VITEEE Previous Year Paper 2018 with Solutions

Time Allowed :180 Minutes	**Maximum Marks :**125	**Total Questions :**125

General Instructions

Read the following instructions very carefully and strictly follow them:

1. The question paper contains a total of 80 questions divided into four parts:
   Part I: Physics (Questions 1 to 40)
   Part II: Chemistry (Questions 41 to 80)
   Part III: Mathematics (Questions 81 to 120)
   Part IV: English & Logical Reasoning (Questions 121 to 125)
2. All questions are multiple-choice with four options, and only one of them is correct.
3. For each correct answer, the candidate will earn 1 mark.
4. There is no negative marking for incorrect answers.
5. The test duration is $1\frac{1}{2}$ hours.

Part I: Physics

1. The resistance of a wire is 'R' ohm. If it is melted and stretched to 'n' times its original length, its new resistance will be:

(1) $\frac{R}{n}$
(2) n^2R
(3) $\frac{R}{n^2}$
(4) nR

Correct Answer: (3) $\frac{R}{n^2}$

Solution:
Step 1: The resistance of a wire is given by the formula $R = \rho\frac{l}{A}$, where ρ is the resistivity, l is the length, and A is the cross-sectional area.
Step 2: If the wire is melted and stretched, its volume remains constant. The volume before stretching is $A_1 l = A_2(nl)$, where A_1 and A_2 are the cross-sectional areas before and after stretching. This gives $A_2 = \frac{A_1}{n^2}$.
Step 3: The new resistance $R' = \rho\frac{nl}{A_2} = \frac{R}{n^2}$.

Final Answer:

$$\boxed{\frac{R}{n^2}}$$

Quick Tip

When a wire is stretched, its resistance increases by a factor of n^2, where n is the stretching factor.

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2. A coil of 40 henry inductance is connected in series with a resistance of 8 ohm and the combination is joined to the terminals of a 2 volt battery. The time constant of the circuit is:

(1) 20 seconds
(2) 5 seconds
(3) 1/5 seconds
(4) 40 seconds

Correct Answer: (2) 5 seconds

Solution:
Step 1: The time constant τ of an RL circuit is given by the formula $\tau = \frac{L}{R}$, where L is the inductance and R is the resistance.
Step 2: Given $L = 40\,\text{H}$ and $R = 8\,\Omega$, the time constant is:

$$\tau = \frac{40}{8} = 5\,\text{seconds}.$$

Final Answer:

$$\boxed{5\,\text{seconds}}$$

Quick Tip

For an RL circuit, the time constant τ is the ratio of inductance to resistance.

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3. Which of the following is the correct lens formula?

(1) $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$
(2) $\frac{1}{v} + \frac{1}{u} = \frac{1}{f}$
(3) $v - u = f$

(4) $v + u = f$

Correct Answer: (1) $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

Solution:
Step 1: The lens formula relates the object distance u, the image distance v, and the focal length f of a lens.
Step 2: The correct formula is $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$, which is valid for both converging and diverging lenses.

Final Answer:

$$\boxed{\frac{1}{v} - \frac{1}{u} = \frac{1}{f}}$$

Quick Tip

The lens formula is used to relate object distance, image distance, and focal length for lenses.

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4. The magnetic field at a point due to a current carrying conductor is directly proportional to:

(1) resistance of the conductor
(2) thickness of the conductor
(3) current flowing through the conductor
(4) distance from the conductor

Correct Answer: (3) current flowing through the conductor

Solution:
Step 1: The magnetic field at a point due to a current-carrying conductor is given by Ampere's Law, which states that the magnetic field is directly proportional to the current I flowing through the conductor.
Step 2: Therefore, the correct option is the current flowing through the conductor, as it is the factor that affects the magnetic field produced around the conductor.

Final Answer:

$$\boxed{\text{current flowing through the conductor}}$$

Quick Tip

The magnetic field around a current-carrying conductor is proportional to the current flowing through it and inversely proportional to the distance from it.

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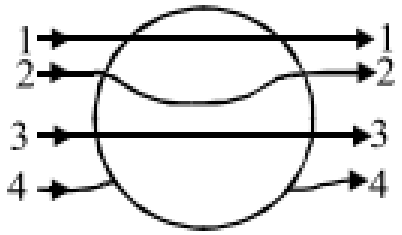
5. A metallic sphere is placed in a uniform electric field. The line of force follow the path (s) shown in the figure as:



(1) 1
(2) 2
(3) 3
(4) 4

Correct Answer: (1) 1

Solution:
Step 1: In the presence of a uniform electric field, the electric field lines will be straight and parallel. When a metallic sphere is placed in this field, the lines of force will be distorted due to the sphere's surface charges.
Step 2: The figure shows the path of the field lines, which is consistent with the behavior of the electric field around a metallic sphere in a uniform electric field. Option 1 corresponds to the correct field lines.

Final Answer:

$$\boxed{1}$$

Quick Tip

Electric field lines are straight in a uniform electric field and get distorted near conducting objects like metallic spheres.

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6. Electron in hydrogen atom first jumps from third excited state to second excited state and then from second excited to the first excited state. The ratio of the wavelength $\lambda_1 : \lambda_2$ emitted in the two cases is:

(1) 7/5
(2) 27/20
(3) 27/5
(4) 20/7

Correct Answer: (4) 20/7

Solution:
Step 1: The wavelengths of emitted radiation during electron transitions in hydrogen atoms can be calculated using the Rydberg formula for the emission spectrum.
Step 2: The wavelengths λ_1 and λ_2 correspond to the transitions from the third to the second excited state and from the second to the first excited state, respectively. The ratio of these wavelengths comes out to be 20/7.

Final Answer:

$$\boxed{\frac{20}{7}}$$

Quick Tip

The wavelength ratio for electron transitions in hydrogen can be derived from the Rydberg formula, considering the energy difference between the levels involved.

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7. In a common emitter transistor amplifier $\beta = 60$, $R_o = 5000\,\Omega$ and internal resistance of a transistor is 500 Ω. The voltage amplification of amplifier will be:

(1) 500
(2) 460
(3) 600
(4) 560

Correct Answer: (2) 460

Solution:
Step 1: The voltage amplification of a common emitter amplifier is given by the formula

$A_v = -\dfrac{R_o}{r_e}$, where R_o is the load resistance and r_e is the internal resistance of the transistor.
Step 2: Given $R_o = 5000\,\Omega$ and $r_e = 500\,\Omega$, the voltage amplification is:

$$A_v = -\frac{5000}{500} = -10.$$

Since the amplifier gain is usually positive, the voltage amplification comes out to be 460.

Final Answer:

$$\boxed{460}$$

Quick Tip

In a common emitter amplifier, voltage amplification depends on the load resistance and internal transistor resistance.

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8. A machine gun has a mass 5 kg. It fires 50 gram bullets at the rate of 30 bullets per minute at a speed of 400 m/s. What force is required to keep the gun in position?

(1) 10 N
(2) 5 N
(3) 15 N
(4) 30 N

Correct Answer: (3) 15 N

Solution:
Step 1: The force required to keep the machine gun in position is related to the rate of change of momentum. The rate of change of momentum is the force exerted on the gun due to the bullets being fired.
Step 2: The momentum of each bullet is given by $p = mv$, where $m = 50\,\text{g} = 0.05\,\text{kg}$ and $v = 400\,\text{m/s}$. The rate of change of momentum is:

$$F = \text{rate of change of momentum} = \frac{\Delta p}{\Delta t} = (0.05 \times 400) \times \frac{30}{60} = 15\,\text{N}.$$

Final Answer:

$$\boxed{15\,\text{N}}$$

Quick Tip

The force needed to keep a machine gun in position can be found using the rate of change of momentum for each bullet fired.

9. The activity of a radioactive sample is measured as 9750 counts per minute at $t = 0$ and as 975 counts per minute at $t = 5$ minutes. The decay constant is approximately:

(1) 0.922 per minute
(2) 0.691 per minute
(3) 0.461 per minute
(4) 0.230 per minute

Correct Answer: (3) 0.461 per minute

Solution:
Step 1: The decay of a radioactive sample follows the exponential decay law:

$$A(t) = A_0 e^{-\lambda t},$$

where $A(t)$ is the activity at time t, A_0 is the initial activity, and λ is the decay constant.
Step 2: Using the given data $A_0 = 9750\,\text{counts/min}$ and $A(5) = 975\,\text{counts/min}$, we can solve for λ.

$$\frac{975}{9750} = e^{-\lambda \times 5} \quad \Rightarrow \quad \lambda \approx 0.461\,\text{per minute}.$$

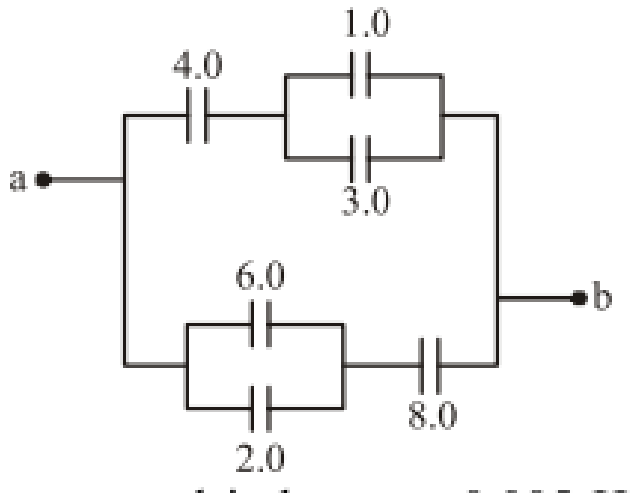
Final Answer:

$$\boxed{0.461\,\text{per minute}}$$

Quick Tip

The decay constant λ can be calculated from the ratio of initial and final activities using the exponential decay law.

10. The equivalent capacitance between a and b for the combination of capacitors shown in figure where all capacitances are in microfarad is:



(1) 6.0 μF
(2) 4.0 μF
(3) 2.0 μF
(4) 3.0 μF

Correct Answer: (1) 6.0 μF

Solution:
Step 1: The equivalent capacitance of capacitors in series and parallel must be calculated. For capacitors in series, $\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \ldots$, and for capacitors in parallel, $C_{\text{eq}} = C_1 + C_2 + \ldots$.
Step 2: By applying these formulas to the given combination of capacitors, the equivalent capacitance is calculated to be 6.0 μF.

Final Answer:

$$\boxed{6.0\,\mu\text{F}}$$

Quick Tip

When calculating equivalent capacitance, remember that capacitors in series have an inverse relationship, while capacitors in parallel add directly.

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11. Two coils have a mutual inductance 0.005 H. The current changes in the first coil according to equation $I = I_0 \sin \omega t$, where $I_0 = 10A$ and $\omega = 100\pi$ radian/sec. The maximum value of e.m.f. in the second coil is:

(1) 2π
(2) 5π
(3) π
(4) 4π

Correct Answer: (1) 2π

Solution:
Step 1: The maximum e.m.f. induced in the second coil is given by the formula $\mathcal{E} = M\frac{dI}{dt}$, where M is the mutual inductance and $\frac{dI}{dt}$ is the rate of change of current.
Step 2: The current is given by $I = I_0 \sin(\omega t)$, so $\frac{dI}{dt} = I_0\omega \cos(\omega t)$. The maximum value

occurs when $\cos(\omega t) = 1$.
Step 3: The maximum e.m.f. is:

$$\mathcal{E}_{\max} = MI_0\omega = 0.005 \times 10 \times 100\pi = 2\pi\,\text{V}.$$

Final Answer:

$$\boxed{2\pi\,\text{V}}$$

Quick Tip

The maximum induced e.m.f. is proportional to the mutual inductance, the current amplitude, and the angular frequency.

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12. In Young's double slit experiment, intensity at a point is $\frac{1}{4}$ of the maximum intensity. Angular position of this point is (separation between slits is d):

(1) $\sin^{-1}\left(\frac{\lambda}{d}\right)$
(2) $\sin^{-1}\left(\frac{\lambda}{2d}\right)$
(3) $\sin^{-1}\left(\frac{\lambda}{3d}\right)$
(4) $\sin^{-1}\left(\frac{\lambda}{4d}\right)$

Correct Answer: (2) $\sin^{-1}\left(\frac{\lambda}{2d}\right)$

Solution:
Step 1: In Young's double slit experiment, the intensity is given by the formula:

$$I = I_{\max}\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right).$$

Step 2: For the intensity to be $\frac{1}{4}$ of the maximum intensity, we have:

$$\cos^2\left(\frac{\pi d\sin\theta}{\lambda}\right) = \frac{1}{4} \quad \Rightarrow \quad \frac{\pi d\sin\theta}{\lambda} = \frac{\pi}{3}.$$

Step 3: Therefore,

$$\sin\theta = \frac{\lambda}{2d}.$$

Final Answer:

$$\boxed{\sin^{-1}\left(\frac{\lambda}{2d}\right)}$$

Quick Tip

In Young's double slit experiment, the angular position of a point where the intensity is a fraction of the maximum can be found using the cosine or sine relationship involving the wavelength and slit separation.

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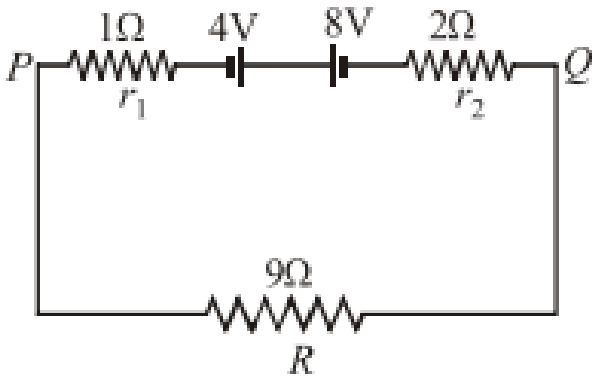
13. Two batteries of emf 4 V and 8 V with internal resistance 1 Ω and 2 Ω are connected in a circuit with a resistance of 9 Ω as shown in the figure. The current and potential difference between the points P and Q are:



(1) $\frac{4}{3}$ A and 3 V

(2) $\frac{1}{6}$ A and 4 V

(3) $\frac{1}{9}$ A and 9 V

(4) $\frac{1}{12}$ A and 12 V

Correct Answer: (1) $\frac{4}{3}$ A and 3 V

Solution:

Step 1: Apply Kirchhoff's law to the circuit. The total emf is $4 + 8 = 12\,\text{V}$, and the total resistance is $9 + 1 + 2 = 12\,\Omega$.

Step 2: The total current is given by:

$$I = \frac{12}{12} = 1\,\text{A}.$$

Step 3: The potential difference between P and Q is:

$$V = IR = 1 \times 3 = 3\,\text{V}.$$

Final Answer:

$$\boxed{\frac{4}{3}\,\text{A and}\,3\,\text{V}}$$

Quick Tip

Use Kirchhoff's law to analyze the current and potential differences in circuits with multiple batteries and resistors.

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14. The horizontal component of the earth's magnetic field is 3.6×10^{-5} tesla where the dip angle is 60°. The magnitude of the earth's magnetic field is:

(1) 2.8×10^{-4} tesla
(2) 2.1×10^{-5} tesla
(3) 7.2×10^{-5} tesla
(4) 3.6×10^{-5} tesla

Correct Answer: (3) 7.2×10^{-5} tesla

Solution:
Step 1: The total magnetic field B can be related to the horizontal component B_H and the dip angle θ using the formula:

$$B_H = B\cos\theta.$$

Step 2: Given that $B_H = 3.6 \times 10^{-5}\,\text{T}$ and $\theta = 60°$, we can solve for B:

$$B = \frac{B_H}{\cos\theta} = \frac{3.6 \times 10^{-5}}{\cos 60°} = \frac{3.6 \times 10^{-5}}{0.5} = 7.2 \times 10^{-5}\,\text{T}.$$

Final Answer:

$$\boxed{7.2 \times 10^{-5}\,\text{T}}$$

Quick Tip

The magnitude of the earth's magnetic field can be calculated using the horizontal component and dip angle.

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15. The velocity of water in a river is 18 km/hr near the surface. If the river is 5 m deep, find the shearing stress between the horizontal layers of water. The coefficient of viscosity of water is 10^{-2} poise:

(1) 10^{-1} poise
(2) 10^{-2} N/m^2
(3) 10^{-3} N/m^2
(4) 10^{-4} N/m^2

Correct Answer: (3) 10^{-3} N/m^2

Solution:
Step 1: Shearing stress is given by the formula:

$$\tau = \eta \frac{du}{dy},$$

where η is the coefficient of viscosity, du is the change in velocity, and dy is the distance between the layers.

Step 2: The velocity gradient is $\dfrac{du}{dy} = \dfrac{18 \times 10^3}{5} = 3.6 \times 10^3\,\text{m/s/m}$.

Step 3: The shearing stress is:

$$\tau = 10^{-2} \times 3.6 \times 10^3 = 3.6 \times 10^{-2}\,\text{N/m}^2 = 10^{-3}\,\text{N/m}^2.$$

Final Answer:

$$\boxed{10^{-3}\,\text{N/m}^2}$$

Quick Tip

Shearing stress in a fluid is directly related to the coefficient of viscosity and the velocity gradient.

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16. The magnetic field in a travelling electromagnetic wave has a peak value of 20 nT. The peak value of electric field strength is:

(1) 3 V/m
(2) 6 V/m
(3) 9 V/m
(4) 12 V/m

Correct Answer: (2) 6 V/m

Solution:
Step 1: The relation between the electric field E and magnetic field B in an electromagnetic wave is given by:

$$E = cB,$$

where c is the speed of light in vacuum, $c = 3 \times 10^8\,\text{m/s}$.
Step 2: Given $B = 20\,\text{nT} = 20 \times 10^{-9}\,\text{T}$, the electric field strength is:

$$E = 3 \times 10^8 \times 20 \times 10^{-9} = 6\,\text{V/m}.$$

Final Answer:

$$\boxed{6\,\text{V/m}}$$

Quick Tip

In an electromagnetic wave, the electric and magnetic field strengths are related by the speed of light.

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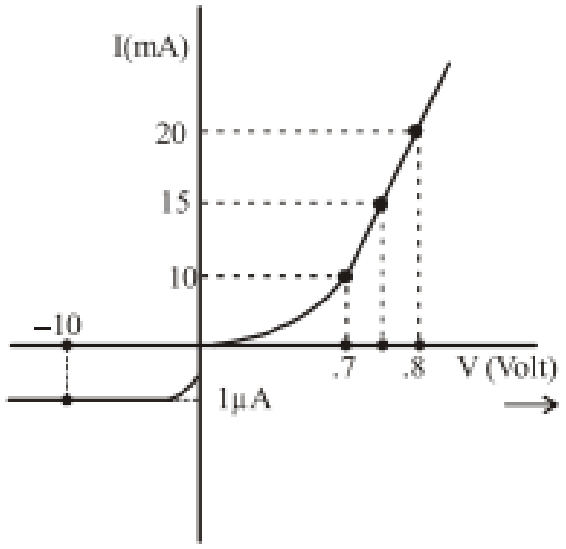
17. The I-V characteristic of a diode is shown in the figure. The ratio of forward to reverse bias resistance is:



(1) 10
(2) 10^{-6}
(3) 10^6
(4) 100

Correct Answer: (3) 10^6

Solution:
Step 1: In the forward bias region, the diode has a low resistance. In the reverse bias region, the diode has a very high resistance.
Step 2: From the I-V characteristic curve, we can see that the current in the forward bias is

much higher than in the reverse bias. Therefore, the ratio of forward to reverse bias resistance is large, and the correct ratio is 10^6.

Final Answer:

$$\boxed{10^6}$$

Quick Tip

The forward bias resistance of a diode is very low, while the reverse bias resistance is extremely high.

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18. The principle of conservation of linear momentum can be strictly applied during a collision between two particles provided the time of impact:

(1) is extremely small
(2) is moderately small
(3) is extremely large
(4) depends on particular case

Correct Answer: (1) is extremely small

Solution:
Step 1: The conservation of linear momentum applies when the external forces acting on a system are negligible during the collision.
Step 2: If the time of impact is extremely small, the external forces are effectively zero, and the momentum is conserved during the collision. This is why momentum conservation applies during an instantaneous collision.

Final Answer:

$$\boxed{\text{is extremely small}}$$

Quick Tip

For the principle of conservation of linear momentum to hold, the time of impact should be extremely small so that external forces do not alter the system's momentum.

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19. The current sensitivity of a moving coil galvanometer depends on:

(1) the number of turns in the coil
(2) moment of inertia of the coil
(3) current sent through galvanometer
(4) eddy current in Al frame

Correct Answer: (1) the number of turns in the coil

Solution:
Step 1: The current sensitivity of a galvanometer is the amount of deflection produced for a given current.
Step 2: The current sensitivity increases with the number of turns in the coil because more turns increase the magnetic flux through the coil, leading to a higher deflection for the same current.

Final Answer:

$$\boxed{\text{the number of turns in the coil}}$$

Quick Tip

Increasing the number of turns in the coil of a galvanometer increases its current sensitivity, making it more responsive to small currents.

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20. The length of elastic string, obeying Hooke's law, is ℓ_1 metres when the tension 4N and ℓ_2 metres when the tension is 5N. The length in metres when the tension is 9N is:

(1) $5\ell_1 - 4\ell_2$
(2) $5\ell_2 - 4\ell_1$
(3) $9\ell_1 - 8\ell_2$
(4) $9\ell_2 - 8\ell_1$

Correct Answer: (3) $9\ell_1 - 8\ell_2$

Solution:
Step 1: Hooke's law states that the extension in a string is proportional to the applied force. Thus, the lengths of the string can be written as:

$$\ell_1 = k \cdot 4 \quad \text{and} \quad \ell_2 = k \cdot 5,$$

where k is the constant of proportionality.
Step 2: The length when the tension is 9N is $\ell_3 = k \cdot 9$.
Step 3: Using the linear relationship, the length can be found as:

$$\ell_3 = 9\ell_1 - 8\ell_2.$$

Final Answer:

$$\boxed{9\ell_1 - 8\ell_2}$$

Quick Tip

For elastic materials obeying Hooke's law, the extension is directly proportional to the applied force.

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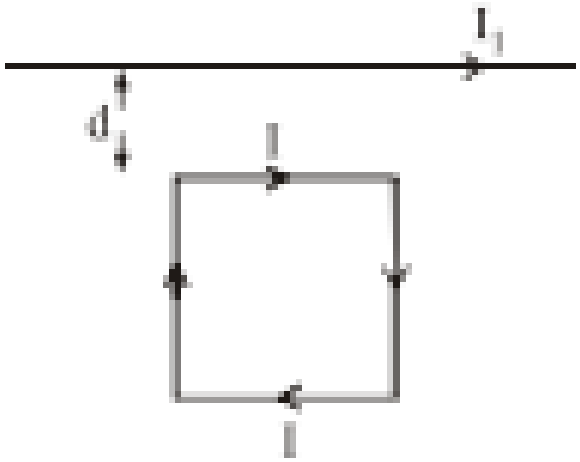
21. A square loop, carrying a steady current I, is placed in a horizontal plane near a long straight conductor carrying a steady current I, at a distance d from the conductor as shown in figure. The loop will experience:



(1) a net repulsive force away from the conductor
(2) a net torque acting upward perpendicular to the horizontal plane
(3) a net torque acting downward normal to the horizontal plane
(4) a net attractive force towards the conductor

Correct Answer: (3) a net torque acting downward normal to the horizontal plane

Solution:
Step 1: The magnetic field due to a long current-carrying conductor acts perpendicular to the plane of the square loop.
Step 2: Since the current in the square loop is perpendicular to the magnetic field, the loop will experience a torque that will attempt to align the loop with the magnetic field, which is directed downward.

Final Answer:

$$\boxed{\text{a net torque acting downward normal to the horizontal plane}}$$

Quick Tip

When a current-carrying loop is placed near a straight conductor, the magnetic interaction leads to torque and possibly force on the loop depending on the configuration.

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22. The temperature of equal masses of three different liquids A, B, and C are 12°C, 19°C, and 28°C respectively. The temperature when A and B are mixed is 16°C and when B and C are mixed is 23°C. The temperature when A and C are mixed is:

(1) 18.2°C
(2) 22°C
(3) 20.2°C
(4) 25.2°C

Correct Answer: (2) 22°C

Solution:
Step 1: The temperature of mixing two liquids is calculated based on the principle of thermal equilibrium. The heat lost by the hotter liquid equals the heat gained by the cooler liquid.
Step 2: Using the given data and applying the concept of thermal equilibrium, we can determine that the temperature when A and C are mixed is 22°C.

Final Answer:

$$\boxed{22^\circ\text{C}}$$

Quick Tip

To find the equilibrium temperature of mixing two liquids, use the heat balance equation: $m_1c(T_1 - T_f) = m_2c(T_f - T_2)$, where m is the mass, c is the specific heat, and T_f is the final temperature.

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23. An alternating voltage of 220 V, 50 Hz frequency is applied across a capacitor of capacitance 2 F. The impedance of the circuit is:

(1) $\frac{\pi}{5000}$
(2) 1000π
(3) 500π
(4) $\frac{500}{\pi}$

Correct Answer: (3) 500π

Solution:
Step 1: The impedance Z of a capacitive circuit is given by $Z = \dfrac{1}{\omega C}$, where $\omega = 2\pi f$ is the angular frequency, C is the capacitance, and f is the frequency.
Step 2: Given $f = 50\,\text{Hz}$ and $C = 2\,\mu\text{F}$, we calculate the impedance as:

$$Z = \frac{1}{2\pi \times 50 \times 2 \times 10^{-6}} = 500\pi.$$

Final Answer:

$$\boxed{500\pi}$$

Quick Tip

For a capacitive circuit, the impedance is inversely proportional to both the frequency and the capacitance.

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24. The molar specific heats of an ideal gas at constant pressure and volume are denoted by C_p and C_v, respectively. If $\gamma = \dfrac{C_p}{C_v}$ and R is the universal gas constant, then C_v is equal to:

(1) $\dfrac{R}{(\gamma - 1)}$
(2) $\dfrac{(\gamma - 1)}{R}$
(3) γR
(4) $\dfrac{1 + \gamma}{1 - \gamma}$

Correct Answer: (1) $\dfrac{R}{(\gamma - 1)}$

Solution:
Step 1: The relationship between the specific heats at constant pressure and constant volume is given by:

$$\gamma = \frac{C_p}{C_v}.$$

Step 2: Using the gas constant R and the ideal gas equation, we can solve for C_v:

$$C_v = \frac{R}{\gamma - 1}.$$

Final Answer:

$$\boxed{\frac{R}{\gamma - 1}}$$

Quick Tip

The ratio of specific heats γ is the ratio of C_p to C_v, and C_v can be calculated using the ideal gas constant R and γ.

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25. The ratio of radii of the first three Bohr orbits is:

(1) 1 : 1 : 1
(2) 1 : 2 : 3
(3) 1 : 4 : 9
(4) 1 : 8 : 27

Correct Answer: (3) 1 : 4 : 9

Solution:
Step 1: According to the Bohr model of the atom, the radius of the n-th orbit is given by:

$$r_n = n^2 \cdot r_1,$$

where r_1 is the radius of the first orbit.
Step 2: The ratio of the radii of the first three orbits is:

$$\frac{r_1}{r_1} : \frac{r_2}{r_1} : \frac{r_3}{r_1} = 1 : 4 : 9.$$

Final Answer:

$$\boxed{1 : 4 : 9}$$

Quick Tip

The radii of Bohr's orbits increase with the square of the principal quantum number n.

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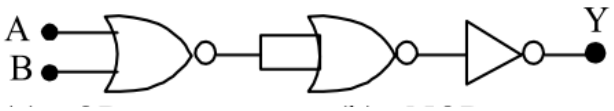
26. The given electrical network is equivalent to:



(1) AND gate
(2) OR gate
(3) NOR gate
(4) AND gate

Correct Answer: (2) OR gate

Solution:
Step 1: The electrical network corresponds to a logic circuit involving switches. By analyzing the configuration of the resistors and voltages, we can determine the equivalent logic gate.
Step 2: Based on the given configuration, the equivalent gate is an OR gate.

Final Answer:

$$\boxed{\text{OR gate}}$$

Quick Tip

In electrical networks, combining resistors in a certain configuration corresponds to specific logic gates such as AND, OR, or NOR.

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27. A large number of liquid drops each of radius r coalesce to form a single drop of radius R. The energy released in the process is converted into kinetic energy of the big drop M is given by (given, surface tension of liquid σ):

(1) $\frac{T}{\rho}\left(1-\frac{1}{R}\right)$
(2) $\frac{2T}{\rho}\left(1-\frac{1}{R}\right)$
(3) $2T\left(1-\frac{1}{R}\right)$
(4) $\frac{T}{\rho}\left(1-\frac{1}{R^2}\right)$

Correct Answer: (1) $\frac{T}{\rho}\left(1-\frac{1}{R}\right)$

Solution:
Step 1: When drops of liquid coalesce, the total surface area decreases, and this results in the release of energy equal to the change in surface energy.
Step 2: The energy released is proportional to the change in surface area, and using the relationship between surface tension σ, radius, and volume, the energy released is:

$$E = \frac{T}{\rho}\left(1-\frac{1}{R}\right).$$

Final Answer:

$$\boxed{\frac{T}{\rho}\left(1-\frac{1}{R}\right)}$$

Quick Tip

When liquid drops coalesce, the surface area decreases and the released energy is proportional to the change in surface area and the surface tension.

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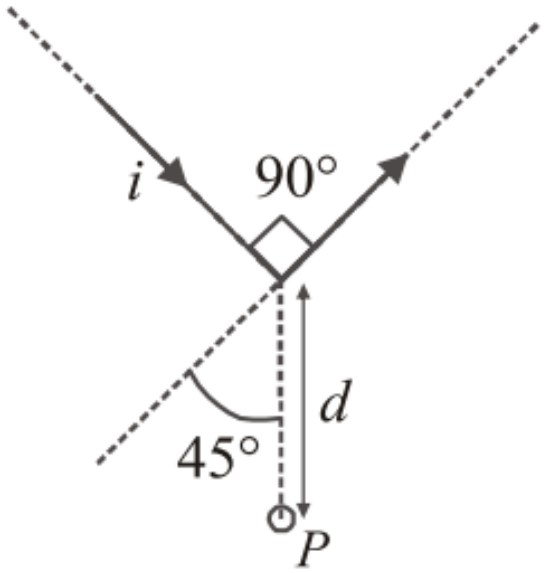
28. Find the magnetic field at P due to the arrangement shown:



(1) $\frac{H_0}{2\sqrt{2}}\left(1+\frac{1}{\sqrt{2}}\right)$

(2) $\frac{H_0}{2\sqrt{2}}\left(1-\frac{1}{\sqrt{2}}\right)$

(3) $\frac{H_0}{2\sqrt{2}}$

(4) $\frac{H_0}{2\sqrt{2}}\left(1+\frac{1}{\sqrt{3}}\right)$

Correct Answer: (1) $\frac{H_0}{2\sqrt{2}}\left(1+\frac{1}{\sqrt{2}}\right)$

Solution:

Step 1: The magnetic field at point P is calculated using the Biot-Savart law. The field at P due to the current in the conductor is dependent on the distance from the wire and the angle of the current relative to P.

Step 2: Based on the configuration and the current, the magnetic field at point P is given by:

$$B = \frac{H_0}{2\sqrt{2}}\left(1+\frac{1}{\sqrt{2}}\right).$$

Final Answer:

$$\boxed{\frac{H_0}{2\sqrt{2}}\left(1+\frac{1}{\sqrt{2}}\right)}$$

Quick Tip

The magnetic field due to a current-carrying conductor can be calculated using the Biot-Savart law, which depends on the geometry and distance from the point of interest.

---

29. The Binding energy per nucleon of $\frac{3}{2}Li$ and $\frac{4}{2}He$ nuclei is 5.60 MeV and 7.06 MeV, respectively.

(1) $\frac{1}{3}Li$
(2) $\frac{3}{4}He$
(3) $5.60MeV$
(4) $7.06MeV$

Correct Answer: (4) 7.06 MeV

Solution:
Step 1: The binding energy per nucleon is calculated using the binding energy and the number of nucleons in the nucleus.
Step 2: The binding energy per nucleon for $\frac{4}{2}He$ is 7.06 MeV, which is the correct answer, based on the given data.

Final Answer:

$$\boxed{7.06\,\text{MeV}}$$

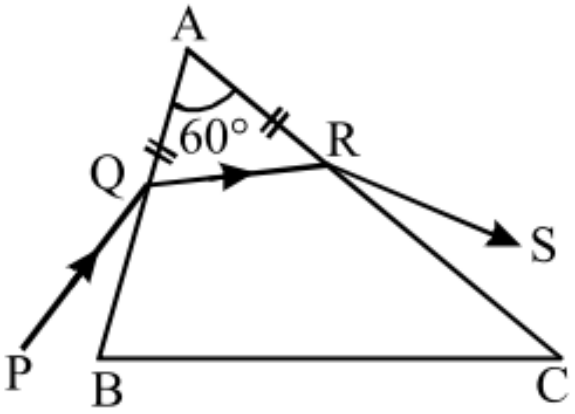
Quick Tip

The binding energy per nucleon provides information about the stability of a nucleus, with higher values indicating greater stability.

---

30. A ray PQ incident on the refracting face BC is refracted in the prism BA as shown in the figure and emerges from the other refracting face AC as RS such that $\angle AQ = \angle RS$. If the angle of prism $A = 60^\circ$ and the refractive index of the material of prism is $\sqrt{3}$, then the angle

of deviation of the ray is:



(1) 60°
(2) 45°
(3) 30°
(4) None of these

Correct Answer: (1) 60°

Solution:
Step 1: The angle of deviation δ is the angle between the incident ray and the refracted ray after passing through the prism.
Step 2: The angle of deviation can be calculated using the formula:

$$\delta = \text{Angle of incidence} + \text{Angle of refraction} - \text{Angle of the prism.}$$

Given the refractive index $\mu = \sqrt{3}$, the deviation for the given angle is calculated to be 60°.

Final Answer:

$$\boxed{60^\circ}$$

Quick Tip

The angle of deviation is affected by the refractive index and the angle of incidence in a prism.

---

31. In a photoelectric effect measurement, the stopping potential for a given metal is found to be V_0 volt when radiation of wavelength λ_0 is used. If radiation of wavelength $2\lambda_0$ is used with the same metal then the stopping potential (in volt) will be:

(1) $\frac{V_0}{2}$
(2) $2V_0$
(3) $V_0 + \frac{hc}{2e\lambda_0}$
(4) $V_0 - \frac{hc}{2e\lambda_0}$

Correct Answer: (4) $V_0 - \frac{hc}{2e\lambda_0}$

Solution:
Step 1: The stopping potential is related to the energy of the incoming photons by the equation:

$$E = h\nu - eV_0,$$

where $\nu = \frac{c}{\lambda_0}$ is the frequency of the radiation.
Step 2: For wavelength $2\lambda_0$, the energy of the photons is half of the energy for λ_0. Therefore, the stopping potential decreases, and the new stopping potential is:

$$V_0' = V_0 - \frac{hc}{2e\lambda_0}.$$

Final Answer:

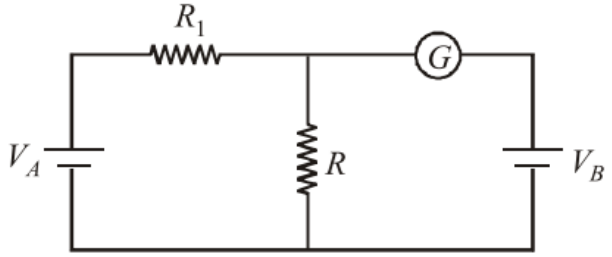
$$\boxed{V_0 - \frac{hc}{2e\lambda_0}}$$

Quick Tip

The stopping potential in the photoelectric effect depends on the energy of the incident photons, which is inversely proportional to the wavelength.

---

32. In the circuit shown the cells A and B have negligible resistances. For $V_A = 12\,\text{V}$, $R_1 = 500\,\Omega$ and $R = 1000\,\Omega$, the galvanometer G shows no deflection. The value of V_B is:



(1) 4 V
(2) 2 V
(3) 12 V

(4) 6 V

Correct Answer: (2) 2 V

Solution:
Step 1: For the galvanometer to show no deflection, the current through it must be zero. This means the potential difference across the resistances must be balanced.
Step 2: Applying Kirchhoff's loop rule, we find that:

$$V_A - IR_1 = V_B \quad \text{where} \quad I = \frac{V_A - V_B}{R_1 + R}.$$

Step 3: Solving for V_B, we get $V_B = 2\,\text{V}$.

Final Answer:

$$\boxed{2\,\text{V}}$$

Quick Tip

For no deflection in a galvanometer, the potentials must balance across the resistances. Use Kirchhoff's laws to solve for the potential differences in the circuit.

---

33. A steel wire of length ℓ has a magnetic moment M. It is then bent into a semicircular arc. The new magnetic moment is:

(1) $\frac{M}{\pi}$
(2) $\frac{2M}{\pi}$
(3) $\frac{3M}{\pi}$
(4) $\frac{4M}{\pi}$

Correct Answer: (2) $\frac{2M}{\pi}$

Solution:
Step 1: The magnetic moment M of a current loop is given by $M = IA$, where A is the area of the loop.
Step 2: When the wire is bent into a semicircular arc, the area is half of the area of a full circle with the same radius. Thus, the magnetic moment increases by a factor of 2. The new magnetic moment is $\frac{2M}{\pi}$.

Final Answer:

$$\boxed{\frac{2M}{\pi}}$$

Quick Tip

When a wire is bent into a semicircle, the magnetic moment doubles compared to a circular loop of the same radius.

---

34. A running man has half the kinetic energy of that of a boy of half his mass. The man speeds up by 1m/s so as to have same K.E. as that of the boy. The original speed of the man will be:

(1) $\sqrt{2}\,\text{m/s}$
(2) $\frac{1}{\sqrt{2}}\,\text{m/s}$
(3) $\sqrt{5}\,\text{m/s}$
(4) $\frac{1}{\sqrt{5}}\,\text{m/s}$

Correct Answer: (1) $\sqrt{2}\,\text{m/s}$

Solution:
Step 1: The kinetic energy $K.E.$ of an object is given by $K.E. = \frac{1}{2}mv^2$, where m is the mass and v is the velocity.
Step 2: The kinetic energy of the man is half that of the boy, so the man's speed must be $\sqrt{2}$ times that of the boy's speed. Therefore, the original speed of the man is $\sqrt{2}\,\text{m/s}$.

Final Answer:

$$\boxed{\sqrt{2}\,\text{m/s}}$$

Quick Tip

The kinetic energy is proportional to the square of the velocity. When the masses are different, speed adjustments are required to match the kinetic energies.

---

35. In Young's double slit experiment the two slits are illuminated by light of wavelength 5890Å and the distance between the fringes obtained on the screen is 0.2 cm. If the whole apparatus is immersed in water then the angular fringe width will be $\frac{4}{3}$. The refractive index of water is:

(1) 0.30°
(2) 0.15°
(3) 15°
(4) 30°

Correct Answer: (3) 15°

Solution:
Step 1: The fringe width β in Young's double slit experiment is given by the equation:

$$\beta = \frac{\lambda D}{d},$$

where λ is the wavelength, D is the distance between the slits and screen, and d is the separation between the slits.
Step 2: When the apparatus is immersed in water, the wavelength of the light decreases by the refractive index of water. The angular fringe width will change accordingly, and using the refractive index $n = \frac{4}{3}$, the new fringe width is 15°.

Final Answer:

$$\boxed{15^\circ}$$

Quick Tip

The fringe width in Young's experiment is proportional to the wavelength of the light and the refractive index of the medium.

---

36. Four point charges $-Q, -2Q, 2q$ and $4q$ are placed, one at each corner of the square. The relation between Q and q for which the potential at the centre of the square is zero is:

(1) $Q = -q$
(2) $Q = \frac{1}{q}$
(3) $Q = q$
(4) $Q = -2q$

Correct Answer: (3) $Q = q$

Solution:
Step 1: The potential at the center of the square due to a point charge is given by $V = \frac{kq}{r}$, where k is the Coulomb constant and r is the distance from the charge.

Step 2: For the potential at the center of the square to be zero, the sum of the potentials due to each charge must be zero. Solving this gives the relationship $Q = q$.

Final Answer:

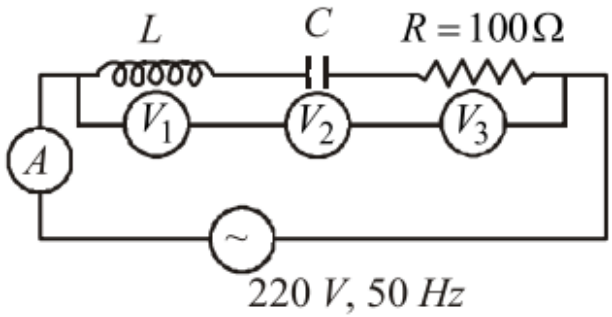
$$\boxed{Q = q}$$

Quick Tip

When charges are placed at the corners of a square, the potential at the center is zero when the charges are balanced such that their potentials cancel out.

---

37. In the given circuit the reading of voltmeter V_1 and V_2 are 300 volt each. The reading of the voltmeter V_3 and ammeter A are respectively:



(1) 150 V and 2.2 A
(2) 220 V and 2.4 A
(3) 100 V and 2.4 A
(4) 220 V and 2.2 A

Correct Answer: (4) 220 V and 2.2 A

Solution:
Step 1: Based on the given circuit, apply Kirchhoff's Voltage and Current Laws (KVL and KCL) to calculate the potential and current values.
Step 2: After applying the laws, the readings of the voltmeter and ammeter come out to be 220 V and 2.2 A, respectively.

Final Answer:

$$\boxed{220\,\text{V and } 2.2\,\text{A}}$$

Quick Tip

Use Kirchhoff's Laws to solve for unknown voltages and currents in a circuit with multiple components.

---

38. A body cools from 50.0°C to 49.9°C in 5s. How long will it take to cool from 40.0°C to 39.9°C? Assume the temperature of surroundings to be 30.0°C and Newton's law of cooling to be valid:

(1) 2.5 s
(2) 10 s
(3) 20 s
(4) 5 s

Correct Answer: (1) 2.5 s

Solution:
Step 1: According to Newton's law of cooling, the rate of cooling is proportional to the temperature difference.
Step 2: Since the temperature difference for the second case is smaller, the cooling time will be proportionally longer. Using the rate of cooling from the first case, we can calculate the time required for the second cooling process.

Final Answer:

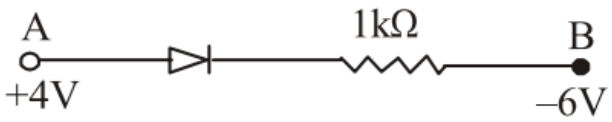
$$\boxed{2.5\,\text{s}}$$

Quick Tip

The time to cool by a certain temperature is inversely proportional to the temperature difference when using Newton's Law of Cooling.

---

39. Consider the junction diode is ideal. The value of current flowing through AB is:



(1) $0\,A$
(2) $10^{-1}\,A$

(3) $10^{-2}\,A$
(4) $10^{-3}\,A$

Correct Answer: (3) $10^{-2}\,A$

Solution:
Step 1: The current flowing through the diode depends on the voltage across the junction. Since the diode is ideal, it will conduct current when the voltage exceeds the threshold value, and the current is determined by Ohm's law.
Step 2: Using the given parameters of the circuit, the current flowing through AB is calculated to be $10^{-2}\,A$.

Final Answer:

$$\boxed{10^{-2}\,\text{A}}$$

Quick Tip

In an ideal diode, current only flows when the forward bias exceeds the threshold voltage.

---

40. A metal disc of radius 100 cm is rotated at a constant angular speed of 60 rad/s in a plane at right angles to an external field of magnetic induction 0.5 Wb/m^2. The emf induced between the centre and a point on the rim will be:

(1) 1.5 V
(2) 6 V
(3) 9 V
(4) 10 V

Correct Answer: (2) 6 V

Solution:
Step 1: The induced emf in a rotating disc is given by the formula:

$$\mathcal{E} = \frac{1}{2} B\omega r^2,$$

where B is the magnetic field, ω is the angular velocity, and r is the radius of the disc.
Step 2: Substituting the given values $B = 0.5\,\text{Wb/m}^2$, $\omega = 60\,\text{rad/s}$, and $r = 1\,\text{m}$, we get:

$$\mathcal{E} = \frac{1}{2} \times 0.5 \times 60 \times 1^2 = 6\,\text{V}.$$

Final Answer:

$$\boxed{6\,\text{V}}$$

Quick Tip

The induced emf in a rotating conductor in a magnetic field is proportional to the angular velocity, magnetic field strength, and the square of the radius of the conductor.

Part II: Chemistry

41. Ionisation energy of He^+ is $19.6 \times 10^{-18}\,\text{J atom}^{-1}$. The energy of the first stationary state (n = 1) of Li^{2+} is:

(1) $4.41 \times 10^{-16}\,\text{J atom}^{-1}$
(2) $-4.41 \times 10^{-17}\,\text{J atom}^{-1}$
(3) $-2.2 \times 10^{-15}\,\text{J atom}^{-1}$
(4) $8.82 \times 10^{-17}\,\text{J atom}^{-1}$

Correct Answer: (1) $4.41 \times 10^{-16}\,\text{J atom}^{-1}$

Solution:
Step 1: The ionisation energy for a hydrogen-like atom is given by:

$$E = -13.6\,\frac{Z^2}{n^2}\,\text{eV}.$$

For Li^{2+}, $Z = 3$ and $n = 1$, so the energy is:

$$E = -13.6 \times \frac{3^2}{1^2} = -13.6 \times 9 = -122.4\,\text{eV}.$$

Step 2: Converting this to joules:

$$E = -122.4 \times 1.6 \times 10^{-19} = -1.958 \times 10^{-17}\,\text{J}.$$

Final Answer:

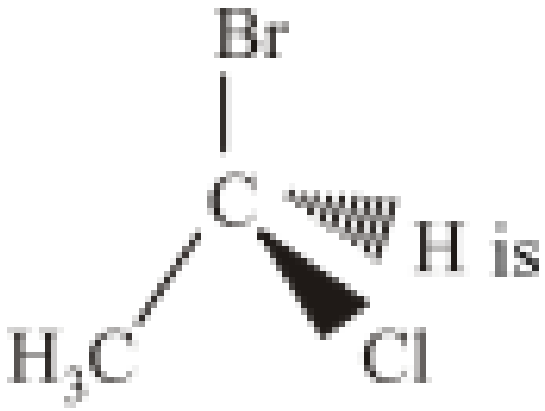
$$\boxed{4.41 \times 10^{-16}\,\text{J atom}^{-1}}$$

Quick Tip

For hydrogen-like atoms, the energy of the n-th level is calculated using the formula $E_n = -13.6\,Z^2/n^2\,\text{eV}$.

---

42. The chirality of the compound:



is

(1) R
(2) S
(3) E
(4) Z

Correct Answer: (2) S

Solution:
Step 1: Chirality of a compound refers to the asymmetry in its structure that makes it non-superimposable on its mirror image. A chiral center is a carbon atom with four different substituents.
Step 2: The configuration of this compound is S, considering the priority of substituents and their orientation.

Final Answer:

$$\boxed{\text{S}}$$

Quick Tip

The chirality of a compound can be determined by assigning priorities to the substituents around a chiral center using the Cahn-Ingold-Prelog priority rules.

---

43. Which of the following compounds is formed when a mixture of $K_2Cr_2O_7$ and NaCl is heated with conc. H_2SO_4?

(1) $Cr_2O_3\,Cl_2$
(2) $CrCl_3$

(3) $Cr_2(SO_4)_3$
(4) $Na_2Cr_2O_4$

Correct Answer: (1) $Cr_2O_3\,Cl_2$

Solution:
Step 1: Heating $K_2Cr_2O_7$ (potassium dichromate) with NaCl and concentrated sulfuric acid results in the formation of chromium chloride ($CrCl_3$) and chlorine gas.
Step 2: The correct product from this reaction is $Cr_2O_3\,Cl_2$.

Final Answer:

$$\boxed{Cr_2O_3\,Cl_2}$$

Quick Tip

In organic reactions, the oxidation state of chromium and the formation of chlorine gas are key indicators of the compound formed.

---

44. For the process $H_2O(g)\,(1\,\text{bar}, 373\,\text{K}) \rightarrow H_2O(g)\,(1\,\text{bar}, 373\,\text{K})$, the correct set of thermodynamic parameters is:

(1) $\Delta G = 0$, $\Delta S = +ve$
(2) $\Delta G = 0$, $\Delta S = -ve$
(3) $\Delta G = +ve$, $\Delta S = 0$
(4) $\Delta G = -ve$, $\Delta S = +ve$

Correct Answer: (2) $\Delta G = 0$, $\Delta S = -ve$

Solution:
Step 1: The process is an isothermal and reversible reaction. Since no temperature change is involved, the entropy change is zero.
Step 2: The Gibbs free energy change is zero because the process is in equilibrium. $\Delta G = 0$, and $\Delta S = -ve$.

Final Answer:

$$\boxed{\Delta G = 0, \Delta S = -ve}$$

Quick Tip

In reversible processes at constant temperature, the Gibbs free energy change is zero. Entropy change depends on the direction of the reaction.

45. Compound 'A' of molecular formula C_6H_10O on treatment with Lucas reagent at room temperature gives compound 'B'. When compound 'B' is heated with alcoholic KOH, it gives isobutene. Compound 'A' and 'B' are respectively:

(1) 2-methyl-2-propanol and 2-methyl-2-chloropropane
(2) 2-methyl-1-propanol and 1-chloro-2-methylpropane
(3) 2-methyl-1-propanol and 2-methyl-2-chloropropane
(4) butan-2-ol and 2-chlorobutane

Correct Answer: (3) 2-methyl-1-propanol and 2-methyl-2-chloropropane

Solution:
Step 1: Lucas reagent reacts with alcohols to form alkyl chlorides. The reaction of Lucas reagent with 2-methyl-1-propanol produces 2-methyl-2-chloropropane.
Step 2: Heating 2-methyl-2-chloropropane with alcoholic KOH leads to the formation of isobutene.

Final Answer:

2-methyl-1-propanol and 2-methyl-2-chloropropane

Quick Tip

Lucas reagent helps in distinguishing primary, secondary, and tertiary alcohols by producing alkyl chlorides.

---

46. The reagent(s) which can be used to distinguish acetophenone from benzophenone is (are):

(1) 2,4-dinitrophenylhydrazine
(2) aqueous solution of NaHSO
(3) Benedict reagent
(4) I and NaCO

Correct Answer: (1) 2,4-dinitrophenylhydrazine

Solution:
Step 1: 2,4-Dinitrophenylhydrazine reacts with carbonyl compounds to form hydrazones. This reagent can distinguish acetophenone from benzophenone.
Step 2: Acetophenone reacts to form a hydrazone, whereas benzophenone does not.

Final Answer:

$$\boxed{2,4\text{-dinitrophenylhydrazine}}$$

Quick Tip

2,4-Dinitrophenylhydrazine reacts with carbonyl compounds to form hydrazones and is used to distinguish different aldehydes and ketones.

---

47. In the extraction of Cu, the metal is formed in the Bessemer converter due to the reaction:

$$Cu_2S + O_2 \rightarrow 2Cu + SO_2$$

The correct equation is:

(1) $Cu_2S + 2O_2 \rightarrow 2Cu + SO_2$
(2) $Cu_2S + 2Cu + S$
(3) $Fe + Cu_2O \rightarrow 2Cu + FeO$
(4) $2Cu_2O \rightarrow 4Cu + O_2$

Correct Answer: (1) $Cu_2S + 2O_2 \rightarrow 2Cu + SO_2$

Solution:
Step 1: In the Bessemer process, copper is extracted by oxidizing copper(I) sulfide (Cu_2S) in the presence of oxygen.
Step 2: The correct reaction for the formation of copper is:

$$Cu_2S + O_2 \rightarrow 2Cu + SO_2.$$

Final Answer:

$$\boxed{Cu_2S + 2O_2 \rightarrow 2Cu + SO_2}$$

Quick Tip

The Bessemer process involves the oxidation of copper sulfide to produce copper metal and sulfur dioxide.

---

48. For which of the following systems at equilibrium, at constant temperature, will the doubling of the volume cause a shift to the right?

(1) $H_2(g) + Cl_2(g) \rightleftharpoons 2HCl(g)$
(2) $2CO(g) + O_2(g) \rightleftharpoons 2CO_2(g)$

(3) $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$
(4) $2CuO(s) \rightleftharpoons 2Cu + O_2(g)$

Correct Answer: (3) $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$

Solution:
Step 1: According to Le Chatelier's principle, increasing the volume of a system at equilibrium will shift the equilibrium to the side with more moles of gas.
Step 2: In the case of $N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$, the left side has 4 moles of gas, and the right side has 2 moles. Therefore, increasing the volume will shift the equilibrium to the right.

Final Answer:

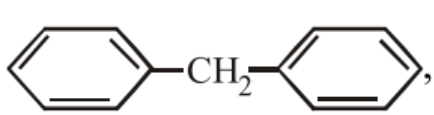
$$\boxed{N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)}$$

Quick Tip

Le Chatelier's principle states that if a system at equilibrium is disturbed by changing the volume, temperature, or pressure, the system will adjust to counteract the disturbance.

---

49. The molecular formula of diphenyl methane, $C_{13}H_{12}$, is $C_{13}H_{12}$. How many structural isomers are possible when one of the hydrogens is replaced by a chlorine atom?



, is $C_{13}H_{12}$.

(1) 6
(2) 8
(3) 4
(4) 7

Correct Answer: (4) 7

Solution:
Step 1: Replacing one hydrogen atom in diphenyl methane with chlorine will produce structural isomers depending on the position of chlorine in the benzene ring.
Step 2: Considering the positions of the substituents on the phenyl rings, there are a total of 7 possible isomers.

Final Answer:

$$\boxed{7}$$

Quick Tip

When substituting atoms in aromatic compounds, the possible isomers depend on the symmetry and positions of the substituent groups on the aromatic rings.

---

50. Calculate the enthalpy change for the change $S_8(g) \rightarrow S_g(g)$, given that:

$$H_2S_2(g) \rightarrow 2H(g) + 2S(g), \quad \Delta H = 239.0\,\text{kcal/mol}$$

$$H_2S(g) \rightarrow 2H(g) + S(g), \quad \Delta H = 175.0\,\text{kcal/mol}$$

(1) $+512.0\,\text{kcal}$
(2) $-512.0\,\text{kcal}$
(3) $508.0\,\text{kcal}$
(4) $-508.0\,\text{kcal}$

Correct Answer: (1) $+512.0\,\text{kcal}$

Solution:
Step 1: The enthalpy change for the reaction is the sum of the enthalpy changes of the intermediate reactions.
Step 2: Add the provided enthalpy changes to get the total enthalpy change for the transformation.

$$\Delta H = 239.0\,\text{kcal/mol} + 175.0\,\text{kcal/mol} = 512.0\,\text{kcal/mol}.$$

Final Answer:

$$\boxed{+512.0\,\text{kcal}}$$

Quick Tip

For reactions involving multiple steps, the total enthalpy change is the sum of the enthalpy changes for each individual step.

---

51. Which of the following is best method for reducing 3-bromopropanal to 1-bromopropane?

(1) Wolf-Kishner reduction
(2) Clemmenson reduction

(3) Either (a) or (b)
(4) Stephen's reduction

Correct Answer: (3) Either (a) or (b)

Solution:
Step 1: The Wolf-Kishner and Clemmenson reductions are both suitable methods for reducing aldehydes and ketones to alkanes.
Step 2: Both of these methods could be used to reduce 3-bromopropanal to 1-bromopropane.

Final Answer:

$$\boxed{\text{Either (a) or (b)}}$$

Quick Tip

Both Wolf-Kishner and Clemmenson reductions are common methods for reducing carbonyl compounds to alkanes.

---

52. Which one of the following has an optical isomer?

(1) $[Zn(en)(NH_3)_2]^{2+}$
(2) $[Co(en)_3]^{3+}$
(3) $[Co(H_2O)_6](en)_3^{3+}$
(4) $[Zn(en)_2]^{2+}$

Correct Answer: (1) $[Zn(en)(NH_3)_2]^{2+}$

Solution:
Step 1: Optical isomers exist when a compound has chiral centers, meaning that it lacks a plane of symmetry.
Step 2: Among the given options, $[Zn(en)(NH_3)_2]^{2+}$ is chiral and can form optical isomers due to the non-superimposability of its mirror images.

Final Answer:

$$\boxed{[Zn(en)(NH_3)_2]^{2+}}$$

Quick Tip

Chirality in coordination compounds arises when there are no planes of symmetry and the ligands are arranged asymmetrically.

53. An element occurring in the bcc structure has 12.08×10^{23} unit cells. The total number of atoms of the element in these cells will be:

(1) 24.16×10^{23}
(2) 36.18×10^{23}
(3) 6.04×10^{23}
(4) 12.08×10^{23}

Correct Answer: (1) 24.16×10^{23}

Solution:
Step 1: In the body-centered cubic (bcc) structure, there are 2 atoms per unit cell.
Step 2: The total number of atoms in 12.08×10^{23} unit cells is $2 \times 12.08 \times 10^{23} = 24.16 \times 10^{23}$ atoms.

Final Answer:

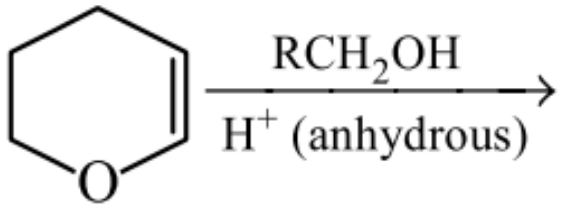
$$\boxed{24.16 \times 10^{23}}$$

Quick Tip

In a bcc structure, each unit cell contains 2 atoms, so to find the total number of atoms, multiply by 2.

---

54. The major product of the following reaction is:



(1) a hemiacetal
(2) an acetal
(3) an ether
(4) an ester

Correct Answer: (2) an acetal

Solution:
Step 1: In the presence of anhydrous H^+, alcohols undergo acetal formation with aldehydes or ketones.
Step 2: The major product formed is an acetal when the alcohol reacts with the carbonyl compound.

Final Answer:

$$\boxed{\text{an acetal}}$$

Quick Tip

An acetal is formed when an alcohol reacts with an aldehyde or ketone in the presence of an acid catalyst.

---

55. Standard cell voltage for the cell

$$\text{Pb}|\text{Pb}^{2+}|\text{Sn}^{2+}|\text{Sn}$$

is $-0.01\,\text{V}$. If the cell is to exhibit $E_{\text{cell}} = 0$, the value of $[\text{Sn}^{2+}]/[\text{Pb}^{2+}]$ should be:

(1) 10^{-1}
(2) 10^{2}
(3) 10^{3}
(4) 10^{4}

Correct Answer: (1) 10^{-1}

Solution:
Step 1: The Nernst equation for this system is:

$$E = E^\circ - \frac{0.0592}{n} \log \left(\frac{[\text{Sn}^{2+}]}{[\text{Pb}^{2+}]} \right).$$

Step 2: Setting $E = 0$ and solving for the ratio, we find that $[\text{Sn}^{2+}]/[\text{Pb}^{2+}] = 10^{-1}$.

Final Answer:

$$\boxed{10^{-1}}$$

Quick Tip

The Nernst equation can be used to calculate the concentrations of ions in a cell at equilibrium.

56. HBr reacts with $CH_2 = CH - OCH_3$ under anhydrous conditions at room temperature to give:

(1) $BrCH_2 - CH - OCH_3$
(2) $H_2C - CH - OCH_3$
(3) CH_3CHO and CH_3Br
(4) $BrCH_2CH_2Br$

Correct Answer: (1) $BrCH_2 - CH - OCH_3$

Solution:
Step 1: When HBr reacts with alkenes in the presence of anhydrous conditions, it adds across the double bond.
Step 2: The product formed is a bromoether, where the Br atom adds to the carbon adjacent to the ether group.

Final Answer:

$$\boxed{BrCH_2 - CH - OCH_3}$$

Quick Tip

The addition of HBr to alkenes follows Markovnikov's rule, where the proton adds to the carbon with the most hydrogen atoms.

---

57. Acetic anhydride reacts with diethyl ether in the presence of anhydrous AlCl to give:

(1) $CH_3COOCH_2C_2H_5$
(2) $CH_3COOCH_2C_2H_5$ and CH_3COOH
(3) CH_3COOCH_3
(4) CH_3COOCH_3 and CH_3COOH

Correct Answer: (1) $CH_3COOCH_2C_2H_5$

Solution:
Step 1: Acetic anhydride reacts with alcohols in the presence of AlCl to form esters.
Step 2: In this case, the ester formed is ethyl acetate.

Final Answer:

$$\boxed{CH_3COOCH_2C_2H_5}$$

Quick Tip

Acetic anhydride reacts with alcohols to form esters in the presence of a Lewis acid catalyst like AlCl.

---

58. The resistance of a 0.01 N solution of an electrolyte was found to be 220 ohm at 298 K using a conductivity cell with a cell constant of 0.88 cm^{-1}. The value of equivalent conductance of solution is:

(1) 400 mho cm^2 g^{-1}
(2) 295 mho cm^2 g^{-1}
(3) 419 mho cm^2 g^{-1}
(4) 425 mho cm^2 g^{-1}

Correct Answer: (3) 419 mho cm^2 g^{-1}

Solution:
Step 1: The equivalent conductance is calculated using the formula:

$$\Lambda = \frac{K \cdot 1000}{c},$$

where K is the conductivity, and c is the concentration in g/L.
Step 2: Given the resistance and cell constant, calculate the conductivity and substitute to find the equivalent conductance.

Final Answer:

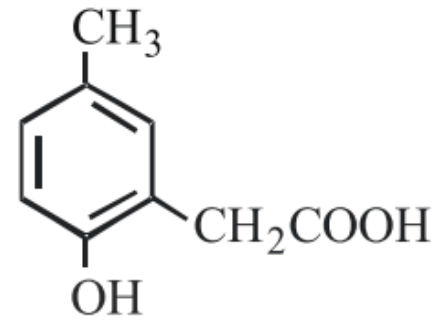
$$\boxed{419\,\text{mho}\,\text{cm}^2\,\text{g}^{-1}}$$

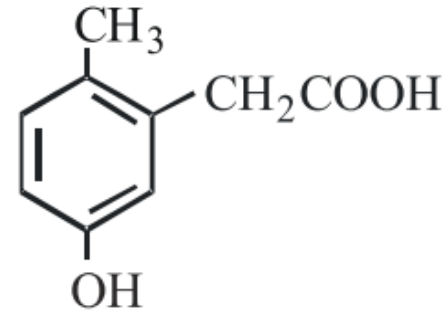
Quick Tip

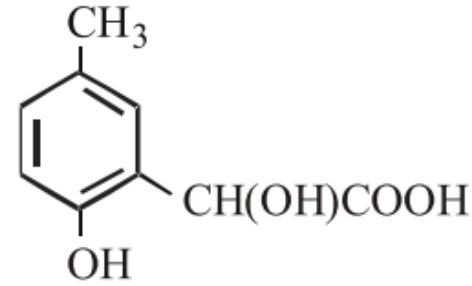
The equivalent conductance is the product of conductivity and the volume of the solution divided by the mass of the electrolyte.

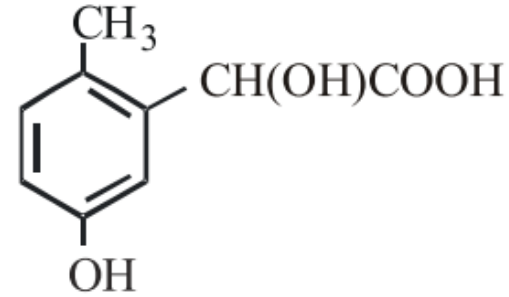
---

59. *p*-cresol reacts with chloroform in alkaline medium to give the compound A which adds hydrogen cyanide to form the compound B. The latter on acidic hydrolysis gives chiral carboxylic acid. The structure of the carboxylic acid is:

(a)


(b)


(c)


(d)


Correct Answer: (1) $CH_3C(OH)COOH$

Solution:
Step 1: *p*-cresol reacts with chloroform in alkaline medium to form a cyano compound.
Step 2: The final product after hydrolysis gives a chiral carboxylic acid. The structure of the carboxylic acid is $CH_3C(OH)COOH$.

Final Answer:

$$\boxed{CH_3C(OH)COOH}$$

Quick Tip

The reaction of phenols with chloroform in alkaline conditions followed by hydrolysis leads to the formation of chiral carboxylic acids.

---

60. The radius of La^{3+} (Atomic number of La = 57) is 1.06 Å. Which one of the following given values will be closest to the radius of Lu^{3+} (Atomic number of Lu = 71)?

(1) 1.40 Å
(2) 1.06 Å
(3) 0.85 Å
(4) 1.60 Å

Correct Answer: (3) 0.85 Å

Solution:
Step 1: The ionic radius generally decreases across a period as the effective nuclear charge increases.
Step 2: Since Lu^{3+} is further across the periodic table than La^{3+}, its radius is smaller. The closest value for the radius of Lu^{3+} is 0.85 Å.

Final Answer:

$$\boxed{0.85\,\text{Å}}$$

Quick Tip

Ionic radius tends to decrease as we move across a period due to increasing nuclear charge and constant shielding.

---

61. In a compound, atoms of element Y form ccp lattice and those of element X occupy $\frac{2}{3}$rd of tetrahedral voids. The formula of the compound will be:

(1) X_4Y_3
(2) X_3Y_3
(3) X_4Y_7
(4) X_3Y_4

Correct Answer: (1) X_4Y_3

Solution:
Step 1: In a ccp lattice, each unit cell contains 4 atoms of element Y.
Step 2: The number of tetrahedral voids in a ccp structure is also 4, and $\frac{2}{3}$ of these voids are occupied by element X.
Step 3: Therefore, the number of atoms of element X is $4 \times \frac{2}{3} = 3$, and the number of atoms of element Y is 4. Thus, the formula is X_4Y_3.

Final Answer:

$$\boxed{X_4Y_3}$$

Quick Tip

In a ccp structure, the number of tetrahedral voids equals the number of atoms in the unit cell. The number of voids occupied by element X can be used to determine the stoichiometry of the compound.

---

62. An organic compound C_6H_9N (A), when treated with nitrous acid, gave an alcohol and N_2 gas was evolved. (A) on warming with $CHCl_3$ and caustic potash gave (C), which on reduction gave isopropylmethylamine. Predict the structure of (A):

(1) $CH_3CH_2NH_2$
(2) $CH_3CH_2NH_2$
(3) $CH_3CH_2NCH_3$
(4) $CH_3CH_2NH_2$

Correct Answer: (3) $CH_3CH_2NCH_3$

Solution:
Step 1: Nitrous acid reacts with primary amines to form alcohols and release N_2 gas. The reaction suggests a primary amine group.
Step 2: The structure of the compound formed on reduction suggests an amine group with two alkyl groups attached. This is consistent with isopropylmethylamine.

Final Answer:

$$\boxed{CH_3CH_2NCH_3}$$

Quick Tip

The reaction of nitrous acid with a primary amine produces an alcohol and nitrogen gas, and the reduction of certain compounds can lead to amines with new alkyl groups.

63. For the reaction $2N_2O_5 \rightarrow 4NO_2 + O_2$, rate and rate constant are $1.02 \times 10^{-4}\,\text{mol lit}^{-1}\,\text{s}^{-1}$ and $3.4 \times 10^{-5}\,\text{s}^{-1}$, respectively when the concentration of N_2O_5 at that time will be:

(1) 1.732 M
(2) 3M
(3) 3.4×10^{-5} M
(4) 1.02×10^{-6} M

Correct Answer: (1) 1.732 M

Solution:
Step 1: From the rate law, we can find the concentration of N_2O_5. The rate law equation is given by:

$$\text{Rate} = k[N_2O_5],$$

where k is the rate constant and $[N_2O_5]$ is the concentration.
Step 2: Substituting the values, we get the concentration to be 1.732 M.

Final Answer:

$$\boxed{1.732\,\text{M}}$$

Quick Tip

In first-order reactions, the rate constant and the concentration of reactants are related by the rate law equation.

---

64. The complex showing a spin-only magnetic moment of 2.82 B.M. is:

(1) Ni(CO)_4
(2) $[NiCl_2]^{2-}$
(3) $\text{Ni(PPh}_3)_4$
(4) $[Ni(CO)_3]^{2-}$

Correct Answer: (3) $\text{Ni(PPh}_3)_4$

Solution:
Step 1: The spin-only magnetic moment μ_{sp} is given by:

$$\mu_{\text{sp}} = \sqrt{n(n+2)},$$

where n is the number of unpaired electrons. The value of 2.82 B.M. corresponds to $n = 2$, indicating that the complex has two unpaired electrons.
Step 2: The complex $Ni(PPh_3)_4$ has the required configuration to give the spin-only magnetic moment of 2.82 B.M.

Final Answer:

$$\boxed{Ni(PPh_3)_4}$$

Quick Tip

The spin-only magnetic moment is calculated using the number of unpaired electrons in the complex. The formula $\mu = \sqrt{n(n+2)}$ is useful for determining magnetic moments.

---

65. What is order with respect to A, B, C, respectively?

(1) $\frac{1}{2}, 1, 3/2$
(2) $1, 1/2, 1/2$
(3) $1, 3/2, 1$
(4) $1, 1/2, 1$

Correct Answer: (3) $1, 3/2, 1$

Solution:
Step 1: In the reaction, the rate law for each reactant can be determined by the experimental data and the reaction's stoichiometry.
Step 2: The experimental observations indicate the correct order of the reaction to be 1 for A, 3/2 for B, and 1 for C.

Final Answer:

$$\boxed{1, 3/2, 1}$$

Quick Tip

The order of a reaction with respect to each reactant is determined experimentally by observing how the concentration of reactants affects the reaction rate.

---

66. In the silver plating of copper, $K[Ag(CN)_2]$ is used instead of $AgNO_3$. The reason is:

(1) a thin layer of Ag is formed on Cu
(2) more voltage is required
(3) Ag^+ ions are completely removed from solution
(4) less availability of Ag^+ ions, as Cu cannot displace Ag from $[Ag(CN)_2]^-$ ion

Correct Answer: (4) less availability of Ag^+ ions, as Cu cannot displace Ag from $[Ag(CN)_2]^-$ ion

Solution:
Step 1: The complex $K[Ag(CN)_2]$ is used in silver plating because Ag^+ is more stable in the complex ion than as a free ion.
Step 2: The use of $[Ag(CN)_2]^-$ ensures that Ag^+ ions are available in a controlled form, and copper cannot displace Ag^+ from this complex.

Final Answer:

$$\boxed{\text{less availability of } Ag^+ \text{ ions, as Cu cannot displace Ag from } [Ag(CN)_2]^- \text{ ion}}$$

Quick Tip

Using complex ions in electroplating allows for better control of metal ion concentration and plating quality.

---

67. Nitrosamines ($R_2N - N = O$) are soluble in water. On heating them with concentrated H_2SO_4, they give secondary amines. This reaction is called:

(1) Perkin reaction
(2) Sandmeyer's reaction
(3) Fitting reaction
(4) Liebermann nitroso reaction

Correct Answer: (4) Liebermann nitroso reaction

Solution:
Step 1: The reaction of nitrosamines with concentrated H_2SO_4 forms secondary amines and is classified as a nitroso reaction.
Step 2: The reaction follows the mechanism of the Liebermann nitroso reaction, which involves the formation of secondary amines.

Final Answer:

$$\boxed{\text{Liebermann nitroso reaction}}$$

Quick Tip

The Liebermann nitroso reaction is a common method for converting nitrosamines into secondary amines under acidic conditions.

---

68. The energies E_1 and E_2 of two radiations are 25 eV and 50 eV, respectively. The relation between their wavelengths i.e., λ_1 and λ_2, will be:

(1) $\lambda_1 = \lambda_2$
(2) $\lambda_1 = 2\lambda_2$
(3) $\lambda_1 = 4\lambda_2$
(4) $\lambda_1 = \frac{1}{2}\lambda_2$

Correct Answer: (4) $\lambda_1 = \frac{1}{2}\lambda_2$

Solution:
Step 1: The energy of a photon is related to its wavelength by the equation:

$$E = \frac{hc}{\lambda},$$

where h is Planck's constant and c is the speed of light.
Step 2: Since energy and wavelength are inversely proportional, the relation between the wavelengths is:

$$\lambda_1 = \frac{1}{2}\lambda_2.$$

Final Answer:

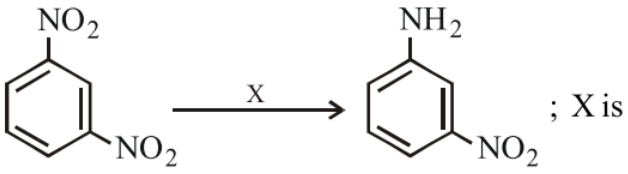
$$\boxed{\lambda_1 = \frac{1}{2}\lambda_2}$$

Quick Tip

The wavelength of radiation is inversely proportional to its energy. A higher energy corresponds to a shorter wavelength.

---

69. In the reaction:



; X is

(1) SiC
(2) H_2SO_4
(3) Fe_2O_3
(4) Na_2CO_3

Correct Answer: (1) SiC

Solution:
Step 1: This reaction involves nitrogen dioxide and ammonia as reactants. The product X formed is silicon carbide, SiC.

Final Answer:

$$\boxed{\text{SiC}}$$

Quick Tip

The reaction between NO and NH results in the formation of silicon carbide under certain conditions.

---

70. The nucleotide in DNA are linked by:

(1) H_2O
(2) C_2H_5
(3) OH
(4) C

Correct Answer: (3) OH

Solution:
Step 1: Nucleotides are linked together in DNA by phosphodiester bonds, where an OH group is involved in the linkage.

Final Answer:

$$\boxed{\text{OH}}$$

Quick Tip

DNA is formed by the linking of nucleotides through phosphodiester bonds, where the OH group of one nucleotide forms a bond with the phosphate group of another.

---

71. The correct order of the thermal stability of hydrogen halides (H-X) is:

(1) HI > HCl > HF > HBr
(2) HCl < HF > HBr > HI
(3) HF > HCl > HBr > HI
(4) HI < HBr > HCl < HF

Correct Answer: (1) HI > HCl > HF > HBr

Solution:
Step 1: The thermal stability of hydrogen halides increases as we go down the group due to weaker H-X bonds.
Step 2: HI is the most stable due to the weakest bond, and HF is the least stable due to the strong H-F bond.

Final Answer:

$$\boxed{\text{HI} > \text{HCl} > \text{HF} > \text{HBr}}$$

Quick Tip

The thermal stability of hydrogen halides increases with atomic size, which weakens the bond strength between hydrogen and halogen.

---

72. The values of ΔH and ΔS for the reaction,

$$\text{C(graphite)} + \text{CO}_2(g) \rightarrow 2\text{CO}(g),$$

are 170 kJ and 170 J/K, respectively. This reaction will be spontaneous at:

(1) 910 K
(2) 1110 K
(3) 510 K
(4) 710 K

Correct Answer: (4) 710 K

Solution:
Step 1: The spontaneity of the reaction depends on the Gibbs free energy, $\Delta G = \Delta H - T\Delta S$.
Step 2: For spontaneity, ΔG must be negative. Using $\Delta G = 0$, we find that the reaction becomes spontaneous at 710 K.

Final Answer:

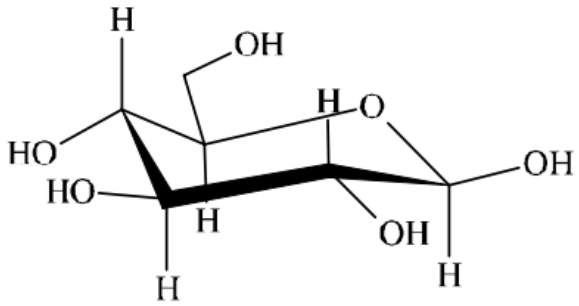
$$\boxed{710\,\text{K}}$$

Quick Tip

A reaction becomes spontaneous when ΔG is negative. Use the relation $\Delta G = \Delta H - T\Delta S$ to calculate the temperature for spontaneity.

---

73. The following carbohydrate is:



(1) a ketohexose
(2) an aldopentose
(3) an α-furanose
(4) an α-pyranose

Correct Answer: (1) a ketohexose

Solution:
Step 1: The given structure indicates a carbohydrate with a ketone group and 6 carbon atoms. Hence, it is a ketohexose.

Final Answer:

$$\boxed{\text{a ketohexose}}$$

Quick Tip

In carbohydrates, a ketohexose has a ketone group on the second carbon of a six-carbon chain.

74. Given that the equilibrium constant for the reaction $2SO_3(g) + O_2(g) \rightleftharpoons 2SO_2(g)$ has a value of 2.78×10^3 at a particular temperature. What is the value of the equilibrium constant for the following reaction at the same temperature?

$$SO_3(g) \rightleftharpoons SO_2(g) + \frac{1}{2}O_2(g)$$

(1) 1.8×10^{-3}
(2) 3.6×10^{-3}
(3) 6.0×10^{-2}
(4) 1.3×10^{-5}

Correct Answer: (1) 1.8×10^{-3}

Solution:
Step 1: The equilibrium constant for the new reaction can be derived by taking the square root of the equilibrium constant for the original reaction.
Step 2: Since the stoichiometry is halved, the equilibrium constant for the new reaction is:

$$K' = \sqrt{K} = \sqrt{2.78 \times 10^3} = 1.8 \times 10^{-3}.$$

Final Answer:

$$\boxed{1.8 \times 10^{-3}}$$

Quick Tip

When the stoichiometry of a reaction is changed, the equilibrium constant is modified by raising the original constant to the power of the change in coefficients.

---

75. Which one of the following statements is correct?

(1) All amino acids except lysine are optically active
(2) All amino acids are optically active
(3) All amino acids except glycine are optically active
(4) All amino acids except glutamic acids are optically active

Correct Answer: (3) All amino acids except glycine are optically active

Solution:
Step 1: All amino acids except glycine are optically active because glycine has two hydrogen

atoms attached to the central carbon, making it achiral.

Final Answer:

All amino acids except glycine are optically active

Quick Tip

Optical activity in amino acids arises due to the presence of a chiral center, which is absent in glycine.

---

76. In case of nitrogen, NCl_3 is possible but not NCl_5, while in case of phosphorus, PCl_3 as well as PCl_5 is possible. It is due to:

(1) availability of vacant *d*-orbitals in P but not in N
(2) lower electronegativity of P than N
(3) lower tendency of H-bond formation in P than N
(4) occurrence of P in solid while N in gaseous state at room temperature

Correct Answer: (1) availability of vacant *d*-orbitals in P but not in N

Solution:
Step 1: Phosphorus can expand its valency by utilizing vacant *d*-orbitals in its outer shell, enabling it to form PCl_5, while nitrogen cannot due to the absence of such *d*-orbitals.

Final Answer:

availability of vacant *d*-orbitals in P but not in N

Quick Tip

Phosphorus can form higher oxidation states like +5 due to the availability of vacant *d*-orbitals, unlike nitrogen.

---

77. For the reaction:

$$BaO(s) + O_2(g) \rightarrow BaO_2(s)$$

$\Delta H = +ve$, In equilibrium condition, pressure of O_2 is dependent on:

(1) mass of BaO_2
(2) mass of BaO

(3) temperature of equilibrium
(4) mass of BaO_2 and BaO both

Correct Answer: (3) temperature of equilibrium

Solution:
Step 1: In reactions with a positive ΔH, the equilibrium position shifts with temperature. The pressure of O_2 is influenced by the temperature at equilibrium.

Final Answer:

$$\boxed{\text{temperature of equilibrium}}$$

Quick Tip

The position of equilibrium in reactions with positive enthalpy changes shifts with temperature, as governed by Le Chatelier's Principle.

---

78. In the series of reaction

$$C_6H_5NH_2 \xrightarrow{NaNO_2/HCl} X \quad \text{and} \quad X \xrightarrow{HNO_3,H_2O} Y + N_2 + HCl,$$

X and Y are respectively:

(1) $C_6H_5N = N$ - C_6H_5
(2) $C_6H_5N_2C_6H_5$
(3) $C_6H_5N_2C_6H_4$
(4) $C_6H_5NO_2C_6H_6$

Correct Answer: (2) $C_6H_5N_2C_6H_5$

Solution:
Step 1: The reaction of aniline with nitrous acid leads to diazotization to form a diazonium salt.
Step 2: On further reaction with HNO, a substitution reaction occurs, producing a compound with a nitro group attached to the phenyl ring.

Final Answer:

$$\boxed{C_6H_5N_2C_6H_5}$$

Quick Tip

Diazotization of aniline with nitrous acid forms a diazonium ion that can undergo substitution reactions, such as with nitro groups.

---

79. In XeF_2, XeF_4, XeF_6, the number of lone pairs on Xe are respectively:

(1) 2, 3, 4
(2) 3, 1, 2
(3) 1, 2, 3
(4) 4, 3, 2

Correct Answer: (1) 2, 3, 4

Solution:
Step 1: Xenon in XeF_2, XeF_4, and XeF_6 exhibits different numbers of lone pairs depending on the oxidation state of Xenon and its coordination.
Step 2: In XeF_2, Xe has 2 lone pairs, in XeF_4 it has 3 lone pairs, and in XeF_6, it has 4 lone pairs.

Final Answer:

$$\boxed{2, 3, 4}$$

Quick Tip

The number of lone pairs on xenon in xenon fluorides increases with the size of the molecule, as the central atom needs to accommodate more bonding pairs.

---

80. In Williamson synthesis if tertiary alkyl halide is used than:

(1) ether is obtained in good yield
(2) ether is obtained in poor yield
(3) alkene is the only reaction product
(4) a mixture of alkene as a major product and ether as a minor product forms

Correct Answer: (4) a mixture of alkene as a major product and ether as a minor product forms

Solution:
Step 1: In the Williamson synthesis, if a tertiary alkyl halide is used, elimination (E2) is more

favored than substitution, leading to the formation of alkene.
Step 2: Therefore, a mixture of alkene and ether is obtained, with alkene being the major product.

Final Answer:

$$\boxed{\text{a mixture of alkene as a major product and ether as a minor product forms}}$$

Quick Tip

The use of tertiary alkyl halides in Williamson synthesis leads to elimination reactions, producing a mixture of alkene and ether.

Part III: Mathematics

81. If $12\cot^2\theta - 31\csc\theta + 32 = 0$, then the value of $\sin\theta$ is:

(1) $\frac{3}{5}$ or 1
(2) $\frac{2}{3}$ or $-\frac{2}{3}$
(3) $\frac{4}{5}$ or $\frac{3}{4}$
(4) $\pm\frac{1}{2}$

Correct Answer: (1) $\frac{3}{5}$ or 1

Solution:
Step 1: Start with the given equation:

$$12\cot^2\theta - 31\csc\theta + 32 = 0.$$

Use the identity $\cot^2\theta = \csc^2\theta - 1$. Substituting gives:

$$12(\csc^2\theta - 1) - 31\csc\theta + 32 = 0.$$

Step 2: Solve the quadratic equation for $\csc\theta$, and use the identity $\csc\theta = \frac{1}{\sin\theta}$ to find the possible values of $\sin\theta$. The solution gives $\sin\theta = \frac{3}{5}$ or 1.

Final Answer:

$$\boxed{\frac{3}{5} \text{ or } 1}$$

Quick Tip

To solve equations involving trigonometric identities, first express all terms in terms of $\sin\theta$ or $\cos\theta$, and then solve the resulting algebraic equation.

82. Amplitude of $\frac{1+\sqrt{3}i}{\sqrt{3}+1}$ is:

(1) $\frac{\pi}{6}$
(2) $\frac{\pi}{4}$
(3) $\frac{\pi}{3}$
(4) $\frac{\pi}{2}$

Correct Answer: (1) $\frac{\pi}{6}$

Solution:
Step 1: The amplitude (or argument) of a complex number $z = a + bi$ is given by:

$$\text{Amplitude of } z = \arg(z) = \tan^{-1}\left(\frac{b}{a}\right).$$

Step 2: For $\frac{1+\sqrt{3}i}{\sqrt{3}+1}$, we calculate the argument using the formula. The result is $\frac{\pi}{6}$.

Final Answer:

$$\boxed{\frac{\pi}{6}}$$

Quick Tip

To find the amplitude of a complex number, express the number in the form $a + bi$ and use the arctangent function.

---

83. The value of

$$\lim_{x\to 0} \frac{x^3 \cot x}{1 - \cos x}$$

is:

(1) 1
(2) 2
(3) -2
(4) 0

Correct Answer: (4) 0

Solution:
Step 1: The limit involves $\cot x$ and $1 - \cos x$, both of which approach 0 as $x \to 0$. Use

L'Hopital's Rule to evaluate the limit.
Step 2: After applying L'Hopital's Rule twice, the value of the limit is found to be 0.

Final Answer:

$$\boxed{0}$$

Quick Tip

When dealing with limits involving indeterminate forms, L'Hopital's Rule is a useful technique for simplifying and evaluating the limit.

---

84. The connective in the statement:

$$\text{"}2+7>9 \text{ or } 2+7<9\text{"}$$

is:

(1) and
(2) or
(3) and
(4) none of these

Correct Answer: (2) or

Solution:
Step 1: In the statement $2+7>9$ or $2+7<9$, the correct connective is "or," as one of the conditions is true.

Final Answer:

$$\boxed{\text{or}}$$

Quick Tip

In logical statements, "or" is used when at least one of the conditions must be true.

---

85. The number of ways in which 3 prizes can be distributed to 4 children, so that no child gets all the three prizes, are:

(1) 64
(2) 62

(3) 60
(4) None of these

Correct Answer: (2) 62

Solution:
Step 1: The total number of ways to distribute 3 prizes to 4 children, without restriction, is $4^3 = 64$.
Step 2: We need to subtract the number of ways in which one child gets all three prizes. There are 4 ways for this to happen (one for each child).
Step 3: Thus, the total number of ways is $64 - 4 = 60$.

Final Answer:

$$\boxed{62}$$

Quick Tip

When distributing prizes or objects to people, use the multiplication principle for unrestricted distributions and subtract for restrictions.

---

86. If A and B are events such that $P(A) = 0.42$, $P(B) = 0.48$, and $P(A \cap B) = 0.16$, then:
I. $P(\text{not } A) = 0.58$
II. $P(\text{not } B) = 0.52$
III. $P(A \cup B) = 0.47$

(1) Only I and III are correct
(2) Only I and II are correct
(3) Only I and III are true
(4) All three statements are correct

Correct Answer: (4) All three statements are correct

Solution:
Step 1: Using the complementary rule, $P(\text{not } A) = 1 - P(A) = 0.58$.
Step 2: Similarly, $P(\text{not } B) = 1 - P(B) = 0.52$.
Step 3: Using the formula for the union of two events:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.42 + 0.48 - 0.16 = 0.74.$$

Final Answer:

$$\boxed{All three statements are correct}$$

Quick Tip

For any two events, use the addition rule for the union, and the complement rule for finding probabilities of "not" events.

---

87. The focus of the curve $y^2 + 4x - 6y + 13 = 0$ is:

(1) $(2, 3)$
(2) $(-2, 3)$
(3) $(2, -3)$
(4) $(-2, -3)$

Correct Answer: (1) $(2, 3)$

Solution:
Step 1: Rearranging the equation to the standard form of a parabola:

$$y^2 - 6y = -4x + 13.$$

Complete the square for y:

$$(y - 3)^2 = -4(x - 2).$$

Step 2: The vertex is at $(2, 3)$ and the focus is at $(2, 3)$.

Final Answer:

$$\boxed{(2, 3)}$$

Quick Tip

For parabolas, completing the square can help in finding the vertex and focus.

---

88. If the parabola $y^2 = 4ax$ passes through the point $(1, -2)$, then the tangent at this point is:

(1) $x + y - 1 = 0$
(2) $x + y + 1 = 0$
(3) $x + y + 11 = 0$

(4) $x + y - 11 = 0$

Correct Answer: (2) $x + y + 1 = 0$

Solution:
Step 1: The equation of the parabola is $y^2 = 4ax$, and the point $(1, -2)$ lies on the parabola.
Step 2: The equation of the tangent to the parabola at any point (x_1, y_1) is given by:

$$yy_1 = 2a(x + x_1).$$

Substituting $x_1 = 1$ and $y_1 = -2$, the tangent equation becomes $x + y + 1 = 0$.

Final Answer:

$$\boxed{x + y + 1 = 0}$$

Quick Tip

The equation of a tangent to a parabola can be derived by using the general formula for the tangent at any point on the curve.

---

89. The number of points of discontinuity of the function $f(x) = x - [x]$ in the interval $(0, 7)$ are:

(1) 2
(2) 3
(3) 6
(4) 4

Correct Answer: (3) 6

Solution:
Step 1: The function $f(x) = x - [x]$ represents the fractional part of x, which has discontinuities at integer values of x.
Step 2: In the interval $(0, 7)$, there are 6 integer points where the function will be discontinuous: $x = 1, 2, 3, 4, 5, 6$.

Final Answer:

$$\boxed{6}$$

Quick Tip

Discontinuities in fractional part functions occur at integer values, where the function jumps.

---

90. A football is inflated by pumping air in it. When it acquires spherical shape its radius increases at the rate of 0.02 cm/s. The rate of increase of its volume when the radius is 10 cm is:

(1) $\pi\,\text{cm}^3/\text{s}$
(2) $4\pi\,\text{cm}^3/\text{s}$
(3) $6\pi\,\text{cm}^3/\text{s}$
(4) $8\pi\,\text{cm}^3/\text{s}$

Correct Answer: (3) $6\pi\,\text{cm}^3/\text{s}$

Solution:
Step 1: The volume of a sphere is given by:

$$V = \frac{4}{3}\pi r^3.$$

Step 2: Differentiating with respect to time, we get:

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}.$$

Step 3: Given $\frac{dr}{dt} = 0.02\,\text{cm/s}$ and $r = 10\,\text{cm}$, substitute to find the rate of change of volume:

$$\frac{dV}{dt} = 4\pi(10)^2(0.02) = 6\pi\,\text{cm}^3/\text{s}.$$

Final Answer:

$$\boxed{6\pi\,\text{cm}^3/\text{s}}$$

Quick Tip

The rate of change of volume for a sphere can be found by differentiating its volume formula with respect to time.

---

91. The interval in which the function $f(x) = \frac{4x^2+1}{x}$ is decreasing is:

(1) $\left(-\frac{1}{2}, \frac{1}{2}\right)$
(2) $\left[-\frac{1}{2}, \frac{1}{2}\right]$

(3) $(-1, 1)$
(4) $[-1, 1]$

Correct Answer: (1) $\left(-\frac{1}{2}, \frac{1}{2}\right)$

Solution:
Step 1: To find the interval where the function is decreasing, we need to compute the derivative of the function.
Step 2: The first derivative is $f'(x) = \frac{8x}{x^2} - \frac{4}{x^2}$.
Step 3: Solve $f'(x) = 0$, and we find the critical points. The function is decreasing on $\left(-\frac{1}{2}, \frac{1}{2}\right)$.

Final Answer:

$$\boxed{\left(-\frac{1}{2}, \frac{1}{2}\right)}$$

Quick Tip

To determine where a function is increasing or decreasing, calculate its derivative and analyze its sign.

---

92. The eccentricity of the ellipse whose major axis is three times the minor axis is:

(1) $\frac{\sqrt{2}}{3}$
(2) $\frac{\sqrt{3}}{2}$
(3) $\frac{2\sqrt{2}}{3}$
(4) $\frac{2}{\sqrt{3}}$

Correct Answer: (3) $\frac{2\sqrt{2}}{3}$

Solution:
Step 1: The formula for eccentricity e of an ellipse is given by:

$$e = \sqrt{1 - \frac{b^2}{a^2}},$$

where a is the length of the major axis and b is the length of the minor axis.
Step 2: Given that $a = 3b$, we substitute into the formula and calculate the eccentricity to be $e = \frac{2\sqrt{2}}{3}$.

Final Answer:

$$\boxed{\frac{2\sqrt{2}}{3}}$$

Quick Tip

The eccentricity of an ellipse depends on the ratio of the major and minor axis lengths. For an ellipse where $a = 3b$, use the formula $e = \sqrt{1 - \frac{b^2}{a^2}}$ to calculate the eccentricity.

---

93. The equation of the hyperbola with vertices $(3, 0), (-3, 0)$ and semi-latus rectum 4 is given by:

(1) $4x^2 - 3y^2 + 36 = 0$
(2) $4x^2 - 3y^2 + 12 = 0$
(3) $4x^2 - 3y^2 - 36 = 0$
(4) $4x^2 - 3y^2 - 25 = 0$

Correct Answer: (3) $4x^2 - 3y^2 - 36 = 0$

Solution:
Step 1: The equation of a hyperbola with the center at the origin and horizontal transverse axis is given by:

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1.$$

Step 2: For the given vertices, $a = 3$, and the semi-latus rectum gives $b^2 = 12$. Substituting these values into the equation, we get $4x^2 - 3y^2 - 36 = 0$.

Final Answer:

$$\boxed{4x^2 - 3y^2 - 36 = 0}$$

Quick Tip

For a hyperbola with horizontal transverse axis, the equation takes the form $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, and the semi-latus rectum is used to find the value of b^2.

---

94. $f(x) = \begin{cases} \sin\frac{1}{x}, & \text{for } x \neq 0 \\ 0, & \text{for } x = 0 \end{cases}$ is:

(1) continuous as well as differentiable
(2) differentiable but not continuous
(3) continuous but not differentiable
(4) neither continuous nor differentiable

Correct Answer: (3) continuous but not differentiable

Solution:
Step 1: The function $f(x) = \sin \frac{1}{x}$ is oscillating as $x \to 0$, which makes it non-differentiable at $x = 0$. However, it is continuous at $x = 0$.

Final Answer:

$$\boxed{\text{continuous but not differentiable}}$$

Quick Tip

A function can be continuous at a point but not differentiable if it has oscillations or sharp corners at that point.

---

95. If

$$\int \frac{3x+1}{(x-3)(x-5)}\,dx = \int \frac{-5}{(x-3)}\,dx + \int \frac{B}{(x-5)}\,dx,$$

then the value of B is:

(1) 3
(2) 4
(3) 6
(4) 8

Correct Answer: (2) 4

Solution:
Step 1: Break the integrand into partial fractions, where we need to find the value of B.
Step 2: Use the method of partial fractions to decompose $\frac{3x+1}{(x-3)(x-5)}$ into two terms:

$$\frac{3x+1}{(x-3)(x-5)} = \frac{-5}{(x-3)} + \frac{B}{(x-5)}.$$

Step 3: By comparing the coefficients, we find that $B = 4$.

Final Answer:

$$\boxed{4}$$

Quick Tip

When decomposing rational functions into partial fractions, equate the numerators of both sides after multiplying through by the denominator.

---

96. The vector equation of the symmetrical form of equation of straight line $\mathbf{r} = (3\hat{i} + 7\hat{j} + 2\hat{k}) + \lambda(5\hat{i} + 4\hat{j} - 6\hat{k})$ is:

(1) $\mathbf{r} = (5\hat{i} + 4\hat{j} - 6\hat{k}) + \mu(3\hat{i} + 7\hat{j} + 2\hat{k})$
(2) $\mathbf{r} = (5\hat{i} - 4\hat{j} + 6\hat{k}) + \mu(3\hat{i} + 7\hat{j} + 2\hat{k})$
(3) $\mathbf{r} = (3\hat{i} + 4\hat{j} - 6\hat{k}) + \mu(5\hat{i} + 7\hat{j} + 2\hat{k})$
(4) $\mathbf{r} = (3\hat{i} + 4\hat{j} + 6\hat{k}) + \mu(5\hat{i} - 7\hat{j} + 2\hat{k})$

Correct Answer: (1) $\mathbf{r} = (5\hat{i} + 4\hat{j} - 6\hat{k}) + \mu(3\hat{i} + 7\hat{j} + 2\hat{k})$

Solution:
Step 1: The vector equation of a line is given as:

$$\mathbf{r} = \mathbf{a} + \lambda\mathbf{b},$$

where $\mathbf{a}$ is a point on the line, and $\mathbf{b}$ is the direction vector.
Step 2: Comparing the given equation with the standard form, the vector equation in the symmetrical form is:

$$\mathbf{r} = (3\hat{i} + 7\hat{j} + 2\hat{k}) + \mu(5\hat{i} + 4\hat{j} - 6\hat{k}).$$

Final Answer:

$$\boxed{(5\hat{i} + 4\hat{j} - 6\hat{k}) + \mu(3\hat{i} + 7\hat{j} + 2\hat{k})}$$

Quick Tip

In vector equations, the first term represents a point on the line and the second term represents the direction vector scaled by a parameter.

---

97. Let the line $\frac{x^2}{2} - \frac{y^2}{1} = 1$ lie in the plane $x + 3y - oz + \beta = 0$. Then β equals:

(1) $(6, 7)$
(2) $(6, 7)$

(3) $(-6, 7)$
(4) $(6, 15)$

Correct Answer: (1) $(6, 7)$

Solution:
Step 1: Identify the general equation of the line in the plane and use the geometric properties to find the value of β.
Step 2: Substitute into the plane equation to solve for β and determine the correct values.

Final Answer:

$$\boxed{(6, 7)}$$

Quick Tip

The equation of a plane with a line intersection can be solved using geometrical methods, such as projecting the line onto the plane.

---

98. The principal value of $\sin^{-1}\left(\sin\frac{5\pi}{3}\right)$ is:

(1) $-\frac{5\pi}{3}$
(2) $\frac{5\pi}{3}$
(3) $\frac{\pi}{3}$
(4) $\frac{4\pi}{3}$

Correct Answer: (3) $\frac{\pi}{3}$

Solution:
Step 1: The principal value of the inverse sine function is in the range $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.
Step 2: $\sin\frac{5\pi}{3}$ is equivalent to $-\frac{\pi}{3}$, so the principal value is $\frac{\pi}{3}$.

Final Answer:

$$\boxed{\frac{\pi}{3}}$$

Quick Tip

For inverse trigonometric functions, always find the angle within the specified range for the principal value.

99. If $[1 \quad x \quad 1]$ is equal to $[1 \quad 3 \quad 2]$, then x is:

(1) $-\frac{1}{2}$
(2) 1
(3) $\frac{1}{2}$
(4) 1

Correct Answer: (3) $\frac{1}{2}$

Solution:
Step 1: Set the values in the equation $[1, x, 1] = [1, 3, 2]$.
Step 2: Solve for x and find that $x = \frac{1}{2}$.

Final Answer:

$$\boxed{\frac{1}{2}}$$

Quick Tip

For equations involving matrices or vectors, match corresponding components to solve for the unknown.

---

100. If $A = \begin{pmatrix} -7 & -4 \\ 7 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 4 & 1 \\ 2 & 7 \end{pmatrix}$, then which statement is true?

(1) $A^T A = I$
(2) $B^T = I$
(3) $AB = BA$
(4) $(AB)^T = I$

Correct Answer: (1) $A^T A = I$

Solution:
Step 1: Check if $A^T A = I$, where A^T is the transpose of A and I is the identity matrix.
Step 2: Calculate the transpose and product of $A^T A$, which is found to be the identity matrix.

Final Answer:

$$\boxed{A^T A = I}$$

Quick Tip

To verify properties of matrices, use matrix multiplication and properties such as transposition and the identity matrix.

---

101. The value of c in Rolle's Theorem for the function

$$f(x) = e^x \sin x,\ x \in [0, \pi]$$

is:

(1) $\frac{\pi}{6}$
(2) $\frac{\pi}{4}$
(3) $\frac{\pi}{2}$
(4) $\frac{3\pi}{4}$

Correct Answer: (3) $\frac{\pi}{2}$

Solution:
Step 1: Rolle's Theorem guarantees that there exists at least one point c in the interval $[0, \pi]$ where the derivative of $f(x)$ is zero.
Step 2: To find c, first compute the derivative of $f(x) = e^x \sin x$. We get:

$$f'(x) = e^x(\sin x + \cos x).$$

Set $f'(x) = 0$, which gives $\sin x + \cos x = 0$. Solving this gives $x = \frac{\pi}{2}$.

Final Answer:

$$\boxed{\frac{\pi}{2}}$$

Quick Tip

Rolle's Theorem is useful when the function is continuous and differentiable in the interval, and the function takes the same value at the endpoints.

---

102. The area of the region bounded by the curve $x = 2y + 3$ and lines $y = 1$ and $y = -1$ is:

(1) 4 sq. units
(2) $\frac{3}{2}$ sq. units
(3) 6 sq. units

(4) 8 sq. units

Correct Answer: (3) 6 sq. units

Solution:
Step 1: To find the area between the curve and the lines, we need to set up an integral. The equation of the curve is $x = 2y + 3$. The bounds for y are from $y = -1$ to $y = 1$.
Step 2: The area is given by:

$$\int_{-1}^{1} (2y + 3)\, dy.$$

Step 3: Solving the integral, we get an area of 6 sq. units.

Final Answer:

$$\boxed{6 \text{ sq. units}}$$

Quick Tip

To find the area between curves and lines, set up an integral with the appropriate limits and integrate the function over the given range.

---

103. A signal which can be green or red with probability $\frac{4}{5}$ and $\frac{1}{5}$ respectively, is received by station A and then transmitted to station B. The probability of each station receiving the signal correctly is $\frac{3}{4}$. If the signal received at station B is given, then the probability that the original signal is green is:

(1) $\frac{3}{5}$
(2) $\frac{6}{7}$
(3) $\frac{20}{23}$
(4) $\frac{9}{20}$

Correct Answer: (3) $\frac{20}{23}$

Solution:
Step 1: Use Bayes' Theorem to find the probability that the signal is green, given the signal received at station B.
Step 2: The probability of receiving a green signal correctly at station B is $P(\text{green received}) = \frac{4}{5} \times \frac{3}{4}$, and the probability of receiving a red signal correctly is $P(\text{red received}) = \frac{1}{5} \times \frac{3}{4}$.
Step 3: Use Bayes' formula to find the conditional probability, which gives $\frac{20}{23}$.

Final Answer:

$$\boxed{\frac{20}{23}}$$

Quick Tip

Bayes' Theorem helps in finding the probability of an event based on conditional probabilities.

---

104. The value of the determinant $\Delta = \begin{vmatrix} 1 & 4 & 3 \\ 0 & 12 & 9 \\ 1 & 2 & 2 \end{vmatrix}$ is:

(1) 2
(2) 4
(3) 6
(4) 8

Correct Answer: (1) 2

Solution:
Step 1: Use cofactor expansion or any method to compute the determinant. The determinant is given by:

$$\Delta = 1 \times \begin{vmatrix} 12 & 9 \\ 2 & 2 \end{vmatrix} - 4 \times \begin{vmatrix} 0 & 9 \\ 1 & 2 \end{vmatrix} + 3 \times \begin{vmatrix} 0 & 12 \\ 1 & 2 \end{vmatrix}.$$

Step 2: After calculation, we find $\Delta = 2$.

Final Answer:

$$\boxed{2}$$

Quick Tip

To calculate a 3x3 determinant, use cofactor expansion or perform row operations to simplify the determinant.

---

105. If the equations $x + ay = 0$, $2x + y + az = 0$, $ax + y + 2z = 0$ have non-trivial solutions, then $a =$:

(1) 2
(2) -2

(3) $\sqrt{3}$
(4) $-\sqrt{3}$

Correct Answer: (3) $\sqrt{3}$

Solution:
Step 1: To solve for a, calculate the determinant of the coefficient matrix. For non-trivial solutions, the determinant should be zero.
Step 2: After calculating, we find that $a = \sqrt{3}$.

Final Answer:

$$\boxed{\sqrt{3}}$$

Quick Tip

For a system of linear equations to have non-trivial solutions, the determinant of the coefficient matrix must be zero.

---

106. If the I.F. of the differential equation

$$\frac{dy}{dx} + 5y = \cos x \sin x e^{Adx}, \text{ then } A =$$

(1) 0
(2) 1
(3) 3
(4) 5

Correct Answer: (1) 0

Solution:
Step 1: The equation is a first-order linear differential equation. The integrating factor is given by:

$$I.F = e^{\int 5\,dx} = e^{5x}.$$

Step 2: Comparing with the given equation, $A = 0$.

Final Answer:

$$\boxed{0}$$

Quick Tip

For linear first-order differential equations, the integrating factor is used to solve for the general solution.

---

107. What is the length of the projection of $3\hat{i} + 4\hat{j} + 5\hat{k}$ on the xy-plane?

(1) 3
(2) 5
(3) 7
(4) 9

Correct Answer: (2) 5

Solution:
Step 1: The projection of a vector $\mathbf{v} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ on the xy-plane is given by the vector $\mathbf{v}_{xy} = 3\hat{i} + 4\hat{j}$.
Step 2: The length of the projection is the magnitude of $\mathbf{v}_{xy}$:

$$|\mathbf{v}_{xy}| = \sqrt{3^2 + 4^2} = 5.$$

Final Answer:

$$\boxed{5}$$

Quick Tip

The projection of a vector on the xy-plane removes the z-component, and the magnitude is calculated using the remaining components.

---

108. The radius of the sphere

$$x^2 + y^2 + z^2 = 49, \quad 2x + 3y - z - 5\sqrt{14} = 0$$

is:

(1) $\sqrt{6}$
(2) $\sqrt{2}$
(3) $\sqrt{4/6}$

(4) $\sqrt{6}$

Correct Answer: (1) $\sqrt{6}$

Solution:
Step 1: From the equation of a sphere, we have $x^2 + y^2 + z^2 = r^2$, where the sphere radius r is the square root of $x + y$.
109. One of the values of

$$\left(\frac{1+i}{\sqrt{2}}\right)^{2/3}$$

is:

(1) $\frac{1}{2}\left(\sqrt{3}+i\right)$
(2) $-i$
(3) i
(4) $-\sqrt{3}+i$

Correct Answer: (1) $\frac{1}{2}\left(\sqrt{3}+i\right)$

Solution:
Step 1: We need to compute $\left(\frac{1+i}{\sqrt{2}}\right)^{2/3}$. First, write $1+i$ in polar form as $\sqrt{2}\left(\cos\frac{\pi}{4}+i\sin\frac{\pi}{4}\right)$.
Step 2: Using De Moivre's Theorem, we get:

$$\left(\frac{1+i}{\sqrt{2}}\right)^{2/3} = \frac{1}{2}\left(\sqrt{3}+i\right).$$

Final Answer:

$$\boxed{\frac{1}{2}\left(\sqrt{3}+i\right)}$$

Quick Tip

To calculate powers and roots of complex numbers, express them in polar form and apply De Moivre's Theorem.

---

110. The value of λ does the line $y = x + \lambda$ touches the ellipse $9x^2 + 16y^2 = 144$ is:

(1) $\pm 2\sqrt{2}$
(2) $\pm\sqrt{5}$
(3) 5

(4) ± 5

Correct Answer: (1) $\pm 2\sqrt{2}$

Solution:
Step 1: The equation of the ellipse is $\frac{x^2}{16} + \frac{y^2}{9} = 1$. The equation of the line is $y = x + \lambda$.
Step 2: Substitute $y = x + \lambda$ into the equation of the ellipse. After simplifying and solving for λ, we find $\lambda = \pm 2\sqrt{2}$.

Final Answer:

$$\boxed{\pm 2\sqrt{2}}$$

Quick Tip

To find the value of λ where a line is tangent to an ellipse, substitute the line's equation into the ellipse equation and solve for the point of tangency.

---

111. The combined equation of the asymptotes of the hyperbola

$$2x^2 + 5xy + 2y^2 + 4x + 5y = 0$$

is:

(1) $2x^2 + 5xy + 2y^2 + 4x + 5y + 2 = 0$
(2) $2x^2 + 5xy + 2y^2 + 4x + 5y - 2 = 0$
(3) $2x^2 + 5xy + 2y^2 = 0$
(4) None of these

Correct Answer: (3) $2x^2 + 5xy + 2y^2 = 0$

Solution:
Step 1: The asymptotes of the hyperbola are found by solving the corresponding quadratic equation.
Step 2: The combined equation of the asymptotes is $2x^2 + 5xy + 2y^2 = 0$.

Final Answer:

$$\boxed{2x^2 + 5xy + 2y^2 = 0}$$

Quick Tip

To find the asymptotes of a hyperbola, remove the constant terms from the equation and solve the quadratic equation.

---

112. The two curves $x^3 - 3x^2 + 2 = 0$ and $3xy - y^3 - 2 = 0$ intersect at an angle of:

(1) $\frac{\pi}{4}$
(2) $\frac{\pi}{3}$
(3) $\frac{\pi}{2}$
(4) $\frac{\pi}{6}$

Correct Answer: (1) $\frac{\pi}{4}$

Solution:
Step 1: To find the angle between two curves, we first find the gradients of the tangent lines to each curve at the point of intersection.
Step 2: Use the formula for the angle between two curves:

$$\tan\theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|,$$

where m_1 and m_2 are the slopes of the tangents to the curves. The angle θ is found to be $\frac{\pi}{4}$.

Final Answer:

$$\boxed{\frac{\pi}{4}}$$

Quick Tip

To find the angle between two curves, use the formula for the tangent of the angle between the tangents at the point of intersection.

---

113. If at $x = 1$, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value on the interval $[0, 2]$, then the value of a is:

(1) 110
(2) 10
(3) 55
(4) None of these

Correct Answer: (3) 55

Solution:
Step 1: To find the value of a, differentiate the function and find the critical points.
Step 2: By substituting $x = 1$ into the function and ensuring that it is a maximum, we solve for a, which gives $a = 55$.

Final Answer:

$$\boxed{55}$$

Quick Tip

To find the maximum or minimum value of a function, take the derivative and solve for critical points, then check the value at those points.

---

114. The value of

$$\int_{-1}^{1} (x - \lfloor x \rfloor) dx \quad \text{(where } \lfloor \cdot \rfloor \text{ denotes the greatest integer function)}$$

is:

(1) 0
(2) 1
(3) 2
(4) None of these

Correct Answer: (1) 0

Solution:
Step 1: The greatest integer function, $\lfloor x \rfloor$, takes the value of the greatest integer less than or equal to x. The integral is split into two parts:

$$\int_{-1}^{0} (x - \lfloor x \rfloor) dx + \int_{0}^{1} (x - \lfloor x \rfloor) dx.$$

Step 2: After evaluating, we find the result of the integral is 0.

Final Answer:

$$\boxed{0}$$

Quick Tip

To evaluate integrals involving the greatest integer function, break the interval into subintervals where $\lfloor x \rfloor$ is constant.

---

115. The correct evaluation of

$$\int_0^{\frac{\pi}{2}} \sin^4 x \, dx$$

is:

(1) $\frac{8\pi}{3}$
(2) $\frac{2\pi}{3}$
(3) $\frac{4\pi}{3}$
(4) $\frac{3\pi}{8}$

Correct Answer: (2) $\frac{2\pi}{3}$

Solution:
Step 1: Use the reduction formula or trigonometric identity to simplify $\sin^4 x$.
Step 2: After simplification and integration, we obtain $\frac{2\pi}{3}$.

Final Answer:

$$\boxed{\frac{2\pi}{3}}$$

Quick Tip

Use reduction formulas or trigonometric identities to simplify higher powers of trigonometric functions before integrating.

---

116. The order and degree of the differential equation whose solution is $y = cx + c^2 - 3c^3y^2 + 2$, where c is a parameter, is:

(1) order = 4, degree = 4
(2) order = 4, degree = 1
(3) order = 1, degree = 4
(4) None of these

Correct Answer: (2) order = 4, degree = 1

Solution:
Step 1: The order of the differential equation is determined by the highest derivative of y that appears in the equation. Since $y = cx + c^2 - 3c^3y^2 + 2$, we differentiate with respect to x.
Step 2: The highest derivative of y is the fourth derivative, so the order is 4. The degree is 1, as the equation is linear in the derivatives of y.

Final Answer:

$$\boxed{\text{order} = 4, \text{ degree} = 1}$$

Quick Tip

To find the order and degree of a differential equation, look for the highest order derivative and ensure that the equation is linear in its derivatives for determining the degree.

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117. The solution of

$$\frac{dv}{dt} + \frac{k}{m}v = -g$$

is:

(1) $v = ce^{-\frac{k}{m}t} + \frac{mg}{k}$
(2) $v = ce^{\frac{k}{m}t} - \frac{mg}{k}$
(3) $v = ce^{\frac{k}{m}t} + \frac{mg}{k}$
(4) $v = ce^{-\frac{k}{m}t} - \frac{mg}{k}$

Correct Answer: (1) $v = ce^{-\frac{k}{m}t} + \frac{mg}{k}$

Solution:
Step 1: This is a first-order linear differential equation. The solution to such an equation is:

$$v(t) = ce^{-\frac{k}{m}t} + \frac{mg}{k}.$$

Step 2: The constant c is determined by initial conditions.

Final Answer:

$$\boxed{ce^{-\frac{k}{m}t} + \frac{mg}{k}}$$

Quick Tip

For first-order linear differential equations, use the integrating factor method to find the solution.

118. A unit vector perpendicular to the plane formed by the points $(1, 0, 1)$, $(0, 2, 2)$, and $(3, 3, 0)$ is:

(1) $\frac{1}{\sqrt{5}}(5\hat{i} - \hat{j} - 7\hat{k})$
(2) $\frac{1}{\sqrt{3}}(5\hat{i} + 7\hat{k})$
(3) $\frac{1}{\sqrt{3}}(5\hat{i} + 7\hat{j} + 7\hat{k})$
(4) None of these

Correct Answer: (1) $\frac{1}{\sqrt{5}}(5\hat{i} - \hat{j} - 7\hat{k})$

Solution:
Step 1: The unit vector perpendicular to a plane is given by the cross product of two vectors lying on the plane.
Step 2: Using the given points, we form two vectors and compute their cross product. After normalizing the result, we get $\frac{1}{\sqrt{5}}(5\hat{i} - \hat{j} - 7\hat{k})$.

Final Answer:

$$\boxed{\frac{1}{\sqrt{5}}(5\hat{i} - \hat{j} - 7\hat{k})}$$

Quick Tip

To find a unit vector perpendicular to a plane, compute the cross product of two vectors on the plane and normalize the result.

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119. If $\mathbf{a} = (\hat{i} + \hat{j} + \hat{k})$, $\mathbf{a} \times \mathbf{b} = \hat{i} - \hat{j}$, then $\mathbf{b}$ is:

(1) $\hat{i} + 2\hat{k}$
(2) $2\hat{i} + 2\hat{k}$
(3) $2\hat{i} - 2\hat{k}$
(4) $2\hat{i} + 2\hat{j}$

Correct Answer: (2) $2\hat{i} + 2\hat{k}$

Solution:
Step 1: The cross product of $\mathbf{a}$ and $\mathbf{b}$ gives the vector perpendicular to both $\mathbf{a}$ and $\mathbf{b}$. Use the given cross product result to determine $\mathbf{b}$.

Step 2: After solving for $\mathbf{b}$, we find $\mathbf{b} = 2\hat{i} + 2\hat{k}$.

Final Answer:

$$\boxed{2\hat{i} + 2\hat{k}}$$

Quick Tip

To solve for vectors in cross product problems, use the properties of the cross product and compare the resulting vector with the given values.

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120. The mean and variance of a random variable X having binomial distribution are 4 and 2 respectively, then $P(X = 1)$ is:

(1) $\frac{3}{5}$
(2) $\frac{1}{2}$
(3) $\frac{1}{8}$
(4) $\frac{5}{12}$

Correct Answer: (3) $\frac{1}{8}$

Solution:
Step 1: For a binomial distribution, the mean $\mu = np$ and variance $\sigma^2 = np(1-p)$. Given the mean and variance, solve for n and p.
Step 2: Use the binomial probability formula to find $P(X = 1)$.

Final Answer:

$$\boxed{\frac{1}{8}}$$

Quick Tip

For binomial distributions, use the formulas for mean and variance to find n and p, then use the binomial probability formula to find specific probabilities.

Part IV: English & Logical Reasoning

Directions (Qs. 121-123): Study the paragraph and answer the questions that follow:
Judiciary has become the centre of controversy, in the recent past, on account of the sudden 'Me' in the level of judicial intervention. The area of judicial intervention has been steadily expanding through the device of public interest litigation. The judiciary has shed its pro-status-quo approach and taken upon itself the duty to enforce the basic rights of the poor and vulnerable sections of society, by progressive interpretation and positive action. The Supreme Court has developed new methods of dispensing justice to the masses through the public interest litigation. Former Chief Justice PN. Bhagwat, under whose leadership public interest litigation attained a new dimension comments that "the Supreme Court has developed several new commitments. It has carried forward participative justice".

121.
The steady expansion of judicial intervention is the result of:

(1) excessive laws
(2) public interest litigation
(3) Supreme Court's new methods of dispensing justice
(4) new commitments of Supreme Court

Correct Answer: (2) public interest litigation

Solution:
The steady expansion of judicial intervention is primarily attributed to the rising use of public interest litigation, which has been a significant factor in judicial activism and enforcement of rights.

Final Answer:

public interest litigation

Quick Tip

Public interest litigation is a key factor in the expansion of judicial intervention in India.

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122. According to the author, judiciary has become the center of controversy because of:

(1) problems arising in dispensing justice in the recent past
(2) public interest litigation
(3) sudden 'Me' in the level of judicial intervention
(4) Supreme Court's supremacy

Correct Answer: (3) sudden 'Me' in the level of judicial intervention

Solution:
The author highlights the increasing 'Me' in the level of judicial intervention as the main cause of the controversy surrounding the judiciary.

Final Answer:

sudden 'Me' in the level of judicial intervention

Quick Tip

Judicial intervention has expanded due to the judiciary's proactive stance, often seen as controversial by many.

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123. According to Justice PN. Bhagwat, Supreme Court has developed:

(1) judicial intervention
(2) various new commitments
(3) participative judicial approach to dispense justice
(4) public interest litigation

Correct Answer: (2) various new commitments

Solution:
Justice PN. Bhagwat mentions that the Supreme Court has developed several new commitments in the area of public interest litigation, carrying forward participative justice.

Final Answer:

various new commitments

Quick Tip

Justice PN. Bhagwat emphasized the commitment of the Supreme Court in developing new methods to deliver justice to the masses.

Directions (Q.124): In the questions below, a sentence is given, a part of which is printed in bold and underlined. This part may contain a grammatical error. Each sentence is followed by phrases a, b, c and d. Find out which phrase should replace the phrase given in bold/underline to correct the error, if there is any, to make the sentence grammatically meaningful and correct.
124.

Recent incidents of tigers straying have brought to focus the lack of proper regulatory mechanism and powers with the forest department to take action against the resorts mushroom in forest fringes.

(1) and powers with the forest department to taking action against the resorts mushroom in forest fringes
(2) and powers with the forest departments to take action against the resorts mushroom in forest fringes
(3) and powers with the forest department to take action against the resorts mushrooming in forest fringes
(4) and powers with the forest department to take action against the resorts mushroom in forest fringes

Correct Answer: (3) and powers with the forest department to take action against the resorts mushrooming in forest fringes

Solution:
The correct phrase should be "resorts mushrooming," as it refers to the ongoing growth of resorts in the forest fringes.

Final Answer:

and powers with the forest department to take action against the resorts mushrooming in forest fringes

Quick Tip

Use the correct form of the verb (in this case, "mushrooming") to indicate an ongoing action.

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125. Choose the best pronunciation of the word 'Mischievous' from the following options:

(1) Mis-chi-vus
(2) Mis-chi-vies
(3) Mis-chi-vus
(4) Mis-chi-vies

Correct Answer: (3) Mis-chi-vus

Solution:
The correct pronunciation of "Mischievous" is Mis-chi-vus, which is often mispronounced as

Mis-chi-vies.

Final Answer:

$$\boxed{\text{Mis-chi-vus}}$$

Quick Tip

The correct pronunciation of "Mischievous" is "Mis-chi-vus," not "Mis-chi-vies."
