AIIMS Paramedical Mathematics
Sample Paper – 5

Duration: 30 Minutes **Maximum Marks: 30**

Instructions

- This paper contains **30** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics portion of **AIIMS Paramedical** entrance.
- Each correct answer carries **+1 mark**; each incorrect answer attracts a penalty of $-\frac{1}{3}$ mark; an unattempted question carries **0** marks.
- Only **one** option is correct. Choose carefully.
- Syllabus level: **Class 11 & 12 NCERT Mathematics**
- Use of mobile phones, calculators, or electronic gadgets is strictly prohibited.

Q1. If $A = \{x \in \mathbb{R} : -2 \leq x < 3\}$ and $B = \{x \in \mathbb{R} : 0 < x \leq 5\}$, then the set $A \cap B$, expressed as an interval, is

(A) $[-2, 5]$

(B) $(0, 3)$

(C) $[0, 3)$

(D) $(-2, 5]$

Q2. Let $f(x) = 2x + 3$ and $g(x) = x^2 - 1$. The value of the composite function $(f \circ g)(2)$ is

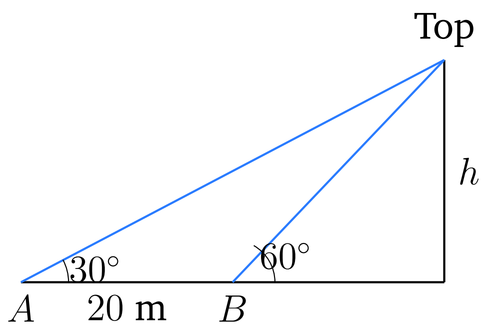
(A) 9

(B) 11

(C) 7

(D) 13

Q3. From a point on level ground the angle of elevation of the top of a vertical tower is $30°$. On walking 20 m straight towards the tower the angle of elevation becomes $60°$. The height of the tower is



(A) 10 m

(B) $10\sqrt{2}$ m

(C) $10\sqrt{3}$ m

(D) $20\sqrt{3}$ m

Q4. The general solution of the trigonometric equation $\tan\theta = \sqrt{3}$ is (where $n \in \mathbb{Z}$)

(A) $\theta = n\pi + \frac{\pi}{6}$

(B) $\theta = 2n\pi \pm \frac{\pi}{3}$

(C) $\theta = n\pi - \frac{\pi}{3}$

(D) $\theta = n\pi + \frac{\pi}{3}$

Q5. The polar (modulus–argument) form of the complex number $z = 1 + i$ is

(A) $2\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

(B) $\sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$

(C) $\sqrt{2}\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$

(D) $\sqrt{2}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$

Q6. A committee of 4 members is to be formed from 5 men and 4 women. The number of ways to form the committee such that it contains exactly 2 men and 2 women is

(A) 60

(B) 40

(C) 120

(D) 36

Q7. In the binomial expansion of $(1+x)^8$, the greatest binomial coefficient is

(A) 28

(B) 56

(C) 70

(D) 40

Q8. The sum to infinity of the geometric progression $6+4+\frac{8}{3}+\frac{16}{9}+\cdots$ is

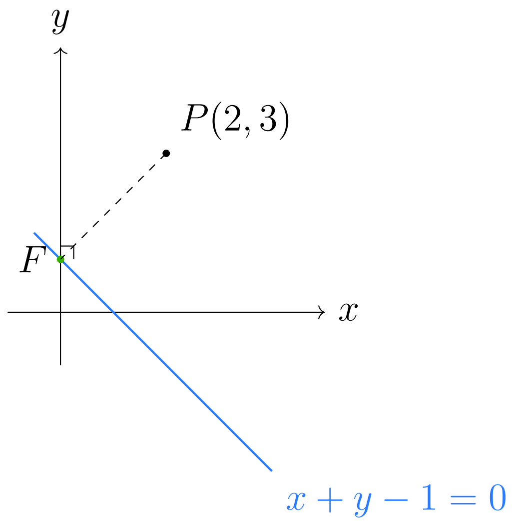
(A) 9

(B) 12

(C) 16

(D) 18

Q9. The foot of the perpendicular drawn from the point $P(2,3)$ to the line $x+y-1=0$ has coordinates



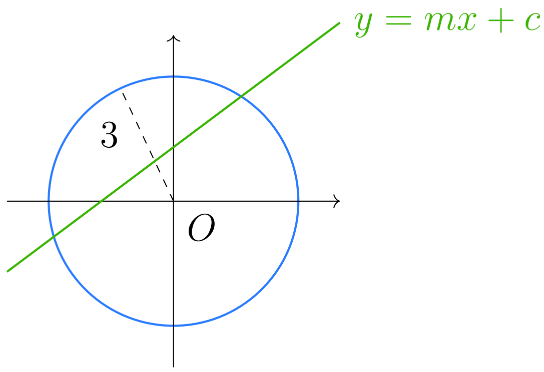
(A) $(0, 1)$

(B) $(1, 0)$

(C) $(2, -1)$

(D) $\left(\frac{1}{2}, \frac{1}{2}\right)$

Q10. The line $y = mx + c$ is a tangent to the circle $x^2 + y^2 = 9$ if



(A) $c = 3\sqrt{1 + m^2}$

(B) $c^2 = 9(1 + m^2)$

(C) $c^2 = 9m^2$

(D) $c = 9(1 + m^2)$

Q11. The value of the limit $\lim_{x \to 0} \frac{5^x - 1}{x}$ is

(A) 5

(B) 1

(C) $\log_e 5$

(D) 0

Q12. If $y = x^x \ (x > 0)$, then $\dfrac{dy}{dx}$ equals

(A) $x\, x^{x-1}$

(B) $x^x \log_e x$

(C) x^x

(D) $x^x(1 + \log_e x)$

Q13. If every element of the second row of a 3×3 determinant Δ is multiplied by 4, then the value of the new determinant is

(A) 4Δ

(B) Δ

(C) 12Δ

(D) 64Δ

Q14. The matrix $A = \begin{pmatrix} x & 2 \\ 3 & 4 \end{pmatrix}$ is singular when x equals

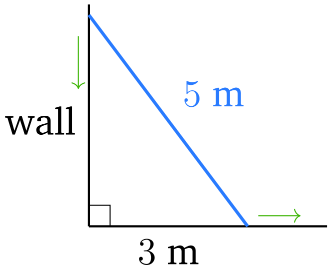
(A) $\dfrac{2}{3}$

(B) $\dfrac{3}{2}$

(C) 6

(D) $-\dfrac{3}{2}$

Q15. A 5 m long ladder leans against a vertical wall. Its foot slides away from the wall at 2 m/s. When the foot is 3 m from the wall, the rate at which the top of the ladder slides down is



(A) $\frac{8}{3}$ m/s

(B) $\frac{2}{3}$ m/s

(C) $\frac{3}{2}$ m/s

(D) $\frac{4}{3}$ m/s

Q16. The function $f(x) = x^2 - 4x + 5$ is strictly increasing on the interval

(A) $(-\infty, 2)$

(B) $(-\infty, 0)$

(C) $(0, 2)$

(D) $(2, \infty)$

Q17. The value of the integral $\int \frac{dx}{x^2 + 16}$ is (where C is the constant of integration)

(A) $\frac{1}{4} \tan^{-1} \frac{x}{4} + C$

(B) $\frac{1}{16} \tan^{-1} \frac{x}{4} + C$

(C) $4 \tan^{-1} \frac{x}{4} + C$

(D) $\tan^{-1} \frac{x}{4} + C$

Q18. Using integration by parts, $\int \log_e x \, dx$ equals (where C is the constant of integration)

(A) $\frac{1}{x} + C$

(B) $x \log_e x - x + C$

(C) $x \log_e x + x + C$

(D) $\log_e x - x + C$

Q19. The value of the definite integral $\int_0^{\pi} \sin x \, dx$ is

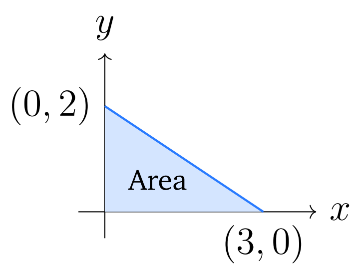
(A) 0

(B) 1

(C) 2

(D) -2

Q20. The area of the region bounded by the line $2x + 3y = 6$ and the coordinate axes (in the first quadrant) is



(A) 6 sq units

(B) 9 sq units

(C) $\frac{9}{2}$ sq units

(D) 3 sq units

Q21. The solution of the differential equation $\frac{dy}{dx} = \frac{y}{x}$ (with $x, y > 0$) is (where C is an arbitrary constant)

(A) $y = Cx$

(B) $y = C + x$

(C) $xy = C$

(D) $y^2 = Cx$

Q22. A bacterial culture grows so that the rate of increase of its population N is proportional to N. If the population doubles in 3 hours, then the time taken for it to become 8 times the original is

(A) 6 hours

(B) 9 hours

(C) 12 hours

(D) 24 hours

Q23. The vectors $\vec{a} = 2\hat{i} + \lambda\hat{j} + \hat{k}$ and $\vec{b} = \hat{i} - 2\hat{j} + 3\hat{k}$ are perpendicular when λ equals

(A) $\frac{1}{2}$

(B) $-\frac{1}{2}$

(C) $\frac{5}{2}$

(D) 5

Q24. The vectors $\vec{a} = \hat{i} + \hat{j}$, $\vec{b} = \hat{j} + \hat{k}$ and $\vec{c} = p\hat{i} + \hat{k}$ are coplanar when p equals

(A) 0

(B) -1

(C) 2

(D) 1

Q25. Consider the two lines $L_1 : \frac{x-1}{2} = \frac{y}{3} = \frac{z+1}{4}$ and L_2 passing through $(1, 0, -1)$ with direction ratios $(4, 6, 8)$. The two lines are

(A) coincident (the same line)

(B) parallel but distinct

(C) intersecting at exactly one point

(D) skew

Q26. The shortest distance between the two parallel lines $\vec{r} = \hat{i} + \hat{j} + t(\hat{i} + \hat{j} + \hat{k})$ and $\vec{r} = 2\hat{i} + 2\hat{j} + s(\hat{i} + \hat{j} + \hat{k})$ is

(A) $\sqrt{2}$

(B) $\sqrt{\frac{2}{3}}$

(C) $\frac{1}{\sqrt{3}}$

(D) $\sqrt{3}$

Q27. The probability that a student passes Mathematics is 0.7 and that he passes Physics is 0.6. If the two events are independent, the probability that he passes both subjects is

(A) 1.3

(B) 0.13

(C) 0.42

(D) 0.88

Q28. A random variable X takes the values $0, 1, 2$ with probabilities $\frac{1}{4}, \frac{1}{2}, \frac{1}{4}$ respectively. The variance of X is

(A) 1

(B) $\frac{3}{4}$

(C) $\frac{1}{4}$

(D) $\frac{1}{2}$

Q29. The standard deviation of the five observations $2, 4, 6, 8, 10$ is

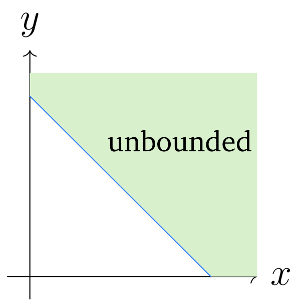
(A) $2\sqrt{2}$

(B) 4

(C) $\sqrt{2}$

(D) 8

Q30. In a linear programming problem, the feasible region of a maximisation problem is shown below. The objective function may fail to attain a maximum value when the feasible region is



(A) bounded and convex

(B) bounded but non-convex

(C) unbounded

(D) a single point

Detailed Solutions

Q1.

Solution

Concept — Intersection of intervals: The intersection $A \cap B$ contains exactly those real numbers that belong to both sets. We take the overlap of the two intervals, keeping each endpoint only if it lies in both sets.

Step 1 — Write the sets as intervals: $A = [-2, 3)$ and $B = (0, 5]$.

Step 2 — Overlap the left endpoints: The larger lower bound is 0. Since $0 \notin A \cap B$ is decided by B, and $0 \notin B$ (as B is open at 0), the left end is open: $(0, \ldots$.

Step 3 — Overlap the right endpoints: The smaller upper bound is 3. Since $3 \notin A$ (as A is open at 3), the right end is open: $\ldots, 3)$.

Step 4 — Combine: Hence $A \cap B = (0, 3)$.

Why other options are wrong:

- Option A: $[-2, 5]$ is the union span, not the intersection.
- Option C: $[0, 3)$ wrongly includes 0, which is excluded by B.
- Option D: $(-2, 5]$ ignores the overlap entirely.

Final Answer: $A \cap B = (0, 3) \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q1

Q2.

Solution

Concept — Composite function: $(f \circ g)(x) = f(g(x))$ means first apply g, then feed the result into f.

Step 1 — Evaluate the inner function: $g(2) = 2^2 - 1$.

Step 2 — Simplify: $g(2) = 4 - 1 = 3$.

Step 3 — Apply the outer function: $f(3) = 2(3) + 3$.

Step 4 — Simplify: $f(3) = 6 + 3 = 9$.

Why other options are wrong:

- Option B: 11 comes from computing $(g \circ f)(2)$ incorrectly.
- Option C: 7 is $f(2)$, forgetting to apply g first.

- Option D: 13 uses $g(2) = 5$, an arithmetic slip.

Final Answer: $(f \circ g)(2) = 9 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q2

Q3.

Solution

Concept — Height and distance: Let the tower height be h and let B be the foot of the tower. Using the tangent of each angle of elevation gives two equations in h and the horizontal distances.

Step 1 — Set up the near triangle: Let BC (foot of tower to the second point) $= d$. From the 60° point, $\tan 60^\circ = \dfrac{h}{d}$, so $h = d\sqrt{3}$.

Step 2 — Set up the far triangle: From the first point (which is 20 m further), $\tan 30^\circ = \dfrac{h}{d+20}$, so $\dfrac{1}{\sqrt{3}} = \dfrac{h}{d+20}$, giving $h\sqrt{3} = d + 20$.

Step 3 — Substitute $d = \dfrac{h}{\sqrt{3}}$: $h\sqrt{3} = \dfrac{h}{\sqrt{3}} + 20$.

Step 4 — Clear the fraction (multiply by $\sqrt{3}$): $3h = h + 20\sqrt{3}$.

Step 5 — Solve for h: $2h = 20\sqrt{3} \Rightarrow h = 10\sqrt{3}$ m.

Why other options are wrong:

- Option A: 10 m drops the $\sqrt{3}$ factor.
- Option B: $10\sqrt{2}$ uses 45° data, not the given angles.
- Option D: $20\sqrt{3}$ forgets to divide by 2.

Final Answer: $h = 10\sqrt{3}\ \text{m} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q3

Q4.

Solution

Concept — General solution of $\tan\theta = \tan\alpha$**:** If $\tan\theta = \tan\alpha$, then $\theta = n\pi + \alpha$ for $n \in \mathbb{Z}$. The tangent function has period π, so the solutions repeat every π.

Step 1 — Identify the reference angle: $\tan\theta = \sqrt{3} = \tan\frac{\pi}{3}$, so $\alpha = \frac{\pi}{3}$.

Step 2 — Apply the general formula: $\theta = n\pi + \frac{\pi}{3}$, $n \in \mathbb{Z}$.

Why other options are wrong:

- Option A: $\frac{\pi}{6}$ has $\tan = \frac{1}{\sqrt{3}}$, not $\sqrt{3}$.
- Option B: the $2n\pi\pm$ form is for cosine, not tangent.
- Option C: the sign of the reference angle is wrong ($-\frac{\pi}{3}$ gives $\tan = -\sqrt{3}$).

Final Answer: $\theta = n\pi + \frac{\pi}{3} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q4

Q5.

Solution

Concept — Polar form of a complex number: For $z = a + ib$, the modulus is $r = \sqrt{a^2 + b^2}$ and the argument θ satisfies $\cos\theta = \frac{a}{r}$, $\sin\theta = \frac{b}{r}$. Then $z = r(\cos\theta + i\sin\theta)$.

Step 1 — Compute the modulus: Here $a = 1,\ b = 1$, so $r = \sqrt{1^2 + 1^2} = \sqrt{2}$.

Step 2 — Find the argument: $\cos\theta = \frac{1}{\sqrt{2}}$ and $\sin\theta = \frac{1}{\sqrt{2}}$, both positive (first quadrant), so $\theta = \frac{\pi}{4}$.

Step 3 — Write the polar form: $z = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$.

Why other options are wrong:

- Option A: modulus is $\sqrt{2}$, not 2.
- Option C: $\frac{\pi}{3}$ is the wrong argument.
- Option D: $\frac{\pi}{6}$ is the wrong argument.

Final Answer: $z = \sqrt{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right) \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q5

Q6.

Solution

Concept — Selection with category restrictions: When a committee needs a fixed number from each group, multiply the number of ways of choosing from each group (combinations, since order does not matter).

Step 1 — Choose 2 men from 5: $\binom{5}{2} = \frac{5 \times 4}{2} = 10.$

Step 2 — Choose 2 women from 4: $\binom{4}{2} = \frac{4 \times 3}{2} = 6.$

Step 3 — Multiply the independent choices: Total $= 10 \times 6 = 60.$

Why other options are wrong:

- Option B: 40 comes from $\binom{5}{2} \times 4$, treating women as a single choice.
- Option C: 120 double counts by treating selections as ordered.
- Option D: 36 uses $\binom{4}{2} \times \binom{4}{2}$, the wrong group size.

Final Answer: 60 ways $\Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q6

Q7.

Solution

Concept — Greatest binomial coefficient: In $(1+x)^n$ the binomial coefficients are $\binom{n}{0}, \binom{n}{1}, \ldots, \binom{n}{n}$. For even n the greatest is the middle one, $\binom{n}{n/2}$.

Step 1 — Identify n: Here $n = 8$ (even), so the middle coefficient is $\binom{8}{4}$.

Step 2 — Evaluate $\binom{8}{4}$: $\binom{8}{4} = \frac{8 \times 7 \times 6 \times 5}{4 \times 3 \times 2 \times 1} = \frac{1680}{24} = 70.$

Why other options are wrong:

- Option A: $28 = \binom{8}{2}$, not the middle term.
- Option B: $56 = \binom{8}{3}$, not the maximum.
- Option D: 40 is not a binomial coefficient of $(1+x)^8$.

Final Answer: Greatest coefficient $= 70 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q7

Q8.

Solution

Concept — Sum of an infinite GP: If $|r| < 1$, the sum to infinity of a geometric progression with first term a and common ratio r is $S_\infty = \frac{a}{1-r}$.

Step 1 — Identify a and r: First term $a = 6$. The ratio $r = \frac{4}{6} = \frac{2}{3}$ (and $\frac{8/3}{4} = \frac{2}{3}$ confirms it).

Step 2 — Check convergence: $|r| = \frac{2}{3} < 1$, so the infinite sum exists.

Step 3 — Apply the formula: $S_\infty = \frac{6}{1-\frac{2}{3}} = \frac{6}{\frac{1}{3}}$.

Step 4 — Simplify: $S_\infty = 6 \times 3 = 18$.

Why other options are wrong:

- Option A: 9 uses $r = \frac{1}{3}$ by mistake.
- Option B: 12 uses $r = \frac{1}{2}$.
- Option C: 16 comes from an arithmetic slip in $1 - r$.

Final Answer: $S_\infty = 18 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q8

Q9.

Solution

Concept — Foot of the perpendicular: The foot F on the line $ax + by + c = 0$ from $P(x_1, y_1)$ satisfies $\frac{x - x_1}{a} = \frac{y - y_1}{b} = -\frac{ax_1 + by_1 + c}{a^2 + b^2}$.

Step 1 — Identify the data: Line $x + y - 1 = 0$ gives $a = 1, b = 1, c = -1$; point $P(2, 3)$.

Step 2 — Compute the common ratio: $ax_1+by_1+c = 2+3-1 = 4$ and $a^2+b^2 = 2$, so the ratio is $-\frac{4}{2} = -2$.

Step 3 — Find x: $x - 2 = a(-2) = -2$, so $x = 0$.

Step 4 — Find y: $y - 3 = b(-2) = -2$, so $y = 1$.

Step 5 — Verify: $(0, 1)$ satisfies $x + y - 1 = 0$ since $0 + 1 - 1 = 0$. ✓

Why other options are wrong:

- Option B: $(1, 0)$ lies on the line but is not the perpendicular foot from P.
- Option C: $(2, -1)$ does not satisfy the line equation here.
- Option D: $\left(\frac{1}{2}, \frac{1}{2}\right)$ is the midpoint of the axes' intercepts, not the foot.

Final Answer: Foot $= (0, 1) \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q9

Q10.

Solution

Concept — Tangency condition: The line $y = mx + c$ touches the circle $x^2 + y^2 = r^2$ when the perpendicular distance from the centre to the line equals the radius r.

Step 1 — Distance from centre to line: The centre is $(0, 0)$ and the line is $mx - y + c = 0$. Distance $= \frac{|c|}{\sqrt{m^2 + 1}}$.

Step 2 — Set distance = radius: Here $r = 3$, so $\frac{|c|}{\sqrt{m^2 + 1}} = 3$.

Step 3 — Square both sides: $c^2 = 9(m^2 + 1)$, i.e. $c^2 = 9(1 + m^2)$.

Why other options are wrong:

- Option A: $c = 3\sqrt{1 + m^2}$ ignores the case $c < 0$ (it should be $|c|$ / a squared relation).
- Option C: $c^2 = 9m^2$ drops the $+1$ term.
- Option D: $c = 9(1 + m^2)$ forgets to take the square root of r^2.

Final Answer: $c^2 = 9(1 + m^2) \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q10

Q11.

Solution

Concept — Standard exponential limit: A standard result is $\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$ for $a > 0$.

Step 1 — Match the form: Here $a = 5$, so the limit is $\lim_{x \to 0} \frac{5^x - 1}{x}$.

Step 2 — Apply the standard limit: The value is $\log_e 5$.

Why other options are wrong:

- Option A: 5 would be the base, not the natural log of the base.
- Option B: 1 is the limit of $\dfrac{e^x - 1}{x}$ (base e), not base 5.
- Option D: 0 ignores the indeterminate $\frac{0}{0}$ structure.

Final Answer: Limit $= \log_e 5 \Rightarrow \boxed{\text{C}}$

Answer: (C) **Go Back to Q11**

Q12.

Solution

Concept — Logarithmic differentiation: When the variable appears in both the base and the exponent, take the natural log of both sides and differentiate implicitly.

Step 1 — Take logs: $y = x^x \Rightarrow \log_e y = x \log_e x$.

Step 2 — Differentiate both sides w.r.t. x: $\dfrac{1}{y}\dfrac{dy}{dx} = 1 \cdot \log_e x + x \cdot \dfrac{1}{x}$ (product rule on the right).

Step 3 — Simplify the right side: $\dfrac{1}{y}\dfrac{dy}{dx} = \log_e x + 1$.

Step 4 — Solve for $\dfrac{dy}{dx}$: $\dfrac{dy}{dx} = y(1 + \log_e x) = x^x(1 + \log_e x)$.

Why other options are wrong:

- Option A: $x\,x^{x-1}$ treats the exponent as a constant (power rule), which is invalid here.
- Option B: $x^x \log_e x$ misses the $+1$ from differentiating $x \log x$.
- Option C: x^x would be the derivative only if the bracket were 1.

Final Answer: $\dfrac{dy}{dx} = x^x(1 + \log_e x) \Rightarrow \boxed{\text{D}}$

Answer: (D) **Go Back to Q12**

Q13.

Solution

Concept — Scalar multiple of a row: If every element of one row (or column) of a determinant is multiplied by a constant k, the value of the whole determinant is multiplied by k.

Step 1 — Identify the factor: Here a single row is multiplied by 4, so $k = 4$.

Step 2 — Apply the property: New determinant $= 4\Delta$.

Why other options are wrong:

- Option B: Δ would be correct only if no row were scaled.
- Option C: 12Δ wrongly multiplies by 4×3 (number of entries).
- Option D: $64\Delta = 4^3\Delta$ applies as if every row were scaled, which is not the case.

Final Answer: New value $= 4\Delta \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q13

Q14.

Solution

Concept — Singular matrix: A square matrix is singular when its determinant is zero, so it has no inverse.

Step 1 — Compute the determinant: For $A = \begin{pmatrix} x & 2 \\ 3 & 4 \end{pmatrix}$, $\det A = (x)(4) - (2)(3) = 4x - 6$.

Step 2 — Set the determinant to zero: $4x - 6 = 0$.

Step 3 — Solve for x: $4x = 6 \Rightarrow x = \frac{6}{4} = \frac{3}{2}$.

Why other options are wrong:

- Option A: $\frac{2}{3}$ inverts the fraction.
- Option C: 6 comes from solving $x = 6$ without dividing by 4.
- Option D: $-\frac{3}{2}$ has the wrong sign.

Final Answer: $x = \frac{3}{2} \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q14

Q15.

Solution

Concept — Related rates (sliding ladder): Let x be the distance of the foot from the wall and y the height of the top. Since the ladder length is constant, $x^2 + y^2 = 25$. Differentiate with respect to time.

Step 1 — Differentiate the constraint: $2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 0$, so $x\frac{dx}{dt} + y\frac{dy}{dt} = 0$.

Step 2 — Find y when $x = 3$: $y = \sqrt{25-9} = \sqrt{16} = 4$ m.

Step 3 — Substitute known values: $\frac{dx}{dt} = 2$ m/s, so $3(2) + 4\frac{dy}{dt} = 0$.

Step 4 — Solve for $\frac{dy}{dt}$: $4\frac{dy}{dt} = -6 \Rightarrow \frac{dy}{dt} = -\frac{3}{2}$ m/s.

Step 5 — Interpret: The negative sign shows the top slides down; its speed is $\frac{3}{2}$ m/s.

Why other options are wrong:

- Option A: $\frac{8}{3}$ swaps x and y in the substitution.
- Option B: $\frac{2}{3}$ inverts the ratio.
- Option D: $\frac{4}{3}$ uses $y = 3, x = 4$ incorrectly.

Final Answer: Top slides down at $\frac{3}{2}$ m/s $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q15

Q16.

Solution

Concept — Increasing function via the derivative: A differentiable function is strictly increasing where its first derivative is positive.

Step 1 — Differentiate: $f(x) = x^2 - 4x + 5 \Rightarrow f'(x) = 2x - 4$.

Step 2 — Solve $f'(x) > 0$: $2x - 4 > 0 \Rightarrow x > 2$.

Step 3 — Write the interval: f is strictly increasing on $(2, \infty)$.

Why other options are wrong:

- Option A: $(-\infty, 2)$ is where $f'(x) < 0$ (decreasing).

- Option B: $(-\infty, 0)$ is a subset of the decreasing region.
- Option C: $(0, 2)$ also lies in the decreasing region.

Final Answer: Increasing on $(2, \infty) \Rightarrow \boxed{D}$

Answer: (D) Go Back to Q16

Q17.

Solution

Concept — Standard integral: $\int \frac{dx}{x^2 + a^2} = \frac{1}{a} \tan^{-1} \frac{x}{a} + C$.

Step 1 — Identify a: Here $a^2 = 16$, so $a = 4$.

Step 2 — Apply the formula: $\int \frac{dx}{x^2 + 16} = \frac{1}{4} \tan^{-1} \frac{x}{4} + C$.

Why other options are wrong:

- Option B: $\frac{1}{16}$ uses a^2 instead of a in front.
- Option C: 4 inverts the coefficient $\frac{1}{a}$.
- Option D: omits the $\frac{1}{a}$ factor entirely.

Final Answer: $\frac{1}{4} \tan^{-1} \frac{x}{4} + C \Rightarrow \boxed{A}$

Answer: (A) Go Back to Q17

Q18.

Solution

Concept — Integration by parts: $\int u\,dv = uv - \int v\,du$. Take $u = \log_e x$ and $dv = dx$.

Step 1 — Choose parts: $u = \log_e x \Rightarrow du = \frac{1}{x} dx$; $dv = dx \Rightarrow v = x$.

Step 2 — Apply the formula: $\int \log_e x\,dx = x \log_e x - \int x \cdot \frac{1}{x}\,dx$.

Step 3 — Simplify the remaining integral: $\int x \cdot \frac{1}{x}\,dx = \int 1\,dx = x$.

Step 4 — Combine: $\int \log_e x\,dx = x \log_e x - x + C$.

Why other options are wrong:

- Option A: $\frac{1}{x}$ is the derivative of $\log x$, not the integral.
- Option C: $+x$ has the wrong sign on the second term.
- Option D: $\log_e x - x$ omits the factor x on the first term.

Final Answer: $x \log_e x - x + C \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q18

Q19.

Solution

Concept — Definite integral via antiderivative: $\int_a^b f(x)\,dx = F(b) - F(a)$ where $F' = f$.

Step 1 — Find the antiderivative: $\int \sin x\,dx = -\cos x$.

Step 2 — Apply the limits: $\int_0^{\pi} \sin x\,dx = \big[-\cos x\big]_0^{\pi} = -\cos\pi - (-\cos 0)$.

Step 3 — Substitute values: $-(-1) - (-(1)) = 1 + 1 = 2$.

Why other options are wrong:

- Option A: 0 would arise from a symmetric odd integrand, which $\sin x$ is not over $[0, \pi]$.
- Option B: 1 uses limits $[0, \frac{\pi}{2}]$.
- Option D: -2 has the wrong overall sign.

Final Answer: $\int_0^{\pi} \sin x\,dx = 2 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q19

Q20.

Solution

Concept — Area of the triangle cut off by a line and the axes: The line meets the axes at its intercepts, forming a right triangle with the origin. Area $= \frac{1}{2} \times (x\text{-intercept}) \times (y\text{-intercept})$.

Step 1 — Find the x-intercept: Put $y = 0$ in $2x + 3y = 6$: $2x = 6 \Rightarrow x = 3$.

Step 2 — Find the y-intercept: Put $x = 0$: $3y = 6 \Rightarrow y = 2$.

Step 3 — Compute the area: Area $= \frac{1}{2} \times 3 \times 2 = 3$ sq units.

Why other options are wrong:

- Option A: 6 is the product of the intercepts without the factor $\frac{1}{2}$.
- Option B: 9 uses wrong intercepts.
- Option C: $\frac{9}{2}$ uses intercepts 3 and 3 incorrectly.

Final Answer: Area $= 3$ sq units $\Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q20

Q21.

Solution

Concept — Variable separable equations: Rearrange so each variable is on its own side, then integrate both sides.

Step 1 — Separate the variables: $\frac{dy}{dx} = \frac{y}{x} \Rightarrow \frac{dy}{y} = \frac{dx}{x}$.

Step 2 — Integrate both sides: $\int \frac{dy}{y} = \int \frac{dx}{x} \Rightarrow \log_e y = \log_e x + \log_e C$.

Step 3 — Combine the logs: $\log_e y = \log_e(Cx)$.

Step 4 — Exponentiate: $y = Cx$.

Why other options are wrong:

- Option B: $y = C + x$ would solve $\frac{dy}{dx} = 1$.
- Option C: $xy = C$ solves $\frac{dy}{dx} = -\frac{y}{x}$.
- Option D: $y^2 = Cx$ does not satisfy the given equation.

Final Answer: $y = Cx \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q21

Q22.

Solution

Concept — Exponential growth: If $\frac{dN}{dt} \propto N$, then $N = N_0 e^{kt}$. Doubling in a fixed time means each such interval multiplies the population by 2.

Step 1 — Use the doubling time: The population doubles every 3 hours.

Step 2 — Reach 8 times: Since $8 = 2^3$, three doublings are needed.

Step 3 — Compute the time: Three doublings $\times\, 3$ hours each $= 9$ hours.

Why other options are wrong:

- Option A: 6 hours gives only $2^2 = 4$ times.
- Option C: 12 hours gives $2^4 = 16$ times.
- Option D: 24 hours far overshoots.

Final Answer: 9 hours $\Rightarrow \boxed{\text{B}}$

Answer: (B) **Go Back to Q22**

Q23.

Solution

Concept — Perpendicular vectors: Two vectors are perpendicular when their dot product is zero.

Step 1 — Form the dot product: $\vec{a} \cdot \vec{b} = (2)(1) + (\lambda)(-2) + (1)(3)$.

Step 2 — Simplify: $\vec{a} \cdot \vec{b} = 2 - 2\lambda + 3 = 5 - 2\lambda$.

Step 3 — Set equal to zero: $5 - 2\lambda = 0$.

Step 4 — Solve: $2\lambda = 5 \Rightarrow \lambda = \frac{5}{2}$.

Why other options are wrong:

- Option A: $\frac{1}{2}$ drops the constant terms.
- Option B: $-\frac{1}{2}$ has the wrong sign and value.
- Option D: 5 forgets to divide by 2.

Final Answer: $\lambda = \frac{5}{2} \Rightarrow \boxed{\text{C}}$

Answer: (C) **Go Back to Q23**

Q24.

Solution

Concept — Coplanar vectors: Three vectors are coplanar when their scalar triple product $[\vec{a}\,\vec{b}\,\vec{c}] = 0$, i.e. the determinant of their components vanishes.

Step 1 — Write the determinant: $\begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ p & 0 & 1 \end{vmatrix} = 0.$

Step 2 — Expand along the first row: $1\,(1\cdot 1 - 1\cdot 0) - 1\,(0\cdot 1 - 1\cdot p) + 0.$

Step 3 — Simplify: $1(1) - 1(-p) = 1 + p.$

Step 4 — Set equal to zero: $1 + p = 0 \Rightarrow p = -1.$

Why other options are wrong:

- Option A: 0 gives determinant $1 \neq 0$, so not coplanar.
- Option C: 2 gives determinant $3 \neq 0$.
- Option D: 1 gives determinant $2 \neq 0$.

Final Answer: $p = -1 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q24

Q25.

Solution

Concept — Relation between two lines in space: Compare direction ratios and a common point. Proportional direction ratios mean the lines are parallel; if they also share a point, they coincide.

Step 1 — Compare direction ratios: L_1 has $(2, 3, 4)$ and L_2 has $(4, 6, 8) = 2(2, 3, 4)$, so the direction ratios are proportional — the lines are parallel.

Step 2 — Check a common point: L_1 passes through $(1, 0, -1)$ (set each fraction to 0). L_2 also passes through $(1, 0, -1)$.

Step 3 — Conclude: Same direction and a shared point means the two lines are coincident (the same line).

Why other options are wrong:

- Option B: parallel but distinct would require no common point.
- Option C: intersecting at one point needs non-parallel directions.

- Option D: skew lines are neither parallel nor intersecting.

Final Answer: The lines are coincident $\Rightarrow \boxed{A}$

Answer: (A) Go Back to Q25

Q26.

Solution

Concept — Distance between parallel lines: For $\vec{r} = \vec{a}_1 + t\vec{b}$ and $\vec{r} = \vec{a}_2 + s\vec{b}$, the shortest distance is $d = \dfrac{|(\vec{a}_2 - \vec{a}_1) \times \vec{b}|}{|\vec{b}|}$.

Step 1 — Find $\vec{a}_2 - \vec{a}_1$: $\vec{a}_1 = \hat{i} + \hat{j}$, $\vec{a}_2 = 2\hat{i} + 2\hat{j}$, so $\vec{a}_2 - \vec{a}_1 = \hat{i} + \hat{j}$.

Step 2 — Compute the cross product with $\vec{b} = \hat{i} + \hat{j} + \hat{k}$: $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{vmatrix} = \hat{i}(1 \cdot 1 - 0 \cdot 1) - \hat{j}(1 \cdot 1 - 0 \cdot 1) + \hat{k}(1 \cdot 1 - 1 \cdot 1)$.

Step 3 — Simplify the cross product: $= \hat{i}(1) - \hat{j}(1) + \hat{k}(0) = \hat{i} - \hat{j}$.

Step 4 — Magnitudes: $|\hat{i} - \hat{j}| = \sqrt{1^2 + (-1)^2} = \sqrt{2}$ and $|\vec{b}| = \sqrt{1^2 + 1^2 + 1^2} = \sqrt{3}$.

Step 5 — Compute the distance: $d = \dfrac{\sqrt{2}}{\sqrt{3}} = \sqrt{\dfrac{2}{3}}$.

Why other options are wrong:

- Option A: $\sqrt{2}$ forgets to divide by $|\vec{b}| = \sqrt{3}$.
- Option C: $\dfrac{1}{\sqrt{3}}$ uses 1 instead of $\sqrt{2}$ in the numerator.
- Option D: $\sqrt{3}$ uses $|\vec{b}|$ as the numerator by mistake.

Final Answer: $d = \sqrt{\dfrac{2}{3}} \Rightarrow \boxed{B}$

Answer: (B) Go Back to Q26

Q27.

Solution

Concept — Multiplication theorem for independent events: If A and B are independent, $P(A \cap B) = P(A) \cdot P(B)$.

Step 1 — Identify the probabilities: $P(\text{Maths}) = 0.7$, $P(\text{Physics}) = 0.6$.

Step 2 — Multiply (independence): $P(\text{both}) = 0.7 \times 0.6$.

Step 3 — Simplify: $0.7 \times 0.6 = 0.42$.

Why other options are wrong:

- Option A: 1.3 adds the probabilities (and exceeds 1, impossible).
- Option B: 0.13 is a decimal-placement error.
- Option D: 0.88 uses the addition rule $P(A) + P(B) - P(A)P(B)$ (probability of passing at least one).

Final Answer: $P(\text{both}) = 0.42 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q27

Q28.

Solution

Concept — Variance of a discrete random variable: $\text{Var}(X) = E(X^2) - [E(X)]^2$, where $E(X) = \sum x_i p_i$ and $E(X^2) = \sum x_i^2 p_i$.

Step 1 — Compute $E(X)$: $E(X) = 0 \cdot \frac{1}{4} + 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} = 0 + \frac{1}{2} + \frac{1}{2} = 1$.

Step 2 — Compute $E(X^2)$: $E(X^2) = 0^2 \cdot \frac{1}{4} + 1^2 \cdot \frac{1}{2} + 2^2 \cdot \frac{1}{4} = 0 + \frac{1}{2} + 1 = \frac{3}{2}$.

Step 3 — Apply the variance formula: $\text{Var}(X) = \frac{3}{2} - (1)^2 = \frac{3}{2} - 1 = \frac{1}{2}$.

Why other options are wrong:

- Option A: 1 is the mean $E(X)$, not the variance.
- Option B: $\frac{3}{4}$ comes from an arithmetic slip in $E(X^2)$.
- Option C: $\frac{1}{4}$ omits one of the probability contributions.

Final Answer: $\text{Var}(X) = \frac{1}{2} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q28

Q29.

Solution

Concept — Standard deviation: $\sigma = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n}}$, where $\bar{x}$ is the mean of the data.

Step 1 — Find the mean: $\bar{x} = \frac{2+4+6+8+10}{5} = \frac{30}{5} = 6.$

Step 2 — Compute the deviations and their squares: $(2-6)^2 = 16$, $(4-6)^2 = 4$, $(6-6)^2 = 0$, $(8-6)^2 = 4$, $(10-6)^2 = 16$.

Step 3 — Sum the squared deviations: $16 + 4 + 0 + 4 + 16 = 40$.

Step 4 — Divide by n and take the root: $\sigma = \sqrt{\frac{40}{5}} = \sqrt{8} = 2\sqrt{2}$.

Why other options are wrong:

- Option B: 4 is the mean deviation magnitude, not the SD.
- Option C: $\sqrt{2}$ divides by n^2 by mistake.
- Option D: 8 is the variance σ^2, not the standard deviation.

Final Answer: $\sigma = 2\sqrt{2} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q29

Q30.

Solution

Concept — Existence of optimum in LPP: If the feasible region is bounded, an optimal value of the objective function is always attained at a corner point. If the feasible region is unbounded, the objective function may not attain a maximum (it can increase without limit).

Step 1 — Recall the boundedness theorem: A bounded feasible region always yields both a maximum and a minimum at the vertices.

Step 2 — Consider the unbounded case: When the region extends infinitely (as shaded in the figure), the objective function can keep increasing, so a maximum may fail to exist.

Step 3 — Match to the question: The maximisation may fail precisely when the feasible region is unbounded.

Why other options are wrong:

- Option A: a bounded, convex region always attains the maximum.
- Option B: feasible regions of linear constraints are always convex, so this case does not arise.
- Option D: a single point trivially gives that point as the optimum.

Final Answer: The maximum may fail to exist when the region is unbounded $\Rightarrow$ $\boxed{C}$

Answer: (C) Go Back to Q30

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	B	2	A	3	C	4	D	5	B
6	A	7	C	8	D	9	A	10	B
11	C	12	D	13	A	14	B	15	C
16	D	17	A	18	B	19	C	20	D
21	A	22	B	23	C	24	D	25	A
26	B	27	C	28	D	29	A	30	C