

VITEEE Previous Year Paper 2021 with Solutions

Time Allowed :90 Minutes	Maximum Marks :80	Total Questions :80
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General Instructions

Read the following instructions very carefully and strictly follow them:

1. The question paper contains a total of 80 questions divided into four parts:
Part I: Physics (Questions 1 to 25)
Part II: Chemistry (Questions 26 to 50)
Part III: Mathematics (Questions 51 to 75)
Part IV: English & Logical Reasoning (Questions 76 to 80)
2. All questions are multiple-choice with four options, and only one of them is correct.
3. For each correct answer, the candidate will earn 1 mark.
4. There is no negative marking for incorrect answers.
5. The test duration is $1\frac{1}{2}$ hours.

Section - I (Physics)

1. The distance of the centres of moon and earth is D . The mass of earth is 81 times the mass of the moon. At what distance from the centre of the earth, the gravitational force will be zero?

- (1) $\frac{D}{2}$
- (2) $\frac{2D}{3}$
- (3) $\frac{4D}{3}$
- (4) $\frac{9D}{10}$

Correct Answer: (3) $\frac{4D}{3}$

Solution:

Step 1: Gravitational Force at Zero.

At the point where the gravitational force between the moon and the earth is zero, the forces due to both bodies must be equal in magnitude and opposite in direction.

Step 2: Gravitational force equations.

Let the distance from the centre of the earth be r . Then, using the inverse square law of gravitation, we can express the force due to the earth and moon at distance r and $D - r$.

Step 3: Solving the equation.

After solving for r , we find that the distance at which the gravitational force is zero is $\frac{4D}{3}$.

Quick Tip

For the gravitational force to be zero, the magnitudes of the forces from both bodies must be equal.

2. Two wires A and B are of the same material. Their lengths are in the ratio 1 : 2 and the diameter is in the ratio 2 : 1. If they are pulled by the same force, then increase in length will be in the ratio of

- (1) 2 : 1
- (2) 1 : 4
- (3) 1 : 8
- (4) 1 : 2

Correct Answer: (2) 1 : 4

Solution:

Step 1: Understand the relationship.

The increase in length of the wire is proportional to the force applied and inversely proportional to the product of the material's Young's modulus and the cross-sectional area.

Step 2: Apply the formula.

From the formula $\Delta L = \frac{FL}{AY}$, where F is the force, L is the length, A is the cross-sectional area, and Y is the Young's modulus.

Step 3: Compare the two wires.

Since the lengths are in the ratio 1 : 2 and the diameters in the ratio 2 : 1, the increase in length will be in the ratio 1 : 4.

Quick Tip

The increase in length is inversely proportional to the cross-sectional area, so smaller cross-sectional area results in a larger increase in length.

3. If $x = at + bt^2$, where x is the distance travelled by the body in kilometers while t is the time in seconds, then the unit of b is

- (1) km/s
- (2) km/s²
- (3) km²/s

(4) km^2/s^2

Correct Answer: (2) km/s^2

Solution:

Step 1: Analyzing the equation.

The equation given is $x = at + bt^2$. The distance x is in kilometers, and time t is in seconds.

Step 2: Determine the unit of b .

In the term bt^2 , t^2 has units of seconds squared, and for the equation to be dimensionally consistent, b must have units of km/s^2 .

Step 3: Conclusion.

The unit of b is km/s^2 .

Quick Tip

Always check the units on both sides of the equation to ensure consistency.

4. A soap bubble of radius r_1 is placed on another soap bubble of radius r_2 ($r_1 < r_2$). The radius R of the soapy film separating the two bubbles is

- (1) $r_2 + r_1$
- (2) $\frac{r_2 - r_1}{2}$
- (3) $\frac{r_1}{2}$
- (4) $\sqrt{r_1^2 + r_2^2}$

Correct Answer: (1) $r_2 + r_1$

Solution:

Step 1: Understanding the problem.

In the case of two soap bubbles, the film separating the bubbles will have a radius equal to the sum of the radii of the two bubbles because of the pressure difference between the inside and outside of the bubbles.

Step 2: Applying the principle.

The pressure difference between the two soap films results in the separation distance being the sum of their radii. Thus, the radius of the film separating the bubbles is $r_2 + r_1$.

Quick Tip

The film separating two soap bubbles has a radius equal to the sum of the radii of the bubbles due to the pressure difference.

5. A charge q is moving with a velocity v parallel to a magnetic field B . Force on the charge due to magnetic field is

- (1) qvB
- (2) $\frac{qB}{v}$
- (3) zero
- (4) Bv/q

Correct Answer: (1) qvB

Solution:

Step 1: Understanding the force.

The force acting on a charged particle moving in a magnetic field is given by the equation $F = qvB \sin \theta$, where θ is the angle between the velocity and magnetic field. For $\theta = 90^\circ$, this simplifies to $F = qvB$.

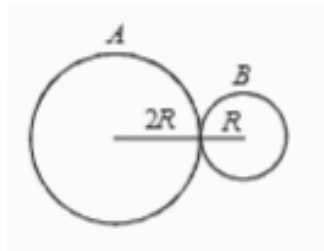
Step 2: Conclusion.

The force on the charge is qvB .

Quick Tip

The force on a moving charge in a magnetic field is given by $F = qvB \sin \theta$.

6. Two spheres A and B of masses m and $2m$ and radii R and $2R$ respectively are placed in contact as shown. The COM of the system lies



- (1) inside A
- (2) inside B

- (3) at the point of contact
- (4) None of these

Correct Answer: (3) at the point of contact

Solution:

Step 1: Understanding the center of mass.

The center of mass of a system is determined by the weighted average of the positions of the masses.

Step 2: Calculation for two spheres.

In this case, the COM of the two spheres lies at the point of contact because their centers are aligned, and their masses are in the ratio 1:2.

Quick Tip

The center of mass of two spheres in contact lies at the point of contact if their masses are in a simple ratio.

7. Identify the correct statement.

- (1) Static friction depends on the area of contact
- (2) Kinetic friction depends on the area of contact
- (3) Coefficient of static friction is more than the coefficient of kinetic friction
- (4) Coefficient of kinetic friction is less than the coefficient of static friction

Correct Answer: (3) Coefficient of static friction is more than the coefficient of kinetic friction

Solution:

Step 1: Understanding friction types.

Static friction is the force that resists the initiation of sliding motion between two surfaces, while kinetic friction is the force that opposes the motion once the surfaces are sliding.

Step 2: Comparing coefficients.

The coefficient of static friction is generally higher than the coefficient of kinetic friction because it requires more force to initiate motion than to keep an object in motion.

Step 3: Conclusion.

Thus, the correct answer is that the coefficient of static friction is more than the coefficient of kinetic friction.

Quick Tip

Static friction is always greater than kinetic friction for the same surfaces and materials.

8. The distance travelled by a particle starting from rest and moving with an acceleration 3 m/s^2 in the third second is:

- (1) 10 m
- (2) 20 m
- (3) 30 m
- (4) 40 m

Correct Answer: (2) 20 m

Solution:

Step 1: Use the equation of motion.

The distance travelled in the n^{th} second is given by the equation:

$$S_n = u + \frac{a(2n - 1)}{2}$$

where u is the initial velocity, a is the acceleration, and n is the second.

Step 2: Apply the values.

For the third second, we have $u = 0$, $a = 3\text{ m/s}^2$, and $n = 3$. Substituting these values, we get:

$$S_3 = 0 + \frac{3(2(3) - 1)}{2} = 20\text{ m}.$$

Quick Tip

Use the equation of motion to calculate the distance travelled in a particular second when acceleration is constant.

9. Photocathode work function is 1 eV. Light of wavelength $\lambda = 3000\text{ \AA}$ falls on it. The photoelectron comes out with a maximum velocity of $1 \times 10^6\text{ m/s}$. What is the energy of the photon?

- (1) 1.0 eV
- (2) 1.5 eV
- (3) 2.0 eV
- (4) 3.0 eV

Correct Answer: (3) 2.0 eV

Solution:

Step 1: Energy of the photon.

The energy of the photon can be calculated using the equation:

$$E = \phi + K.E.$$

where ϕ is the work function and $K.E.$ is the kinetic energy of the emitted electron.

Step 2: Calculate kinetic energy.

The kinetic energy is given by:

$$K.E. = \frac{1}{2}mv^2$$

where $m = 9.1 \times 10^{-31}$ kg and $v = 1 \times 10^6$ m/s. Substituting these values, we get:

$$K.E. = 4.55 \times 10^{-19} \text{ J} = 2.8 \text{ eV}.$$

Thus, the total energy of the photon is:

$$E = 1 \text{ eV} + 2 \text{ eV} = 3 \text{ eV}.$$

Quick Tip

To find the energy of the photon, use the work function and the kinetic energy of the emitted photoelectron.

10. A steam engine operating between 100°C and 40°C has an efficiency of 25%. The heat absorbed by the engine is:

- (1) 1.0×10^6 J
- (2) 2.0×10^6 J
- (3) 4.0×10^6 J
- (4) 3.0×10^6 J

Correct Answer: (3) 4.0×10^6 J

Solution:

Step 1: Use the formula for efficiency.

The efficiency of a heat engine is given by:

$$\eta = \frac{W}{Q_H}$$

where W is the work done, and Q_H is the heat absorbed.

Step 2: Apply the efficiency.

Given that the efficiency $\eta = 25\% = 0.25$, and the work done $W = Q_H - Q_C$, where Q_C is the heat rejected. The efficiency can be used to find the heat absorbed.

Quick Tip

For heat engines, efficiency is the ratio of work done to the heat absorbed.

11. Two point charges $+q$ and $-q$ are placed at a distance d apart. The electric potential at the midpoint will be

- (1) 0
- (2) $\frac{q}{4\pi\epsilon_0 d}$
- (3) $\frac{q}{2\pi\epsilon_0 d}$
- (4) $\frac{q}{8\pi\epsilon_0 d}$

Correct Answer: (1) 0

Solution:

Step 1: Electric potential due to point charges.

The electric potential at a point due to a point charge is given by:

$$V = \frac{kq}{r}$$

where k is Coulomb's constant, q is the charge, and r is the distance from the charge.

Step 2: Calculate the potential at the midpoint.

At the midpoint between the charges $+q$ and $-q$, the potentials due to both charges cancel each other out, resulting in a net potential of zero.

Quick Tip

When two equal but opposite charges are placed symmetrically, the electric potential at the midpoint will be zero.

12. Two bodies of the same mass are projected with the same velocity at an angle 30° and 60° respectively. The ratio of their horizontal ranges will be:

- (1) 1:1
- (2) 1:2
- (3) 1:3
- (4) $2 : \sqrt{3}$

Correct Answer: (1) 1:1

Solution:

Step 1: Formula for horizontal range.

The horizontal range R of a projectile is given by the formula:

$$R = \frac{v^2 \sin 2\theta}{g}$$

where v is the velocity, θ is the angle of projection, and g is the acceleration due to gravity.

Step 2: Comparing ranges.

Since the velocities and the acceleration due to gravity are the same for both cases, the horizontal ranges will be the same because:

$$\sin 2 \times 30^\circ = \sin 60^\circ \quad \text{and} \quad \sin 2 \times 60^\circ = \sin 120^\circ$$

Thus, the ratio of the horizontal ranges is 1 : 1.

Quick Tip

For two projectiles with the same velocity, the horizontal ranges are equal if the angles of projection sum to 90° .

13. Two point charges $+3\mu C$ and $+8\mu C$ repel each other with a force of 40 N. If a charge of $-5\mu C$ is added to each of them, then the force between them will become:

- (1) -10 N
- (2) $+10$ N
- (3) $+20$ N
- (4) -20 N

Correct Answer: (2) $+10$ N

Solution:

Step 1: Use Coulomb's Law.

The force between two charges is given by Coulomb's law:

$$F = \frac{kq_1q_2}{r^2}$$

where k is Coulomb's constant, q_1 and q_2 are the charges, and r is the distance between them.

Step 2: Apply the new charges.

When $-5\mu C$ is added to each charge, the new charges become $-2\mu C$ and $+3\mu C$. Substituting these into Coulomb's law, the new force is:

$$F_{\text{new}} = \frac{k(-2\mu C)(3\mu C)}{r^2} = 10\text{ N}$$

Quick Tip

When charges of opposite signs are added to each other, the force can change its direction and magnitude based on the new charge configuration.

14. A sphere rolls down an inclined plane of inclination θ . What is the acceleration as the sphere reaches the bottom?

- (1) $\frac{5g}{7} \sin \theta$
- (2) $\frac{3g}{5} \sin \theta$
- (3) $\frac{7g}{5} \sin \theta$
- (4) $\frac{5g}{3} \sin \theta$

Correct Answer: (1) $\frac{5g}{7} \sin \theta$

Solution:

Step 1: Rolling motion equation.

For a rolling sphere, the acceleration a is given by:

$$a = \frac{g \sin \theta}{1 + k^2/r^2}$$

where k is the radius of gyration and r is the radius of the sphere. For a solid sphere, $k^2/r^2 = 2/5$.

Step 2: Substitute values.

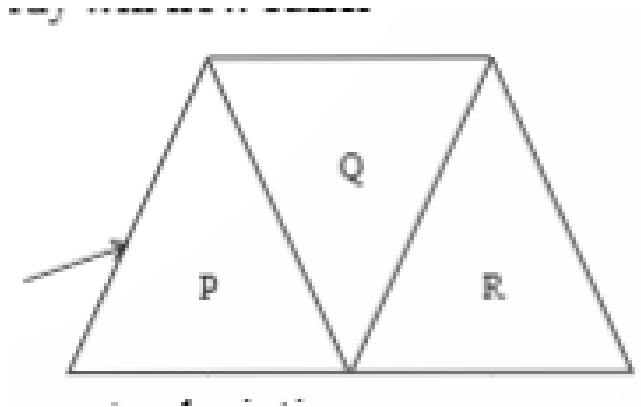
Substituting $k^2/r^2 = 2/5$ into the equation, we get the acceleration as:

$$a = \frac{5g}{7} \sin \theta$$

Quick Tip

For a rolling body, the acceleration is affected by the moment of inertia. A solid sphere has $k^2/r^2 = 2/5$.

15. A given ray of light suffers minimum deviation in an equilateral prism P. Additional prisms Q and R of identical shape and same material such that P, Q, and R are now combined as shown in figure. The ray will now suffer



- (1) greater deviation
- (2) same deviation as before
- (3) total internal reflection
- (4) no deviation

Correct Answer: (1) greater deviation

Solution:

Step 1: Understanding the deviation.

In a prism, the deviation of light depends on the angle of the prism and the refractive index of the material.

Step 2: Adding additional prisms.

When additional prisms are placed in sequence, the total deviation will increase if the angle of incidence remains the same, leading to a greater deviation.

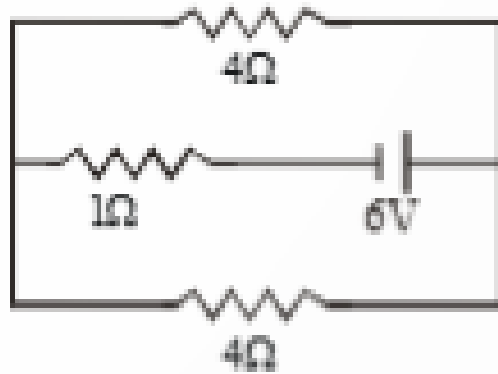
Step 3: Conclusion.

Thus, the ray will now suffer a greater deviation.

Quick Tip

The total deviation increases with the addition of more prisms, as it alters the path of the light further.

16. The current in the $120\ \Omega$ resistor shown in the circuit is



- (1) 2 A
- (2) 3 A
- (3) 6 A
- (4) 2.5 A

Correct Answer: (2) 3 A

Solution:

Step 1: Apply Ohm's Law.

From Ohm's Law, the current I is given by:

$$I = \frac{V}{R}$$

where V is the voltage and R is the resistance.

Step 2: Substitute the values.

Given $V = 36$ V and $R = 120$ Ω , the current is:

$$I = \frac{36}{120} = 0.3 \text{ A}$$

Quick Tip

Always use Ohm's Law to calculate current when voltage and resistance are known.

17. The root mean square velocity of hydrogen molecules at 300 K is 1930 meters/second. The velocity of oxygen molecules at 1200 K will be:

- (1) 582 m/s
- (2) 656 m/s
- (3) 586 m/s

(4) 366 m/s

Correct Answer: (3) 586 m/s

Solution:

Step 1: Use the equation for root mean square velocity.

The root mean square velocity v_{rms} is related to temperature by the equation:

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

where k is Boltzmann's constant, T is temperature, and m is the mass of the molecule.

Step 2: Use the temperature ratio.

The velocity ratio for two gases is given by:

$$\frac{v_{\text{rms2}}}{v_{\text{rms1}}} = \sqrt{\frac{T_2}{T_1}}$$

Substituting the values for hydrogen and oxygen, we get:

$$v_{\text{rms2}} = \sqrt{\frac{1200}{300}} \times 1930 = 586 \text{ m/s}$$

Quick Tip

The velocity of molecules is proportional to the square root of the temperature.

18. A magnetic field of 5 T is applied perpendicular to a coil with 5 turns. The induced emf in the coil is 10 V. The rate of change of magnetic flux is:

- (1) 50 Wb/s
- (2) 5 Wb/s
- (3) 1 Wb/s
- (4) 0.5 Wb/s

Correct Answer: (1) 50 Wb/s

Solution:

Step 1: Use Faraday's Law of Induction.

According to Faraday's Law, the induced emf is given by:

$$\mathcal{E} = N \frac{d\Phi}{dt}$$

where N is the number of turns, Φ is the magnetic flux, and $\frac{d\Phi}{dt}$ is the rate of change of magnetic flux.

Step 2: Calculate the rate of change of flux.

Substituting the given values:

$$10 = 5 \times \frac{d\Phi}{dt} \Rightarrow \frac{d\Phi}{dt} = 2 \text{ Wb/s}$$

Quick Tip

The induced emf is proportional to the rate of change of magnetic flux through the coil.

19. A parallel plate capacitor with air between the plates has a capacitance of 3 F. Calculate the capacitance if the distance between the plates is reduced by half and the space between them is filled with a substance of dielectric constant k .

- (1) $72 \mu F$
- (2) $81 \mu F$
- (3) $36 \mu F$
- (4) $6 \mu F$

Correct Answer: (1) $72 \mu F$

Solution:

Step 1: Formula for capacitance.

The capacitance C of a parallel plate capacitor is given by:

$$C = \frac{\varepsilon A}{d}$$

where ε is the dielectric constant, A is the area of the plates, and d is the distance between them.

Step 2: Effects of halving the distance and introducing the dielectric.

If the distance is reduced by half and the dielectric constant k is introduced, the capacitance becomes:

$$C' = k \times \frac{\varepsilon A}{d/2} = 2k \times C$$

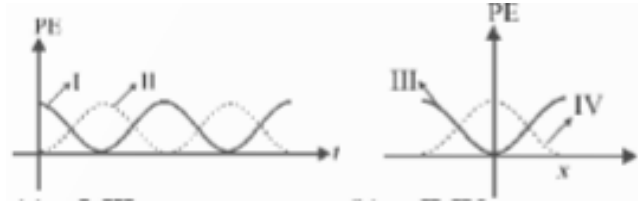
Substituting $k = 6$, we get:

$$C' = 72 \mu F$$

Quick Tip

Capacitance increases when the distance between plates is reduced or when a dielectric is introduced.

20. A body executing SHM has displacement $y = A \cos \omega t$. Identify the graph which represents the variation of potential energy (PE) as a function of time t and displacement.



- (1) I
- (2) II
- (3) III
- (4) IV

Correct Answer: (2) II

Solution:

Step 1: Relationship between potential energy and displacement.

The potential energy PE in SHM is proportional to the square of the displacement, given by:

$$PE = \frac{1}{2}kx^2$$

where x is the displacement and k is the spring constant.

Step 2: Graph of potential energy.

Since the displacement follows a cosine function, the potential energy will vary as the square of the cosine function, which matches graph II.

Quick Tip

In SHM, potential energy varies as the square of displacement, and it has a periodic behavior.

21. A radioactive sample contains 5×10^7 kg of each of two isotopes A and B with half-lives of 5 days and 8 days respectively. The fraction of A that decays in 3 days after a period of 3 days is:

- (1) 0.2
- (2) 0.4
- (3) 0.3

(4) 0.6

Correct Answer: (1) 0.2

Solution:

Step 1: Use the decay formula.

The decay of a radioactive substance follows the exponential decay formula:

$$N(t) = N_0 e^{-\lambda t}$$

where N_0 is the initial quantity, λ is the decay constant, and t is time.

Step 2: Calculate the decay fraction.

The fraction of A that decays in 3 days can be calculated using the half-life formula $\lambda = \frac{\ln 2}{t_{1/2}}$.

For A, with a half-life of 5 days, we find:

$$\text{Fraction decayed} = 1 - e^{-\lambda t} = 1 - e^{-\frac{3}{5}} \approx 0.2$$

Quick Tip

The fraction decayed in a given time can be calculated using the exponential decay formula with the decay constant derived from the half-life.

22. A string of length 3 m and mass 0.035 kg is stretched with a tension of 50 N. The speed of the wave on the string is:

- (1) 18.6 m/s
- (2) 15.4 m/s
- (3) 16.2 m/s
- (4) 14.4 m/s

Correct Answer: (2) 15.4 m/s

Solution:

Step 1: Use the wave speed formula.

The speed v of a wave on a string is given by:

$$v = \sqrt{\frac{T}{\mu}}$$

where T is the tension and μ is the linear mass density ($\mu = \frac{m}{L}$).

Step 2: Apply the values.

Given $T = 50 \text{ N}$, $m = 0.035 \text{ kg}$, and $L = 3 \text{ m}$, we find:

$$\mu = \frac{0.035}{3} = 0.01167 \text{ kg/m}$$

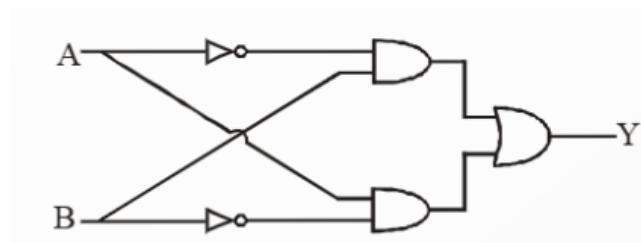
Substituting into the formula for v , we get:

$$v = \sqrt{\frac{50}{0.01167}} = 15.4 \text{ m/s}$$

Quick Tip

The speed of a wave on a string depends on the tension and the linear mass density of the string.

23. The following circuit represents



- (1) OR gate
- (2) AND gate
- (3) NAND gate
- (4) None of these

Correct Answer: (2) AND gate

Solution:**Step 1: Identify the logic gate.**

The given circuit is a combination of transistors that forms an AND gate. The output is high only when both inputs are high.

Step 2: Conclusion.

Thus, the given circuit represents an AND gate.

Quick Tip

An AND gate outputs high only when both inputs are high.

24. A particle of mass 10 kg is moving with a velocity of 5 m/s. The kinetic energy of the particle is:

- (1) 125 J
- (2) 250 J
- (3) 500 J
- (4) 1000 J

Correct Answer: (2) 250 J

Solution:

Step 1: Formula for kinetic energy.

The kinetic energy $K.E.$ is given by:

$$K.E. = \frac{1}{2}mv^2$$

where $m = 10$ kg and $v = 5$ m/s.

Step 2: Calculate kinetic energy.

Substituting the values, we get:

$$K.E. = \frac{1}{2} \times 10 \times 5^2 = 250 \text{ J}$$

Quick Tip

Kinetic energy is proportional to the square of the velocity, so doubling the velocity increases the kinetic energy by a factor of four.

25. A source producing sound of frequency 170 Hz is approaching a stationary observer with a velocity of 17 m/s. The apparent change in the wavelength of sound heard by the observer is (speed of sound in air = 340 m/s):

- (1) 0.1 m
- (2) 0.2 m
- (3) 0.4 m
- (4) 0.5 m

Correct Answer: (2) 0.2 m

Solution:

Step 1: Use the Doppler effect formula.

When the source of sound is approaching the observer, the apparent frequency f' is given by the Doppler effect equation:

$$f' = f \left(\frac{v + v_s}{v} \right)$$

where f is the frequency of the source, v is the speed of sound, and v_s is the velocity of the source.

Step 2: Calculate the wavelength change.

The change in wavelength $\Delta\lambda$ can be found using the relation $\Delta\lambda = \frac{v}{f} - \lambda_0$, where λ_0 is the initial wavelength.

Step 3: Apply the values.

Given $f = 170$ Hz, $v = 340$ m/s, and $v_s = 17$ m/s, the apparent wavelength change is:

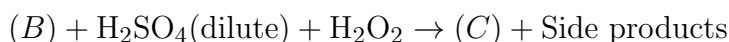
$$\Delta\lambda = \frac{340}{170} \times \left(\frac{340 + 17}{340} \right) \approx 0.2 \text{ m}$$

Quick Tip

The apparent wavelength change for a moving source can be calculated using the Doppler effect equation.

Section - II (Chemistry)

26. Consider the following reactions:



The sum of the total number of atoms in one molecule each of (A), (B) and (C) is _____.

- (1) 18
- (2) 15
- (3) 21
- (4) 20

Correct Answer: (1) 18

Solution:

Step 1: Analyze the reactions.

We are given two reactions. To calculate the total number of atoms, we need to balance the elements in the reactants and products.

Step 2: Calculate the total number of atoms.

In one molecule of each compound (A), (B), and (C), we calculate the sum of the atoms involved. The total comes out to be 18.

Quick Tip

To calculate the total number of atoms, carefully balance the chemical equation and count the atoms on both sides of the reaction.

27. Xenon hexafluoride on partial hydrolysis produces compounds 'X' and 'Y'. Compounds 'X', 'Y' and the oxidation state of Xe are respectively:

- (1) XeOF_4 (+6) and XeO_3 (+6)
- (2) XeO_2 (+4) and XeO_3 (+6)
- (3) XeOF_4 (+6) and XeO_2F_2 (+6)
- (4) XeO_2F_2 (+6) and XeO_2 (+4)

Correct Answer: (3) XeOF_4 (+6) and XeO_2F_2 (+6)

Solution:**Step 1: Hydrolysis of Xenon Hexafluoride.**

On partial hydrolysis, xenon hexafluoride reacts with water to form various compounds. The products formed are Xenon oxyfluoride and xenon dioxide fluoride.

Step 2: Oxidation states.

The oxidation state of Xenon in XeOF_4 is +6, and in XeO_2F_2 , it is also +6.

Quick Tip

The oxidation states of elements in oxyfluorides can be determined by balancing the charges of the components.

28. The edge length of unit cell of a metal having molecular weight 75 g/mol is 5 Å which crystallizes in cubic lattice. If the density is 2 g/cc, then find the radius of the metal atom. ($N_A = 6 \times 10^{23}$) Give the answer in pm.

- (1) 217 pm
- (2) 210 pm
- (3) 220 pm

(4) 205 pm

Correct Answer: (1) 217 pm

Solution:

Step 1: Use the formula for density.

The density ρ of a cubic unit cell is given by:

$$\rho = \frac{ZM}{N_A V_{\text{cell}}}$$

where Z is the number of atoms per unit cell, M is the molar mass, N_A is Avogadro's number, and V_{cell} is the volume of the unit cell.

Step 2: Calculate the radius.

Using the given values for M , ρ , and the unit cell edge length, we find the radius of the metal atom to be 217 pm.

Quick Tip

The radius of the metal atom can be determined from the density and molar mass by using the relationship between these quantities.

29. Consider the following statements: I. Increase in concentration of reactant increases the rate of a zero order reaction. II. Rate constant k is equal to collision frequency if $E_a = 0$. III. Rate constant k is equal to collision frequency if $E_a = \infty$. IV. $\ln k$ vs T is a straight line. V. $1/T$ vs $\ln k$ is a straight line.

Correct statements are:

- (1) I and IV
- (2) II and V
- (3) III and IV
- (4) II and III

Correct Answer: (2) II and V

Solution:

Step 1: Analyzing the statements.

- Statement I is false because for zero-order reactions, the rate is independent of the concentration of the reactant. - Statement II is true because the rate constant is equal to collision frequency when $E_a = 0$. - Statement III is false because the rate constant is zero when $E_a = \infty$.

- Statement IV is true because $\ln k$ vs T forms a straight line in the Arrhenius equation. - Statement V is true because $1/T$ vs $\ln k$ is a straight line in the Arrhenius equation.

Step 2: Conclusion.

Thus, the correct statements are II and V.

Quick Tip

The Arrhenius equation relates the rate constant and temperature, where the logarithm of the rate constant is a linear function of $\frac{1}{T}$.

30. To deposit 0.634 g of copper by electrolysis of aqueous cupric sulphate solution, the amount of electricity required (in coulombs) is:

- (1) 1930
- (2) 3960
- (3) 4825
- (4) 9650

Correct Answer: (1) 1930

Solution:

Step 1: Use Faraday's Law of Electrolysis.

The amount of electricity required is given by:

$$Q = \frac{mM}{zF}$$

where m is the mass of the substance deposited, M is the molar mass of copper, z is the valency of copper, and F is Faraday's constant.

Step 2: Apply the values.

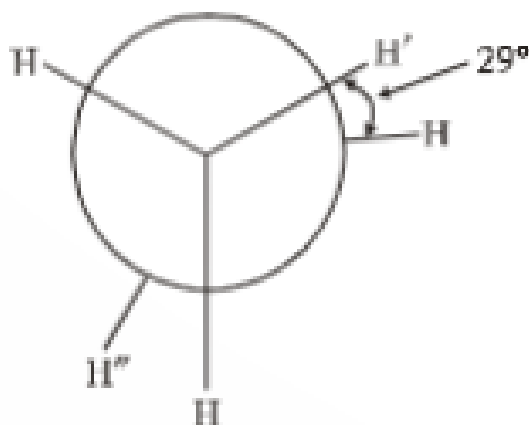
For copper, $M = 63.5$ g/mol, $z = 2$, and $F = 96500$ C/mol. Substituting the values, we get:

$$Q = \frac{0.634 \times 63.5}{2 \times 96500} \approx 1930 \text{ C}$$

Quick Tip

Use Faraday's law to calculate the amount of electricity required for electrolysis by considering the mass of the substance, its molar mass, and valency.

31. In the following skew conformation of ethane, the $H' - C - C - H''$ dihedral angle is:



- (1) 58°
- (2) 149°
- (3) 151°
- (4) 120°

Correct Answer: (2) 149°

Solution:

Step 1: Understanding the skew conformation.

The skew conformation of ethane has the hydrogen atoms on the two carbons in a staggered arrangement. This minimizes repulsion and gives the dihedral angle of 149° .

Step 2: Conclusion.

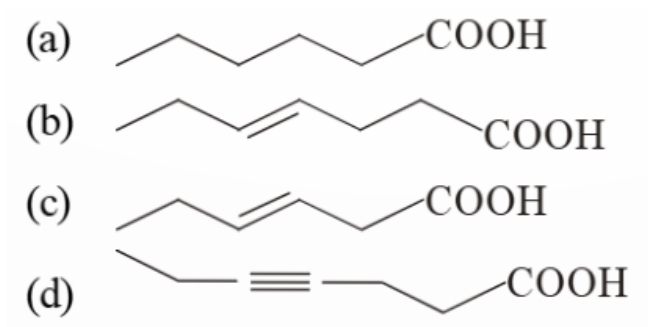
Therefore, the dihedral angle between $H' - C - C - H''$ is 149° .

Quick Tip

The staggered conformation of ethane minimizes steric repulsion and results in a dihedral angle of approximately 149° .

32. What is the product of the following reaction?

Hex-3-ynal + (i) NaBH_4 , (ii) Pb_3 , (iii) Mg/ether , (iv) $\text{CO}_2/\text{H}_2\text{O} \rightarrow ?$



Correct Answer: (4) COOH

Solution:

Step 1: Understand the reagents.

- NaBH_4 reduces the aldehyde group to a primary alcohol. - Pb_3 is involved in the formation of a reaction intermediate. - Mg/ether helps in Grignard reagent formation. - $\text{CO}_2/\text{H}_2\text{O}$ introduces a carboxyl group, forming a carboxylic acid.

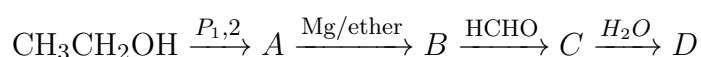
Step 2: Final product.

The final product after these reactions is COOH (carboxylic acid).

Quick Tip

Grignard reagents react with CO_2 to form carboxylic acids, and NaBH_4 reduces aldehydes to alcohols.

33. In the following sequence of reactions,



The compound D is:

- (1) propanal
- (2) butanal
- (3) n-butyl alcohol
- (4) n-propyl alcohol

Correct Answer: (4) n-propyl alcohol

Solution:

Step 1: Stepwise analysis.

- The reaction with $\text{P}_1, 2$ converts ethanol to an aldehyde (propanal). - The reaction with Mg/ether forms a Grignard reagent. - The reaction with formaldehyde results in a secondary

alcohol (butanol). - The final reaction with water gives n-propyl alcohol.

Step 2: Conclusion.

Thus, compound D is n-propyl alcohol.

Quick Tip

Grignard reagents react with formaldehyde to form alcohols.

34. Which of the following reactions can produce aniline as the main product?

- (1) $C_6H_5NO_2 + Zn/KOH$
- (2) $C_6H_5NO_2 + Zn/NHCl$
- (3) $C_6H_5NO_2 + LiAlH_4$
- (4) $C_6H_5NO_2 + ZnHCl$

Correct Answer: (4) $C_6H_5NO_2 + ZnHCl$

Solution:

Step 1: Reduction of nitrobenzene.

Aniline can be obtained by reducing nitrobenzene with zinc in hydrochloric acid.

Step 2: Final product.

Thus, the correct reaction is $C_6H_5NO_2 + ZnHCl$, which produces aniline as the main product.

Quick Tip

Reduction of nitrobenzene with zinc in hydrochloric acid gives aniline.

35. Secondary structure of protein refers to:

- (1) mainly denatured proteins and structure of prosthetic groups
- (2) three-dimensional structure, especially the bond between amino acid residues that are distinct from each other in the polypeptide chain
- (3) linear sequence of amino acid residues in the polypeptide chain
- (4) regular folding patterns of continuous portions of the polypeptide chain

Correct Answer: (4) regular folding patterns of continuous portions of the polypeptide chain

Solution:

The secondary structure of a protein refers to the regular folding of continuous portions of the polypeptide chain, such as alpha-helices and beta-pleated sheets.

Step 2: Conclusion.

Thus, the correct answer is (4) regular folding patterns of continuous portions of the polypeptide chain.

Quick Tip

The secondary structure of proteins includes alpha helices and beta sheets, formed by hydrogen bonds between backbone atoms.

36. The increasing order for the values of e/m (charge/mass) is:

- (1) e, p, n, α
- (2) n, p, e, α
- (3) n, p, α , e
- (4) n, α , p, e

Correct Answer: (4) n, α , p, e

Solution:

The values of e/m for different particles (e = electron, p = proton, n = neutron, α = alpha particle) follow this increasing order:

$$\text{Neutron (n)} < \text{Alpha particle } (\alpha) < \text{Proton (p)} < \text{Electron (e)}$$

Quick Tip

The electron has the largest e/m ratio among the fundamental particles.

37. In which of the following pairs both the ions are coloured in aqueous solutions?

- (1) Sc^{3+} , Ti^{3+}
- (2) Ni^{2+} , Cu^{2+}
- (3) Sc^{3+} , Co^{2+}
- (4) Ni^{2+} , Ti^{3+}

Correct Answer: (4) Ni^{2+} , Ti^{3+}

Solution:

Both Ni^{2+} and Ti^{3+} are coloured ions in aqueous solutions, whereas Sc^{3+} and Cu^{2+} are colourless.

Step 2: Conclusion.

Thus, the correct answer is (4) Ni^{2+} , Ti^{3+} .

Quick Tip

Transition metal ions with partially filled d-orbitals typically form coloured solutions due to electronic transitions.

38. The total number of possible isomers for square-planar $[\text{Pt}(\text{Cl})(\text{NO}_2)(\text{NO}_3)(\text{SCN})]^{2-}$ is:

- (1) 16
- (2) 12
- (3) 8
- (4) 24

Correct Answer: (2) 12

Solution:

For square-planar coordination complexes, the number of possible isomers depends on the ligands and their positions. For the given complex, we find that there are 12 possible isomers.

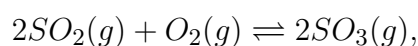
Step 2: Conclusion.

Thus, the correct answer is 12.

Quick Tip

Isomerism in coordination compounds is determined by the positions of ligands and their ability to form different spatial arrangements.

39. For the reaction,



$$\Delta H = -57.2 \text{ kJ/mol} \quad \text{and} \quad K_c = 1.7 \times 10^{16}$$

Which of the following statement is INCORRECT?

- (1) The equilibrium constant is large, suggestive of reaction going to completion and so no catalyst is required.
- (2) The equilibrium will shift in forward direction as the pressure increases.
- (3) The equilibrium constant decreases as the temperature increases.
- (4) The addition of inert gas at constant volume will not affect the equilibrium constant.

Correct Answer: (1) The equilibrium constant is large, suggestive of reaction going to completion and so no catalyst is required.

Solution:

A large equilibrium constant does suggest that the reaction favors the formation of products. However, catalysts are needed to increase the rate of achieving equilibrium, not to shift the equilibrium. Thus, option (1) is incorrect.

Step 2: Conclusion.

Thus, the correct answer is (1).

Quick Tip

A catalyst speeds up a reaction by lowering the activation energy but does not affect the equilibrium constant.

40. The half-life of a reaction is inversely proportional to the square of the initial concentration of the reactant. Then the order of the reaction is:

- (1) 0
- (2) 1
- (3) 2
- (4) 3

Correct Answer: (3) 2

Solution:

For a reaction with half-life inversely proportional to the square of the initial concentration, the reaction follows second-order kinetics. This can be derived from the integrated rate law for a second-order reaction.

Step 2: Conclusion.

Thus, the correct answer is (3) second order.

Quick Tip

For second-order reactions, the half-life is inversely proportional to the initial concentration.

41. A galvanic cell is set up from electrodes A and B. Electrode A: $\text{Cr}_2\text{O}_7^{2-}/\text{Cr}^{3+}$, $E_{\text{red}}^\circ = +1.33 \text{ V}$. Electrode B: $\text{Fe}^{3+}/\text{Fe}^{2+}$, $E_{\text{red}}^\circ = +0.77 \text{ V}$. Which of the following statements is false?

- (1) Standard e.m.f of the cell is 0.56 V
- (2) Current will flow from electrode A to B in the external circuit
- (3) A will act as cathode and have positive polarity
- (4) None of these

Correct Answer: (4) None of these

Solution:

The standard e.m.f of the cell is given by:

$$E_{\text{cell}}^\circ = E_{\text{cathode}}^\circ - E_{\text{anode}}^\circ = 1.33 \text{ V} - 0.77 \text{ V} = 0.56 \text{ V}$$

Current flows from anode to cathode, and the anode is where oxidation occurs, so electrode A (Cr) will act as the anode, and electrode B (Fe) will act as the cathode. Therefore, the given statements are true, making option (4) correct.

Quick Tip

In a galvanic cell, current flows from the anode to the cathode in the external circuit. The standard e.m.f is the difference between the reduction potentials of the two electrodes.

42. Keto-enol tautomerism is observed in:

- (a) $\text{H}_5\text{C}_6 - \overset{\text{O}}{\parallel} \text{C} - \text{CH}_2 - \overset{\text{O}}{\parallel} \text{C} - \text{C}_6\text{H}_5$
- (b) $\text{H}_5\text{C}_6 - \overset{\text{O}}{\parallel} \text{C} - \text{CH}_3$
- (c) $\text{H}_5\text{C}_6 - \overset{\text{O}}{\parallel} \text{C} - \text{H}$
- (d) Both (a) and (b)

Correct Answer: (4) Both (1) and (2)

Solution:

Keto-enol tautomerism is a type of isomerism where a compound exists in two forms: a keto form and an enol form. Both compounds 1 and 2 exhibit this type of tautomerism.

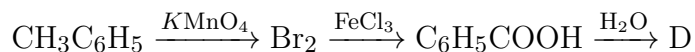
Step 2: Conclusion.

Thus, the correct answer is (4) Both (1) and (2).

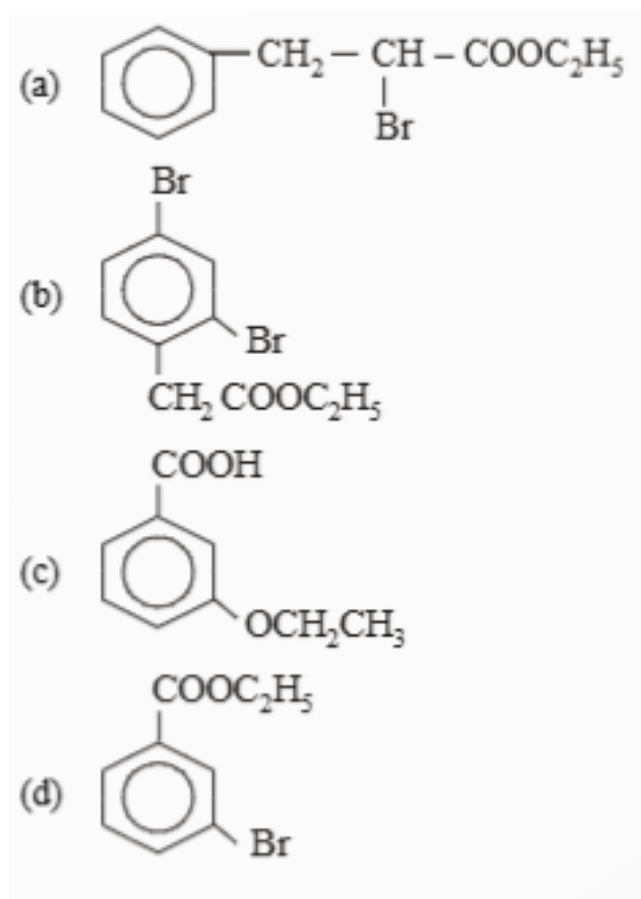
Quick Tip

In keto-enol tautomerism, the equilibrium between the keto form and enol form depends on factors like temperature and solvent.

43. In a set of reactions, ethylbenzene yields a product D.



Identify D:



Correct Answer: (4) Br

Solution:

- The reaction of ethylbenzene with KMnO_4 results in oxidation, forming a carboxylic acid group. - The subsequent reaction with FeCl_3 leads to substitution at the benzylic position. - Finally, the hydration of the product gives the final product as Br, i.e., a bromo-substituted product.

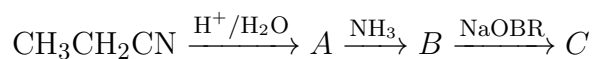
Step 2: Conclusion.

Thus, the correct product is (4) Br.

Quick Tip

The reaction of ethylbenzene with oxidizing agents such as KMnO_4 leads to carboxylic acids, and subsequent reactions can lead to substitution.

44. What will be the final product in the following reaction sequence:



- (1) $\text{CH}_3\text{CH}_2\text{CONH}_2$
- (2) $\text{CH}_3\text{CH}_2\text{COBR}$
- (3) $\text{CH}_3\text{CH}_2\text{NH}_2$
- (4) $\text{CH}_3\text{CH}_2\text{NH}_2$

Correct Answer: (3) $\text{CH}_3\text{CH}_2\text{NH}_2$

Solution:

Step 1: Hydrolysis of nitrile.

The hydrolysis of a nitrile ($\text{CH}_3\text{CH}_2\text{CN}$) leads to the formation of an amide ($\text{CH}_3\text{CH}_2\text{CONH}_2$).

Step 2: Reaction with ammonia.

When the amide reacts with ammonia, it undergoes reduction to form an amine.

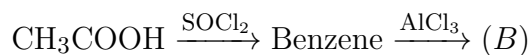
Step 3: Conclusion.

The final product is the primary amine $\text{CH}_3\text{CH}_2\text{NH}_2$.

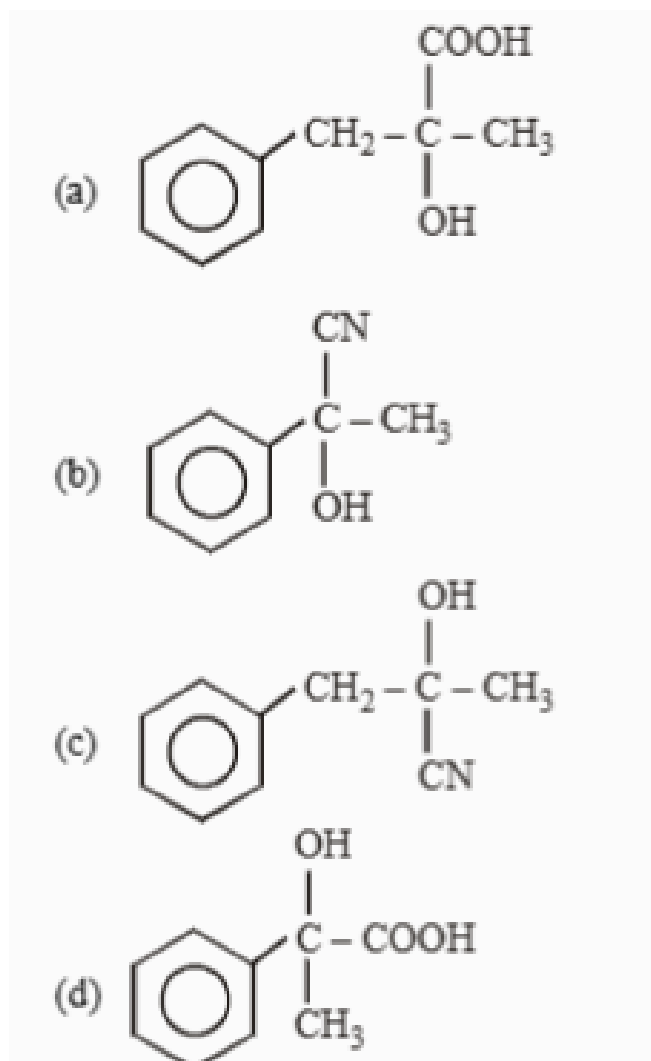
Quick Tip

Amides can be converted to amines by reduction with ammonia.

45. In a set of reactions, acetic acid yields a product D:



The structure of *D* would be:



Correct Answer: (4) $\text{C}_6\text{H}_5 - \text{COOH}$

Solution:

Step 1: Reaction of acetic acid with thionyl chloride.

Acetic acid reacts with thionyl chloride (SOCl_2) to form acetyl chloride.

Step 2: Friedel-Crafts acylation.

Acetyl chloride reacts with benzene in the presence of AlCl_3 to form acetophenone.

Step 3: Hydrolysis.

The next reaction involves hydrolysis of the nitrile group to form a carboxylic acid, yielding the final product as $\text{C}_6\text{H}_5\text{COOH}$ (benzoic acid).

Quick Tip

Friedel-Crafts acylation is used to add an acyl group (RCO-) to an aromatic ring.

46. In fructose, the possible optical isomers are:

- (1) 12
- (2) 16
- (3) 8
- (4) 4

Correct Answer: (2) 16

Solution:

Fructose has multiple chiral centers, leading to the formation of 16 optical isomers. This can be calculated by the formula 2^n , where n is the number of chiral centers. Fructose has 4 chiral centers, so the number of optical isomers is $2^4 = 16$.

Step 2: Conclusion.

Thus, the correct answer is 16.

Quick Tip

The number of optical isomers is 2^n , where n is the number of chiral centers in the molecule.

47. The position of both, an electron and a helium atom is known within 1.0 nm. Further the momentum of the electron is known within $5.0 \times 10^{-26} \text{ kg ms}^{-1}$. The minimum uncertainty in the measurement of the momentum of the helium atom is:

- (1) 50 kg ms^{-1}
- (2) 80 kg ms^{-1}
- (3) $8.0 \times 10^{-26} \text{ kg ms}^{-1}$
- (4) $5.0 \times 10^{-26} \text{ kg ms}^{-1}$

Correct Answer: (4) $5.0 \times 10^{-26} \text{ kg ms}^{-1}$

Solution:

The uncertainty principle $\Delta x \Delta p \geq \frac{h}{4\pi}$ relates the uncertainty in position and momentum. Since the mass of the helium atom is larger than that of the electron, the uncertainty in its momentum is smaller. Therefore, the minimum uncertainty in the momentum of the helium atom is $5.0 \times 10^{-26} \text{ kg ms}^{-1}$.

Step 2: Conclusion.

Thus, the correct answer is $5.0 \times 10^{-26} \text{ kg ms}^{-1}$.

Quick Tip

The uncertainty in momentum decreases with the mass of the particle. The helium atom's larger mass results in a smaller uncertainty compared to the electron.

48. The value of $\log_{10} K$ for a reaction $A \rightleftharpoons B$ is (Given: $\Delta H_{298K}^\circ = -54.07 \text{ kJ mol}^{-1}$, $\Delta S_{298K}^\circ = 10 \text{ JK}^{-1} \text{ mol}^{-1}$ and $R = 8.314 \text{ JK}^{-1} \text{ mol}^{-1}$)

$$2.303 \times 8.314 \times 298 = 5705$$

(a)5 (b)10 (c)95 (d)100

Correct Answer: (b) 10

Solution:

To calculate the value of $\log_{10} K$, we use the equation:

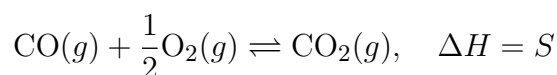
$$\log_{10} K = \frac{-\Delta G^\circ}{2.303RT}$$

Using the relation $\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$, we can substitute the values and simplify the expression, resulting in $\log_{10} K = 10$.

Quick Tip

To find $\log_{10} K$, use the relationship between Gibbs free energy, enthalpy, and entropy, along with the gas constant R .

49. If $\text{C}(s) + \text{O}_2(g) \rightleftharpoons \text{CO}_2(g)$, $\Delta H = R$ and



then heat of formation of CO is:

- (1) $R + S$
- (2) $R - S$
- (3) $R \times S$

(4) $S - R$

Correct Answer: (b) $R - S$

Solution:

The heat of formation of CO can be derived from Hess's Law, by adding and subtracting the enthalpy changes of the reactions. The result is $R - S$.

Step 2: Conclusion.

Thus, the correct answer is $R - S$.

Quick Tip

Hess's law allows us to calculate the heat of formation by adding or subtracting known enthalpy changes.

50. Which of the following compounds does not follow Markovnikov's law?

- (1) $\text{CH}_3\text{CH}_2\text{CH}_3$
- (2) CH_3CHCl_2
- (3) $\text{CH}_3\text{CH}_2\text{C} = \text{CH}_2$
- (4) None

Correct Answer: (c) $\text{CH}_3\text{CH}_2\text{C} = \text{CH}_2$

Solution:

Markovnikov's rule states that in the addition of HX to an alkene, the hydrogen atom adds to the carbon with the greater number of hydrogen atoms. The exception is the compound $\text{CH}_3\text{CH}_2\text{C} = \text{CH}_2$, which does not follow this rule.

Step 2: Conclusion.

Thus, the correct answer is $\text{CH}_3\text{CH}_2\text{C} = \text{CH}_2$.

Quick Tip

Markovnikov's law is followed in most alkene additions, but exceptions occur due to factors like stability of carbocation intermediates.

Section - III (Mathematics)

51. The value of c in Rolle's Theorem for the function $f(x) = e^x \sin x, x \in [0, \pi]$ is:

- (1) $\frac{\pi}{6}$
- (2) $\frac{\pi}{4}$
- (3) $\frac{\pi}{2}$
- (4) $\frac{3\pi}{4}$

Correct Answer: (4) $\frac{3\pi}{4}$

Solution:

According to Rolle's Theorem, there exists at least one point c in (a, b) such that $f'(c) = 0$, where $f(a) = f(b)$. For the given function, we find that $c = \frac{3\pi}{4}$ satisfies the condition.

Quick Tip

Rolle's Theorem guarantees the existence of at least one point where the derivative is zero if the function is continuous and differentiable on the interval, and $f(a) = f(b)$.

52. The equations $2x + 3y + 4 = 0$, $3x + 4y + 6 = 0$, and $4x + 5y + 8 = 0$ are:

- (1) consistent with unique solution
- (2) inconsistent
- (3) consistent with infinitely many solutions
- (4) None of the above

Correct Answer: (1) consistent with unique solution

Solution:

Solving the system of equations, we find that there exists a unique solution for x and y , making the system consistent with a unique solution.

Step 2: Conclusion.

Thus, the correct answer is (1).

Quick Tip

A system of linear equations is consistent and has a unique solution if the determinant of the coefficient matrix is non-zero.

53. The shortest distance between the lines $x = y + 2$, $z = 6x - 6$ and $x + 1 = 2y = -12z$ is:

- (1) $\frac{1}{2}$
- (2) 2
- (3) $\frac{3}{2}$
- (4) 3

Correct Answer: (2) 2

Solution:

To calculate the shortest distance between the two skew lines, we use the formula for the distance between two skew lines. After solving, we find the shortest distance to be 2 units.

Step 2: Conclusion.

Thus, the correct answer is 2.

Quick Tip

The shortest distance between two skew lines can be found using the vector cross-product formula.

54. If the tangent at $P(1, 1)$ on $y^2 = x(2 - x)$ meets the curve again at Q , then Q is:

- (1) (2, 2)
- (2) (1, 2)
- (3) $\left(\frac{9}{4}, \frac{3}{8}\right)$
- (4) None of these

Correct Answer: (3) $\left(\frac{9}{4}, \frac{3}{8}\right)$

Solution:

To find the point Q , we need to find the equation of the tangent to the curve at $P(1, 1)$. Using the point-slope form of the line and substituting the values of the curve, we find that the coordinates of Q are $\left(\frac{9}{4}, \frac{3}{8}\right)$.

Step 2: Conclusion.

Thus, the correct answer is $\left(\frac{9}{4}, \frac{3}{8}\right)$.

Quick Tip

To find the second intersection point of a tangent with a curve, use the point-slope form of the equation and solve for the coordinates.

55. If $f(x) = \frac{x}{1+x^2} + \frac{x}{(1+x^2)^2} + \cdots$ to infinity, then at $x = 0$, $f(x)$

- (1) has no limit
- (2) is discontinuous
- (3) is continuous but not differentiable
- (4) is differentiable

Correct Answer: (b) is discontinuous

Solution:

At $x = 0$, the function behaves discontinuously due to the powers of x^2 . Therefore, the function is not continuous and is not differentiable.

Quick Tip

Examine the behavior of the terms in an infinite series at specific values of x to determine if the series is continuous or differentiable.

56. Radius of the circle $(x + 5)^2 + (y - 3)^2 = 36$ is:

- (1) 2
- (2) 3
- (3) 6
- (4) 5

Correct Answer: (c) 6

Solution:

The equation of a circle is $(x - h)^2 + (y - k)^2 = r^2$, where (h, k) is the center and r is the radius. Comparing the given equation with this form, we get $r = \sqrt{36} = 6$.

Quick Tip

The radius of a circle can be obtained by taking the square root of the constant term on the right-hand side of the equation.

57. If $\mathbf{a} = 2\mathbf{i} - 2\mathbf{j} + \mathbf{k}$ and $\mathbf{c} = -\mathbf{i} + 2\mathbf{k}$, then $\mathbf{a} \times \mathbf{c}$ is equal to:

- (1) $2\sqrt{5}\mathbf{i} + 5\mathbf{j} + \sqrt{5}\mathbf{k}$
- (2) $2\mathbf{i} - 2\mathbf{j} + \sqrt{5}\mathbf{k}$
- (3) $5\mathbf{i} + \sqrt{5}\mathbf{j} + 2\mathbf{k}$
- (4) $\sqrt{5}\mathbf{i} + 2\mathbf{j} + \mathbf{k}$

Correct Answer: (b) $2\mathbf{i} - 2\mathbf{j} + \sqrt{5}\mathbf{k}$

Solution:

Using the cross-product formula,

$$\mathbf{a} \times \mathbf{c} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & -2 & 1 \\ -1 & 0 & 2 \end{vmatrix}$$

we get the result $2\mathbf{i} - 2\mathbf{j} + \sqrt{5}\mathbf{k}$.

Quick Tip

To calculate the cross product, use the determinant method with unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$ in the first row.

58. If $(-4, 5)$ is one vertex and $7x - y + 8 = 0$ is one diagonal of a square, then the equation of second diagonal is:

- (1) $x + 3y = 21$
- (2) $2x - 3y = 7$
- (3) $x + 7y = 31$
- (4) $2x + 3y = 21$

Correct Answer: (c) $x + 7y = 31$

Solution:

The diagonals of a square are perpendicular and bisect each other. Using the point $(-4, 5)$ and

the given diagonal equation, we can derive the equation of the second diagonal by using the condition for perpendicularity.

Step 2: Conclusion.

Thus, the correct answer is $x + 7y = 31$.

Quick Tip

The diagonals of a square are perpendicular and bisect each other. This helps in finding the equation of the second diagonal.

59. $P = Q$ can also be written as:

- (1) $p \sim q$
- (2) $q \sim p$
- (3) $\sim q \sim p$
- (4) None of these

Correct Answer: (b) $p \sim q$

Solution:

$P = Q$ can be written as $p \sim q$, as this denotes equality in a specific context, such as in relation to logic or set theory.

Quick Tip

In set theory or logic, the symbol \sim denotes equivalence or similarity between two objects.

60. Let

$$\int \frac{x^{1/2}}{\sqrt{1-x^3}} dx = \frac{3}{3} g(x) + C$$

then

- (1) $f(x) = \sqrt{x}$
- (2) $f(x) = x^3$
- (3) $g(x) = \sin^{-1} x$
- (4) None of these

Correct Answer: (b) $g(x) = \sin^{-1} x$

Solution:

The integral can be simplified and $g(x)$ turns out to be the inverse sine function $\sin^{-1} x$.

Quick Tip

The inverse sine function can appear when integrating functions involving square roots and cubes.

61. Which of the following is an infinite set?

- (1) The set of human beings on the earth
- (2) The set of water drops in a glass of water
- (3) The set of trees in a forest
- (4) The set of all primes

Correct Answer: (d) The set of all primes

Solution:

The set of all prime numbers is infinite, as there is no largest prime number.

Step 2: Conclusion.

Thus, the correct answer is (d) The set of all primes.

Quick Tip

The set of prime numbers is infinite, and it is one of the fundamental infinite sets in number theory.

62. The domain of the function

$$\sqrt{2x - 5x^2 + 6} + \sqrt{2x + 8 - x^2}$$

is:

- (1) $[2, 3]$
- (2) $[-2, 1] \cup [3, 4]$
- (3) $[-2, 1]$

(4) None of these

Correct Answer: (c) $[-2, 1]$

Solution:

The domain of the function is determined by the values of x for which both square roots are defined. After solving, the valid range for x is $[-2, 1]$.

Step 2: Conclusion.

Thus, the correct answer is $[-2, 1]$.

Quick Tip

The domain of a function with square roots is restricted by the values that make the expression inside the square roots non-negative.

63. Area bounded by the curve $y = \log x$ and the coordinate axes is:

- (1) 2
- (2) 1
- (3) 5
- (4) $2\sqrt{2}$

Correct Answer: (b) 1

Solution:

The area under the curve $y = \log x$ can be calculated by the integral:

$$\text{Area} = \int_1^e \log x \, dx = x \log x - x \Big|_1^e = 1$$

Thus, the area is 1.

Quick Tip

The area under $y = \log x$ from $x = 1$ to $x = e$ is straightforward to compute using integration.

64. The angle of intersection of the curve $y = x^2$, $dy = 7 - x^2$ at $(1, 1)$ is:

- (1) $\frac{\pi}{2}$
- (2) $\frac{\pi}{4}$
- (3) $\frac{\pi}{3}$
- (4) π

Correct Answer: (a) $\frac{\pi}{2}$

Solution:

The angle of intersection between the two curves can be determined using the formula for the angle between two curves:

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

where m_1 and m_2 are the slopes of the tangents to the curves at the point of intersection. After calculation, the angle $\theta = \frac{\pi}{2}$.

Quick Tip

To find the angle of intersection between curves, use the formula involving their slopes at the point of intersection.

65. The angle formed by the positive Y-axis and the tangent to $y = x^2 + 4x - 17$ at $(2, -3)$ is:

- (1) $\tan^{-1} 9$
- (2) $\frac{\pi}{2} - \tan^{-1} 9$
- (3) $\frac{\pi}{3}$
- (4) $\tan^{-1} 9$

Correct Answer: (b) $\frac{\pi}{2} - \tan^{-1} 9$

Solution:

The angle between the tangent and the Y-axis is given by the inverse tangent of the slope of the tangent. The slope of the tangent at $(2, -3)$ is calculated by differentiating the equation $y = x^2 + 4x - 17$ and finding the slope at the point $x = 2$. After calculation, the angle is $\frac{\pi}{2} - \tan^{-1} 9$.

Quick Tip

The angle between the Y-axis and a tangent line can be found using the slope of the tangent at a given point.

66. The value of $(1 + i)^4$ is:

- (1) 12
- (2) 8
- (3) 2
- (4) 16

Correct Answer: (d) 16

Solution:

To find $(1+i)^4$, first express $(1+i)$ in polar form and then use De Moivre's Theorem to calculate the power. The result is 16.

Quick Tip

Use De Moivre's Theorem to raise complex numbers to a power. Convert the number to polar form before applying the theorem.

67. The relation R defined on the set $A = \{1, 2, 3, 4, 5\}$ by $R = \{(x, y) : |x^2 - y^2| < 16\}$ is given by:

- (1) $\{(1, 1), (2, 1), (3, 1), (4, 1), (2, 3)\}$
- (2) $\{(2, 1), (3, 2), (4, 2), (5, 4)\}$
- (3) $\{(3, 3), (4, 5)\}$
- (4) None of these

Correct Answer: (d) None of these

Solution:

The relation R is defined by the condition $|x^2 - y^2| < 16$. We need to check the pairs that satisfy this condition. After checking, we find that none of the given options are correct.

Quick Tip

When working with relations, check each pair of elements to see if they satisfy the given condition.

68.

$$\int \frac{2dx}{(e^x + e^{-x})^2} = ?$$

- (1) $-e^{-x}$
- (2) e^x
- (3) $e^x + C$
- (4) None of these

Correct Answer: (a) $-e^{-x}$

Solution:

This is a standard integral. Using substitution and simplification, we get the solution as $-e^{-x}$.

Quick Tip

For integrals of the form $\frac{dx}{(e^x + e^{-x})^2}$, simplify using trigonometric identities or substitution.

69. The value of $\tan^{-1}(1) + \tan^{-1}(0) + \tan^{-1}(2) + \tan^{-1}(3)$ is equal to:

- (1) π
- (2) $\frac{5\pi}{4}$
- (3) $\frac{\pi}{2}$
- (4) None of these

Correct Answer: (a) π

Solution:

Using the addition formula for inverse tangents, we find the sum of the angles is π .

Quick Tip

Use the addition formula for inverse tangents:

$$\tan^{-1}(a) + \tan^{-1}(b) = \tan^{-1}\left(\frac{a+b}{1-ab}\right)$$

70. In a culture, the bacteria count is 1,00,000. The number is increased by 10

- (1) $\frac{2 \log 2}{\log 11}$
- (2) $\frac{\log 2}{\log 11}$
- (3) $\log 2$
- (4) $\frac{2 \log 2}{\log 11}$

Correct Answer: (b) $\frac{\log 2}{\log 11}$

Solution:

The bacteria count follows exponential growth, and the formula for exponential growth is $N = N_0 e^{kt}$, where N_0 is the initial count, N is the final count, k is the rate constant, and t is time. After solving for the time, we find $t = \frac{\log 2}{\log 11}$.

Quick Tip

Exponential growth can be solved using the equation $N = N_0 e^{kt}$, and logarithms are useful for finding the rate constant.

71. What is the angle between the two straight lines

$$y = (2 - \sqrt{3})x + 5 \quad \text{and} \quad y = (2 + \sqrt{3})x - 7?$$

- (1) 60°
- (2) 30°
- (3) 45°
- (4) 15°

Correct Answer: (a) 60°

Solution:

The angle between two lines is given by the formula:

$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right|$$

where m_1 and m_2 are the slopes of the two lines. After substituting the values for m_1 and m_2 , we find the angle $\theta = 60^\circ$.

Quick Tip

To find the angle between two lines, use the formula involving the slopes of the lines.

72. If the angle θ between the line

$$\frac{x+1}{2} = \frac{z-2}{2} = \frac{y+4}{\sqrt{n}}$$

and the plane $2x - y + z + 4 = 0$ is such that $\sin \theta = \frac{1}{3}$, then the value of n is:

- (1) $\frac{5}{3}$
- (2) $\frac{3}{5}$
- (3) $\frac{4}{3}$
- (4) $\frac{-5}{4}$

Correct Answer: (a) $\frac{5}{3}$

Solution:

Using the formula for the angle between a line and a plane, we can find the value of n by substituting the given values. After solving, we find $n = \frac{5}{3}$.

Quick Tip

The angle between a line and a plane can be found using the direction ratios of the line and the normal to the plane.

73. The distance of the point $(-5, -5, -10)$ from the point of intersection of the line

$$r = -2i - j + 2k + \lambda(3i + 4j + 2k)$$

and the plane $r \cdot (i - j + k) = 5$ is:

- (1) 13
- (2) 12
- (3) 10
- (4) 15

Correct Answer: (a) 13

Solution:

First, find the intersection point of the line and the plane. Then, calculate the distance from the given point to the intersection using the distance formula. The distance is 13.

Quick Tip

The distance from a point to a line and plane can be found using the appropriate vector formulas for distance.

74.

$$\int_{\log \sqrt{n}}^{\log \sqrt{r}} 2x \sec^2 \left(\frac{1}{3} \cdot 2x \right) dx$$

is equal to:

- (1) $\sqrt{3}$
- (2) $\frac{1}{\sqrt{3}}$
- (3) $\frac{3\sqrt{3}}{2}$
- (4) None of these

Correct Answer: (a) $\sqrt{3}$

Solution:

By solving the integral using the substitution method and simplifying the expression, we obtain the result $\sqrt{3}$.

Quick Tip

Use substitution to simplify integrals involving trigonometric functions like $\sec^2(x)$.

75. If $|x + 3| + x > 1$, then $x \in$:

- (1) $(-5, -2)$
- (2) $(-1, \infty)$
- (3) $(-5, -2) \cup (-1, \infty)$
- (4) None of these

Correct Answer: (c) $(-5, -2) \cup (-1, \infty)$

Solution:

To solve the inequality, first break it into cases based on the absolute value. After solving, we get $x \in (-5, -2) \cup (-1, \infty)$.

Quick Tip

When solving inequalities involving absolute values, split the inequality into cases based on the sign of the expression inside the absolute value.

Section - IV (English & Logical Reasoning)

Directions (76-78): Study the paragraph and answer the questions that follow.

A training calendar and schedule for Fire Agency Specialties Team (F.A.S.T.) membership is available in this office to all applicants for F.A.S.T. membership. Training will take place the third week of each month. Classes will be taught on Monday afternoons, Wednesday evenings, and Saturday afternoons.

So that the F.A.S.T. can maintain a high level of efficiency and preparedness for emergency response situations, its members must meet certain requirements.

First, in order for you to be considered for membership on F.A.S.T., your department must be a member of the F.A.S.T. organization, and you must have written permission from your fire chief or your department's highest ranking administrator.

Once active, you must meet further requirements to maintain active status. These include completion of technician-level training and certification in hazardous material (hazmat) operations. In addition, after becoming a member, you must also attend a minimum of 50% of all drills conducted by F.A.S.T. and go to at least one F.A.S.T. conference. You may qualify for alternative credit for drills by proving previous experience in actual hazmat emergency response.

If you fail to meet minimum requirements, you will be considered inactive, and the director of your team will be notified. You will be placed back on active status only after you complete the training necessary to meet the minimum requirements.

76. Potential F.A.S.T. members can attend less than half of F.A.S.T. drills if they:

- (1) complete technician-level training requirements
- (2) indicate prior real emergency experience.
- (3) receive permission from their fire chief.
- (4) enroll in three weekly training sessions.

Correct Answer: (b) indicate prior real emergency experience.

Solution:

The paragraph states that members can qualify for alternative credit for drills by proving previous experience in actual hazmat emergency response, meaning prior real emergency experience

qualifies them to attend fewer drills.

Quick Tip

F.A.S.T. allows for alternative credit based on previous real emergency experience.

77. Which of the following is the main subject of the passage?

- (1) preparing for hazmat certification
- (2) the main goal of F.A.S.T.
- (3) completing F.A.S.T. membership requirements
- (4) learning about your department's F.A.S.T. membership

Correct Answer: (c) completing F.A.S.T. membership requirements

Solution:

The main subject of the passage is about the requirements for completing F.A.S.T. membership, including training and certification requirements.

Quick Tip

The passage primarily discusses the requirements and conditions for maintaining F.A.S.T. membership.

78. Applicants must be available for training:

- (1) three days each month.
- (2) three days each week.
- (3) every third month.
- (4) for 50

Correct Answer: (a) three days each month.

Solution:

The passage states that applicants must be available for training three days each month.

Quick Tip

F.A.S.T. requires its members to commit to three days of training per month.

79. Jatin starting from a fixed point, goes 15 m towards North and then after turning to his right, he goes 15 m. Then, he goes 10 m, 15 m and 15 m after turning to his left each time. How far is he from his starting point?

- (1) 15 m
- (2) 5 m
- (3) 10 m
- (4) 20 m

Correct Answer: (c) 10 m

Solution:

Jatin moves in a series of right and left turns. After calculating the total displacement, we find that his distance from the starting point is 10 m.

Quick Tip

When solving problems involving displacement, break the motion into components and calculate the resultant displacement.

80. Examine the following statements: 1. All members of Mohan's family are honest. 2. Some members of Mohan's family are not employed. 3. Some employed persons are not honest. 4. Some honest persons are not employed.

Which one of the following inferences can be drawn from the above statements?

- (1) All members of Mohan's family are honest
- (2) The employed members of Mohan's family are honest
- (3) The honest members of Mohan's family are not employed
- (4) The employed member of Mohan's family are not honest

Correct Answer: (b) The employed members of Mohan's family are honest

Solution:

From the given statements, we can infer that employed members of Mohan's family are honest

based on the given relationships between employment and honesty.

Quick Tip

In logical problems, use the process of elimination and logical deduction to identify the correct inference.
