AIIMS Paramedical Mathematics Sample Paper – 10

Duration: 30 Minutes **Maximum Marks: 30**

Instructions

- This paper contains **30** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics portion of **AIIMS Paramedical** entrance.
- Each correct answer carries **+1 mark**; each incorrect answer carries a penalty of $-\frac{1}{3}$ mark; an unattempted question carries **0** mark.
- Only **one** option is correct. Choose carefully.
- Syllabus level: **Class 11 & 12 NCERT Mathematics**
- Use of mobile phones, calculators, or electronic gadgets is strictly prohibited.

Q1. If $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A = \{2, 4, 6, 8\}$ and $B = \{1, 2, 3, 4\}$, then $A' \cap B$ equals

(A) $\{2, 4\}$

(B) $\{5, 7\}$

(C) $\{1, 3\}$

(D) $\{1, 2, 3, 4\}$

Q2. The range of the function $f(x) = |x - 3|$ defined for all real x is

(A) $[0, \infty)$

(B) $(-\infty, \infty)$

(C) $[3, \infty)$

(D) $(-\infty, 0]$

Q3. If $\cos\theta = \frac{7}{25}$ and $0 < \theta < \frac{\pi}{2}$, then using the half-angle formula $\cos^2\frac{\theta}{2} = \frac{1+\cos\theta}{2}$, the value of $\cos\frac{\theta}{2}$ is

(A) $\frac{3}{5}$

(B) $\frac{1}{5}$

(C) $\frac{16}{25}$

(D) $\frac{4}{5}$

Q4. The positive value of x satisfying $\tan^{-1}(2x) + \tan^{-1}(3x) = \frac{\pi}{4}$ is

(A) $\frac{1}{3}$

(B) $\frac{1}{6}$

(C) 1

(D) $\frac{1}{4}$

Q5. The roots of the quadratic equation $x^2 - 4x + 13 = 0$ are

(A) $2 \pm i$

(B) $4 \pm 3i$

(C) $2 \pm 3i$

(D) $-2 \pm 3i$

Q6. The number of four-digit numbers that can be formed using the digits $1, 2, 3, 4, 5$ without repetition is

(A) 120

(B) 625

(C) 24

(D) 20

Q7. The sum of all the binomial coefficients in the expansion of $(1+x)^{12}$ is

(A) 12

(B) 144

(C) 2048

(D) 4096

Q8. The sum of the series $\sum_{n=1}^{10} n(n+1)$, that is $1\cdot 2 + 2\cdot 3 + \cdots + 10\cdot 11$, is

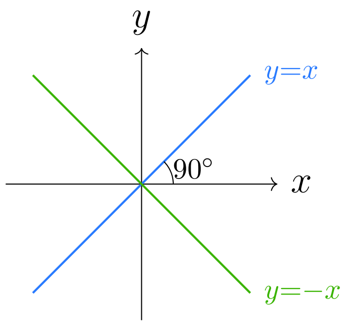
(A) 385

(B) 440

(C) 550

(D) 495

Q9. The angle between the pair of straight lines represented by $x^2 - y^2 = 0$ is



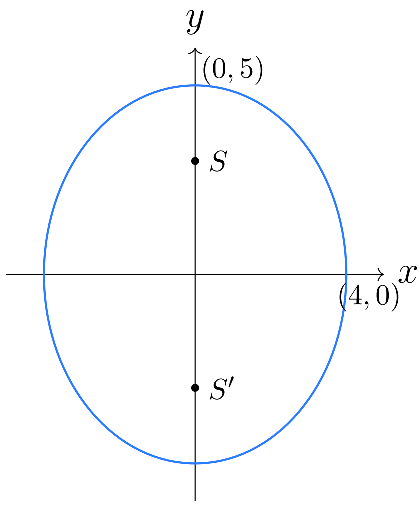
(A) 90°

(B) 45°

(C) 60°

(D) 30°

Q10. The eccentricity of the conic $\dfrac{x^2}{16} + \dfrac{y^2}{25} = 1$ is



(A) $\frac{4}{5}$

(B) $\frac{5}{4}$

(C) $\frac{3}{5}$

(D) $\frac{3}{4}$

Q11. The value of $\lim_{n\to\infty}\left(1+\frac{3}{n}\right)^n$ is

(A) e

(B) 1

(C) $e^{1/3}$

(D) e^3

Q12. If $y = \sin x$, then the second derivative $\frac{d^2y}{dx^2}$ is

(A) $\cos x$

(B) $-\sin x$

(C) $\sin x$

(D) $-\cos x$

Q13. The system $x + y = 5$, $x - y = 1$ written as $AX = B$ has solution $X = A^{-1}B$ given by

(A) $x = 3,\ y = 2$

(B) $x = 2,\ y = 3$

(C) $x = 4,\ y = 1$

(D) $x = 1,\ y = 4$

Q14. If A and B are square matrices of the same order with $|A| = 3$ and $|B| = -2$, then $|AB|$ is

(A) 1

(B) 5

(C) -6

(D) 6

Q15. The radius of a sphere is measured as 9 cm with a possible error of 0.03 cm. The approximate percentage error in the calculated volume is

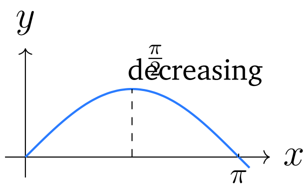
(A) 0.1%

(B) 1%

(C) 0.33%

(D) 3%

Q16. The function $f(x) = \sin x$ on the interval $\left(\frac{\pi}{2}, \pi\right)$ is



(A) increasing

(B) constant

(C) neither increasing nor decreasing

(D) decreasing

Q17. The value of $\int \frac{\sec^2 x}{\tan x}\,dx$ is (where C is the constant of integration)

(A) $\log|\tan x| + C$

(B) $\tan^2 x + C$

(C) $\sec x \tan x + C$

(D) $\log|\sec x| + C$

Q18. Using partial fractions, $\int \frac{dx}{x(x^2+1)}$ equals (where C is the constant of integration)

(A) $\log|x| + \frac{1}{2}\log(x^2+1) + C$

(B) $\frac{1}{2}\log(x^2+1) + C$

(C) $\log|x| - \frac{1}{2}\log(x^2+1) + C$

(D) $\log|x| - \log(x^2+1) + C$

Q19. Using the property $\int_0^{nT} f(x)\,dx = n\int_0^{T} f(x)\,dx$ for a periodic f of period T, the value of $\int_0^{4\pi} |\sin x|\,dx$ is

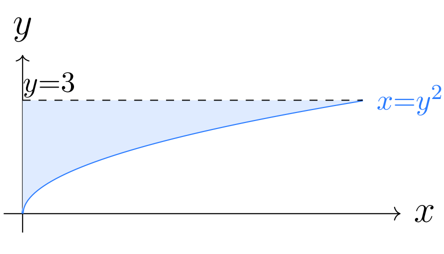
(A) 4

(B) 8

(C) 2

(D) 16

Q20. The area bounded by the curve $x = y^2$ and the y-axis between $y = 0$ and $y = 3$ is



(A) 3

(B) $\frac{27}{2}$

(C) 18

(D) 9

Q21. After rationalising, the differential equation $\frac{dy}{dx} = \sqrt{1 + \left(\frac{dy}{dx}\right)}$ written free of radicals has degree

(A) 2

(B) 1

(C) 3

(D) $\frac{1}{2}$

Q22. A population grows according to $\frac{dP}{dt} = kP$. If the population doubles in 5 years, then in terms of the growth constant, P at any time t is

(A) $P_0 + kt$

(B) $P_0 t^k$

(C) $P_0 e^{kt}$

(D) $P_0 e^{-kt}$

Q23. If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$, then $\vec{a}$ and $\vec{b}$ are

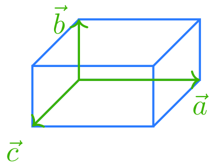
(A) parallel

(B) equal in magnitude

(C) anti-parallel

(D) perpendicular

Q24. The volume of the parallelepiped whose coterminous edges are $\vec{a} = \hat{i}$, $\vec{b} = \hat{j}$ and $\vec{c} = 2\hat{k}$ is



(A) 1

(B) 2

(C) 0

(D) 4

Q25. The vector form of the line $\dfrac{x-1}{2} = \dfrac{y+3}{-1} = \dfrac{z}{4}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$ with

(A) $\vec{a} = \hat{i} - 3\hat{j},\ \vec{b} = 2\hat{i} - \hat{j} + 4\hat{k}$

(B) $\vec{a} = 2\hat{i} - \hat{j} + 4\hat{k},\ \vec{b} = \hat{i} - 3\hat{j}$

(C) $\vec{a} = \hat{i} + 3\hat{j},\ \vec{b} = 2\hat{i} - \hat{j} + 4\hat{k}$

(D) $\vec{a} = \hat{i} - 3\hat{j},\ \vec{b} = 2\hat{i} + \hat{j} + 4\hat{k}$

Q26. The perpendicular distance of the point $(1, 2, 3)$ from the x-axis (the line $\vec{r} = \lambda\hat{i}$) is

(A) $\sqrt{5}$

(B) 1

(C) $\sqrt{13}$

(D) $\sqrt{14}$

Q27. For three events, $P(A) = 0.3$, $P(B) = 0.4$, $P(C) = 0.2$, and the events are mutually exclusive. Then $P(A \cup B \cup C)$ is

(A) 0.5

(B) 0.7

(C) 0.24

(D) 0.9

Q28. For a binomial distribution with $n = 12$ and $p = \frac{1}{3}$, the mean and variance are respectively

(A) 4 and $\frac{8}{3}$

(B) 4 and 4

(C) $\frac{8}{3}$ and 4

(D) 12 and 4

Q29. For the data $4, 7, 10, 13, 16, 19, 22$, the range and the lower quartile Q_1 are respectively

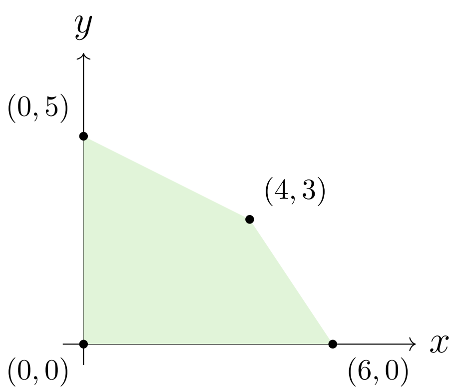
(A) 18 and 10

(B) 18 and 7

(C) 22 and 7

(D) 15 and 10

Q30. A factory ships goods from a warehouse with x units to outlet P and y units to outlet Q. To maximise the profit $Z = 4x + 3y$ over the feasible region with corner points $(0, 0)$, $(6, 0)$, $(4, 3)$ and $(0, 5)$, the maximum value of Z is



(A) 15

(B) 24

(C) 25

(D) 27

Detailed Solutions

Q1.

Solution

Concept — Complement and intersection: The complement A' contains every element of the universal set U that is not in A; the intersection keeps elements common to both sets.

Step 1 — Find A': remove $A = \{2, 4, 6, 8\}$ from $U = \{1, 2, 3, 4, 5, 6, 7, 8\}$, giving $A' = \{1, 3, 5, 7\}$.

Step 2 — Intersect with B: $B = \{1, 2, 3, 4\}$, so $A' \cap B$ keeps elements in both $\{1, 3, 5, 7\}$ and $\{1, 2, 3, 4\}$.

Step 3 — List the common elements: $\{1, 3\}$.

Why other options are wrong:

- $\{2, 4\}$: these belong to A, not A'.
- $\{5, 7\}$: in A' but not in B.
- $\{1, 2, 3, 4\}$: this is B itself, ignoring the complement.

Final Answer: $\{1, 3\} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q1

Q2.

Solution

Concept — Range of a modulus function: The absolute value of any real number is never negative, so $|x - 3| \geq 0$.

Step 1 — Note the lowest value: $|x - 3| = 0$ when $x = 3$, so the minimum output is 0.

Step 2 — Note the upper behaviour: as x moves away from 3, $|x - 3|$ grows without bound.

Step 3 — State the range: the outputs cover all values from 0 upward, i.e. $[0, \infty)$.

Why other options are wrong:

- $(-\infty, \infty)$: a modulus is never negative.
- $[3, \infty)$: confuses the output with the shift value 3.
- $(-\infty, 0]$: gives negative values, impossible for $|x - 3|$.

Final Answer: $[0, \infty) \Rightarrow \boxed{A}$

Answer: (A) Go Back to Q2

Q3.

Solution

Concept — Half-angle formula: $\cos^2\frac{\theta}{2} = \frac{1+\cos\theta}{2}$, and since $0 < \theta < \frac{\pi}{2}$ the angle $\frac{\theta}{2}$ is in the first quadrant, so $\cos\frac{\theta}{2} > 0$.

Step 1 — Substitute $\cos\theta = \frac{7}{25}$: $\cos^2\frac{\theta}{2} = \frac{1+\frac{7}{25}}{2}$.

Step 2 — Simplify the numerator: $1 + \frac{7}{25} = \frac{32}{25}$.

Step 3 — Divide by 2: $\cos^2\frac{\theta}{2} = \frac{32}{25} \cdot \frac{1}{2} = \frac{16}{25}$.

Step 4 — Take the positive square root: $\cos\frac{\theta}{2} = \frac{4}{5}$.

Why other options are wrong:

- $\frac{3}{5}$: this is $\sin\frac{\theta}{2}$, not $\cos\frac{\theta}{2}$.
- $\frac{1}{5}$: an arithmetic slip in the numerator.
- $\frac{16}{25}$: this is $\cos^2\frac{\theta}{2}$, before the square root.

Final Answer: $\cos\frac{\theta}{2} = \frac{4}{5} \Rightarrow \boxed{D}$

Answer: (D) Go Back to Q3

Q4.

Solution

Concept — Sum formula for arctangent: $\tan^{-1}u + \tan^{-1}v = \tan^{-1}\frac{u+v}{1-uv}$ when $uv < 1$.

Step 1 — Combine the left side: with $u = 2x$ and $v = 3x$, $\tan^{-1}\frac{2x+3x}{1-(2x)(3x)} = \frac{\pi}{4}$.

Step 2 — Simplify the argument: $\frac{5x}{1-6x^2} = \tan\frac{\pi}{4} = 1$.

Step 3 — Clear the fraction: $5x = 1 - 6x^2$.

Step 4 — Form the quadratic: $6x^2 + 5x - 1 = 0$.

Step 5 — Factorise: $(6x - 1)(x + 1) = 0$, so $x = \frac{1}{6}$ or $x = -1$.

Step 6 — Pick the positive root: the required positive value is $x = \frac{1}{6}$.

Why other options are wrong:

- $\frac{1}{3}, \frac{1}{4}$: do not satisfy $5x = 1 - 6x^2$.
- 1: gives $\tan^{-1} 2 + \tan^{-1} 3$, far larger than $\frac{\pi}{4}$.

Final Answer: $x = \frac{1}{6} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q4

Q5.

Solution

Concept — Quadratic with complex roots: For $ax^2 + bx + c = 0$ with negative discriminant, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ gives complex conjugate roots.

Step 1 — Identify coefficients: $a = 1$, $b = -4$, $c = 13$.

Step 2 — Compute the discriminant: $b^2 - 4ac = 16 - 52 = -36$.

Step 3 — Take the square root: $\sqrt{-36} = 6i$.

Step 4 — Apply the formula: $x = \frac{4 \pm 6i}{2} = 2 \pm 3i$.

Why other options are wrong:

- $2 \pm i$: uses $\sqrt{-4}$ instead of $\sqrt{-36}$.
- $4 \pm 3i$: forgets to divide by $2a = 2$.
- $-2 \pm 3i$: wrong sign on the real part.

Final Answer: $2 \pm 3i \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q5

Q6.

Solution

Concept — Permutation without repetition: The number of ways to arrange r items chosen from n distinct items is ${}^nP_r = \dfrac{n!}{(n-r)!}$.

Step 1 — Identify n and r: $n = 5$ digits, $r = 4$ positions, no repetition.

Step 2 — Apply the formula: ${}^5P_4 = \dfrac{5!}{(5-4)!} = \dfrac{120}{1} = 120.$

Step 3 — Note all digits are non-zero: since none of $1, 2, 3, 4, 5$ is 0, every arrangement is a valid four-digit number.

Why other options are wrong:

- $625 = 5^4$: allows repetition.
- $24 = 4!$: only permutes four fixed digits.
- 20: equals ${}^5P_2$, the wrong count.

Final Answer: 120 numbers $\Rightarrow \boxed{A}$

Answer: (A) Go Back to Q6

Q7.

Solution

Concept — Sum of binomial coefficients: Putting $x = 1$ in $(1+x)^n$ gives $\sum_{r=0}^{n} \binom{n}{r} = 2^n$.

Step 1 — Set $x = 1$: $(1+1)^{12} = 2^{12}$.

Step 2 — Evaluate the power: $2^{12} = 4096$.

Step 3 — State the sum: the sum of all the coefficients is 4096.

Why other options are wrong:

- 12: simply the exponent.
- $144 = 12^2$: an unrelated square.
- $2048 = 2^{11}$: uses the wrong exponent.

Final Answer: $4096 \Rightarrow \boxed{D}$

Answer: (D) Go Back to Q7

Q8.

Solution

Concept — Sum of $n(n+1)$: $\sum_{n=1}^{N} n(n+1) = \sum n^2 + \sum n = \frac{N(N+1)(2N+1)}{6} + \frac{N(N+1)}{2}$.

Step 1 — Set $N = 10$ and compute $\sum n^2$: $\frac{10 \cdot 11 \cdot 21}{6} = \frac{2310}{6} = 385$.

Step 2 — Compute $\sum n$: $\frac{10 \cdot 11}{2} = 55$.

Step 3 — Add the two sums: $385 + 55 = 440$.

Why other options are wrong:

- 385: only $\sum n^2$, missing $\sum n$.
- $550, 495$: arithmetic errors in combining the two parts.

Final Answer: $440 \Rightarrow \boxed{B}$

Answer: (B) Go Back to Q8

Q9.

Solution

Concept — Pair of straight lines: $x^2 - y^2 = 0$ factors as $(x-y)(x+y) = 0$, giving the two lines $y = x$ and $y = -x$.

Step 1 — Find the slopes: $y = x$ has slope $m_1 = 1$; $y = -x$ has slope $m_2 = -1$.

Step 2 — Check the product: $m_1 m_2 = (1)(-1) = -1$.

Step 3 — Conclude the angle: a product of slopes equal to -1 means the lines are perpendicular, so the angle is 90°.

Why other options are wrong:

- 45°: the angle each line makes with the axis, not between them.
- $60^\circ, 30^\circ$: do not match the slopes ± 1.

Final Answer: $90^\circ \Rightarrow \boxed{A}$

Answer: (A) Go Back to Q9

Q10.

Solution

Concept — Eccentricity of an ellipse: When the larger denominator is under y^2, the major axis is vertical with $a^2 = 25$, $b^2 = 16$, and $e = \sqrt{1 - \frac{b^2}{a^2}}$.

Step 1 — Identify a^2 and b^2: $a^2 = 25$ (larger), $b^2 = 16$.

Step 2 — Form the ratio: $\frac{b^2}{a^2} = \frac{16}{25}$.

Step 3 — Subtract from 1: $1 - \frac{16}{25} = \frac{9}{25}$.

Step 4 — Take the square root: $e = \sqrt{\frac{9}{25}} = \frac{3}{5}$.

Why other options are wrong:

- $\frac{4}{5}$: uses $\frac{b}{a}$ rather than the eccentricity formula.
- $\frac{5}{4}$: greater than 1, impossible for an ellipse.
- $\frac{3}{4}$: uses $\sqrt{1 - b^2/a^2}$ with the axes swapped.

Final Answer: $e = \frac{3}{5} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q10

Q11.

Solution

Concept — The e limit: $\lim_{n\to\infty} \left(1 + \frac{a}{n}\right)^n = e^a$.

Step 1 — Match the form: here $a = 3$, so the limit equals e^3.

Step 2 — Justify briefly: writing the exponent as $\frac{n}{3} \cdot 3$, the inner part tends to e and the outer power 3 gives e^3.

Why other options are wrong:

- e: ignores the factor 3 in the numerator.
- 1: wrongly treats the base as tending to 1 with bounded power.
- $e^{1/3}$: inverts the constant 3.

Final Answer: $e^3 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q11

Q12.

Solution

Concept — Higher-order derivative: Differentiating $\sin x$ twice returns a negated sine.

Step 1 — First derivative: $\frac{dy}{dx} = \cos x$.

Step 2 — Second derivative: $\frac{d^2y}{dx^2} = \frac{d}{dx}(\cos x) = -\sin x$.

Why other options are wrong:

- $\cos x$: this is only the first derivative.
- $\sin x$: this is the fourth derivative.
- $-\cos x$: this is the third derivative.

Final Answer: $-\sin x \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q12

Q13.

Solution

Concept — Matrix method: Write the system as $AX = B$ and solve, equivalently by elimination.

Step 1 — Write the equations: $x + y = 5$ and $x - y = 1$.

Step 2 — Add the equations: $(x + y) + (x - y) = 5 + 1$, giving $2x = 6$, so $x = 3$.

Step 3 — Back-substitute: $3 + y = 5$, so $y = 2$.

Why other options are wrong:

- $x = 2, y = 3$: swaps the values.
- $x = 4, y = 1$: does not satisfy $x - y = 1$.
- $x = 1, y = 4$: does not satisfy $x + y = 5$ with $x - y = 1$.

Final Answer: $x = 3,\ y = 2 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q13

Q14.

Solution

Concept — Determinant of a product: $|AB| = |A|\,|B|$.

Step 1 — Substitute the values: $|AB| = (3)(-2)$.

Step 2 — Multiply: $|AB| = -6$.

Why other options are wrong:

- 1: comes from $|A| + |B|$ wrongly.
- 5: adds the magnitudes.
- 6: forgets the negative sign of $|B|$.

Final Answer: $|AB| = -6 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q14

Q15.

Solution

Concept — Percentage error in volume: For $V = \frac{4}{3}\pi r^3$, $\frac{dV}{V} = 3\,\frac{dr}{r}$, so the percentage error in volume is three times that in the radius.

Step 1 — Find the relative error in r: $\frac{dr}{r} = \frac{0.03}{9} = \frac{1}{300}$.

Step 2 — Multiply by 3: $\frac{dV}{V} = 3 \cdot \frac{1}{300} = \frac{1}{100}$.

Step 3 — Convert to percentage: $\frac{1}{100} = 1\%$.

Why other options are wrong:

- 0.1%: misplaces a decimal.
- 0.33%: this is the error in r alone, not multiplied by 3.
- 3%: forgets to divide 0.03 by 9.

Final Answer: $1\% \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q15

Q16.

Solution

Concept — Increasing/decreasing test: A function is decreasing where its derivative is negative.

Step 1 — Differentiate: $f(x) = \sin x \Rightarrow f'(x) = \cos x$.

Step 2 — Sign on the interval: for $x \in \left(\frac{\pi}{2}, \pi\right)$, the cosine is negative, so $f'(x) < 0$.

Step 3 — Conclude: since $f'(x) < 0$ throughout, $\sin x$ is decreasing on $\left(\frac{\pi}{2}, \pi\right)$.

Why other options are wrong:

- increasing: would need $\cos x > 0$, true only on $\left(0, \frac{\pi}{2}\right)$.
- constant: $\sin x$ is not constant here.
- neither: the derivative keeps a fixed sign, so the behaviour is definite.

Final Answer: decreasing $\Rightarrow$ $\boxed{\text{D}}$

Answer: (D) Go Back to Q16

Q17.

Solution

Concept — Substitution: Put $t = \tan x$, so $dt = \sec^2 x\, dx$, turning the integral into $\int \frac{dt}{t}$.

Step 1 — Substitute: $t = \tan x$, $dt = \sec^2 x\, dx$.

Step 2 — Rewrite the integral: $\int \frac{\sec^2 x}{\tan x}\, dx = \int \frac{dt}{t}$.

Step 3 — Integrate: $\int \frac{dt}{t} = \log|t| + C$.

Step 4 — Substitute back: $\log|\tan x| + C$.

Why other options are wrong:

- $\tan^2 x$: results from a wrong substitution choice.
- $\sec x \tan x$: not an antiderivative of this integrand.
- $\log|\sec x|$: that is $\int \tan x\, dx$.

Final Answer: $\log|\tan x| + C \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q17

Q18.

Solution

Concept — Partial fractions: Write $\frac{1}{x(x^2+1)} = \frac{A}{x} + \frac{Bx+C}{x^2+1}$ and match coefficients.

Step 1 — Set up the identity: $1 = A(x^2+1) + (Bx+C)x$.

Step 2 — Solve for the constants: putting $x = 0$ gives $A = 1$; comparing x^2 terms gives $A + B = 0 \Rightarrow B = -1$; comparing x terms gives $C = 0$.

Step 3 — Rewrite the integral: $\int \left(\frac{1}{x} - \frac{x}{x^2+1} \right) dx$.

Step 4 — Integrate term by term: $\log|x| - \frac{1}{2}\log(x^2+1) + C$.

Why other options are wrong:

- $\log|x| + \frac{1}{2}\log(x^2+1)$: wrong sign on the second term.
- $\frac{1}{2}\log(x^2+1)$: drops the $\log|x|$ term.
- $\log|x| - \log(x^2+1)$: missing the factor $\frac{1}{2}$.

Final Answer: $\log|x| - \frac{1}{2}\log(x^2+1) + C \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q18

Q19.

Solution

Concept — Periodic-function property: $|\sin x|$ has period π, and over one period $\int_0^{\pi} |\sin x|\, dx = 2$.

Step 1 — Count the periods: the interval $[0, 4\pi]$ contains 4 full periods of length π.

Step 2 — Evaluate one period: $\int_0^{\pi} \sin x\, dx = [-\cos x]_0^{\pi} = 1 - (-1) = 2$.

Step 3 — Multiply by the number of periods: $4 \times 2 = 8$.

Why other options are wrong:

- 4: counts only 2 periods.
- 2: the value over a single period.
- 16: doubles the correct answer.

Final Answer: $8 \Rightarrow \boxed{B}$

Answer: (B) Go Back to Q19

Q20.

Solution

Concept — Area with respect to the y-axis: When a curve is given as $x = g(y)$, the area between it and the y-axis is $\int_c^d x\,dy$.

Step 1 — Set up the integral: area $= \int_0^3 y^2\,dy$.

Step 2 — Integrate: $\int y^2\,dy = \frac{y^3}{3}$.

Step 3 — Apply the limits: $\left[\frac{y^3}{3}\right]_0^3 = \frac{27}{3} - 0 = 9$.

Why other options are wrong:

- 3: integrates y instead of y^2.
- $\frac{27}{2}$: divides by 2 instead of 3.
- 18: forgets to divide by 3 on part of the term.

Final Answer: area $= 9$ square units $\Rightarrow \boxed{D}$

Answer: (D) Go Back to Q20

Q21.

Solution

Concept — Degree of a differential equation: The degree is the power of the highest-order derivative once the equation is made free of radicals and fractions of derivatives.

Step 1 — Square both sides: from $\frac{dy}{dx} = \sqrt{1 + \frac{dy}{dx}}$, squaring gives $\left(\frac{dy}{dx}\right)^2 = 1 + \frac{dy}{dx}$.

Step 2 — Rearrange: $\left(\frac{dy}{dx}\right)^2 - \frac{dy}{dx} - 1 = 0.$

Step 3 — Read the degree: the highest power of the highest-order derivative $\frac{dy}{dx}$ is 2, so the degree is 2.

Why other options are wrong:

- 1: ignores the squaring needed to remove the radical.
- 3: too high; the squared term is the largest power.
- $\frac{1}{2}$: degree must be a positive integer.

Final Answer: degree $= 2 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q21

Q22.

Solution

Concept — Exponential growth model: $\frac{dP}{dt} = kP$ is separable and integrates to an exponential law.

Step 1 — Separate variables: $\frac{dP}{P} = k\,dt$.

Step 2 — Integrate both sides: $\log P = kt + \log P_0$.

Step 3 — Exponentiate: $P = P_0 e^{kt}$.

Why other options are wrong:

- $P_0 + kt$: linear growth, not exponential.
- $P_0 t^k$: a power law, not from $\frac{dP}{dt} = kP$.
- $P_0 e^{-kt}$: decay, but the population is growing ($k > 0$).

Final Answer: $P = P_0 e^{kt} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q22

Q23.

Solution

Concept — Magnitudes of sum and difference: $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b}$ and $|\vec{a}-\vec{b}|^2 = |\vec{a}|^2 + |\vec{b}|^2 - 2\vec{a}\cdot\vec{b}$.

Step 1 — Set the two equal: $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$ implies $|\vec{a}+\vec{b}|^2 = |\vec{a}-\vec{b}|^2$.

Step 2 — Expand and subtract: the squared magnitudes differ only by $4\vec{a}\cdot\vec{b}$, so $4\vec{a}\cdot\vec{b} = 0$.

Step 3 — Conclude: $\vec{a}\cdot\vec{b} = 0$, meaning $\vec{a}$ and $\vec{b}$ are perpendicular.

Why other options are wrong:

- parallel / anti-parallel: would give a non-zero dot product.
- equal in magnitude: not required by the condition.

Final Answer: perpendicular $\Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q23

Q24.

Solution

Concept — Scalar triple product: The volume of a parallelepiped with edges $\vec{a}, \vec{b}, \vec{c}$ is $|\vec{a}\cdot(\vec{b}\times\vec{c})|$.

Step 1 — Write the determinant: with $\vec{a} = (1,0,0)$, $\vec{b} = (0,1,0)$, $\vec{c} = (0,0,2)$, the box product is $\begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{vmatrix}$.

Step 2 — Evaluate the diagonal determinant: $1\cdot 1\cdot 2 = 2$.

Step 3 — Take the magnitude: volume $= |2| = 2$.

Why other options are wrong:

- 1: the value for the unit cube, ignoring the factor 2 in $\vec{c}$.
- 0: would require coplanar edges.
- 4: doubles the correct volume.

Final Answer: volume $= 2$ cubic units $\Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q24

Q25.

Solution

Concept — Cartesian to vector form: The line $\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c}$ becomes $\vec{r} = \vec{a} + \lambda\vec{b}$ with $\vec{a} = (x_1, y_1, z_1)$ and $\vec{b} = (a, b, c)$.

Step 1 — Read the fixed point: from $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z}{4}$, the point is $(1, -3, 0)$, so $\vec{a} = \hat{i} - 3\hat{j}$.

Step 2 — Read the direction ratios: denominators are $2, -1, 4$, so $\vec{b} = 2\hat{i} - \hat{j} + 4\hat{k}$.

Step 3 — Combine: $\vec{r} = (\hat{i} - 3\hat{j}) + \lambda(2\hat{i} - \hat{j} + 4\hat{k})$.

Why other options are wrong:

- swapping $\vec{a}$ and $\vec{b}$: reverses the roles of point and direction.
- $\vec{a} = \hat{i} + 3\hat{j}$: wrong sign on the y-coordinate.
- $\vec{b} = 2\hat{i} + \hat{j} + 4\hat{k}$: wrong sign on the $\hat{j}$ direction ratio.

Final Answer: $\vec{a} = \hat{i} - 3\hat{j},\ \vec{b} = 2\hat{i} - \hat{j} + 4\hat{k} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q25

Q26.

Solution

Concept — Distance from a point to the x-axis: The foot of the perpendicular from (x_0, y_0, z_0) to the x-axis is $(x_0, 0, 0)$, so the distance is $\sqrt{y_0^2 + z_0^2}$.

Step 1 — Identify the point: $(x_0, y_0, z_0) = (1, 2, 3)$.

Step 2 — Drop the x-coordinate: distance $= \sqrt{y_0^2 + z_0^2} = \sqrt{2^2 + 3^2}$.

Step 3 — Evaluate: $\sqrt{4+9} = \sqrt{13}$.

Why other options are wrong:

- $\sqrt{5}$: uses $1^2 + 2^2$, the wrong coordinates.
- 1: keeps only the x-coordinate.
- $\sqrt{14}$: the distance from the origin, not from the x-axis.

Final Answer: $\sqrt{13} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q26

Q27.

Solution

Concept — Union of mutually exclusive events: If A, B, C are pairwise disjoint, then $P(A \cup B \cup C) = P(A) + P(B) + P(C)$.

Step 1 — Note the overlaps vanish: mutual exclusivity means all intersection terms are 0.

Step 2 — Add the probabilities: $P(A \cup B \cup C) = 0.3 + 0.4 + 0.2$.

Step 3 — Compute the sum: $0.3 + 0.4 + 0.2 = 0.9$.

Why other options are wrong:

- 0.5: adds only two of the three.
- 0.7: omits $P(C)$.
- 0.24: multiplies the probabilities instead of adding.

Final Answer: $0.9 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q27

Q28.

Solution

Concept — Binomial mean and variance: For n trials with success probability p, mean $= np$ and variance $= npq$, where $q = 1 - p$.

Step 1 — Compute the mean: $np = 12 \times \frac{1}{3} = 4$.

Step 2 — Find q: $q = 1 - \frac{1}{3} = \frac{2}{3}$.

Step 3 — Compute the variance: $npq = 12 \times \frac{1}{3} \times \frac{2}{3} = \frac{8}{3}$.

Why other options are wrong:

- 4 and 4: variance must be $npq < np$, not equal.
- $\frac{8}{3}$ and 4: swaps mean and variance.
- 12 and 4: uses n as the mean.

Final Answer: mean $= 4$, variance $= \frac{8}{3} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q28

Q29.

Solution

Concept — Range and quartiles: Range = largest − smallest; for an ordered list of 7 values, Q_1 is the $\frac{7+1}{4} = 2\text{nd}$ value.

Step 1 — Compute the range: largest = 22, smallest = 4, so range = $22 - 4 = 18$.

Step 2 — Locate Q_1: the position is $\frac{n+1}{4} = \frac{8}{4} = 2$, i.e. the 2nd value in $4, 7, 10, 13, 16, 19, 22$.

Step 3 — Read Q_1: the 2nd value is 7.

Why other options are wrong:

- range 22: uses the largest value alone.
- $Q_1 = 10$: takes the 3rd value instead of the 2nd.
- range 15: a subtraction slip.

Final Answer: range $= 18$, $Q_1 = 7 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q29

Q30.

Solution

Concept — Corner-point method: The optimum of a linear objective over a bounded feasible region occurs at a corner point; evaluate Z at each.

Step 1 — Evaluate $Z = 4x + 3y$ at each corner: $(0,0):\ 0$; $(6,0):\ 24$; $(4,3):\ 16 + 9 = 25$; $(0,5):\ 15$.

Step 2 — Compare the values: the values are $0, 24, 25, 15$.

Step 3 — Pick the maximum: the largest is 25, at $(4,3)$.

Why other options are wrong:

- 15: value at $(0,5)$.
- 24: value at $(6,0)$.
- 27: not attained at any corner.

Final Answer: maximum $Z = 25 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q30

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	A	3	D	4	B	5	C
6	A	7	D	8	B	9	A	10	C
11	D	12	B	13	A	14	C	15	B
16	D	17	A	18	C	19	B	20	D
21	A	22	C	23	D	24	B	25	A
26	C	27	D	28	A	29	B	30	C