AIIMS Paramedical Mathematics

Sample Paper – 4

Duration: 30 Minutes **Maximum Marks: 30**

Instructions

- This paper contains **30** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics portion of **AIIMS Paramedical** entrance.
- Each correct answer carries **+1 mark**. A penalty of $-\frac{1}{3}$ **mark** is deducted for each incorrect answer; unattempted questions carry **0** marks.
- Only **one** option is correct. Choose carefully.
- Syllabus level: **Class 11 & 12 NCERT Mathematics**
- Use of mobile phones, calculators, or electronic gadgets is strictly prohibited.

Q1. For two finite sets A and B, $n(A) = 20$, $n(B) = 15$ and $n(A \cap B) = 7$. The value of $n(A \cup B)$ is

(A) 42

(B) 35

(C) 28

(D) 22

Q2. The function $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = 3x - 5$ is

(A) one-one and onto

(B) one-one but not onto

(C) onto but not one-one

(D) neither one-one nor onto

Q3. The value of $\sin 75^\circ$ is

(A) $\dfrac{\sqrt{6}-\sqrt{2}}{4}$

(B) $\dfrac{\sqrt{6}+\sqrt{2}}{4}$

(C) $\dfrac{\sqrt{3}+1}{2}$

(D) $\dfrac{\sqrt{3}-1}{2\sqrt{2}}$

Q4. The value of $\tan^{-1}\dfrac{1}{2}+\tan^{-1}\dfrac{1}{3}$ is

(A) $\dfrac{\pi}{2}$

(B) $\dfrac{\pi}{3}$

(C) $\dfrac{\pi}{6}$

(D) $\dfrac{\pi}{4}$

Q5. If ω is a non-real cube root of unity, then the value of $1+\omega^2+\omega^4$ is

(A) 3

(B) 1

(C) 0

(D) -1

Q6. The number of distinct arrangements (words, with or without meaning) that can be formed using all the letters of the word **BANANA** is

(A) 720

(B) 60

(C) 120

(D) 360

Q7. The term independent of x in the expansion of $\left(x^2+\dfrac{1}{x}\right)^6$ is

(A) 15

(B) 20

(C) 6

(D) 1

Q8. The sum of the squares of the first n natural numbers, $1^2 + 2^2 + \cdots + n^2$, equals

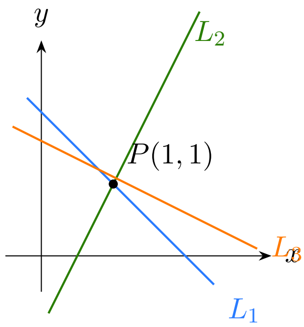
(A) $\frac{n(n+1)}{2}$

(B) $\left[\frac{n(n+1)}{2}\right]^2$

(C) $\frac{n(n+1)(2n+1)}{3}$

(D) $\frac{n(n+1)(2n+1)}{6}$

Q9. Three straight lines $L_1 : x + y = 2$, $L_2 : 2x - y = 1$ and $L_3 : x + 2y = k$ are concurrent (they pass through a single common point), as shown. The value of k is



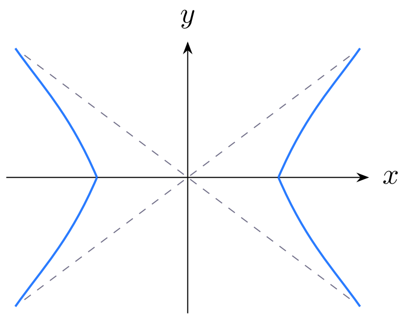
(A) 3

(B) 2

(C) 4

(D) 1

Q10. For the hyperbola $\frac{x^2}{16} - \frac{y^2}{9} = 1$, whose asymptotes are sketched below, the equations of the asymptotes are



(A) $y = \pm\frac{4}{3}x$

(B) $y = \pm\frac{3}{4}x$

(C) $y = \pm\frac{9}{16}x$

(D) $y = \pm\frac{16}{9}x$

Q11. The value of $\lim_{x\to 0}\frac{e^{3x}-1}{x}$ is

(A) 0

(B) 1

(C) 3

(D) $\frac{1}{3}$

Q12. If $x^2 + y^2 = 25$, then $\frac{dy}{dx}$ at the point $(3, 4)$ is

(A) $\frac{3}{4}$

(B) $\frac{4}{3}$

(C) $-\frac{4}{3}$

(D) $-\frac{3}{4}$

Q13. If A is a matrix of order 3×4 and B is a matrix of order 4×2, then the order of the product matrix AB is

(A) 3×2

(B) 4×4

(C) 2×3

(D) 3×4

Q14. Using Cramer's rule for the system $2x + 3y = 8$, $x - y = -1$, the value of x is

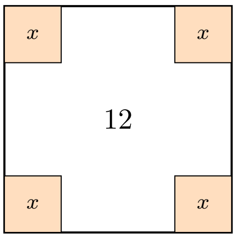
(A) 2

(B) 1

(C) 3

(D) -1

Q15. An open box is made from a square sheet of side 12 cm by cutting equal squares of side x from the four corners and folding up the flaps, as shown. The value of x that maximises the volume of the box is



(A) 4 cm

(B) 3 cm

(C) 2 cm

(D) 6 cm

Q16. The function $f(x) = x^3 - 6x^2 + 9x + 2$ has a local maximum at

(A) $x = 3$

(B) $x = 0$

(C) $x = 2$

(D) $x = 1$

Q17. The value of $\int_0^1 (3x^2 + 2x)\, dx$ is

(A) 2

(B) 1

(C) 3

(D) $\frac{5}{2}$

Q18. $\int \frac{1}{1+x^2}\, dx$ equals

(A) $\log|1 + x^2| + C$

(B) $\frac{1}{1+x^2} + C$

(C) $\tan^{-1} x + C$

(D) $\sin^{-1} x + C$

Q19. Using the property of definite integrals, the value of $\int_{-2}^{2} x^3 \cos x\, dx$ is

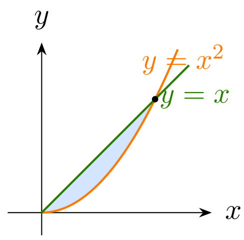
(A) 4

(B) 0

(C) 8

(D) 2

Q20. The area of the region bounded by the parabola $y = x^2$ and the line $y = x$, shown shaded below, is



(A) 1

(B) $\frac{1}{2}$

(C) $\frac{1}{3}$

(D) $\frac{1}{6}$

Q21. The integrating factor of the linear differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$ is

(A) x

(B) $\frac{1}{x}$

(C) $\log x$

(D) e^x

Q22. The general solution of the differential equation $\frac{dy}{dx} = 2x$ is

(A) $y = 2x + C$

(B) $y = x^2 + C$

(C) $y = 2x^2 + C$

(D) $y = \frac{x^2}{2} + C$

Q23. The vectors $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = 4\hat{i} + 6\hat{j} + \lambda\hat{k}$ are collinear (parallel). The value of λ is

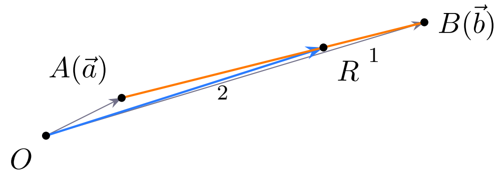
(A) 2

(B) 4

(C) 8

(D) 6

Q24. The position vector of the point R that divides the line segment joining the points A (position vector $\vec{a}$) and B (position vector $\vec{b}$) internally in the ratio $2 : 1$, as shown, is



(A) $\dfrac{\vec{a}+\vec{b}}{2}$

(B) $\dfrac{2\vec{a}+\vec{b}}{3}$

(C) $\dfrac{\vec{a}+\vec{b}}{3}$

(D) $\dfrac{\vec{a}+2\vec{b}}{3}$

Q25. The direction ratios of the line joining the points $P(1,2,3)$ and $Q(4,5,9)$ are

(A) 3, 3, 6

(B) 5, 7, 12

(C) 4, 5, 9

(D) 3, 3, 3

Q26. The line $\dfrac{x}{1}=\dfrac{y}{1}=\dfrac{z}{0}$ makes an angle θ with the plane $z=0$ (the xy-plane). The value of θ is

(A) 90°

(B) 0°

(C) 45°

(D) 30°

Q27. For two events A and B, $P(A)=0.5$, $P(B)=0.3$ and $P(A\cap B)=0.1$. Then $P(A\cup B)$ equals

(A) 0.9

(B) 0.8

(C) 0.7

(D) 0.6

Q28. A random variable X takes the values $0,1,2$ with probabilities $0.2,\ 0.5,\ 0.3$ respectively. The mean (expected value) of X is

(A) 1.5

(B) 0.8

(C) 1.0

(D) 1.1

Q29. The variance of the data $2,\ 4,\ 6,\ 8,\ 10$ is

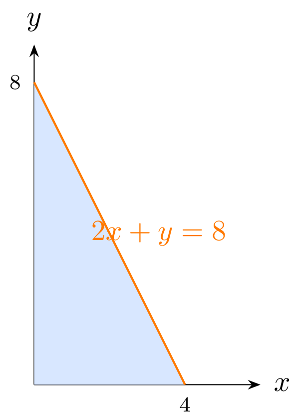
(A) 8

(B) 6

(C) 10

(D) 4

Q30. A factory makes x units of product P and y units of product Q. Each unit of P needs 2 hours and each unit of Q needs 1 hour on a machine available for at most 8 hours; both x and y must be non-negative. The feasible region for these constraints is shown shaded. The set of constraints is



(A) $2x + y \leq 8,\ x \leq 0,\ y \leq 0$

(B) $x + 2y \leq 8,\ x \geq 0,\ y \geq 0$

(C) $2x + y \leq 8,\ x \geq 0,\ y \geq 0$

(D) $2x + y \geq 8,\ x \geq 0,\ y \geq 0$

Detailed Solutions

Q1.

Solution

Concept — Cardinality of a union: For any two finite sets, the inclusion-exclusion principle gives

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

Step 1 — List the given values: We are given

$$n(A) = 20, \qquad n(B) = 15, \qquad n(A \cap B) = 7.$$

Step 2 — Substitute into the formula:

$$n(A \cup B) = 20 + 15 - 7.$$

Step 3 — Simplify:

$$n(A \cup B) = 35 - 7 = 28.$$

Why other options are wrong:

- 42: this is $n(A) + n(B) + n(A \cap B)$, adding the overlap instead of subtracting it.
- 35: this is $n(A) + n(B)$, forgetting to remove the common elements.
- 22: this subtracts twice the intersection, $20 + 15 - 2(7)$, which is incorrect.

Final Answer: $n(A \cup B) = 28 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q1

Q2.

Solution

Concept — One-one and onto for a linear map: A function $f(x) = ax + b$ with $a \neq 0$ on $\mathbb{R}$ is both injective (one-one) and surjective (onto).

Step 1 — Test one-one: Suppose $f(x_1) = f(x_2)$. Then

$$3x_1 - 5 = 3x_2 - 5.$$

Adding 5 to both sides gives $3x_1 = 3x_2$, so $x_1 = x_2$. Hence f is one-one.

Step 2 — Test onto: Take any $y \in \mathbb{R}$. Solve $y = 3x - 5$:

$$x = \frac{y+5}{3},$$

which is a real number for every y. So every y has a pre-image and f is onto.

Step 3 — Conclusion: Being both one-one and onto, f is a bijection.

Why other options are wrong:

- "one-one but not onto": the map clearly covers all of $\mathbb{R}$, so it is onto.
- "onto but not one-one": a non-constant linear map cannot send two different inputs to the same output.
- "neither": contradicts both checks above.

Final Answer: f is one-one and onto $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q2

Q3.

Solution

Concept — Sine of a sum: Write $75^\circ = 45^\circ + 30^\circ$ and use

$$\sin(A+B) = \sin A \cos B + \cos A \sin B.$$

Step 1 — Substitute the angles:

$$\sin 75^\circ = \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ.$$

Step 2 — Insert standard values:

$$\sin 75^\circ = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \frac{1}{2}.$$

Step 3 — Combine the terms:

$$\sin 75^\circ = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}.$$

Step 4 — Rationalise: Multiply numerator and denominator by $\sqrt{2}$:

$$\sin 75^\circ = \frac{(\sqrt{3}+1)\sqrt{2}}{2\cdot 2} = \frac{\sqrt{6}+\sqrt{2}}{4}.$$

Why other options are wrong:

- $\dfrac{\sqrt{6}-\sqrt{2}}{4}$: this equals $\sin 15^\circ$, not $\sin 75^\circ$.
- $\dfrac{\sqrt{3}+1}{2}$: arithmetic slip; the correct denominator is $2\sqrt{2}$ before rationalising.
- $\dfrac{\sqrt{3}-1}{2\sqrt{2}}$: uses a minus sign, which corresponds to $\sin 15^\circ$.

Final Answer: $\sin 75^\circ = \dfrac{\sqrt{6}+\sqrt{2}}{4} \Rightarrow \boxed{\text{B}}$

Answer: (B) **Go Back to Q3**

Q4.

Solution

Concept — Addition formula for arctangent: When $xy < 1$,

$$\tan^{-1} x + \tan^{-1} y = \tan^{-1}\left(\frac{x+y}{1-xy}\right).$$

Step 1 — Identify x and y: Here $x = \frac{1}{2}$ and $y = \frac{1}{3}$, and $xy = \frac{1}{6} < 1$.

Step 2 — Compute the numerator:

$$x + y = \frac{1}{2} + \frac{1}{3} = \frac{3+2}{6} = \frac{5}{6}.$$

Step 3 — Compute the denominator:

$$1 - xy = 1 - \frac{1}{6} = \frac{5}{6}.$$

Step 4 — Form the ratio:

$$\frac{x+y}{1-xy} = \frac{5/6}{5/6} = 1.$$

Step 5 — Take the arctangent:

$$\tan^{-1}(1) = \frac{\pi}{4}.$$

Why other options are wrong:

- $\frac{\pi}{2}$: this is the value of $\tan^{-1} x + \tan^{-1} \frac{1}{x}$ for $x > 0$, a different pairing.
- $\frac{\pi}{3}$ and $\frac{\pi}{6}$: their tangents are $\sqrt{3}$ and $\frac{1}{\sqrt{3}}$, not 1.

Final Answer: The sum is $\frac{\pi}{4} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q4

Q5.

Solution

Concept — Cube roots of unity: The non-real cube roots ω satisfy

$$\omega^3 = 1, \qquad 1 + \omega + \omega^2 = 0.$$

Step 1 — Reduce ω^4: Since $\omega^3 = 1$,

$$\omega^4 = \omega^3 \cdot \omega = 1 \cdot \omega = \omega.$$

Step 2 — Rewrite the expression:

$$1 + \omega^2 + \omega^4 = 1 + \omega^2 + \omega.$$

Step 3 — Apply the identity: The terms regroup as

$$1 + \omega + \omega^2 = 0.$$

Why other options are wrong:

- 3: this would hold only if every term equalled 1, which is false for non-real ω.
- 1: ignores the cancellation given by $1 + \omega + \omega^2 = 0$.
- -1: drops the constant term 1 from the sum.

Final Answer: $1 + \omega^2 + \omega^4 = 0 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q5

Q6.

Solution

Concept — Permutations of letters with repetition: The number of distinct arrangements of n letters, where letters repeat $p, q, \ldots$ times, is

$$\frac{n!}{p!\,q!\cdots}.$$

Step 1 — Count the letters of BANANA: There are 6 letters in total: A appears 3 times, N appears 2 times, B appears 1 time.

Step 2 — Apply the formula:

$$\text{Number} = \frac{6!}{3!\,2!\,1!}.$$

Step 3 — Evaluate factorials:

$$6! = 720, \qquad 3! = 6, \qquad 2! = 2, \qquad 1! = 1.$$

Step 4 — Divide:

$$\frac{720}{6 \cdot 2 \cdot 1} = \frac{720}{12} = 60.$$

Why other options are wrong:

- 720: this is $6!$, treating all letters as distinct.
- 120: divides only by $3!$, ignoring the two N's.
- 360: divides only by $2!$, ignoring the three A's.

Final Answer: 60 arrangements $\Rightarrow$ B

Answer: (B) Go Back to Q6

Q7.

Solution

Concept — General term of a binomial expansion: For $(a+b)^n$, the general term is

$$T_{r+1} = \binom{n}{r} a^{n-r} b^r.$$

Step 1 — Write the general term: With $a = x^2$, $b = \frac{1}{x}$, $n = 6$,

$$T_{r+1} = \binom{6}{r} (x^2)^{6-r} \left(\frac{1}{x}\right)^r.$$

Step 2 — Collect the power of x:

$$T_{r+1} = \binom{6}{r} x^{2(6-r)} x^{-r} = \binom{6}{r} x^{12-3r}.$$

Step 3 — Set the exponent to zero: For the term independent of x,

$$12 - 3r = 0 \;\Rightarrow\; r = 4.$$

Step 4 — Evaluate the coefficient:

$$\binom{6}{4} = \frac{6!}{4!\,2!} = 15.$$

Why other options are wrong:

- 20: this is $\binom{6}{3}$, which corresponds to $r = 3$ (exponent 3, not 0).
- 6: this is $\binom{6}{1}$ or $\binom{6}{5}$, the wrong term.
- 1: this is $\binom{6}{0}$, the first term, which has x^{12}.

Final Answer: The constant term is $15 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q7

Q8.

Solution

Concept — Sum of squares formula: A standard NCERT result states

$$\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}.$$

Step 1 — Recall the related sums: The linear sum is $\frac{n(n+1)}{2}$ and the sum of cubes is $\left[\frac{n(n+1)}{2}\right]^2$; the sum of squares is a separate formula.

Step 2 — Verify with $n = 3$: Direct addition gives

$$1^2 + 2^2 + 3^2 = 1 + 4 + 9 = 14.$$

Step 3 — Check the formula at $n = 3$:

$$\frac{3 \cdot 4 \cdot 7}{6} = \frac{84}{6} = 14.$$

The formula reproduces 14, confirming the choice.

Why other options are wrong:

- $\frac{n(n+1)}{2}$: that is the sum of the first n natural numbers, giving 6 at $n = 3$.
- $\left[\frac{n(n+1)}{2}\right]^2$: that is the sum of cubes, giving 36 at $n = 3$.
- $\frac{n(n+1)(2n+1)}{3}$: wrong denominator; it gives 28 at $n = 3$.

Final Answer: $\frac{n(n+1)(2n+1)}{6} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q8

Q9.

Solution

Concept — Concurrency of lines: Three lines are concurrent when they share a common point. Find the intersection of two of them, then make the third pass through it.

Step 1 — Solve L_1 and L_2: From

$$x + y = 2, \qquad 2x - y = 1,$$

add the two equations:

$$3x = 3 \;\Rightarrow\; x = 1.$$

Step 2 — Find y: Substitute $x = 1$ into $x + y = 2$:

$$1 + y = 2 \;\Rightarrow\; y = 1.$$

So the common point is $P(1, 1)$.

Step 3 — Make L_3 pass through P: Substitute $(1, 1)$ into $x + 2y = k$:

$$k = 1 + 2(1) = 3.$$

Why other options are wrong:

- $2,\ 4,\ 1$: substituting $(1, 1)$ gives $x + 2y = 3$ exactly, so no other value makes the lines meet at P.

Final Answer: $k = 3 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q9

Q10.

Solution

Concept — Asymptotes of a hyperbola: For $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$, the asymptotes are the lines

$$y = \pm\frac{b}{a}x.$$

Step 1 — Read off a and b: Comparing with the given equation,

$$a^2 = 16 \Rightarrow a = 4, \qquad b^2 = 9 \Rightarrow b = 3.$$

Step 2 — Form the asymptote slopes:

$$\frac{b}{a} = \frac{3}{4}.$$

Step 3 — Write the equations:

$$y = \pm\frac{3}{4}x.$$

Why other options are wrong:

- $y = \pm\frac{4}{3}x$: this inverts the ratio, using $\frac{a}{b}$ instead of $\frac{b}{a}$.
- $y = \pm\frac{9}{16}x$ and $y = \pm\frac{16}{9}x$: these use b^2/a^2 or a^2/b^2 without taking square roots.

Final Answer: $y = \pm\frac{3}{4}x \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q10

Q11.

Solution

Concept — Standard exponential limit: A key NCERT limit is

$$\lim_{x\to 0}\frac{e^x - 1}{x} = 1.$$

Step 1 — Adjust for the factor 3: Multiply and divide by 3 so the exponent and denominator match:

$$\frac{e^{3x} - 1}{x} = 3\cdot\frac{e^{3x} - 1}{3x}.$$

Step 2 — Substitute $t = 3x$: As $x \to 0$, $t \to 0$, and

$$\lim_{x\to 0}\frac{e^{3x} - 1}{3x} = \lim_{t\to 0}\frac{e^t - 1}{t} = 1.$$

Step 3 — Combine:

$$\lim_{x\to 0}\frac{e^{3x} - 1}{x} = 3\cdot 1 = 3.$$

Why other options are wrong:

- 0: substituting $x = 0$ gives $\frac{0}{0}$, an indeterminate form, not 0.

- 1: forgets the factor of 3 that comes from the exponent.
- $\frac{1}{3}$: inverts the factor; the correct adjustment multiplies by 3.

Final Answer: The limit equals $3 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q11

Q12.

Solution

Concept — Implicit differentiation: Differentiate both sides with respect to x, treating y as a function of x.

Step 1 — Differentiate the relation: From $x^2 + y^2 = 25$,

$$2x + 2y\frac{dy}{dx} = 0.$$

Step 2 — Solve for $\frac{dy}{dx}$:

$$2y\frac{dy}{dx} = -2x \;\Rightarrow\; \frac{dy}{dx} = -\frac{x}{y}.$$

Step 3 — Evaluate at $(3, 4)$:

$$\frac{dy}{dx} = -\frac{3}{4}.$$

Why other options are wrong:

- $\frac{3}{4}$: drops the negative sign that comes from moving $2x$ across.
- $\frac{4}{3}$ and $-\frac{4}{3}$: invert $\frac{x}{y}$ to $\frac{y}{x}$.

Final Answer: $\left.\frac{dy}{dx}\right|_{(3,4)} = -\frac{3}{4} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q12

Q13.

Solution

Concept — Order of a matrix product: If A is $m \times n$ and B is $n \times p$, then AB is defined and has order $m \times p$ (inner dimensions must agree).

Step 1 — Check compatibility: A is 3×4 and B is 4×2. The inner numbers (4 and 4) match, so AB exists.

Step 2 — Read off the outer dimensions: The product takes the rows of A and the columns of B:

$$(3 \times 4)(4 \times 2) \to 3 \times 2.$$

Why other options are wrong:

- 4×4: uses the inner dimension, which is cancelled in the product.
- 2×3: this is the order of BA if it existed, not AB.
- 3×4: this is just the order of A.

Final Answer: AB has order $3 \times 2 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q13

Q14.

Solution

Concept — Cramer's rule: For $a_1x + b_1y = c_1$, $a_2x + b_2y = c_2$,

$$x = \frac{D_x}{D}, \qquad D = \begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix}.$$

Step 1 — Compute D: With $a_1 = 2, b_1 = 3, a_2 = 1, b_2 = -1$,

$$D = \begin{vmatrix} 2 & 3 \\ 1 & -1 \end{vmatrix} = 2(-1) - 3(1) = -2 - 3 = -5.$$

Step 2 — Compute D_x: Replace the x-column by the constants 8 and -1:

$$D_x = \begin{vmatrix} 8 & 3 \\ -1 & -1 \end{vmatrix} = 8(-1) - 3(-1) = -8 + 3 = -5.$$

Step 3 — Divide:

$$x = \frac{D_x}{D} = \frac{-5}{-5} = 1.$$

Why other options are wrong:

- 2: this is the value of y for this system, not x.
- 3 and -1: sign or substitution errors in D_x.

Final Answer: $x = 1 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q14

Q15.

Solution

Concept — Maximising a volume: Express the volume as a function of one variable, differentiate, and set the derivative to zero.

Step 1 — Write the volume: The base is $(12-2x)$ by $(12-2x)$ and the height is x:

$$V(x) = x(12-2x)^2.$$

Step 2 — Differentiate: Using the product rule,

$$\frac{dV}{dx} = (12-2x)^2 + x \cdot 2(12-2x)(-2).$$

Factor out $(12-2x)$:

$$\frac{dV}{dx} = (12-2x)\big[(12-2x)-4x\big] = (12-2x)(12-6x).$$

Step 3 — Set the derivative to zero:

$$(12-2x)(12-6x) = 0 \;\Rightarrow\; x = 6 \text{ or } x = 2.$$

Step 4 — Reject the invalid root: If $x = 6$ the base side $12-2x = 0$, giving zero volume. So the maximum is at

$$x = 2.$$

Why other options are wrong:

- 4 cm and 3 cm: do not satisfy $\frac{dV}{dx} = 0$.

- 6 cm: gives a degenerate box of zero volume.

Final Answer: $x = 2$ cm maximises the volume $\Rightarrow$ $\boxed{\text{C}}$

Answer: (C) Go Back to Q15

Q16.

Solution

Concept — Second derivative test: At a critical point, $f'' < 0$ indicates a local maximum and $f'' > 0$ a local minimum.

Step 1 — Find the critical points:

$$f'(x) = 3x^2 - 12x + 9 = 3(x^2 - 4x + 3) = 3(x-1)(x-3).$$

Setting $f'(x) = 0$ gives $x = 1$ and $x = 3$.

Step 2 — Compute $f''(x)$:

$$f''(x) = 6x - 12.$$

Step 3 — Test $x = 1$:

$$f''(1) = 6(1) - 12 = -6 < 0,$$

so $x = 1$ is a local maximum.

Step 4 — Test $x = 3$ (for contrast):

$$f''(3) = 6(3) - 12 = 6 > 0,$$

so $x = 3$ is a local minimum.

Why other options are wrong:

- $x = 3$: this is the local minimum, not maximum.
- $x = 0$ and $x = 2$: these are not critical points of f.

Final Answer: Local maximum at $x = 1 \Rightarrow$ $\boxed{\text{D}}$

Answer: (D) Go Back to Q16

Q17.

Solution

Concept — Fundamental theorem of calculus: Integrate term by term, then evaluate at the limits.

Step 1 — Integrate the polynomial:

$$\int (3x^2 + 2x)\,dx = x^3 + x^2.$$

Step 2 — Apply the limits 0 to 1:

$$\left[x^3 + x^2\right]_0^1 = (1^3 + 1^2) - (0 + 0).$$

Step 3 — Simplify:

$$(1 + 1) - 0 = 2.$$

Why other options are wrong:

- 1: integrates only one of the two terms.
- 3: adds the original coefficients instead of integrating.
- $\frac{5}{2}$: arises from an antiderivative error such as $x^3 + \frac{x^2}{2}$ miscombined.

Final Answer: The integral equals $2 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q17

Q18.

Solution

Concept — Standard integral: A basic NCERT result is

$$\int \frac{dx}{1 + x^2} = \tan^{-1} x + C.$$

Step 1 — Recall the derivative: We know

$$\frac{d}{dx}\left(\tan^{-1} x\right) = \frac{1}{1 + x^2}.$$

Step 2 — Reverse the differentiation: Integration undoes this, so

$$\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C.$$

Why other options are wrong:

- $\log|1+x^2|+C$: this is the integral of $\frac{2x}{1+x^2}$, which has an x in the numerator.
- $\frac{1}{1+x^2}+C$: this is the integrand itself, not its integral.
- $\sin^{-1} x + C$: this is the integral of $\frac{1}{\sqrt{1-x^2}}$, a different form.

Final Answer: $\tan^{-1} x + C \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q18

Q19.

Solution

Concept — Integral of an odd function over a symmetric interval: If $f(-x) = -f(x)$, then

$$\int_{-a}^{a} f(x)\,dx = 0.$$

Step 1 — Test the parity of the integrand: Let $f(x) = x^3 \cos x$. Then

$$f(-x) = (-x)^3 \cos(-x) = -x^3 \cos x = -f(x),$$

since x^3 is odd and $\cos x$ is even.

Step 2 — Apply the symmetry property: The integrand is odd and the limits are symmetric about 0, so

$$\int_{-2}^{2} x^3 \cos x\,dx = 0.$$

Why other options are wrong:

- $4, 8, 2$: any non-zero value contradicts the odd-function symmetry, which guarantees exact cancellation.

Final Answer: The integral equals $0 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q19

Q20.

Solution

Concept — Area between two curves: The area between $y = f(x)$ (upper) and $y = g(x)$ (lower) from $x = p$ to $x = q$ is $\int_p^q (f - g)\, dx$.

Step 1 — Find the intersection points: Solve $x^2 = x$:

$$x^2 - x = 0 \;\Rightarrow\; x(x-1) = 0 \;\Rightarrow\; x = 0,\ 1.$$

Step 2 — Identify the upper curve: On $0 < x < 1$, $x > x^2$, so the line $y = x$ lies above the parabola $y = x^2$.

Step 3 — Set up and integrate:

$$A = \int_0^1 (x - x^2)\, dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1.$$

Step 4 — Evaluate:

$$A = \frac{1}{2} - \frac{1}{3} = \frac{3-2}{6} = \frac{1}{6}.$$

Why other options are wrong:

- $\frac{1}{2}$ and $\frac{1}{3}$: these are the separate areas $\int x\, dx$ and $\int x^2\, dx$, not their difference.
- 1: ignores the integration entirely.

Final Answer: Area $= \frac{1}{6} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q20

Q21.

Solution

Concept — Integrating factor: For $\frac{dy}{dx} + P(x)y = Q(x)$, the integrating factor is

$$\text{I.F.} = e^{\int P\, dx}.$$

Step 1 — Identify $P(x)$: Comparing the given equation, $P(x) = \frac{1}{x}$.

Step 2 — Integrate P:

$$\int \frac{1}{x}\, dx = \log x.$$

Step 3 — Exponentiate:

$$\text{I.F.} = e^{\log x} = x.$$

Why other options are wrong:

- $\frac{1}{x}$: this is $e^{-\log x}$, using the wrong sign of the integral.
- $\log x$: this is $\int P\,dx$, before exponentiating.
- e^x: would require $P(x) = 1$, not $\frac{1}{x}$.

Final Answer: I.F. $= x \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q21

Q22.

Solution

Concept — Direct integration: A differential equation of the form $\frac{dy}{dx} = g(x)$ is solved by integrating both sides.

Step 1 — Separate and integrate:

$$dy = 2x\,dx \ \Rightarrow \ \int dy = \int 2x\,dx.$$

Step 2 — Carry out the integration:

$$y = x^2 + C.$$

Step 3 — Verify by differentiating:

$$\frac{dy}{dx} = \frac{d}{dx}(x^2 + C) = 2x,$$

which matches the original equation.

Why other options are wrong:

- $y = 2x + C$: differentiates to 2, not $2x$.
- $y = 2x^2 + C$: differentiates to $4x$.
- $y = \frac{x^2}{2} + C$: differentiates to x.

Final Answer: $y = x^2 + C \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q22

Q23.

Solution

Concept — Condition for collinear vectors: Two vectors are parallel when their corresponding components are proportional, $\vec{b} = \lambda\vec{a}$.

Step 1 — Compare the first components:

$$\frac{4}{2} = 2,$$

so the scaling factor is 2.

Step 2 — Confirm with the second components:

$$\frac{6}{3} = 2,$$

consistent with the same factor.

Step 3 — Apply to the third component:

$$\lambda = 2 \times 4 = 8.$$

Why other options are wrong:

- 2 and 4: these are the scaling factor and the original k-component, not the scaled value.
- 6: this is the j-component of $\vec{b}$, unrelated to the k-component.

Final Answer: $\lambda = 8 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q23

Q24.

Solution

Concept — Section formula (internal division): The point dividing $A(\vec{a})$ and $B(\vec{b})$ internally in the ratio $m : n$ has position vector

$$\vec{r} = \frac{m\vec{b} + n\vec{a}}{m+n}.$$

Step 1 — Insert the ratio: Here $m = 2$, $n = 1$:

$$\vec{r} = \frac{2\vec{b} + 1 \cdot \vec{a}}{2 + 1}.$$

Step 2 — Simplify:

$$\vec{r} = \frac{\vec{a} + 2\vec{b}}{3}.$$

Step 3 — Interpret: Since the point is nearer B (ratio $2 : 1$ from A), $\vec{b}$ carries the larger weight, as expected.

Why other options are wrong:

- $\frac{\vec{a} + \vec{b}}{2}$: this is the midpoint, ratio $1 : 1$.
- $\frac{2\vec{a} + \vec{b}}{3}$: this weights $\vec{a}$ more, corresponding to ratio $1 : 2$.
- $\frac{\vec{a} + \vec{b}}{3}$: the denominators sum correctly but the weights are wrong.

Final Answer: $\vec{r} = \frac{\vec{a} + 2\vec{b}}{3} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q24

Q25.

Solution

Concept — Direction ratios from two points: The direction ratios of the line through $P(x_1, y_1, z_1)$ and $Q(x_2, y_2, z_2)$ are the differences $x_2 - x_1$, $y_2 - y_1$, $z_2 - z_1$.

Step 1 — Subtract coordinates:

$$4 - 1 = 3, \qquad 5 - 2 = 3, \qquad 9 - 3 = 6.$$

Step 2 — State the direction ratios:

$$(3,\ 3,\ 6).$$

Why other options are wrong:

- $5,\ 7,\ 12$: these add the coordinates instead of subtracting.
- $4,\ 5,\ 9$: these are just the coordinates of Q, not the differences.

- 3, 3, 3: uses the wrong z-difference.

Final Answer: Direction ratios $(3, 3, 6) \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q25

Q26.

Solution

Concept — Angle between a line and a plane: If $\vec{b}$ is the line's direction and $\vec{n}$ the plane's normal, then

$$\sin\theta = \frac{|\vec{b}\cdot\vec{n}|}{|\vec{b}|\,|\vec{n}|}.$$

Step 1 — Read off the vectors: The line $\frac{x}{1} = \frac{y}{1} = \frac{z}{0}$ has direction $\vec{b} = (1, 1, 0)$; the plane $z = 0$ has normal $\vec{n} = (0, 0, 1)$.

Step 2 — Compute the dot product:

$$\vec{b}\cdot\vec{n} = 1\cdot 0 + 1\cdot 0 + 0\cdot 1 = 0.$$

Step 3 — Find the angle:

$$\sin\theta = \frac{0}{|\vec{b}|\,|\vec{n}|} = 0 \;\Rightarrow\; \theta = 0^\circ.$$

The line lies parallel to the xy-plane.

Why other options are wrong:

- 90°: would require the line to be along the normal $\vec{n} = (0, 0, 1)$.
- 45° and 30°: correspond to a non-zero z-component in the direction vector.

Final Answer: $\theta = 0^\circ \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q26

Q27.

Solution

Concept — Addition theorem of probability:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

Step 1 — Substitute the given probabilities:

$$P(A \cup B) = 0.5 + 0.3 - 0.1.$$

Step 2 — Simplify:

$$P(A \cup B) = 0.8 - 0.1 = 0.7.$$

Why other options are wrong:

- 0.9: adds the intersection instead of subtracting it.
- 0.8: forgets to remove $P(A \cap B)$.
- 0.6: subtracts twice the intersection.

Final Answer: $P(A \cup B) = 0.7 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q27

Q28.

Solution

Concept — Mean of a probability distribution: The expected value is

$$E(X) = \sum x_i \, p_i.$$

Step 1 — Form each product:

$$0 \times 0.2 = 0, \quad 1 \times 0.5 = 0.5, \quad 2 \times 0.3 = 0.6.$$

Step 2 — Add the products:

$$E(X) = 0 + 0.5 + 0.6 = 1.1.$$

Step 3 — Sanity check: The probabilities sum to $0.2 + 0.5 + 0.3 = 1$, so the distribution is valid.

Why other options are wrong:

- 1.5: this would be the mean of $0, 1, 2$ with equal weighting upward, not the given weights.
- 0.8 and 1.0: arithmetic slips in summing $x_i p_i$.

Final Answer: $E(X) = 1.1 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q28

Q29.

Solution

Concept — Variance of a data set:

$$\sigma^2 = \frac{1}{n}\sum(x_i - \bar{x})^2.$$

Step 1 — Find the mean:

$$\bar{x} = \frac{2+4+6+8+10}{5} = \frac{30}{5} = 6.$$

Step 2 — Compute the squared deviations:

$$(2-6)^2 = 16,\ (4-6)^2 = 4,\ (6-6)^2 = 0,\ (8-6)^2 = 4,\ (10-6)^2 = 16.$$

Step 3 — Sum the squared deviations:

$$16 + 4 + 0 + 4 + 16 = 40.$$

Step 4 — Divide by $n = 5$:

$$\sigma^2 = \frac{40}{5} = 8.$$

Why other options are wrong:

- 6: this is the mean, not the variance.
- 10: divides the sum of squared deviations by 4 instead of 5.
- 4: takes only half the squared-deviation sum.

Final Answer: Variance $= 8 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q29

Q30.

Solution

Concept — Formulating LPP constraints: Translate each resource limit into an inequality, and add the non-negativity restrictions on the decision variables.

Step 1 — Machine-time constraint: Product P uses 2 hours each ($2x$) and Q uses 1 hour each (y); total is at most 8 hours:

$$2x + y \leq 8.$$

Step 2 — Non-negativity: Quantities produced cannot be negative:

$$x \geq 0, \qquad y \geq 0.$$

Step 3 — Match with the shaded region: The boundary line is $2x + y = 8$, meeting the axes at $(4, 0)$ and $(0, 8)$, exactly as drawn; the feasible region lies in the first quadrant below the line.

Why other options are wrong:

- $2x + y \leq 8,\ x \leq 0,\ y \leq 0$: wrong direction on non-negativity; the region would be in the third quadrant.
- $x + 2y \leq 8, \ldots$: swaps the coefficients, giving intercepts $(8, 0)$ and $(0, 4)$, not the drawn line.
- $2x + y \geq 8, \ldots$: shades the region above the line, opposite to the figure.

Final Answer: $2x + y \leq 8,\ x \geq 0,\ y \geq 0 \Rightarrow \boxed{C}$

Answer: (C) Go Back to Q30

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
2	A	5	C	7	A	9	A	11	C
13	A	15	C	17	A	18	C	21	A
23	C	25	A	27	C	29	A	30	C