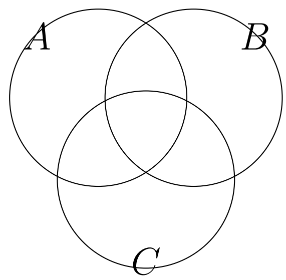
AIIMS Paramedical Mathematics
Sample Paper – 6

Duration: 30 Minutes **Maximum Marks: 30**

Instructions

- This paper contains **30** Multiple Choice Questions (Single Correct Answer), modelled on the Mathematics portion of **AIIMS Paramedical** entrance.
- Each correct answer carries **+1 mark**; each incorrect answer carries a penalty of $-\frac{1}{3}$ mark; an unattempted question carries **0** mark.
- Only **one** option is correct. Choose carefully.
- Syllabus level: **Class 11 & 12 NCERT Mathematics**
- Use of mobile phones, calculators, or electronic gadgets is strictly prohibited.

Q1. For three sets $A = \{x : x \in \mathbb{N},\ x \leq 6\}$, $B = \{x : x \in \mathbb{N},\ 3 \leq x \leq 8\}$ and $C = \{x : x \in \mathbb{N},\ 5 \leq x \leq 9\}$, the number of elements in $A \cup B \cup C$ is



(A) 9

(B) 8

(C) 7

(D) 10

Q2. If $f : \mathbb{R} \to \mathbb{R}$ is defined by $f(x) = \dfrac{3x - 2}{5}$, then $f^{-1}(x)$ is

(A) $\dfrac{5x + 3}{2}$

(B) $\dfrac{5x+2}{3}$

(C) $\dfrac{3x+5}{2}$

(D) $\dfrac{2x+5}{3}$

Q3. In a triangle ABC, if $a = 8$, $b = 7$ and $\cos C = \dfrac{11}{14}$, then the length of side c is

(A) 7

(B) 6

(C) 5

(D) 4

Q4. The value of $\tan^{-1}\dfrac{1}{2} + \tan^{-1}\dfrac{1}{3}$ is

(A) $\dfrac{\pi}{3}$

(B) $\dfrac{\pi}{6}$

(C) $\dfrac{\pi}{2}$

(D) $\dfrac{\pi}{4}$

Q5. Using De Moivre's theorem, the value of $\left(\cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}\right)^6$ is

(A) 1

(B) i

(C) $-i$

(D) -1

Q6. The number of diagonals of a convex polygon having 12 sides is

(A) 66

(B) 54

(C) 48

(D) 44

Q7. The sum of all the coefficients in the expansion of $(2x - 3y)^5$ is

(A) -1

(B) 1

(C) 32

(D) -32

Q8. If the 3rd and 7th terms of a harmonic progression are $\frac{1}{5}$ and $\frac{1}{9}$ respectively, then its 10th term is

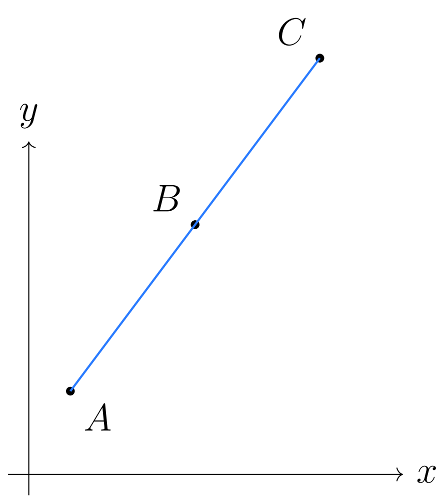
(A) $\frac{1}{10}$

(B) $\frac{1}{11}$

(C) $\frac{1}{12}$

(D) $\frac{1}{13}$

Q9. The points $A(1, 2)$, $B(4, 6)$ and $C(7, k)$ are collinear. The value of k is



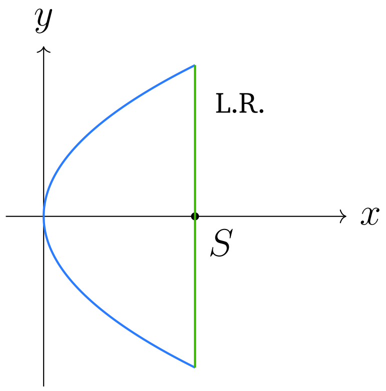
(A) 8

(B) 9

(C) 11

(D) 10

Q10. The length of the latus rectum of the parabola $y^2 = 16x$ is



(A) 8

(B) 16

(C) 4

(D) 32

Q11. The value of $\lim_{x\to 2^+} [x]$, where $[\cdot]$ denotes the greatest integer function, is

(A) 2

(B) 1

(C) 3

(D) does not exist

Q12. If $x = a\cos\theta$ and $y = b\sin\theta$, then $\dfrac{dy}{dx}$ is

(A) $\dfrac{b}{a}\cot\theta$

(B) $-\dfrac{a}{b}\cot\theta$

(C) $-\dfrac{b}{a}\cot\theta$

(D) $\dfrac{a}{b}\tan\theta$

Q13. If $A = \begin{pmatrix} 2 & 4 \\ 6 & 8 \end{pmatrix}$ is written as $P + Q$, where P is symmetric and Q is skew-symmetric, then Q equals

(A) $\begin{pmatrix} 0 & 5 \\ -5 & 0 \end{pmatrix}$

(B) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

(C) $\begin{pmatrix} 2 & 5 \\ 5 & 8 \end{pmatrix}$

(D) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$

Q14. The area of the triangle with vertices $(0, 0)$, $(4, 0)$ and $(0, 4)$, found using a determinant, is

(A) 12

(B) 8

(C) 16

(D) 6

Q15. The slope of the tangent to the curve $y = x^3 - 3x + 5$ at the point $(2, 7)$ is

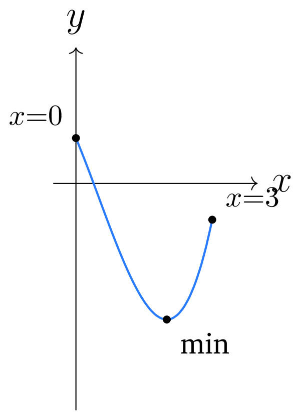
(A) 9

(B) 6

(C) 12

(D) 3

Q16. The absolute maximum value of $f(x) = 2x^3 - 3x^2 - 12x + 5$ on the interval $[0, 3]$ is



(A) -15

(B) 0

(C) 5

(D) -4

Q17. Using a rationalising substitution, $\int \frac{1}{1+\sqrt{x}}\,dx$ equals (where C is the constant of integration)

(A) $2\sqrt{x} + \log|1+\sqrt{x}| + C$

(B) $\sqrt{x} - \log|1+\sqrt{x}| + C$

(C) $2\log|1+\sqrt{x}| + C$

(D) $2\sqrt{x} - 2\log|1+\sqrt{x}| + C$

Q18. By partial fractions, $\int \frac{1}{(x-1)(x+2)}\,dx$ equals (where C is the constant of integration)

(A) $\frac{1}{3}\log\left|\frac{x+2}{x-1}\right| + C$

(B) $\frac{1}{3}\log\left|\frac{x-1}{x+2}\right| + C$

(C) $\log\left|\frac{x-1}{x+2}\right| + C$

(D) $3\log\left|\frac{x-1}{x+2}\right| + C$

Q19. Using the property $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$, the value of $\int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\,dx$ is

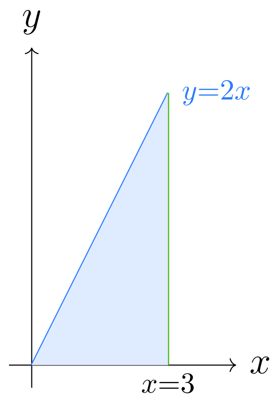
(A) $\frac{\pi}{4}$

(B) $\frac{\pi}{2}$

(C) $\frac{\pi}{3}$

(D) 1

Q20. The area of the triangle bounded by the line $y = 2x$, the line $x = 3$ and the x-axis, computed by integration, is



(A) 6

(B) 18

(C) 9

(D) 12

Q21. The order and degree of the differential equation $\left(\frac{d^2y}{dx^2}\right)^3 + \left(\frac{dy}{dx}\right)^2 + \sin\left(\frac{dy}{dx}\right) + 1 = 0$ are respectively

(A) 3, 2

(B) 2, 2

(C) 3, 3

(D) 2, 3

Q22. The solution of the homogeneous differential equation $\frac{dy}{dx} = \frac{x+y}{x}$ is (where C is an arbitrary constant)

(A) $y = x\log|x| + Cx^2$

(B) $y = x\log|x| + Cx$

(C) $y = \log|x| + C$

(D) $y = x^2\log|x| + C$

Q23. The angle between the vectors $\vec{a} = \hat{i} + \hat{j}$ and $\vec{b} = \hat{j} + \hat{k}$ is

(A) $\frac{\pi}{3}$

(B) $\frac{\pi}{4}$

(C) $\frac{\pi}{6}$

(D) $\frac{\pi}{2}$

Q24. The area of the triangle whose two adjacent sides are given by $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ is

(A) $\frac{\sqrt{35}}{4}$

(B) $\sqrt{35}$

(C) $\frac{\sqrt{35}}{2}$

(D) $\frac{3\sqrt{5}}{2}$

Q25. The foot of the perpendicular from the point $(1, 2, 3)$ to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{6}$ has x-coordinate

(A) $\frac{26}{7}$

(B) $\frac{13}{49}$

(C) 1

(D) $\frac{52}{49}$

Q26. The equation of the plane passing through the three points $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$ is

(A) $x + y + z = 0$

(B) $x + y + z = 1$

(C) $x + y + z = 3$

(D) $2x + 2y + 2z = 1$

Q27. A card is drawn at random from a well-shuffled pack of 52 cards. The probability that it is either a king or a heart is

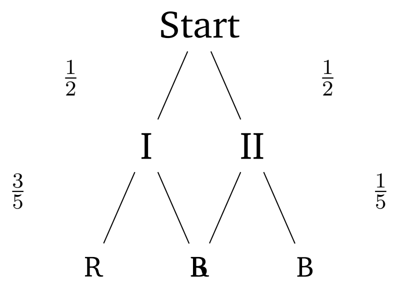
(A) $\frac{17}{52}$

(B) $\frac{1}{13}$

(C) $\frac{1}{4}$

(D) $\frac{4}{13}$

Q28. Bag I contains 3 red and 2 black balls; Bag II contains 1 red and 4 black balls. A bag is chosen at random and a ball is drawn. Using the tree diagram, the probability that the drawn ball is red is



(A) $\frac{3}{5}$

(B) $\frac{1}{5}$

(C) $\frac{2}{5}$

(D) $\frac{4}{5}$

Q29. The mean deviation about the mean for the data $4, 7, 8, 9, 10, 12, 13, 17$ is

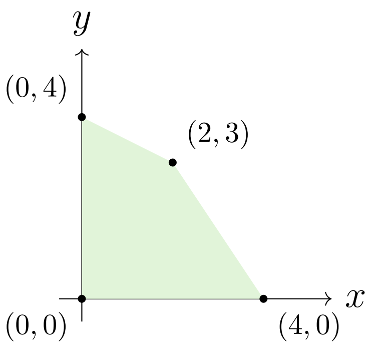
(A) 2.75

(B) 3.0

(C) 3.25

(D) 2.5

Q30. Maximise $Z = 4x + 5y$ subject to the corner points of the feasible region $(0, 0)$, $(4, 0)$, $(2, 3)$ and $(0, 4)$. The maximum value of Z occurs and equals



(A) 23

(B) 20

(C) 16

(D) 21

Detailed Solutions

Q1.

Solution

Concept — Union of sets: List every element of each set, then collect the distinct elements appearing in at least one set.

Step 1 — Write the sets in roster form: $A = \{1, 2, 3, 4, 5, 6\}$.

Step 2 — Next set: $B = \{3, 4, 5, 6, 7, 8\}$.

Step 3 — Third set: $C = \{5, 6, 7, 8, 9\}$.

Step 4 — Take the union: $A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$.

Step 5 — Count the distinct elements: there are 9 elements.

Why other options are wrong:

- 8: drops one of the listed values by miscounting.
- 7: counts only the overlap region.
- 10: wrongly includes 10, which is not in any set.

Final Answer: $|A \cup B \cup C| = 9 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q1

Q2.

Solution

Concept — Inverse function: Put $y = f(x)$, solve for x in terms of y, then swap symbols.

Step 1 — Set $y = f(x)$: $y = \dfrac{3x - 2}{5}$.

Step 2 — Clear the denominator: $5y = 3x - 2$.

Step 3 — Isolate x: $3x = 5y + 2$.

Step 4 — Divide by 3: $x = \dfrac{5y + 2}{3}$.

Step 5 — Replace y by x: $f^{-1}(x) = \dfrac{5x + 2}{3}$.

Why other options are wrong:

- $\dfrac{5x + 3}{2}$ and $\dfrac{3x + 5}{2}$: come from dividing by the wrong coefficient.

- $\frac{2x+5}{3}$: swaps the 2 and the 5.

Final Answer: $f^{-1}(x) = \frac{5x+2}{3} \Rightarrow \boxed{B}$

Answer: (B) Go Back to Q2

Q3.

Solution

Concept — Law of cosines: $c^2 = a^2 + b^2 - 2ab\cos C$.

Step 1 — Substitute the data: $c^2 = 8^2 + 7^2 - 2(8)(7)\cdot\frac{11}{14}$.

Step 2 — Evaluate the squares: $c^2 = 64 + 49 - 2(8)(7)\cdot\frac{11}{14}$.

Step 3 — Simplify the product: $2(8)(7) = 112$, and $112\cdot\frac{11}{14} = 8\cdot 11 = 88$.

Step 4 — Combine: $c^2 = 64 + 49 - 88 = 25$.

Step 5 — Take the square root: $c = 5$.

Why other options are wrong:

- $7, 6, 4$: result from arithmetic slips in the cosine term.

Final Answer: $c = 5 \Rightarrow \boxed{C}$

Answer: (C) Go Back to Q3

Q4.

Solution

Concept — Sum formula: $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{1-xy}$ when $xy < 1$.

Step 1 — Identify x, y: $x = \frac{1}{2}$, $y = \frac{1}{3}$, and $xy = \frac{1}{6} < 1$.

Step 2 — Numerator: $x + y = \frac{1}{2} + \frac{1}{3} = \frac{5}{6}$.

Step 3 — Denominator: $1 - xy = 1 - \frac{1}{6} = \frac{5}{6}$.

Step 4 — Form the ratio: $\frac{5/6}{5/6} = 1$.

Step 5 — Apply arctan: $\tan^{-1}(1) = \frac{\pi}{4}$.

Why other options are wrong:

- $\frac{\pi}{3}, \frac{\pi}{6}, \frac{\pi}{2}$: not equal to $\tan^{-1} 1$.

Final Answer: value $= \frac{\pi}{4} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q4

Q5.

Solution

Concept — De Moivre's theorem: $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$.

Step 1 — Apply with $\theta = \frac{\pi}{6}$, $n = 6$: the value is $\cos\frac{6\pi}{6} + i\sin\frac{6\pi}{6}$.

Step 2 — Simplify the angle: $\frac{6\pi}{6} = \pi$.

Step 3 — Evaluate cosine: $\cos\pi = -1$.

Step 4 — Evaluate sine: $\sin\pi = 0$.

Step 5 — Combine: $-1 + i(0) = -1$.

Why other options are wrong:

- 1: would need $n\theta = 2\pi$.
- $i, -i$: would need $n\theta = \frac{\pi}{2}$ or $\frac{3\pi}{2}$.

Final Answer: value $= -1 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q5

Q6.

Solution

Concept — Diagonals of a polygon: A polygon with n sides has $\frac{n(n-3)}{2}$ diagonals.

Step 1 — Substitute $n = 12$: number $= \frac{12(12-3)}{2}$.

Step 2 — Simplify the bracket: $12 - 3 = 9$.

Step 3 — Multiply: $12 \times 9 = 108$.

Step 4 — Divide by 2**:** $\frac{108}{2} = 54$.

Why other options are wrong:

- $66 = \binom{12}{2}$: counts all line segments, not just diagonals.
- $48, 44$: arithmetic slips.

Final Answer: number of diagonals $= 54 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q6

Q7.

Solution

Concept — Sum of coefficients: Substitute $x = 1$ and $y = 1$ into the expansion to get the sum of all coefficients.

Step 1 — Put $x = y = 1$**:** sum $= (2(1) - 3(1))^5$.

Step 2 — Simplify inside: $2 - 3 = -1$.

Step 3 — Raise to the fifth power: $(-1)^5 = -1$.

Why other options are wrong:

- 1: would be $(-1)^4$ or an even power.
- $32, -32$: come from using only the coefficient 2.

Final Answer: sum of coefficients $= -1 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q7

Q8.

Solution

Concept — Harmonic progression: The reciprocals of an HP form an AP. Work with the reciprocals.

Step 1 — Reciprocate the given terms: the AP has 3rd term 5 and 7th term 9.

Step 2 — Use $a_n = a + (n-1)d$**:** $a + 2d = 5$ and $a + 6d = 9$.

Step 3 — Subtract the equations: $4d = 4$, so $d = 1$.

Step 4 — Find a: $a + 2(1) = 5 \Rightarrow a = 3$.

Step 5 — Tenth AP term: $a_{10} = 3 + 9(1) = 12$.

Step 6 — Reciprocate back: the 10th HP term $= \frac{1}{12}$.

Why other options are wrong:

- $\frac{1}{10}, \frac{1}{11}, \frac{1}{13}$: use a wrong a or d.

Final Answer: 10th term $= \frac{1}{12} \Rightarrow \boxed{C}$

Answer: (C) Go Back to Q8

Q9.

Solution

Concept — Collinearity: Three points are collinear when the slope between any two pairs is equal.

Step 1 — Slope of AB: $\frac{6-2}{4-1} = \frac{4}{3}$.

Step 2 — Slope of BC: $\frac{k-6}{7-4} = \frac{k-6}{3}$.

Step 3 — Equate the slopes: $\frac{k-6}{3} = \frac{4}{3}$.

Step 4 — Solve: $k - 6 = 4$, so $k = 10$.

Why other options are wrong:

- $8, 9, 11$: do not satisfy the slope condition.

Final Answer: $k = 10 \Rightarrow \boxed{D}$

Answer: (D) Go Back to Q9

Q10.

Solution

Concept — Latus rectum: For $y^2 = 4ax$, the length of the latus rectum is $4a$.

Step 1 — Compare forms: $y^2 = 16x$ matches $y^2 = 4ax$.

Step 2 — Find a: $4a = 16$, so $a = 4$.

Step 3 — Length of latus rectum: $4a = 16$.

Why other options are wrong:

- 4: that is the value of a, not $4a$.
- 8: that is $2a$.
- 32: doubles the correct value.

Final Answer: latus rectum $= 16 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q10

Q11.

Solution

Concept — Greatest integer function: $[x]$ is the largest integer not exceeding x; it is constant on $[n, n+1)$.

Step 1 — Approach from the right: for x slightly greater than 2, say $x = 2.001$, we have $2 \le x < 3$.

Step 2 — Apply the floor: on $[2, 3)$, $[x] = 2$.

Step 3 — Take the limit: $\lim_{x \to 2^+} [x] = 2$.

Why other options are wrong:

- 1: that is the left-hand limit.
- 3: x has not reached 3.
- "does not exist": the right-hand limit alone is well defined.

Final Answer: right-hand limit $= 2 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q11

Q12.

Solution

Concept — Parametric derivative: $\dfrac{dy}{dx} = \dfrac{dy/d\theta}{dx/d\theta}$.

Step 1 — Differentiate x: $\dfrac{dx}{d\theta} = -a\sin\theta$.

Step 2 — Differentiate y: $\dfrac{dy}{d\theta} = b\cos\theta$.

Step 3 — Form the ratio: $\dfrac{dy}{dx} = \dfrac{b\cos\theta}{-a\sin\theta}$.

Step 4 — Simplify: $= -\dfrac{b}{a}\cot\theta$.

Why other options are wrong:

- $\dfrac{b}{a}\cot\theta$: misses the negative sign from $dx/d\theta$.
- $-\dfrac{a}{b}\cot\theta$: inverts the ratio b/a.
- $\dfrac{a}{b}\tan\theta$: wrong ratio and wrong trig function.

Final Answer: $\dfrac{dy}{dx} = -\dfrac{b}{a}\cot\theta \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q12

Q13.

Solution

Concept — Skew-symmetric part: For $A = P + Q$ with P symmetric and Q skew-symmetric, $Q = \frac{1}{2}(A - A^T)$.

Step 1 — Write A^T: $A^T = \begin{pmatrix} 2 & 6 \\ 4 & 8 \end{pmatrix}$.

Step 2 — Compute $A - A^T$: $\begin{pmatrix} 2-2 & 4-6 \\ 6-4 & 8-8 \end{pmatrix} = \begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix}$.

Step 3 — Halve it: $Q = \frac{1}{2}\begin{pmatrix} 0 & -2 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

Why other options are wrong:

- $\begin{pmatrix} 0 & 5 \\ -5 & 0 \end{pmatrix}$: wrong magnitude.

- $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$: correct entries but the signs are transposed.
- $\begin{pmatrix} 2 & 5 \\ 5 & 8 \end{pmatrix}$: that is the symmetric part P, not Q.

Final Answer: $Q = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q13

Q14.

Solution

Concept — Area by determinant: Area $= \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$.

Step 1 — Substitute the vertices: with $(0,0), (4,0), (0,4)$,

$$\text{Area} = \tfrac{1}{2}\left|0(0-4) + 4(4-0) + 0(0-0)\right|.$$

Step 2 — Evaluate each bracket: $0(-4) = 0$, $4(4) = 16$, $0(0) = 0$.

Step 3 — Add the terms: $0 + 16 + 0 = 16$.

Step 4 — Take half the absolute value: $\frac{1}{2}|16| = 8$.

Why other options are wrong:

- 16: forgets the factor $\frac{1}{2}$.
- $12, 6$: arise from sign or arithmetic slips in the determinant.

Final Answer: Area $= 8$ square units $\Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q14

Q15.

Solution

Concept — Slope of tangent: The slope at a point is $\frac{dy}{dx}$ evaluated there.

Step 1 — Differentiate: $\frac{dy}{dx} = 3x^2 - 3$.

Step 2 — Substitute $x = 2$: slope $= 3(2)^2 - 3$.

Step 3 — Evaluate the square: $3(4) - 3$.

Step 4 — Simplify: $12 - 3 = 9$.

Why other options are wrong:

- $6, 12, 3$: come from forgetting the constant term or a sign.

Final Answer: slope $= 9 \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q15

Q16.

Solution

Concept — Absolute extrema on $[a, b]$: Compare f at the critical points inside and at the endpoints.

Step 1 — Differentiate: $f'(x) = 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x - 2)(x + 1)$.

Step 2 — Critical points in $[0, 3]$: $x = 2$ (since $x = -1$ is outside).

Step 3 — Evaluate endpoints and critical point: $f(0) = 5$, $f(2) = 16 - 12 - 24 + 5 = -15$, $f(3) = 54 - 27 - 36 + 5 = -4$.

Step 4 — Pick the largest: the values are $5, -15, -4$; the maximum is 5 at $x = 0$.

Why other options are wrong:

- -15: that is the absolute minimum at $x = 2$.
- -4: value at $x = 3$.
- 0: not attained.

Final Answer: absolute maximum $= 5 \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q16

Q17.

Solution

Concept — Rationalising substitution: Put $t = \sqrt{x}$ to remove the radical.

Step 1 — Substitute: let $t = \sqrt{x}$, so $x = t^2$ and $dx = 2t\,dt$.

Step 2 — Rewrite the integral: $\int \frac{2t}{1+t}\,dt$.

Step 3 — Split the fraction: $\frac{2t}{1+t} = 2 - \frac{2}{1+t}$.

Step 4 — Integrate term by term: $\int \left(2 - \frac{2}{1+t}\right) dt = 2t - 2\log|1+t| + C$.

Step 5 — Back-substitute $t = \sqrt{x}$: $= 2\sqrt{x} - 2\log|1+\sqrt{x}| + C$.

Why other options are wrong:

- The others miss the factor 2 on the log or on the $\sqrt{x}$ term.

Final Answer: $2\sqrt{x} - 2\log|1+\sqrt{x}| + C \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q17

Q18.

Solution

Concept — Partial fractions: Write $\frac{1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}$.

Step 1 — Clear denominators: $1 = A(x+2) + B(x-1)$.

Step 2 — Put $x = 1$: $1 = A(3)$, so $A = \frac{1}{3}$.

Step 3 — Put $x = -2$: $1 = B(-3)$, so $B = -\frac{1}{3}$.

Step 4 — Integrate: $\frac{1}{3}\log|x-1| - \frac{1}{3}\log|x+2| + C$.

Step 5 — Combine the logs: $= \frac{1}{3}\log\left|\frac{x-1}{x+2}\right| + C$.

Why other options are wrong:

- Inverted ratio or missing the $\frac{1}{3}$ factor.

Final Answer: $\frac{1}{3}\log\left|\frac{x-1}{x+2}\right| + C \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q18

Q19.

Solution

Concept — King property: Replacing x by $a-x$ and adding gives a self-cancelling integral.

Step 1 — Let I be the integral: $I = \int_0^{\pi/2} \frac{\sin x}{\sin x + \cos x}\,dx.$

Step 2 — Replace $x \to \frac{\pi}{2} - x$: $I = \int_0^{\pi/2} \frac{\cos x}{\cos x + \sin x}\,dx.$

Step 3 — Add the two forms: $2I = \int_0^{\pi/2} \frac{\sin x + \cos x}{\sin x + \cos x}\,dx = \int_0^{\pi/2} 1\,dx.$

Step 4 — Evaluate: $2I = \frac{\pi}{2}.$

Step 5 — Solve for I: $I = \frac{\pi}{4}.$

Why other options are wrong:

- $\frac{\pi}{2}$: forgets to divide by 2.
- $\frac{\pi}{3}$, 1: do not follow from the property.

Final Answer: $I = \frac{\pi}{4} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q19

Q20.

Solution

Concept — Area by integration: Integrate the bounding function over the relevant interval.

Step 1 — Set up the integral: Area $= \int_0^3 2x\,dx.$

Step 2 — Integrate: $\int 2x\,dx = x^2.$

Step 3 — Apply limits: $[x^2]_0^3 = 3^2 - 0^2.$

Step 4 — Evaluate: $9 - 0 = 9.$

Why other options are wrong:

- $6, 18, 12$: come from wrong limits or forgetting the antiderivative.

Final Answer: Area $= 9$ square units $\Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q20

Q21.

Solution

Concept — Order and degree: Order is the highest derivative; degree is the power of that highest derivative only when the equation is polynomial in all derivatives.

Step 1 — Identify the highest derivative: $\frac{d^2y}{dx^2}$, so the order is 2.

Step 2 — Check polynomial form: the term $\sin\left(\frac{dy}{dx}\right)$ is transcendental in $\frac{dy}{dx}$, but the equation is still polynomial in the *highest* derivative $\frac{d^2y}{dx^2}$.

Step 3 — Read the degree: the highest derivative appears to power 3, so the degree is 3.

Step 4 — State the pair: order $= 2$, degree $= 3$.

Why other options are wrong:

- $3, 2$ and $3, 3$: misread the order.
- $2, 2$: misreads the power of the second derivative.

Final Answer: order 2, degree $3 \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q21

Q22.

Solution

Concept — Homogeneous DE: Substitute $y = vx$ so $\frac{dy}{dx} = v + x\frac{dv}{dx}$.

Step 1 — Rewrite the right side: $\frac{x+y}{x} = 1 + \frac{y}{x} = 1 + v$.

Step 2 — Substitute: $v + x\frac{dv}{dx} = 1 + v$.

Step 3 — Cancel v: $x\frac{dv}{dx} = 1$.

Step 4 — Separate and integrate: $dv = \frac{dx}{x} \Rightarrow v = \log|x| + C.$

Step 5 — Back-substitute $v = \frac{y}{x}$**:** $\frac{y}{x} = \log|x| + C$, so $y = x\log|x| + Cx$.

Why other options are wrong:

- Constants attached to x^2 or missing the factor x in the constant term.

Final Answer: $y = x\log|x| + Cx \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q22

Q23.

Solution

Concept — Angle between vectors: $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|}.$

Step 1 — Dot product: $\vec{a}\cdot\vec{b} = (1)(0) + (1)(1) + (0)(1) = 1.$

Step 2 — Magnitudes: $|\vec{a}| = \sqrt{1+1} = \sqrt{2}$, $|\vec{b}| = \sqrt{1+1} = \sqrt{2}.$

Step 3 — Form the cosine: $\cos\theta = \frac{1}{\sqrt{2}\cdot\sqrt{2}} = \frac{1}{2}.$

Step 4 — Find the angle: $\theta = \cos^{-1}\frac{1}{2} = \frac{\pi}{3}.$

Why other options are wrong:

- $\frac{\pi}{4}$: would need $\cos\theta = \frac{1}{\sqrt{2}}$.
- $\frac{\pi}{6}, \frac{\pi}{2}$: do not match $\cos\theta = \frac{1}{2}$.

Final Answer: $\theta = \frac{\pi}{3} \Rightarrow \boxed{\text{A}}$

Answer: (A) Go Back to Q23

Q24.

Solution

Concept — Area of triangle from vectors: Area $= \frac{1}{2}|\vec{a} \times \vec{b}|$.

Step 1 — Cross product:

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & -1 & 2 \end{vmatrix}.$$

Step 2 — Components: $\hat{i}(1 \cdot 2 - (-1)(-1)) - \hat{j}(2 \cdot 2 - (-1)(1)) + \hat{k}(2(-1) - 1 \cdot 1)$.

Step 3 — Simplify: $\hat{i}(2 - 1) - \hat{j}(4 + 1) + \hat{k}(-2 - 1) = \hat{i} - 5\hat{j} - 3\hat{k}$.

Step 4 — Magnitude: $|\vec{a} \times \vec{b}| = \sqrt{1 + 25 + 9} = \sqrt{35}$.

Step 5 — Halve it: Area $= \frac{\sqrt{35}}{2}$.

Why other options are wrong:

- $\sqrt{35}$: forgets the factor $\frac{1}{2}$.
- $\frac{\sqrt{35}}{4}$: halves twice.

Final Answer: Area $= \frac{\sqrt{35}}{2} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q24

Q25.

Solution

Concept — Foot of perpendicular: A general point on the line is $(2t, 3t, 6t)$; choose t so the connecting vector is perpendicular to the direction $(2, 3, 6)$.

Step 1 — Connecting vector: from $(1, 2, 3)$ to $(2t, 3t, 6t)$ is $(2t - 1, 3t - 2, 6t - 3)$.

Step 2 — Perpendicularity: dot with $(2, 3, 6)$ equals zero:

$$2(2t - 1) + 3(3t - 2) + 6(6t - 3) = 0.$$

Step 3 — Expand: $4t - 2 + 9t - 6 + 36t - 18 = 0$.

Step 4 — Combine: $49t - 26 = 0$, so $t = \frac{26}{49}$.

Step 5 — Find the x-coordinate: $x = 2t = \frac{52}{49}$.

Why other options are wrong:

- $\frac{26}{7}$: misses a factor.
- $\frac{13}{49}$: equals $t/2$, not $2t$.
- 1: the original point's x, not the foot.

Final Answer: x-coordinate $= \frac{52}{49} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q25

Q26.

Solution

Concept — Intercept form: A plane with intercepts a, b, c is $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

Step 1 — Read the intercepts: the points $(1,0,0), (0,1,0), (0,0,1)$ give $a = b = c = 1$.

Step 2 — Write the plane: $\frac{x}{1} + \frac{y}{1} + \frac{z}{1} = 1$.

Step 3 — Simplify: $x + y + z = 1$.

Why other options are wrong:

- $x + y + z = 0$: passes through the origin, not the given points.
- $x + y + z = 3$: would require intercepts of 3.
- $2x + 2y + 2z = 1$: gives intercepts of $\frac{1}{2}$.

Final Answer: $x + y + z = 1 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q26

Q27.

Solution

Concept — Addition rule: $P(K \cup H) = P(K) + P(H) - P(K \cap H)$.

Step 1 — Probability of a king: $P(K) = \frac{4}{52}$.

Step 2 — Probability of a heart: $P(H) = \frac{13}{52}$.

Step 3 — Overlap (king of hearts): $P(K \cap H) = \frac{1}{52}$.

Step 4 — Apply the rule: $P = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52}$.

Step 5 — Simplify: $\frac{16}{52} = \frac{4}{13}$.

Why other options are wrong:

- $\frac{17}{52}$: forgets to subtract the overlap.
- $\frac{1}{13}, \frac{1}{4}$: count only kings or only hearts.

Final Answer: $P = \frac{4}{13} \Rightarrow \boxed{\text{D}}$

Answer: (D) Go Back to Q27

Q28.

Solution

Concept — Total probability: Sum over each bag the (probability of choosing the bag) $\times$ (probability of red from it).

Step 1 — Bag I branch: $\frac{1}{2} \times \frac{3}{5} = \frac{3}{10}$.

Step 2 — Bag II branch: $\frac{1}{2} \times \frac{1}{5} = \frac{1}{10}$.

Step 3 — Add the branches: $P(\text{red}) = \frac{3}{10} + \frac{1}{10} = \frac{4}{10}$.

Step 4 — Simplify: $\frac{4}{10} = \frac{2}{5}$.

Why other options are wrong:

- $\frac{3}{5}, \frac{1}{5}$: use only one bag.
- $\frac{4}{5}$: doubles the result.

Final Answer: $P(\text{red}) = \dfrac{2}{5} \Rightarrow \boxed{\text{C}}$

Answer: (C) Go Back to Q28

Q29.

Solution

Concept — Mean deviation about the mean: $\text{M.D.} = \dfrac{1}{n}\sum |x_i - \bar{x}|$.

Step 1 — Find the mean: sum $= 4 + 7 + 8 + 9 + 10 + 12 + 13 + 17 = 80$, and $\bar{x} = \dfrac{80}{8} = 10$.

Step 2 — Absolute deviations: $|4 - 10| = 6$, $|7 - 10| = 3$, $|8 - 10| = 2$, $|9 - 10| = 1$, $|10 - 10| = 0$, $|12 - 10| = 2$, $|13 - 10| = 3$, $|17 - 10| = 7$.

Step 3 — Sum the deviations: $6 + 3 + 2 + 1 + 0 + 2 + 3 + 7 = 24$.

Step 4 — Divide by $n = 8$: $\dfrac{24}{8} = 3.0$.

Why other options are wrong:

- $2.75, 3.25, 2.5$: come from arithmetic slips in the deviations.

Final Answer: $\text{M.D.} = 3.0 \Rightarrow \boxed{\text{B}}$

Answer: (B) Go Back to Q29

Q30.

Solution

Concept — Corner-point method: The optimum of a linear objective on a bounded feasible region occurs at a vertex.

Step 1 — Evaluate $Z = 4x + 5y$ at $(0, 0)$: $Z = 0$.

Step 2 — At $(4, 0)$: $Z = 4(4) + 5(0) = 16$.

Step 3 — At $(2, 3)$: $Z = 4(2) + 5(3) = 8 + 15 = 23$.

Step 4 — At $(0, 4)$: $Z = 4(0) + 5(4) = 20$.

Step 5 — Pick the maximum: the values are $0, 16, 23, 20$; the maximum is 23 at $(2, 3)$.

Why other options are wrong:

- 20: value at $(0, 4)$.
- 16: value at $(4, 0)$.
- 21: not attained at any vertex.

Final Answer: maximum $Z = 23 \Rightarrow \boxed{\text{A}}$

Answer: (A) **Go Back to Q30**

Answer Key

Q	Ans	Q	Ans	Q	Ans	Q	Ans	Q	Ans
1	A	2	B	3	C	4	D	5	D
6	B	7	A	8	C	9	D	10	B
11	A	12	C	13	D	14	B	15	A
16	C	17	D	18	B	19	A	20	C
21	D	22	B	23	A	24	C	25	D
26	B	27	D	28	C	29	B	30	A