AIIMS Paramedical Mathematics

Sample Paper – 1

Duration: 30 Minutes **Maximum Marks: 30**

Instructions

- This paper contains **30** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics portion of **AIIMS Paramedical** entrance.
- Each correct answer carries **+1 mark**. Each incorrect answer carries a penalty of $-\frac{1}{3}$ mark; an unattempted question scores **0**.
- Only **one** option is correct. Choose carefully.
- Syllabus level: **Class 11 & 12 NCERT Mathematics**
- Use of mobile phones, calculators, or electronic gadgets is strictly prohibited.

Q1. If $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{2, 4, 6, 8, 10\}$, then the number of elements in $A \cup B$ is:

(A) 7

(B) 8

(C) 9

(D) 11

Q2. On the set $A = \{1, 2, 3\}$, consider the relation

$$R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 1), (2, 3), (3, 2)\}.$$

The relation R is:

(A) Reflexive and transitive but not symmetric

(B) Symmetric and transitive but not reflexive

(C) Reflexive and symmetric but not transitive

(D) An equivalence relation

Q3. The value of $\dfrac{\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ}{1}$ is:

(A) 1

(B) $\dfrac{1}{2}$

(C) $\dfrac{\sqrt{3}}{2}$

(D) 0

Q4. The general solution of the equation $\sin\theta = \dfrac{1}{2}$ is (where $n \in \mathbb{Z}$):

(A) $\theta = 2n\pi + \dfrac{\pi}{6}$

(B) $\theta = n\pi + \dfrac{\pi}{6}$

(C) $\theta = 2n\pi \pm \dfrac{\pi}{6}$

(D) $\theta = n\pi + (-1)^n\dfrac{\pi}{6}$

Q5. The modulus and argument of the complex number $z = 1 + i\sqrt{3}$ are respectively:

(A) $1,\ \dfrac{\pi}{3}$

(B) $2,\ \dfrac{\pi}{3}$

(C) $2,\ \dfrac{\pi}{6}$

(D) $\sqrt{2},\ \dfrac{\pi}{4}$

Q6. In how many ways can the letters of the word **DELHI** (all distinct) be arranged in a row?

(A) 120

(B) 60

(C) 24

(D) 720

Q7. The term independent of x in the expansion of $\left(x + \frac{1}{x}\right)^6$ is:

(A) 15

(B) 6

(C) 20

(D) 1

Q8. The sum of the first 20 terms of the arithmetic progression $3, 7, 11, 15, \ldots$ is:

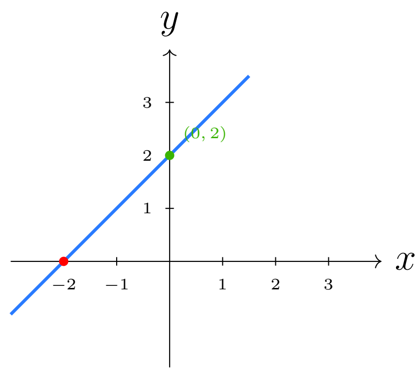
(A) 410

(B) 400

(C) 760

(D) 820

Q9. The equation of the straight line passing through $(0, 2)$ with slope 1 is shown below. Its x-intercept is:



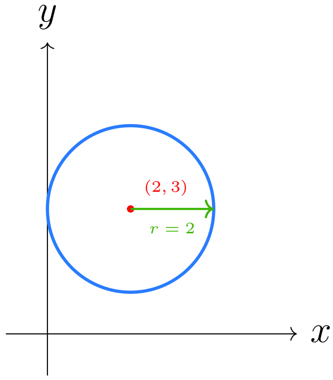
(A) -1

(B) -2

(C) 2

(D) 1

Q10. For the circle $x^2 + y^2 - 4x - 6y + 9 = 0$ shown below, the centre and radius are:



(A) Centre $(2, 3)$, radius 2

(B) Centre $(-2, -3)$, radius 2

(C) Centre $(2, 3)$, radius 4

(D) Centre $(4, 6)$, radius 3

Q11. The value of $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$ is:

(A) 0

(B) 2

(C) 4

(D) Does not exist

Q12. The function $f(x) = \begin{cases} x + 3, & x \neq 1 \\ k, & x = 1 \end{cases}$ is continuous at $x = 1$ if k equals:

(A) 1

(B) 4

(C) 3

(D) 0

Q13. If $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 0 \\ 1 & 2 \end{bmatrix}$, then the element in the first row, first column of AB is:

(A) 2

(B) 8

(C) 6

(D) 4

Q14. The value of the determinant $\begin{vmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{vmatrix}$ is:

(A) 18

(B) 6

(C) 30

(D) 0

Q15. The radius of a circle increases at 3 cm/s. The rate of increase of its area when the radius is 5 cm is:

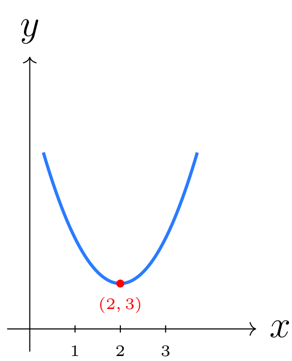
(A) 15π cm^2/s

(B) 25π cm^2/s

(C) 30π cm^2/s

(D) 10π cm^2/s

Q16. The function $f(x) = x^2 - 4x + 7$ (graphed below) attains its minimum value at $x =$:



(A) 0

(B) 2

(C) 4

(D) 7

Q17. The value of $\int 2x\, e^{x^2}\, dx$ is (where C is the constant of integration):

(A) $2e^{x^2} + C$

(B) $x^2 e^{x^2} + C$

(C) $\frac{e^{x^2}}{2} + C$

(D) $e^{x^2} + C$

Q18. The value of $\int x\, e^x\, dx$ is (where C is the constant of integration):

(A) $(x-1)e^x + C$

(B) $(x+1)e^x + C$

(C) $x\, e^x + C$

(D) $\frac{x^2}{2} e^x + C$

Q19. Using the property $\int_0^a f(x)\, dx = \int_0^a f(a-x)\, dx$, the value of $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\, dx$ is:

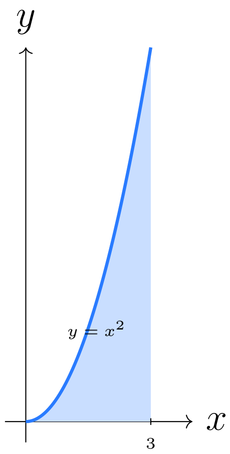
(A) $\frac{\pi}{2}$

(B) 1

(C) $\frac{\pi}{4}$

(D) 0

Q20. The area bounded by the curve $y = x^2$, the x-axis and the lines $x = 0$ and $x = 3$ (shaded below) is:



(A) 27

(B) 9

(C) 18

(D) 3

Q21. The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + y = 0$ are respectively:

(A) 2 and 3

(B) 3 and 2

(C) 2 and 2

(D) 3 and 3

Q22. The general solution of the differential equation $\frac{dy}{dx} = \frac{x}{y}$ is (where C is an arbitrary constant):

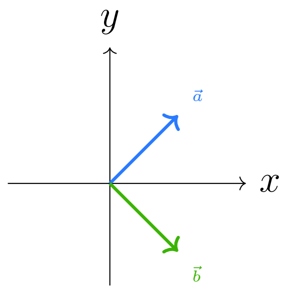
(A) $y^2 + x^2 = C$

(B) $y = x + C$

(C) $xy = C$

(D) $y^2 - x^2 = C$

Q23. The angle between the vectors $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = \hat{i} - \hat{j}$ (drawn below) is:



(A) $0°$

(B) $45°$

(C) $90°$

(D) $180°$

Q24. If $\vec{a} = 2\hat{i}$ and $\vec{b} = 3\hat{j}$, then the magnitude $|\vec{a} \times \vec{b}|$ is:

(A) 6

(B) 5

(C) 0

(D) $\sqrt{13}$

Q25. The direction cosines of a line whose direction ratios are $1, 2, 2$ are:

(A) $1, 2, 2$

(B) $\frac{1}{5}, \frac{2}{5}, \frac{2}{5}$

(C) $\frac{1}{9}, \frac{2}{9}, \frac{2}{9}$

(D) $\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$

Q26. The equation of the plane passing through the point $(1, 0, 0)$ with normal vector $\hat{i} + \hat{j} + \hat{k}$ is:

(A) $x + y + z = 0$

(B) $x + y + z = 1$

(C) $x + y + z = 3$

(D) $x - y - z = 1$

Q27. A single die is rolled once. The probability of getting an even number is:

(A) $\frac{1}{2}$

(B) $\frac{1}{3}$

(C) $\frac{2}{3}$

(D) $\frac{1}{6}$

Q28. If $P(A) = 0.6$, $P(B) = 0.5$ and $P(A \cap B) = 0.3$, then the conditional probability $P(A \mid B)$ is:

(A) 0.5

(B) 0.3

(C) 0.6

(D) 0.8

Q29. The mean of the observations $4, 8, 10, 12, 16$ is:

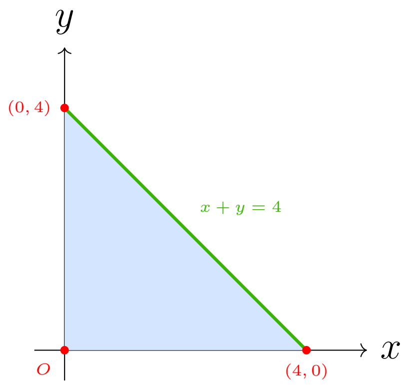
(A) 8

(B) 12

(C) 9

(D) 10

Q30. For the linear programming problem, maximise $Z = 3x + 2y$ subject to $x + y \leq 4$, $x \geq 0$, $y \geq 0$. The shaded feasible region with its corner points is shown below. The maximum value of Z is:



(A) 8

(B) 10

(C) 12

(D) 14

Detailed Solutions

Q1.

Solution

Concept — Union of sets: $A \cup B$ contains every element that lies in A, in B, or in both, with each element counted only once.

Step 1 — List the elements of each set: $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{2, 4, 6, 8, 10\}$.

Step 2 — Combine without repetition: $A \cup B = \{1, 2, 3, 4, 5, 6, 8, 10\}$.

Step 3 — Count the elements: the combined set has 8 distinct members.

Why other options are wrong:

- Option (A) 7: drops one of the shared elements incorrectly.
- Option (C) 9: counts an element twice.
- Option (D) 11: this is $|A| + |B| = 6 + 5$, ignoring the overlap $\{2, 4, 6\}$.

Final Answer: $|A \cup B| = 8 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 1

Q2.

Solution

Concept — Properties of relations: a relation is reflexive if (a, a) holds for every a, symmetric if $(a, b) \Rightarrow (b, a)$, and transitive if (a, b) and $(b, c) \Rightarrow (a, c)$.

Step 1 — Reflexive check: $(1, 1), (2, 2), (3, 3)$ are all present, so R is reflexive.

Step 2 — Symmetric check: $(1, 2)$ has its mirror $(2, 1)$, and $(2, 3)$ has its mirror $(3, 2)$; every off-diagonal pair is matched, so R is symmetric.

Step 3 — Transitive check: $(1, 2) \in R$ and $(2, 3) \in R$ would require $(1, 3) \in R$, but $(1, 3) \notin R$. The transitive chain breaks, so R is not transitive.

Conclusion: R is reflexive and symmetric but not transitive.

Why other options are wrong:

- Option (A): claims it is not symmetric, but every mirror pair is present.
- Option (B): claims it is not reflexive, yet all three (a, a) pairs are present.
- Option (D): an equivalence relation must be transitive, and R fails transitivity since $(1, 3) \notin R$.

Final Answer: R is reflexive and symmetric but not transitive $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 2

Q3.

Solution

Concept — Compound angle identity: $\sin A \cos B + \cos A \sin B = \sin(A + B)$.

Step 1 — Recognise the form: the numerator is $\sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ = \sin(60^\circ + 30^\circ)$.

Step 2 — Add the angles: $60^\circ + 30^\circ = 90^\circ$, so the expression equals $\sin 90^\circ$.

Step 3 — Evaluate: $\sin 90^\circ = 1$.

Why other options are wrong:

- Option (B) $\frac{1}{2}$: this is $\sin 30^\circ$, not the full sum.
- Option (C) $\frac{\sqrt{3}}{2}$: this is $\sin 60^\circ$ alone.
- Option (D) 0: would require the angle sum to be 0° or 180°.

Final Answer: $\sin 90^\circ = 1 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 3

Q4.

Solution

Concept — General solution of $\sin\theta = \sin\alpha$**:** the complete solution is $\theta = n\pi + (-1)^n\alpha$, where $n \in \mathbb{Z}$.

Step 1 — Identify α**:** since $\sin\dfrac{\pi}{6} = \dfrac{1}{2}$, we take $\alpha = \dfrac{\pi}{6}$.

Step 2 — Apply the standard formula: $\theta = n\pi + (-1)^n\dfrac{\pi}{6}$.

Why other options are wrong:

- Option (A): captures only some of the solutions, missing the obtuse-angle family.
- Option (B): $n\pi + \frac{\pi}{6}$ omits the alternating sign and is incorrect.
- Option (C): $2n\pi \pm \frac{\pi}{6}$ is the form for $\cos\theta = \cos\alpha$, not sine.

Final Answer: $\theta = n\pi + (-1)^n\dfrac{\pi}{6} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 4

Q5.

Solution

Concept — Modulus and argument: for $z = a + ib$, $|z| = \sqrt{a^2 + b^2}$ and $\arg z = \tan^{-1}\frac{b}{a}$ (first quadrant when $a, b > 0$).

Step 1 — Identify parts: here $a = 1$ and $b = \sqrt{3}$.

Step 2 — Compute modulus: $|z| = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{1+3} = \sqrt{4} = 2$.

Step 3 — Compute argument: $\arg z = \tan^{-1}\frac{\sqrt{3}}{1} = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$.

Why other options are wrong:

- Option (A): modulus 1 ignores the imaginary part.
- Option (C): argument $\frac{\pi}{6}$ uses $\tan^{-1}\frac{1}{\sqrt{3}}$, the reciprocal ratio.
- Option (D): $\sqrt{2}, \frac{\pi}{4}$ corresponds to $1 + i$, a different number.

Final Answer: $|z| = 2,\ \arg z = \frac{\pi}{3} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 5

Q6.

Solution

Concept — Permutations of distinct objects: n distinct items can be arranged in $n!$ ways.

Step 1 — Count the letters: DELHI has 5 letters, all distinct.

Step 2 — Apply the factorial: number of arrangements $= 5! = 5 \times 4 \times 3 \times 2 \times 1$.

Step 3 — Evaluate: $5! = 120$.

Why other options are wrong:

- Option (B) 60: this is $\frac{5!}{2}$, which would apply if a letter repeated.
- Option (C) 24: this is $4!$, treating only four letters.
- Option (D) 720: this is $6!$, one letter too many.

Final Answer: $5! = 120 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 6

Q7.

Solution

Concept — General term of a binomial expansion: the $(r+1)$th term of $\left(x+\frac{1}{x}\right)^n$ is $T_{r+1}=\binom{n}{r}x^{n-r}\left(\frac{1}{x}\right)^r=\binom{n}{r}x^{n-2r}$.

Step 1 — Set the power of x to zero: for the term independent of x, we need $n-2r=0$ with $n=6$.

Step 2 — Solve for r: $6-2r=0\Rightarrow r=3$.

Step 3 — Compute the coefficient: $\binom{6}{3}=\dfrac{6!}{3!\,3!}=20$.

Why other options are wrong:

- Option (A) 15: equals $\binom{6}{2}$ or $\binom{6}{4}$, the wrong term.
- Option (B) 6: equals $\binom{6}{1}$, $r=1$.
- Option (D) 1: equals $\binom{6}{0}$, the leading term.

Final Answer: term independent of x is $\binom{6}{3}=20\Rightarrow$ $\boxed{\text{C}}$

Answer: (C) Go Back to Q 7

Q8.

Solution

Concept — Sum of an AP: $S_n=\dfrac{n}{2}\big[2a+(n-1)d\big]$, where a is the first term and d the common difference.

Step 1 — Identify the parameters: $a=3$, $d=7-3=4$, $n=20$.

Step 2 — Substitute into the formula: $S_{20}=\dfrac{20}{2}\big[2(3)+(20-1)(4)\big]$.

Step 3 — Simplify inside the bracket: $2(3)=6$ and $19\times4=76$, so the bracket is $6+76=82$.

Step 4 — Multiply out: $S_{20}=10\times82=820$.

Why other options are wrong:

- Option (A) 410: this is half the correct sum, from forgetting the leading factor.
- Option (B) 400: would arise from using $d=2$ instead of 4.
- Option (C) 760: drops a term, using 19 in place of 20 at the wrong step.

Final Answer: $S_{20}=820\Rightarrow$ $\boxed{\text{D}}$

Answer: (D) Go Back to Q 8

Q9.

Solution

Concept — Equation and intercept of a line: the line through $(0, c)$ with slope m is $y = mx + c$; its x-intercept is found by setting $y = 0$.

Step 1 — Write the equation: with $m = 1$ and $c = 2$, the line is $y = x + 2$.

Step 2 — Find the x-intercept: set $y = 0$: $0 = x + 2 \Rightarrow x = -2$.

Step 3 — Read the graph: the blue line cuts the x-axis at $(-2, 0)$, confirming the intercept.

Why other options are wrong:

- Option (A) -1: solves $0 = x + 1$, a wrong intercept.
- Option (C) 2: this is the y-intercept, not the x-intercept.
- Option (D) 1: sign and value both incorrect.

Final Answer: x-intercept $= -2 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 9

Q10.

Solution

Concept — Circle from general form: for $x^2 + y^2 + 2gx + 2fy + c = 0$, the centre is $(-g, -f)$ and radius is $\sqrt{g^2 + f^2 - c}$.

Step 1 — Match coefficients: comparing $x^2 + y^2 - 4x - 6y + 9 = 0$ gives $2g = -4 \Rightarrow g = -2$, $2f = -6 \Rightarrow f = -3$, $c = 9$.

Step 2 — Centre: $(-g, -f) = (2, 3)$.

Step 3 — Radius: $\sqrt{(-2)^2 + (-3)^2 - 9} = \sqrt{4 + 9 - 9} = \sqrt{4} = 2$.

Why other options are wrong:

- Option (B): centre $(-2, -3)$ flips the sign of $-g, -f$.
- Option (C): radius 4 forgets to subtract c.
- Option (D): centre $(4, 6)$ uses $2g, 2f$ directly without halving.

Final Answer: centre $(2, 3)$, radius $2 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 10

Q11.

Solution

Concept — Limit of a $0/0$ form: factor and cancel the common term before substituting.

Step 1 — Factor the numerator: $x^2 - 4 = (x-2)(x+2)$.

Step 2 — Cancel: $\frac{(x-2)(x+2)}{x-2} = x + 2$ for $x \neq 2$.

Step 3 — Substitute the limit: $\lim_{x\to 2}(x+2) = 2 + 2 = 4$.

Why other options are wrong:

- Option (A) 0: would require the numerator's factor not to cancel cleanly.
- Option (B) 2: substitutes only one part of $x + 2$.
- Option (D): the limit does exist after cancellation.

Final Answer: the limit equals $4 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 11

Q12.

Solution

Concept — Continuity at a point: f is continuous at $x = a$ when $\lim_{x\to a} f(x) = f(a)$.

Step 1 — Find the limit: as $x \to 1$, $f(x) = x + 3 \to 1 + 3 = 4$.

Step 2 — Match the function value: continuity requires $f(1) = k = 4$.

Why other options are wrong:

- Option (A) 1: this is the value of x, not the limit.
- Option (C) 3: this is the constant term alone, dropping the x.
- Option (D) 0: leaves a jump discontinuity at $x = 1$.

Final Answer: $k = 4 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 12

Q13.

Solution

Concept — Matrix multiplication: the (i, j) entry of AB is the dot product of row i of A with column j of B.

Step 1 — Pick the right row and column: row 1 of A is $(1,\ 2)$; column 1 of B is $(2,\ 1)^\top$.

Step 2 — Multiply elementwise and add: $(1)(2) + (2)(1) = 2 + 2$.

Step 3 — Total: the $(1, 1)$ entry is 4.

Why other options are wrong:

- Option (A) 2: uses only the first product, dropping $(2)(1)$.
- Option (B) 8: comes from multiplying the wrong column.
- Option (C) 6: a miscount of the cross terms.

Final Answer: $(AB)_{11} = 4 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 13

Q14.

Solution

Concept — Determinant of a triangular matrix: for a lower (or upper) triangular matrix, the determinant equals the product of the diagonal entries.

Step 1 — Identify the structure: the matrix is lower triangular with diagonal $1, 3, 6$.

Step 2 — Multiply the diagonal: $1 \times 3 \times 6 = 18$.

Why other options are wrong:

- Option (B) 6: uses only the last diagonal entry.
- Option (C) 30: multiplies the wrong entries (5×6).
- Option (D) 0: would need a zero on the diagonal.

Final Answer: the determinant is $18 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 14

Q15.

Solution

Concept — Related rates: differentiate $A = \pi r^2$ with respect to time to link $\frac{dA}{dt}$ and $\frac{dr}{dt}$.

Step 1 — Differentiate the area: $\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$.

Step 2 — Substitute given values: $r = 5$ cm and $\frac{dr}{dt} = 3$ cm/s give $\frac{dA}{dt} = 2\pi(5)(3)$.

Step 3 — Evaluate: $2 \times 5 \times 3 = 30$, so $\frac{dA}{dt} = 30\pi$ cm^2/s.

Why other options are wrong:

- Option (A) 15π: forgets the factor of 2.
- Option (B) 25π: uses πr^2 directly rather than its derivative.
- Option (D) 10π: drops the radius value.

Final Answer: $\frac{dA}{dt} = 30\pi$ cm^2/s $\Rightarrow \boxed{\text{C}}$

Answer: (C) **Go Back to Q 15**

Q16.

Solution

Concept — Maxima and minima: at a turning point $f'(x) = 0$; a positive second derivative signals a minimum.

Step 1 — Differentiate: $f'(x) = 2x - 4$.

Step 2 — Set to zero: $2x - 4 = 0 \Rightarrow x = 2$.

Step 3 — Confirm minimum: $f''(x) = 2 > 0$, so $x = 2$ gives a minimum (the vertex of the parabola in the graph).

Why other options are wrong:

- Option (A) 0: not a root of $f'(x)$.
- Option (C) 4: this is the coefficient, not the critical point.
- Option (D) 7: this is the constant term $f(0)$, not the location of the minimum.

Final Answer: minimum occurs at $x = 2 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q 16

Q17.

Solution

Concept — Integration by substitution: choose u so that du matches the rest of the integrand.

Step 1 — Substitute: let $u = x^2$, so $du = 2x\,dx$.

Step 2 — Rewrite the integral: $\int 2x\,e^{x^2}\,dx = \int e^u\,du$.

Step 3 — Integrate and back-substitute: $\int e^u\,du = e^u + C = e^{x^2} + C$.

Why other options are wrong:

- Option (A): the stray factor 2 should be absorbed by the substitution.
- Option (B): incorrectly multiplies by x^2.
- Option (C): the $\frac{1}{2}$ would appear only if the $2x$ were absent.

Final Answer: $\int 2x\,e^{x^2}\,dx = e^{x^2} + C \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 17

Q18.

Solution

Concept — Integration by parts: $\int u\,dv = uv - \int v\,du$.

Step 1 — Choose parts: let $u = x$ (so $du = dx$) and $dv = e^x\,dx$ (so $v = e^x$).

Step 2 — Apply the formula: $\int x\,e^x\,dx = x\,e^x - \int e^x\,dx$.

Step 3 — Finish the integral: $= x\,e^x - e^x + C = (x-1)e^x + C$.

Why other options are wrong:

- Option (B): sign error gives $(x+1)$ instead of $(x-1)$.
- Option (C): omits the $-\int e^x\,dx$ correction term.
- Option (D): treats x as if integrated, producing $\frac{x^2}{2}$.

Final Answer: $(x-1)e^x + C \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 18

Q19.

Solution

Concept — King property of definite integrals: $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$ lets symmetric integrands collapse.

Step 1 — Name the integral: let $I = \displaystyle\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\,dx$.

Step 2 — Apply the property with $a = \frac{\pi}{2}$: replacing x by $\frac{\pi}{2} - x$ swaps $\sin \leftrightarrow \cos$, giving $I = \displaystyle\int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x}\,dx$.

Step 3 — Add the two forms: $2I = \displaystyle\int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x}\,dx = \int_0^{\pi/2} 1\,dx = \frac{\pi}{2}$.

Step 4 — Solve for I: $I = \dfrac{\pi}{4}$.

Why other options are wrong:

- Option (A) $\frac{\pi}{2}$: this is $2I$, not I.
- Option (B) 1: ignores the π factor.
- Option (D) 0: the integrand is positive throughout.

Final Answer: $I = \dfrac{\pi}{4} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 19

Q20.

Solution

Concept — Area under a curve: the area between $y = f(x)$ and the x-axis from $x = a$ to $x = b$ is $\int_a^b f(x)\,dx$.

Step 1 — Set up the integral: area $= \displaystyle\int_0^3 x^2\,dx$.

Step 2 — Integrate: $\int x^2\,dx = \dfrac{x^3}{3}$.

Step 3 — Apply the limits: $\left[\dfrac{x^3}{3}\right]_0^3 = \dfrac{27}{3} - \dfrac{0}{3} = 9$.

Why other options are wrong:

- Option (A) 27: forgets to divide by 3.
- Option (C) 18: uses the wrong antiderivative.
- Option (D) 3: divides 9 by 3 once too often.

Final Answer: area $= 9$ square units $\Rightarrow$ $\boxed{\text{B}}$

Answer: (B) Go Back to Q 20

Q21.

Solution

Concept — Order and degree: the order is the highest derivative present; the degree is the power of that highest derivative after the equation is made polynomial in derivatives.

Step 1 — Find the highest derivative: the highest is $\frac{d^2y}{dx^2}$, so the order is 2.

Step 2 — Read its power: that term appears as $\left(\frac{d^2y}{dx^2}\right)^3$, so the degree is 3.

Why other options are wrong:

- Option (B): swaps order and degree.
- Option (C): degree 2 misreads the cube as a square.
- Option (D): order 3 misreads the second derivative as a third.

Final Answer: order 2, degree $3 \Rightarrow$ $\boxed{\text{A}}$

Answer: (A) Go Back to Q 21

Q22.

Solution

Concept — Variable separable equations: collect all y-terms on one side and x-terms on the other, then integrate.

Step 1 — Separate the variables: $\frac{dy}{dx} = \frac{x}{y} \Rightarrow y\,dy = x\,dx$.

Step 2 — Integrate both sides: $\int y\,dy = \int x\,dx \Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + C_1$.

Step 3 — Simplify: multiply by 2: $y^2 = x^2 + C$, i.e. $y^2 - x^2 = C$.

Why other options are wrong:

- Option (A): wrong sign, giving $y^2 + x^2 = C$.
- Option (B): treats the equation as if $\frac{dy}{dx} = 1$.
- Option (C): $xy = C$ solves a different separable equation.

Final Answer: $y^2 - x^2 = C \Rightarrow$ $\boxed{\text{D}}$

Answer: (D) Go Back to Q 22

Q23.

Solution

Concept — Angle from the dot product: $\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}$.

Step 1 — Compute the dot product: $\vec{a}\cdot\vec{b} = (1)(1) + (1)(-1) = 1 - 1 = 0$.

Step 2 — Interpret a zero dot product: since the numerator is 0, $\cos\theta = 0$.

Step 3 — Find the angle: $\theta = \cos^{-1}(0) = 90^\circ$, matching the perpendicular vectors in the figure.

Why other options are wrong:

- Option (A) 0°: would need parallel vectors.
- Option (B) 45°: would need a positive, non-zero dot product.
- Option (D) 180°: would need oppositely directed vectors.

Final Answer: $\theta = 90^\circ \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 23

Q24.

Solution

Concept — Magnitude of a cross product: $|\vec{a}\times\vec{b}| = |\vec{a}|\,|\vec{b}|\sin\theta$, and for perpendicular axis vectors $\sin\theta = 1$.

Step 1 — Compute the cross product: $\vec{a}\times\vec{b} = (2\hat{\imath})\times(3\hat{\jmath}) = 6(\hat{\imath}\times\hat{\jmath}) = 6\hat{k}$.

Step 2 — Take the magnitude: $|6\hat{k}| = 6$.

Why other options are wrong:

- Option (B) 5: adds the components $2+3$ instead of multiplying.
- Option (C) 0: would require parallel vectors.
- Option (D) $\sqrt{13}$: uses $\sqrt{2^2+3^2}$, the formula for a resultant, not a cross product.

Final Answer: $|\vec{a}\times\vec{b}| = 6 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q 24

Q25.

Solution

Concept — Direction cosines from ratios: if direction ratios are a, b, c, the direction cosines are $\frac{a}{\sqrt{a^2+b^2+c^2}}$, and similarly for b and c.

Step 1 — Compute the magnitude: $\sqrt{1^2+2^2+2^2} = \sqrt{1+4+4} = \sqrt{9} = 3$.

Step 2 — Divide each ratio by 3: direction cosines are $\frac{1}{3}, \frac{2}{3}, \frac{2}{3}$.

Why other options are wrong:

- Option (A): leaves the ratios undivided, so they are not cosines.
- Option (B): divides by 5 instead of 3.
- Option (C): divides by 9 instead of $\sqrt{9} = 3$.

Final Answer: direction cosines $\frac{1}{3}, \frac{2}{3}, \frac{2}{3} \Rightarrow \boxed{D}$

Answer: (D) Go Back to Q 25

Q26.

Solution

Concept — Equation of a plane: a plane with normal (a, b, c) through point (x_0, y_0, z_0) is $a(x-x_0)+b(y-y_0)+c(z-z_0)=0$.

Step 1 — Insert the data: normal $(1,1,1)$, point $(1,0,0)$ gives $1(x-1)+1(y-0)+1(z-0)=0$.

Step 2 — Simplify: $x-1+y+z=0 \Rightarrow x+y+z=1$.

Why other options are wrong:

- Option (A): $x+y+z=0$ passes through the origin, not $(1,0,0)$.
- Option (C): $x+y+z=3$ would pass through $(1,1,1)$.
- Option (D): wrong signs on the normal components.

Final Answer: $x+y+z=1 \Rightarrow \boxed{B}$

Answer: (B) Go Back to Q 26

Q27.

Solution

Concept — Classical probability: $P(E) = \dfrac{\text{favourable outcomes}}{\text{total outcomes}}$.

Step 1 — List the sample space: a die shows $\{1, 2, 3, 4, 5, 6\}$, so total outcomes $= 6$.

Step 2 — Count favourable outcomes: the even numbers are $\{2, 4, 6\}$, i.e. 3 outcomes.

Step 3 — Compute the probability: $P(\text{even}) = \dfrac{3}{6} = \dfrac{1}{2}$.

Why other options are wrong:

- Option (B) $\frac{1}{3}$: counts only two favourable outcomes.
- Option (C) $\frac{2}{3}$: counts four favourable outcomes.
- Option (D) $\frac{1}{6}$: counts a single outcome.

Final Answer: $P(\text{even}) = \dfrac{1}{2} \Rightarrow \boxed{\text{A}}$

Answer: (A) **Go Back to Q 27**

Q28.

Solution

Concept — Conditional probability: $P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$, provided $P(B) \neq 0$.

Step 1 — Insert the given values: $P(A \mid B) = \dfrac{0.3}{0.5}$.

Step 2 — Divide: $\dfrac{0.3}{0.5} = 0.6$.

Why other options are wrong:

- Option (A) 0.5: divides by the wrong probability.
- Option (B) 0.3: stops at $P(A \cap B)$ without dividing.
- Option (D) 0.8: comes from $\frac{0.3+0.5}{1}$, an invalid manipulation.

Final Answer: $P(A \mid B) = 0.6 \Rightarrow \boxed{\text{C}}$

Answer: (C) **Go Back to Q 28**

Q29.

Solution

Concept — Arithmetic mean: the mean is the sum of the observations divided by their number.

Step 1 — Add the observations: $4 + 8 + 10 + 12 + 16 = 50.$

Step 2 — Count them: there are 5 observations.

Step 3 — Divide: mean $= \frac{50}{5} = 10.$

Why other options are wrong:

- Option (A) 8: a wrong total or count.
- Option (B) 12: this is one of the data values, not the mean.
- Option (C) 9: divides an incorrect sum.

Final Answer: mean $= 10 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q 29

Q30.

Solution

Concept — Corner-point method: the optimum of a linear objective over a bounded feasible region occurs at a corner (vertex) of that region.

Step 1 — Identify the corner points: from the shaded region the vertices are $O(0, 0)$, $(4, 0)$ and $(0, 4)$.

Step 2 — Evaluate $Z = 3x + 2y$ at each corner: at $O(0, 0)$, $Z = 0$; at $(4, 0)$, $Z = 3(4) + 2(0) = 12$; at $(0, 4)$, $Z = 3(0) + 2(4) = 8$.

Step 3 — Pick the largest: the maximum value is 12, attained at $(4, 0)$.

Why other options are wrong:

- Option (A) 8: this is the value at $(0, 4)$, not the maximum.
- Option (B) 10: not the value of Z at any corner point.
- Option (D) 14: exceeds the value at every feasible corner.

Final Answer: maximum $Z = 12 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q 30

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	C	3	A	4	D	5	B
6	A	7	C	8	D	9	B	10	A
11	C	12	B	13	D	14	A	15	C
16	B	17	D	18	A	19	C	20	B
21	A	22	D	23	C	24	A	25	D
26	B	27	A	28	C	29	D	30	C