AIIMS Paramedical Mathematics Sample Paper – 3

Duration: 30 Minutes **Maximum Marks: 30**

Instructions

- This paper contains **30** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics portion of **AIIMS Paramedical** entrance.
- Each correct answer carries **+1 mark**; each incorrect answer carries **a penalty of** $-\frac{1}{3}$ **mark**; an unattempted question carries **0 marks**.
- Only **one** option is correct. Choose carefully.
- Syllabus level: **Class 11 & 12 NCERT Mathematics**
- Use of mobile phones, calculators, or electronic gadgets is strictly prohibited.

Q1. For sets A and B inside a universal set U, De Morgan's law states that $(A \cup B)'$ equals which of the following?

(A) $A' \cup B'$

(B) $A \cap B$

(C) $A' \cap B'$

(D) $A' \cup B$

Q2. The domain of the real function $f(x) = \dfrac{1}{\sqrt{4 - x^2}}$ is

(A) $[-2, 2]$

(B) $(-2, 2)$

(C) $(-\infty, 2)$

(D) $[2, \infty)$

Q3. If $\sin\theta = \dfrac{3}{5}$ and θ lies in the first quadrant, then the value of $\sin 2\theta$ is

(A) $\frac{12}{5}$

(B) $\frac{7}{25}$

(C) $\frac{6}{5}$

(D) $\frac{24}{25}$

Q4. The number of solutions of the equation $2\sin x = 1$ in the interval $[0,\ 2\pi]$ is

(A) 2

(B) 1

(C) 3

(D) 4

Q5. The value of i^{4n+3}, where n is a positive integer and $i = \sqrt{-1}$, is

(A) 1

(B) i

(C) $-i$

(D) -1

Q6. In how many ways can 6 different persons be seated around a circular table?

(A) 720

(B) 120

(C) 24

(D) 60

Q7. The coefficient of x^2 in the binomial expansion of $(1+x)^6$ is

(A) 6

(B) 20

(C) 30

(D) 15

Q8. If the arithmetic mean (AM) and geometric mean (GM) of two positive numbers are 10 and 8 respectively, then the two numbers satisfy

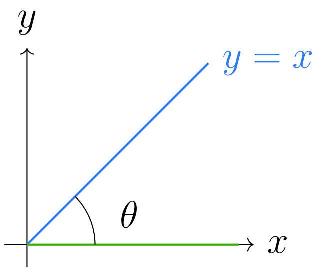
(A) AM $\geq$ GM, and the numbers are 16 and 4

(B) AM $<$ GM, and the numbers are 16 and 4

(C) the numbers are 10 and 8

(D) the numbers cannot be real

Q9. The acute angle between the two straight lines $y = x$ and $y = 0$ (the x-axis) is shown below.



The measure of θ is

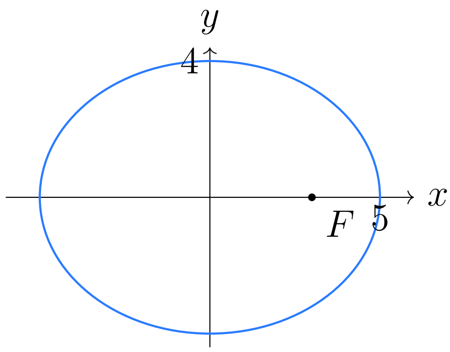
(A) 30°

(B) 60°

(C) 45°

(D) 90°

Q10. The eccentricity of the ellipse shown below, with equation $\dfrac{x^2}{25} + \dfrac{y^2}{16} = 1$, is



(A) $\frac{4}{5}$

(B) $\frac{3}{5}$

(C) $\frac{2}{5}$

(D) $\frac{1}{5}$

Q11. The value of $\lim_{x\to\infty} \dfrac{3x^2+2x+1}{5x^2-x+4}$ is

(A) 0

(B) ∞

(C) $\frac{2}{1}$

(D) $\frac{3}{5}$

Q12. If $y=(2x+3)^4$, then $\dfrac{dy}{dx}$ equals

(A) $8(2x+3)^3$

(B) $4(2x+3)^3$

(C) $2(2x+3)^4$

(D) $8(2x+3)^4$

Q13. The inverse of the matrix $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ is

(A) $\begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix}$

(B) $\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$

(D) $\begin{bmatrix} -1 & 1 \\ 1 & -2 \end{bmatrix}$

Q14. For the matrix $A = \begin{bmatrix} 3 & 2 \\ 4 & 5 \end{bmatrix}$, the adjoint of A is

(A) $\begin{bmatrix} 3 & -2 \\ -4 & 5 \end{bmatrix}$

(B) $\begin{bmatrix} 5 & -2 \\ -4 & 3 \end{bmatrix}$

(C) $\begin{bmatrix} 5 & 2 \\ 4 & 3 \end{bmatrix}$

(D) $\begin{bmatrix} 3 & -4 \\ -2 & 5 \end{bmatrix}$

Q15. Using differentials, the approximate increase in the area of a circle when its radius increases from 5 cm to 5.02 cm is (take area $= \pi r^2$)

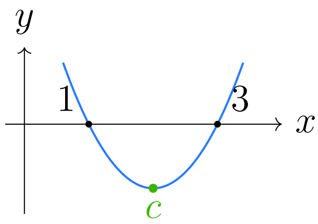
(A) 0.5π cm^2

(B) 0.1π cm^2

(C) 0.4π cm^2

(D) 0.2π cm^2

Q16. For the function $f(x) = x^2 - 4x + 3$ on $[1, 3]$, Rolle's theorem guarantees a point c where $f'(c) = 0$. The graph and that point are shown below.



The value of c is

(A) 2

(B) 1

(C) 3

(D) $\frac{3}{2}$

Q17. The value of the integral $\int x\, e^x\, dx$ is (where C is the constant of integration)

(A) $x\, e^x + C$

(B) $e^x(x+1) + C$

(C) $e^x(x-1) + C$

(D) $\frac{x^2}{2}e^x + C$

Q18. The value of the integral $\int \frac{dx}{\sqrt{1-x^2}}$ is (where C is the constant of integration)

(A) $\cos^{-1} x + C$

(B) $\sin^{-1} x + C$

(C) $\tan^{-1} x + C$

(D) $\sec^{-1} x + C$

Q19. Expressed as the limit of a sum, $\int_0^1 x\, dx$ equals $\lim_{n\to\infty} \frac{1}{n^2} \sum_{r=1}^{n} r$. Its value is

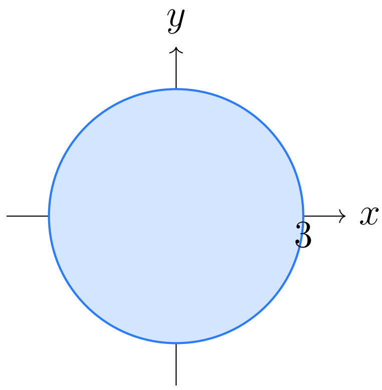
(A) 1

(B) $\frac{1}{3}$

(C) 2

(D) $\frac{1}{2}$

Q20. The area of the region enclosed by the circle $x^2 + y^2 = 9$, shown shaded below, is



(A) 9π

(B) 6π

(C) 3π

(D) 18π

Q21. The differential equation obtained by eliminating the arbitrary constant A from the family of curves $y = A\,e^x$ is

(A) $\frac{dy}{dx} = x$

(B) $\frac{dy}{dx} + y = 0$

(C) $\frac{dy}{dx} = y$

(D) $\frac{dy}{dx} = e^x$

Q22. A population grows so that its rate of increase is proportional to its current size, giving $\frac{dP}{dt} = kP$. The general solution of this variable-separable equation is

(A) $P = kt + C$

(B) $P = C\,e^{kt}$

(C) $P = C\,e^{-kt}$

(D) $P = \dfrac{C}{t}$

Q23. A unit vector in the direction of $\vec{a} = 3\hat{i} + 4\hat{j}$ is

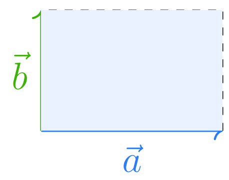
(A) $3\hat{i} + 4\hat{j}$

(B) $\dfrac{3\hat{i} + 4\hat{j}}{7}$

(C) $\dfrac{3\hat{i} + 4\hat{j}}{25}$

(D) $\dfrac{3\hat{i} + 4\hat{j}}{5}$

Q24. The area of the parallelogram whose adjacent sides are $\vec{a} = 2\hat{i}$ and $\vec{b} = 3\hat{j}$, shown below, is



(A) 6

(B) 5

(C) 12

(D) $\sqrt{13}$

Q25. The angle between the lines with direction ratios $\langle 1, 0, 0\rangle$ and $\langle 0, 1, 0\rangle$ in space is

(A) 0°

(B) 45°

(C) 90°

(D) 60°

Q26. The angle between the planes $x + y + z = 1$ and $x - y + z = 5$ has cosine equal to

(A) $\frac{2}{3}$

(B) $\frac{1}{3}$

(C) $\frac{1}{\sqrt{3}}$

(D) 0

Q27. Bag I has 2 white and 3 black balls; Bag II has 4 white and 1 black ball. A bag is chosen at random and one ball is drawn. By the total probability theorem, the probability that the drawn ball is white is

(A) $\frac{2}{5}$

(B) $\frac{4}{5}$

(C) $\frac{1}{2}$

(D) $\frac{3}{5}$

Q28. A fair coin is tossed 4 times. Using the binomial distribution, the probability of getting exactly 2 heads is

(A) $\frac{3}{8}$

(B) $\frac{1}{4}$

(C) $\frac{1}{2}$

(D) $\frac{1}{8}$

Q29. The mode of the data set $4,\ 5,\ 5,\ 6,\ 7,\ 5,\ 8,\ 6$ is

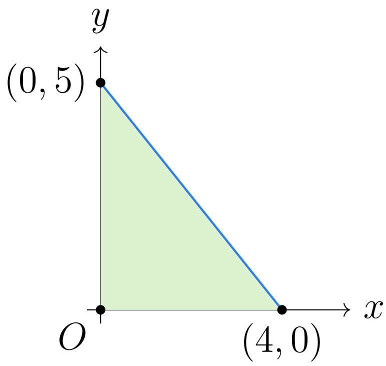
(A) 6

(B) 4

(C) 5

(D) 7

Q30. For the linear programming problem, minimise $Z = 3x + 2y$ over the feasible region shown below, whose corner points are $(0, 5)$, $(4, 0)$ and $(0, 0)$.



The minimum value of Z is

(A) 10

(B) 0

(C) 12

(D) 15

Detailed Solutions

Q1.

Solution

Concept — De Morgan's Laws: The complement of a union of two sets equals the intersection of their complements.

Step 1 — State the law: De Morgan's first law is

$$(A \cup B)' = A' \cap B'.$$

Step 2 — Reasoning: An element lies in $(A \cup B)'$ exactly when it is in neither A nor B.

Step 3 — Conclusion: "Not in A" means in A' and "not in B" means in B', so the element is in $A' \cap B'$.

Why other options are wrong:

- $A' \cup B'$ is the complement of $A \cap B$, a different identity.
- $A \cap B$ and $A' \cup B$ are not complements of $A \cup B$ at all.

Final Answer: $(A \cup B)' = A' \cap B' \Rightarrow \boxed{\text{C}}$

Answer: (C) **Go Back to Q1**

Q2.

Solution

Concept — Domain of a function: The domain is the set of all x for which $f(x)$ is a real number.

Step 1 — Identify restrictions: We need the denominator $\sqrt{4 - x^2}$ to be real and non-zero.

Step 2 — Require positive radicand: This forces

$$4 - x^2 > 0.$$

Step 3 — Solve the inequality: Rearranging gives $x^2 < 4$, that is $-2 < x < 2$.

Step 4 — Exclude endpoints: At $x = \pm 2$ the denominator is 0, so they are excluded; the domain is the open interval $(-2, 2)$.

Why other options are wrong:

- $[-2, 2]$ wrongly includes the endpoints where the function is undefined.
- $(-\infty, 2)$ and $[2, \infty)$ contain values where $4 - x^2 \leq 0$.

Final Answer: Domain $= (-2, 2) \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q2

Q3.

Solution

Concept — Multiple-angle formula: The double-angle identity is $\sin 2\theta = 2 \sin\theta \cos\theta$.

Step 1 — Find $\cos\theta$: Since $\sin\theta = \frac{3}{5}$ and θ is in the first quadrant,

$$\cos\theta = \sqrt{1 - \frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}.$$

Step 2 — Apply the identity:

$$\sin 2\theta = 2 \cdot \frac{3}{5} \cdot \frac{4}{5}.$$

Step 3 — Simplify:

$$\sin 2\theta = \frac{24}{25}.$$

Why other options are wrong:

- $\frac{12}{5}$ and $\frac{6}{5}$ exceed 1, impossible for a sine value.
- $\frac{7}{25}$ is $\cos 2\theta$, not $\sin 2\theta$.

Final Answer: $\sin 2\theta = \frac{24}{25} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q3

Q4.

Solution

Concept — Trigonometric equations: Count the angles in the given interval that satisfy the equation.

Step 1 — Isolate the sine: From $2\sin x = 1$ we get

$$\sin x = \tfrac{1}{2}.$$

Step 2 — Find base solutions: In $[0, 2\pi]$, $\sin x = \frac{1}{2}$ at $x = \frac{\pi}{6}$ and $x = \frac{5\pi}{6}$.

Step 3 — Count solutions: Both lie inside $[0, 2\pi]$, so there are exactly 2 solutions.

Why other options are wrong:

- 1 misses the second-quadrant solution $\frac{5\pi}{6}$.
- 3 and 4 over-count; no further angles in $[0, 2\pi]$ give $\sin x = \frac{1}{2}$.

Final Answer: 2 solutions $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q4

Q5.

Solution

Concept — Powers of iota: The powers of i cycle with period 4: $i,\ -1,\ -i,\ 1$.

Step 1 — Split the exponent: Write

$$i^{4n+3} = i^{4n} \cdot i^{3}.$$

Step 2 — Evaluate i^{4n}: Since $i^4 = 1$, we have $i^{4n} = (i^4)^n = 1$.

Step 3 — Evaluate i^3: We know $i^3 = i^2 \cdot i = -1 \cdot i = -i$.

Step 4 — Combine:

$$i^{4n+3} = 1 \cdot (-i) = -i.$$

Why other options are wrong:

- 1 is i^{4n}, i is i^{4n+1}, and -1 is i^{4n+2}.

Final Answer: $i^{4n+3} = -i \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q5

Q6.

Solution

Concept — Circular permutations: The number of ways to arrange n distinct objects around a circle is $(n-1)!$.

Step 1 — Identify n: Here $n = 6$ persons.

Step 2 — Apply the formula:

$$(n-1)! = (6-1)! = 5!.$$

Step 3 — Compute:

$$5! = 5 \times 4 \times 3 \times 2 \times 1 = 120.$$

Why other options are wrong:

- $720 = 6!$ counts linear arrangements, not circular.
- $24 = 4!$ and 60 use the wrong value of n.

Final Answer: 120 ways $\Rightarrow$ B

Answer: (B) Go Back to Q6

Q7.

Solution

Concept — Binomial coefficient: In $(1+x)^n$ the coefficient of x^k is $\binom{n}{k}$.

Step 1 — Identify n and k: Here $n = 6$ and $k = 2$.

Step 2 — Write the coefficient:

$$\binom{6}{2} = \frac{6!}{2!\,4!}.$$

Step 3 — Compute:

$$\binom{6}{2} = \frac{6 \times 5}{2 \times 1} = 15.$$

Why other options are wrong:

- $6 = \binom{6}{1}$ is the coefficient of x^1.
- $20 = \binom{6}{3}$ and 30 do not match x^2.

Final Answer: Coefficient $= 15 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q7

Q8.

Solution

Concept — AM–GM relation: For two positive numbers, AM $\geq$ GM, with equality only when the numbers are equal.

Step 1 — Set up equations: Let the numbers be a and b. Then

$$\frac{a+b}{2} = 10 \quad \Rightarrow \quad a+b = 20,$$

$$\sqrt{ab} = 8 \quad \Rightarrow \quad ab = 64.$$

Step 2 — Solve: a and b are roots of $t^2 - 20t + 64 = 0$, giving $t = 16$ or $t = 4$.

Step 3 — Check inequality: Here $10 \geq 8$, so AM $\geq$ GM holds, and the numbers are 16 and 4.

Why other options are wrong:

- AM $<$ GM never holds for positive reals.
- The numbers are 16 and 4, not 10 and 8; and they are real.

Final Answer: AM $\geq$ GM with numbers $16, 4 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q8

Q9.

Solution

Concept — Angle between two lines: If a line has slope m, the angle it makes with the x-axis satisfies $\tan\theta = m$.

Step 1 — Find the slope: The line $y = x$ has slope $m = 1$.

Step 2 — Use the tangent relation:

$$\tan\theta = 1.$$

Step 3 — Solve for θ: The acute angle with $\tan\theta = 1$ is

$$\theta = 45^\circ.$$

Why other options are wrong:

- 30° gives $\tan\theta = \frac{1}{\sqrt{3}}$ and 60° gives $\sqrt{3}$.
- 90° would mean a vertical line.

Final Answer: $\theta = 45^\circ \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q9

Q10.

Solution

Concept — Eccentricity of an ellipse: For $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ with $a > b$, $e = \sqrt{1 - \frac{b^2}{a^2}}$.

Step 1 — Identify a^2 and b^2: Here $a^2 = 25$ and $b^2 = 16$.

Step 2 — Substitute:

$$e = \sqrt{1 - \frac{16}{25}} = \sqrt{\frac{9}{25}}.$$

Step 3 — Simplify:

$$e = \frac{3}{5}.$$

Why other options are wrong:

- $\frac{4}{5}$ uses b/a wrongly; $\frac{2}{5}$ and $\frac{1}{5}$ come from arithmetic slips.

Final Answer: $e = \frac{3}{5} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q10

Q11.

Solution

Concept — Limit at infinity: For a ratio of polynomials of equal degree, the limit is the ratio of leading coefficients.

Step 1 — Divide by x^2:

$$\frac{3x^2+2x+1}{5x^2-x+4} = \frac{3+\frac{2}{x}+\frac{1}{x^2}}{5-\frac{1}{x}+\frac{4}{x^2}}.$$

Step 2 — Take the limit: As $x \to \infty$, every term with x in the denominator vanishes:

$$\lim_{x\to\infty} = \frac{3+0+0}{5-0+0}.$$

Step 3 — Simplify:

$$= \frac{3}{5}.$$

Why other options are wrong:

- 0 and ∞ would occur only if the degrees differed.
- $\frac{2}{1}$ uses the wrong coefficients.

Final Answer: Limit $= \frac{3}{5} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q11

Q12.

Solution

Concept — Chain rule: $\frac{d}{dx}[u(x)]^n = n\,[u(x)]^{n-1} \cdot u'(x).$

Step 1 — Identify inner function: Let $u = 2x+3$, so $u' = 2$, and $y = u^4$.

Step 2 — Differentiate the outer power:

$$\frac{dy}{dx} = 4u^3 \cdot u'.$$

Step 3 — Substitute:

$$\frac{dy}{dx} = 4(2x+3)^3 \cdot 2 = 8(2x+3)^3.$$

Why other options are wrong:

- $4(2x+3)^3$ forgets the inner derivative 2.
- The options keeping power 4 never differentiated the outer power.

Final Answer: $\frac{dy}{dx} = 8(2x+3)^3 \Rightarrow \boxed{A}$

Answer: (A) Go Back to Q12

Q13.

Solution

Concept — Inverse of a 2×2 matrix: For $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, $A^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Step 1 — Compute the determinant:

$$\det A = (2)(1) - (1)(1) = 1.$$

Step 2 — Swap and negate: The adjoint is $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$.

Step 3 — Divide by determinant: Since $\det A = 1$,

$$A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}.$$

Why other options are wrong:

- $\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix}$ fails to swap the diagonal entries.
- The remaining matrices have wrong signs or entries.

Final Answer: $A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} \Rightarrow \boxed{C}$

Answer: (C) Go Back to Q13

Q14.

Solution

Concept — Adjoint of a 2×2 matrix: For $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, adj $A = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$.

Step 1 — Read entries: Here $a = 3,\ b = 2,\ c = 4,\ d = 5$.

Step 2 — Swap the diagonal: Place $d = 5$ top-left and $a = 3$ bottom-right.

Step 3 — Negate off-diagonal: Take $-b = -2$ and $-c = -4$:

$$\text{adj } A = \begin{bmatrix} 5 & -2 \\ -4 & 3 \end{bmatrix}.$$

Why other options are wrong:

- $\begin{bmatrix} 3 & -2 \\ -4 & 5 \end{bmatrix}$ forgets to swap the diagonal entries.
- $\begin{bmatrix} 5 & 2 \\ 4 & 3 \end{bmatrix}$ does not negate the off-diagonal terms.

Final Answer: adj $A = \begin{bmatrix} 5 & -2 \\ -4 & 3 \end{bmatrix} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q14

Q15.

Solution

Concept — Approximation using differentials: For small Δr, $\Delta A \approx \dfrac{dA}{dr}\,\Delta r$.

Step 1 — Differentiate the area: With $A = \pi r^2$,

$$\frac{dA}{dr} = 2\pi r.$$

Step 2 — Substitute values: At $r = 5$ and $\Delta r = 0.02$,

$$\Delta A \approx 2\pi(5)(0.02).$$

Step 3 — Compute:

$$\Delta A \approx 0.2\pi \text{ cm}^2.$$

Why other options are wrong:

- 0.5π, 0.1π, 0.4π come from using a wrong Δr or derivative.

Final Answer: $\Delta A \approx 0.2\pi$ cm^2 $\Rightarrow$ $\boxed{\text{D}}$

Answer: (D) Go Back to Q15

Q16.

Solution

Concept — Rolle's theorem: If f is continuous on $[a, b]$, differentiable on (a, b), and $f(a) = f(b)$, then $f'(c) = 0$ for some $c \in (a, b)$.

Step 1 — Check endpoints: $f(1) = 1 - 4 + 3 = 0$ and $f(3) = 9 - 12 + 3 = 0$, so $f(1) = f(3)$.

Step 2 — Differentiate:

$$f'(x) = 2x - 4.$$

Step 3 — Solve $f'(c) = 0$:

$$2c - 4 = 0 \quad \Rightarrow \quad c = 2.$$

Why other options are wrong:

- 1 and 3 are endpoints, not interior points.
- $\frac{3}{2}$ does not satisfy $f'(c) = 0$.

Final Answer: $c = 2 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q16

Q17.

Solution

Concept — Integration by parts: $\displaystyle\int u\,dv = uv - \int v\,du.$

Step 1 — Choose parts: Let $u = x$ (so $du = dx$) and $dv = e^x dx$ (so $v = e^x$).

Step 2 — Apply the formula:

$$\int x\,e^x\,dx = x\,e^x - \int e^x\,dx.$$

Step 3 — Integrate the remaining term:

$$= x\,e^x - e^x + C = e^x(x-1) + C.$$

Why other options are wrong:

- $e^x(x+1)+C$ uses the wrong sign on the second term.
- $xe^x + C$ omits $-\int e^x dx$; the last option misapplies the rule.

Final Answer: $\int xe^x dx = e^x(x-1) + C \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q17

Q18.

Solution

Concept — Trigonometric substitution: Put $x = \sin\theta$ so that $\sqrt{1-x^2} = \cos\theta$.

Step 1 — Substitute: With $x = \sin\theta$, $dx = \cos\theta\,d\theta$, the integral becomes

$$\int \frac{\cos\theta\,d\theta}{\cos\theta}.$$

Step 2 — Simplify:

$$= \int d\theta = \theta + C.$$

Step 3 — Back-substitute: Since $\theta = \sin^{-1} x$,

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x + C.$$

Why other options are wrong:

- $\cos^{-1} x + C$ differs only by a constant but the standard antiderivative is $\sin^{-1} x$.
- $\tan^{-1} x$ and $\sec^{-1} x$ arise from different integrands.

Final Answer: $\sin^{-1} x + C \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q18

Q19.

Solution

Concept — Definite integral as a limit of a sum: The Riemann sum converges to the definite integral.

Step 1 — Use the sum formula:

$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}.$$

Step 2 — Substitute:

$$\frac{1}{n^2} \cdot \frac{n(n+1)}{2} = \frac{n+1}{2n}.$$

Step 3 — Take the limit:

$$\lim_{n\to\infty} \frac{n+1}{2n} = \frac{1}{2}.$$

Why other options are wrong:

- 1 and 2 ignore the factor $\frac{1}{2}$; $\frac{1}{3}$ is the value of $\int_0^1 x^2\,dx$.

Final Answer: $\int_0^1 x\,dx = \frac{1}{2} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q19

Q20.

Solution

Concept — Area of a circle: A circle $x^2 + y^2 = r^2$ encloses area πr^2.

Step 1 — Identify the radius: From $x^2 + y^2 = 9$, $r^2 = 9$, so $r = 3$.

Step 2 — Apply the area formula:

$$\text{Area} = \pi r^2 = \pi(3)^2.$$

Step 3 — Compute:

$$\text{Area} = 9\pi.$$

Why other options are wrong:

- 6π uses $2\pi r$ (circumference) instead of area.
- 3π uses r not r^2; 18π doubles the correct value.

Final Answer: Area $= 9\pi \Rightarrow \boxed{A}$

Answer: (A) Go Back to Q20

Q21.

Solution

Concept — Formation of a differential equation: Differentiate the family and eliminate the arbitrary constant.

Step 1 — Start with the family:

$$y = A\,e^{x}.$$

Step 2 — Differentiate:

$$\frac{dy}{dx} = A\,e^{x}.$$

Step 3 — Eliminate A: Since $Ae^{x} = y$, the right side equals y, so

$$\frac{dy}{dx} = y.$$

Why other options are wrong:

- $\frac{dy}{dx} + y = 0$ matches $y = Ae^{-x}$, not Ae^{x}.
- $\frac{dy}{dx} = x$ and $\frac{dy}{dx} = e^{x}$ still contain x or fail to eliminate A.

Final Answer: $\frac{dy}{dx} = y \Rightarrow \boxed{C}$

Answer: (C) Go Back to Q21

Q22.

Solution

Concept — Variable separable equations: Separate variables and integrate both sides.

Step 1 — Separate variables:

$$\frac{dP}{P} = k\,dt.$$

Step 2 — Integrate both sides:

$$\ln|P| = kt + c_1.$$

Step 3 — Exponentiate:

$$P = e^{kt+c_1} = C\,e^{kt},$$

where $C = e^{c_1}$.

Why other options are wrong:

- $P = kt + C$ would solve $\frac{dP}{dt} = k$, not kP.
- Ce^{-kt} describes decay; $\frac{C}{t}$ is unrelated.

Final Answer: $P = C\,e^{kt} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q22

Q23.

Solution

Concept — Unit vector: A unit vector along $\vec{a}$ is $\hat{a} = \dfrac{\vec{a}}{|\vec{a}|}$.

Step 1 — Find the magnitude:

$$|\vec{a}| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5.$$

Step 2 — Divide by the magnitude:

$$\hat{a} = \frac{3\hat{\imath} + 4\hat{\jmath}}{5}.$$

Step 3 — State result: This vector has length 1 in the direction of $\vec{a}$.

Why other options are wrong:

- $3\hat{\imath} + 4\hat{\jmath}$ has magnitude 5, not 1.
- Dividing by 7 or 25 uses a wrong magnitude.

Final Answer: $\hat{a} = \dfrac{3\hat{\imath} + 4\hat{\jmath}}{5} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q23

Q24.

Solution

Concept — Area of a parallelogram: The area equals $|\vec{a} \times \vec{b}|$.

Step 1 — Compute the cross product:

$$\vec{a} \times \vec{b} = (2\hat{\imath}) \times (3\hat{\jmath}) = 6(\hat{\imath} \times \hat{\jmath}) = 6\hat{k}.$$

Step 2 — Take the magnitude:

$$|\vec{a} \times \vec{b}| = 6.$$

Step 3 — State the area: The parallelogram (a 2×3 rectangle here) has area 6 square units.

Why other options are wrong:

- 5 adds the side lengths; $\sqrt{13}$ is the diagonal length.
- 12 doubles the correct value.

Final Answer: Area $= 6 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q24

Q25.

Solution

Concept — Angle between two lines in space: $\cos\theta = \dfrac{a_1a_2 + b_1b_2 + c_1c_2}{\sqrt{a_1^2 + b_1^2 + c_1^2}\,\sqrt{a_2^2 + b_2^2 + c_2^2}}$.

Step 1 — Compute the dot product: For $\langle 1, 0, 0\rangle$ and $\langle 0, 1, 0\rangle$,

$$(1)(0) + (0)(1) + (0)(0) = 0.$$

Step 2 — Find the cosine: Each magnitude is 1, so

$$\cos\theta = \frac{0}{1 \cdot 1} = 0.$$

Step 3 — Solve for θ:

$$\theta = 90^\circ.$$

Why other options are wrong:

- 0° would need parallel lines; 45° and 60° require nonzero dot products.

Final Answer: $\theta = 90^\circ \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q25

Q26.

Solution

Concept — Angle between two planes: It is the angle between their normal vectors $\vec{n}_1$ and $\vec{n}_2$.

Step 1 — Read the normals: For $x + y + z = 1$, $\vec{n}_1 = \langle 1, 1, 1 \rangle$; for $x - y + z = 5$, $\vec{n}_2 = \langle 1, -1, 1 \rangle$.

Step 2 — Dot product and magnitudes:

$$\vec{n}_1 \cdot \vec{n}_2 = (1)(1) + (1)(-1) + (1)(1) = 1,$$

$$|\vec{n}_1| = |\vec{n}_2| = \sqrt{3}.$$

Step 3 — Compute the cosine:

$$\cos\theta = \frac{1}{\sqrt{3} \cdot \sqrt{3}} = \frac{1}{3}.$$

Why other options are wrong:

- $\frac{2}{3}, \frac{1}{\sqrt{3}}$ come from arithmetic errors; 0 would mean perpendicular planes.

Final Answer: $\cos\theta = \frac{1}{3} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q26

Q27.

Solution

Concept — Total probability theorem: $P(W) = P(B_1)P(W \mid B_1) + P(B_2)P(W \mid B_2)$.

Step 1 — Probabilities of choosing bags: Each bag is equally likely: $P(B_1) = P(B_2) = \frac{1}{2}$.

Step 2 — Conditional probabilities: Bag I gives $P(W \mid B_1) = \frac{2}{5}$; Bag II gives $P(W \mid B_2) = \frac{4}{5}$.

Step 3 — Combine:

$$P(W) = \tfrac{1}{2} \cdot \tfrac{2}{5} + \tfrac{1}{2} \cdot \tfrac{4}{5} = \tfrac{1}{5} + \tfrac{2}{5} = \tfrac{3}{5}.$$

Why other options are wrong:

- $\frac{2}{5}$ and $\frac{4}{5}$ use only one bag.
- $\frac{1}{2}$ ignores the differing white-ball counts.

Final Answer: $P(W) = \frac{3}{5} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q27

Q28.

Solution

Concept — Binomial distribution: $P(X = r) = \binom{n}{r} p^r (1-p)^{n-r}$.

Step 1 — Identify parameters: Here $n = 4$, $p = \frac{1}{2}$, $r = 2$.

Step 2 — Substitute:

$$P(X = 2) = \binom{4}{2} \left(\tfrac{1}{2}\right)^2 \left(\tfrac{1}{2}\right)^2.$$

Step 3 — Compute:

$$= 6 \cdot \tfrac{1}{16} = \frac{6}{16} = \frac{3}{8}.$$

Why other options are wrong:

- $\frac{1}{4}, \frac{1}{2}, \frac{1}{8}$ skip the binomial coefficient $\binom{4}{2} = 6$.

Final Answer: $P(X = 2) = \frac{3}{8} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q28

Q29.

Solution

Concept — Mode: The mode is the value that occurs most frequently.

Step 1 — Tally frequencies: In $4, 5, 5, 6, 7, 5, 8, 6$: the value 5 appears 3 times, 6 appears 2 times, others once.

Step 2 — Pick the highest frequency: The maximum frequency is 3, belonging to 5.

Step 3 — State the mode: The mode is 5.

Why other options are wrong:

- 6 occurs only twice; 4 and 7 occur once each.

Final Answer: Mode $= 5 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q29

Q30.

Solution

Concept — LPP optimisation: The optimum of a linear objective over a bounded feasible region occurs at a corner point.

Step 1 — Evaluate $Z = 3x + 2y$ at corners:

$$Z(0, 0) = 0, \quad Z(4, 0) = 12, \quad Z(0, 5) = 10.$$

Step 2 — Compare values: The smallest value among $0, 12, 10$ is 0.

Step 3 — Identify the minimum: The minimum of Z is 0 at $(0, 0)$.

Why other options are wrong:

- 10 and 12 are values at other corners, not the minimum.
- 15 is not attained at any corner.

Final Answer: $Z_{\text{min}} = 0 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q30

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	C	2	B	3	D	4	A	5	C
6	B	7	D	8	A	9	C	10	B
11	D	12	A	13	C	14	B	15	D
16	A	17	C	18	B	19	D	20	A
21	C	22	B	23	D	24	A	25	C
26	B	27	D	28	A	29	C	30	B