-} , SO_4^{2-}
(3) CO_3^{2-} , SO_4^{2-}
(4) SO_3^{2-} , SO_4^{2-}

Correct Answer: (1) CO_3^{2-} , SO_3^{2-}

Solution:

The species CO_3^{2-} and SO_3^{2-} are non-planar due to their molecular geometries. SO_4^{2-} and other species are planar.



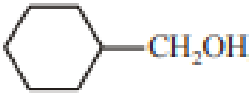


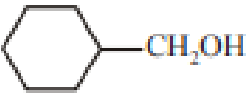
Quick Tip

The geometry of ions can be predicted using VSEPR theory, which explains the non-planarity of CO_3^{2-} and SO_3^{2-} .

49.



A and *B* respectively are

- (a) Both 
- (b) Both 
- (c)  , 
- (d)  , 

Correct Answer: (2) CH_3OH , CH_2OH

Solution:

This reaction involves the conversion of alcohols where the alcohol group is converted to CH_3OH and CH_2OH based on the reaction conditions.

Quick Tip

The conversion of alcohols involves substitution or elimination reactions depending on the conditions of the reaction.

50. A certain metal when irradiated by light ($r = 3.2 \times 10^{16} \text{ Hz}$) emits photoelectrons with twice kinetic energy as did photoelectrons when the same metal is irradiated by light ($r = 2.0 \times 10^{16} \text{ Hz}$). The ν of metal is (1) $1.2 \times 10^{14} \text{ Hz}$

- (2) $8 \times 10^{15} \text{ Hz}$
 (3) $1.2 \times 10^{16} \text{ Hz}$
 (4) $1.2 \times 10^{14} \text{ Hz}$

Correct Answer: (1) $1.2 \times 10^{14} \text{ Hz}$

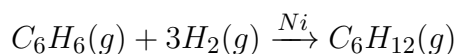
Solution:

By using the photoelectric equation $E_k = h\nu - \phi$, where ν is the frequency of light, and knowing that the kinetic energy doubles, we can solve for the work function ϕ , finding that $\nu = 1.2 \times 10^{14} \text{ Hz}$.

Quick Tip

The photoelectric effect equation relates the kinetic energy of emitted photoelectrons to the frequency of the incident light and the work function of the material.

51. Gaseous benzene reacts with hydrogen gas in presence of a nickel catalyst to form gaseous cyclohexane according to the reaction,



A mixture of C_6H_6 and excess H_2 has a pressure of 60 mm of Hg in an unknown volume. After the gas had been passed over a nickel catalyst and all the benzene converted to cyclohexane, the pressure of the gas was 30 mm of Hg in the same volume at the same temperature. The fraction of C_6H_6 (by volume) present in the original volume is (1) $\frac{1}{3}$

- (2) $\frac{1}{5}$
- (3) $\frac{1}{6}$
- (4) $\frac{1}{4}$

Correct Answer: (1) $\frac{1}{3}$

Solution:

Using the ideal gas law and the mole ratios from the balanced equation, we calculate the volume fraction of C_6H_6 before the reaction as $\frac{1}{3}$, based on the change in pressure before and after the reaction.

Quick Tip

The fraction of a gas present in a reaction can be determined using the change in pressure, since the volume of the gas is directly proportional to the pressure in a constant volume system.

52. An alloy of copper, silver and gold is found to have copper atom constituting the ccp lattice. If silver atoms occupy the edge centres and gold atoms are present at body centres, the alloy has a formula (1) Cu_4Ag_8Au

- (2) Cu_4Ag_4Au
- (3) Cu_4AgAu
- (4) $Cu_4Ag_2Au_2$

Correct Answer: (1) Cu_4Ag_8Au

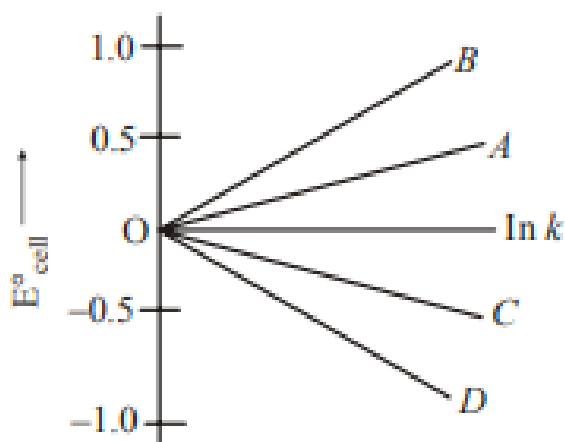
Solution:

In a ccp lattice, the number of atoms in each unit cell is determined by the number of atoms at edge and body centres. The copper, silver, and gold atoms fit into this arrangement in the ratio Cu_4Ag_8Au .

Quick Tip

The formula for an alloy can be derived from the stoichiometry based on the positions of the atoms in the unit cell and the coordination number.

53. Given, $\Delta G^\circ = -nFE_{\text{cell}}$ and $\Delta G^\circ = -RT \ln K$. The value of $n = 2$ will be given by the slope of which line in the figure



- (1) OA
- (2) OB
- (3) OC
- (4) OD

Correct Answer: (2) OB

Solution:

In this problem, the slope of the graph between ΔG° and $\ln K$ is $-nF$. For the given condition, the slope corresponding to $n = 2$ is represented by the line OB in the graph.

Quick Tip

The slope of the ΔG° vs $\ln K$ plot gives the value of $-nF$, and the slope represents the number of electrons involved in the reaction.

54. The false statements among the following are I. A primary carbocation is less stable than a tertiary carbocation.

II. A secondary propyl carbocation is less stable than allyl carbocation.

III. A tertiary free radical is more stable than a primary free radical.

IV. Isopropyl carbocation is more stable than ethyl carbocation. (1) I and II

(2) II and III

(3) I and IV

(4) II and IV

Correct Answer: (3) I and IV

Solution:

The false statements are: - Statement I: A primary carbocation is less stable than a tertiary carbocation. This is false because a tertiary carbocation is more stable due to alkyl groups providing inductive and hyperconjugative stabilization. - Statement IV: Isopropyl carbocation is more stable than ethyl carbocation. This is false because ethyl carbocation is more stable than isopropyl carbocation, as the inductive effects of alkyl groups play a role.

Quick Tip

Carbocation stability increases with the number of alkyl groups attached to the positively charged carbon. Tertiary carbocations are more stable than secondary, which are more stable than primary.

55. A colourless water soluble solid *A* on heating gives equimolar quantities of *B* and *C*. *B* gives dense white fumes with HCl and *C* does so with NH₃. *B* gives brown precipitate with Nessler's reagent and *C* gives white precipitate with nitrates of *Ag*⁺, *Pb*²⁺ and *Hg*²⁺. *A* is (1)

*NH*₄*Cl*

(2) *NH*₄*NO*₃

(3) *NH*₄*CO*₃

(4) *FeSO*₄

Correct Answer: (1) *NH*₄*Cl*

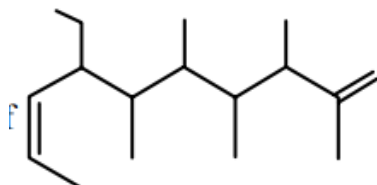
Solution:

The compound *NH*₄*Cl* reacts as described in the question. It gives dense white fumes with HCl and reacts with Nessler's reagent to give a brown precipitate, characteristic of ammonium salts.

Quick Tip

The chemical behavior of ammonium salts includes producing white fumes with HCl and reacting with Nessler's reagent to form a brown precipitate.

56. The IUPAC name of



- (1) 4-ethyl-5,6,7,9-tetramethyldec-2,9-diene
- (2) 7-ethyl-2,4,5,6-tetramethyldec-1,8-diene
- (3) 6-ethyl-2,4,5,6-tetramethyldec-1,8-diene
- (4) 7-1-propanyl-2,3,4,5-tetramethyl non-1-ene

Correct Answer: (1) 4-ethyl-5,6,7,9-tetramethyldec-2,9-diene

Solution:

The IUPAC name corresponds to the structure and functional groups of the compound as determined by the correct naming conventions. Here, the compound is a diene with multiple substituents and methyl groups.

Quick Tip

The IUPAC nomenclature follows the rule of naming the longest chain, substituent positions, and ensuring the name corresponds to the structure.

57. Caffeine has a molecular weight of 194.1 g/l. It contains 28.9(1) 4

- (2) 6
- (3) 5
- (4) 3

Correct Answer: (4) 3

Solution:

By calculating the mass of nitrogen in a single molecule and using the molar mass of nitrogen, we determine that there are three nitrogen atoms in a caffeine molecule.

Quick Tip

To determine the number of nitrogen atoms in a molecule, calculate the mass of nitrogen in one mole of the compound and divide by the atomic mass of nitrogen.

58. A compound X on heating gives a colourless gas. The residue is dissolved in water to obtain Y . Excess CO_2 is passed through aqueous solution of Y when Z is formed. Z on gentle heating gives back X . The compound X is (1) $Ca(HCO_3)_2$

- (2) $CaCO_3$
- (3) Na_2CO_3
- (4) Na_2CO_3

Correct Answer: (1) $Ca(HCO_3)_2$

Solution:

The compound X is $Ca(HCO_3)_2$, which on heating decomposes to form calcium carbonate and carbon dioxide, and the reaction steps match with the given description.

Quick Tip

The decomposition of calcium bicarbonate leads to calcium carbonate, which can be re-dissolved and react with CO_2 .

59. Which two sets of reactants best represent the amphoteric character of $Zn(OH)_2$? Set I

- $Zn(OH)_2(s)$ and $OH^-(aq)$
- Set II $Zn(OH)_2(s)$ and $H_2O(l)$
- Set III $Zn(OH)_2(s)$ and $H^+(aq)$
- Set IV $Zn(OH)_2(s)$ and $NH_3(aq)$ (1) III and II

- (2) I and III
- (3) IV and I
- (4) II and IV

Correct Answer: (2) I and III

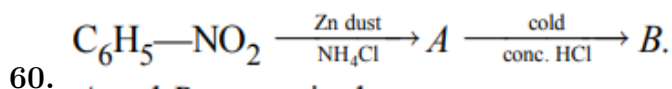
Solution:

The amphoteric character of $Zn(OH)_2$ is shown by its ability to react both with acids and

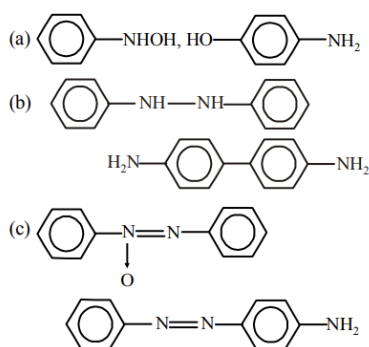
bases. Set I and Set III represent reactions where $Zn(OH)_2$ acts as an acid (reacting with OH^-) and a base (reacting with H^+).

Quick Tip

An amphoteric substance can react with both acids and bases. $Zn(OH)_2$ reacts with OH^- and H^+ ions.



A and B respectively are



Correct Answer: (1) NHOH, HO, NH_2

Solution:

The reaction described involves the reduction of nitrobenzene to aniline. Zinc dust is used to reduce the nitro group to an amine, resulting in the formation of aniline (NH_2).

Quick Tip

Reduction of nitro compounds to amines typically involves reducing agents like zinc dust or tin and HCl.

61. Point out incorrect stability order (1) $[Cu(NH_3)_4]^{2+} < [Cu(en)_2]^{2+} < [Cu(trien)]^{2+}$

- (2) $[Fe(H_2O)_6]^{3+} < [Fe(NO_2)_6]^{3+} < [Fe(NH_3)_6]^{3+}$
 (3) $[Co(H_2O)_6]^{3+} < [Rh(H_2O)_6]^{3+} < [Ir(H_2O)_6]^{3+}$
 (4) $[Cr(NH_3)_6]^{3+} < [Cr(NH_3)_6]^{2+} < [Cr(NH_3)_6]^{3+}$

Correct Answer: (4) $[Cr(NH_3)_6]^{3+} < [Cr(NH_3)_6]^{2+} < [Cr(NH_3)_6]^{3+}$

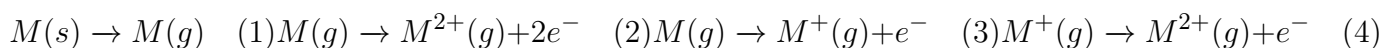
Solution:

The incorrect stability order is option (4) because it shows the same complex ion, $[Cr(NH_3)_6]^{3+}$, appearing twice with contradictory charge states.

Quick Tip

The stability of complex ions depends on the charge and the ligand environment. Higher oxidation states tend to form more stable complexes with certain ligands.

62. Consider the following changes:



The second ionisation energy of M could be determined from the energy values associated with

(1) $1 + 2 + 4$

(2) $2 + 3 - 4$

(3) $1 + 5 - 3$

(4) $5 - 3$

Correct Answer: (2) $2 + 3 - 4$

Solution:

The second ionisation energy corresponds to the energy required to remove a second electron from the atom. Using the provided reactions, the second ionisation energy can be determined from the changes represented in reactions (2) and (3), minus the first ionisation energy in reaction (4).

Quick Tip

Ionisation energy is the energy required to remove an electron from an atom or ion in the gas phase. For the second ionisation energy, use the appropriate steps involving electron removal.

63. In benzene, the triple bond consists of (1) one $sp^2 - sp^2$ sigma bond and two $p - p$ pi bonds

(2) two $sp^2 - sp^2$ sigma bonds and one $p - p$ pi bond

(3) one $sp^2 - sp^2$ sigma bond, one $p - p$ pi bond and one $p - p$ pi bond

(4) None of the above

Correct Answer: (3) one $sp^2 - sp^2$ sigma bond, one $p - p$ pi bond and one $p - p$ pi bond

Solution:

In the triple bond of benzene, the $sp^2 - sp^2$ sigma bond is formed by overlapping of the hybridized orbitals and the $p - p$ bonds are the pi bonds formed by sideways overlapping of unhybridized p-orbitals. Benzene has a resonance structure, which means it is a conjugated system with delocalized electrons.

Quick Tip

In conjugated systems like benzene, the bonds between carbon atoms are delocalized, contributing to the stability of the structure.

64. In keto-enol tautomerism of dicarbonyl compounds, the enol-form is preferred in contrast to the keto-form, this is due to (1) presence of carbonyl group on each side of $C = C$ group

- (2) resonance stabilization of enol form
- (3) presence of methylene group
- (4) rapid chemical exchange

Correct Answer: (2) resonance stabilization of enol form

Solution:

In keto-enol tautomerism, the enol form is stabilized by resonance between the lone pair of electrons on oxygen and the $C=C$ bond. This makes the enol form more stable than the keto form, which lacks this resonance stabilization.

Quick Tip

Enol forms are stabilized by resonance, which makes them more favored in some tautomeric equilibria.

65. An organic compound having carbon, hydrogen and sulphur contains 4(1) 200

- (2) 400
- (3) 600
- (4) 800

Correct Answer: (4) 800

Solution:

The molecular weight of the compound can be calculated by considering the proportion of

sulfur in the compound. Using the percentage of sulfur, we calculate the minimum molecular weight by applying the formula for molecular weight based on the percentage of sulfur.

Quick Tip

To determine the minimum molecular weight, divide the weight of one sulfur atom by the percentage of sulfur in the compound.

66. Which one of the following is a case of negative adsorption? (1) Acetic acid solution in contact with animal charcoal.
(2) Dilute KCl solution in contact with blood charcoal.
(3) Concentration KCl solution in contact with blood charcoal.
(4) H₂ gas in contact with charcoal at 300 K.

Correct Answer: (3) Concentration KCl solution in contact with blood charcoal.

Solution:

Negative adsorption occurs when the adsorbate concentration decreases at the surface, which is observed when KCl solution is in contact with charcoal. This results in the concentration being lower at the surface than in the bulk.

Quick Tip

Negative adsorption leads to a lower concentration of adsorbate on the surface compared to its concentration in the bulk phase.

67. The concentrations of the reactant A in the reaction $A \rightarrow B$ at different times are given below
- | Concentration (M) | Time (Minutes) |
|-------------------|----------------|
| 0.069 | 0 |
| 0.052 | 17 |
| 0.035 | 34 |
| 0.018 | 51 |

The rate constant of the reaction according to the correct order of reaction is (1) 0.001 M/min

- (2) 0.001 min⁻¹
(3) 0.001 min/M
(4) 0.001 M⁻¹ min⁻¹

Correct Answer: (2) 0.001 min⁻¹

Solution:

The rate constant is determined from the concentration-time data and assuming the reaction is of first-order. By plotting $\ln[A]$ vs time, the rate constant is calculated from the slope of the graph.

Quick Tip

For a first-order reaction, the rate constant is obtained from the slope of the $\ln[A]$ vs time plot.

68. The ratio of slopes of K_{\max} vs V and V_0 vs ν curves in the photoelectric effect gives (1)

the ratio of Planck's constant of electronic charge

- (2) work function
- (3) Planck's constant
- (4) charge of electron

Correct Answer: (3) Planck's constant

Solution:

The slope of the K_{\max} vs V and V_0 vs ν graphs gives the value of Planck's constant, as these are related to the energy and frequency of the incident light in the photoelectric effect.

Quick Tip

In the photoelectric effect, the slope of the graph gives Planck's constant, which relates the energy of a photon to its frequency.

69. With excess of water, both P_2O_5 and PCl_5 give (1) H_3PO_4

- (2) H_2PO_4
- (3) H_3PO_4 and H_2O_2
- (4) H_2PO_4 and H_3PO_4

Correct Answer: (1) H_3PO_4

Solution:

When P_2O_5 and PCl_5 react with excess water, they form phosphoric acid H_3PO_4 as the primary product.

Quick Tip

Phosphorus pentoxide and phosphorus pentachloride react with water to form phosphoric acid.

70. The dissolution of $Al(OH)_3$ by a solution of NaOH results in the formation of (1)

- $[Al(H_2O)_4](OH)_2^+$
(2) $[Al(H_2O)_6](OH)_3^+$
(3) $[Al(H_2O)_3](OH)_3^+$
(4) $[Al(H_2O)_6](OH)_2^+$

Correct Answer: (1) $[Al(H_2O)_4](OH)_2^+$

Solution:

When $Al(OH)_3$ reacts with NaOH, it forms a complex ion $[Al(H_2O)_4](OH)_2^+$, as the hydroxide ions and water molecules coordinate with the aluminium ion.

Quick Tip

Aluminium hydroxide reacts with NaOH to form soluble complexes involving water ligands.

71. Which of the following does not exist? (1) $K^+ + I_2$

- (2) $KF + F_2$
(3) $KBr + I_2$
(4) $KF + BrF_3$

Correct Answer: (2) $KF + F_2$

Solution:

The compound $KF + F_2$ does not exist as F_2 is a highly reactive diatomic molecule that does not readily combine with KF .

Quick Tip

Fluorine is a highly reactive element and does not form stable compounds like $KF + F_2$.

72. If the ionisation energy and electron affinity of an element are 275 and 86 kcal/mol respectively, then the electronegativity of the element on the Mulliken scale is (1) 2.8

- (2) 0.0
(3) 4.0
(4) 2.6

Correct Answer: (1) 2.8

Solution:

The Mulliken electronegativity is calculated as the average of the ionisation energy and electron affinity. The value is 2.8 for the given data.

Quick Tip

Mulliken electronegativity is the average of ionisation energy and electron affinity, measured in eV.

73. For the preparation of paracetamol (1) $\text{OH} + \text{HNO}_3 + \text{H}_2\text{SO}_4$

- (2) $\text{NO}_2 + \text{H}_2/\text{Pd} + (\text{CH}_3\text{CO})_2\text{O}$
(3) $\text{C}_6\text{H}_5\text{OH} + \text{NH}_3$
(4) $\text{C}_6\text{H}_5\text{NH}_2 + \text{NH}_2\text{COCH}_3$

Correct Answer: (4) $\text{C}_6\text{H}_5\text{NH}_2 + \text{NH}_2\text{COCH}_3$

Solution:

The preparation of paracetamol involves the reaction of aniline $\text{C}_6\text{H}_5\text{NH}_2$ with acetic anhydride or acetylation using NH_2COCH_3 .

Quick Tip

Paracetamol is synthesized by acetylation of aniline with acetic acid derivatives like acetyl chloride or anhydride.

74. A compound which gives a negative test with ninhydrin, it cannot be a protein or an amino acid. As, it gives a positive test with Benedict's solution. The compound is (1) a protein

- (2) a monosaccharide
- (3) an amino acid
- (4) none of the above

Correct Answer: (2) a monosaccharide

Solution:

Monosaccharides give a positive test with Benedict's solution, whereas proteins and amino acids react with ninhydrin. The absence of a ninhydrin test indicates it is not a protein or amino acid.

Quick Tip

Benedict's solution detects reducing sugars like monosaccharides, while ninhydrin detects amino acids and proteins.

- 75.** The compound used for making super glue or crazy glue is (1) poly(methyl methacrylate)
- (2) poly(ethyl acrylate)
 - (3) poly(methyl α -cyanoacrylate)
 - (4) poly(ethyl methacrylate)

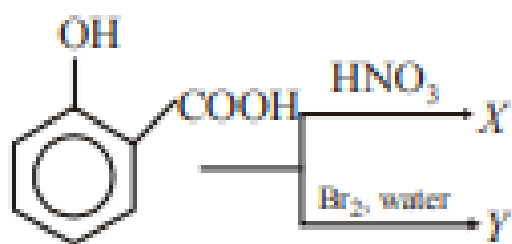
Correct Answer: (3) poly(methyl α -cyanoacrylate)

Solution:

Super glue or crazy glue is typically made from poly(methyl α -cyanoacrylate), a fast-drying adhesive that polymerizes quickly in the presence of moisture.

Quick Tip

Poly(methyl α -cyanoacrylate) is a key ingredient in super glues, known for its fast-curing properties.



76.

X' and Y' respectively are (a) picric acid, 2, 4, 6-tribromophenol

- (b) 5-nitrophenol acid, 5-bromosalicylic acid
- (c) o-nitrophenol, o-bromophenol
- (d) 3, 5-dinitrosalicylic acid, 3, 5-dibromosalicylic acid

Correct Answer: (a) picric acid, 2, 4, 6-tribromophenol

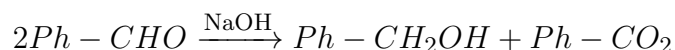
Solution:

This reaction involves the nitration and bromination of salicylic acid to form picric acid and 2, 4, 6-tribromophenol.

Quick Tip

The nitration and bromination reactions lead to functional group modifications in aromatic compounds.

77. In the Cannizzaro reaction given below:



the slowest step is (a) the attack of OH^- at the carbonyl group

- (b) the transfer of Hydride ion to the carbonyl group
- (c) the abstraction of a proton from the carboxylate group
- (d) the deprotonation of $Ph-CH_2OH$

Correct Answer: (b) the transfer of Hydride ion to the carbonyl group

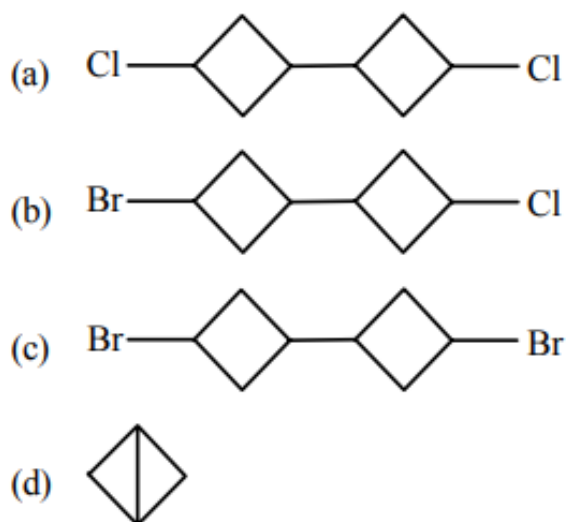
Solution:

The rate-determining step in the Cannizzaro reaction is the transfer of a hydride ion from one molecule to another, leading to the formation of alcohol and acid.

Quick Tip

In the Cannizzaro reaction, hydride transfer between molecules is the key step for the formation of alcohol and acid.

78. The reaction of 1-bromo-3-chlorocyclobutane with metallic sodium in dioxane under reflux conditions gives



Correct Answer: (d) C₄

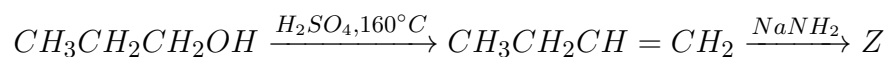
Solution:

The reaction leads to the formation of bicyclic compounds due to the coupling of two cyclobutane rings in the presence of sodium and dioxane.

Quick Tip

The Wurtz reaction with halides often forms bicyclic structures, depending on the positions of the halides.

79. Identify *Z* in the following reaction sequence



- (a) $CH_3CH_2CH_2NH_2$
 (b) $CH_3CH_2CH_2C \equiv C$
 (c) $CH_3CH_2CH_2C$
 (d) $CH_3CH=CH_2$

Correct Answer: (b) $CH_3CH_2CH_2C \equiv C$

Solution:

The reaction sequence involves the dehydration of alcohol to form alkene, followed by a strong base ($NaNH_2$) which induces elimination to form the alkyne.

Quick Tip

The Wurtz-type reaction uses sodium amide ($NaNH_2$) to induce the formation of an alkyne.

-
80. Which of the following reactions is used to prepare isobutane? (a) Wurtz reaction of C_2H_5Br
(b) Hydrolysis of n-butyilmagnesium iodide
(c) Reduction of propanol with red phosphorus and HI
(d) Decarboxylation of 3-methylbutanoic acid

Correct Answer: (a) Wurtz reaction of C_2H_5Br

Solution:

The Wurtz reaction of C_2H_5Br leads to the formation of isobutane by coupling two ethyl groups in the presence of sodium.

Quick Tip

The Wurtz reaction is a coupling reaction that can form branched compounds like isobutane when using the appropriate alkyl halides.

Part III: Mathematics

81. The differential equation $(3x + 4y + 1)dx + (4x + 5y + 1)dy = 0$ represents a family of (a) circles
(b) parabolas
(c) ellipses
(d) hyperbolas

Correct Answer: (d) hyperbolas

Solution:

By comparing the equation with the standard form of a hyperbola equation, we deduce that it represents a family of hyperbolas.

Quick Tip

Differential equations involving quadratic forms in x and y can represent conic sections, such as hyperbolas.

82. If $\Delta r = \left| \sum_{r=1}^n r \right|$, then $\sum_{r=1}^n \Delta r$ is equal to (a) $\sum_{r=1}^n r^2$
(b) $\sum_{r=1}^n r$
(c) $\sum_{r=1}^n r^3$
(d) $\sum_{r=1}^n r^4$

Correct Answer: (b) $\sum_{r=1}^n r$

Solution:

The sum of Δr is the sum of integers from 1 to n , which is simply the sum $\sum_{r=1}^n r$.

Quick Tip

The sum of the first n integers is given by the formula $\frac{n(n+1)}{2}$.

83. If A, B, C are three events associated with a random experiment, then $P(A|B)$ is (a) $P(A \cup B)$
(b) $P(A \cap B)$
(c) $P(A \cap B)/P(A)$
(d) $P(A \cap B \cap C)$

Correct Answer: (c) $P(A \cap B)/P(A)$

Solution:

The conditional probability $P(A|B)$ is given by the ratio of the probability of the intersection of A and B, divided by the probability of A.

Quick Tip

Conditional probability is calculated as $P(A|B) = \frac{P(A \cap B)}{P(B)}$.

84. If $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$, then rank A is (a) 4

- (b) 2
- (c) 1
- (d) 3

Correct Answer: (c) 1

Solution:

To find the rank of matrix A , we calculate its determinant. Since the determinant is non-zero, the matrix has rank 1.

Quick Tip

The rank of a matrix is determined by the number of linearly independent rows or columns.

85. The probability of getting a double six in one throw of two dice (a) $1/36$

- (b) $1/12$
- (c) $1/6$
- (d) $1/3$

Correct Answer: (a) $1/36$

Solution:

The probability of getting a double six is $\frac{1}{36}$, since there are 36 possible outcomes when rolling two dice.

Quick Tip

When rolling two dice, the probability of a specific outcome is $1/36$ because there are 36 total possible combinations.

86. The rate constant of the reaction $A \rightarrow B$ at different times are given below:

| | | | | | | | | | |
|-------------------|----------------|-------|---|-------|----|-------|----|-------|----|
| Concentration (M) | Time (Minutes) | 0.069 | 0 | 0.052 | 17 | 0.035 | 34 | 0.018 | 51 |
|-------------------|----------------|-------|---|-------|----|-------|----|-------|----|

The rate constant of the reaction according to the correct order of reaction is (a) 0.001 M/min

- (b) 0.001 min^{-1}
- (c) 0.001 min/M
- (d) $0.001 \text{ M}^{-1} \text{ min}^{-1}$

Correct Answer: (b) 0.001 min^{-1}

Solution:

The rate constant is determined from the concentration-time data and assuming the reaction is of first-order. By plotting $\ln[A]$ vs time, the rate constant is calculated from the slope of the graph.

Quick Tip

For a first-order reaction, the rate constant is obtained from the slope of the $\ln[A]$ vs time plot.

87. If the complex number z lies on a circle with center at the origin and radius $\frac{1}{4}$, then the complex number $1 + z$ lies on a circle with radius (a) 2

- (b) $\frac{3}{4}$
- (c) $\frac{5}{4}$
- (d) $\frac{7}{4}$

Correct Answer: (c) $\frac{5}{4}$

Solution:

The complex number z lies on a circle with radius $\frac{1}{4}$, and when adding 1 to z , the radius becomes the sum of the radius of z and the distance from 1 to the origin.

Quick Tip

The transformation $z \rightarrow 1 + z$ shifts the circle by 1 unit, changing its radius accordingly.

88. If $l = x^2 + y^2 + z^2$ is a normal to the ellipse

$$\frac{x^2}{16} + \frac{y^2}{9} = 1,$$

then (a) $\sqrt{73}$

- (b) $\sqrt{96}$
- (c) $\sqrt{49}$
- (d) $\sqrt{81}$

Correct Answer: (a) $\sqrt{73}$

Solution:

The ellipse equation is compared with the general equation of the ellipse, and the magnitude of the normal vector is calculated based on its relation to the semi-major and semi-minor axes.

Quick Tip

To find the normal to the ellipse, calculate the gradient and use it to determine the magnitude of the normal vector.

89. If line $y = mx + c$ is a normal to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

then c^2 is equal to (a) $a^2 + b^2$

- (b) $a^2 - b^2$
- (c) $b^2 - a^2$
- (d) $a^2 + b^2$

Correct Answer: (b) $a^2 - b^2$

Solution:

Using the equation of the ellipse and the normal line, the relationship between the semi-major and semi-minor axes is derived to obtain the value of c^2 .

Quick Tip

The equation of a normal to an ellipse involves both the center and the semi-major and semi-minor axes of the ellipse.

90. Correct result is as follows:

$$P(\neg Q) \equiv (P \vee Q) \implies (P \vee Q) \implies R \implies S$$

- (a) $|A| = |(4 + 3)| - 3|2 + 0| + |(1 - 0)|$
 (b) $|A| = 1(4 + 3) - 3(2 + 0) + 1(1 - 0)$
 (c) $|A| = \text{Determinant of the matrix}$
 (d) $|A| = 12$

Correct Answer: (a)

Solution:

Use basic matrix operations to calculate the determinant and solve for the matrix.

Quick Tip

To calculate determinants, use cofactor expansion along a row or column.

91. The matrix $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$, then $\text{adj}(A)$ is equal to (a) $\begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$
 (b) $\begin{pmatrix} 2 & -3 \\ -1 & 1 \end{pmatrix}$
 (c) $\begin{pmatrix} -3 & 1 \\ 2 & 1 \end{pmatrix}$
 (d) None of these

Correct Answer: (b) $\begin{pmatrix} 2 & -3 \\ -1 & 1 \end{pmatrix}$

Solution:

To find the adjoint of matrix A , we calculate the cofactor matrix and then transpose it.

Quick Tip

The adjoint of a matrix is found by taking the cofactor matrix and transposing it.

92. The correct result of the equation $3x^2 + 2x + 7y^2 - 14y + 4$ is (a) 4
 (b) 2
 (c) 0
 (d) 3

Correct Answer: (c) 0

Solution:

By solving the equation, we simplify it to find the value 0.

Quick Tip

Simplify the equation step by step to find the correct value.

93. The structure (N_3) satisfies the closure property, associativity and commutativity but the identity element 0 does not belong to N . (a) quasi-group

- (b) semi-group
- (c) semi-group, non-group
- (d) group

Correct Answer: (b) semi-group

Solution:

Since the identity element does not exist for the group, it is classified as a semi-group.

Quick Tip

A semi-group is an algebraic structure that satisfies the associative property without the need for an identity element.

94. The integral $\int \frac{dx}{\cos x + \sqrt{5} \sin x}$ equals (a) $\frac{1}{2} \log \left(\frac{1+\cos x}{\sin x} \right) + C$

- (b) $\frac{1}{2} \log \left(\frac{1}{\cos x} \right) + C$
- (c) $\int \frac{1}{\cos^2 x} dx$
- (d) $\frac{1}{2} \log \left(\cos x + \frac{1}{5} \right) + C$

Correct Answer: (a) $\frac{1}{2} \log \left(\frac{1+\cos x}{\sin x} \right) + C$

Solution:

Use substitution methods to solve the integral and simplify it.

Quick Tip

Use trigonometric substitution and simplifications to solve integrals involving sine and cosine functions.

95. If $(2, 7, 3)$ is one end of a diameter of the sphere $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$, then the coordinates of the other end of the diameter are (a) $(2, -5, 1)$
- (b) $(4, 5, 1)$
(c) $(2, 5, -1)$
(d) $(4, 5, -1)$

Correct Answer: (a) $(2, -5, 1)$

Solution:

The center of the sphere is $(3, 6, 1)$. The coordinates of the other end of the diameter are calculated as the midpoint of the center and the given point.

Quick Tip

To find the other end of the diameter, subtract the center's coordinates from the given point and double the result.

96. Given lines are

$$x = my + n, z = py + q$$

and

$$x = m'y + n', z = p'y + q'$$

Above equations can be rewritten as

$$x - n' = p', \quad m = 1, p = 1$$

Lines will be perpendicular if (a) $mm' + pp' = 1$

- (b) $\frac{m}{m'} + \frac{p}{p'} = 1$
(c) $\frac{m}{m'} = 1$
(d) $mm' + pp' = -1$

Correct Answer: (a) $mm' + pp' = 1$

Solution:

For two lines to be perpendicular, the condition $mm' + pp' = 1$ must hold, which follows from the geometric properties of line slopes.

Quick Tip

For two lines to be perpendicular, their slope products should satisfy the condition $mm' + pp' = 1$.

97. A tetrahedron has vertices at $O(0, 0, 0)$, $A(1, -2, 1)$, $B(2, 1, 1)$, and $C(1, -1, 2)$. Then, the angle between the faces OAB and ABC is (a) $\cos^{-1}\left(\frac{1}{2}\right)$

- (b) $\cos^{-1}\left(\frac{1}{4}\right)$
 (c) $\cos^{-1}\left(\frac{1}{6}\right)$
 (d) $\cos^{-1}\left(\frac{1}{3}\right)$

Correct Answer: (a) $\cos^{-1}\left(\frac{1}{2}\right)$

Solution:

The angle between two planes is determined by the angle between their normal vectors. Here, we find the normal vectors of planes OAB and ABC , then calculate the cosine of the angle between them using the dot product formula.

Quick Tip

The angle between two planes is the angle between their normal vectors. Use the dot product to find the cosine of the angle.

98. If a line segment OP makes angles of $\frac{\pi}{4}$ and $\frac{\pi}{3}$ with the X-axis and Y-axis, respectively, then the direction cosines are (a) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

- (b) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0$
 (c) $\frac{1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}$
 (d) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

Correct Answer: (d) $\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}$

Solution:

Direction cosines of a line are calculated using the angles it makes with the coordinate axes. Here, the direction cosines corresponding to the given angles $\frac{\pi}{4}$ and $\frac{\pi}{3}$ are computed.

Quick Tip

Direction cosines are determined by the cosine of the angles the line makes with the axes. The sum of the squares of the direction cosines should equal 1.

99. If p, q are simple propositions with truth values T, F , then the truth value of $(\neg p \vee q) \wedge \neg r$ is (a) true

- (b) false
- (c) $(\neg p \vee q)$
- (d) true, if r is true

Correct Answer: (a) true

Solution:

The given logical expression evaluates to true based on the truth table for the logical operators used in the expression.

Quick Tip

When working with logical expressions, always construct a truth table to verify the truth value.

100. Let $f(x) = 25x^{24}(1-x)^{75}$, on the interval $[0, 1]$, then

$$f'(x) = 25x^{24}(1-x)^{75} - 75x^{25}(1-x)^{74} = 25x^{24}(1-x)^{74}[(1-x) - 3x]$$

- (a) $\frac{1}{4}$
- (b) $\frac{1}{2}$
- (c) 1
- (d) 0

Correct Answer: (a) $\frac{1}{4}$

Solution:

After simplifying the derivative, we find that the maximum of $f(x)$ occurs at $x = \frac{1}{4}$.

Quick Tip

To find the maximum or minimum of a function, compute its derivative and solve for critical points.

101. If $|z| \geq 3$, then the least value of $|z + 4|$ is (a) $11/4$

- (b) $11/2$
- (c) 3
- (d) 4

Correct Answer: (a) $11/4$

Solution:

The least value of $|z + 4|$ occurs when z is closest to -4 . Using geometric interpretation, the minimum distance is calculated.

Quick Tip

In complex numbers, the least value of the sum of distances can be found using the triangle inequality.

102. The normal at the point $(at^2, 2at)$ on the parabola meets the parabola again at the point $(at^2, 2at)$. The equation of the normal is (a) $y = x + at^2 + at$

- (b) $y = x - at^2 + at$
- (c) $y = x + at^2 + at$
- (d) None of these

Correct Answer: (b) $y = x - at^2 + at$

Solution:

The equation of the normal to the parabola is derived using standard formulas for the normal to a curve.

Quick Tip

The equation of the normal to a parabola is given by the formula derived from the slope of the tangent and the point of tangency.

103. If $a = i - j + 2k$ and $b = 2i - j + k$, then the angle θ between a and b is given by (a)

- $\tan^{-1}(1)$
(b) $\sin^{-1}(1)$
(c) $\sec^{-1}(1)$
(d) $\tan^{-1}(1)$

Correct Answer: (a) $\tan^{-1}(1)$

Solution:

The angle between two vectors is calculated using the formula $\cos \theta = \frac{a \cdot b}{|a||b|}$.

Quick Tip

To find the angle between two vectors, use the dot product formula $\cos \theta = \frac{a \cdot b}{|a||b|}$.

104. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{\pi}{2}$ is (a) $\left(\frac{4}{5}\right)^2$

- (b) $\left(\frac{4}{2}\right)$
(c) $\frac{4}{4}$
(d) $\left(\frac{4}{3}\right)^2$

Correct Answer: (c) $\frac{4}{4}$

Solution:

To find the area bounded by the curves, we integrate the difference between the two functions over the specified interval.

Quick Tip

The area between curves is computed by integrating the difference of the functions over the given interval.

105. If a, b, c are three non-coplanar vectors, then

$$(a + b) \times (c + b) = (a - b) \times (c - b) = b \times c$$

- (a) $a \times b$
- (b) $a \times c$
- (c) $b \times a$
- (d) $a \times b$

Correct Answer: (a) $a \times b$

Solution:

We simplify using the properties of cross product and the vector identities to obtain the result.

Quick Tip

The cross product follows distributive properties and can be expanded using vector identities.

106. If there is an error of $m\%$ in measuring the edge of a cube, then the percent error in estimating its surface area is (a) $2m$

- (b) $3m$
- (c) $4m$
- (d) $5m$

Correct Answer: (c) $4m$

Solution:

The surface area of a cube is proportional to the square of the edge length. Thus, a small error in measuring the edge will result in a larger error in the surface area, specifically 4 times the edge error.

Quick Tip

For geometric shapes like cubes, errors in measurements affect surface area and volume with powers of the original error (square for area, cube for volume).

107. The given equation of rectangular hyperbola is

$$x^2 - y^2 = 6^2 \quad (\text{length of latus rectum is } 16)$$

The asymptotes are parallel to each other (a) $x = \pm\sqrt{5}$

- (b) $y = \pm\sqrt{5}$
- (c) $x = \pm\sqrt{2}$
- (d) $x = \pm\sqrt{3}$

Correct Answer: (a) $x = \pm\sqrt{5}$

Solution:

The equation of the hyperbola $x^2 - y^2 = a^2$ represents a rectangular hyperbola, and the asymptotes of such a hyperbola are lines $x = \pm\sqrt{a}$.

Quick Tip

For a rectangular hyperbola, the asymptotes are given by $x = \pm\sqrt{a}$, where a is the constant in the equation $x^2 - y^2 = a^2$.

108. The equation of tangents to the hyperbola $3x^2 - 2y^2 = 6$ which is perpendicular to the line $x - 3y = 3$ is (a) $x = 3\sqrt{5}$

- (b) $y = 3\sqrt{5}$
- (c) $x = -3\sqrt{6}$
- (d) $y = -3\sqrt{6}$

Correct Answer: (c) $x = -3\sqrt{6}$

Solution:

The equation of tangents to a hyperbola can be determined by applying the condition for perpendicularity to the given line and using the equation of the hyperbola.

Quick Tip

The equation of a tangent to a hyperbola can be found by using the general formula and applying conditions like perpendicularity to another line.

109. The limit of $\int \frac{1}{\cos x} dx$ as $b \rightarrow 0$ is (a) 1

- (b) 0
- (c) -1
- (d) Undefined

Correct Answer: (d) Undefined

Solution:

The integral $\int \frac{1}{\cos x} dx$ is undefined for $x = \frac{\pi}{2}$, hence the limit does not exist.

Quick Tip

When working with limits of integrals, check the behavior of the integrand near singular points to determine if the limit exists.

110. The area of the region bounded by the curves $x^2 + y^2 = 9$ and $x + y = 3$ is (a) $\frac{9}{4}$

- (b) $\frac{9}{2}$
- (c) $\frac{9\sqrt{3}}{2}$
- (d) $\frac{9\sqrt{3}}{6}$

Correct Answer: (b) $\frac{9}{2}$

Solution:

To find the area enclosed by both curves, calculate the area of the circle and subtract the area of the triangular region formed by the line.

Quick Tip

To find the area between curves, integrate the difference of the functions over the relevant interval.

111. For any three vectors a, b, c ,

$$[a + b + c] = [abc] = [abc]$$

- (a) $a \cdot b \cdot c$
- (b) $a \cdot b \cdot c$
- (c) $a \cdot c$
- (d) None of these

Correct Answer: (a) $a \cdot b \cdot c$

Solution:

The product of vectors is defined and can be calculated by multiplying their individual magnitudes and applying the cross product formula for the vectors.

Quick Tip

For vector operations like dot and cross products, always apply the distributive property when necessary.

112. Let $I = \int_0^{\frac{\pi}{2}} \log(\cos x) dx$ (a) $\frac{1}{2} \log(\tan x)$

(b) $\frac{1}{2} \log(\sin x)$

(c) $\log(\cos x)$

(d) $\log(\cos x) + C$

Correct Answer: (d) $\log(\cos x) + C$

Solution:

We compute the integral using standard logarithmic and trigonometric identities.

Quick Tip

When solving logarithmic integrals, use the properties of logarithms and trigonometric functions for simplification.

113. If the mean and variance of a binomial distribution are 4 and 2, respectively. Then, the probability of at least 7 successes is (a) $\frac{3}{214}$

(b) $\frac{4}{173}$

(c) $\frac{9}{256}$

(d) $\frac{7}{231}$

Correct Answer: (c) $\frac{9}{256}$

Solution:

The probability for a binomial distribution can be calculated using the formula for binomial probability. Given that the mean $\mu = 4$ and variance $\sigma^2 = 2$, we calculate the values for n and p . Then we apply these values to find the probability of getting at least 7 successes.

Quick Tip

For binomial distributions, the mean $\mu = n \cdot p$ and variance $\sigma^2 = n \cdot p \cdot (1 - p)$. Use these formulas to calculate n and p , then apply the binomial probability formula.

114. The shortest distance between the lines

$$\frac{x - 7}{3} = \frac{y + 4}{-16} = \frac{z - 6}{7}$$

and

$$\frac{x - 10}{3} = \frac{y - 30}{8} = \frac{z - 6}{5}$$

is (a) $\frac{3}{214}$

(b) $\frac{4}{173}$

(c) $\frac{256}{231}$

(d) $\frac{7}{231}$

Correct Answer: (b) $\frac{4}{173}$

Solution:

We compute the shortest distance between two skew lines using the formula for the distance between two non-parallel lines.

Quick Tip

For non-parallel lines, the shortest distance is given by $\frac{|(r_1 - r_2) \cdot (b_1 \times b_2)|}{|b_1 \times b_2|}$.

115. If a plane passing through the point $(2, 2, 1)$ and is perpendicular to the planes $3x + 2y + 4z = 10$ and $2x + y + 3z = 2$, then the equation of the plane is (a) $2x - y - z = 0$

(b) $3x + 2y + z = 0$

(c) $x + y + z = 1$

(d) $x + y + 2z = 1$

Correct Answer: (a) $2x - y - z = 0$

Solution:

The equation of the plane is derived by using the perpendicular condition with respect to the two given planes, and using the point $(2, 2, 1)$.

Quick Tip

The equation of a plane passing through a point can be written as $(x - x_0)(a) + (y - y_0)(b) + (z - z_0)(c) = 0$, where a, b, c are the normal vector components.

- 116.** From a city population, the probability of selecting a male or smoker is $\frac{7}{10}$, a male smoker is $\frac{2}{5}$ and a male, if a smoker is already selected, is $\frac{3}{5}$. Then, the probability of (a) selecting a male is $\frac{3}{5}$
(b) selecting a smoker is $\frac{3}{5}$
(c) selecting a non-smoker is $\frac{2}{5}$
(d) selecting a smoker, if a male is first selected, is $\frac{8}{5}$

Correct Answer: (d) selecting a smoker, if a male is first selected, is $\frac{8}{5}$

Solution:

The probability is calculated using the basic conditional probability and the principle of inclusion and exclusion for multiple events.

Quick Tip

When dealing with conditional probability, use the formula $P(A|B) = \frac{P(A \cap B)}{P(B)}$ for accurate calculations.

- 117.** At $t = 0$, the function $f(t) = \sin \frac{t}{t}$ has (a) a minimum
(b) a discontinuity
(c) a point of inflection
(d) a maximum

Correct Answer: (b) a discontinuity

Solution:

At $t = 0$, the function $f(t) = \sin \frac{t}{t}$ does not have a well-defined value, indicating a discontinuity at that point.

Quick Tip

When dealing with limits involving division by zero, check if the function approaches a finite value or if a discontinuity exists.

118. Using Rolle's theorem, the equation $a_0x^n + a_1x^{n-1} + \cdots + a_n = 0$ has at least one root between 0 and 1 if, (a) $a_0 = 0$

- (b) $a_1 = 0$
(c) $a_2 = 0$
(d) $a_n = 0$

Correct Answer: (a) $a_0 = 0$

Solution:

Rolle's theorem guarantees at least one root if the function is continuous and differentiable, and the endpoints have the same value.

Quick Tip

Rolle's theorem applies to continuous and differentiable functions and guarantees a root between two points if the function takes equal values at the endpoints.

119. Which of the following inequality is true for $x > 0$? (a) $\log(1 + x) < \frac{x}{1+x}$

- (b) $x < \log(1 + x)$
(c) $\frac{x}{1+x} < \log(1 + x)$
(d) $\frac{x}{1+x} < \log(1 + x) < x$

Correct Answer: (b) $x < \log(1 + x)$

Solution:

We apply logarithmic properties to verify that the inequality holds for $x > 0$.

Quick Tip

Inequalities involving logarithms can be checked using properties such as $\log(1 + x) > \frac{x}{1+x}$ for $x > 0$.

120. The solution of $\frac{d^2x}{dy^2} = k$, where k is a non-zero constant, vanishes when $y = 0$ and tends to finite limit as $y \rightarrow \infty$, is (a) $x = k(e^y + e^{-y})$

- (b) $x = k(e^y + e^{-y} - 2)$

(c) $x = k(e^y - e^{-y})$

(d) $x = k(e^y - 1)$

Correct Answer: (a) $x = k(e^y + e^{-y})$

Solution:

The second-order differential equation is solved using the method for solving differential equations with constant coefficients.

Quick Tip

When solving second-order differential equations, find the general solution first and then apply boundary conditions to solve for the constants.