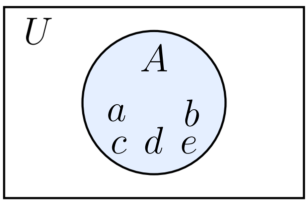
AIIMS Paramedical Mathematics Sample Paper – 2

Duration: 30 Minutes **Maximum Marks: 30**

Instructions

- This paper contains **30** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics portion of **AIIMS Paramedical** entrance.
- Marking scheme: $+1$ mark for each correct answer, **a penalty of** $-\frac{1}{3}$ **mark** for each incorrect answer, and 0 for each unattempted question.
- Only **one** option is correct. Choose carefully.
- Syllabus level: **Class 11 & 12 NCERT Mathematics**
- Use of mobile phones, calculators, or electronic gadgets is strictly prohibited.

Q1. If a set A has 5 elements, how many subsets of A contain at least one element?



(A) 32

(B) 31

(C) 30

(D) 16

Q2. On the set $\mathbb{Z}$ of integers, the relation $R = \{(a, b) : a - b \text{ is divisible by } 3\}$ is:

(A) reflexive and symmetric but not transitive

(B) symmetric and transitive but not reflexive

(C) reflexive, symmetric and transitive

(D) neither reflexive nor symmetric

Q3. The value of $\cos 75°$ is:

(A) $\frac{\sqrt{6}-\sqrt{2}}{4}$

(B) $\frac{\sqrt{6}+\sqrt{2}}{4}$

(C) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ only

(D) $\frac{1}{2}$

Q4. The principal value of $\sin^{-1}\left(-\frac{1}{2}\right)$ is:

(A) $\frac{7\pi}{6}$

(B) $\frac{5\pi}{6}$

(C) $\frac{\pi}{6}$

(D) $-\frac{\pi}{6}$

Q5. If $z = 3 + 4i$, then the value of $z \cdot \bar{z}$ is:

(A) 7

(B) 5

(C) 25

(D) $7 + 24i$

Q6. In how many ways can a committee of 3 persons be chosen from a group of 7 persons?

(A) 21

(B) 35

(C) 210

(D) 343

Q7. The middle term in the expansion of $\left(x + \frac{1}{x}\right)^6$ is:

(A) 20

(B) 15

(C) $20x^2$

(D) 6

Q8. The sum of the geometric series $2 + 6 + 18 + \cdots$ up to 5 terms is:

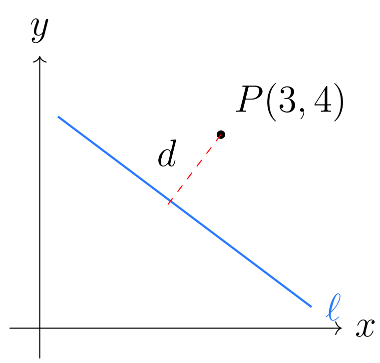
(A) 162

(B) 200

(C) 364

(D) 242

Q9. The distance of the point $P(3, 4)$ from the line $3x + 4y - 5 = 0$ is:



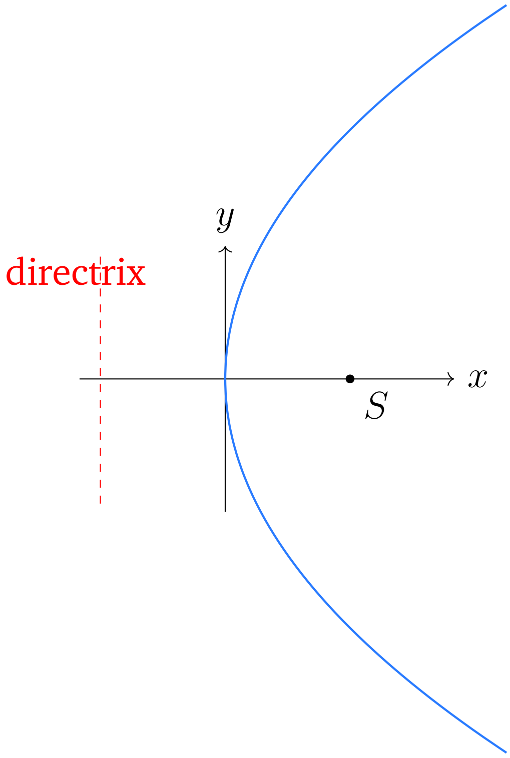
(A) 2

(B) $\frac{19}{25}$

(C) 4

(D) $\frac{5}{22}$

Q10. For the parabola $y^2 = 12x$, the focus and directrix are:



(A) Focus $(0, 3)$, directrix $y = -3$

(B) Focus $(3, 0)$, directrix $x = -3$

(C) Focus $(12, 0)$, directrix $x = -12$

(D) Focus $(-3, 0)$, directrix $x = 3$

Q11. The value of $\lim_{x \to 0} \frac{\sin 5x}{3x}$ is:

(A) $\frac{5}{3}$

(B) $\frac{3}{5}$

(C) 1

(D) 5

Q12. The function $f(x) = |x - 2|$ is:

(A) differentiable everywhere

(B) discontinuous at $x = 2$

(C) differentiable at $x = 2$ but not continuous there

(D) continuous everywhere but not differentiable at $x = 2$

Q13. If A is a square matrix, then $A - A^T$ is always:

(A) a symmetric matrix

(B) an identity matrix

(C) a skew-symmetric matrix

(D) a diagonal matrix

Q14. If A is a 3×3 matrix with $\det(A) = 4$, then $\det(2A)$ equals:

(A) 8

(B) 32

(C) 16

(D) 64

Q15. The function $f(x) = x^2 - 6x + 5$ is increasing on the interval:

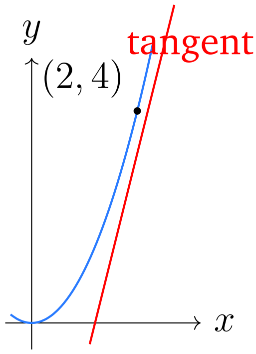
(A) $(-\infty, 3)$

(B) $(-\infty, \infty)$

(C) $(0, 3)$

(D) $(3, \infty)$

Q16. The slope of the tangent to the curve $y = x^2$ at the point $(2, 4)$ is:



(A) 4

(B) 2

(C) $\frac{1}{4}$

(D) 8

Q17. $\int \frac{1}{x^2-1}\,dx$ equals (where C is the constant of integration):

(A) $\frac{1}{2}\log\left|\frac{x+1}{x-1}\right| + C$

(B) $\log|x^2-1| + C$

(C) $\frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + C$

(D) $\tan^{-1} x + C$

Q18. $\int \sec^2 x\,dx$ equals (where C is the constant of integration):

(A) $\sec x \tan x + C$

(B) $\tan x + C$

(C) $-\cot x + C$

(D) $\sec x + C$

Q19. The value of $\int_{-2}^{2} x^3 \cos x\,dx$ is:

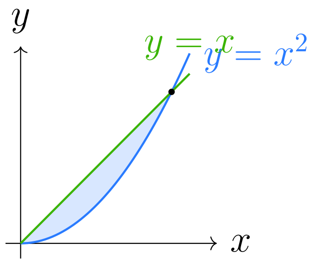
(A) 4

(B) 8

(C) 2

(D) 0

Q20. The area bounded by the curve $y = x^2$ and the line $y = x$ (between their points of intersection) is:



(A) $\frac{1}{3}$

(B) $\frac{1}{2}$

(C) $\frac{1}{6}$

(D) 1

Q21. Which of the following differential equations is homogeneous?

(A) $\frac{dy}{dx} = \frac{x+y}{x}$

(B) $\frac{dy}{dx} = x + y + 1$

(C) $\frac{dy}{dx} = x^2 + y$

(D) $\frac{dy}{dx} = \frac{1}{x} + y$

Q22. The integrating factor of the differential equation $\frac{dy}{dx} + \frac{2}{x}y = x^2$ is:

(A) x

(B) e^{2x}

(C) $\log x$

(D) x^2

Q23. The scalar triple product $[\hat{i} \ \hat{j} \ \hat{k}]$ is equal to:

(A) 0

(B) 1

(C) 3

(D) -1

Q24. The projection of the vector $\vec{a} = 2\hat{i} + 3\hat{j} + 2\hat{k}$ on the vector $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ is:

(A) $\frac{10}{3}$

(B) $\sqrt{6}$

(C) $\frac{10}{\sqrt{6}}$

(D) $\frac{6}{\sqrt{10}}$

Q25. The vector equation of the line passing through the point $(1, 2, 3)$ and parallel to the vector $2\hat{i} + \hat{j} - \hat{k}$ is:

(A) $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + \hat{j} - \hat{k})$

(B) $\vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

(C) $\vec{r} = \lambda(\hat{i} + 2\hat{j} + 3\hat{k})$

(D) $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + \hat{j} + \hat{k})$

Q26. The distance of the point $(1, 2, 2)$ from the plane $2x + y + 2z + 3 = 0$ is:

(A) $\frac{11}{9}$

(B) $\frac{9}{11}$

(C) 3

(D) $\frac{11}{3}$

Q27. If A and B are independent events with $P(A) = 0.3$ and $P(B) = 0.4$, then $P(A \cap B)$ is:

(A) 0.7

(B) 0.12

(C) 0.58

(D) 0.1

Q28. Bag I contains 3 red and 2 black balls; Bag II contains 1 red and 4 black balls. A bag is chosen at random and a ball is drawn. If the ball is red, the probability that it came from Bag I is:

(A) $\frac{1}{4}$

(B) $\frac{1}{2}$

(C) $\frac{3}{4}$

(D) $\frac{2}{3}$

Q29. The median of the data $7,\ 12,\ 3,\ 9,\ 15,\ 5,\ 11$ is:

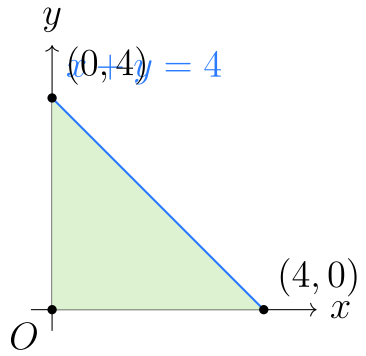
(A) 9

(B) 11

(C) 12

(D) 8

Q30. Maximize $Z = 3x + 4y$ subject to $x + y \leq 4,\ x \geq 0,\ y \geq 0$. The maximum value of Z is:



(A) 12

(B) 16

(C) 14

(D) 24

Detailed Solutions

Q1.

Solution

Concept — Subsets of a set: A set with n elements has exactly 2^n subsets, one of which is the empty set $\varnothing$.

Step 1 — Total subsets: Here $n = 5$, so the total number of subsets is

$$2^5 = 32.$$

Step 2 — Remove the empty set: The only subset with no element is $\varnothing$. So subsets containing at least one element number

$$32 - 1 = 31.$$

Why other options are wrong:

- Option A (32): counts the empty set as well.
- Option C (30): incorrectly removes two subsets.
- Option D (16): equals 2^4, the wrong power.

Final Answer: 31 subsets $\Rightarrow$ $\boxed{\text{B}}$

Answer: (B) Go Back to Q1

Q2.

Solution

Concept — Equivalence relation: A relation is an equivalence relation if it is reflexive, symmetric and transitive.

Step 1 — Reflexive: For any a, $a - a = 0$ is divisible by 3, so $(a, a) \in R$. Reflexive holds.

Step 2 — Symmetric: If $a - b$ is divisible by 3, then $b - a = -(a - b)$ is also divisible by 3. Symmetric holds.

Step 3 — Transitive: If $3 \mid (a - b)$ and $3 \mid (b - c)$, then

$$(a - b) + (b - c) = a - c$$

is divisible by 3. Transitive holds.

Why other options are wrong:

- Options A, B, D: each denies a property that actually holds, as shown above.

Final Answer: reflexive, symmetric and transitive $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q2

Q3.

Solution

Concept — Difference formula: $\cos(A-B) = \cos A\cos B + \sin A\sin B$, and $75°$ can be split usefully.

Step 1 — Split the angle: Write $75° = 45° + 30°$ and use

$$\cos(A+B) = \cos A\cos B - \sin A\sin B.$$

Step 2 — Substitute values:

$$\cos 75° = \cos 45°\cos 30° - \sin 45°\sin 30°.$$

$$= \frac{1}{\sqrt{2}}\cdot\frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}}\cdot\frac{1}{2} = \frac{\sqrt{3}-1}{2\sqrt{2}}.$$

Step 3 — Rationalise: Multiply numerator and denominator by $\sqrt{2}$:

$$\frac{\sqrt{3}-1}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}-\sqrt{2}}{4}.$$

Why other options are wrong:

- Option B: this is $\cos 15°$, the sum form, not $\cos 75°$.
- Option C: correct before rationalising, but the listed final form is A.
- Option D: $\frac{1}{2} = \cos 60°$, a different angle.

Final Answer: $\cos 75° = \dfrac{\sqrt{6}-\sqrt{2}}{4} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q3

Q4.

Solution

Concept — Principal value of $\sin^{-1}$**:** The range of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

Step 1 — Identify the reference angle: We need θ with $\sin\theta = -\frac{1}{2}$ in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$. Since $\sin\frac{\pi}{6} = \frac{1}{2}$,

$$\sin\left(-\frac{\pi}{6}\right) = -\frac{1}{2}.$$

Step 2 — Check the range: $-\frac{\pi}{6}$ lies in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, so it is the principal value.

Why other options are wrong:

- Options A, B: $\frac{7\pi}{6}$ and $\frac{5\pi}{6}$ lie outside the principal range.
- Option C: $+\frac{\pi}{6}$ gives $\sin = +\frac{1}{2}$, wrong sign.

Final Answer: $\sin^{-1}\left(-\frac{1}{2}\right) = -\dfrac{\pi}{6} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q4

Q5.

Solution

Concept — Modulus and conjugate: For $z = a + bi$, $z\bar{z} = |z|^2 = a^2 + b^2$.

Step 1 — Identify parts: Here $a = 3$ and $b = 4$.

Step 2 — Compute:

$$z\bar{z} = a^2 + b^2 = 3^2 + 4^2 = 9 + 16 = 25.$$

Why other options are wrong:

- Option A (7): equals $a + b$, not $a^2 + b^2$.
- Option B (5): equals $|z|$, not $|z|^2$.
- Option D $(7 + 24i)$: equals z^2, not $z\bar{z}$.

Final Answer: $z\bar{z} = 25 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q5

Q6.

Solution

Concept — Combinations: The number of ways to choose r from n is $\binom{n}{r} = \dfrac{n!}{r!\,(n-r)!}$.

Step 1 — Set up: We need $\binom{7}{3}$.

Step 2 — Compute:

$$\binom{7}{3} = \frac{7\times 6\times 5}{3\times 2\times 1} = \frac{210}{6} = 35.$$

Why other options are wrong:

- Option A (21): equals $\binom{7}{2}$.
- Option C (210): equals $7\times6\times5$ without dividing by $3!$ (a permutation count).
- Option D (343): equals 7^3, allowing repetition and order.

Final Answer: $\binom{7}{3} = 35 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q6

Q7.

Solution

Concept — Middle term: In $(a+b)^n$ with n even, there is one middle term, the $\left(\frac{n}{2}+1\right)$th term, $T_{n/2+1} = \binom{n}{n/2} a^{n/2} b^{n/2}$.

Step 1 — Find the term number: Here $n = 6$, so the middle term is the 4th term, T_4, with $r = 3$.

Step 2 — Apply the general term: $T_{r+1} = \binom{6}{3} x^{6-3} \left(\dfrac{1}{x}\right)^3$.

$$= \binom{6}{3} x^3 \cdot x^{-3} = \binom{6}{3}.$$

Step 3 — Evaluate:

$$\binom{6}{3} = \frac{6\times 5\times 4}{3\times 2\times 1} = 20.$$

Why other options are wrong:

- Option B (15): equals $\binom{6}{2}$ or $\binom{6}{4}$, the wrong term.
- Option C ($20x^2$): the powers of x actually cancel, leaving a constant.

- Option D (6): equals n, not the coefficient.

Final Answer: middle term $= 20 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q7

Q8.

Solution

Concept — Sum of a GP: For a GP with first term a and common ratio $r \neq 1$, $S_n = \dfrac{a(r^n - 1)}{r - 1}$.

Step 1 — Identify a and r: Here $a = 2$ and $r = \dfrac{6}{2} = 3$, with $n = 5$.

Step 2 — Substitute:

$$S_5 = \frac{2\,(3^5 - 1)}{3 - 1} = \frac{2\,(243 - 1)}{2}.$$

Step 3 — Simplify:

$$S_5 = \frac{2 \times 242}{2} = 242.$$

Why other options are wrong:

- Option A (162): only the 5th term $(2 \cdot 3^4 = 162)$, not the sum.
- Options B, C: arithmetic slips; 364 would be the sum to 6 terms divided wrongly.

Final Answer: $S_5 = 242 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q8

Q9.

Solution

Concept — Distance from a point to a line: For $ax+by+c = 0$ and point (x_1, y_1),

$$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}.$$

Step 1 — Substitute: With $a = 3$, $b = 4$, $c = -5$ and $(x_1, y_1) = (3, 4)$:

$$d = \frac{|3(3) + 4(4) - 5|}{\sqrt{3^2 + 4^2}}.$$

Step 2 — Compute numerator and denominator:

$$= \frac{|9 + 16 - 5|}{\sqrt{9 + 16}} = \frac{|20|}{\sqrt{25}} = \frac{20}{5}.$$

Step 3 — Simplify:

$$d = \frac{20}{5} = 4.$$

Why other options are wrong:

- Option A (2): halves the numerator wrongly.
- Option B ($\frac{19}{25}$): uses 25 (the square root not taken) as the denominator with a numerator slip.
- Option D ($\frac{5}{22}$): an inverted, incorrect value.

Final Answer: $d = \frac{20}{5} = 4 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q9

Q10.

Solution

Concept — Standard parabola: For $y^2 = 4ax$ (opening right), focus is $(a, 0)$ and directrix is $x = -a$.

Step 1 — Find a: Compare $y^2 = 12x$ with $y^2 = 4ax$:

$$4a = 12 \Rightarrow a = 3.$$

Step 2 — Write focus and directrix: Focus $= (a, 0) = (3, 0)$ and directrix $x = -a$, i.e. $x = -3$.

Why other options are wrong:

- Option A: treats the parabola as opening upward.
- Option C: uses $a = 12$ instead of $a = 3$.
- Option D: swaps focus and directrix signs.

Final Answer: Focus $(3, 0)$, directrix $x = -3 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q10

Q11.

Solution

Concept — Standard trig limit: $\lim_{u \to 0} \frac{\sin u}{u} = 1$.

Step 1 — Adjust to standard form:

$$\lim_{x \to 0} \frac{\sin 5x}{3x} = \lim_{x \to 0} \frac{\sin 5x}{5x} \cdot \frac{5x}{3x}.$$

Step 2 — Evaluate each factor:

$$= \left(\lim_{x \to 0} \frac{\sin 5x}{5x}\right) \cdot \frac{5}{3} = 1 \cdot \frac{5}{3} = \frac{5}{3}.$$

Why other options are wrong:

- Option B $(\frac{3}{5})$: the reciprocal of the correct ratio.
- Option C (1): ignores the coefficients 5 and 3.
- Option D (5): forgets to divide by 3.

Final Answer: limit $= \frac{5}{3} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q11

Q12.

Solution

Concept — Differentiability of $|x - a|$: A modulus function is continuous everywhere but has a corner at $x = a$, where the left and right derivatives differ.

Step 1 — Continuity: $|x - 2|$ is continuous for all x (no break in the graph).

Step 2 — Derivative from each side at $x = 2$:

$$\text{left slope} = -1, \qquad \text{right slope} = +1.$$

Since these differ, f is not differentiable at $x = 2$.

Why other options are wrong:

- Option A: differentiability fails precisely at $x = 2$.
- Option B: the function is continuous at $x = 2$.
- Option C: a function differentiable at a point must be continuous there, so this is impossible.

Final Answer: continuous everywhere but not differentiable at $x = 2 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q12

Q13.

Solution

Concept — Skew-symmetric matrix: A matrix M is skew-symmetric if $M^T = -M$.

Step 1 — Take transpose of $A - A^T$: Using $(X - Y)^T = X^T - Y^T$ and $(A^T)^T = A$:

$$(A - A^T)^T = A^T - A.$$

Step 2 — Relate to original:

$$A^T - A = -(A - A^T).$$

So $(A - A^T)^T = -(A - A^T)$, which is exactly the skew-symmetric condition.

Why other options are wrong:

- Option A: symmetric would need $M^T = +M$; here it is $-M$.
- Options B, D: no reason $A - A^T$ is identity or diagonal in general.

Final Answer: $A - A^T$ is skew-symmetric $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q13

Q14.

Solution

Concept — Scalar multiple of a determinant: For an $n \times n$ matrix, $\det(kA) = k^n \det(A)$.

Step 1 — Identify n and k: Here $n = 3$ and $k = 2$.

Step 2 — Apply the rule:

$$\det(2A) = 2^3 \det(A) = 8 \times 4 = 32.$$

Why other options are wrong:

- Option A (8): forgot to multiply by $\det(A)$.
- Option C (16): used k^2 instead of k^3.
- Option D (64): used $k^4 \det(A)$ form.

Final Answer: $\det(2A) = 32 \Rightarrow \boxed{\text{B}}$

Answer: (B) **Go Back to Q14**

Q15.

Solution

Concept — Increasing function: f is increasing where $f'(x) > 0$.

Step 1 — Differentiate:

$$f(x) = x^2 - 6x + 5 \;\Rightarrow\; f'(x) = 2x - 6.$$

Step 2 — Solve $f'(x) > 0$:

$$2x - 6 > 0 \;\Rightarrow\; x > 3.$$

So f is increasing on $(3, \infty)$.

Why other options are wrong:

- Option A: $(-\infty, 3)$ is where f is decreasing.
- Option B: a parabola is not increasing on all of $\mathbb{R}$.
- Option C: $(0, 3)$ lies in the decreasing region.

Final Answer: increasing on $(3, \infty) \Rightarrow \boxed{\text{D}}$

Answer: (D) **Go Back to Q15**

Q16.

Solution

Concept — Slope of tangent: The slope of the tangent to $y = f(x)$ at a point is $f'(x)$ evaluated there.

Step 1 — Differentiate:

$$y = x^2 \;\Rightarrow\; \frac{dy}{dx} = 2x.$$

Step 2 — Evaluate at $x = 2$:

$$\left.\frac{dy}{dx}\right|_{x=2} = 2(2) = 4.$$

Why other options are wrong:

- Option B (2): used $x = 1$ or dropped the factor.
- Option C ($\frac{1}{4}$): that is the normal-related reciprocal, not the tangent slope.
- Option D (8): doubled the slope incorrectly.

Final Answer: slope $= 4 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q16

Q17.

Solution

Concept — Partial fractions: $\dfrac{1}{x^2-1} = \dfrac{1}{(x-1)(x+1)}$ splits as $\dfrac{A}{x-1} + \dfrac{B}{x+1}$.

Step 1 — Find A and B: Solving gives $A = \frac{1}{2}$, $B = -\frac{1}{2}$, so

$$\frac{1}{x^2-1} = \frac{1}{2}\left(\frac{1}{x-1} - \frac{1}{x+1}\right).$$

Step 2 — Integrate term by term:

$$\int \frac{1}{x^2-1}\,dx = \frac{1}{2}\big(\log|x-1| - \log|x+1|\big) + C.$$

Step 3 — Combine logs:

$$= \frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + C.$$

Why other options are wrong:

- Option A: the ratio is inverted.
- Option B: integrating gives a $\frac{1}{2}$ factor and a ratio, not $\log|x^2-1|$.
- Option D: $\tan^{-1}x$ comes from $\frac{1}{x^2+1}$, a different denominator.

Final Answer: $\frac{1}{2}\log\left|\frac{x-1}{x+1}\right| + C \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q17

Q18.

Solution

Concept — Standard antiderivative: $\frac{d}{dx}(\tan x) = \sec^2 x$, so $\int \sec^2 x\,dx = \tan x + C$.

Step 1 — Recognise the derivative: The integrand $\sec^2 x$ is exactly the derivative of $\tan x$.

Step 2 — Write the integral:

$$\int \sec^2 x\,dx = \tan x + C.$$

Why other options are wrong:

- Option A: $\sec x \tan x$ is the derivative of $\sec x$, not the antiderivative wanted.
- Option C: $-\cot x$ is the integral of $\csc^2 x$.
- Option D: $\sec x$ is not an antiderivative of $\sec^2 x$.

Final Answer: $\tan x + C \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q18

Q19.

Solution

Concept — Odd-function property: If $g(-x) = -g(x)$, then $\int_{-a}^{a} g(x)\,dx = 0$.

Step 1 — Test parity of the integrand: Let $g(x) = x^3 \cos x$. Then

$$g(-x) = (-x)^3 \cos(-x) = -x^3 \cos x = -g(x).$$

So g is odd.

Step 2 — Apply the property: The limits -2 to 2 are symmetric, hence

$$\int_{-2}^{2} x^3 \cos x \, dx = 0.$$

Why other options are wrong:

- Options A, B, C: any nonzero value ignores that an odd function integrated over a symmetric interval cancels.

Final Answer: integral $= 0 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q19

Q20.

Solution

Concept — Area between curves: Area $= \displaystyle\int_a^b \left(y_{\text{upper}} - y_{\text{lower}}\right) dx.$

Step 1 — Find intersection points: Set $x^2 = x$:

$$x^2 - x = 0 \;\Rightarrow\; x(x-1) = 0 \;\Rightarrow\; x = 0,\, 1.$$

Step 2 — Identify upper curve: On $(0,1)$, $y = x$ lies above $y = x^2$.

Step 3 — Integrate:

$$A = \int_0^1 (x - x^2)\, dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1.$$

$$= \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}.$$

Why other options are wrong:

- Option A ($\frac{1}{3}$): integrated only x^2.
- Option B ($\frac{1}{2}$): integrated only x.
- Option D (1): no subtraction performed.

Final Answer: area $= \dfrac{1}{6} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q20

Q21.

Solution

Concept — Homogeneous DE: $\frac{dy}{dx} = F(x, y)$ is homogeneous if $F(\lambda x, \lambda y) = F(x, y)$, i.e. F can be written as a function of $\frac{y}{x}$.

Step 1 — Test option A:

$$\frac{dy}{dx} = \frac{x+y}{x} = 1 + \frac{y}{x},$$

which depends only on $\frac{y}{x}$. Homogeneous.

Step 2 — Reject the rest: Options B, C, D contain extra constants or pure powers of x that cannot be reduced to a function of $\frac{y}{x}$ alone.

Why other options are wrong:

- Option B: $x + y + 1$ has a $+1$ term breaking homogeneity.
- Option C: $x^2 + y$ mixes degrees.
- Option D: $\frac{1}{x} + y$ is not a function of $\frac{y}{x}$.

Final Answer: option A is homogeneous $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q21

Q22.

Solution

Concept — Integrating factor: For $\frac{dy}{dx} + P(x)\,y = Q(x)$, the integrating factor is $\text{IF} = e^{\int P\,dx}$.

Step 1 — Identify $P(x)$: Here $P(x) = \frac{2}{x}$.

Step 2 — Integrate P:

$$\int \frac{2}{x}\,dx = 2\log x = \log x^2.$$

Step 3 — Exponentiate:

$$\text{IF} = e^{\log x^2} = x^2.$$

Why other options are wrong:

- Option A (x): would come from $P = \frac{1}{x}$.
- Option B (e^{2x}): treats P as constant 2.
- Option C ($\log x$): forgot to exponentiate.

Final Answer: $\text{IF} = x^2 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q22

Q23.

Solution

Concept — Scalar triple product: $[\vec{a}\,\vec{b}\,\vec{c}] = \vec{a}\cdot(\vec{b}\times\vec{c})$, and for the standard unit vectors it equals the determinant of the identity arrangement.

Step 1 — Use the cross product: $\hat{j}\times\hat{k} = \hat{i}$.

Step 2 — Take the dot product:

$$[\,\hat{i}\;\hat{j}\;\hat{k}\,] = \hat{i}\cdot(\hat{j}\times\hat{k}) = \hat{i}\cdot\hat{i} = 1.$$

Why other options are wrong:

- Option A (0): would mean the vectors are coplanar, but they are mutually perpendicular.
- Option C (3): confuses the magnitude sum with the product.
- Option D (-1): the orientation $\hat{i},\hat{j},\hat{k}$ is right-handed, giving $+1$.

Final Answer: $[\,\hat{i}\;\hat{j}\;\hat{k}\,] = 1 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q23

Q24.

Solution

Concept — Projection of $\vec{a}$ on $\vec{b}$: $\text{proj} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$.

Step 1 — Compute the dot product:

$$\vec{a}\cdot\vec{b} = (2)(1) + (3)(2) + (2)(1) = 2 + 6 + 2 = 10.$$

Step 2 — Compute $|\vec{b}|$:

$$|\vec{b}| = \sqrt{1^2 + 2^2 + 1^2} = \sqrt{6}.$$

Step 3 — Divide:

$$\text{proj} = \frac{10}{\sqrt{6}}.$$

Why other options are wrong:

- Option A ($\frac{10}{3}$): used $|\vec{b}| = 3$ instead of $\sqrt{6}$.
- Option B ($\sqrt{6}$): used $|\vec{b}|$ alone, ignoring the dot product.
- Option D ($\frac{6}{\sqrt{10}}$): swapped numerator and the radical.

Final Answer: projection $= \dfrac{10}{\sqrt{6}} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q24

Q25.

Solution

Concept — Vector equation of a line: A line through point $\vec{a}$ with direction $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\vec{b}$.

Step 1 — Write the position vector of the point: $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$.

Step 2 — Write the direction vector: $\vec{b} = 2\hat{i} + \hat{j} - \hat{k}$.

Step 3 — Combine:

$$\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + \hat{j} - \hat{k}).$$

Why other options are wrong:

- Option B: swaps the point and the direction.
- Option C: omits the fixed point $\vec{a}$.
- Option D: uses $+\hat{k}$ instead of $-\hat{k}$ in the direction.

Final Answer: $\vec{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(2\hat{i} + \hat{j} - \hat{k}) \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q25

Q26.

Solution

Concept — Distance from point to plane: For $ax + by + cz + d = 0$ and point (x_1, y_1, z_1),

$$D = \frac{|ax_1 + by_1 + cz_1 + d|}{\sqrt{a^2 + b^2 + c^2}}.$$

Step 1 — Substitute: With $a = 2, b = 1, c = 2, d = 3$ and $(1, 2, 2)$:

$$D = \frac{|2(1) + 1(2) + 2(2) + 3|}{\sqrt{2^2 + 1^2 + 2^2}}.$$

Step 2 — Compute:

$$= \frac{|2 + 2 + 4 + 3|}{\sqrt{4 + 1 + 4}} = \frac{11}{\sqrt{9}} = \frac{11}{3}.$$

Why other options are wrong:

- Option A ($\frac{11}{9}$): used 9 instead of $\sqrt{9} = 3$ in the denominator.
- Option B ($\frac{9}{11}$): inverted the fraction.
- Option C (3): dropped the numerator.

Final Answer: $D = \frac{11}{3} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q26

Q27.

Solution

Concept — Independent events: If A and B are independent, $P(A \cap B) = P(A)\,P(B)$.

Step 1 — Multiply the probabilities:

$$P(A \cap B) = 0.3 \times 0.4.$$

Step 2 — Compute:

$$= 0.12.$$

Why other options are wrong:

- Option A (0.7): equals $P(A) + P(B)$, used the addition rule wrongly.
- Option C (0.58): equals $P(A \cup B)$ for independent events.

- Option D (0.1): an arithmetic slip in the product.

Final Answer: $P(A \cap B) = 0.12 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q27

Q28.

Solution

Concept — Bayes' theorem: $P(B_1 \mid R) = \dfrac{P(B_1)P(R \mid B_1)}{P(B_1)P(R \mid B_1) + P(B_2)P(R \mid B_2)}$.

Step 1 — Prior and likelihoods: $P(B_1) = P(B_2) = \frac{1}{2}$. From Bag I, $P(R \mid B_1) = \frac{3}{5}$; from Bag II, $P(R \mid B_2) = \frac{1}{5}$.

Step 2 — Apply Bayes:

$$P(B_1 \mid R) = \frac{\frac{1}{2} \cdot \frac{3}{5}}{\frac{1}{2} \cdot \frac{3}{5} + \frac{1}{2} \cdot \frac{1}{5}}.$$

Step 3 — Simplify:

$$= \frac{\frac{3}{10}}{\frac{3}{10} + \frac{1}{10}} = \frac{\frac{3}{10}}{\frac{4}{10}} = \frac{3}{4}.$$

Why other options are wrong:

- Option A $(\frac{1}{4})$: the complementary probability $P(B_2 \mid R)$.
- Option B $(\frac{1}{2})$: the prior, ignoring the evidence.
- Option D $(\frac{2}{3})$: a miscomputed ratio.

Final Answer: $P(B_1 \mid R) = \dfrac{3}{4} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q28

Q29.

Solution

Concept — Median: For an odd number of ordered observations, the median is the middle value.

Step 1 — Arrange in ascending order:

$$3,\ 5,\ 7,\ 9,\ 11,\ 12,\ 15.$$

Step 2 — Locate the middle: There are 7 values, so the median is the 4th term.

$$\text{Median} = 9.$$

Why other options are wrong:

- Option B (11): the 5th term, off by one position.
- Option C (12): the 6th term.
- Option D (8): not even in the data set.

Final Answer: median $= 9 \Rightarrow \boxed{A}$

Answer: (A) Go Back to Q29

Q30.

Solution

Concept — LPP corner-point method: The optimum of a linear objective over a bounded feasible region occurs at a corner (vertex).

Step 1 — Identify the corners: The region $x + y \leq 4$, $x \geq 0$, $y \geq 0$ has vertices

$$O(0, 0), \quad (4, 0), \quad (0, 4).$$

Step 2 — Evaluate $Z = 3x + 4y$ at each:

$$Z(O) = 0, \quad Z(4, 0) = 12, \quad Z(0, 4) = 16.$$

Step 3 — Pick the maximum: The largest value is 16 at $(0, 4)$.

Why other options are wrong:

- Option A (12): value at $(4, 0)$, not the maximum.
- Option C (14): not attained at any corner.
- Option D (24): exceeds the feasible region's reach.

Final Answer: maximum $Z = 16 \Rightarrow \boxed{B}$

Answer: (B) Go Back to Q30

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	C	3	A	4	D	5	C
6	B	7	A	8	D	9	C	10	B
11	A	12	D	13	C	14	B	15	D
16	A	17	C	18	B	19	D	20	C
21	A	22	D	23	B	24	C	25	A
26	D	27	B	28	C	29	A	30	B